Financial Analysts Journal

ISSN: 0015-198X (Print) 1938-3312 (Online) Journal homepage: https://www.tandfonline.com/loi/ufaj20

Optimal Currency Hedging for International Equity Portfolios

Jacob Boudoukh, Matthew Richardson, Ashwin Thapar & Franklin Wang

To cite this article: Jacob Boudoukh, Matthew Richardson, Ashwin Thapar & Franklin Wang (2019) Optimal Currency Hedging for International Equity Portfolios, Financial Analysts Journal, 75:4, 65-83, DOI: 10.1080/0015198X.2019.1628556

To link to this article: https://doi.org/10.1080/0015198X.2019.1628556

Financial Analysts Journal | A Publication of **CFA Institute**
https://doi.org/10.1080/0015198X.2019.1628556

OPEN ACCESS

Research

Optimal Currency Hedging for International Equity Portfolios

Jacob Boudoukh, Matthew Richardson, Ashwin Thapar, and Franklin Wang

Jacob Boudoukh is professor of finance at the Arison School of Business, Interdisciplinary Center Herzliya, and a consultant at AQR Capital Management, Greenwich, CT. Matthew Richardson is the Charles E. Simon Professor of Applied Economics at the Leonard N. Stern School of Business, New York University, a research associate at the National Bureau of Economic Research, and a consultant at AQR Capital Management, Greenwich, CT. Ashwin Thapar is a managing director at AQR Capital Management, Greenwich, CT. Franklin Wang is a vice president at AQR Capital Management, Greenwich, CT.

This study explores optimal currency exposures in international equity portfolios through the lens of a modified mean–variance optimization framework. We decomposed the optimal currency portfolio into a "hedge portfolio" that uses a dynamic risk model to minimize equity volatility and an "alpha-seeking portfolio" based on the well-documented currency styles of value, momentum, fundamental momentum, and carry. This method is an integrated and economically intuitive approach to currency management that simultaneously provides lower risk and higher returns than either hedged or unhedged benchmarks. Crucially, the solution is practical, with realistic and implementable leverage, turnover, and tail-risk characteristics.

CE Credits: 1

The currency-hedging policies of international equity portfolios have been subject to much debate by academics and practitioners, yet a consensus has failed to emerge on an optimal approach.[1] Many investors choose simply to ignore the currency component of these portfolios—in an *eVestment* universe of 104 MSCI EAFE Index managers totaling $419 billion of assets, as of June 2018, only 18.3% pursued active currency management.[2] Perhaps this inaction stems from uncertainty about whether hedging can truly offset risk or from concerns that hedging programs that reduce risk may also create a drag on returns.

Although academics and theorists have pointed to mean–variance optimization (MVO) as a potential framework for combining risk-and-return considerations, its practical relevance has remained questionable because of its opacity and tendency to generate unrealistic portfolios with high leverage, turnover, and tail risk. We attempt to bridge the gap between theory and practice by presenting a "modified portfolio mean–variance optimization" (*MPMVO*) framework that is specifically adapted to improve the practical relevance of a mean–variance approach. The result of our work is a transparent and robust hedging solution, one that both lowers portfolio risk *and* raises realized returns, with manageable turnover, leverage, and tail properties.

MPMVO decomposes the MVO portfolio into three distinct components: (1) a pure equity piece, taken as given, (2) a "hedging" portfolio designed to minimize equity volatility, and (3) a currency "alpha" portfolio that is focused on generating high standalone risk-adjusted returns. The underlying theory determines the optimal mix

We would like to thank the Financial Analysts Journal *editors, Stephen J. Brown and Daniel Giamouridis, David Gallagher, and Shaun Fitzgibbons, Antti Ilmanen, Ronen Israel, Michael Katz, Thomas Maloney, Lasse Pedersen, Han Tian, Benjamin Weill, and two anonymous referees for valuable comments. We especially thank Thomas Lee for helpful comments and research assistance.*

of the two currency components that are added to the initial equity portfolio. We studied various approaches to hedging and alpha seeking and found that hedging is best achieved by using a full-blown *optimized* approach that takes advantage of differing abilities of currencies to hedge both equity risk and each other. As an example, an effective hedge determined by the optimizer consists of a short position in the Canadian dollar, offset by long positions in the US dollar—a currency that, in spite of being correlated with the Canadian dollar, has an opposite relationship with equity returns.

Conversely, alpha seeking is best achieved via what we term in this article "robust portfolio construction"—that is, using simple long–short currency portfolio strategies based on investment styles of carry, momentum, fundamental momentum, and value that have been well documented in the academic literature.[3] The benefit of *MPMVO* is that it allows the flexibility to pursue these differentiated approaches to hedging and alpha. As we demonstrate in the section "Currency Hedging: Theory," the methodological choice for the currency alpha portfolio did not interfere with the volatility reduction mandate of the hedging portfolio. Thus, we could take advantage of a rich and detailed information set about currency risk to reduce volatility but still avoid the major pitfall of standard MVO—namely, the interaction of low-signal-to-noise expected return estimates with highly correlated asset returns.[4]

Our empirical work focused on investors in G-10 countries holding an MSCI World Index ex-home-country portfolio over the period 1981–2017.[5] Whereas full hedging reduced international equity volatility from a cross-sectional average of 16.5% to an average of 14.9%, our optimized hedging portfolio yielded further improvements—to an average of 12.7%, a 23% reduction from unhedged equity volatility. However, this risk reduction came at the expense of returns. Perhaps unsurprisingly, the optimization attempted to offset equity risk by taking short positions in high-carry currencies that correlate positively with equity markets (but also have higher expected returns) with corresponding long positions in currencies with the opposite characteristics. This negative loading on carry, a known predictor of currency returns, highlights the importance of including expected return information in forming optimal currency portfolios.

MPMVO in this study sought to balance lower risk with higher returns, with the return part being achieved from an optimal allocation to an alpha portfolio based on well-documented styles, including carry. As a result, the persistent short carry exposure of the minimum-variance portfolio was undone while the value, momentum, and fundamental momentum positions, which are generally not, on average, correlated with equities, were placed on top of existing currency hedges. The resulting portfolio still had lower average volatility than either the unhedged or fully hedged portfolios (14.6% versus 16.5% and 14.9%). But with the addition of the alpha portfolio, the Sharpe ratio almost doubled—from 0.38 to 0.68. In effect, the *MPMVO* framework judiciously chose to hedge international equities with currencies that did not offer expected return opportunities and took diversifying positions in currencies that did. An important aspect is that the technique also incorporates implementation considerations, including leverage, turnover, and tail risk, so the theoretical results are practically relevant.

Of particular interest is that we document a novel empirical result—namely, that currency-optimized international equity portfolios are highly similar for all G-10 investors. This result is intuitive; the equity component of international portfolios is similar across most home countries, so given the same set of currencies at their disposal, G-10 investors should arrive at nearly identical optimal solutions. In other words, findings of different hedge ratios for different home countries relate purely to starting (unhedged) currency exposures being different or constraints applied relative to those starting points (e.g., a constant hedge ratio for all foreign currencies). In the absence of these constraints, desired outcomes converge. To this end, we also introduce a single optimal global equity portfolio that is identical for all global investors.

Currency Hedging: Theory

The general mean–variance problem, as applied to hedging, was solved by Anderson and Danthine (1981), with specific application for currencies appearing in, among others, Glen and Jorion (1993), Jorion (1994), Gagnon, Lypny, and McCurdy (1998), Ang and Bekaert (2002), De Roon, Nijman, and Werker (2003), Campbell, Serfaty-de Medeiros, and Viceira (2010), Schmittmann (2010), Topaloglou, Vladimirou, and Zenios (2011), Opie and Dark (2015), Christensen and Varneskov (2018), and Opie and Riddiough (2019). The practical relevance of full-blown mean–variance analysis is limited, however, because of both its opacity and its tendency to produce unintuitive portfolios as a result of overfitting of inputs that are inherently uncertain and ignoring investor preferences beyond mean and volatility.

In this section, we provide a step-by-step overview of the mean–variance optimal portfolio with a novel decomposition into separate components focused on hedging and alpha generation. This decomposition provides investors with the tools to address the aforementioned challenges associated with MVO.

Without loss of generality, consider a US investor holding a portfolio of unhedged world equities, denoted *EQU*, with return $\mathbf{w}_t'\mathbf{R}_{t+1}^{\$}$, where $\mathbf{R}_{t+1}^{\$}$ is the $N \times 1$ vector of dollar returns on world equities and $\mathbf{w}_t$ is the $N \times 1$ vector of period t portfolio weights. A typical question asked in this case is whether the investor should hedge some fraction of the currency exposure of this portfolio. The expected return on the *EQU* portfolio can be written as

$$E_t\left[R_{pt+1}^{\$}\right] \equiv \mathbf{w}_t' E_t\left[\mathbf{R}_{t+1}^{\$}\right] \approx \mathbf{w}_t' E_t\left[\mathbf{R}_{t+1}^{*}\right] + \mathbf{w}_t' E_t\left[\mathbf{R}_{t+1}^{FX}\right], \quad (1)$$

where $\mathbf{R}_{t+1}^{*}$ is the $N \times 1$ vector of world equity returns in the foreign currency and $\mathbf{R}_{t+1}^{FX}$ is the $N \times 1$ vector of exchange rate returns.[6] The US investor is implicitly holding two portfolios—a currency-hedged equity portfolio, *EQ*, with return $\mathbf{w}_t'\mathbf{R}_{t+1}^{*}$, and a basket of foreign currencies, *B*, with return $\mathbf{w}_t'\mathbf{R}_{t+1}^{FX}$—that is, *EQU* = *EQ* + *B*. By shorting the basket of foreign currencies through borrowing at the short-term foreign interest rate, R_{ft}^{*}, and lending at the US interest rate, $R_{ft}^{\$}$, US investors can effectively rid themselves of direct exposure to exchange rate risk and earn the hedged return stream of *EQ*.[7]

Let the return to any currency portfolio *s* be $\mathbf{w}_{s,t}'\mathbf{R}_{t+1}^{FX}$. In addition to the previously defined choices of unhedged equity (*EQU*) or hedged equity with no direct currency exposure (*EQ*), the investor may also choose currency weights w_s to form any portfolio *P* with return $R_P = R^{EQ} + \mathbf{w}_s'\mathbf{R}^{FX}$. A natural objective in choosing w_s would be to maximize expected returns of R_P for a given level of volatility risk. In the following material, we show that the desired currency portfolio can be decomposed into the variance-minimizing currency portfolio, *FXHEDGE*, with weights $\mathbf{w}_{hedge}$, and a second currency portfolio, *FXALPHA*, with weights $\mathbf{w}_{alpha}$ chosen to have a maximal standalone Sharpe ratio.

