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Graphs with cyclomatic number three having panconnected square, II

Gek L. Chia[a], Wanida Hemakul[b], and Sirirat Singhun[c]

[a]Department of Mathematical and Actuarial Sciences, Universiti Tunku Abdul Rahman, Sungai Long, Malaysia; [b]Department of Mathematics and Computer Science, Faculty of Science, Chulalongkorn University, Bangkok, Thailand; [c]Department of Mathematics, Faculty of Science, Ramkhamhaeng University, Bangkok, Thailand

ABSTRACT
We define three large families of graphs which contain $\mathcal{F}_3, \mathcal{F}_4$ and $\mathcal{F}_7$, respectively as subfamily. Necessary and sufficient conditions for those graphs in these three large families whose squares are panconnected are determined.

KEYWORDS
Cyclomatic number; square of a graph; panconnected

1. Introduction and preliminaries

Recall that the *cyclomatic number* of a connected graph G is defined to be $|E(G)|-|V(G)|+1$ where $V(G)$ and $E(G)$ are the vertex set and the edge set of G, respectively. Graphs with cyclomatic number no more than 2 have been completely determined in [3] and [1]. In the previous paper [2], it was shown that if G has cyclomatic number 3, and G^2 is panconnected, then G belongs to one of the eight families $\mathcal{F}_1, \mathcal{F}_2, \ldots, \mathcal{F}_8$ of graphs (defined in [2]). Moreover, three large families of graphs that contain $\mathcal{F}_1, \mathcal{F}_2$ and $\mathcal{F}_5$ as subfamily were defined. Necessary and sufficient conditions for those graphs in each of these families whose squares are panconnected are determined. In this paper, we do the same for another three families $\mathcal{F}_3, \mathcal{F}_4$ and $\mathcal{F}_7$. We follow most of the notations and terminologies of [2] except for those that are defined or reformulated here.

Let $G(m_1), \ldots, G(m_r)$ denote r SF graphs. If $r \geq 2$, let z_i be a vertex in the cycle of $G(m_i)$ and let $G(m_1, \ldots, m_r)$ denote a *bouquet of r SF graphs* which is the graph obtained by identifying $z_1, \ldots, z_r$ into a single vertex z. If $r=1$, we take $z_1 = z$. Let $\mathfrak{r}$ denote the r-multiset whose elements are $m_1, \ldots, m_r$ and for brevity of notation, we write $G(\mathfrak{r})$ for the graph $G(m_1, \ldots, m_r)$. The graph $G(\mathfrak{r})$ is said to be *broken* if for every $i = 1, 2, \ldots, r$, there exists a vertex w in $G(m_i)$ where $A_w = \emptyset$ and $w \neq z$.

Suppose that $s \geq 3$ is an integer. Let θ_s be a multigraph with 2 vertices, say x and y, together with s multiple edges. Suppose that $n_i \geq 2$ is an integer for each $i = 1, 2, \ldots, s$. Let $\theta(n_1, n_2, \ldots, n_s)$ denote the graph obtained by replacing the edges of θ_s with paths $\mathcal{P}_{n_1}, \mathcal{P}_{n_2}, \ldots, \mathcal{P}_{n_s}$ on $n_1, n_2, \ldots, n_s$ vertices respectively. Note that if $n_s = 2$, then we require that $n_1, n_2, \ldots, n_{s-1} \geq 3$. Let an s-strip cactus graph $\Theta(n_1, n_2, \ldots, n_s)$ where $s \geq 3$ denote any graph obtained by joining each vertex v of $\theta(n_1, n_2, \ldots, n_s)$ to a new set of independent vertices A_v. That is, A_v is the pendant set of v. Again let $\mathfrak{s}$ denote the s-multiset whose elements are $n_1, \ldots, n_s$ and we write $\Theta(\mathfrak{s})$ instead of $\Theta(n_1, n_2, \ldots, n_s)$. Also, we call any vertex $v \in V(\Theta(\mathfrak{s}) - \{x, y\})$ an *s-vertex* if v is a vertex in some path P_{n_i} of $\Theta(\mathfrak{s})$.

