Journal of Spatial Science

ISSN: 1449-8596 (Print) 1836-5655 (Online) Journal homepage: https://www.tandfonline.com/loi/tjss20

Reality measure of the published GPS satellite ephemeris uncertainties

Masoud Shirazian, Iraj Jazireeyan & Mohammad Bagherbandi

To cite this article: Masoud Shirazian, Iraj Jazireeyan & Mohammad Bagherbandi (2020): Reality measure of the published GPS satellite ephemeris uncertainties, Journal of Spatial Science, DOI: 10.1080/14498596.2020.1746702

To link to this article: https://doi.org/10.1080/14498596.2020.1746702

Published online: 23 Apr 2020.

JOURNAL OF SPATIAL SCIENCE
https://doi.org/10.1080/14498596.2020.1746702

Reality measure of the published GPS satellite ephemeris uncertainties

Masoud Shirazian[a], Iraj Jazireeyan[b] and Mohammad Bagherbandi[c]

[a]Department of Geomatics Engineering, Shahid Rajaee Teacher Training University, Tehran, Iran; [b]Faculty of Geodesy & Geomatics Engineering, Khaje Nasir Toosi University, Tehran, Iran; [c]Faculty of Engineering and Sustainable Development, University of Gävle, Gävle, Sweden

ABSTRACT
The international GNSS service (IGS) started publishing the precise ephemeris files in the form of the standard products #3, version C (the sp3c files) in which the GPS satellite orbits and clocks and their uncertainties were available since 2004. Incorporating these uncertainties into the GPS observation equations results in a better stochastic model of the processing system. The reality of these uncertainties is questioned and studied in this paper. Precise point positioning (PPP) model, statistical tests and variance component estimation (VCE) techniques are employed for this study. The results confirm the efficiency of the proposed method in the assessment of reality of the published ephemeris uncertainties.

KEYWORDS
PPP; VCE; precise ephemeris; uncertainties

1. Introduction

Satellite ephemeris, published by the IGS (see Hilla, 2010), used to play the role of fixed known constraints in the system of the GPS observation equations (Beutler et al. 1999 and Dow et al. 2009). After they began to add the ephemeris uncertainties (see JCGM-100, 2008 for details on the term "Uncertainty") to the precise ephemeris files, the use of these values for the GPS processing became possible. Shirazian (2006, 2013a) showed that incorporation of these values improves the stochastic model of the GPS-PPP. Nevertheless, how realistic these values are, is a question that should be properly answered. Griffiths and Ray (2009) and Griffiths (2018) tried to answer this question by considering the discontinuity in the IGS ephemeris or by the elimination of more systematic errors and modifying their models. These modifications were implemented in reprocessing of the IGS data which led to the new realisations of the International Terrestrial Reference Frame (ITRF), namely ITRF2008 and ITRF2014. The details of these frames and their attributed IGS ephemeris files could be found in Griffiths (2018), Rebischung *et al.* (2016), Altamimi *et al.* (2016) and Altamimi *et al.* (2011).

Our approach to answer the question about the ephemeris uncertainties is to elaborate on the observation weight matrix (which is the inverse of the observation covariance matrix), and using an overall model test (see Teunissen 2000), infer whether the covariance matrix is realistic enough or not.

CONTACT Mohammad Bagherbandi modbai@hig.se

In this paper, the inference is conducted through three steps:

(1) GPS undifferenced phase observation equations are formed while the satellite ephemeris entries are used as fixed constraints. Then, the overall model test is carried out.
(2) The satellite coordinates and clocks are used as weighted constraints and then the overall model test is repeated for this case.
(3) Utilising VCE techniques, the weight matrix is balanced in advance, and then after the adjustment of the observation equations with the new weight matrix, the overall model test carried out once again.

Comparing the results of the overall model tests shows that after applying the variance components, the resulting weight matrix turns out to be more realistic. Note that the reason for using undifferenced observation is that the satellite ephemeris takes part in the equations directly and their effect on the estimated parameters is not reduced by differencing. Another restriction is that the method is tested only on the static positioning mode because in this case the number of degrees of freedom of the system of observation equations is big enough for such tests. The method of this study is explained in the sequel.