To show this decomposition, we first define excess equity returns, $r_{t+1} \equiv R_{t+1} - R_{ft}$, and excess currency returns, $r_{t+1}^{FX} \equiv \mathbf{R}_{t+1}^{FX} + \left(\mathbf{R}_{ft}^{*} - \mathbf{R}_{ft}^{\$}\right)$. Additionally, we denote α_{EQ} as the expected excess return for hedged equities; $\boldsymbol{\alpha}_{FX}$ as the $N \times 1$ vector of currency expected excess returns; $\boldsymbol{\Sigma}_{FX}$ as the $N \times N$ variance–covariance matrix of currency excess returns, var[r^{FX}]; σ_{EQ} as the volatility of hedged equity excess returns; and $\boldsymbol{\Sigma}_{EQFX}$ as the $N \times 1$ covariance vector between r^{EQ} and $\mathbf{r}^{FX}$. For ease of notation, we have dropped here any time subscripts, but in application, these parameters could, of course, be time varying. (The proofs of the results are provided in Appendix A.)

We solve for the weights, w_s, that maximize the portfolio *P* expected excess return, α_P, for a given level of risk (i.e., variance)—that is, the weights that place the portfolio on the mean–variance frontier:

$$MAX_{w_s}\, \alpha_P = \alpha_{EQ} + \mathbf{w}_s'\boldsymbol{\alpha}_{FX} \quad (2)$$

subject to

$$\mathbf{w}_s'\boldsymbol{\Sigma}_{FX}\mathbf{w}_s + \sigma_{EQ}^2 + 2\mathbf{w}_s'\boldsymbol{\Sigma}_{EQFX} = \sigma_P^2.$$

First, the currency portfolio resulting in the maximal portfolio Sharpe ratio combines hedging currency demand, $\mathbf{w}_{hedge}$, with alpha-seeking currency demand, $\mathbf{w}_{alpha}$:

$$\mathbf{w}_{opt} = \boldsymbol{\Sigma}_{FX}^{-1}\left(-\boldsymbol{\Sigma}_{EQFX} + k\boldsymbol{\alpha}_{FX}\right), \quad (3)$$

↙ Hedging: $\mathbf{w}_{hedge}$ ↘ Alpha seeking: $\mathbf{w}_{alpha}$

where

$$k = \sqrt{\frac{\sigma_P^2 - \left(\sigma_{EQ}^2 - \boldsymbol{\Sigma}_{EQFX}'\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\Sigma}_{EQFX}\right)}{\boldsymbol{\alpha}_{FX}'\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\alpha}_{FX}}} = \sqrt{\frac{\sigma_P^2 - \sigma_{MINVAR}^2}{\boldsymbol{\alpha}_{FX}'\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\alpha}_{FX}}},$$

where the hedging currency portfolio, *FXHEDGE*, with weights $\mathbf{w}_{hedge} = -\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\Sigma}_{EQFX}$, combined with the equity portfolio, *EQ*, is the minimum-variance portfolio (denoted *MINVAR*). (See Proof A in Appendix A.)

Second, the *MINVAR* portfolio has mean and variance of returns as follows: $\alpha_{MINVAR} = \alpha_{EQ} - \boldsymbol{\alpha}_{FX}'\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\Sigma}_{EQFX}$ and $\sigma_{MINVAR}^2 = \sigma_{EQ}^2 - \boldsymbol{\Sigma}_{EQFX}'\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\Sigma}_{EQFX}$. Note that the *MINVAR* portfolio is uncorrelated with any standalone FX (foreign exchange) portfolio. (See Proof B in Appendix A.)

Third, the alpha-seeking currency portfolio, *FXALPHA*, with weights $\mathbf{w}_{alpha} = \boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\alpha}_{FX}$, is the standalone mean–variance optimal currency portfolio with both the mean and variance of returns equal to the same value, $\boldsymbol{\alpha}_{FX}'\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\alpha}_{FX}$.

Fourth, $\mathbf{w}_{opt}$ is the combined currency portfolio that maximizes the Sharpe ratio of portfolio returns $R_P = R_{EQ} + \mathbf{w}_{opt}'\mathbf{R}_{FX}$. It places relative weight $k_{opt} = \sigma_{MINVAR}^2/\alpha_{MINVAR}$ on *FXALPHA* such that $\mathbf{w}_{opt} = \mathbf{w}_{hedge} + k_{opt}\mathbf{w}_{alpha}$. (See Proof C in Appendix A.) Therefore, combined optimal portfolio *MPMVO* results from combining the currency portfolio with hedged equity—that is, $MPMVO = EQ + FXHEDGE + k_{opt}FXALPHA$.

We illustrate these portfolios in **Figure 1**. The investor starts with the hedged equity portfolio represented by point *EQ* in Figure 1 with currency position $w_s = 0$. Although, by design, this portfolio has no explicit currency exposure, it does not necessarily have the lowest volatility, because some currencies might have the potential to provide a natural hedge for the equity. The investor may add a currency-hedging portfolio, *FXHEDGE*, with positions $\mathbf{w}_{hedge} = -\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\Sigma}_{EQFX}$, to form the *ex ante* minimum-variance portfolio $MINVAR = EQ + FXHEDGE$, which lies on the mean–variance frontier. An investor focused on Sharpe ratios, however, can do better than *MINVAR* by accepting higher expected volatility for greater expected returns. The combined currency portfolio required to do so would equal the *FXHEDGE* portfolio plus an optimal weight of k_{opt} on the *FXALPHA* portfolio $\left(\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\alpha}_{FX}\right)$, where $k_{opt} = \sigma_{MINVAR}^2/\alpha_{MINVAR}$.[8] This point, labeled *MPMVO* in Figure 1, is the point on the mean–variance frontier where the tangency line (the solid green line), emanating from the origin, intersects the mean–variance frontier, and this point places optimal weight on the *FXALPHA* portfolio.

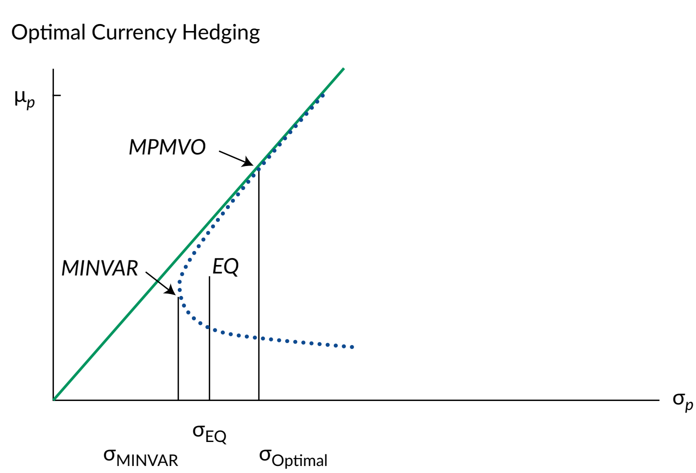
The resulting decomposition, $MPMVO = EQ + FXHEDGE + k_{opt}FXALPHA$, cleanly separates *MPMVO* into portfolios with distinct objectives: *FXHEDGE* minimizes variance in the context of equities (i.e., $EQ + FXHEDGE = MINVAR$), and k_{opt} is an optimal weight on an *FXALPHA* portfolio with a high standalone Sharpe ratio.

MPMVO is differentiated from traditional mean–variance optimization in at least two ways. First, the decomposition reduces opacity by giving the investor insight into *why* the optimal portfolio holds the positions that it does (e.g., whether currency positions are motivated by hedging or alpha seeking). Second, each portfolio has a clearly defined objective that provides the investor with the flexibility to use any hedging or alpha portfolio that works best in practice. In particular, the investor may choose approaches that involve fitting fewer parameters or include additional preferences on such dimensions as leverage, turnover, skewness, and kurtosis.

Therefore, in the remainder of the article, we evaluate the trade-off in out-of-sample performance between two implementation approaches: (1) estimating all risk and return parameters specified in this section (which we denote the *Optimized* portfolio) and (2) estimating only a few key parameters (which we denote the *Robust* portfolio). We do the evaluation separately for *FXHEDGE* and *FXALPHA*. In doing so, we show outperformance of the *Optimized* approach to *FXHEDGE* and, conversely, outperformance of the *Robust* approach to *FXALPHA*.

The decomposition of *MPMVO* into *EQ*, *FXHEDGE*, and k_{opt}FXALPHA explicitly gives the investor the

Figure 1. Optimal Currency Hedging



flexibility to implement these different approaches and then recombine the portfolios to produce an implementable portfolio with high risk-adjusted returns and other desirable characteristics, which we discuss in later sections.

Data

All the analysis that follows was conducted on weekly data from the period between January 1981 and December 2017 for all G-10 currencies: the Australia dollar (AUD), Canadian dollar (CAD), euro (EUR),[9] Japanese yen (JPY), Norwegian krone (NOK), New Zealand dollar (NZD), Swedish krona (SEK), Swiss franc (CHF), British pound (GBP), and US dollar (USD). Equity returns were computed from country index returns from Datastream. We present results for *International* portfolios for each home country, weighted by market capitalizations over the MSCI World ex-home-country universe, and we present a single *Global* portfolio that is weighted by market caps over the full MSCI World universe.[10] Excess currency returns were computed by using spot exchange rates from the Bank of England and three-month LIBOR rates from Bloomberg, supplemented by three-month T-bill returns on short-term government debt from Global Financial Data.

For both *International* and *Global* portfolios, equity returns have been transformed to hedged equity returns through shorting the currency basket at MSCI market-cap weights. All returns are in excess of cash and gross of fees and transaction costs.

Table 1 provides the annualized means and volatilities of the excess return on an equally weighted basket of G-10 currencies and the *Global* equity index. Table 1 also provides the minimum and maximum correlations between currencies and correlation with the hedged *Global* equity index (results for hedged *International* equity indexes across home countries are highly similar). The minimum and maximum correlations show a distinctive and rich cross-sectional pattern—namely, positive correlations within regions (such as Europe, Australasia, or the Americas) and low or negative correlations elsewhere. In addition, the correlations diverge meaningfully in a range of −0.3 to 0.3. Australia and New Zealand have the currencies with the highest excess returns (respectively, 1.5% and 2.8%); Japan and the euro have produced the lowest returns (respectively, −1.1% and −1.3%). Intuitively, we see a tendency for the higher-return currencies to have higher correlations with the market. Note the important aspect that, although the realized excess currency returns seem second order compared with the *Global* equity mean return of 6.0%, the currencies have significant volatility, averaging 7.3%, roughly half that of *Global* equity.

