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Fundamental Components for Lamina Emergent Mechanisms

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FUNDAMENTAL COMPONENTS FOR LAMINA EMERGENT MECHANISMS

by

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GRADUATE COMMITTEE APPROVAL

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ABSTRACT

FUNDAMENTAL COMPONENTS FOR LAMINA

EMERGENT MECHANISMS

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Master of Science

This thesis introduces lamina emergent mechanisms (LEMs) and presents components that can be used as building blocks to create LEMs capable of more complex motion. As the name suggests, lamina emergent mechanisms are fabricated out of planar materials (the lamina) but their motion is out of that plane (emergent). Lamina emergent mechanisms can provide benefits that include reduced manufacturing costs and minimal volume when in the planar state. The compact initial state of LEMs is beneficial in reducing shipping costs, especially in volume critical applications. LEMs also exhibit the potential benefits of compliant mechanisms, namely increased precision, reduced weight, reduced wear, and part count reduction. The LEM components presented in this thesis include flexible segments, fundamental mechanisms, and a new complaint joint, the lamina emergent torsional (LET) Joint.

The flexible segments are developed through changes in geometry, boundary/loading conditions, and material. The fundamental mechanisms presented have motion similar to planar change-point four-bar and six-bar mechanisms, and spherical change-point mechanisms.

The LET Joint is presented as a compliant joint suited for applications where large angular rotation is desired, but high off-axis stiffness is not as critical. The joint is introduced and the equations necessary for determining the force-deflection characteristics and stress are presented. Since the LET Joint can be fabricated from a single planar layer, it is well suited for macro and micro applications. Illustrative examples of the LET Joint are provided with devices fabricated from materials as diverse as steel, polypropylene, and polycrystalline silicon.

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Chapter 1

Introduction

Space travel, surgical procedures, and applications where shipping costs need to be minimized, would benefit from mechanisms that are compact during transportation or positioning, then expand once at the desired location. There also exists a need for mechanisms that can perform their function in applications with limited space. These applications could include electronics, automotive, biomedical, search and rescue equipment and microelectromechanical systems (MEMS). To be competitive in today's market these mechanisms must also be designed for low cost manufacturing.

To meet these needs a new class of mechanisms is proposed: lamina emergent mechanisms (LEMs). As the name suggests, these mechanisms are fabricated out of planar material (a lamina) but their motion is out of that plane (emergent).

For the successful development of LEMs, an organized logical design approach should be followed. The following six steps are proposed: 1) understand the fundamentals governing flexibility, 2) form components, 3) create interaction strategies, 4) develop modeling and analysis approaches, 5) synthesize mechanisms, and 6) design, fabricate, and test demonstrators. This thesis focuses on the first and second step in this process: understand the fundamentals governing flexibility and form components. These components can be further subdivided into two categories: flexible segments and fundamental mechanisms.

1.1 Outline of the Thesis

The remainder of this chapter discusses the relation of LEMs to other mechanisms, and gives a background of the relevant work as it pertains to LEMs. Chapter 2 discusses steps one and two in the design approach presented above. Chapter 3 develops the lamina emergent torsional (LET) Joint. The equations necessary to model the motion and stress a

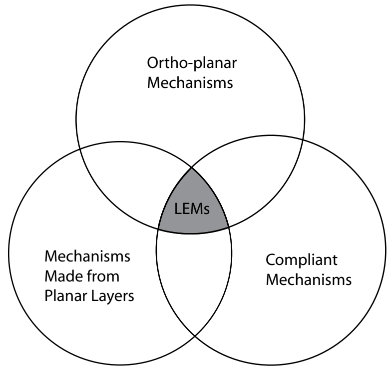


Figure 1.1: Venn diagram of LEMs, ortho-planar mechanisms, compliant mechanisms, and mechanisms made from planar layers.

LET Joint are provided. Chapter 3 also includes an example of the design and testing of a spring steel LET Joint and provides examples of LET Joints implemented in various macro and micro mechanisms. Chapter 4 reviews the work presented and gives some suggestions for future work.

1.2 Background

To aid in understanding where lamina emergent mechanisms fit with respect to other mechanisms, Figure 1.1 is a venn diagram illustrating the relationship between LEMs, ortho-planar mechanisms, compliant mechanisms, and mechanisms made from planar layers. LEMs are a subset of ortho-planar mechanisms, compliant mechanisms, and mechanisms made from planar layers. Depending on their design LEMs could also be a metamorphic mechanism. A brief description of these categories as related to LEMs will be given.

1.2.1 Ortho-planar Mechanisms

Ortho-planar mechanisms are defined by the ability to have all of their links in a single plane while their motion is out of that plane. Their main advantage is the compact planar state they achieve with all of their links collinear. Parise et al. [2] investigated the requirements for ortho-planar behavior of mechanisms. They defined a compressed and uncompressed planar state. The two states receive their name based on the amount of

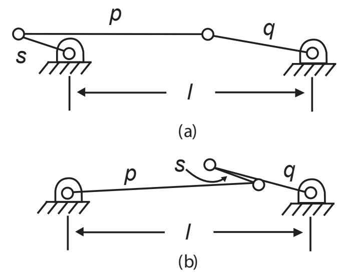


Figure 1.2: (a) With l adjacent to s an uncompressed change-point occurs, (b) with l opposite s a compressed change-point planar state occurs [2].

space they take in the link length direction. A compressed mechanism will only require the length of its longest link. An uncompressed mechanism will require more space than than the length of its longest link. They also proposed conjectures which determine, based on link lengths, whether a mechanism will enter a compressed or uncompressed planar state. The conjecture for four-bars states: For a four-bar where $s+l=p+q$, if the longest link (l) is adjacent to the shortest link (s) an uncompressed change-point configuration is possible. If l is opposite of s then a compressed change-point mechanism is possible [2]. Figure 1.2 illustrates this conjecture. Both of these configurations are possible for a LEM with motion that resembles that of a four-bar mechanism. In their paper Parise et al. [2] also propose analogous postulates for five-bar mechanisms.

The main disadvantage of ortho-planar mechanisms is that they are change-point mechanisms; this change-point occurs in the planar state. For a change-point mechanism to be successfully implemented in a product it needs to enter the same configuration every time [2–5].

1.2.2 Compliant Mechanisms

LEMs utilize ortho-planar mechanisms' advantages and couple them with the advantages of compliant mechanisms. Compliant mechanisms are defined as mechanisms which gain some or all of their their motion through the deflection of their members rather than only through traditional pin joints and hinges [6]. Compliant mechanisms exhibit advantages over their rigid-body counterparts in particular applications. These advantages include increased precision, reduced weight, reduced wear, and part count reduction, which leads to less assembly, more reliable operation, and reduced cost. Due to the deflection of their members compliant mechanisms have the ability to store energy, thus eliminating the need for springs. Many compliant mechanisms are monolithic in fabrication, which is ideal for lamina emergent mechanisms.

The main challenges presented by compliant mechanisms are their nonlinear behavior and coupled motion and energy equations. If a compliant mechanism already exists, nonlinear FEA can be used to analyze the mechanism. However, nonlinear FEA is neither convenient nor practical for synthesis of new mechanisms. The pseudo-rigid-body-model (PRBM) has been developed as a simple and efficient way to deal with the coupled equations and nonlinear deflection of compliant mechanisms. The PRBM is useful in analysis of existing mechanisms, and is more effective in synthesis of new mechanisms. In the PRBM, compliant segments are replaced by rigid links and torsional springs that exhibit the same force-deflection characteristics as the compliant segment. The torsional springs and associated joints are placed at the approximate location of rotation, which location is determined by the boundary conditions. The spring constants are determined by the boundary conditions, geometric and material properties. Many PRBMs have been developed for different types of compliant segments.

1.2.3 Mechanisms Made from Planar Layers

Planar layers are often used in manufacturing. Processes such as forming and others allow three-dimensional shapes to be made from sheet materials. Origami and pop-up mechanisms found in pop-up books have been studied to varying extents [7]. The advantages of lamina emergent mechanism derived from their being manufactured in planar layers include their laminar construction and compact storage.

The laminar construction of LEMs provides them benefits in manufacturing and performance. Due to their initial planar state, LEMs can be manufactured out of sheet material. This provides lamina emergent mechanisms a considerable advantage due to the low cost and availability of sheet material. Manufacturing options for sheet material are numerous and many are economical [3]. Some of the more popular of these process include stamping, fine blanking, water jet cutting, laser cutting, plasma cutting, and wire electrical discharge machining (EDM).

Many applications benefit from mechanisms which are compact in one of their states. Reasons for this could include but are not limited to shipping costs, applications where available space is limited, and applications where devices are needed to be small during positioning then expand at the location of function. LEMs offer opportunities to meet these requirements due to their planar initial state.

1.2.4 Metamorphic Mechanisms

Some LEMs are change-point mechanisms. Change-point mechanisms are characterized by the ability to go into alternate configurations when exiting the planar state [8]. LEMs should be designed in a way that this uncertainty is controlled and predictable motion occurs every time.

One way to avoid the challenges of change-point mechanisms is through the use of metamorphic mechanisms. A metamorphic mechanism is a "mechanism whose number of effective links changes as it moves from one configuration to another" [9]. Many metamorphic mechanisms are fabricated in a planar change-point state, then through a metamorphic process are able to become a non-change-point mechanism. Metamorphic mechanisms can be grouped into more specific categories. These include but are not limited to foldable/erectable mechanisms, deployable structures, and compliant ortho-planar metamorphic mechanisms [3].

