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Building Procedural Fluency from Conceptual Understanding in Equivalence of

Fractions: A Content Analysis of a Textbook Series

Mark S. Nance

A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of

Master of Arts

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ABSTRACT

Building Procedural Fluency from Conceptual Understanding in Equivalence of
Fractions: A Content Analysis of a Textbook Series

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Master of Arts

During the last several decades, mathematics reform has emphasized the goal of ensuring that students develop both conceptual and procedural understanding in mathematics. The current mathematics reform, *Common Core State Standards for Mathematics* (National Governors Association and the Council of Chief State School Officers [NGA Center & CCSSO], 2010), promotes this goal, with procedural knowledge building upon a strong conceptual base. This study uses content analysis to investigate the extent and ways in which Houghton Mifflin Harcourt's *Go Math! K-8* (HMH, 2016) supports teachers in building procedural fluency from conceptual understanding when teaching equivalence of fractions.

Krippendorf's (1980) framework for content analysis guided this study. I identified *a priori* codes, and allowed for emergent codes, that characterize quality mathematical instruction. Careful analysis of the teacher editions of the textbook series revealed that, if the teacher instructions are to be followed with fidelity, students are not given opportunities to create and share their own strategies for solving tasks designed to help them learn equivalence of fractions. Neither are they given opportunities to make connections among strategies. All connections are introduced by the teacher. Although the teacher editions promote transitions from visual models to algorithms, they provide inconsistent use of problem-solving practice tasks and equal-sharing problems, two methods that are strongly supported by the research literature for developing procedural fluency from conceptual understanding in equivalence of fractions. Finally, the teacher materials include multiple instances in which the same or similar language and terms are used for mathematical and pedagogical practices found in mainstream research and professional literature, yet the practices were to be implemented in ways contrary to mainstream interpretations.

Overall, *Go Math! K-8* (HMH, 2016) provided little support to teachers in helping students build procedural fluency from conceptual understanding. A teacher-driven, rather than student-driven, approach to instruction was emphasized, thus minimizing opportunities for students to engage in the kinds of tasks and discourse recommended in the literature. The ways in which mathematical language and terms were implemented contrary to mainstream research interpretations can easily cause confusion among educators. The dearth of authentic problem-solving practice was inconsistent with quality mathematics instruction that supports students' conceptual and procedural understanding.

Keywords: conceptual understanding, latent content analysis, mathematics instruction, procedural fluency, reform perspective, textbook analysis

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CHAPTER 1

Introduction

Students' deep and connected understanding of mathematics is becoming increasingly

important to the improvement and development of many areas within our society, including the

economy, job growth, and science investigations (National Research Council, 2001). The

National Council of Teachers of Mathematics (NCTM, 2000) expressed that people who have

mathematical understanding will have an advantage in shaping the future. With mathematical

understanding becoming increasingly essential, teachers have the responsibility to prepare

students to be more mathematically competent, indicating the need for improved mathematics

instruction.

The last several decades of educational policy have seen many reforms proposed and

implemented for mathematics instruction. In the 1950s, after the creation of the National

Science Foundation, the call for improvements in the teaching and learning of mathematics was

pressed by Congress because of the lack of competition between the United States space program

with the space program of the Soviet Union during the Space Race (Stotsky, 2007; Walmsley,

2007). Later, in the mid-to-late 1970s, a push for a back-to-basics approach, or a command of

procedural fluency, was adopted because students were not showing proficiency in the basics of

mathematical computation according to national and international assessments (Walmsley,

2007).

The next major national call for improved mathematics instruction began with another

investigation into the state of American mathematics education. T. H. Bell, the Secretary of

Education during the 1980s, created the National Commission on Excellence in Education that

reported on the issues facing students, teachers, and administrators regarding mathematics and

other educational topics. The continued need for educational changes became widely apparent in 1983, when the National Commission on Excellence in Education (NCEE) made public its report entitled *A Nation at Risk* (NCEE, 1983; Walmsley, 2007).

Based on a study of research papers; testimonies from administrators, teachers and parents; and analyses of current educational issues, the commission's report centered around the nature of teaching and resulting achievement. An area of the deficiencies in the nature of teaching was mathematics instruction and, consequentially, the poor performance of students in mathematics. The NCEE (1983) described mathematics education as "being eroded by a rising tide of mediocrity that threatens our very future as a Nation and a people" (p. 9). The report also stated that the "curricula have been homogenized, diluted, and diffused to the point that they no longer have a central purpose. In effect, we have a cafeteria-style curriculum in which the appetizers and desserts can easily be mistaken for the main courses" (p. 26). The commission reported that in the majority of schools the teaching of mathematics was "haphazard and unplanned" (p. 30). The assessment revealed the "ineffective" (p. 29) use of teachers' instructional time and showed deficits in teacher content knowledge. The commission also summarized that the educational process had "disturbing inadequacies" (p. 21).

The lack of comprehensive mathematical knowledge, mathematical purpose, and student and teacher mathematical knowledge pointed out in *A Nation at Risk* (NCEE, 1983) became a catalyst for a series of national reform documents that described needed changes in mathematics instruction (Walmsley, 2007). NCTM created standards documents, beginning with *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). Subsequent NCTM standards documents addressed instruction (1991) and assessment (1995), leading to the publication of *Principles and Standards for School Mathematics* (2000), which was designed to

guide mathematics education into the new millennium. In 2001 the National Research Council (NRC) published *Adding It Up: Helping Children Learn Mathematics*. Singularly and in combination, these documents described how students could develop mathematical proficiency and comprehensive understanding of mathematics concepts. They also gave teachers potential guidelines to facilitate the development of students' understanding.

In order to bring clarity and understanding of the nature of mathematics, the teaching of mathematics, and greater student achievement, each document encouraged teachers to re-evaluate their use of traditional rote memorization in developing procedural fluency, such as drill-and-kill practices and step-by-step algorithms, and to promote more conceptual mathematics understanding among all students. This deviation from rote memorization reinforced the view that memorization of algorithms is not conducive to students' understanding of mathematics (Baumann, 2009). NCTM (2000) and NRC (2001) clarified how students demonstrate proficiency in mathematics instruction; they suggested that mathematically proficient students have the capability to problem solve, reason mathematically, connect ideas and methods, communicate what they are thinking, and represent their understanding. NCTM (2000) described students proficient in mathematics as those who develop a conceptual understanding along with factual knowledge and procedural facility. The same document also suggested that being flexible with one's knowledge is important in order to apply what is learned and connect it with another method or concept.

Similarly, the NRC (2001) described mathematical proficiency using five "intertwined" (p. 117) strands that, when implemented together, demonstrate mathematical understanding. The intertwined strands described proficiency as these five abilities:

1. *Conceptual understanding* or comprehension of mathematical concepts, operations and relations (p. 116)

2. *Procedural fluency* in performing procedures with flexibility, accurately, effectively and appropriately (p. 116)

3. *Strategic competence* in representing, solving, and formulating mathematical problems (p.116)

4. *Adaptive reasoning,* shown by communicating logical thought, reflection, explanation and justification of a problem (p. 116)

5. A *productive disposition* about the usefulness and worthwhileness of mathematics as one diligently works to build self-efficacy (p. 116)

More recent researched-based documents have also expressed the need for students to have both conceptual and procedural knowledge in mathematics (Crooks & Alibali, 2014; Fazio & Siegler, 2011; NCTM, 2014; Rittle-Johnson & Schneider, 2014). Heibert and Lefevre (1986) described students with conceptual understanding as having the ability to see relationships among various mathematical representations and models. As students develop conceptual knowledge, they can choose appropriate representations depending on the problem, make deeper mathematical connections, and have the capacity to check for reasonability of their solutions (NRC, 2001; Siegler et al., 2010).

Procedural fluency is best developed when it is built upon conceptual understanding (Cuoco, Goldenburg, & Mark, 2010; Empson & Levi, 2011; NCTM, 2014; Stein, Smith, Henningsen, & Silver 2009). Students who follow the rules and procedures of mathematics efficiently have strong procedural fluency when efficiency of procedures is rooted in conceptual understanding (NRC, 2001). Procedural fluency is shown to build from conceptual

understanding through appropriate practice (NRC, 2001). Practice is defined as opportunities to problem solve by using procedures that are based on the underpinnings of conceptual understanding (Isaacs & Carroll, 1999; NCTM, 2014; NRC, 2001). When students understand the procedures of mathematics, they become more versant in the structure of mathematics and can predict mathematical patterns in different standardized methods (NRC, 2001).

A deficiency in either conceptual understanding or procedural fluency has been found to have adverse effects on mathematical proficiency (Heibert & Lefevre, 1986; Rittle-Johnson & Schneider, 2014). NCTM (2000) emphasized that teachers must teach in a way that encourages a balance of conceptual and procedural knowledge. Lappan, Fey, Fitzgerald, Friel, and Phillips (2006), based on the earlier work of Schroyer and Fitzgerald (1985), suggested that an inquiry-based instructional model of Launch, Explore, Summarize would create the balanced opportunity that children require to show mathematical proficiency.

Although a balance of conceptual and procedural knowledge is needed, as stated earlier, students gain a greater understanding of mathematics when procedural knowledge is based on conceptual knowledge (Stein et al., 2009; Empson & Levi, 2011). NCTM's *Principles to Actions* (2014) described what teachers and students should do in order to build procedural knowledge from conceptual knowledge. Teachers can encourage student-led strategies by allowing students more opportunities to use their own reasoning to solve problems. Teachers can also facilitate student-led discussion in which students can explain and question each other about how they solved a problem. When teachers include meaningful opportunities for students to make connections between visual representations and procedures, procedural fluency begins to build upon conceptual understanding. Finally, NCTM (2014) suggested that teachers provide practice to guide students in developing procedures from concepts.

NCTM (2014) gave teacher suggestions but also included what students must be capable of doing in order to build procedural fluency from conceptual understanding. *Principles to Actions* recommended that students should be capable of explaining their solutions to justify their understanding of the method they used to solve the problem. Additionally, students must be flexible with the strategies and methods used and determine which strategies and methods are most appropriate as they analyze a problem. Students should also be able to generalize problems and decide if strategies can be transferred to a larger class of problems. Lastly, NCTM (2014) specified that students show they are building procedural fluency from conceptual understanding when they use procedures efficiently and accurately.

NCTM (2000; 2014) emphasized the need for students to develop both conceptual understanding and procedural fluency as they learn mathematics. Additionally, citing international competitiveness and college and career readiness, the call for students to understand mathematics both conceptually and procedurally was reinforced in 2009 when the National Governors Association created the Common Core State Standards Initiative for English language arts and mathematics (Polikoff, 2015). The *Common Core State Standards for Mathematics* (CCSS-M) comprises two sections: content standards and mathematical practice standards. These standards emphasize rigor, which is defined as understanding mathematics conceptually, procedurally, and showing application through real-world mathematics (National Governors Association and the Council of Chief State School Officers [NGA Center & CCSSO], 2010). The writers of the CCSS-M organized the standards to progress from conceptual knowledge to procedural knowledge across grade levels. This progression across grade levels also requires curriculum creators to make sure their resources match the expectations of CCSS-M.

Statement of the Problem

As more states adopted the standards found in the CCSS-M, textbook publishers and other companies that provide educational resources repeatedly claimed that they had adapted their curricula to align with the standards (Sawchuk, 2012). Textbooks have increasingly become a strong influence on how mathematics is taught and paced in the classroom (Jitendra, Deatline-Buchman, & Sczesniak, 2005). Although textbooks are used to varying degrees amongst teachers, some districts and schools expect that teachers will follow the textbook either with fidelity or as their main resource for teaching mathematics (Ball & Feiman-Nesmer, 1988). One important characteristic of alignment to the CCSS-M is rigor; the CCSS-M definition of rigor is recognized as pursuing "conceptual understanding, procedural skills and fluency, and application with equal intensity" (NGA Center & CCSSO, 2010, Key Shifts in Mathematics section, subsection 3). Thus, textbook alignment to the standards is vital if teachers are to meet the demands of teaching the textbook with fidelity while implementing the CCSS-M expectation for rigor.

Most current textbooks claim to adhere to the CCSS-M expectation for rigor. However, criticisms of textbooks include (a) their misalignment to CCSS-M standards, (b) the addition of prescribed scripts for each lesson, and (c) the lack of opportunities for students to present their own thinking (Jitendra et al., 2005; Polikoff, 2015; Sood & Jitendra, 2007). Schoenfeld (2002) presented a different critique: He stated that the majority of textbooks have been developed to meet the needs of larger states to turn a greater profit. Additionally, Schoenfeld reported that the names of textbook authors are often emphasized as a selling point, but chapters and content are frequently composed by various authors in a "piecework fashion" (p. 262).

It is important to determine if mathematics textbooks are designed to meet teachers' needs as a resource that connects conceptual knowledge and procedural fluency. Teachers must be aware of the strengths and limitations of the textbook in providing instructional support for implementing the teaching and learning standards for CCSS-M. Determining whether or not a textbook meets such needs requires analysis of specific domains of mathematics.

Fractions constitute a domain that transcends multiple grade levels. Students can demonstrate conceptual knowledge of fractions by recognizing fractions as quantities or values (Carpenter, Fennema, & Romberg, 1993; Empson & Levi, 2011; Fazio & Siegler, 2011; Siegler et al., 2010; Siegler, Thompson, & Schneider, 2011). Empson and Levi (2011) stated that conceptually cognizant students use their prior knowledge to build meaningful relationships among fractional ideas. Many of these connections are developed by learning fractions through equal-sharing problems across a multiple-year domain span. Teachers should help students build conceptual knowledge of fractions to "ensure a deep and enduring understanding of fractions and fractions arithmetic" (Fazio & Siegler, 2011, p. 23).

In this study, one textbook series, the series adopted for use in the district in which I am employed, will be examined: Houghton Mifflin Harcourt's (HMH) *Go Math! K-8* (2016). As stated previously, the role of understanding mathematics both conceptually and procedurally is critical in developing students' mathematical essence. One vital topic within fractions with which many students struggle is fraction equivalence (Empson & Levi, 2011; Fazio & Siegler, 2011). Because conceptual understanding and procedural fluency in equivalence of fractions are central to students' understanding of fractions, this topic, addressed in the core of grades 3-6, was the focus of my research.

Research Question

      The following question guided my research.

      Regarding equivalence of fractions, to what extent and in what ways does *Go Math! K-8* (HMH, 2016) provide teachers instructional support in building procedural fluency from conceptual understanding, grades 3-6?

Definition of Terms

      Several key terms used in this paper are defined below. While some of the terms have widely accepted definitions, others vary greatly in the definition used. For the latter category, the terms have been specifically defined for the purpose of this study.

      Conceptual understanding. Comprehension of mathematical concepts, operations, and relations (NRC, 2001).

      Equal-sharing problem. "A problem that involves a total number of items to be distributed to a given number of groups, usually people" (Empson & Levi, 2011, p. 8).

      Intention(s). What teachers and students are expected to learn, achieve, or do when using the textbook.

      Mathematical modeling. Using mathematics to solve authentic real-world problems that students may encounter in their own lives (Hirsch & McDuffie, 2016).

      Modeling mathematics. Using representations, such as visuals and models, to express mathematical ideas (Hirsch & McDuffie, 2016).

      Practice. Engaging in problem-solving tasks in which both conceptual understanding and procedural fluency are required to solve a problem (Isaacs & Carroll, 1999).

Problem solving. "Engaging in a task for which the solution method is not known in advance" (NCTM, 2000, p. 52); such a task is considered a **problem**.

Procedural fluency. "Performing procedures with flexibility, accuracy, efficiency, and appropriateness to the problem" (NRC, 2001, p. 116).

Student discourse. Discussion amongst students that builds "shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments" (NCTM, 2014, p. 10).

CHAPTER 2

Review of the Literature

Many researchers have emphasized the need for all students to have access to quality mathematics instruction. Representative of these ideas is Schoenfeld's (2002) statement that "conversations about the mathematical needs of American students must focus not only on what mathematics the students should learn, but also on how we as a nation can insure that all students have the opportunity to learn it" (p. 13). Because curricula available to students can have a powerful influence on their access to instruction (Ma, 1999; Schoenfeld, 2002), I have chosen to examine one element of the textbook series used in my district. This element, equivalence of fractions, is central to the fractions curriculum of the 3-6 grade span for which I provide leadership. The research question that guides my study is as follows

> Regarding equivalence of fractions, in what ways, and to what extent, does *Go Math! K-8* (HMH, 2016) provides teachers instructional support in building procedural knowledge from conceptual knowledge, grades 3-6?

