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Probability distributions of shotcrete parameters for reliability-based analyses of rock tunnel support

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ABSTRACT

A common support measure for underground excavations in jointed rock masses to support loose blocks is to apply a thin shotcrete layer to the periphery of the excavation and systematically install rockbolts into the surrounding rock mass. In this support system, large blocks are carried by the rockbolts and small blocks are carried by the thin shotcrete layer. To verify the shotcrete layer's load-bearing capacity and to stringently account for the large uncertainties incorporated in the variables involved in determining its capacity, analytical calculations in combination with reliability-based methods can be used. However, a lack of knowledge exists regarding the magnitude and uncertainty of shotcrete characteristics (thickness, adhesion, flexural tensile strength, residual flexural tensile strength, and compressive strength), making it difficult to apply reliability-based methods. A statistical quantification of these characteristics is therefore important to facilitate reliability-based methods in design and verification of shotcrete support. In this paper, we illustrate how shotcrete support against small loose blocks can be viewed as a correlated conditional structural system and how this system can be analyzed using reliability-based methods. In addition, we present a unique amount of data for the aforementioned variables, which are all incorporated in the design and verification of a shotcrete layer's ability to sustain loads from small loose blocks. Based on the presented data, we statistically quantify and propose suitable probability distributions for each variable. Lastly, we illustrate how the proposed probability distributions can be used in the design process to calculate the probability of exceeding the shotcrete's load-bearing capacity. Both the probabilistic quantification and the defined correlated conditional structural system along with the illustrative calculation example are followed by a discussion of their implications.

1. Introduction

In the design and construction of underground excavations in rock, there are a number of failure modes that need to be considered (Terzaghi, 1946; Hoek et al., 1995; Martin et al., 1999; Palmstrom and Stille, 2007). One common failure mode in jointed rock mass is falling or sliding of loose blocks into the underground opening, for which a common support measure is to apply a thin shotcrete layer in combination with systematically installed rockbolts. In this support system, large blocks are mainly supported by the rockbolts while smaller blocks, which may exist between the rockbolts, are mainly carried by the thin shotcrete layer. The shotcrete layer's load-bearing capacity is to a large extent governed by the existence of sufficient adhesion in the rock–shotcrete interface along the circumference of the block. If the adhesion is sufficient, the load-bearing capacity is determined by the shotcrete's direct shear capacity; on the other hand, if the adhesive capacity is insufficient, the shotcrete debonds (completely) from the rock surface, so that the load-bearing capacity instead is governed by the shotcrete's flexural capacity or its capacity to resist punching shear around the face plates of the bolts (Barrett and McCreath, 1995).

In the design of shotcrete support, structural safety is commonly verified with a deterministic design approach (e.g. Hoek and Brown, 1980). However, there are considerable uncertainties incorporated in the variables governing the shotcrete's support capacity; these uncertainties are, for example, related to the irregular surface of the tunnel periphery, the variation in rock mass and shotcrete characteristics as well as in the rock–shotcrete interface characteristics, the applied thickness of the shotcrete layer to the tunnel surface (Malmgren and Nordlund, 2008), and the calculation model used (Nilsson, 2003). Therefore, if a constant reliability level of the structure is to be ensured, which the target reliability provided in Eurocode (CEN, 2002) implies, using a deterministic design approach to ensure adequate structural safety is unsuitable, since the sensitivity of a limit state to the variability of the load and the resistance is not constant (Johansson et al., 2016; Matarawi and Harrison, 2016; Bjureland et al., 2017a).

To overcome this, an alternative approach, accepted in the Eurocodes (CEN, 2002), that can be used in the design process is reliability-based methods. Such methods have previously been applied to falling or sliding of loose blocks (e.g. Kohno, 1989; Mauldon, 1990, 1995; Hatzor, 1992, 1993; Kuszmaul, 1994, 1999; Starzec and Andersson, 2002; Jimenez-Rodriguez et al., 2006; Jimenez-Rodriguez and Sitar, 2007; Bagheri, 2011; Low and Einstein, 2013; Matarawi and Harrison, 2017), and many other rock engineering design problems (Kohno et al., 1992; Low, 1997; Celestino et al., 2006; Li and Low,

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2010; Lü and Low, 2011; Lü et al., 2011, 2013; Langford, 2013; Langford and Diederichs, 2013; Low and Phoon, 2015; Matarawi and Harrison, 2015; Bjureland et al., 2017b; Napa-García et al., 2017). In reliability-based methods, uncertainties are accounted for by defining probability distributions for all relevant uncertain input variables. Adequate structural safety is ensured by verifying that the calculated probability of failure (probability of limit state violation), p_f, is less than the acceptable target probability, $p_{f.target}$.

However, even though different aspects of the strength and design of shotcrete support have been studied by a number of authors (Holmgren, 1979, 1987, 1992; Hahn, 1983; Franzen, 1992; Barrett and McCreath, 1995; Nilsson, 2003; Ansell, 2004; Malmgren, 2005; Malmgren and Nordlund, 2006; Bernard, 2008, 2011, 2015; Saw et al., 2013; Son, 2013; Bryne et al., 2014) a lack of knowledge exists regarding the magnitude and uncertainty of shotcrete characteristics. A statistical quantification of these characteristics is therefore important to facilitate the use of reliability-based design methods.

In this paper, we illustrate how shotcrete support against small loose blocks can be viewed as a correlated conditional structural system and how this system can be analyzed using reliability-based methods. In addition, we present a unique amount of data for a number of variables required in the design and verification of a shotcrete layer's ability to sustain loads from loose blocks. The studied parameters are adhesion of the rock–shotcrete interface, a, (354 tests), the thickness of the applied shotcrete layer, t, (6068 tests), the shotcrete's flexural tensile capacity, $f_{ctm,fl}$, (344 tests), the shotcrete's residual flexural tensile capacity, $f^{re}_{ctm.fl}$, (344 tests), and the compressive strength of the shotcrete, f_c, (690 tests). For all variables, data were collected as a part of the control program in constructing a new railway tunnel in Stockholm, Sweden (The Stockholm City Line). The collected data are herein used for quantification of the magnitude and uncertainty of shotcrete characteristics. To address the issue of using the data in a verification calculation, an illustrative calculation example is performed, in which the data in combination with the defined correlated conditional structural system is used to calculate the p_f of a shotcrete layer's ability to sustain possible loose blocks that can exist between the rockbolts. Both the probabilistic quantification and the defined correlated conditional structural system along with the illustrative calculation example are followed by a discussion of their implications.

2. Shotcrete support capacity against falling or sliding of small loose blocks

2.1. Overview

Taking the reliability-based approach, failure of the shotcrete layer needs to be analyzed as a system. In Fig. 1, we present a fault tree that describes this system based on the failure modes detailed in the introduction.

It should be noted that in addition to these idealized failure modes, shotcrete can in practice fail through a combination of the idealized failure modes, which can affect its load-bearing capacity. In this paper, however, we have chosen the common procedure of considering the idealized failure modes separately, see e.g. Barrett and McCreath (1995), Holmgren (1992), and Lindfors et al. (2015). This choice along with the idealization of the failure modes introduces model uncertainty into the analysis. Quantifying this uncertainty is outside the scope of this paper.

