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E-super vertex magic labeling of cartoon flowers and wounded flowers

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ABSTRACT
In this paper, we define a new class of graph called the cartoon flower and then show that E-super vertex magic labeling does not exist for the class of cartoon flower graphs. We further define the wounded cartoon flower graphs and establish some sufficient conditions for the graph not to be E-super vertex magic. We also give examples of some wounded cartoon flowers that admit an E-super vertex magic labeling and some others that do not.

KEYWORDS
E-super vertex magic; E-super vertex magic labeling; cartoon flower

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1. Introduction

In this paper, we consider only simple undirected finite graphs. Let G be a simple undirected finite graph with $|V(G)| = p$ and $|E(G)| = q$. We use Marr and Wallis [8] for general graph theoretic notations.

A labeling of a graph is a mapping that carries the set of vertices and/or edges into a set of numbers. An excellent survey on graph labeling can be found in Gallian [1].

A graph is said to be magic if the set of edges of the graph can be labeled by distinct positive integers such that the sum of the labels of all edges incident with any vertex is the same. See Sedlacek [9] for magic labeling.

A labeling is said to be a vertex magic total labeling if there is a bijection from the union of the vertex set and the edge set to the set of consecutive integers $\{1, 2, , ..., p+q\}$ with the property that for every u in the vertex set, the sum of label of u and the label of edges incident with u is equal to k for some constant k. See MacDougall et al. [4]. A vertex magic total labeling is said to be super vertex magic if the labels of the vertices set is $\{1, 2, ..., p\}$. See MacDougall et al. [5].

A vertex magic total labeling is said to be E-super vertex magic if the labels of the edge set is $\{1, 2, ..., q\}$. A graph G is called E-super vertex magic if it admits an E-super vertex magic labeling. Swaminathan [10] referred to this labeling as super vertex magic labeling, but later Marimuthu [6] named this labeling as E-super vertex magic total labeling to avoid the confusion. The formal definition of E-super vertex magic is given below.

Definition 1. A graph G of order p and size q is said to be *E*-super vertex magic if there exists a bijective function $f : V(G) \cup E(G) \to \{1, 2, ..., p+q\}$ such that for every $u \in V(G)$, $f(u) + \sum_{v \in N(u)} f(uv) = k$ for some constant k and $f(E(G)) = \{1, 2, ..., q\}$. This k is known as magic constant and the function f is said to be E-super vertex magic labeling of G.

Many authors have presented interesting results for E-super vertex magic graphs and have investigated many families of graphs for E-super vertex magic labeling. For example, see [2, 3, 7, 11]. The following theorems will be useful to prove some results presented in this paper.

Theorem 2. [10] *If a non-trivial graph G of order p and size q is E-super vertex magic, then the magic constant k is given by* $k = q + \frac{p+1}{2} + \frac{q(q+1)}{p}$.

Theorem 3. [10] *An n-cycle is E-super vertex magic if and only if n is odd.*

2. Cartoon flower graph

In this section, we first define n-cartoon flower graph for $n \geq 3$ and then show that this class of graph is not E-super vertex magic.

Definition 4. An n-cartoon flower graph is a graph, denoted by $CF(n)$, with the set of vertices and set of edges as follows:

$$V(CF(n)) = \{v_1, ..., v_{2n}\}$$

and

$$E(CF(n)) = \{v_i v_{i+1} | 1 \leq i < n\} \cup \{v_n v_1\} \cup \{v_i v_{i+n}, v_{i+1} v_{i+n} | 1 \leq i < n\} \cup \{v_n v_{2n}, v_1 v_{2n}\}.$$

Since $n = 3$ is the smallest value for which the cartoon flower is defined, in this paper, we assume that $n \geq 3$. The graphs of cartoon flowers $CF(3)$ and $CF(4)$ are shown in Figure 1(a) and (b), respectively.

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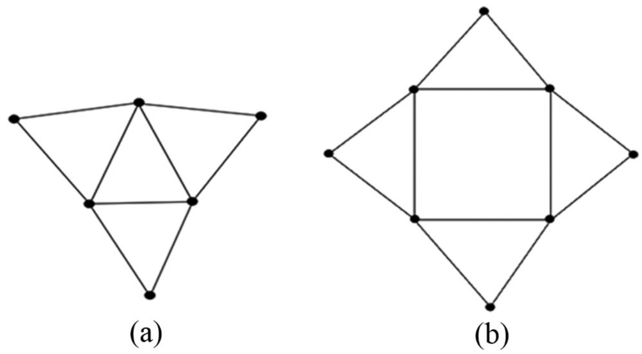

Figure 1. Cartoon flowers $CF(3)$ and $CF(4)$.

