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# An analytical study of tunnel–pile interaction

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ABSTRACT

Tunnelling under the foundations of structures is becoming more common because of the lack of available space for infrastructure, both above and below ground. The interaction between newly constructed tunnels and existing piled foundations is an important issue. This paper presents results obtained using a computationally efficient analytical approach which aims to estimate the effect that constructing a new tunnel will have on an existing pile. The method uses a spherical cavity expansion analysis to evaluate the end-bearing capacity of the pile, and cylindrical cavity contraction to estimate the decrease in the confining pressure and resulting reduction in pile capacity caused by tunnel volume loss. The paper extends previously published work using this method by considering the effect of tunnel location on the tunnel–pile interactions, examining different possible assumptions of soil stiffness used in the analysis, and by considering the effect that tunnel cavity contraction has on the friction along the pile shaft.


## 1. Introduction

The need for effective Civil infrastructure in cities is paramount. As populations grow and the demand on infrastructure systems increases, the need to further develop already congested underground space in many urban areas will become unavoidable. This will result in new underground construction activities taking place ever closer to existing structures and buried infrastructure. The resulting interaction between construction activities and the affected Civil assets must be considered in the design process.

Tunnels are arguably the most popular medium to large-scale underground structures in crowded urban areas. They are used to minimise the volume of traffic on the surface and can also have environmental benefits (e.g. traffic noise reduction). Tunnelling inevitably causes some ground movements which can have detrimental effects on buried and above-ground infrastructure and buildings. There has been considerable research conducted on the subject of evaluating the shape of tunnelling induced ground movements (Peck, 1969; O'Reilly and New, 1982; Mair et al., 1993; Marshall et al., 2012) and in determining the effects these movements have on man-made assets (Attewell et al., 1986; Klar et al., 2005; Vorster et al., 2005; Klar et al., 2008; Marshall et al., 2010; Zhang et al., 2012). In general, the potential for harmful interaction between tunnel construction and Civil assets is greatest for shallow tunnels, which suggests that a deeper tunnel is preferable. The cost of tunnelling varies considerably depending on local site conditions, however in general it increases with depth due to the additional cost of construction of associated excavations (e.g. shafts for tunnel boring machine launch, ventilation, and escalators/lifts). A careful decision must therefore be made at the design stage with respect to the optimum depth for new tunnel construction.

Piled foundations are particularly sensitive to the effects of tunnelling. Piles risk a reduction of their end-bearing capacity and shaft friction resistance due to the displacements and ground stress redistributions that occur as a result of tunnelling. A variety of research has been conducted on the subject, ranging from field studies (Kaalberg et al., 2005; Pang et al., 2006; Selemetas et al., 2006), experimental work (Bezuijen and Van der Schrier, 1994; Loganathan et al., 2000; Jacobsz et al., 2004; Marshall and Mair, 2011), and numerical or analytical modelling (Chen et al., 1999; Kitiyodom et al., 2005; Lee and Ng, 2005; Cheng et al., 2007). Analytical methods provide an efficient way for studying soil–structure interaction problems such as the effect of tunnelling on piles.

This paper studies the effect of a newly constructed tunnel on an existing pile using the analytical approach introduced by Marshall (2012, 2013). Some critical points from the original method are examined and some new ideas are presented which are intended to achieve a more sensible and thorough analysis approach. The paper includes data obtained using the original method of Marshall (2012) to elucidate the important effect that depth and the relative horizontal and vertical distance between the pile and tunnel have on results. Next, the selection and influence of soil stiffness used in the tunnel–pile interaction analysis are illustrated, and a new method for accounting for the effect of

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**Nomenclature**

| | |
|---|---|
| $a$ | current radius of a cavity |
| $a_0$ | original cavity radius |
| $C$ | cohesion |
| $c$ | current radius of the plastic zone around a pile or a tunnel |
| $c_0$ | original distance from pile tip to elastic–plastic interface |
| $c_1$ | parameter used to calculate $G_0$ |
| $D_p$ | pile diameter |
| $d_{tp}$ | distance from tunnel axis to pile tip |
| $d_{lp}$ | distance from tunnel lining to pile tip |
| $E$ | Young's modulus |
| $G$ | soil shear modulus |
| $G_0$ | small strain shear stiffness |
| $G_{0,mod}$ | modified shear stiffness due to the effect of pile installation |
| $G_{0,tun}$ | shear stiffness calculated at tunnel depth |
| $I_d$ | relative density |
| $I_R$ | relative dilatancy index |
| $K_0$ | the coefficient of at-rest lateral earth pressure |
| $k$ | cavity expansion parameter: spherical $k = 2$; cylindrical $k = 1$ |
| $L$ | embedded pile length |
| $n$ | parameter used to calculate $G_0$ |
| $N_q$ | bearing capacity factor |
| $p'$ | mean effective stress or confining pressure |
| $p'_0$ | initial isotropic stress at tunnel or pile tip |
| $p'_{0,mod}$ | modified pressure |
| $p'_{0,pile}$ | confining pressure at pile tip |
| $p'_{0,tun}$ | confining pressure at tunnel depth |
| $p'_{mid}$ | confining pressure half-way between pile tip and tunnel lining |
| $p'_{V_l}$ | confining pressure after tunnel volume loss |
| $p_a$ | atmospheric pressure (100 kPa) |
| $p'_{lim}$ | limiting stress for spherical cavity expansion |
| $P$ | cavity pressure |
| $P_a$ | current cavity pressure when cavity radius $= a$ |
| $q_b$ | end-bearing capacity of pile |
| $q_{b,0}$ | end-bearing capacity of pile before tunnel volume loss |
| $q_{b,V_l}$ | reduced end-bearing capacity of pile after tunnel volume loss |
| $Q$ | total load capacity of the pile |
| $Q_0$ | total load capacity of pile before tunnel volume loss |
| $Q_{V_l}$ | total load capacity of pile after tunnel volume loss |
| $r_p$ | pile radius |
| $r_t$ | tunnel radius |
| $R_{q_b}$ | pile end-bearing capacity reduction factor |
| $R_Q$ | pile capacity reduction factor |
| $R_{Q,S}$ | pile capacity reduction factor including effect on pile shaft |
| $S$ | parameter used to calculate $G_0$ |
| $S_t$ | ratio of radial effective stress near pile tip at failure to $q_b$ |
| $V_l$ | volume loss due to tunnelling, in % |
| $z$ | depth to any point below the ground surface |
| $z_p$ | depth to pile tip |
| $z_t$ | depth of tunnel axis |
| $\alpha_c$ | parameter used in calculation of $q_b$ |
| $\beta_s$ | ratio of shaft shear stress to vertical effective stress of soil |
| $\beta_{min}$, $\beta_{max}$ | minimum and maximum values of $\beta_s$ |
| $\delta_s$ | soil-shaft friction angle |
| $\theta$ | parameter used in calculation of $\alpha_c$ |
| $\phi$ | soil friction angle |
| $\phi'_{cv}$ | critical state friction angle |
| $\gamma$ | soil unit weight |
| $\mu_s$ | a parameter to calculate $\beta_s$ |
| $v$ | Poisson's ratio |
| $\sigma'$ | normal effective stress |
| $\sigma'_r$ | radial stress |
| $\sigma'^e_r$ | radial stress in elastic zone |
| $\sigma'^p_r$ | radial stress in plastic zone |
| $\sigma'_v$ | vertical stress |
| $\sigma'_\theta$ | circumferential stress |
| $\sigma'^e_\theta$ | circumferential stress in elastic zone |
| $\sigma'^p_\theta$ | circumferential stress in plastic zone |
| $\tau_s$ | shaft shear stress |
| $\overline{\tau_s}$ | average shear stress on pile shaft |
| $\overline{\tau_{s,0}}$ | average shear stress on pile shaft before tunnel volume loss |
| $\overline{\tau_{s,V_l}}$ | average shear stress on pile shaft after tunnel volume loss |
| $\psi$ | soil dilation angle |

