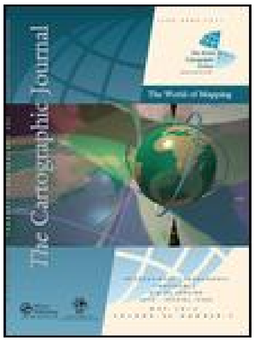

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REFEREED PAPER

Ambiguous Use of Geographical Information Systems for the Rectification of Large-Scale Geometric Maps

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ABSTRACT
Unlike modern maps, geometric maps lack a coordinate system and contain unsystematic geometric inaccuracies. This paper illuminates four aspects concerning the problem of uniting geographical information technology with old geometric maps. These are as follows: first, the origin of and geometric qualities in the representation of objects in geometric maps; second, the distortions originating from measurement techniques; third, the assumption that it is possible to find points that are the same over time for rectification in Geographic Information System (GIS); and, fourth, the extrapolation of unsystematic geometric distortions when using GIS techniques without any knowledge of the present unsystematic distortions in a map. The article presents the background of Swedish geometric maps and a hypothetical example is used to present the principle problems of using GIS techniques to rectify geometric maps. The conclusion of the paper is that systematic and unsystematic geometric distortions need to be identified and handled separately.

KEYWORDS
Large-scale geometric maps; old mapping techniques; unsystematic geometric distortions; geographic information systems (GIS)

Introduction

Historical geographers and landscape researchers in the twentieth century found historical maps to be useful for investigating and explaining the transformation of land use and rural landscapes during the last few centuries (Jongepier *et al*., 2016). The information that is found in historical maps is a very rich and useful source of information regarding former settlement structures and land use in past times. Older historical geographical studies (Helmfrid, 2007) often used these maps for retrogressive (Karsvall, 2013) studies of how settlements were structured before the maps were drawn. By instead adopting a retrospective perspective, finding out what remnants of old times exist in the contemporary landscape using modern maps and historical maps can be likened to separating the time layers of a landscape and it becomes easy to identify which elements in the contemporary landscape have historical origins in different from time periods (Statuto *et al*. 2017). The last decades have witnessed a development where these sources have been used more extensively by archaeologists and ecologists to determine the relationships among older settlement structures, land use activities, and the biodiversity that is produced in agricultural landscapes over time (Skånes, 1996; Cousins, 2001; Vuorela *et al*. 2001; Lagerås, 2013). This also means that the maps have been used in an increasingly spatially precise way over time (Shaffer and Levin 2015). Geographical information systems (GIS) have become the common way of rectifying, storing and analysing information from historical maps.

Overlay techniques using GIS have become the usual analysis methods for the historical dimensions of landscapes. The main assumption with most overlay analysis is that it is possible to demarcate real landscape changes from the geometric distortions that are inherent in historical maps. Most researchers are aware of the systematic geometric errors in old maps since they often lack coordinates and have angular deviations and scale differences. However, few are aware of how the unsystematic distortions that are inherent in most geometric maps can lead to the creation of new extrapolated ‘distortions’, which result from the regression and interpolation techniques that are most often used for rectification. How will it affect the end result and what are the possible errors that are added to change analysis? There is not yet a straight forward way to delineate real spatial changes from different types of geometric distortions using modern GIS tools. This is an obstacle if the rectified maps are used in change analysis at a detailed scale level and with a precise aim.

The problem of geometric distortions has different origins. The most important is how mapmaking was conducted and this changes over a long time span for which there are large-scale maps that are available in

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many countries. This is described later in this paper. In addition to knowledge about the inherent qualities and origins of maps, there is also the 'point-to-point problem' when combining historical maps with modern reference maps and coordinate systems in GIS. In an early study, Stone and Gemmell (1977: 8) concluded that

> [t]he very fact that of the selection of only those points which can be found on the modern map introduces bias, and perhaps the historical map is particularly inaccurate in respect of the points which cannot be located on the modern map.

In another study about assessing map qualities, Vuorela *et al.* (2001) point to the problem of appropriately selecting the ground control points while using GIS for the rectification of large-scale maps in Finland. They mention that the maps that were used contained 'nonlinearities', which were difficult to fully compensate. Nevertheless, they do not explain the origins of the nonlinearities behind the geometric problem or explain the shortcomings of the used technique. Instead, they just point to the fact that the oldest maps are the hardest to fit to the modern maps that are used as ground troughs.

This conclusion is similar to Jongepier *et al.* (2016), who discuss the problem and conclude that 'Local distortions may also play a large role. Maps proved to contain clusters of large displacement vectors, exceptional outliers (for instance more illustrative depictions of large cities), or higher accuracy for typical elements such as fortresses' (p.129). They also state that a qualitative assessment of the inherent geometric qualities and/or deepened knowledge about the historical production process of mapmaking is needed. There has recently become a growing interest in assessing the accuracy of historical maps in general, but these tests are most often built on the assumption that it is possible to identify points that remain the same over time (Jenny and Hurni, 2011; Vervust *et al.*, 2018).