Minimum-Variance Currency Hedging

As described in the first section, as an alternative to holding an unhedged equity portfolio (*EQU*) or a fully hedged equity portfolio (*EQ*), given an $N \times N$ covariance matrix of currency returns ($\mathbf{\Sigma}_{FX}$) and an

Table 1. Currency Return Summary Statistics, 1981–2017

Currency	Mean Return	Volatility	Minimum Correlation with Other Currency	Maximum Correlation with Other Currency	Correlation with *Global EQ*
AUD	1.5%	8.5%	−0.50 (EUR)	0.46 (NZD)	0.29
EUR	−1.3	5.4	−0.50 (CAD)	0.58 (CHF)	−0.17
CAD	0.1	6.9	−0.50 (EUR, CHF)	0.45 (USD)	0.21
JPY	−1.1	9.5	−0.31 (AUD)	0.16 (USD)	−0.26
NOK	0.0	6.0	−0.41 (USD)	0.39 (EUR)	0.08
NZD	2.8	9.2	−0.45 (EUR)	0.46 (AUD)	0.21
SEK	−0.6	6.5	−0.33 (USD)	0.37 (NOK)	0.09
CHF	−0.4	7.6	−0.50 (CAD)	0.58 (EUR)	−0.29
GBP	−0.4	6.8	−0.26 (AUD)	0.15 (EUR)	−0.03
USD	−0.7	7.0	−0.41 (NOK)	0.45 (CAD)	−0.17
Global EQ	6.0	14.6	−0.29	0.29	1.00

$N \times 1$ vector of covariance between currencies and the *International*/*Global* equity basket ($\mathbf{\Sigma}_{EQFX}$), an investor may choose the minimum-variance portfolio (*MINVAR)* by choosing a custom currency portfolio (*FXHEDGE*) with weights $\mathbf{w}_{hedge} = -\mathbf{\Sigma}_{FX}^{-1}\mathbf{\Sigma}_{EQFX}$. In order to apply dynamic estimates of the covariance matrixes, we used an exponentially weighted dynamic risk model with a 130-day center of mass estimated on weekly returns overlapping daily, with 25% correlation shrinkage.[11]

Here, we provide our empirical evaluation of the two approaches to implementing *FXHEDGE* in practice. Recall that in the *Optimized* approach, all elements of $\mathbf{\Sigma}_{FX}$ and $\mathbf{\Sigma}_{EQFX}$ are estimated to form a hedging portfolio (which may include both long and short positions), but the *Robust* approach uses a single *EQ* market-cap-weighted basket of foreign currencies (so it requires only a single currency volatility and correlation estimate). Many approaches could be applied to *Robust* hedging, but ours has the economic interpretation of an investor choosing a single (time-varying) hedge ratio to apply to all foreign currencies. In other words, the *Robust* hedging formula simplifies to $\mathbf{w}_{hedge} = hB$, where $h = -\rho_{EQ,FXB}\left(\sigma_{EQ}/\sigma_{FXB}\right)$, the negated beta in a regression of the hedged equity basket on the foreign currency basket.

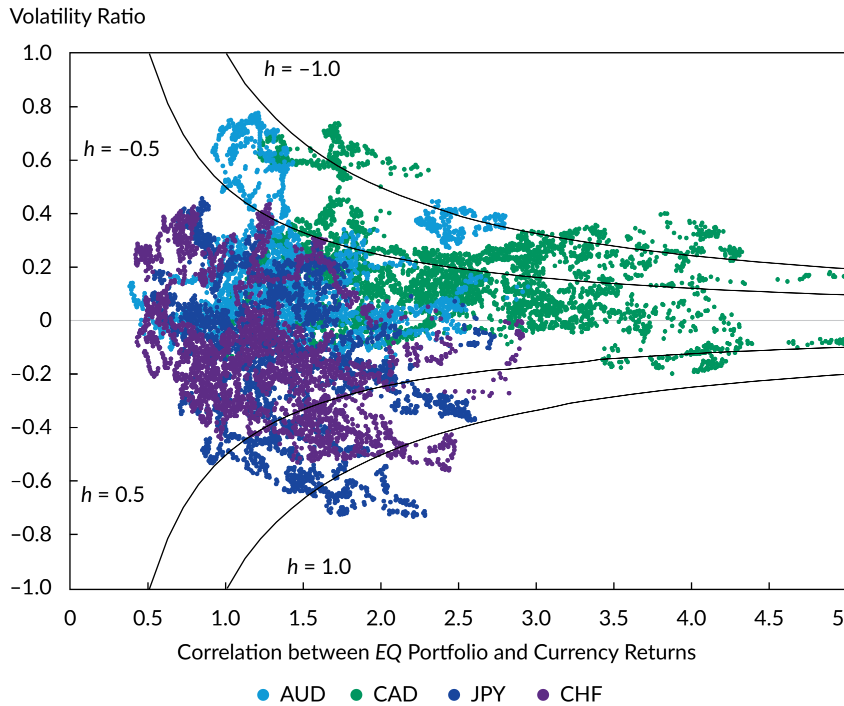
Both hedging approaches exploit the key ingredient of the correlation between currencies and equities, but the *Optimized* approach has more flexibility to exploit different correlation properties of different currencies with equities and between themselves.[12] The *Optimized* portfolio must dominate *ex ante*, but whether it will outperform out-of-sample when many more parameters must be estimated is an empirical question. The relevant analysis follows.

First, to develop some intuition for how the correlation between currencies and equities translates to optimal hedging, consider the aforementioned one-currency formula of $h = -\rho\left(\sigma_R/\sigma_{R_B}\right)$. For a correlation of zero, a variance-minimizing investor has no desire for currency exposure (i.e., prefers to be fully hedged) because any currency risk simply adds noise (volatility) to his or her portfolio. If correlations are positive, the investor wants to be net short (more than fully hedged) the currency risk; a negative correlation justifies long currency risk. To justify a fully unhedged position, one must observe a beta of the hedged equity basket to the currency basket at least as negative as –1. **Figure 2** depicts the month-by-month optimal currency exposure for four individual foreign currencies—measured by $h = -\rho_{EQ,FXB}(\sigma_{EQ}/\sigma_{FXB})$ standalone for each currency—from the perspective of a US investor over the 1981–2017 sample period.

Note, first, that most positions deviate meaningfully from the $h = 0$ line, indicating that investors may be able to reduce risk by taking some currency exposure. Second, for each currency, the optimal level of exposure varies significantly over time, which supports the case for an *active* hedging approach. Third, patterns vary meaningfully by currency. Optimally, the investor generally shorts the Australian and Canadian dollars for most of the sample period, which is consistent with these currencies having positive unconditional correlations with equities (shown in Table 1). At the same time, positive Japanese yen and Swiss franc exposure is justified, which is consistent with their negative correlations. Figure 2, consistent with the rich cross-sectional pattern of equity exposures for various foreign currencies depicted in Table 1, supports the full flexibility of *Optimized* hedging.

Table 2 shows our evaluation of the volatility-reducing efficacy of the two approaches compared with benchmarks of unhedged equities (*EQU*) and hedged equities (*EQ*). We report volatility individually and on average for *International* (MSCI World ex-home-country) portfolios from the perspective of each G-10 home country and results for a *Global* portfolio (home country agnostic). To start, the first two columns illustrate consistent and compelling evidence for the volatility-reducing benefits of simply full hedging. With it, volatility fell for 9 out of 10 foreign currencies, with an average percentage reduction of 10%, from 16.5% to 14.9%. Next, the column titled "*MINVAR* (*Robust*)" reports the results for our approach that dynamically varied desired currency exposure but used a fixed basket of currencies. Compared with a fully hedged portfolio, active basket hedging led to a further volatility reduction for all G-10 investors, with average volatility dropping from 14.9% to 14.3%. This finding supports the potential benefit of targeting an optimal hedge ratio by using time-varying correlation. "*MINVAR* (*Optimized*)" reports results for the fully flexible approach that varied desired exposure for each currency in each time period. Volatility reduction is still stronger, with significant drops for all G-10 investors. The reduced amount averages 12.7%, equaling a percentage reduction, respectively, of 23%, 15%, and 11% relative to the benchmarks of unhedged, fully hedged, and *Robust* hedging.

Figure 2. Standalone Optimal Hedge Ratios for Selected Currencies over Time, 1981–2017



Notes: Hedging the MSCI World equity portfolio (ex US) from the perspective of a US investor. The currencies are the Australian dollar, Canadian dollar, Japanese yen, and Swiss franc. A fully unhedged position corresponds to h = 1; a full hedge is h = 0. The contour lines represent the combination of correlation r and volatility ratio σ_R/σ_S that matches the specific value of currency exposure h.

Table 2. Volatility of Portfolio Returns with Various Currency-Hedging Approaches, 1981–2017

Portfolio	*EQU*	*EQ*	*MINVAR (Robust)*	*MINVAR (Optimized)*
International (Australia)	15.2%	14.9%	13.7%	12.7%
International (Eurozone)	16.5	14.6	14.3	12.6
International (Canada)	14.2	14.9	13.8	12.7
International (Japan)	19.6	15.2	14.6	13.1
International (Norway)	15.6	14.8	14.3	12.6
International (New Zealand)	16.2	14.8	14.0	12.6
International (Sweden)	15.7	14.8	14.2	12.6
International (Switzerland)	18.3	14.9	14.6	12.7
International (United Kingdom)	16.5	15.0	14.8	12.9
International (United States)	17.6	15.5	15.1	13.1
International average	16.5	14.9	14.3	12.7
Global		14.7		12.4

Notes: The four hedging scenarios are unhedged (*EQU*), fully hedged (*EQ*), dynamic hedging using a *Robust* basket approach [*MINVAR* (*Robust*)], and dynamic hedging using a fully flexible approach [*MINVAR* (*Optimized*)]. All correlation estimates were shrunk by 25%.

Interestingly, the *MINVAR* (*Optimized*) volatility, ranging from 12.6% to 13.1%, is almost identical across G-10 home countries. This result is not by accident. The optimal currency position, $\mathbf{w}_{hedge} = -\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\Sigma}_{EQFX}$, differs only to the extent that each investor faces a different *International* equity portfolio (each excluding the investor's own market). Because most home countries represent a small weight in the global portfolio, *International* equity portfolios have highly similar compositions. Optimal hedging is also similar for *International* equity portfolios across home countries, both in construction and performance. To this end, Table 2 shows results for a *Global* (MSCI World) equity portfolio that included market-cap weights in all developed equity indexes, fully currency hedged as a starting point and then paired with optimal global currency exposures. This portfolio is identical for all investors regardless of home country.[13]

Currency Alpha Seeking

In this section, we explore alternative approaches to creating *FXALPHA*, a currency portfolio using expected return information from well-known investment styles to seek a high standalone Sharpe ratio. We started by evaluating the historical efficacy and interactions of these styles, and consistent with our analysis in the previous section, we also evaluated the trade-off between an *Optimized* approach and a *Robust* approach. The *Optimized* portfolio again used estimates for all risk and return parameters to form an *ex ante* optimal portfolio, and the *Robust* portfolio relied on fewer parameters. Recall from the section "Currency Hedging: Theory" that because all standalone currency portfolio returns are *ex ante* uncorrelated with *MINVAR*, the choice of *FXALPHA* is independent of *MINVAR*. Therefore, in spite of preferring an *Optimized* approach to *MINVAR*, we retained full flexibility in choosing our approach to *FXALPHA*. In this section, we evaluate our two options from the perspective of out-of-sample risk-adjusted returns and additional desirable characteristics.

