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# Characterization of classes of graphs with large general position number 

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#### Abstract

Getting inspired by the famous no-three-in-line problem and by the general position subset selection problem from discrete geometry, the same is introduced into graph theory as follows. A set $S$ of vertices in a graph $G$ is a general position set if no element of $S$ lies on a geodesic between any two other elements of $S$. The cardinality of a largest general position set is the general position number $\operatorname{gp}(G)$ of $G$. The graphs $G$ of order $n$ with $\operatorname{gp}(G) \in\{2, n, n-1\}$ were already characterized. In this paper, we characterize the classes of all connected graphs of order $n \geq 4$ with the general position number $n-2$.


## KEYWORDS

Diameter; girth; general position set; general position number

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## 1. Introduction

The general position problem in graphs was introduced by P. Manuel and S. Klavžar [6] as a natural extension of the well-known century old Dudeney's no-three-in-line problem and the general position subset selection problem from discrete geometry [3, 4, 9]. The general position problem in graph theory was introduced in [6] as follows. A set $S$ of vertices in a graph $G$ is a general position set if no element of $S$ lies on a geodesic between any two other elements of $S$. A largest general position set is called a $g p$-set and its size is the general position number ( $g p$-number, in short), $\operatorname{gp}(G)$, of $G$.

The same concept was in use two years earlier in [2] under the name geodetic irredundant sets. The concept was defined in a different method, see the preliminaries below. In [2] it is proved that for a connected graph of order $n$, the complete graph of order $n$ is the only graph with the largest general position number $n$; and $\operatorname{gp}(G)=n-1$ if and only if $G=K_{1}+\cup_{j} m_{j} K_{j}$ with $\sum m_{j} \geq 2$ or $G=K_{n}-\left\{e_{1}, e_{2}, \ldots, e_{k}\right\}$ with $1 \leq k \leq n-2$, where $e_{i}$ 's all are edges in $K_{n}$ which are incident to a common vertex $v$. In [6], certain general upper and lower bounds on the $g p$-number are proved. In the same paper it is proved that the general position problem is NP-complete for arbitrary graphs. The $g p$-number for a large class of subgraphs of the infinite grid graph, for the infinite diagonal grid, and for Beneš networks were obtained in the subsequent paper [7]. Anand et al. [1] gives a characterization of general position sets in arbitrary graphs. As a consequence, the $g p$-number of graphs of diameter 2 , cographs, graphs with at least one universal vertex, bipartite graphs and their complements were obtained. Subsequently, $g p$-number for the complements of trees, of grids, and of
hypercubes were deduced in [1]. Recently, in [5] a sharp lower bound on the $g p$-number is proved for Cartesian products of graphs. In the same paper the $g p$-number for joins of graphs, coronas over graphs, and line graphs of complete graphs are determined. Recent developments on general position number can be seen in [8].

## 2. Preliminaries

Graphs used in this paper are finite, simple and undirected. The distance $d_{G}(u, v)$ between $u$ and $v$ is the minimum length of an $u, v$-path. An $u, v$-path of minimum length is also called an $u, v$-geodesic. The maximum distance between all pairs of vertices of $G$ is the diameter, $\operatorname{diam}(G)$, of $G$. A subgraph $H$ of a graph $G$ is isometric subgraph if $d_{H}(u, v)=$ $d_{G}(u, v)$ for all $u, v \in V(H)$. The interval $I_{G}[u, v]$ between vertices $u$ and $v$ of a graph $G$ is the set of vertices that lie on some $u$, $v$-geodesic of $G$. For $S \subseteq V(G)$ we set $I_{G}[S]=$ $\cup_{u, v \in S} I_{G}[u, v]$. We may simplify the above notation by omitting the index $G$ whenever $G$ is clear from the context.

A set of vertices $S \subseteq V(G)$ is a general position set of $G$ if no three vertices of $S$ lie on a common geodesic in $G$. A $g p$-set is thus a largest general position set. Call a vertex $v \in T \subseteq$ $V(G)$ to be an interior vertex of $T$, if $v \in I[T-\{v\}]$. Now, $T$ is a general position set if and only if $T$ contains no interior vertices. In this way general position sets were introduced in [2] under the name geodetic irredundant sets. The maximum order of a complete subgraph of a graph $G$ is denoted by $\omega(G)$. Let $\eta(G)$ be the maximum order of an induced complete multipartite subgraph of the complement of G. Finally, for $n \in \mathbb{N}$ we will use the notation $[n]=\{1, \ldots, n\}$.