Foldable/erectable mechanisms can be designed for various levels of complexity. A simple example would be a cardboard box. The box begins as a flat sheet of cardboard with a few creases in it for the joints. In this state it has many degrees of freedom. As each flap and side is put in place the degrees of freedom are reduced until the box has no degrees of freedom and is a structure [9, 10]. Deployable structures can also be metamorphic mechanisms. During their transition between compact and deployed states the length between joints must change; this has been defined as a change in number of effective links [9]. Compliant ortho-planar metamorphic mechanisms (COPMM) exhibit the closest similarity to LEMs. As their name suggests, COPMMs gain at least some of their mobility from the

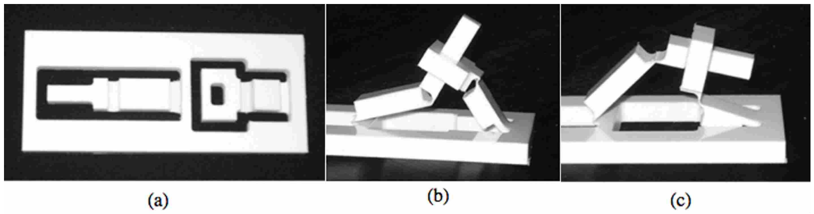


Figure 1.3: COPMM bistable light switch in its (a) planar state, and (b and c) after its metamorphic operation [3].

deflection of their members (compliant). They also can have all of their links simultaneously located in the same plane (ortho-planar). During operation their effective number of links will also change (metamorphic mechanism). The COPMM bistable light switch shown in Figure 1.3 illustrates a metamorphic transformation [3].

1.2.5 MEMS

In addition to macro applications microelectromechanical systems (MEMS) are an ideal place to implement LEMs. Due to limited manufacturing options, MEMS devices are usually fabricated out of planar layers. Although most MEMS devices have their motion in the plane, there has been research done in making and assembling ortho-planar structures [11, 12] and also in ortho-planar mechanisms. Unique among the ortho-planar mechanisms are the Micro Helico-Kinematic Platform, the Spherical Bistable Mechanism, and the Three-degree-of-freedom Platform [5]. These mechanisms use spherical kinematics to achieve their out of plane motion. Analysis has also been done for standard four-bar, slider-crank four-bar and compliant slider-crank mechanisms that all have ortho-planar motion [2]. An increased understanding of LEMs could lead to new MEMS applications with ortho-planar behavior.

Chapter 2

Fundamentals of Lamina Emergent Mechanisms

Mechanisms capable of complex motion can often be formed by a systematic combination of fundamental mechanisms. This chapter illustrates various flexible segments and their use in fundamental mechanisms. The mechanisms are grouped into three categories: closed loop planar, open loop planar, and spherical mechanisms. Highlighted in these groups are LEMs with motion that resembles planar and spherical four-bar mechanisms, and Watt and Stephenson six-bars.

2.1 Flexible Segments for LEMs

Useful and functional LEM components will have some segments with more compliance or flexibility than other segments. In order to form these fundamental mechanisms it is necessary that the compliant segments are understood, and behave with predictable motion. The linear deflection of a cantilever beam will be used as an example to illustrate how different parameters affect the compliance of a segment. The trends shown in this simple linear deflection example hold true for the larger non-linear deflections that lamina emergent mechanisms undergo. A cantilever beam with length L, thickness h, width b, and force F point loaded at the end is shown in Figure 2.1. The maximum deflection the cantilever beam can undergo before failure is given by

$$\delta_{max} = \frac{2SL^2}{3Eh} \tag{2.1}$$

where S is the yield strength, and E is Young's modulus. The vertical deflection for a given force F, is found by

$$\delta = \frac{FL^3}{3EI} \tag{2.2}$$

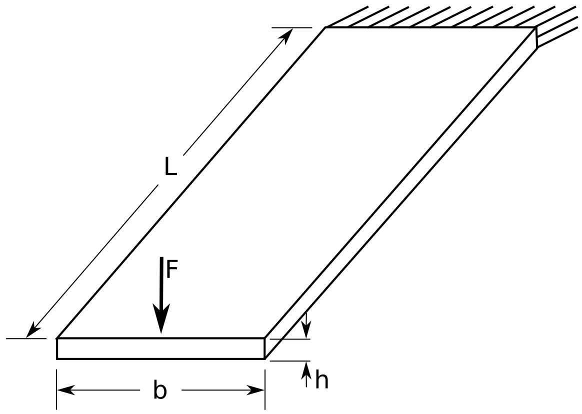


Figure 2.1: Cantilever beam

The moment of inertia for the cantilever beam is

$$I = \frac{bh^3}{12} \tag{2.3}$$

where b and h are the width and thickness of the beam as illustrated in Figure 2.1.

These equations can be broken into geometry, boundary conditions, and material. This is most easily seen by examining equation (2.1), where geometry corresponds to (L^2/h), boundary conditions to $(2/3)$ and material to (S/E). Each of these will be briefly discussed.

2.1.1 Geometry

Modifying the geometry is perhaps the most common method to change the stiffness of a segment. There are four main ways the geometry can be changed. The first three are done by changing certain dimensions of the segment: width, thickness, or length. The final approach is to change the segment's moment of inertia, or cross sectional shape.

Width

Changing the width, the dimension b in Figure 2.1, has a linear effect on the flexibility of the segment. As b changes linearly the moment of inertia I also changes linearly, which is in turn proportional to δ (see equation (2.2)).

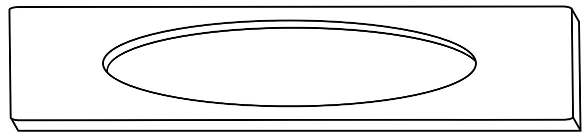


Figure 2.2: LEM segment with increased flexibility due to inside width reduction.

The width of a segment may be changed by removing material from the sides of the segment (Figure 2.5 illustrates this in the extreme case), the interior of the segment (Figure 2.2) or some combination of these changes. The reduction in width does not need to be symmetric. If the width of the segment becomes thin and the length of the segment is small, the compliant section resembles a three-degree-of-freedom joint. This is desirable if flexibility in many directions is important.

If the desired motion is solely out-of-plane, then reducing the width by removing material from the inside would yield the same out-of-plane flexibility but give better in-plane and torsional stiffness (see Figure 2.2).

Thickness

Changing the thickness of a member has a more dramatic effect on flexibility. The h term in equation (2.3) has a cubic effect on the segment's flexibility. Thus reducing the thickness by half will increase the flexibility eight times. Because LEMs are fabricated out of sheet material changing their thickness is not as straight forward as changing their width. Many of the manufacturing process proposed for fabricating LEMs are not able to create geometry of varying thicknesses out of sheet material. Thus a secondary operation would be required to reduce the thickness of certain segments, which is often undesirable because it will increase manufacturing costs and time to manufacture. However, a solution can be achieved with some manufacturing processes. For example if the LEMs were being manufactured by a punching process one of the punches could be set to compress or score the material. Figure 2.3 illustrates how this reduction in thickness may look in a LEM. Creating a reduction of thickness by this method will create a stress concentration along the scored segment. Care must be taken when designing a LEM using this technique so as to prevent premature failure.

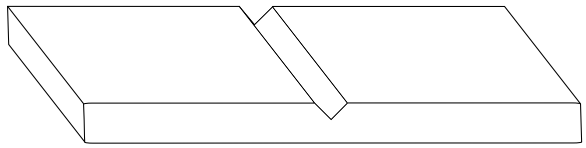


Figure 2.3: LEM segment with increased flexibility due to a reduction in thickness.

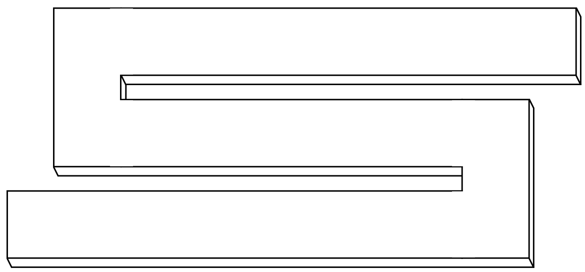


Figure 2.4: LEM segment with increased flexibility due to a length increase because of a switchback.

Length

Length is similar to thickness in that it also has a cubic effect on the flexibility. Referring again to the cantilever example, equation (2.2) shows the vertical displacement δ is proportional to L^3. Therefore, doubling the length will make the beam eight times more flexible.

There are a variety of ways the length of a segment can be increased. The trivial solution would be to simply make the segment longer in a straight line. The segment could also follow an arc between the end points making it slightly longer than a straight line between those points. Another method to increase its length while still maintaining a compact form is to use ”switchbacks,” as illustrated in Figure 2.4.

Cross Sectional Shape

It is also possible to increase or decrease a segment’s flexibility by changing the cross sectional shape. A well known example of this is a carpenters tape. The thin metal strip is in the shape of a partial circle rather than flat. This change of shape makes the tape stiffer when put in bending. Another example of cross sectional shape changes increasing stiffness is implemented in cardboard boxes. The rigidity of the box, transverse to the corrugation direction is greatly increased due to its shape.

2.1.2 Boundary Conditions

The flexibility of a segment can also be varied by changing its boundary conditions. This can be achieved by either changing its end constraints or the way in which the segment is loaded.

End Constraint

Changing the end constraints of a segment can have a large effect on the flexibility of a segment. The most common end constraints to lamina emergent mechanisms are fixed-free, fixed-guided, and fixed-fixed. Another type of end constraint, that can be approximated, is a pin joint.

For each of these end constraint examples the equation for vertical deflection, given a vertical force, F, at the end of a rectangular cantilever beam will be given. The equations for vertical deflection are of the same form in each of these examples. The generalized form is

$$\delta = \frac{FL^3}{cEI} \tag{2.4}$$

where δ is the vertical deflection, L is the length of the beam, E is Young's modulus of the material, I is the moment of inertia, and c is the scale factor which depends on the end constraints.

For the fixed-free condition the maximum vertical deflection is given by equation (2.2). Comparing equations (2.2) & (2.4), it can be seen for a fixed-free end constraint c is equal to 3. Compared to a fixed-guided section, which will be discussed next, the fixed-free loading is four times more flexible. Fixed-free provides more flexibility than fixed-guided or fixed-fixed but it may be more difficult to implement.

The scale factor, c, for fixed-guided segments is 12. This means it is four times less flexible, or stiffer, than the same beam with a fixed-free end constraint.

