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An Investigation of Positive Engagement, Continuously Variable Transmissions

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AN INVESTIGATION OF POSITIVE ENGAGEMENT, CONTINUOUSLY VARIABLE TRANSMISSIONS

by

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GRADUATE COMMITTEE APPROVAL

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As chair of the candidate's graduate committee, I have read the thesis of Ryan R. Dalling in its final form and have found that (1) its format, citations, and bibliographical style are consistent and acceptable and fulfill university and department style requirements; (2) its illustrative materials including figures, tables, and charts are in place; and (3) the final manuscript is satisfactory to the graduate committee and is ready for submission to the university library.

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ABSTRACT

AN INVESTIGATION OF POSITIVE ENGAGEMENT,

CONTINUOUSLY VARIABLE TRANSMISSIONS

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Master of Science

A Positive Engagement, Continuously Variable Transmission (PECVT) allows for a continuously variable transmission ratio over a given range using positively engaged members, such as gear teeth, to transmit torque. This research is an investigation of PECVTs to establish a classification system and governing principles that must be satisfied for an embodiment to overcome the non-integer tooth problem. Results of an external patent search are given as examples of different concepts and PECVT embodiments that have been employed to negate the effects of the non-integer tooth problem. To classify all published and unpublished PECVT embodiments, a classification system is developed, based on how particular PECVT embodiments overcome the non-integer tooth problem. Two classes of PECVTs are defined: 1) the problem correction class and 2) the alternate device class. General principles that must

be satisfied for a promising PECVT embodiment to exist in each class of PECVTs are also developed. These principles, along with the classification system, are the major contribution of this research. The principles describe what an embodiment in each of the PECVT classes must accomplish to negate the effects of the non-integer tooth problem.

A product development phase integrated with TRIZ methodology is implemented to generate several concepts that satisfy the newly developed general principles and the product specifications that were also created. A screening and scoring process is used to eliminate less promising concepts and to find the most viable PECVT embodiment. An embodiment that only operates at preferred transmission ratios, where no meshing problems exist, proves to be the most promising concept based on the results of this methodology. The embodiment also utilizes cams and a differential device to provide the needed correction to the orientation of the driving members when misalignment occurs. This misalignment only occurs while transitioning between preferred operating ratios.

A case study of the final embodiment developed by Vernier Moon Technologies and Brigham Young University is presented and analyzed to show how the final concepts ensure proper engagement without the effects of the non-integer tooth problem. The final embodiment is not the optimal solution but represents a conceptual design of an embodiment that satisfies the governing principles. The classification system and the governing principles that have been established are valid for all PECVT embodiments and will be valuable in future research. Future work yet to be conducted for this research, including an involutometry analysis, is discussed as well as other recommendations.

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1 Introduction

This chapter will discuss the background and purpose of this research. It will also present the functional decomposition of a standard transmission and give a brief introduction to Positive Engagement, Continuously Variable Transmissions (PECVT). A description of the non-integer tooth problem, which exists in the majority of known PECVT embodiments, will also be provided. The final sections of this chapter will discuss the objectives and methods of this research, along with certain delimitations.

1.1 Background

The primary purpose of an automotive transmission is to transmit mechanical power from the engine to the applied load at the wheels. The mechanical power produced by the engine is transmitted in the form of the engine's rotational speed and torque. The transmission couples this power from the engine to the load at the wheels by providing a speed (RPM) and torque ratio change. Although the power provided by the engine is conserved while being transmitted to the load, the transmission is able to transmit the desired RPM and torque values to the load by using different gear ratios. A basic flow diagram of a functional decomposition of a transmission is shown in Figure 1.1. A functional decomposition begins by breaking the primary function of the product down into smaller, more specific functions that describe what a product must do to achieve its primary function. This is continued until each function is small enough to work with; after which, a function diagram is created. It is important not to specify a specific method for obtaining or performing the function, as each function should include only what the desired outcome is and not how the function is to be achieved [1]. The importance of this functional decomposition will be discussed later in Chapter 3.

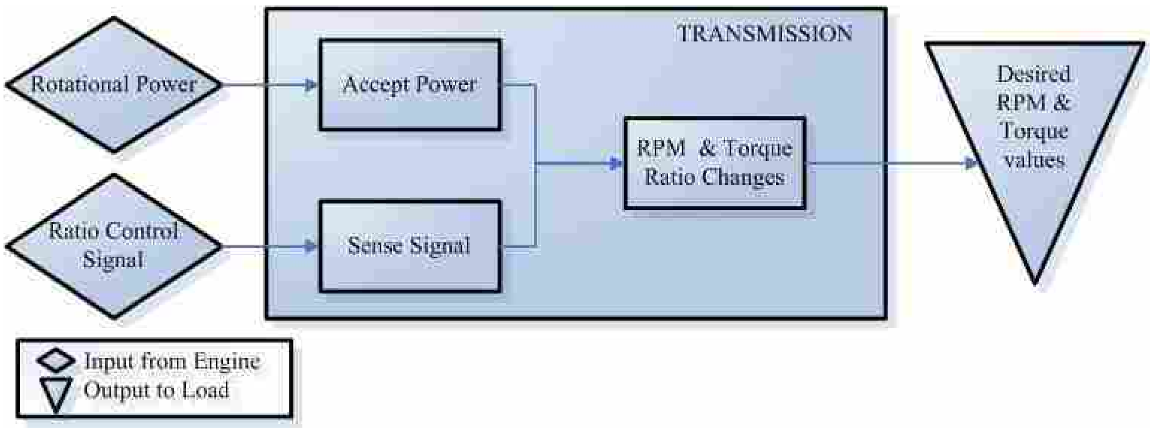

Figure 1.1: Functional Decomposition of a Traditional Transmission

Generally, a transmission receives as inputs some form of rotational power along with a ratio control signal and varies the RPM and torque ratios to obtain a desired output RPM and applied torque value at the load. This RPM ratio is also called the transmission ratio, or drive ratio [2]. For any given engine speed converted by the transmission to the load, there exists a certain transmission ratio which will allow the engine to operate at maximum torque, power, or efficiency. Therefore, if the transmission ratio were allowed to continuously change to that desired ratio as the engine output speed continually changes, the engine could operate at optimal torque, power, or efficiency states depending on which characteristic is desired for certain operating conditions. Because a standard transmission possesses a finite number of gear ratios (usually 5 ratios) as seen in Figure 1.2, the engine is required to vary its speed in order to provide a continuously varying output from the transmission to the load at the wheels. As a result, the engine is limited to providing maximum performance or efficiency over small ranges of output speed [3]. In addition, the torque applied to the gear sets from the load must be disconnected in a standard transmission when changing from one gear ratio to the next, also decreasing the overall efficiency and performance of the engine and automobile as a whole.

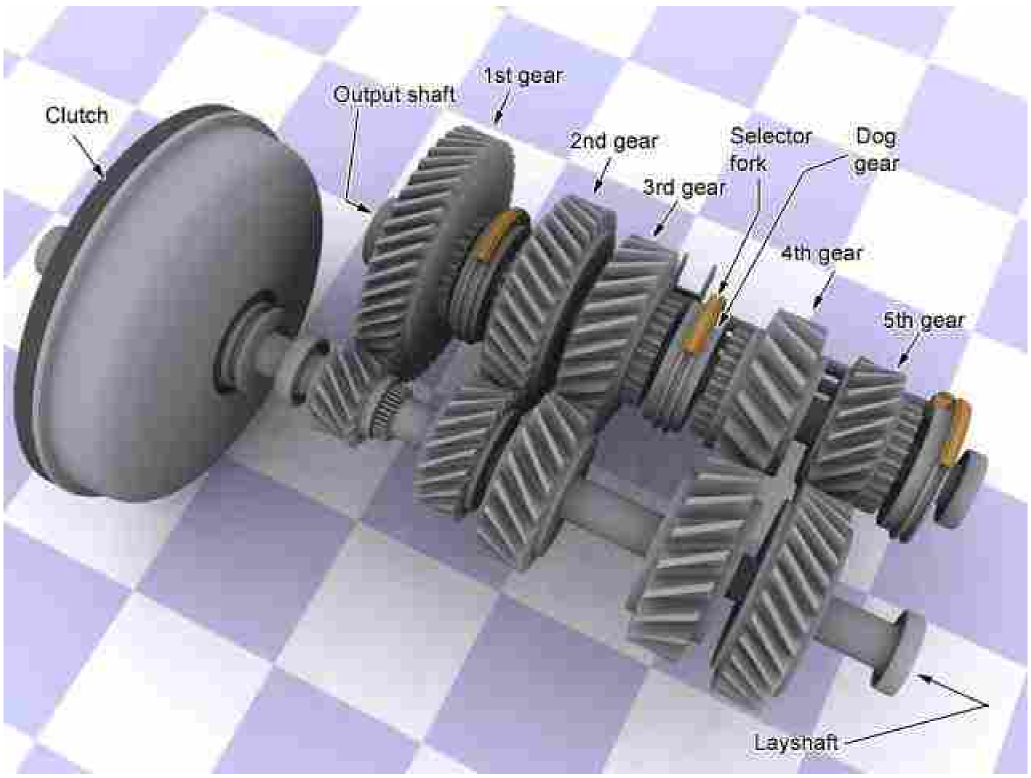

Figure 1.2: Basic Components of a Standard Manual Transmission [4]

A Continuously Variable Transmission (CVT) is a transmission that allows for a continuously variable RPM and torque ratio change over a finite operating range. In traditional manual and automatic transmissions, this continuous ratio change is not possible due to a finite number of gear ratios that exist in the system. Through its ability to continuously change the transmission ratio without disconnecting the load, a CVT allows for a continuously variable output, while also allowing an engine to operate at optimum RPM ranges or at maximum torque, regardless of the output speed of the transmission, thus maximizing vehicle efficiency and performance [5].

Many different categories of CVTs exist today in various applications based on basic design components that will be further discussed in Chapter 2. Many CVTs currently used in ATVs and automobiles rely upon friction driven belts and pulleys to transmit mechanical power from a power source (engine) to the load (wheels). These friction-drive CVTs, however, are limited to lower torque applications where the torque applied from the load does not exceed the maximum allowable torque transmitted by the belts and pulleys through friction. In applications where higher torque values are applied by the load, the need for positive engagement, geared transmissions found in automatic and manual transmissions exists to increase the maximum allowable transmitted torque. In addition, because many traditional CVTs rely on friction to transmit torque, internal components often experience excessive wear, which reduces efficiencies and causes failure in the transmission's functionality. These disadvantages have lead researchers to develop a new category of CVTs that do not rely on friction to provide continuously variable speed and torque ratios. This category is known as the Positive Engagement Continuously Variable Transmission (PECVT).

1.2 PECVT Introduction

The main function of a PECVT is to vary the RPM and torque ratios of the transmission in a continuous manner, as in a traditional CVT, by using positively engaged members, as shown in Figure 1.3. The common gear set shown in the figure only provides one set gear ratio; however, a PECVT will use the same positively engaged method of transmitting torque while achieving a range of continuously variable gear ratios.

Figure 1.3: Example of Positively Engaged Members Used in PECVT Embodiments (Retrieved from www.gcseinappliedscience.com)

While conserving popular attributes possessed in traditional CVTs, a PECVT utilizes positively engaged gears or other members to transmit torque (without disconnecting the load from the engine). This allows the transmission to transmit more torque than friction-drive CVTs and reduces component wear. In addition, the engine is allowed to operate at maximum performance or efficiency ranges made possible through a continually variable transmission ratio provided by the PECVT. Because of these attributes, the PECVT could be used in more automotive applications than the traditional CVT, and if proven viable, the PECVT could also prove to be beneficial in broader applications than those of the automobile industry.

Many PECVT embodiments currently exist in various designs and are published in patents and other literature; however, an ideal embodiment that meets all functional specifications, or desired characteristics, of a PECVT has not yet been realized. It is also necessary to note that in each of the previously published embodiments in different literature sources, there are inherent challenges or limitations that exist which inhibit the particular embodiments from being ideal. The major challenge which exists in the majority of published PECVT embodiments, is a gear tooth engagement problem known as the non-integer tooth problem [3]. An investigation of different PECVT embodiments that may eliminate the aforementioned non-integer tooth problem has been proposed at Brigham Young University in association with a research sponsor. A functional decomposition of a standard PECVT is shown in Figure 1.4, which describes the primary function of a PECVT as described previously.

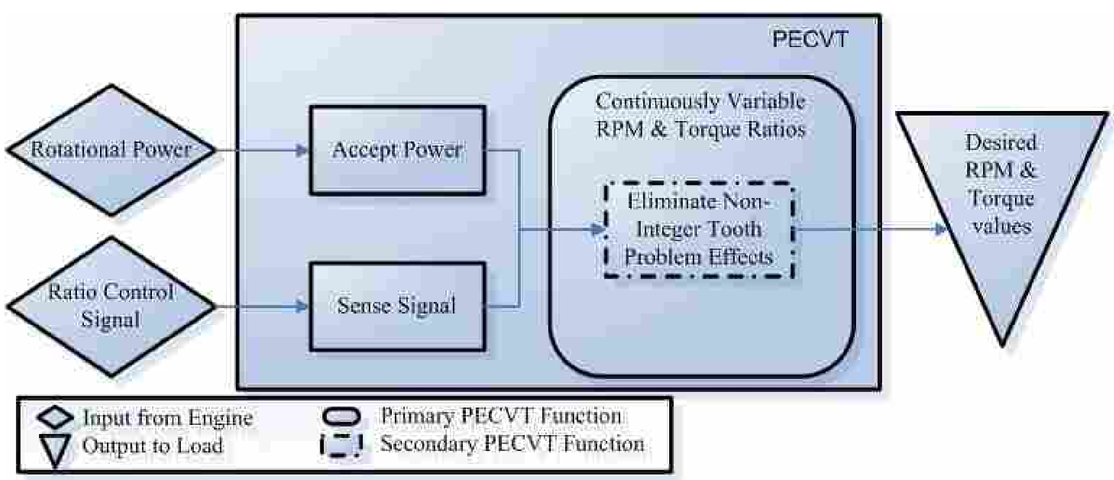

Figure 1.4: Functional Decomposition of a Standard PECVT

The primary function of the PECVT is to provide a continuously variable RPM and Torque Ratio to the load using positively engaged members. The secondary function is to eliminate the effects of the non-integer tooth problem for proper engagement to occur.

1.3 The Non-Integer Tooth Problem

In transmission embodiments that use a single mechanical input, the transmission functions much like a common gear pair relationship like that shown in Figure 1.3. This type of embodiment is similar to traditional manual and automatic transmissions in that one gear acts as the input drive gear while the other gear acts as the driven or output gear. To achieve an RPM ratio change in single input embodiments, the diameter of at least one of the two gears in the gear pair must change. The RPM ratio is defined as:

$$\frac{RPM_{in}}{RPM_{out}} = \frac{D_2}{D_1} \tag{1.1}$$

Where:

D_1 = Pitch diameter of the drive, or input, gear

D_2 = Pitch diameter of the driven, or output, gear

Since the output RPM is generally lower than the input, this is commonly called the *gear reduction*. The pitch diameter, D, of any gear in general is defined as [2]:

$$D = \frac{N}{P_d} \tag{1.2}$$

Where:

N = Number of teeth on the gear

P_d = Diametral pitch of the gear

In order to change the diameter of a gear, either the number of teeth on the gear (N) or the diametral pitch of the gear (P_d) must change, as seen in equation 1.2. To ensure proper meshing in traditional manual and automatic transmissions, the change in diameter and effectively the change in RPM ratio are achieved by switching to a gear pair in which the ratio of teeth numbers on the drive to driven gears has been increased. This is accomplished by rerouting the power through a different gear pair. However, some published PECVT embodiments use the equally viable method of changing the diametral pitch to achieve the desired change in diameter.

A meshing problem called the non-integer tooth problem exists in PECVTs that attempt to achieve a step-less change in the transmission ratio by varying the number of teeth or the diametral pitch of a gear. This problem, along with many of its characteristics, is described in detail in research performed by Brian Andersen [3]. A full detailed account, with all of the characteristics of the non-integer tooth problem, will not be provided in this research. However, a brief description of the problem is valuable and useful in understanding the objectives of this research, which will be given later in this chapter. The non-integer tooth problem is best understood by considering an example of the problem shown in Figure 1.5 below.

Figure 1.5 shows two different drive sprockets (or gears) with a driven chain that represents the driven members. The two gears have different diameters, D_1 and D_2, representing two different transmission ratios, each of which requires the same circular pitch to mesh properly with the constant pitch chain (a). Each gear's circumference is a function of its diameter, D_1 for drive gear 1 (b) and D_2 for drive gear 2 (c). Drive gear 1 (b) has a circumference such that when divided by its circular pitch (a) results in an

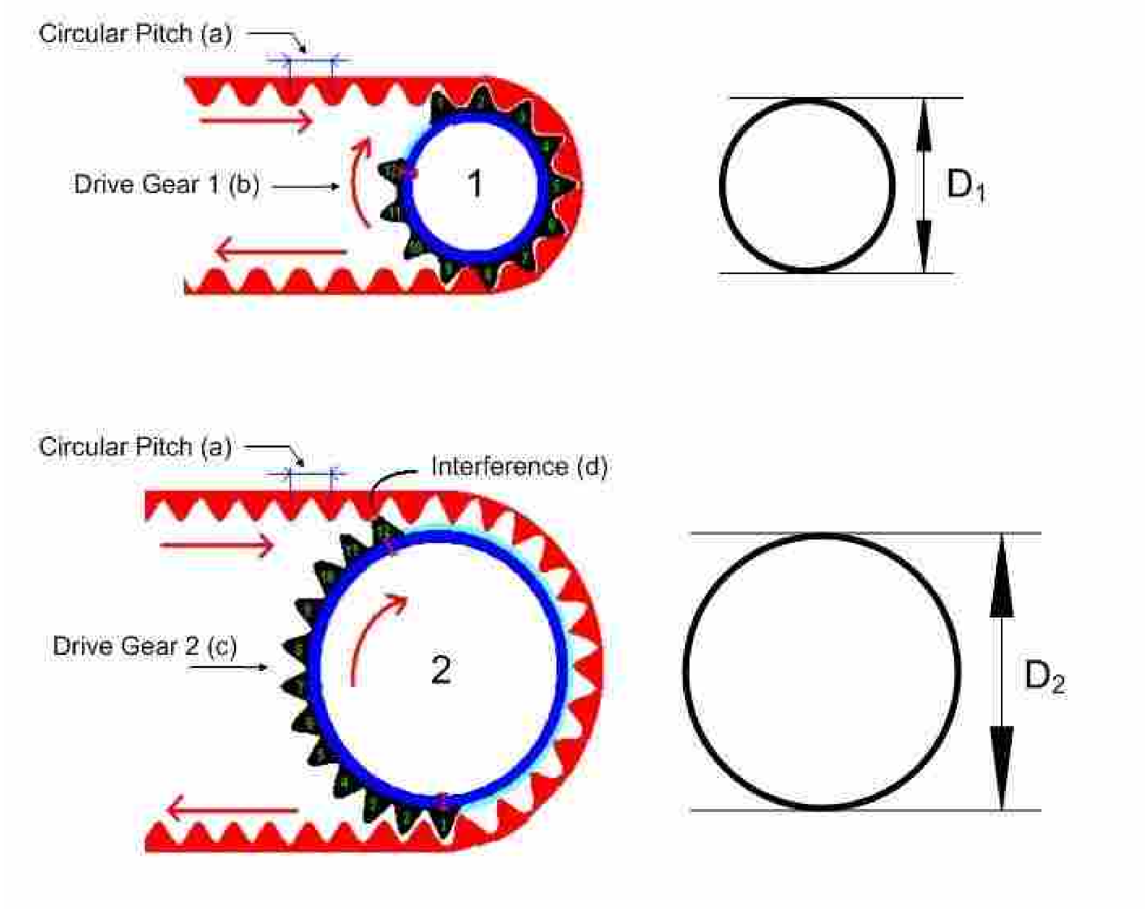

Figure 1.5: Non-integer Tooth Problem That Exists in PECVTs Requiring a Correction (Retrieved from http://cvt.com.sapo.pt)

integer number of teeth on the gear. Therefore, drive gear 1 (b) meshes properly with the chain. Drive gear 2 (c) has a circumference such that when divided by its circular pitch (a) results in a non-integer number of teeth, or a partial tooth, on the gear. This partial tooth causes a collision (d) to occur when the gear teeth attempt to reengage with the chain. Therefore, drive gear 2 (c) will not mesh properly when engaged with the chain. This meshing problem is known as the non-integer tooth problem.

By definition, a PECVT can continuously vary the transmission ratio in a step-less manner over a given range, thus providing an infinite number of gear ratios between that range. To create an infinite number of gear ratios, there would need to be a continuous increase or decrease in the number of teeth of a particular gear, within an engaged gear pair, to effectively change the gear diameter. This continuous change in the number of teeth requires that teeth be added to the gear in non-integer increments. If a gear was created with a circumference that, when divided by the circular pitch, results in a non-integer number of teeth on the gear, the non-integer tooth problem occurs. Figure 1.6 shows the effect of adding teeth to a gear in non-integer increments as a partial tooth is formed.

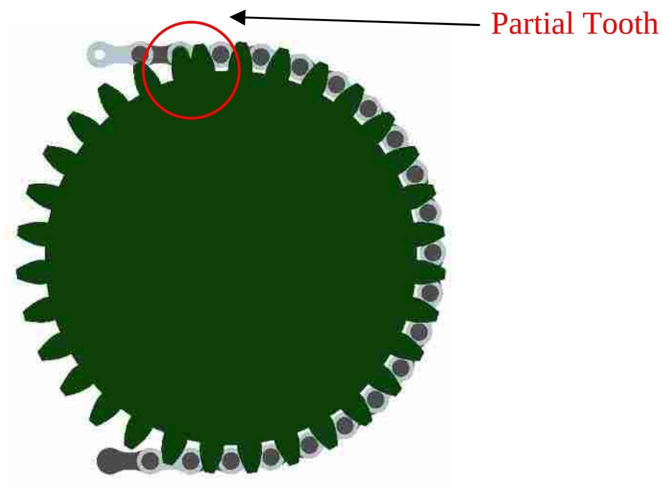

Figure 1.6: A Gear with a Pitch Diameter that Results in a Non-Integer Number of Teeth [3]

1.4 PECVT Embodiment Description

A conceptual design of a PECVT embodiment is shown in Figure 1.7. This embodiment will be discussed in detail in chapter 5 as a case study of a possible final embodiment. It is presented here for additional understanding of how the non-integer tooth problem manifests itself in similar PECVT embodiments.

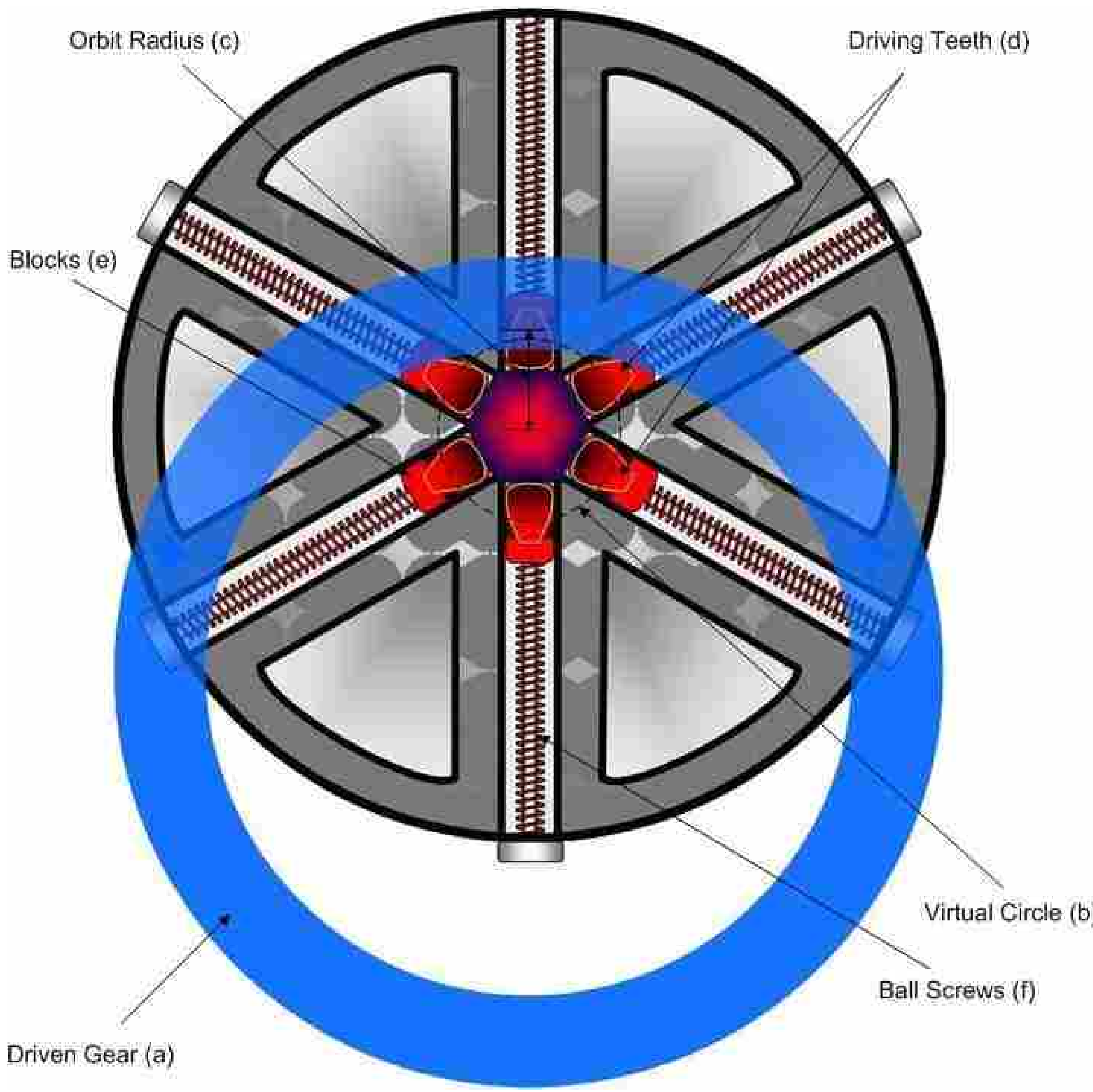

Figure 1.7: Example of PECVT Embodiment

Comparable to the sprocket and chain example shown previously, the embodiment in Figure 1.7 is composed of an effective input gear, or virtual circle (b), represented by 6 individual driving teeth (d) and an output, or driven, gear (a). The driving teeth (d) rotate about the central axis of the virtual circle (b) providing an input to the transmission. Torque is transmitted from the driving teeth (d) to the driven gear (a) in a positively engaged manner (which is not shown in the figure due to the transparency of the driven gear).

To achieve a variable ratio, each driving tooth (d) is attached to individual blocks (e), which are connected to ball screws (f). The ball screws (f) will collectively translate the driving teeth (d) in and out radially, effectively changing the diameter of the virtual input gear, or virtual circle (b). As the effective input gear, or virtual circle (b), increases or decreases in diameter, the transmission ratio also varies in a continuous manner as at least one driving tooth (d) is always engaged with the driven gear (a) at any given time. As the diameter of the virtual circle (b) increases, the driven gear (a) is forced to translate vertically to maintain engagement with the driving teeth (d), while at the same time rotating about its own central axis.

The non-integer tooth problem manifests itself during transition of the driving teeth (d) as the ball screws (f) collectively change the diameter of the virtual circle (b), effectively varying the transmission ratio. During this transition, the virtual drive gear, or virtual circle (b), possesses characteristics of a gear with a continuously variable diameter. However, in several instances during the translation of the driving teeth (d) to different radial locations, the virtual circle (b) possesses a diameter where its circumference is not equally dividing by the pitch of the driving teeth (d). When the virtual circle (b) possesses this circumference, the driving gears (d) will not mesh properly with the driven gear (a) due to the non-integer tooth problem. The case study in Chapter 5 will explain in detail how this embodiment overcomes the non-integer tooth problem.

1.5 Research Objectives

This research investigated possible solutions to overcome the non-integer tooth problem that exists in many Positive Engagement, Continuously Variable Transmissions. The objectives of this research are to:

- Establish governing principles and functional specifications that must be satisfied in PECVT embodiments for solutions to the non-integer tooth problem to exist
- Develop a classification system of classes and families, based upon these principles, to categorize all existing embodiments of PECVTs found in patents and from other literature review sources
- Demonstrate a methodology for evaluating PECVT classes and families according to the established functional specifications that will be created
- Classify, develop, and analyze new innovative concepts that are created during this research in attempt to find the most promising PECVT embodiment.

The following steps were used to meet these objectives:

1. For each class of PECVT, identify the governing principles and functional specifications for possible solution embodiments
2. Generate and develop concepts of possible solutions to the non-integer tooth problem in the PECVT classes
3. Screen and score the generated concepts to help identify the most promising embodiment
4. Create a model of the best embodiment solution to demonstrate its effectiveness in meeting the governing principles and functional specifications.

1.6 Research Approach

To achieve the objectives of this research, a brief description of the history of CVTs is provided in Chapter 2. In addition, a PECVT classification system is developed to group different PECVT embodiments into classes and families based on the methods used to attempt to overcome the non-integer tooth problem. Chapter 2 also defines certain principles that need to be satisfied for promising PECVT embodiments to exist in each of the classes and families. Several patented PECVT embodiments are categorized into the classification system with the advantages and disadvantages of each embodiment given.

With the classification system and general operating principles defined, a product development process can be implemented to find the most promising PECVT embodiment. A product development process is a series of steps used to develop the design of a particular product or alternatively, a viable solution to a specific problem. This process includes many phases that carry an idea from initial stages to a widespread production of a product [1]. One of the phases in this process is called the concept development phase and is discussed in Chapter 3 of this thesis. The concept development phase is a conceptual design phase, rather than a detail design phase, meaning that the complete and final specifications are not included in this research. The conceptual design phase, once completed, was very helpful in completing the objectives of this research.

Another popular problem solving method that has proven to be effective in many technical problem solving situations is TIPS or TRIZ. "TIPS" is the acronym for "Theory of Inventive Problem Solving," and "TRIZ" is the acronym for the same phrase in Russian, the language in which the methodology was conceived [6]. TRIZ is intended to improve design concepts by using a more structured methodology than brainstorming or other creative thinking techniques. This methodology can be very useful when attempting to improve systems where creative conceptual design solutions are needed to solve a problem instead of using solutions already known to the industry [7]. A greater explanation of how TRIZ was implemented in this research is also detailed in Chapter 3.

By integrating TRIZ techniques into the concept development phase, the less effective trial and error approaches often used in the concept generation step were replaced with a more systemic approach to aid in carrying out the objectives of this research. Chapter 3 also details the manner in which these two methodologies were integrated.

Chapter 4 presents a discussion of the results obtained by applying the methodology described in Chapter 3 to the PECVT family.

Chapter 5 presents the most promising PECVT embodiment found to satisfy the general principles and product specifications set forth in previous chapters. The effectiveness of the final design in satisfying these principles is also shown.

Chapter 6 discusses conclusions of the selection of the final embodiment as well as provides recommendations for future work in this area of research.

1.7 Delimitations

Other phases in the product design and development process including the detail design phase were not addressed in this research. Only a conceptual investigation of PECVT embodiments was conducted through the concept development phase. Only the final concept chosen was analyzed to show its theoretical feasibility. A physical prototype was not constructed nor tested as part of this research. Further, more detailed analyses must be conducted to prove functionality and feasibility of the final embodiment.

2 CVT Technology and PECVT Development

This chapter provides a brief history of the development of CVT technology over the last few decades. Certain CVT categories are introduced and detailed according to the design principles that define them. The advantages and disadvantages of the design principles are also listed. This chapter provides the background and motivation for further development into a separate category of CVTs: PECVTs. The customer needs and product specifications of an ideal PECVT embodiment are also identified to better understand its necessary operating principles.

A PECVT classification system used to organize and analyze all PECVT embodiments are also introduced. A representative population of patented PECVT embodiments with a detailed functional description of some of these is provided to aid in developing the PECVT classification system. A few well-defined governing principles are also provided as tools to evaluate PECVT concepts and embodiments during this investigation.

2.1 CVT History

A CVT is defined as a transmission that allows for a continuously variable RPM ratio, output shaft to input shaft, over a given range. A CVT, therefore, can take on an infinite number of transmission ratios within the range. As mentioned in Chapter 1, this characteristic of CVTs allows the engine to operate at maximum efficiency or power. The idea of CVTs has been around for hundreds of years. Leonardo da Vinci sketched the first known designs of a CVT in 1490 [8]. The idea was not seriously developed, however, until the late 1800's and early 1900's. In 1886, the first CVT patent was filed, but it wasn't until the early to mid 1900's that CVTs were being utilized in automobiles and other applications. In the late 1930's, General Motors patented their first automated ratio-controlled traction drive CVT. This was the first of many patented traction drive CVTs that appeared over the following years [9]. A Dutch automaker (DAF) first started using CVTs in automobiles in the late 1950s, but due to limitations caused by their reliance on friction, CVTs were unsuited for internal combustion engines that produced more than about 100 horsepower [8]. Other automakers experienced similar results during this time.

As engine horsepower increased during the mid 1900's, the focus of CVT development shifted toward the development of transmissions with higher torque transfer capabilities. The movement nearly halted CVT development for automobiles in favor of fixed ratio transmissions, such as today's manual and automatic transmissions, which can meet the needs of high-torque producing engines. Only in the last few decades have higher-torque CVTs for commercial applications appeared [10]. The need for higher torque transfer capabilities of transmissions fostered development of improved CVTs, some of which do not rely on friction. Currently, the development of CVTs can be classified into several categories: belt-driven systems, rolling contact systems, hydrostatic systems, electric systems, and fully mechanical systems which use positively engaged members and variable geometry to achieve output [11]. It is useful to describe these and other categories in which many CVTs are classified as background and motivation for this research.

2.2 CVT Categories

The development of CVTs has resulted in numerous embodiment designs with many different operating principles used for each design. By understanding the basic advantages and limitations of these operating principles, evaluation of new or similar CVT designs is greatly facilitated. Distinct CVT categories exist based on the operating principles that are utilized by different CVT embodiments. To provide a foundation for this research, basic CVT categories are described along with advantages and limitations of each category type. If the proper category can be found for any particular CVT embodiment, then the characteristics of that embodiment can be understood without knowledge of all the mechanical details or a complete understanding of the functionality of the embodiment [12]. The five basic CVT categories described are: Friction Drive, Traction Drive, Hydrostatic, Electric, and Positive Engagement.

2.2.1 Friction Drive CVTs

A friction drive CVT is one that uses static friction between the driving and driven member to transmit torque. The most common type of friction drive CVT is the variable diameter pulley V-belt system. This CVT consists of a driving and driven pulley that are coupled by a friction-driven composite V-belt. Each pulley consists of two separate sheaves that are allowed to spread or contract to vary the diameter at which the belt rides on the pulley. This change in the radial location of contact of the belt on the pulley is equivalent to changing the effective diameter of the pulley and, essentially, the transmission ratio of the CVT. Pulleys are used in pairs. As one spreads, creating a smaller pulley, the other contracts to maintain belt tension and to create a larger pulley. By allowing the sheaves to continuously vary the distance between them, a continuously variable ratio change results. An example of this type of friction driven CVT is shown in Figure 2.1.