In this paper, we make use of the following results.

[^0]Theorem 2.1. [2] Let $G$ be a connected graph of order $n$ and diameter $d$. Then $\operatorname{gp}(G) \leq n-d+1$.

Theorem 2.2. [2] For any cycle $C_{n}(n \geq 5), \operatorname{gp}\left(C_{n}\right)=3$.
We recall the characterization of general position sets from [1], for which we need some additional information. Let $G$ be a connected graph, $S \subseteq V(G)$, and $\mathcal{P}=\left\{S_{1}, \ldots, S_{p}\right\}$ a partition of $S$. Then $\mathcal{P}$ is distance-constant if for any $i, j \in$ $[p], i \neq j$, the distance $d(u, v)$, where $u \in S_{i}$ and $v \in S_{j}$ is independent of the selection of $u$ and $v$. If $\mathcal{P}$ is a distanceconstant partition, and $i, j \in[p], i \neq j$, then let $d\left(S_{i}, S_{j}\right)$ be the distance between a vertex from $S_{i}$ and a vertex from $S_{j}$. Finally, we say that a distance-constant partition $\mathcal{P}$ is intransitive if $d\left(S_{i}, S_{k}\right) \neq d\left(S_{i}, S_{j}\right)+d\left(S_{j}, S_{k}\right)$ holds for arbitrary pairwise different $i, j, k \in[p]$.

Theorem 2.3. [1] Let $G$ be a connected graph. Then $S \subseteq$ $V(G)$ is a general position set if and only if the components of $G[S]$ are complete subgraphs, the vertices of which form an in-transitive, distance-constant partition of $S$.

Theorem 2.4. [1] If $\operatorname{diam}(G)=2$, then $\operatorname{gp}(G)=$ $\max \{\omega(G), \eta(G)\}$.

## 3. The characterization

In the following, we characterize all connected graphs $G$ of order $n \geq 4$ with the $g p$ - number $n-2$. Since the complete graph $K_{n}$ is the only connected graph of order $n$ with the g $p$-number $n$, by Theorem 2.1, we need to consider only graphs with diameter 2 or 3 . First, we introduce four families of graphs with the diameter 3; and four families of graphs with the diameter 2 .

Let $\mathcal{F}_{1}$ be the collection of all graphs obtained from the cycle $C: u_{1}, u_{2}, u_{3}, u_{4}, u_{1}$ by adding $k$ new vertices $v_{1}, v_{2}, \ldots, v_{k}(k \geq 1)$ and joining each $v_{i}, i \in[k]$ to the vertex $u_{1}$. Graphs from the family $\mathcal{F}_{1}$ are presented in Figure 1.

Let $\mathcal{F}_{2}$ be the collection of all graphs obtained from the path $P_{2}: x, y$ and complete graphs $K_{n_{1}}, K_{n_{2}}, \ldots, K_{n_{r}}(r \geq 1)$, $K_{m_{1}}, K_{m_{2}}, \ldots, K_{m_{s}}(s \geq 1)$ and $K_{l_{1}}, K_{l_{2}}, \ldots, K_{l_{t}}$ (possibly complete graphs of this kind may be empty), by joining both $x$ and $y$ to all vertices of $K_{l_{1}}, K_{l_{2}}, \ldots, K_{l_{t}}$; joining $x$ to all vertices of $K_{n_{1}}, K_{n_{2}}, \ldots, K_{n_{r}} ;$ and joining $y$ to all vertices of $K_{m_{1}}, K_{m_{2}}, \ldots, K_{m_{s}}$. Graphs from the family $\mathcal{F}_{2}$ are presented in Figure 2. Trees with diameter 3 are called double stars and they belong to the class $\mathcal{F}_{2}$.

Let $\mathcal{F}_{3}$ be the collection of all graphs obtained from the path $P_{4}: u, x, y, v$ and a complete graph $K_{r}(r \geq 1)$ by joining both $u$ and $x$ to all vertices of $K_{r}$ and joining $y$ to a subset $S$ of vertices of $V\left(K_{r}\right)$ (possibly $S$ may be empty or $S=V\left(K_{r}\right)$ ). Graphs from the family $\mathcal{F}_{3}$ are presented in Figure 3.