2. GPS precise point positioning

GPS precise point positioning (PPP) is a processing technique which uses measurements from a single receiver to provide accurate position. Many contributions, e.g. Zumberge *et al*. (1997) and Kouba and Héroux (2001,) initiated and studied PPP and its various aspects. In this paper, the phase-only PPP method, explained by Shirazian (2013a) is used as the mathematical model. Undifferenced ionosphere-free linear combinations of L1 and L2 phase data (L3) are used as raw observations. The observation equation for this observation, which is obtained after some simplification reads:

$$\phi_r^s = \rho_r^s + c(dt_r - dt^s) + T_r^s + \lambda a_r^s + \eta + \epsilon_\phi \tag{1}$$

ϕ_r^s is the L3 carrier phase from satellite s to receiver r, λ is the L3 wavelength (10.3 cm), a_r^s is phase ambiguity, ρ_r^s is the satellite-receiver range, T_r^s is the tropospheric slant delay, dt_r and dt^s are the receiver and satellite clock errors, c is the speed of light and ϵ_ϕ is measurement noise including multipath (Navstar, 2004). The term η contains corrections for various systematic effects (e.g. relativistic effect, tropospheric refraction, satellite and receiver antenna offsets, solid Earth tide and phase wind-up effect) with the amplitude of more than 1 cm that are to be computed from known models (see Shirazian 2013a and IERS Conventions 2010 for more details).

The linearised observation equations in the form of the Gauss–Markov model, for each epoch, are:

$$\mathrm{E}\left\{\underbrace{\begin{pmatrix}\Delta\phi_r^1\\ \Delta\phi_r^2\\ \vdots\\ \Delta\phi_r^m\end{pmatrix}}_{\Delta y_k}\right\} = \underbrace{\begin{pmatrix}-e_r^{1T} & \lambda & 0 & \cdots & 0 & c & M_w(z_r^1)\\ -e_r^{2T} & 0 & \lambda & \cdots & \vdots & c & M_w(z_r^2)\\ \vdots & \vdots & \vdots & \ddots & 0 & \vdots & \vdots\\ -e_r^{mT} & 0 & 0 & \cdots & \lambda & c & M_w(z_r^m)\end{pmatrix}}_{A_k}\underbrace{\begin{pmatrix}\Delta x_r\\ a_r^1\\ a_r^2\\ \vdots\\ a_r^m\\ dt_{rk}\\ D_{wk}\end{pmatrix}}_{\Delta x_k};\ \mathrm{D}\{\Delta y_k\} = q_{y_k} \tag{2}$$

The vector Δy_k consists of the original L3 observations minus computed observations from Equation (1), $E\{.\}$ and $D\{.\}$ are the mathematical expectation and dispersion operators, $e_r^s = \left(e_{r_1}^s, e_{r_2}^s, e_{r_3}^s\right)^T$ is the unit satellite-receiver vector at epoch k, $M_w\left(z_r^s\right)$ is the Neill's wet mapping function when z_r^s is the satellite zenith angle (Niell 1996), D_{wk} is the wet part of zenith tropospheric delay and q_{y_k} is the diagonal covariance matrix of the observations, of which the diagonal entries are the inverse of cosine of the zenith angle of satellites.

