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## Experimental and numerical analysis of flow hydraulics in triangular and rectangular piano key weirs

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### ABSTRACT

Piano key weir is a new type of spillways designed to improve the discharge capacity of dams. Generally, increasing the upstream hydraulic load in piano key weirs results in reduced discharge capacity of the weir. Accordingly, the present study investigated the effects of triangular notch on the discharge coefficient of piano key weirs. The 3D flow field over the piano key weirs was simulated in FLOW-3D software in order to study the flow hydraulics and compare the discharge rates, and the effect of each model on the flow field over the weirs and discharge coefficient was investigated. The results suggested that data of the numerical model were appropriately consistent with that of the laboratory model. According to the results, the discharge coefficient of the triangular piano key weirs was 25% higher than that of the rectangular piano key weirs. It was also observed that changing the notch shape of the piano key weir increased the discharge coefficient of the piano key weir by 36% and 13% for the heights of 5 cm of 7.5 cm, respectively.

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discharge coefficient; FLOW-  
3D numerical model

### Introduction

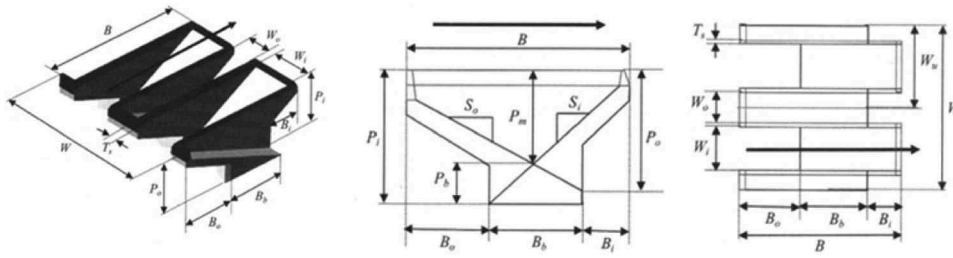
A weir is a structure used to avoid excessive storage of water in the reservoir and transfer the excess water, especially during the flood times, from upstream to the downstream areas and rivers. Weirs are one of the main and important structures of a dam. Considering their sensitive role, the weirs are required to be readily operational for utilization and discharge of the floodwaters and protection of the dam and the associated installations (Beyrami, 1997). Hydraulic efficiency of the free over-fall weirs for a given head is directly related to the length of the weir, and the discharge coefficient ( $C_d$ ) of this type of weirs can be calculated using Equation (1) (Henderson, 1966).

$$Q = \frac{2}{3} C_d \sqrt{2g} L H_t^{3/2} \quad (1)$$

where  $C_d$  is the discharge coefficient,  $g$  is the gravity acceleration,  $H_t$  is the total upstream head (i.e. the sum of the piezometric head of the crest ( $H$ ) and the kinetic energy ( $V^2/2g$ )), and  $L$  is the length of the weir.

One of the drawbacks of the conventional weirs is their low discharging capacity due to the limited width available when deploying this type of spillways. In the third decade of the last century, labyrinth weirs were developed as an effective solution to enhance the hydraulic performance of this type of weirs. Labyrinth weirs usually consist of plain vertical walls, offering a significantly better performance compared to linear weirs. This type of weirs is easy to build as they consist

of plain vertical walls. However, the flows approaching this type of weirs, especially the bottom flows, are intensively compressed after entering the area between two vertical walls of the side crests, consequently causing the upstream and downstream crests to demonstrate a poor hydraulic performance. Furthermore, the major disadvantage of this type of weirs lies in the large foundation area required for their installation on the concrete dams. Piano key weir is a new form of non-linear weirs introduced by HydroCoop of France, and Hydraulic and Environment Laboratory of the University of Biskra in Algeria. In this type of weir, unlike the labyrinth weirs, the mouths are alternately sloped inward and outward the reservoir. Construction of this type of weirs needs a smaller space compared to the labyrinth weirs and, therefore, this type of weirs is smaller in size. This advantage facilitates the use of this type of weirs in the crest of concrete dams. They increase the capacity of the reservoir and are economically feasible, and require lower maintenance costs. The first piano key weir was built in 2006 over the Golours Dam in France (Laugier, 2007). Later, Blanc and Lempérière (2001) extended the piano key weirs to enhance the performance of labyrinth weirs. The piano key weirs can be used in places where length or width of the spillway course is limited, e.g. at the end of concrete gravity dams or narrow channels where there is no enough space for designing the traditional labyrinth weirs. Piano key weir is a type of labyrinth weir enhanced for places with limited width. As shown in