We start by describing the methodology for the *Robust* and *Optimized* approaches. The *Robust* approach to forming *FXALPHA* ranks assets on the basis of carry, momentum, fundamental momentum, and value factors.[14] Style portfolios were built directly with weights proportional to these ranks—an approach consistent with the academic literature. Conversely, the *Optimized* approach took raw factor values as conditional expected returns ($\boldsymbol{\alpha}_{FX}$) for each currency and solved for the optimal *FXALPHA* portfolio, $\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\alpha}_{FX}$, by using the dynamic covariance matrix defined in the section "Minimum-Variance Currency Hedging."[15]

Table 3 documents relative performance of the two approaches. For ease of comparison, we used the same covariance matrix to scale both the *Robust* and *Optimized* portfolios to target 10% *ex ante* volatility in each period.[16] Although the *Optimized* approach may be expected to outperform because of an ability to directly use covariance matrix information, a concern is that optimization, in attempting to take advantage of small differences in noisy α_{FX} estimates for highly correlated currency pairs, may produce unstable and highly levered portfolios. To evaluate this trade-off, we looked at results for risk to reward (Sharpe and Sortino ratios), other dimensions of risk (skew, maximal drawdown, and correlation with global equities), and portfolio characteristics (leverage and turnover). Note that currency alpha seeking is independent of home country and the *Global* versus *International* distinction, implying a single *Robust* portfolio and a single *Optimized* portfolio.

Table 3. Currency Alpha Seeking, 1981–2017

Factor/Portfolio	Mean Return	Volatility	Sharpe Ratio	Sortino Ratio	Skew	Maximum Drawdown	Correlation with *EQ*	Leverage	Turnover
Carry	5.5%	10.6%	0.51	0.76	−1.7	−35%	0.25	1.4	2.3
Momentum	3.7	11.0	0.34	0.54	−0.4	−36%	−0.05	1.4	6.9
Value	3.8	10.6	0.36	0.66	0.3	−38%	−0.07	1.5	3.8
Fundamental momentum	7.2	10.4	0.69	1.25	3.2	−31%	−0.04	1.6	10.2
Robust full model	9.2	10.5	0.88	1.43	−0.4	−25%	0.04	1.5	8.5
Optimized full model	8.8	10.5	0.84	1.34	−1.8	−33%	0.03	2.3	10.7

Note: Both leverage and turnover are one sided in a long–short portfolio.

For brevity, we show in Table 3 results for just the *Robust* approach at the single-style level but results for both the *Robust* and *Optimized* approaches for the multistyle portfolio. The style portfolios each generated meaningful excess returns (i.e., Sharpe ratios between 0.34 and 0.69) over the long term. Although correlations tended to be low across styles, two exceptions (not reported in the table) are a meaningful negative correlation between value and momentum (i.e., –0.40) and a positive correlation between momentum and fundamental momentum (i.e., 0.33). As expected, because of the benefit of diversification, the multistyle portfolio, which put equal weights on each style, outperformed individual styles. Its Sharpe ratio of 0.88 is higher than even the maximum among individual styles, and it achieved better characteristics from the perspective of the Sortino ratio or maximum drawdown. Comparing *Robust* and *Optimized*, we see that the *Optimized* approach consistently underperformed across dimensions. The Sharpe and Sortino ratios are slightly worse (Sharpe: 0.84 versus 0.88; Sortino: 1.34 versus 1.43), and maximum drawdowns are greater (–33% versus –25%). More meaningful are the data for leverage and turnover. For the *Optimized* approach, they are 53% and 26% higher, respectively, further reducing the practical feasibility of this approach. The conclusion is that whereas an *Optimized* approach outperforms in the variance-minimization context, a *Robust* approach to alpha seeking appears to be superior (as judged by this particular implementation of the *Optimized* approach).[17]

Modified Portfolio Mean–Variance Optimization (MPMVO)

As described in the first section, *MPMVO* is a decomposition of the mean–variance optimal portfolio into a hedged equity component (*EQ*), a variance-minimizing currency-hedging portfolio (*FXHEDGE*), and an optimal allocation, k_{opt}, to a standalone maximum-Sharpe-ratio currency portfolio (*FXALPHA*).[18] The first two pieces combine to form the minimum-variance portfolio (*MINVAR*), and the allocation $k_{opt} = \sigma^2_{MINVAR} / \alpha_{MINVAR}$ to *FXALPHA* is intended to enhance returns.[19] The benefit of *MPMVO*, therefore, is in allowing the flexibility to use different approaches for hedging versus alpha generation; in previous sections, we motivated the benefit of this flexibility in light of the outperformance of an *Optimized* approach to forming *MINVAR* versus the outperformance of the *Robust* approach to forming *FXALPHA*.

Table 4 reports the empirical results for the aggregate *MPMVO* and its components relative to a benchmark of hedged equities (given our differentiated approach). Consistent with prior sections, we looked at both *International* (MSCI World ex-home-country) portfolios and the single *Global* (MSCI World) equity portfolio. Because of the highly similar results across home countries, for *International* portfolios, we report in Table 4 a result averaged across home countries rather than results by country. In the following text, we focus on the *Global* equity portfolio, which is identical across home countries.

As previously noted, Panel B of Table 4 shows the *MINVAR* portfolio reducing volatility relative to a fully hedged *EQ* portfolio (from 14.7% to 12.4%). Interestingly, however, it does not improve either the Sharpe or the Sortino ratio (Sharpe: 0.41 versus 0.41; Sortino: 0.64 versus 0.65) as a result of a commensurate drop in expected returns. This result is driven by the *FXHEDGE* component of *MINVAR* having a negative mean return of –0.9%. In other words, investors holding the *FXHEDGE* portfolio give up expected return in exchange for lower equity exposure.[20] To capture the risk–return trade-off, we followed the methodology outlined in Campbell and Thompson (2008) to compute the certainty equivalents.[21] For example, *MINVAR* has lower risk but also lower returns; an investor with a high degree of risk aversion (γ = 10) would prefer *MINVAR* over *EQ* for *MINVAR*'s lower volatility and would be willing to sacrifice 2.1% per year in returns. An investor with relatively low risk aversion (γ = 3) would not prefer *MINVAR* and, in fact, would be indifferent between it and *EQ* because of *MINVAR*'s lower returns.

Consider now adding the alpha-seeking FX portfolio (*FXALPHA*) with optimal weight k_{opt} to form an optimal *combined* portfolio. For our sample period, k_{opt}*FXALPHA* had strong standalone performance: average returns of 6.7% with a volatility of 7.2% and thus Sharpe and Sortino ratios of, respectively, 0.92 and 1.51. Because, by construction, this portfolio is *ex ante* uncorrelated with the *MINVAR* portfolio, k_{opt}*FXALPHA* has the potential to improve the overall Sharpe ratio of the *MPMVO* portfolio. The results confirm this presumption: As the last column in Panel B of Table 4 shows, the addition of k_{opt}*FXALPHA* to *MINVAR* raised the Sharpe ratio from 0.41 to 0.84 and the Sortino ratio from 0.65 to 1.34. Relative to *EQ*, the fully optimized portfolio achieves both higher return and lower risk. Given a quadratic utility function, an investor would always prefer *MPMVO* over *EQ*; the certainty equivalent of *MPMVO* over

Table 4. Performance of Modified Portfolio Mean–Variance Optimization, 1981–2017

Measure	*EQU*	*EQ*	*FXHEDGE*	*MINVAR*	k_{opt}*FXALPHA*	*MPMVO*
A. International average						
Mean return	6.3%	6.1%	−0.9%	5.2%	6.9%	12.0%
Volatility	16.5%	14.9%	7.3%	12.7%	7.4%	14.6%
Sharpe ratio	0.38	0.41	−0.13	0.41	0.93	0.83
Sortino ratio	0.61	0.63	−0.21	0.63	1.52	1.32
Skew	−0.1	−0.4	0.4	−0.4	−0.5	−0.5
Max. drawdown	−63%	−56%	−63%	−57%	−17%	−44%
Certainty equivalent, γ = 3	−0.5%	0.0%	−4.5%	0.0%	3.3%	6.1%
Certainty equivalent, γ = 10	−2.3%	0.0%	1.5%	2.1%	9.2%	6.5%
B. Global						
Mean return		6.0%	−0.9%	5.2%	6.7%	11.9%
Volatility		14.7%	7.2%	12.4%	7.2%	14.2%
Sharpe ratio		0.41	−0.12	0.41	0.92	0.84
Sortino ratio		0.64	−0.19	0.65	1.51	1.34
Skew		−0.4	0.5	−0.4	−0.5	−0.5
Max. drawdown		−56%	−63%	−53%	−17%	−44%
Certainty equivalent, γ = 3		0.0%	−4.4%	0.0%	3.1%	6.0%
Certainty equivalent, γ = 10		0.0%	1.2%	2.1%	8.8%	6.5%

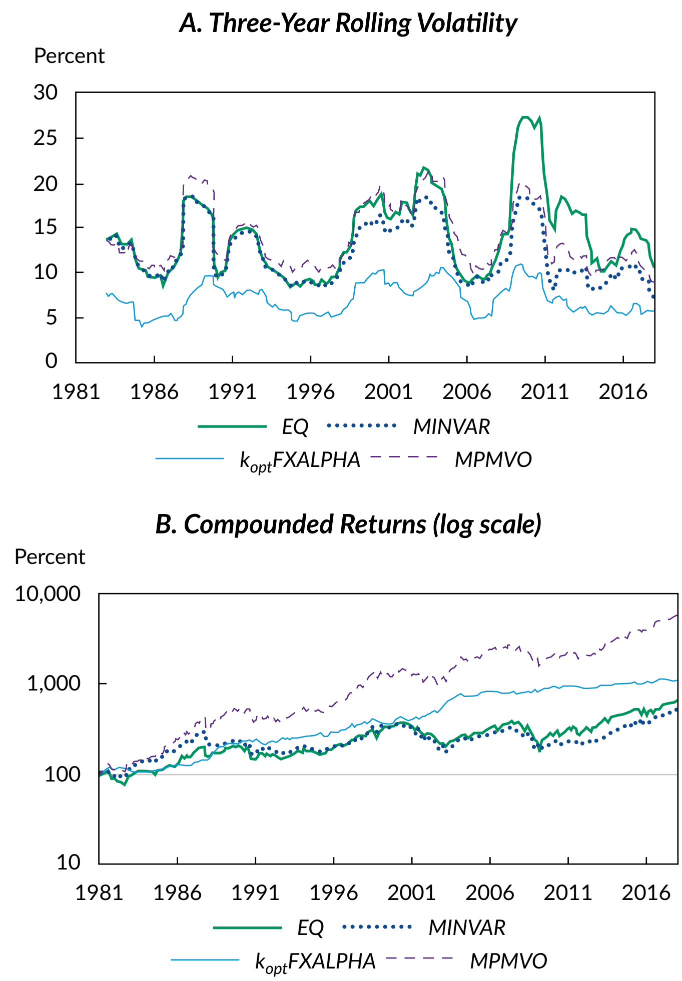
Notes: Results for the *Global* equity portfolio using the *MPMVO* from two perspectives: (1) average of the *International* equity portfolios (ex home country) across 10 home countries and (2) the *Global* equity portfolio including all countries. The *Global* portfolio combines *MINVAR* with k_{opt}*FXALPHA*. *MINVAR* is a combination of a fully hedged MSCI World portfolio (*EQ*) with an optimized variance-minimizing currency basket (*FXHEDGE*). *FXALPHA* is a Sharpe-ratio-maximizing currency basket based on the investment styles of carry, momentum, fundamental momentum, and value in a *Robust* rank-proportional approach. Specifically, the portfolio is built directly with weights proportional to the ranks of each currency style. The weight is given by $k_{opt} = \sigma^2_{MINVAR} / \alpha_{MINVAR} = \sigma_{EQ} \left\{ \left[1 - \left(\rho_{FXHEDGE,EQ} \right)^2 \right] \Big/ \left(SR_{EQ} - \rho_{FXALPHA,EQ} SR_{FXALPHA} \right) \right\}$, where $\rho_{FXHEDGE,EQ}$ and $\rho_{FXALPHA,EQ}$ are the correlations between the *FXHEDGE* and *FXALPHA* portfolio returns and hedged equity *EQ*. Both leverage and turnover are one sided in a long–short portfolio. The "Certainty equivalent" is computed as $CER = \mu_p - \mu_{benchmark} - 0.5\gamma\left(\sigma^2_p - \sigma^2_{benchmark}\right)$, with *EQ* used as the benchmark portfolio and γ = 3, 10.