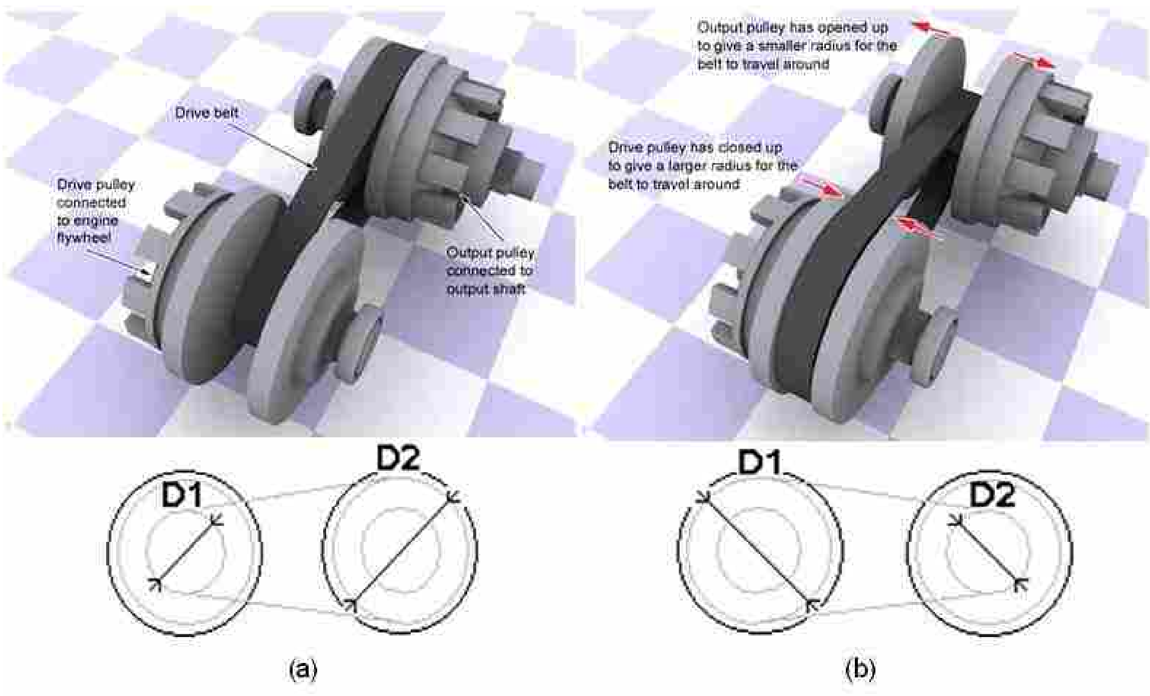

Figure 2.1: Variable Diameter Pulley V-belt CVT; (a) Driving Pulley Diameter is Smaller than Driven Pulley Diameter (b) Driving Pulley Diameter is Larger than Driven Pulley Diameter (Retrieved from http://cvt.com.sapo.pt/scvt/index.htm) [4]

There are several variable diameter pulley v-belt CVT embodiments that exist in the friction drive category. One of these embodiments, which is very popular and is currently used commercially in automobiles, is the metal push belt design. This design increases the torque capacity and efficiency of the standard composite V-belt designs by increasing the coefficient of friction between the metal belt and the pulley sheaves, which reduces the inherent slipping of the belt. The metal push belt design was first designed by van Doorne Transmissie Company. Currently, the push belt design is found in many different vehicles from companies such as Subaru, Nissan, Ford, and Honda [13]. The basic driving and driven segments of this type of CVT are shown in Figure 2.2.

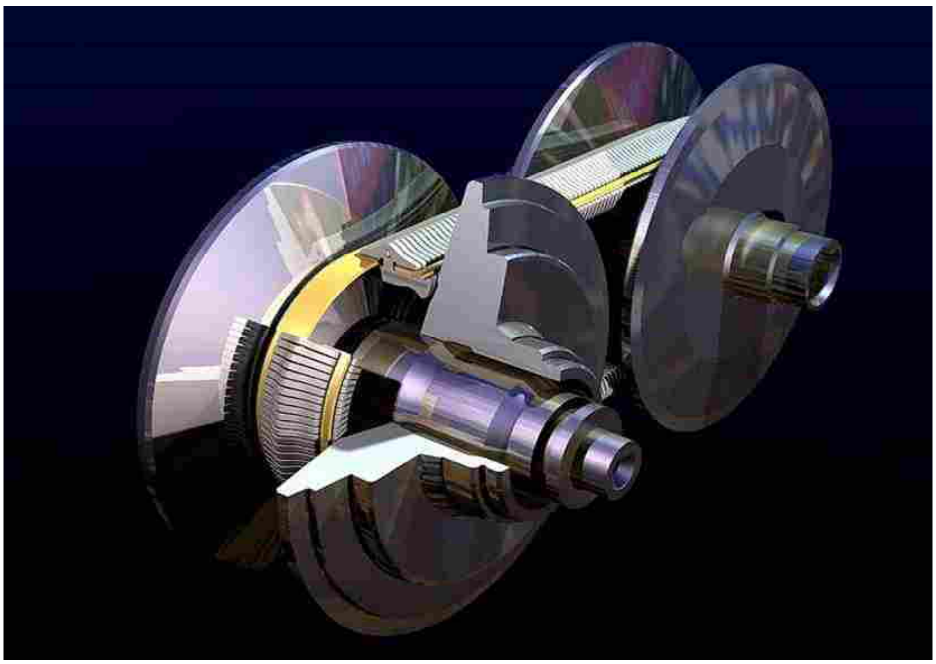

Figure 2.2: Basic Embodiment of Metal Push Belt CVT (Retrieved from www.cvt.co.nz/cvt_how_does_it_work.htm)

This embodiment is composed of segmented, thick stamped metal V-shaped blocks that have cutouts on both sides to accommodate stacker steel bands that hold the V-shaped blocks in place. The blocks, along with the steel bands, make up the CVT belt, which can be seen in Figure 2.3. The metal push belt CVT functions similar to the variable diameter pulley rubber V-belt designs. The V-shaped blocks decrease the amount of slipping that occurs between the disks and the sheaves, while the metal bands allow the belt to handle high torque loads. The amount of torque that can be transmitted by this CVT is dependant on the tensile strength of the steel bands as the belt is squeezed between the two sheaves [13].

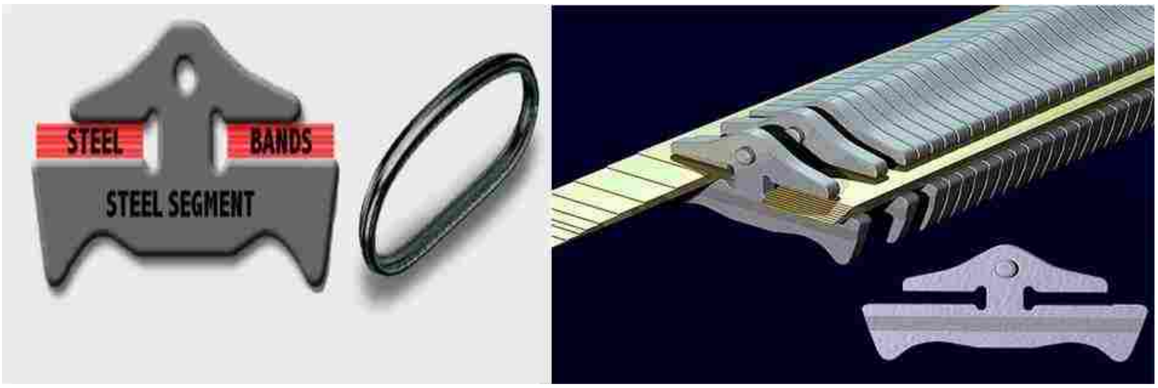

Figure 2.3: Metal Discs and Steel Bands that Compose the Belt of a Metal Push Belt CVT (Retrieved from www.cvt.co.nz/cvt_how_does_it_work.htm)

As previously stated, the advantages of the metal push belt are the increased torque capacity and efficiency over the rubber v-belt drives. To date, these CVTs have been used in vehicles that produce an engine torque under 150 foot-pounds [13]. Andersen states that this category of CVTs can reach levels of between 80-90% efficiency. Some disadvantages of the metal push belt CVT are the increased part count of the steel discs, which also require the use special transmission fluid to reduce the wear between the metal belt and the metal sheaves. Because the metal belt must maintain its static contact with the pulley sheaves in the presence of transmission fluid, the contact stresses are much greater than those in the standard rubber v-belt drive CVTs [3].

2.2.2 Traction CVTs

Traction drive CVTs are similar to friction drive CVTs in that power must be transmitted between two surfaces using friction; however, traction drives use a smooth rolling contact between two smooth surfaces instead of static contact that exists between members in friction drives. A common example of a traction drive CVT is called a Toroidal CVT. The Toroidal CVT uses two toroidal-shaped disks, coupled by power rollers which transmit torque from the driving disk to the driven disk. Currently, several Toroidal CVTs are used in commercial applications like Nissan's Extroid Toroidal CVT, which can be seen in Figure 2.4a.

(a) ( )

Figure 2.4: (a) Nissan's Extroid Toroidal CVT (b) Example of 1:1Transmissino Ratio in Extroit (Retrieved from http://www.auto-innovations.com/site/dossier/dextroid.html)

The power rollers used in the Toroidal CVT, and the majority of traction drive CVTs, couple an input rotating member to an output rotating member. The location on the disks, where roller contact takes place, determines the gear ratio of the transmission. If the power rollers are not tilted, and remain parallel with the axes of the two rotating disks, the transmission ratio is 1:1 as seen in Figure 2.4b. This is because the effective diameter of the input disk, caused by the location of contact between the disk and roller, is equal to the effective diameter created by the location of the contact point on the output member. If the power rollers are tilted away from the input disk, such that the effective diameter of the input created by the contact point is smaller than the effective diameter of the output, then the output will rotate faster than the input (Figure 2.5a). If the power rollers are tilted toward the input disk, such that the effective diameter of the input created by the contact point is larger than the effective diameter of the output, then the output will rotate slower than the input (Figure 2.5b). In this way, a continuously variable ratio change can occur between the input and output disks by varying the degree of tilt on the power rollers.

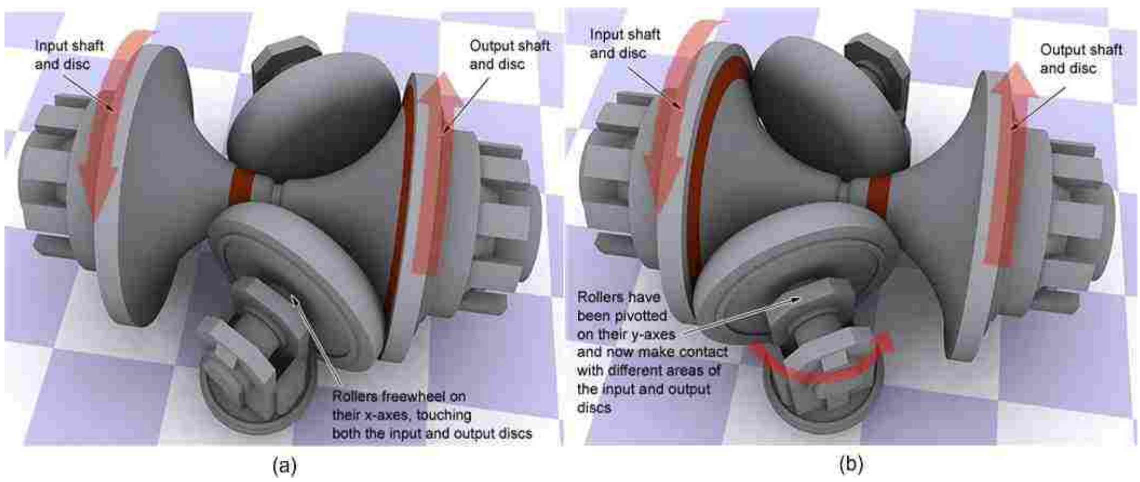

Figure 2.5: Basic Toroidal CVT, A Common Commercially Used Traction Drive CVT; (a) Power Rollers are Tilted Away from the Input Disk (b) Power Rollers are Tilted Toward the Input Disk [4]

There are two major forces that exist in traction drive CVTs. One of these forces is the normal force that the disks exert on the rollers; the other force is the tangential force that is applied to the disks by the rotation of the rollers. The traction coefficient is defined as the tangential force divided by the normal force and determines the efficiency of power transfer between the two members, analogous to the coefficient of friction in friction drive CVTs.

Because the two members (disk and roller) are in rolling contact, lubrication is necessary to reduce wear between the two power transmitting members. The lubrication must also act as a medium for power transfer since there must exist a thin fluid film between the disks and power rollers. If hydrodynamic fluid, also known as regular transmission fluid, is used as the lubricating fluid, normal forces can exceed the shearing resistance of the fluid and cause shearing of the fluid. This results in direct metal to metal contact of the members, which will greatly damage their surfaces due to wear. For this reason, a special elastohydrodynamic lubricant (EHL) is used that possesses a higher traction coefficient than hydrodynamic fluids. That is, EHLs allow the traction drive CVT to increase its torque capacity above that found in hydrodynamic traction drive CVTs. When two rolling contact members are in contact under high loading, the EHL momentarily obtains properties like those of a solid, exactly at the contact point, due to the extremely high contact stresses. This allows the EHL to transmit torque from one member to the next, as if it were part of the torque carrying system. As the rotation of the disks carries the EHL outside of the contact region, the EHL immediately becomes a liquid and regains its original properties [8]. This can be seen in Figure 2.6.

The major advantages of toroidal CVTs is their ability to transmit much higher torque than friction drive CVTs while operating at very high torque transmitting efficiencies (average of 91.6%) due to the high traction coefficient created by the EHL [13]. However, often times a secondary hydraulic pump is required to maintain high contact forces between driving and driven members, which decreases the overall efficiency of the CVT [14]. Other disadvantages that traction drive CVTs possess is their higher weight and increased complexity over friction belt type designs. They require

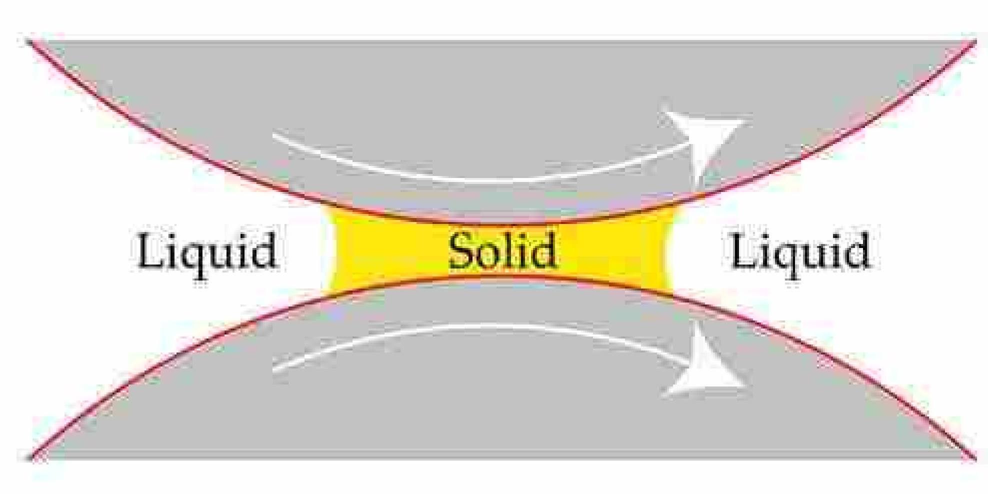

Figure 2.6: The EHL Momentarily Becomes a Solid on the Contact area of the Rolling Contact Members [8]

high precision control to maintain proper contact, and also require expensive special lubricants (EHLs) to resist the extremely high contact stresses that are generated [3].

2.2.3 Hydrostatic CVTs

Hydrostatic Transmissions are another type of CVT that moves away from traditional low torque friction driven CVTs and more toward higher torque applications. Hydrostatic CVTs use high pressure oil, up to 5000 psi, to transmit power. This type of transmission consists of a hydraulic pump and motor with hydraulic lines coupling the two. The pump receives rotary power from the engine and transmits this power in the form of pressure and volumetric flow rate through a high pressure line to the hydraulic motor. The hydraulic motor then converts the hydraulic power back to mechanical rotational power, as the output of the CVT. The low pressure line carries the oil back to the pump to complete the closed cycle [12]. The basic embodiment of hydrostatic CVTs is shown in Figure 2.7.

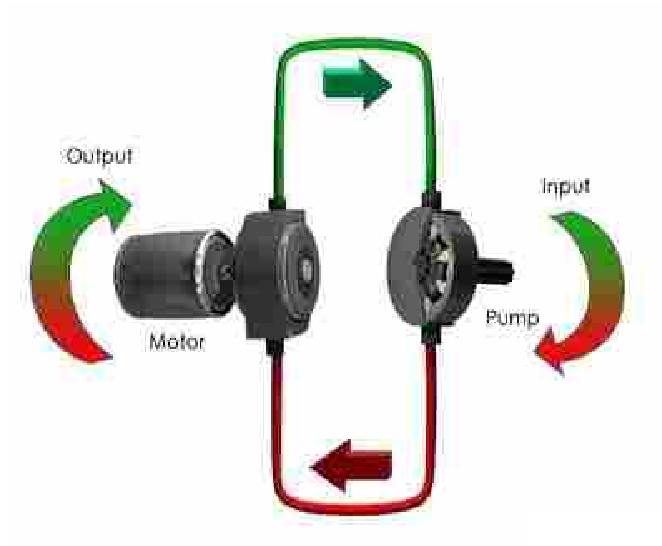

Figure 2.7: Basic Hydrostatic CVT Embodiment Setup (Retrieved from http://auto.howstuffworks.com/cvt4.htm)

By varying the amount of oil displacement, or volumetric flow rate, provided by the pump in a continuous manner, the hydraulic power, and thus the mechanical power provided by the motor, is also continuously variable. A hydrostatic motor-drive transmission developed at John Deere, shown in Figure 2.8, employs a hydrostatic CVT unit and is currently used commercially.

Some major advantages of hydrostatic CVTs are their friction independent method of power transfer and high torque capacity. These features allow this category of CVT embodiments to be used in many automotive and other applications. Disadvantages of hydrostatic CVTs include size, weight, cost, and inefficiencies. Andersen states that these efficiencies are around 60-80% [3]. These inefficiencies will be further discussed later.

Figure 2.8: John Deere's Hydrostatic Motor-drive Transmission (Retrieved from http://www.deere.com/en_US/rg/productsequipment/productcatalog/tms/hmd/hydrostatic/series/index.html)

2.2.4 Electric CVTs

Electric CVTs function similarly to hydrostatic CVTs by using a DC generator and a DC motor. The generator converts the mechanical power provided by the engine into electrical power in the form of voltage and current. This electrical power is transmitted to the electric motor by a control system which can continuously vary the amount of electrical power that is transmitted. The motor then converts the continuously variable voltage and current back into mechanical power to achieve a step-less transmission ratio change [12]. The basic setup of electrical CVTs has been provided in Figure 2.9.

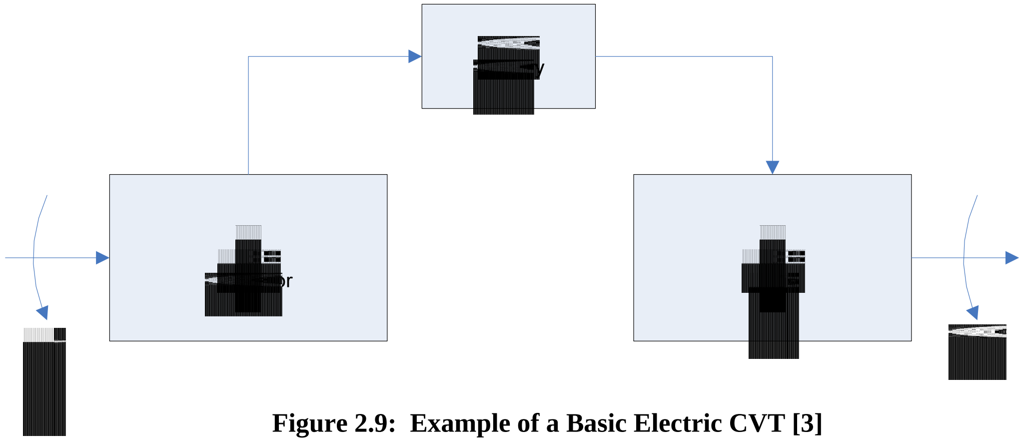

Figure 2.9: Example of a Basic Electric CVT [3]

The advantages of electrical CVTs are their ability to transmit high torque like the hydrostatic CVTs, and based on the control circuitry, the output ratio can be controlled very precisely to allow the engine to operate at its maximum efficiency or performance. By using electric motors as a direct drive source, gear train inefficiencies and weight are also reduced. The efficiency of a DC Motor-Generator varies between 72% and 81%, which is relatively low compared to the metal push belt and traction drive CVTs [15]. The reason for this relatively low efficiency in the motor-generator set is caused by forcing the motor to operate above or below its maximum efficiency rated load range. An electric motor is designed to run between 50% and 100% of a rated load. Maximum efficiency typically occurs at 75% of the rated load. For example, a 100 hp motor has a rated load range of 50-100 hp and its peak efficiency is at 75 hp. When a motor operates below 50% load, the efficiency significantly decreases. While it’s true that larger motors have a larger range of acceptable efficiencies, the task of requiring an infinitely variable output speed from an electric motor to provide a continuous RPM ratio change reduces efficiencies of the motor over certain output ranges [16].

2.2.5 Ratcheting CVTs

Ratcheting CVTs are transmissions that achieve a continuously variable RPM ratio by converting a rotational motion into an oscillating motion transmitting only the forward stroke through the use of overrunning clutches. The continuous variable ratio is achieved by the ability to alter the geometry to vary the forward stroke. An overrunning clutch is a mechanism that transmits rotational power in one direction but is allowed to spin freely in the opposite direction. Although there are many embodiment configurations that currently exist, there are some common features found in all ratcheting CVTs. Ratcheting CVTs transmit torque through a rigidly connected input and output (not relying on friction), produce a pulsating output ripple for a constant input velocity, and employ overrunning clutches to compensate the negative portion of the oscillating output motion [10].

One type of ratcheting CVT used commercially is the Zero-Max Adjustable Speed Drive shown in Figure 2.10. The power link (c) is connected eccentrically to the input shaft (a). This causes the power link (c) to oscillate back and forth for every rotation of the input shaft (a). The control link (b) determines the amount or magnitude of oscillation that takes place in the power link (c). The power link (c) is connected to output shaft (e) through an overrunning clutch (d), which only transmits torque from the power link (c) to the output shaft (e) in one direction. The overrunning clutch (d) allows the negative oscillation of the power link (c) without affecting the output shaft (e). The Zero-Max uses 4 mechanisms, shown in the figure, in parallel but out of phase, to achieve a more continuous output. Each of the 4 mechanisms is 90° out of phase, such that the power links (c) deliver 4 sequential pulses to the output shaft (e) for each rotation of the input shaft (a). This results in a continuously oscillating output. By adjusting the location of the control link (b), the magnitude of oscillation changes, as well as the amount of output that each power link (c) provides to the output shaft (e). In this way the RPM ratio of the transmission is able to vary in a continuous manner throughout the adjustment range of the control link (b) [17].

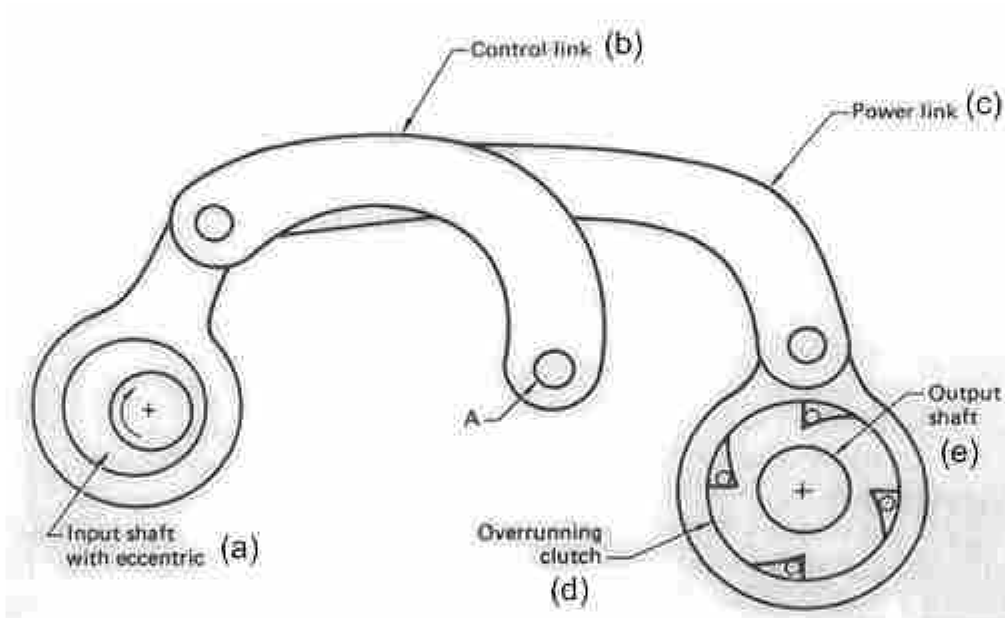

Figure 2.10: Zero-Max Adjustable Speed Drive Ratcheting CVT [17]

The major advantage of ratcheting CVTs is their ability to transmit high torque by using positive engagement to couple the input to the output. Because they operate independent of friction, the ratcheting CVT operates with less wear and at much higher efficiencies (90-95%) than other CVTs embodiments in its forward stroke [3]. One disadvantage of ratcheting CVTs is the oscillating output which is inherent with most ratcheting CVTs that employ overrunning clutches. These clutches also have internal gear teeth that allow them to free-wheel and lock at discrete locations throughout the oscillations of the power link, which keeps this type of CVT from providing a fully continuous variable RPM ratio [14]. Other overrunning clutches found in ratcheting CVTs rely on friction to function properly.

2.3 CVT Category Summary

The five most commonly known CVT categories have been presented in this chapter. Examples of CVT embodiments in each category, with their respective advantages and disadvantages, have also been provided. A summary table has been created by Andersen showing the advantages and disadvantages for each category in Table 2.1.

Table 2.1: Advantages and Disadvantages of Different CVT Categories [3]

Transmission Type	Characteristics				
	Torque Capability	Wear	Output	Complexity	Ratio Range
Hydrostatic	High	Low	Non-Oscillating	Low	High
Friction	Low	High	Non-Oscillating	Low	Moderate
Traction	Moderate	Moderate	Non-Oscillating	Low/Moderate	High
Variable Geometry	Moderate	Low	Oscillating	Moderate	Moderate
Electric	High	Low	Non-Oscillating	Low	High

2.3.1 CVT Efficiency

Often, the efficiency of CVTs is questioned when determining whether a CVT would be a good candidate for a particular application. This is because the efficiencies of automatic and especially manual transmission are, for the most part, higher than many commercially used CVTs. However, it is important to understand the difference between torque transmitting efficiency and overall vehicle efficiency. All efficiency values that have been provided in Chapter 2 were not given in the summary table because they refer to the torque transmitting efficiency of the CVT within the transmission. It is true that higher efficiencies in a particular CVT embodiment are preferred; however, this should not be a critical factor in determining if a CVT is more beneficial than a standard automatic or manual transmission for a particular application. Some engineers have said, referring to the efficiency of certain CVTs,

> "… It is important to remember that the most important feature this type of transmission [CVT] brings to the power train is optimizing the engine's performance and efficiency across the whole spectrum of operating conditions. The efficiency of a CVT, measured as an isolated component on a dynamometer test stand, is between 85-90 percent, lower than that of an automatic transmission measured under the same conditions. However, when an evaluation is carried out on the complete power train system - engine, transmission, and axle- the CVT-configured power train demonstrates much lower fuel consumption compared to an automatic-configured power train." [13]

While standard transmissions possess high transmission efficiency, the efficiency lost through shifting and running the engine at less than optimal efficiency ranges causes the overall vehicle efficiency to be less than many commercially used CVTs. Therefore, a CVTs overall efficiency can only be determined when placed in a specific application and tested.

2.3.2 CVT Conclusions

There are many advantages of using CVTs over standard manual and automatic transmissions. One major advantage is higher overall vehicle efficiency through a continuously varying RPM ratio. As seen in the Table 2.1, there are many advantages and disadvantages found within the different CVT categories which make some CVT embodiments more promising than others in certain applications. The purpose of this research is to investigate the feasibility of producing a CVT embodiment that captures many of the advantages from each of the CVT categories without, however, introducing the disadvantages associated with previously designed CVTs. This ideal CVT embodiment would therefore have high efficiency, high torque capacity, precise speed control, and low wear; in addition, the embodiment would not rely on friction, not provide an oscillatory output, and not be complex or too costly. Andersen proposed a new category of CVT in his research that possesses these attributes and characteristics called the Positive Engagement, Continuously Variable Transmission category.

2.4 PECVT Category

A PECVT uses positively engaged gears or other members to transmit torque allowing the transmission to transmit more torque than friction and traction drive CVTs and also would theoretically reduce component wear. In addition, a PECVT allows the engine to operate at its maximum performance or efficiency ranges through a continually variable transmission ratio, thus providing a constant, non-oscillating output. A PECVT is a purely mechanical device and does not use non-mechanical power sources that may have significant power losses, such as hydrostatic and electrical CVTs [14]. Such a transmission would be ideal and would indeed change the course of power transmission if a feasible embodiment were discovered and developed. A more in-depth investigation of this PECVT category will be provided in the following chapters. The customer needs and product specifications of an ideal PECVT embodiment will also be provided so as to later develop the characteristics and general principles of the ideal embodiment in the PECVT category.

2.4.1 PECVT Customer Needs

Customer needs are a list of desired attributes or functions that a certain product must possess to fulfill its purpose or improve its functionality; in this case, the product is a PECVT. The customers of a PECVT are all those who are both directly or indirectly affected by it. The customer needs should be expressed in terms of what functions the device or product has to do and not on *how* the device will accomplish these functions. In addition to creating the list of PECVT customer needs, importance ratings should also be placed on each of these needs to help establish their importance and priority in fulfilling the device's overall function. The ideal PECVT embodiment should satisfy the customer needs tabulated and rated in Table 2.2.

Table 2.2: PECVT Customer Needs

Number	Customer Need	Importance
1	The transmission ratio is continuously variable.	1
2	The transmission does not transmit power through friction.	1
3	The transmission provides positive engagement of the input and output.	1
4	The transmission provides continuous engagement of the input and output.	1
10	The transmission is able to vary ratio under load.	1
16	The transmission does not produce an oscillating output.	2
5	The transmission can transmit high torque.	2
6	The transmission is highly efficient.	2
9	The transmission is not complex.	3
7	The transmission is light weight.	3
8	The transmission is made of standard parts.	3
12	The transmission is retrofit-able in current applications.	3
11	The transmission can provide a large ratio range.	4
13	The transmission is simple to control.	4
14	The transmission is capable of high rpm's.	4
15	The transmission does not produce excessive vibrations.	4

This list of PECVT customer needs is divided into 4 groups based on their importance ratings according to how well they contribute to the PECVT's overall desired functionality. The customer needs of a number one importance will be referred to as the

PECVT's primary needs while those with a number two importance are the secondary needs, etc. These importance ratings are based on the author's heuristics and can therefore vary for different designers; however, customer needs 1, 2, and 3 must be satisfied for an embodiment to be classified and defined as a PECVT.

2.4.2 Metrics

A list of metrics based on the PECVT customer needs is shown in Table 2.3. The list also shows which PECVT customer needs are represented by each metric. It also provides a marginal and ideal value for each metric as produced by the PECVT embodiment. Although most metrics provide numeric values of how well they satisfy customer needs, it should be noticed that all primary PECVT customer needs are represented by binary metrics. Binary metrics do not specify particular values; instead, they simply specify whether or not the customer need is being met.

2.4.3 PECVT Product Specifications

Product specifications represent specific, measurable characteristics that are tied to the original needs. Once again, they do not specify how to address customer needs, but they do detail precisely what the product or device has to do in measurable ways. By creating a list of metrics to satisfy the PECVT customer needs, it becomes clearer as to the type of concepts that needs to be considered and developed to solve the non-integer tooth problem.

Table 2.3: Metrics for PECVT Product Specifications (Values for an ATV [18])*

Metric Number	Need Number	Metric	Importance	Units	Marginal Value	Ideal Value
1	1, 4	Continuously Variable Ratio	1	Binary	Yes	Yes
2	2	Friction Dependent	1	Binary	No	No
3	3	Positive Engagement	1	Binary	Yes	Yes
4	4, 10	Continuous Engagement	1	Binary	Yes	Yes
5	5	Max Torque	2	ft-lbs	30	40
6	6	Efficiency	2	%	90	95
7	7, 12	Weight	3	lbs	<30*	<20
8	8	Number of Non-standard Parts	3	#	<5	0
9	9	Number of Parts	3	#	<100	<50
10	10	Able to Vary Ratio under Load	1	Binary	Yes	Yes
11	11	Ratio Range	4	Δ# :1	2.5	3
12	12	Able to be Retrofit in Current Apps.	3	Binary	Yes	Yes
13	13	Number of Control Sources	4	#	1	1
14	14	Max RPM	4	#	>5000	>7000
15	15	Kinematic Interference	4	Binary	No	No
16	16	Oscillating Output	2	Binary	No	No

2.5 PECVT Classes

In order to define the general principles that need to be satisfied for functional PECVT embodiments to exist, it is necessary to create a classification system in which all published PECVT embodiments can be organized. Different designs of PECVT embodiments have been created and published in numerous patents, each with a detailed physical descriptions being provided. Some of these designs are very complex and are difficult to understand even with the use of diagrams and written descriptions. Therefore, analogous to the CVT categories, it is useful to create and define different PECVT classes into which the majority of PECVT embodiments can fall. This classification system will foster understanding concerning the characteristics of the non-integer tooth problem as well as other advantages and disadvantages that might exist within each class. Knowing the advantages and disadvantages of each class will aid in the evaluation of both published and unpublished or proposed embodiments.

It is proposed that two different classes of PECVTs be created to include all PECVT embodiments. As discussed in section 1.2, the primary function of a PECVT is to continuously vary and control the RPM and torque ratios of input to output, while the secondary function is to overcome the non-integer tooth problem. Different classes of PECVTs were created based on the methods used to solve the non-integer tooth problem (the secondary function, see page 6), since these methods differentiate the different PECVT embodiments. Although there could be many classes of PECVT embodiments based on the heuristics of different designers, it is proposed that these two PECVT classes be created to encompass the vast majority of published PECVT embodiments, as well as other innovative embodiments that have not yet been published. If the proper class can be found for any particular PECVT embodiment, then the characteristics of that embodiment can be understood without knowledge of the mechanical details or a complete understanding of the functionality of the embodiment.

The two PECVT classes are shown in the flow diagram in Figure 2.11 and are defined as the problem correction class and the problem elimination class. It is important to note that this classification system is not general to all transmissions, such as traditional CVTs and manual transmissions, but only to the PECVT category.

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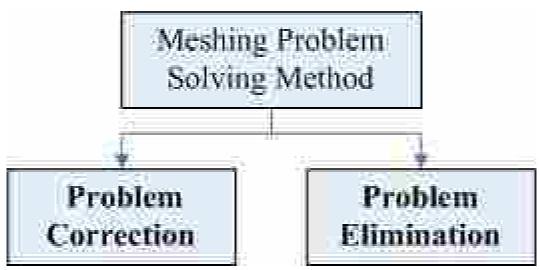

Figure 2.11: PECVT Classes Based on Methods for Solving the Non-Integer Tooth Problem

An explanation of these classes will be given below, along with the advantages and disadvantages associated with each class. The PECVT families created within these classes will also be provided and defined, as well as examples and descriptions of published embodiments within these classes and families, which were extracted from published patents.

2.5.1 Problem Correction Class

As discussed in Chapter 1, in transmission embodiments where a single mechanical input is employed, the transmission functions much like a common gear pair relationship like that shown in Figure 1.3. This type of embodiment is similar to traditional manual and automatic transmissions in that one gear acts as the input drive gear and the other gear acts as the driven or output gear. To achieve a continuously variable RPM ratio change in single input embodiments, the diameter of one of the two gears in the gear pair must also change in a continuous manner. This is accomplished by allowing the number of teeth on the gear (N) or the diametral pitch of the gear (P_d) to also change in a continuous manner as seen in equation 1.2.

In traditional manual and automatic transmissions, a change in diameter is achieved by engaging different gears that have increased the number of teeth on the drive or driven gear; however, these ratio changes are not continuous, but discrete for each different gear used, and will not provide a *continuously* variable RPM ratio. In most published PECVTs, the diametral pitch, not the number of teeth, is varied to achieve a continuously variable gear diameter. If the number of teeth were to continuously vary to allow for a continuously variable diameter, then teeth would need to be added to the gear in non-integer increments; however, producing non-integer numbers of teeth on a gear is infeasible and does not exist in most embodiments. Therefore, the most common method for achieving a continuously variable transmission ratio is to continuously vary the diametral pitch of a gear by using different gears, so that the fixed number of teeth remains evenly spaced around the gear circumference, as explained in Chapter 1. When the diametral pitch is varied in a continuous manner, the pitch diameter of the gear also changes in a continuous manner as seen in equation 1.2, resulting in a continuously variable RPM ratio between the driving and driven gear.

Chapter 1 described the nature of the non-integer tooth problem and showed that when the circumference of a gear in a gear pair is not evenly divisible by the circular pitch of the gear, then the non-integer tooth problem exists as a partial tooth. In the case of PECVTs that maintain a fixed number of teeth on a driving gear and continuously vary the gear's diametral pitch, the non-integer tooth problem occurs, not as a partial tooth on a gear, but as a mismatch of circular pitches. The circular pitch of one gear in a gear pair must be equal to or a factor of the circular pitch of the other for proper meshing of the gear teeth of both gears to occur. Because the circular pitch of a driving gear in a PECVT has to be continuously varied to achieve a continuously variable transmission ratio, there are times when the pitch of the driving gear is not an integer factor of the constant pitch of the driven gear and proper engagement will not occur as a result of the non-integer tooth problem. If the driving gear can be reoriented so that its continuously variable circular pitch is an integer factor of the circular pitch of the driven gear, then proper meshing will occur between the gear pair, even though their circular pitches are not equivalent.

