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Hybrid PMC (HPMC) fault model and diagnosability of interconnection networks

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ABSTRACT
System level diagnosis, an area pioneered by Preparata, Metze and Chien, has been a dominant area of research in the broader area of fault-tolerant computing since the proposition of the PMC model. In this paper, we study the fault diagnosis problem for systems in hybrid fault circumstances where both node and link faults may occur. Under our diagnosis model, a test involves two adjacent processors and the link between them. We propose the definition of consistent faulty pairs and distinguishable faulty pairs. Given an assignment of testing links, we establish necessary and sufficient conditions for distinguishable faulty pairs. Finally, we introduce parameters to measure the fault diagnosis capability of multiprocessor systems under this model and as an application the fault diagnosis capability of hypercubes under this model has been evaluated.

KEYWORDS
Interconnection networks; PMC model; hybrid PMC model; multiprocessor systems; fault diagnosis; diagnosability; graph theory

1. Introduction

High performance computing has been receiving increasing attentions in recent years. Nowadays some super computers even have hundreds of thousands of processors. With so many processors and communication links, it's inevitable that processor or link faults occur in such systems. The demand for high availability requires automatic fault diagnosis for the maintenance of such systems. In 1967 [17], F.P. Preparata, G. Metze and R.T. Chien introduced a graph-theoretical model for the purpose of automatic fault diagnosis in multiprocessor systems, the model was later called PMC model after the last names of the three authors.

Under the PMC model, it is assumed that only node faults can occur. But in real circumstances, both node and link faults may exist. So it's important to study the fault diagnosis of multiprocessor systems under hybrid fault circumstances. Based on the PMC model, we introduce a graph-theoretic model to adapt to hybrid fault patterns. With this model, it's possible to explore the fault diagnosis in circumstances with both node and link faults. We characterize diagnosable fault patterns in this model. Fault diagnosis capability measure is also proposed in this paper and the fault diagnosis capability of hypercubes under the hybrid diagnosis model is studied.

The rest of this paper is organized as follows: In Section 2, we first introduce the basics of the PMC model and then introduce the hybrid PMC(HPMC) model. In Section 2, we also introduce the notion of distinguishable fault pairs. In Section 3, we first define h-restricted vertex diagnosability and r-restricted edge diagnosability to measure the diagnosis capability of interconnection networks. In Section 3, we also presented a necessary and sufficient condition for two hybrid fault patterns to be distinguishable and present certain basic results on the diagnosability of interconnection networks assuming the nodes and links of the networks are used as testing processors and test links. In Section 4, we present results on the diagnosability of n-dimensional hypercubes under the HPMC model. We conclude in Section 5.

2. Hybrid fault diagnosis model

In this section, we present a graph-theoretic model for fault diagnosis in hybrid fault circumstances. The model is based on the famous PMC model [17]. So we first introduce the basics of the PMC model, then we propose the hybrid PMC model for fault diagnosis in hybrid fault circumstances.

2.1. Notations and terminologies

For notations and terminologies not defined here, we follow [3]. Given a simple undirected graph $G=(V,E)$, $V(G)$ and $E(G)$ are used to denote the vertex set and edge set of G, respectively. If there is an edge $e=uv$ in G, u, v are said to be adjacent to each other and e is said to be incident to both u and v. Given u in $V(G)$, we use $N(u)$(resp. $NE(u)$) to denote the set of all its adjacent vertices (resp. incident edges) in G. $d(u)$ is defined to be the number of vertices in $N(u)$, called the degree of u. $\delta(G)$ is defined to be the minimum degree over all vertices in G.

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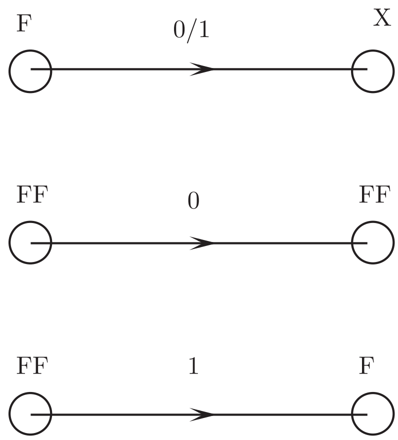

Figure 1. PMC Model: F-Faulty, FF- Fault-free, X- Faulty or Fault-free.