$$q_{y_k} = \sigma_0^2\begin{pmatrix}{}^1\!/\cos^2 z_r^1 & 0 & \cdots & 0\\ 0 & {}^1\!/\cos^2 z_r^2 & \cdots & \vdots\\ \vdots & \vdots & \ddots & 0\\ 0 & 0 & \cdots & {}^1\!/\cos^2 z_r^m\end{pmatrix}_k \tag{3}$$

where σ_0 is the observation noise at zenith direction which is around 1 cm (see Shirazian 2013a). As one can use the satellite orbits and clocks in the form of weighted known parameters (Δx_k^s) for positioning purposes, they must be incorporated into the system of observation equations. Then, Equation (2) must be converted to the following equation:

$$\mathrm{E}\left\{\begin{pmatrix}\Delta y_k\\ \Delta x_k^s\end{pmatrix}\right\} = \begin{pmatrix}A_k & A_k^s\\ 0 & I\end{pmatrix}\begin{pmatrix}\Delta x_k\\ \Delta x_k^s\end{pmatrix};\ \mathrm{D}\left\{\begin{pmatrix}\Delta y_k\\ \Delta x_k^s\end{pmatrix}\right\} = \begin{pmatrix}q_{y_k} & 0\\ 0 & Q_k^s\end{pmatrix} \tag{4}$$

where A_k^s, the relevant design matrix of orbits and clock at epoch k, converting orbit and clock errors into the range domain and Q_k^s is the relevant covariance matrix of the orbits and clocks at epoch k and I is the unity matrix of appropriate size. Finally, for the static solution, the covariance matrix for all epochs reads:

$$Q_y = \mathrm{blkdiag}\left(\underbrace{q_{y_1}, q_{y_2}, \cdots, q_{y_n}}_{q_y}, \underbrace{Q_1^s, Q_2^s, \cdots, Q_n^s}_{Q^s}\right) \tag{5}$$

where n is the number of epochs and 'blkdiag' denotes block diagonal matrix. The Gauss–Markov model for the static solution reads then:

$$\mathrm{E}\left\{\begin{pmatrix}\Delta y\\ \Delta x^s\end{pmatrix}\right\} = \begin{pmatrix}A & A^s\\ 0 & I\end{pmatrix}\begin{pmatrix}\Delta x\\ \Delta x^s\end{pmatrix};\ \mathrm{D}\left\{\begin{pmatrix}\Delta y\\ \Delta x^s\end{pmatrix}\right\} = Q_y \tag{6}$$

where $\Delta y = (\Delta y_1, \ \ldots\ , \Delta y_n)^T$, $\Delta x = (\Delta x_1, \ \ldots\ , \Delta x_n)^T$ and $\Delta x^s = \left(\Delta x_1^s, \ \ldots\ , \Delta x_n^s\right)^T$.

Table 1. The results of comparing four different VCE methods.

	MINQUE	BIQUE	HELMERT	LS-VCE
σ_1	0.85634223413487	0.85661304428063	0.85634228018342	0.85661304428062
σ_2	0.54245294349512	0.54240155717123	0.54245293635716	0.54240155717124

3. Variance component estimation (VCE)

In general, the Gauss–Markov model with p variance components is given as:

$$E\{\mathcal{Y}\} = \mathcal{A}\mathcal{X}; \quad \mathcal{Q}_y = \sum_{i=1}^{p} \sigma_i \mathcal{Q}_i \tag{7}$$

where $\sigma_1, \ldots, \sigma_p$ are unknown variance components and $\mathcal{Q}_1, \ldots, \mathcal{Q}_p$ are known cofactor matrices. The estimation of the variance components is the goal of VCE, provided that $E\{\mathcal{Y} - \mathcal{A}\mathcal{X}\} = E\{e\} = 0$ and the cofactor matrices $\mathcal{Q}_1, \ldots, \mathcal{Q}_p$ are linearly independent. In Equations (5), if $Q_y = \underbrace{blkdiag(q_y, 0)}_{Q_1} + \underbrace{blkdiag(0, Q^s)}_{Q_2}$, one can infer that the VCE principals are possible to apply on the PPP model of Equation (6). Having the variance components, the covariance matrix could be balanced and using this new covariance matrix, the more realistic estimates could be obtained.

Various VCE methods are introduced by different scholars, e.g. MINQUE method by Rao (1971), BIQUE method by Sjöberg (1983), Helmert method by Helmert (1924), LS method by Teunissen (1988), etc. (see Amiri-Simkooei 2007 for more details).