**Figure 1.** Geometric parameters of the piano key weir.

Figure 1, the piano key weir has a plain rectangular labyrinth crest with sloped inlet and outlet key floors. Generally, the parameters related to the geometric and hydraulic characteristics, and the shape of the piano key weirs include: (1) the angle between the sidewall and the main direction of the flow ( $\alpha$ ) (using this parameter, the weir plan can vary from the rectangular shape ( $\alpha = 0$ ) to the trapezoidal shape ( $\alpha \neq 0$ ), and finally to the triangular form ( $\alpha = \alpha \text{ max}$ )); (2) longitudinal magnification or total length to width ratio of the weir ( $L/Wt$ ); (3) cycle width to crest height ratio ( $w/p$ ) in the labyrinth weir; (4) hydraulic load to weir height ratio ( $H_0/P$ ) (since increasing the hydraulic load reduces the discharge coefficient, the highest efficiency of the weir takes place in low hydraulic load); (5) ratio of inner width of the notch to the cycle width ( $A/w$ ); (6) height of the weir crest ( $P$ ) and thickness of the weir wall ( $T$ ).

Karimi, Attari, Saneie, and Jalili Ghazizadeh (2017) experimentally investigated the discharge coefficient in piano key side weirs. In this study, the discharge coefficient of nine type-C PKSW with different geometries were measured under free and submerged flow conditions and compared with rectangular sharp-crested side weirs (RSW).

Effects of approach Froude number, upstream head, and crest length were investigated. Results showed that the discharge coefficients of PKSWs were significantly higher than RSWs. Moreover, PKSWs with longer crests have higher discharge coefficients, but this effect diminishes at high upstream heads and high Froude numbers. Belzner, Merkel, Gebhardt, and Thorenz (2017) studied and compared piano key and labyrinth weirs (case study: waterways in Germany). This study provides a general review on the study of piano key and labyrinth weirs for free and submerged flows. Five different types were put into physical and numerical analyses subject to boundary conditions for the normal level of water in the rivers. The results suggested that the submergence sensitivity of the trapezoidal and rectangular labyrinth weirs is slightly higher than that of the piano key weir and triangular labyrinth weir, and that the triangular labyrinth weir has the lowest efficiency amongst all types of weirs. Bremer and Oertel (2017) used a numerical model to study the effect of wall thickness on the discharge coefficient of piano key

weirs. The results showed that the 3D models studied in FLOW-3D provide an efficient analysis of the free flow in piano key weirs. According to the results, the efficiency of PKW<sub>0.05</sub> (minimum wall thickness) is 40% higher than that of PKW (with 4 times thicker walls) at lower discharge rates. Delgado, Paulina, and Camino (2015) used experimental methods to study the discharge coefficient of the trapezoidal labyrinth weirs using hydrodynamic structures installed at the upstream notch of the weir. They conducted their experiment in a 30-cm laboratory flume. The length of the crest of the hydrodynamic structures used in this study was 6 cm, 12 cm, and 18 cm, and three cycles were installed over the upstream notch of the trapezoidal labyrinth weir. The results showed that the structures used in this study, as a hydrodynamic method, had a positive effect on the discharge coefficient and discharge rate of the weir. It should be noted that amongst the hydrodynamic structures used in this study, the one with a crest length of 18 cm achieved the highest hydraulic efficiency. It is important to know that the construction method of these structures is costly and should be taken into consideration in the design.

