2011-10-03

Teachers' Decisions to Use Student Input During Class Discussion

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A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of

Master of Arts

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December 2011

ABSTRACT

Teachers Decisions to Use Student Input During Class Discussion

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One of the most vital decisions that teachers make during classroom discussion is whether to and how to validate the thinking that students present to the class. In this thesis I describe a study that addressed the issues that are associated with the decisions that teachers make in regards to validating students' ideas. Through qualitative research I explored these issues through videotaping an expert teacher, taking field notes, and conducting interviews. I share a description of what it looks like for one middle school mathematics teacher to make different decisions to use student input during class. The expert teacher in this study chose to use student input more than any other decision that she could have made. This study can help pre-service teachers and teachers learn to use student input.

Keywords: teacher decisions, class discussion, student input

Contents

List of Figures

List of Tables

Chapter 1: Introduction

Many sources in literature claim that using student thinking during class discussion is beneficial (Doerr, 2006; Franke & Kazemi, 2001; Peterson & Leatham, 2009). Class discussion is beneficial when students can participate in social endeavors about mathematics because the communication helps students to learn and gain mathematical knowledge (Cobb, Wood, Yackel, & McNeal, 1992; Hiebert & Wearne, 1993; Lo, Wheatley, & Smith, 1994). As students are given a chance to communicate their thinking, they are given an opportunity to bump up against mathematical ideas that don't go along with their already constructed thoughts (Hiebert, 1992; Simon, 1995) and thus alter the knowledge that they have previously constructed. As students explain their thinking to others during class discussion the explanation of ideas helps to solidify understanding (Chamberlin, 2005; Fraivillig, Murphy, & Fuson, 1999; Rosenthal, 1995; Wood, 1998). The student who is explaining is forced to verbalize their mathematical thinking so that other students can understand. Thus the student who is explaining their thinking deepens their mathematical understanding.

Although many in literature have claimed that using student thinking during class discussion is beneficial it has also been described as a challenge for teachers (Doerr, 2006; Franke & Kazemi, 2001; Peterson & Leatham, 2009; Sherin, 2002). One factor that makes conducting classroom discussion difficult is the many tasks teachers have to attend to (e.g. keeping students on task, listening to what students say, deciding if what a student says has mathematical merit, making sure the goals of the lesson are reached) (McCrone, 2005; National Council of Teachers of Mathematics, 2007; Simon, 1995; Simon & Schifter, 1991). Another challenge in orchestrating class discussion is there are more student mathematical ideas presented than can be discussed (Ball, 1993; Leikin & Dinur, 2007; Sherin, 2002; van Zee &

Minstrell, 1997). Adding to the challenge of conducting class discussion is many teachers have not seen a good example of how to conduct whole-class discussion (Nathan & Knuth, 2003; Tyminski, 2010).

While it has been stated by many that it is beneficial to use student thinking during class discussion part of the challenge is currently there is no complete model for what a good class discussion using student thinking looks like. Although Stein, Engle, Smith, and Hughes (2008) presented a model of five practices that can help teachers to respond to student thinking during class discussion, the authors claimed that their model was incomplete and more research should be done in the area of teachers' decisions during class discussion. In discussing a classroom model Hiebert and Wearne (1993) mentioned that classroom discourse is an aspect of the classroom model that needs to be accounted for when representing the relationship between teaching and learning. Thus there is a call for more research on class discussion that includes discourse and teacher's decisions.

A way to alleviate the challenge of not having a model for class discussion is to have more research on teachers' decisions during class discussion. The reason why more research on teacher's decisions during class discussion would help with creating a model for class discussion is a teacher's decisions during class discussion determine the direction that the discussion takes. A major role of a teacher is to assist students in learning how to communicate about mathematics (Lo, et al., 1994; Nassaji & Wells, 2000; Wood, 1998). Part of the role of the teacher is to listen to and elicit student thinking. A teacher must decide what to do with student thinking (Franke & Kazemi, 2001). With student thinking the teacher can help form classroom discussion (McCrone, 2005). Thus a teacher helps to determine how a discussion is conducted in the

classroom. Therefore, there is a need to further study teachers' decisions that are made during whole-class discussion.

My study will help add to a model of how to conduct class discussion by focusing specifically on the decisions that a teacher makes in response to student thinking.

Chapter 2: Theoretical Framework and Literature Review

A teacher decision has many definitions and can occur in various forms. In the subsequent paragraphs the following will be given: a description of the definition for a teacher decision that I used during data collection and analysis and a description of the lens I looked through to analyze teacher decisions.

Decisions

A decision in general is making a choice to do or not do an action whether conscious or subconscious (Tyminski, 2010). A decision is difficult to define because distinguishing between whether or not a situation warranted a decision can be debated. Decisions are sometimes a deliberate selection (Clark & Peterson, 1986); however, not all decisions are deliberate. For example, some decisions may be made out of routine.

One specific type of decision is an interactive decision. An interactive decision occurs as teachers interact with their students (Borko & Shavelson, 1990). When a teacher interacts with his or her students often the teacher must choose to execute a particular action (Clark & Peterson, 1986; Schoenfeld, 2008). These interactive decisions differ from other decisions that a teacher makes because often the teacher is required to make a decision without having time to reflect on the decision (Borko & Shavelson, 1990).