To test and compare different methods of VCE, a 24-hours set of GPS data of the 19th of June 2005 for the Brussels' IGS station is selected and processed. To fix the satellite coordinates and clocks, we used the IGS sp3 c files (final orbits and clocks) of the aforementioned days. The number of degrees of freedom is 395. The comparison results are shown in Table 1.

For this comparison, $Q_y = \sigma_1 Q_1 + \sigma_2 Q_2$, with unknown variance components σ_1 and σ_2 is the stochastic model, and the unknown variance components are estimated through the abovementioned VCE methods. As one can see in Table 1, different estimates of the variance components are very close to each other. Among the above methods, BIQUE and LS-VCE possess the best property (Amiri-Simkooei 2007) and, therefore, are better estimators. The BIQUE method is employed in this paper due to its simplicity in implementation. See the Appendix for more information about BIQUE.

4. The overall model test

A test on the variance factor of unit weight $\hat{\sigma}_0^2$, called overall model test (or Bartlett's test), is done to determine if the selected weight matrix is acceptable or there are blunders in the observation vector. An overview of the test is given below (see Kuang 1996, Section 5.3.1):

We require to test the null hypothesis $H_0 : \mathrm{E}\{y\} = Ax$ versus the alternative hypothesis $H_A : \mathrm{E}\{y\} \in \mathbb{R}^m$. From these hypotheses, one can infer these new hypotheses for the overall model test:

$$\begin{cases} H_0 : \mathrm{E}\left\{\hat{\sigma}_0^2\right\} = \sigma_0^2 = 1 \\ H_A : \mathrm{E}\left\{\hat{\sigma}_0^2\right\} \neq \sigma_0^2 = 1 \end{cases} \quad (8)$$

The test statistic is $t = df\hat{\sigma}_0^2 = \hat{e}^T Q_y^{-1}\hat{e}$, where $\hat{e} = y - \hat{y}$ and df is the number of degrees of freedom. The distribution of t is $t \sim \chi^2_{1-\alpha}(df)$, where χ^2 denotes the Chi-square distribution and $1-\alpha$ is the significance level. Equally, one can write $\hat{\sigma}_0^2 \sim \mathrm{F}(1-\alpha,\ df,\ \infty)$; where F denotes the Fisher's distribution (Teunissen 2000). As a two-tailed test, one accepts the null hypothesis H_0 if $k_{\alpha/2} \leq \hat{\sigma}_0^2 \leq k_{1-\alpha/2}$ where $k_\alpha = \mathrm{F}(\alpha,\ df,\ \infty)$, is the critical value at the significance level α and rejects H_0 otherwise. This two-tailed test is selected for the numerical study of this paper.

5. Numerical study

5.1. Data specifications

Data from 14 IGS stations (shown in Table 2) observed for different full days (24 hours) are processed. The geographical distribution of the points is shown in Figure 1. The IGS final orbit and clock files are used throughout the PPP processing. These files are partially (up to 2010) produced before the IGS reprocessing campaigns and the rest belong to the reprocessed ones. The data are processed at the observation time interval of 15 min. The results are tabulated and discussed in the sequel.

5.2. The analysis strategy and results

The data analysis in this paper consists of three steps:

(1) Static PPP solution of the station coordinates while the satellite coordinates and clocks are fixed (their uncertainties are not incorporated into the system of observation equations),
(2) Static PPP solution of the station coordinates with the satellite ephemeris uncertainties incorporated into the observation equations,
(3) Repetition of step 2 after multiplying cofactor matrices Q_1 and Q_2 by the estimated variance components σ_1 and σ_2 (through the BIQUE method) and balancing the covariance matrix.

At the end of each step, the overall model test (a two-tailed test at a confidence level of 95%) is conducted to assess the influence of the defined covariance matrix of that step. The analysis results are shown in Table 2.

To determine the effect of the IGS reprocessing contributions, the above-mentioned method is tested on the data of 2008 used in Table 2 and their corresponding reprocessed ephemeris files. The results are shown in Table 3.