PECVT embodiments that use a correction device to correct the orientation of a continuously variable diameter gear as described above belong to the PECVT correction class. The reorientation of the driving or driven gear (whichever possesses the characteristics of a continuously variable diameter gear), so that its circular pitch is a factor of the other, is the driving principle behind the embodiments of this PECVT correction class. Due to this pitch matching principle, the problem correction class is further addressed below by describing the two families of PECVT embodiments commonly found within this class that satisfy this principle. The two PECVT correction class families are defined by the methods used to reorient the pitch varying gear to satisfy the matching pitch principle. Therefore, the two families that were created are the one-way clutch family and the alternate device family.

A common device used to reorient either a driving or driven gear in an engaged gear pair is a one-way clutch, or an overrunning clutch. As described earlier in this chapter, a one-way clutch allows a particular gear to rotate in one direction without carrying torque but immediately begins to transmit torque when rotated in the opposite direction. Many other alternate corrective devices exist which could be implemented to correct the orientation of a misaligned gear in a gear pair that are not used as commonly as one-way clutches. Because one-way clutches are so prevalent in many PECVT designs, this corrective device can stands alone as a family of PECVT embodiments that use one-way clutches.

The alternate device family categorizes all embodiments that use alternatives (other that one-way clutches) to reorient the drive or driven gear. Since it would be tedious and very complex to create PECVT families for each of the many corrective devices used in PECVTs, the alternate device family was created to include all other correcting embodiments and simplify the classification system.

2.5.1.1 One-way Clutch Family

For proper meshing to occur in this PECVT family, a correction needs to be made to the orientation of either the drive or driven gears or members by employing the use of a one-way clutch, or an overrunning clutch as previously described. The manner and magnitude of this correction have been analyzed to fully understand the characteristics needed in a proposed solution to the non-integer tooth problem. Andersen's research has detailed the nature of the correction needed in this class of PECVTs [3].

In several PECVT embodiments employing a one-way clutch, individual input gears come into and out of engagement with an output gear at different phases to allow for a continuously variable gear ratio. These input gears employ one-way clutches and usually function as an effective input gear. These embodiments also vary the diameter of the effective input gear (similar to Figure 1.7 while maintaining continual engagement with the output gear. The correction needed to overcome the non-integer tooth problem in one-way clutch embodiments should occur as a realignment of the individual input members before they are to be engaged with the output, or driven, gear. This realignment could be the result of partially rotating the misaligned gear about its own central axis relative to the engaged gear while it is not engaged. Figure 2.12 shows how the reorientation of the driving gear corrects for the non-integer tooth problem just before it begins to engage with the output gear. In the figure, the driven, or output gear, is represented using a chain, which essentially transmits torque through positive engagement to an output sprocket (not shown) in the same way as if the chain were replaced with an output gear.

This type of correction allows the gear teeth of the input gear to realign and engage properly with the output chain as shown in this type of embodiment. This correction needs to be able to occur continuously (every time a new input gear begins to engage with the output gear) to negate the continuous misalignment caused and accumulated by the non-integer tooth problem [3].

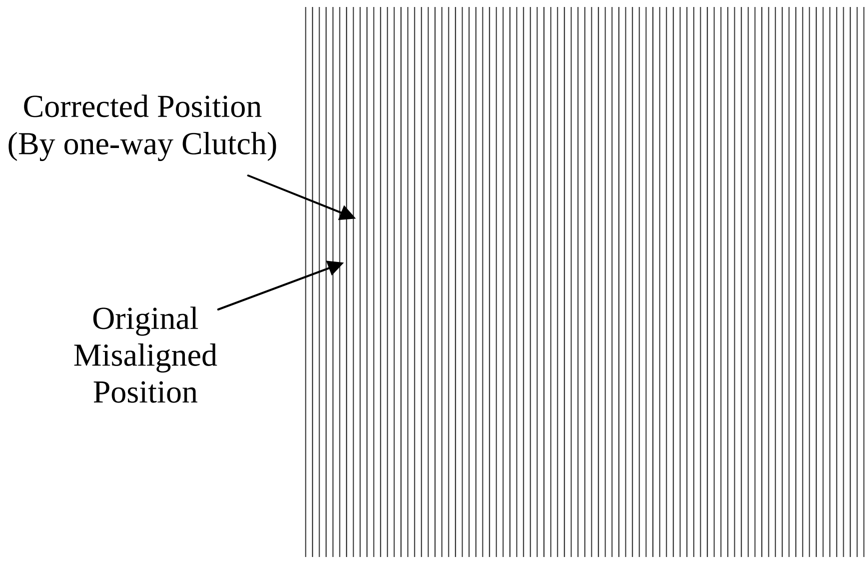

Figure 2.12: Realignment of Driving Gear by Partial Rotation caused by a One-way Clutch [3]

It is also necessary to understand the amount of correction needed for the realignment to be able to design or implement a device that can control certain magnitudes of correction. The maximum correction needed to realign the driving gear for proper engagement is the circular pitch of that gear, if the correction is to be made in one direction. If the corrective mechanism has the capability to correct in either direction, then the maximum required correction is one half of the circular pitch.

As previously mentioned, many PECVT embodiment designs employ the use of a one-way clutch in order to match the pitches of the driving and driven members linking the input to the output. A published example of an embodiment in the one-way clutch family is given below with the advantages and also inherent challenge that arise when utilizing one-way clutches.

U.S.Patent No. 4,660,427 (Fenton) Issued: 1987

Class: Problem Correction

This embodiment also represents a large number of patents with similar embodiments belonging to this class. The embodiment consists of a several spokes (a), or arms, connected to a spoke shaft (b), that extend radially and have gears (c) on the end of the spokes (a). A driven gear (d) is positioned around the spokes (a) so that only one spoke meshes with the driven gear (d) at one time over a given range. This range is determined by the angular spacing (e) of the spokes where meshing occurs. A driving gear (f) is situated outside of the spoke shaft (b), but inside the gears (c) at the ends of the spokes (a). The driving gear (f) has equally spaced openings (g) around its circumference for the spokes (a) to exit. The spoke shaft (b) can move within the driving gear (f) so that the two parts are not concentric. By offsetting the shaft (b) from the center of the driving gear (f), the angular spacing (e) of the spokes change as the driving gear (f) rotates as seen in Figure 2.13.

If the spoke shaft (b) lies between the center of the driving gear (f) and the point of engagement with the driven gear (d), the angular spacing (e) between the spokes (a) is larger at the point of engagement than if the spoke shaft (b) lies on the opposite side of the driving gear (f). At certain positions of the spoke shaft (b), the arc distance between adjacent gears (c) are such that they are not evenly divisible by the constant pitch of the driven gear (d). It is also important to note that this arc distance is also the changing circular pitch of the effective drive gear, composed of all the individual gears (c), which is apparent in all correction class embodiments. There is no correctional device in this patent that will correct for the non-integer tooth problem that exists at these positions;

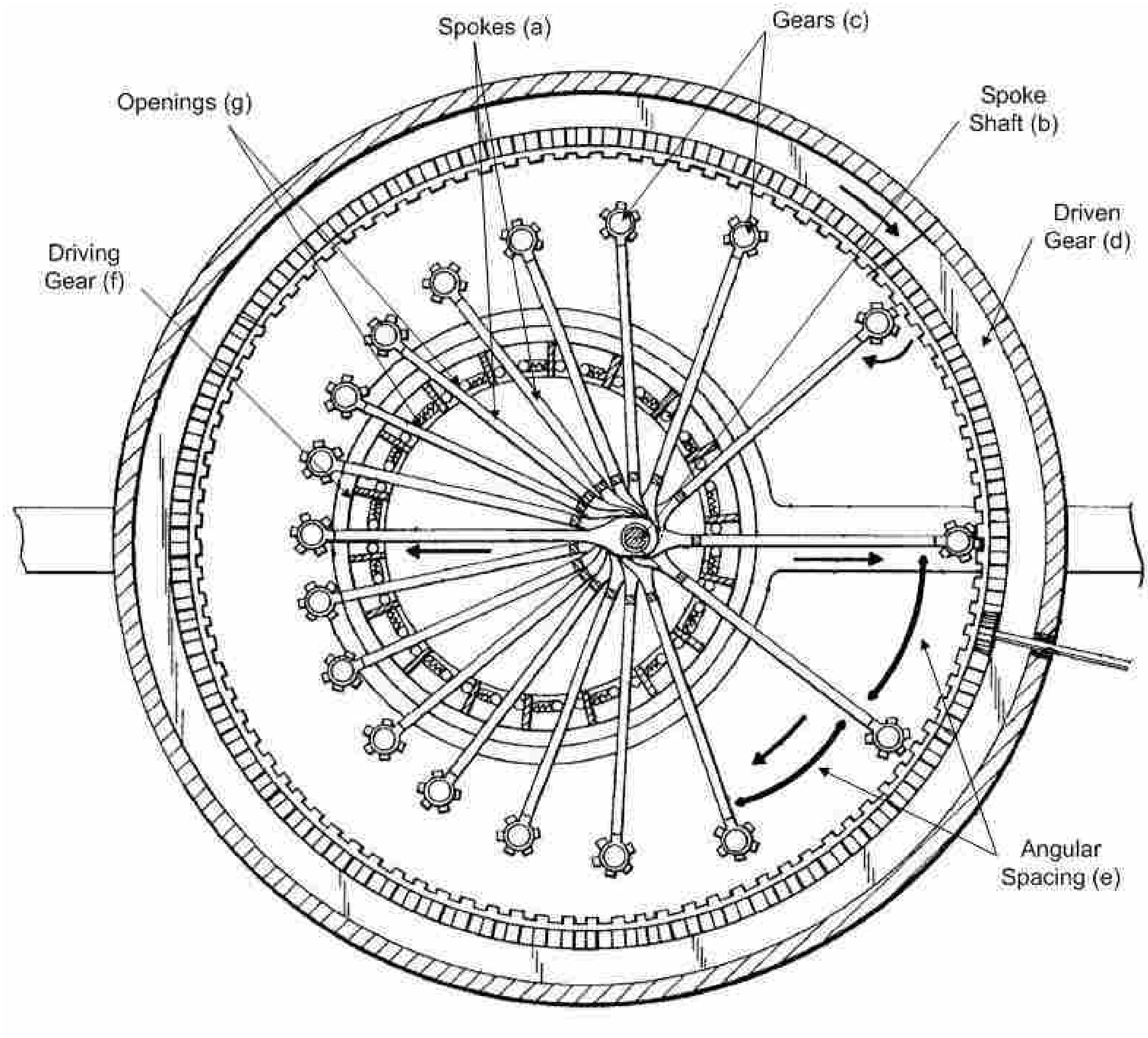

Figure 2.13: Embodiment of Problem Correction Class with Eccentric Gear Spokes (Retrieve from U.S. Patent No. 4660427)

however, employing one-way clutches at each of the gears (c) would allow the gears to free-wheel to the correct orientation satisfying the matching pitch principle and resulting in proper meshing of the gears (c) with the driven gear (d). Also, because the angular velocity of the spokes (a) is constantly changing through one rotation of the driving gear (f), if the spoke shaft (b) is not concentric with the driving gear (f), the transmission's output would also have an oscillating output. This happens because each spoke gear (c) is engaged with the output, or driven gear (d) over a given arc length in which its angular velocity will change.

One major advantage of one-way clutch embodiments is the ease of reorienting the drive gear when engaging with the driven gear as the one-way clutch allows the gear to self-adjust as it begins to engage. Also, if the amount of correction needed for proper engagement changes throughout the range of the transmission, the clutch is able to adjust its amount of correction to satisfy the matching pitch principle at any point of engagement. The wide commercial use of one-way clutches also makes them an attractive device. The various embodiments of this family have been tested under a number of different operating conditions and applications and can handle high torques. One disadvantage of the one-way clutch family is the slight oscillating output which is inherent with some embodiments that employ one-way clutches. These clutches have internal gear teeth that allow them to free-wheel and lock at discrete locations throughout the reorientation of the drive gear, which keeps these embodiments from providing a completely non-oscillating output.

2.5.1.2 Alternate Device Family

There are several devices that are and could be used to reorient the drive or driven gears of a PECVT embodiment to satisfy the pitch matching principle. Some of these devices are compliant members, springs, cams, tracks, etc. Of all the embodiments found in the alternate device family, these corrective devices are used to solve the non-integer tooth problem by satisfying the pitch matching principle as previously described. Some of these embodiments that belong to the alternate device family are described below along with their general advantages and disadvantages. These embodiments were chosen based on promising aspects found within the employed device to overcome the non-integer tooth problem.

U.S.Patent No. 6575856 (Andersen) Issued: 2003

Class: Problem Correction

The Anderson CVT shown in Figure 2.14 is an example of a pitch varying embodiment belonging to the correction class and the other device family of PECVTs. The drive and driven cones of the Andersen CVT have a constant number of teeth, so as the diameter of the cones changes (from one end to the other), the diametral pitch continually varies, resulting in a continuously variable ratio change.

Figure 2.14: Andersen CVT, an Embodiment of the Other Device Family of PECVTs [21]

Because the constant circular pitch of the chain, which connects the driving cone to a driven cone, does not always match the varying circular pitch of the two cones, a correction is needed for proper meshing of the teeth on the chain with the teeth on the cones to occur. The correction to this meshing problem associated with this pitch varying PECVT is very similar to the correction needed for the problem associated with the one way clutch family of PECVTs. The gear pair will not mesh properly when the changing circular pitch of one gear is not equal to or a multiple of the circular pitch of the constant diameter gear, or in this case, the constant pitch chain. This requires some correction to effectively change the pitch of the diameter-varying gear to a multiple of the constant-diameter gear for the matching pitch principle to be satisfied and proper meshing to occur. If the pitch of both gears is changed so that the varying pitches match, while maintaining an equal number of teeth on both gears, then the effective gear ratio does not change, even though both gears are increasing or decreasing in diameter.

The nature of the correction of this PECVT embodiment is also similar to that of the one way clutch correction, where the teeth can either be rotated or translated depending on the embodiment. The maximum correction required in embodiments of the alternate device family of PECVTs is also the same as that of the embodiments of the one way clutch family of PECVTs. The maximum correction distance in both families of PECVTs in this class is the smallest circular pitch of the two gears in the gear pair. In the Anderson CVT, a combination of translation and rotation is used to correct the problem. The maximum correction is one half of the constant pitch of the chain because the correction can be made in both directions in this case. This correction method used in the Andersen CVT is shown in Figure 2.15.

Floating sprocket bars running axially along the circumference of the cones represent engaged teeth that are misaligned relative to the chain. The sprocket bars are allowed to translate tangentially and rotate on an embedded spring to provide the desired correction, but they are confined to a small range of motion in order to transmit torque once properly engaged. Like the embodiments belonging to the one-way clutch family, the floating sprocket bars of the Anderson CVT allow for a varying amount of correction

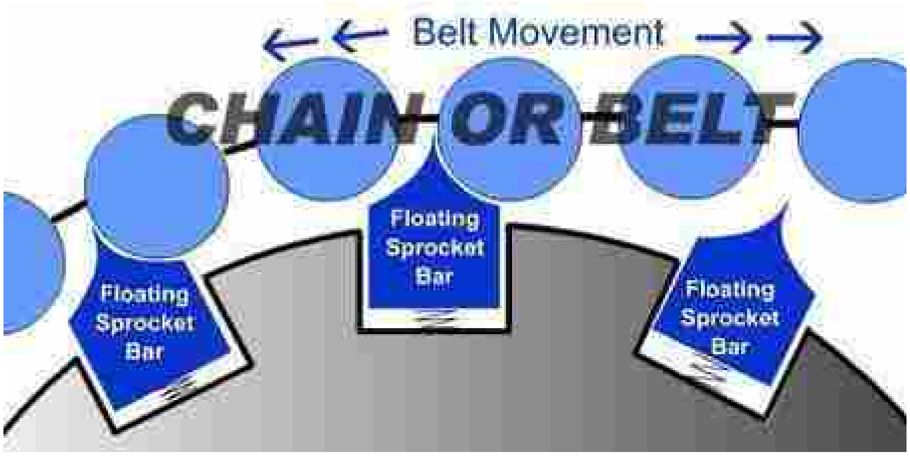

Figure 2.15: Example of Correction Method used to Correct the Circular Pitch [21]

to be applied throughout the range of the transmission, which is a big advantage of this embodiment. However, the sprocket bars (teeth) do not transmit torque until they have translated to the end of that confined region. Because the confined correction region of the sprocket bars is constant and the amount of correction needed continually varies depending on the diameter of the cone, if the amount of correction needed is less than the confined correction space, the engaged sprocket bar will not transmit torque until it has translated through the remaining correctional region. This occurrence results in an oscillatory output and does not meet the product specifications of an ideal PECVT embodiment as defined in section 2.4.3 [21].

U.S.Patent No. 3.867851 (Gregory et al.) Issued: 1975

Class: Problem Correction

This PECVT embodiment is expressed as an effective sprocket and chain drive system. It contains sprockets (a) held by a carrier mechanism that controls the radial position of the sprockets (a), thus allowing the effective diameter (c) of the PECVT to be changed, and in so doing, the transmission ratio. The sprocket's effective diameter (c) is allowed to increase or decrease as the individual sprockets (a) move in or out, radially, together as shown in Figure 2.16.

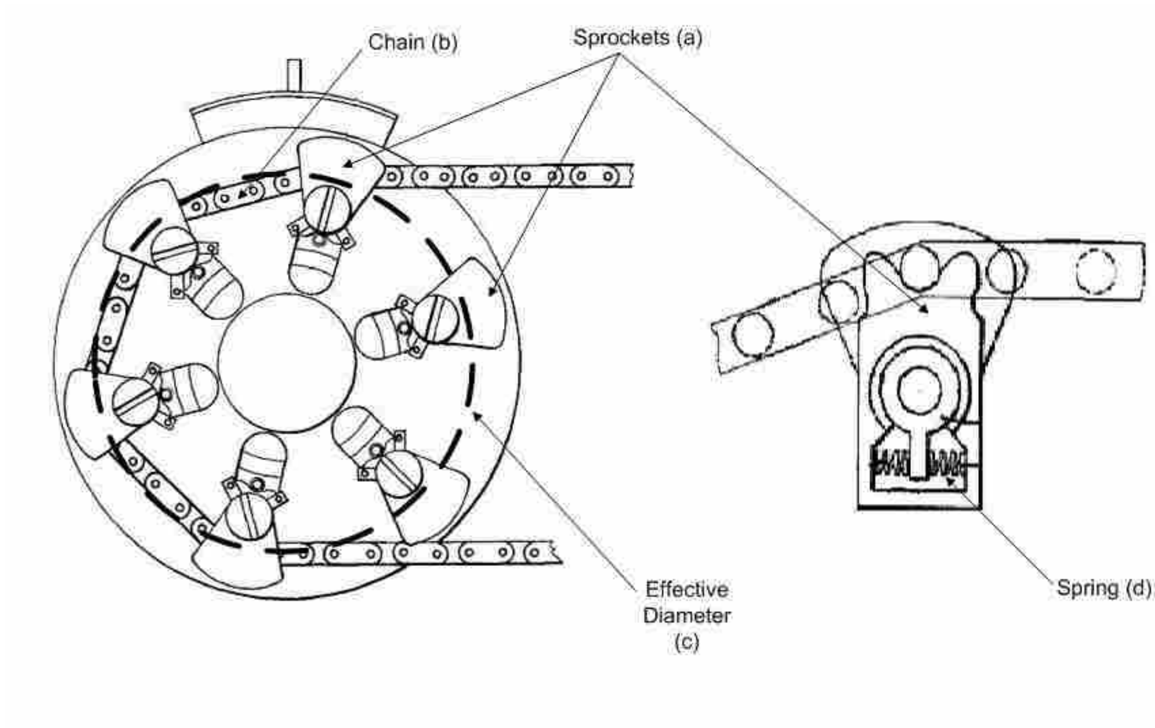

Figure 2.16: Problem Correction Class Embodiment (Retrieved from U.S. Patent No. 3867851)

This embodiment is likewise subject to the non-integer tooth problem because the distance between the sprockets (a) can change in a continuous manner. This continuous change would allow the distance between the sprockets (a), specifically where the sprockets (a) would mesh with a chain (b), to assume values not evenly divisible by the pitch of the chain (b). This would cause slack to occur in the chain (b) as the effective diameter (c) of the PECVT decreased, and the chain would skip off the sprockets (a) when the effective diameter (c) increased. In this embodiment, the sprockets (a) are allowed to rotate slightly in either direction with the use of a spring (d) to allow proper meshing to occur. The individual sprockets (a) must adjust different amounts to satisfy the pitch matching principle for each of the effective gear diameters (c) that do not yield an integer number of teeth on the circumference of the effective sprocket diameter (c). The limitation to the correction device in this patent is found in the characteristics of the spring (d). There is nothing that will stop the deflection of the spring (d), once the sprocket (a) is engaged with the chain (b), so that the sprocket (a) will transmit torque. Only when the spring (d) is fully deflected will torque be transmitted. This will cause the undesired oscillating output which does not meet our desired product specifications.

2.5.2 Problem Elimination Class

Unlike the embodiments belonging to the problem correction class, embodiments classified under the problem elimination class use a device, mechanism, or specific method to eliminate the non-integer tooth problem to ensure proper engagement. Embodiments in this class do not need any type of realignment correction because the characteristics of the members or devices that are engaged eliminate any misalignment prior to engagement. These embodiments can best be understood by creating two families that represent the different methods of eliminating the non-integer tooth problem. These families are the tooth conforming family and the feedback family.

2.5.2.1 Tooth Conforming Family

It is common in many problem elimination class embodiments to use mechanisms that allow driven teeth to conform to the driving teeth with which they mesh. For example, in place of actual teeth, mechanisms that can be positively engaged similar to actual gear teeth can be used in place of an actual gear. When the actual tooth comes into contact with these special mechanisms, a virtual tooth conforms to the actual gear tooth and torque can be transferred. Embodiments that use this method to eliminate misalignment problems are classified under the tooth conforming family. There is no need for realignment of any members for this family of PECVTs because a virtual tooth conforms to the actual gear tooth with exactly the right orientation needed. There are many innovative mechanisms or embodiments that can be used to eliminate the non-integer tooth problem, while still utilizing positively engaged members, which clearly need to be further explored in the PECVT concept generation. A few examples of published PECVT embodiments belonging to the tooth conforming family are given below with general advantages and disadvantages of the embodiments belonging to this family.

U.S.Patent No. 6,055,880 (Gogovitza) Issued: 2000

Class: Problem Elimination

This embodiment consists of two gear pairs, a drive (a) and driven set (b), which are each engaged with an effective ring gear called a transfer ring (c). The transfer ring (c) consists of several plates (d) situated and connected inside a ring (c) which engages with both the drive gear sets (a) and driven gear sets (b) shown in Figure 2.17.

The RPM ratio is able to continuously vary as the transfer ring moves axially relative to the shafts (e) of the gear pairs (a and b). As the conical shaped gear pairs (a and b) increase or decrease in diameter, their circular and diametral pitches also change allowing for a continuously variable ratio. However, the two gears in the driving (a) and driven gear pairs (b) do not actually engage with each other. The individual plates (d) in the transfer ring (c) are displaceable back and forth parallel to the transfer ring axis (f), but are not allowed to completely translate outside of the ring. When the two drive (a)

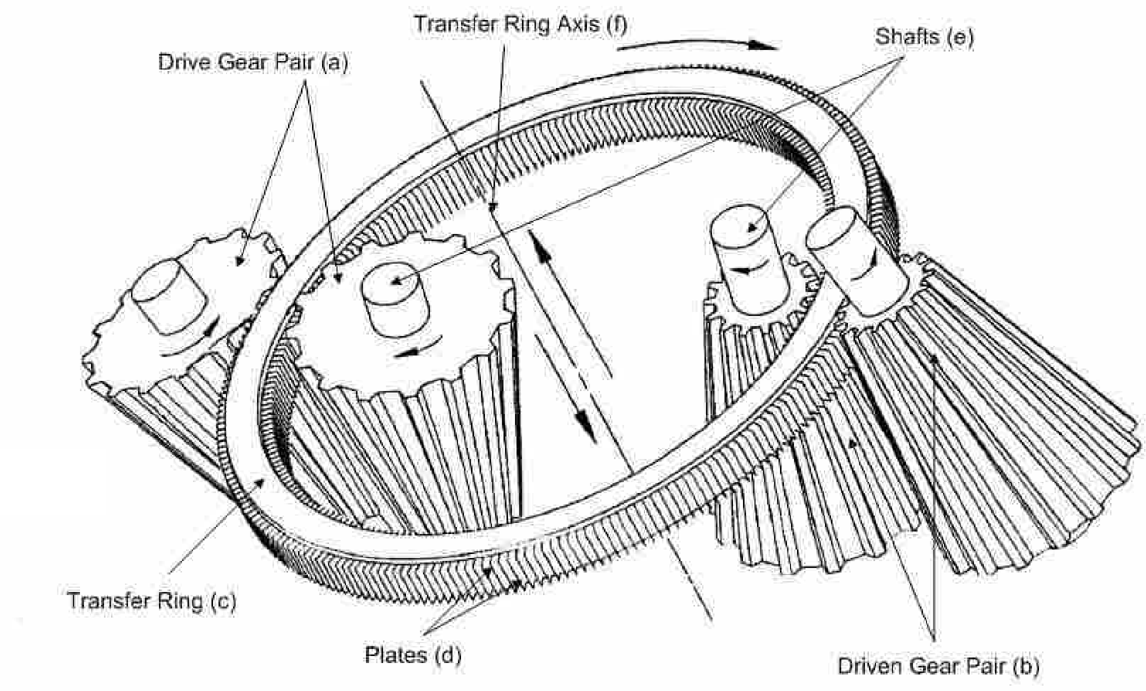

Figure 2.17: Problem Elimination Class Embodiment (Retrieved from U.S. Patent No. 6055880)

and driven gears (b) rotate, the plates (d) in the transfer ring (c) conform to the gear teeth to effectively engage the two gears (a and b). This method of positive engagement by virtual conforming teeth is shown in Figure 2.18 and is very common among published patents in the problem elimination class because it eliminates the non-integer tooth problem. Since the orientation of the two gears always guarantees that the top land (g) of one gear line up with the bottom land (h) of the other gear, there are no meshing problems between the two gears in either gear pair.

The meshing of the two drive gears (a) with the transfer ring (c) causes rotation of the ring (c) which then transfers torque to the driven gear set (b). The driven gear set (b) also meshes with the transfer ring (c) in the same manner as the drive gears (a). In this manner, the individual plates (d) form teeth between the gear sets (a and b) when needed

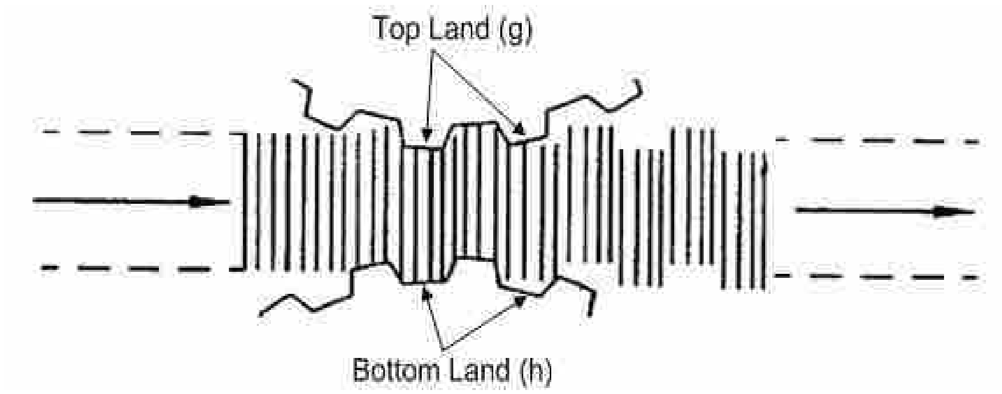

Figure 2.18: Method of Positive Engagement in Problem Elimination Class (Retrieved from U.S. Patent No. 6055880)

and do not require any correction when meshing with the next set of gears. The biggest limitation to this concept is the lack of robustness of the individual plates (d) held and restricted to a certain amount of parallel displacement in the transfer ring (c). The amount of torque that the engaged plates (d) are able to carry would be minimal compared to the amount of torque applied by the high torque vehicle applications which is desired.

U.S.Patent No. 6,964,630 (Magyari) Issued: 2005

Class: Problem Elimination

This embodiment consists of a pair of conical members (a and b) that can rotate on their geometric axes (c) relative to one another and are engaged at one point common between their surfaces. One member is the driving member (a) that rotates and transmits torque to the output, or driven member (b). On the surface of both members are small torque carrying needles (d) that extend outward and allow the two members (a and b) to positively engage with each other. As the two members change the angle between their two central axes (c), the RPM ratio can continuously vary while maintaining positive engagement. Figure 2.19 shows the two members (a and b) along with their method of engagement through the torque needles (d).

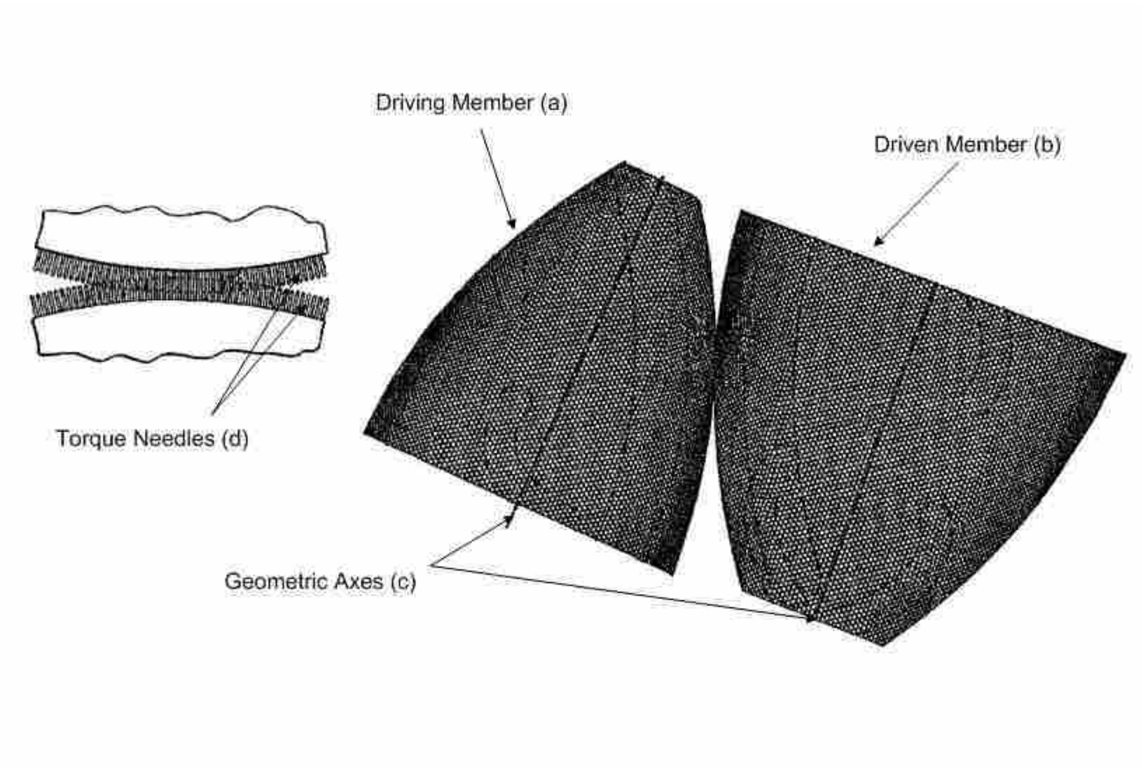

Figure 2.19: Embodiment of Problem Elimination Class Using Torque Transmitting Needles (Retrieved from U.S. Patent No. 6964630)

Because there are no actual teeth located on the two members (a and b), the non-integer tooth problem does exist, and proper meshing takes place as the needles (d) of one member (a) push past the slightly flexible needles (d) of the other member (b) in a positive engagement manner. Again, the amount of torque the needles (d) are able to carry before slipping is not enough for high torque applications. Also, the robustness of this problem elimination concept, and others, need to be improved for proper functionality to take place under a wider range of operating conditions.

2.5.2.2 Feedback Family

A second family of PECVTs in the problem elimination class was created to classify embodiments that use a positively engaged feedback drive train from the output, combined with the mechanical input, to conceivably achieve a continuously variable RPM ratio change, while eliminating the non-integer tooth problem. The feedback embodiments are most effective when using differential devices, which are able to take the rotational difference between the transmission's input and output to provide a separate rotational input to be used within the transmissions components. A better understanding of this concept will be gained by studying the example of the feedback family embodiment below.

The feedback family consists of embodiments that use only one mechanical input source from the engine and use the transmission's output as another input source to achieve a continuously variable ratio similar to hydrostatic and electric CVTs. These embodiments generally do no require the driving or driven gear to change their diameter to achieve a continuously variable transmission ratio because the input and output gears are always engaged in a fixed ratio manner. Instead, by allowing the continuously variable feedback source to provide the continuously variable transmission ratio, the non-integer tooth problem is eliminated. Other methods of using intelligence from the input or output of the transmissions will also be explored for further concepts to be developed in this family during the concept generation step. An example of a patented embodiment belonging to the feedback family is described below to demonstrate the functional principles associated with this family.

U.S.Patent No. 4,235,125 (Perlin) Issued: 1980

Class: Problem Elimination

This embodiment utilizes two differential systems that are coupled together to conceivably vary the RPM ratio as shown in Figure 2.20.

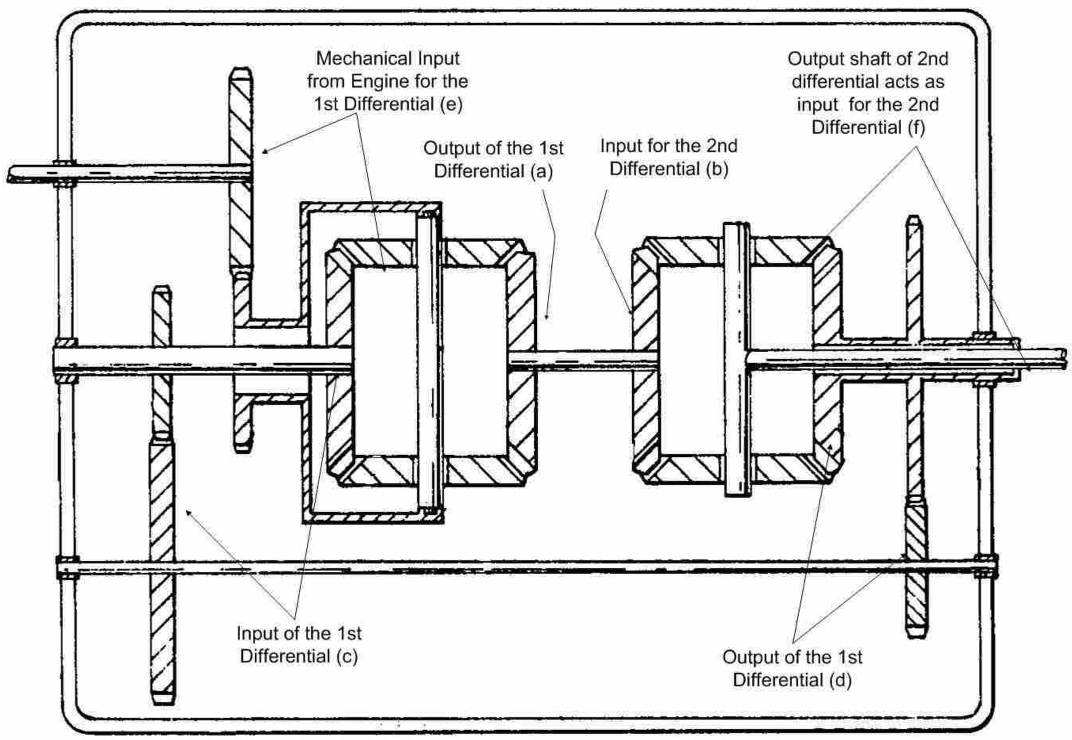

Figure 2.20: Two Differential Systems Coupled Together to Form a Feedback Family embodiment (Retrieved from U.S. Patent No. 4235125)

A differential system is essentially a 2 degree of freedom mechanism as two inputs determine the single output. With two differentials in this embodiment, the transmission becomes a 4 degree of freedom system; however, since the output of the first differential (a) is the input for the second differential (b), one degree of freedom is absorbed in the system. Also, one of the inputs of the first differential system (c) is the same as the output of the second differential system (d), which absorbs another degree of freedom. The transmission thus becomes a 2 degree of freedom system with 2 inputs. One input, for the first differential system, is the mechanical input from the engine (e); the other input is actually obtained from the output shaft of the transmission (f), which acts as the input for the second differential system. Since the output shaft serves as an input, the output RPM ratio is allowed to continuously vary depending on the output torque experienced by the transmission. In this manner, the transmission is allowed to continuously vary its RPM ratio; however, there is no control over when and how much the ratio should change.