This means that it is necessary to increase our knowledge both about the origin and character of geometric distortions and the consequences of the inherent preconditions of using GIS in the rectification processes. The possible extrapolation of the distortions using the techniques that are often used in the rectification process also needs to be clarified. Deepening the knowledge about both historical mapmaking and limitations with current GIS techniques is the main purpose of the research that is presented in this paper.

This paper consists of four parts. First, a brief historical overview of the first geometric mapping project lays the foundation for large-scale geometric mapping in Sweden. These geometric maps present different representation forms and have different anticipated accuracies. Second, an overview of the measurement techniques that were used in Sweden in the seventeenth to twentieth centuries is given. Third, the rectification in a geographic information system of six maps of the medieval town of Eksjö using existing street corners dating back to the oldest map is conducted and assessed. This case was selected because it is seldom the case that we know exactly the point-to-points, as in the unique example of the street corners of Eksjö. More often than not, the landscape has changed dramatically. Fields and fences have disappeared over the centuries, and the selection of control points needs to be done by approximation. Because of this principal problem, fourth, a hypothetical case simulating how the existence of unsystematic geometric distortions affects the rectified map is shown. These hypothetical maps show how the use of GIS techniques extrapolate unsystematic distortions instead of eliminating errors while conducting rectification using the most common methods. This means that the differences that never existed locally appear in the resulting rectified map.

Geometric mapping in Sweden since 1628

The first large-scale geometric mapping project in Sweden was initiated by King Gustav II Adolf in 1628 (Baigent, 1990). The king declared that all villages and towns should be mapped, and the maps should be sent to the castle in Stockholm. The king is explicit in the first instruction, saying that they should be so thorough that he would be able to see all of Sweden's villages and towns from them.

These first large-scale geometric maps came to include the majority of Sweden's villages and towns (Tollin, 2005). Whilst the original objectives are still unclear, a widely held opinion is that the maps were produced to provide a detailed register for taxation purposes (Lönborg, 1903; Johnsson, 1965). Others have dismissed this claim and instead promote the idea that it was only one part of an even-larger mapping project (Helmfrid, 1959). In a paper that was published in 1959, Helmfrid shows how taxation continued to be organized and collected through older praxis even *after* the maps had been produced. He argues that 'during an epoch which patronized science and a concern for the nations image abroad in the context of its new role as an important power, one need not look to purely practical goals in explaining something such as a modern mapping of the country' (Helmfrid, 1959).

This new visual and cartographic perspective was a probable reason for the founding of the National Board of Land Survey, as well as for the production of the geometrical maps (Wästfelt, 2007; Harley, 2009). The ambition can be associated with the development of a new way of both seeing and mapping the world and that the precise use to which the visual and cartographic elements of the maps would be directed was unknown at the time of their commissioning (Wästfelt, 2007). If one considers the growing land survey over a longer time perspective, it can be seen that Sweden was already one of the leading nations in the field of large-scale geometric cartography at the end of the seventeenth century (Helmfrid, 1959), while countries such as Spain had already produced a large number of cartographic maps by the late sixteenth century.

It is still unknown to what extent the first maps were actually used in contemporary times, but by the end of the seventeenth century, the next generation of younger geometric maps were used as legal documents in disputes over land. With the appropriation of the lands from the nobility during the 1680s, maps came to be used as a more effective reference material for demarcating units of land that were to be passed over to the Crown. The National Board of Land Surveys eventually grew into an organization that carried out a number of large land-partitioning reforms wherein maps would have been of significant instrumental and documentary importance.

When the first geometrical maps were produced in the seventeenth century, they were probably thought of as being sufficiently good representations to serve the kings for whom they were produced (Wästfelt, 2007). As mentioned earlier, such maps represented a new method of measuring and representing the country. A consequence of this is that, today, these maps must be understood not only as representations of landscapes and towns but also as commentaries over the time and the society that produced them. The map's function at this time was not just as a cartographic representation of remote places; they also acted as representational objects for belongings in the hands of the king (Jonsson 2001). Map-making techniques have varied over the centuries, as have their respective thematic foci. However, common to all periods has been the fact that map-making has been carried out as a conscious process with clearly defined frameworks that were consistently applied within any specific era. The measurement techniques that were used and the perceptions of the actual objects that were measured have changed considerably between the seventeenth and twenty-first centuries.