pile installation on soil stiffness is presented that gives a more sensible approach than the original method from Marshall (2012). A method for estimating the effect of tunnel cavity contraction on pile shaft friction is also proposed. Finally, data obtained using the original analysis method are compared against new results, and a recommendation is provided, based on analysis of available geotechnical centrifuge experiment data, on how to obtain a conservative evaluation of tunnel–pile separation or safe tunnel volume loss in order to avoid large pile displacements.

## 2. Cavity expansion methods

Fig. 1 shows the problem that is considered in this paper and the main geometric parameters considered. A tunnel of radius $r_t$ is constructed beneath the tip of an existing pile with radius $r_p$. Distance $d$ is measured along the path connecting the pile tip to the tunnel axis. The shortest distance from the centreline of the pile to the axis of the tunnel is given by $d_{tp}$; the distance from pile to the tunnel lining is given by $d_{lp}$. The paper focuses on driven or jacked piles which cause a significant impact on ground stresses around the pile tip. The analysis could, however, be applied to

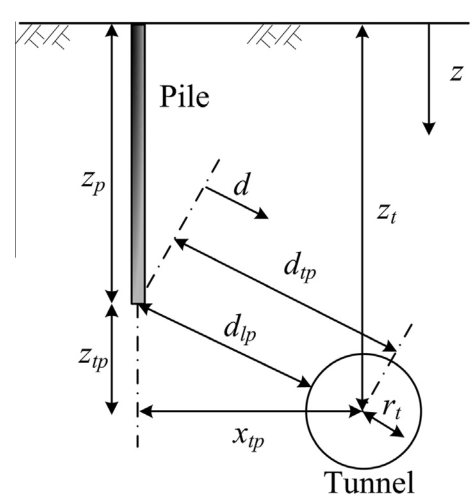


**Fig. 1.** View of the analysis problem.

bored piles in which the effect of pile installation on the ground is minimal. The volume loss induced by tunnelling affects the pile and may reduce its load-carrying capacity. The analysis aims to evaluate the distance between the pile and the tunnel, $d_{tp}$, that is required to ensure the pile does not suffer a significant reduction of its capacity (which could result in large pile displacements).

The cavity expansion method has been used for the study of a wide variety of geotechnical problems since its early application to pressuremeter test interpretation by Gibson and Anderson (1961). These include the study of in situ soil testing (Salgado and Prezzi, 2007; Mo et al., 2014), deep foundations (Randolph et al., 1994), and tunnels and underground excavations (Mair and Taylor, 1993; Yu and Rowe, 1999). Yu (2000) provided a thorough review of the method and its various applications.

The adopted analysis considers an enlarging or contracting cavity of initial radius $a_0$ in an infinite soil mass, as illustrated in Fig. 2. The cavity can be either cylindrical or spherical in shape. Various assumptions can be applied in the method; the description here applies to the analysis undertaken by Marshall (2012). The soil is assumed to be elastic–perfectly plastic with a non-associated Mohr–Coulomb yield criterion. Prior to the formation of the cavity, isotropic stress conditions are assumed and given by $p'_0$. Initially, the cavity pressure $P$ is equal to this isotropic stress. As the cavity pressure increases to $P_a$ and the cavity expands to radius $a$, a plastic zone forms around it that extends to radius $c$ from the cavity centre. The yielded soil mass is surrounded by elastic soil that extends to infinity. The radial and circumferential stresses within the ground are given by $\sigma'_r$ and $\sigma'_\theta$, respectively. The cavity contraction problem, where the cavity size decreases, can be considered in a similar manner.

## 3. Basic analysis procedure

The analyses undertaken as part of this work followed the general approach set out in Marshall (2012, 2013). The cavity expansion analysis for the interaction between tunnel construction and an existing driven or jacked pile can be summarised as follows. The analysis consists of 4 stages. (1) The end-bearing capacity of the pile is evaluated following the method of Randolph et al. (1994) whereby a spherical cavity expansion analysis is used to evaluate the limiting cavity pressure, $p'_{lim}$, and the end-bearing pressure of the pile, $q_b$. (2) The change in ground stress around the installed pile and the effect of pile installation on the ground stress profile is evaluated from the spherical cavity expansion analysis in (1). (3) A cylindrical cavity contraction analysis is used to evaluate the effect of tunnel volume loss (cavity contraction) on the stresses within the ground between the tunnel and the pile. (4) The reduction in pile end-bearing capacity is evaluated based on the altered stress conditions within the ground (due to (3)) at the tip of the pile. Stage (3) of the analysis incorporates an estimation of the effect of pile installation on soil stiffness; this aspect of the analysis is examined closely in this paper.