One big advantage of feedback family embodiments is the ability to maintain constant, positive engagement of the input and output members of the embodiments suitable for high torque applications without lending themselves to become subject to the non-integer tooth problem. The lack of control over the continuously variable transmission ratio, however, is a huge disadvantage of similarly functioning embodiments belonging to this family.

2.5.3 Classification System Summary

A classification system has been developed to classify and organize all published and unpublished PECVT embodiments that have been published and that are not yet published. With this system, principles that must be satisfied for the most promising solutions of PECVT meshing problems to exist can now be further established according to each class. A summary of the PECVT classes and families created are shown in Figure 2.21.

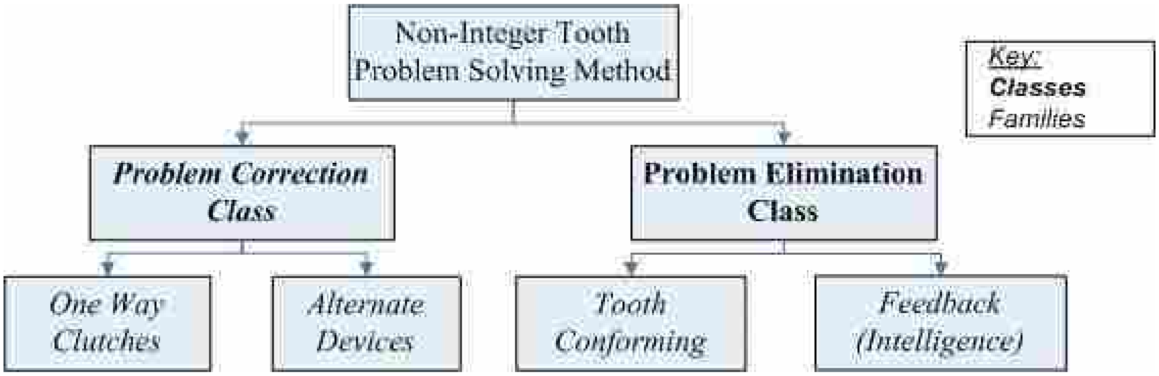

Figure 2.21: PECVT Classes and Families

2.6 External Patent Search

Studying, analyzing, and classifying published embodiments of PECVTs developed by other engineers can aid in creating innovative ideas for generating new concepts. Comparing attributes of benchmark embodiments to the product specifications needed in an ideal PECVT embodiment will also be helpful in generating more ideal embodiments. This section will provide insight obtained from an external patent search, which was conducted on a representative sample of all published PECVT embodiments. By examining these patents, not only were methods and ideas extracted and examined on how to solve the non-integer tooth problem, but the information also helped guide this work to avoid infringement on existing patents. Explanations of some of these embodiments have already been discussed above to add understanding to the newly created classification system. A detailed description of every patented embodiment analyzed will not be given; however, those that were detailed above demonstrated the common methods used to achieve certain characteristics in many of the patented embodiments.

The patent search was conducted using the US patent office in finding over 100 different published PECVT patents. Of these patents, 37 were selected and analyzed, with the results tabulated in Appendix A, showing which product specification metrics were satisfied for each patent. These 37 patents were carefully selected to represent the whole population of PECVT embodiments as many embodiments that were not selected were very similar in design and functionality to the selected embodiments. Only product specifications with importance levels of 1 and 2 were calculated for each of the patents. The latter specifications (3 and 4) are difficult to calculate and are not included in the patents; therefore, these specifications were disregarded. However, these specifications can be estimated when comparing concepts that satisfy the same product specifications of importance levels of 1 and 2.

2.6.1 The Problem Correction Class Patents

Embodiments belonging to the problem correction class satisfied nearly all primary and secondary specifications. The product specification that was most commonly unsatisfied by any of the problem correction embodiments was the non-oscillatory output. It appears that any mechanism or device which incorporated problem correction techniques thus far to correct the non-integer tooth problem inherently introduces the undesired oscillating output. Nearly all devices used in these embodiments are some type of one-way clutch, or other mechanisms, which inherently introduces an oscillating output.

Principles: The reorientation of the driving or driven gear (whichever possesses the characteristics of a continuously variable diameter gear) must occur so that its circular pitch is equal to or a factor of the circular pitch of the gear or member with which it is engaged. This is called the matching pitch principle. Furthermore, if the ideal PECVT embodiment belongs to the problem correction class, a device needs to be devised and implemented in such a way that a constant output is not being traded for positive, continuous engagement when a correction is applied to satisfy the matching pitch principle.

2.6.2 Problem Elimination Class Patents

The patented embodiments found in this class satisfied all primary and secondary product specifications without other major performance trade-offs or inefficiencies. Embodiments in the tooth conforming family eliminate the non-integer tooth problem and avoid meshing problems; however, the methods or devices used appear to be extremely complex or contain numerous parts. Complexity and part count, although product specifications of level 4 importance (See Appendix A), are still important specifications to consider. When the complexity and the part count of certain embodiments increase, the manufacturing and functional feasibility of those embodiments decrease. Many tooth conforming family embodiments also appear to lack robustness, meaning these embodiments will not function properly in high torque applications, which violate one of the product specifications.

In addition, embodiments of the feedback family appear to be limiting in ability to change and control the RPM ratio. Without the aid of an additional input source, sufficient intelligence does not exist merely from the transmission's output to continuously vary and maintain control of the RPM ratio change. In many of the patented embodiments, the RPM ratio does not vary at all, resulting in a fixed output to input ratio through multiple differential devices.

Principles: If an ideal PECVT embodiment belongs to the problem elimination class, the devices and methods used to eliminate the problem and ensure proper meshing need to be less complex and more robust for high torque applications in the tooth conforming family. A device that gathers more intelligence from the transmission's parameters in order to vary and, more importantly, control the RPM ratio would be another alternative for a promising embodiment in the feedback family.

2.6.3 Useful Mechanisms and Methods from Patents

In addition to using patent information to establish principles that need to be satisfied for promising PECVT embodiments to exist, other information can also be helpful in future concept generation methods. The methods and devices used in these and other analyzed patents to overcome the non-integer tooth problem and achieve a continuously variable RPM ratio are included in Table 2.4. For each class of PECVTs, some patented embodiments were previously described in detail to show how to overcome the non-integer tooth problem using their respective methods and devices shown in Table 2.4.

These and other notable methods and concepts used in published embodiments could prove promising when combined and integrated differently in new embodiments. Distinct embodiments employ these methods in different ways to achieve different desired results.

Table 2.4: Methods and Devices used in Analyzed Patents

Class	Family	Patent Number	Methods and Mechanisms Used
CVT	Electric/Hydrostatic	4,854,190	Input from Variable Speed Electric Motor
		John Deere	Input from Variable Displacement Hydraulic Pump
	Friction	3,899,941	Fluid Brake Input
		4,610,184	Fluid Shear
		5,169,359	Friction Brake Input
		4,625,588	Friction Brake Input
		4,805,489	Friction Brake Input
		6,033,332	Hydraulic Braking Input
Problem Correction	One-Way Clutch	5,440,945	One-Way Clutches
		4,644,828	One-Way Clutches
		4,373,926	One-Way Clutches
		4,277,986	One-Way Clutches
		4,181,043	One-Way Clutches
		4,909,101	One-Way Clutches
		4,697,469	One-Way Clutches
		4,660,427	One-Way Clutches
		5,454,766	One-Way Clutches
		3,359,813	One-Way Clutches
		3,867,851	One-Way Clutches
		2,199,051	One-Way Clutches
		5,516,132	One-Way Clutches
		4,763,544	One-Way Clutches and Torque Converter
	Alternate Device	4,680,985	Deformable Teeth
		5,036,716	Still Has Problem
		6,835,153	One-Way Clutches
		6,066,061	Differentials
Problem Elimination	Tooth Conforming	4,852,569	Slot ring cable chain
		2,026,928	Small Plates / Teeth Conform where needed
		6,055,880	Small Plates / Teeth Conform where needed
		3,175,410	Small Plates / Teeth Conform where needed
		2,970,494	Fluid Piston / Spring Loaded Teeth
		6,964,630	Small meshing pins, No teeth
		Fixed Pitch	Power Sprocket
	Feedback	4,327,604	One-Way Clutches/Double Planetary System
		4,235,125	Two Differential Units
		6,053,840	Double Planetary System

2.6.4 Conclusions of External Patent Search

From the results tabulated in Appendix A, the strengths and weaknesses of each class of PECVT are exposed as they present themselves in different embodiments. In summary, Table 2.5 lists the advantages and disadvantages of each of the newly developed classes based on published PECVT embodiments. From this information, the trade-offs for embodiments of all classes are better understood and will be helpful in the concept generation phase of this investigation. By creating this classification system and conducting the patent search to analyze previously published embodiments, principles that must be satisfied for the existence of a promising PECVT embodiment have also been established.

Table 2.5: Trade-Offs Between Different PECVT Classes

	Advantages	**Disadvantages**
Problem Correction	High RPM Ratio Range	Oscillating Output
	Many Functional Specs Satisfied	Feasibly Difficult
	Continuous Engagement	
Problem Elimination	High Efficiency	Feasibly Difficult
	No Meshing Problems	Not Robust
	Continuous Engagement	Lack of Ratio Control

3 Concept Development Methods

This chapter will describe the methodology that will be used to investigate new, unpublished PECVT concepts. Two different methodologies, TRIZ and the concept development phase of the product design and development process, are described along with a detailed explanation of the steps that will be followed later. The manner in which these two methodologies will be implemented in this research will also be provided. The purpose of implementing these methodologies is to find the most viable solution to the non-integer tooth problem by narrowing the concepts to one final embodiment that will satisfy the product specifications and the governing principles shown in Chapter 2.

3.1 TRIZ Methodology

As briefly described in section 1.6, TRIZ is a problem solving methodology popular in solving scientific and engineering problems. The methodology was developed by Genrich Altshuller in Russia around 1946 [7]. Altshuller searched over 40,000 patents and discovered the following four important problem solving principles as a result [19]:

1. There are five levels of invention.
2. Inventive problems contain at least one contradiction, that is, solutions to a primary problem generally create an inherent secondary problem.

3. The same inventive principles are used in many designs and can therefore be considered solution patterns in solving similar designs.
4. There are standard patterns of evolution in design.

These four principles form the basis of TRIZ, and because understanding and implementing these principles will generally lead to a more systemic method of problem solving, they are discussed below in detail.

3.1.1 Five Levels of Invention

The methodology known as TRIZ was developed as a result of the findings of Altshuller due to his extensive patent search. Since (and including) the patent search conducted by Altshuller, over 1.5 million patents have been studied and a discovery was made that in many similar problems of different technical fields, the problems presented had been solved using the same inventive principles. The solutions to the problems were organized into five levels [7].

1. Routine design problems solved by methods well known within the specialty. (No invention needed)
2. Minor improvements to an existing system, by methods known within the industry. (Usually some compromise, or trade-off of desired solution made)
3. Fundamental improvement to an existing system, by methods known outside the industry. (Contradictions resolved, this will be discussed later)
4. A new generation that uses a new principle to perform the primary functions of the system. (Solution found more in science than in technology)
5. A rare scientific discovery or pioneering invention of essentially a new system.

In addition to discovering these 5 levels of invention, Altshuller also evaluated the percentage of patents that are found in each of these levels along with an approximation of how many possible solutions one might need to create or consider before a final solution might be found. A summary of these five levels along with these two statistics are given in Table 3.1.

Table 3.1: Altshuller's Five Levels of Invention (http://www.mazur.net/triz/)

Level	Degree of inventiveness	% of solutions	Source of knowledge	Approximate # of solutions to consider
1	Apparent solution	32%	Personal knowledge	10
2	Minor improvement	45%	Knowledge within company	100
3	Major improvement	18%	Knowledge within the industry	1000
4	New concept	4%	Knowledge outside the industry	100,000
5	Discovery	1%	All that is knowable	1,000,000

TRIZ is especially effective when dealing with problems in levels 3 and 4 where solutions already known in industry are not always the optimal solution. Due to the degree of inventiveness needed in these two levels as shown in Table 3.1, often times the implementation of a major improvement or a new concept as a solution to one aspect of the system tends to create a problem elsewhere in the system (i.e. Today's solutions create tomorrow's problems). This is what Altshuller describes as a physical or technical contradiction [7].

3.1.2 Contradictions

A physical contradiction is when an element of the product or system has two opposing requirements that it is subject to. A technical contradiction occurs when an improvement of a certain characteristic or attribute of the product or system causes a different characteristic to deteriorate within the system. In many problem solving methods, the solution to contradicting problems result in trade-offs from choosing either one solution or another; however, TRIZ proposes a methodology where these contradictions are eliminated and the solutions found that might satisfy the contradicting requirements [7].

3.1.3 Solution Patterns

Due to the discoveries of Altshuller and the need to eliminate contradiction problems instead of compromising them, 40 inventive principles were created and found to help solve a very wide variety of technical problems when applied to the design process. These principles were commonly found in the 1.5 million patents searched by Altshuller and coworkers that helped solve the contradictions. These principles were found to eliminate contradictions when applied to the design of many different products due to a common solution pattern that similar designs possessed. These principles that determine the design pattern are listed in Table 3.2 [7].

A detailed description of each of these principles will not be given; however, upon inspection, many of these principles are understood. The next task in discovering the solution pattern of a particular product is to know which principles to implement.

One useful tool to guide a designer in knowing which of the 40 inventive principles to apply is the contradiction matrix also developed by Altshuller. The matrix is composed of 39 rows and 39 columns, using the same 39 commonly conflicting design parameters listed as both column headings and row labels of that matrix. This list of 39 commonly conflicting parameters were also created as a result of Altshuller's patent

Table 3.2: The 40 Inventive Principles Used to Eliminate Design Contradictions

Inventive Principles of TRIZ	
1. Segmentation	21. Rushing through
2. Extraction	22. Convert harm into benefit
3. Local quality	23. Feedback
4. Asymmetry	24. Mediator
5. Combining	25. Self-service
6. Universality	26. Copying
7. Nesting	27. An inexpensive short-lived object instead of an expensive durable one
8. Counterwieght	28. Replacement of a mechanical system
9. Prior counteraction	29. Use of a pneumatic or hydraulic construction
10. Prior action	30. Flexible film or thin membranes
11. Cushion in advance	31. Use of porous material
12. Equipotentiality	32. Change the color
13. Inversion	33. Homogeneity
14. Spheroidality	34. Rejecting and regenerating parts
15. Dynamicity	35. Transformation of physical and chemical states of an object
16. Partial or overdone action	36. Phase transition
17. Moving to a new dimension	37. Thermal expansion
18. Mechanical vibration	38. Use strong oxidizers
19. Periodic action	39. Inert environment
20. Continuity of useful action	40. Composite materials

search. The parameters were created to help the designer discover the contradictions that might exist in the design of a particular product. These parameters represent the characteristics that a designer might want to improve in a system, and at the same time they represent the attributes that might be inherently affected by that improvement (a contradiction). These parameters are listed in Table 3.3 [7].

The matrix is used by locating the matrix row that represents the feature or parameter that needs to be improved in the design and then by locating the column heading representing the feature that might be negatively affected by proposed improvement. By examining the cell in the matrix that is common to both the row and column selected, certain inventive principles that might be used to solve the contradiction are shown as numbers, which numbers represent the principles shown in Table 3.2. The

Table 3.3: The 39 Conflicting Design Parameters used in the Contradiction Matrix

Engineering parameters commonly used in TRIZ			
1.	Weight of moving object	21.	Power
2.	Weight of nonmoving object	22.	Waste of energy
3.	Length of moving object	23.	Waste of substance
4.	Length of nonmoving object	24.	Loss of information
5.	Area of moving object	25.	Waste of time
6.	Area of nonmoving object	26.	Amount of substance
7.	Volume of moving object	27.	Reliability
8.	Volume of nonmoving object	28.	Accuracy of measurement
9.	Speed	29.	Accuracy of manufacturing
10.	Force	30.	Harmful factors acting on object
11.	Tension, pressure	31.	Harmful side effects
12.	Shape	32.	Manufacturability
13.	Stability of object	33.	Convenience of use
14.	Strength	34.	Repairability
15.	Durability of moving object	35.	Adaptability
16.	Durability of nonmoving object	36.	Complexity of device
17.	Temperature	37.	Complexity of control
18.	Brightness	38.	Level of automation
19.	Energy spent by moving object	39.	Productivity
20.	Energy spent by nonmoving object		

proposed principles suggested by the contradiction matrix are those that were found to correct the same contradictions that existed in the patents examined by TRIZ creators.

For example, suppose that a designer wishes to increase the length of an airplane wing to provide more lift. If the wing length is increased, then the weight of the wing will increase as an inherent result. To solve this contradiction and find a possible design that could increase the wing length without affecting the weight of the wing, a contradiction matrix could be constructed to find out which inventive principle to use. A portion of a contradiction matrix is shown in Figure 3.1, which shows an example of improving the length of an object while trying not to change its volume.

Worsening Feature / Improving Feature		Weight of moving object	Weight of stationary object	Length of moving object	Length of stationary object	Area of moving object	Area of stationary object	Volume of moving object	Volume of stationary object
		1	2	3	4	5	6	7	8
1	Weight of moving object	+	-	15, 8, 29,34	-	29, 17, 38, 34	-	29, 2, 40, 28	-
2	Weight of stationary object	-	+	-	10, 1, 29, 35	-	35, 30, 13, 2	-	5, 35, 14, 2
3	Length of moving object	8, 15, 29, 34	-	+	-	15, 17, 4	-	7, 17, 4, 35	-
4	Length of stationary object		35, 28, 40, 29	-	+	-	17, 7, 10, 40	-	35, 8, 2,14
5	Area of moving object	2, 17, 29, 4	-	14, 15, 18, 4	-	+	-	7, 14, 17, 4	
6	Area of stationary object	-	30, 2, 14, 18	-	26, 7, 9, 39	-	+	-	

Figure 3.1: A Portion of the TRIZ Contradiction Matrix [20]

The parameter that the designer wants to improve is located as the third parameter in the row label, representing the third parameter of Table 3.3, the length of a moving object. The inherent effect of the volume is then located in the column heading as the seventh parameter, also equivalent to the seventh parameter of Table 3.3, the volume of a moving object. By locating the common cell in the matrix of the two conflicting parameters, inventive principles numbers 7, 17, 4, and 35 are those suggested to eliminate the contradiction and achieve the desired design results. These numbers represent the principles shown in Table 3.2 and suggest that nesting, temperature, length of nonmoving object, and adaptability might be used to eliminate this contradiction because the solution to problems with similar contradictions in the past were solved using these principles. Again, a detailed description of these principles will not be given until the matrix is applied to the PECVT. Only an understanding of how to use the contradiction matrix is desired at this time.

3.1.4 Patterns of Evolution

The last of the four important principles discovered by Altshuller was that a standard pattern of evolution exists for all technical systems. By studying the pattern of evolution of more-fully developed products from other technical fields, Altshuller found that the same patterns can be used to predict and even invent new products in a technical field that is not as developed. The evolution of design in one field is related to the evolution of design in other related fields, and knowing that process of evolution can help facilitate the process of finding solutions that eliminate contradictions.

3.1.5 Application of TRIZ Principles

TRIZ possesses a unique feature in its methodology different from other types of problem solving techniques. Most problem solving methods use a trial-and-error approach when generating and developing concepts, that is, the concepts are in some cases randomly mixed and matched with other concepts in hopes of finding a viable end solution. The goal of this trial-and-error approach is to achieve a final solution by combining solution fragments with little being done to avoid trade-offs between desired solutions. Figure 3.2 shows the unique problem solving technique used in TRIZ by applying Altshuller's principles as opposed to other problem solving methods.

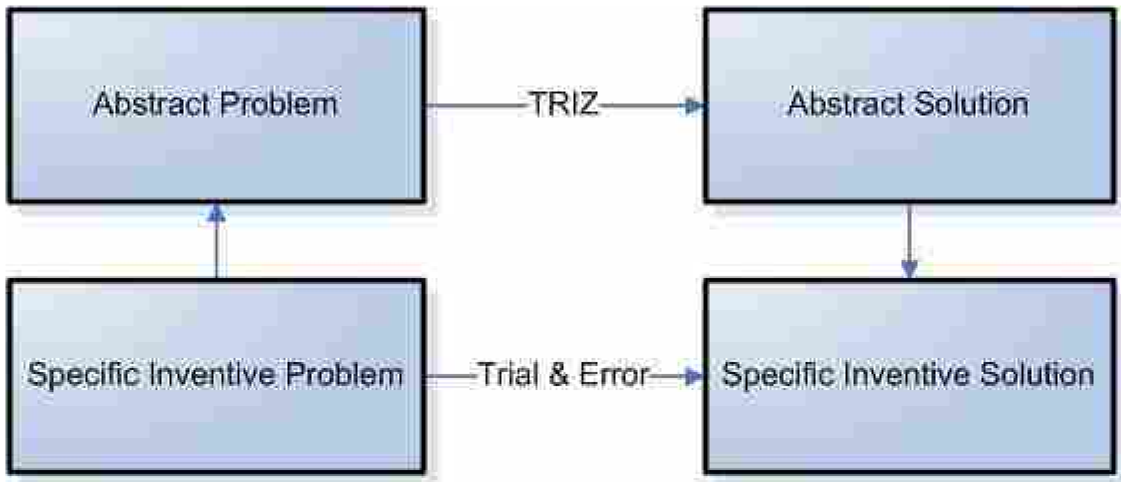

Figure 3.2: TRIZ Problem Solving Technique versus Trial and Error Approach (http://www.insytec.com/TRIZApproach.hrm)

Figure 3.2 shows that when faced with a specific inventive problem, it is not always best to try to develop a specific inventive solution. Often times this trial and error approach will lead to undesired trade-offs, or contradictions, in the solution. As an alternative, a designer should use TRIZ principles to identify any contradictions that the product itself contains. This will lead to the development of an abstract problem that can be solved by using the TRIZ contradiction matrix and Altshuller's 40 inventive principles. The solution of this process will be an abstract or general solution to that type of contradiction and should help to eliminate any contradictions in the system. The specific inventive solution is then obtained by using the suggested inventive principles to modify the product or system. An example of this is shown in Figure 3.3 as it applies in designing an airplane wing.

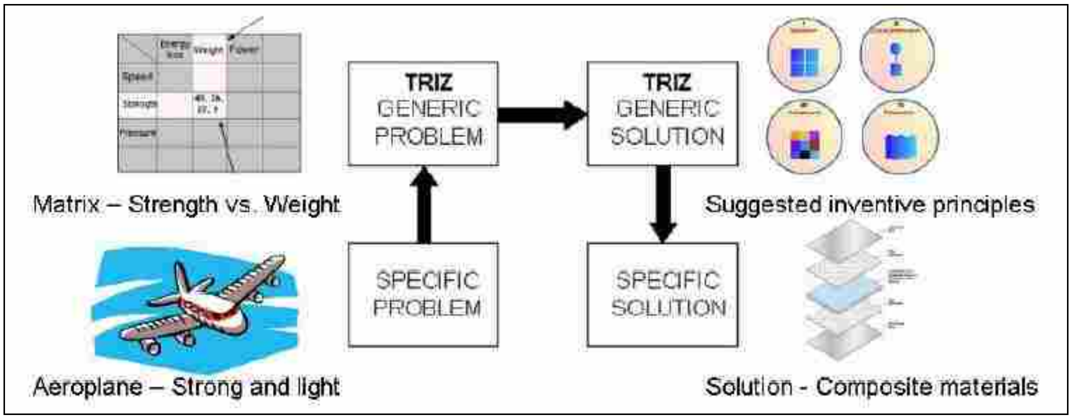

Figure 3.3: Example of Unique Problem Solving Process of TRIZ (Retrieved from http://www.musif.mt.tuiasi.ro/icms/isms2k5/papers/2k5052.pdf)

By using the example of designing an airplane wing that is stronger without increasing its weight, we can see the effectiveness of using the TRIZ contradiction matrix. After identifying the contradiction of strength and weight (Generic Problem) and using the contradiction matrix to identify the inventive principles of segmentation and composite materials from the list of 40 inventive principles (Generic Solution) to use to eliminate that contradiction, a specific solution to the problem can be formed by converting the generic solution to a specific solution. The final solution is to construct the wing using a composite material, which should allow the wing to be both stronger and lighter. In this way, the contradiction was eliminated and an optimal solution found.

3.2 Concept Development Phase

The concept development phase is one of several phases included in the product design and development process. This chapter details the methodology of this phase that will be used in investigating possible solutions to the non-integer tooth problem. The concept development phase can be seen in Figure 3.4 with its steps described below. The steps of this phase were taken from Product Design and Development by Ulrich and Eppinger, a widely used book in product development [1].

Figure 3.4: Seven Basic Steps of the Concept Development Phase [1]

The concept development phase consists of seven steps; however, the last two steps will not be implemented in this research as all research objectives can be met by implementing only these first five steps.

3.2.1 Customer Needs

Before designing any product or device, it is important to understand the needs of all users of the device for whom it is to be designed. Customer needs are a list of desired attributes or functions that a certain product must possess to fulfill its purpose or improve its functionality. Whether or not the purpose is fulfilled is decided by all who are affected directly or indirectly by the product. The customer needs should be expressed in terms of the function(s) that the product has to *do* and not on the process of doing the function. The needs should then be organized by classifying them as primary or secondary needs. After organization of the customer needs, it is important to examine each need to obtain information about its relative importance. This allows the designer to make correct decisions as to the trade-offs that need to be made during product design.

After this examination, a numerical importance weighting for each of the needs should be provided. The customer needs were already established in Chapter 2 to help define the operating principles of the desired PECVT.

3.2.2 Product Specifications

Product specifications represent measurable ways of meeting each customer need. As noted in Chapter 2, they do not specify *how* to address customer needs, but they do detail precisely what the product has to do. A product specification takes the language of the customer and converts it into the language of the engineer or designer. By creating a list of metrics, or measurable characteristics, the concept generation portion of the concept development process becomes clearer as to the type of concepts that need to be identified, considered and/or developed. When developing this list of metrics, it can be useful to collect benchmarking information for types of existing products that have already been designed. By setting ideal and marginal target metric values for the new product being developed, the product can posses the attributes needed to exceed previous and similar products. These two metric values (ideal and marginal) act as bounds for which a competitive market product should perform [1]. The product specifications were also established in Chapter 2 to help define the operating principles of a PECVT.

3.2.3 Concept Generation

The concept generation process of the concept development phase consists of five steps to ensure that the entire concept design space has been explored. A thorough concept generation usually signifies that the full space of alternatives has been explored and a greater possibility of generating viable concepts. These five steps of the concept generation process are shown in Figure 3.5.

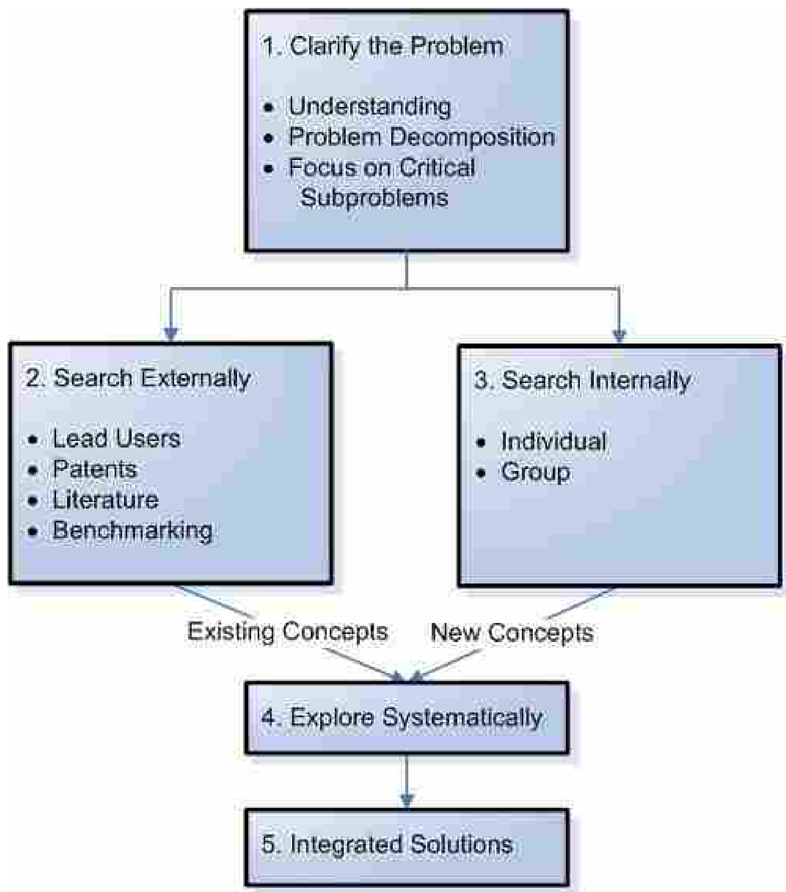

Figure 3.5: Concept Generation Steps [1]

This process reduces the concept generation task into smaller, simpler sub-tasks and provides a systemic approach to generating concepts. The process is universal, can be applied to almost any product, and can be developed or refined to each designer's problem-solving styles or methods. These steps simply serve as an outline to what should be done to more fully explore the entire design space. A description of these steps and how they are to be performed are detailed below. It is in this step of the concept development phase that TRIZ techniques can be implemented for a more powerful approach to generating possible concepts. The TRIZ methodology can be implemented simultaneously with this step of the concept development phase and the two methods should complement each other in a synergetic manner.

3.2.3.1 Clarify Problem

The seeds of a solution to problems are often found within the problem itself. By studying the attributes of a problem in different circumstances in which it might exist, the problem and possible solutions will be better understood. This increased understanding will facilitate the development of governing principles and functional specifications for possible solutions to the problem. Clarifying the problem entails developing a general understanding of the problem and breaking it down in sub-problems. The process of dividing the problem in smaller sub-problems is known as problem decomposition and is a very useful tool in concept development. The problem decomposition can be conducted in different manners based on the type of results desired. Functional decomposition, decomposition by sequence of user actions, and decomposition by key customer needs are all possible methods used to break the problem into sub-problems [1]. In Chapter 1, the functional decomposition of PECVTs has already been described. Using this decomposition as well as the research conducted by Andersen will aid in increasing the probability of fully understanding the problem and ensuring that the entire design region has been searched for good solutions.

3.2.3.2 Search Externally

In order to build upon existing concepts, an external search should be conducted for similar concepts, designs, and ideas. An external patent search, for example, can be very useful in collecting existing concepts from other researchers and in learning from their design methods. An explanation of why certain products originated from certain designs might also be extracted in these patents to help understand how others have solved similar design challenges. Relevant literature should also be thoroughly searched as another possible source of information. Also, finding out how competitors have dealt with similar problems can breed new thoughts and spark innovative ideas when generating concepts for similar products. Benchmarking becomes very useful in this aspect as much can be learned from the design process of other engineers and designers with different design experience and backgrounds [1].

3.2.3.3 Search Internally

An internal search is one that focuses on finding solutions within a certain design team. Internal searches might allow an individual to come up with a completely new concept that hasn't been developed before, which is often times the solution when dealing with innovative designs like a new PECVT. Brainstorming for different concepts among a research group or any other group can prove to be helpful, especially when many people combine their creativity and thinking synergistically with others. The designer should continually be searching for new concepts throughout the entire concept development phase as different design steps will have different effects on thoughts and ideas. After conducting an internal search, ideas and concepts of both the external and internal searches can be pooled together for further examination and categorization [1].

3.2.3.4 Explore Systematically

The concepts collected in both the external and internal searches can be sorted and explored in a systematic method so as to begin selecting the best concepts for further consideration. A classification system can be developed to group certain concepts based on how they solve a particular contradiction or problem. Many of these concepts will be incomplete solution fragments, but when combined with other fragments, may form a complete and functional solution [1].

3.2.4 Concept Selection

Concept selection is the process of evaluating the list of generated concepts based on the customer needs and product specifications as described and detailed above. Each concept should be evaluated by comparing its strengths and weaknesses to other generated concepts after which the best concepts are selected for further exploration, development, and testing or validation. This process can be iterative in that the number of concepts selected might not converge immediately after the first evaluation; however, the process will reduce the number of concepts to, generally, more promising solutions. A popular method for choosing a concept involves using decision matrices, which allows the rating of each concept according to a specified selection criteria based on previously established customer needs and measurable engineering functional specifications. This method contains two stages in selecting a final concept: Concept Screening and Concept Scoring. Both of these stages follow a similar five step process:

1. Prepare the selection matrix
2. Rate the concepts
3. Rank the concepts
4. Combine and improve the concepts
5. Select one or more concepts

These steps will be described below as they apply to both stages of the concept selection process. It is also important to note the basic goals of each of these two stages. The goal of the concept screening stage is primarily to reduce the number of total concepts down to only a few of the most viable solutions. The concept scoring stage then further explores each of the remaining concepts for a more in depth evaluation to reduce the number concepts to the most viable and dominant concept [1].

3.2.4.1 Concept Screening

As mentioned above, the concept screening process, or Pugh concept selection method, follows five steps in a systemic manner to reduce the number of concepts to a select few and to improve on each of these remaining concepts.

The selection matrix is composed of possible design concepts listed in the column headings and selection criteria listed in the row labels of the first column. The selection criteria should be chosen in such a way as to distinguish between different concepts. Also, because the criteria are all equally weighted, it is important not to choose criteria that aren't very relevant to the product specifications. If unimportant criteria are selected in this stage, the differences in the concepts with respect to the more important criteria are not revealed. One of the most important things to consider when creating the selection matrix is to choose a concept that will be known as the reference concept. All other concepts will be rated relative to this reference concept. The concept should be a concept that is well understood, whether it be a benchmark solution or one of the concepts being evaluated.

Step two in the screening stage is to rate concepts in comparison to the reference concept for each of the listed criteria. Each concept receives either a "better than," a “same as,” or a “worse than” rating for all the criteria. Each concept receives a score of (1) for a “better than” rating, (-1) for a “worse than” rating, and (0) for a “same as” rating. When rating the concepts becomes difficult relative to a certain criterion, which is often the case, objective metrics can be helpful in measuring the concept’s performance with respect that criterion. For example, a good approximation for manufacturability ease could be the number of parts contained and/or their required dimensional tolerances, which are objective metrics for manufacturability. An example of a screening matrix is shown in Table 3.4 with three different concepts being screened. Concepts A, B, and C are being evaluated according to how well they meet the selection criteria, which in this example are similar to the criteria that will be used for a PECVT.

Table 3.4: An Example of a Screening Matrix to be used in the Concept Selection

	Concepts		
Selection Criteria	**A (Reference)**	**B**	**C**
Does not produce an oscillating output	0	1	1
Can transmit high torque	0	0	1
Highly efficient	0	1	-1
Feasible	0	-1	0
Robust	0	0	1
Net Score	0	1	2
Rank	**3**	**2**	**1**

Ranking the concepts is the next step and consists of summing up the rating scores from the previous step for each concept. After summing up the scores, each concept will have a total score and will be ranked according to that score showing how well they meet the criteria compared to the reference concept. In Table 3.4, concept C had a net score of 2 while concepts B and A had net scores of 1 and 0 respectively.

Therefore, concept C receives a rank of 1, concept B receives a rank of 2, and concept A (the benchmark or reference concept) receives a rank of 3.

After rating and ranking has occurred, each concept should be examined to see if certain features are causing a low ranking on a generally good concept. Some concepts could be combined together to conserve their good attributes and negate the negative attributes so as to increase their overall ranking. The matrix can then be modified as improved or combined concepts are generated, and another round of screening can be conducted. If these steps are completed in a systematic manner, the more viable concepts should yield the higher ranking, and these more dominant concepts can then be selected to move onto the concept scoring stage [1].

3.2.4.2 Concept Scoring

The concept scoring process takes on the same basic steps as the concept screening but differs in the depth of analysis and selection. Concept scoring consists of a more refined comparison of the remaining concepts with respect to the criteria.

Creating the selection matrix in the concept scoring process is much like that of the screening process shown in Table 3.4 only certain weighting factors are added to the criteria found in the matrix giving more importance to the more critical criteria. Another strategy that could be implemented in this stage is using different concepts as the reference concept for the different design criteria. Each concept's ranking score is obtained by adding up the products of the weighting factors and the rating score for each of the criteria.

After scoring, the different concepts can be combined or changed to achieve a better overall concept as was done in the concept screening stage. The final selection should not only be based upon the highest overall ranking score, but should take into account the trade-offs associated between different weighting scores of the criteria. More freedom is given to the designer to select a lower rank scoring concept than the highest scoring concept when these trade-offs are understood. Usually only one final design is chosen to begin validation and testing; however, more than one can be selected if further development is desired on the concepts before selecting the final concept [1].