The development of geometrical mapping has its origin in the combination of visual representation forms and primarily in the development of the measurement table. Geometric mapping is, so to say, 'free floating' in relation to the coordinate grid, but it is grounded upon geometric and mathematical principles, making it possible to create detailed maps corresponding to the objects that are mapped. Interesting to note is that this development corresponds to the expansion of the use of 'central perspective', which maintained that one could visually describe reality from a freely chosen point in space. By representing the three-dimensional world in a two-dimensional form, *sight* assumes priority over our other senses. This has been conceptualized as a distancing [a 'stepping back'] of the self from the tactile and material world (Linde, 1999). Common to both geometric cartography and the 'central perspective' is a foundation built upon geometric principles (Lönborg, 1903). The geometric two-dimensional cartographic representations from above, in relation to reality, are just a part of these first maps. They often consist of buildings that are illustrated in central perspective (Figure 1), though other objects (such as streams and hillsides) were reproduced in a stereotyped fashion whilst other details that were considered to be of lesser importance, such as shorelines and distant fences in the forest, were often sketched onto maps by hand (Svärdsson, 1928). This can be seen in Figure 1. Note especially that the buildings and trees that are shown are illustrated in a central perspective. Note also the straight lines of fences, which were not measured in detail by the surveyor. The level of detail is higher in regard to the arable fields and their location and delineation, which were measured using the table.

An awareness of this practice is of central importance when analysing the spatiality of objects. It also raises the question as to the level of spatial accuracy that is possible in gauging an object's contemporary position based upon evidence from seventeenth century maps. For the areas of a map that have been produced using the measurement table and the following geometrical measurements, it is possible to identify divergences from the knowledge of how the map was produced and how these divergences arose from the flaws in the measurement techniques. If we consider the historical maps over a longer time perspective, it can be said that land surveys move progressively towards a more unified framework and praxis whereby all maps are based upon measurement alone.

Equipped with this knowledge, the earliest geometrical maps are flawed and contain several kinds of geometrical divergences that must be understood and rectified if they are to offer a source material of comparable quality to today's landscape in a GIS.

The geometrical divergences in the earliest geometrical maps can be split into three different groups:

- Differences in measurement techniques producing different geometric accuracies between different map series;
- Mixed forms of cartographic representation, measurement illustrations and stereotyped symbols; and
- Paper shrinkage variations due to paper type or quality.

Drawing styles changed over time, and in the oldest maps, lines have often been badly drawn. It was common praxis to join the various points that had been measured with straight lines, which gave rise to a generalization of the levels of detail.

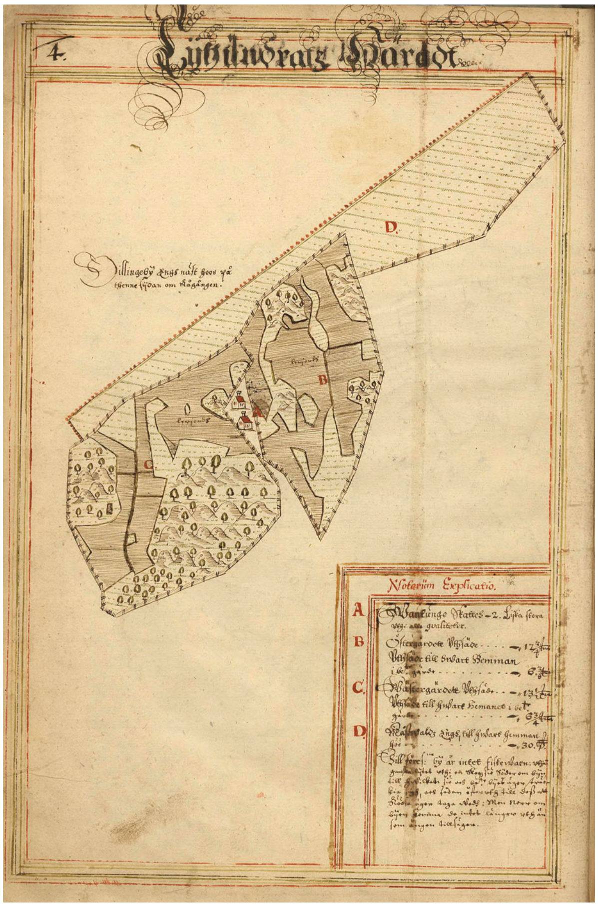

Figure 1. Geometric map of Vantunge hamlet, 1639. (A8:4 1639 Sven Månsson. Lantmäteristyrelsens arkiv).

Paper shrinkage is a source of geometrical divergence that affects different generations of maps differently, but the resulting distortions are most often smaller than other kinds of distortions. However, such errors tend to lie in a direction related to how the fibres of the paper lie.

In the next section of this paper, the main focus is on how the differences in measurement techniques created unsystematic errors. A case study is given for the town of Eksjö in which six historical maps are analysed in relation to each other to measure the changes in the geometrical precision between the maps. The ways in which the changes in the measurement techniques over time can explain the findings of errors between the six maps will be addressed.

Measurement techniques in Swedish large-scale geometric mapping between the seventeenth and twentieth centuries

The oldest geometrical maps can in some ways be likened to a jigsaw puzzle in that the pieces have been *forced* together despite the lack of a close fit. The errors affecting a map's geometrical component are of two kinds, systematic geometrical divergences (e.g. paper shrinkage, scale and orientation) and unsystematic geometrical divergences. Unsystematic divergences occur in an uneven fashion over a map's surface, which means that for some areas of the map they can be minor in proportion, whilst in other areas they can be considerable or different.