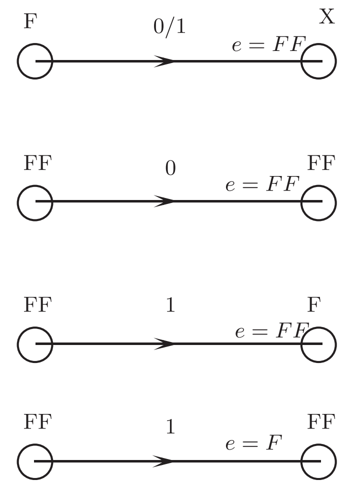

Figure 2. Test results under the Hybrid PMC model.

2.2. Basics of the PMC model

System level diagnosis is an important approach for the fault diagnosis of multiprocessor systems. In this approach, diagnosis is performed by mutual tests of processors in the system. The set of all test results is called a syndrome of the system. Then based on the assumptions on the test results, the faulty processors are located according to the syndrome of the system. Different definitions of tests and different assumptions on test results lead to different diagnosis models [5, 6, 8, 12–14, 21, 23].

Proposed by Preparata, Metze and Chien [17], PMC model is the most famous and most widely studied model in the system level diagnosis of multiprocessor systems. Under the PMC model, it's assumed that there is no link faults and only adjacent processors can test the status of each other. All node faults are permanent, and a node fault can always be detected by a fault-free vertex. Under the PMC model, a test can be represented by an ordered pair (u, v) where u is the tester and v is the testee. The result of the test (u, v) is denoted by $r(u, v)$. It is 0 if u evaluates v as fault-free and 1 if u evaluates v as faulty. Under the PMC model, it is assumed that test result $r(u, v)$ is reliable if and only if the tester u is fault-free. That is, if u is fault-free, then $r(u,v) = 0$ means that v is fault-free and $r(u,v) = 1$ means that v is faulty; If the tester u is faulty, then the status of v is irrelevant to the test result $r(u, v)$. See Figure 1 where the arrow points to the testee.

Given a multiprocessor system, the set of all test results is called a syndrome of the multiprocessor system. Under the PMC model, a fault set F is said to be consistent with a syndrome σ if σ can arise in the circumstance that all nodes in F are faulty and all nodes not in F are fault-free. Since the test result of a faulty tester is unreliable, a fault set can be consistent with many syndrome. The set of all syndromes consistent with F is denoted by $\sigma(F)$. Two faulty sets F_1, F_2 are distinguishable if and only if $\sigma(F_1) \cap \sigma(F_2) = \emptyset$. Otherwise, they are indistinguishable. Since the test result of a faulty vertex is unreliable, $V(G)$ is consistent with any syndrome of G. Thus to locate faulty vertices, it's often assumed that there exists an upper bound for the number of faulty vertices. A multiprocessor system G is t-diagnosable if and only if all the faulty vertices can be guaranteed to be located provided that the number of faulty vertices does not exceed t. The diagnosability of G is the maximum integer t such that G is t-diagnosable. The diagnosability of a multiprocessor system measures its fault diagnosis capability. The diagnosability of many interconnection networks have been explored [1, 4, 7, 9, 10, 16, 18, 19].

2.3. HPMC: PMC model based hybrid fault diagnosis model

Like in PMC model, we assume that in the HPMC model test happens between adjacent processors. That is, a test $(u, v; e)$ can be assigned if and only if u, v are the two end-vertices of the edge e for a given multiprocessor system G. Unlike the PMC model, the test result of a test $(u, v; e)$ may be impacted by the status of all of the 3 elements in it. In the following, we define test in the new model and make justified assumptions to develop the model to adapt to the hybrid fault circumstances. See Figure 2 for test results in HPMC model.

- Definition of test
  Similar to the PMC model, under the HPMC model, it is assumed that a test $t(u, v; e)$ involves two adjacent processors u, v and the edge $e = uv$ between them. In the test $t(u, v; e)$, u is called the tester, v is called the testee and e is called the test edge.
- Assumptions:
  1. The links incident with any faulty processors are good. The assumption is justified based on the following analysis:
  (a) When a link has a faulty end-vertex, it's of no use;
  (b) When a faulty processor is replaced or removed, all its incident links have to be rechecked or removed. So it's not necessary to determine their status.
  (c) When a link has a faulty end-vertex, it's not possible to determine which causes the failure of the test, so it's impossible to determine the status of both.
  2. The test result of a good tester is reliable and test result of a faulty tester is unreliable.