Crookston, Paxson, and Savage (2012) studied the hydraulic performance of the trapezoidal labyrinth weirs for higher hydraulic load ratios. They carried out their physical model experiments on a rectangular flume and their numerical model experiments using FLOW-3D software and ultimately compared the results of the two methods (physical and numerical). The results showed that FLOW-3D can appropriately predict the relationship between the discharge rate and the hydraulic load of the trapezoidal labyrinth weir obtained from experiments on the physical model, including the hydraulic load upstream of the weir with respect to its crest. Furthermore, the hydraulic design diagrams developed by Crookston (2010) can be acceptable for a hydraulic load ratio of 2 or higher.

## Materials and methods

### Dimensional analysis

Dimensional analysis is one of the basic tools of experimental studies aiming to calculate the dimensionless

quantities. Accordingly, as the first step, we must identify the parameters affecting the discharge coefficient of trapezoidal labyrinth weirs and piano key weirs, and then use Buckingham's  $\pi$  theory to determine the dimensionless parameters, using which their effect on the discharge coefficient of the weirs can be investigated so as to find logical relationships between them.

### Dimensional analysis of piano key weirs

Generally, the quantities affecting the discharge coefficient of piano key weirs can be presented as follows:

$$f(W_c, H_c, L_c, S_c, A_c, n_c, T, P, P_i, P_o, W_i, W_o, B_i, B_o, B, S_i, S_o, w, W_t, L_{pk}, n_{pk}, Q, V_m, \gamma_m, H_0, g, \mu, \rho, \sigma, \nu) = 0 \quad (2)$$

The parameters affecting the geometry of the channel include its width ( $W_c$ ), depth ( $H_c$ ), length ( $L_c$ ), slope ( $S_c$ ), cross-sectional area ( $A_c$ ), and roughness ( $n_c$ ). The geometric parameters of the weir are thickness of the weir crest ( $T$ ), weir height from crest to the bottom at the upstream ( $P$ ), crest height at the inlet key ( $P_i$ ), crest height at the outlet key ( $P_o$ ), width of the inlet key ( $W_i$ ), width of the outlet key ( $W_o$ ), overhang length of the inlet key ( $B_i$ ), overhang length of the outlet key ( $B_o$ ), length of the sidewall ( $B$ ), slope of the inlet floor ( $S_i$ ), slope of the outlet floor ( $S_o$ ), width of the cycle ( $W$ ), total width of the weir ( $W_t$ ), length of the weir crest ( $L_{pk}$ ), and number of the keys ( $n_{pk}$ ). The parameters related to the flow and the fluid properties are similar to those of the trapezoidal labyrinth weir (previous dimensional analysis). Using the dimensional analysis techniques and combining the parameters of Equation (2), the final relationship between the dimensionless parameters affecting the discharge coefficient of the piano key weirs in this study would be expressed by Equation (3):

$$c_d = f(H_0/p, n_{pk}) \quad (3)$$

## Experimental and numerical models

### Experimental model

The experimental studies in the present research were conducted at Applied Hydraulic Laboratory of the Irrigation Department of Water and Power Industries Higher Education Center in Ahwaz-Khuzestan. The laboratory flume used in the present study was manufactured by the Gant Company of Germany. In terms of geometric specifications, this flume has a rectangular cross-section with a length of 12.5 m and a width of 30 cm. At the two ends of the flume, there are reservoirs that direct the water in and out of the flume. Walls of the flume are transparent and made of 10-mm glass, which is suitable for viewing the water surface profile and hydraulic phenomena. The floor of the flume is made of polished metal. The device was equipped with a mechanical jack to adjust the slope to create and study subcritical, critical, and supercritical flow regimes. Four tanks (each with a capacity of 1100 liters of water) were placed under the main body of the flume. The tanks were connected to each other in series to supply the water required in the flume (Figure 2).

After setting up the laboratory models, water was pumped to the laboratory flume and the required parameters, including the level of water and discharge rate, were measured. Prior to the drop in the water level, the height of the upstream water was measured by a  $\pm 1$  mm point gauge, and the discharge rate was measured by a digital discharge rate measuring device with a percentage error of 5%. A PVC baffle was placed at the flume inlet in order to stabilize the upstream flow and reduce fluctuations at the water surface. In this study, a total of 41 discharge rates ranging from 5 to 70 m<sup>3</sup>/h in free and submerged states were generated in the flume. It should be noted that, considering the discharging system of the flume, all the experiments were conducted without controlling the downstream.