Let $X_3(\mathfrak{r}, \mathfrak{s})$ denote the graph obtained from $G(\mathfrak{r})$ and $\Theta(\mathfrak{s})$ by identifying the vertex z with the vertex x. Clearly, $\mathcal{F}_3$ is the set of all graphs $X_3(\mathfrak{r}, \mathfrak{s})$ with $r=1$ and $s=3$.

Let $X_4(\mathfrak{r}, \mathfrak{s})$ denote any graph obtained from $G(\mathfrak{r})$ and $\Theta(\mathfrak{s})$ by identifying the vertex z with any s-vertex of $\Theta(\mathfrak{s})$. Clearly, $\mathcal{F}_4$ is the set of all graphs $X_4(\mathfrak{r}, \mathfrak{s})$ with $r=1$ and $s=3$.

In the next two sections, we completely classify those graphs in $X_3(\mathfrak{r}, \mathfrak{s})$ and $X_4(\mathfrak{r}, \mathfrak{s})$ having panconnected square. Again, we use Fleischner's theorem (see [4]) that states that the square of a connected graph is panconnected if and only if it is Hamilton-connected.

In the last section, we define a large family of graphs, namely $X_7(\mathfrak{r}, \mathfrak{s}, w, z)$, that contains $\mathcal{F}_7$ as a subfamily. We obtain necessary and sufficient conditions for some special cases of graphs in this family whose squares are panconnected.

2. $X_3(\mathfrak{r}, \mathfrak{s})$

Lemma 1. *Suppose $2 \notin \mathfrak{s}$ and assume that $\Theta(\mathfrak{s})$ has at most two paths without vertices of degree 2. Let J_x denote the graph obtained from $\Theta(\mathfrak{s})^2$ by deleting all vertices in $\{x\} \cup A_x$. Then there is an (a,b)-Hamilton path $P(a,b)$ in J_x where a and b are some vertices in $N(x)$.*

Proof. Suppose $\mathcal{P}_{n_i} = w_{i,1} w_{i,2} \cdots w_{i,n_i}$ for each $i \in \{1, 2, \ldots, s\}$. Also assume that $w_{i,1} = x$ and $w_{i,n_i} = y$.

Suppose $\mathcal{P}_{n_1}, \mathcal{P}_{n_2}, \ldots, \mathcal{P}_{n_{s-2}}$ are $s-2$ paths of $\Theta(\mathfrak{s})$ which each contains a vertex of degree 2, $w_{i,j}$. Let

CONTACT Sirirat Singhun sin_sirirat@ru.ac.th Department of Mathematics, Faculty of Science, Ramkhamhaeng University, Bangkok, Thailand.

$P_i(n_i) = w_{i,2}P_{w_{i,2}}w_{i,3}P_{w_{i,3}}\ldots P_{w_{i,j-1}}w_{i,j}P_{w_{i,j+1}}w_{i,j+1}\ldots P_{w_{i,n_i-1}}$ w_{i,n_i-1} if i is odd and let

$P_i(n_i) = w_{i,n_i-1}P_{w_{i,n_i-1}}w_{i,n_i-2}P_{w_{i,n_i-2}}\ldots P_{w_{i,j+1}}w_{i,j}\ P_{w_{i,j-1}}w_{i,j-1}\ldots$ $P_{w_{i,2}}w_{i,2}$ if i is even.

Consider two cases of s.

Case (1): s is odd. Let

$N_1 = w_{s,n_s-1}P_{w_{s,n_s-2}}w_{s,n_s-3}P_{w_{s,n_s-4}}\cdots w_{s,4}P_{w_{s,3}}w_{s,2}\ P_{w_{s,2}}w_{s,3}\cdots$ $P_{w_{s,n_s-3}}w_{s,n_s-2}P_{w_{s,n_s-1}}$ if n_s is odd and let

$N_1 = w_{s,n_s-1}P_{w_{s,n_s-2}}w_{s,n_s-3}\cdots w_{s,3}P_{w_{s,2}}w_{s,2}P_{w_{s,3}}w_{s,4}\cdots\ P_{w_{s,n_s-3}}$ $w_{s,n_s-2}P_{w_{s,n_s-1}}$ if n_s is even.

Case (2): s is even.

Let $N_1 = w_{s,2}P_{w_{s,2}}w_{s,3}P_{w_{s,3}}\cdots P_{w_{s,n_s-2}}w_{s,n_s-1}P_{w_{s,n_s-1}}$.