The method involves superposition of results from two separate elastic–plastic analyses, and therefore can only be regarded as providing an approximation of the real interaction between the tunnel and pile. Also, it is assumed that the installation of the pile has little effect on the confining stress at the location of the tunnel. This is likely to be adequate for most practical scenarios however it should be recognised as a feature of the method. A summary of the analysis and the relevant equations is provided as Appendix A. Readers should also refer to Marshall (2012, 2013) for further details. This type of analysis could be used to evaluate the effect of tunnelling on bored piles (where pile installation has little effect on the ground) by omitting stage (2) of the analysis. In this case, if an estimation of the effect of tunnelling on pile shaft friction is required then an appropriate relationship between radial and vertical stress along the pile shaft should be adopted, as described by Fleming et al. (1992).

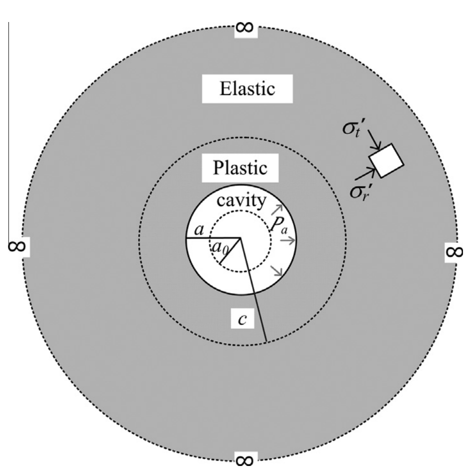


**Fig. 2.** Cavity expansion in yielding soil.

## 4. Effect of tunnel and pile depth

The results presented in Marshall (2012) illustrate the sensitivity of the analysis to the tunnel–pile separation, given by $d_{tp}$. The analysis results are also sensitive to the depth at which the tunnel and pile are located; the variation of results with $d_{tp}$ will vary depending on this depth. To illustrate this feature of the analysis, the cases presented in Fig. 3 are examined using the original Marshall (2012) analysis method. In Case 1, the relative tunnel–pile separation is increased by moving the tunnel laterally away from the pile so that the depth of the pile and tunnel remain constant but $d_{tp}$ increases. In Case 2, the pile tip depth remains constant and the tunnel–pile separation is increased by increasing the depth of the tunnel. In Case 3, the tunnel–pile separation is kept constant while the depth of both the tunnel and pile are increased. All cases considered the following parameters: tunnel radius $r_t$ = 3 m, pile radius $r_p$ = 0.5 m, tunnel volume loss $V_l$ = 5%, critical state friction angle $\phi'_{cv}$ = 30°, soil unit weight $\gamma$ = 18 kN/m$^3$, relative density $I_d$ = 0.8, at rest earth pressure coefficient $K_0$ = 0.5, Poisson's ratio $\nu$ = 0.2. All other parameters, including soil stiffness, friction angle, and dilation angle were determined using the methods outlined in Marshall (2012, 2013) and in Appendix A.

The results of the Marshall (2012) analysis of the 3 cases are presented in Fig. 4. The results relate to the pile end-bearing capacity reduction factor, $R_{q_b}$, which was defined by Marshall (2012) as:

$$R_{q_b} = \frac{q_{b,V_l}}{q_{b,0}} \tag{1}$$

where $q_b$ is the end-bearing capacity of the pile calculated in stage (1) of the analysis and $q_{b,V_l}$ is the reduced end-bearing capacity caused by tunnel cavity contraction calculated in stage (4). Note that a value of $R_{q_b}$ = 1 indicates that the tunnel has no effect on the pile end-bearing capacity; a lower value of $R_{q_b}$ indicates that the tunnel causes a reduction in the pile end bearing capacity. Comparing Case 1 with Case 2, for a given tunnel–pile separation, Case 1 results in a lower value of $R_{q_b}$ than Case 2. The effect of moving the tunnel deeper in Case 2 has a beneficial effect on the interaction analysis. This beneficial effect of depth is also demonstrated by considering Case 3, where the tunnel–pile separation is kept constant but the depth of both the tunnel and pile are increased. Fig. 4 shows that as depth increases for Case 3, the value of $R_{q_b}$ increases. This beneficial effect of depth on the results of the analysis is due to