3.2.5 Test Product Concept

The purpose of concept testing is to compare how well the selected concept actually satisfies all of the customer needs and product specifications. In this step of the concept development phase, many times it is appropriate and helpful to build a simple prototype of the concept to be able to communicate the basic geometry and functions of the final concept. Testing the concept using a prototype allows the designer to validate results obtained through mathematical models predicting certain function characteristics. Through experimental runs conducted with the prototype, experimental data can be compared to the model results to ensure that all calculations and design factors have been calculated and analyzed correctly. Also, learning will always occur when dealing with real hardware. In the research of this thesis, only numeric testing and validation will be conducted in preparation of a prototype to follow in future research.

3.3 Method Integration

Two methods of problem solving, TRIZ and the concept development phase, have been discussed in this chapter as methods of improving the probability of obtaining an optimal solution to the non-integer tooth problem for PECVT design. Each of these methods has certain advantages and disadvantages depending on the type of product or system being analyzed and the depth or extent of the investigation. Due to the nature of the non-integer tooth problem encountered in all PECVT embodiments, there exist certain challenges in developing solutions to this problem, which appear to be difficult and non-intuitive. If a combination of the two problem-solving methods is created to combine the advantages of both methods, the likelihood of finding a promising solution will increase. By implementing the innovative tools and principles of TRIZ into the concept development phase of the product design process, concept generation will be greatly enhanced with the power of generating concepts that eliminate contradictions associated with the non-integer tooth problem. By using this integrated methodology, increased confidence that the entire design region has been enveloped to find the most viable solution is achieved. Figure 3.6 shows the basic structure of this integrated product development method.

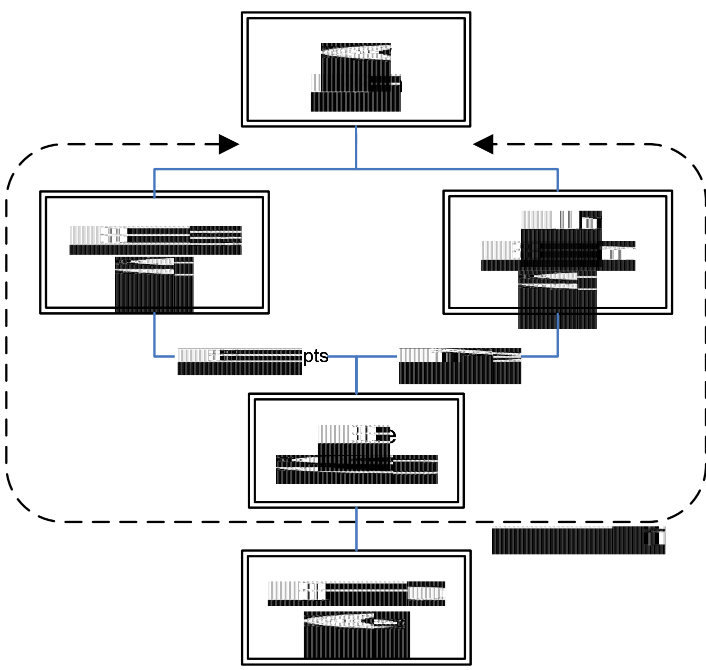

Figure 3.6: Concept Generation Phase Integrated with TRIZ Principles

4 Concept Generation Results

This chapter presents the results obtained from implementing the next steps of the integrated concept development methodologies on PECVTs as explained in Chapter 3. The customer needs and product specifications of PECVTs have already been addressed in Chapter 2 to aid in the formation of the classification system. The results of the next step, concept generation, in the methodology are presented in this section.

4.1 PECVT Concept Generation

By following the outlined methodology of generating concepts, a thorough examination of the design field was conducted to include a wide range of concept ideas. The first step of the concept generation process is to clearly create a functional decomposition for the purpose of identifying or defining the problem. This was completed and recorded in Chapter 1 and has been helpful throughout the realization of the concept generation phase. The non-integer tooth problem has also been clearly defined as it appears in each class of PECVT. Further information about the non-integer tooth problem can be found in Chapter 2 and in the research conducted by Andersen [3].

4.1.1 TRIZ Methodology Results

Before generating PECVT concepts to overcome the non-integer tooth problem, a TRIZ contradiction matrix was constructed with all possible design contradictions that might exist during the process. The author, along with Andersen, determined the parameters to include in the row headings of the matrix based on which parameters need to be improved in an ideal PECVT embodiment. In a like manner, the column headings were also chosen based on the most probable inherent effects of the parameter improvements. The inventive principles suggested by TRIZ were applied to eliminate particular contradictions as shown in Table 4.1. The numbers in the matrix cells for all combinations of contradicting parameters represent the inventive principles listed in Table 3.2. For example, if increasing the shape feature of the embodiment is negatively affecting its speed, then inventive principles 35, 15, 34, or 18 from Table 3.2 should be used to eliminate that contradiction.

Table 4.1: TRIZ Contradiction Matrix for Possible PECVT Embodiments

	Worsening Feature → Improving Feature ↓	Speed	Shape	Stability of the object's composition	Duration of action of moving object	Loss of Information	Measurement accuracy	Object-generated harmful factors	Ease of manufacture	Adaptability or versatility	Device complexity	Difficulty of detecting and measuring
		9	**12**	**13**	**15**	**24**	**28**	**31**	**32**	**35**	**36**	**37**
12	**Shape**	35, 15, 34, 18	+	33, 1, 18, 4	14, 26, 9, 25		28, 32, 1	35, 1	1, 32, 17, 28	1, 15, 29	16, 29, 1, 28	15, 13, 39
13	**Stability of the object's composition**	33, 15, 28, 18	22, 1, 18, 4	+	13, 27, 10, 35		13	35, 40, 27, 39	35, 19	35, 30, 34, 2	2, 35, 22, 26	35, 22, 39, 23
15	**Duration of action of moving object**	3, 35, 5	14, 26, 28, 25	13, 3, 35	+	10	3	21, 39, 16, 22	27, 1, 4	1, 35, 13	10, 4, 29, 15	19, 29, 39, 35
23	**Loss of substance**	10, 13, 28, 38	29, 35, 3, 5	2, 14, 30, 40	28, 27, 3, 18		16, 34, 31, 28	10, 1, 34, 29	15, 34, 33	15, 10, 2	35, 10, 28, 24	35, 18, 10, 13
24	**Loss of Information**	26, 32			10	+		10, 21, 22	32			35, 33
35	**Adaptability or versatility**	35, 10, 14	15, 37, 1, 8	35, 30, 14	13, 1, 35		35, 5, 1, 10		1, 13, 31	+	15, 29, 37, 28	1
36	**Device complexity**	34, 10, 28	29, 13, 28, 15	2, 22, 17, 19	10, 4, 28, 15		2, 26, 10, 34	19, 1	27, 26, 1, 13	29, 15, 28, 37	+	15, 10, 37, 28
37	**Difficulty of detecting and measuring**	3, 4, 16, 35	27, 13, 1, 39	11, 22, 39, 30	19, 29, 39, 25	35, 33, 27, 22	26, 24, 32, 28	2, 21	5, 28, 11, 29	1, 15	15, 10, 37, 28	+

All possible features that might need to be improved are listed in the improving feature column as features 12, 13, 15, 23, 24, 35, 36, and 37. The worsening feature row contains all features that might be inherently worsened when trying to make aforementioned improvements. These worsening features are listed as features 9, 12, 13, 15, 24, 28, 31, 32, 35, 36, and 37. With these features, any contradiction that is formed by matching an improving feature to a worsening feature should yield the recommended inventive principles. The multiple numbers found within the cells of the matrix represent the inventive principles to be used to eliminate the contradictions. Because of the generality of different design contradiction combinations, the most often occurring inventive principles suggested in the matrix were found and recorded for future use in the concept generation of possible PECVT embodiments. Table 4.2 shows the 7 most occurring inventive principles to help eliminate trade-offs associated with all possible design contradictions provided in Table 4.1 for solving the non-integer tooth problem.

Table 4.2: Top 7 Inventive Principles to Implement During Concept Generation

No.	Inventive Principle
35	Parameter Changes
1	Segmentation
10	Preliminary Action
28	Mechanics Substitution
15	Dynamics
13	The other way around
29	Pneumatics and Hydraulics

It is now important to understand how to apply the inventive principle to a particular design. To more fully understand what should be done to eliminate the undesired contradiction, Table 4.3 contains a description of these most occurring suggested inventive principles. An example of implementing these principles can also be seen in the table.

Table 4.3: Description of 7 Suggested Inventive Principles from Contradiction Matrix

No.	Inventive Principle	Description
35	Parameter Changes	Change the degree of flexibility. Change an object's physical state (e.g. to a gas, liquid, or solid).
1	Segmentation	Divide an object into independent parts. Increase the degree of fragmentation or segmentation.
10	Preliminary Action	Perform, before it is needed, the required change of an object (either fully or partially). Pre-arrange objects such that they can come into action without losing time for their delivery.
28	Mechanics Substitution	Replace a mechanical means with a sensory means. Use electric, magnetic and electromagnetic fields to interact with the object.
15	Dynamics	Allow the characteristics of an object or process to change to be optimal or to find an optimal operating condition. Divide an object into parts capable of movement relative to each other.
13	The other way around	Invert the action(s) used to solve the problem (e.g. instead of cooling an object, heat it). Make movable parts (or the external environment) fixed, and fixed parts movable.
29	Pneumatics and Hydraulics	Use gas and liquid parts of an object instead of solid parts.

Two principles (inventive principle #28 and #29) suggest that different power sources other than mechanical sources be used as a solution. This is applied in the hydrostatic and electric CVTs function to eliminate the non-integer tooth problem; however, these two principles would not yield promising solutions because of the goal to achieve a step-less transmission ratio with only one mechanical input source from the engine.

The two classes of PECVT embodiments will generally differ in the design contradictions that will exist when generating concepts. For this purpose, condensed contradiction matrices for each class were created to more accurately apply the correct inventive principle to the appropriate class. These condensed contradiction matrices are listed in Table 4.4 (Problem Correction Class Contradictions) and (Problem Elimination Class Contradictions) as they apply to each class.

Table 4.4: TRIZ Contradiction Matrices for Problem Correction Class

	Worsening Feature → / Improving Feature ↓	Adaptability or versatility	Device complexity
		35	36
12	Shape	1, 15, 29	16, 29, 1, 28

	Worsening Feature → / Improving Feature ↓	Speed
		9
35	Adaptability or versatility	35, 10, 14
36	Device complexity	34, 10, 28

Table 4.5: TRIZ Contradiction Matrices for Problem Elimination Class

	Worsening Feature → / Improving Feature ↓	Device complexity
		36
35	Adaptability or versatility	15, 29, 37, 28

	Worsening Feature → / Improving Feature ↓	Object-generated harmful factors
		31
36	Device complexity	19, 1

The matrices on the left represent the primary contradictions while the matrices on the right represent the secondary contradiction. These matrices represent two iterations of TRIZ methodology, that is, the worsening features caused in the left matrices (primary contradictions) become the improving features in the right matrices (secondary contradictions). This occurs to take into account certain cases when the primary contradiction cannot be eliminated. If this happens, then the primary worsening feature will indeed be negatively affected, and a secondary contradiction matrix will need to be constructed showing the primary worsening feature as the secondary improving feature. The secondary matrix will then attempt to solve the next contradiction. Therefore, not only are inventive principles suggested for the primary contradictions, but also for secondary contradictions that might exist if the primary contradictions are not eliminated.

In the problem correction class, the matching pitch principle needs to be satisfied without affecting the embodiments output. For this reason, the contradiction matrix in Table 4.4 shows a contradiction between shape versus adaptability and complexity. The secondary matrix shows that if adaptability and complexity need to be improved, then the contradiction that speed will be negatively affected also needs to be eliminated. By examining the inventive principles suggested in the two contradiction matrices for the problem correction class, it appears that segmentation and preliminary action approaches (principles #1 and #10) will be useful in generating concepts that overcome the non-integer tooth problem contradictions. Segmentation is especially effective when relative movement is needed between two engaged members as is the case with embodiments belonging to the one-way clutch family. The devices used to eliminate the non-integer tooth problem in the problem elimination class need to be flexible and adaptable, especially in the teeth conforming family. The closest contradiction matrix that can be constructed to satisfy this principle is to improve the embodiment's adaptability and flexibility without increasing the complexity of the system. The secondary matrix consists of improving the complexity of the embodiment without generating harmful factors such as lack of robustness. These contradiction matrices of the problem elimination class are shown in Table 4.5 and suggest that using Dynamics and Segmentation approaches (principles #15 and #1) in generating concepts for this class may overcome the non-integer tooth problem contradictions.

In conclusion, the three most suggested principles for eliminating any contradictions while satisfying the governing principles of the most promising PECVT are segmentation, preliminary action, and dynamics. Segmentation suggests dividing parts into smaller independent parts and increasing the degree of fragmentation. Preliminary action suggests to perform a required corrective action before it is needed or to prearrange items before they come into contact or action with another item. Dynamics suggests allowing the characteristics of an object or process to become optimal or to remain in an optimal position. These principles are very apparent in many of the concepts already generated and found in the patents analyzed in Chapter 2. These principles should also be implemented when generating concepts for possible PECVT embodiments as suggested by TRIZ methodology.

4.1.2 Concept Generation Results

This section will discuss general concepts of several embodiments that have been generated as a result of the different methodologies described in Chapter 3 and the TRIZ contradiction matrices previously analyzed. A list of several concepts will be provided with illustrations, when applicable, and a brief physical description. There are two main characteristics that will be described for each embodiment to help classify and understand its functionality: the method of solving the non-integer tooth problem and the device, mechanism, or method used to ensure proper meshing. Because the PECVT classes were created based on the method of solving the non-integer tooth problem in a particular embodiment, this characteristic determines in which class the concept will be classified. In this section, the list of generated concepts will be presented according to their respective classes. Due to the non-integer tooth problem that arises in many PECVT embodiments, a description of the device, mechanism, or method used to ensure proper meshing of the gears in the embodiment will also be provided. These concepts are only conceptual embodiments of the principles that were established in Chapter 2.

4.1.2.1 Diameter-Changing Approaches

Before a list of concepts is given, a brief discussion is needed concerning the different diameter changing approaches. Diameter changing approaches do not deal with the non-integer tooth problem or correction mechanisms, but only dictate the method in which a PECVT can change the diameter of either the input or output members to achieve a ratio change. The list of different generated concepts will function regardless of which diameter changing approach is chosen; therefore, the diameter changing approaches function almost independently of the different concepts. A brief description of three of the main approaches used in previously published PECVTs will be provided since certain advantages can be gained by using the most applicable diameter changing approach with the final concept. Again, any of the three diameter changing approaches could be used interchangeably with the list of different concepts. The three approaches are the equal segmentation approach, the unequal segmentation approach, and the cone approach.

The equal segmentation approach uses the TRIZ segmentation principle to achieve a ratio change as shown in Figure 4.1 by replacing a drive gear (a) with several separate driving gears (b) equally spaced radially about a common center point (c), to effectively function as the original drive or driven gear (a). These smaller gears (b) can move in and out radially to change the diameter of the effective gear (a). As the gears (b) orbit about a common center point (d), the result of their kinematic motion is the same as if the original drive or driven gear (a) were rotating about its own axis (c).

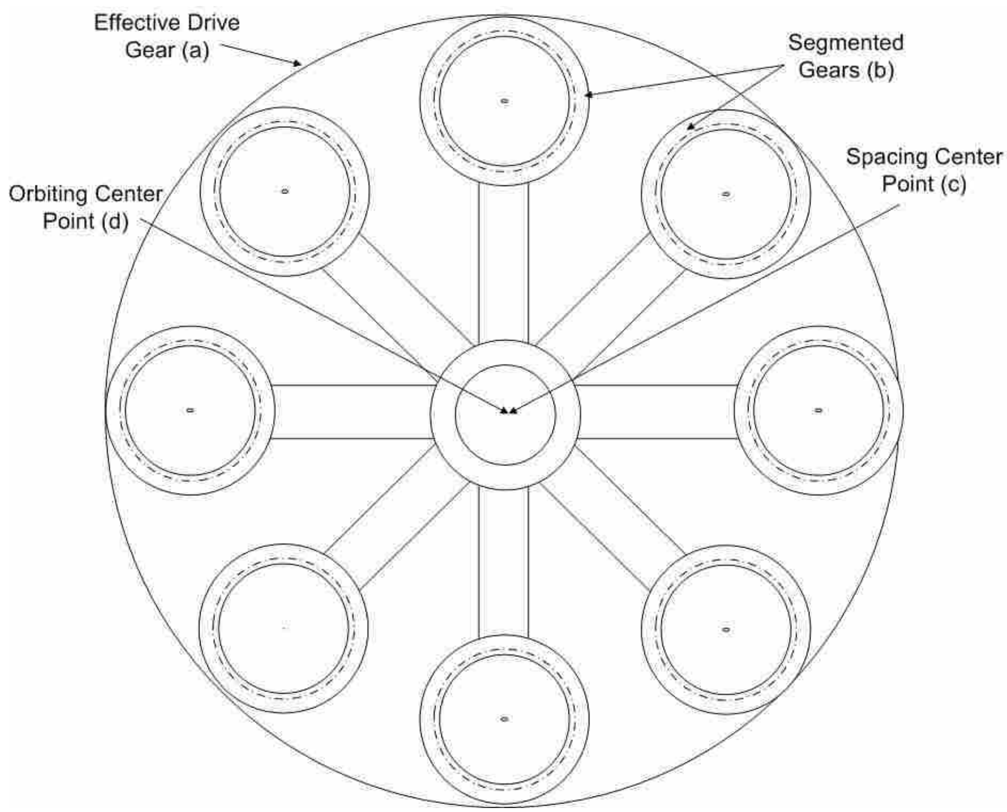

Figure 4.1: Equal Segmentation Approach

The unequal segmentation approach is similar to the equal segmentation approach; however, this approach does not restrict the smaller gears (b) to orbit about their common center point (c). As shown in Figure 4.2, even though the smaller gears (b) remain equally spaced radially from their common center point (c), the gears (b) orbit about some point (d) other than the common center point (c).

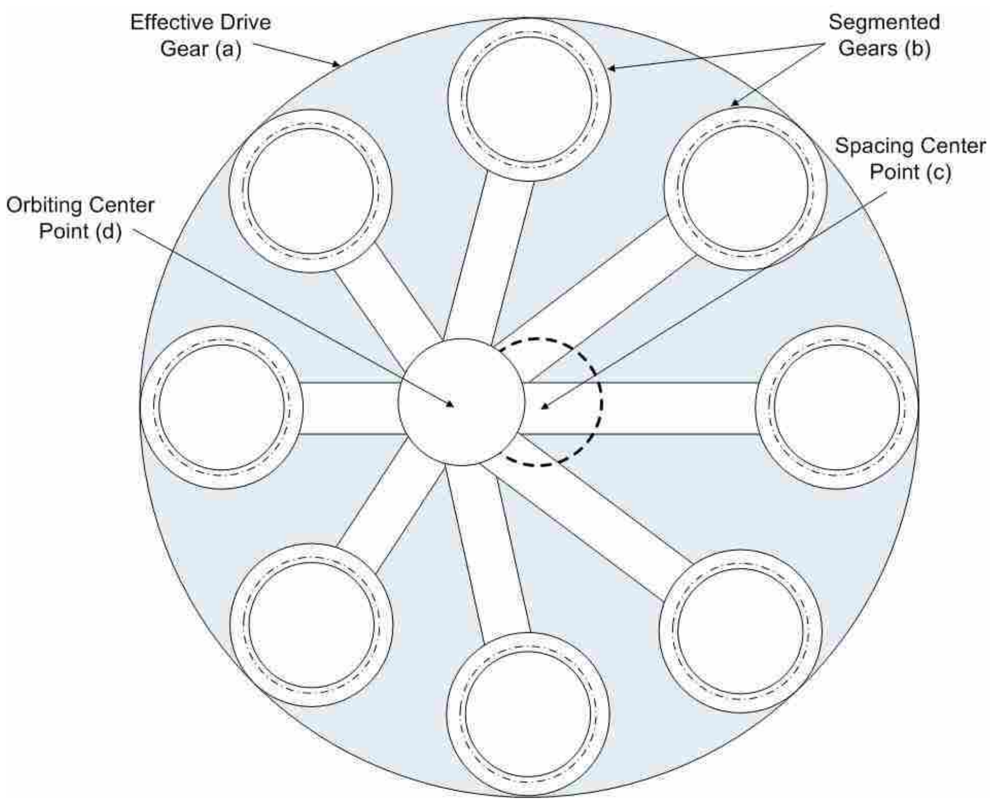

Figure 4.2: Unequal Segmentation Approach

This causes the resulting pitch line velocity of the smaller gears (b) to vary as they orbit about a point not located in the center of the effective drive gear (a). By placing an output gear at different locations along the variable speed circumference, different output speeds can be achieved.

The third approach involves the meshing of two cones placed side by side acting as the input and output members respectively. These cones are coupled together through engagement of another gear. The diameter changes in this approach by simply varying the axial location of the meshing of the cones. Even though this diameter changing approach acts as an entire embodiment by itself through two engaging inverted cones as seen in Figure 2.14 and Figure 2.17, this approach can also be used in other embodiments and concepts to change the diameter of the drive or driven gear.

It should be noted that these three ratio changing approaches can be implemented in both classes of PECVTs and do not change the results of the concept generation section. The main focus of this concept generation will be centered on how well each concept satisfies the established principles for a promising embodiment to exist according to how the concept overcomes the non-integer tooth problem. For simplicity, each of the concepts presented in this chapter will be described by using the diameter changing approach described and shown in Figure 4.1 (equal segmentation approach). By using only this one approach, only one functional description of how the embodiment changes its diameter is required to provide for all of the different concepts.

An embodiment using the equal segmentation approach is shown in Figure 4.3. This approach consists of a fixed central reference gear (A) whose axis is co-axial with the major axis of the transmission. An input arm, or drive gear carrier (B), is connected to the axis of the reference gear (A), allowing it to rotate around the axis of, and relative to, the reference gear (A). The input arm (B) is the input to the transmission from an external power source. Connected to the input arm is a drive gear (C), which the input arm (B) causes to orbit about the reference gear (A). The drive gear (C) is connected to the reference gear (A) through a gear pair relationship, which means that the rotation of the drive gear (C) about its axis has a fixed relationship to the rotation of the reference gear about its axis (this would be accomplished through a gear set between the reference gear (A) and the drive gear (C), not shown in Figure 4.3). Also connected to the axis of the reference gear (A) is a stationary arm (D), which remains fixed (does not rotate) and supports a driven gear (E), which also acts as the output of the transmission. When the input arm (B) is rotating about the axis of the reference gear (A), the drive gear (C) orbits around the reference gear (A) at an angular velocity equal to that of the input to the transmission. This orbiting motion also causes the drive gear (C) to rotate about its own axis in the direction of the orbit motion. The angular velocity, at which the drive gear (C) rotates, relative to its orbit, is also dependent on the gear ratio of the gear set connecting the reference gear (A) and the drive gear (C).

The drive gear (C) shown in Figure 4.3 connects the input portion of the transmission to the driven portion of the transmission. This is accomplished as the drive gear (C) orbits past and meshes with the driven gear (E). The contact and meshing of the drive (C) and driven (E) gears are what cause rotation of the driven (output) gears (E). Because the drive (C) and driven gears (E) are allowed to move in and out radially, their effective meshing diameter, also known as the Virtual Circle, can increase and decrease in infinite increments.

As the orbit radius (F) increases, the virtual circle increases, and the effective pitch line velocity of the drive gear (C) increases. Because the resulting pitch line velocities of both the drive gears (C) and driven gears (E) must be equal for proper meshing to occur, the pitch line velocity of driven gears (E) increases causing an increase in rotation of the driven gears (E) in a continuous manner resulting in an infinitely variable output.

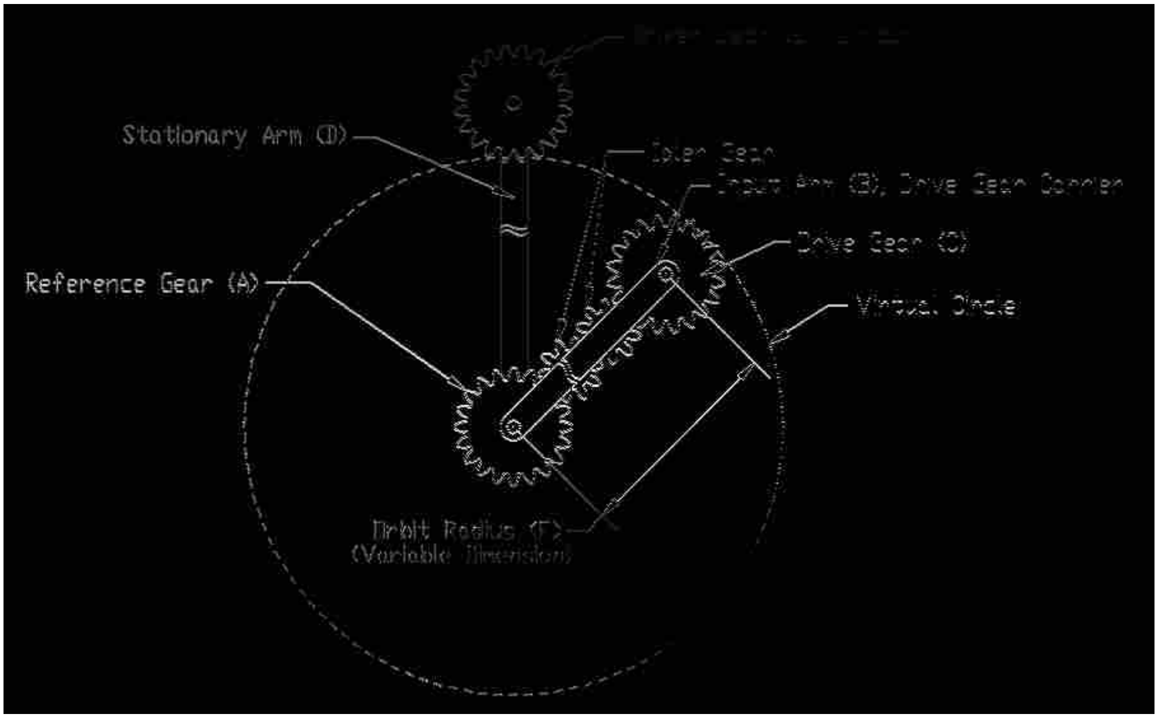

Figure 4.3: Example of Diameter Changing Approach used in the PECVT Concept Generation [3]

Figure 4.4 consists of the same embodiment as that shown in Figure 4.3 but with five driven gears (E) and four drive gears (C). This arrangement is called a Vernier Relationship, which consists of having a different number of drive gears (C) than driven gears (E) so as to maintain a constant engagement between the input and output of the transmission. Through this relationship, at least one drive gear (C) will be engaged with a given driven (E) gear at any given time.

Using this approach described in detail, all of the generated concepts describing the device, mechanism, or method used to ensure proper meshing can be presented in the subsequent sections without describing how different approaches would be implemented. The main characteristics and features of each concept will not change greatly according to the diameter approach used in a particular embodiment. Certainly, there exist other

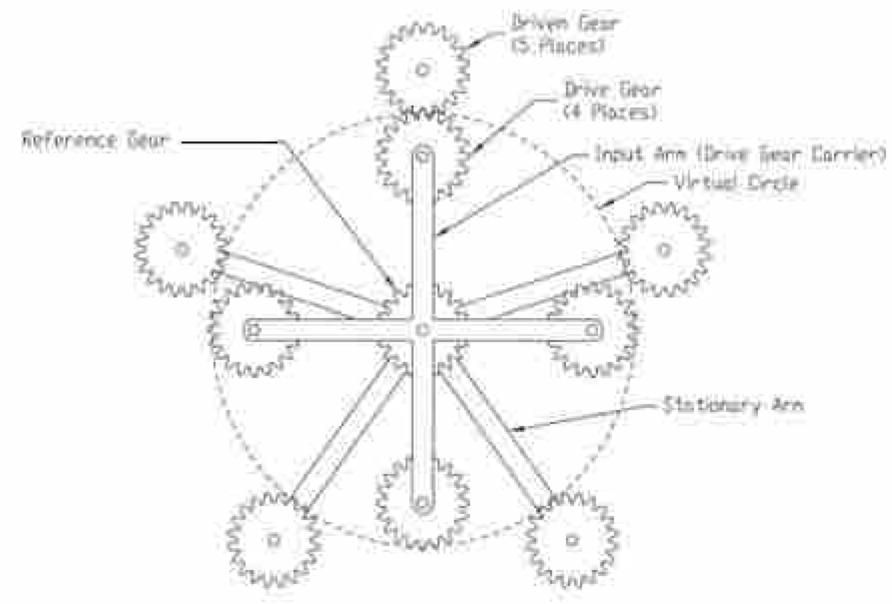

Figure 4.4: Reference Gear Feedback Concept [3]

approaches, apart from the three approaches that have been described, which could also be considered when selecting the final concept and determining the best approach. Figure 4.3 shows the equal segmentation diameter changing approach that was described in detail, and the list of generated concepts that follows will be presented using this approach and will reference this figure.

4.1.2.2 Problem Correction Class Embodiments

Embodiment using Electric Actuator to provide Correction

One possible method of assuring proper meshing and overcoming the non-integer tooth problem is to provide a correction to each of the drive gears before they engage with a driven gear. This concept also uses the TRIZ preliminary action principle #10. The concept consists of electric actuators (a) attached to each drive gear (b) as shown in

Figure 4.5. Because the actuator (a) is only used as a corrective device and not as an input device, it is classified in the problem correction class.

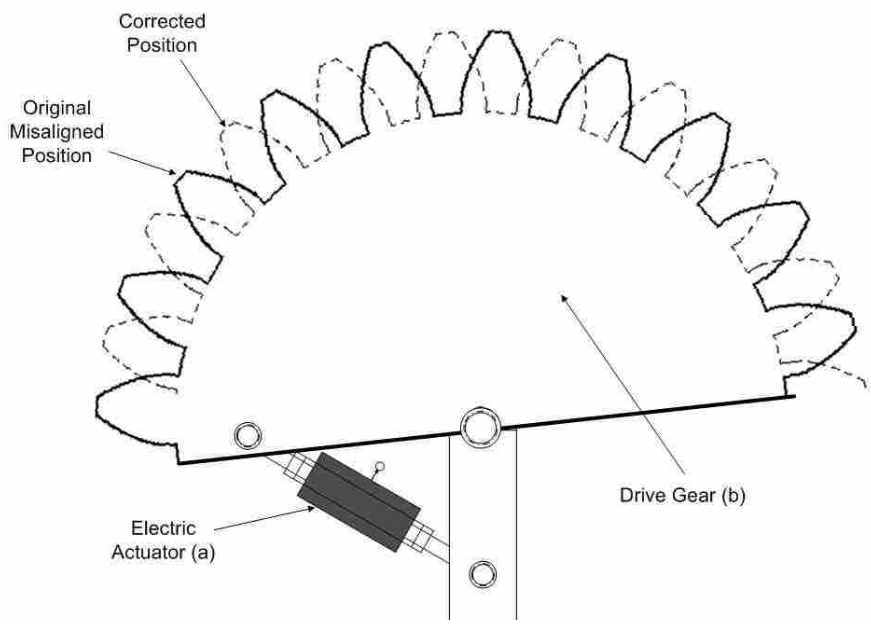

Figure 4.5: Drive Gear Correction using Electric Actuator

Due to the nature of the non-integer tooth problem within the ratio RPM range provided by the transmission, the magnitude of the correction would need to change each time the drive gear (b) is to be engaged with a new driven gear. This would require the actuator (a) to provide different amounts of correction for every engagement of the drive gear. One advantage of the electric actuator concept is its ability to apply quick corrections to the driving gears through an electric current, thus satisfying the matching pitch principle. The major disadvantage includes the difficulty in changing the correction magnitude for every revolution of the driving gear. Also, there is great difficulty in providing power to moving electric actuators.

Embodiment using Helical Gears to Provide Correction

Like the electric actuator concept, this concept also provides a rotational correction to each of the drive gears while not engaged; however, the mechanisms used to achieve this correction are helical gears. If two helical gears are meshed with one another and one of these is displaced axially with respect to the other, a relative rotation of one gear with respect to the other occurs as shown in Figure 4.6. The magnitude of this rotation is dependent upon the diametral pitch of the gears and also the amount of axial displacement.

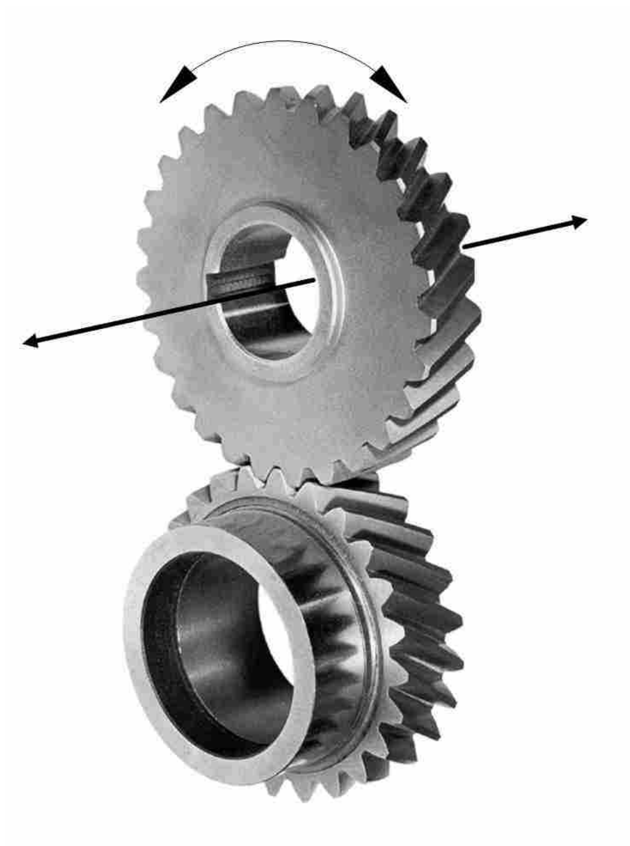

Figure 4.6: Concept using Helical Gears to Provide Correction to the Driving Gear (Retrieved from www.arrowgear.com/images/helical_gears.JPG)

There are numerous methods of implementing this concept in different embodiments. One obvious method is to include a helical gear pair at each driving gear location to provide a correction to the driving gears before they are to engage with the driven gears. Some type of actuation could be provided to cause the axial displacement the helical gear, which magnitude would also change each time the driving gear reengages. The helical gear could also be corrected by the driven gear itself when the driven gear begins to engage with the driving gear, which is connected to the correctable helical gears. If this latter approach is taken, a mechanical stop could be provided for the axial displacement of the helical gear, once properly engaged, so that the driving spur gear, to which the helical unit is attached, would begin to transmit torque after the matching pitch principle was met. This autonomous correction is a major advantage to this concept since no power would be required to correct the mechanism. Another advantage is that through simple linear translation, a more accurate correction can be applied to the driving gear than through pure rotation. However, the major disadvantage for this concept is also the difficulty in changing the magnitude of the correction needed for proper meshing of the driving gear.

Embodiment using Corrective Cam-Follower Systems

One possible problem-correcting embodiment utilizes the use of cam-follower systems to correct the orientation of the driving teeth either by rotation or translation. The manner in which the cam is implemented into an embodiment can vary substantially according to the design of the embodiment. Since this is the case, only one method will be suggested in this section although there are many other possible methods. Because the driving gears are directly connected to the reference gear as shown in Figure 4.4 through an unseen drive train, any rotation provided to the stationary reference gear would also provide a rotation, or correction, to the driving gears. Also, because the amount of correction needed for proper meshing to take place is different for each driving gear, the reference gear could be duplicated, so that each drive gear is connected to a separate reference gear through a different drive train. By segmenting the reference gear, a relative movement between driving gears could be provided to ensure proper meshing. Separate cam-follower systems could provide the needed correction to the individual reference gears using the method suggested in Figure 4.7, which is just one possible use of cams to provide the needed correction.