In either case, let

$$N_2 = P_{w_{s-1,n_{s-1}-1}}w_{s-1,n_{s-1}-1}P_{w_{s-1,n_{s-1}-2}}w_{s-1,n_{s-1}-2}\cdots P_{w_{s-1,2}}w_{s-1,2}.$$

Then we take $a = w_{1,2}$ and $b = w_{s-1,2}$ and a required Hamilton path $P(a,b)$ is given by

$$P_1(n_1)P_yP_2(n_2)\ldots P_{s-2}(n_{s-2})N_1yN_2.$$

□

Theorem 1. *Let G be the graph $X_3(\mathfrak{r},\mathfrak{s})$ where $2 \notin \mathfrak{s}$. Then G^2 is panconnected if and only if $G(\mathfrak{r})$ is broken and $\Theta(\mathfrak{s})$ has at most two paths without vertices of degree 2.*

Proof. If $G(\mathfrak{r})$ is not broken or $\Theta(\mathfrak{s})$ has three or more paths without vertices of degree 2, then replace $\Theta(\mathfrak{s})$ or $G(\mathfrak{r})$, respectively by the complete graph on two vertices. Then respectively we get a non-broken bouquet of r S F graphs or an s-stripe cactus graph having three or more paths without vertices of degree 2. Either case implies that G^2 is not panconnected (by Theorem 4 or 7 of [2]).

To prove the sufficiency, we shall show that for any two vertices u and v in G, there is a Hamilton path $P(u,v)$ in G^2 from u to v.

Case (1): $u \in V(G(\mathfrak{r}))$, $v \in V(\Theta(\mathfrak{s}))$ and $u,v \neq z$.

Then we take $P(u,v) = P_1(u,z)P_2(z,v)$ to be the Hamilton path in G^2 where $P_1(u,z)$ (respectively $P_2(z,v)$) is a Hamilton path in $G(\mathfrak{r})^2$ (respectively $\Theta(\mathfrak{s})^2$) from u to z by Theorem 4 (respectively from z to v by Theorem 7) of [2].

Case (2): $u,v \in V(G(\mathfrak{r}))$.

Then there is a (u,v)-Hamilton path $P_1(u,v)$ in the subgraph $G(\mathfrak{r})^2$ (by Theorem 4 of [2]).

By Lemma 1, there is an (a,b)-Hamilton path $P_2(a,b)$ in $\Theta(\mathfrak{s})^2 - (\{z\} \cup A_z)$ where a and b are some vertices in $N(z) \cap V(\Theta(\mathfrak{s}))$.

We can obtain the required Hamilton path $P(u,v)$ in the following way. If $P_1(u,v)$ contains a subpath of the form zP_z or zw (where $w \in N(z) \cap V(G(\mathfrak{r}))$), then we replace it by $zP_2(a,b)P_z$ or $zP_2(a,b)w$, respectively. If $P_1(u,v)$ contains a subpath of the form $w_1P_zw_2$ (where $w_1,w_2 \in N(z) \cap V(G(\mathfrak{r}))$), we replace it by $w_1P_2(a,b)P_zw_2$.

Case (3): $u,v \in V(\Theta(\mathfrak{s}))$.

Then there is a (u,v)-Hamilton path $P_1(u,v)$ in the subgraph $\Theta(\mathfrak{s})^2$ by Theorem 7 of [2].

By Lemma 1(ii) of [2], there is an (a,b)-Hamilton path $P_2(a,b)$ in $G(\mathfrak{r})^2 - (\{z\} \cup A_z)$ where a and b are some vertices in $N(z) \cap V(G(\mathfrak{r}))$.

We can obtain the required Hamilton path $P(u,v)$ in the following way. If $P_1(u,v)$ contains a subpath of the form zP_z or zw (where $w \in N(z) \cap V(\Theta(\mathfrak{s}))$), then we replace it by $zP_2(a,b)P_z$ or $zP_2(a,b)w$, respectively. If $P_1(u,v)$ contains a subpath of the form $w_1P_zw_2$ (where $w_1,w_2 \in N(x) \cap V(\Theta(\mathfrak{s}))$), we replace it by $w_1P_2(a,b)P_zw_2$.