2.2. Analytical calculations for determination of shotcrete support capacity

To account for the possible failure modes, analytical calculations can be used (e.g. Barrett and McCreath, 1995; Nilsson, 2003). The shotcrete's adhesive capacity, R_a, to sustain loose blocks can then be calculated as (Barrett and McCreath, 1995):

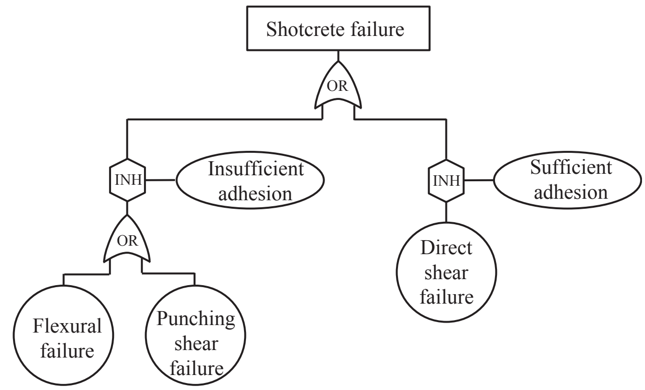

Fig. 1. Fault tree representing the idealized failure system of the shotcrete layer. Hexagons are inhibit gates in which both the basic events (i.e. flexural failure, punching shear failure, direct shear failure) and the conditional events (i.e. insufficient or sufficient adhesion) must occur in order to pass through the gates. Basic events are represented by circles and conditional events are represented by ovals.

$$R_a = a\delta O \tag{1}$$

in which δ is the width of the load bearing zone along the circumference, O, of the block (Fig. 2a). Sufficient a is maintained if the R_a is larger than the potential weight, W, of the loose block:

$$W = V\gamma_r \tag{2}$$

in which V is the volume of the block and γ_r is the unit weight of the rock mass.

If a is sufficient, the failure will be governed by the shotcrete's direct shear capacity, $R_{d.sh}$, (Barrett and McCreath, 1995):

$$R_{d.sh} = f_{sh} tO, \tag{3}$$

in which f_{sh} is the direct shear strength of the shotcrete (Fig. 2b). The $R_{d.sh}$ is sufficient if it is larger than W.

If a is not sufficient and the shotcrete debonds from the rock surface, the shotcrete must instead support the block through its punching shear resistance, $R_{p.sh}$ and its bending moment capacity, R_{fl}, In the former case, failure of the shotcrete occurs at the location of the rockbolts where shear forces are at their maximum, i.e. the rockbolts' face plates punch through the shotcrete layer when the shotcrete is exposed to a load, (Barrett and McCreath, 1995) (Fig. 2c). Such failure does not occur through a vertical line along the circumference of a block, as does direct shear failure; failure instead occurs at an inclined plane along the circumference of the face plate. However, we follow the common practice of assuming that failure occurs along an equivalent vertical plane situated at a distance of $(2b + t)/2$ from the rockbolts, where b is the equivalent radius of the face plate (Holmgren, 1992; Barrett and McCreath, 1995). Under this assumption, the $R_{p.sh}$ can be calculated as (Holmgren, 1992):

$$R_{p.sh} = f_{sh}\pi t(2b + t) \tag{4}$$

Similar to $R_{d.sh}$, the $R_{p.sh}$ is sufficient if it exceeds W.

It should be noted that in analyzing punching shear failure, it is commonly assumed that the load is evenly distributed on the shotcrete surrounding a rockbolt (see the gray surface in Fig. 2c). If the loose block has a high stiffness, the shotcrete must therefore be exposed to a load from a loose block in all directions surrounding the rockbolts. It is thus common practice to assume that that one block exist in each of the four spaces surrounding the rockbolt and that a quarter of each block is carried by the shotcrete surrounding one rockbolt (see the gray surface in Fig. 2c) (Lindfors et al., 2015). This assumption does, however, require that the blocks are small enough to fit between the rockbolts (as indicated by the placement of the blocks in relation to the rockbolts in Fig. 2c compared to Fig. 2a, b, and d). Alternatively, it can be assumed

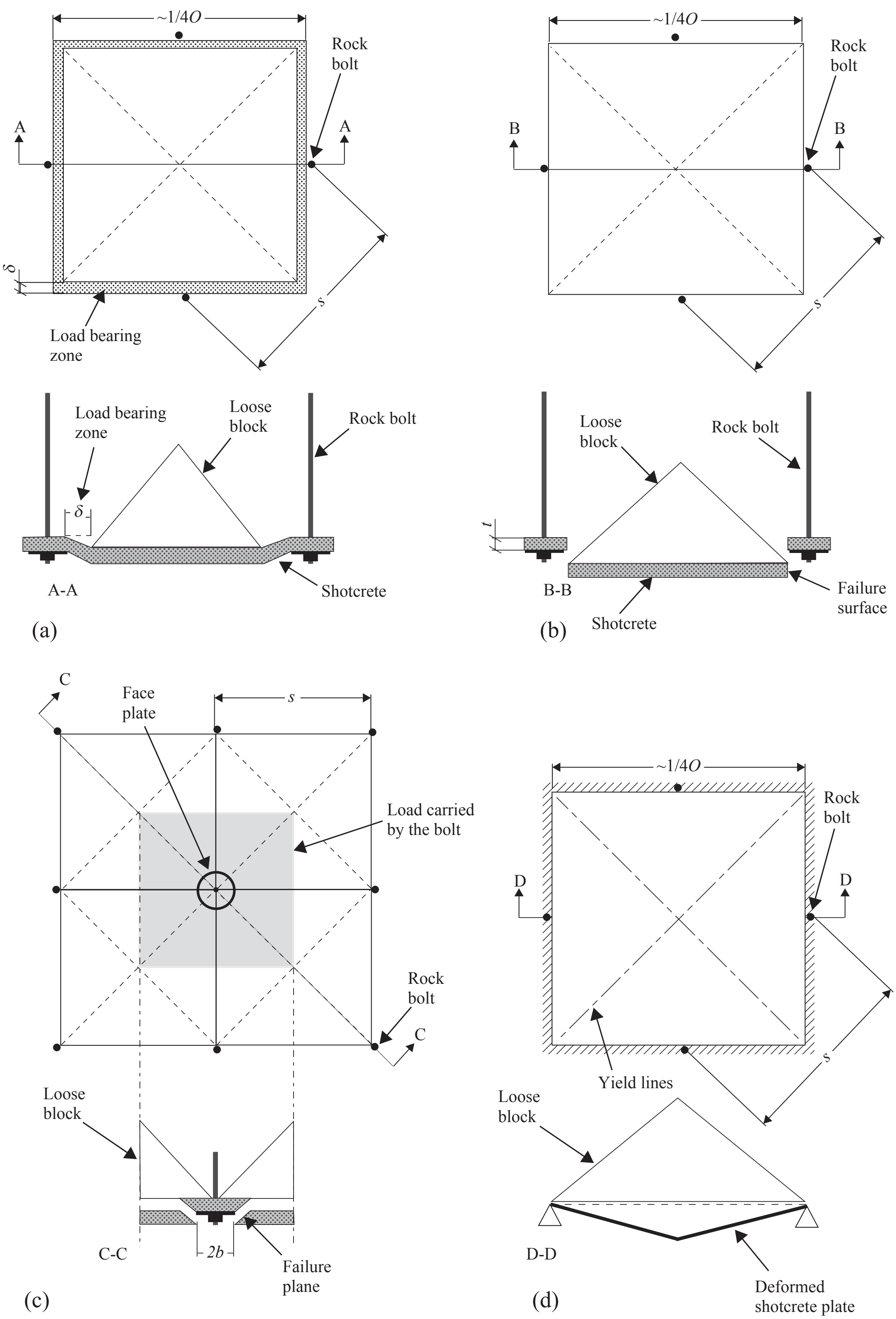

Fig. 2. (a) Adhesive failure model; (b) Direct shear failure model; (c) Punching shear failure model; (d) Flexural failure model.

that one block with the same volume as the blocks illustrated in Fig. 2a, b, and d exist on two sides of the rockbolts and that one quarter of each block is carried by the shotcrete layer surrounding the rockbolt. However, the first option is the conservative choice since the block volume is larger and it is therefore the assumption used in this paper.