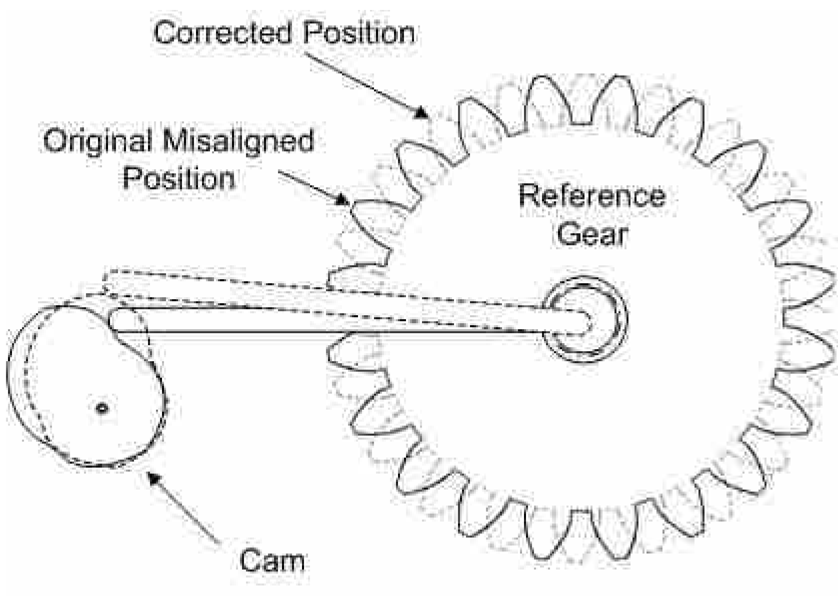

Figure 4.7: Embodiment using Cams to Provide Correction to Driving Gears

Other challenges arise with this embodiment due to the need to use different cam profiles as the orbit radius changes or as the magnitude of the correction changes from one rotation to the next. If this concept is chosen as a possible solution after the scoring process, these and other challenges will need to be addressed. The major advantage of using corrective cams is the ability to provide a fully mechanical correction through the mechanical input in satisfying the matching pitch principle. A disadvantage of using the corrective cams is the need for a continuously changing cam profile to be able to provide the amount of correction needed for each rotation, violating the constant output principle.

4.1.2.3 Problem Elimination Class Embodiments

Constant Tooth and Pitch Embodiment

Because the non-integer tooth problem is a result of increasing either the diametral pitch or the number of teeth of a drive or driven gear in a gear pair, a gear pair that could change ratio without changing these two parameters would eliminate the non-integer tooth problem. Figure 4.8 shows a possible embodiment of such a gear mechanism.

By radially changing the meshing location of the output gear with respect to the input gear, different gear ratios are achieved due to the varying pitch line velocity of the larger input gear at different radial locations of the output gear. In order to achieve this function, the teeth of the drive gear (input gear) would have to change orientation relative to one another when rotating past the driven gear in order to match the circular pitch of the input gear to the constant circular pitch of the driven gear at all radial mesh locations. The major advantage to this embodiment is the lack of correction needed to ensure proper engagement; however, difficulty arises as the individual teeth need to continuously change their pitch through the engagement region. The complexity of changing the circular pitch of the input gear teeth makes the concept less feasible than other concepts. The moving output gear also provides challenges to the feasibility of this embodiment.

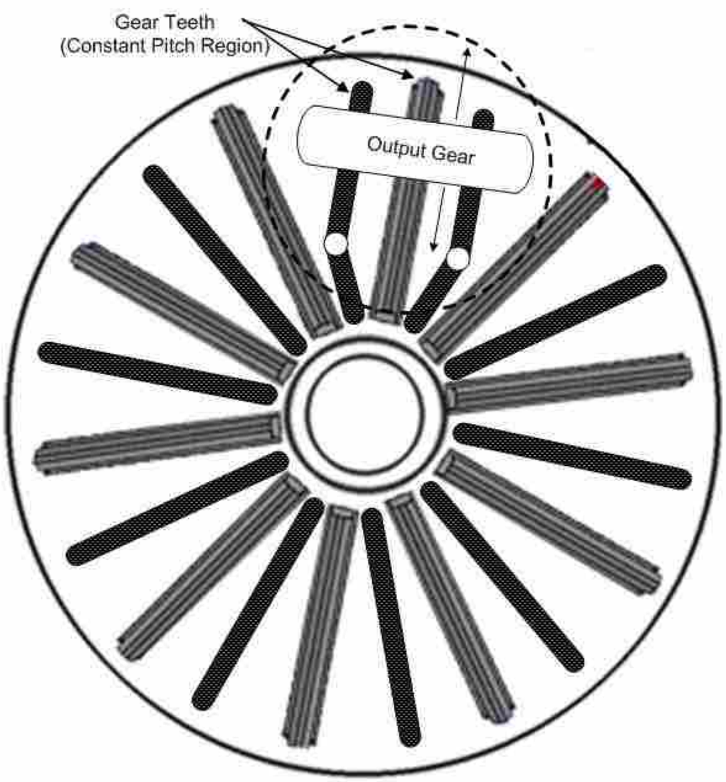

Figure 4.8: Possible Constant Tooth and Pitch Embodiment

Shear-Thickening Fluid Embodiment

The use of shear-thickening fluid as a possible method of eliminating the non-integer tooth problem was also suggested as a feasible concept. The shear-thickening fluid could also be replaced by a magneto-rheological fluid with viscosity properties that change based on the application of an electric current. A possible embodiment that implements shear-thickening fluid is shown in Figure 4.9.

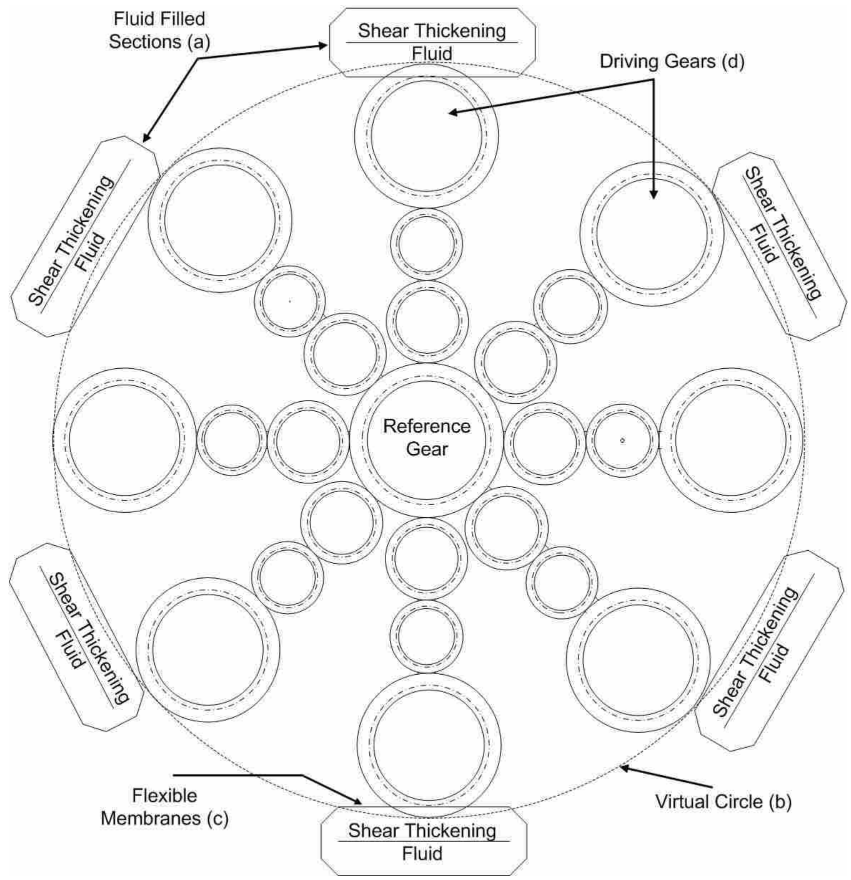

Figure 4.9: Possible Shear Thickening Fluid Embodiment

The driven gears, in this case, would be replaced by fluid filled sections (a) similarly located around the virtual circle (b) that are able to expand or retract radially. A flexible membrane (c) would be used to enclose the fluid on the side of the sections (a) where the drive gears (d) come into mesh with that section (a). Also, the reference gear of the embodiment would be able to rotate relative to its own axis. The membrane (c) of the fluid filled section (a) would conform to the teeth of the drive gear (d) as the drive gear (d) orbits about the reference gear and meshes with the sections (a). At this point an electric current would be applied to instantaneously solidify the fluid with which the driving gear (d) is engaged. Because the effective driven teeth created in the fluid by the application of the electric current do not rotate or translate, another method of obtaining the transmissions output needs to be developed. As the viscosity of the fluid rapidly increases, the teeth of the drive gear (d) will transmit their rotational motion and torque to the reference gear, which is now allowed to rotate in this embodiment. The reference gear becomes the new output gear in this case.

The major advantage of this concept is that it eliminates the need to correct the orientation of any gear because the shear thickening fluid allows effective driven teeth to conform where and when they are needed to mesh properly with the drive gears (d). One disadvantage to this embodiment is the complexity and cost of utilizing a magnet-rheological fluid. Also, because the sections with flexible membranes need to withstand high torques, lack of robustness becomes an issue as described in the problem elimination principle.

Embodiment using Small Spikes with Resilient Material

This concept looks and functions similarly to the shear thickening concept in Figure 4.9. Instead of fluid filled sections like those used in the Shear Thickening concept, the sections are made of a type of resilient material. The driving members in this embodiment contain several small penetrating mechanisms in place of gear teeth that penetrate the material as the drive members orbit past the segmented sections shown in Figure 4.10. Although not shown, the penetrating mechanisms are located entirely around the entire circumference of the driving gear to ensure constant engagement.

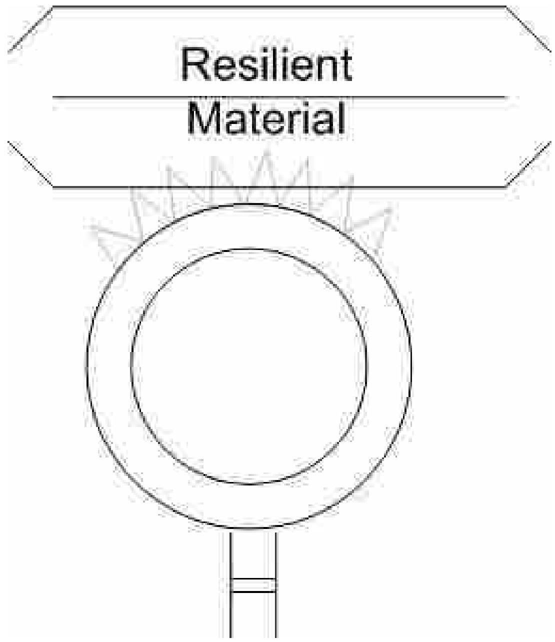

Figure 4.10: Method of Torque Transfer with Spikes and Resilient Material

This meshing of the driving members with the material will cause a no-slip effect at the point of tangency and will transmit all motion and torque from the driving member to the central reference gear, which will again function as the output of the transmission in this concept. Due to the properties of the resilient elastomer like material, the driving members will never have a misalignment problem at any point of engagement, thus eliminating the non-integer tooth problem. Another advantage is that the resilient material will rebound to a near original state once the penetrating drive members are retracted from the section after engagement. Other concerns are with robustness and torque capacity. Robustness becomes an issue because the resilient material will deteriorate over time. Also, the penetrating mechanisms have to be thin enough to easily penetrate the material, but also wide enough to withstand higher torques, which would be an issue with the robustness of the design as mentioned in the problem elimination principle.

Sprocket/Chain Embodiment with Tension Rollers

One possible embodiment that would eliminate the non-integer tooth problem involves the use of sprockets and a chain to ensure proper meshing. The concept consists of a drive (a) and driven (b) portion of the embodiment which are connected by use of a chain (c). Each portion is composed of a segmented gear similar to that shown in Figure 4.1 using sprockets (d and e) in place of the gears. If the driven sprockets (d) are spaced at a particular orbit radius where the non-integer tooth problem does not exist, then the chain (c) meshes properly with the sprockets (d). The sprockets (e), representing the driving portion of the embodiment, are allowed to change the effective diameter of the driving portion (a) and the effective transmission ratio. In addition, adjustable tension rollers (f) are also added to the driving portion (a) of the embodiment between each driving sprocket (e) that is meshed with the chain (c) as shown in Figure 4.11.

As the circumference of the virtual circle (g) changes, the distance between adjacent driving sprockets (e) is not always divisible by the pitch of the chain; as a result, there will be a meshing problem every time a driving sprocket (e) is engaged with the chain (c). This is one way the non-integer tooth problem presents itself in this embodiment. The main function of the tension rollers (f) is to reconcile the non-integer tooth problem by always ensuring that the distance from one sprocket (e) to the roller (f) to an adjacent sprocket (e) is always divisible by the pitch of the chain. With these adjustable rollers (f), the non-integer tooth problem will be eliminated regardless of the orbit radius of the drive gears (e). Other advantages of the tension roller concept are the

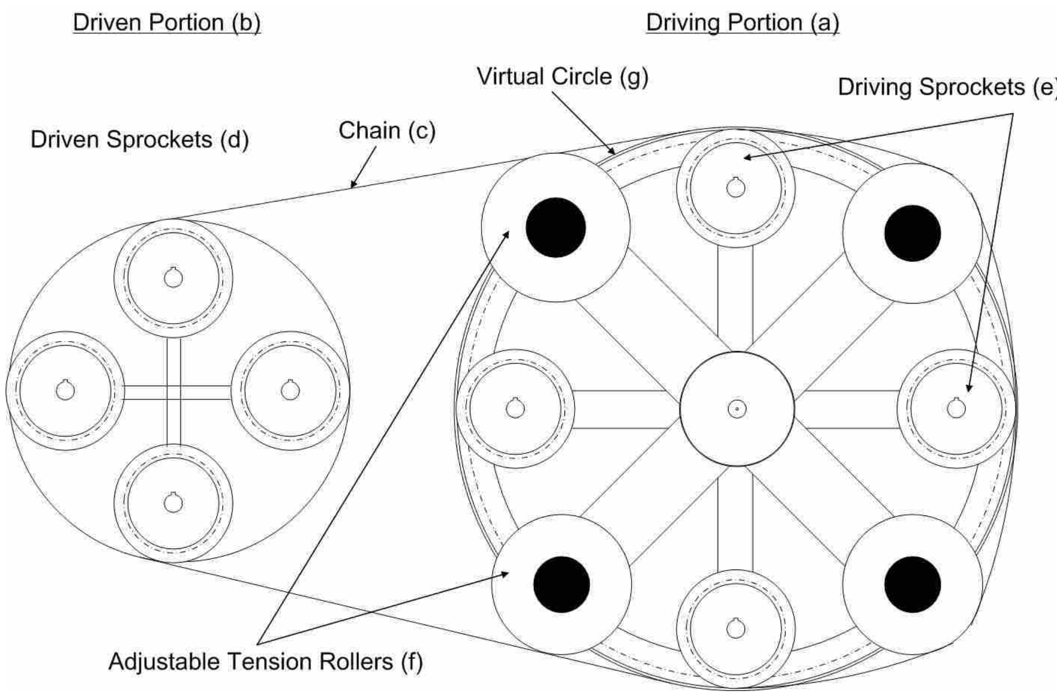

Figure 4.11: Sprocket and Chain Embodiment Using Tension Rollers

robustness of the design and high torque capacity. Disadvantages include size and a need for high precision shifting mechanisms.

Feedback from Output using Differentials on Driven Gear

Another possible concept consists of using differential gears between the engaged and non-engaged driven gears to provide relative rotational movement between them, thus eliminating the non-integer tooth problem. Both the TRIZ segmentation principle #1 and the preliminary action principle #10 are utilized in this design. To implement the differential gears in a somewhat feasible design, the diameter changing approach shown in Figure 4.4 can be duplicated and placed side by side out of plane on the same central axis. The input arms of the two different systems in each plane would utilize a differential system between them to cause the driving gears on one plane to orbit in the opposite direction of those in the other plane. In this way, engagement between input and output gears alternate from one gear pair in one plane to the other. Differential gears systems would also be placed between each of the driven gears from one plane and the driven gears of the second plane so that as a rotation to the driven gear is made in one plane, the equivalent driven gear in the other plane makes an equal rotation in the opposite rotational direction.

Even though it can be said that a correction takes place between the driven gears, the main feature of this concept is that feedback from the driven gears (output) of one plane causes a correction (preliminary action) to take place to the unengaged driven gears in the other plane before engagement occurs. If this concept is further developed, other design issues would also have to be overcome and a deeper analysis conducted. The major advantage to this embodiment is that no input sources would be needed to assist the shifting process. However, because of the many differential systems that would be needed, the analysis of this transmission becomes very complex. This causes disadvantages such as size and numerous moving parts which usually introduces efficiency losses.

Embodiment that Operates at Preferred Locations

One of the final three TRIZ principles suggested to eliminate design contradiction in PECVTs is Dynamics. Dynamics refers to finding an optimal location where the mechanism functions properly and allow the mechanism to run at that location (Table 4.3). By combining this principle with other previously mentioned concepts, new combined concepts may be developed that are more feasible that individual concepts. One notable improvement occurs with the concepts from the problem correction class. If the transmission is forced to operate the majority of time at locations where the non-integer tooth problem does not exist (a), then the corrections needed to ensure proper meshing would only have to be applied while the transmission is transitioning from one proper meshing location to another. This greatly eliminates the number of corrections needed by a corrective device to correct the orientation of different members of the embodiments, which only occurs while transversing through a range of RPM and torque ratios (b). The transmission can continuously vary the RPM and torque ratios throughout the entire range of the transmission; however, the transmission generally operates at a specified set of operating ratios, and as a result, no correction is needed while operating at these preferred locations. The embodiment behind this concept is shown in Figure 4.12.

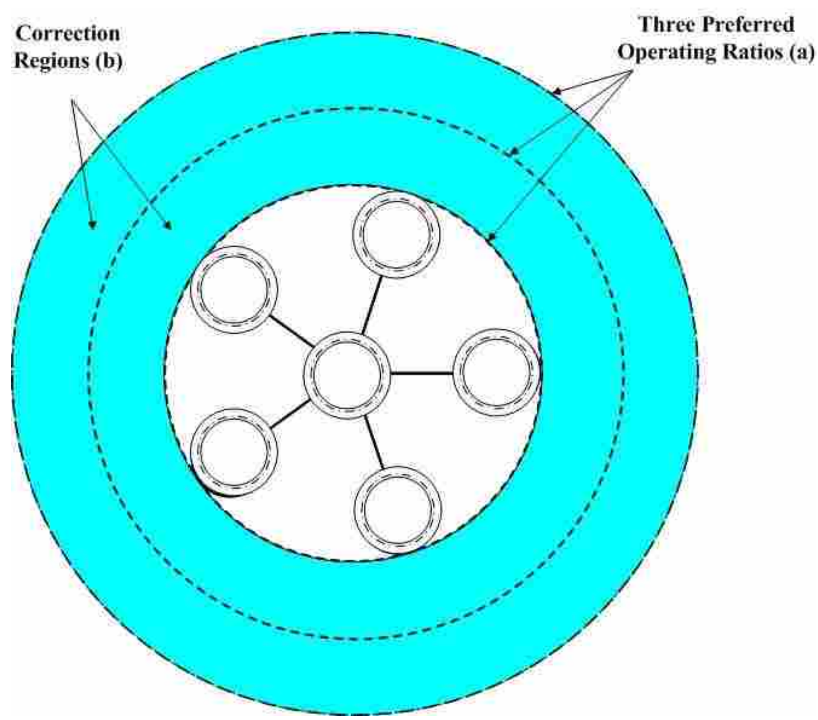

Figure 4.12: Preferred Ratio Embodiment

Although this embodiment would not be able to operate over an extended period of time at all transmissions ratios, it would still be able to continuously vary the transmission ratio and maintain constant, positive engagement at all times. The advantage of this embodiment is that only during transition from one preferred ratio to the next would a correction need to be made. Because of this characteristic, correction devices that were before infeasible to implement now become feasible as only a finite number of corrections are needed between operating ratios. The major disadvantage of this embodiment is that the engine's RPM output will not be able to continually operate at its most efficient and optimal range, which was one of the major reasons for the development of a PECVT. Although this embodiment possesses characteristics not typical of a traditional PECVT, it does not violate any of the requirements which define a PECVT which are found within the customer needs.

4.1.3 Concept Generation Summary

A number of possible concepts have been presented with their noted advantages and disadvantages according to their PECVT class. Using these concepts and other concepts obtained by the external patent search in Chapter 2, the concept selection process can be conducted. Table 4.6 has been created to summarize the advantages and disadvantages of all the concepts that will be used in the concept selection process as well as general concepts that have been used in the previously published patents. The concepts that will be analyzed were chosen based on design space representation, basic feasibility, creativity, knowledge obtained from the patent search, and the engineering judgment of the author and other engineers.

Table 4.6: Summary of Advantages and Disadvantages of Existing and Generated Concepts

Concept	Advantages	Disadvantages
Electric Actuator Correction	• Quick correction capability	• Difficult to change correction magnitude • Multiple motors needed • Numerous moving parts • Powering moving actuators
Helical Gear Correction	• Only a linear translation motion needed for correction • Good correction accuracy • Autonomous correction option	• Possible need of electric actuator or stop • Difficult to change correction magnitude
Cammed Correction	• Smooth correction transmission • Can use input for rotation • Already proven concept	• Cam surface describes only one correction path (more than one is needed for PECVT)
Constant Tooth and Pitch Embodiment	• If feasible, no need for correction	• May be infeasible • Teeth would move like a chain to obtain constant pitch • Moving output gear
Shear Thickening/ Magneto-Rheological Fluid	• No correction needed	• Expensive • Low torque capability • Not Robust
Resilient Material and Spikes	• Eliminates need for correction	• Limited torque capacity • Perishable after many cycles • Not robust
Tension Rollers w/ Sprocket and Chain	• Eliminates need for correction • Robust • High torque capability	• Slightly oscillating output during shifts • Larger Embodiment
Feedback using Differentials between Driven Gears	• Uses the non-integer tooth problem to make correction • Continuous correction • No need of outside power source	• Greatly increases part count • Size
Preferred Meshing Location Embodiment	• Correction is no longer continuous in nature • Finite number of corrections to be made between entire range of the transmission • Allows feasibility of otherwise infeasible corrective devices	• RPMs will not always be at optimum values • Slightly increased complexity of design

Table 4.6 Continued

Power Sprocket and Multi-Chain Approach	• Allows for variable correction magnitudes • One of the few non-oscillating output embodiments in the problem correction class	• Power Sprocket in not robust • Numerous parts
Variable Source Embodiments in General	• No meshing problems • Low part count • Robust	• Low efficiency
Frictional Control Embodiments in General	• No meshing problems	• Low efficiency • Low torque capability • Not much better than standard friction drive CVT
One-way Clutches Between Driven Gears	• Allow for variable correction magnitudes • Robust and proven mechanism	• Oscillating output

5 Concept Selection Results and Validation

This chapter will present the results of the concept selection process. After the results of the concept selection process are given, one concept will be chosen as the most viable solution to the non-integer tooth problem. This concept is only a conceptual embodiment that meets the relevant governing principles that were previously established and should not be considered as an optimal solution. A more refined description of the final concept will be given along with a final embodiment that will include the final concept and diameter changing approach. Evidence as to how the final concept satisfies functional principles and product specifications will also be provided.

5.1 Concept Selection Results

After considering the advantages and disadvantages of each of the concepts generated in chapter 4, the concept screening process was performed as described in Chapter 3. The results are shown in Table 5.1. To be presented here as a PECVT concept, the primary customer needs of all presented concepts were shown to be satisfied in the concept description. This list of concepts was narrowed down to the four most viable concepts according to the selection criteria, which were chosen based on the functional specifications developed from the secondary and tertiary customer needs of PECVTs which were provided in Chapter 2. The remaining concepts are D, G, I, and K, which represent the cam-follower correction, the tension rollers with sprocket and chain, the feedback using differentials, and the preferred meshing location concept, respectively.

These concepts were then analyzed with a weighted scoring process based on the same selection criteria. The weights corresponding to the selection criteria were determined based on the main criteria lacking in previously published embodiments examined in this research. Since the oscillating output and the seemingly infeasible designs were the most occurring limitations in the previously published embodiments, these criteria were given larger weights. In other words, the criteria that are more difficult to satisfy are assigned higher weights to reward designs that are capable of satisfying the difficult criteria. It is difficult to determine the feasibility of some embodiments as this criterion is somewhat subjective. The main metric used in determining the feasibility of an embodiment are number of corrections required for proper meshing to occur. Since the problem elimination concepts do not require any corrections and because the value of this metric changes, based on the specific application of the concepts in the final embodiment, a specific metric was not assigned to each concept. As a result, the score given to each concept is based somewhat on the author’s heuristics and upon the feasibility of similar previously published embodiments.

The results of this process are shown in Table 5.2. According to the selection criteria and the weighted scores, the preferred meshing location embodiment was chosen as the most viable concept. The cam-follower correction concept and the feedback using differentials concepts ranked second and third, respectively. A combination of some of these final concepts might also prove more promising than any one concept alone by pooling together all of the advantages of the individual concepts.

Table 5.1: Concept Screening Process Results

Selection Criteria	Concepts						
	A (Reference) One way Clutches Between Driven Gears	B Helical Gear Correction	C Electric Actuator Correction	D Cammed Correction	E Constant Tooth and Pitch Embodiment	F Shear Thickening/ Magneto-Rheological Fluid	G Tension Rollers w/ Sprocket and Chain
Does not produce an oscillating output	0	+	+	+	0	+	-
Can transmit high torque	0	0	0	+	-	-	+
Highly efficient	0	-	-	0	-	-	0
Not complex	0	0	-	+	-	-	0
Made of standard parts	0	0	0	0	-	-	+
Retrofit-able in current applications	0	0	0	0	0	0	-
Feasible	0	0	-	-	-	0	+
Robust	0	0	-	+	-	-	0
Net Score	0	0	-3	3	-6	-4	1
Rank	5	5	9	2	11	10	3
Continue	NO	NO	NO	YES	NO	NO	YES

Selection Criteria	Concepts					
	F Shear Thickening/ Magneto-Rheological Fluid	G Tension Rollers w/ Sprocket and Chain	H Feedback using Reference Gear	I Feedback using Differentials between Driven Gears	J Resilient Material and Spikes	K Preferred Meshing Location Embodiment
Does not produce an oscillating output	+	-	+	+	0	+
Can transmit high torque	-	+	0	0	-	0
Highly efficient	-	0	0	0	0	+
Not complex	-	0	0	-	+	-
Made of standard parts	-	+	0	0	-	0
Retrofit-able in current applications	0	-	0	0	0	+
Feasible	0	+	-	+	0	+
Robust	-	0	0	0	-	+
Net Score	-4	1	0	1	-2	4
Rank	10	3	5	3	8	1
Continue	NO	YES	NO	YES	NO	YES

Table 5.2: Concept Scoring Process Results

(Reference Concept Criteria in Bold)		A Cammed Correction		B Tension Rollers w/ Sprocket and Chain		C Feedback using Differentials between Driven Gears		D Preferred Meshing Location Embodiment	
Selecection Criteria	**Weight**	Rating	Weighted Score	Rating	Weighted Score	Rating	Weighted Score	Rating	Weighted Score
Does not produce an oscillating output	**25%**	3	0.75	1	0.25	**3**	0.75	3	0.75
Can transmit high torque	**15%**	3	0.45	3	0.45	**3**	0.45	3	0.45
Not complex	**10%**	3	0.3	2	0.2	**3**	0.3	2	0.2
Made of standard parts	**5%**	2	0.1	3	0.15	**3**	0.15	3	0.15
Retrofit-able in current applications	**5%**	3	0.15	2	0.1	**3**	0.15	4	0.2
Feasible	**25%**	4	1	**3**	0.75	2	0.5	5	1.25
Robust	**15%**	3	0.45	3	0.45	**3**	0.45	4	0.6
Total Weighted Score		**3.2**		**2.35**		**2.75**		**3.6**	
Rank		**2**		**4**		**3**		**1**	

5.2 Final Embodiment Description

The preferred meshing location embodiment does not have a specified corrective device implemented into the concept to make corrections while transitioning between optimal locations. Therefore, the cam-follower correction concept, which ranked second in the concept scoring process, can be combined with the preferred meshing locations embodiment to produce a more complete solution. These two concepts together are believed to achieve the most promising final embodiment. A description of such an embodiment will be given below and represents the current embodiment being developed and analyzed by Vernier Moon Technologies and Brigham Young University.

5.2.1 Functional Description

Combining the preferred meshing location and the cam-follower correction concepts will result in a very unique transmission embodiment. The embodiment will function as a PECVT that operates only at preferred transmission ratios where the non-integer tooth problem does not exist (problem elimination class). In his research work, Andersen defines these locations where the non-integer tooth problem does not exist as Case 1 locations [3]. Therefore, the only correction needed for proper engagement to occur, in this embodiment, is while transitioning between adjacent Case 1 locations. The distance between these locations will be discussed later in this section. By constraining the time in which the transmission will make this transition, a discrete number of corrections between the drive or driven members and magnitudes of those corrections can be calculated using the kinematic equations derived in this chapter. The less time required to transition, the fewer corrections are needed to reorient the teeth entering engagement. By utilizing corrective cam-follower systems to reorient the engaging teeth during these transitions between preferred gear ratios (Case 1 locations), a finite number of corrections can be applied to the engaging members by a specific cam profile (problem correction class). Therefore, the cam profile is dependant upon the speed that the transmission is able to make the transition between adjacent operating ratios by increasing or decreasing the orbit radius of the embodiment. These and other design factors will be further discussed later in this chapter using a case study. Therefore, the following characteristics describe the most promising embodiment obtained from the concept selection process according to the satisfied general principles that were established in Chapter 2:

- A discrete number of operating ratios are available in the embodiment
- There is continuous engagement of the input and output
- The transmission ratio is continuously variable between operating ratios
- The transmission ratio is able to vary under load
- There is virtually no oscillating output during transmission operation
- The embodiment can transmit high torque values
- The embodiment is adequately efficient
- The embodiment uses both problem correction and problem elimination concepts to overcome the non-integer tooth problem.

The embodiment possesses characteristics of an embodiment belonging both to the problem correction class and to the problem elimination class. Because it possesses characteristics of both classes, the embodiment appears to be more functionally feasible as described earlier. This claim is somewhat subject and based on author heuristics;

however, because the embodiment is not required to provide a continuous correction, the feasibility of utilizing a mechanism to provide a correction less than 1% of the operating time, during transitions, seems significantly higher that other problem correction embodiments mentioned earlier.

Although there are a discrete number of operating ratios available in this embodiment, there is still continuously variable ratio change throughout the entire range of the transmission. These changes occur very rapidly between the operating ratios, but because the input and the output of the transmission are continuously engaged throughout the transmission, the embodiment is still defined as a PECVT. This arrangement is not optimal; however, based on the criteria upon which the concepts were selected, especially the highly weighted feasibility criterion, this concept scored higher than traditional PECVT concepts. Therefore, selection of a non-traditional functioning PECVT is justified by the high priority placed on concept feasibility. As mentioned previously, the feasibility of a concept that uses a device that has to provide a constantly changing correction magnitude for all transmission ratios is very low.

5.2.2 Preliminary Design

It is now important to reintroduce the different diameter changing approaches, discussed in Chapter 4, which can be used to achieve a ratio change for the final embodiment. Until now, only the equal segmentation approach has been used to describe the new concepts; however, it should be determined which of the approaches will best function in the final embodiment to eliminate any undesired characteristics or tradeoffs. The approach that does not increase the complexity or infeasibility of the final embodiment, while maintaining robustness, should be chosen as the final embodiment.

In this preliminary design of the most promising embodiment, the equal segmentation approach is again coincidentally used. It appears that a similar equal segmentation approach to that described in Figure 4.3 satisfies the desired goals of the diameter changing approach better than either of the two approaches discussed in Section 4.1.2.1. The feasibility of an embodiment decreases when attempting to integrate teeth that are corrected using a cam-follower system and the inverted cone diameter changing approach. When using the unequal segmentation approach shown in Figure 4.2, difficulty arises in providing accurate relocation of the output gear to the location that corresponds to the desired operating ratio. A separate control source to position the output gear would be required, and the complexity of the embodiment greatly increases. Difficulty in maintaining a constant output also arises with this approach because the pitch line velocities are never the same over even the smallest range.

Other segmentation approaches were also considered while developing the final embodiment. Some alterations were made to the approach shown in Figure 4.3 to reduce the number of required corrections needed when transitioning between Case 1 locations. Because a correction is needed every time a drive gear orbits past a driven gear, an embodiment with six driving gears and seven driven gears would need a total of 42 corrections for every rotation of the drive gear carrier arm. To reduce the number of corrections, a segmentation approach like that shown in Figure 4.1 was applied only to the driving gear, so that only one un-segmented driven gear (d) exists in the embodiment. This approach reduces the number of corrections from 42 to 6 when the embodiment possesses a six driving teeth (a) configuration like that shown in Figure 5.1, greatly increasing the feasibility of the embodiment. The driving gears were also replaced by individual drive teeth (a) to simplify the design as only one tooth, not an entire gear, is necessary to carry the load during engagement. Now, only one correction would need to be made every time a driving tooth (a) comes into mesh with the driven teeth (c).

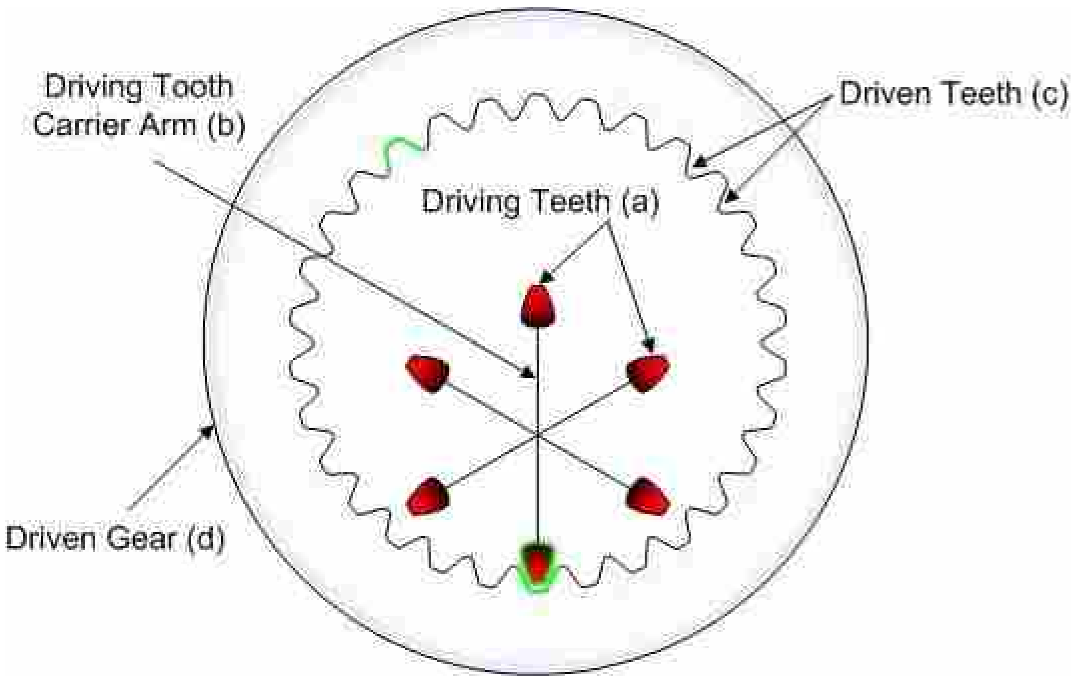

Figure 5.1: An Example of the Diameter Changing Approach used in the Final Embodiment

If the transitioning to adjacent Case 1 locations is desired to occur in one rotation of the driving gear carrier arm (b) (input), then each of the driving teeth (a) would need only one correction to mesh properly with the driven gear (d). Six cam-follower systems can then be applied, one for each driving tooth (a), to individually control the orientation of each driving tooth (a) during transitioning. By creating an independent cam profile for each cam, the predetermined amount of correction can be specifically applied to each driving gear (a), ensuring proper engagement. A basic example of this embodiment is shown in Figure 5.2. This is the same embodiment shown and briefly described in Chapter 1.