The cause of this problem is the two constituent elements of the measurement technique itself, both of which can affect the result:

- The measurement from the station points; and
- The construction of the connections between measurement points and/or the grid network.

The principles guiding survey procedures and the organization of station points have altered over time. This is one of the explanations for the variations in maps from different time periods. How, then, was measurement carried out in the seventeenth century?

In the seventeenth century, the technique that was used was called the double intersection. By located the same measurement points from two stations, it was possible to generate intersections that resulted in crossings marked with new points, which were then connected in such a way that they delineated the object, such as a field (Figure 2). However, from the latter part of the seventeenth century and throughout the eighteenth century, another technique was in use, namely, polar measurement. In contrast to the positioning of an object by way of intersections, this technique entailed the measurement of the angle and distance from station points. This change in approach probably contributed to the divergences that arose during the actual measurement procedures. The double intersection technique was subsequently reintroduced and became the standard technique throughout the nineteenth century (Svärdsson, 1928).

The second factor that affects divergence is the organization of the station points. In first period, when creating a new station point, the land surveyor used the double intersection. This meant that land surveyors always checked the new station location from at least two other positions. However, in the eighteenth century, it was standard practise to randomly place station points using polar measurement in the landscape, which were connected by only the length and angle measurements. This approach gave rise to an open grid net, which meant that the errors in angle and distance measurements could be exacerbated by transposing them from one area of the map to another. Such divergences become visible to the map reader between station points. A fixed grid net was first introduced in conjunction with the major land partitioning that began in 1757. This system involved

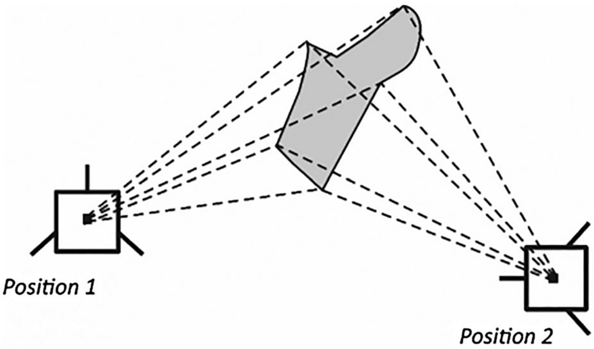

Figure 2. Double intersection. The crossing or meeting point of sightlines from two different positions of the measuring tables was the method originally used when making large-scale maps. Looking at the map, bot straight lines and more freely drawn curves was common way to connect the measured points (compare Figure 1).

positioning fixed station points on a known baseline, which succeeded in reducing errors between the various station points. This greatly assisted in improving the geometrical accuracy of the maps that were produced from the middle of the eighteenth century onwards (Svärdsson 1928).

Today's geodetic surveys, which use total stations, are, to a considerable extent, further developments of the techniques that were established in the seventeenth century. During the seventeenth century, just as the case today, mapping was done by positioning the measuring table at certain station points that were in known relation to each other. One major difference, however, is that currently one always measures the distance from the instrument/set-up point to the measured object itself.

The preceding discussion allows us to make several important observations. The geometrical divergences that can be found in the large-scale historical maps have arisen because of inaccuracies in station transfers. On a detailed level, errors have also occurred during the measurement of the object. Divergences resulting from poor correspondence between station points have been of the greatest importance. A consequence is that whilst the reproduction of an area surrounding one or several station points may be precise, this area in relation to the rest of the map may not be quite as precise. An amplification of the angle error could also be compounded by repeated transference in the open grid net, especially during the eighteenth century. Consequently, the various measurement methods that have been used mean that the geometrical digressions and errors vary not only within a specific map but also among different generations of maps. In seventeenth century maps, we find a good representation of arable land. In nineteenth century maps, the geometrical representation is accurate for the maps in their entirety. The largest geometrical divergences are found in the maps that were produced in the later part of the seventeenth century and during the early eighteenth century, a situation which, at least partly, is a direct consequence of the respective techniques that were used during these periods (see Table 1 for an summary of techniques).

Seen in its totality, it is rarely possible to trace the cause of a specific geometrical divergence. It is more often the case that the errors that we can now identify in maps and town plans have arisen from a combination of the various error types that were classified above.

Visualizing and quantifying geometric distortions in historical maps

Geometrical divergence has been the focus of recent cartographic research (Vervust *et al.*, 2018). Whilst this research has focused on a different scale level – cartographic small-scale maps – there are nevertheless certain parallels to the problems that are addressed in this discussion.

In the 1990s, Gustav Forstner developed a method for constructing what was called 'distortion grids', which was followed by Jenny and Hurni's development of MapAnalyst (2011) and Vervust's differential distortion analysis (2018). With the aid of a distortion grid, Jenny and Hurni (2011) show how it has become possible to visualize the extent of the compensation of systematic geometrical errors between two-point sets. Forstner contends that

Table 1. Summary of measurement techniques in Swedish land-surveying praxis between seventeenth and nineteenth century.