The following literature review organizes relevant research for this study. First, research related to conceptual knowledge and procedural knowledge and how the two come together is discussed. The next section addresses conceptual and procedural knowledge within the domain of fractions, specifically in regard to equivalence. The CCSS-M reform (NGA Center & CCSSO, 2010) is described by introducing rigor, or building procedural fluency from conceptual understanding. Finally, the literature review explains how mathematics textbooks have adapted to include the CCSS-M content and practice standards for mathematics.

Conceptual Understanding

Researchers have consistently acknowledged the importance of conceptual understanding in mathematics when referring to student mathematical learning and understanding (Crooks & Alibali, 2014; NRC, 2001; Rittle-Johnson, Siegler, & Alibali, 2001). Conceptual knowledge has been defined as the ability to understand mathematical concepts and how they relate and connect to one another (Heibert & Lefevre, 1986). Rittle-Johnson et al. (2001) added to the definition by including the need to be "flexible" (p. 347) in one's understanding of the principles that are found within each mathematical domain. This flexibility can support students in generalizing mathematical principles.

Developing conceptual knowledge within mathematics has many benefits. The National Research Council (2001) reported that when students think conceptually, they can model "mathematical situations in different ways and know how different representations can be useful for different purposes" (p. 119). Such situations could occur when comparing fractions. Students with conceptual understanding may find it more appropriate to compare fractions using a number-line model when the context of the problem is measurement, while a context of sharing an object may be more adequately represented with a picture model (Fazio & Siegler, 2011).

The NRC (2001) also proposed that students who have conceptual understanding do not need to learn as many discrete ideas as do other students in mathematics. Instead, they are able to find the deeper connections that build relationships among mathematical ideas and algorithms. Students who learn mathematics conceptually are more capable of seeing the interrelatedness of facts and principles than are students who learn by memorizing individual algorithms and rules. Conceptual understanding has a vital role in allowing students to generalize, rather than memorize, mathematical principles (Crooks & Alibali, 2014). This ability to generalize

mathematical principles also supports students in finding the underlying principles that are used when learning specific mathematical algorithms or procedures.

Researchers have also found that students with conceptual understanding in mathematical domains are able to check for reasonability in their calculations (NRC, 2001; Siegler et al., 2010). Reasonability supports students in avoiding simple errors that can create discrepancies between solutions (NRC, 2001). For example, if they are multiplying the factors 8 and ½ and they arrive at an answer of 8/16, they would recognize the equivalence between the ½ (factor) and 8/16 (solution), thus realizing that an error had been made.

Heibert and Lefevre (1986) explained that in order to develop conceptual understanding students must begin building relationships from previous knowledge or among other pieces of information that they can connect. Conceptual understanding continues to grow as new information is linked to previous information. NCTM (2000) emphasized that conceptual understanding is also developed and reinforced by everyday experiences that are connected to prior learning and that students' conceptual understanding can be strengthened by engaging in tasks that are specifically designed to deepen and link to previous understanding. Furthermore, NCTM (2000) indicated that promoting classroom interactions, creating rules or conjectures, analyzing one's own thinking along with the thinking of others, and developing reasoning skills can also enhance students' abilities to develop conceptual knowledge.

Procedural Fluency

Hiebert and Lefevre (1986) defined procedural knowledge in two distinct ways: understanding formal visual representations and symbols, and completing mathematical problems using rules and algorithms. "The first part is sometimes called the form of mathematics" (p. 6). In other words, one can recognize the symbols of mathematics and

understand the "syntactic rules" (p. 6) that one must know in order to use these symbols correctly.

Heibert and Lefevre (1986) described the second part of procedural knowledge as "knowledge that consists of rules, algorithms, or procedures used to solve mathematical tasks" (p. 6). The NRC's (2001) definition of procedural knowledge included the ability to differentiate between the use of various procedures in order to use the most appropriate one based on the problem type. Being accurate and efficient in the use of procedures is also important (NRC, 2001). As procedures become more flexible, accurate, and efficient, procedural fluency emerges. For the purpose of this research I focused on Heibert and Lefevre's second definition of procedural knowledge.

Students who cultivate procedural knowledge perceive mathematics as a subject with structure (NRC, 2001). When the structure of mathematics is understood, students recognize and use patterns within step-by-step processes. As students become more versed in procedural knowledge, then the structures of mathematical algorithms become more predictable (NRC, 2001). Understanding the patterns, organization, and predictability in mathematical procedures can be a useful and important tool when solving worthwhile mathematical tasks: problems "that promote reasoning, problem solving, contain multiple entry points, and varied solution strategies" (NCTM, 2014, p. 17).

Where Students Begin

Historically, there have been differing opinions on the "causal relations" (p. 7) between conceptual and procedural knowledge (Rittle-Johnson & Schneider, 2014): Concept-first views maintain that conceptual understanding develops first, while procedures-first views support the initial development of procedural knowledge. Rittle-Johnson and Schneider (2014) suggested

that people who support an "inactivation view" (p. 6) believe that both knowledges are isolated from each other; an iterative view stresses that procedural fluency builds off conceptual understanding and vice versa. Current research indicates that students learn most effectively by developing conceptual understanding first, then building procedural fluency from that foundation of conceptual understanding (Cuoco et al., 2010; Empson & Levi, 2011; NCTM, 2014; Stein et al., 2009).

Importance of Gaining Conceptual Understanding and Procedural Fluency

Researchers acknowledge that in order to be proficient in mathematics one must have both conceptual and procedural knowledge (Crooks & Alibali, 2014; Fazio & Siegler, 2011 Hiebert & Lefevre, 1986; NCTM, 2000, 2014; NRC, 2001). When students are lacking in either conceptual knowledge or procedural knowledge, they have a limited mathematical understanding (Heibert & Lefevre, 1986; Rittle-Johnson & Schneider, 2014). Heibert and Lefevre (1986) stated that a lack of understanding can be shown in two ways. First, students who have an "intuitive" (p. 9) feel for mathematics but are unable to solve mathematical problems may not have a complete understanding of the mathematics involved. A second group of students who also lack mathematical understanding can solve mathematical problems easily but can explain neither how nor why the procedures work. Ma (1999) added that students must know what they are doing mathematically and be able to explain the reasoning behind their processes.

Bossé and Bahr (2008) addressed the importance of student retention of learning: "Retention of concepts along with the procedures which apply to, and can be employed in expanding upon, those concepts is vital to learning" (p. 20). When students are taught primarily procedures in the classroom, the procedures are less understood and more easily forgotten (NCTM, 2000). However, Schoenfeld (1988) pointed out that when a balance of conceptual

understanding and procedural fluency is taught then more connections are made. When more connections are present, students are able to retain the information in long-term memory.

Similarly, NRC (2001) described the benefits of conceptual understanding interconnected with procedural fluency. The process of gaining conceptual knowledge enables students to remember computational algorithms and use them with flexibility. As procedures become automatic, students are able to focus on deeper aspects of a problem and build new understanding. Students can also reflect on why a specific algorithm, or pattern within an algorithm, works and apply that understanding to increase their current conceptual knowledge.

Development of Procedural Fluency From Conceptual Understanding

NCTM's *Principles and Standards for School Mathematics* (2000) suggested that in order for students to develop mathematical understanding there must be a balance of conceptual and procedural learning. Balance in learning can occur when students are able to use their knowledge flexibly by applying the appropriate methods and reasoning within each problem setting. Students may also show balance when they are able to make connections between their conceptual understanding and the algorithms they use. *Principles and Standards for School Mathematics* explained that procedural knowledge and conceptual knowledge can be developed through "problem solving, reasoning, and argumentation" (p. 21). However, teachers should also be aware that students come to school with a "considerable knowledge base" (p. 21) in mathematical understanding that they have gained from personal experiences. This knowledge base can be used to help bridge the gap between conceptual and procedural knowledge.

A prevalent instructional model that advocates opportunities for students to engage in problem solving, reasoning, and argumentation is the inquiry-based Launch, Explore, Summarize design framework (Lappan et al., 2006; Schroyer & Fitzgerald, 1985) also referred to as Launch,

Explore, Discuss by Hendrickson, Hilton, and Bahr (2008). This instructional model enables students to be immersed in an environment where they can both construct their own knowledge and invent or rediscover mathematical patterns, thus creating active engagement among students (Schroyer & Fitzgerald, 1985).

Schroyer and Fitzgerald (1985) expounded on what takes place during a Launch, Explore, Summarize teaching design. During the Launch, students review concepts that have already been encountered and clarify when necessary. New concepts are then introduced in an informal way, usually through a story or experience that focuses on familiar terms. Along with a possible review of concepts, the Launch portion of the lesson introduces a worthwhile mathematical task. Verbal explanation illustrating the task is also included in order to make sure that all students understand the problem. This portion of the lesson is usually rather short to allow students ample time to work on the task. The teacher does not give instructions on how to solve the task, but encourages students to find their own methods to solve the task.

After the task has been clarified, students enter the Explore phase of the lesson, during which they are given time to analyze and solve the task (Hendrickson et al., 2008; Schroyer & Fitzgerald, 1985). Additionally, Schroyer and Fitzgerald (1985) recommended that students interact with each other in teacher-created groups in order to enrich their learning experience. However, students can also work individually to represent their own thinking. When students finish with the task, challenging supplementary tasks can be given that revolve around the same topic in order to delve more deeply into the concept. Once the majority of students has completed the initial task introduced during the Launch, the Explore phase transitions to the Summarize, or Discuss, phase.

The purpose of the Summarize phase is to bring clarity to student-driven mathematical ideas (Schroyer & Fitzgerald, 1985). Clarity is accomplished as students present and describe their solutions through student discourse. When misconceptions occur, students can advocate their position by explaining their understanding and subsequent solutions; students justify their own processes and solutions by describing the rules and patterns they have observed while completing the task. Once these student-driven conjectures, representations, or strategies are found to be valid, students can then apply them to new situations or new mathematical tasks. This process of student discussion opens possibilities for students to create new understanding, clarify misconceptions, and examine their own thinking (Hendrickson et al., 2008), while also increasing students' retention and flexibility.

During the Summarize phase, students are encouraged to share their own examples in order of concrete thinking to abstract thinking (Schroyer & Fitzgerald, 1985). Progressing from concrete visual representations to abstract procedures can help students build procedural fluency from conceptual understanding (NCTM, 2014). The concrete-representational-abstract (CRA) continuum follows Piaget's stages of cognitive development theory (Ojose, 2008). The concrete operational stage comes before the pictorial-representational and formal operational stages. Concrete operational thinking, often accomplished with the use of manipulatives, can allow students to make connections with more formal representations and lead to procedures such as standard algorithms and numeric computation. As teachers follow the CRA continuum in allowing students to share their ideas, retention is enhanced because more connections can be made between and among strategies.

Embedded in each instructional phase, regardless of which instructional framework is being used, student discourse that honors student thinking is crucial (Bahr & Bahr, 2017; Herbel-

Eisenmann & Breyfogle, 2005; NCTM, 2014). Herbel-Eisenmann and Breyfogle (2005) stated

that for discourse to be effective, teachers must analyze for "patterns of interaction" (p. 484) in

their questioning to ensure that students are able to clarify and explain their own mathematical

thinking. Often, teachers will incorporate a "funneling-interaction pattern" (p. 486) when

discussions take place. A funneling-interaction pattern directs student thinking toward

explaining how the teacher modeled or explained a path to solution with no inclusion of student

thinking (Herbel-Eisenmann & Breyfogle, 2005). Throughout this thesis I refer to a funneling-

interaction pattern of student discourse as *structured discourse*.

      Teacher and student actions. NCTM's *Principles to Actions* (2014), a highly research-

based guide for ensuring mathematics success for all, outlined what teachers and students can do

to develop students' mathematical thinking both conceptually and procedurally. In order for

procedural fluency to build on conceptual understanding, teachers need to provide students with

opportunities to create their own reasoning strategies when they are engaging in a mathematical

task (Carpenter et al., 2003; Empson & Levi, 2011; NCTM, 2014). Teachers should also invite

students to lead discussions in which they explain (a) the procedures used to solve a problem, (b)

why their procedural steps make sense, and (c) how their method connects to other students'

strategies (Carpenter, Franke, & Levi, 2003; NCTM, 2000, 2014). The teacher's attentive

listening as students present their strategies encourages students to pay attention to their own

processes and ideas and the ideas of other students (Empson & Levi, 2011). As students present

their strategies, teachers facilitate meaningful connections between students' procedures to help

students understand particular applications of the different procedures; some procedures are

more effective than others in solving various types of problems (NCTM, 2014). Teachers help

students strengthen these connections through visual models in order to support general

procedures. After making connections, teachers provide opportunities for students to practice appropriate and effective procedures in various problem-solving situations (Empson & Levi, 2011; NCTM, 2000, 2014).

The work of Isaacs and Carroll (1999) supported the thesis currently held by many (e.g., Carpenter, Fennema, Franke, Levi, & Empson, 1999; NCTM, 2014) that practice is best accomplished as an integral part of students' work on worthwhile problem-solving tasks. However, when practice is treated as a separate component, it should not be premature (Isaacs & Carroll, 1999): If students practice before they have a conceptual foundation, it can lead to "becoming quicker at immature approaches" (p. 511). Practice as a separate component should be brief, engaging, purposeful, and distributed. Isaacs and Carroll (1999) described games, such as flash cards and choral drills, to be practice opportunities when appropriate. Although the definitions of what counts as practice range greatly, this research will define practice as showing elements of both conceptual understanding and procedural fluency. Isaacs and Carroll included rote memorization activities in their possibilities for practice; however, such practice has automaticity as its primary goal for procedural fluency whereas fluency requires using both conceptual and procedural knowledge (NCTM, 2000; 2014).

With automaticity not being the central goal of mathematics instruction, understanding of mathematical principles must come before students' construction of strategies and procedures; such construction then leads from conceptual understanding to procedural fluency (Carpenter et al., 2003; NCTM, 2014). Students must also be able to use mathematical strategies flexibly in order to choose the specific procedures, or methods, that would be the most appropriate in solving the problem (NCTM, 2000, 2014). For example, when comparing the fractions ½ and $^1/_3$, students with conceptual understanding could reason through which is larger based on the

same digits in the numerator and using this understanding to compare the meaning and size of the denominator. On the other hand, students without this flexibility may choose a cross-multiplication procedure without revealing an understanding of fractional concepts. Both sets of students could arrive at the correct answer, but only students with a conceptual understanding of fractional concepts could appropriately explain their reasoning.

Problem solving. The process of building procedural knowledge upon conceptual understanding is developed through problem solving (NCTM, 2014). True problem solving requires students to rely on knowledge previously acquired to solve a task they initially do not know how to solve (NCTM, 2000). Smith and Stein (1998) detailed the characteristics of tasks at differing levels of cognitive demand and devised a four-level guide for examining tasks for instructional use. Smith and Stein's leveled guide to examining tasks for cognitive demand included (a) lower-level demands based on memorization, (b) lower-level demands requiring procedures to be completed without connections, (c) higher-level demands involving procedures that are used with connections, and (d) higher-level demands that are described as doing mathematics (See Appendix A for Characteristics of Mathematical Tasks at Four Levels of Cognitive Demand). Although each level has its appropriate place in mathematics instruction, "for students to learn mathematics with understanding, they must have opportunities to engage on a regular basis with tasks that focus on reasoning and problem solving and make possible multiple entry points and varied strategies" (NCTM, 2014, p. 23). Smith and Stein suggested that higher-level demand tasks promote such opportunities for problem solving.

NCTM (2014) emphasized that "procedural fluency follows and builds on a foundation of conceptual understanding, strategic reasoning, and problem solving" (p. 42). When procedures are connected to concepts with which students are already familiar, students are better able to

remember the procedures and apply those procedures to previously unencountered situations, demonstrating procedural fluency. Exploring informal problem-solving strategies develops students' conceptual understanding: As students reason through their informal strategies, teachers can recognize the concepts that students understand or do not understand based on their justifications. Students' informal strategies lead to general methods of problem solving within a mathematical operation; such general methods then become tools to solve problems and develop into flexible procedures that can be used purposefully.

Teacher Content Knowledge

An additional characteristic that provides students with the capability to strengthen their conceptual understanding of mathematics and progress to procedural fluency is the knowledge that a teacher brings into the classroom (Ma, 1999; Schoenfeld, 2006). The knowledge that a teacher brings into the classroom is an important factor in the type of instructional approach that the teacher uses. Shulman (1986) separated teacher content knowledge into two main categories: mathematical content knowledge and pedagogical content knowledge. He defined mathematics content knowledge as the teacher's understanding of mathematics concepts. Pedagogical content knowledge is the teacher's knowledge in teaching mathematics concepts in an understandable way (Shulman, 1986; Ball, Thames, & Phelps, 2008). Schoenfeld (2006) and Ball et al. (2008) included a teacher's ability to plan for and resolve students' misconceptions as part of pedagogical content knowledge.