The R_{fl} can be calculated using different approaches, depending on e.g. the shotcrete type (i.e. plain or fiber reinforced). If plain shotcrete is used, one approach is to estimate the R_{fl} based on its elastic limit, which is reached when $f_{ctm,fl}$ is exceeded and cracking of the shotcrete first occurs (Banton et al., 2004). The R_{fl} per meter width of the shotcrete layer can then be calculated as (e.g. Barrett and McCreath, 1995; Banton et al., 2004):

$$R_{fl} = \frac{f_{ctm,fl}\, t^2}{6}. \tag{5}$$

If fiber reinforced shotcrete is used, a common approach is to estimate the R_{fl} by accounting for the increased toughness introduced by the fibers as (Holmgren, 1992):

$$R_{fl} = 0.9\frac{R_{10/5} + R_{30/10}}{200}\frac{f_{ctm,fl}\, t^2}{6}, \tag{6}$$

in which $R_{10/5}$ and $R_{30/10}$ are flexural toughness factors (ASTM, 1997). In principle, these flexural toughness factors adjust the moment capacity of the shotcrete material to account for the residual strength of the shotcrete (i.e. they provide information regarding how the shotcrete performs compared to an elastic perfectly plastic shotcrete) (Holmgren, 1992). For an elastic perfectly plastic material, both $R_{10/5}$ and $R_{30/10}$ are equal to 100. The factor 0.9 is introduced to account for the overestimation of R_{fl} that Eq. (6) otherwise yields at small deflections for a shotcrete with a relatively high residual strength (Holmgren, 1992). The R_{fl} is sufficient if it is larger than the potential bending moment, M, in the shotcrete caused by the load from the loose block.

It should be noted here that the actual M to which the shotcrete is subjected is difficult to calculate analytically, because it depends on a number of factors, such as the stiffness of the shotcrete–rockbolt system (Barrett and McCreath, 1995) and the relative stiffness of the loose block to the shotcrete layer. There are, however, analytical methods, such as yield line theory, available for calculation of the maximum M that the shotcrete layer can withstand, assuming that the shotcrete layer acts as a reinforced concrete slab (Holmgren, 1992). According to yield line theory, a yield pattern (i.e. a crack pattern) of the studied slab (i.e. the shotcrete layer) is assumed. As the shotcrete layer is exposed to the load from the loose block, deformations occur solely in the assumed yield lines as the shotcrete yields. At this point, M is calculated by balancing the external energy induced by the loose block acting on the slab with the internal energy dissipated within the yield lines (Kennedy and Goodchild, 2004). One benefit of using yield line theory to calculate M is that the theory allows for redistribution of moments within the shotcrete layer as it yields, and thus the residual moment capacity of the shotcrete can be accounted for, as in Eq. (6). Another benefit is that tabulated theoretical solutions are available for a number of different plate cases, e.g. simply supported one or two way slabs (Holmgren, 1992). A drawback, however, is that such idealizations and assumptions must be made in order to use the theory for the case studied, although this is the case for all analytical methods.

In this paper, we idealize the shotcrete between four rockbolts to a flat, simply supported, reinforced concrete slab with a yield line pattern in accordance with Fig. 2d, caused by a uniformly distributed load q. By doing so, the support around all edges of the block is equal, i.e. the same as the adhesive fixity along all edges of the idealized slab. Given these assumptions the M can be calculated as (Kennedy and Goodchild, 2004):

$$M = \frac{q(\sqrt{2s^2})^2}{24}, \tag{7}$$

in which s is the center to center distance between the rockbolts.

2.3. Reliability-based design and verification of shotcrete support

The parameters in Eqs. (1)–(7) and the respective calculation models incorporate a number of uncertainties that can be accounted for with reliability-based methods. In reliability-based methods, uncertainties are accounted for by defining one or several limits between safe and unsafe behavior, i.e. limit state functions, G_i, that contain all relevant uncertain input parameters, $\boldsymbol{X}$. Uncertainties in the calculation model can be accounted for by introducing an additional random variable into the limit state functions (see e.g. JCSS, 2001; Krounis, 2016; Spross and Gasch, 2019). However, since the magnitude of this uncertainty is unknown, it is not accounted for in this paper. A similar approach was used by (e.g. Krounis et al., 2017). The limit states for the shotcrete's capacity to sustain loose blocks through either $R_{d.sh}$, R_{fl}, or $R_{p.sh}$ are:

$$G_{d.sh} = R_{d.sh} - W, \tag{8}$$

$$G_{fl} = R_{fl} - M, \tag{9}$$

and

$$G_{p.sh} = R_{p.sh} - W, \tag{10}$$

respectively, which are all conditioned on the occurrence of the events, h_a, of sufficient or insufficient a. The h_a can be defined as:

$$h_a = R_a - W. \tag{11}$$

The shotcrete support thereby acts as a correlated conditional structural system because all limit states contain a loose block and since they are all conditioned on the event h_a (Consequently, G_{fl} and $G_{p.sh}$ are relevant for blocks that are large enough to make a insufficient and $G_{d.sh}$ only relevant for small blocks, for which the a is sufficient) (Fig. 1) The effect of this will thus be that the probability of violating $G_{d.sh}$ will be smaller than the probability of violating $G_{d.sh}$ for an uncorrelated, unconditional system and the probability of violating G_{fl} and $G_{p.sh}$ will be greater than the probability of violating G_{fl} and $G_{p.sh}$ for that same uncorrelated unconditional system.

The probability of exceeding the shotcrete's capacity for the correlated conditional structural system is found by evaluating the multi-dimensional integral over the unsafe regions, D_i, (Melchers, 1999):

$$p_{f.sys} = P[G_i(\boldsymbol{X}) \le 0] = \int_{\cup\cap D_i \in X} \cdots \int f_{\mathrm{X}}(\boldsymbol{X})d\boldsymbol{x}. \tag{12}$$

It should be observed that the integral in Eq. (12) is in many cases very difficult, or even impossible, to solve analytically. Therefore, it is preferable to use an approximate or numerical method to solve the integral. This is exemplified in Section 5, where Monte Carlo simulations are used.

3. Project Stockholm City Line

3.1. Project site

As is evident from Eq. (1) and Eqs. (3)–(6), the shotcrete's capacity to carry loads is mainly governed by the magnitude of a, t, f_{sh}, $f_{ctm,fl}$, and to some extent $f^{re}_{ctm.fl}$ (if R_{fl} is calculated using Eq. (6)). The magnitudes of these parameters therefore need to be established. In addition, to account for the uncertainty incorporated in these parameters, through the use of e.g. Eq. (12), the parameters need to be defined in terms of representative probability distributions. Another important parameter is δ; however, since δ was not quantified in the control program, its statistical quantification is outside the scope of this paper.

To define probability distributions for each parameter, data were collected from the executed controls at one of the construction sites of a recently constructed commuter-train tunnel in Stockholm, Sweden, namely the Stockholm City Line. The project consists of two 6-kilometer long parallel rock tunnels, and two new underground commuter-train stations. The tunnel is situated approximately 10–45 m below the ground surface.

Table 1
Predefined support classes for the Stockholm City Line (modified after Lindfors et al. (2009) and Kettunen Linder and Kilic (2011)).

Rock class (–)	RMR (–)	Distance between rockbolts (m)	Thickness of steel-fiber reinforced shotcrete (mm)							
			Span class 1 (< 6 m)		Span class 2 (6– < 9 m)		Span class 3 (9– < 15 m)		Span class 4 (15–20 m)	
			Roof	Walls	Roof	Walls	Roof	Walls	Roof	Walls
A	70–100	Selective[a]	50	0	50	0	50	0	50	0
B	50–69	1.7[b]	50	0	50	0	75	50	100	50
C	30–49	1.5	75	50	75	50	100	75	100	75
D	< 30	Determined on a case by case basis								

[a] For span class 4, rockbolts were installed systematically in the roof with $s = 2$ m.
[b] For span classes 1 and 2, rockbolts were installed selectively in the walls.

3.2. Geological conditions and rock support

The rock mass at the site consists mainly of good-quality granite and gneiss with a mapped rock mass rating, RMR (Bieniawski, 1989), in the range of approximately 55–95 (Kjellström, 2015). The support used in the project varied along the length of the tunnel. The support used in a certain section of the tunnel was selected based on the predefined support classes (see Table 1), the continuously observed RMR, and the width of the tunnel at the relevant cross section (Lindfors et al., 2009).