That is, for a test $t(u, v; e)$, if u is good, the test passed if and only if v, e are both good. If u is bad, the test result is irrelevant to the status of v, e. See Figure 2 for illustration. This assumption is similar to the corresponding assumption in the PMC model. When a link fails, the test must fail, but considering assumption 1, the justification of this assumption is obvious.

Based on these assumptions, we propose the definition of consistent faulty pairs in hybrid fault circumstances.

Definition 2.1. Given a multiprocessor system G, we call (F, S) a consistent faulty pair of G if all vertices in F cannot be incident to any edge in S.

Similar to the PMC model, a syndrome in the HPMC model is the set of all the test results. A syndrome σ is said to be consistent to a faulty pair (F, S) if the syndrome can arise in the circumstance that all vertices in F and all edges in S are faulty and all the other vertices and edges are faulty-free. Since the test result of a faulty tester is unreliable, syndromes consistent with a faulty pair (F, S) may not be unique. We use $\sigma(F, S)$ to denote the set of all syndromes consistent with (F, S). Given a syndrome σ of a multiprocessor system G under the HPMC model, the purpose of diagnosis is to find a desired consistent faulty pair (F, S) consistent with σ.

3. Fundamental properties of HPMC model

In this section we define the notion of distinguishable faulty pairs and present a necessary and sufficient condition for distinguishable faulty pairs.

Definition 3.1. Two faulty pairs (F_1, S_1) and (F_2, S_2) are distinguishable if and only if $\sigma(F_1, S_1) \cap \sigma(F_2, S_2) = \emptyset$. Otherwise, they are indistinguishable.

The diagnosis of a multiprocessor system is the process of locating faulty processors and faulty links according to a syndrome of the system. If all processors in a multiprocessor system are faulty, then any syndrome can arise. So under the HPMC model any syndrome is consistent with $(V(G), S)$ where S is any edge subset of G. For a syndrome σ, we use $FS(\sigma)$ to denote the set of all faulty pairs consistent with it. In this circumstance, how to judge which faulty pair is the desired faulty pair is a challenging problem. Under the PMC model, some researchers suppose that there exists an upper bound for the number of faulty vertices. A system is called t-diagnosable under the PMC model if all the faulty processors can be guaranteed to be located provided that the number of faulty processors does not exceed t. The diagnosability of a system G is the maximum integer t such that G is t-diagnosable. Under the HPMC model, we propose the definition of (t, s)-diagnosable systems and h-restricted vertex diagnosability and r-restricted edge diagnosability to measure the diagnosis capability of interconnection networks.

Definition 3.2. Let t, s be two positive integers, a multiprocessor system G is (t, s) diagnosable if and only any two distinct faulty pairs (F_1, S_1) and (F_2, S_2) with $|F_1|, |F_2| \leq t$ and $|S_1|, |S_2| \leq s$ are distinguishable.

Definition 3.3. Given a multiprocessor system G and two integers h, r, the h-restricted vertex diagnosability of G is the maximum integer t such that G is (t, h)-diagnosable, denoted by $t_h^e(G)$; Similarly, the r-restricted edge diagnosability of G is the maximum integer s such that G is (r, s)-diagnosable, denoted by $s_r^v(G)$.

The h-restricted vertex diagnosability and r-restricted edge diagnosability under the HPMC model can be viewed as a generalization of the diagnosability under the PMC model, and thus can reflect the fault diagnosis capability of interconnection networks under the HPMC model. The following lemma gives a relation of the two measures:

Lemma 3.4. *Given an interconnection network G and two positive integers h, r, if $t_h(G) \geq r$, then $s_r(G) \geq h$.*

Proof. If $t_h(G) \geq r$, then G is (r, h) diagnosable, so $s_r(G) \geq h$. □

To locate faulty processors and faulty links, we need to characterize distinguishable faulty pairs. The following lemma characterizes two distinguishable faulty pairs.