Ma (1999) characterized the knowledge that a teacher contributes to the mathematics classroom as comprising "knowledge packages" (p. 17): These knowledge packages consist of (a) what content means, (b) how mathematical concepts relate to other mathematical concepts, (c) algorithmic skill and how it is founded in conceptual knowledge, and (d) how all of these

knowledges interrelate. Ma suggested that as a mathematics teacher develops these knowledges the teacher also develops a profound understanding of fundamental mathematics, or PUFM. Schoenfeld (2006) described PUFM as being not only fundamentally mathematical, but also fundamentally pedagogical: When teachers develop PUFM, they are able to make meaning of mathematics, find deeper connections, and provide opportunities for students to do the same.

A wide range of research showed that, unfortunately, many teachers do not develop adequate teacher knowledge in mathematical content and pedagogical practices, or PUFM (Ball & Feinman-Nesmer, 1988; Lamon, 2007; Ma, 1999; Schoenfeld, 2002; Shulman, 1986). Lamon (2007) and Schoenfeld (2002) pointed out that inadequate teacher knowledge makes it difficult for teachers to help students develop understanding of both concepts and procedures.

One of the suggestions that Ma (1999) proposed to build PUFM was to study intensely the curriculum being used; she found that teachers who studied the curriculum intensively were able to make more connections between mathematical ideas and guide students to make similar connections. Through intensive study of curriculum materials, teachers showed a greater command of mathematical ideas and pedagogical practices and, therefore, PUFM.

Schoenfeld (2006) recommended that building teacher knowledge in content and pedagogy and therefore PUFM should start with a greater focus on teacher professional development. The NRC (2002) advocated that teachers should take more of a student learning approach, because a learner is still a learner, no matter what age. Teachers should focus their own learning on ways students think and the students' mathematical ways of working (Schoenfeld, 2006). Learning mathematics as they work with students provides teachers the opportunity to look at mathematical ways of working that will cultivate mathematical understanding in a conceptually rich way (Ma, 1999).

Conceptual to Procedural in Fractions

When analyzing how procedural fluency builds from conceptual understanding, I chose a specific domain within mathematics: fractions. As with other mathematical domains and concepts, many students have an incomplete understanding of fractions because they did not first develop a conceptual understanding before progressing to the procedures of fractional mathematics (Carpenter et al., 1993; Fazio & Siegler, 2011; Lamon, 2007; Siegler et al., 2010). Fazio and Siegler (2011) related the weakness of conceptual understanding to situations in which students either view fractions as symbols that have no meaning or consider the numerator and denominator as separate quantities rather than seeing the relationships between the two. Carpenter et al. (1993) supported these claims by recognizing that students who have not made the transition to thinking of rational numbers as quantities are still focusing on each symbol as an individual part, without understanding the relation to the whole. For example, a student who lacks conceptual understanding of the holistic nature of fractions, who focuses only on the individual numbers, would likely reason that that 1/4 is larger than 1/3.

In order for students to develop conceptual understanding, they must understand that fractions represent a number or value (Carpenter et al., 1993; Empson & Levi, 2011; Fazio & Siegler, 2011; Siegler et al., 2010; Siegler et al., 2011). Empson and Levi (2011) claimed that students can be taught to ignore the conceptual meaning of fractions when exposed only to the typical models that schools often use to focus on procedures, such as using shapes to show part-whole fractions and having students write shaded parts over total parts. Siegler et al. (2011) concluded that narrow-minded models stem from the overuse of part-whole interpretations of fractions. In other words, students may visualize 1/5 as one of five pieces of pizza, but not as an actual quantity. In order to develop conceptual understanding and recognize that fractions are

numbers, researchers have suggested that students be given more exposure to problems where fractions are shown as quantities of measurement (Fazio & Siegler, 2011; Siegler et al., 2010; Siegler et al., 2011). Using measurement questions begets representations such as number lines to help students see fractions as quantities of parts of a whole, equal to a whole, or larger than a whole. Siegler et al. (2010) stated that providing measurement questions and number lines in fractions instruction strengthens students' abilities to recognize equivalent fractions; this line of instruction also helps students understand that fractions share the same number properties as whole numbers, which aids the expansion into different number systems.

Conceptually, students must be able to understand that fractions are quantities, but they must also be able to connect their fractional thinking with familiar contexts (Empson & Levi, 2011; Fazio and Siegler, 2011; Lamon, 2007; NRC, 2001; Siegler et al., 2010). Empson and Levi (2011) stated that all learning is built upon previously understood ideas: As students are given opportunities to connect new concepts with prior knowledge, they can create an informal understanding of fractions, which then leads to the development of proportional reasoning. Fazio and Siegler (2011) also suggested that giving students real-life contextual problems will facilitate their making connections between their problem-solving capabilities and their background knowledge of the context.

Many researchers have found that because of common life experiences, students already understand what it means to share equally; as a result, teachers are encouraged to pose equal-sharing fraction problems to connect with students' intuitive knowledge (Empson & Levi, 2011; Fazio & Siegler, 2011; Kieren, 1993; Lamon, 2007; NRC, 2001; Siegler et al., 2010). Fazio and Siegler (2011) gave examples of how children's equal-sharing understanding progresses through their experiences. Children who are 4 years old are able to partition whole number quantities

equally amongst small groups. By 5 years of age, children can take a single object and split it up

to share equally with others, and by age 6 they can match up equivalent quantities using shapes

(Fazio & Siegler, 2011). Assuming that students come with background in concrete models,

Empson and Levi (2011) recommended that teachers should support students' movement from

concrete models to more abstract fraction notation as they engage in discourse using equal-

sharing problems.

Equal-sharing problems are useful both in introducing the concept of fractions and in

helping students make connections with other related concepts: (a) dividing a single whole into

different parts, (b) recognizing equivalent fractions, (c) comparing and ordering fractions, and

(d) developing early proportional reasoning concepts (Siegler et al., 2010). These connections

can be built and strengthened by giving students opportunities to create drawings and

representations based on story problems that include splitting and sharing. Equal-sharing

problems can also be extended to discuss comparing unlike fractions and creating equivalent

fractions, as the teacher concentrates on parts of the student solutions where the concepts can be

deepened.

Empson and Levi (2011) gave several examples of equal-sharing tasks that can be used to

deepen fractional knowledge. A problem such as, "Four children want to share 10 brownies so

that everyone gets exactly the same amount. How much brownie can each child have?" (p. 6) can

be used to encourage students to show various representations of dividing multiple wholes in

equal amounts. A teacher could then require a student to compare fractions using this equal-

sharing problem: "Who gets more clay: a child at a table where 4 children are sharing 1 box of

modeling clay equally or a child at a table where 3 children are sharing 1 box of modeling clay

equally?" (p. 142). Other equal-sharing problems can be created to require thinking in equivalences, rates, and proportional reasoning.

Teachers should focus on developing conceptual understanding of fractions in order to build procedural fluency (Fazio & Siegler, 2011). Students are often taught computational procedures, and sometimes tricks, to help them find equivalent fractions, but they often lack understanding of why the procedures work (Empson & Levi, 2011; Fazio & Siegler, 2011). Hallett, Nunes, and Bryant (2010) described a simple task teachers can ask students to solve to determine if they have developed a conceptual understanding of equivalence or if they rely on procedural methods with little or no understanding of why the procedures work. When students are given a task that requires them to add ½ and ¼, they may show conceptual understanding by recognizing that ½ is equivalent to 2 one-fourth pieces; they would then add all the fourths together. If the students rely on a memorized procedure to create a common denominator for two fractions in order to compare their quantities, for example, multiplying the numerator and the denominator by the same number, then the students may lack conceptual understanding. However, it must be emphasized that allowing students to discuss how they solved a problem is key to recognizing their total understanding (Empson & Levi, 2011).

Rigor of CCSS-M Promotes Conceptual to Procedural

The Common Core State Standards Initiative, which developed CCSS-M, was created, in part, for the purpose of including rigor in the mathematics curriculum (NGA Center & CCSSO, 2010). CCSS-M defines rigor as a balanced approach in pursuit of conceptual understanding, procedural fluency, and application of mathematical knowledge. Consequently, students learning from such an approach should develop a deeper understanding of mathematical concepts and be able to connect ideas throughout coherent topics across grade levels.

Such rigor can be found in the content standards by analyzing the topic of Numbers & Operations-Fractions section in CCSS-M (NGA Center & CCSSO, 2010). In third grade, students are expected to understand that two fractions are equivalent when they are the same size as long as they pertain to the same whole. The conceptual development of equivalence of fractions progresses as students compare two fractions based on the same numerator or denominator when the fractions refer to the same whole. This comparison is founded on the ability to reason through the meaning of the numerator and denominator and to justify the reasoning using visual models, which can include pictorial representations, number lines, and fraction notation (NCTM, 2000). The NGA Center and CCSSO (2010) continued a conceptual construction through using reasoning, justification, and comparison to build procedural fluency by generating equivalent fractions.

While the sequence of building procedural fluency from conceptual understanding is shown within the standard of equivalence in third grade, CCSS-M also builds on equivalence of fractions across grade levels (NGA Center & CCSSO, 2010). In fourth grade, students extend their understanding of fraction equivalence by comparing fractions that have different numerators and denominators; fractional models are still used, and students must be able to generate equivalent fractions. In fifth grade, students connect their established understanding of equivalence to their knowledge of addition and subtraction of fractions with unlike denominators; students must generate equivalent fractions and recognize that they can be used in order to add or subtract like terms. The sixth grade standards transition from fractions to ratios with contexts that extend students' understanding and use of equivalence.

Throughout third through sixth grades, as children continue to expand their understanding of the whole number system, they begin to build their understanding of fractional quantities in

relation to the rational number system. The NGA Center and CCSSO (2010) provides

opportunities to make these connections through a sequential process of learning whole number

division operations, followed by rational number fractional operations.

Standards for Mathematical Practice

Included in CCSS-M alongside the content standards are the Standards for Mathematical

Practice, which are mathematical ways of working that all teachers are required to help all their

students develop (NGA Center & CCSSO, 2010). These practices will impact students'

mathematical proficiencies throughout their mathematics education. NGA Center and CCSSO

(2010) created these practice standards based on the NCTM process standards (2000) and the

NRC strands of mathematical proficiency (2001), as previously mentioned. The standards for

mathematical practice comprise the following expectations.

Students will develop the ability to

- Make sense of problems and persevere in solving them.

- Reason abstractly and quantitatively.

- Construct viable arguments and critique the reasoning of others.

- Model with mathematics.

- Use appropriate tools strategically.

- Attend to precision.

- Look for and make use of structure. (NGA Center & CCSSO, 2010)

Students who make sense of problems, and persevere in solving them, make meaning of a

problem and look for different ways to find a solution (NGA Center & CCSSO, 2010). Students

may demonstrate this practice by using manipulatives and other objects to help explain their

thinking and visualize a solution. Students may also consistently ask themselves if their method

"makes sense" (p. 1) throughout their process.

When students reason abstractly and quantitatively, they "decontextualize" (p. 2) the

problem by showing flexibility in their computations through manipulating numbers and

mathematical symbols (NGA Center & CCSSO, 2010). However, students also "contextualize"

(p. 2) by acknowledging the referents of the problem and connecting their method to the specific

situations found in the original problem.

NGA Center and CCSSO (2010) indicated that students who construct viable arguments

and critique the reasoning of others are able to "justify their conclusions" (p. 2) and

communicate how they solved the problem. They are also able to construct meaningful

explanations when others disagree or challenge their results. Students who have developed this

mathematical practice are able to analyze the work of others and find connections, strengths, or

weaknesses in the reasoning of others.

Modeling with mathematics is a practice for which the title and definition have created

conflicting messages or become confused with the definition of mathematical modeling. NGA

Center and CCSSO (2010) defined *modeling with mathematics* as connecting mathematical

problems to real-world situations. Hirsch and McDuffie (2016) noted that, unfortunately, CCSS-

M makes no explicit distinction between *modeling mathematics* and *mathematical modeling*: the

terms are used interchangeably "to mean both modeling mathematics and mathematical

modeling" (p. 1). However, in a volume designed in part to clarify the use of the terms modeling

mathematics and mathematical modeling (Hirsch & McDuffie, 2016), the difference between the

two terms is made explicit. Hirsch and McDuffie (2016) defined modeling mathematics as using

representations to express "a mathematical concept or idea" (p. 4) and mathematical modeling as

the translation of an "authentic real-world" (p. 5) problem "into mathematical form" (p. 5). For the sake of this thesis, I will focus on the definition given by the NGA Center and CCSSO (2010) and Hirsch and McDuffie's (2016) definition of mathematical modeling as a focus on real-world problems that derive from student experience. The NGA Center and CCSSO (2010) described possible early examples of real-world connections as young children's representations of a situation using a mathematical equation, and as middle grade students' use of their understanding of ratio and proportion to figure out a problem in their community. The reader will note that a word problem assigned to children to solve algorithmically, even if the context appears to be connected to the real world, does not necessarily constitute a mathematical problem, nor does it involve modeling mathematics.

When students are able to use mathematical tools strategically, they select calculators, manipulatives, number lines, protractors, digital technology, or other appropriate mathematical resources to assist in representing and solving a mathematical problem (NGA Center & CCSSO, 2010). Often, students' use of these tools deepens their understanding and explanations of mathematical concepts.

The NGA Center & CCSSO (2010) described the mathematical practice of attending to precision primarily as the ability to communicate one's mathematical ideas clearly and precisely. The ability to be clear and precise is found not only in students' efficiency and accuracy in solving a problem, but also in giving clear mathematical definitions, in understanding the meaning of mathematical symbols used, and in describing to others how they solved the problem. Attending to precision, in a broader mathematical sense, refers to making sure that one's work is carefully completed and accurate (Kinseth, 2018). In this thesis, attending to

precision as a mathematical practice will follow the definition given by NGA Center and CCSSO (2010).

When students can look for and make use of structure, they are able to find patterns and structure in mathematical problem solving (NGA Center & CCSSO, 2010). Students show examples of these patterns and structures when using mathematical properties, creating algebraic expressions, and decomposing and recomposing numbers.

Finally, the last mathematical practice included in the practice standards is to look for and express regularity in repeated reasoning (NGA Center & CCSSO, 2010). This practice is defined as being able to recognize repetition of general methods and applying the repetition to look for shortcuts in solving a problem. An example of this practice could be that students notice the repetition of division when 25 is divided by 11 as a repeated decimal. Rather than continuing to divide, they can create a shortcut and represent their answer with a fraction or mark the repeating decimal.

Textbooks Claim to Follow CCSS-M

Developers of curricular materials such as textbooks typically express a fundamental goal of supporting national and state academic standards, thus claiming to offer students many opportunities to develop conceptual knowledge (Jitendra et al., 2005; Remillard, 2005; Sood & Jitendra, 2007). As standards have become more widely accepted and have raised expectations for student understanding, the standards of accuracy and quality in textbooks have also been raised (Sood & Jitendra, 2007). Consequently, over the years, textbooks have become a strong influence on instructional practices and how mathematics is structured and paced (Jitendra et al., 2005). According to Remillard (2005), many researchers and educators have found that the use of textbooks and commercial curricula encourage improvement in teaching.

Although textbooks can enhance teaching and give a sense of what is needed to be taught, there have also been many criticisms. One of these criticisms is the directness with which the teaching is prescribed (Sood & Jitendra, 2007). Polikiff (2015) and Sood and Jitendra (2007) reported that traditionally, such direct instruction has emphasized memorized learning and following procedures and generally lacks in the development of conceptual understanding. Polikoff also found that textbooks claimed to be aligned with CCSS-M curricula but were substantially misaligned. The results of research done by Jitendra et al. (2005) revealed that such curricula with standards-based claims exposed students to questions that provided opportunities to reason and make connections less than 50% of the time. These researchers found that application questions were present in many mathematics textbooks, but students were not given opportunities to generate their own representations that they would then be able to explain and discuss; such opportunities for student discourse are considered central to establishing the conceptual mastery required for procedural fluency (Herbel-Eisenmann & Breyfogle, 2005; NCTM, 2014).

In an earlier study on how teachers use textbooks, Freeman and Porter (1989) found inconsistency in teachers' adherence to the content of the textbook. No policies were found on how teachers should use the textbook, but schools and districts varied in their views of fidelity with which textbooks should be used, both in district expectations and in classroom practices (Ball & Feiman-Nesmer, 1988). However, it is worth noting that Freeman and Porter concluded that teachers who followed the textbook more closely tended to have an emphasis on conceptual knowledge and application of the content. On the other hand, teachers who deviated from the textbook had a heavy emphasis on practicing procedural skills. The reader will note that the references cited regarding the role and value of textbooks are prior to 2000. I have yet to find

current research that confirms these conclusions. Nevertheless, if teachers who emphasize conceptual understanding follow textbooks more closely, then my research can be informative to teachers and administrators in determining to what extent and in what ways the curriculum materials build procedural fluency from conceptual understanding.