The rockbolts used for all rock types were grouted 25 mm reinforcement bars, B500 BT, with lengths of 2.4–5.0 m depending on the span class (see Table 1 for the definition of different span classes). The cement used in the shotcrete was of the Swedish type "Anläggningscement" (CEM I 42.5 N – SR 3 MH/LA), which is a cement commonly used for civil engineering purposes (Bryne et al., 2014). The prescribed recipe for the shotcrete is presented in Table 2. The strength class of the shotcrete was C32/40 (CEN, 2004b) and the exposure class was set to XS3/XF4 (see CEN (2004a)) with a maximum water–cement ratio of 0.40. The minimum allowable air temperature for application of shotcrete was +5 °C.

3.3. Measurements and tests conducted during construction

Measurements and tests were conducted in accordance with the requirements defined in governing standards. The measurement and test methods used, the extent of the measurements and tests, the governing standards, and the defined requirements for each of the quantified parameters are presented in Table 3.

For evaluation of a, cores were drilled through the shotcrete layer and the rock substrate. Each core was cross-cut, after which steel dolles were glued onto the rock end surfaces of the core and the specimens were subjected to an increasing stress until they fractured, all in accordance with EN 14488-4:2005 (CEN, 2005a).

For evaluation of f_c, cube specimens were sawn from a test panel and subjected to load from plates, in accordance with EN 14488-1:2005 (CEN, 2005b) and EN 12504-1:2009 (CEN, 2009) (see Table 3). For evaluation of $f_{ctm,fl}$ and $f^{re}_{ctm.fl}$, prismatic beam specimens sawn from a test panel, prepared in accordance with EN 14488-1:2005 (CEN, 2005b), were subjected to bending moment by applying a load through upper and lower rollers in accordance with EN 14488-3:2006 (CEN, 2006a) (Table 3). Deflections were controlled to obtain the specimens load-deflection response (CEN, 2006a)

However, f_{sh} was not measured on the Stockholm City Line; we therefore use the equation proposed by Bernard (2008):

$$f_{sh} = 0.42\sqrt{f_c} \tag{13}$$

Table 2
Shotcrete used in the Stockholm City Line (Kettunen Linder and Kilic, 2011).

Ingredients	Quantity [kg/m^3]
Cement	520
Water	208
Aggregates, 0–8 mm	394
Steel fibers	55
Alkali-free accelerator	Varying dosage
Air-entrainer	Varying dosage
Superplasticizer	Varying dosage
Retarder	Varying dosage
Density	2300

4. Quantification of shotcrete support variables

4.1. Overview

In the forthcoming sections, the parameters in Table 3 are presented in histograms and quantified in terms of their respective probability distribution, mean, standard deviation, and coefficient of variation. In conjunction with the statistical quantifications, a discussion is provided for each specific parameter. To optimize the bin-width used in the histograms against the number of available test results and the magnitude of their variability, the bin-width was calculated using Scott's rule (Scott, 1979) for t and Freedman–Diaconis rule (Freedman and Diaconis, 1981) for a, $f_{ctm,fl}$, $f^{re}_{ctm.fl}$, and f_c. The goodness of fit was analyzed using one-sample Kolmogorov-Smirnov tests (Massey, 1951) and an ocular assessment.

The probability distribution goodness of fit test was limited to the normal and lognormal distribution, because both distribution types can be objectively assessed based on the statistical moments of the input data; the goodness of fit assessment can thus be done stringently with a minimum of subjective interpretations of, for example, acceptable ranges.

All individual measurements were assumed to be independent of each other. For each parameter, the validity of this assumption was analyzed using Welch's test (Welch, 1947) by comparing the sample means calculated directly from all available measurements and tests (i.e. the results presented in this paper) with sample means calculated via the mean values of each measurement and test series (i.e. the test series mentioned in Table 3). The comparison showed that for all parameters there is no significant difference between the analyzed mean values (with a significance level, α, of 0.05), which makes the assumption of independence reasonable.

4.2. Shotcrete thickness

4.2.1. Statistical quantification

For an evaluation of t, the data from the two test methodologies, i.e. measurements in drilled holes at random locations and in installation holes for rockbolts, were compared for each required t, t_{req} (50 mm, 75 mm, and 100 mm), using Welch's test (Welch, 1947) on the sample means. The tests showed that there was no significant difference (at significance level 0.05) between the sample means obtained from the

Table 3
Measurements conducted in the control program along with the governing standard for each of the quantified parameters. The t_{req} is the required shotcrete thickness.

Parameter	Symbol	Measurements performed	Extent of Measurements	Governing standard	Requirements
Shotcrete thickness	t	Drilled holes in random locations	1 series/500 m^2 of tunnel surface[c]	EN 14488-6:2006 CEN (2006b)	Average of one test series must exceed t_{req} and all individual samples must exceed 80% of t_{req}.
		Drilled holes for rockbolts	> 20% of all holes/ 500 m^2 of tunnel surface[d]	Not a standardized test method	Average of each set of four rockbolts must exceed t_{req}
Adhesion[e]	α	Axially loaded pull-out test	1 series/1000–1250 $m^2$of tunnel surface[f]	EN 14488-4:2005 CEN (2005a)	Average of one test series must exceed 0.5 MPa
Flexural tensile capacity	$f_{ctm,fl}$	Prismatic beam specimens subjected to load from rollers.	1 series/500–2000 m^2 of tunnel surface.[f]	EN 14488-3:2006 CEN (2006a)	Average of one test series must exceed 4.0 MPa and all individual samples must exceed 3.2 MPa.
Residual flexural tensile capacity	$f^{re}_{ctm,fl}$ [g]	Prismatic beam specimens subjected to load from rollers	1 series/500–2000 m^2 of tunnel surface.[f]	EN 14488–3:2006 (CEN, 2006a)	Average of one test series must exceed 3.0 MPa and all individual samples must exceed 2.4 MPa
Compressive strength	f_c	Cube specimens subjected to load from plates	1–2 series/250–1000 m^2 of tunnel surface[f]	EN 12504–1:2009[h] CEN (2009)	Average of one test series must exceed 40 MPa

[c] One series contained five holes spaced 600 ± 50 mm apart in two lines of three holes at right angles sharing the center hole.
[d] Initially all holes. If satisfactory results were achieved a reduction was allowed, although never less than 20% of all holes per 500 m^2 of tunnel surface.
[e] Scaling of the rock surface was performed mechanically with a subsequent cleaning of the rock surface.
[f] One series contained three samples. Samples were sawn from a test panel in accordance with EN 14488-1:2005 (CEN, 2005b).
[g] In this paper, $f^{re}_{ctm,fl}$ corresponds to f_2, the residual flexural tensile strength corresponding to normal deformations (CEN, 2006a). According to (CEN, 2006a) f_1, f_2, and f_4 (i.e. the residual flexural tensile strength corresponding to low, normal, and high deformations, respectively) shall be evaluated. However, on the Stockholm City Line, only f_2 was evaluated.
[h] Evaluation was performed in accordance with BBK 04, 9.1.2 (Swedish National Board of Housing, 2004).

two measurement methodologies when t_{req} equaled 50 or 75 mm ($p = 0.059$ and $p = 0.51$, respectively). For t_{req} 100 mm, however, Welch's test showed a significant difference ($p < 0.001$) between the mean values obtained from the two measurement methodologies (the mean value was 120 mm for the measurements conducted in the installation holes for rockbolts as compared to 132 mm for the measurements conducted in the holes drilled in random locations).

The main reason for this difference is that the measurements conducted in the installation holes for rockbolts contain measurements from one 15-meter long section of the tunnel in which a relatively thin shotcrete t was measured (thin in relation to the t_{req} of 100 mm). The measurements in holes drilled at random locations, on the other hand, do not contain data from this particular section and therefore a higher sample mean value was found. Important to note, however, is that the range of data are the same for both measurement methodologies. In addition to the inclusion of the measurements from this particular section of the tunnel, the measurements conducted in the installation holes for rockbolts show a relatively large portion of thickness measurements, approximately 25% of the measurements, that exactly correspond to t_{req} and a relatively small portion of thickness measurements, approximately 55% of the measurements, that exceed the t_{req}. In all other thickness measurements, approximately 10% of all values correspond to t_{req} and 75–85% exceed the t_{req}, which, again, results in a higher sample mean value for the measurements in holes drilled in random locations.