	Method	Organization of station points
Seventeenth century	Double intersection	Open polygon
Early eighteenth century	Polar measurements	Open polygons
Late eighteenth century	Polar measurement and double intersection	Fixed grid net
Nineteenth century	Double intersection	Fixed grid net

his method is, to a certain extent, subjective and dependent upon factors such as being able to identify corresponding points in both the historic map and the modern map (Forstner, 1998).

Important for this paper is that no one has yet solved the problem how to quantify and visualize the unsystematic distortions. To show how we can find the extent of unsystematic distortions in historical maps, a case with six maps for the town of Eksjö has been studied. In this unique case, it was possible to find a point-to-point correspondence between historical buildings and street corners in a complete series of maps dating back to 1697. It is unusual to have a case where the points can be identified and trusted to be exactly the same for over 300 years, which makes it possible to quantify the actual geometric distortions.

Rectification of historical maps for subsequent analysis of geometric distortions of six maps over the town of Eksjö

Rectifying large-scale geometric maps that cover villages in rural areas often means that very few, if any, points can be identified between the historical map and the reference map. In this part of the paper, the historical maps of Eksjö will therefore be presented and analysed.

The modern cadastral map from the planning office in the town of Eksjö, which exist both as a paper map and as a GIS file, is used as the reference map. This map exhibits a high degree of geometric accuracy (<0.01 m), which suggests that the divergences that have been identified derive, in principle, from the geometrical divergences in the series of historical maps.

To enable a closer examination of the geometrical properties of these maps, all of the maps have been rectified in ArcGIS on the basis of a point-to-point transformation of five known house corners (as identified by personal correspondence with town architect Lennart Grandelius). The regression model in ArcGIS calculates the standard deviation in relation to a modulated systematic transformation. This means that it is possible to obtain an indication of the size and location of the unsystematic geometrical divergences between the five points. All five of the house corners are situated in the central northern area of the town. This procedure was used to examine both the nature of the geometrical divergences between these five points and the ways in which the different maps relate to each other. These points are not optimally located because they are close to a straight line. The strength of this case is rather that the house corners are exactly the same.

The points are numbered from north to south, where 1 is situated furthest north and 5 furthest south. The table presents the root mean square errors (RMSEs) for each respective map and the residuals (divergences for the individual points) for each of the respective points that are used in the transformation. A close look at the RMS values (i.e. the average divergence for each respective map) reveals that the RMS values first increase and then decrease over time. This means that the geometrical divergences generally decrease the closer that one moves in time to today. In this case, where we know the points, it also shows that the oldest maps have geometrical divergences that are unsystematic, but as mentioned above, we do not know anything about the distortion between the analysed points. The values also show that the largest divergences are to be found in the map from 1798, and this finding corresponds with our knowledge that is presented about the changes in measurement techniques.

In summary, it can be stated that there are local geometric unsystematic distortions in the old large-scale maps and that these problems arise partly from the fact that the maps contain geometrical divergences that cannot be identified in advance. What cannot be seen from Table 2, however, is the degree to which the areas lying *between* and *surrounding* the five points have been affected by the transformation. To show this in principle, a hypothetical example has been constructed.

Hypothetical example showing the limitations with rectifying geometric maps containing unsystematic geometric distortions with GIS

The problem of information transference from historical maps into a GIS stems largely from the actual meeting of old source material and modern technology. The internal logic of the modern measurement technique and the geographical information technique both presume that errors that are detected in the map material are, as a rule, systematic in character and evenly distributed and that points that are stable over time in the different maps can be unproblematically located. The oldest geometrical maps are characterized, however, by geometrical variations that occur *within* each map generation and each individual map.

The vital question is about how one should go about identifying and isolating the geometrical divergences from actual spatial changes. The ideal solution would be to successfully compensate for the geometrical divergences whilst at the same time retaining the ‘real’ spatial changes between the points in time. These two aims, therefore, stand in opposition to each other.

Table 2. Table showing RMS and residual point in between the points used for rectification of the maps of Eksjö town.

Year	RMS	Residual point 1	2	3	4	5
1697	2.49	6.18	3.36	5.88	0.33	2.21
1798	3.61	4.58	6.15	1.08	0.59	0.47
1876	1.95	0.76	3.40	3.65	0.57	0.50
1922	0.94	0.9	0.39	1.79	0.02	0.53
1937	1.06	0.98	0.71	1.75	0.62	0.80
2000	0	0	0	0	0	0

Because we cannot travel back in time to re-visit landscapes from the past, and since it is very seldom the case as in Eksjö that we know the exact corners (points), there is no simple key to solving the problem. However, certain knowledge and a certain awareness can be utilized to prevent us from drawing incorrect conclusions.

As a way of simply illustrating the GIS techniques' shortcomings with respect to the rectification of unsystematic geometric distortions, a hypothetical example has been constructed and processed in ArcGIS and MapAnalyst whereby three different objects (an area, a line, and a group of points) are used to characterize the objects and geometry in a typical old historical village map. In such a map (compare Figure 1), it is common to find a field, a fence and groups of objects that are all geometrically correspondent over time, but contain unsystematic geometric distortions in between each other.