Summary

Developing conceptual understanding and procedural fluency is imperative to understanding mathematics (Heibert & Lefevre, 1986). The ability to make connections and create rules for solutions are two ultimate goals of understanding mathematics. Using a Launch, Explore, Discuss instructional model, as described by Schroyer and Fitzgerald (1985) provides students with the opportunities to build procedural fluency from conceptual understanding.

Fundamental Issues

Throughout the literature review, seven fundamental issues related to building procedural fluency from conceptual understanding were repeated, and these issues yielded the codes for this content analysis of this review of the *Go Math! K-8* textbook series. Issues one through five are general pedagogical practices recommended by NCTM (2014); issue six is domain-specific for fractions and issue seven yielded an emergent code. Each code is listed as follows, accompanied by a brief summary of the fundamental issue underlying the code.

1. Student-generated strategies. Teachers provide students opportunities to solve problems by creating their own strategies (Hendrickson et al., 2008; Ma, 1999; NCTM, 2014; Schoenfeld, 2002).

2. Student discourse. Teachers give students the opportunity to explain their own problem-solving methods (Bahr & Bahr, 2017; Hendrickson et al., 2008; Herbel-Eisenmann & Breyfogle, 2005; NCTM, 2014).

3. Connecting student strategies to general methods. Teachers guide students to connect their strategies with standard algorithmic methods (Hendrickson et al., 2008; NCTM, 2014).

4. Visual models. Teachers progress from concrete models to abstract algorithms (NCTM, 2014).

5. Practice. Teachers provide students with meaningful opportunities to practice problem solving (Isaacs & Carroll, 1999; NCTM, 2014; Smith & Stein, 1998).

6. Equal sharing. Understanding of equal sharing is a fundamental issue for students' progression from conceptual understanding to procedural fluency with regard to fractions (Empson & Levi, 2011; Fazio & Siegler, 2011; Siegler et al., 2010).

7. Teacher content knowledge. The textbook analyzed provided professional development resources to deepen a teacher's understanding of mathematical content as it relates to pedagogy and necessary student skills (HMH, 2016; Ma, 1999; Schoenfeld, 2006).

Each of these fundamental issues is important in building procedural fluency from conceptual understanding with equal sharing as a specific issue that is pertinent to equivalence of fractions. The implementation of each code is explained in chapter three.

CHAPTER 3

Method

For the purpose of examining a textbook series regarding to what extent and in what ways the resources provide teacher support in building procedural fluency from conceptual understanding, I used a content analysis research method (Thayer, Evans, McBride, Queen, & Spyridakis, 2007). I analyzed the textbooks, applying several *a priori* codes derived from the literature while allowing for emergent codes. I focused my analysis on the domain of fractions, specifically equivalence, because understanding of equivalence is central to developing understanding of fractions as quantities, operations within fractions, and ideas of ratio and proportion (Lamon, 2007). I examined the development of equivalence by looking for evidence of teacher actions identified in *Principles to Actions* (NCTM, 2014). The presence of equal-sharing problems was also addressed because the work of multiple researchers (Empson & Levi, 2011; Fazio & Siegler, 2011; Kieren, 1993; Lamon, 2007; NRC, 2001; Siegler et al., 2010) has shown that equal-sharing problems provide an opportunity for students to visualize and connect simplistic reasoning and deeper understanding with equivalent amounts. An emergent code regarding teacher content knowledge and its role in professional development was analyzed according to research from Ma (1999) and Schoenfeld (2006). Prior to presenting the details of the methodology, I first describe myself as the researcher. After detailing the methodology, limitations of the study are discussed.

The Researcher

With an undergraduate degree in elementary education, I have been teaching mathematics as an integral part of my classroom teaching for my entire 9-year professional career. I completed a graduate level 2-year mathematics endorsement, a professional development

program that engages participants in an in-depth examination of both content knowledge and pedagogy in mathematics. As a member of numerous committees, at both school and district levels, I have been involved in many decisions and professional development activities regarding mathematics, including (a) adopting mathematics textbooks, (b) creating grade-level pacing guides, and (c) helping teachers develop inquiry-based pedagogical skills. These experiences and opportunities have influenced my perspective on effective mathematics instruction in the direction of inquiry.

My role as a researcher is heavily influenced by my background and perspective, yet it is also shaped by the environment in which I work. The district in which I am employed requires mathematics textbooks to be used with fidelity as a guide for pacing and pedagogy. With these administrative instructions, my analysis of the alignment of curricular materials with effective mathematics instruction should help to inform teachers, administrators, and others regarding effective use of the textbook.

Content Analysis as the Research Methodology

In order to analyze the alignment of mathematics textbooks with effective instruction, I used a qualitative content analysis. Thayer et al. (2007) described a content analysis as "a research method that empirically examines the characteristics of messages" (p. 268). Often, a content analysis can include tallies marks of specific words or phrases to produce quantitative data. However, Thayer et al. go on to describe a qualitative latent analysis, which involves interpreting meaning of text using previously agreed-upon *a priori* codes or emergent codes that emerge through analysis of the text.

In conducting this content analysis, I used a six-question structure proposed by Krippendorf (1980) and reinforced by Stemler (2001). Stemler summarized the six questions:

1. What data will be analyzed? (p. 2)

2. How are they defined? (p. 2)

3. What is the population from which they are drawn? (p. 2)

4. What is the context relative to which the data are analyzed? (p. 3)

5. What are the boundaries of the analysis? (p. 3)

6. What is the target of the inferences? (p. 3)

The questions of (a) what data are to be analyzed, (b) how the data are defined, and (c) what is the population from which they are drawn are answered in the Data Sources section. Two of the remaining questions Krippendorf proposed are "What is the context relative to that which the data are to be analyzed?" (p. 51), and "What are the boundaries of the analysis?" (p. 51). These questions have been considered in the development of the codes (see Coding the Data section). This chapter describes (a) the boundaries of the analysis, (b) the definition of the codes, and (c) the selection of the data to be analyzed. Finally, the target of the inferences needs to be known. This question is answered at the end of the methods chapter through stating how this research can benefit teachers, administrators, and textbook publishers.

Data Sources

I used qualitative content analysis to determine the ways in which the 2015 edition of *Go Math! K-8* mathematics textbook series published by Houghton Mifflin Harcourt (HMH) attends to building procedural fluency through conceptual understanding. *Go Math! K-8* is a curriculum designed for instruction in elementary school and middle school with the stated purpose of enhancing students' abilities to think critically and apply what they are learning (HMH, 2016). According to a report by Kane, Owens, Marinell, Thal, and Staiger (2016), students who used *Go Math! K-8* (edition not revealed) had higher achievement scores than the scores of students who

used other textbooks or no textbook at all. The report also stated that, when compared with 19 other textbooks, *Go Math! K-8* ranked in the top three "in terms of focus, coherence, rigor, and mathematical practice-content connections" (p. 22).

HMH's *Go Math! K-8* (2016) provides teacher editions for all grade levels and a student write-in book for grades K-5. The teacher edition includes instructions for teachers and a copy of the student write-in book, including answers. Because the student books are included in the teacher edition, examples of student tasks will be referenced as coming from the teacher editions. Teacher instruction in the teacher edition includes (a) questions to ask, (b) what to do if students do not understand, and (c) explanations of the content to build teacher understanding. Additionally, I also included the *Go Math! Teacher Planning Guide* (Burger, Dixon, Larsen, Sandoval-Martinez, & Leinwand, 2015) in this research. This document, referred to in this paper as the planning guide, includes sections describing CCSS-M mathematical practices for the purpose of building teacher content knowledge. The planning guide consists of grade level documents with the same program overview for each of the grades K-5, and includes specified mathematical practices connected to each lesson in each grade. Although *Go Math! K-8* (HMH, 2016) provides teachers with online resources; I did not include them in this analysis because they are ancillary to the program implementation.

The instructional framework for grades 3-5 of *Go Math! K-8* (*Go math! G3,* 2015; *Go math! G4*, 2015; *Go math! G5*, 2015) follows the 5E model--Engage, Explore, Explain, Elaborate, and Evaluate--originally created for science instruction. The 5E model appears throughout the series. The 5E outline of the teacher editions has subsections and tasks within each of the 5E sections of every lesson of every grade level examined. For example, the Engage section is meant to prepare students for the task by providing an essential question that informs

children what they should be able to answer by the end of the lesson. Additionally, subsections titled Making Connections, Learning Activity, and Literacy in Mathematics prepare students to discuss previous learning, vocabulary, and background knowledge. *Go Math! K-8* (HMH, 2016) also includes either an Investigate or Unlock the Problem task that begins each Explore section. The teacher is instructed to work through these questions with the students and provide strategies and learning models to help solve the task. The Explain part of the 5E model includes two subsections, Share and Show and On Your Own, instructing students to use the learning model from the previous Explore section to solve tasks. Next, Go Deeper and Think Smarter tasks bridge the Explore and Elaborate sections as they are found in both sections. *Go Math! K-8* (HMH, 2016) describes Go Deeper and Think Smarter tasks as requiring critical thinking skills to help students learn the content more thoroughly (HMH, 2016). Finally, the Evaluate section asks students to answer the essential questions from the beginning of the lesson; the teacher edition most often suggests that students use a math journal to write their responses.

The *Grade 6 Teacher Edition* (Burger, Larsen, Dixon, Leinwand, Kanold, & Sandoval-Martinez, 2014) follows the same 5E model for instruction as is found in the teacher editions for grades 3-5, with some adaptations for grade 6. The Engage section contains only two subsections: Essential Question and Motivate the Lesson. The Explore section is a set of instructions to explain to the students. The Explain section provides questioning strategies and a subsection titled Your Turn that is similar to the Share and Show subsection in grades 3-5. The Elaborate and Evaluate portions focus on providing practice for students with the inclusion of Higher Order Thinking (HOT) tasks in the Evaluate section (Burger et al., 2014).

Data Analysis

Following latent content analysis methodology as described by Thayer et al. (2007), I analyzed the teacher materials by interpreting the text to determine *intentions*, my term for messages about what teachers and students were expected to learn, achieve, or do when using the textbook. I interpreted the meaning of the text within my background of experience according to the language used and the suggested verbal progression of the curriculum. For cases in which the intentions were not clear, I consulted with two experienced university mathematics education professors to work toward clarity and agreement. In this way I addressed the first part of my research question: In what ways does *Go Math! K-8* support teachers in building procedural fluency from conceptual understanding within the topic of equivalence? I used *a priori* codes drawn from the literature that describes what teachers and students should be doing in order to build procedural knowledge based on conceptual understanding. At the same time, I allowed for the creation of emergent codes for procedural fluency based on conceptual understanding that did not fit my *a priori* codes: Emergent codes allowed for messages unanticipated by the researcher to be revealed during the analysis of the text. Using codes associated with teacher and student actions aligned well with NCTM's *Principles to Actions* (2014), which outlined the actions that teachers and students should demonstrate in building procedural fluency from conceptual understanding.

In addition to supporting the qualitative analysis of the textbook through codes, content analysis also allowed for the use of simple frequencies (Stemler, 2001). I used frequency counts of codes to answer to what extent the textbook provides instructional support for teachers in building procedural fluency from conceptual understanding. Initially for each code, data analysis

was conducted by myself and a university professor to establish interrater agreement. I

proceeded the analysis individually, but checked on agreement throughout.

  Coding the data. The teacher and student text of the teacher editions for each grade

level were analyzed first for the presence of problem solving, using the following definition:

"engaging in a task for which the solution method is not known in advance" (NCTM, 2000, p.

52). The implementation of the problem-solving task was reanalyzed according to each of the

teacher actions (NCTM, 2014), which served as *a priori* codes. Next, each problem was

reexamined to determine the presence, or absence, of equal-sharing as defined by Empson and

Levi (2011): problems that "[involve] a total number of items to be distributed to a given number

of groups, usually people" (p. 8). This analysis was especially helpful in revealing the manner in

which the textbook attends to current research recommendations in developing equivalence of

fractions from conceptual understanding leading to procedural fluency.

  Finally, an emergent code of teacher content knowledge was considered as each lesson

contained a professional development section that focused on building teacher content

knowledge through the use of the *Standards for Mathematical Practice* (NGA Center & CCSSO,

2010). These professional development lessons were analyzed for frequencies of each

mathematical practice. Additionally, questions provided to teachers to ask students in the

planning guide were analyzed for alignment with intentions of the questions and description of

the coinciding mathematical practice.

  Teacher action codes. These codes were focused on teacher actions suggested by

Principles to Actions (NCTM, 2014). These teacher actions should consistently be included in

instruction, allowing students to build procedural fluency from conceptual understanding.

Providing student-generated methods. For evidence of this code, I analyzed the textbook for open tasks with instructions for students to solve the problem using their own strategy. Tasks in which students are encouraged to solve in multiple ways or that have varying solutions were taken into account.

Student discourse. The kinds of questions for students the textbook provides teachers were the basis for how I determined whether this directive was found. I looked for any questions that invited students to go into depth on their strategies or to explain the processes they use in solving the problem. Opportunities for discourse were analyzed by how the teacher was directed to lead students to discuss and expound on their reasoning for their developed strategies. If the teachers followed a funneling-discourse pattern (Herbel-Eisenmann & Breyfogle, 2005) and provide the strategies and opportunities for discourse on those strategies, then such examples were referred to as structured discourse. If student-generated strategies were not present, I analyzed the textbook for structured discourse between teacher to student, student to student, and inconclusive. Inconclusive was defined as opportunities where discussion was suggested, but the parties to be involved in the discussion were not clarified. Closed questions without any follow up were not taken into account, as meaningful discussion as defined by NCTM (2014) does not take place with closed questions unless follow-up questions ensue.

Connecting student strategies to general methods. The progression from student-driven strategies to more efficient strategies greatly depends on the opportunities for exploration and multiple entry points, as evaluated by the student-generated methods code. When the directive for students to formulate their own strategies was present, then the progression that the textbook encouraged should contribute to students' transition from concrete models to abstract representations. In the teacher editions examined, the movement to more efficient procedures

was based on the progression from visual models to numeric procedures. If student-generated

methods were not present to allow for focusing on this progression, I reported on the connections

that the textbook offers and labels as connections.

Using visual models to enhance understanding. The textbook progression, grades 3-6,

was analyzed for the methods used to introduce and develop equivalence. The sequence of how

the teacher edition used representations in the lower grade and transitions to algorithmic

procedures was considered, along with meaningful connections made to enhance conceptual and

procedural understanding.

Practice. Taking into consideration that true practice requires problem solving in which

connections take place (Isaacs & Carroll, 1999), I analyzed tasks according to the categories

established by Smith and Stein (1998) for either higher-level demand (connections with

procedures or doing mathematics) or lower-level demand (memorization or procedures without

connections). Exercises that were deemed lower-level demand were not considered problem-

solving practice as neither conceptual understanding nor procedural understanding was required.

Student action codes. Although my plans included analysis of teacher materials from *Go

Math! K-8* (Burger et al., 2014; *Go math! G3,* 2015; *Go math! G4*, 2015; *Go math! G5*, 2015)

using student action codes as identified in NCTM's *Principles to Action* (2014), I found no

examples of student work and conversation. Therefore, I was unable to analyze for student

actions. However, the teacher actions codes appear to be sufficient in indicating if the textbook is

designed to progress student thinking from a conceptual understanding to a procedural

understanding, as identified in *Principles to Actions*, as teacher actions impact the intended

outcomes of student thinking and learning (NCTM, 2014).

Equal-sharing code. Empson and Levi (2011) defined an equal-sharing problem as a problem that "involves a total number of items to be distributed to a given number of groups, usually people" (p. 8). As I analyzed the mathematical tasks in the textbook that are to be posed to students, I looked for tasks in which students are intended (a) to distribute a quantity into a number of groups, (b) to give every group the same amount, and (c) to exhaust all of the material in the context of equal-sharing problems.

The inclusion of equal-sharing problems in coding stems from the decision to focus on the domain of fractions. This code would not be prevalent in other domains as it pertains to domain content rather than pedagogical implementation.

Teacher content knowledge. During early stages of data collection for the *a priori* codes, teacher content knowledge emerged as an aspect of professional development provided within the textbook series. Grade-level planning guides accompanying *Go Math! K-8* (HMH, 2016) specified descriptions for each of the standards for mathematical practice and connected mathematical practices with the topic for each lesson. Also incorporated in every lesson was a section on professional development for teachers, designed to build teacher content knowledge using mathematical practice standards (Burger et al., 2015). Each section on professional development referred to a mathematical practice standard either directly, by specifically labeling the practice, or indirectly, through the messages and intentions of the text.