As one can see in Table 2, the IGS stations using the data of years 2006, 2008, 2010, 2012, 2015 and 2016 are processed. In all cases, the overall model test fails when the ephemeris uncertainties are not incorporated into the system of observation equations. After incorporating them, in more than 38% of cases the test passes. This means that incorporating the uncertainties improves the stochastic model of the PPP process which confirms the conclusion of Shirazian (2013b). In Step 3, after applying the estimated variance components on

Table 2. The IGS stations processed and the date of the observations and their corresponding results (DOY means Day of Year).

				Step 1		Step 2		Step 3			
Year	Station-DOY	$k_{0.025}$	$k_{0.975}$	$\hat{\sigma}_0^2$	Test result	$\hat{\sigma}_0^2$	Test result	σ_1	σ_2	$\hat{\sigma}_0^2$	Test result
2006	ANKR-120	0.870	1.139	6.680	Fail	0.880	Pass	3.550	0.408	1.001	Pass
	BRUS-258	0.866	1.144	6.241	Fail	0.640	Fail	1.385	0.534	0.998	Pass
	BRUS-311	0.865	1.145	4.085	Fail	0.498	Fail	1.423	0.332	0.997	Pass
	BRUS-338	0.870	1.139	6.258	Fail	0.555	Fail	1.035	0.495	0.999	Pass
	BZRG-258	0.867	1.143	5.589	Fail	0.626	Fail	0.814	0.595	1.000	Pass
	BZRG-311	0.868	1.141	10.056	Fail	1.050	Pass	4.031	0.639	0.993	Pass
	BZRG-338	0.871	1.138	7.462	Fail	0.675	Fail	1.771	0.540	0.998	Pass
	DRAO-120	0.873	1.135	5.880	Fail	0.754	Fail	2.172	0.505	0.992	Pass
	DRAO-258	0.874	1.134	3.491	Fail	0.468	Fail	1.542	0.270	0.995	Pass
	DRAO-311	0.875	1.134	7.754	Fail	0.913	Pass	5.100	0.314	0.977	Pass
	DUBO-311	0.870	1.139	7.109	Fail	0.734	Fail	2.635	0.473	0.994	Pass
	DUBO-338	0.877	1.131	8.803	Fail	0.870	Fail	4.184	0.451	0.988	Pass
	FLIN-120	0.873	1.135	6.798	Fail	0.893	Pass	3.072	0.495	0.997	Pass
	FLIN-311	0.875	1.133	8.094	Fail	0.875	Pass	1.700	0.751	0.998	Pass
	GRAZ-004	0.873	1.136	6.338	Fail	0.900	Pass	3.970	0.343	0.984	Pass
	GRAZ-009	0.866	1.144	6.641	Fail	0.755	Fail	0.312	0.832	0.999	Pass
	GRAZ-338	0.867	1.143	4.638	Fail	0.551	Fail	0.964	0.478	0.998	Pass
	POTS-312	0.872	1.136	8.115	Fail	0.818	Fail	1.663	0.695	0.999	Pass
	PRDS-258	0.873	1.135	3.364	Fail	0.472	Fail	1.904	0.206	0.990	Pass
	PRDS-311	0.876	1.132	9.490	Fail	1.154	Fail	5.707	0.487	0.967	Pass
	PRDS-338	0.875	1.133	7.109	Fail	0.632	Fail	1.725	0.501	0.996	Pass
	QUIN-120	0.875	1.134	7.476	Fail	0.953	Pass	3.028	0.587	0.993	Pass
	QUIN-258	0.871	1.138	3.494	Fail	0.424	Fail	0.781	0.358	1.000	Pass
	QUIN-311	0.870	1.139	7.190	Fail	0.753	Fail	3.225	0.405	0.997	Pass
	SASK-120	0.874	1.135	6.594	Fail	0.847	Fail	2.486	0.548	0.996	Pass
	SASK-258	0.871	1.138	3.603	Fail	0.486	Fail	1.562	0.290	0.994	Pass
	SASK-311	0.860	1.151	5.315	Fail	0.487	Fail	0.490	0.486	1.000	Pass
	SOL1-120	0.872	1.137	7.972	Fail	1.024	Pass	4.296	0.471	0.986	Pass
2008	DRAO-003	0.882	1.126	4.435	Fail	0.510	Fail	0.983	0.435	0.999	Pass
	GRAZ-003	0.875	1.133	4.673	Fail	0.447	Fail	0.801	0.394	1.000	Pass
	SASK-003	0.879	1.129	4.743	Fail	0.536	Fail	1.299	0.410	0.996	Pass
	TEHN-003	0.883	1.124	5.483	Fail	0.539	Fail	1.662	0.380	0.996	Pass
2010	TEHN-051	0.877	1.131	5.194	Fail	0.912	Pass	2.085	0.603	0.998	Pass
2012	TEHN-014	0.879	1.129	5.620	Fail	1.018	Pass	1.698	0.828	0.999	Pass
2015	QUIN-044	0.881	1.126	5.020	Fail	0.964	Pass	1.175	0.897	1.000	Pass
	TEHN-044	0.881	1.126	5.694	Fail	1.095	Pass	1.115	1.089	1.000	Pass
2016	ARUC-086	0.878	1.130	6.569	Fail	1.437	Fail	2.470	1.093	0.997	Pass
	TEHN-086	0.880	1.128	7.597	Fail	1.682	Fail	3.041	1.216	0.997	Pass