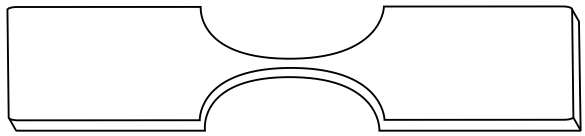


Figure 2.5: An example of how a pin joint could be approximated in LEMs.

The fixed-fixed end constraint can be used if a rigid segment is desired. Using a fixed-fixed end constraint will not insure a completely rigid segment, but it will significantly increase the stiffness. The scale factor, c, for fixed-fixed is 192. This makes a fixed-fixed beam 16 times stiffer than a fixed-guided beam, and 64 times stiffer than a fixed-free beam.

Pin joints are often used in mechanisms because they allow large rotation in one direction but limit motion in all other directions. This can be approximated in LEMs by reducing the width of a short segment. Figure 2.5 illustrates one possible configuration. As the width of the segment becomes small, its rotation will resemble that of a pin joint. That rotation, however, can be about any of its three axis unless it is constrained to only rotate in the desired direction. Another configuration that gives improved in-plane stiffness while maintaining the same out-of-plane flexibility, removes the material from the center, as was illustrated in Figure 2.2.

Loading

How a segment is loaded also affects its deflection characteristics. The most expected loading types in LEMs are bending, axial, torsion, and combined loading.

All of the examples given thus-far, have loaded the flexible segment in bending. Achieving pure bending is difficult and most bending will also be accompanied by axial or other loading. To achieve the desired deflection with the minimum input force, a LEM should be designed to put its flexible segment into bending.

If axial loading is applied along with bending the flexibility of the segment will change. Tension can cause axial stiffening and is one means of making a segment behave as if it were more stiff. On the other hand, compression tends to cause a segment to behave as if it were more flexible.

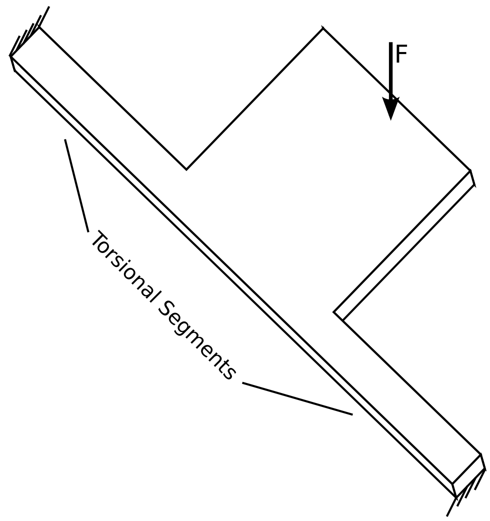


Figure 2.6: An example of how LEMs could be loaded in torsion.

Torsion can also be used to achieve out-of-plane motion. Torsion is achieved by applying a moment about the longitudinal axis of a segment. An example of how this can be approximated in LEMs is shown in Figure 2.6. The induced rotation, θ, and max shear stress, τ, for a given torque, T are,

$$\theta = \frac{TL}{KG} \tag{2.5}$$

and

$$\tau_{max} = \frac{T}{Q} \tag{2.6}$$

L is the length of the torsional segment, and K and Q are factors that are geometry specific. Most torsional hinges in LEMs will be rectangular in shape. K and Q for a rectangular cross section of width w and thickness t are

$$K = wt^3 \left[\frac{1}{3} - 0.21 \frac{t}{w} \left(1 - \frac{t^4}{12w^4} \right) \right] \tag{2.7}$$

and

$$Q = \frac{w^2 t^2}{3w + 1.8t} \tag{2.8}$$

The width dimension is always considered the larger of the two, with the thickness being the smaller. The max shear stress occurs at the mid point of the wider side, w, on the surface.

In the case where w is equal to t, the torsional hinge becomes square and equations (2.7) & (3.15) reduce to

$$K = \frac{w^4}{7.11} \tag{2.9}$$

and

$$Q = \frac{w^3}{4.8} \tag{2.10}$$

with the max shear stress located at the mid point of each of the sides [13].

As the width of the torsional section increases, maintaining a constant thickness, the max shear stress is reduced.

Similar to beams in bending, if more flexibility is desired (a larger rotation of θ), reducing the thickness provides the most efficient result. Fortunately, in torsion the thickness of the torsion bar is considered the thinner of the width and thickness. Thus the thickness of a torsional segment can be reduced by making its width thinner than the thickness of the sheet material from which it is fabricated.

2.1.3 Material

In addition to changing the geometry and the boundary conditions, the choice of material will also affect the flexibility. The S/E ratio in equation (2.1) corresponds to the choice of material. Materials that are better suited for LEMs have a higher S/E ratio. As the ratio of S/E increases, the flexibility also increases. When choosing a material it is possible to choose a material that is both flexible and strong.

2.2 Closed-chain Planar Mechanisms

With an understanding of the principles governing the design of flexible segments it is now possible to create basic LEM's with predictable deflection characteristics.

This section focuses on closed-chain planar mechanisms, including mechanisms whose motion resembles that of four-bars and six-bars with one degree of freedom. The following sections will discuss open-chain planar mechanisms and spherical mechanisms.

Closed-chain planar mechanisms are perhaps the most commonly used mechanisms due to their well understood analysis and predictable motion. Much research and time has been spent investigating planar kinematics. Many resources are available dedicated to such purposes including [8, 14–19]. The pseudo-rigid-body model allows the designer to take advantage of all of this knowledge when synthesizing LEMs for a specific application.

2.2.1 Four-bar

Four-bar mechanisms are perhaps the simplest and most common closed-chain planar mechanisms. They have been studied extensively, much has been published about them, and their synthesis is well understood. For this reason they are often the mechanism of choice for a designer.

Grashof laid the ground work for the most widely used classification system for four-bars [8], which can be described by the mathematical equation of Grashof's criterion:

$$s+l \leq p+q \tag{2.11}$$

where $s, l, p,$ and q are the shortest, longest, and the two intermediary link lengths, respectively. If equation (2.11) is satisfied the resulting mechanisms is classified as a Grashof mechanism and at least one of the links will be able to complete a full revolution relative to it's adjoining links. If the inequality is not satisfied, the mechanism is classified as non-Grashof.

Barker's [1] classification of four bars follows Grashof's but he further breaks them down into fourteen classes. Four Grashof, Four non-Grashof, and six change-point mechanisms. The PRBM of all LEMs will follow the special case of Grashof's equation where $s+l = p+q$. According to Barker's classification system they would fall into one of the six change-point mechanism categories shown in Table 2.1. Although the PRBM of LEMs fall into one of Barker's six categories, their motion will not completely follow the description of that type of mechanism. Because LEMs are compliant mechanisms, their motion derives from the deflection of their members rather than pin joints. Due to this, cranks that rotate a full 360 degrees are not feasible in most cases.

Table 2.1: Barker's complete classification of change-point four-bar mechanisms [1]

C.P. Class	Characteristic Length	Name	Symbol
1	$R_1 = s$	c.p. crank-crank-crank	CPCCC
2	$R_2 = s$	c.p. crank-rocker-rocker	CPCRR
3	$R_3 = s$	c.p. rocker-crank-rocker	CPRCR
4	$R_4 = s$	c.p. rocker-rocker-crank	CPRRC
5	two equal pairs	double change point	CP2X
6	$R_1 = R_2 = R_3 = R_4$	triple change point	CP3X

An example of a change point crank-rocker-rocker (CPCRR) four-bar with torsional hinges is illustrated in Figure 2.7. Equations (2.5)-(2.8) and planar kinematics can be used to analyze the motion of this four-bar.

2.2.2 Six-bar

Although four-bar mechanisms are the most common examples of mechanisms with one degree-of-freedom motion, it is also possible to achieve this controlled motion with mechanisms of more links without requiring additional inputs. This section will present mechanisms resembling one-degree-of-freedom six-bar mechanisms. These will be grouped into the familiar Watt and Stephenson categories based on their pseudo-rigid-body model.

Watts

Watts mechanisms are characterized by having their two ternary links connected together. Within this Watt's category there are two inversions of the six bar mechanism. An inversion of a mechanisms occurs when a different link is connected to ground. Figure 2.8 illustrates a LEM with motion that resembles a Watts six-bar mechanism. Using its pseudo-rigid-body model and planar kinematics is motion can be analyzed.

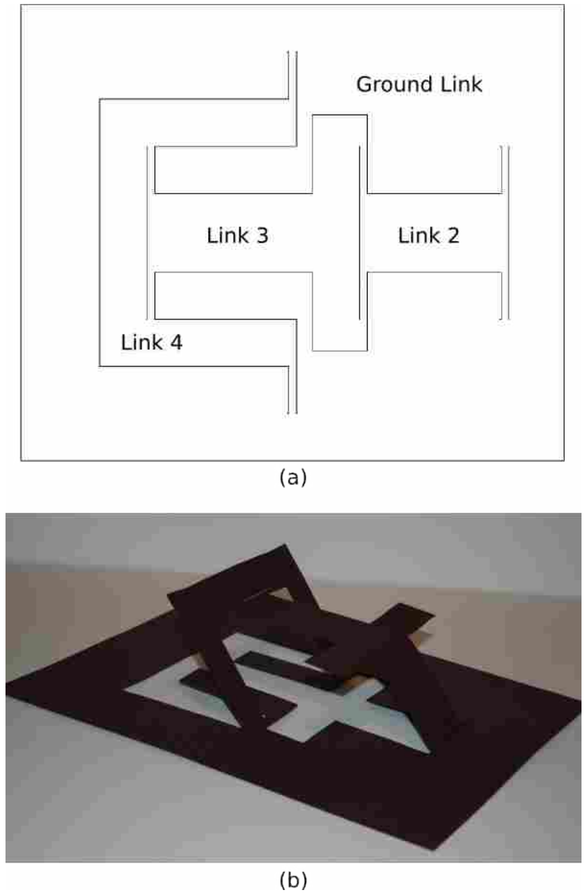


Figure 2.7: A CPCRR four-bar LEM with torsional hinges.