Since neither of the aforementioned reasons for seeing the difference in sample means for t_{req} of 100 mm indicates that the two measurement methodologies actually yield different measurement results, we have chosen to treat the data from all three values of t_{req} equally and thereby consider the two measurement methodologies to yield mean values that are not significantly different for each t_{req}. Thus, for the evaluation of the suitable probabilistic distributions for t, the data from the two test methodologies were compiled into one dataset for each t_{req}. The quantified results for each dataset of t_{req} can be seen in Fig. 3a–c, and Table 4.

The most suitable distribution for t for all t_{req} (50 mm, 75 mm, and 100 mm) was found to be a lognormal distribution ($D_n = 0.067$, $D_n = 0.11$, and $D_n = 0.13$, respectively, all of which correspond to $\alpha < 0.001$). The lognormal distribution was seen as a suitable distribution in spite of the low significance levels found in the Kolmogorov-Smirnov test, because the significance level is greatly affected by a single large deviation from the theoretical distribution, here in the form of the peaks that can be seen in at least one bin in each of the histograms (Fig. 3a–c). Similar probability distributions for t were found by Björkman and Jabbar (2016) and Sunesson (2017).

4.2.2. Discussion

As seen in Table 4, the mean applied shotcrete thickness was found to be approximately 20–60% higher than the requirement. Notably, the shotcrete layer is approximately 20–30 mm thicker than t_{req}, regardless of the magnitude of t_{req}. The results show a relatively large variability in the measured thicknesses, which might be an effect of the application process and the skill of the operator (Malmgren et al., 2005). The tendency to apply too much shotcrete agrees with the results found in previous research, in which it was concluded that for a specified t_{req} of 60 mm, e.g. the applied t was found to be on average 72 mm with a standard deviation of 27 mm (Ansell, 2010).

The tendency to apply too much shotcrete is probably due to the formulated requirements in the design documents, and thus the construction approach used by the contractor. On the Stockholm City Line, t_{req} was (similar to previous case studies (Ellison, 2000)) defined as a fixed value, with some allowance for values lower than the requirement (see Table 3). This results in an applied t that is larger than t_{req}, largely owing to the cost effectiveness for the contractor to apply a thicker shotcrete layer than required in the first round of application, simply to avoid having to return later to apply an extra layer. As a comparison, in

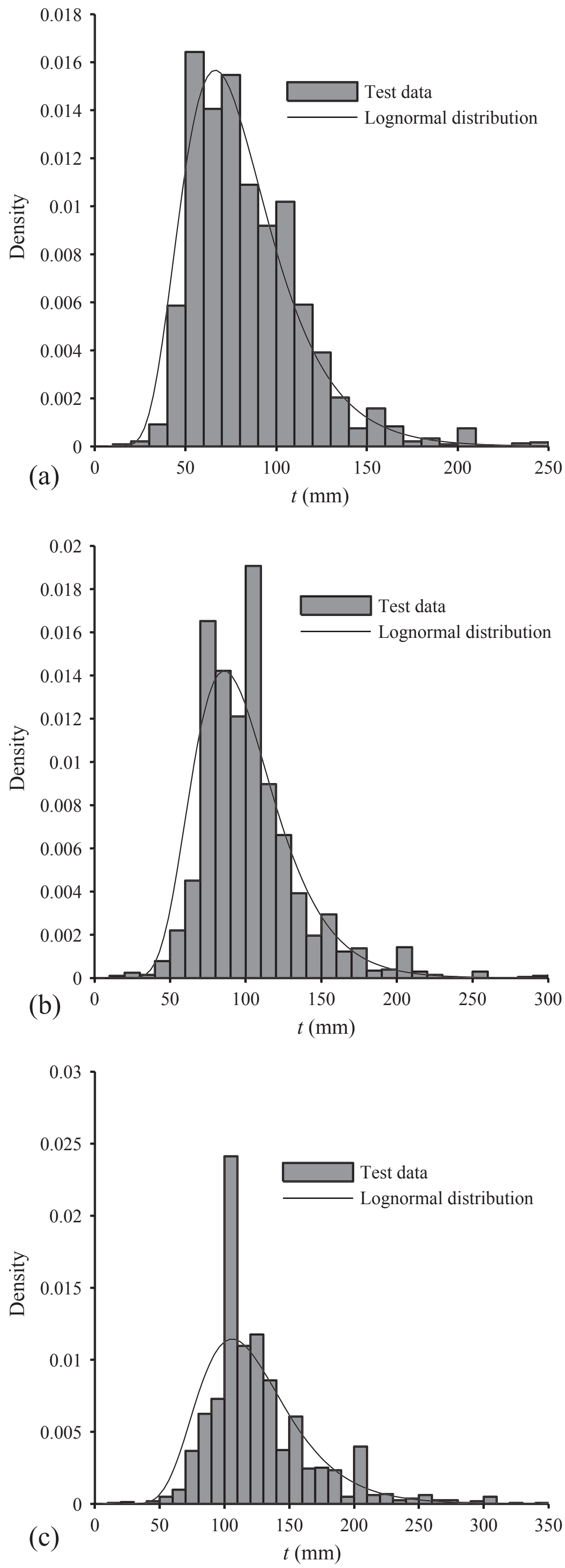

Fig. 3. Shotcrete thickness data for (a) t_{req} = 50 mm, (b) t_{req} = 75 mm, and (c) t_{req} = 100 mm.

a project in which the specified requirement was defined as a range, a mean value close to the midrange was found (Malmgren et al., 2005).

A histogram of t normalized against t_{req}, accounting for all measurements can be seen in Fig. 4 (n = 6068). The sample mean and standard deviation equal 1.4 and 0.5, respectively. As can be seen in the figure, the data show a tendency to be lognormally distributed when combined; however, the fit is not as good as for the separated data (D_n = 0.069, corresponding to α < 0.001). This is due to the above-mentioned tendency to apply approximately 20–30 mm more shotcrete than demanded, regardless of t_{req}. Therefore, we believe that suitable probability distributions should be evaluated separately for each t_{req}.

4.3. Adhesion in the rock–shotcrete interface

4.3.1. Statistical quantification

A histogram of the evaluated a of the shotcrete to the rock surface can be seen in Fig. 5 (n = 354). The sample mean and standard deviation are equal to 0.81 MPa and 0.32 MPa, respectively (Björkman and Jabbar, 2016). A normal distribution was found to describe the data relatively well (D_n = 0.080, which corresponds approximately to α = 0.02).

4.3.2. Discussion

One issue with assigning a normal distribution to a is that the large variability causes the normal distribution to yield a substantial probability of receiving negative values for a, and thus the normal distribution does not describe the uncertainties in a completely correct manner. This can be accounted for by truncating the distribution at zero; however, with respect to the analyzed system, the normal distribution is acceptable to use without truncation because the probability of obtaining negative values does not noticeably affect the calculated probability of shotcrete failure.

The identified large variability in a agrees well with the results found in previously conducted research. During the construction of a road tunnel in Stockholm, Sweden, the mean and standard deviation of the tested a were calculated to be 1.37 MPa and 0.71 MPa, respectively (Ellison, 2000). The calculated mean and standard deviation of a in LKAB's mine in Malmberget, Sweden, were 0.40 MPa and 0.38 MPa, respectively (Malmgren et al., 2005).

The variation in a might be due to a number of different factors (Malmgren et al., 2005). One such factor might be the type of rock to which the shotcrete is applied, owing to the mineral content of the rock and its possible foliation and coarseness (e.g. Hahn, 1983). In weak, highly jointed, porous, coarse, or non-favorably foliated rock, it is likely that debonding does not solely occur in the rock–shotcrete interface (Karlsson, 1980), which results in an evaluated a that is relatively small. In Sweden, this is usually accounted for by disregarding the effect of adhesion in the structural analysis when RMR is less than 50 (Lindfors et al., 2015).