I analyzed and evaluated the professional development section of each lesson of the teacher editions for the mathematical practice standards found therein. This analysis includes the frequencies for each mathematical practice standard in the professional development section of each lesson. I also analyzed the intentions of questions included in the planning guide (Burger et al., 2015), to be used in developing the mathematical practice standards, for alignment with

descriptions of mathematical practice standards provided by NGA Center & CCSSO (2010).

These intentions, or my interpretation of the meaning of the text (Thayer et al., 2007), were

compared for alignment with descriptions provided by CCSS-M (NGA Center & CCSSO, 2010).

Table 1 provides the rubric used for determining alignment of CCSS-M descriptions and

intentions of the planning guide questions.

Table 1

Rubric for Alignment of Descriptions of Mathematical Practices (MP) and Planning Guide Questions

Scale Score		
0	1	2
The intentions of the planning guide questions do not appear to align with the description of the specific standard for mathematical practice provided by CCSS-M and repeated in the planning guide of *Go Math! K-8*.	The intentions of the planning guide questions appear to align with the description of the specific standard for mathematical practice provided by CCSS-M and repeated in the planning guide of *Go Math! K-8*. However, some questions in the planning guide show inconsistencies with the specific standard.	The intentions of the planning guide questions appear to align with the description of the specific standard for mathematical practice provided by CCSS-M and repeated in the planning guide of *Go Math! K-8*, with no observed inconsistencies.

This rubric was evaluated and agreed upon by two university professors who specialize in

mathematics education. These professors and I tested the rubric separately during analysis of

planning guide questions to check for interrater agreement. The rubric appears to yield

consistent results when checking for alignment of mathematical practices and planning guide

questions as there was 100% interrater agreement.

Determining the frequencies. Included in the frequency counts are (a) the quantity of

occurrences of applicable teacher actions codes, (b) all subsets of the practice code, (c) the equal-

sharing code, and (d) emergent codes. For the frequency counts, codes were organized by the context within which they occurred, including, but not limited to, grade level.

Interpreting the data. I read and reviewed the qualitative data derived from the *a priori* and emergent codes within the contexts, including grade levels, in which they occurred. I analyzed the data for themes and outliers within those contexts to bring meaning to the data. The findings that emerged from this analysis are reported in Chapter 4, and the discussion of the findings is reported in Chapter 5. These conclusions informed my research questions as to what extent and in what ways *Go Math! K-8* (HMH, 2016) provides instructional support in building procedural fluency from conceptual understanding.

The compiled data from the frequency counts were also used to find patterns and draw conclusions. These patterns and conclusions further informed my response as to what extent the textbook provides instructional support to the teacher in building procedural fluency from conceptual understanding.

Limitations

When considering the focus of this content analysis, there are some limitations to the study that the reader must take into account. First, it must be understood that the content analysis (a) investigated only a single mathematics textbook series, *Go Math! K-8* (HMH, 2016); (b) focused on one mathematics domain, fractions in grades 3-5; and (c) examined only the domain of equivalence of fractions. The reasoning behind this funneled approach was to allow for an in-depth analysis of the important ideas and concepts that surround the domain of equivalence. Rather than developing a general overview by focusing on a broad band of content, I sought for a depth of understanding and clarity on how a fundamental concept, equivalence, progresses from a conceptual to a procedural understanding within the textbook series.

Secondly, because of the nature of this research, generalizability is not expected. Plano Clark and Creswell (2008) stated that generalizability of the results of a study cannot be connected to an entire population, in this case, the entire textbook series. Although the focus of this study revolved around equivalence, the pattern that each lesson followed in other units and domains appeared consistent. Each lesson, in every mathematical domain and grade level, had the same sectional setup, or in other words, the same subheadings within the 5E model of Engage, Explore, Explain, Elaborate, and Evaluate. With this thinking in mind, the pattern of teacher action codes found might be expected to remain consistent with teacher actions codes found in different mathematical domains of the same textbook series.

Third, the latent content analysis methodology used in this study for analyzing teacher materials involves analysis at an interpretive level. Much of the text analyzed consisted of directions to teachers, which were written to be readily interpreted by classroom teachers. However, in instances in which I as a classroom teacher/researcher found multiple interpretations to arise, I relied on the professional literature, research, and university professors until an agreed-upon interpretation within the context given was reached.

Finally, three of the five teacher action codes provided by *Principles to Actions* (NCTM, 2014) are dependent on student-generated methods: (a) student-generated strategies, (b) student discourse, and (c) connecting student-generated strategies to general methods. Therefore, when student-generated methods were not present in the textbook series, it would become unnecessary to check for the remaining teacher action codes of student discourse and connecting student strategies with general methods and practice. Therefore, when evidence of teachers allowing for student-generated methods was not present, I provided evidence and information on how the

textbook series handled teacher actions regarding student discourse, making connections, and practice.

Keeping these limitations in mind, this study could prove to be useful to assist in instructional planning for teachers, grade level teams that use this textbook series, and various administrators by presenting a clear view as to the extent the mathematics textbook aligns with current research and best practice in one topic. Teachers can use this research to guide students to understand fractions by building procedural fluency from conceptual understanding.

The hope is that this study will alert textbook selection committees to look beyond the publisher's labels related to CCSS-M (NGA Center & CCSSO, 2010) alignment and examine for themselves the quality of mathematics instruction that is likely to take place when a specific textbook series is implemented with fidelity. Furthermore, this research can support textbooks committees in providing meaningful feedback to textbook publishers when analyzing a textbook for quality mathematical instruction.

CHAPTER 4

Findings

The purpose of this research was to analyze the extent and ways in which *Go Math! K-8* (HMH, 2016) provides teachers with the instructional support for building procedural fluency from conceptual understanding. *A priori* codes were developed from NCTM's *Principles to Actions* (2014) and research on equal-sharing problems (Empson & Levi, 2011; Fazio & Siegler, 2011; Siegler et al., 2010). In addition, one code emerged during the analysis: teacher content knowledge (Ma, 1999; Schoenfeld, 2006).

This chapter details the findings from analyses of data collected according to the *a priori* and emergent codes, namely (a) student-generated methods, (b) student discourse, (c) connecting student strategies to general methods, (d) visual models, (e) practice, (f) equal-sharing problems, and (g) teacher content knowledge. Data for several of these codes were not evidenced in the textbook series; for those instances, this chapter also examines the publisher interpretations and intentions in implementing similar concepts.

Student-Generated Methods

Incorporating student-generated methods into regular classroom instruction is important, even essential, to quality mathematics instruction (Hendrickson et al., 2008; Herbel-Eisenmann & Breyfogle, 2005; NCTM, 2014; Schroyer & Fitzgerald, 1985). Allowing for student-generated methods provides students opportunities to reason through their own thinking to solve a mathematical task. I analyzed each task in each grade level for teacher directions instructing students to use their own strategies and multiple methods to solve a task.

Nearly all of the lessons examined in this study included opportunities for students to use multiple methods to solve a problem, but the methods were introduced by the teacher at the

beginning of the lesson, not generated by students. The only example that related to student-generated problem-solving methods was in the Assessing Prior Knowledge section of the Chapter 6 introduction of *Go Math! G4* (2015) teacher edition. There the teacher is instructed to "brainstorm problem-solving strategies that they could use to solve the problem, such as drawing a picture, using a model, or reducing fractions to their simplest form" (p. 325). However, since this directive is given to the teacher, it is inconclusive if the brainstorming should be done by the teacher in preparation, by the students to generate methods, or if the intended instruction was to facilitate a class discussion.

Student Discourse

Providing students opportunities to justify and explain their strategies to other students is an important teacher action that *Principles to Actions* encouraged (NCTM, 2014). Student discourse should center on student-generated strategies that respect student thinking and allow time to communicate their own understanding of the process toward solving the task (Bahr & Bahr, 2017; Herbel-Eisenmann & Breyfogle, 2005; NCTM, 2014).

While *Go Math! K-8* (HMH, 2016) provided multiple structured discourse opportunities for students to engage in discussion, these discussion opportunities do not appear to fit the criteria for student discourse as defined by NCTM (2014), in which students discuss and build a common understanding of mathematical ideas as student strategies are analyzed and compared. Each of the discussion opportunities appearing in the data collected was classified as structured discourse, featuring teacher-driven strategies and methods rather than student-driven discourse during which students develop and explain their own strategies to solve mathematics problems. These structured discourse opportunities are represented in Table 2 below, showing occurrences when teachers asked their students to explain mathematical strategies and methods. The

occurrences were categorized as either teacher-to-student structured discourse or student-to-student structured discourse, depending on the two parties who were to discuss. The inconclusive category consisted of discussions that were encouraged where the two parties were not explicitly stated.

Table 2

Occurrences of Structured Student Discourse

Grade Level	Teacher-Student	Student-Student	Inconclusive
Grade 3	20	10	16
Grade 4	22	7	15
Grade 5	19	9	19
Grade 6	8	1	0

As the table shows, the analysis revealed far more examples of teacher-to-student structured discourse than student-driven discourse, with a large number of inconclusive discussion opportunities. This pattern held true for all grade levels, particularly in the Share and Show section of the Explain phase in each lesson, where the majority of the inconclusive discussion opportunities were found. In these sections, students were consistently instructed to explain their thinking using a prescribed strategy to solve the problem, but it was unclear to whom students should explain, rendering these discussion opportunities inconclusive as to whether they were student-driven or structured discourse. The Making Connections section at the beginning of each lesson encouraged students to engage in a structured conversation with the teacher and each other. These discussion opportunities primarily focused on building background knowledge, reviewing terminology, and reviewing content; therefore, they were excluded from the counted data set, as those questions did not pertain to NCTM discourse criteria relating to students' mathematical strategies and methods.

Within the analyzed data, the majority of the structured discourse opportunities were teacher-to-student, wherein the teacher was directed to ask a question to elicit student responses. In one grade 3 example, the teacher was directed to "discuss with students which strategy could be used and why the strategy is appropriate for the problem" (Go math! G3, 2015, p. 528). A grade 4 example initiated structured discourse between the teacher and student: "What other strategy could you have used to solve this problem? Explain how you would use it" (Go math! *G4*, 2015, p. 352). This example was considered structured discourse as the teacher had modeled two strategies at the beginning of the lesson. Grades 5 and 6 contained similar structured discourse prompts, such as "How does each part of the number line relate to each part of the fraction model?" (*Go math! G5*, 2015, p. 491) and "How can you use a double number line to make a prediction?" (Burger et al., 2014 p. 163). Although expected student answers were provided, the teacher edition suggested that answers would vary.

Examples of student-to-student structured discourse were found in all grade level texts, with the number of discussion opportunities decreasing in the grade 6 teacher edition. For example, the grade 3 teacher edition *(Go math! G3*, 2015) contained 10 student-to-student discussion prompts, such as "have students look at Exercise 9 and discuss with a partner if the size of the parts in the circle are the same size" (p. 463). Similar numbers of student-to-student discussion questions were found in the teacher editions for grades 4 and 5. One grade 4 example instructed the teacher to invite partners to model a problem and "then discuss the problems, models, and equations" (*Go math! G4*, 2015, p. 386), while a grade 5 example instructed, "Have students use their MathBoard [whiteboard] to demonstrate to a partner the answer to the essential question" (*Go math! G5*, 2015, p. 506). The solitary example of structured discourse between students in grade 6 was found in the Cooperative Learning portion of the Differentiate

Instruction section, where the teacher is directed to "have pairs compare their work and explain their steps" (Burger et al., 2014, p. 158) after solving a unit rates problem.

While no examples of student discourse and discussion based on student-generated strategies as described by NCTM were found in *Go Math! K-8* grades 3-6 fraction units (Burger et al., 2014; *Go math! G3,* 2015; *Go math! G4*, 2015; *Go math! G5*, 2015), each chapter did contain opportunities for structured discourse, both teacher-to-student discussion and student-to-student discussion. Although the opportunities for student-to-student discourse are meager throughout grades 3-5, the data revealed these opportunities are almost nonexistent in grade 6.

Connecting Student Strategies to General Methods

To effectively build procedural fluency from conceptual understanding, students need to generate their own problem-solving strategies and then connect their own strategies to other students' strategies (Carpenter et al., 2003; NCTM, 2000, 2014). Although many kinds of connections are discussed in the literature, the textbook series was evaluated for connections to student-generated strategies as described by NCTM (2014) and, if present, student progression from concrete to abstract conceptual understanding demonstrated.

The chapters analyzed provided zero examples of problems, questions, or instructions allowing students to create their own strategies and make connections to other students' strategies: Instead, strategies were included in the suggested lesson development for the teacher to present to the students. The lack of student-driven strategies negated the need to analyze the concrete-to-abstract progression of these methods. Although connections as defined by NCTM (2014) are not present, it is important to recognize the interpretation of connections that were found in the teacher materials.

While lacking student-driven strategies and connections among these strategies, the teacher editions did feature a Making Connections section in grades 3-5; this section provided opportunities for students to review previous content, recall important vocabulary, and build background knowledge for context in upcoming lessons. The Making Connections section was not included in the grade 6 teacher edition.

The following summary provides an analysis of the relevant data regarding textbook authors' intentions about connections and how connections were interwoven throughout the lessons. To review previous lesson content and evaluate knowledge gained, the grade 3 teacher edition contained several prompts for teachers to ask students to describe what they knew about fractions (*Go math! G3,* 2015). The grade 4 and grade 5 teacher editions encouraged review with prompts such as, "Ask students to tell what they know about benchmarks" (*Go math! G4*, 2015, p. 365) and "Invite students to tell what they know about adding and subtracting fractions and mixed numbers" (*Go math! G5*, 2015, p. 401), where each suggested topic recalled the previous lesson's content.

The Making Connections section also focused on vocabulary discussion. The grade 3 teacher edition started fraction vocabulary discussions with prompts such as, "What does the word whole mean?" (*Go math! G3,* 2015, p. 461) and "Name the parts of a fraction" (*Go math! G3,* 2015, p. 519). The grade 4 teacher edition asked, "What do the numerator and denominator tell you?" (*Go math! G4*, 2015, p. 359B) and grade 5 suggested "In a fraction, which part is the numerator?" (*Go math! G5*, 2015, p. 351B) with the textbook definition of numerator being the top number and the denominator being the bottom number.

The third topic found in the Making Connections section was an invitation to build background knowledge about a topic to be presented in the first problem. For example, the grade

3 teacher edition invited students to "talk about a time when they shared something" (*Go math! G3,* 2015, p. 443B); the grade 4 and grade 5 teacher editions invited students to share their knowledge about ants (*Go math! G4*, 2015) and gas tanks (*Go math! G5*, 2015) to build background knowledge.

Additionally, some lessons featured a Making Connections section in the middle of the lesson development. "Help[ing] students make connections to the arrays that they used in multiplication" (*Go math! G3,* 2015, p. 489) was encouraged in a lesson on finding part of a group using unit fractions in grade 3. A grade 4 lesson on investigating adding and subtracting parts of a whole encouraged students to make connections, with teacher assistance, that one can only "join or separate parts that refer to the same-size whole" (*Go math! G4*, 2015, p. 386); the teacher helped the students come to that realization by pointing the information out to them. During one lesson, grade 5 students were encouraged to compare students' work in a Make Connections section (*Go math! G5*, 2015). It should be noted that the student work was provided by the textbook rather than having students figure out a problem and share their work with each other.

The grade 6 teacher edition (Burger et al., 2014) did not include the Make Connections section, but each lesson did feature a Connect Vocabulary section in the Explain portion of the suggested lesson development. This section was specifically designated for English Language Learners and asked the teacher to remind the students of specific vocabulary terms, such as *ratio* and *equivalent*, and definitions important to the lesson.

While these forms of connections can be valuable, no evidence of connecting student strategies to general methods was found, since in the analyzed sections students were never invited to develop their own strategies to solve the mathematics problems. *Go Math! K-8*

(HMH, 2016) did provide sections on Making Connections that centered on vocabulary review, concept review, and building contextual background. Further, a few examples of comparing student work, connecting methods, and connecting concepts were included. The inclusion of these results provides data pertaining to what ways *Go Math! K-8* interprets connections and how those interpretations differ from connections that are intended to help students build procedural fluency from conceptual understanding.

Visual Models

Visual models refer to drawings, pictures, diagrams, physical objects, fraction notation or other representations that students can see to further their connections within mathematics (NCTM, 2000). Students build procedural fluency from conceptual understanding by connecting algorithmic thinking to previously learned visual models (NCTM, 2014). The teacher editions were analyzed regarding the progression of visual models to algorithmic procedures and to what extent visuals were purposefully connected to general algorithmic methods.

The visual models in *Go Math! G3* (2015) lessons on equivalence were all drawings. Students were shown how to cut, or divide, a visual representation, a drawing, to demonstrate how the model of the whole and the divided model were representative of the same amount. In the same lesson, when partitioning a line segment into equal parts, the text suggested connecting to a previous lesson that used fraction strip drawings to create a number line.