Stephenson

Stephenson six-bars have their ternary links separated by a binary link. This configuration allows for three inversions of the Stephenson six-bar mechanism. A LEM whose pseudo-rigid-body model is one of these inversions is illustrated in Figure 2.9. This LEM uses a reduction in thickness, through scoring, to increase flexibility at the proper locations.

2.3 Open-chain Mechanisms

As the focus of this chapter is on single degree-of-freedom mechanisms, open-chain mechanisms are mentioned only to show that these are also possible with lamina emergent mechanisms. Figure 2.10 illustrates an example of a two-link open-chain lamina emergent mechanism with a torsional hinge as one joint and the other joint resembling a spherical, three-degree-of-freedom, joint.

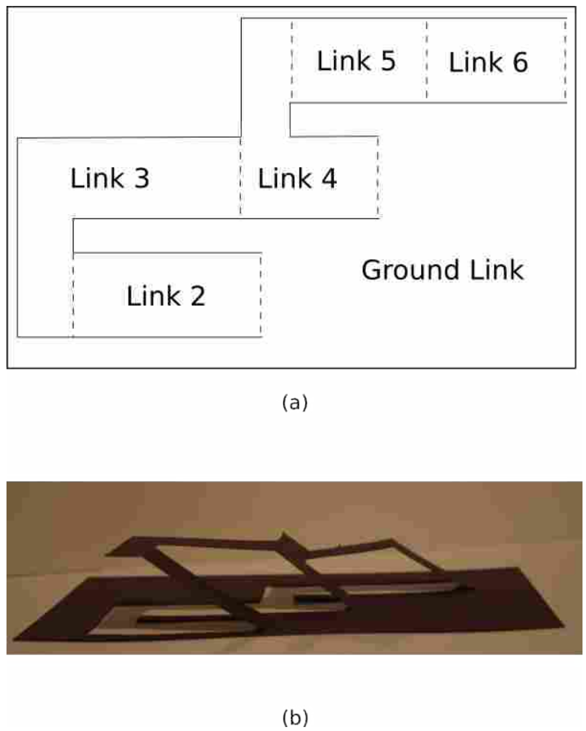


Figure 2.8: A LEM with motion that resembles a Watt six-bar mechanism

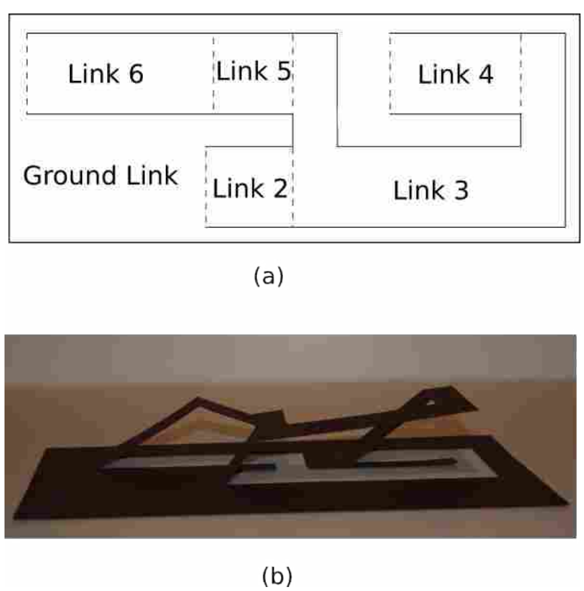


Figure 2.9: A LEM with motion that resembles a Stephenson six-bar mechanism

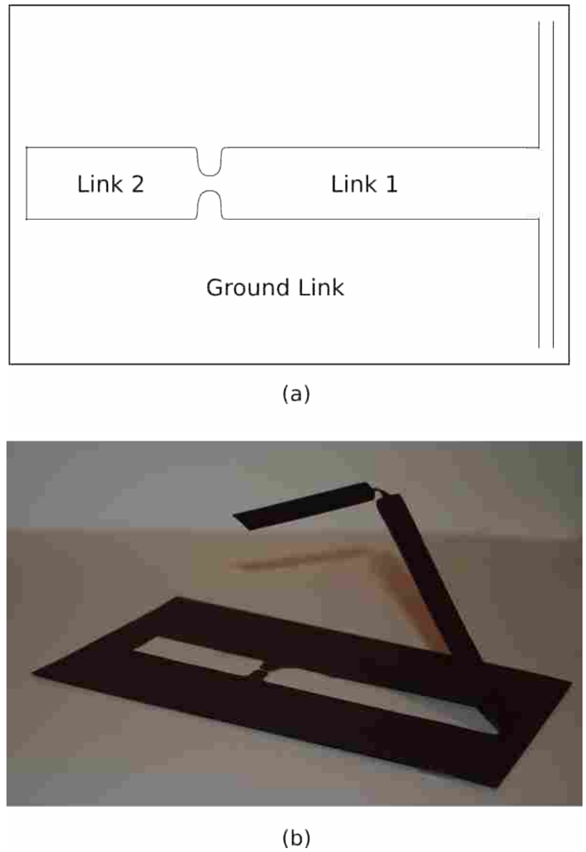


Figure 2.10: A two link open-chain mechanism, utilizing a torsional hinge and a joint with motion similar to a spherical joint.

2.4 Spherical Mechanisms

LEMs with motion that resembles that of spherical mechanisms also show promise, particularly for applications with a need to convert an in-plane rotation to an out-of-plane rotation or if 3D rotation is desired in space about a point (on the initial plane for LEMs) [5, 20]

Spherical mechanisms are more general than planar mechanisms because a plane can be considered a sphere of infinite radius. There are many similarities between spherical and planar analysis.

There are many good resources with more in-depth discussions of spherical mechanism analysis and synthesis including spherical trigonometry [21–27].

Spherical mechanisms are characterized by having all of the joint axes intersect at a single point. This condition must also remain true throughout the motion of the mechanism.

Visually this can be thought of as the center of a sphere with links located on the surface of the sphere. However, this is only to aid in visualizing the mechanisms, the links can be any shape so long as their joint axis's all intersect at a single point. This gives the designer additional freedom when synthesizing a new mechanism.

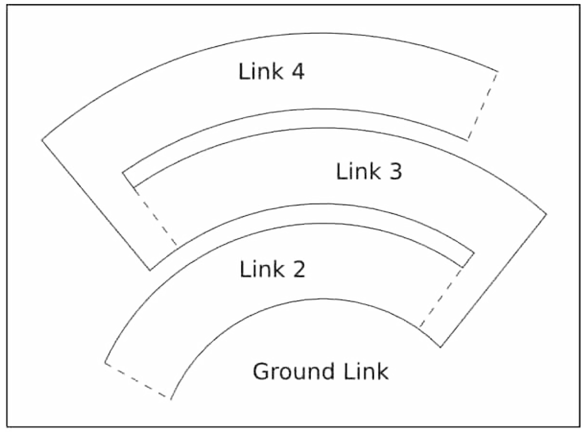
The relationship between link lengths and angles can be determined using spherical trigonometry, which is not described here. The main difference between planar and spherical trigonometry is that the surface is now curved instead of planar. The spherical equivalent of a straight line is a circle. Because links follow an arc instead of lying in a plane it is more convenient to measure link lengths as an angle rather than the arc length of the link. The main advantage of measuring link lengths as angles is it allows the mechanisms to be scaled to any size without changing performance. The angle between links is the dihedral angle between the two planes containing the great circles of which the links are a part.

Pure sliders do not exist in spherical mechanisms, but a similar counterpart does. If one end of a link is fixed to the point that the axis's of all of the other links pass through, and the other is on the surface of the virtual sphere then that joint will act as a slider traversing part of an arc.

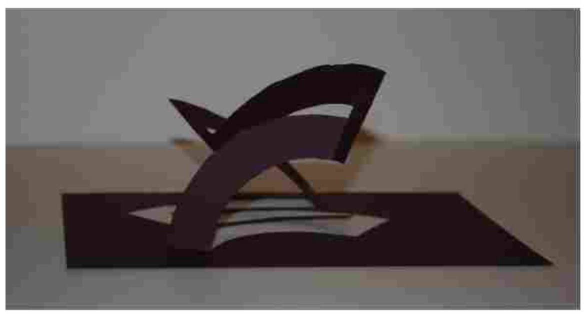
The following sections illustrate with an example a lamina emergent spherical four-bar mechanism, and a lamina emergent spherical slider-crank mechanism.

2.4.1 Four-bar

A lamina emergent spherical four-bar mechanism is illustrated in Figure 2.11. In order to achieve a design that can be made from a lamina, each link consists of an arc from concentric circles of different radii. In order to make a four-bar three concentric circles are needed. The link layout has a similar appearance to the switchback design presented earlier. However, in this case the switchbacks are not to increase flexibility but to allow for multiple links to lie in the same plane, all with joint axes that intersect at a common point in that plane (i.e. the center of those spheres).



(a)



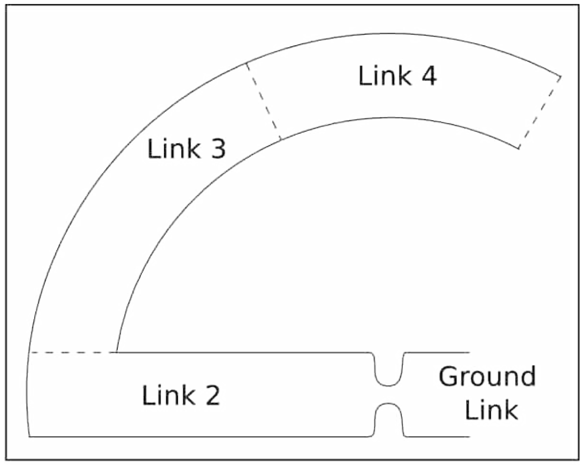
(b)

Figure 2.11: A lamina emergent spherical four-bar mechanism

Grashof's rules can apply to spherical four-bars [27]. The only additional requirement is that the sum of any two link lengths must be less than 180° [23]. In essence, this states that the four bar is contained within one hemisphere.