the covariance matrix, the resulting covariance matrix turns out to be better and all overall model tests pass. If we accept that this new stochastic model is the most realistic one, we can infer that the satellite ephemeris uncertainties are pessimistic. Looking at the column of σ_2 in Table 2, one can find out that all the estimated variance components corresponding to the ephemeris uncertainties up to the year 2008 are much less than one (~0.47 in average). This means that the estimated uncertainties are pessimistic. From year 2010 to 2016, this σ_2 becomes closer to one which means that the uncertainties are more realistic. This is also the case for reprocessed ephemeris in Table 3. This result is consistent with the contributions lead to the ITRF2008 and ITRF2014 (e.g. Griffiths 2018 or Rebischung *et al.* 2016) and confirms that our presented three-step method is an efficient tool to measure the reality of the published ephemeris uncertainties.

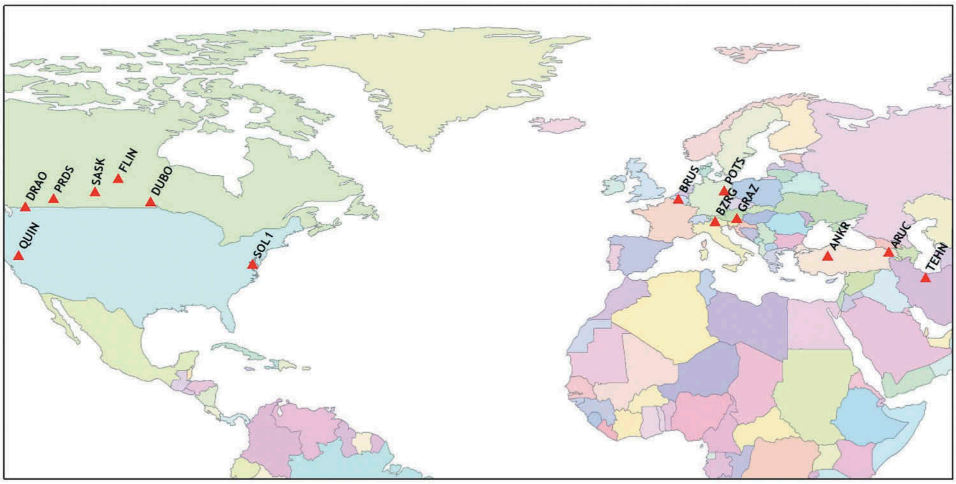

Figure 1. Geographical distribution of the used IGS stations.