EQ is 6.0% for γ = 3 and 6.5% for γ = 10. Higher returns and lower risk are also reflected in a smaller drawdown than for *EQ*—namely, −56% versus −44%.

Panel A of **Figure 3** contains graphs of the rolling realized volatility of *EQ*, *MINVAR*, k_{opt}*FXALPHA*, and *MPMVO* over the full sample period. Consistent with its objective as a low-risk portfolio, *MINVAR* consistently (85% of the time) realized lower volatility than *EQ*. And Panel B of Figure 3 shows that it outperformed *EQ* during the financial crisis of 2007–2009. *MPMVO* also realized slightly lower volatility than *EQ*, on average, although its volatility was greater than that of *MINVAR*.

Panel A of Figure 3 also shows that the return volatility of currency portfolios *MINVAR* and k_{opt}*FXALPHA* appear to move in unison with the return volatility of *EQ*. This point is important because one of the contributions of the *MPMVO* approach is determining optimal risk weights on hedging and alpha seeking. Although the optimal time-varying risk budget for hedging and alpha seeking would differ, they both scale with forecast equity volatility. In particular, the optimal dynamic risk target of *FXHEDGE* is $\sigma_{FXHEDGE} = -\rho_{EQ,FXHEDGE}\sigma_{EQ}$ (see the proofs in Appendix A)—that is, the product of forecast equity volatility and the negated correlation between the hedge portfolio and hedged equities.[22] Meanwhile,

Figure 3. Rolling *Ex Ante* and *Ex Post* Portfolio Risk and Returns, 1981–2017



Note: Time series of return volatility and cumulative return performance of the hedged *EQ*, *MINVAR*, *FXALPHA*, and *MPMVO* portfolios.

the optimal dynamic risk target of *FXALPHA* is

$$k_{opt}\sigma_{FXALPHA} = \sigma_{EQ}\left[\frac{1-(\rho_{FXHEDGE,EQ})^2}{(SR_{EQ}/SR_{FXALPHA})-\rho_{FXALPHA,EQ}}\right]$$

(see the proof in Appendix B). In other words, more alpha seeking is desirable if (1) $\rho_{FXHEDGE,EQ}$ is close to zero, (2) *FXALPHA* has a relatively high Sharpe ratio compared with *EQ*, or (3) *FXALPHA* is more correlated with *EQ*.

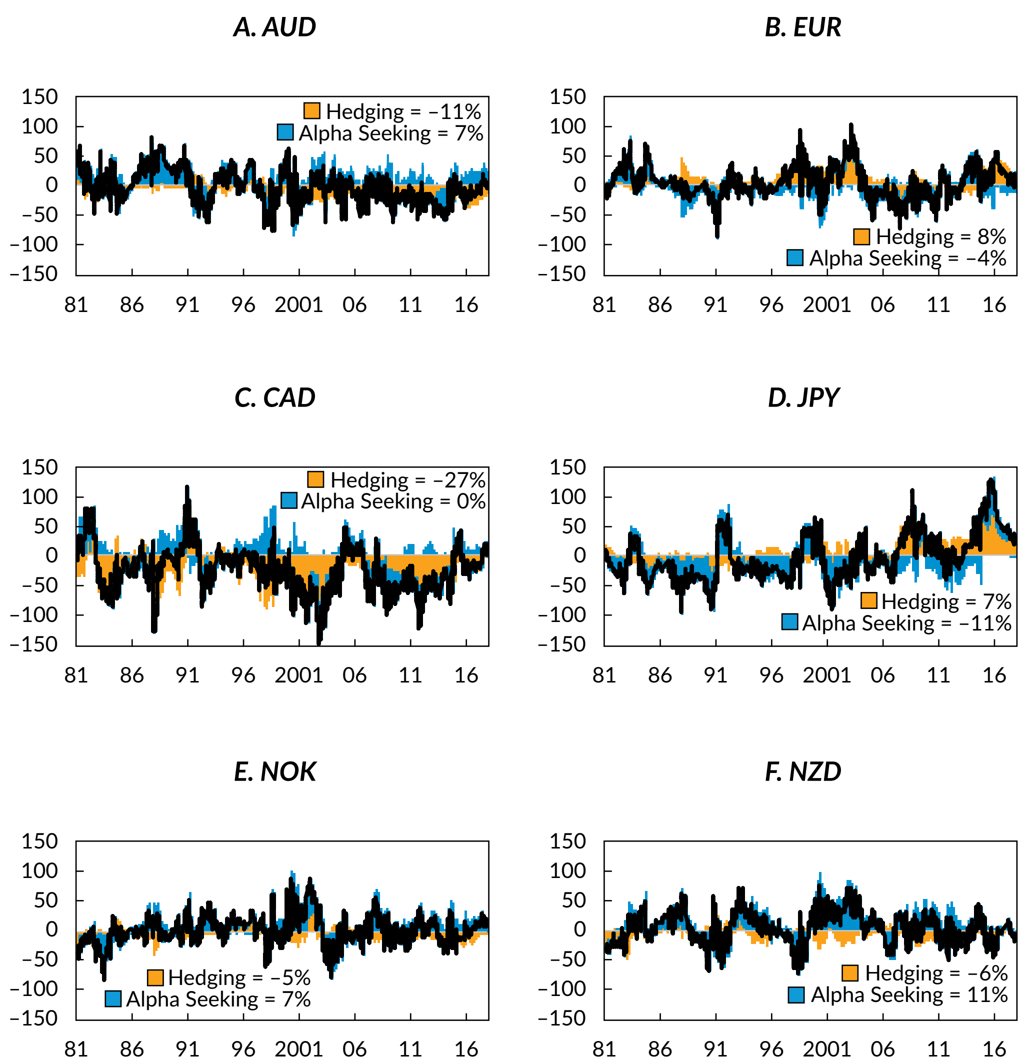
Turning to the returns in Panel B of Figure 3, we can clearly see the superior returns of *MPMVO*. Indeed, the *MPMVO* portfolio realized higher returns than *EQ* 81% of the time (based on three-year rolling returns). This performance, coupled with the aforementioned lower realized volatility than *EQ*, makes *MPMVO* a compelling option from a risk-and-return perspective. Of some note is the pattern that, although k_{opt}*FXALPHA*'s cumulative return flattens out after 2003, it nevertheless still increases throughout the later period. This fact, combined with *MINVAR*, produces the consistency of *MPMVO* throughout the sample. We discuss subsample results in more detail in the next section.

With this clear view of the potential benefits of *MPMVO*, a useful next step is to decompose its currency positions into the components driven by hedging versus alpha seeking. Using the US dollar as the base asset,[23] **Figure 4** graphs each currency's combined optimal weight (the black line), broken down by its contributions from *FXHEDGE* (yellow shading) and *FXALPHA* (blue shading) weights.

Some clear patterns emerge. Consider, first, the hedging demand component associated with the *MINVAR* portfolio (i.e., *FXHEDGE*). Although all positions are time varying, this investor is short on average, among other currencies, Australia (–11%), Canada (–27%), and New Zealand (–6%). These short positions coincide with these currencies' unconditional positive correlations with *Global* equities, documented in Table 1. In contrast, the currency returns of the Eurozone, Japan, Switzerland, and the United States are all negatively correlated with *Global* equities and, therefore, typically have a positive hedging demand (respectively, 8%, 7%, 19%, and 24%). Not surprisingly, these hedging positions appear to be related to the carry trade—with higher-yielding currencies generally having positive correlations and lower yielders generally having negative correlations. Carry, however, only partially explains correlation. Indeed, the Canadian and US dollars appear to have the greatest hedging benefits, yet they are not currencies typically associated with the carry trade. Additionally, the time variation of the hedging demand weights also points to time-varying correlations between currencies and equities that are likely unrelated to carry.

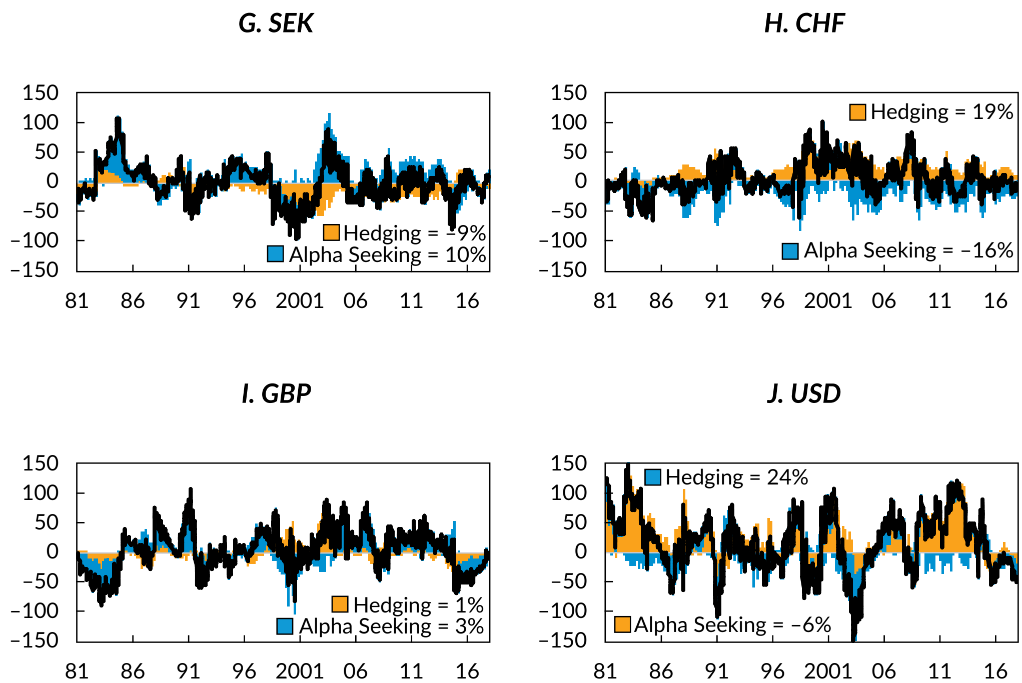
Now consider the alpha-seeking component of currencies, k_{opt}*FXALPHA*. In contrast to the hedging weights, these weights are relatively dynamic for most currencies. Notable exceptions are New Zealand (11%) and Australia (7%), which tend to have positive weights, on average, whereas Japan (–11%) and Switzerland (–16%) are more negative. Carry, the most persistent among the styles we included, again appears to be a driver—this time, in the opposite direction to that of *FXHEDGE*.