Note that the set $\{v_iv_{i+1} | 1 \le i < n\} \cup \{v_nv_1\}$ forms an n-cycle. We call any edge of this n-cycle as a ring edge. The set $\{v_iv_{i+1}, v_iv_{i+n}, v_{i+1}v_{i+n} | 1 \le i < n\} \cup \{v_nv_1, v_nv_{2n}, v_1v_{2n}\}$ forms petals of the flower. We refer to any vertex from the set $\{v_{n+1}, ..., v_{2n}\}$ as a petal corner and refer to the edges $v_iv_{i+n}, v_{i+1}v_{i+n}$ for $1 \le i < n$, along with v_nv_{2n} and v_1v_{2n} as petal edges. Every edge of the n-cycle is forming a triangular petal and no two petals share any edge.

The n-cycle has n vertices and n petals have n petal corners hence the order of the graph $CF(n)$ is $2n$. Since the n-cycle has n ring edges and every petal has 2 petal edges, the size of graph is $3n$. Now we are ready to prove the following theorem.

Theorem 5. *An E-super vertex magic labeling does not exist for any cartoon flower graph CF(n).*

Proof. For the sake of contradiction, assume that there exists an E-super vertex magic labeling f of $CF(n)$ with the magic constant k. Using the fact that $|V(CF(n))| = 2n, |E(CF(n))| = 3n$ and by Theorem 2,

$$k = 3n + \frac{2n+1}{2} + \frac{3n(3n+1)}{2n}$$
$$= \frac{17n+4}{2}.$$

Note that since k is an integer, n must be even. To get a contradiction, we calculate the minimum and maximum sum of labels of $2n$ petal edges in any E-super vertex magic label of $CF(n)$, denoted by S_{min} and S_{max}, respectively.

Since there are only two edges incident at every petal corner v_{n+i} for $1 \le i \le n$, the magic constant at v_{n+i} is given by $f(v_{n+i}) + f(v_iv_{i+n}) + f(v_{i+1}v_{i+n}) = k$ for $1 \le i < n$ and at v_{2n} is given by $f(v_{2n}) + f(v_nv_{2n}) + f(v_1v_{2n}) = k$. Thus

$$f(v_iv_{i+n}) + f(v_{i+1}v_{i+n}) = k - f(v_{n+i}) \quad \text{for} \quad 1 \le i < n$$
$$f(v_nv_{2n}) + f(v_1v_{2n}) = k - f(v_{2n}).$$

Therefore, the largest possible label for the petal corner results as the smallest sum of labels for both petal edges. Since the vertex label set is $f(V(CF(n))) = \{3n+1, 3n+2, ..., 5n\}$, S_{min} is obtained by labeling the n petal corners as, $f(\{v_{n+i} | 1 \le i \le n\}) = \{4n+1, 4n+2, ..., 5n\}$.

$$S_{min} = \sum_{i=1}^{n-1} [f(v_iv_{i+n}) + f(v_{i+1}v_{i+n})] + [f(v_nv_{2n}) + f(v_1v_{2n})]$$
$$= \sum_{i=1}^{n} [k - f(v_{n+i})]$$
$$= \sum_{i=1}^{n} k - \sum_{i=1}^{n} f(v_{n+i})$$
$$= n(k) - [(4n+1) + (4n+2) ... + (4n+n)]$$
$$= n\left(\frac{17n+4}{2}\right) - \left[n(4n) + \frac{n(n+1)}{2}\right]$$
$$= \frac{8n^2+3n}{2}.$$

The maximum sum of $2n$ petal edges labels is obtained by labeling petal edges as the largest possible labels. Since the edge label set is $f(E(CF(n))) = \{1, 2, ..., 3n\}$, S_{max} is obtained by labeling the $2n$ petal edges as

$$f(\{v_iv_{i+n}, v_{i+1}v_{i+n} | 1 \le i < n\} \cup \{v_nv_{2n}, v_1v_{2n}\})$$
$$= \{n+1, n+2, ..., 3n\}.$$