The *Go Math! G4* (2015) equivalence lesson was introduced by directly showing the same visual representations as were used in grade 3. However, the grade 4 lesson extended the notion of equivalence of the two representations by multiplying the numerator and the denominator by the same factor, thus demonstrating the use of the Multiplicative Identity Property, or multiplying a fraction by an expression for one, such as 1/3 x 2/2 = 2/6; 1/3 and 2/6

being equivalent fractions (*Go math! G4*, 2015). This example shows progression from a visual representation to an algorithm to find equivalent fractions.

The grade 5 teacher edition included a specific lesson on common denominators and equivalent fractions in which students were provided with two strategies to create common denominators and equivalent fractions (*Go math! G5*, 2015). First, students were guided by the teacher to write out a list of multiples of each denominator, look for the multiples that the denominators had in common, and then multiply the numerator and the denominator by the same factor to create a common denominator. Second, students were told to multiply the numerator and denominator by the denominator of the second fraction. Later, these two strategies were reinforced when students were expected to add and subtract fractions with different denominators. There were no visuals provided during this lesson or other lessons that indicated the grade 5 teacher edition moved from visual models toward abstract strategies to represent equivalence.

Finally, in the grade 6 lesson (Burger et al., 2014) on ratios, teachers were instructed to explain the multiplicative relationship between two ratios to justify equivalence of the two. Later, teachers were advised to remind students to "multiply or divide both terms of the ratio by the same number to find equivalent ratios" (p. 151). This reminder was specifically provided to avoid common mistakes that students make. An algorithmic step-by-step procedure was given to create equivalent ratios, with no connections to conceptual understanding through the use of visual models.

Beginning in grade 3, visual representations were relied on to provide students with an understanding of equivalence. In grade 4, the visual representations were connected to a strategy that included multiplying the numerator and denominator by the same number. However, the

conceptual knowledge present, if any, in making connections is unclear. The visuals were not

apparent in grade 5 and 6, but the same algorithmic thinking, multiply or divide the numerator

and denominator by the same number, was provided to create equivalent fractions and ratios.

Practice

Engaging in practice in which both conceptual understanding and procedural fluency are

applied is important to gaining problem-solving skills (Isaacs and Carroll, 1999). Meaningful

practice takes place when questions and tasks require problem solving. The practice tasks in *Go*

Math! K-8 (HMH, 2016) were analyzed using Smith and Stein's (1998) Levels of Cognitive

Demand and were categorized as higher-level demand or lower-level demand tasks (see

Appendix A). Although the textbooks were not explicit in labeling which tasks were designed

for practice, tasks were categorized as practice when they appeared after students had an initial

experience of discussing and learning about a new concept or procedure (NRC, 2001).

Table 3

Practice Task Demand Levels

Grade Level	Low Level Demand	High Level Demand
Grade 3 Chapters	290	61
Grade 4 Chapters	498	102
Grade 5 Chapters	513	121
Grade 6 Chapters	261	39

The majority of the higher-demand practice tasks came from the Think Smarter and Go

Deeper questions during the Explain and Elaborate phase. Tasks in both of these sections

typically required students to make connections with multiple strategies and to engage in the

conceptual ideas of the mathematics to work toward a procedural method. Although not every

Go Deeper or Think Smarter task was found to be higher-level demand, there were

approximately two Go Deeper and two Think Smarter questions in every lesson. The grade 6

teacher edition did not include Go Deeper questions or Think Smarter questions, but instead

featured Higher Order Thinking (HOT) questions. In grades 3-5, Go Deeper and Think Smarter

questions were reserved for the students who had correctly answered beginning of the lesson

questions, indicating that not all students would be expected to solve these tasks. However, in

the grade 6 teacher edition (Burger et al., 2014), there was no indication for teachers to stop the

students from continuing even if they were having difficulties solving the tasks. Therefore, it is

presumed that all students in grade 6 would have the experience of being asked to solve HOT

questions.

Based on Smith and Stein's characteristics of higher-level demand tasks and lower-level

demand tasks (1998), the analysis revealed a total of 290 lower-level demand tasks and 61

higher-level demand tasks in grade 3, translating to a nearly 5:1 ratio of lower- to higher-level

demand tasks. As shown in Table 3, grade 4 followed a similar 5:1 split between lower-level

and higher-level demand questions. Grade 5 revealed an approximate 4:1 ratio, while grade 6

followed a more drastic split between lower and higher-level demand questions at a ratio

approaching 7:1. Most of the lower-level demand tasks were procedures without connections to

conceptual ideas, whereas the higher-level demand tasks required conceptual understanding and

explanation.

An example of a higher-level demand task found as a Think Smarter practice item in the

grade 3 textbook follows.

Zach has a piece of pie that is ¼ of a pie. Max has a piece of that is ½ of a pie.

Max's piece is smaller than Zach's piece. Explain how this could happen. Draw

a picture to show your answer. (*Go math! G3,* 2015, p. 522)

This task was considered to be a higher-level demand because there were underlying conceptual ideas that required some cognitive effort to figure out and students needed to connect their reasoning to a visual model they created. A lower-level demand word problem from grade 3 was, "Carlos finished ⅝ of his art project on Monday. Tyler finished ⅞ of his art project on Monday. Who finished more of his art project on Monday?" (*Go math! G3,* 2015, p. 515). This task was categorized as lower-level demand because it did not require any explanation and was primarily focused on producing a correct answer, with no expressed requirement designed to develop mathematical understanding (See Appendix B for Sample Tasks for Four Levels of Cognitive Demand).

The grade 4 teacher edition (*Go math! G4,* 2015) had a similar almost 5:1 ratio of lower-level demand tasks to higher-level demand tasks, with 498 lower-level and 102 higher-level demand tasks. An example of a higher-level demand task, found in a lesson about comparing fractions, can be represented in multiple ways and required students to examine the constraints of the task: "Give an example of fractions that you would compare by finding common denominators, and an example of fractions you would compare by finding common numerators" (*Go math! G4*, 2015, p. 365). The same lesson featured a series of eight lower-level demand problems in which students were required to write the mathematical symbols of greater than, less than, or equal to in comparing two fractions given to them such as 1/3 O 1/4.

The ratio of lower-level to higher-level demand tasks decreased slightly to roughly 4:1 in the grade 5 teacher edition, featuring 513 lower-level demand questions and 121 higher-level demand questions. A Think Smarter example included a diagram of a road map with different fractional distance values and landmarks labeled.

On one afternoon, Mario walks from his house to the library. That evening, Mario walks from the library to the mall, and then to Kyle's house. Describe how you can use the [commutative and associative] properties to find how far Mario walks. (*Go math! G5*, 2015, p. 410) This problem showed a higher-level demand because students were asked both to solve the problem and to explain their solution. Previous questions in the lesson modeled the solution with explicit pathways, but the problem can still be classified as higher-demand following Smith and Stein's guidelines (see appendices A and B). The lesson on fraction multiplication featured an example of a lower-level demand problem: "Karen raked ⅗ of the yard. Minni rakes ⅓ of the amount Karen raked. How much of the yard did Minni rake?" (*Go math! G5*, 2015, p. 455). This problem (a) showed an expectation of a procedure to solve the problem based on the strategy modeled in the beginning of the lesson, (b) required no explanation, and (c) required no connections of conceptual understanding to procedural fluency.

The grade 6 textbook featured a higher ratio of lower-level demand questions to higher-level demand questions than was found in the lower grades: 261 lower-level demand questions and 39 higher-level demand questions approaches a 7:1 ratio. As an example, the lesson on applying greatest common factor and least common multiple to fractions operations contained 37 consecutive lower-level demand tasks in which students were given two fractions to add, subtract, or multiply (Burger et al., 2014, pp. 82-83). The same lesson later contained a higher-level demand HOT question regarding error analysis: "To find the product 3/7 x 4/9, Cameron simplified 3/7 to 1/7 and multiplied the fractions 1/7 and 4/9 to find the product 4/63. What is Cameron's error?" (Burger et al., 2014, p. 84). This problem required cognitive effort and understanding of the procedures, but the underlying conceptual understanding can be demonstrated in the explanation of the error.

The analysis of the 351 practice items in grade 3, 600 in grade 4, 634 in grade 5, and 300 in grade six showed that a major discrepancy between lower- and higher-level demand tasks existed. Based on data collected, the overall ratio of lower-level demand to higher-level demand practice items provided to students was approximately 5:1. As the grade levels progressed, the ratio of lower-level to higher-level problems increased; thus as the grades progressed, students presumably would have proportionally fewer opportunities to engage in practice that promotes building procedural fluency from conceptual understanding.

Equal-Sharing Problems

Empson and Levi (2011) and Seigler et al. (2010) emphasized the importance of using equal-sharing problems to guide students from their beginning conceptual understanding of fractions toward more advanced procedural fluency through the development of fractional thinking. Equal-sharing problems were found in varying frequencies in the *Go Math! K-8* (HMH, 2016) curriculum as shown in Table 4.

Table 4

Equal-Sharing Problems

Grade Level Chapters	Equal-Sharing Problems
Grade 3	32
Grade 4	0
Grade 5	24
Grade 6	3

Equal-sharing problems were most prevalent in the grade 3 teacher edition (*Go math! G3*, 2015), in which one lesson (p. 449A) was dedicated entirely to equal-sharing problems. That lesson alone contained 15 of the total 32 equal-sharing problems at the textbook. In other lessons, many of the equal-sharing problems were a direct review or an assessment question

associated with the individual lesson. One of the examples observed in Lesson 8.2 of grade 3 was, "Four friends share 2 small pizzas. What are two ways that they could be divided equally? How much pizza will each friend get?" (*Go math! G3,* 2015, p. 449). This problem allows for multiple solutions, including one modeled strategy of drawing a picture.

Although the grade 4 teacher edition (*Go math! G4*, 2015) included topics such as equivalence, comparing fractions, and the beginning development of the representation of whole numbers and fractions together, zero examples of equal-sharing problems were found in these lessons or in the additional lessons in the fraction chapters. This decrease in equal-sharing problems from grade 3 to grade 4 showed a lack of continuity and connection regarding equal-sharing problems between grades 3 and 5.

Lesson 8.3 of the grade 5 teacher edition included 24 equal-sharing problems connecting fractions to division, instructing students to solve problems with contexts such as, "Six friends share 4 small pizzas equally" (*Go math! G5*, 2015, p. 505). To solve these problems, students were directed by the teacher to draw lines on a rectangular model provided on the page to visualize the equal-sharing problem. Nine of the ten remaining equal-sharing problems in Chapter 8 were review and assessment problems connected to that lesson. There were no examples of equal-sharing problems in the remaining chapters of the grade 5 *Go Math!* teacher edition.

Only three equal-sharing problems were found in the three chapters associated with fractions in the grade 6 teacher edition, and all three of the examples were found in a lesson on dividing fractions (Burger et al., 2014, p. 85). One problem given to students was, "Six people share ⅗ pound of peanuts equally. What fraction of a pound of peanuts does each person receive?" (Burger et al., 2014, p. 89). At the beginning of the lesson, students were asked, "Five

people share ½ pound of cheese equally. How much cheese does each person receive?" (Burger et al., 2014, p. 85). However, this problem was one for which the strategy and process were modeled for them; therefore, students did not use their own ways of thinking to solve the problem. The final equal-sharing problem found in the grade 6 teacher edition was, "A pitcher contains 2/3 quart of lemonade. If an equal amount is poured into each of the 6 glasses, how much lemonade will each glass contain?" (Burger et al., 2014, p. 89).

The data revealed that the majority of the equal-sharing problems were concentrated in individual lessons within a chapter unit. The huge disparities in the presence of equal-sharing problems was obvious, as grade 4 and grade 6 contained very few to zero examples throughout the entire equivalence portion of the fraction chapters.

Teacher Content Knowledge

To support students' progression from conceptual understanding to procedural fluency, teachers must have a solid foundation in PUFM, which includes both content and pedagogical knowledge (Ma, 1999; Schoenfeld, 2006). Many teachers have inadequate teacher content knowledge, preventing them from teaching the depth of mathematics that students need (Ball & Feinman-Nesmer, 1988; Lamon, 2007; Ma, 1999; Schoenfeld, 2002; Shulman, 1986). Ma (1999) and Schoenfeld (2006) suggested that teachers should deepen and develop their own mathematical skills by intensely studying the curriculum materials. Additionally, Ma (1999) recommended that teachers engage in professional development focused on students' ways of working mathematics and how to enable students to lead conversations about their own mathematical strategies.

Each lesson of *Go Math! K-8* (HMH, 2016) contained a professional development section that focused on a student mathematical practice to build teacher content knowledge and to help

teachers learn better how students engage meaningfully in mathematics. The content of these sections was analyzed for alignment to the eight standards for mathematical practices (NGA Center & CCSSO, 2010). The analysis centered on mathematical practices because *Go Math K-8* (2016) used these practices to develop teacher content knowledge. While the teacher editions included connections to various mathematical practices throughout most lessons, this analysis examined only those included in the professional development sections to focus on the evidence of building teacher content knowledge (*Go math!,* 2012).

As the professional development in *Go Math! K-8* (HMH, 2016) focused on the mathematical practice standards to build teacher content knowledge (*Go math!*, 2012), it is important to understand the descriptions of the mathematical practices. The teacher materials contained a curricular planning guide that described mathematical practices and listed suggested questions to guide students in developing each of the mathematical practices (Burger et al., 2015). These descriptions and questions were analyzed for intended outcome and alignment with CCSS-M (NGA Center & CCSSO, 2010) descriptions and standards by applying the rubric shown in Table 1 in the Methods chapter.

Some mathematical practice standards received greater focus than others, based on the number of times the mathematical practice was referenced either directly or indirectly in the professional development sections. For the two most frequently occurring mathematical practices, this analysis includes descriptions and examples of each as well as suggested sample questions appearing in the planning guide (Burger et al., 2015). For the six practices occurring less frequently in the teacher editions, this analysis includes only a summative overview of the frequencies of mathematical practice standards and a discussion of the agreement between practices and the intentions of planning guide questions.

Table 5 lists the number of occurrences of each mathematical practice standard from the

professional development portion the lessons. The table also includes the grade-level

occurrences for each mathematical practice.

Table 5

Number of Occurrences of Standards for Mathematical Practices (MP) By Grade Level Teacher Edition for Equivalence of Fractions

MP	Description	Grade 3	Grade 4	Grade 5	Grade 6	Total
1	Make sense of problems and persevere in solving them	2	0	2	1	5
2	Reason abstractly and quantitatively	4	7	6	0	17
3	Construct viable arguments and critique the reasoning of others	2	1	5	1	9
4	Model with mathematics	9	12	7	7	35
5	Use appropriate tools strategically	2	1	1	1	5
6	Attend to precision	0	0	2	1	3
7	Look for and make use of structure	0	3	4	1	8
8	Look for and express regularity in repeated reasoning	0	1	2	0	3

Note. The standards for mathematical practices are in reference to NGA Center & CCSSO, 2010, Standards for Mathematical Practice as applied to equivalence of fractions.

The analysis of the total number of occurrences of individual mathematical practices and the

relation that number had to the total occurrences led to two conclusions: two of the Standards for

Mathematical Practice (NGA Center & CCSSO, 2010), MP2 and MP4, accounted for approximately 60% of the references to mathematical practices, while approximately 40% of occurrences were distributed among the remaining six mathematical practices.

The majority of the professional development sections examined in grade 3 (*Go math! G3*, 2015) focused on model with mathematics (MP4), followed by reasoning abstractly and quantitatively (MP2). No examples of attend to precision (MP6), look for and make use of structure (MP7), or express regularity in repeated reasoning (MP8) were noted in the professional development sections of the grade 3 curriculum regarding equivalence.

Evidence of a similar pattern in the data was found in grade 4 (*Go math! G4*, 2015), including the continuation of professional development focused on model with mathematics (MP4) and reason abstractly and quantitatively (MP2). Although no examples of look for and make use of structure (MP7) were apparent in grade 3, multiple examples were found in the grade 4 textbook. No evidence of the mathematical practice attend to precision (MP6) was found in any of the professional development sections of equivalence in grade 4.

Grade 5 (*Go math! G5*, 2015) lessons showed a more balanced approach to building teacher content knowledge through mathematical practices with each mathematical practice either referenced or implied. Model with mathematics (MP4) continued to be the predominantly occurring practice, but reason abstractly and quantitatively (MP2), construct viable arguments and critique the reasoning of others (MP3), and look for and make use of structure (MP7) were similarly included in the professional development sections.