Theorem 3.5. *Given a multiprocessor systems $G(V, E)$, $F_1, F_2 \subset V(G)$ and $S_1, S_2 \subset E(G)$. Two distinct consistent faulty pairs (F_1, S_1), (F_2, S_2) are distinguishable under the HPMC model if and only if one of the following four conditions are satisfied.*

- *A) There exists an edge $e = uv$ satisfying*

$$u \in F_1 - F_2 \tag{1}$$

$$v \in V - F_1 \cup F_2 \tag{2}$$

$$e \notin S_2. \tag{3}$$

- *B) There exists an edge $e = uv$ satisfying*

$$u \in F_2 - F_1 \tag{a}$$

$$v \in V - F_1 \cup F_2 \tag{b}$$

$$e \notin S_1 \tag{c}$$

- *C) There exists an edge $e = uv$ satisfying*

$$e \in S_1 - S_2 \tag{4}$$

$$u, v \notin F_2. \tag{5}$$

- *D) There exists an edge $e = uv$ satisfying*

$$e \in S_2 - S_1 \tag{d}$$

$$u, v \notin F_1 \tag{e}$$

Proof. 1) Sufficiency:

We prove sufficiency by showing that when condition A, B, C or D is satisfied, then the results of a particular test $t(v, u; e)$ are different in the two fault circumstances and thus (F_1, S_1) and (F_2, S_2) are distinguishable.

Sufficiency of A:

By (1) and (2), $t(v,u;e)=1$ since $v\notin F_1$ and $u\in F_1$ when $(F_1,\ S_1)$ is the fault circumstance.

By (1), (2) and (3), $t(v,u;e)=0$ when $(F_2,\ S_2)$ is the fault circumstance since $v,u\ \notin\ F_2$ and $e\ \notin\ S_2$.

Sufficiency of C:

By (4) $e\in S_1$, then $u,v\ \notin\ F_1$ since by our assumption that the end vertices of a faulty link is good. So $t(v,u;e)=1$ when $(F_1,\ S_1)$ is the fault circumstance.

By (4) and (5), when $(F_2,\ S_2)$ is the fault circumstance $t(v,u;e)=0$ since $v,u\ \notin\ F_2$ and $e\ \notin\ S_2$.

The sufficiency of B and D can be proved similarly.

2) Necessity:

Suppose (F_1,S_1) and (F_2,S_2) are distinguishable, in the following we will show that at least one of the above 4 conditions holds. Since (F_1,S_1) and (F_2,S_2) are distinguishable, there must exist a test $t(v,u;e)$ whose result $r(v,u;e)$ must be different in the two fault circumstances $(F_1,\ S_1)$ and $(F_2,\ S_2)$.

- Case 1) $r(v,u;e)=1$ in $(F_1,\ S_1)$ and $r(v,u;e)=0$ in $(F_2,\ S_2)$.
  Since $r(v,u;e)$ must be 0 in the fault circumstance $(F_2,\ S_2)$, $u,v\notin F_2$ and $e\notin S_2$.
  Since $r(v,u;e)$ must be 1 in $(F_1,\ S_1)$, $v\notin F_1$, for otherwise the test outcome maybe 0. Since $r(v,u;e)=1$ and $v\notin F_1$, either $u\in F_1$ or $e\in S_1$.
  If $u\in F_1$, this is condition A;
  If $e\in S_1$, this is condition C.
- Case 2) $r(v,u;e)=0$ in $(F_1,\ S_1)$ and $r(v,u;e)=1$ in $(F_2,\ S_2)$.
  Since $r(v,u;e)$ must be 0 in the fault circumstance $(F_1,\ S_1)$, $u,v\notin F_1$ and $e\notin S_1$.
  Since $r(v,u;e)$ must be 1 in $(F_2,\ S_2)$, $v\notin F_2$ otherwise the test outcome maybe 0. Since in $(F_2,\ S_2)$ $r(v,u;e)=1$ and $v\notin F_2$, either $u\in F_2$ or $e\in S_2$.
  If $u\in F_2$, this is condition B;
  If $e\in S_2$, this is condition D. □

In the following Lemma we present some basic results about the h-restricted vertex diagnosability and r-restricted edge diagnosability of an interconnection network G.

Lemma 3.6. *Given a multiprocessor G with minimum degree $\delta(G)$ and m edges, let $t(G)$ be the diagnosability of G under the PMC model, we have*

1. $t_0(G)=t(G), t_h(G)\le\delta(G)-h$ *for* $0\le h\le\delta(G)$.
2. $s_0(G)=m, s_1(G)\le\delta(G)-2$ *if G has two adjacent vertices both of degree* $\delta(G)$.