Therefore, S_{max} is given by

$$S_{max} = \sum_{i=1}^{n-1} [f(v_iv_{i+n}) + f(v_{i+1}v_{i+n})] + [f(v_nv_{2n}) + f(v_1v_{2n})]$$
$$= \sum_{i=1}^{2n} (n+i)$$
$$= \sum_{i=1}^{2n} n + \sum_{i=1}^{2n} i$$
$$= 2n(n) + \frac{2n(2n+1)}{2}$$
$$= 4n^2 + n.$$

Clearly, $S_{min} \le S_{max}$, we get

$$\frac{8n^2+3n}{2} \le 4n^2 + n$$
$$8n^2 + 3n \le 8n^2 + 2n$$
$$n \le 0.$$

This is a contraction to the fact that n is a positive integer. Hence $CF(n)$ is not E-super vertex magic. □

3. Wounded cartoon flower

In this section, we consider the graphs obtained by plucking a few petals from cartoon flower graph. We provide some examples of wounded cartoon flowers that are and some that are not E-super vertex magic.

Definition 6. A wounded n-cartoon flower with t-petals graph is a graph, denoted by $WF(n, t)$, obtained by adding t petals to an n-cycle.

In other words, $WF(n, t)$ is the graph obtained by plucking $n - t$ petals from a cartoon flower $CF(n)$. Note that for $t = n$, the flower is not wounded, that is, it is the cartoon flower

Table 1. Magic constants of $WF(n, t)$.

$t \downarrow \backslash n \rightarrow$	3	4	5	6
0	9	$11\frac{1}{2}$	14	$16\frac{1}{2}$
1	15	$17\frac{2}{5}$	$19\frac{5}{6}$	$22\frac{2}{7}$
2	$21\frac{1}{5}$	$23\frac{1}{2}$	$25\frac{6}{7}$	$28\frac{1}{4}$
3	–	$29\frac{5}{7}$	32	$34\frac{1}{3}$
4	–	–	$38\frac{2}{9}$	$40\frac{1}{2}$
5	–	–	–	$46\frac{8}{11}$

$CF(n)$. Thus in this paper, we assume that $0 \leq t < n$. The n-cycle has n vertices and t petals have t petal corners, hence $|V(WF(n,t))| = n + t$. Since the n-cycle has n ring edges and every petal has two petal edges, $|E(WF(n,t))| = n + 2t$.

Theorem 7. *For any wounded cartoon flower graph WF(n, t), the magic constant (if exists) is given by*

$$k = (2n + 5t + 1) + \frac{n+t}{2} + \frac{1}{2} + \frac{t^2 + t}{n+t}. \tag{1}$$

Proof. To calculate the magic constant k, substitute $|V(WF(n,t))| = n + t$ and $|E(WF(n,t))| = n + 2t$ in the formula given in Theorem 2. Hence

$$\begin{aligned} k &= (n + 2t) + \frac{n+t+1}{2} + \frac{(n+2t)(n+2t+1)}{n+t} \\ &= (n+2t) + \frac{n+t}{2} + \frac{1}{2} + \frac{n^2 + 4nt + 4t^2 + n + 2t}{n+t}. \end{aligned}$$

Using the division algorithm, the last fraction in the expression above can be rewritten as $(n + 3t + 1) + \frac{t^2+t}{n+t}$. Hence the result. □

Using Equation (1), we calculate the magic constants of wounded cartoon flower $WF(n, t)$ for $(n,t) \in \{(n,t) | 3 \leq n \leq 6, 0 \leq t < n\}$ and are shown in Table 1. Since we are not interested in calculating values of k for $t \geq n$, we use a"-"symbol for these entries in the table.

Theorem 8. *The wounded cartoon flower graph $WF(n, 0)$ is E-super vertex magic if and only if n is odd.*

Proof. Using Theorem 3 and the fact that $WF(n, 0)$ is an n-cycle, we get the result. □

Theorem 9. *The wounded cartoon flower graph WF(3, t) is not E-super vertex magic for $t > 0$.*

Proof. For the sake of contradiction, assume that there exists an E-super vertex magic labeling f of $WF(3, 1)$.

Since there is only one petal, there is exactly one petal corner and is of degree 2. Since there are 2 ring edges with no attached petals, there is exactly one vertex on the cycle of degree 2. Since the petal could be attached to any of the ring edges, assume that $V(WF(3,1)) = \{v_1, v_2, v_3, v_4\}$ and $E(WF(3,1)) = \{v_1v_2, v_2v_3, v_3v_1, v_1v_4, v_2v_4\}$. See Figure 2(a).