There are many times during a lesson that interactive decisions take place. The focus of my study was when interactive decisions were made during whole class discussion. The reasons I chose to focus on conducting class discussions are because it presented a challenge for me in my personal teaching and it presents a challenge for other teachers as well (Doerr, 2006; Franke & Kazemi, 2001; Peterson & Leatham, 2009; Sherin, 2002). Lo et al. (1994) described "mathematics class discussion" (p. 32) as activity in a classroom which has student explanations

of tasks and student-to-student communication, with the teacher's job being to assist student-to-student exchange of ideas rather than to clarify or assess. Taking this definition of mathematics class discussion, whole class discussion will be defined as moments during the lesson where attention of all members in the class would be focused on explanations, justifications, descriptions, and conversations of mathematical ideas and concepts together as a class.

Although all of the decisions that a teacher makes are important and can lead to the outcome of a lesson, the only decisions that were focused on in my study were the decisions that a teacher makes during class discussion and specifically the decisions that a teacher makes in response to student input. One of the common themes in the literature is that decision making occurs when things don't go according to plan (Shavelson & Stern, 1981); thus the teacher has the decision to either continue the lesson or change it. While the decisions that I categorized probably fit into the categories of the teacher either continuing the lesson or changing it, I wanted to dig deeper to the specific types of decisions that the teacher makes and why.

In order to dig deeper I looked at a decision to use student verbalizations, which builds on the idea of an interactive decision. Student verbalizations are often termed "thinking"; however the term "thinking" in mathematics education is hard to identify and can sometimes lead to confusion. For this reason I have decided to focus on what I will term student input. Student input is any mathematical verbal idea that a student presents during class discussion. Thus a decision to use student input is reflected in a choice that a teacher must make in response to that input.

When I collected and analyzed data, I looked for a change in speaker during class discussion to help me determine whether or not a decision to use student input had occurred. A change in speaker occurred when the speaker switched from one student to another student, from

a student to the teacher (Ms. Cook[1]), or Ms. Cook to a student. The change in speakers resulted in the teacher deciding that the student verbalization needed to be addressed by the teacher (or not) or could be addressed by other students.

Lens

As was discussed in the rationale, a teacher's decisions affect the direction the class discussion follows. Thus all types of decisions that a teacher uses in regards to student input are important and lead to the questions: What exactly does a teacher making a decision to use student input look like? How, when, and why does a teacher use student input?

Along with teacher decisions there were four variables that were used as a lens to answer the research questions given above: student input[2], teacher responses, why the teacher responds the way they do, and time. During the lesson I looked for a variety of student input, teacher responses, and time during the lesson in order to provide adequate data for analysis. The variable of why the teacher responds the way they do to student input is not a variable that can be selected for analysis because it depends solely on the response of the teacher during the post-lesson interview. The data were analyzed according to connections between these four variables (as illustrated in Figure 1). In the following sections I describe my view of these connections and discuss what existing research has found thus far related to them.

[1] Ms. Cook and all student names are pseudonyms.
[2] More description of student input, teacher decisions, and time will be given in the methodology chapter.

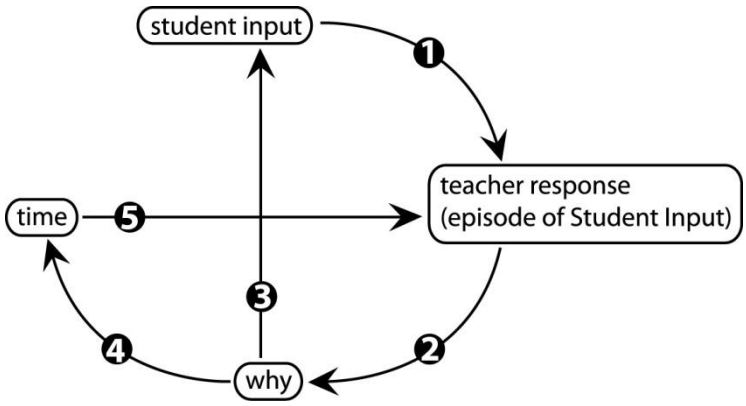

Figure 1

Connection 1. Connection number one is the connection between student input and teacher response. This connection was analyzed to see if there was any correlation between the different types of student input and the different types of teacher response. For example, 90% of the time student input that is a question may relate to the teacher response being the teacher merely talking about the question. Thus in the lesson observation there was a variety of student input and a variety of teacher responses in order to see if there were connections between the types of input and the teacher responses.

Others have also studied this connection between student input and teacher response. One of the main ideas associated with connection one is what a teacher does in response to student input. With all types of student input a teacher can make the same decisions. In response to student input a teacher could choose to request an explanation from the student, make an interpretation as the teacher, request other students to make arguments against the student input, or request an explanation from other students (Lampert, 2001).

Another important aspect of connection one is student input that seems to align with the teacher's goals for the lesson and what the teacher does in response to that input. When student input appears to align with the teacher's goals that teacher can make a few different decisions.

One of the decisions is to let students discuss the idea, then interrupt at a point, say the idea is not relevant at the moment, and ask for more ideas to be shared (Speer & Wagner, 2009). Another decision is to present an idea and have the students discuss the idea (Speer & Wagner, 2009).