Let $\mathcal{F}_{4}$ be the collection of all graphs obtained from the path $P_{3}: x, y, v$ and complete graphs $K_{q}, K_{n_{1}}, K_{n_{2}}, \ldots, K_{n_{r}}(r \geq$ 1), $K_{m_{1}}, K_{m_{2}}, \ldots, K_{m_{s}}(s \geq 1)$ by joining $x$ to all vertices of $K_{n_{1}}, K_{n_{2}}, \ldots, K_{n_{r}} ;$ joining $x$ and $v$ to all vertices of $K_{m_{1}}, K_{m_{2}}, \ldots, K_{m_{s}}$; joining $x$ and $y$ to all vertices of $K_{q}$. Graphs from the family $\mathcal{F}_{4}$ are presented in Figure 4.


Figure 1. Family $\mathcal{F}_{1}$.


Figure 2. Family $\mathcal{F}_{2}$.


Figure 3. Family $\mathcal{F}_{3}$.


Figure 4. Family $\mathcal{F}_{4}$.


Figure 5. Family $\mathcal{F}_{5}$.

Next, we introduce four families of graphs with diameter 2.

Let $\mathcal{F}_{5}$ be the collection of all graphs obtained from the complete graph $K_{n-2}(n \geq 5)$ by adding two new vertices $u$ and $v$, joining $u$ to all vertices of non-empty subset $S$ of $V\left(K_{n-2}\right)$ of size at most $n-3$; and joining $v$ to all vertices of non-empty subset $T$ of $V\left(K_{n-2}\right)$ of size at most $n-3$. The set $S$ must intersect with the set $T$ so that, the diameter of each graph from the family $\mathcal{F}_{5}$ is 2 . Graphs from the family $\mathcal{F}_{5}$ are presented in Figure 5.

Let $\mathcal{F}_{6}$ be the collection of all graphs obtained from the family $\mathcal{F}_{5}$ by adding the edge $u v$. Moreover; in this case, the set $S$ may be disjoint with the set $T$. Graphs from the family $\mathcal{F}_{6}$ are presented in Figure 6.

Let $\mathcal{F}_{7}$ be the collection of all graphs obtained from the complete graphs $K_{n_{1}}, K_{n_{2}}, \ldots, K_{n_{r}}(r \geq 2)$ by adding two new vertices $x$ and $y$, joining $x$ to a non-empty subset $S_{i}$ of $V\left(K_{n_{i}}\right)$ for all $i \in[r]$; and $y$ to a non-empty subset $T_{i}$ of $V\left(K_{n_{i}}\right)$ for all $i \in[r]$ (the edges are in a way that for any $u \in V\left(K_{n_{i}}\right)$ and $v \in V\left(K_{n_{j}}\right)$ with $i \neq j$ must have a common neighbor). Moreover, for some $i \in[r]$; the set $S_{i}$ must


Figure 6. Family $\mathcal{F}_{6}$.


Figure 7. Family $\mathcal{F}_{7}$.
intersect with the set $T_{i}$ so that, the diameter of each graph from the family $\mathcal{F}_{7}$ is 2 . Graphs from the family $\mathcal{F}_{7}$ are presented in Figure 7. It is clear that both $C_{4}$ and $C_{5}$ belong to class $\mathcal{F}_{7}$.

Let $\mathcal{F}_{8}$ the collection of all graphs obtained from the family $\mathcal{F}_{7}$ by adding the edge $x y$. In this case, the set $S_{i}$ may be disjoint with the set $T_{i}$ for all $i \in[r]$. Graphs from the family $\mathcal{F}_{8}$ are presented in Figure 8.

Theorem 3.1. Let $G$ be a connected graph of order $n \geq 4$, then $\operatorname{gp}(G)=n-2$ if and only if $G$ belongs to the family $\cup_{i=1}^{8} \mathcal{F}_{i}$.

Proof . First, suppose that $G$ is a connected graph of order $n$ with $\operatorname{gp}(G)=n-2$. Then it follows from Theorem 2.1 that $\operatorname{diam}(G)$ is either 2 or 3 . We consider the following two cases.

Case 1: $\operatorname{diam}(G)=3$. If $G$ is a tree, then $G$ is a double star and hence it belongs to $\mathcal{F}_{2}$. So, assume that $G$ has cycles. Let $\operatorname{girth}(G)$ denotes the length of a shortest cycle in $G$.