2.4.2 Slider-crank

Although pure translation does not exist in spherical mechanisms, if one of the links is pinned at the center of the sphere the motion of that link will behave similar to a slider. Figure 2.12 illustrates a lamina emergent spherical slider-crank mechanism. Lusk et al. has demonstrated at the MEMS level that a slider-crank with traditional joints (rather than compliant) can be combined with a Young mechanisms to create a spherical bistable slider crank mechanism [28].



(a)

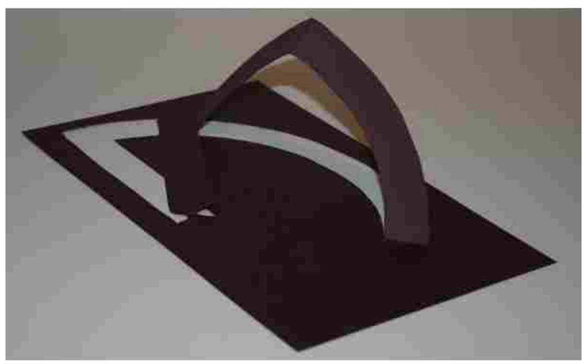

(b)

Figure 2.12: A lamina emergent spherical slider-crank mechanism

2.5 Conclusion

Lamina emergent mechanisms are mechanisms created from planar material with motion out of that plane. They exhibit many benefits including reduced manufacturing costs and minimal volume when in the planar state. In addition to presenting the principles governing flexible segments, six fundamental LEMs are presented as components or building blocks for LEMs capable of more complex motion. These were grouped into three categories, planar mechanisms, open chain mechanisms, and spherical mechanisms.

Chapter 3

Lamina Emergent Torsional Joint

Many applications benefit from mechanisms not assembled with traditional joints (e.g. pin joints, sliders, cams) and compliant components may be utilized to generate motion via elastic deflection of flexures. Part of the challenge in designing mechanisms with compliant joints, or flexures, comes in finding a suitable joint which gives the range of motion and force-deflection characteristics required. This thesis introduces the Lamina Emergent Torsional (LET) Joint as an addition to the compliant joints available to engineers designing complaint mechanism systems. The LET Joint, shown in Figures 3.1 and 3.2, is made from a single planer layer but provides rotational motion out of that plane, it is capable of large angular deflection, and is well suited to both macro and micro applications. Under some conditions the LET Joint can exhibit motion in directions not desired if certain off-axis loading of the joint occurs (e.g. compression/extension of the joint may be possible if the actuation force has an axial component). The symmetry of the joint allows each individual torsional bar to go through less than the total joint motion, thus reducing the stress in each torsional member.

3.1 Compliant Joints

Compliant joints, or flexures, gain their motion through the deflection of the joint material, rather than multiple joint surfaces rotating and/or sliding relative to each other. Because compliant joints are monolithic they have no backlash from joint clearances or friction from contacting surfaces. Energy is stored in the material as the joint flexes. This energy can then be used advantageously by designing the joint to exhibit desired force-deflection characteristics without additional springs. Compliant joints are often used when creating multi-stable mechanisms capable of having two or more equilibrium positions.

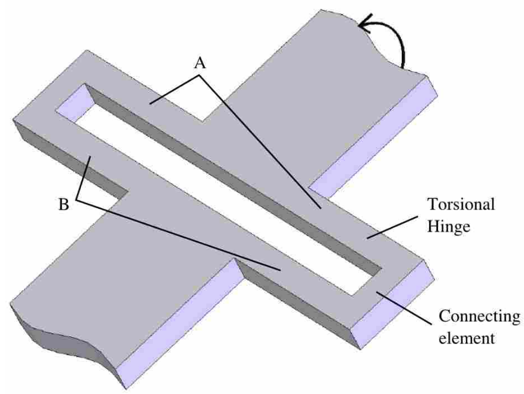


Figure 3.1: An example of an outside LET Joint.

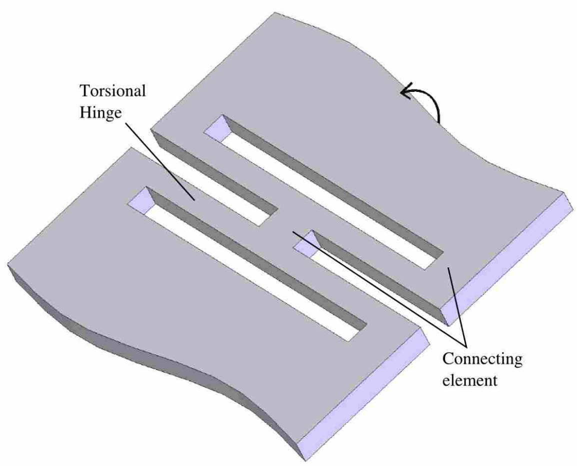


Figure 3.2: An example of an inside LET Joint.

Although compliant joints exhibit many benefits, they are often limited in their range of motion, and are generally harder to design, than their rigid body counterparts, due to the coupled motion and energy equations that govern their behavior. Much has been published to aid in the design of compliant joints [6, 29, 30], including works investigating the design of flexures, or notch type compliant joints [29, 30]. One approach for compliant mechanism design is the use of the pseudo-rigid-body model (PRBM). The PRBM analyzes compliance by replacing the compliant segments with rigid links and torsional springs at the proper locations [6]. Traditional kinematics, along with the known torsional spring reactions, can then be used to analyze the motion and energy storage of the mechanism.

Benefits in addition to those of compliant mechanisms can be gained if the compliant joint is manufactured from planar layers. This is due to the low cost and economical manufacturing methods associated with sheet goods. For micro applications, planar fabrication makes the joint compatible with micro fabrication methods. Joints fabricated from planar layers can be very compact due to their planar initial state. Compliant mechanisms manufactured from sheet goods with motion out of the plane of manufacture have been classified as "lamina emergent mechanisms" (LEMs) [31]. The LET Joint exhibits the advantages of LEMs.

Much work has been done in developing compliant joints for micro and macro applications, only a few of which are highlighted here. Trease et al. [32] investigated large displacement compliant joints, where they proposed a new joint for translation and another for rotation. They also benchmarked four large-displacement compliant translation joints and ten large-displacement rotational joints.

Many compliant joints have been designed to simulate the rotational motion of a pin joint. The cross-axis flexural pivot uses two crossed bands to provide the rotational motion. The stresses and motion are well defined for small deflections [33–36]. The split tube flexure investigated by Goldfarb and Speich [37] allows for rotation about the tubes longitudinal axis while maintaining most of its rigidity about the other axis.

Sometimes it is also useful to have a pin joint in the middle of a link, such as is the case with scissors, pliers, and pantographs. The Q-Joint, or quadrilateral joint, provides a compliant solution for this challenge [38].

Often compliant joints are susceptible to buckling and other undesired motion if placed in compression. Guerinot et al. [39] describes how the principles of isolation and inversion can help to overcome challenges of compliant joints in compression.

MEMS frequently use compliant and non-compliant joints to achieve their desired motion. Some of the non-compliant options available when using multilayer surface micromachining methods include the in-plane pin joint, the out-of-plane pin joint, and the scissor hinge [40–44]. Although these joints continue to be successfully used, they suffer from backlash inherent in the typical micro-fabrication processes. Many compliant joints have also been used successfully in MEMS, including torsional hinges, long flexible segments, folded-beam suspensions, and others.

The LET Joint provides designers with a joint capable of large angular motion while maintaining its ability to be implemented in a wide variety of macro and micro applications. This is due to its laminar construction and fully compliant nature.

3.2 Lamina Emergent Torsional (LET) Joint

When designing for motion out of the plane of fabrication it is advantageous to design the linkages and associated joints so that the forces on the out-of-plane members are balanced, thus reducing the off-axis loading and parasitic motion. This can be done with torsional hinges by combining two torsional hinges, with one on each side of the link as shown in Figure 3.1. In this configuration the two torsional hinges labeled “A” in Figure 3.1 act as springs in parallel. The torsional hinges labeled “B” also are in parallel. Sets A and B then act as springs in series.

The two parallel sets of torsional hinges are connected by connecting elements (Figure 3.1). As the hinge actuates, these short segments will be put into bending, while the remainder of the joint undergoes torsion. The significance of this bending in the overall motion of the joint will depend on the stiffness of those connecting elements.

It is also possible to design a LET Joint where the torsional hinges do not extend beyond the width of the link, as shown in Figure 3.2. In designing the LET Joint with the torsional hinges on the inside, new connecting elements are introduced that are put into bending during hinge motion. This type of joint is an “inside” LET Joint.

Either the outside (Figure 3.1) or inside (Figure 3.2) configuration of the LET Joint can be used for connecting links to ground or for a link-to-link connection, as shown in Figure 3.3. It may not always be advantageous to use both sets of springs (“A” and “B” in Figure 3.1). A “half” LET Joint, which only uses two torsional hinges connected in parallel, can also be used.

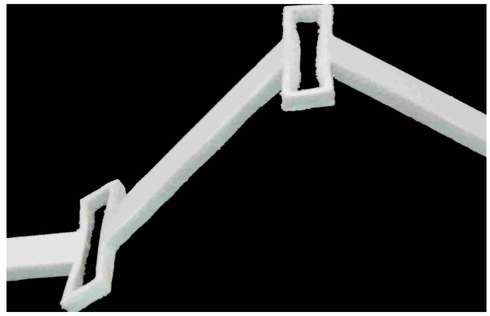


Figure 3.3: An example of a LET Joint connecting one link to another link.