Proof. By the definition of $t_s(G)$, it's obvious that $t_0(G)=t(G)$. Let u be a vertex of G with $d(u)=\delta(G)$. Suppose $N(u)=\{u^1,u^2,\ldots u^{\delta(G)}\}, NE(u)=\{e_1,e_2,\ldots e_{\delta(G)}\}$ Z $N(u)=\{u^1,u^2,\ldots u^{\delta(G)}\}, NE(u)=\{e_1,e_2,\ldots e_{\delta(G)}\}$ where $e_i=(u,u^i)$. Let $F_1=\{u,u^{h+1},u^{h+2},\ldots u^{\delta(G)}\}, F_2=\{u^{h+1},u^{h+2},\ldots u^{\delta(G)}\}$, $S_1=\emptyset, S_2=\{e_1,e_2,\ldots e_h\}$. Then it's clear that both $(F_1,\ S_1)$ and $(F_2,\ S_2)$ are consistent faulty pairs. And by Lemma 3.5, the two pairs are indistinguishable. Thus $t_h(G)\le\delta(G)-h$.

2) When there is no faulty vertices, the status of any faulty edge can be determined by the test result involving it, so $s_0(G)=m$; Let $e=(u,v)$ where $d(u)=d(v)=\delta(G)$. Suppose $NE(u)=\{e,e_1,e_2,\ldots e_{\delta(G)-1}\}$ and $NE(v)=\{e,f_1,f_2,\ldots f_{\delta(G)-1}\}$. Let $F_1=\{u\},F_2=\{v\},S_1=\{f_1,f_2,\ldots f_{\delta(G)-1}\}$, $S_2=\{e_1,e_2,\ldots e_{\delta(G)-1}\}$. It's clear that the two consistent faulty pairs $(F_1,\ S_1)$ and $(F_2,\ S_2)$ are indistinguishable, so $s_1(G)\le\delta(G)-2$. □

The above lemma shows that when we explore the h-restricted vertex diagnosability of G, we only need to explore the case of $h\le\delta(G)$. And the 1-restricted edge diagnosability of G may not equal its 1-restricted vertex diagnosability. We may need quite distinct methods to explore the r-restricted edge diagnosability of G.

4. Fault diagnosis capability of hypercubes under the HPMC model

In this section, we will explore the diagnosis capability of hypercubes under hybrid fault circumstances.

4.1. Preliminaries of hypercubes

Hypercubes are one of the most famous interconnection networks, due to its many attractive properties such as high symmetry, good graph embedding and etc. The properties of hypercubes have been extensively studied [11, 15, 20, 22, 24, 25]. Hypercubes have been used in the design of large multiprocessor systems [2, 15].

Each vertex of an n-dimensional hypercube Q_n can be labeled with an n-bit binary string, two vertices u, v are adjacent if and only if their labels differ in exactly 1 bit position. Thus Q_n has 2^n vertices and the degree of each vertex is n. For a vertex u in Q_n, we use u^i to denote the neighbor of u which differ with u only in the i-th bit position. That is, $u^i=u_1\cdots u_{i-1}\overline{u_i}u_{i+1}\cdots u_n$ where $\overline{u_i}$ means the complement of u_i. By the definition of n-cube, two vertices have common neighbors iff their labels differ in exactly 2 bit positions. So any pair of vertices either have no common neighbors or have exactly 2 common neighbors. Thus the girth of an n-dimensional hypercube $g(Q_n)$ is 4 when $n\ge2$.

In [4], Caruso et al. have determined that the diagnosability of n-dimensional hypercube is n when $n\ge1$.

Lemma 4.1. *For $n\ge1$, the diagnosability of an n-cube Q_n under the PMC model is n.*

4.2. H-restricted vertex diagnosability and 1-restricted edge diagnosability of hypercubes

Theorem 4.2. *Let Q_n be an n-dimensional hypercube, $1\le h\le n-2$, then $t_h^e(Q_n)=n-h$.*

Proof. 1) Let $u=0^n, F_1=\{u^{h+1},\ldots u^n\}, F_2=\{u,u^{h+1},\ldots u^n\}$, $S_1=\{uu_1,uu_2,\ldots uu_h\}$ $S_2=\emptyset,$. It's obvious that $(F_1,\ S_1)$ and $(F_2,\ S_2)$ are both consistent faulty pair with $|\max(|S_1,S_2|)|=$

h. By Lemma 3.5, (F_1, S_1), (F_2, S_2) are indistinguishable. Thus $t_h^e(Q_n) \le \max(|F_1|, |F_2|)| - 1 = n - h$.