Maybe even more important than student input that appears to align with the teacher's goal are types of student input that are not aligned with goals for the lesson, unanticipated by the teacher, or are non-standard. In relation to student input that may not seem to be aligned with the teacher's goal for the lesson a teacher has a few decisions they can make. One of the decisions that a teacher can make is to try to understand what the student presented and then tell students that they can pursue the idea outside of class (Speer & Wagner, 2009). Another decision that can be made is the teacher presents a problem without acknowledging undesired suggestions (Speer & Wagner, 2009).

Unanticipated student input can cause a teacher to pause in their teaching, but there are many different decisions that a teacher can do to respond to the input. Often during class discussion a student presents an idea that was not anticipated by the teacher and the teacher cannot follow the plan that she had made before hand (McClain, 2002). This illustrates a specific type of student input, which is a variable of connection number one. When a student first presents an idea that is not anticipated a teacher can step out of the conversation and let students discuss the idea that was presented while the teacher tries to make sense of the situation (McClain, 2002). Another decision is the teacher rephrases what the student has presented in terms of how the teacher interpreted what the student said (McClain, 2002). Another decision that can be made in association with unanticipated student input is to not address the issue at the time, but to come back to the student idea at a later time (McClain, 2002).

Finally there is the issue of non-standard student input. Non-standard student input is input that is not aligned with mathematical convention (Ball, 1993). For example in study conducted by Ball (1993) a student presented an idea of what was termed Sean numbers. A Sean number is the idea that a number can be both even and odd. For example, six is both even and odd because it can be split into two groups and three groups, two is even and three is odd. As part of the results Ball said that often teachers must debate about whether or not it is worthwhile to validate non-standard ideas. In the situation of the Sean numbers Ball decided to validate Sean's idea. Thus in relation to a non-standard student input a teacher can decide to validate or not validate the student input. In relation to the student input and the teacher response it is important to know why the teacher responds the way that they do, because sometimes without knowing why the teacher made their decision the decision may not make sense.

Connection 2. The next connection that was analyzed was number two: the connection between the teacher responses and why the teacher did what she did. This connection was of most interest because it helped to answer the question of why a teacher decides to use student input. In this connection the only variable that could be controlled in terms of collection was teacher response. The variable "why" was not a variable that could be seen during observation, but came from the post-lesson interviews.

Many studies have focused on why teachers make decisions in response to student input. One of the reasons presented for why teachers made the decisions they did was the decisions were guided by the teacher's goals or "agenda" for the class even though the agenda or plan for the lesson was often revised in reaction to what students would do and present (McClain, 2002; Schoenfeld, 2010). The teacher from my study, Ms. Cook, said that her goals or agenda also played a role in why she made the decisions that she did. Another reason that has been given for

why teachers make the decisions they do is in order to give students an opportunity to do mathematics like mathematicians do (Ball, 1993). This reason for the response to use student input was because of other goals outside of the lesson that the teacher had.

Many teachers work hard to establish the environment that they have in their classrooms. Thus another reason given for why teachers make the decisions they do is in order to introduce students to productive mathematical habits and in order to create the mathematical community that the teacher wants to exist (Schoenfeld, 2010). Along with the environment in the classroom teachers have to keep track of time in their classes. Therefore another reason for making a decision is to keep the discussion at a good pace (Lampert, 2001).

A number of reasons for teacher decisions were given above. In my study I hope to be able to add to the list given by associating the reasons for the decision with a specific type of decision that the teacher made.

Connection 3. Connection number three is the connection between why and student input. This connection was presented because the reason why a teacher makes a decision may be in response to the type of student input. For example, if the student input was a solution Ms. Cook may have decided to incorporate the solution into the discussion in order to correct or clarify the solution. Thus the variable of why was analyzed to see if the type of student input had an impact on why Ms. Cook did what she did. While none of the reviewed literature directly talked about the connection between student input and why decisions were made, student input was discussed in connection number one and why was discussed in connection number two.

Connection 4. Connection number four is the connection between *why* and *time*. I use the variable *time* in two different ways. One way that I will refer to time is in relation to time constraints. For example, Ms. Cook may decide not to incorporate the student input into the class

discussion because there are only five minutes left of the class and Ms. Cook wanted to hand out a homework assignment. Thus when I refer to time in this manner from here on out I will refer to it as *time constraints*. The other way that I will refer to time is in reference to the time of the lesson (opener, homework questions, launch of task, and conclusion of task). This is when the purpose of goal for the different lesson segments plays a role into what Ms. Cook decides. For example, if during the opener a student presents an idea that is related to the mathematics that is being discussed, but is beyond the scope of what Ms. Cook wanted to talk about during the opener, Ms. Cook may decide not to pursue the idea. This reference to time will be referred to as the *lesson segment* from here on. The lesson segments can easily be traced. I made sure to have a variety of episodes of student input that came from the different lesson segments of the opener, homework questions, the launch of the task, and the conclusion of the task.