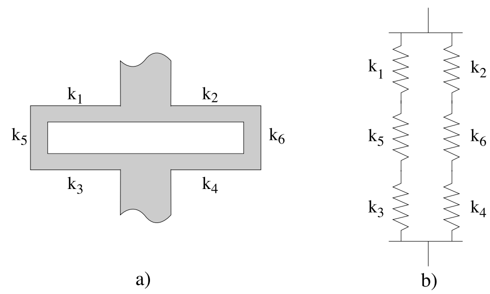


Figure 3.4: a) The LET Joint and b) its analogous linear coil spring model illustrating how the springs combine in series and parallel.

3.2.1 Stiffness Modeling

The force-deflection characteristic for a LET Joint is found by combining all of the individual spring constants into a single equivalent spring constant, k_{eq}, such that

$$T = k_{eq}\theta \tag{3.1}$$

where T is the total torque on the joint, k_{eq} is the equivalent spring constant and θ is the angle of rotation of the joint, in radians.

To find the equivalent spring constant, k_{eq}, the elements in the LET Joint will be combined using the appropriate spring system. Figure 3.4 shows the LET Joint with its individual spring constants labeled and the analogous spring system. Combining k_1, k_3, k_5

in series and k_2, k_4, k_6 in series, then combining those in parallel yields

$$k_{eq} = \frac{k_1k_3k_5}{k_1k_3 + k_1k_5 + k_3k_5} + \frac{k_2k_4k_6}{k_2k_4 + k_2k_6 + k_4k_6} \tag{3.2}$$

Most applications will utilize a symmetric LET Joint where the torsional joints are equal ($k_1 = k_2 = k_3 = k_4$) and the connecting elements in bending are equal ($k_5 = k_6$). For this case, equation (3.2) reduces to

$$k_{eq} = \frac{2k_T^2 k_B}{k_T^2 + 2k_T k_B} \tag{3.3}$$

where k_T refers to the joints in torsion and k_B to those in bending.

If all of the torsional joints are equal ($k_1 = k_2 = k_3 = k_4$) and k_5 and k_6 can be considered rigid, because their spring stiffness is much larger than that of the torsional joints, then equation (3.2) reduces to

$$k_{eq} = k_T \tag{3.4}$$

where k_T is the spring constant of one of the torsional joints. This result is convenient because the equivalent spring constant is the spring constant of one of the springs but the deflection required by each torsional hinge was cut in half, also reducing the stress by half.

A similar procedure leads to the equivalent spring constant for the inside LET Joint. Figure 3.5 shows the inside LET Joint with its analogous spring system. The general form for the inside LET Joint's equivalent spring constant is

$$\frac{1}{k_{eq}} = \frac{1}{\frac{k_1k_5}{k_1+k_5} + \frac{k_2k_6}{k_2+k_6}} + \frac{1}{k_9} + \frac{1}{\frac{k_3k_7}{k_3+k_7} + \frac{k_4k_8}{k_4+k_8}} \tag{3.5}$$

If all of the torsional springs are equal ($k_1 = k_2 = k_3 = k_4$) and all of the segments in bending equal ($k_5 = k_6 = k_7 = k_8 = k_9$), then equation (3.5) becomes

$$k_{eq} = \frac{k_T k_B}{5k_T + 4k_B} \tag{3.6}$$

with k_T referring to the joints in torsion and k_B to those in bending.

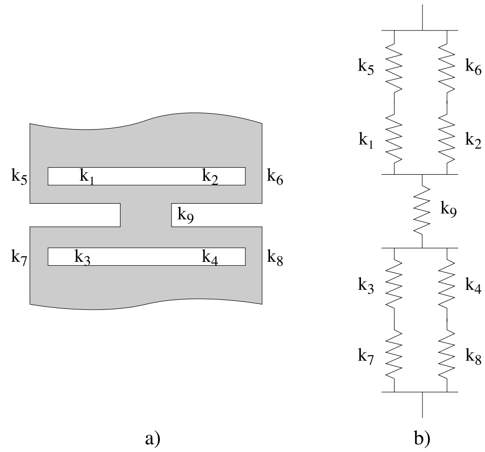


Figure 3.5: a)The LET Joint and b) its analogous linear coil spring model illustrating how the springs combine in series and parallel.

If the links put into bending can be considered rigid and all of the torsional hinges have the same spring constant, then equation (3.5) reduces to

$$k_{eq} = k_T \tag{3.7}$$

which is the same result as that for the outside LET Joint in equation (3.4).

The spring constant for each individual torsional hinge can be found by

$$k_T = \frac{K_i G}{L_i} \tag{3.8}$$

where L_i is the length of the torsional segment, G is the modulus of rigidity, and K_i is a parameter associated with the cross sectional geometry. K_i is analogous to J, the polar moment of inertia, for circular cross sections.

Multiple equations have been proposed to model rectangular cross-sectional geometries in torsion[1] . The elasticity solution in terms of an infinite series was given by

[1]This section describing the different ways to model the rectangular cross-section of a torsion beam was contributed by Dr. Guimin Chen.

Saint-Vennant [45] as

$$K_i = wt^3 \left(\frac{1}{3} - \frac{64}{\pi^5} \frac{t}{w} \sum_{i=1,3,5,\cdots}^{\infty} \frac{\tanh \frac{iw\pi}{2t}}{i^5} \right) \tag{3.9}$$

where, ‘tanh’ is the hyperbolic tangent function.

Roark and Young [13] gave an approximate equation for K_i, which can be expressed as

$$K_i = wt^3 \left[\frac{1}{3} - 0.21 \frac{t}{w} \left(1 - \frac{t^4}{12w^4} \right) \right] \tag{3.10}$$

Lobontiu [46] simplified Roark and Young’s equation by neglecting the high-power term reducing it to

$$K_i = wt^3 \left[\frac{1}{3} - 0.21 \frac{t}{w} \right] \tag{3.11}$$

It should be noted that all the equations above require that the width dimension, w, is always larger than the thickness t ($w \geq t$). This is especially important when a variable cross-sectional beam is used, that would require the equation’s variables to switch mid-beam, such as a tapered bar. Figure 3.6 shows that the errors between the series solution and the errors of equation (3.10) are less than 0.5 percent. The accuracy of equation (3.11) decreases as t/w approaches 1.

Hearn [47] proposed the following approximate equation:

$$K_i = \frac{2t^3w^3}{7t^2 + 7w^2} \tag{3.12}$$

Because of the symmetric relation between t and w, this equation is not dependent on the relative magnitude of t and w. However, this advantage comes at a loss of accuracy in the equation (with errors up to 14 percent), as shown in Figure 3.6.

Equation (3.10) is straight forward and accurate, and will be used for the remainder of the thesis. Given equations (3.8) and (3.10), the spring constant for each individual torsional spring can be found.

The flexible segments put into bending can be modeled using the pseudo-rigid-body model (PRBM) as small-length flexural pivots [48]. As a small-length flexural pivot,

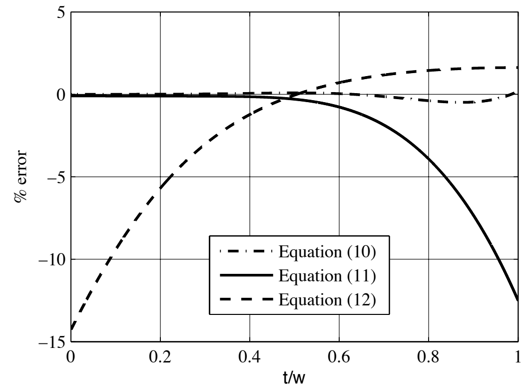


Figure 3.6: Errors compared to the series solution. The benchmark of the comparison was achieved by adding the first 500 terms, i.e. $i = 1, 3, 5, \cdots, 999$ in the series together, which is more than enough to get high precision results because the series converges rapidly.

its individual spring constant, k_B, can be found as

$$k_B = \frac{EI_B}{L_B} \tag{3.13}$$

where E is the modulus of elasticity, I_B is the beam's moment of inertia, and L_B is the length of the segment in bending.

These individual spring constants are then used to find an equivalent spring constant using the most appropriate of equations (3.2) to (3.4). This equivalent spring constant can be used with equation (3.1) to find the force-deflection characteristics of the LET Joint.

3.2.2 Stress

The stress in each of the compliant segments can also be determined. The shear stress in a non-circular torsion bar can be modeled as

$$\tau_{max} = \frac{T_i}{Q} \tag{3.14}$$

where T_i is the torque in the individual torsion bar and Q is a geometry dependent factor. For a rectangle, Q is

$$Q = \frac{w^2 t^2}{3w + 1.8t} \tag{3.15}$$

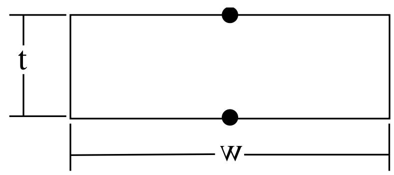


Figure 3.7: Cross sectional view of a rectangular torsion beam with the max shear stress points shown.

Once again, the width dimension is always considered larger than the thickness (i.e. $w \geq t$). The max shear stress occurs on the surface of the beam at the mid point of the longer side, w, as shown by the dots in Figure 3.7.

To find T_i, the torque through each of the complaint segments must be determined. Springs in series experience the same load. For the outside and inside LET Joints the following naming convention will be used for determining T_i: a subscript of R refers to the right-hand side of the joint, an L to the left-hand side of the joint. For an outside LET Joint, k_R would represent the equivalent spring constant for the right-hand side of the joint (i.e. k_2, k_4, and k_6 joined in series). k_L equals k_1, k_3, and k_5 joined in series. T_L is the torque through springs k_1, k_3, and k_5, thus $T_1 = T_3 = T_5 = T_L$. T_R is torque through springs k_2, k_4, and k_6 thus $T_2 = T_4 = T_6 = T_R$. The fraction of the total torque T that T_L and T_R experience are given by

$$T_L = \frac{k_L T}{k_{eq}} \tag{3.16}$$

$$T_R = \frac{k_R T}{k_{eq}} \tag{3.17}$$

If the outside LET Joint is symmetric (i.e. the segments in torsion are equal and the segments in bending are equal) the torque through the left and right half (T_L and T_R) will be half of the total torque (T).