This completes the proof. □

Theorem 2. *Let G be the graph $X_3(\mathfrak{r},\mathfrak{s})$ where $2 \in \mathfrak{s}$.*

(i) *Suppose $s = 3$. Then G^2 is panconnected if and only if $G(\mathfrak{r})$ is broken and $\Theta(\mathfrak{s})$ has a vertex $u \neq z$ with $A_u = \emptyset$.*

(ii) *Suppose $s \geq 4$. Then G^2 is panconnected if and only if $G(\mathfrak{r})$ is broken and $\Theta(\mathfrak{s})$ has at most two paths (not including xy) without vertices of degree 2.*

Proof. The necessity is established using similar argument as was done in the proof of Theorem 1.

For the sufficiency, we delete the edge xy from G. Then for (i), the resulting graph is a broken bouquet of $r+1$ SF graphs $G(\mathfrak{r}')$ where $\mathfrak{r}' = \mathfrak{r} \cup \{m_{r+1}\}$ and hence has a panconnected square by Theorem 4 of [2]. As for (ii) the resulting graph satisfies the conditions of Theorem 1 and hence its square is panconnected. Either case implies that G^2 is also panconnected. □

3. $X_4(\mathfrak{r},\mathfrak{s})$

Recall that $X_4(\mathfrak{r},\mathfrak{s})$ is obtained from $G(\mathfrak{r})$ and $\Theta(\mathfrak{s})$ by identifying the vertex z with some s-vertex, say w of $\Theta(\mathfrak{s})$. Throughout this section, when we say that $\Theta(\mathfrak{s})$ is a subgraph of $X_4(\mathfrak{r},\mathfrak{s})$, we shall mean that $\Theta(\mathfrak{s})$ is obtained from $X_4(\mathfrak{r},\mathfrak{s})$ by replacing $G(\mathfrak{r})$ with a non-empty set A_w together with all edges joining w and A_z.

Theorem 3. *Let G be the graph $X_4(\mathfrak{r},\mathfrak{s})$ where $2 \notin \mathfrak{s}$. Then G^2 is panconnected if and only if $G(\mathfrak{r})$ is broken and $\Theta(\mathfrak{s})$ has at most two paths without vertices of degree 2.*

Proof. The necessity is established as was done in the proof of Theorem 1.

For the sufficiency, let u and v be any two vertices in G. If $u \in V(G(\mathfrak{r})), v \in V(\Theta(\mathfrak{s}))$ and $u,v \neq z$, then apply the same technique in Case (1) of the proof of Theorem 1, we can get a (u,v)-Hamilton path in G^2.

If $u,v \in V(\Theta(\mathfrak{s}))$, then apply the same technique in Case (3) of the proof of Theorem 1, and again we can get a (u,v)-Hamilton path in G^2.

We are left with the case $u,v \in V(G(\mathfrak{r}))$. For this, all we need is to show that there is an (a,b)-Hamilton path $P(a,b)$ in $\Theta(\mathfrak{s})^2 - (\{z\} \cup A_z)$ (where a and b are some vertices in $N(z) \cap V(\Theta(\mathfrak{s}))$) and then apply the same technique in Case (2) of the proof of Theorem 1 to get a (u,v)-Hamilton path in G^2.

Suppose $z \in \mathcal{P}_{n_1} = w_1 w_2 \cdots w_{n_1}$ where $w_1 = x, w_{n_1} = y$ and $w_j = z$.

Let J denote the graph $\Theta(\mathfrak{s}')$ where $\mathfrak{s}' = \mathfrak{s} - \{n_1\}$. Then J is a subgraph of G.

Suppose P_{n_1} has no vertices of degree 2. Then J^2 has an (x, y)-Hamilton path $P_1(x, y)$ (because J has at most two paths without vertices of degree 2 or J is a broken *SF* graph). Then the Hamilton path $P(a, b)$ is given by

$$w_{j-1}P_{w_{j-1}}w_{j-2}P_{w_{j-2}}\cdots w_2P_{w_2}P_1(x,y)P_{w_{n_1-1}}$$
$$w_{n_1-1}P_{w_{n_1-2}}w_{n_1-2}\cdots P_{w_{j+1}}w_{j+1}$$

where $a = w_{j-1}$ and $b = w_{j+1}$.