From Table 1, it is clear that the magic constant of $WF(3, 1)$ is 15. The magic constant at vertices v_4 and v_3 gives us the following two equations.

$$\begin{aligned} f(v_4) + f(v_1v_4) + f(v_2v_4) = 15 \\ f(v_3) + f(v_2v_3) + f(v_3v_1) = 15. \end{aligned}$$

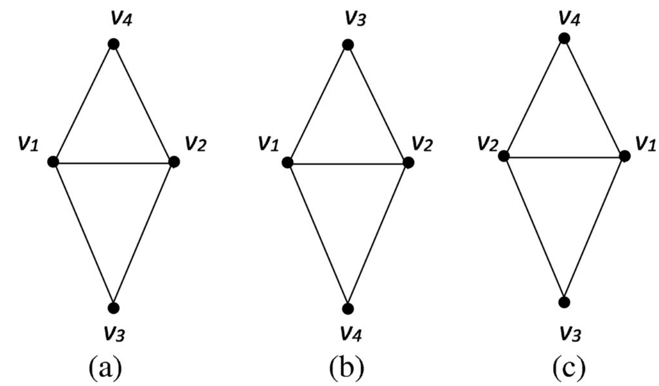

Figure 2. Isomorphic graphs of $WF(3, 1)$.

Since $f(E(WF(3,1))) = \{1,2,3,4,5\}$ and $f(V(WF(3,1))) = \{6,7,8,9\}$, the magic constant 15 can be achieved at v_4 and v_3 from any two disjoint decompositions of 15 from the following:

$9+5+1, 9+4+2, 8+5+2, 8+4+3, 7+5+3$ and $6+5+4$.

There are two such disjoint pairs: $9+4+2$, $7+5+3$ and $9+5+1$, $8+4+3$. First consider $9+4+2$, $7+5+3$. For this pair, $f(v_4)$ has two choices, 9 or 7. Since the graph obtained by interchanging the names of vertices v_4 and v_3 (shown in Figure 2(b)) is isomorphic to the original graph (shown in Figure 2(a)), any value to $f(v_4)$ from the choices is fine. So assign $f(v_4) = 9$ and $f(v_3) = 7$. The $f(v_1v_4)$ has two choices, 4 and 2. Since the graph obtained by interchanging the names of vertices v_1 and v_2 (shown in Figure 2(c)) is isomorphic to the original graph (shown in Figure 2(a)), assign $f(v_1v_4) = 4$ and $f(v_2v_4) = 2$. This results into two choices for $f(v_2v_3)$, 5 or 3.

Case (i) $f(v_2v_3) = 5$.

This implies that $f(v_3v_1) = 3$ and the label for the remaining edge is $f(v_1v_2) = 1$. Since $f(v_1) + f(v_1v_4) + f(v_1v_2) + f(v_3v_1) = 15$, we get $f(v_1) = 7$. However, $f(v_3) = 7$. This is a contraction to the fact that f is bijective.

Case (ii) $f(v_2v_3) = 3$.

This implies that $f(v_3v_1) = 5$ and $f(v_1v_2) = 1$. Since $f(v_2) + f(v_2v_4) + f(v_1v_2) + f(v_2v_3) = 15$, we get $f(v_2) = 9$. However, $f(v_4) = 9$. In this case also we get a contraction to the fact that f is bijective.

Now consider $9+5+1$, $8+4+3$. Since the graphs shown in Figure 2(a,b) are isomorphic, assign $f(v_4) = 9$ and $f(v_3) = 8$. Since the graphs shown in Figure 2(a,c) are isomorphic, assign $f(v_1v_4) = 5$ and $f(v_2v_4) = 1$. This results into two choices for $f(v_2v_3)$, 4 or 3.

Case (I) $f(v_2v_3) = 4$.

This implies that $f(v_3v_1) = 3$ and $f(v_1v_2) = 2$. Thus $f(v_1) = 5$. However, $f(v_1v_4) = 5$. This is a contraction to the fact that f is bijective.

Case (II) $f(v_2v_3) = 3$.