2) Next we use contradiction to prove that $t_h^e(Q_n) \ge n - h$. Suppose not, that is, $t_h^e(Q_n) < n - h$. So there exist two indistinguishable consistent faulty pairs (F_1, S_1) and (F_2, S_2) with $\max(|S_1|, |S_2|) \le h$ and $\max(|F_1|, |F_2|) \le n - h$. Since($F_1$, S_1) and (F_2, S_2) are two distinct consistent faulty pairs, either $F_1 \Delta F_2 \ne \emptyset$ or $S_1 \Delta S_2 \ne \emptyset$.

Case 1) $F_1 \Delta F_2 \ne \emptyset$. Then either $F_1 - F_2$ or $F_2 - F_1$ has at least 2 vertices or $\max\{|F_1 - F_2|, |F_2 - F_1|\} = 1$.

Subcase 1. 1) $|F_1 - F_2| = 1, |F_2 - F_1| = 1$.

Suppose $F_1 - F_2 = \{u\}, F_2 - F_1 = \{v\}$. For any vertex w in $N_{V-F_1 \cup F_2}(u)$, (u, w) must be in S_2 since (F_1, S_1), (F_2, S_2) are indistinguishable consistent pairs. Since $|S_2| \le h$, u has at least $n - h$ vertices in F_2. Considering $|F_2| \le n - h$, we have $F_2 \subset N(u)$ and $|F_2| = n - h$. Similarly, we can prove that $|F_1| = n - h$ and $F_1 \subset N(v)$. Thus u, v are adjacent and they have $n - h - 1 > 1$ common neighbors. This is impossible since $g(Q_n) = 4$.

Subcase 1. 2) $|F_1 - F_2| = 1, |F_2 - F_1| = 0$ or $|F_1 - F_2| = 0$, $|F_2 - F_1| = 1$.

Without loss of generality, suppose $|F_1 - F_2| = 1, |F_2 - F_1| = 0$. Suppose $F_1 - F_2 = \{u\}$, then as in the above analysis, we can prove that $|F_2| = n - h$, so $|F_1| = n - h + 1$ contradicting $|F_1| \le n - h$.

Subcase 1. 3) $|F_1 - F_2| \ge 2$ or $|F_2 - F_1|\} \ge 2$.

Without loss of generality, let $|F_1 - F_2| \ge 2$. Suppose $u, v \in F_1 - F_2$. For any vertex $w \in N_{V-F_1 \cup F_2}(u, v)$, the edge between w and u or v must be in S_2 since (F_1, S_1) and (F_2, S_2) are indistinguishable consistent pairs. So $|N_{F_1 \cup F_2}(u, v)| = |N(u, v)| - |N_{V-F_1 \cup F_2}(u, v)| \ge 2n - 2 - h$. Thus $|F_1 \cup F_2| \ge 2 + (2n - 2 - h) = 2n - h > |F_1| + |F_2|$, a contradiction.

Case 2) $S_1 \Delta S_2 \ne \emptyset$

Without loss of generality, suppose $S_1 - S_2 \ne \emptyset$. Suppose $e = (u, v) \in S_1 - S_2$. Since (F_1, S_1) and (F_2, S_2) are indistinguishable consistent pairs, $u, v \notin F_1$ and at least one of u, v is in F_2. That is, $F_1 \Delta F_2 \ne \emptyset$, so as in case 1) we obtain a contradiction. □

Theorem 4.3. *Let Q_n be an n-dimensional hypercube, $n - 1 \le h \le n$, then $t_h^e(Q_n) = 0$.*

Proof. Let $u = 0^n$ and $v = 0^{n-1}1$ be two vertices. Let $e = uv$, suppose the other n – 1 incident edges of u and v are $e_1, e_2, \ldots, e_{n-1}$ and $f_1, f_2, ..f_{n-1}$, respectively. Let $F_1 = \{u\}$, $S_1 = \{f_1, f_2, \ldots, f_{n-1}\}$ $F_2 = \{v\}, S_2 = \{e_1, e_2, \ldots, e_{n-1}\}$, then according to Theorem 3.5, (F_1, S_1) and (F_2, S_2) are indistinguishable. Since $|S_1| = |S_2| = n - 1, t_{n-1}^e(Q_n) < 1$ and it has to be 0. Since $t_n^e(Q_n) \le t_{n-1}^e(Q_n), t_n^e(Q_n) = 0$ too. □

Theorem 4.4. *$S_1^v(Q_n) = n - 2$ for $n \ge 2$.*

Proof. 1) By Lemma 3.6 $S_1(Q_n) \le n - 2$.