Table 3. Repetition of some of the IGS stations processing, using reprocessed ephemeris files.

				Step 1		Step 2		Step 3			
Year	Station-DOY	$k_{0.025}$	$k_{0.975}$	$\hat{\sigma}_0^2$	Test result	$\hat{\sigma}_0^2$	Test result	σ_1	σ_2	$\hat{\sigma}_0^2$	Test result
2008	DRAO-003	0.897	1.129	5.109	Fail	0.968	Pass	1.672	0.753	0.998	Pass
	GRAZ-003	0.875	1.133	4.812	Fail	0.940	Pass	1.369	0.790	0.999	Pass
	SASK-003	0.879	1.129	4.940	Fail	0.976	Pass	1.804	0.717	0.997	Pass
	TEHN-003	0.877	1.131	5.794	Fail	1.131	Pass	1.242	1.093	1.000	Pass

6. Conclusions

A GPS observation data series of 38 days of 14 different IGS stations was processed to measure the reality of the published satellite ephemeris uncertainties. The results confirm the previous contributions mentioning that incorporation of these uncertainties improves the stochastic model of the PPP process. The major conclusion indicates that the published ephemeris uncertainties are pessimistic about two times of their realistic values and after the IGS reprocessing campaign, released in 2017, they became more realistic. This is also worth mentioning that the presented three-step method in this paper is an efficient tool to verify the reality of the published GPS ephemeris by the IGS (or other organisations).

Other complications about the ephemeris uncertainties may come from neglecting the correlations between estimated ephemeris by different analysis centres. This issue is worth studying in the future to estimate more realistic ephemeris uncertainties. The same method is applicable to the other GNSS constellations (GLONASS, GALILEO or BeiDou systems) which should be conducted in the future studies.

Disclosure statement

No potential conflict of interest was reported by the author(s).

References

Altamimi, Z., *et al*., 2016. ITRF2014: a new release of the International Terrestrial Reference Frame modeling nonlinear station motions. *Journal of Geophysical Research: Solid Earth*, 121 (8), 6109–6131.

Altamimi, Z., Collilieux, X., and Métivier, L., 2011. ITRF2008: an improved solution of the international terrestrial reference frame. *Journal of Geodesy*, 85 (8), 457–473. doi:10.1007/s00190-011-0444-4

Amiri-Simkooei, A.R., 2007. Least-squares estimation of variance components: theory and GPS applications. Ph. D. Thesis, Delft Univ. of Tech., delft, The Netherlands. ISBN 978-90-804147-5–4.

Beutler, G., *et al*., 1999. The International GPS Service (IGS): an interdisciplinary service in support of Earth sciences. *Advances in Space Research*, 23 (4), 631–635. doi:10.1016/S0273-1177(99)00160-X

Dow, J.M., Neilan, R.E., and Rizos, C., 2009. The International GNSS Service in a changing landscape of Global Navigation Satellite Systems. *Journal of Geodesy*, 83, 191–198. doi:10.1007/s00190-008-0300-3

Griffiths, J., 2018. Combined orbits and clocks from IGS second reprocessing. *Journal of Geodesy*, 93 (2), 177–195. doi:10.1007/s00190-018-1149-8

Griffiths, J. and Ray, J.R., 2009. On the precision and accuracy of IGS orbits. *Journal of Geodesy*, 83 (3–4), 277–287. doi:10.1007/s00190-008-0237-6

Helmert, F.R., 1924. *Die ausgleichungsrechnung nach der methode der kleinsten quadrate. 3*. Leipzig/ Berlin: AUFL.

Hilla, S., 2010. *The Extended Standard Product 3 Orbit Format (SP3-c)*. Available from: http://igscb.jpl.nasa.gov/igscb/data/format/sp3c.txt

IERS Conventions, 2010. *Gérard Petit and Brian Luzum (eds.)*. (IERS Technical Note; 36). Frankfurt am Main: Verlag des Bundesamts für Kartographie und Geodäsie, 179. ISBN 3-89888-989–6.