Generally, the weirs used in the present study were designed and built as per recommended by the earlier studies. Accordingly, four weirs including two



Figure 2. General image of the flume and equipment used in the laboratory.

**Table 1.** Specifications of the weir models used in this study.

Name of the weir	Number of cycles	L(cm) Total length of weir crest	W(cm) Total width of weir crest	P(cm) Total height of weir crest	W/P
Piano key triangular	2	221.5	15	7.6	3.039
Piano key triangular	2	221.2	15	5.1	4.54
Piano key rectangular	2	221.5	15	7.5	3.08
Piano key rectangular	2	221.2	15	5	4.64

rectangular piano key weirs and two triangular piano key weirs were used in the experiments (Table 1). In terms of geometric specifications, the rectangular piano key weirs consisted of two cycles with heights of 5 cm and 5.7 cm, and the triangular piano key weirs consisted of two cycles with heights of 1.5 cm and 6.7 cm. Figure 3 shows an image of the dimensions of the prepared piano key weirs with a various number of cycles. Table 2 shows the specifications of the prepared models. The said weirs were made of Plexiglas with a thickness of 5 mm, as this material can be machined decently and accurately. It is necessary to mention that during the installation of weirs, their horizontal level was checked by a bubble level. The perpendicularity of the walls on the flume floor was also checked both during the installation and after directing the water to the weir.

### Numerical model

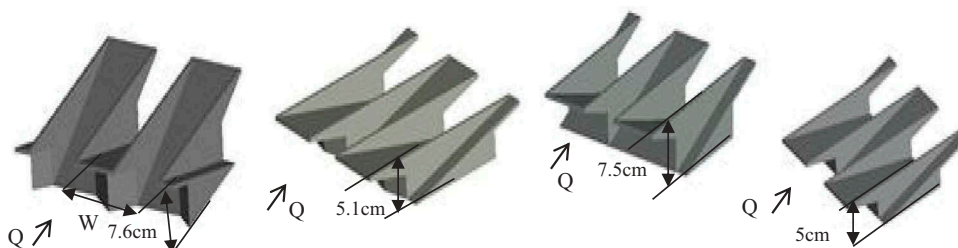
FLOW-3D software is a suitable model with extensive application in the analysis of complicated fluid problems such as free-surface transient three-dimensional flow problems with complicated geometries. This software uses the finite volume method along with regular rectangular grids. Due to the use of the finite volume model in a regular grid, the form of the discrete equations used in the software is similar to those in the finite difference method. Usually, the two-equation models are used to simulate turbulence in hydraulic problems. In this study, the RNG model was used to obtain time-averaged Reynolds equations. Moreover, the FLOW-3D software was also used to solve the numerical problems and the transient governing equations were numerically solved using the finite volume model. In this software, the Fractional Area-Volume Obstacle Representation (FAVOR) algorithm was used to define the geometry in the finite volume method. This algorithm considers the in-field obstacles in the computational cells as a fractional value between 0 and 1, so that if the entire cell is filled by the

obstacles, the fractional value of area-value would be equal to 1. Free surface of the flow is determined using the volume of fluid (VOF) algorithm. Velocity and pressure terms are implicitly coupled with the continuity and momentum equations using the pressure and velocity values at earlier times. In this software, the resulting quasi-implicit equations are solved iteratively using reduction techniques. In this article, the GMRES technique is used as the implicit solver of pressure (Maroosi, Roshan, & Sarkardeh, 2014).

The FLOW-3D numerical model generates a three-dimensional structural grid consisting of cuboid cells for the considered field. Accordingly, a three-dimensional model was developed in AutoCAD based on the specifications of the laboratory models. The results were then imported to FLOW-3D to generate the grid using VOF and FAVOR and determine the boundaries and the computational network. After importing the geometric data into the software and determining the boundaries of the main and secondary channels, the considered area was meshed using VOF and FAVOR methods. The optimum dimensions of the grid were selected based on the accuracy and time allocated to the calculations, and the field grid was adjusted in a way that the grid lines were orthogonal. In this study, an average of 2065012 cells was selected for the three mesh blocks each with dimensions of 25x25x25cm. After generating the computational grid, the boundary conditions, and the initial conditions, the flow of water was simulated (Figure 4).