Figure 4. Optimal Currency Hedging Weights by Currency, 1981–2017



(continued)

Figure 4. Optimal Currency Hedging Weights by Currency, 1981–2017 (continued)



Notes: Optimal currency exposure (*FXHEDGE*) in the hedging (*MINVAR*) portfolio and the alpha-seeking (k_{opt}*FXALPHA*) portfolio and their averages. The yellow and blue areas represent, respectively, hedging and alpha seeking; the black line represents the combined optimal currency exposures.

Combining the two components into the full optimal currency positions yields interesting results. Weights are again relatively dynamic, but some patterns do emerge. A persistent short position in Canada contrasts strikingly to the situation in Australia and New Zealand. The short hedging positions of Australia and New Zealand are now offset by their alpha-seeking positions. Indeed, the investor would hold mostly a small positive weight in the New Zealand dollar (5%) because the positive average return view of carry overwhelms a negative average hedging view. The investor would also hold a small positive position in Swiss francs (3%), however, even though New Zealand dollars and Swiss francs are on the opposite sides of the carry trade. Their strong negative unconditional correlation with *each other* (i.e., −0.41) makes them natural hedges for each other and, therefore, desirable to hold as a package.

Implementation Considerations

The results in Table 4 suggest sizable benefits to the *MPVMO* approach, but a reasonable question is whether these exposures are practical to implement and consistent over time. Focusing on the *Global* equity portfolio, Panel A of **Table 5** documents the impact on the results of three implementation restrictions: (1) changing the rebalancing frequency from monthly to quarterly, (2) restricting the *ex ante* volatility of *MPMVO* to be no higher than that of *EQ* in every period, and (3) restricting the leverage of the active currency portfolio (including both *FXHEDGE* and k_{opt}*FXALPHA* components) to be ≤100% in any given period. Leverage is measured as the sum of the long and short positions divided by 2.

First, the rebalancing constraint has little impact on the results other than the expected reduction in turnover: The Sharpe and Sortino ratios of *MPMVO* drop, respectively, from 0.84 to 0.78 and from 1.34 to 1.23, with almost no change in volatility, skew, or maximum drawdown. The annualized turnover of the currency portfolio, however, as measured by the sum of the absolute value of the change in currency positions (per unit of the equity index) falls from 6.6 to 3.5 times.

Second, with respect to the volatility-constrained portfolio, *MPMVO*'s realized volatility drops from 14.2% to 13.5%, albeit with no substantive effect on skewness or drawdown. Restricting the *MPMVO*

Table 5. Robustness Checks

Measure	MPMVO	Quarterly Rebalancing	Volatility Capped	Leverage Capped
A. Robustness analysis for MPMVO, 1981–2017				
Mean return	11.9%	11.1%	9.5%	8.7%
Volatility	14.2%	14.3%	13.5%	15.6%
Skew	−0.49	−0.47	−0.48	−0.57
Sharpe ratio	0.84	0.78	0.71	0.56
Sortino ratio	1.34	1.23	1.12	0.88
Max. drawdown	−44%	−44%	−43%	−42%
Leverage	1.3	1.3	1.1	0.8
Turnover	6.6	3.5	5.3	5.4
Breakeven transaction cost	0.88%	1.45%	0.66%	0.51%
	Full Sample	1981–1998	1999–2017	1999–2017, +EM[a]
B. Subsample analysis for MPMVO				
Mean return	11.9%	15.3%	8.5%	8.1%
Volatility	14.2%	13.8%	14.5%	13.7%
Skew	−0.49	−0.77	−0.30	−0.10
Sharpe ratio	0.84	1.11	0.59	0.59
Sortino ratio	1.34	1.77	0.94	0.96
Max. drawdown	−44%	−33%	−44%	−46%
Leverage	1.3	1.3	1.3	1.7
Turnover	6.6	6.6	6.7	7.5
Breakeven transaction cost	0.88%	1.42%	0.38%	0.27%

Notes: Robustness results for the *Global* equity portfolio using the *MPMVO* approach to optimize currency hedging with a perturbation to rebalancing frequency, common-sense portfolio constraints, and different sample periods.

[a]Eight emerging market currencies added as their data became available.

volatility to be less than that of *EQ* does bind in certain periods, however, leading to a small reduction in the Sharpe and Sortino ratios (from, respectively, 0.84 to 0.71 and 1.34 to 1.12), although leverage and turnover also drop.

Third, the leverage constraint often binds because the leverage of the currency portfolio is, on average, 1.3. This restriction causes a large drop in the Sharpe and Sortino ratios—to 0.56 and 0.88—but with much lower leverage. Of some note is that this constrained *MPMVO* still outperforms the *EQ* and *EQU* portfolios described in Table 4.

As a measure of robustness, Panel B of Table 5 documents risk-and-return characteristics of *MPMVO* in two distinct subperiods: 1981–1998 (the pre-euro period) and 1999–2017. For the more current period, we report the results not only for G-10 currencies but also for eight emerging market (EM) currencies.[24] *MPMVO*'s performance was better in the sample's first half, with a Sharpe ratio of 1.11, compared with the second half's 0.59, and a Sortino ratio of 1.77, compared with the second half's 0.94. This drop is consistent with that of *EQ*, whose Sharpe ratios are 0.54 and 0.32 for the two subperiods. The addition of eight EM currencies to the mix in the second half of the sample had surprisingly little impact on the results. Other than a slight increase in leverage and turnover, little change affected any of the risk–return characteristics of *MPMVO*.

Finally, we present breakeven transaction costs—excess returns earned by FX positions in a portfolio divided by turnover—to evaluate the possibility that transaction costs will reduce the attractiveness of the active hedging strategy in practice. Estimates for the breakeven point range from 27 bps to 142 bps per dollar traded—much higher than most practitioners are likely to face in practice.

Conclusion

What is the optimal way to hedge the currency risk of international equity portfolios? We have shown that the optimal portfolio (in a mean–variance sense) can be decomposed into a hedged equity component, a variance-minimizing currency-hedging portfolio, and an optimal allocation to a standalone maximum-Sharpe-ratio currency portfolio. The optimal solution has a convenient interpretation in mean–variance space; the investor chooses a combination of two long–short currency portfolios—that is, a hedging one that minimizes risk of the equity portfolio and a speculative one that puts some weight on the standalone mean–variance optimal currency portfolio. The decomposition allows the practitioner to choose various methodologies for construction of the two long–short currency portfolios, thus avoiding many of the pitfalls of mean–variance optimization. We call this portfolio the modified mean–variance optimal portfolio, or *MPMVO*.

We empirically documented the evolution from unhedged equity to fully hedged equity to dynamic hedging of the equity by using a currency basket to a minimum-variance optimal hedge to the *MPMVO*. Notably, although currency hedging of global equity portfolios reduced volatility in most cases, it often did so at the expense of expected returns. Indeed, Sharpe ratios rarely improved. In contrast, the *MPMVO* approach judiciously chose currencies to balance views on risk and return and, in the process, doubled the Sharpe ratio of hedged equity returns.

Appendix A. Proof of "Currency Hedging: Theory" Results

Proof A. Consider the following maximization problem:

$$\max_{\mathbf{w}_s}\left(\alpha_{EQ}+\mathbf{w}_s'\boldsymbol{\alpha}_{FX}\right)=\alpha_P$$

subject to

$$\mathbf{w}_s'\boldsymbol{\Sigma}_{FX}\mathbf{w}_s+\sigma_{EQ}^2+2\mathbf{w}_s'\boldsymbol{\Sigma}_{EQFX}=\sigma_P^2.$$

The first-order condition is

$$\boldsymbol{\alpha}_{FX}-2\lambda\left(\mathbf{w}_s'\boldsymbol{\Sigma}_{FX}+\boldsymbol{\Sigma}_{EQFX}\right)=0.$$

Solving for $\mathbf{w}_{opt}$, we get

$$\mathbf{w}_{opt}=\frac{\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\alpha}_{FX}}{2\lambda}-\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\Sigma}_{EQFX}.$$

Substituting $\mathbf{w}_{opt}$ into $\mathbf{w}_s'\boldsymbol{\Sigma}_{FX}\mathbf{w}_s+\sigma_{EQ}^2+2\mathbf{w}_s'\boldsymbol{\Sigma}_{EQFX}=\sigma_P^2$, solving for λ, and substituting in λ back into $\mathbf{w}_{opt}$ yields

$$\mathbf{w}_{opt}=\boldsymbol{\Sigma}_{FX}^{-1}\left(k\boldsymbol{\alpha}_{FX}-\boldsymbol{\Sigma}_{EQFX}\right),$$

where

$$k=\sqrt{\frac{\sigma_P^2-\sigma_{EQ}^2+\boldsymbol{\Sigma}_{EQFX}'\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\Sigma}_{EQFX}}{\boldsymbol{\alpha}_{FX}'\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\alpha}_{FX}}}.$$

Proof B. The *MINVAR* (*EQ* + *FXHEDGE*) portfolio places a value of k = 0 in w_{opt}. The *MINVAR* portfolio has the following mean and variance of returns:

$$\alpha_{MINVAR}=\alpha_{EQ}-\boldsymbol{\alpha}_{FX}'\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\Sigma}_{EQFX},$$

and

$$\sigma_{MINVAR}^2=\sigma_{EQ}^2-\boldsymbol{\Sigma}_{EQFX}'\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\Sigma}_{EQFX}.$$