Others have talked about the dilemma of time when talking about why to make decisions while teaching. In determining whether or not to validate student input that is not standard one must take into account the time that it would take to explore such an idea (Ball, 1993). It is also important to consider cost when making a decision. Cost is what educational effect the decision has on the lesson or the time the decision might take to implement (Schoenfeld, 2008). Thus time does affect a teacher's responses.

Connection 5. The last connection (number five) is the connection between teacher response and the lesson segment. Connections four and five are closely related because the way the teacher responds to student input may be dependent on time constraints or the lesson segment. Connection five was important because depending on the lesson segment the teacher might make a different decision than if the student input occurred during a different lesson segment. For example, if the lesson segment was the beginning of the lesson the teacher may

have decided to merely talk about student input. Thus the lesson segment was noted along with the teacher response to become aware of patterns that might occur between the lesson segment and the type of response to student input.

Chapter 3: Methodology

The layout of the methodology section will follow the chronological order in which data were collected. The first section is a description of what I was looking for in the class periods that I observed. The second section is a description of the type of study that was conducted and the participants in the study. The next section is the preparation that took place before data was collected. The fourth section is how data was actually collected. The final section is the analysis of the data.

Description of What I Looked for in Classroom Observations

Student Input. As was mentioned earlier, student input is identified as instances when a mathematical verbalization is made by a student during class discussion. Student input can appear in many different forms (Nathan & Knuth, 2003). The categorization I am about to describe was not a lens I used to look at the data, but rather an organization tool. The categorization helped me make sure that I had enough information so that when I analyzed I could have a good description. I took the part of the categorization of student input that was presented in Nathan and Knuth (2003) and built on the ideas. Although the authors not only categorized student input, but also input from the teacher to spark student input, I am only focusing on the student input and not necessarily what the teacher did to spark the student input. The categories of student input that I took from Nathan and Knuth are what they termed "ask question-Math", "make presentation to the class", and "response to an open invitation." (p. 184). I however termed the categorizations of student input as *student questions, student solutions,* and *student answers*, respectively. More details will be given below as to what constitute these types of student input. I also felt there might be another category of student input: *student incomplete statements*. I meant for the categorization of student input to be all

inclusive; however as I collected data I realized that I needed to add one more category: *student comment*. Thus in the subsequent paragraphs I will describe in more detail these different forms into which student input was categorized.

Student questions. Again *student questions* were taken from what Nathan and Knuth (2003) "ask question-Math" (p. 184). *Student questions* are when a student has a mathematical inquiry during the discussion. For example, during one of the observed lessons about corresponding parts of congruent figures a *student question* was asked after another student presented a solution for why two triangles were congruent and the corresponding parts were congruent. After the student presented the solution another student asked, "How did you find that P and Q are the same?" The *student question* can either be directed to the teacher, another student, or the class as a whole.

Solution method. Student *solution methods* were taken from what Nathan and Knuth (2003) termed "make presentation to the class" (p. 184). A *solution method* occurs during class discussion when a student presents how he or she arrived at their particular answer. For example, in one of the observed lessons that was about the sum and measure of the interior angles of a polygon a student presented her solution for how she figured out the sum of the interior angles of a pentagon. As the student was presenting her work she said, "So I drew it and it made 3 triangles and there were 5 of the original sides. And then here there are 4 and 6. And then…" A *solution method* may be a partial explanation of what the student did to arrive at an answer, but the *solution method* is more than just the answer to the problem.

Student answers. *Student answers* were developed from what Nathan and Knuth (2003) termed "response to an open invitation" (p. 184). *Student answers* are when the student input is merely a solution to a question or problem with no explanation. For example, during one of the

observed lessons where the topic was the properties of parallelograms the following exchange took place between a student and the teacher:

Ms. Cook: Where's my transversal up there?
Luke: AD to BC

That is an example of an answer because the student merely gives the answer to the question with no explanation.

Incomplete Statements. *Incomplete statements* were added to the categorization during the pilot study. I found that *incomplete statements* were a prevalent type of student input during the pilot study. *Incomplete statements* are when the student input is an unfinished assertion. For example, during one of the observed lessons about the properties of kites a student said, "Um, I'm not sure. I thought it was always true, but…" This is an example of an incomplete statement because the student does not finish what he started to say. Sometimes *incomplete statements* might be confused with *student answers* or *students solutions*. The distinction is *student answers* and *student solutions* are complete statements, where *incomplete statements* are when a student stops their response midsentence.

Student comment. As I collected data I realized that my categorization did not encompass all types of student input. Thus during my study I added the category of *student comment* in order for my categorization of student input to be all inclusive. A *student comment* is a mathematical comment that does not fit into the other categories of student input. An example is given during an observed lesson about quadrilaterals on the coordinate plane. A student made the following comment during a discussion about alternate interior angles inside a quadrilateral: "The same is true on the other side." This is an example of a *student comment*, because the comment is not a question like *student questions* are; the comment is not a *solution method*, but

merely an addition to a student solution; the comment is not in response to a question like *student answers*; and the comment is a complete statement unlike *incomplete statements*.