### Experiments

In order to compare the experimental and numerical results, a predefined research plan was prepared. Among all the experimented models, the best weir models in terms of discharge coefficient meeting a specific geometrical criterion (height of the weirs) were selected and simulated in order to make a more accurate comparison. In total, 21 three-dimensional

**Figure 3.** Design and dimensions of triangular and rectangular piano key weir models used in the present study.

**Table 2.** Discharge coefficient results and errors compared in numerical model and laboratory conditions ( $Q = 25\text{m}^3/\text{h}$ ).

Name of the weir	Experimental results		Numerical simulation results		Error percentage	
	Hu(cm)	Hd(cm)	Hu(cm)	Hd(cm)	Hu	Hd
PKW-T-P7.6	10	3.9	9.36	3.34	6.4	14.35
PKW-T-P5.1	8	4	7.47	3.67	6.63	8.25
PKW-R-P7.5	9.9	4	9.6	3.85	3.03	3.75
PKW-R-P5	8	3.9	7.52	3.25	6	16.67

simulations were performed in this study. After creating the geometry of the weirs and importing it into FLOW-3D, the model was meshed and the boundary and inlet conditions were set. Accordingly, 41 experiments and 21 simulations were conducted under physical model and numerical model conditions (62 in total).

## Results and discussion

After performing the experiments and collecting the information, we used the general weir equation (Equation 1) to analyze the discharge rate of the piano key weir. The measured data were used to determine the discharge rate and the total hydraulic load of the weirs placed along the channel.

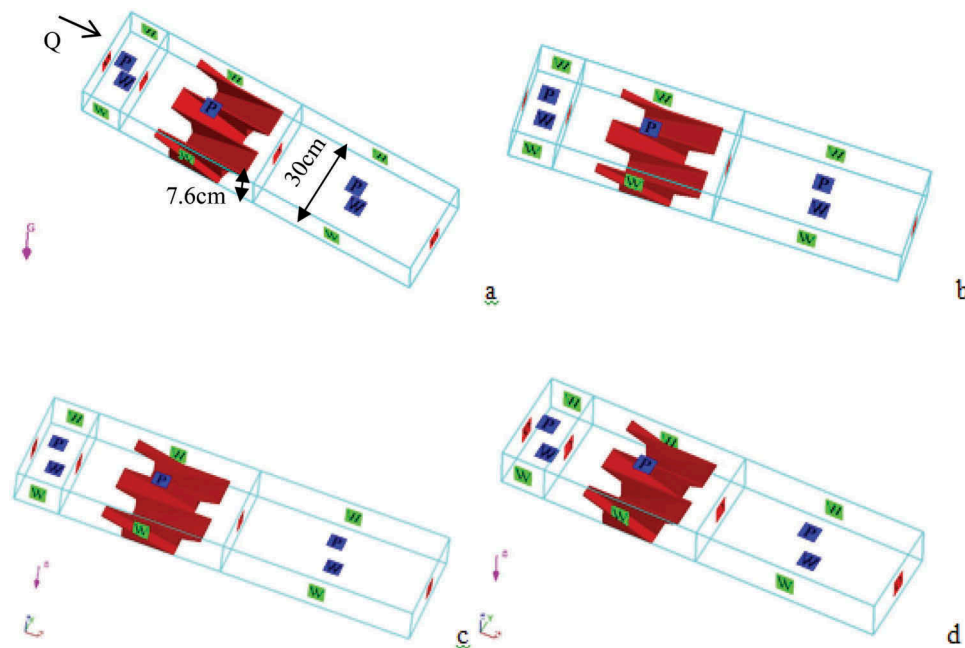
### Hydraulic performance of piano key weir