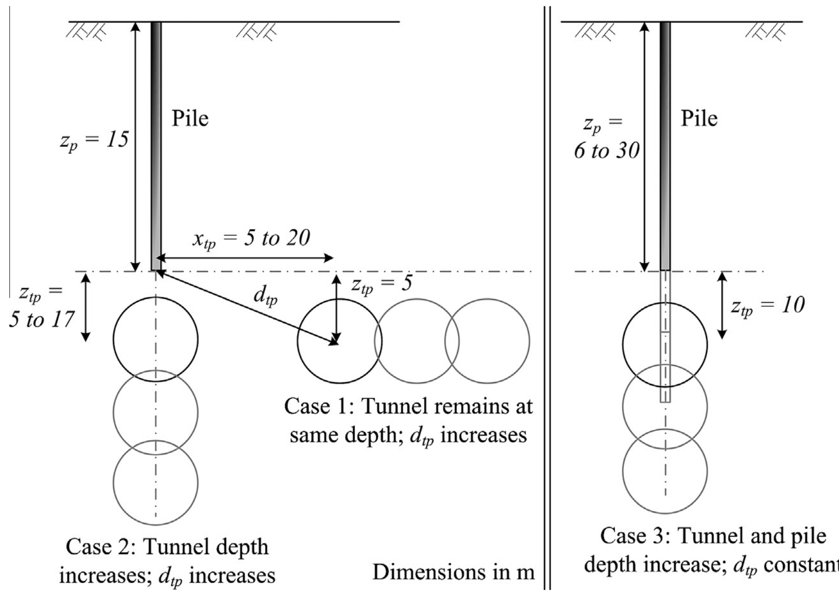


**Fig. 3.** Cases considered in evaluating the effect of tunnel and pile depth.

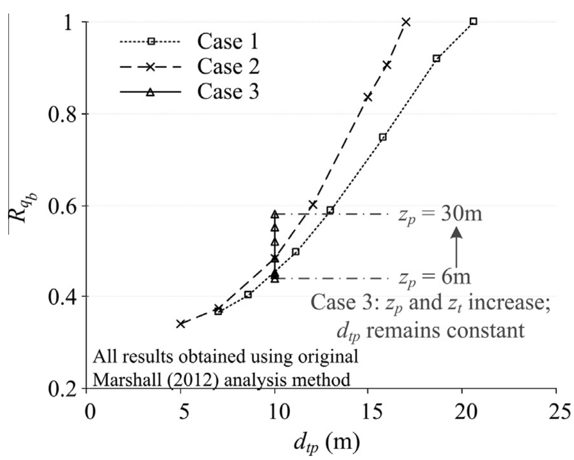


**Fig. 4.** Results of analysis of cases presented in Fig. 3.

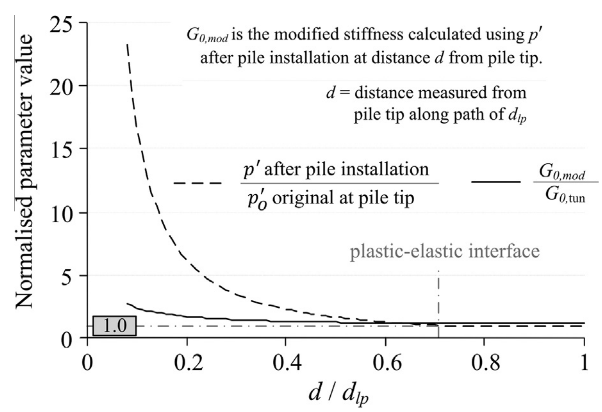


**Fig. 5.** Variation of confining pressure and modified stiffness with tunnel–pile separation.

the increase in the isotropic confining pressure, $p_0'$, which is based on the depth of the tunnel and pile tip at different stages of the analysis. The method calculates a friction and dilation angle based on the relative dilatancy term, $I_R$, (Bolton, 1986, 1987) which is a function of the confining pressure (friction and dilation angles increase with an increase in $p_0'$). The shear stiffness of the soil, $G$, also increases with confining stress in the analysis, but this has the effect of decreasing the value of $R_{q_b}$. The increase in strength and dilation with $p_0'$ evidently has a greater effect on the resulting value of $R_{q_b}$ than the effect of increased stiffness. The results in Fig. 4 illustrate the importance of considering the specific geometry of the interaction problem when using the Marshall (2012) analysis approach.

## 5. Effect of modified soil stiffness

The small strain shear stiffness of the soil, $G_0$, is used in the pile end-bearing capacity analysis as a representative value of soil stiffness (Randolph et al., 1994) (see Appendix A). In order to evaluate $G_0$, the mean effective stress or confining pressure, $p'$, is needed. Fig. 5 shows the variation of mean effective stress in the soil after pile installation (based on $p'_{lim}$ from the cavity expansion analysis), normalised by the original value of $p_0'$ at the tip of the pile. The distance, $d$, is measured from the location of the pile tip along the path of $d_{lp}$ (refer to Fig. 1); $d$ is normalised by $d_{lp}$ to indicate the relative distance between the pile tip and the tunnel lining. The minimum value of $d/d_{lp}$ plotted in Fig. 5 corresponds to a value just greater than the radius of the pile (which is the final radius of the cavity considered in the expansion analysis). The data plotted in Fig. 5 considers Case 2 from Fig. 3 with $z_p$ = 15 m and $z_{tp}$ = 10 m. The mean effective stress reduces at an exponential rate and the normalised value reaches unity at the plastic–elastic interface. For the parameters considered, this occurs at $d/d_{lp}$ = 0.7.

Also plotted in Fig. 5 is the modified value of small strain shear stiffness, $G_{0,mod}$, used in Marshall (2012), calculated using the value of $p'$ illustrated in Fig. 5 at a given distance $d$ from the pile tip, and normalised by the value of $G_0$ at the depth of the tunnel ($G_{0,tun}$). The $G_{0,mod}$ term was used in the original analysis to account for the effect of pile installation on soil stiffness between the tunnel and the pile. Due to the way that $G_{0,mod}$ was calculated, the normalised term does not go to unity if it is calculated at the plastic–elastic interface. In the Marshall (2012) method, $G_{0,mod}$ was calculated based on a modified mean effective stress: $p'_{0,mod} = p'_{0,tun} + p'_{mid}$, where $p'_{0,tun}$ is the isotropic confining pressure at the depth of the tunnel and $p'_{mid}$ is the value of $p'$ due to pile installation at a distance equivalent to half-way between the pile tip and the tunnel lining ($d/d_{lp}$ = 0.5 in Fig. 5).

There may be a more sensible and conservative approach to calculate $G_{0,mod}$ than that proposed in Marshall (2012). The Marshall (2012) method added the value of $p'$ at the mid-point between the pile tip and the tunnel lining to the value of confining stress at the depth of the tunnel axis in order to evaluate $p'_{0,mod}$ and $G_{0,mod}$ in stage (3) of the analysis. It is suggested here that a more rational approach is to normalise the calculated confining pressure at the mid-point using the initial confining pressure at the pile tip ($p'_{0,pile}$) and then to factor the confining pressure at the tunnel axis by this value. This 'new method' of calculating $p'_{0,mod}$ is represented by Eq. (2) and results in a more rational trend of $G_{0,mod}/G_{0,tun}$, as illustrated in Fig. 6a where the value for this 'new method' is 1.0 when $G_{0,mod}$ is calculated based on $p'$ at the plastic–elastic interface (the Marshall (2012) line from Fig. 5 is presented again in Fig. 6a for comparison).

$$p'_{0,mod} = \frac{p'_{mid}}{p'_{0,pile}} \times p'_{0,tun} \tag{2}$$

In Fig. 6b, the values of $R_{q_b}$ calculated based on the modified stiffness values in Fig. 6a are compared against results obtained when no modification to $G_0$ is made (i.e. $G_0$ is calculated based on the confining pressure at the depth of the tunnel axis). The 'no modification' line represents an example of results which could apply to bored piles in which pile installation does not have a significant effect on the in situ soil conditions. In Marshall (2012), the value of $p'_{0,mod}$ is calculated at the mid-point between the pile tip and the tunnel lining ($d/d_{lp}$ = 0.5). Fig. 6b shows that for the case considered, there is very little difference between the values of $R_{q_b}$ between the two methods of modifying $G_0$ (based on $d/d_{lp}$ = 0.5 in Fig. 6b). In Fig. 6b, the 'new method' is noted to agree with the unmodified method at the plastic–elastic interface, which seems more reasonable than the Marshall (2012) trend. The use of this 'new method' of evaluating soil stiffness to determine the effect of tunnelling on pile capacity is compared against the original results of Marshall (2012) in a later section.