Teacher Decisions. NCTM (2007) said that mathematics teachers should "orchestrate discourse by…listening carefully to students' ideas and deciding what to pursue in depth from among the ideas that students generate during a discussion" (p. 45). This idea of listening to students and then deciding what to pursue is an element of what will be termed an episode of student input. An episode of student input begins when either a student verbalizes a question or mathematical idea during class discussion or a teacher presents previous student input; the episode continues until discussion of the student input is left.[3] An example of an episode is given in the following excerpt from an observed lesson:

Teacher: Ok if you have a property of a parallelogram that you would like to share, please raise your hand, but don't shout out. Mike?
Mike: The opposite angles are congruent.
Teacher: Ok that is always true. Luke?
Luke: Opposite sides are parallel.

The episode of student input starts with Mike's comment. The episode ends as the teacher goes to Luke for another property of parallelograms and leaves the property that she was discussing with Mike. Thus the teacher moving to a new idea marks the end of the episode.

A teacher has many possible decisions to make in response to student input. According to Sparks-Langer, Pasch, Starko, Moody and Gardner (2000) there are many responses that a teacher might have to student input which range from praising a student to probing them for more information. These responses to student input will be termed teacher's decisions.

[3] The main focus of episodes in my research was when students present input and not when the teacher brings up previous student input. For example when a student presents one of the categories of students input that was mentioned earlier as opposed to the teacher presenting student input that had already been presented.

In relation to episodes of student input a teacher has to make decisions about what ideas to validate and what ideas not to validate (Ball, 1993). For the purposes of my study the responses to student input a teacher makes will be categorized into three broad categories of decisions a teacher can make. The three categories of decisions that a teacher can make, that were used as an organizational tool, were established during a pilot study[4] I conducted. Initially the categorization of episodes of how teachers use student input was not meant to be all inclusive, but after data collection the categorization proved to be all inclusive. The beginning framework of categories of what a teacher can do are *not talk about student input (NT)*, *teacher talk about student input (TT)*, or *run with the idea of the student input (RW)*. The categories were used as an initial categorization at the beginning of my study. Through my study the categories were refined and a presentation of this refinement can be found in the results section.

Not talk about student input. To *not talk about student input* entails either not acknowledging student input or simply acknowledging a student's input and either postponing the discussion for another time or merely moving on with the lesson. One example of *not talk* is given by what Mehan (1979) termed "repeating elicitations"(p. 288). Repeating elicitations is when a student gives an incorrect answer in response to a teacher question and the teacher does not acknowledge the student's answer, but asks the question again for a different student to answer. This is an example of *not talking about student input* because the teacher does not acknowledge the student input and moves onto a different student's input. Another example of *not talk* about student input is when a teacher merely repeats what the student input was and the teacher moves on. This is an example of *not talking about student input* because nothing is said (by the teacher or the students) beyond the initial student input.

[4] For a more detailed description of the pilot study see Appendix A.

Teacher talk about student input. To have *teacher talk* in response to student input is to acknowledge student input and to only have the teacher talk about the student input. An example of *teacher talk* is when, in a lesson about slope and y-intercept, a student asks the teacher about two lines that intersect and if that is the same thing as a y-intercept. The teacher responds by talking about the difference between intercept and intersect without turning to the class for discussion, but by merely talking about the difference herself. Peressini and Knuth (1998) described that often times it is easier for a *teacher to talk* instead of listening to and trying to figure out what students are saying, thus the desire to merely talk about student input without letting students be involved in the talk.

Run with student input. To *run with* the idea of the student input is to acknowledge the student input and to incorporate the student input into class discussion by eliciting participation from other students. Eliciting the participation of other students may be done through asking questions to the class (Nassaji & Wells, 2000), asking for clarification (Wood, 1998), asking what others think (Nathan & Knuth, 2003), etc. Wood (1998) described something similar to running with student input which she termed "focusing" (p. 172). Focusing is the teacher's decision to step out of the class discussion to let the students discuss the mathematics while still being ready to jump in if necessary to slow the pace of the conversation or to add clarification. This type of *running with* might not even need verbal acknowledgement from the teacher; the acknowledgement might be merely letting the students continue with their conversation (Rittenhouse, 1998).

van Zee and Minstrell (1997) also described something similar to *running with student input* which they termed a "reflective toss" (p. 229). A reflective toss is when a student has input, the teacher grabs the sense of the input, and throws the accountability of thinking back to the

students for further discussion. How the teacher decides to toss the thinking back to students can vary. van Zee and Minstrell illustrated that a teacher could ask a single question, ask a series of questions, make a statement, or make a series of statements.

An example of *running with student input* comes from a lesson about graphing. A student asks the teacher what the key elements of graphing are. The teacher then decides to pose the question to the class and the class continues the discussion by brainstorming what key elements are needed to graph. This situation is an example of running with student input because the teacher decided to use the student input posed by the student as a catalyst for class discussion.

There is a possibility that episodes are embedded within other episodes. From the beginning of an episode to the end if the student input is *run with* then there are other students and/or the teacher that will participate in the discussion. When other students participate in the discussion this is the start of another episode of student input. The episode that had already occurred may not have ended thus causing the new episode of student input to be embedded in the initial student input that was presented. There will be a better explanation of how the complexity of the embedded episodes was handled in the description of the coding of the transcript.