#### Discharge coefficient diagrams

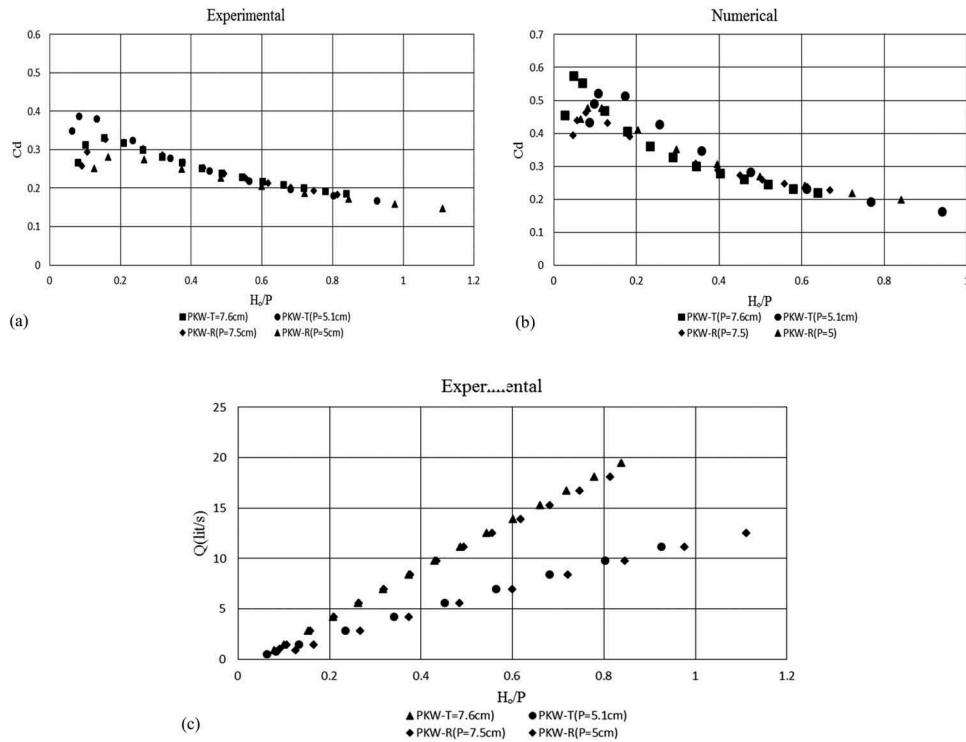
Figure 5 shows that increasing the hydraulic load results in a decreased discharge coefficient. Considering the piano key weirs at a hydraulic load ratio of 0.17, it is observed that the two weirs have

almost the same hydraulic performance. However, by increasing the hydraulic load ratio up to 0.2 and higher, the effect of factors such as interference of flow blades, development of local submergence region, and energy drop is clearly seen on hydraulic performance (especially discharge coefficient) of the piano key weir with a height of 5.7 cm, compared to the one with a height of 5 cm.

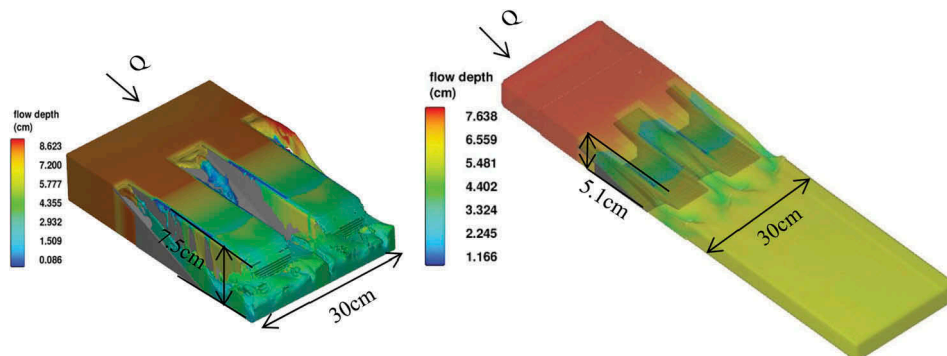
Moreover, the effect of the said factors can be observed in the tangible and significant difference in the reduced discharge coefficient under all hydraulic loads compared to the other types of weirs. Figure 5 shows that under small hydraulic loads, the discharge coefficient of the weirs is increased to the maximum level, such that the piano key weirs have the highest discharge coefficient under all hydraulic loads. On the other hand, the gradual increase of hydraulic loads shall result in reduced discharge coefficient for all types of weirs, which can be due to interference of the flow blades, local submergence, and its increased dimensions at the beginning of the notches of the inlet and outlet keys. Therefore, these conditions have a negative effect on the performance of the piano key weirs. The results suggested that changing the notch of the piano key weir can increase the discharge coefficient by 85%. Figures 5b and 6 are related to the numerical model of the piano key weirs. The figures show that the numerical model is highly capable of simulating the piano key weirs. Having compared the results of numerical and physical models, we observed that the numerical model offers an appropriate performance in simulating the mentioned weirs and that the results of the numerical model were in agreement with those of the laboratory model.