Let $C$ be any shortest cycle in $G$. Then it is clear that $C$ is an isometric subgraph of $G$. This shows that if $S$ is a general position set in $G$, then $S \cap V(C)$ is a general position set in $C$. Hence it follows from Theorem 2.2 that any general position


Figure 8. Family $\mathcal{F}_{8}$.
set of $G$ contains at most three vertices from the cycle $C$. Now, since $\operatorname{gp}(G)=n-2$, we have that the length of $C$ is at most 5 and so $\operatorname{girth}(G) \leq 5$.

Next, we claim that there is no connected graph of order $n$ with $\operatorname{girth}(G)=5$ and $\operatorname{gp}(G)=n-2$. For, assume the contrary that there is a connected graph of order $n$ with $\operatorname{girth}(G)=5$ and $\operatorname{gp}(G)=n-2$. Let $C: u_{1}, u_{2}, u_{3}, u_{4}, u_{5}, u_{1}$ be a shortest cycle of length 5 in $G$. Since $\operatorname{girth}(G)=5$, it follows that the vertices from $N\left(u_{i}\right)$ are independent for all $i \in[5]$. Also, as above we have that any general position set of $G$ has at most three vertices from the cycle $C$. Let $S$ be a general position set in $G$. Since $\operatorname{gp}(G)=n-2$, we have that $S=V(G) \backslash\left\{u_{i}, u_{j}\right\}$. If $u_{i}$ and $u_{j}$ are sucessive vertices in $C$, then it follows that the induced subgraph of $S$ has a $P_{3}$, which is impossible. Hence without loss of generality, we may assume that $i=1$ and $j=3$. So $S=V(G) \backslash\left\{u_{1}, u_{3}\right\}$. Now, since $u_{2}, u_{4}, u_{5} \in S$ and $N\left(u_{i}\right)$ is independent, by Theorem 2.3, it follows that $\operatorname{deg}\left(u_{i}\right) \leq 3$ for $i=2,4,5$. Now we claim that $\operatorname{deg}\left(u_{2}\right)=\operatorname{deg}\left(u_{4}\right)=\operatorname{deg}\left(u_{5}\right)=2$. Otherwise, we may assume that $\operatorname{deg}\left(u_{2}\right)=3$ and let $x$ be the neighbour of $u_{2}$ different from $u_{1}$ and $u_{3}$. Since girth $(G)=5$, it follows that $x$ is not adjacent with the remaining vertices of $C$. Now, since $u_{2}, u_{5}, x \in S$, by Theorem 2.3, $d\left(u_{5}, x\right)=d\left(u_{5}, u_{2}\right)=2$. Let $P: u_{5}, y, x$ be a $u_{5}, x$-geodesic of length 2 . Then it is clear that $y \notin V(C)$ and so $y \in S$. This leads to the fact that induced subgraph of $S$ has a $P_{3}$, impossible in a general position set. Hence $\operatorname{deg}\left(u_{2}\right)=2$. Similarly $\operatorname{deg}\left(u_{4}\right)=$ $\operatorname{deg}\left(u_{5}\right)=2$.

Now, if $N\left(u_{1}\right) \neq \emptyset$, then $u_{5} \in I\left[x, u_{4}\right]$ for all $x \in N\left(u_{1}\right)$ (otherwise $S$ contains an induced $P_{3}$ ), impossible. Hence $N\left(u_{1}\right)=\emptyset$. Similarly, $N\left(u_{2}\right)=\emptyset$. Hence $G \cong C_{5}$. But $\operatorname{gp}\left(C_{5}\right)=3=n-2$ and $\operatorname{diam}(G)=\operatorname{diam}\left(C_{5}\right)=2$. Hence there is no connected graph of order $n$ with $\operatorname{diam}(G)=3$, $\operatorname{girth}(G)=5$ and $\operatorname{gp}(G)=n-2$. Hence $\operatorname{girth}(G)$ is at most 4.