Type of Study and Participants

My study was a case study of one teacher. The use of a case study was decided upon because it gave access to an expert teacher's classroom and helped to paint a picture of how an expert teacher makes decisions to use student input while providing for others "a sense of "being there" by providing a highly detailed, contextualized analysis" (VanWynsberghe & Khan, 2007, p. 4) of episodes of student input. Only one teacher was chosen because it is easier to focus on the detail with only one teacher. If there were more teachers more lessons would need to be

observed and that would be beyond the scope of this study. Therefore, the focus for my study was on a junior high school mathematics teacher who was teaching Algebra, Algebra 2, and Geometry and has experience creating a classroom environment that lends to class discussion. The expert teacher, Ms. Cook has a task-based teaching style that produces a lot of class discussion, which is a major focus of my study.

Ms. Cook is an expert teacher because she has a lot of experience establishing a classroom that uses student input. Ms. Cook's classroom has a lot of participation from students in their mathematical class discussions. Students offer solutions, questions, and comments without having to be persuaded by Ms. Cook.

Data collection began during October of the 2010-2011 school year at a local junior high school. The methodology of my study was designed to answer the following research questions: What exactly does an expert teacher making a decision to use student input look like? How, when, and why does an expert teacher use student input?

The answers to the research questions were formed through preparation before data collection; data collection consisting of pre-lesson interview, lesson observation, and post-lesson interview; and data analysis. There was a triangulation of different methods (interview, field notes, video tape, etc.) of collecting data in order to gain a more complete record of what occurred in the classroom (Jick, 1979; Patton, 2002). Data collection and analysis took place in a cyclic process (Clarkson, 2000; Golombek, 1998; Peterson & Clark, 1978) that occurred five times in which five lessons were observed and video-taped along with pre and post-lesson interviews. Five lessons were used for the following reasons: in order to provide at least ten episodes (two from each lesson) of each category (NT, TT, and RW) to be discussed with the teacher and used in analysis, the results of the pilot study showed that there would be at least one

of every type of teacher decision to talk about in each lesson, and to give a variety of lessons to observe. A detailed description of preparation before data collection, data collection, and data analysis will be given below.

Preparation to Collect Data

Preparation took place before the data collection began. I met with Ms. Cook before data collection began in order to plan which days lessons would be observed and in order to give the teacher permission slips for her students. Through the pilot study I realized there are certain lessons and class periods that would not be beneficial to my study, because those lessons would not produce enough or any class discussion to analyze and thus were avoided. The lessons that were avoided were review lessons, tests, shortened class periods, class periods that occur immediately before or after a holiday from school, etc.

Also through the pilot study I was able to learn the type of lesson that I wanted to observe. Lessons in which new mathematical material was discussed were desired in order to make certain that mathematical classroom discussion occurred. There was a plan to observe different lessons through units (e.g. beginning of unit lesson, end of unit lesson) in order to get a variety of classes for analysis and to get a better idea of how the teacher uses student input in different situations. Thus in the meeting with Ms. Cook before data collection we worked out a plan to observe a variety of lessons. There were five lessons that were observed in the middle of the second quarter. The five lessons followed a unit from beginning to the day before the review. Thus the lessons ranged from launching the unit, the middle of the unit, and the conclusion of the unit.

Through the pilot study I also realized how difficult it may be to conduct interviews. Thus I practiced interviewing with a different teacher before data collection commenced in order

to be ready for interviews during data collection. I used the same setup that is explained later in this section with a local secondary teacher that was not the teacher from my study. I did this in order to practice interviewing and to troubleshoot possible dilemmas that could arise. The interview protocol that I used to practice interviewing answered all of the questions that I had for the teacher. Thus I used the same interview protocol in the actual study. The interview protocol also worked well with the teacher that I conducted the study with.

Data Collection

Pre-lesson interview. Data collection started by holding a pre-lesson interview with Ms. Cook. Through the pilot study I learned that aside from the lesson goals it was also helpful to know what the unit goals are, because there might be instances when the teacher uses student input that doesn't go with the lesson goal, but might go with the unit goal. Thus in the interview before the lesson along with asking what the lesson goals were I asked what the unit goals were. This was useful because in knowing what the lesson and unit goals were it helped me to predict why and how the teacher was using student input. During the pre-lesson interview the protocol was to ask the teacher what the unit goals were, what the lesson goals were, what task was used for the lesson, and what the teacher hoped students would do in response to the task questions. Also during the pre-lesson interview I took notes at the top of the field note form (Appendix B) of what the goals were and anything else that the teacher shared about the lesson.

Lesson. Next the lesson was videotaped by a research assistant while I took field notes. The lesson was video-taped to be used for data analysis, which will be described in detail later. During the lesson I took field notes of the lesson on the field note form found in Appendix B. The field notes were useful in linking the video to episodes that were discussed with the teacher.

This idea was taken from Clarke (2001) as the field notes for his research were linked to the time of the video for quick reference.

When I found episodes of interest I marked the time that was in sync with the video, marked the codes for the episode and the student input, and wrote a brief description on the field note form (Appendix B). During the lesson I looked for one or two compelling episodes of each of the three categories of episodes (not talk (NT), teacher talk (TT), and run with (RW)). One to three of each NT, TT, and RW episodes were discussed in the post-lesson interview in order to have sufficient data for each episode and to determine if there was a pattern to how and why the teacher decided to use student input. The field note form was also used to write down possible questions I wanted to ask the teacher during the post-lesson interview. Another use of the field notes was to start analysis of the lesson (Patton, 2002).