However, data from the Stockholm City Line project show that for the two rock types present, granite and gneiss, no relation can be identified between the type of rock mass and the measured a, and a value of a up to 1.5 MPa has been obtained with an RMR of less than 50 (Fig. 6). It can be seen in Fig. 5, however, that for cases where low values of a have been obtained, failure has to a relatively large extent partly occurred in the rock mass. This indicates that the local rock mass quality close to the rock–shotcrete interface nonetheless has some effect on the measured a, although this can generally not be captured by the RMR value.

These results show that the uncertainty in a can be described using a normal distribution and that no correlation between RMR and a can be seen.

4.4. Compressive, flexural tensile, and residual flexural tensile strength distributions

4.4.1. Statistical quantification

A histogram of the quantified f_c, $f_{ctm,fl}$, and $f_{ctm,fl}^{re}$ can be seen in Fig. 7 (n = 690), Fig. 8 (n = 344), and Fig. 9 (n = 344), respectively.

Table 4
Quantified statistical parameters for shotcrete thickness and suggested distributions.

Requirement (mm)	Number of samples, n (–)	Sample mean (mm)	Sample standard deviation (mm)	Coefficient of variation (%)	Probability distribution (–)
50	2405	81	31	38	Lognormal
75	2040	100	32	32	Lognormal
100	1813	123	42	34	Lognormal

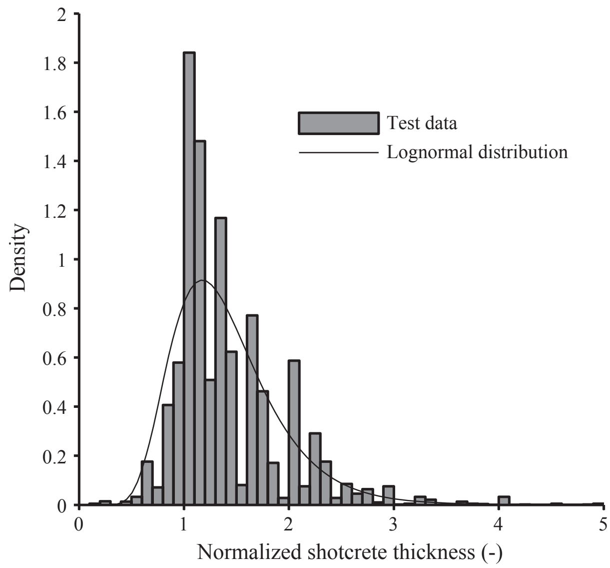

Fig. 4. Histogram and probability density function for t normalized against t_{req}.

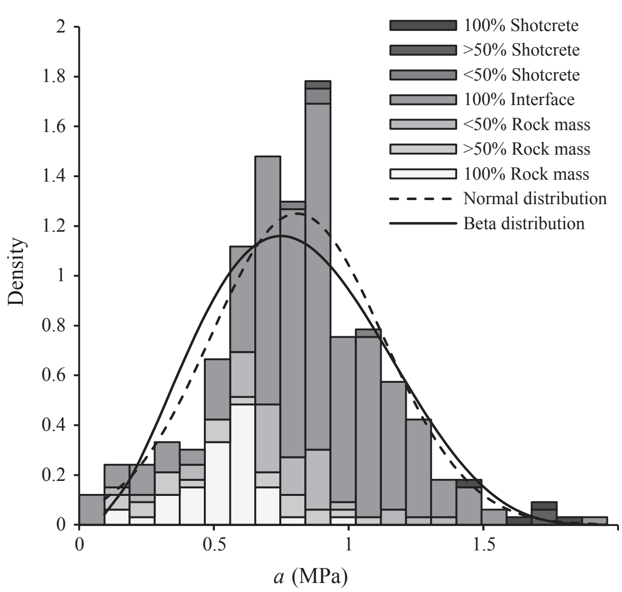

Fig. 5. Probability density plot for a. The dataset for a has been divided into subsets depending on the mapped failure surface from the test. For example, the data denoted as > 50% shotcrete correspond to tests in which more than 50% of the failure occurred in the shotcrete instead of in the shotcrete–rock interface.

For f_c, the sample mean and standard deviation are equal to 59 MPa and 8.1 MPa, respectively. For $f_{ctm,fl}$ and $f^{re}_{ctm.fl}$, the sample mean and standard deviation are equal to 6.8 MPa and 0.84 MPa and 3.8 MPa and 0.80 MPa, respectively. As can be seen in Figs. 7–9, the normal distribution describes the uncertainty in f_c, $f_{ctm,fl}$, and $f^{re}_{ctm.fl}$ relatively well (D_n = 0.050, D_n = 0.071, and D_n = 0.054, respectively, which approximately corresponds to $\alpha \approx 0.05$ for f_c and $f_{ctm,fl}$ and to $\alpha \approx 0.2$ for $f^{re}_{ctm.fl}$).

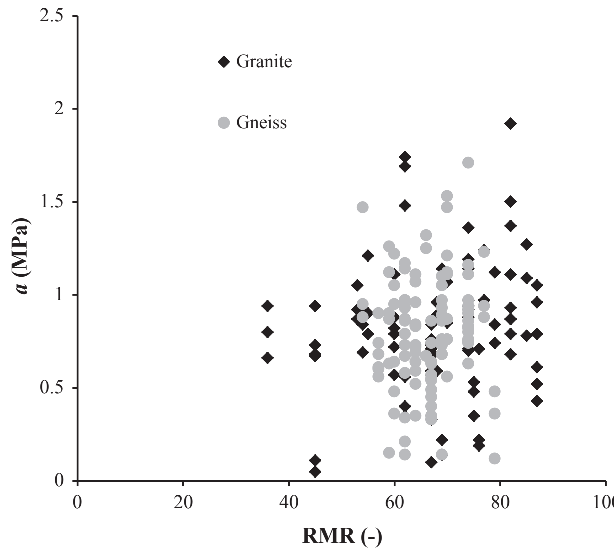

Fig. 6. Measured a for granite and gneiss and their corresponding RMR values.

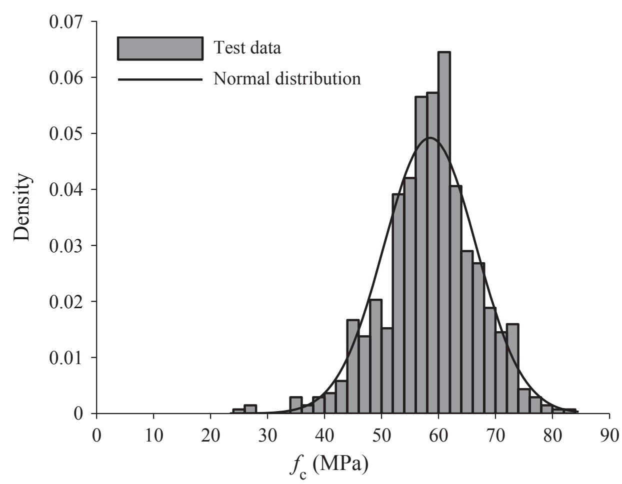

Fig. 7. Histogram and probability distribution for f_c.

4.4.2. Discussion

Finding the normal distribution to be a suitable fit agrees with the expected results (Neville and Brooks, 1987; Neville, 1995). An interesting aspect to note, however, is that the 5th percentiles of the measured f_c and $f_{ctm,fl}$, (which are 45.0 MPa and 5.4 MPa, respectively) exceed the 5th percentile values suggested in Eurocode 2 (CEN, 2004a) for C32/40, which are 40.0 MPa for f_c and 2.1 MPa for $f_{ctm,fl}$ (or 3.1–3.3 MPa for $f_{ctm,fl}$ if consideration is taken to the magnitude of t). In addition, the measured f_c, $f_{ctm,fl}$, and $f^{re}_{ctm.fl}$ largely exceed the formulated requirements of 40 MPa, 4.0 MPa, and 3.0 MPa, respectively (see Table 3).