Now, assume that $\operatorname{girth}(G)=4$ and let $C$ : $u_{1}, u_{2}, u_{3}, u_{4}, u_{1}$ be a shortest cycle of length 4 in $G$. Since $\operatorname{diam}(G)=3$, we have that $G \nVdash C_{4}$. Now, we may assume that $u_{1} \in V(C)$ be a vertex such that $\operatorname{deg}\left(u_{1}\right) \geq 3$ and let $x$ be a neighbour of $u_{1}$ such that $x \notin V(C)$. Since $S$ is a general position set and $|S|=n-2$, we have that $S$ contains exactly 2 vertices from $C$. We claim that $u_{1} \notin S$. For otherwise assume that $u_{1} \in S$. Since $|S|=n-2$ and $x, u_{1} \in S$, it follows from Theorem 2.3 that $u_{2}, u_{4} \notin S$ and $u_{3} \in S$. This shows that the path $x, u_{1}, u_{2}, u_{3}$ must be a $x, u_{3}$ - geodesic
(otherwise, since $|S|=n-2, S$ contains an induced $P_{3}$. Hence $d\left(x, u_{3}\right) \neq d\left(u_{1}, u_{3}\right)$, which is impossible in a general position set. Hence $u_{1} \notin S$.

Now, we claim that $u_{1}$ is the unique vertex in $C$ with degree at least 3 . Assume the contrary that there exists $u_{j} \in$ $C$ with $j \neq 1$ and $\operatorname{deg}\left(u_{j}\right) \geq 3$. Then as above we have that $u_{j} \notin S$. Now, if $u_{i}$ and $u_{j}$ are adjacent vertices in $C$, then we can assume that $j=2$. It follows from the fact that $S$ is a general position set of size $n-2, \quad d\left(u_{3}, x\right)=3$ and $u_{3}, u_{4}, u_{1}, x$ is a geodesic in $G$, where $x$ is a neighbour of $u_{1}$ such that $x \notin V(C)$. This shows that the vertices $x, u_{4}, u_{3}, x$ lie on a common geodesic, a contradiction. Similarly if $u_{1}$ and $u_{j}$ are non adjacent vertices in $C$ then $u_{j}=u_{3}$ and $u_{2}$, $u_{4}$ belong to $S$. Moreover, as above $S$ is a general position set of size $n-2$, we have that $x, y \in S$ and $d(x, y)=4$, where $x \in N\left(u_{1}\right) \backslash V(C)$ and $y \in N\left(u_{3}\right) \backslash V(C)$, which is impossible. Thus $u_{1}$ is the unique vertex in $C$ with $\operatorname{deg}\left(u_{1}\right) \geq 3$. Also, since $\operatorname{girth}(G)=4$, we have that $N\left(u_{1}\right)$ induces an independent set. Hence the graph belongs to $\mathcal{F}_{1}$.