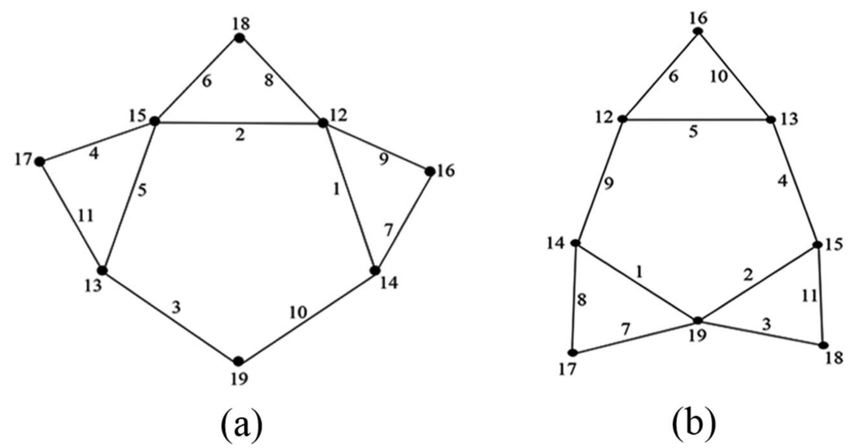

Figure 3. Labeling of non-isomorphic wounded flowers *WF*(5, 3).

This implies that $f(v_3v_1) = 4$ and $f(v_1v_2) = 2$. Thus $f(v_1) = 4$. However, $f(v_3v_1) = 4$. In this case also we get a contraction to the fact that f is bijective.

Hence there is no E-super vertex magic labeling for *WF*(3,1).

It is clear from Table 1 that the magic constant of *WF*(3, 2) is not an integer. Hence no E-super vertex magic labeling exists for *WF*(3, 2). □

Note that from Table 1, the magic constant of wounded flower *W*(*n*, *t*) for $(n, t) \in \{(4, t), (6, t) | 0 \leq t < n\}$ does not exist. Hence *WF*(4, *t*) and *WF*(6, *t*) are not E-super vertex magic for any value of *t*.

Theorem 10. *The wounded cartoon flower WF(5, t) is E-super vertex magic only for t = 0 and 3.*

Proof. From Table 1, it is clear that the only values of *t* for which the magic constant of *WF*(5, *t*) to be an integer are 0 and 3. Since 5 is an odd integer, Theorem 8 implies that *WF*(5, 0) is E-super vertex magic.

There are two non-isomorphic structures of wounded cartoon flower *WF*(5, 3).

Structure (I) All three petals are consecutive, that is, there is no ring edge between these three petals, leaving two consecutive ring edges without petals. The labeling of this structure is shown in Figure 3(a).

Structure (II) Two of the petals are consecutive and the third one has one ring edge between the other two consecutive petals. The labeling of this structure is shown in Figure 3(b).

Hence both the structures of *WF*(5, 3) are E-super vertex magic. □

Note that the magic constants of both the wounded cartoon flowers *WF*(3, 1) and *WF*(5, 3) are integers. However, *WF*(3, 1) is not E-super vertex magic and *WF*(5, 3) is E-super vertex magic. Thus, even if the magic constant k exists, that is, k is an integer, it is not necessary that the wounded cartoon flower is E-super vertex magic.

4. Sufficient conditions for no E-super vertex magic labeling

The last three terms in Equation (1) determine whether the magic constant for any *WF*(*n*, *t*) exists or not. In fact, careful examination of these three terms established some sufficient conditions for the wounded cartoon flower *WF*(*n*, *t*) to be not E-super vertex magic. We state these results in this section.

Theorem 11. *Let $t > 0$. If n and t have the opposite parity and $t^2 < n$, then WF(n, t) is not E-super vertex magic.*

Proof. Since n and t have the opposite parity, $(n + t + 1)$ is an even integer. Therefore, the sum of fractions $\frac{n+t}{2} + \frac{1}{2}$ is an integer. From Equation (1), the magic constant k in this case is given by

$$k = \left\{ (2n + 5t + 1) + \frac{n+t}{2} + \frac{1}{2} \right\} + \frac{t^2 + t}{n + t}$$
$$= \{\text{an integer}\} + \frac{t^2 + t}{n + t}.$$

Adding t to both side of the given inequality $t^2 < n$, we get $t^2 + t < n + t$. Thus $\frac{t^2+t}{n+t} < 1$. Since $t \neq -1$ and $t \neq 0$, this fraction cannot be zero. Since there is no integer between 0 and 1, the fraction $\frac{t^2+t}{n+t}$ is not an integer. Hence the magic constant k is not an integer. Thus *WF*(*n*, *t*) is not E-super vertex magic. □

Note that if n is odd then n and 0 have opposite parity and $0^2 < n$. However, Theorem 8 implies that the wounded cartoon flower $WF(n, 0)$ is E-super vertex magic. Thus $t > 0$ in the theorem above is required.