Note that the *MINVAR* portfolio is uncorrelated with any standalone FX portfolio:

$$\begin{aligned}\operatorname{cov}\left(\mathbf{w}_s'\mathbf{R}_{FX},MINVAR\right)&\equiv\operatorname{cov}\left(\mathbf{w}_s'\mathbf{R}_{FX},R_{EQ}-\boldsymbol{\Sigma}_{EQFX}'\boldsymbol{\Sigma}_{FX}^{-1}\mathbf{R}_{FX}\right)\\&=\mathbf{w}_s'\boldsymbol{\Sigma}_{EQFX}-\mathbf{w}_s'\boldsymbol{\Sigma}_{FX}\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\Sigma}_{EQFX}\\&=0.\end{aligned}$$

Proof C. To derive the optimal k, we need to choose k to maximize the Sharpe ratio of all the possible mean–variance-efficient portfolios. Note that the mean–variance-efficient weights on currencies and equity, respectively, are

$$w(k)=\begin{pmatrix}-\Sigma_{FX}^{-1}\Sigma_{EQFX}\\1\end{pmatrix}+k\begin{pmatrix}-\Sigma_{FX}^{-1}\alpha_{FX}\\0\end{pmatrix}.$$

Because the *MINVAR* portfolio is the first component and is uncorrelated with all FX portfolios, the portfolio expected return and variance are

$$\alpha_P(k) = \alpha_{MINVAR} + k\boldsymbol{\alpha}'_{FX}\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\alpha}_{FX}$$

and

$$\sigma_P^2(k) = \sigma_{MINVAR}^2 + k^2\boldsymbol{\alpha}'_{FX}\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\alpha}_{FX}.$$

The Sharpe ratio is, therefore,

$$\frac{\alpha_P}{\sigma_P} = \frac{\alpha_{MINVAR} + kA}{\left(\sigma_{MINVAR}^2 + k^2 A\right)^{1/2}},$$

where scalar

$$A = \boldsymbol{\alpha}'_{FX}\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\alpha}_{FX}.$$

The first-order condition with respect to k is

$$\frac{\left(\sigma_{MINVAR}^2 + k^2 A\right)^{1/2} A - \left(\alpha_{MINVAR} + kA\right)\left(\sigma_{MINVAR}^2 + k^2 A\right)^{-1/2} kA}{\sigma_{MINVAR}^2 + k^2 A} = 0,$$

implying

$$k_{opt} = \frac{\sigma_{MINVAR}^2}{\alpha_{MINVAR}}.$$

Appendix B. Proof of “Modified Portfolio Mean–Variance Optimization” Theoretical Results

Using the fact that *MINVAR* is uncorrelated with any FX portfolio, we have

$$\text{cov}(FX, EQ) = -\text{cov}(FX, FXHEDGE)$$

for any *FX* portfolios. Substituting in *FXHEDGE* for *FX* yields

$$\text{cov}(FXHEDGE, EQ) = -\sigma_{FXHEDGE}^2$$

or

$$\sigma_{FXHEDGE} = -\rho_{FXHEDGE,EQ}\sigma_{EQ}.$$

Substituting in *FXALPHA* for *FX* yields

$$\begin{aligned}\sigma_{EQ}\rho_{FXALPHA,EQ} &= -\sigma_{FXHEDGE}\rho_{FXHEDGE,FXALPHA} \\ &= \rho_{FXHEDGE,EQ}\rho_{FXHEDGE,FXALPHA}\sigma_{EQ}\end{aligned}$$

or

$$\rho_{FXALPHA,EQ} = \rho_{FXHEDGE,EQ}\rho_{FXHEDGE,FXALPHA}.$$

Given that *FXALPHA* has a mean and variance of $\boldsymbol{\alpha}'_{FX}\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\alpha}_{FX}$, its Sharpe ratio is

$$\begin{aligned}SR_{FXALPHA} &= \sqrt{\boldsymbol{\alpha}'_{FX}\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\alpha}_{FX}} \\ &= \sigma_{FXALPHA}.\end{aligned}$$

Applying mean–variance-efficient mathematics in FX space, we find the expected returns on any FX portfolio (including *FXHEDGE*) as its beta with the MVO currency portfolio (i.e., *FXALPHA*) times the expected return on *FXALPHA*; that is,

$$\begin{aligned}\alpha_{FXHEDGE} &= \frac{\text{cov}(FXHEDGE, FXALPHA)}{\text{var}(FXALPHA)}\alpha_{FXALPHA} \\ &= \sigma_{FXHEDGE}\rho_{FXHEDGE,FXALPHA}SR_{FXALPHA}.\end{aligned}$$

Plugging these equations into

$$k_{opt} = \frac{\sigma_{MINVAR}^2}{\alpha_{MINVAR}}$$

yields

$$k_{opt} = \sigma_{EQ}\left(\frac{1 - \rho_{FXHEDGE,EQ}^2}{SR_{EQ} - \rho_{FXALPHA,EQ}SR_{FXALPHA}}\right).$$

Editor’s Note

This article was externally reviewed using our double-blind peer-review process. When the article was accepted for publication, the authors thanked the reviewers in their acknowledgments. David Gallagher was one of the reviewers for this article.

Submitted 3 October 2018

Accepted 20 May 2019 by Stephen J. Brown

Notes

1. The academic literature is more supportive of the benefits to currency hedging than are practitioners. See, for example, Perold and Schulman (1988); Black (1989); Glen and Jorion (1993); Jorion (1994); Ang and Bekaert (2002); Campbell, Serfaty-de Medeiros, and Viceira (2010); De Roon, Nijman, and Werker (2003); Schmittmann (2010); Topaloglou, Vladimirou, and Zenios (2011); Opie and Dark (2015); Boudoukh, Katz, Richardson, and Thapar (2016); Christensen and Varneskov (2018); Opie and Riddiough (2019). For an alternative view, see Froot (1993) and LeGraw (2015).

2. EAFE stands for Europe, Australasia, and Far East. This universe is restricted to those funds that report their approach to currency.

3. Our definitions are provided later in the article. For papers on carry, see, among others, Burnside, Eichenbaum, and Rebelo (2011); Lustig, Roussanov, and Verdelhan (2011); Menkhoff, Sarno, Schmeling, and Schrimpf (2012a); Jurek (2014); and related papers by Corte, Riddiough, and Sarno (2016) and Colacito, Croce, Gavazzoni, and Ready (2018). For works on momentum, see, among others, Okunev and White (2003); Menkhoff, Sarno, Schmeling, and Schrimpf (2012b); and Moskowitz, Ooi, and Pedersen (2012). For papers on fundamental momentum, see Bhojraj and Swaminathan (2006); Ang and Chen (2010); Brooks (2017); and Dahlquist and Hasseltoft (2017). And for works on value, see, for example, Froot and Ramadorai (2005) and Menkhoff, Sarno, Schmeling and Schrimpf (2017). A few papers have examined the performance of combining styles in the context of currencies—Pojarliev and Levich (2008, 2011); Asness, Moskowitz, and Pedersen (2013); Kroencke, Schindler, and Schrimpf (2014); Barroso and Santa-Clara (2015); and Opie and Riddiough (2019).

4. See Jorion (1985) in a similar context and, more generally, Ledoit and Wolf (2004) and DeMiguel, Garlappi, and Uppal (2009), among many others.

5. In this article, we report our investigation of the currency hedging of international equity portfolios, but the same approach can be applied to a more diverse set of financial assets, including fixed income.

6. Note that the approximation sign in Equation 1 results from dropping the cross-product of local equity and currency returns, which tends to be close to zero.

7. To short the spot currencies, our approach uses the short-term interest rate market as opposed to the forward market. Our empirical analysis started in 1981, when forward price data were not available. By covered interest rate parity, a one-to-one mapping exists between the two methodologies. Specifically, returns on the dollar world equity portfolio, $\mathbf{w}_t'\mathbf{R}_{t+1}^{\$}$, convert approximately into $\mathbf{w}_t'\mathbf{R}_{t+1}^{*} + \mathbf{w}_t'\left(\mathbf{F}_t^{*} - \mathbf{s}_t^{*}\right)$, where **F** and **s** are the respective forward and spot exchange rates. Note that some evidence has been discovered since the financial crisis of 2007–2009 of violations of covered interest rate parity (see Du, Tepper, and Verdelhan 2018).

8. Interestingly, because the *MINVAR* portfolio returns are by construction uncorrelated with any FX standalone portfolio return, any new FX risk relative to the *MINVAR* portfolio comes from additional standalone volatility of the MVO currency portfolio.

9. For the period before 1999, we substituted for the euro a synthetic euro basket constructed by the Bank of England consisting of legacy European currencies.

10. The home country of the international equity portfolio in question is denoted in parentheses; for example, "*International* (United States)" reflects the international equity portfolio from the perspective of a US investor. For *Global*, the equity component of the portfolio is, by definition, identical for all investors. For simplicity, all *Global* results are presented hedged to US dollars, although this choice did not meaningfully affect any results.

11. Using weekly overlapping returns is a simple way to mitigate potential asynchronicity in daily data across equity and currency markets. Shrinking correlations 25% from their point estimates toward zero is a standard approach to improving the stability of results because inversion of the covariance matrixes is required in the mean–variance optimization. Shrinkage was applied to both *Robust* and *Optimized* hedging.

12. As an illustration, the returns on Canadian and US currencies are positively correlated with each other (i.e., 0.45) but have opposite correlations with *Global* equities (0.21 for Canada and –0.17 for the United States). Both a long position in the US dollar and a short position in the Canadian dollar reduce the risk of *Global* equities but also diversify the risk of the currency portfolio because of the currencies' positive correlation with each other. In contrast, Australia and Japan also have opposite correlations with *Global* equities (respectively, 0.29 and –0.26), but a long position in the Japanese yen and a short position in the US dollar do not diversify currency risk because these currencies are negatively correlated (–0.31).

13. To a first order, hedged *Global EQ* returns are identical for all investors. Conversely, *Global EQU* returns would differ by home country because unhedged currency exposures would differ. Similarly, the *MINVAR* (*Robust*) portfolio differs across home countries as the "hedge basket" (long foreign currency versus short home currency) differs, whereas the *MINVAR* (*Optimized*) portfolio, with the flexibility to implement any long and short currency positions, would be identical for countries. For this reason, we do not show *Global* results for *EQU* or *MINVAR* (*Robust*).

14. For three of the styles, we used standard measurement methodologies: Carry is the 3-month interest rate, momentum is the prior 12-month return on the currency relative to the basket, and value is the negated prior five-year change in the real exchange rate. For fundamental momentum, we used Brooks's (2017) construction of the signal, which is a composite of business cycle measures (lagged one-year change in GDP growth and inflation forecasts), international trade (past one-year changes

in spot FX rates measured against an export-weighted basket), monetary policy (previous one-year change in the two-year yield), and past one-year equity returns.

15. Specifically, we took the vector of raw values for each factor, subtracted the cross-sectional mean, and scaled to have a cross-sectional standard deviation of 1.0. The scaling was required to make factor raw values comparable across factors. These scaled raw values are the expected returns for each currency for each factor. For the multistyle portfolio, we took an equal-weighted average of factor-level expected returns.