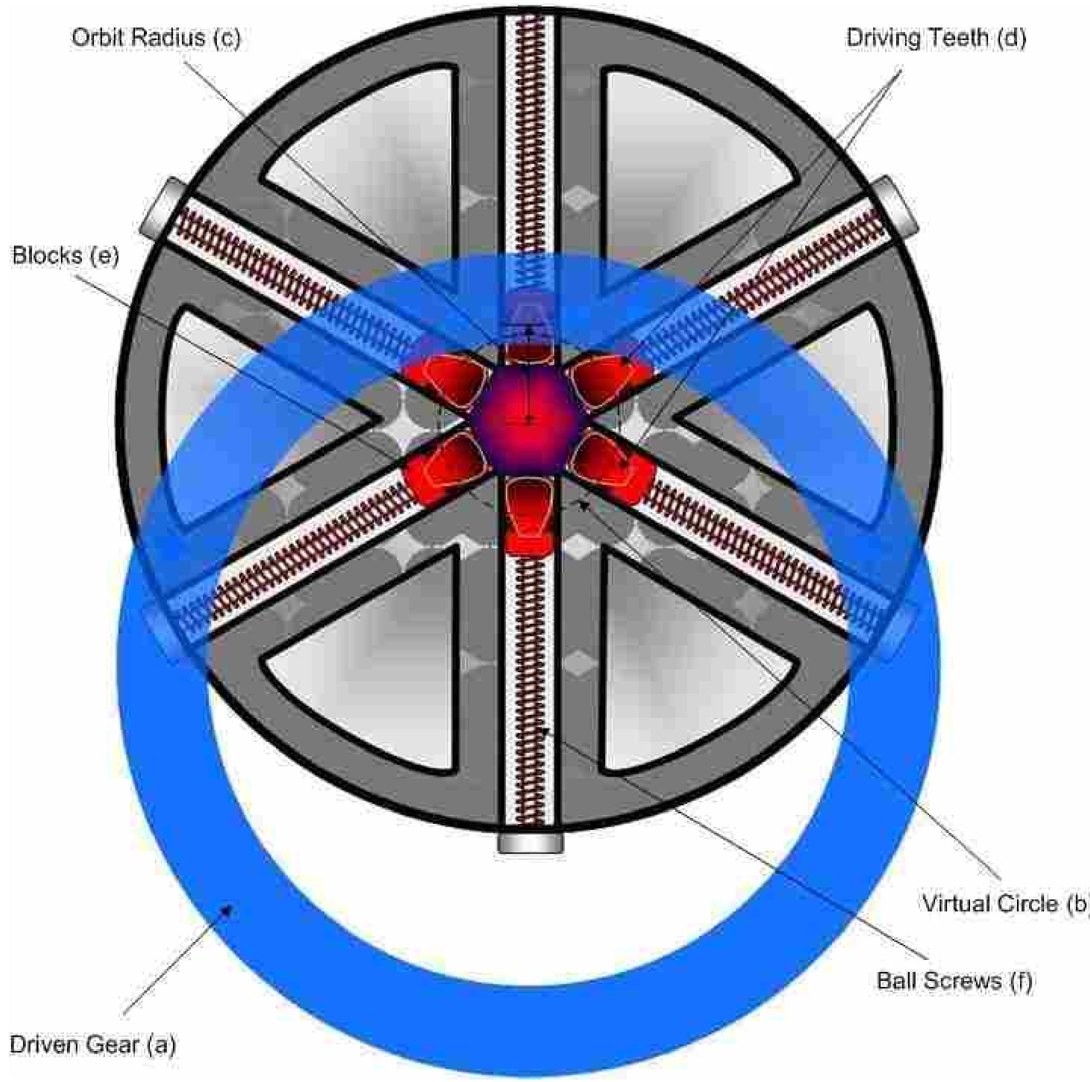

Figure 5.2: PECVT Final Concept Embodiment

The lower ring represents the driven gear (a), which was made transparent for visual purposes. This driven gear (a), or output gear, translates upward during transitioning to a larger Case 1 location, as the orbit radius (c) of the driving teeth (d) increases, thereby reducing the center distance of the virtual circle of the driving teeth (d) and the driven gear (a). The cam-follower systems are situated in the blocks (e) connecting the ball screws (f) to the driving teeth (d). The cam profile changes, that is, the cam follower (not shown) will correct the tooth orientation, as the teeth change their radial location during transitions. In this manner, the correction provided to the driving teeth occurs during the transition between Case 1 locations.

5.2.3 Kinematic Analysis

Up to this point, it has been assumed that the final concepts and devices used in the final embodiment have the ability to correct the non-integer tooth problem and ensure proper meshing over a given transmission range, while satisfying all established principles of the most promising embodiment as listed in section 5.2.1. Even though these assumptions appear valid, it is important to construct a mathematical model to test and prove, theoretically, the kinematic functionality of the final embodiment to demonstrate the feasibility of the embodiment. This validation also provides a starting point for future work, such as proving the functionality of the entire embodiment, developing a detailed CAD model, and building and testing a physical prototype.

Because the transmission will spend the vast majority of its operating time at certain Case 1 locations, it is necessary to derive the equations that show where these locations exist in terms of the transmission's parameters. Figure 5.3 shows an example of the first Case 1 location in this embodiment.

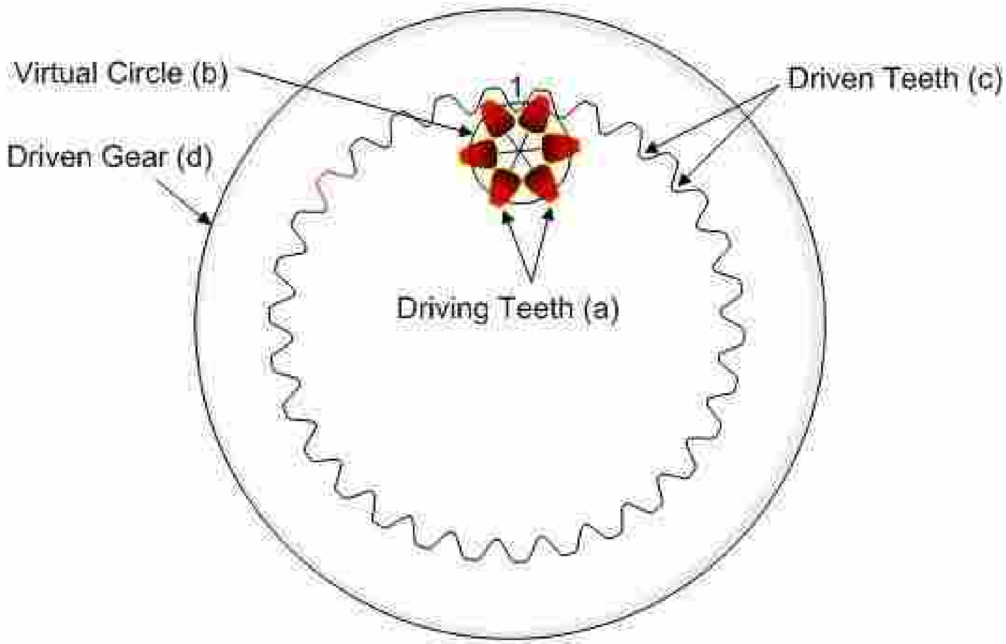

Figure 5.3: Example of First Case 1 Location (Operating Ratio) at the Smallest Orbit Radius

For this embodiment, the number of driven teeth (c) between the driving members (a) for Case 1 locations is an integer amount, in this case, the drive teeth (a) mesh properly with the driven teeth (b) and the number of driven teeth (b) between driving members (a) is 1 tooth. This integer principle must be satisfied for Case 1 locations to exist. To transition to the next Case 1 location, the orbit radius of the driving members must increase so that exactly one more integer tooth, two teeth in this case, will mesh between the two driving members (a) as seen in Figure 5.4.

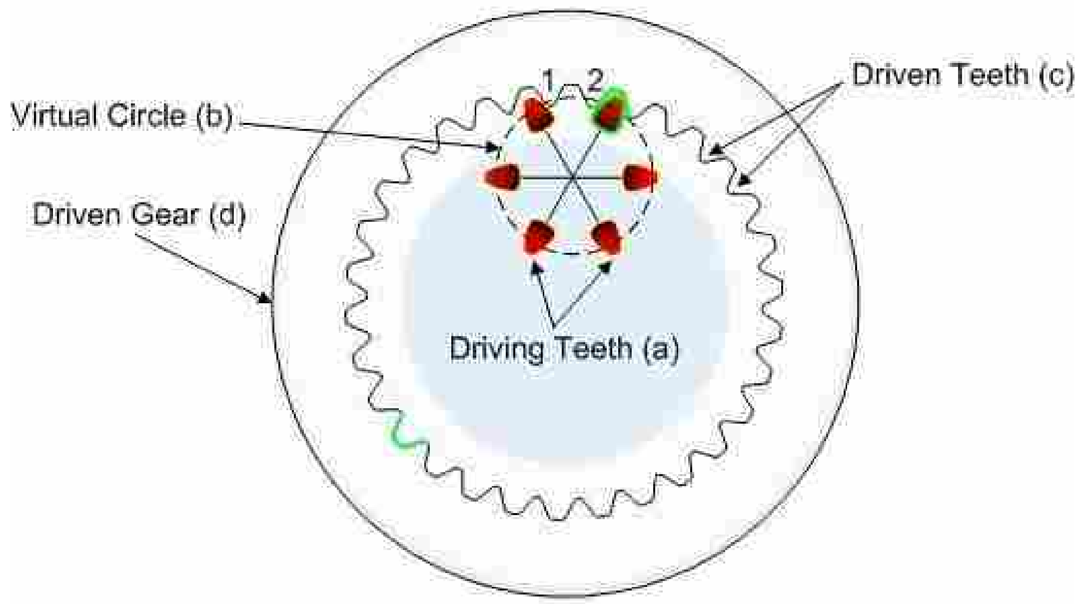

Figure 5.4: Example of Second Case 1 Location at Adjacent Operating Ratio

Therefore, for this configuration the first Case 1 position occurs when the virtual circle (b) contains 6 teeth and the second Case 1 position occurs when the virtual circle (b) contains 6 real teeth with 6 virtual teeth, or 12 teeth. The increase in the number of teeth on the virtual circle (b) from the first Case 1 to the next Case 1 is exactly six. This relationship is shown in equation 5.1 and is valid for all similar segmentation diameter changing approaches:

$$\Delta N = (Drv)(Drn) \quad \textbf{(5.1)}$$

Where:

ΔN= Change in number of teeth on virtual circle of adjacent Case 1 locations

Drv = Number of driving members (teeth)

Drn = Number of driven gears

From this relationship, the incremental distance can be derived, in terms of the transmission's orbit radius, R_{Orbit}, between adjacent Case 1 locations. Using the basic gear design equation shown in equation 1.2 as a model, N is replaced by ΔN and D is replaced by $2*\Delta R_{Orbit}$. The resultant equations, equation 5.2 and equation 5.3) appear as follows:

$$(2)(\Delta R_{Orbit}) = \frac{\Delta N}{P_d} \tag{5.2}$$

Or

$$\Delta R_{Orbit} = \frac{(Drv)(Drn)}{(2P_d)} \tag{5.3}$$

The Case 1 locations are therefore defined, in terms of the orbit radius, R_{Orbit}, in equation 5.4 as:

$$R_{Orbit} = \frac{(Drv)(Drn)(I)}{(2P_d)} \tag{5.4}$$

Where:

I=1,2,3….n

n = Total number of operating ratios available in the embodiment

It is important to note that the radial difference, ΔR_{Orbit}, between adjacent Case 1 locations is constant for all Case 1 locations. This is due to the linear relationship between the change in orbit radius, ΔR_{Orbit}, and the increase of the number of teeth, ΔN, on the circumference of the virtual circle between adjacent Case 1 locations, as seen in equation 5.2. This will be an important factor when introducing the magnitudes of corrections that need to be provided by the cam-follower systems.

5.2.3.1 Correction Equations

With the previous relationships, the amount of correction needed at each point of tooth engagement to correct for the non-integer tooth problem needs to be derived. By carefully examining the effects of the changing orbit radius at the point of contact between the driving and driven teeth, the magnitude of non-integer tooth problem and the magnitude of the needed correction can be derived. If the change in orbit radius takes place when the engaged teeth are located at the pitch point of the gear set then both teeth will move vertically upward and no separation takes place at the point of contact between the teeth (Figure 5.5(a)). However, if the teeth are located somewhere other that the pitch point, 20 degrees in the counter-clockwise direction for example, as shown in Figure 5.5(b), then a small separation occurs between the teeth. This is because the driven tooth will translate vertically, and the driving tooth will translate outward, radially, when the orbit radius is increased.

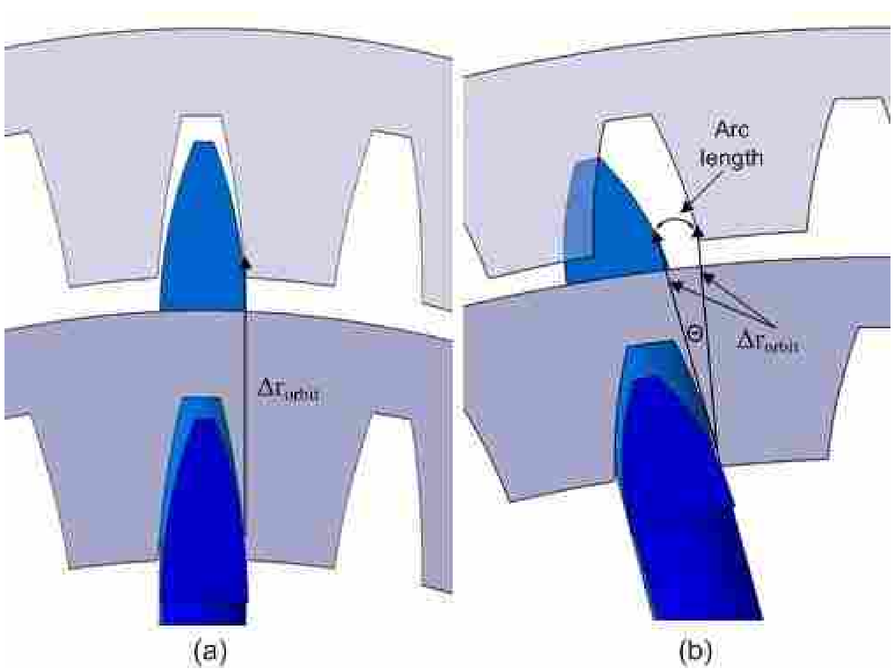

Figure 5.5: A Differential Segment of the Non Integer Tooth Problem in the Final Embodiment Caused by an Increase in the Orbit Radius

In order for the driving and driven teeth to maintain proper engagement, a correction needs to be applied to the driving tooth in the amount of the arc distance shown in Figure 5.5. The magnitude of the correction, in arc distance, is related to magnitude of the change in the orbit radius, Δr_{orbit}, and also the difference, Θ (in radians), of the direction of motion of the driving and driven tooth at that instant. Because the value of Θ is constantly changing as the driving teeth orbit about their central axis, the amount of correction, for each differential change in the orbit radius, needed by the driving teeth will also continuously vary. By summing the total differential arc lengths over the range of motion of a particular driving tooth, the total amount of correction needed by any tooth can be calculated. The equation that sums these discrete correction amounts is defined as the total amount of correction, C, in arc length:

$$C = \int_{\theta_1}^{\theta_2} (\Delta r_{orbit}) d\Theta \qquad \textbf{(5.5)}$$

Where:

Δr_{orbit} = The instantaneous change in the orbit radius.

The change in orbit radius is a function of the distance between adjacent Case 1 locations, ΔR_{Orbit}, and the speed at which the transition takes place. Thus, if the transmission is designed to make the transition in one rotation of the driving teeth, then Δr_{orbit} is defined in equation 5.6 as:

$$\Delta r_{orbit} = \frac{\Delta R_{Orbit} \Theta}{2\pi} \qquad \textbf{(5.6)}$$

Where:

Θ = The angular displacement, in radians, of the driving teeth during transition

Therefore, the resultant equation is:

$$C = \int_{\theta_1}^{\theta_2} \left(\frac{\Delta R_{Orbit}\Theta}{2\pi} \right) d\Theta \qquad \textbf{(5.7)}$$

Recall that equation 5.7 was derived from Figure 5.5 where the engaged teeth were located on the left side of the pitch point. If the engaged teeth are located on the right side of the pitch point, then the value of Θ is negative. In other words, the correction needed to ensure proper engagement on the right side of the pitch point is in the opposite direction than that on the left side of the pitch point. After calculating the integral, the amount, *C*, in arc length of correction is:

$$C = \frac{\Delta R_{Orbit}(\theta_2^{\ 2} - \theta_1^{\ 2})}{4\pi} \qquad \textbf{(5.8)}$$

Where:

ΔR_{Orbit} = The radial distance between adjacent Case 1 locations (equation 5.3)

θ_1 = The distance, in degrees, of the driving tooth from the pitch point at the time in which the misalignment occurs

θ_2 = The distance, in degrees, of the driving tooth from the pitch point at the time in which the misalignment has not yet occurred or ceases to occur

C = Is a positive value when summing the correction on the left side of the pitch point and a negative value when summing the correction on the right side of the pitch point

By multiplying equation 5.8 by "$(2\pi/360)^2$," the values of θ_1 and θ_2 are converted to degrees instead of radians while the amount of correction needed for any particular driving tooth is still in terms of arc length. The term is squared because the conversion takes place after the integral was performed. Therefore, the correction needed for the non-integer tooth problem, C, in arc length is:

$$C = \frac{\Delta R_{Orbit} \pi (\theta_2^{\ 2} - \theta_1^{\ 2})}{360^2} \tag{5.9}$$

With equation 5.9, the amount of correction needed for all driving teeth can be calculated for this embodiment by defining the range of motion of a driving tooth while the orbit radius is changing. In addition, the following conclusions can also be made about the amount of correction needed by a particular driving tooth during a Case 1 transition:

- There is a specific amount of correction, C_i, that a tooth requires to ensure a proper initial engagement with the driven tooth.
- There exists a continuous correction, C_a, that the same driving tooth requires while engaged and orbiting in the angle of approach (while engaged on the left side of the pitch point, see Figure 5.6).
- There is similar continuous correction, C_r, in the opposite direction, required by the same driving tooth while engaged and orbiting in the angle of recession (while engaged on the right side of the pitch point, see Figure 5.6).

If cams are the only device used to correct the non integer tooth problem during Case 1 transitions, then the cams will need to provide a correction to the orientation of the driving teeth prior to engagement (C_i) and during engagement (C_a and C_r). This provides the following challenges:

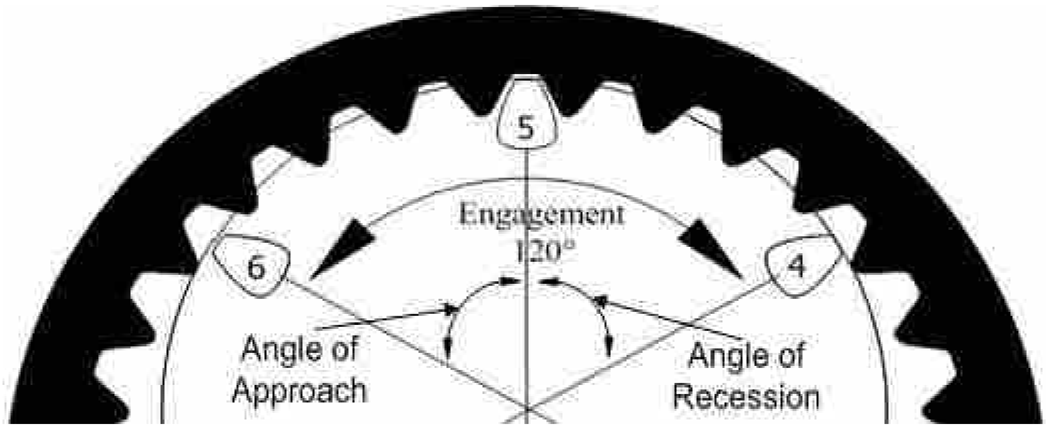

Figure 5.6: Angles of Approach and Recession as Part of Total Engagement Angle

- Because the cams would be providing a correction to the teeth during engagement, high loads would be transferred through the teeth to the cams and cam-followers while correcting.
- Because two adjacent driving members will at some time carry the load, not only does the proper amount of correction to the teeth need to be provided, but the acceleration of the corrections provided to the two engaged members by the cams would also need to be equal so that there are no infinite jerk values in the cam profile.
- The six cam profiles of the driving members in this embodiment would not be created independent of one another.
- The difficulty in constructing feasible cam profiles would greatly increase, possibly to the point of infeasibility due to this interdependence.

The cam-follower system can, however, account for the initial engagement correction, C_i, without the previously mentioned challenges. Therefore, it is apparent that some other device should be implemented to make the needed corrections during engagement of driving tooth. If the corrections, C_a and C_r, were provided using another device, the following benefits would result:

- The six cams used for each respective driving member could dwell, that is, not provide a correction during engagement of the respective driving members. This results in matching the velocity and acceleration of all corrections provided by the cams for all driving members during engagement.
- All cam profiles could be constructed independent of one another, greatly increasing the feasibility of the embodiment.
- The cams would not provide varying corrections during engagement, where high loading conditions exist.

From the concept scoring results, the use of differentials between driven gears had the second highest ranking and could be a viable concept for this application. Through discussion with the design group, it was determined to implement a differential device into the final embodiment to negate the effects of the non-integer tooth problem during engagement.

Differential devices allow for relative movement between two different gears that rotate at different angular velocities. To negate the effects of the non-integer tooth problem during engagement, the system in Figure 5.2 can be altered to include a differential device. A second driven gear (b) could be duplicated and placed axially along side the current system with three of the original driving members (teeth #2, #4, and #6) driving one driven gear (a) and three driving members (teeth #1, #3, and #5) driving the other (b). A differential device can be placed between the two driven gears (a and b) to allow relative motion between them when two driving members are engaged, one with each driven gear. The new system is shown in Figure 5.7.

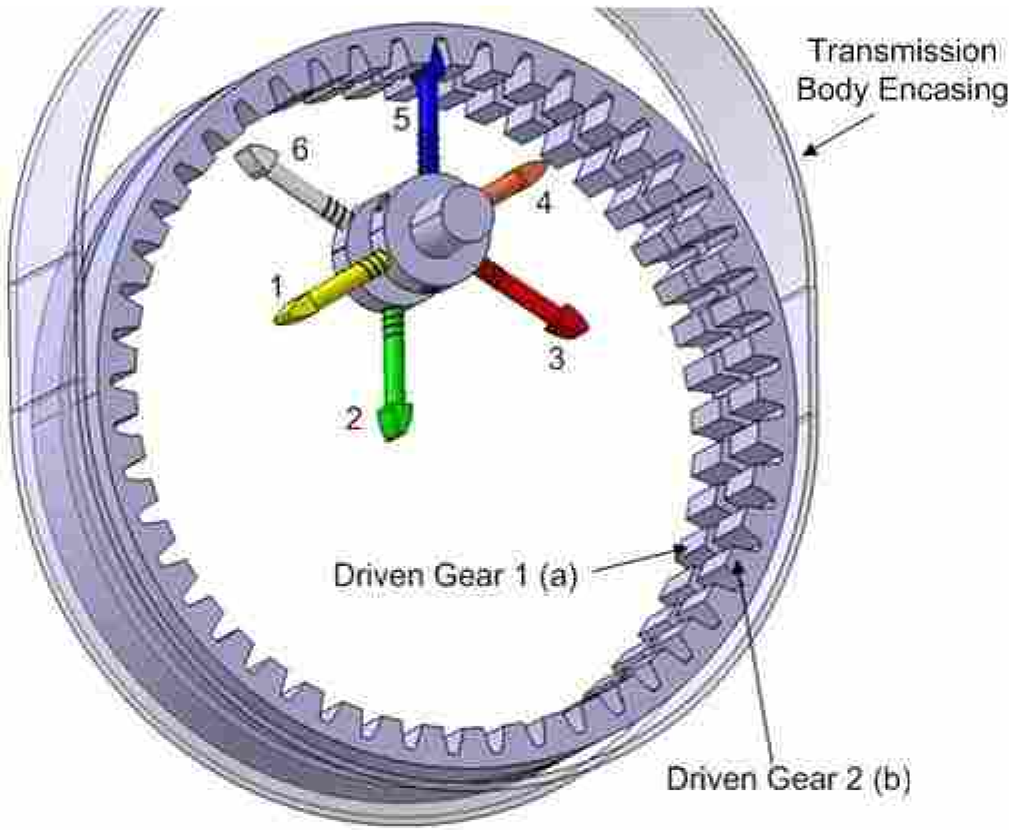

Figure 5.7: Final Embodiment with Two Driven Gears and Differential Device

The differential device does not alone guarantee proper meshing at all orbit radii; however, it does allow a self-correction to be made to the orientation of the driven gears when more than one driving member is engaged at one time. The values of those self-corrections will be important when predicting the amount of correction (C_i) that the cams are required to provide to the driving tooth. This is due to the relative motion of the driven gears, which will be a factor later in the analysis.

To ensure constant engagement of the driving teeth to the driven teeth for all orbit radii of the transmission, occasions must exist where more than one driving member is engaged with the driven gear at the same time. In fact, the contact ratio is a measure of the number of teeth that are in contact, or that are engaged, with the driven gear at one time. Under no circumstances should this contact ratio be under 1.1 [22]. If the contact ratio is between 1 and 2, there is always more than one tooth engaged. In the final embodiment, the contact ratio changes as a function of the orbit radius. When the transmission is operating at its largest orbit radius, the lowest value that the contact ratio could be is 1.1. When the orbit radius of the embodiment is at its smallest radius, the contact ratio is much greater because more engagement takes place between the driven gear and the now smaller virtual driving gear. Since we assume in this embodiment that there are always two driving teeth engaged at the same time, the new final embodiment with the differential will correct the non-integer tooth problem during engagement by allowing relative motion of the driven gears equal to the magnitude of the correction, C_a and C_r. Combined with the correcting cam-follower systems, these two devices (cams and differential) eliminate the effects of the non-integer tooth problem and can be designed to ensure proper engagement during the all Case 1 transitions.

Since the cam-follower systems need only account for the initial engagement correction, C_i, equation 5.9 can be re-written to define the magnitude of this correction:

$$C_i = \frac{\Delta R_{Orbit} \pi (\theta_2{}^2 - \theta_1{}^2)}{360^2} \qquad \textbf{(5.10)}$$

Where:

$\theta_1 =$ The angle of approach, which is defined as the location, in degrees, of the drive tooth from the pitch point at the time of initial engagement

θ_2 = The initial location of the driving tooth, in degrees, from the pitch point when the Case 1 transition begins

Certainly the Correction, C_i, should never exceed the circular pitch, P_c, of the drive gear; therefore, subtracting off full tooth widths, when accumulated, will provide the minimum amount of correction needed at any given orbit radius. For example, suppose that at a particular orbit radius, equation 5.10 yields a correction of C_i=0.3 in. If the width of one driving tooth is 0.2 in., then only a correction of 0.1 in. of the drive gear is needed for proper engagement to occur. The correction equation, therefore, becomes equation 5.11 after expanding ΔR_{Orbit} from equation 5.3:

$$C_i = P_c\left(\frac{(Drv)(Drn)(\theta_2^{\,2}-\theta_1^{\,2})}{(2)360^2} - I\right) \qquad \textbf{(5.11)}$$

Where:

P_c = The circular pitch of the driven gear, or π/P_d

I = 1, 2, 3…p to minimize $|C_i|$

p = Total number of accumulated tooth width misalignments at the largest θ_2

With any given values of an embodiment, all parameters in equation 5.11 remain constant while transitioning between all Case 1 locations, except θ_1, which is dependant upon the orbit radius, R_{Orbit}. It is not in the scope of the research at this time to calculate how θ_1 is affected by R_{Orbit}, but rather to prove that the final embodiment is capable of overcoming the non-integer tooth problem. The assumption is then made that θ_1 is independent of the orbit radius until future work is done on a more detailed analysis. Therefore, the amount of correction for a particular tooth to have a proper initial engagement is constant, in terms of arc length, throughout the transitioning of all Case 1 locations of the transmission. For this reason, the correction is defined in terms of the arc length. The cam-follower systems will actually provide the correction to the driving teeth by translation of the tooth, not rotation. In addition, only one cam profile is needed per drive gear to correct the non-integer tooth problem for initial engagement, C_i, during any Case 1 to Case 1 transition. Therefore, the cam profile is duplicated between each Case 1 location for each driving member. In future work where θ_1 is not constant for Case 1 transitions, the cam profiles will vary slightly from one Case 1 transition to the next.

In summary, the following important conclusion can be made from the derivation of the previous correction equations:

1- The change in orbit radius, ΔR_{Orbit}, of the transmission between Case 1 locations, or operating ratios, is constant throughout the operating range of the transmission.

2- The amount of correction, C_a and C_r, are self-corrected by allowing relative movement of the two identical driven gears through the use of a differential device between them. The magnitudes of these corrections are both defined by equation 5.9.

3- The amount of correction, C_i, required by the cam profile designs can be easily evaluated using a single equation (5.11) dependant upon θ_1, θ_2, Drv, Drn, and P_c (based on the θ_1 assumption previously described).

4- The amount of correction, in arc length, needed by a particular driving tooth is the same when transitioning from any Case 1 location to an adjacent Case 1 location, that is, the correction is not dependant upon the orbit radius but only on the difference of the orbit radius and the nearest Case 1 location (based on the θ_1 assumption).

5.2.3.2 Equations of Motion

The derivation of the kinematic equations governing the motion of the transmission is also necessary in analyzing the functionality of the final embodiment. The resultant pitch line velocity of the drive member at the point of engagement with the driven gear is due to the orbit of the driving tooth about the central axis of the transmission. The drive gear pitch line velocity, $V_{P.L., drive}$, is:

$$V_{P.L.,\ drive} = (\omega_{drive})(R_{Orbit}) \tag{5.12}$$

Where:

ω_{drive} = Angular velocity of the driving member (Input from motor)

R_{Orbit} = Radius of the Virtual Circle

The Driven Gear Pitch-Line Velocity ($V_{P.L., driven}$) is:

$$V_{P.L.,\ driven} = (\omega_{driven})(R_{driven}) \tag{5.13}$$

Where:

ω_{driven} = Angular velocity of the driven gear (Output)

R_{driven} = Radius of the driven gear

The final equation of motion relates the pitch line velocities of the driving member and driven member at the point of tangency on the virtual circle since they must be equal for proper meshing to occur [3]. This equation is derived from equations 5.12 and 5.13:

$$V_{P.L.,\ drive} = V_{P.L.,\ driven} \tag{5.14}$$

$$(\omega_{drive})(R_{Orbit}) = (\omega_{driven})(R_{driven}) \tag{5.15}$$

$$\omega_{driven} = \frac{(\omega_{drive})(R_{orbit})}{R_{driven}} \tag{5.16}$$

Now, by substituting ω_{in} for ω_{drive} and ω_{out} for ω_{driven}, the following equation relates the input angular velocity of the transmission, ω_{in}, to the output angular velocity of the transmission, ω_{out},:

$$\omega_{out} = \frac{(\omega_{in})(R_{Orbit})}{R_{driven}} \qquad \textbf{(5.17)}$$

5.3 Case Study

These principles and concepts are now ready to be analyzed in the final embodiment to provide evidence of their feasibility. The following section introduces a case study of the final concepts as they are found in a preliminary design.

5.3.1 Input Parameters

The final embodiment that will be used in this case study, shown in Figure 5.7, consists of 2 concentric driven gears connected out-of-plane through a differential mechanism so that relative movement will occur. The embodiment also consists of six driving teeth, also separated out of plane so that three driving gears, offset 120 degrees apart, are located in each driven gear planes and orbit about the same axis. Similarly structured in a simplified embodiment shown in Figure 5.8, driving teeth 1, 3, and 5 are located in the same plane as one of the driven gears while teeth 2, 4, and 6 are located in the plane of the other driven gear. The three driving teeth in one plane are also offset 60 degrees from the driving teeth in the adjacent plane, such that all driving teeth are equally spaced at 60 degrees about the same central axis. Again, it is assumed that exactly two driving teeth are always engaged at the same time through this analysis. This assumption allows us to validate the equations and understand the nature of correction without going into complex analysis details. This is another assumption that could be changed in future research.

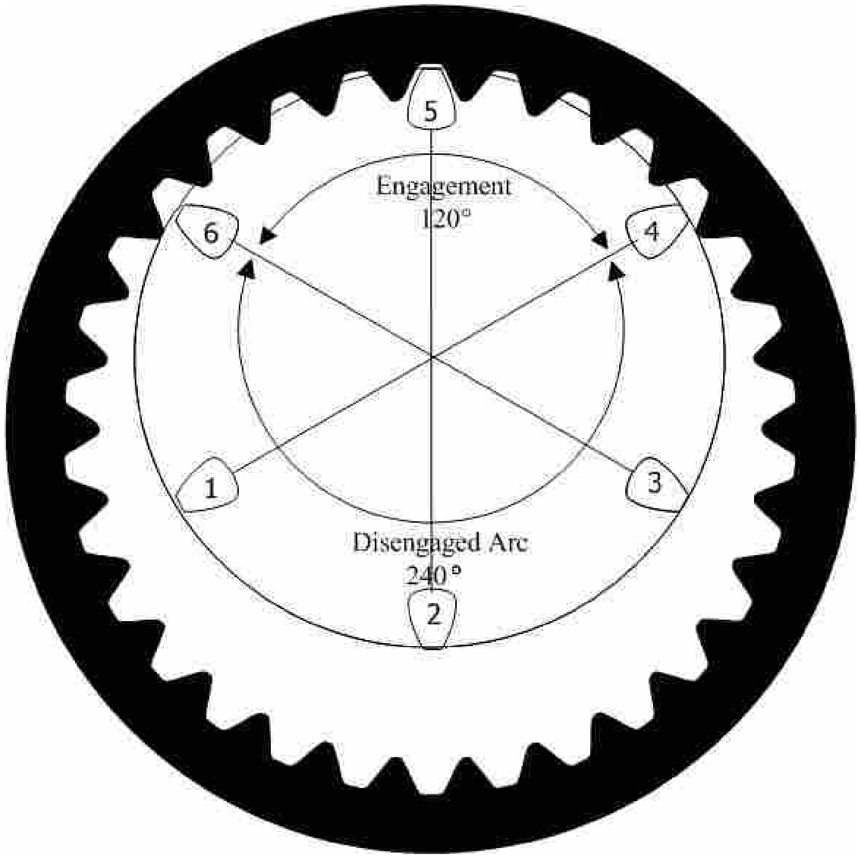

Figure 5.8: Simplified Case Study Embodiment of Preliminary Design

The configuration of the driving teeth is such that no relative movement takes place between them. Only relative movement between driven gears can occur through the differential device. This relative movement is equal to the misalignment caused by the non-integer tooth problem during engagement, C_a and C_r.

The case study will show proper meshing of the transmission while shifting from the fifth Case 1 location, I_o, to the sixth Case 1 location, I_f in a linear manner. In other words, the orbit radius of the driving teeth will increase at a constant rate determined by the input angular velocity, ω_{in}, and the number of rotations, R, of the driving teeth to complete the transition. Parameters I_o, I_f, and ω_{in} were chosen arbitrarily and have no effect on the output, while parameter R=1 was chosen to simplify the analysis of the case study. The transition will occur during 1 rotation of the driving teeth.

Even though the driven gear is segmented into two separate ring gears, only one driven member, Drv, is considered for the equations since they are interconnected. From this configuration, the change in the number of teeth on the virtual circle between operating ratios, ΔN, in equation 6.1 is equal to 6 teeth. The change in orbit radius between operating ratios, ΔR_{Orbit}, defined in equation 6.3 is 0.1875 in. These and other basic input parameters needed to show proper kinematic meshing of the final embodiment are shown in Table 5.3. The additional dimensioning input values, $R_{Orbit,o}$, $R_{Orbit,f}$, and P_d, were chosen in order to minimize the size of the transmission while maintaining a standard transmission ratio range which will be shown later.

Table 5.3: Case Study Input Parameters

Input Parameters	Values
Driven Gear Pitch Diameter, D_{driven}	6 in.
Initial Orbit Radius, $R_{Orbit,o}$	0.9375 in.
Fianal Orbit Radius, $R_{Orbit,f}$	1.125 in.
Change in Orbit Radius, ΔR_{Orbit}	0.1875 in.
Diametric Pitch, P_d	16
Number of Drive Gears, Drv	6
Number of Driven Gears, Drn	1
Input RPM, ω_{in}	1000
Initial Case 1 Integer, I_o	5
Final Case 1 Integer, I_f	6
Input Rotations, R	1

5.3.2 Output Parameters

With the input parameter values, a few important output parameters can be calculated, including the output velocity range, number of operating ratios, and change in transmission ratios. The transmission ratio, T*R*, is described using Equation 5.18 by:

$$TR = \frac{\omega_{in}}{\omega_{out}} \tag{5.18}$$

Where:

ω_{in} = Input RPM of driving teeth (See Figure 5.2 “d”)

ω_{out} = Output RPM of driven gear (See Figure 6.2 “a”)

Table 5.4 shows the respective operating ratios throughout the range of the orbit radii, R_{Orbit}, starting at the fourth Case 1 location and ending at the fourteenth Case 1 location. This range provides from a 4:1 to a 1:1 ratio, which is similar to the range used currently in standard transmissions.

Table 5.4: Transmission Ratios Throughout Transmission Range

Case 1 Integer, I	R_{Orbit}	TR
4	0.7500	4.00
5	**0.9375**	**3.20**
6	**1.1250**	**2.67**
7	1.3125	2.29
8	1.5000	2.00
9	1.6875	1.78
10	1.8750	1.60
11	2.0625	1.45
12	2.2500	1.33
13	2.4375	1.23
14	2.6250	1.14

The smallest Case 1 location in this embodiment will be constrained where the number of driven teeth between the driving members is 4 (the fourth Case 1 location). Therefore, the smallest orbit radius for the final embodiment is 0.75 in. The largest orbit radius will be 2.6250 in. at the fourteenth Case 1 location. This allows for 11 different transmission ratios between which the transmission is able to operate. The transmission ratio range for this case study starts at 4:1 and ends at 1.14:1.