**Figure 4.** Boundary conditions applied to piano key weirs used in this study. (a) Triangular piano key weir with a height of 7.6 cm. (b) Triangular piano key weir with a height of 5.1 cm. (c) Piano key weir with a height of 5 cm. (d) Piano key weir with a height of 7.5 cm.



**Figure 5.** Discharge coefficient ( $c_d$ ) diagram with respect to  $h_a/p$  for piano key weirs in laboratory and numerical model conditions, and comparison of numerical and experimental results.



**Figure 6.** General pattern of simulated flow on triangular and rectangular piano key weirs with heights of 5.7 and 6.7 cm.

## Conclusion

The present study investigated the hydraulic performance of the triangular and rectangular piano key weirs with two cycles placed along the channel in laboratory and compared the results with the numerical model. Accordingly, three dimensionless parameters, namely the hydraulic load to weir crest height ratio ( $H_a/P$ ), number of cycles ( $n$ ), and shape factor ( $Se$ ), were investigated. Moreover, the results of the laboratory model were also compared to those of the numerical model. The results of the study can be summarized as follows:

(a) Having compared the flow coefficient in piano key weirs, it can be concluded that the flow coefficient of piano key weir with a triangular notch in all pre-submergence conditions is greater than that of the plain (rectangular) piano key weir. The largest difference was observed at the

ratio of  $w/p = 4.54$ , where the discharge coefficient in piano key weir was 36% greater than that of the rectangular weir.

(b) Triangular piano key weir with a height of 6.7 cm experiences maximum discharge coefficient in a wider range of hydraulic load ratios. This can be considered an advantage for this weir over the plain piano key weirs, meaning that the weir has reached full aeration in this scenario and the flow over the spillway is of the desired quality as it is well mixed with air.

(c) Having compared the results of the two experiments (physical and numerical), it can be concluded that the FLOW-3D software can accurately predict the relationship between the discharge rate and the hydraulic load of the piano key weirs obtained from experiments on the physical model, including the upstream hydraulic load to crest length ratio.

## Notation

Here is a list of abbreviations used in this study:

B	=	length of the side wall
Bo	=	Length of the outlet key
Bi	=	Length of the inlet key
Cd	=	Dimensionless discharge coefficient
g	=	Gravity
H	=	Piezometric height
Ht	=	Piezometric height (head of upstream weir plus kinetic energy) $\psi$
L	=	Weir Crest
Hu	=	Head of upstream water
Hd	=	Head of downstream water
L	=	Total length of weir crest
Lpk	=	Length of piano key weir
Npk	=	Number of piano keys
n	=	Number of weir cycles
P	=	Total height of weir crest
PKW	=	Piano key weir
PKSW	=	Piano key side weir
RSW	=	Rectangular side weir
PMF	=	Maximum possible flood
Q	=	Discharge
TL	=	Trapezoid piano key weir
Si	=	Slope of the inlet key
So	=	Slope of the outlet key
Ts	=	Thickness of the weir wall
V	=	Flow velocity
W	=	Total width of the weir crest
Wi	=	Width of the inlet key
Wo	=	Width of the outlet key

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