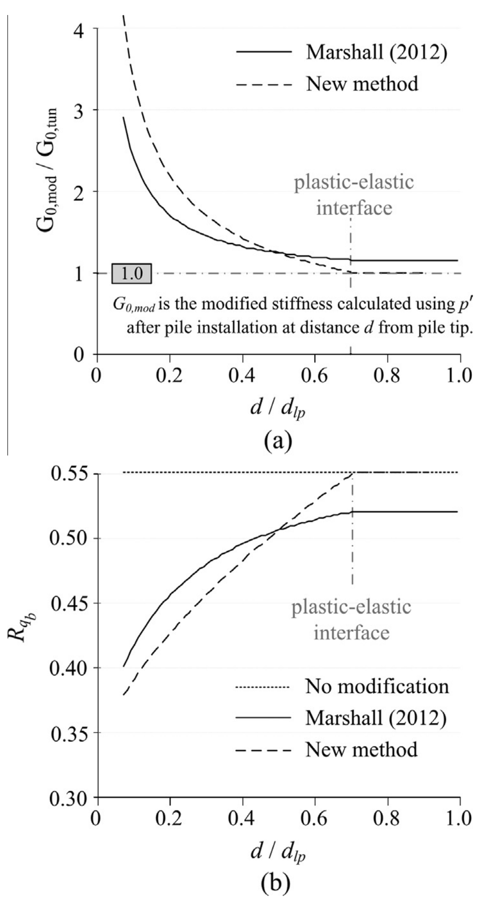


**Fig. 6.** Comparison of results for (a) modified soil stiffness and (b) $R_{q_b}$.

## 6. The effect of tunnel volume loss on pile shaft friction

The original analysis of Marshall (2012) did not consider the effect of tunnelling on shaft friction but did include the contribution of shaft friction to the total pile capacity. The $\beta$-method described in Randolph et al. (1994) was used to estimate the distribution of shear stress along the pile shaft, as defined by Eq. (3).

$$\beta_s(z) = \frac{\tau_s}{\sigma'_v} = \beta_{min} + (\beta_{max} - \beta_{min}) \exp\left[-\mu_s(L - z)/D_p\right] \tag{3}$$

where $\beta_{min}$ = 0.2, $\beta_{max} = S_t N_q \tan(\delta_s), N_q = q_b/\sigma'_v$ ($\sigma'_v$ is vertical effective stress at pile tip), $S_t = 2exp[-7tan(\phi'_{cv})], D_p$ = pile diameter, $\mu_s$ = 0.05, $q_b$ is calculated using the cavity expansion analysis outlined in Randolph et al. (1994), $L$ is the embedded pile length, and $z$ is measured from the surface. The profile of shear stress ($\tau_s$) can be integrated along the pile length in order to calculate the total shaft friction contribution to the pile capacity. It should be noted that Eq. (3) is based on driven piles. If this type of analysis were to be used for bored piles then a modified profile of horizontal stress along the pile length should be adopted (refer to Fleming et al. (1992) for guidance on this topic).

A pile capacity reduction factor, $R_Q$, was defined in Marshall (2012) as:

$$R_Q = \frac{Q_{V_l}}{Q_0} = \frac{q_{b,V_l} D_p + 4\overline{\tau_{s,0}} L}{q_{b,0} D_p + 4\overline{\tau_{s,0}} L} \tag{4}$$

where $\overline{\tau_s}$ is the equivalent average shear stress which provides the same total shaft load as the distributed shaft shear stress, and the subscripts 0 and $V_l$ indicate the initial and post tunnel volume loss values, respectively. Note that the $\overline{\tau_{s,0}}$ term is included in the numerator, indicating that the value of $Q_{V_l}$ does not include for the effect of tunnel volume loss on shaft friction.

A method for evaluating the effect of tunnelling on shaft friction using the cavity contraction analysis is now proposed. In stage (3) of the basic analysis procedure, a distribution of radial and circumferential stresses within an initially isotropic stress field is calculated. These stresses are used to evaluate the change in mean effective stress at the location of the pile tip so that a reduced end-bearing capacity of the pile can be calculated in stage (4). In a similar fashion, the change in mean effective stress along the length of the pile axis may be used to estimate the effect of tunnel volume loss on pile shaft friction. Fig. 7 shows contours of mean effective stress after tunnel volume loss ($p'_{V_l}$) normalised by the initial isotropic stress calculated at the depth of the tunnel axis ($p'_{0,tun}$). The plotted data were obtained using the same parameters as the cases in Fig. 4 but with $z_p$ = 15 m, $z_{tp}$ = 10 m, and using the 'new method' for evaluating $p'_{0,mod}$ (Eq. (2)). Two cases are shown, where the lateral offset between the tunnel and pile are (a) 0 m and (b) 15 m. Fig. 7b shows that the length of the pile falls outside the elastic–plastic interface where there is no change in the value of mean effective stress ($p'_{V_l}/p'_{0,tun} = 1$). In Fig. 7a, however, a large portion of the pile shaft is shown to be within the zone where the value of mean effective stress is reduced due to the tunnel cavity contraction.