2) We use contradiction to prove that $S_1(Q_n) \ge n - 2$. Suppose $S_1(Q_n) < n - 2$, that is, there exist two indistinguishable consistent pairs (F_1, S_1), (F_2, S_2) with $|F_1|, |F_2| \le 1, |S_1|, |S_2| \le n - 2$. Since $S_0(Q_n) = n * 2^{n-1}$, at least one of F_1, F_2 is not empty. Without loss of generality, suppose $F_1 = \{u\}$.

Subcase 2. 1) $F_2 = \emptyset$. In this case any edge in $NE(u)$ must be in S_2 since (F_1, S_1), (F_2, S_2) are indistinguishable. Thus $n - 2 \ge |S_2| \ge n$, a contradiction.

Subcase 2. 2) $F_2 = F_1$. By Lemma 3.6, this is impossible.

Subcase 2. 3) $F_2 = \{v\}$, any edge in $NE(u)$ other than uv must be in S_2 since (F_1, S_1), (F_2, S_2) are indistinguishable. Thus $|S_2| \ge n - 1$, contradicting our assumption that $|S_2| \le n - 2$. □

Theorem 4.5. *$S_r^v(Q_n) = n - r$ for $n > r \ge 2$.*

Proof. 1) Let $u = 0^n, F_1 = \{u^1, u^2, \ldots u^{r-1}\}$, $F_2 = \{u, u^1, u^2, \ldots u^{r-1}\}, S_1 = \{uu^r, uu^{r+1}, \ldots uu^n\}, S_2 = \emptyset$,. It's obvious that ($F_1$, S_1) and (F_2, S_2) are both consistent faulty pair with $|\max(|F_1, F_2|)| = r$. By Lemma 3.5, (F_1, S_1), (F_2, S_2) are indistinguishable. Thus $S_r^v(Q_n) \le \max(|S_1|, |S_2|)| - 1 = n - r$.

2) Let $h = n - r$, then $h \le n - 2$ since $r \ge 2$. By Theorem 4.2, $t_h^e(Q_n) = n - h = r$. By Theorem 3.6, $S_r^v(Q_n) \ge h = n - r$.

By 1) and 2), we have $S_r^v(Q_n) = n - r$ for $n > r \ge 2$. □

5. Conclusions

In this paper, we make two contributions: (1) To adapt to the hybrid faulty circumstances, we generalize the well-known PMC model to define the HPMC model. Based on a justified assumption that all incident edges of a faulty vertex is good, we propose the definition of consistent faulty pairs and characterize distinguishable consistent faulty pairs. Thus the foundation of HPMC model is established. (2) We then generalize the notion of diagnosability to establish the h-restricted vertex diagnosability and r-restricted edge diagnosability for measuring the fault diagnosis capability of interconnection networks under hybrid fault circumstances. We then explore the h-restricted vertex diagnosability and r-restricted edge diagnosability of hypercubes. The result is important for understanding the fault diagnosis capability of hypercubes under hybrid fault circumstances. The method used here may be used to explore the two parameters of other interconnection networks under the HPMC model.

Here we give some advice for possible future work:

1. the r-vertex restricted diagnosability of hypercubes when $r \ge 2$ may be explored.
2. Similar methods used here may be used to explore the h-edge restricted diagnosability and r-vertex-restricted diagnosability of other interconnection networks.
3. The HPMC model may be used to the fault diagnosis in wireless sensor networks.

Disclosure statement

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References

[1] Ahlswede, R, Aydinian, H. (2008). On diagnosability of large multiprocessor networks. *Discrete Appl. Math.* 156(18): 3464–3474.

[2] Arlanskas, R. (1988). ipsc/2 system: a second generation hypercube. Paper presented at the Proceedings of the 3rd ACM Conference on Hypercube Concurrent Computers and Applications, Pasadena, CA, USA, January 27–30, pp. 38–42.

[3] Bondy, J. A, Murty, U. S. R. (1976). *Graph Theory with Applications*. New York: North Holland.

[4] Caruso, A., Chessa, S., Maestrini, P, Santi, P. (2002). Diagnosability of regular systems. *J. Algorithms* 45(2):126–143.