Now, consider $\operatorname{girth}(G)=3$ and $\operatorname{diam}(G)=3$. Let $P$ : $u, x, y, v$ be a $u, v$ - shortest path in $G$ of length 3. Then $S$ contains atmost 2 vertices from $V(P)$. Since $|S|=n-2$, we have that $S$ contains exactly two vertices from $V(P)$. We consider the following four cases.
Subcase 1.1: $u, v \in S$. Then $x, y \notin S$. Moreover, $S=$ $V(G) \backslash\{x, y\}$. Now, let $z$ be any neighbour of $u$. Since $S$ is a general position set of size $n-2$, it follows that $I[z, v] \subseteq$ $V(P)$. This shows that $d(z, v) \leq 3$. If $d(z, v)=2$, then $z$ must be adjacent with $y$ and so $u, z, y, v$ is a $u, v^{-}$geodesic, which contradicts the fact that $S$ is a general position set. Hence $d(z, v)=3$ and since $I[z, v] \subseteq V(P)$, we have that $z$ is adjacent with $x$ but it is not adjacent with $y$. Similarly, we have that any neighbour of $v$ is adjacent with $y$ but nonadjacent with $x$. Now, assume that $z$ be any vertex in $G$ such that $z \notin V(P)$ and $z$ is non-adjacent with both $u$ and $v$. Then as in the previous case, we have that $I[z, v] \subseteq V(P)$. Also, we have $d(z, v) \in\{2,3\}$ and $d(z, u) \in\{2,3\}$. Hence it follows that $z$ is adjacent to $x$ or $y$ or both. Also, by Theorem 2.3, we have that the components of $S$ are in-transitive distance-constant cliques. Hence the graph reduces to the class $\mathcal{F}_{2}$.
Subcase 1.2: $u, x \in S$. Then $y, v \notin S$ and $S=V(G) \backslash\{y, v\}$. Now, let $z$ be any vertex in $G$ such that $z \notin V(P)$. Then, we have that $I[z, u] \subseteq V(P)$. Moreover, by Theorem 2.3, $d(z, u)=d(z, x)$. If $d(z, x)=2$, then $I[z, x] \subseteq V(P)$, we have that $z$ is adjacent to $y$. But in this case $d(z, u)$ cannot be equal to 2 . Similarly, if $d(z, x)=3$ then $z$ is adjacent with $v$ but not $y$. Then it is clear that $d(z, u) \neq 3$. Hence it follows that $d(z, u)=d(z, x)=1$. Again by Theorem 2.3, $V(G) \backslash$ $\{y, v\}$ induces a clique. Hence the graph reduces to the class $\mathcal{F}_{3}$.
Subcase 1.3: $u, y \in S$. Then $x, v \notin S$ and $S=V(G) \backslash\{x, v\}$. Now, for any $z \notin V(P)$, we have that $I[z, y] \subseteq V(P)$ and $I[z, u] \subseteq V(P)$. Thus $d(z, y) \leq 3$ for all $z \notin V(P)$. If $d(z, y)=$ 3, then $z$ must be adjacent to $u$ and so by Theorem 2.3, $d(u, y)=3, \quad$ a contradiction. Thus $d(z, y) \in\{1,2\}$. If $d(z, y)=1$, then again by Theorem 2.3, we have that
$d(u, z)=2$ and so $z$ must be adjacent to $x$. Moreover, $\{z \notin$ $V(P): d(z, y)=1\}$ induces a clique. Now, if $d(z, y)=2$, then by using the same argument, we have that $z$ is either adjacent to $x$ or $z$ is adjacent to both $x$ and $v$. Hence the graph reduces to class $\mathcal{F}_{4}$.
Subcase 1.4: $x, y \in S$. Then $u, v \notin S$ and $S=V(G) \backslash\{u, v\}$. Now, for any $z \notin V(P)$, as in the previous case we have that $I[z, x] \subseteq V(P)$ and $I[z, y] \subseteq V(P)$. Moreover, by Theorem 2.3, $d(z, x)=d(z, y)$. Now, if $d(z, x) \neq 1$, then $d(z, y) \neq 1$. This shows that $z$ must be adjacent to both $u$ and $v$, which is impossible. Hence $d(z, x)=d(z, y)=1$. Hence it follows from Theorem 2.3, $V(G) \backslash\{u, v\}$ induces a clique. Moreover, since both $x$ and $y$ belong to $S$, it is clear that $d(u, z)=d(v, z)=2$ for all $z \notin V(P)$. Hence in this case the graph reduces to the family $\mathcal{F}_{2}$.
Case 2: $\operatorname{diam}(G)=2$. Then by Theorem 2.4, we have $\operatorname{gp}(G)=\max \{\omega(G), \eta(G)\}=n-2$. We consider the following two subcases.
Subcase 2.1: $\omega(G) \geq \eta(G)$. Then $\operatorname{gp}(G)=\omega(G)=n-2$. Let $K$ be a clique of order $n-2$ and let $u, v \in V(G)$ be such that $u, v \notin V(K)$. Then it is clear that $1 \leq \operatorname{deg}(u) \leq n-3$ and $1 \leq \operatorname{deg}(v) \leq n-3$. Now, if $u$ and $v$ are adjacent in $G$, then $G$ belongs to the family $\mathcal{F}_{6}$. Otherwise, $G$ belongs to the family $\mathcal{F}_{5}$.
Subcase 2.2: $\eta(G)>\omega(G)$. Then $\operatorname{gp}(G)=\eta(G)=n-2$. This shows that the complement of $G$ has complete mulipartite subgraph $H$ of order $n-2$. Thus the components of the induced subgraphs of $H$ in $G$ are cliques, say $K_{n_{1}}, K_{n_{2}}, \ldots, K_{n_{r}}$. Moreover $d(u, v)=2$ for all $u \in V\left(K_{n_{i}}\right)$ and $v \in V\left(K_{n_{j}}\right)$. Now, let $x$ and $y$ be the vertices in $G$ such that $x, y \notin V(H)$. Then it is clear that the graph reduces to the family $\mathcal{F}_{8}$, when $x$ and $y$ are adjacent in $G$. Otherwise it belongs to the family $\mathcal{F}_{7}$.

On the other hand, if $G$ belongs to the family $\cup_{i=1}^{8} \mathcal{F}_{i}$, by Theorems 2.1 and 2.3, one can easily verify that $\operatorname{gp}(G)=$ $n-2$. This completes the proof.

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