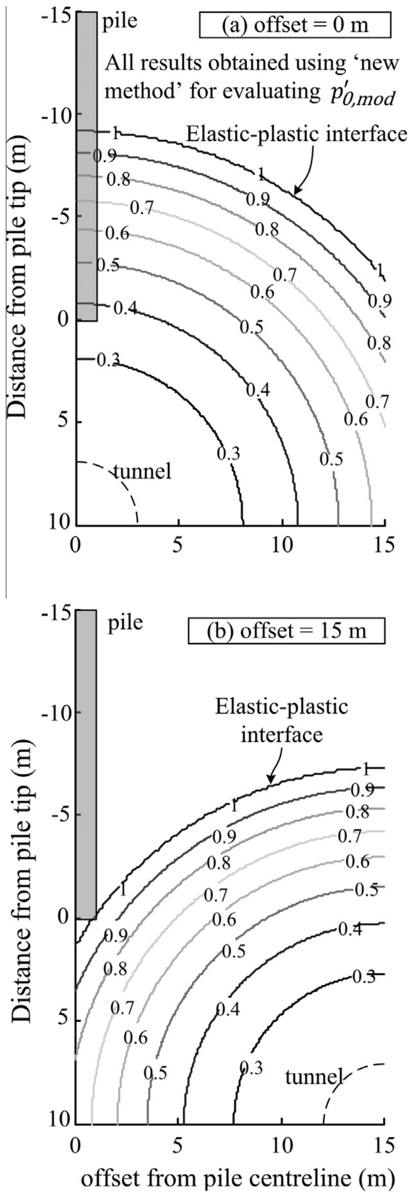


**Fig. 7.** Contours of $p'_{V_l}/p'_{0,tun}$.

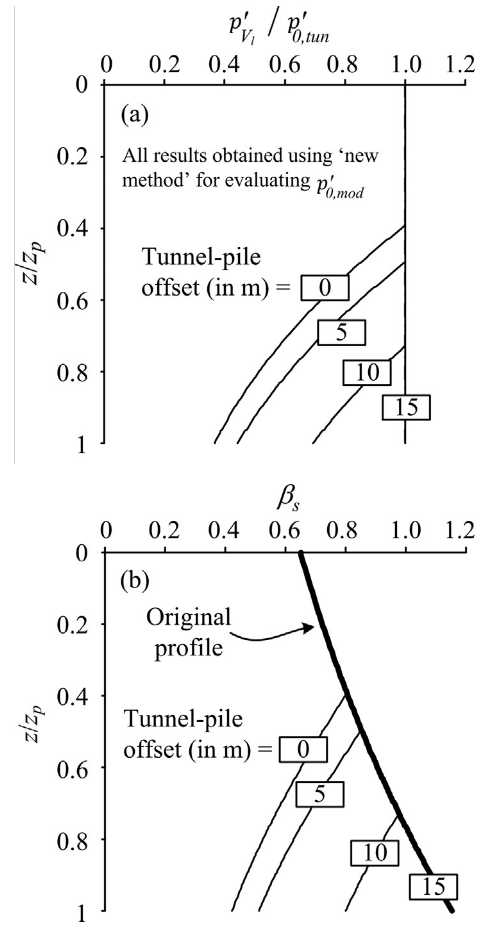


**Fig. 8.** Profiles along pile length: (a) $p'_{V_l}/p'_{0,tun}$ and (b) $\beta_s$.

As discussed in Randolph et al. (1994), the $\beta_s$ function essentially describes a profile of horizontal effective stress along the pile length. A given change in horizontal stress will therefore result in a proportional change in the value of $\beta_s$. It is suggested here that in order to obtain an estimate of the effect of tunnel cavity contraction on the pile shaft shear stress, the profile of $\beta_s$ may be scaled by the ratio of $p'_{V_l}/p'_{0,tun}$ along the pile axis (as illustrated in Fig. 7). This is an approximation since it involves the superposition of the mean effective stresses from the isotropic tunnel cavity contraction analysis on the original horizontal effective stress state along the length of the pile, which would realistically not be isotropic ($K_0$ is assumed to be 0.5 in the current analysis).

The profiles of $p'_{V_l}/p'_{0,tun}$ and $\beta_s$ for different values of tunnel–pile offsets are shown in Fig. 8 (all other parameters are consistent with cases from Fig. 7). The data in Fig. 8a illustrate the sections of the pile in which mean effective stress is affected by the tunnel cavity contraction (i.e inside the elastic–plastic interface where $p'_{V_l}/p'_{0,tun} < 1$). When the original profile of $\beta_s$ is factored by the values in Fig. 8a, the new profiles shown in Fig. 8b are obtained. The data shows that as the lateral offset between the tunnel and pile increases, the length of the pile affected by the tunnel decreases; the $\beta_s$ profile is unaffected when the offset is 15 m. The modified profile of $\beta_s$ can be used to determine a new distribution of shear stress along the pile shaft using Eq. (3). The contribution of pile shaft shear stress to the total pile capacity after tunnel volume loss, $Q_{V_l}$, can then be calculated. A new pile capacity reduction factor which accounts for the effect of the tunnel cavity contraction on both pile end-bearing capacity and shaft friction is defined by Eq. (5). The term $\overline{\tau_{s,V_l}}$ that now appears in the numerator (compared to Eq. (4)) accounts for the effect of tunnel cavity contraction on shaft friction. The following section compares results obtained using this method for evaluating the effect of tunnelling on pile capacity against the original method from Marshall (2012).

$$R_{Q,S} = \frac{Q_{V_l}}{Q_0} = \frac{q_{b,V_l} D_p + 4\overline{\tau_{s,V_l}} L}{q_{b,0} D_p + 4\overline{\tau_{s,0}} L} \tag{5}$$

## 7. Comparison of results with Marshall (2012)

This section compares analysis results from the original Marshall (2012) method against those obtained by considering the 'new method' for evaluating $p'_{0,mod}$ (Eq. (2)) as well as the effect of tunnelling on shaft friction (Eq. (5)). The geotechnical centrifuge experiments of tunnel–pile interaction provided by Marshall (2009) and Jacobsz (2002) which were analysed in Marshall (2012) are considered again here. Fig. 9 shows the results obtained from the three analyses. Note that the shaft analysis ($R_{Q,S}$) also incorporates the 'new method' for evaluating $p'_{0,mod}$. Each set of