It should be noted that there is no direct relation between f_c and $f_{ctm,fl}$ and $f^{re}_{ctm.fl}$ because factors such as the shape of the aggregates affect the development of the strength differently (Neville, 1995). A difference in the variability can therefore be expected for $f_{ctm,fl}$ and

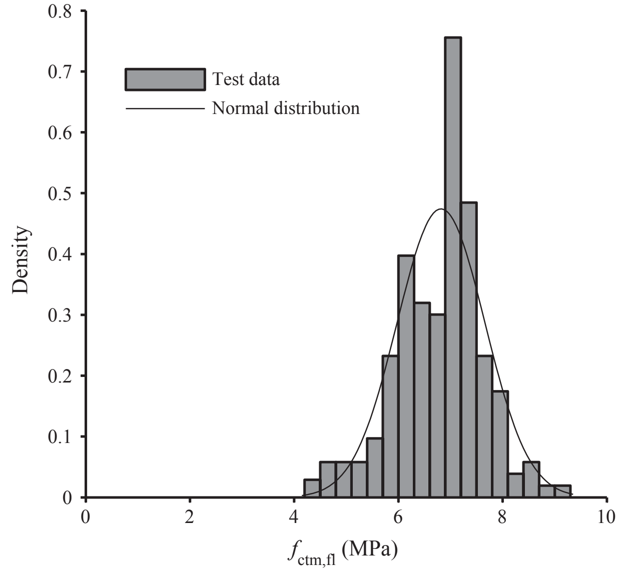

Fig. 8. Histogram and probability distribution for $f_{ctm,fl}$.

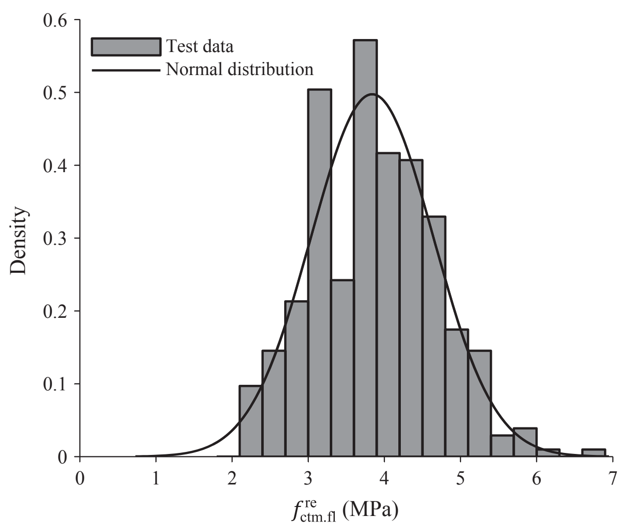

Fig. 9. Histogram and probability distribution for $f^{re}_{ctm.fl}$.

$f^{re}_{ctm.fl}$ compared to f_c.

Based on these results, it can be concluded that the most suitable probability distribution type for f_c, $f_{ctm,fl}$, and $f^{re}_{ctm.fl}$ is the normal distributions. Moreover, f_c and $f_{ctm,fl}$ in this case exceed the values suggested in Eurocode 2 (CEN, 2004a) as well as the formulated requirements in the project. From a design perspective, a relatively high strength is not an issue, because it is on the conservative side. However, similarly to t, from an economic and long-term sustainability perspective, the f_c, $f_{ctm,fl}$, and $f^{re}_{ctm.fl}$ obtained should preferably coincide with the values used in the design.

4.5. Suggested distributions for the quantified shotcrete support characteristics

A summary of the suggested distributions is presented in Table 5.

5. Discussion on the applicability of the presented probability distributions in calculation of shotcrete failure probability

5.1. Illustrative example

To illustrate and discuss the applicability of using the presented data as a part of reliability-based shotcrete support verification, a calculation example with an assumed block located between rockbolts is presented in the following.

There is a fundamental difference between design and verification of shotcrete support, and how the uncertain parameters should be treated. In the design process, prior to excavation of the tunnel, a tunnel support system is usually designed based on calculations and scarce (or experience-based) data for all input variables for predefined rock mass classes. Requirements corresponding to the values used in design are also formulated for each critical input parameter. During construction, the rock mass quality is then observed and, based on the observed rock mass quality, the suitable support system is chosen. To verify that the quality of the constructed support fulfills the formulated requirements, controls are performed in accordance with the defined control program and the support's capacity is verified by comparing the results of the controls with the formulated requirements. Depending on the results of this comparison, updated calculations based on the knowledge gained though the control program might be necessary to judge whether the chosen design is satisfactory. The process of performing an initial design of the structure, observe the behavior of the structure during construction, and verify that the structure fulfills the formulated support capacity requirements, follows the main principles of the original version of the observational method (Peck, 1969).

As this example illustrates the use of the presented parameters as a part of shotcrete support verification, both the quantified magnitude and probability distribution can be used directly. If calculations are made during the design phase, the magnitude of the parameters must conform to the formulated requirements.

The input data used in the example are presented in Table 6. The calculated probability of failure of the shotcrete system, $p_{f.sys}$, (see Fig. 1 for the definition of failure of the shotcrete system) was calculated by approximating the integral in Eq. (12) using crude Monte Carlo simulations. The number of simulations was set to 10 000 000. In each simulation, all parameters were randomly generated based on their defined distribution and the event $h_a < 0$, and the limit states functions $G_{d.sh}$, $G_{p.sh}$, and G_{fl} were evaluated. A simulation was counted as failure if (1) the event $h_a < 0$ occurred in the same simulation as the limit state $G_{p.sh}$, or G_{fl} were violated or (2) the event $h_a < 0$ did not occur in the same simulation as the limit state $G_{d.sh}$ was violated; thus, the correlation between the event h_a, and the limit states $G_{d.sh}$, $G_{p.sh}$, and G_{fl} was accounted for directly (Melchers, 1999). Model uncertainty was not accounted for. The calculations of R_a, $R_{d.sh}$, $R_{p.sh}$, and R_{fl} were conducted using Eq. (1), and Eqs. (3)–(5), respectively. Eq. (5) was used instead of Eq. (6), i.e. the R_{fl} was calculated at first crack) because the data obtained from the control program were not sufficient for evaluation of $R_{10/5}$ and $R_{30/10}$. The load acting on the shotcrete was calculated using Eqs. (2) and (7). The f_{sh} was estimated using Eq. (13) and the quantified f_c. The volume of the block was approximated based on the volume of a pyramid-shaped block with a base area that fits within the spacing of the rockbolts (Fig. 2a, b, and d). To account for some uncertainty in the assumed block volume, the apical angle of the pyramid-shaped block was set to vary within the range of 60 and 120 degrees. For a normally distributed variability in block volume, this corresponds to the values given in Table 6. Similar block volumes have been used in previous research (Barrett and McCreath, 1995; Bjureland et al., 2017a). For simplicity, the same volume was used in the calculations for all failure modes, except for punching shear. In punching shear, the block volume was calculated in accordance with the suggestion made in Section 2.2. The calculated probability of limit violation for each limit state can be seen in Fig. 10.

Table 5
Statistical parameters for each of the quantified variables along with the suggested distribution.