Most of the professional development examples in grade 6 (Burger et al., 2014) referred to model with mathematics (MP4). In a surprising change from the focus in grades 3-5, no examples of reason abstractly and quantitatively (MP2) were found in the grade 6 professional

development section of the teacher edition. It is worth noting that, unlike professional development sections for other grades, each of the professional development sections for grade 6 referenced and included a label of a specific mathematical practice integration.

Overall, model with mathematics (MP4) was the most frequently referenced practice, accounting for 35 of the 82 occurrences or approximately 40% of all references in professional development sections of the teacher editions analyzed. The publishers of *Go Math! K-8* (HMH, 2016) aligned their description with the CCSS-M definition for model with mathematics as "applying the mathematics they know to solve problems arising in everyday life, society, and the workplace" (Burger et al., 2015, p. 27). One such grade 4 professional development section stated that the "lesson provides students opportunities to apply mathematics to situations they might meet in everyday life" (*Go math! G4*, 2015, p. 351A); however, the example continued with creating models of dividing rectangles, with no context given, to make generalizations of what takes place when finding equivalent fractions. The intention of the professional development appears to be misaligned with the description of model with mathematics (MP4) as indicated both in the planning guide (Burger et al., 2015) and CCSS-M (NGA Center & CCSSO, 2010).

An example of the mathematical practice model with mathematics (MP4) in the grade 6 teacher guide showed a similar structure and misalignment, using multiple models to apply to MP4. The example explains,

> This lesson provides an opportunity to address this mathematical practice
> standard [model with mathematics]. It calls for students to communicate
> mathematical ideas using multiple representations as appropriate. In Explore the
> Activity, students use bar models to model the division of fractions. Using the

model, they are able to see that dividing a fraction by a fraction can result in a whole number quotient, a concept that many students do not find intuitive. (Burger et al., 2014, p. 86)

The planning guide implemented in grades K-5 included sample questions designed for teachers to use to support the learning and understanding of mathematical practices (Burger et al., 2015). The rubric analysis (See Table 1) considered the alignment of all planning guide questions in the *Go Math! K-8* planning guide to the descriptions of mathematical practices. Model with mathematics (MP4) is again defined as using real-world problems to apply what students know. The suggested questions that correlate to this mathematical practice were, "Why is this a good model for this problem?" (p. 23), and "How would you change your model if...?" (p. 23), again sending the message that the model or representation is the main idea rather than the real-world problem and application. Consequently, model with mathematics (MP4) was given a 0 on the rubric score, signifying that the description of MP4 was not consistent with the intended outcomes of the planning guide questions.

In the professional development sections of the teacher editions (Burger et al., 2014; *Go math! G3,* 2015; *Go math! G4*, 2015; *Go math! G5*, 2015) and in the planning guide the description of model with mathematics (MP4) centered around real-world problems and applications. However, examples asked students to show multiple representations and visual models, illustrating a disconnect between the intent of the professional development questions and the described mathematical practice.

The second-most frequently occurring practice in the professional development sections was reason abstractly and quantitatively (MP2). This practice accounted for 17 of the 82 total references, or approximately 20%. Again, *Go Math! K-8* (HMH, 2016) followed the exact

definition of this mathematical practice as given by the NGA Center and CCSSO: Students can make sense of the quantities and the connection the quantities have to the problem (Burger et al., 2015; NGA Center & CCSSO, 2010). In other words, students can "decontextualize" (p. 1) and "contextualize" (p. 1) the quantities they are examining (NGA Center & CCSSO, 2010).

One example of a professional development instruction from *Go Math! K-8* that focused on reasoning abstractly and quantitatively (MP2) came from the grade 3 lesson relating to equivalence: "Students need to use abstract reasoning as they make connections between fractions less than one and fractions greater than 1" (*Go math! G3,* 2015, p. 475A). The professional development section described the misconception that fractions are limited to a quantity between zero and one; instead students begin to learn that a fraction is a "relationship between the parts and a whole" (p. 475A). The professional development section continued:

> [Students] have learned that the denominator represents the number of equal
>
> parts that the whole is divided into. Now they must use abstract reasoning as they
>
> apply that knowledge to fractions where the numerator is greater than the
>
> denominator. This type of fraction gives students an opportunity to develop
>
> quantitative reasoning. They must understand the representation of a fraction,
>
> think about the components involved in a fraction, and understand the meaning
>
> the quantities. (*Go math! G3,* 2015, p. 475A)

The description of the meaning of fractions follows the definition of reason abstractly and quantitatively (MP2) by emphasizing the importance of being able to contextualize quantities and decontextualize numbers based on what a fraction represents.

Another MP2 professional development example, from a grade 5 lesson on adding and subtracting fractions (*Go math! G5*, 2015, p. 375A), stated that students will begin to move from

concrete models to more abstract, symbolic methods. Teachers were instructed to guide students from using fraction models to representing fractions and operations with symbols; by doing so, students learn to manipulate fractions by writing equivalent fractions and can rely less on fraction strips to understand equivalence. The aim of this process was that "students conceptualize what symbols mean without having to use the models" (*Go math! G5*, 2015, p. 375A), learning to reason abstractly and quantitatively and therefore indicating that the message from the example matches the description from CCSS-M (NGA Center & CCSSO, 2010).

The *Go Math! Teacher Planning Guide* (Burger et al., 2015) provided questions designed to help teachers build students' skills to reason abstractly and quantitatively (MP2): "How do you know your answer is reasonable?", "What is a situation that could be represented by this equation?", and "Why does that operation represent the situation?" (Burger et al., 2015, p. 23). These questions may support abstract and quantitative reasoning, but they do not appear to contribute to a progression from concrete understanding to abstract thinking (Hendrickson et al., 2008). As such, reasoning abstractly and quantitatively was assigned a rubric score of 1 for alignment because the intentions of the planning guide questions aligned with the mathematical practice but only partially met the depth of the full description of this mathematical practice.

The remaining six mathematical practices occurred a total of 33 times in the professional development sections of each lesson, accounting for only a combined 40% of examples. Because of the low frequency of occurrences, individual examples of these mathematical practices are not described in depth; instead, a comparison of the CCSS-M (NGA Center & CCSSO, 2010) descriptions of mathematical practices and planning guide sample questions are provided in Table 6. The table includes (a) the mathematical practice description, (b) sample planning guide questions, and (c) a rubric score based on alignment between the description and the intent of the

sample questions. Although the questions provided for supporting each mathematical practice varied in number, each question has the intention of helping students develop the correlating mathematical practice. Therefore, the first four questions from the suggested questions for each mathematical practice are listed in the table for illustrative purposes, with a link to view all questions suggested by the planning guide in the associated reference (Burger et al., 2015).

Table 6

Description and Planning Guide Questions: Rubric Score for Alignment

MP	Description	Example Planning Guide Questions	Rubric Score
1	Make sense of problems and persevere in solving them	What is the problem asking? How will you use the information? What other information do you need? Why did you choose that operation?	2
2	Reason abstractly and quantitatively	What is a situation that could be represented by this equation? What operation did you use to represent the situation? Why does that operation represent the situation? What properties did you use to find the operation?	1
3	Construct viable arguments and critique the reasoning of others	Will that method always work? How to you know? What do you think about what she said? Who can tell us about a different method?	2
4	Model with mathematics	Why is that a good model for this problem? How can you use a simpler problem to help you find the answer? What conclusions can you make from your model? How would you change your model	0

if…?

5	Use appropriate tools strategically	What could you use to help you solve the problem? What strategy could you use to make that calculation easier? How would estimation help you solve that problem? Why did you decide to use…?	1
6	Attend to precision	How do you know your answer is reasonable? How can you use math vocabulary in your explanation? How do you know those answers are equivalent? What does that mean?	2
7	Look for and make use of structure	How did you discover that pattern? What other patterns can you find? What rule did you use to make this group? Why can you use that property in this problem?	2
8	Look for and express regularity in repeated reasoning	What do you remember about…? What happens when…? What if you… instead of…? What might be a shortcut for…?	1

Note. Please refer to p. 46 of this document to view the rubric.

For four of the six remaining mathematical practices, the intent and expected outcomes of the suggested planning guide questions aligned with the descriptions of the mathematical practices: Make sense of problems and persevere in solving them (MP1), construct viable arguments and critique the reasoning of others (MP3), attend the precision (MP6), and look for and make use of structure (MP7) clearly aligned and were scored 2 on the rubric. The planning guide questions for two mathematical practices, use appropriate tools strategically (MP5) and

look for and express regularity in repeated reasoning (MP8), aligned only partially with the descriptions of the mathematical practices. While the planning guide included some questions to lead to the development of the mathematical practice, not all questions were consistent with the description of the mathematical practice. For example, one suggested MP5 question led toward conceptual strategies rather than hands-on tools such as calculators and protractors, and MP8 questions lacked situational background for each question. For example, one question referred to what students remembered, but without situational information, this question did not provide enough evidence to assume that students would build the skill of recognizing regularity and repeated reasoning. For these reasons, these two mathematical practices scored 1 on the rubric.

In summation, each lesson on equivalence of fractions in the teacher editions of *Go Math! K-8* (Burger et al., 2014; *Go math! G3,* 2015; *Go math! G4*, 2015; *Go math! G5*, 2015) included professional development instruction to foster teacher content knowledge relating to the established standards for mathematical practice (*Go math!,* 2012). Each grade level curriculum addressed fractional equivalence, though the number and frequency of occurrences of individual mathematical practices varied among the grade levels. The majority of the examples and questions in *Go Math! K-8* supporting documents clearly aligned with the descriptions of the standards for mathematical practice*;* however, the textbook examples and suggested questions in the planning guide were clearly misaligned with the most frequently referenced practice, model with mathematics (MP4).

Summary of Results

This research was conducted to answer the question: Regarding equivalence of fractions, to what extent and in what ways does *Go Math! K-8* (HMH, 2016) provide teachers instructional support to build procedural fluency from conceptual

understanding, grades 3-6? Quantitative data were collected in the form of simple frequency counts from codes that detailed characteristics of mathematical instruction that builds procedural fluency from conceptual understanding. Quantitative data were also collected in relation to the same codes. Data analyzed for some codes were consistent with high-level mathematics instruction while data for other codes were inconsistent with high-level mathematics instruction. The majority of the coded data showed inconsistencies in supporting quality mathematics instruction as evidenced in the following codes: student-generated strategies, student discourse, connecting strategies, practice, equal-sharing problems, and teacher content knowledge.

No examples of student-generated strategies were found in the *Go Math! K-8* (HMH, 2016) teacher editions regarding equivalence of fractions in grades 3-6. The teacher editions provided no opportunities for students to generate their own methods to solve a mathematical problem; therefore, student engagement in meaningful discourse by explaining their strategies to the teacher and other students was also not apparent. Although some examples of promoting student conversations during mathematics lessons were found, there were no examples of students describing their own strategies. As a result, the opportunity for students to connect their own strategies to general methods was also not present.

As a whole, *Go Math! K-8* (Burger, et al., 2014; *Go math! G3,* 2015; *Go math! G4*, 2015; *Go math! G5*, 2015) provided students with visual models in the younger grades and included more algorithmic opportunities as the students progressed by grade level. The instructional materials contained an overall structure that allowed students to progress from visual representations to algorithmic procedures.

Practice through problem solving, as defined by Isaacs and Carroll (1999), requires tasks that promote applying conceptual understanding and procedural fluency. Following Smith and Stein's (1998) 4-level rubric of low-level demand and high-level demand, *Go Math! K-8* (Burger et al., 2014; *Go math! G3,* 2015; *Go math! G4*, 2015; *Go math! G5*, 2015) provided very few high-level tasks, or valid practice problems, for students to experience. The quantity and proportion of high-level demand tasks stayed consistent in grades 3-5 in relation to the number of chapters dedicated to fractions. However, the grade 6 teacher edition revealed a decrease in quantity and proportion of high-level demand tasks.

Developing conceptual understanding and procedural fluency in fractions starts with using and applying equal-sharing problems (Empson & Levi, 2011). Although equal-sharing problems were found in individual lessons in grades 3 and 5, very few were found outside those specific lessons. The dearth of equal-sharing problems was again apparent, as indicated by the complete absence of equal-sharing tasks in grades 4 and 6, showing that students were not given opportunities to build conceptual understanding of equivalence through consistently solving such problems.

Teachers were provided many opportunities to build their teacher content knowledge through professional development opportunities focused on student mathematical practices. The descriptions of the mathematical practices, as found in the professional development sections and planning guide questions, aligned with descriptions from NGA Center and CCSSO (2010). However, the intentions of the instructions given teachers for implementation of the most commonly referenced mathematical practices, reason abstractly and quantitatively (MP2) and model with

mathematics (MP4), only partially aligned or were misaligned with the descriptions

provided by CCSS-M (NGA Center & CCSSO, 2010).

I analyzed the textbook series *Go Math! K-8* (HMH, 2016) for extent and ways it

provides support for teachers in helping students build procedural fluency from

conceptual understanding in equivalence of fractions. I found no instances in which

teachers were given instructions to allow students to solve tasks using self-created or

student-known strategies. The teacher resources did not provide guidance for teachers to

discuss with students the strategies they generated; thus, meaningful discourse related to

their own ways of thinking about tasks was not a textbook consideration. The majority of

the models used in grade 3 (*Go math! G3)* were visual models, and the use of algorithms

to understand equivalence of fractions increased as the grade levels progressed. The

inconsistent use of problem-solving practice and equal-sharing problems showed little

evidence of systematic opportunities for students to build procedural fluency from

conceptual understanding through these means. In my analysis of teacher content

knowledge, I found that multiple mathematical practices, or ways of working

mathematically that students should develop, were implemented into the curriculum in

ways contrary to the descriptions given for them. In the next chapter I discuss overall

messages that can be inferred from these findings.

CHAPTER 5

Discussion

The purpose of this content analysis was to examine a textbook series, *Go Math! K-8* (HMH, 2016) for evidence of supporting teachers in guiding students from conceptual to procedural understanding in mathematics. Using teacher actions, equal-sharing problems, and teacher content knowledge categories of codes, the findings of this study revealed multiple consistencies and inconsistencies between the research-based recommendations reported in the literature review for this study and actual content of the teacher resources. This series was examined for one mathematical domain, fractions, and one major concept within that domain, equivalence. Founded on research reported by NCTM (2014), five summative points of discussion emerged regarding the extent to which and in what ways teachers are supported in building procedural fluency from conceptual understanding in *Go Math! G3* (2015), *Go Math! G4* (2015), *Go Math! G5* (2015), and Burger et al. (2014).

1. Teacher-driven instruction was promoted rather than student-driven instruction.

2. Inconsistencies were found in the meaning of mathematical terms and language used to describe mathematics teaching and learning.

3. Problem-solving opportunities as defined by NCTM (2000) were not prevalent.

4. Equal–sharing problems were used sparingly throughout the development of equivalence.

5. The rationale for the use of the 5E model as an instructional model for mathematics was unclear.

The purpose of this chapter is twofold. First, the findings that emerged through the analysis are discussed and the implications these findings have for future research are reported. Second, implications for mathematics teaching and learning are considered.

Promoting Teacher-Driven Rather Than Student-Driven Instruction

Multiple researchers have stated that mathematics instruction should be student driven rather than teacher driven (Carpenter et al., 2003; Empson & Levi, 2011; Hendrickson et al., 2008; Lappan et al., 2006; NCTM, 2014; Schroyer & Fitzgerald, 1985). When students share their own strategies and communicate their own reasoning, they are able to develop conceptual knowledge and build procedural fluency (NCTM, 2014). *Go Math! K-8* (HMH, 2016) included some messages that were consistent and others that were inconsistent with this recommended emphasis on student-driven instruction.

No examples of student-generated strategies were found in any of the fraction lessons for grades 3-6; students were only given strategies by the teacher to support solving problems with equivalence. Although there was one example of brainstorming strategies to solve a problem, it was inconclusive as to who was doing the brainstorming. A teacher-driven instructional approach was foundational in the creation of each lesson: Teachers were given directives to show the students particular strategies, explain the process of solving a problem, and state which methods would be the most appropriate in each lesson. During one grade 5 lesson, teachers were instructed to "give students an opportunity to invent a strategy that uses circles or a number line instead of fraction strips" (*Go math! G5*, 2015, p. 359). Although students were invited to use their own strategies, guidance was still provided as to the representation to use, connecting more to a teacher-driven expectation. Teachers were also continually directed to work through the

problem at the beginning of a lesson with the students. Such direction indicated that the curriculum promotes a teacher-driven approach rather than a student-driven approach.

Consequently, in the absence of student-driven strategies, students were not able to connect their strategies or have meaningful student discourse providing their reasoning for the methods they used. Students were given opportunities to discuss the process they used to solve a problem; however, students were given the method for solving the problem beforehand. Thus, students were unlikely to discuss their own reasoning and strategies, but rather to reproduce an explanation that was previously given. Student discourse is an important strategy in mathematics (Ma, 1999; Empson & Levi, 2011), and discourse that includes opportunities to share and discuss their own strategies promotes a student-driven approach to mathematics education.