The objects (an area, a line, and three points) that are depicted in the two hypothetical 'maps' (Map A and Map B) in the example are identical (the three points are in the same location in Figure 3). However, the ways in which they are spatially positioned in relation to each other differ. It can thus be said that the objects have both 'position errors' and angle errors, or an unsystematic geometric distortion is distributed in between them, while still holding the correct internal geometry within each object. This is a typical error for the oldest historical maps and can suggest that although certain areal divisions exhibit a high degree of geometric correlation, they do not necessarily correlate with other areal divisions.

To illustrate this line of reasoning, map B has been rectified against map A. The result is a map that has been transformed to a map with incorrect objects on the basis of known pairs of identified points. As seen in Figure 4, there is no perfect match. The explanation for this is that the transformation calculations systematically compensate as much as possible based on three control points. The regression model that is used does not, however, compensate for the unsystematic geometrical divergences that occur in map B. This causes the remaining unsystematic deviations to be spread out over the image, which generates new spatial differences between the pictures.

GIS experts often propose to solve this problem by using more control points to improve the result. Figures 5 and 6 show the results using six control points. Figure 5 uses a first-order polynomial transformation and Figure 6 uses a second-order polynomial transformation. As seen in the Figures, the distortion of the area is now even worse than in Figure 4 with only three control points. On the other hand, the three points in the bottom left hand corner overlap perfectly and, in the second-order transformation, the point pairs match perfectly. What happens is that the relative distribution of the unsystematic geometric distortions becomes uneven in relation to the number and location of points and how they are distributed in the area.

In the rectification of map B, as was the case in Eksjö, the exact locations of the pairs of points correspond. This made it possible to measure and visualize the unsystematic errors. By following this procedure, the remaining divergences were able to be identified (Figures 5 and 6). Suppose that, in this example, map B was used for an area calculation, which, in this case, would inexorably lead to the addition of a foreign source that never actually existed between the maps in question. Furthermore, it is important to note that in many cases, it is permissible to accept such divergences. This is obviously dependent on the specific aims of each respective study.

The advantages of the described GIS methodology are that it is a quick and reliable approach for transforming maps that contain systematic geometrical divergences. There are, however, a number of disadvantages. First, one must reduce the objects or areal divisions into points. Second, it cannot compensate for unsystematic divergences. Third, because one is unaware of any eventual unsystematic divergences that the original map may contain, it is possible to implant erroneous spatial variations between the respective maps.

In Figure 7, a similar rectification as in Figure 6 has been carried out in MapAnalyst, now making it possible to display a distortion grid on top. The distortion grid in Figure 7 shows how the distortions between the six points with a first-order transformation have distorted the hypothetical map B to fit in relation to the reference map A. What is not seen between these two images is how the transformation affected the unsystematic geometry of the object. This result is similar to Figure 5, except that the compensated systematic distortion that is visible in the left of Figure 7 has been separated from the unsystematic geometric distortion that is visible in Figure 5. By using MapAnalyst, the systematic compensation between the pairs of points are visualized, but the unsystematic distortions are not shown.

Figure 3. Illustration of the hypothetical examples in overlay format. Reference map A on top and map B underlying in grey. The three points in the bottom left corner are located on top of each other.

Figure 4. Illustration of the hypothetical example after transformation based on three identified control points (1,2,3) in GIS software with an affine transformation. Note in particular how the boundaries of the area fail to correspond, despite the fact that they initially had identical shapes and dimensions. Also note that only one of the three points corresponds after the transformation compared to Figure 3. The points that are numbered are those that have been used in the transformation.