Now suppose P_{n_1} has a vertex of degree 2. Then we may assume without loss of generality that w_i is a vertex of degree 2 where $2 \leq i < j$. Then $J^2 - A_x$ has an (x, y)-Hamilton path $P_1(x, y)$ (because J has at most two paths without vertices of degree 2 or J is a broken *SF* graph). Then the Hamilton path $P(a, b)$ is given by

$$w_{j-1}P_{w_{j-1}}w_{j-2}P_{w_{j-2}}\cdots P_{w_{i+1}}w_iP_{w_{i-1}}\cdots P_{w_2}w_2P_xP_1(x,y)$$
$$P_{w_{n_1-1}}w_{n_1-1}\cdots P_{w_{j+1}}w_{j+1}.$$

where again, $a = w_{j-1}$ and $b = w_{j+1}$. □

Analogous to Theorem 2, we have the following result whose proof is very similar to the proof of Theorem 2 and hence is omitted.

Theorem 4. *Let G be the graph $X_4(\mathfrak{r}, \mathfrak{s})$ where $2 \in \mathfrak{s}$.*

(i) *Suppose $s = 3$. Then G^2 is panconnected if and only if $G(\mathfrak{r})$ is broken and $\Theta(\mathfrak{s})$ has a vertex $u \neq z$ with $A_u = \emptyset$.*

(ii) *Suppose $s \geq 4$. Then G^2 is panconnected if and only if $G(\mathfrak{r})$ is broken and $\Theta(\mathfrak{s})$ has at most two paths (not including xy) without vertices of degree 2.*

4. $X_7(\mathfrak{r}, \mathfrak{s}, w, z)$

Let X denote the pseudo graph with 4 vertices x_1, x_2, y_1, y_2 where (i) x_1 is adjacent to x_2 and y_1 is adjacent to y_2 and (ii) there are r (respectively s) multiple edges joining x_1 and y_1 (respectively x_2 and y_2). Subdivide the edges of X so that at most one of the r (respectively s) multiple edges is not subdivided. To each vertex v of the resulting graph, we join a new set of independent vertices A_v (which may be empty). Denote the resulting graph by $X_7(\mathfrak{r}, \mathfrak{s}, w, z)$ where $\mathfrak{r} = \{m_1, m_2, \ldots, m_r\}, \mathfrak{s} = \{n_1, n_2, \ldots, n_s\}$ are two multisets, and $w, z \geq 2$ are integers. Here m_i (respectively n_j) denotes the number of vertices in the path $\mathcal{P}_{m_i}$ (respectively $\mathcal{P}_{n_j}$) from x_1 to y_1 (respectively x_2 to y_2). Also, w (respectively z) denotes the number of vertices in the path $\mathcal{P}_w = c_1c_2\cdots c_w$ (respectively $\mathcal{P}_z = d_1d_2\cdots d_z$) from $x_1 = c_1$ to $x_2 = c_w$ (respectively $y_1 = d_1$ to $y_2 = d_z$).

Clearly, $\mathcal{F}_7$ is the set of all graphs $X_7(\mathfrak{r}, \mathfrak{s}, w, z)$ where $r = 2$ and $s = 2$.

Theorem 5. *Let G denote the graph $X_7(\mathfrak{r}, \mathfrak{s}, w, z)$ with $r = 2 = s$ and $z, w \geq 2$.*

(i) *Suppose $2 \in \mathfrak{r}$ and $2 \in \mathfrak{s}$. Then G^2 is panconnected if and only if G has vertex u with $A_u = \emptyset$.*

(ii) *Suppose $2 \in \mathfrak{r}$ and $2 \notin \mathfrak{s}$. Then G^2 is panconnected if and only if G has vertex u with $A_u = \emptyset$ where $u \notin \{x_2, y_2\}$.*

Proof. We prove the necessity part for (i) and (ii) first. Let $\mathcal{P}_{m_1} = a_1a_2\cdots a_{m_1}$ and $\mathcal{P}_{n_1} = b_1b_2\cdots b_{n_1}$ where $x_1 = a_1, a_{m_1} = y_1, x_2 = b_1, b_{n_1} = y_2$, and $m_1, n_1 \geq 3$. We shall show that there is no Hamilton path from x_1 to a_2 in G^2.