In addition to the subscripts used above, T and B will also be used for top and bottom, respectively, in the inside LET Joint. Thus, k_{LT} would refer to the equivalent spring constant for the left top (i.e. k_1 and k_5 joined in series) or k_{RB} for right bottom (i.e. k_4 and k_8 joined in series). T_{RT} refers to the torque through the top right (i.e. the torque through springs k_2 and k_6). k_T is the equivalent spring constant for the four top springs, k_1, k_2, k_5, k_6, and k_B is the equivalent spring constant for the four bottom springs,

k_3, k_4, k_7, k_8. The torque through each of the springs can be found as follows: $T_1 = T_5 = T_{LT}$, $T_2 = T_6 = T_{RT}$, $T_3 = T_7 = T_{LB}$, $T_4 = T_8 = T_{RB}$, and

$$T_9 = T \tag{3.18}$$

$T_{LT}, T_{RT}, T_{LB}, T_{RB}$ are given by

$$T_{LT} = \frac{k_{LT} T}{k_T} \tag{3.19}$$

$$T_{RT} = \frac{k_{RT} T}{k_T} \tag{3.20}$$

$$T_{LB} = \frac{k_{LB} T}{k_B} \tag{3.21}$$

$$T_{RB} = \frac{k_{RB} T}{k_B} \tag{3.22}$$

where T is the total torque applied to the LET Joint.

Given the appropriate torque, T_i, the stress in each of the members in torsion can be calculated using equations (3.14) and (3.15).

For the segments in bending, the stress can be found using

$$\sigma_{max} = \frac{T_i c}{I} \tag{3.23}$$

where T_i is the torque or bending moment on the segment, c is the distance from the neutral plane to the outer surface, and I is the segment's area moment of inertia. The max stress will occur on the outer surface of the segment.

3.2.3 Compression / Extension of the LET Joint

If the forces actuating the LET Joint are not a pure moment or tangential to the beam at all times, parasitic motion of the joint could occur. Although this motion could be in any direction the LET Joint is prone to compression or extension of the joint. Ideally the LET Joint would have low torsional stiffness while maintaining high stiffness in the other directions. Compression/extension of the joint occurs as the torsional segments are placed into bending, as illustrated in Figure 3.8. The four torsional elements placed bending can be

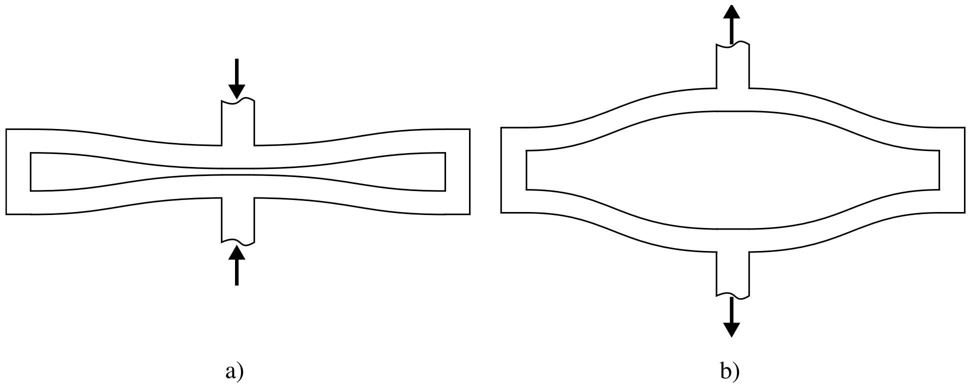


Figure 3.8: a) Compression and b) Extension of a LET Joint.

modeled using the pseudo-rigid-body model (PRBM) as fixed-guided beams [6]. If each of the torsional segments have the same geometry, and the short segments connecting the torsional segments can be considered rigid, the total joint stiffness in compression/extension will be equal to the stiffness of one of the beams. Figure 3.9 shows the PRBM of a fix-guided beam. The spring constant for one of the fix-guided beams, k_{fg}, is

$$k_{fg} = 2\gamma K_\theta \frac{EI}{L} \tag{3.24}$$

where γ and K_θ are often approximated as $\gamma = 0.85$ and $K_\theta = 2.65$ [6], E is the modulus of elasticity, I is the beam's area moment of inertia, and L is the length of the fix-guided beam. The distance the LET joint compresses/extends will be twice the displacement of one of the fix-guided beams. The distance the total joint compresses/extends is

$$d = 2\gamma L \sin(\Theta) \tag{3.25}$$

where Θ can be found using

$$\cos(\Theta)\gamma LP = 4k_{fg}\Theta \tag{3.26}$$

where P is the compressive/tensile force on the joint, L is the length of the beam, and k_{fg} is the spring constant for a fix-guided beam. The compression/extension of the joint will depend on the load direction and joint geometry.

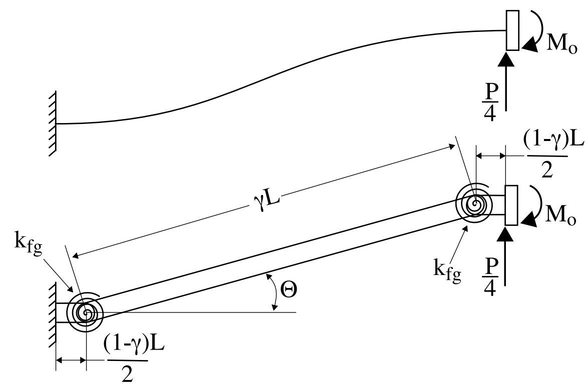


Figure 3.9: Pseudo-rigid-body model of a fix-guided beam

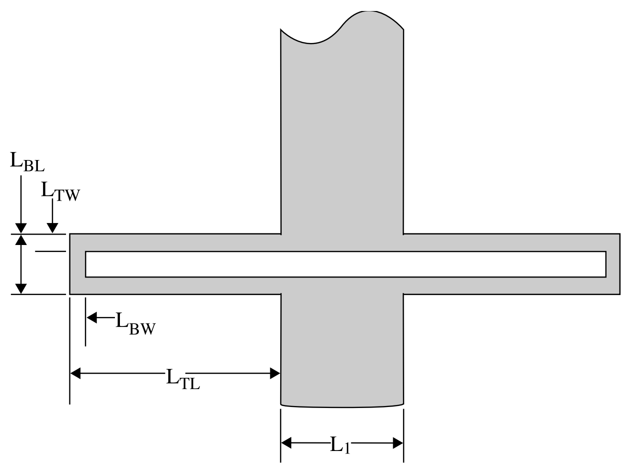


Figure 3.10: Spring steel prototype LET Joint

3.3 Example

To illustrate and verify the proposed LET Joint and model, a spring steel (AISI 1095) LET Joint was designed, fabricated, and tested. Figure 3.10 shows the different LET Joint parameters, and Table 3.1 gives their values.

The following example determines the force-deflection characteristics and the stress in each of the members of the spring steel prototype. Because of the symmetry of the joint only two spring constants need to be calculated, one for the torsional segments and one for the segments in bending. The torsional spring constant for a single torsional element can be found using equations (3.8) and (3.10). The modulus of rigidity (G) was found using

Table 3.1: LET Joint parameters

Variable	cm	in
L_{TL}	2.77	1.09
L_{TW}	0.49	0.19
L_{BL}	1.28	0.50
L_{BW}	0.50	0.20
L_1	2.55	1.01
Thickness	0.081	0.032

a modulus of elasticity of 205 GPa (29,700 ksi) and a poisson ratio of 0.29 ($G = \frac{E}{2(1+\nu)}$), resulting in G=79.37 GPa (11,511 ksi). K_i=7.92×10^{-5} cm^4 (1.90×10^{-6} in^4) is found using L_{TW} and the material thickness, where $L_{TW} = w$ and thickness=t in equation (3.10). Using K_i, G, and the length of the torsion member, L_{TL}, k_T=2.26 N-m/rad (20.04 in-lbs/rad) is calculated using equations (3.8).

k_B is found using equation (3.13) by substituting the modulus of elasticity, bending length (L_{BL}), and the moment of inertia, $I = \frac{wt^3}{12}$=2.23×10^{-5} cm^4 (5.35×10^{-7} in^4), which results in k_B=3.56 N-m/rad (31.51 in-lbs/rad). The two spring constants can then be used in equation (3.3) to calculated the equivalent spring constant of k_{eq}=1.72 N-m/rad (15.20 in-lb/rad). The output torque is now easily found using equation (3.1). At 20 degrees (0.34 rad) of joint rotation, the output torque is 0.60 N-m (5.31 in-lbs).

With the force-deflection now known, it becomes important to determine the joint range of motion prior to yielding. The stress in the torsional segments is found using equations (3.14), (3.15) and either equations (3.16) or (3.17). Using the material thickness and L_{TW}, equation (3.15) yields Q=9.90×10^{-4} cm^3 (6.04×10^{-5} in^3). Due to the elements in parallel of this LET Joint, the appropriate torque (T_i) through the torsional hinge will be half of the total torque applied to the joint. At an angle of 20 degrees, the stress calculated using equation (3.14) is τ_{max}= 303 MPa (43,914 psi). The stress in the sections in bending is found using equation (3.23). T_i is again half of the total torque, the moment of inertia I is the same as calculated above, and c is equal to half of the thickness. Substituting these into equation (3.23) yields σ_{max}=547 MPa (79,325 psi).