Theorem 12. *If n and t have the same parity and $2t^2 + t < n$, then WF(n, t) is not E-super vertex magic.*

Proof. Since n and t have the same parity, $(n + t)$ is an even integer and hence the fraction $\frac{n+t}{2}$ is an integer. From Equation (1), the magic constant k in this case is given by

$$k = \left\{ (2n + 5t + 1) + \frac{n+t}{2} \right\} + \left\{ \frac{1}{2} + \frac{t^2 + t}{n + t} \right\}$$
$$= \{\text{an integer}\} + \left\{ \frac{1}{2} + \frac{t^2 + t}{n + t} \right\}.$$

By given condition on n and t,

$$\begin{aligned} 2t^2 + t &< n \\ 2t^2 + 2t &< n + t \\ \frac{t^2 + t}{n + t} &< \frac{1}{2} \\ \frac{1}{2} + \frac{t^2 + t}{n + t} &< 1. \end{aligned}$$

Since $\frac{t^2+t}{n+t}$ is a non-negative number, the fraction $\frac{1}{2} + \frac{t^2+t}{n+t}$ cannot be zero. Therefore, k is not an integer. Thus $WF(n, t)$ is not E-super vertex magic. □

Theorem 13. *The wounded cartoon flower $WF(t^2, t)$ is not E-super vertex magic for $t > 1$.*

Proof. Substitute $n = t^2$ in Equation (1), we get

$$\begin{aligned} k &= (2t^2 + 5t + 1) + \frac{t^2 + t}{2} + \frac{1}{2} + \frac{t^2 + t}{t^2 + t} \\ &= (2t^2 + 5t + 2) + \frac{t^2 + t + 1}{2}. \end{aligned}$$

Since t^2 and t have the same parity, $(t^2 + t + 1)$ is an odd integer. Thus k is not an integer. □

Note that in the theorem above, the condition $t > 1$ is required as $WF(n, t)$ is defined only for $n \geq 3$.

Theorem 14. *If n is even, then $WF(n, n-2)$ is not E-super vertex magic.*

Proof. Substitute $t = n - 2$ in Equation (1), we get

$$\begin{aligned} k &= (2n + 5(n-2) + 1) + \frac{n + (n-2)}{2} + \frac{1}{2} + \frac{(n-2)^2 + (n-2)}{n + (n-2)} \\ &= (7n - 9) + (n - 1) + \frac{1}{2} + \frac{(n-2)(n-1)}{2n - 2} \\ &= (8n - 10) + \frac{1}{2} + \frac{(n-2)}{2} \\ &= (8n - 10) + \frac{(n-1)}{2}. \end{aligned}$$

Since n is even, $\frac{(n-1)}{2}$ is not an integer. Thus the magic contact k does not exist for $WF(n, n–2)$ if n is even. □

Note that if n is odd, then $WF(n, n–2)$ may or may not be E-super vertex magic. For example, $WF(n, n–2)$ is E-super vertex magic for $n = 5$ (Theorem 10), whereas, $WF(n, n–2)$ is not E-super vertex magic for $n = 3$ (Theorem 9).

Theorem 15. *The wounded cartoon flower $WF(n,1)$ is not E-super vertex magic.*

Proof. We prove this result by considering the following three cases.

Case (i) n is even.

Since n and 1 have the opposite parity and $t^2 = 1 < n$, Theorem 11 is applicable. Thus $WF(n,1)$ is not E-super vertex magic.

Case (ii) n is odd and $n \leq 3$.

The wounded cartoon flowers are defined only for $n \geq 3$, Thus for this case, $n = 3$. By Theorem 9, $WF(3, 1)$ is not E-super vertex magic.

Case (iii) n is odd and $n > 3$.

Since n and 1 have the same parity and $2t^2 + t = 2(1)^2 + 1 = 3 < n$, Theorem 12 is applicable. Thus $WF(n,1)$ is not E-super vertex magic.

Theorem 16. *The wounded cartoon flower $WF(n, 2)$ is not E-super vertex magic for $n \neq 10$.*

Proof. We prove this result by considering the following four cases.

Case (i) n is odd and $n \leq 3$.