Despite research showing that student-driven instruction helps students build procedural fluency from conceptual understanding, *Go Math! K-8* (HMH, 2016) provided a teacher-driven approach to mathematics teaching. This teacher-driven approach was evidenced by (a) the consistent modeling of strategies by teachers, (b) no opportunities for students to model their own strategies, and (c) the absence of student discourse of student-generated strategies.

Inconsistent Mathematical Meanings

Within the lessons and curricular materials on equivalence in grades 3-6 of *Go Math! K-8* (2016) that were examined in this study, terms and descriptions were often used to communicate mathematical interpretations that were inconsistent with those proposed by the same terms and descriptions in the professional and research literature. These inconsistencies appeared to occur most frequently regarding meanings of the Standards for Mathematical Practice (NGA Center & CCSSO), although they occurred elsewhere as well—in the Making Connections at the

beginning of each lesson (Burger, et al., 2014; *Go math! G3,* 2015; *Go math! G4,* 2015; *Go math! G5,* 2015).

Some of the descriptions of standards for mathematical practices conflicted with their implementation. Model with mathematics (MP4) was the most visible of the mathematical practices in building teacher content knowledge through professional development. In the text, model with mathematics (MP4) was specifically described as applying mathematical understanding to a real-world problem that a student may need to solve, similar to the definition given by Hirsch and McDuffie (2016). However, the vast majority of the professional development references and the planning guide support questions referred to MP4 as creating visuals or models to represent mathematical strategies. A professional development section in the grade 5 teacher edition redefined model with mathematics by stating, "Mathematically proficient students can use models to represent and solve problems" (*Go math! G5*, 2015, p. 361). This inconsistency is not isolated to the analyzed text. Hirsch and McDuffie (2016) reported similar confusion in definitions referring to mathematical modeling and modeling mathematics, as many educators confuse the definitions of using visual representations to solve problems and using mathematics to solve real-world problems.

Other mathematical practices were found to have similarly contrasting interpretations of intentions and definitions. Use tools strategically (MP5) is described as students' being able use specific mathematical tools, such as a calculator, protractor, or other hands-on tools to support them in solving a problem (NGA Center & CCSSO, 2010). However, the planning guide stated that students "use concrete models to help them visualize problems" (Burger et al., 2015, p. 28). Attend to precision (MP6) is defined as students use of precise language to communicate their mathematical reasoning (NGA Center & CCSSO, 2010), but in a grade 4 lesson on equivalent

fractions, students are invited to draw models to double-check their work as a way to focus on MP6 (*Go math! G4*, 2015).

One prevalent example of an unaligned intention showed in the use of connections throughout the text. NCTM (2014) described that when students are making connections to build their conceptual and procedural knowledge, they are able to find similarities and differences in student-generated strategies. In *Go Math! K-8* (HMH, 2016), each lesson contained a Making Connections section that could be used to introduce or enhance the lesson. The content of the sections varied greatly, from connecting the previous lessons to the current lesson; to building background knowledge such as, tell me everything you know about spiders and insects (*Go math! G4*, 2015); to reviewing mathematical terminology. However, no sections contained suggestions for connecting student-generated strategies as NCTM (2014) described. For example, in the grade 3 textbook, students were invited to make connections by comparing physical characteristics in each other (such as eye color and hair length) in order to build the skill of comparing to introduce a lesson on comparing fractions (*Go math! G3,* 2015). This example showed a clear misalignment to making mathematical connections to student-generated strategies.

While these different types of connections may be meaningful to students, the different questions and expectations that the text associated with connections are not aligned with the NCTM definition, as the text provided no opportunities for student-driven strategies or connections among them. This misalignment obscured the importance of connections in bridging from conceptual to procedural knowledge through discussion of student-driven strategies while also confusing teachers about the nature of meaningful student connections.

Using language that clearly describes mathematical terms is essential (Hirsch & McDuffie, 2016) especially when the teaching strategies and practices presented are necessary to help students become mathematically proficient. Even though some definitions were accurately represented in the textbook series, it seems appropriate to develop meanings congruent with mainstream professional and research literature so that educators can speak a common language.

A Deficiency of Problem-Solving Opportunities

Researchers have found that building procedural fluency from conceptual understanding requires that students have multiple opportunities to immerse themselves in problem solving (NCTM, 2000, 2014). NCTM (2000) defined problem-solving tasks as tasks that students do not initially know how to solve and that require perseverance in finding a solution. Using this definition, little evidence of authentic problem-solving opportunities was found in the analysis of *Go Math! K-8* (Burger, et al., 2014; *Go math! G3,* 2015; *Go math! G4*, 2015; *Go math! G5,* 2015).

No evidence of student-driven strategies was found; therefore, it can be concluded that students were not given the opportunity to engage in authentic problem solving. Teachers were continually directed to introduce and explain multiple strategies to the students, thus giving students a method to solve a problem. "Read the problem with students and identify the two questions students need to answer. Before working though the example..." (*Go math! G4*, 2015, p. 391) was an example from the grade 4 teacher edition. The beginning of each lesson consistently provided directions on what strategies to introduce, how to show students methods, and how to walk them through the initial problem. Consequently, students did not engage in student-driven strategies and thus, their opportunities for meaningful problem solving were limited.

Isaacs and Carroll (1999) connected meaningful practice with problem-solving opportunities. A deficiency of problem-solving opportunities was represented by a huge discrepancy between low and high cognitively demanding questions as determine by a rubric based on the work of Smith and Stein (1998). The vast majority of practice tasks were direct procedural actions with only numbers and symbols involved. Word problems were also included in the practice section; however, the bulk of the problems did not require any explanations when solving the problem. For example, in a grade 3 lesson on comparing fractions the task stated, "Ben and Antonio both take the same bus to school. Ben's ride is 7/8 mile. Antonio's ride is ¾ mile. Who has the longer bus ride?" (*Go math! G3,* 2015, p. 529). This practice task was considered low demand because there was no justification, and, because students had already been taught how to write equivalent fractions, little reasoning was required. On many occasions, but not all, the Think Smarter and Go Deeper questions provided problem-solving practice problems, but with only one or two of those in each lesson, there was little consistent meaningful practice.

The scarcity of problem solving should also be attributed to the instructions that teachers are given to disallow students to work on the practice sections, including the Think Smarter and Go Deeper practice tasks, if they were not successful in solving the Share and Show portion of the lesson. Such students would not be permitted to attempt the higher-level problems that resemble good practice.

The few opportunities of problem-solving experiences for students stems from three factors in the text: (a) teacher-directed problem-solving methods, (b) a high amount of low-demand questions with relatively few high-demand questions, and (c) teacher instruction to disallow students to continue to the practice section unless they had mastered the procedural

steps. Following the teacher edition in regard to problem solving would not strengthen students' ability to build their procedural fluency from conceptual understanding (NCTM, 2014).

Additional Inconsistencies

Two additional inconsistencies contributed to the misalignment of the text with established and well-researched mathematical teaching practices. Inconsistent use of equal-sharing problems was found through the course of grades 3-6. The use of the 5E instructional framework instead of a research-based mathematics instructional framework was another inconsistency that lacked justification.

Although research has demonstrated that consistent use of equal-sharing problems provides students with opportunities to develop conceptual understanding in equivalence (Empson & Levi, 2011; Fazio & Siegler, 2011; Kieren, 1993; Lamon, 2007; NRC, 2001; Siegler et al., 2010), for grades 3-6 teacher editions (Burger, et al., 2014; *Go math! G3,* 2015; *Go math! G4*, 2015; *Go math! G5*, 2015), there was great disparity in opportunities to work on equal-sharing problems. Grades 3 and 5 contained individual lessons dedicated to equal-sharing problems, while grades 4 and 6 contained no individual equal-sharing lessons and few equal-sharing problems. This situation reveals little of the consistency with equal-sharing problems to be expected if research-based recommendations are followed for developing understanding of equivalence.

The 5E instructional framework is a research-based instructional model for teaching science. The lessons in *Go Math! K-8!* (HMH, 2016) were segmented into the five Es of this model: Engage, Explore, Explain, Elaborate, and Evaluate sections. However, the use of the 5E model in mathematics was neither described nor justified in the teacher edition or online resources, nor in subsequent inquiries to the publishers or their representatives. A Launch-

Explore-Summarize instructional model has shown to be effective in mathematics instruction (Lappan et al., 2006; Schroyer & Fitzgerald, 1985), but the absence of information as to why *Go Math! K-8* implemented the 5E model or to its effectiveness in mathematics makes its choice as a framework unclear.

Implications for Research and Practice

The purpose of this study was to analyze to what extent and ways *Go Math! K-8* (HMH, 2016) supports teachers in building procedural fluency from conceptual understanding. Two implications for future research revealed themselves throughout the study: the importance of providing student-driven instruction while using problem-solving tasks as meaningful practice and the use of the 5E model in mathematics instruction. Implications for teacher practice were also revealed in this study specifically, implementing a student-driven instructional approach using *Go Math! K-8* and the inclusion of high-level demand questions for all students.

Future research. First: the text provided very few opportunities for student-driven methods and high-demand problem solving, despite the emphasis in nearly all of the research in which this study is grounded on the importance of student-driven instruction and problem solving. This stark contrast indicates the need for more research on the creation of textbooks with a student-driven focus. Are there currently textbook materials that provide teachers and students with opportunities for student-driven strategies and instructional approaches? If so, what methods do they use to build procedural fluency from conceptual understanding? If such curricular materials are not available, what research needs to be conducted to contribute to their development? As demonstrated in this content analysis, student-driven strategies create opportunities to build conceptual understanding in order to move to procedural fluency. Without

such curricular materials in mathematics, students may fall short in reaching proficiency in mathematical understanding.

Second, *Go Math! K-8* (HMH, 2016) established the 5E model, a research-based instructional framework for science, as the textbook instructional framework without known basis or precedent. While research suggests that an instructional framework should encourage the growth of both conceptual and procedural knowledge (Lappan et al., 2006; Ma, 1999; Schroyer & Fitzgerald, 1985), no research or explanation was discovered, despite concerted effort as to why this model was chosen for *Go Math! K-8*. The efficacy of the 5E model in mathematics bears further research.

Teacher practice. Not only does this study have implications for future research, it also reveals implications for teacher practice. Such implications stem not only from the findings of this study, but from the research on student-generated strategies, meaningful practice through problem solving, and the implementation of reformed mathematics pedagogy.

Ball and Feiman-Nesmer (1988) described the value of following a textbook series with fidelity for beginning teachers. However, as this study has shown, this textbook series promotes a teacher-driven instructional approach rather than student-driven, revealed primarily by the absence of student-generated strategies. The absence of student-generated strategies directly impacted meaningful student discourse and making connections as describe by NCTM (2014). Teachers can and should include the use of student-driven strategies in their mathematics instruction. This inclusion can begin as simply as allowing students to solve the initial problem in the textbook lesson, rather than providing strategies for the students to use.

A similar implication for teacher practice is evident in the need for high-level demand questions (Smith & Stein, 1998), yet the *Go Math! K-8* textbook series (HMH, 2016) provides

very few opportunities for meaningful practice as defined by Isaacs and Carroll (1999). As stated earlier, teachers are instructed not to allow students to progress in the lesson if they struggle with initial tasks, implying that they will not have the experience of solving any meaningful practice problems. However, ensuring that the initial question of a lesson promotes problem solving allows all students to have immediate access to high demand mathematical problems.

According to a study from Bossé and Bahr (2008), university professors as a group appear to promote and instruct preservice teachers using a reform mathematics perspective. Unfortunately, many preservice and beginning inservice teachers feel pressure to use curricular materials that are mandated by their school leadership (Ball & Feiman-Nesmer, 1988). It is necessary that curricular materials provided by schools align closely with *Principles to Actions* (2014) so as to continue to build and implement a reform mathematics perspective in the classroom. Professional development and other learning opportunities for educators should also align more closely with best practices in mathematics reform (Schoenfeld, 2006).

Conclusion

This study has described the progression of mathematical focuses throughout recent history that have been designed to move students toward becoming mathematically proficient. Currently, one such focus in this progression of proficiency has been directed by CCSS-M (NGA Center & CCSSO, 2010). For rigorous mathematics instruction to take place teachers must help students build procedural knowledge from conceptual understanding (NCTM, 2014; NGA Center & CCSSO, 2010). As a result, teachers and educators have been encouraged to follow mathematical instructional frameworks that allow for instruction to be student-driven (NCTM, 2014).

This study found that although the creators of *Go Math! K-8* (2016) claimed to emphasize learning leading to student development of conceptual and procedural understanding, a teacher-driven approach to instruction was apparent. While there were qualities found in the *Go Math! K-8* teacher materials consistent with allowing students to build procedural fluency from conceptual understanding, the curricular materials were overwhelmingly centered on the teacher as the dispenser of knowledge.

The implications emerging from this study affirm that more research on creating student-driven instructional models for curricular materials is needed. Further research on textbooks that have an instructional framework that centers on student-driven strategies rather than teacher-introduced methods would be particularly enlightening. Also, further research into the reasoning and effectiveness of using the well-known science instructional framework, the 5E model, in mathematics would be beneficial.

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APPENDIX A

Characteristics of Mathematical Tasks at Four Levels of Cognitive Demand.
From Smith and Stein (1998)

Levels of Demands

Lower-level demands (memorization):
- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure
- Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced

Lower-level demands (procedures without connections):
- Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used
- Are focused on producing correct answers instead of on developing mathematical understanding
- Require no explanations or explanations that focus solely on describing the procedure that was used

Higher-level demands (procedures with connections):
- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.

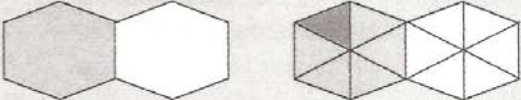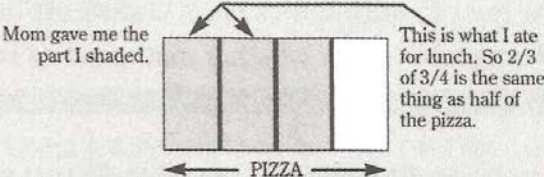*Higher-level demands (doing mathematics):*
- Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships
- Demand self-monitoring or self-regulation of one's own cognitive processes
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required

These characteristics are derived from the work of Doyle on academic tasks (1988) and Resnick on high-level-thinking skills (1987), the *Professional Standards for Teaching Mathematics* (NCTM 1991), and the examination and categorization of hundreds of tasks used in QUASAR classrooms (Stein, Grover, and Henningsen 1996; Stein, Lane, and Silver 1996).

APPENDIX B

Sample Tasks for Four Levels of Cognitive Demand.
From Smith and Stein (1998)

Lower-Level Demands

Memorization
What is the rule for multiplying fractions?

Expected student response:

You multiply the numerator times the numerator and the denominator times the denominator.

or

You multiply the two top numbers and then the two bottom numbers.

Procedures without Connections

Multiply:

$$\frac{2}{3} \times \frac{3}{4}$$

$$\frac{5}{6} \times \frac{7}{8}$$

$$\frac{4}{9} \times \frac{3}{5}$$

Expected student response:

$$\frac{2}{3} \times \frac{3}{4} = \frac{2 \times 3}{3 \times 4} = \frac{6}{12}$$

$$\frac{5}{6} \times \frac{7}{8} = \frac{5 \times 7}{6 \times 8} = \frac{35}{43}$$

$$\frac{4}{9} \times \frac{3}{5} = \frac{4 \times 3}{9 \times 5} = \frac{12}{45}$$

Higher-Level Demands

Procedures with Connections
Find 1/6 of 1/2. Use pattern blocks. Draw your answer and explain your solution.

Expected student response:

First you take half of the whole, which would be one hexagon. Then you take one-sixth of that half. So I divided the hexagon into six pieces, which would be six triangles. I only needed one-sixth, so that would be one triangle. Then I needed to figure out what part of the two hexagons one triangle was, and it was 1 out of 12. So 1/6 of 1/2 is 1/12.

Doing Mathematics
Create a real-world situation for the following problem:

$$\frac{2}{3} \times \frac{3}{4}.$$

Solve the problem you have created without using the rule, and explain your solution.

One possible student response:

For lunch Mom gave me three-fourths of a pizza that we ordered. I could only finish two-thirds of what she gave me. How much of the whole pizza did I eat?

I drew a rectangle to show the whole pizza. Then I cut it into fourths and shaded three of them to show the part Mom gave me. Since I only ate two-thirds of what she gave me, that would be only two of the shaded sections.

Mom gave me the part I shaded.

This is what I ate for lunch. So 2/3 of 3/4 is the same thing as half of the pizza.

PIZZA