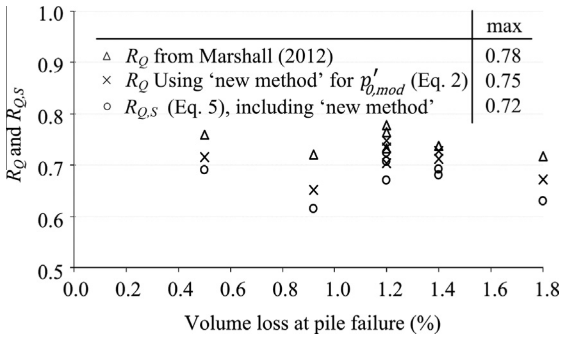


**Fig. 9.** Comparison of results with Marshall (2012).

data points represents the analysis of an individual centrifuge experiment and includes an input of material properties, geometrical conditions, and the known tunnel volume loss at which the pile failed (defined as the moment when the rate of change of the pile displacement showed a significant increase). Including the 'new method' is shown to give a slightly more conservative (lower) evaluation of $R_Q$ than the original method (maximum value of $R_Q$ reduced from 0.78 to 0.75). As illustrated in Fig. 6, the 'new method' tends to give a higher value of soil stiffness, resulting in a lower prediction of base capacity and therefore a lower evaluation of $R_Q$. Likewise, incorporating the effect of shaft friction (and using the 'new method') gives an even lower result, where $R_{Q,S} < R_Q$ and the maximum value of $R_{Q,S}$ is 0.72. Fig. 8 showed that the shaft analysis only reduces the value of $\beta_s$ therefore resulting in a lower value of $Q_{V_l}$ and reducing the value of $R_{Q,S}$ below that of $R_Q$.

The conclusion from Marshall (2012) was that, based on the experimental data available and the analysis proposed, a value of $R_Q$ = 0.85 provides a conservative approximation of a safe volume loss or tunnel–pile separation to avoid pile stability issues and potentially large displacements. This conclusion is still valid when adopting the 'new method' proposed here since $R_Q$ = 0.85 is an even more conservative threshold for this analysis. Including the effect of tunnelling on shaft friction in the analysis provides an even more conservative evaluation of pile capacity reduction. The proposed shaft analysis method involves an approximation whereby the change in mean effective stresses from the tunnel cavity contraction analysis are used to evaluate the change in horizontal stresses along the pile shaft. Given the higher level of conservatism obtained, it is suggested that the validity of this approximation should be evaluated by any individual conducting such an analysis, with appropriate consideration of the various project-specific conditions and risks.

## 8. Conclusions

This paper deals with the problem of tunnel–pile interaction and presented results obtained using analytical cavity expansion/contraction methods. The analysis aims to provide an efficient means of assessing the effect of a newly constructed tunnel on an existing pile. The results presented were obtained using an analysis which generally followed the approach set out by Marshall (2012, 2013), as summarised in Appendix A. The paper illustrated the importance of considering the specific geometry of each case due to the sensitivity of results to the depth of the pile and tunnel. A new method of evaluating the soil stiffness and modified confining stress used in the analysis was proposed. The new method gives a more rational approach since the value of $G_{0,mod}/G_{0,tun}$ goes to unity at the plastic–elastic interface. A method for approximating the effect of tunnel cavity contraction on pile shaft shear stress was also proposed. Results were compared against the experimental data used in Marshall (2012) and it was again found that a value of $R_Q$ = 0.85 is a conservative threshold for determining the safe tunnel volume loss or relative tunnel–pile separation.

The analytical approach presented makes various simplifications with regard to the real tunnel–pile interaction problem. The analysis is aimed at providing a quick and relatively straightforward method for evaluating if tunnel construction is likely to have significant adverse effects on existing piles. The evaluation of a safe value of $R_Q$ or $R_{Q,S}$ is based on data at pile failure (i.e. significant pile displacements initiated). The analysis does not attempt to evaluate the pre-failure displacements that would occur within the soil or to the pile. Whilst these limitations of the analysis should not be ignored, it is felt that the approach does provide a valuable tool for tunnel–pile interaction analysis.

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## Appendix A. Summary of analytical tunnel–pile interaction method

This appendix provides a summary of the analytical tunnel–pile interaction analysis presented in Marshall (2012, 2013). The cavity expansion analyses are based on Yu (2000), with common parameter definitions provided in Eq. (A.12).

(1) Predict the end-bearing pressure, $q_b$, using spherical cavity expansion analysis to evaluate $p'_{lim}$ in Eq. (A.1) (Randolph et al., 1994) where $\alpha_c = \theta + \phi'_{cv}/2$ and $\theta$ = 45° or the penetrometer cone tip angle.

$$q_b = p'_{lim}\left[1 + \tan\left(\phi'_{cv}\right)\tan\left(\alpha_c\right)\right] \tag{A.1}$$

The limit pressure, $p'_{lim}$, is found using Eqs. (A.3)–(A.5) by varying the value of $p'_{lim}$ in Eq. (A.3) until the left and right sides of Eq. (A.4) are equal. Soil stiffness may be evaluated using a variety of methods; in this analysis the small strain shear stiffness, $G_0$, was used based on the method suggested by Randolph et al. (1994):

$$\frac{G_0}{p_a} = S\exp(c_1 I_d)\left(\frac{p'_0}{p_a}\right)^n \tag{A.2}$$

where $S$ = 600, $c_1$ = 0.7, $n$ = 0.43 (Lo Presti, 1987), and $p_a$ is atmospheric pressure (100 kPa).

$$R_{lim} = \frac{(k+\alpha)\left[Y + (\alpha-1)p'_{lim}\right]}{\alpha(1+k)\left[Y + (\alpha-1)p'_0\right]} \tag{A.3}$$

$$\sum_{n=0}^{\infty} A_n(R_{lim}, \mu) = \frac{\chi}{\gamma}(1-\delta)^{\frac{\beta+k}{\beta}} \tag{A.4}$$

$$A_n(R,\mu) = \begin{cases} \frac{\mu^n}{n!}\ln R & \text{if } n = \gamma \\ \frac{\mu^n}{n!(n-\gamma)}\left(R^{n-\gamma} - 1\right) & \text{otherwise} \end{cases} \tag{A.5}$$