16. The choice of risk target is irrelevant for the comparison in this section, but it will be important to having the risk target of the *FXALPHA* portfolio be consistent with that of the $\boldsymbol{\Sigma}_{FX}^{-1}\boldsymbol{\alpha}_{FX}$ portfolio in the section "Modified Portfolio Mean–Variance Optimization." The risk target was dictated by our assumed Sharpe ratio of the *FXALPHA* portfolio, which is 0.3.

17. We report on two approaches here, but we by no means exhausted all possible techniques. An alternative optimized portfolio—for example, one with size constraints, more elaborate variance–covariance shrinkage, or a different expected return formulation (e.g., see Black and Litterman 1992)—could possibly perform better than the approaches we report.

18. That is, the *FXALPHA* currency portfolio is mean–variance optimal within currencies; interactions with the equity component of the portfolio are ignored.

19. Given assumptions about the Sharpe ratios of hedged equities *EQ* and *FXALPHA* and the correlation of the *FXHEDGE* and *FXALPHA* portfolios with equity, k_{opt} can be computed. For the purposes of this study, we assumed the following Sharpe ratios for all *International* and *Global* portfolios: SR_{EQ} = 0.5 and $SR_{FXALPHA}$ = 0.3. The higher Sharpe ratio for equities is for conservativism. An investor may choose his or her own assumptions for the two (either constant or time varying). Note, however, that the greater the assumed *FXALPHA* Sharpe ratio relative to the assumed *EQ* Sharpe ratio, the greater the role alpha seeking will play in the portfolio.

20. A possible explanation is that persistent short positions in Australia and New Zealand and long positions in Japan and Switzerland mean the investor is short the currency carry trade. To evaluate this possibility, we regressed the returns of *MINVAR* (*Optimized*) on both the returns of *EQ* and the simple currency style portfolios described in the section "Currency Alpha Seeking." The loading on *EQ* was found to be less than 1.0, with an average across countries of 0.8. Unsurprisingly, active hedging lowered volatility by offsetting equity risk in the portfolio. At the same time, carry had a meaningful negative loading, with an average beta of −0.23. An important aspect is that once these exposures have been controlled for, the active hedging approach has alphas relative to the amount of equity beta that has been offset.

21. Following Campbell and Thompson (2008), we computed the certainty equivalent as $CER = \mu_p - \mu_{benchmark} - 0.5\gamma\left(\sigma_p^2 - \sigma_{benchmark}^2\right)$ with *EQ* as a benchmark portfolio. We picked γ = 3 for an investor with low risk aversion and γ = 10 for an investor with high risk aversion.

22. This result is intuitive: The objective of *FXHEDGE* is to offset market risk by generating a maximally negatively correlated currency portfolio, and its optimal risk allocation is proportional to how negative this correlation is.

23. A base currency is required as a numeraire for presenting results, but as previously discussed, the solution is a generic one that applies to all investors.

24. The added emerging currencies are those of Brazil, Mexico, Poland, Russia, Singapore, South Africa, South Korea, and Taiwan. Based on the Bank for International Settlements (2001) Triennial Central Bank Survey on foreign exchange and derivative market activities, we picked all emerging market currencies that had a daily trading volume higher than the least liquid developed currency (the New Zealand dollar). The 2001 survey was the first in its series to include substantial data on EM currencies.

References

Anderson, Ronald W., and Jean-Pierre Danthine. 1981. "Cross Hedging." *Journal of Political Economy* 89 (6): 1182–96.

Ang, Andrew, and Geert Bekaert. 2002. "International Asset Allocations with Time-Varying Correlations." *Review of Financial Studies* 15 (4): 1137–87.

Ang, Andrew, and Joseph Chen. 2010. "Yield Curve Predictors of Foreign Exchange Returns." Working paper.

Asness, Clifford S., Tobias Moskowitz, and Lasse H. Pedersen. 2013. "Value and Momentum Everywhere." *Journal of Finance* 68 (3): 929–85.

Bank for International Settlements. 2001. "Triennial Central Bank Survey of Foreign Exchange and Derivatives Market Activity in 2001."

Barroso, Pedro, and Pedro Santa-Clara. 2015. "Beyond the Carry Trade: Optimal Currency Portfolios." *Journal of Financial and Quantitative Analysis* 50 (5): 1037–56.

Bhojraj, Sanjeev, and Bhaskaran Swaminathan. 2006. "Macromomentum: Returns Predictability in International Equity Indices." *Journal of Business* 79 (1): 429–51.

Black, Fischer. 1989. "Universal Hedging: Optimizing Currency Risk and Reward in International Equity Portfolios." *Financial Analysts Journal* 45 (4): 16–22.

Black, Fischer, and Robert Litterman. 1992. "Global Portfolio Optimization." *Financial Analysts Journal* 48 (5): 28–43.

Boudoukh, Jacob, Michael Katz, Matthew Richardson, and Ashwin Thapar. 2016. "Risk without Reward: The Case for Strategic Currency Hedging." White paper, AQR Capital Management.

Brooks, Jordan. 2017. “A Half Century of Macro Momentum.” White paper, AQR Capital Management.

Burnside, Craig, Martin Eichenbaum, and Sergio Rebelo. 2011. “Carry Trade and Momentum in Currency Markets.” *Annual Review of Financial Economics* 3 (1): 511–35.

Campbell, John Y., Karine Serfaty-de Medeiros, and Luis M. Viceira. 2010. “Global Currency Hedging.” *Journal of Finance* 65 (1): 87–121.

Campbell, John Y., and Samuel B. Thompson. 2008. “Predicting Excess Stock Returns Out of Sample: Can Anything Beat the Historical Average?” *Review of Financial Studies* 21 (4): 1509–31.

Christensen, Bent Jesper, and Rasmus Tangsgaard Varneskov. 2018. “Dynamic Global Currency Hedging.” Working paper, Aarhus University.

Colacito, Ric, Mariano M. Croce, Federico Gavazzoni, and Robert Ready. 2018. “Currency Risk Factors in a Recursive Multicountry Economy.” *Journal of Finance* 73 (6): 2719–56.

Corte, Pasquale Della, Steven J. Riddiough, and Lucio Sarno. 2016. “Currency Premia and Global Imbalances.” *Review of Financial Studies* 29 (8): 2161–93.

Dahlquist, Magnus, and Henrik Hasseltoft. 2017. “Economic Momentum and Currency Return.” Swedish House of Finance Research Paper 16-14.

De Roon, Frans A., Theo E. Nijman, and Bas J. M. Werker. 2003. “Currency Hedging for International Stock Portfolios: The Usefulness of Mean–Variance Analysis.” *Journal of Banking & Finance* 27 (2): 327–49.

DeMiguel, Victor, Lorenzo Garlappi, and Raman Uppal. 2009. “Optimal versus Naive Diversification: How Inefficient is the 1/*N* Portfolio Strategy?” *Review of Financial Studies* 22 (5): 1915–53.

Du, Wenxin, Alexander Tepper, and Adrien Verdelhan. 2018. “Deviations from Covered Interest Rate Parity.” *Journal of Finance* 73 (3): 915–57.

Froot, Kenneth A. 1993. “Currency Hedging over Long Horizons.” NBER Working Paper No. 4355 (May).

Froot, Kenneth A., and Tarun Ramadorai. 2005. “Currency Returns, Intrinsic Value, and Institutional-Investor Flows.” *Journal of Finance* 60 (3): 1535–66.

Gagnon, Louis, Gregory J. Lypny, and Thomas H. McCurdy. 1998. “Hedging Foreign Currency Portfolios.” *Journal of Empirical Finance* 5 (3): 197–220.

Glen, Jack, and Philippe Jorion. 1993. “Currency Hedging for International Portfolios.” *Journal of Finance* 48 (5): 1865–86.

Jorion, Philippe. 1985. “International Portfolio Diversification with Estimation Risk.” *Journal of Business* 58 (3): 259–78.

——. 1994. “Mean/Variance Analysis of Currency Overlays.” *Financial Analysts Journal* 50 (3): 48–56.

Jurek, Jakub W. 2014. “Crash-Neutral Currency Carry Trades.” *Journal of Financial Economics* 113 (3): 325–47.

Kroencke, Tim A., Felix Schindler, and Andreas Schrimpf. 2014. “International Diversification Benefits with Foreign Exchange Investment Styles.” *Review of Finance* 18 (5): 1847–83.

Ledoit, Olivier, and Michael Wolf. 2004. “Honey, I Shrunk the Sample Covariance Matrix: Problems in Mean–Variance Optimization.” *Journal of Portfolio Management* 30 (4): 110–19.

LeGraw, Catherine. 2015. “The Case for Not Currency Hedging Foreign Equity Investments: A U.S. Investor’s Perspective.” White paper, GMO (14 April).

Lustig, Hanno, Nikolai Roussanov, and Adrien Verdelhan. 2011. “Common Risk Factors in Currency Markets.” *Review of Financial Studies* 24 (11): 3731–77.

Menkhoff, Lukas, Lucio Sarno, Maik Schmeling, and Andreas Schrimpf. 2012a. “Currency Momentum Strategies.” *Journal of Financial Economics* 106 (3): 660–84.

——. 2012b. “Carry Trades and Global Foreign Exchange Volatility.” *Journal of Finance* 67 (2): 681–718.

——. 2017. “Currency Value.” *Review of Financial Studies* 30 (2): 416–41.

Moskowitz, Tobias J., Yao Hua Ooi, and Lasse H. Pedersen. 2012. “Time Series Momentum.” *Journal of Financial Economics* 104 (2): 228–50.

Okunev, John, and Derek White. 2003. “Do Momentum-Based Strategies Still Work in Foreign Currency Markets?” *Journal of Financial and Quantitative Analysis* 38 (2): 425–47.

Opie, Wei, and Jonathan Dark. 2015. “Currency Overlay for Global Equity Portfolios: Cross-Hedging and Base Currency.” *Journal of Futures Markets* 35 (2): 186–200.

Opie, Wei, and Steven J. Riddiough. 2019. “Global Currency Hedging with Common Risk Factors.” Working paper, University of Melbourne.

Perold, Andre F., and Evan C. Schulman. 1988. “The Free Lunch in Currency Hedging: Implications for Investment Policy and Performance Standards.” *Financial Analysts Journal* 44 (3): 45–50.

Pojarliev, Momtchil, and Richard M. Levich. 2008. “Do Professional Currency Managers Beat the Benchmark?” *Financial Analysts Journal* 64 (5): 18–32.

——. 2011. “Detecting Crowded Trades in Currency Funds.” *Financial Analysts Journal* 67 (1): 26–39.

Schmittmann, Jochen M. 2010. “Currency Hedging for International Portfolios.” IMF Working Paper 10/151 (June).

Topaloglou, Nikolas, Hercules Vladimirou, and Stavros A. Zenios. 2011. “Optimizing International Portfolios with Options and Forwards.” *Journal of Banking & Finance* 35 (12): 3188–201.