The wounded cartoon flowers are defined only for $n \geq 3$, Thus for this case, $n = 3$. By Theorem 9, $WF(3, 2)$ is not E-super vertex magic.

Case (ii) n is odd and $n > 3$.

The smallest odd value for n in this case is 5. Thus $4 < n$. This implies that $t^2 = 2^2 < n$. Since n and 2 have the opposite parity, Theorem 11 is applicable. Thus $WF(n, 2)$ is not E-super vertex magic.

Case (iii) n is even and $n < 10$.

This means that in this case, possible values for n are 4, 6 and 8. From Table 1, it is clear that magic constants of $WF(4, 2)$ and $WF(6, 2)$ do not exist. Substitute $(n, t) = (8, 2)$ in Equation (1), we get $k = 33\frac{1}{10}$. Hence $WF(4, 2)$, $WF(6,2)$ and $WF(8, 2)$ are not E-super vertex magic.

Case (iv) n is even and $n > 10$.

In this case, $2t^2 + t = 2(2)^2 + 2 = 10 < n$. Since n and 2 have same parity, by Theorem 12, $WF(n, 2)$ is not E-super vertex magic. □

Theorem 17. *The only wounded n-cartoon flower with 2-petal, $WF(n, 2)$, is E-super vertex magic for $n = 10$.*

Proof. There are five non-isomorphic structures of wounded cartoon flower $WF(10, 2)$. All of them are E-super vertex magic. The labeling of all non-isomorphic structures of $WF(10, 2)$ are established in Figure 4. □

5. Conclusions

In this paper, new classes of graphs, the cartoon flower and the wounded cartoon flower, were defined. The cartoon flower of any size is not E-super vertex magic. However, depending on the number of petals plucked from the cartoon graph, the wounded cartoon flower can be E-super vertex magic. We presented some conditions on number of petals that guarantee the wounded cartoon flower not to be E-super vertex magic. More conditions may be explored based upon results in this paper and previous work. We also

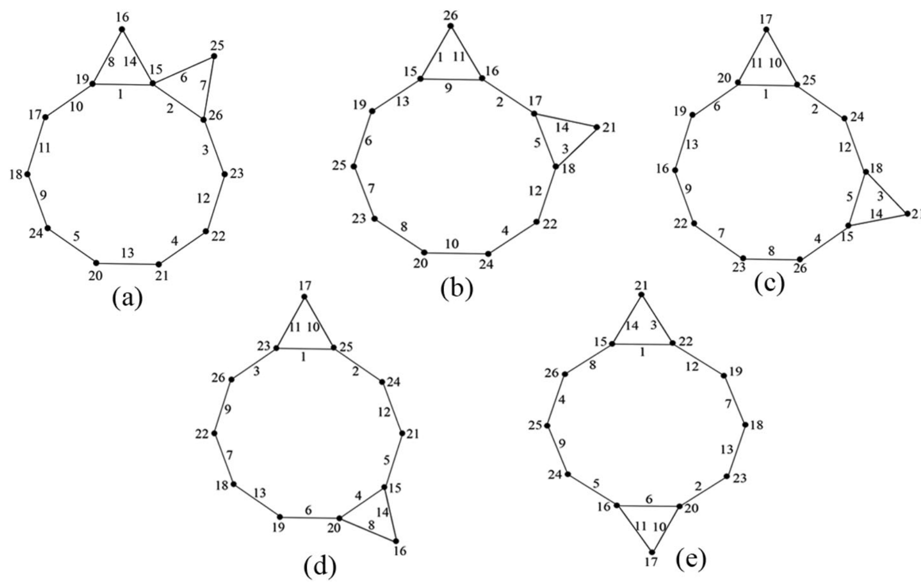

Figure 4. Labeling of non-isomorphic wounded flowers *WF*(10, 2).

presented examples of wounded cartoon flowers for which E-super vertex magic labeling exist as well as examples for which such labeling do not exist even though the corresponding magic constant exists.

Our work also opens another question of determining conditions on petals so that wounded cartoon flowers admit E-super vertex magic labeling.

Theorems 10 and 17 show that all the structures of wounded cartoon flowers, *WF*(5, 3) and *WF*(10, 2), are E-super vertex magic, respectively. Another interesting problem is to examine the statement that if one of the structures of wounded cartoon flower is E-super vertex magic, then all of the structures (non-isomorphic graphs) of wounded flower with same number of ring edges and same number of petals would be E-super vertex magic.

Disclosure statement

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