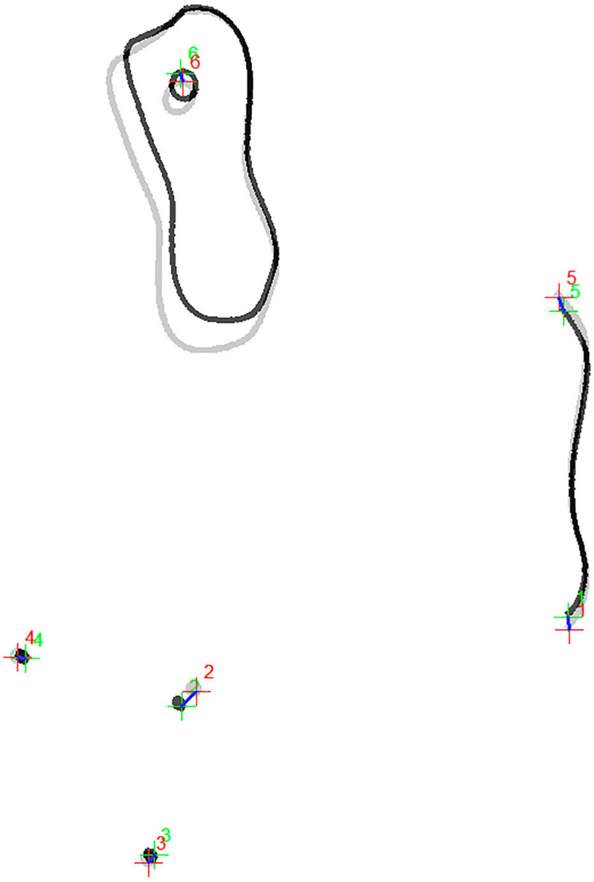

Figure 5. Illustration of the hypothetical example after first-order polynomial transformation (regression model) based on six identified control points (1,2,3,4,5,6) in GIS software. Note in particular how the area is distorted related to the non-systematic errors in between the control points. Also note that the group of three points corresponds relatively better than in Figure 4 after the transformation. The points that are numbered are those that have been used in the transformation.

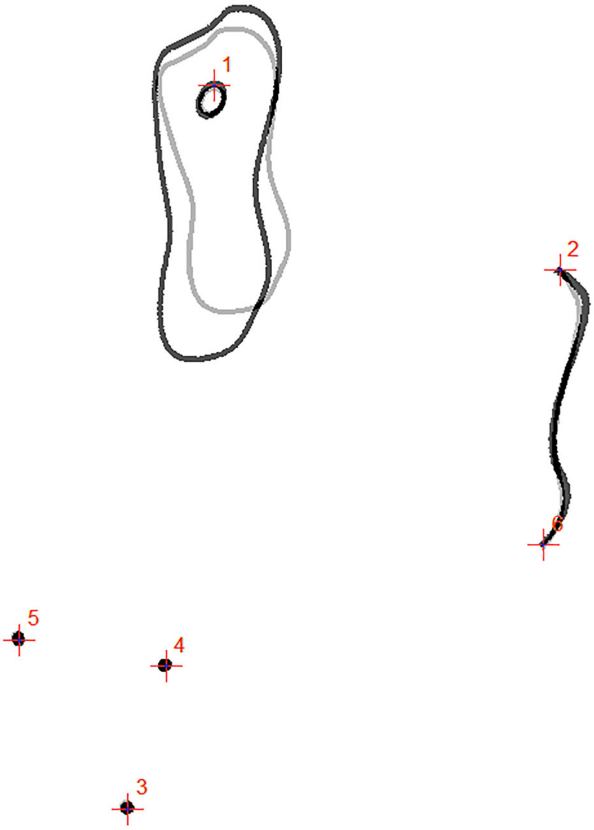

Figure 6. Second-order polynominal transformation (interpolation model) based on six points. Note that the points cover each other completely, and also that the area is now distorted to a larger degree than in Figure 5.

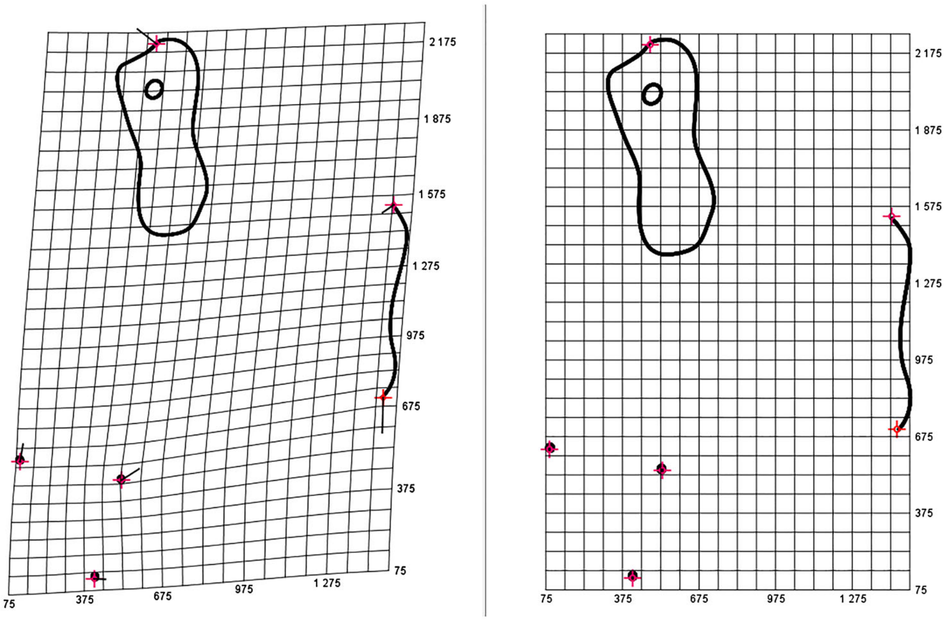

Figure 7. An illustration of rectification results within MapAnalyst with six control points, Map A after rectification to the left, Map B to the right. (the use method is the regression model, same as in Figure 5) and the application of a distortion grid on top.