[5] Das, A., Thulasiraman, K, Agarwal, V. K. (1994). Diagnosis of t/(t + 1)-diagnosable systems. *SIAM J. Comput.* 23(5):895–905.

[6] Hakimi, S. L, Amin, A. T. (1974). Characterization of connection assignment of diagnosable systems. *IEEE Trans. Comput.* C-23(1):86–88. vol

[7] Hsieh, S.-Y, Lee, C.-W. (2008). Diagnosability of two-matching composition networks. In: Hu, X., Wang, J., eds., *Proceedings of the 14th Annual International Conference on Computing and Combinatorics, COCOON '08*. Berlin: Springer-Verlag. pp. 478–486.

[8] Hsu, G.-H, Tan, J. J. (2007). A local diagnosability measure for multiprocessor systems. *IEEE Trans. Parallel Distrib. Syst.* 18(5): 598–607.

[9] Huang, S., Xu, J, Chen, T. (1989). *Characterization and Design of Sequentially t-Diagnosable Systems*. Washington, DC: IEEE pp. 554–559.

[10] Krawczyk, H, Kozlowski, W. E. (1988). On the diagnosability of multicomputer systems with homogeneous and incomplete tests. *IEEE Trans. Comput.* 37(11):1419–1422.

[11] Kung, T.-L., Lin, C.-K., Liang, T., Hsu, L.-H, Tan, J. J. (2009). On the bipanpositionable bipanconnectedness of hypercubes. *Theor. Comput. Sci.* 410(8-10):801–811.

[12] Lai, P.-L., Tan, J. J., Chang, C.-P, Hsu, L.-H. (2005). Conditional diagnosability measures for large multiprocessor systems. *IEEE Trans. Comput.* 54(2):165–175.

[13] Maeng, J, Malek, M. (1981). A comparison connection assignment for self-diagnosis of multicomputer systems. In *Proc. 1981 Symp. on Fault Tolerant Comp*, Portland, ME, USA, June 24–26. pp. 173–175.

[14] Maheshwari, S. N, Hakimi, S. L. (1976). On models for diagnosable systems and probabilistic fault diagnosis. *IEEE Trans. Comput.* 100(3):228–236.

[15] Nugent, S. (1988). The ipsc/2 direct-connect communication technology. In *Proceedings of the Conference on Hypercube Concurrent Computers and Applications*, Pasadena, CA, USA, January 27–30, pp. 51–60.

[16] Otsuka, M, Itakura, H. (1990). Diagnosabilities in system level diagnosis. *IEEJ Trans. EIS.* 110(10):653–660.

[17] Preparata, F. P., Metze, G, Chien, R. T. (1967). On the connection assignment problem of diagnosable systems. *IEEE Trans. Electron. Comput.* 6:848–854.

[18] Shin, Y, Sengupta, A. (1993). Self-diagnosability of multiprocessor systems with hybrid faults: diagnosis by comparison approach. *IEEE Trans. Circuits Syst. I.* 40(5):355–358.

[19] Stephens, J, Raghavan, V. (1993). On single-fault set diagnosability in the PMC model. *IEEE Trans. Comput.* 42(8):981–983.

[20] Wang, D, Wang, Z. (1995). *System-Level Testing Assignment for Hypercubes with Lower Fault Bounds*. Scottsdale, AZ: IEEE, pp. 136–142.

[21] Xu, M., Thulasiraman, K, Hu, X.-D. (2009). Conditional diagnosability of matching composition networks under the PMC model. *IEEE Trans. Circuits Syst. II Exp. Briefs* 56(11):875–879.

[22] Xu, X.-M. H. J.-M., Zhu, Q, Zhou, T. (2005). On restricted connectivity and extra connectivity of hypercubes and folded hypercubes. *J. Shanghai Jiaotong Univ.* 10(2):203–207.

[23] Yang, C.-L, Masson, G. M. (1986). *Efficient Algorithm for Multiprocessor Fault Diagnosis Using the Comparison Approach.* Vienna, Austria: IEEE, pp. 238–243.

[24] Yang, X. (2003). *A Linear Time Fault Diagnosis Algorithm for Hypercube Multiprocessors under the MM* Comparison Model.* Xi'an, China: IEEE, pp. 50–55.

[25] Zhu, Q., Xu, J.-M, Lv, M. (2006). Edge fault tolerance analysis of a class of interconnection networks. *Appl. Math. Comput.* 172(1):111–121.