Let H denote the graph obtained from G^2 by deleting x_1 and a_2.

Let $S = \{a_3, \ldots, a_{m_1-1}, b_2, \ldots, b_{n_1-1}, c_2, \ldots, c_{w-1}, d_2, \ldots, d_{z-1}, y_1, x_2, y_2\}$. Then $|S| = m_1 + n_1 + w + z - 6$ and $H - S$ has at least $m_1 + n_1 + w + z - 4$ components (where equality holds only if $2 \in \mathfrak{s}$). This implies that H has no Hamilton path and hence G^2 has no Hamilton path from x_1 to a_2.

Next, we prove the sufficiency.

(i) Assume that G has a vertex u such that $A_u = \emptyset$. By deleting the edges x_1y_1 and x_2y_2, we obtain a broken *SF* graph. Hence $(G - \{x_1y_1, x_2y_2\})^2$ is panconnected by Corollary 2 of [3]. This implies that G^2 is panconnected.

(ii) Assume that G has a vertex u (which is neither x_2 nor y_2) such that $A_u = \emptyset$. Then by deleting the edge x_1y_1, we obtain a 3-stripe cactus graph with a vertex of degree 2 whose square is panconnected by Theorem 7 of [2]. Therefore, G^2 is also panconnected. □

As for the case $2 \notin \mathfrak{r} \cup \mathfrak{s}$ with $r = 2 = s$ and $w, z \geq 2$, we observe that $G = X_7(\mathfrak{r}, \mathfrak{s}, w, z)$ has a panconnected square if and only if G has a vertex of degree 2. However, the proof is omitted since it is too lengthy.

Of course one could also consider other cases (depending on $\mathfrak{r}, r, \mathfrak{s}, s, w, z$) and determine the necessary and sufficient conditions for which the square of $X_7(\mathfrak{r}, \mathfrak{s}, w, z)$ is panconnected. We leave this to the interested reader. Nevertheless, the following comments might be useful.

Recall the following result in [2] (Proposition 3). Suppose $\Theta(m, n, r)$ has no vertex of degree 2 and $m, n, r \geq 3$. Let H be any graph with $|V(H)| \geq 3$ and let u and v be any two vertices in H. Let G denote any graph obtained from $\Theta(m, n, r)$ and H by identifying x with u and y with v respectively. Then G^2 is not panconnected.

Using this result, we can establish easily the following condition for a large class of graphs $X_7(\mathfrak{r}, \mathfrak{s}, w, z)$ that do not have a panconnected square.

Note that the graph $X_7(\mathfrak{r}, \mathfrak{s}, w, z)$ consists of an r-strip cactus graph $\Theta(\mathfrak{r})$ and an s-strip cactus graph $\Theta(\mathfrak{s})$ together with paths $\mathcal{P}_w$ and $\mathcal{P}_z$ joining these two cactus graphs.

Lemma 2. *Let G denote the graph $X_7(\mathfrak{r}, \mathfrak{s}, w, z)$ where $2 \notin \mathfrak{r} \cup \mathfrak{s}$ and $r, s \geq 3$. If the subgraph $\Theta(\mathfrak{r})$ or $\Theta(\mathfrak{s})$ has 3 or more paths having no vertex of degree 2, then G^2 is not panconnected.*

Recall that if $\Theta(\mathfrak{r})$ is such that $r \geq 3$ and $2 \notin \mathfrak{r}$, then $\Theta(\mathfrak{r})^2$ is panconnected if and only if $\Theta(\mathfrak{r})$ has at most 2 paths without vertices of degree 2 (see [2] Theorem 7).

We shall end this paper by stating without proof the following result. The proof for the necessity part nevertheless follows immediately from Lemma 2.

Theorem 6. *Let G denote the graph $X_7(\mathfrak{r}, \mathfrak{s}, w, z)$ where $2 \notin \mathfrak{r} \cup \mathfrak{s}$ and $r, s \geq 3$. Then G^2 is panconnected if and only if (i) each subgraph $\Theta(\mathfrak{r})$ and $\Theta(\mathfrak{s})$ has a panconnected square and (ii) $\mathcal{P}_w \cup \mathcal{P}_z$ contains a vertex of degree 2.*

Disclosure statement

The authors declare no conflict of interest.

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