The results show that MapAnalyst's distortion grids do not show the unsystematic errors. Instead, the unsystematic distortions will be extrapolated if they are not identified while using the common algorithms for rectification in GIS. To show this, the two maps A and B (Figure 7) have been rectified against each other based on the distortion grid in Figure 7. This step makes it possible to visualize the extrapolation on the inner geometry of the objects. The result in Figure 8 shows that distortion grids, such as that in MapAnalyst, only display the systematic compensation between the points but not the unsystematic distortions that exist in between the objects in the example.

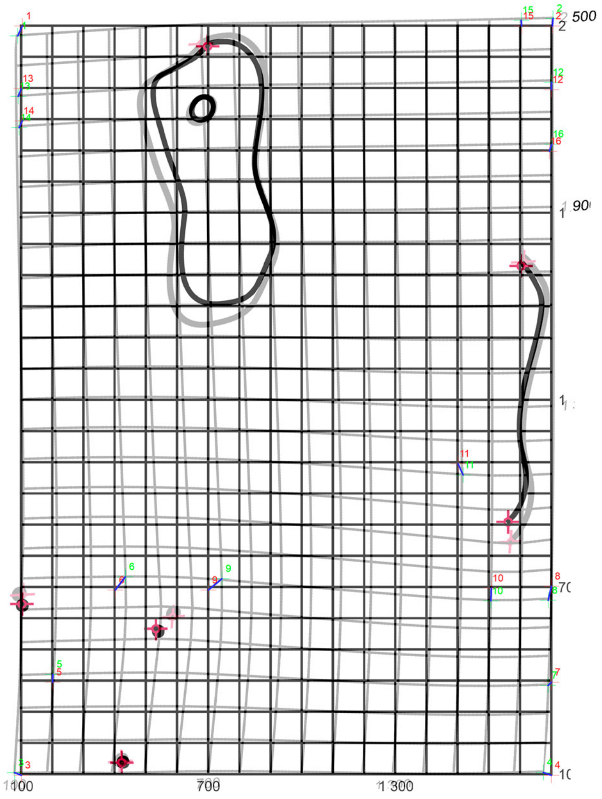

Figure 8. The two distortion grids in Figure 7 rectified against each other. This figure shows that distortion grid in MapAnalyst only display the systematic compensation between the points but not the extrapolated distortions in-between the control points which is seen here and which is similar to Figure 5. Figure 8 hold some smaller distortions between the two grid due to precision in point out the same points in GIS software.

Conclusion

The main conclusion of this paper is that geometrical errors can be distributed both *systematically* and *unsystematically* in geometric maps. The most commonly used method employed by today's geographical information technology for correcting geometrical errors is founded upon the premise that corresponding points can be identified and that the distortion can be either systematically redistributed, in between points, or become interpolated (Vervust *et al.*, 2018). A consequence of this is that an existing unsystematic geometric distortion can be extrapolated in the rectified maps. A discord exists then between the characteristics of the source materials (geometrical properties) on the one hand and the premises and methods that are used in modern geographical information technology on the other.

The identified unsystematic errors in the oldest Swedish geometric maps correspond to the errors coming with the different measurement techniques that are used. The measurement techniques changed over time and maps become more spatially accurate over time. The oldest geometric maps are combinations of different representation forms. The parts that are drawn based on measurement have relatively high geometrical precision due to the practice of double intersection by the land surveyors. However, eighteenth-century maps have lower accuracy, which corresponds to the introduction of the polar measurement methods instead of the double intersection method. This was shown to be correlated with the empirical findings in the rectification of the maps from the town of Eksjö.

The systematic and unsystematic geometric distortions in the oldest maps can be characterized as appearing in between islands of areas of relatively correct geometry. This phenomenon comes from the fact that distortions often appeared in between different station points in the mapping process.

The discord between GIS and geometrical maps from the seventeenth to nineteenth centuries is particularly problematic in the analysis of spatial change, which requires precise geometric accuracy. This paper finally argues that it is necessary to strive towards utilizing our capacity to use former knowledge and analysis of the unsystematic divergences in the oldest maps. Rectification needs to be achieved in a way in which systematic and unsystematic geometric distortions are identified and treated in the rectification process.

The final conclusion is that currently available GIS software does not allow for map transformations at the global (whole-map) level if one also wishes to retain the inner geometry of any local areas or objects that have geometries that differ from the map as a whole (*cf* Jongepier *et al.* 2016). A consequence of using contemporary GIS software in cases where maps contain unsystematic geometrical divergences is that the rectification will be ambiguous. Whilst the results may give the impression of a close correlation, the corrections that are made may also give rise to new and extended extrapolated spatial divergences in the rectified map.

Disclosure statement

No potential conflict of interest was reported by the author.

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Anders Wästfelt is Associate Professor and Lecturer in Human Geography at Stockholm University. Studies of agriculture landscapes and agriculture transformation processes have been his focus both in Sweden and abroad. He has developed methods for local contextual analysis of remotely sensed images and has a special interest in the history of cartography.

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