If a vertical force was acting on the joint as the joint deflects some of the force would contribute to extension of the LET Joint. If the force had a moment arm of 10 cm

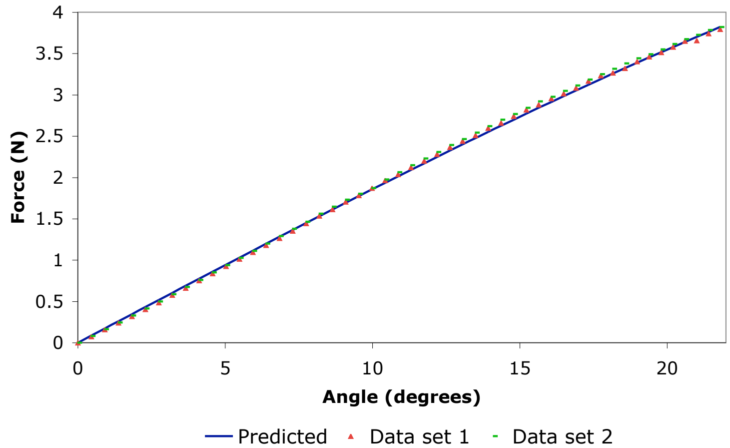


Figure 3.11: Force Deflection plot of the spring steel prototype.

(3.94 in) and the joint was deflected to 20 degrees, the axial force (P) on the LET Joint can be found as follows: $P=\frac{T}{10\text{cm}}\sin(20°)$=2.05 N (0.46 lbs). This can then be used along with equation (3.24) to determine the pseudo-rigid-body angle Θ (illustrated in Figure 3.9). With this angle and the torsion bar length (L_{TL}), equation (3.25) can be used to find the extension of the joint. Following these steps will yield: k_{fg}=2.71$\times$10^2 N-m/rad (2.40$\times$10^3 in-lb/rad), Θ=0.0026 degrees, and the total joint extension d=2.11$\times$10^{-4} cm (8.31$\times$10^{-5} in). Therefore, for this loading condition, geometry, and material, very little joint extension occurs.

3.3.1 Testing

Testing of the spring steel LET Joint was preformed by fixing the link on one end then displacing the link on the other side of the joint using a force transducer at a distance of 15.88 cm (6.25 in) from the center of the LET Joint.

Two sets of data were acquired, and both sets of data and the predicted values are shown in Figure 3.11. The solid line represents the model prediction based on the equations as illustrated in this section. The measured data is represented by the discrete points. As demonstrated in Figure 3.11, the measured data correlates well with the model's prediction.

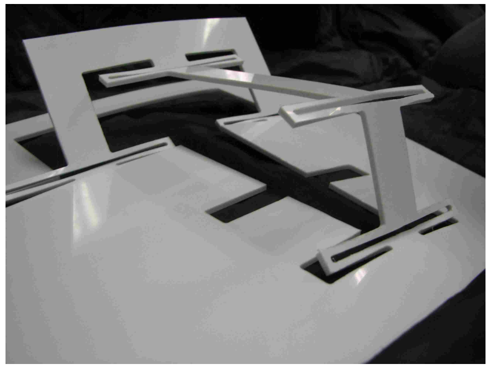


Figure 3.12: An example of a LET Joint used in a mechanism with motion similar to a four-bar mechanism.

3.4 Example Implementation in Mechanisms

The LET Joint is suited for a variety of applications where large angular motion is desired. Figures 3.12-3.16 illustrate the LET Joint being used successfully in a variety of mechanisms. Figure 3.12 illustrates its use in a mechanism with motion similar to a planar four-bar mechanism. Five LET Joints are used. One LET Joint is used for each of three joints of the four-bar joints, but two LET Joints are combined to represent one joint to ground. This approach used symmetric joints in parallel to accommodate for the geometric constraints associated with one of the joints.

Figures 3.13–3.14 utilizes the joint in a spherical compliant mechanism. An array of three spherical slider-crank mechanisms are arranged symmetrically to balance the forces and create the desired geometry. All three spherical slider crank mechanisms can be simultaneously actuated with a single input. Each slider crank mechanism contains three LET Joints for a total of nine LET Joints in the mechanism. Spherical components, including spherical slider-crank mechanisms, have been demonstrated in rigid-body microelectromechanical systems (MEMS) [20, 49, 50].

The LET Joint has also been demonstrated in MEMS. Figure 3.15 illustrates an inside LET Joint that is used to achieve out-of-plane motion. Figure 3.16 uses half LET Joints to create a MEMS four-bar mechanism.

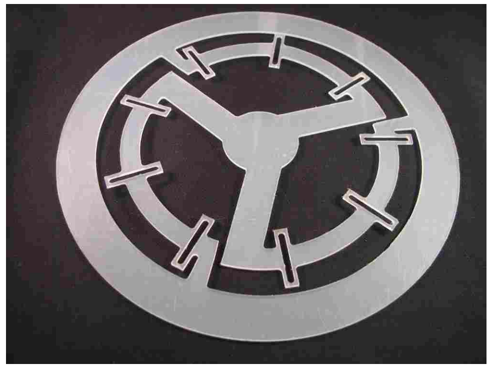


Figure 3.13: The planar position of a spherical mechanism utilizing LET Joints.

Figure 3.14: The actuated position of a LET Joint used in a spherical mechanism.

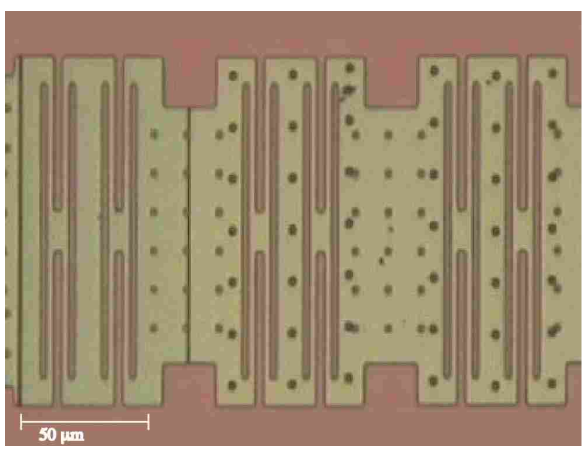


Figure 3.15: An example of an inside LET Joint in MEMS.

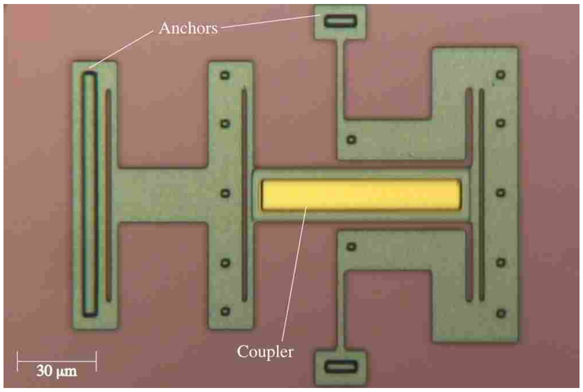


Figure 3.16: An example of a MEMS four-bar mechanism using half LET Joints.

3.5 Conclusion

The LET Joint has been presented as a feasible compliant joint. The inherent off-axis flexibility under some conditions makes it non ideal for some precision engineering devices, but its high flexibility and the ability to fabricate it from a single layer of material make it attractive for many compliant mechanism applications where large deflections are desired. The equations needed to determine the force-deflection characteristics have been presented. The model was illustrated in an example, the model results compared favorably to measured results.

The symmetric nature of the LET Joint allows each torsional hinge to undergo less deflection than the total motion of the joint. This allows for lower stress in the torsional members or a larger rotation with the same stress level when compared to a single torsional hinge. The symmetric design also reduces off-axis loading on adjacent links. LET Joints have been demonstrated in macro and micro applications, including devices made from several different materials.

Chapter 4

Conclusions and Recommendations

4.1 Conclusions

Lamina emergent mechanisms have been presented as mechanisms which show great promise for applications where weight and space requirements are limited. LEMs can also provide reduced manufacturing costs, reduced part count leading to reduced assembly time, and increased performance. The fundamentals governing LEM flexible segments have been presented, along with fundamental mechanisms utilizing these flexible segments. The fundamental mechanisms include closed- and open-chain planar mechanisms, and spherical mechanisms.

The LET Joint has been introduced, as a large-deflection single-layer lamina-emergent fully compliant joint. The equations modeling its motion and stress have been presented along with the possible compression/extension of the joint. The LET Joint has been demonstrated in various mechanisms ranging from the macro to the micro scale. Various materials, including polycrystalline silicon, polypropylene and spring steel have been used. A design example was provided and the prototype tested. The results comparing the force deflection characteristics of the prototype to the model showed a good correlation between the predicted and measured results.

4.2 Applications

Due to their compliant nature, compact form factor, laminar construction and economic manufacturing options, LEMs can be successfully implemented in a variety of applications. Electronic equipment, aerospace components, and many consumer goods would benefit from a mechanisms that is compact when not needed then could emerge from this compact form to perform the desired function. Other devices such as components for search and rescue, mechanisms for outer space or minimally invasive surgical instruments would benefit from LEMs' compact nature during transportation or positioning; the device could deploy once at the desired location. All of these applications would benefit from availability of sheet goods and the many economic manufacturing options. MEMS would also benefit from an increased understanding of lamina emergent mechanisms, as most MEMS are manufactured in planer layers.

4.3 Recommendations

This thesis introduces lamina emergent mechanisms as a new class of mechanisms exhibiting many advantages. As such, there is still much work to be done for LEMs to achieve their potential. Although not a comprehensive list, a few recommendations are made, that if followed will aid in the development of LEMs. Many of of these recommendations are directly correlated to the six step approach for the development of LEMs presented in chapter one: 1) understand the fundamentals governing flexibility, 2) form components, 3) create interaction strategies, 4) develop modeling and analysis approaches, 5) synthesize mechanisms, and 6) design, fabricate, and test demonstrators.

Due to their planar nature, LEMs often exhibit more parasitic motion than desired. Work investigating methods to increase LEMs off-axis stiffness is recommended. A wider selection of LEM joints would also reduce the time required to design a LEM for a specific application. The author recommends compiling a library of joints suitable for LEMs.

As LEMs are incorporated into increasingly more complex applications, an increased understanding of how LEMs interact with other LEMs and their surroundings would also be useful. With this understanding it will then be possible to create arrays of LEMs that work together seamlessly. Another area of importance to the development of LEMs is strategies for their actuation. As arrays of LEMs are developed it will become increasingly important to understand how to actuate the entire array.

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