5.3.3 Non-Integer Tooth Problem Correction Values

If the transition from the fifth Case 1 location to the sixth Case 1 location will occur during one rotation of the transmission's input, then each of the six driving teeth will come into engagement with one of the driven gears only once. The misalignment of each driving tooth will thus be calculated at the time the driving tooth comes into engagement with the driven gear to allow the respective cams to correct the orientation of the driving teeth. The evaluation of equation 5.11 for each of the six driving teeth using the given input parameters, C_i, is shown in Table 5.5.

Table 5.5: Correction Needed For Driving Teeth Caused by Non-Integer Tooth Problem (in.)

Driving Teeth	θ_1	θ_2	I	Correction, C_i (in.)
1	60.00	120	0	0.0491
2	60.00	180	1	-0.0654
3	60.00	240	1	0.0491
4	60.00	300	2	0.0000
5	60.00	360	3	-0.0164
6	60.00	420	4	0.0000

Because the differential device effectively corrects the non-integer tooth problem during engagement, the cams need only provide a correction, C_i. The correction values shown in Table 5.5 do not, however, take into account the effects of relative movement of the driven gears caused by the differential device. Therefore, the values in Table 5.5 need to be adjusted to take into account the effects of the differential device. Equation 5.9 defines the amount of relative movement provided by the differentials due to C_a and C_r. The first teeth that experience the correction during engagement are teeth 5 and 6, which are both engaged at the start of the transition when a contact ratio of 2 is ensured. The relative movement of the driven gear in which the incoming driving tooth will engage is what should be calculated using equation 5.9.

Since driving tooth #1 is the first tooth to be engaged, we will focus on the relative movement of the driven gear caused by engaged driving tooth #5, since these two teeth, along with driving tooth #3 attribute to the motion of that driven gear. Since driving tooth #5 orbits in the angle of recession, equation 5.9 yields a negative correction, that is, the driven gear rotates clock-wise. Again, assuming that the angle of approach and recession are constant at a value of 60 degrees in this example, the magnitude of correction from equation 5.9 (C_r) is 0.0164 in. This amount is added onto the correction (C_i) of driving tooth #1 in Table 5.5. The same actions take place for driving teeth #3 and #5. When calculating the resultant relative movement for driving tooth #2, we look at the effect that driving tooth #6 has on its corresponding driven ring. While the driving tooth #6 moves through the angle of approach, equation 5.9 yields the same magnitude of correction for C_a as C_r, in the opposite direction. Therefore, the driven ring moves counter clock-wise by an amount of .0164 inches. However, driving tooth #2 does not begin engagement until driving tooth #6 orbits through the angle of recession, which causes the driven ring to move back to its original orientation. Therefore, driving teeth

#2, #4, and #6 yield a net correction value (C_a + C_r) of 0 in. The resulting initial correction values (C_i) for the 6 driving teeth, including the relative movement caused by the differential device, is shown in Table 5.

Table 5.6: Resultant Correction for Driving Members (in.)

Driving Teeth	θ1	θ2	(C_a+C_r)	Correction, C_i (in.)
1	60	0	0.0164	0.0654
2	60	0	0	-0.0654
3	60	0	0.0164	0.0654
4	60	0	0	0.0000
5	60	0	0.0164	0.0000
6	60	0	0	0.0000

Notice that driving tooth #5 originally had a correction value of -0.0164 in., which is exactly the amount of the net correction (C_a +C_r). The effect of the relative movement of the driven gear actually negated the effects of the non-integer tooth problem, and there is no resulting correction amount for this tooth. The effects of the non-integer tooth problem can now be eliminated by using the differential device and by applying the necessary correction to the driving teeth by using the cam-follower system on driving teeth #1, #2, and #3. Even though these values represent the corrections that need to be provided by the cam-follower systems from the fifth to sixth Case 1 transition, these values do not change for transitions from any Case 1 location to an adjacent Case 1 location.

5.3.4 Cam Design

The correction amounts in Table 5.6 now need to be converted into feasible cam profiles to finalize the analysis of the case study. Since each of the driving teeth needs a distinct amount of correction, a separate cam profile can be applied to the cam-follower system of each driving tooth. Since the driving teeth #4, #5, and #6 do not require a correction, there is no cam profile needed for these teeth. Also, because the corrections provided by the cam-follower system are needed before the teeth come into engagement and not during engagement, the cam will be designed with dwells. A dwell is defined as no output motion for a specified period of input motion and is important in the cam-follower system that will be used in this embodiment [2].

To eliminate any oscillatory output, no correction should be provided to the teeth by the cam-follower system during engagement. For this engagement period, a dwell will be designed into each of the cam profiles. Because only certain cam positions (start, correction amount, and finish) are defined over the complete interval of cam motion, the type of motion constraint for this problem is a Critical Extreme Position (CEP) constraint. This allows the designer freedom to use any path of motion while moving between critical positions in the design. With this constraint, different types of motion programs could exist which should be identified in this embodiment. Each of the profiles can follow either a rise-dwell-fall (RDF) or rise-dwell-fall-dwell (RDFD) program. Due to the nature of these programs, the boundary conditions of the displacement, velocity, and acceleration functions need to be matched at the different interfaces of the program between the segments in the cams. In other words, the velocity and acceleration functions have to equal zero when the rise meets the dwell, when the dwell meets the fall, and when the fall meets the end of the cycle.

The fundamental law of cam design states that the cam’s motion must be continuous through its first and second derivatives of displacement throughout the entire

360 degrees of rotation of the cam. This also means that the derivative of the acceleration function, or jerk function, needs to have a finite value, not infinity, over the same 360 degrees of motion. In other words, the acceleration function needs to be continuous over the 360 degrees of rotation of the cam. To meet this requirement, each cam displacement function must have third order continuity. This can be shown by examining plots of the four motion equations also known as the s-v-a-j plots. If these conditions are met, then a feasible cam design can be ensured [2].

To ensure that this first motion constraint is met, a proper displacement function must be constructed to connect the extreme positions of the profile and still satisfy the fundamental law of cam design. If a polynomial function is used, then the displacement function must be fifth order or higher to satisfy the law and maintain third order continuity of the acceleration function. An even higher order polynomial will allow the designer to either decrease acceleration values or constrain jerk values, which will be helpful in further cam design. In this case study, a seventh-order polynomial function will be used to create the displacement functions of the cam profiles. The higher order polynomial will allow the profile to match not only the displacement, velocity, and acceleration functions at the extreme positions but also the jerk function at all profile segments, which will eliminate unnecessary vibrations during operation. These jerk constraints are not necessary for proper cam design, and removal of these constraints can result in lower acceleration values if the acceleration values of the cam followers are too high. The program used in the profiles will be a RDF program with the dwell value representing the value of the correction needed to realign the gear tooth.

These boundary conditions of the motion functions should also be equal between the cams of the two engaged teeth when these teeth are entering or leaving engagement. By ensuring that the cam of the tooth entering or leaving engagement is always in its dwell period, this second constraint is met because the velocity and accelerations of the motion functions are zero during the dwell periods. If one overall acceleration function were constructed by taking the piece-wise acceleration functions of the six separate cam profiles during their respective engagement periods, then the acceleration function would appear as a strait line having no slope and a value of zero. This satisfies the fundamental law of cam design and the cam profiles would be feasible.

For this case study, three different cam profiles need to be designed for driving members 2, 3, and 4. To create the profiles, a seventh order polynomial was constructed for the rise and fall segments of each of the driving members using 8 boundary conditions shown in Table 5.7 where C is the respective correction amount needed by the tooth and the value of the dwell period.

Table 5.7: The 8 Boundary Conditions in Developing the Cam Equations of Motion

		s	v	a	j
RISE	**t=0**	0	0	0	0
	t= time at dwell	C	0	0	0
FALL	**t= time after dwell**	C	0	0	0
	t=time after 1 cycle	0	0	0	0

The three cam profile equations for the rises and falls with their respective s-v-a-j diagrams are located in Appendix B [2]. The s-v-a-j diagrams show continuity for all four functions in the three cam profiles that were created. This continuity proves feasibility of the cam designs.

5.3.5 Case Study Conclusion

With the specific embodiment and transmission input parameters used in this case study, the feasibility of correcting the orientation of the driving teeth using a differential device and cam-follower systems has been shown. Different input parameters will yield different corrective values that need to be provided by the cam-follower systems, and a more in depth analysis also needs to be conducted for a different number of driven rings or different contact ratio values; however, similar methods can be applied to meet numerous designs. There were several assumptions made during this analysis for certain specific reasons. The assumptions were made in order to isolate the individual characteristic violations that occur as a result of the non-integer tooth problem. This case study provides sufficient theoretical evidence for the feasibility of the final embodiment and concepts presented in this chapter.

6 Conclusions and Recommendations

6.1 Conclusions

The objectives of this thesis were to identify functional principles that must be satisfied for the most promising PECVT embodiment to exist and to create a classification system to classify all PECVT embodiments. Through the examination and analysis of several patented PECVT embodiments, these objectives were met. The major contribution of this research has been in developing the classification system and creating the functional principles and not in attempting to develop a new PECVT embodiment.

The classification system is composed of two classes: the problem correction class and the problem elimination class. The problem correction class embodiments utilize a variety of mechanisms to correct the orientation of gear teeth to overcome the non-integer tooth problem. The two families in this class are the one-way clutch family, and the alternate device family. The problem elimination class is also composed of two families: the tooth conforming family and the feedback family. This class uses different mechanisms and methods to eliminate the non-integer tooth problem. The governing principles created to assure the functionality and feasibility of all PECVT embodiments are again summarized below:

1) The reorientation of the driving or driven gear (whichever possesses the characteristics of a continuously variable diameter gear) must occur so that its circular pitch is equal to or a factor of the circular pitch of the gear or member with which it is engaged. This is called the matching pitch principle.

2) If the ideal PECVT embodiment belongs to the problem correction class, a device needs to be devised and implemented in such a way that a constant output is not being traded for positive, continuous engagement when a correction is applied to satisfy the matching pitch principle.

3) If an ideal PECVT embodiment belongs to the problem elimination class, the devices and methods used to eliminate the problem and ensure proper meshing need to be less complex and more robust for high torque applications in the tooth conforming family.

4) A device that gathers more intelligence from the transmission's parameters in order to vary and, more importantly, control the RPM ratio would be another alternative for a promising embodiment in the feedback family of the problem elimination class.

These principles provide a basis for evaluating all PECVT embodiments that can be categorized in the newly developed classification system. By understanding these principles and how they are met by different embodiments, one can quickly assess the functional feasibility and compare to other previously published embodiments.

Common functional tradeoffs and commonly used mechanisms in PECVT embodiments for each of the newly developed PECVT classes were also identified in this research. By using TRIZ principles to help eliminate common contradictions and tradeoffs, a concept development effort was implemented to identify the most promising embodiment for a PECVT based on the functional principles needing to be satisfied. An embodiment has been proposed by Vernier Moon Technologies and BYU that is believed to eliminate the effects of the non-integer tooth problem using a cam-follower and a differential device. A case study of this final conceptual embodiment was presented and analyzed to show how the mechanism ensures proper engagement without the effects of the non-integer tooth problem. The mathematical model created in MATLAB (see Appendix C) is very useful in validating the derived equations that predict the magnitude of the correction values that need to be provided by a cam-follower system and differential device for this particular embodiment. Theoretically, implementation of the corrective cam-follower and differential device appears to be feasible based on the MATLAB validation results of the kinematic analysis, which shows the magnitude of the needed corrections. Also, each of the cam profiles have also been calculated with their respective s-v-a-j diagrams to show the feasibility of using the cam-follower system for this proposed embodiment. This embodiment is not the optimal embodiment, but is only a conceptual embodiment that satisfies the governing principles that were established in this research.

There could be other possible embodiments that may exist or that could be developed where the required functional specifications are also satisfied; however, it is believed that by using the outlined methodology, the embodiment developed in this thesis and presented as the case study is one of the most promising embodiments in terms of feasibility, robustness, and other satisfied functional specifications. By applying the same methods described in this research, other embodiments might be more promising based on a different set of assumptions. However, the principles and classification system that have been established are valid for all PECVT embodiments and will be valuable in future research. They both provide a solid base from which others can build.

This research will serve as a starting point for further PECVT research sure to follow in the future, not only for this proposed embodiment, but also for other embodiments classified in different classes. However, the practicalities of designing and building a functional PECVT are still in question and the results of further research will be critical in realizing commercialization of this concept.

6.2 Recommendations

A CAD model, showing the animation of the basic PECVT device and mechanism while transitioning to adjacent Case 1 locations, would be helpful in visually verifying the accuracy of the MATLAB correction results. The model would also serve as the preliminary design for the construction of a physical prototype. A prototype, used to match the analytical results of the mathematical model, would be very valuable in proving the concepts of the final PECVT embodiment. Although a kinematic analysis has been conducted to show the feasibility of the final embodiment, there are other refining analyses yet to be conducted to prove its functionality such as: load and torque analyses of teeth, tolerance analyses, dynamic analyses of moving parts, further cam design, wear analyses, involutometry study, etc.

Currently, the involutometry analysis is being conducted at BYU, as well as a more refined kinematic study. In this research, it was assumed that the involute curve defining the shape of the involute teeth remains constant, regardless of the gear radius. That is, as the orbit radius of the input gear on the final embodiment increases, the input teeth will continue to mesh properly with the output gear while using the same involute curve. However, the involute curve of gear teeth is dependant upon the radius of the gear in which the teeth are located. Therefore, in the final embodiment, the involute curve of the driving teeth needs to continuously change as the orbit radius of the driving teeth increases or decreases. This problem is currently being addressed on the final embodiment, as well as other assumptions that were made and discussed in Chapter 5.

Other devices, such as the correction control device and the orbit radius changing device, should also be developed past the theoretical stage to a design and preliminary prototype to further show functionality of this PECVT.

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Appendix A Patented PECVT Embodiments according to Product Specifications

			Year		1938	1964	1975	1980	1980	1981	1983	1987	1987	1987	1987	1990	1991	1995	1995	1996	2000	2003	2004	
Metric Number	Need Number	Metric	Importance	Units	2,199,051	3,359,813	3,867,851	4,181,043	4,235,125	4,277,986	4,373,926	4,644,828	4,660,427	4,697,469	4,680,985	4,909,101	5,036,716	5,454,766	5,440,945	5,516,132	6,066,061	6575856 Anderson CVT	6,835,153	Fixed Pitch
1	1, 4	Friction Dependent	1	Binary	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
2	2	Positive Engagement	1	Binary	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
3	3	Continuous Engagement	1	Binary	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	?	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes
4	4, 10	Continuously Variable Ratio	1	Binary	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
5	5	Oscilating Output	2	Binary	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
6	6	Able to Vary Ratio under Load	2	Binary	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	?	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
7	7, 12	Max Torque	3	ft-lbs																				
8	8	Efficiency	3	%																				
9	9	Weight	3	lbs																				
10	10	Ratio Range	3	Δ# :1																				
11	11	Number of Control Sources	3	#																				
12	12	Kinematic Interference	3	Binary																				
13	13	Number of Non-standard Parts	4	#																				
14	14	Number of Parts	4	#																				
15	15	Able to be Retrofit in Current Apps.	4	Binary																				
16	16	Max RPM	4	#																				

CLASS	FAMILY	2,199,051	3,359,813	3,867,851	4,181,043	4,235,125	4,277,986	4,373,926	4,644,828	4,660,427	4,697,469	4,680,985	4,909,101	5,036,716	5,454,766	5,440,945	5,516,132	6,066,061	6,575,856	6,835,153	
CVTs	Variable Input																				
	Friction Input																				
Correct Problem	One-way Clutch	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
	Alternate Device																				
Eliminate Problem	Tooth Conforming																				
	Feedback																				

Metric Number	Need Number	Metric	Importance	Units	3,899,941	4,610,184	4,625,588	4,805,489	5,169,359	6,033,332	4,854,190	John Deere	4,327,604	6,053,840	2,026,928	2,970,494	3,175,410	4,852,569	6,055,880	6,964,630
			Year		1975	1986	1986	1989	1992	2000	1989		1982	2000	1932	1961	1965	1989	2000	2005
1	1, 4	Friction Dependent	1	Binary	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	No	No	No	No	No	No	No
2	2	Positive Engagement	1	Binary	No	No	No	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
3	3	Continuous Engagement	1	Binary	No	No	No	No	No	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
4	4, 10	Continuously Variable Ratio	1	Binary	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	Yes	Yes	Yes	Yes	Yes	Yes
5	5	Oscilating Output	2	Binary	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
6	6	Able to Vary Ratio under Load	2	Binary	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
7	7, 12	Max Torque	3	ft-lbs																
8	8	Efficiency	3	%																
9	9	Weight	3	lbs																
10	10	Ratio Range	3	Δ# :1																
11	11	Number of Control Sources	3	#																
12	12	Kinematic Interference	3	Binary																
13	13	Number of Non-standard Parts	4	#																
14	14	Number of Parts	4	#																
15	15	Able to be Retrofit in Current Apps.	4	Binary																
16	16	Max RPM	4	#																

CLASS	**FAMILY**	3,899,941	4,610,184	4,625,588	4,805,489	5,169,359	6,033,332	4,854,190		4,327,604	6,053,840	2,026,928	2,970,494	3,175,410	4,852,569	6,055,880	6,964,630
CVTs	Variable Input							X	X								
	Friction Input	X	X	X	X	X	X										
Correct Problem	One-way Clutch																
	Alternate Device																
Eliminate Problem	Tooth Conforming											X	X	X	X	X	X
	Feedback									X	X						

Appendix B Cam Profile equations for Rises and Falls with s-v-a-j Diagrams

Driving Member #1

Rise:

$$s = h * \left[35 * \left(\frac{\theta_1}{\beta_1} \right)^4 - 84 * \left(\frac{\theta_1}{\beta_1} \right)^5 + 70 * \left(\frac{\theta_1}{\beta_1} \right)^6 - 20 * \left(\frac{\theta_1}{\beta_1} \right)^7 \right]$$

Fall:

$$s = h * \left[-35 * \left(\frac{\theta_2}{\beta_2} \right)^4 + 84 * \left(\frac{\theta_2}{\beta_2} \right)^5 - 70 * \left(\frac{\theta_2}{\beta_2} \right)^6 + 20 * \left(\frac{\theta_2}{\beta_2} \right)^7 + 1 \right]$$

Where:

h = .0654 in.

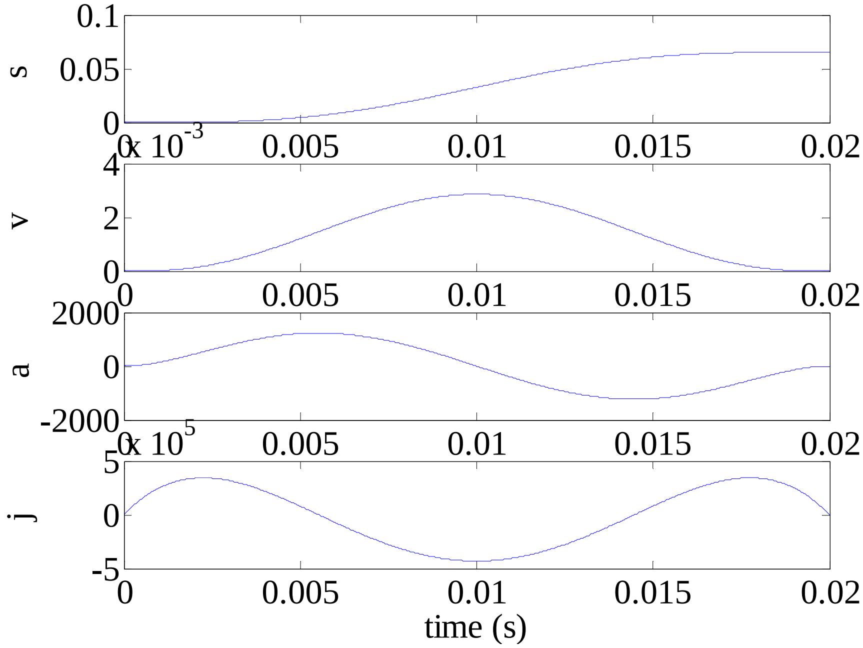
β_1 = 60 degrees

β_2 = 240 degrees

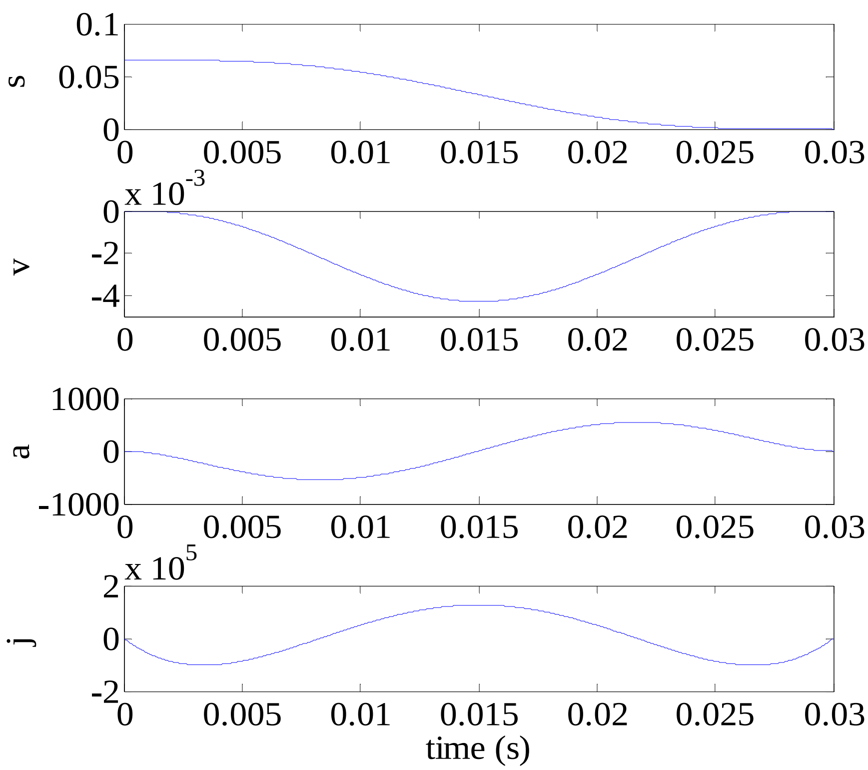
$\theta_1 = 0 \ldots \beta_1$

$\theta_2 = 0 \ldots \beta_2$

S-V-A-J Rise Figure for Driving Member #1



S-V-A-J Fall Figure for Driving Member #1



Driving Member #2

Rise:

$$s = h*\left[35*\left(\frac{\theta_1}{\beta_1}\right)^4 - 84*\left(\frac{\theta_1}{\beta_1}\right)^5 + 70*\left(\frac{\theta_1}{\beta_1}\right)^6 - 20*\left(\frac{\theta_1}{\beta_1}\right)^7\right]$$

Fall:

$$s = h*\left[-35*\left(\frac{\theta_2}{\beta_2}\right)^4 + 84*\left(\frac{\theta_2}{\beta_2}\right)^5 - 70*\left(\frac{\theta_2}{\beta_2}\right)^6 + 20*\left(\frac{\theta_2}{\beta_2}\right)^7 + 1\right]$$

Where:

h = - .0654 in.

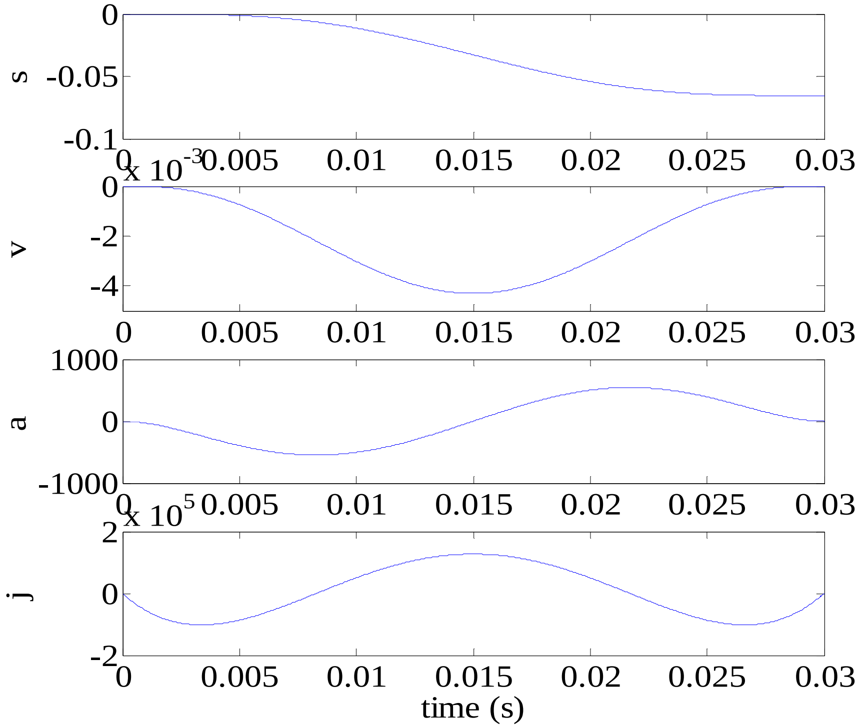
β_1 = 120 degrees

β_2 = 180 degrees

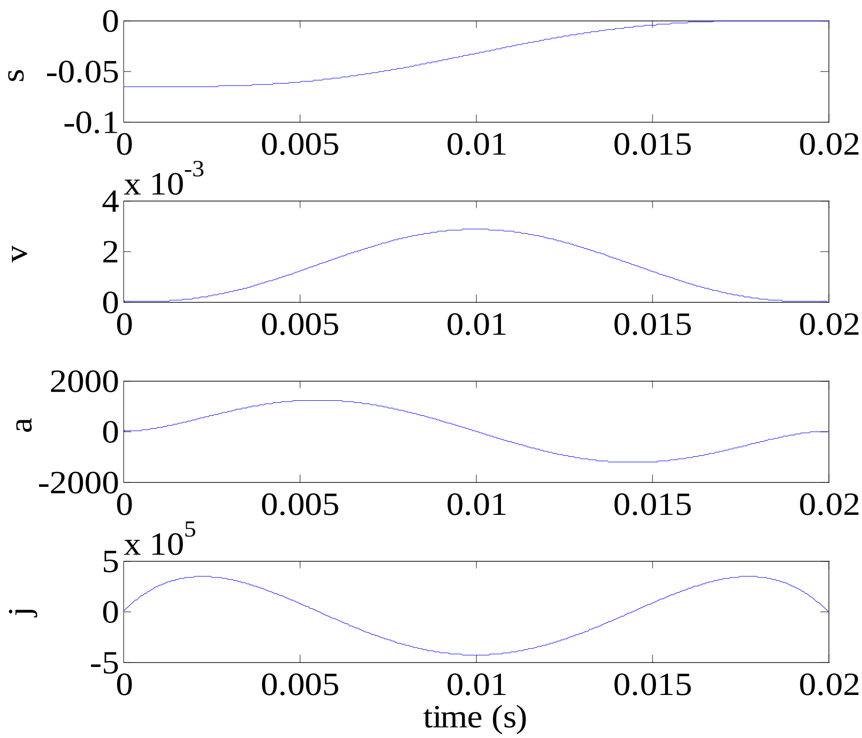
$\theta_1 = 0 \ldots \beta_1$

$\theta_2 = 0 \ldots \beta_2$

S-V-A-J Rise Figure for Driving Member #2



S-V-A-J Fall Figure for Driving Member #2



Driving Member #3

Rise:

$$s = h * \left[35 * \left(\frac{\theta_1}{\beta_1} \right)^4 - 84 * \left(\frac{\theta_1}{\beta_1} \right)^5 + 70 * \left(\frac{\theta_1}{\beta_1} \right)^6 - 20 * \left(\frac{\theta_1}{\beta_1} \right)^7 \right]$$

Fall:

$$s = h * \left[-35 * \left(\frac{\theta_2}{\beta_2} \right)^4 + 84 * \left(\frac{\theta_2}{\beta_2} \right)^5 - 70 * \left(\frac{\theta_2}{\beta_2} \right)^6 + 20 * \left(\frac{\theta_2}{\beta_2} \right)^7 + 1 \right]$$

Where:

h = .0654 in.

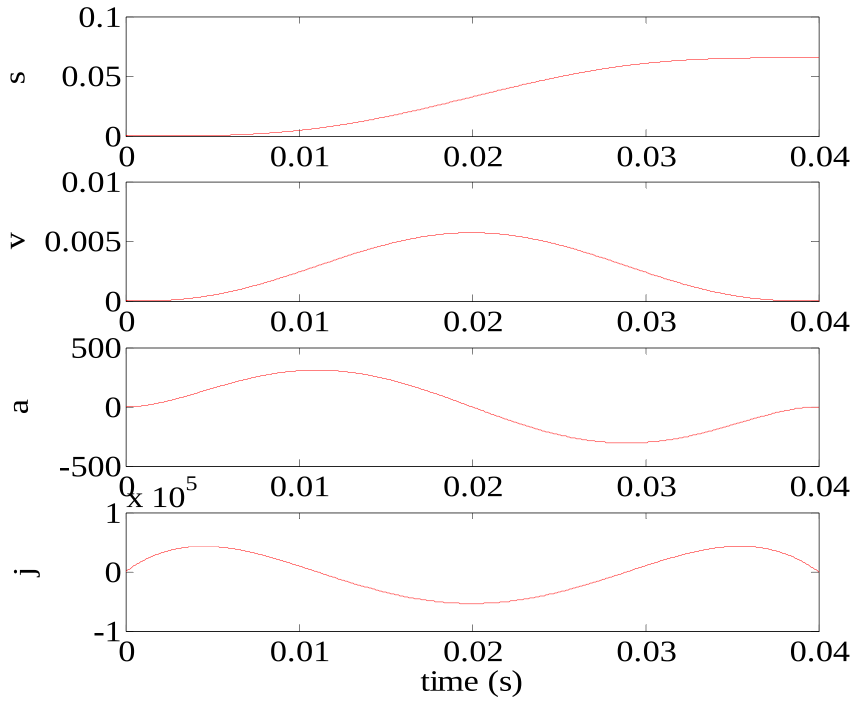
β_1 = 180 degrees

β_2 = 120 degrees

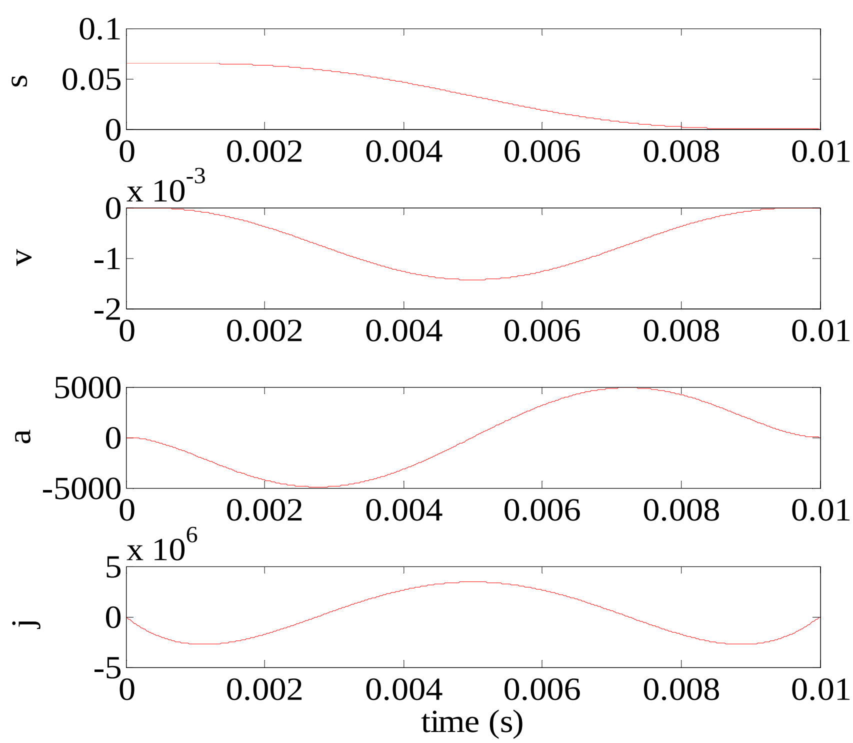
$\theta_1 = 0 \ldots \beta_1$

$\theta_2 = 0 \ldots \beta_2$

S-V-A-J Rise Figure for Driving Member #3



S-V-A-J Fall Figure for Driving Member #3



Appendix C MATLAB Code to Predict the Amount of Correction Needed to Ensure Proper Engagement of the Final Embodiment

```
function thesis1(Ndriven,win,nodrive,nodriven,DP,VC1,VC2,Rotations)

%resolution on the integral
%Points Per Gear (mesh points)
%mesh points on mesh point vector
res=.000001;
PPG=Rotations*nodriven;
mesh_points=PPG*nodrive;%this should be enough to satisfy
2*PPG*nodriven (for 'moon')

%These determine the three gear radii of the embodiment
Rdriven=(Ndriven/DP)/2;


meshpoint_arm=360/(nodrive*nodriven);%number of degrees between mesh
points
tooth_width_deg=360/Ndriven;   %degee spacing of driven gear teeth

win=win*360/60;%win in converted from RPM to degrees/sec

%Theta_arm created an array of all mesh points degrees, ie. 12, 24,
36...
for n=1:mesh_points
theta_arm(n)=meshpoint_arm*n;
end

%theta_arm is now the first n locations of mesh, so we will make...
%a time array of the first n times we need to mesh (in sec)
t=theta_arm/win;

%This is going to calculate the 2 virtual circles and increase the
%orbit from 1 to the next.  Since I define orbitnot and drdt, it does
not matter what these 2 values are being input from the function.
orbitnot=VC1*nodrive*nodriven/(2*DP)
orbit2=VC2*nodrive*nodriven/(2*DP)
drdt=(orbit2-orbitnot)/(360*Rotations/win);%this is for # of
revolutions
```

```
for i=1:mesh_points

%This is for all meshing during the increase of the orbit radius
      if i==1
        timestep = 0:res:t(i);
         theta_old=0;
     else
    timestep = t(i-1):res:t(i);
     theta_old=theta_out(i-1);
      end

Rorbit{i}=orbitnot+(drdt*timestep); %This is a piece of the linear
Rorbit function
wo=((win)*(Rorbit{i}))/(Rdriven); %in degrees per second

%This takes the integral of the time step with the wo to yeild Theta
Out
%These are the locations of the output every mesh point of the input
theta_out(i)=theta_old+trapz(timestep,wo);

end



%These four loops sort the misalligments to the particular drive gears
 for j=1:nodrive
 for i=0:PPG-1  %# of MP attached to each gear

   drive_gears(i+1,j)=(theta_out(j+i*nodrive));
 end
 end



Theta_of_output=drive_gears
pc=pi()/DP;
MESH=drive_gears/tooth_width_deg;%in terms of number of output teeth
for i=1:mesh_points

drive_gears(i)=((((MESH(i)-round(MESH(i)))*tooth_width_deg))*...
(Rdriven/Rorbit{i}(end)))*Rorbit{i}(end)*2*pi()/360;%in terms of
arclength *;
%Correction=(C1-C2);%*Rorbit{i}(end)*2*pi()/360;
end

Correction_of_driving_teeth=drive_gears; %in terms of arclength

%This correction is only the initial correction needed for engagement,
%which is why the initial quantity is subtracted off.
Correction_of_driving_teeth=Correction_of_driving_teeth-
Correction_of_driving_teeth(1)

%This will clear the old plots and assign colors
hold off
a(1)='b';
```

```
a(2)='b';
a(3)='b';
a(4)='r';
a(5)='k';
a(6)='r';
a(7)='k';
a(8)='b';

%This determines the time axis for the plots, I need to work on this
%section a bit more
%t(1)=meshpoint_arm/win

for i=1:nodrive
time{i}=i*t(1):nodrive*t(1):(360*Rotations/win)+i*t(1);
P{i}=polyfit([0; time{i}(1:PPG)'; time{1}(PPG+1)-t(1)], [0;
drive_gears(1:PPG,i); 0],PPG+1);
figure (1)
subplot(1,1,1), plot(time{i}(1:PPG),drive_gears(1:PPG,i),'o')
hold on

end

%This is for the graphing features
x=linspace(0,time{1}(PPG+1)-t(1),1000);
figure(1)
for i=1:nodrive
A=0;
for j=1:PPG+2
A=A+P{i}(PPG-(PPG-j))*x.^(PPG-(j-2));
end
%subplot(1,1,1),plot(x,A,a(i))
end
grid on
```