(2) Assuming an infinite ground mass with an isotropic stress $p'_0$ based on the pile tip depth, and using the determined value of $p'_{lim}$ from stage 1, use spherical cavity expansion to calculate the location of the elastic–plastic interface, $c$ (Eq. (A.6)), and distribution of confining stress, $p'$, resulting from pile installation, where $p' = (\sigma'_r + 2\sigma'_\theta)/3$. Ground stresses resulting from pile installation, where the superscripts $e$ and $p$ refer to elastic and plastic, respectively, are given by Eq. (A.7) (refer also to Fig. 2).

$$\frac{c}{a_0} = \left\{\frac{(k+\alpha)[Y+(\alpha-1)p]}{\alpha(1+k)\left[Y+(\alpha-1)p_0'\right]}\right\}^{\frac{\alpha}{k(\alpha-1)}} \tag{A.6}$$

$$\begin{aligned}
&\text{Plastic zone}: r < c\\
&\sigma_r'^p = \tfrac{Y}{\alpha-1} + Ar^{-\frac{k(\alpha-1)}{\alpha}}\\
&\sigma_\theta'^p = \tfrac{Y}{\alpha-1} + \tfrac{A}{\alpha} r^{-\frac{k(\alpha-1)}{\alpha}}\\
&A2 = -\tfrac{(1+k)\alpha\left[Y+(\alpha-1)p_0'\right]}{(\alpha-1)(k+\alpha)}\, c^{\frac{k(\alpha-1)}{\alpha}}\\
&\text{Elastic zone}: r > c\\
&\sigma_r'^e = -p_0' - Br^{-(1+k)}\\
&\sigma_\theta'^e = -p_0' + \tfrac{B}{k} r^{-(1+k)}\\
&B2 = \tfrac{k\left[Y+(\alpha-1)p_0'\right]}{k+\alpha}\, c^{1+k}
\end{aligned} \tag{A.7}$$

(3) Use a cylindrical cavity contraction analysis to evaluate the effect of tunnel volume loss on ground stresses. An isotropic stress is assumed based on the depth of the tunnel axis. A modified (increased) value of shear stiffness, $G_{mod}$ (based on $p'_{mod}$), is used to account for the effect of pile installation on the soil between the tunnel and pile. For an assumed value of volume loss, $V_l$, and assuming concentric contraction of the tunnel boundary, the final tunnel radius can be found using $a = a_0\sqrt{(1-V_l/100)}$. For cases where a plastic zone forms around the tunnel (most relevant scenario), the cavity pressure $P$ when $r = a$ is required. Using Eq. (A.8) to determine $c_0/c$, the value of $P$ in Eq. (A.9) is iterated until the desired value of $a$ is obtained. Eq. (A.10) is then used to evaluate $c$ and Eq. (A.11) is used to determine the ground stresses.

$$\frac{c_0}{c} = 1 - \frac{(1-\alpha)p_0' + Y}{2(1+k\alpha)G_{mod}} \tag{A.8}$$

$$\begin{aligned}
\frac{a_0}{a} &= \left[1 - X\left(1 - \left(\tfrac{c_0}{c}\right)^{1+k\beta}\right)\right]^{\frac{1}{1+k\beta}}\\
X &= \left\{\tfrac{(1+k\alpha)[Y+(\alpha-1)P]}{(1+k)[Y+(\alpha-1)p_0']}\right\}^{\frac{1+k\beta}{k(1-\alpha)}}
\end{aligned} \tag{A.9}$$

$$\frac{c}{a} = \left\{\frac{(1+k\alpha)[Y+(\alpha-1)P]}{(1+k)[Y+(\alpha-1)p_0']}\right\}^{\frac{1}{k(1-\alpha)}} \tag{A.10}$$

$$\begin{aligned}
&\text{Plastic zone}: r < c\\
&\sigma_r'^p = \tfrac{Y}{\alpha-1} + Ar^{k(\alpha-1)}\\
&\sigma_\theta'^p = \tfrac{Y}{\alpha-1} + A\alpha r^{k(\alpha-1)}\\
&A3 = -\tfrac{(1+k)[Y+(\alpha-1)p_0']}{(\alpha-1)(1+k\alpha)}\, c^{(1-\alpha)k}\\
&\text{Elastic zone}: r > c\\
&\sigma_r'^e = -p_0 - Br^{-(1+k)}\\
&\sigma_\theta'^e = -p_0 + \tfrac{B}{k} r^{-(1+k)}\\
&B3 = \tfrac{k\left[(1-\alpha)p_0' - Y\right]}{1+k\alpha}\, c^{1+k}
\end{aligned} \tag{A.11}$$

(4) A reduced end-bearing capacity based on the change in stresses at the base of the pile due to tunnel volume loss ($\Delta\sigma_r', \Delta\sigma_\theta'$) from stage 3 is calculated using the same process as in stage 1. The confining stress at the pile tip is factored by $R_p = 1 - \Delta p'/p'_{0,tun}$, where $\Delta p' = [(1+v)(\Delta\sigma_r' + \Delta\sigma_\theta')]/3$.

Common parameters used for cavity expansion and contraction analyses:

$$\begin{aligned}
&G = \frac{E}{2(1+v)} \quad M = \frac{E}{1-v^2(2-k)} \quad Y = \frac{2C\cos\phi}{1-\sin\phi} \quad \alpha = \frac{1+\sin\phi}{1-\sin\phi}\\
&\beta = \frac{1+\sin\psi}{1-\sin\psi} \quad \gamma = \frac{\alpha(\beta+k)}{k(\alpha-1)\beta} \quad \delta = \frac{Y+(\alpha-1)p_0'}{2(k+\alpha)G}\\
&\mu = \frac{(1+k)\delta\left[1-v^2(2-k)\right]}{(1+v)(\alpha-1)\beta}\left[\alpha\beta + k(1-2v) + 2v - \frac{kv(\alpha+\beta)}{1-v(2-k)}\right]\\
&\chi = \exp\left\{\frac{(\beta+k)(1-2v)(1+(2-k)v)\left[Y+(\alpha-1)p_0'\right]}{E(\alpha-1)\beta}\right\}
\end{aligned} \tag{A.12}$$

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