Parameter	Symbol	Sample mean	Sample standard deviation	Coefficient of variation	Suggested distribution
Shotcrete thickness	t	81 mm	31 mm	38%	Lognormal
		100 mm	32 mm	32%	
		123 mm	42 mm	34%	
Adhesion	a	0.81 MPa	0.32 MPa	40%	Normal
Flexural tensile capacity	$f_{ctm,fl}$	6.8 MPa	0.84 MPa	12%	Normal
Residual flexural tensile capacity	$f_{ctm.fl}^{re}$	3.8 MPa	0.80 MPa	21%	Normal
Compressive strength	f_c	59 MPa	8.1 MPa	14%	Normal

The results show that $p_{f.sys}$ to a large extent is governed by R_a; if R_a is sufficient (i.e. $h_a > 0$), limit state violation of the shotcrete system is relatively unlikely, owing to the high $R_{d.sh}$. The probability of simultaneous occurrence of $h_a > 0$ ($p_{s.a}$=0.8442) and $G_{d.sh} < 0$ ($p_{f.shd} < 10^{-7}$) is $<10^{-7}$. On the other hand, if R_a is insufficient (i.e. $h_a < 0$), even though still relatively unlikely, the probability of limit violation increases (i.e. $p_{f.sys}$ is larger than the probability of the simultaneous occurrence of $h_a > 0$ and $G_{d.sh} < 0$). It can be observed that a correlation exists between the $p_{f.a}$, $p_{f.fl}$, and $p_{f.shp}$ since the calculated $p_{f.sys}$ is larger than the product of the calculated $p_{f.a}$ and the series system probability of $p_{f.fl}$ and $p_{f.shp}$. In this calculation example, $R_{p.sh}$ is relatively large compared to R_a. However, this result is dependent on the assumptions made in the calculation example and the input parameters used. Therefore, if other assumptions are made and if other input parameters are used the calculated probability of limit violation for each limit state will change.

It should be observed that the calculated results apply, given the assumed block. In practice, the block size probability distribution, which in turn is connected to the rock mass quality, should be taken into account.

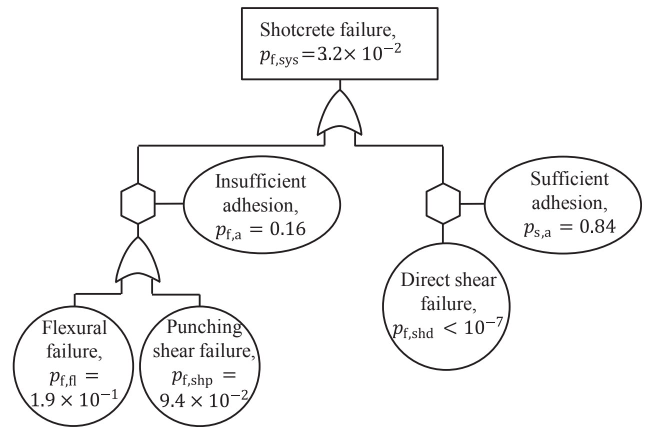

Fig. 10. Fault tree representing the idealized failure system of the shotcrete layer and calculated probability of limit state violation for each failure mode.

5.2. Discussion on reliability-based design and verification

As shown in the calculation example, the presented probability distributions of the parameters can be used in reliability-based analyses of the considered shotcrete support failure modes. However, there are some important aspects that should be further addressed in the existing model.

First, the scale at which each parameter is quantified has an influence on the magnitude, uncertainty, and suitable probability distributions. The measurement methods used on the Stockholm City Line allow for a quantification of each parameter over the entire length of the tunnel, as in the quantification made in this paper. However, the quantified variability in each parameter does not necessarily coincide with the variability of each parameter within the area of a single block, owing to its scale of fluctuation. A deeper understanding of the variability of the parameters within the area of a single block and the scale of fluctuation of each parameter would therefore be preferable.

Second, the common way of treating the mechanical system of a shotcrete support in design against loose blocks, similar to the approach used in the calculation example, is to assume that the shotcrete support capacity is governed by the spatial mean value of each parameter over the entire area of the block, i.e. it acts as a mean value driven system (see e.g. Holmgren, 1992; Barrett and McCreath, 1995; Banton et al., 2004; Lindfors et al., 2015). As such, the spatial variability of each parameter and the effect of this variability on the calculated results are commonly neglected. In reality, however, the spatial variability of each parameter might cause the shotcrete support to act as a weakest link system, implying that the support capacity might be governed by the

Table 6
Input data for the calculation example of shotcrete support verification.

Parameter	Symbol	Unit	Distribution	Sample mean	Sample standard deviation
Shotcrete					
Shotcrete thickness [i]	t	[mm]	Lognormal	100	32
Adhesion	a	[MPa]	Normal	0.81	0.32
Bending tensile capacity	$f_{ctm,fl}$	[MPa]	Normal	6.8	0.84
Compressive strength	f_c	[MPa]	Normal	59.0	8.10
Width of the load bearing zone [j]	δ	[mm]	–	30	–
Rockbolts					
Center to center distance	s	[m]	–	1.7	–
Equivalent radius of face plates	b	[mm]	–	80	–
Rock mass					
Unit weight of rock mass	γ_r	[kN/m^3]	–	27.00	–
Circumference of block	O	[m]	–	6.80	–
Volume of block	V	[m^3]	Normal	5.14	0.61

[i] Corresponds to values retrieved for t_{req} = 75 mm.
[j] The chosen value corresponds to that supported by Hahn and Holmgren (1979); Banton et al. (2004).

lowest value of a certain parameter over the area of the block. The assumption of an averaging system could thus be non-conservative and a deeper understanding of the effect of this assumption is therefore needed.

Third, in the calculation example, δ was defined as a deterministic value, the magnitude of which was based on the small number of tests performed by Hahn and Holmgren (1979). As discussed by e.g. Stille et al. (1988), however, δ must to some extent be correlated to the t of the applied shotcrete layer, since, δ for small values of t is presumably smaller than the 30 mm suggested by Hahn and Holmgren (1979) and Banton et al. (2004), which indicates that some correlation exists. This should preferably be accounted for in the analysis. In addition, to account for the uncertainty incorporated in δ, it should also be defined in terms of the representative probability distribution. However, this would require a large number of additional laboratory tests.

Fourth, as mentioned in chapter 2, all of the idealized single shotcrete failure modes are assumed to be independent of each other. In practice, however, failure of the shotcrete layer can occur through a combination of the idealized single failure modes, which can affect its load carrying capacity. As a consequence, the idealization has a major effect on the uncertainty in the calculation model and further studies on how a combined failure, consisting of a combination of the idealized single failure modes, affects the load carrying capacity are therefore needed to quantify the uncertainty related to the calculation model.

Last, in the calculations performed in the illustrative example, as is also common in deterministic design of shotcrete support, it is assumed that the block is located between four rockbolts, that it is of a certain volume and shape, and that it is actually loose. These assumptions affect the calculated p_f and should therefore ideally be considered. This subject has been the topic of some of the previously conducted research (e.g. Hatzor, 1993; Kuszmaul, 1994, 1999; Mauldon, 1995; Duzgun and Einstein, 2004); however, further efforts are still needed to improve the model.

Based on the above, it can be concluded that further research would be preferable on several issues related to the existing calculation model to improve its accuracy in the reliability-based design and verification of shotcrete support in rock tunnels.

6. Conclusions

In this paper, we present how shotcrete support for small loose blocks in a jointed rock mass can be viewed as a correlated conditional structural system. In addition, we statistically quantify a unique amount of data for the parameters governing the load carrying capacity of shotcrete support (shotcrete thickness, adhesion, flexural tensile strength, residual flexural tensile strength, and compressive strength). This statistical quantification shows that the most suitable probability distribution for shotcrete thickness is a lognormal distribution, whereas for adhesion, flexural tensile strength, residual flexural tensile strength, and compressive strength it is the normal distribution. A calculation example illustrates how the data can be used in a reliability-based analysis of a shotcrete's capacity to support small loose blocks. However, as in all existing calculation models, there are aspects that are regularly not taken into account in the calculations. In the model used in this paper, aspects such as the scale of fluctuation of the parameters governing the support capacity, the mechanical system of the shotcrete support, the magnitude and uncertainty of the area over which the shotcrete adhesion acts, failure consisting of a combination of the idealized failure modes, and the actual existence of a loose block acting on the shotcrete are not accounted for. Therefore, to further improve the accuracy of the existing calculation model in reliability-based calculations in the design and verification of shotcrete support in rock tunnels, these aspects should be studied further.

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Appendix A. Supplementary material

Supplementary data to this article can be found online at https://doi.org/10.1016/j.tust.2019.02.002.

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