JCGM-100, 2008. *Evaluation of measurement data – guide to the expression of uncertainty in measurement*. Joint Committee for Guides in Metrology (JCGM/WG 1). Available from: www.bipm.org

Kouba, J. and Héroux, H., 2001. Precise point positioning using IGS orbit and clock products. *GPS Solutions*, 5 (2), 12–28. doi:10.1007/PL00012883

Kuang, S., 1996. *Geodetic network analysis and optimal design: concepts and applications*. Chelsea, MI: Ann Arbor Press.

Navstar, G. P. S. Joint Program Office. 2004. *Navstar GPS space segment/navigation user interfaces (IS-GPS-200, Revision D). Interface specification, El Segundo (verfügbar unter)*. Available from: http://www.navcen.uscg.gov/gps/modernization/

Niell, A.E., 1996. Global mapping functions for the atmosphere delay at radio wavelengths. *Journal of Geophysical Research*, 101 (B), 3227–3246. doi:10.1029/95JB03048

Rao, C. R., 1971. Estimation of variance and covariance components—MINQUE theory. *Journal of Multivariate Analysis*, 1 (3), 257–275.

Rebischung, P., *et al*., 2016. The IGS contribution to ITRF2014. *Journal of Geodesy*, 90 (7), 611–630. doi:10.1007/s00190-016-0897-6

Shirazian, M., 2006. An attempt on more realistic view of GPS precise point positioning. In *Proceeding of the 3rd ESA workshop on satellite navigation user equipment technologies (NAVITEC 2006)*, 11–13 December 2006. Noordwijk, the Netherlands: ESA/ESTEC.

Shirazian, M., 2013a. Quality description in GPS precise point positioning. Doctoral dissertation, KTH Royal Institute of Technology. ISBN: 973–91–7501–657–3

Shirazian, M., 2013b. Incorporation of the GPS satellite ephemeris covariance matrix into the precise point positioning. *Journal of Geodetic Science*, 3 (3), 143–150. doi:10.2478/jogs-2013-0022

Sjöberg, L.E., 1983. Unbiased estimation of variance-covariance components in condition adjustment with unknowns-a MINQUE approach. *Zeitschrift für Vermessungswesen*, 108 (9), 382–387.
Teunissen, P. J. G., 1988. *Towards a Least-Squares Framework for Adjusting and Testing of both Functional and Stochastic Models Mathematical Geodesy and Positioning Series.* Delft University of Technology, the Netherlands.
Teunissen, P.J.G., 2000. *Testing theory; an introduction*. Delft University Press, 90–93.
Zumberge, J.F., *et al*., 1997. Precise point positioning for the efficient and robust analysis of GPS data from large networks. *Journal of Geophysical Research (B)*, 102, 5005–5017. doi:10.1029/96JB03860

Appendix. A brief expression of BIQUE method in Gauss–Markov model

A reformulation of Equation (7) reads:

$E\{\mathcal{Y}\} = \mathcal{A}\mathcal{X}; \quad \mathcal{Q}y = \sum_{i=1}^{p} \sigma_i \mathcal{Q}_i = E\{ee^T\}$, where e is the residual vector. If the vector of the unknown variance factors is $\sigma = (\sigma_1, \sigma_2, \ldots, \sigma_p)^T$, then the estimator $\hat{\sigma}$ of σ will be:

$\hat{\sigma} = N^{-1}l$,

where the entries of N and l are as follows:

$$N_{ij} = trace\left(R\mathcal{Q}_i R\mathcal{Q}_j\right); \quad i, j = 1, 2, \ldots, p$$

$$l_i = \; e^T R\mathcal{Q}_i Re; \quad i = 1, 2, \ldots, p \text{ and}$$

$$R = \mathcal{Q}y^{-1}\left(I - \mathcal{A}\left(\mathcal{A}^T \mathcal{Q}_y{}^{-1}\mathcal{A}\right)^{-1}\mathcal{A}^T \mathcal{Q}y^{-1}\right)$$