Preparation for post-lesson interview. After the lesson, but before the post-lesson interview I did a few things to prepare for the interview. I had a few hours between the lesson and the interview in order to make sure that there was enough time to prepare. I reviewed the field notes in order to review episodes for discussion. Another task that occurred before the interview was I wrote questions to ask the teacher in addition to the interview protocol (Appendix C). I also ordered the episodes in the way they would be discussed during the interview. Episodes were chosen because the type of episode had not been discussed in previous interviews or more episodes of the category of episode were needed in order to gain a better description of the type of episode.

Post-lesson interview. The 20-30 minute post-lesson (Shimizu, 2002) interview with Ms. Cook occurred on the same day that the lesson was taught in order to gain better insight into what she might have been thinking in regards to the different episodes of student input. I

provided a description of the episode similar to stimulated recall (Leikin & Dinur, 2007; Lyle, 2003; Westerman, 1991) without video of the lesson to remind Ms. Cook of the episode that occurred during the lesson[5]. During the interview Ms. Cook was asked to describe the decisions she made in connection to the different episodes of student input. From the pilot study I found that there were two questions I could ask Ms. Cook no matter what type of episode of student input occurred (1. Why did you decide to, or decide not to discuss this idea? 2. How does this idea relate to the goal of your lesson?). There were certain questions that were asked for the different types of episodes. The protocols that were adapted through the data collection process can be found in Appendix C.

The final question that was asked in the interview related to whether, after having time to reflect on the decisions to use student input, Ms. Cook would do anything different and if so what and why. To wrap up the interview I took the suggestion of Patton (2002) and told Ms. Cook that I asked all the questions I wanted to ask and was there anything else she wanted to add. In the final interview I also asked Ms. Cook what her thoughts were about using student input in her classroom in general.

Data Analysis

The analysis that occurred has elements of discourse analysis. The basic unit of analysis was the switch between speakers, which is part of the bigger discourse of classroom discussion (Wagner & Herbel-Eisenmann, 2008). The analysis of the switch between speakers was necessary in order to identify different types of student input and different categories of episodes of student input. This unit of analysis can be considered discourse analysis because the focus is

[5] Video was prepared to show Ms. Cook if necessary, but Ms. Cook was able to recall episodes with simply the verbal explanation.

on what Gee, Michaels, and O'Conner (1992) termed stanzas. A stanza is a section of text that takes on a particular perspective, thus as the speaker changes the stanza changes as well.

The analysis that occurred in between the lessons in the cycle had a preliminary framework that was adapted as more lessons were observed and analyzed. The analysis began by transcribing the video from the lesson and the post-lesson interview. The transcription was coded into the categories of student input (questions, solution methods, answers, incomplete statements, and student comments), categories of episodes of student input (NT, TT, and RW), why a teacher made the decision she did, and the lesson segment (opener, homework questions, launch of task, and conclusion of task).

Elements of grounded theory appeared in relation to the variable of why a teacher uses or does not use student input. Grounded theory is a methodology for developing theory that is based in data that is cyclically collected and analyzed (Strauss & Corbin, 1994). This cyclical idea was used to find out why Ms. Cook made the decisions she made. One may hypothesize why a teacher made the decision that they did, but this information cannot be determined unless the teacher is asked. Thus the categorization of why Ms. Cook made the decisions she did was built as the data was collected and analyzed.

Analysis before the next cycle of data collection was desired, because otherwise it would have been difficult to keep track of the connections between the four variables (student input, teacher response, time constraints/lesson segments, and why), which were mentioned in the framework chapter. One of the tools that was used to keep track of all the connections was an array that was created using spreadsheet software (Wagner & Herbel-Eisenmann, 2008), like the one seen in Figure 2. In the left hand column are the types of student input and in the first row are the types of episode of student input. The numbers in the boxes represent the lesson segment

when the student input and episode occurred. Different colors were used for the numbers to represent different days of data collection. After each interview I kept track of the student input, episode of student input, and the lesson segment in order to know what types of student input, episodes of student input, and lesson segments were needed during the next observation in order to saturate the data. The chart in figure 2 represents the episodes that were talked about during the first interview. For the second observation I was able to focus more on the episodes that were not discussed during the previous interview. As videos from the observations were coded I kept track of the student input, episode of student input, and the lesson segment in order analyze the data which will be described later. The completed tables from the interview and the lessons after all the observations were performed can be found in Appendix D.

Analysis of videos before the next cycle of data collection began was desired in order to look at episodes that were not used in the interview. This helped in forming of questions that would be asked in future interviews. It was also used to look for things that went unnoticed in the observation, to look for patterns (Patton, 2002) in teacher's decisions and uses of student input, to examine previous experience, and to improve methods before the next observation and interview. I kept track of what type of category was needed to discuss more or anything else that could be learned from the data at that time.

During the time period of data collection and analysis a research colleague coded some of the transcriptions to confirm the interpretation of the transcription (Huntley, Rasmussen, Villarubi, Sangtong, & Fey, 2000). Also during the time period of analysis I continued to meet with my research mentor for discussions and consultations about the research. The coding and analyzing of the classroom lessons and interviews were transcribed as much as possible. The transcription was also coded and analyzed as much as possible before conducting another

	Category of Student Input			
Episode of Student Input		**NT**	**TT**	**RW**
	Question (SQ)		2, 3	1, 1, 2, 2
	Student comment (SC)			
	Solution method (SS)			2, 4, 4
	Answer (SA)			
	Incomplete statement (SI)	4		

1 = opener
2 = homework questions
3 = launch of task
4 = conclusion of task

Figure 2

classroom observation and interview session to see if there were things that need to be done differently in the next session.

After all of the proposed data was collected from the five lessons recoding needed to take place. Since a big part of the framework was being built as data was collected the data needed to be recoded according to the framework that was developed at the end of data collection.

The data was looked at broadly to explore more deeply emerging patterns across the different classroom observations. Each category of teacher decision was sorted to see if there were similar patterns to how Ms. Cook used the student input. The analysis of the categories helped to develop a model for each category of how to use student input. Just as Asiala et al. (1996) explained the importance of having goals in data analysis my goals for analyzing across all collected data were: to answer the research questions (What exactly does a teacher making a decision to use student input look like? How, when, and why does a teacher use student input?) and to look for patterns (Patton, 2002) in the different categories of NT, TT, and RW.

Grounded theory was also used throughout my study by constantly analyzing the data compared to emerging hypothesis (Creswell, 1998). Thus to analyze the data at the end of data collection elements of grounded theory were also used, because I was not too sure what the data would present as it was collected and I did not want to miss any possible connections that might have been made in the data by narrowing my analysis before collection began.

The analysis of the videos from the lessons led to answer the research question: What exactly does a teacher making a decision to use student input look like? Through analysis of the video I anticipated better solidifying the definitions that I had already created for the different categories so that when individuals are observing in a classroom they too can easily identify the different episodes. I also found in analysis that there were categories within the three already determined categories, which will be explained in the results section.

Videos from the lessons and the post-lesson interview were examined to answer the research question: *How* does a teacher use student input? The lesson video allowed me to look back and see what type of episode correlated to different student input. The post-lesson interview also helped to get Ms. Cook's insight into how she thought she used the student input.

Videos from the lessons were examined to answer the research question: *When* does a teacher use student input? During the pilot study a pattern began to emerge in regards to what the teacher decided to do with student input as the lesson proceeded. At the beginning of the lesson the teacher mostly decided to teacher talk about input and towards the end of lesson the teacher chose to run more with student input. This result led me to think that there might be a pattern that emerges of when Ms. Cook makes certain types of decisions through the lesson.

Videos of the post-lesson interviews were used to answer the research question: *Why* does a teacher use student input? I initially had a few speculations of why a teacher might decide

to make decisions about student input. I anticipated that a reason why a teacher might have decided to not talk about student input is because the input did not align with the goal of the lesson, because of time (Tyminski, 2010), or because the teacher feels a need to cover the required material (Schoenfeld, 1998). A teacher might have decided to teacher talk about an episode because the input related to mathematical convention and the teacher wanted to clarify the convention, because of the lesson segment in which the episode occurred (Tyminski, 2010), or because a teacher wanted to help students understand an idea without deviating too much from the lesson (Schoenfeld, 1998). A final speculation I had about a teacher's decisions in regards to student input was a teacher might have decided to run with student input, because the student input aligned with the goals of the lesson or unit, or because the input pertained to a mathematical idea that the teacher thought was important no matter when it arises in conversation.

Chapter 4: Results

In this section there will first be a description of Ms. Cook's classroom. Next an explanation for what it means for Ms. Cook to use student input will be given. Then a description of the finding associated with connections 1, 2, and 3, which were explained in the framework chapter (see Figure 1), will be given. Finally there will be a discussion about the variable of time (time constraints and lesson segments) and the connections between time and the other variables.

Ms. Cook's classroom

Ms. Cook's lessons had a similar setup each day. Each lesson consisted of an opener, which was either review of a previously discussed topic or introduction of the topic for the day's lesson; homework questions; launch of the task; time for students to work on the task; and conclusion of the task. The only lesson segment that will not be reported on is when students had time to work on the task, because no whole class discussion occurred during that portion of the lessons.

As I observed Ms. Cook's class and talked with her during interviews, it became apparent to me that she let student input run her classroom. There was great participation from the students. Ms. Cook had to do very little in terms of asking for students to participate in whole class discussion. In the following excerpt Ms. Cook expressed her feelings about using student input:

> I love using students' ideas. I like presenting their work, having them talk about it, having them ask questions. I think more than anything it helps them start to listen to each other. So they start to say, "Oh, I could learn something from someone else," and it broadens their perspective.

In the above excerpt Ms. Cook spoke about student ideas, which will be referred to as student input. Ms. Cook also described the different ways that student input can be presented either

through her presenting the student work, having the class talk about student work, having students ask questions, or just having students listen to each other. Ms. Cook explained that she likes to use student input because it helps students to listen to each other and to realize that they could learn something from their peers. This helps to illustrate why Ms. Cook uses student input so regularly in her classroom.

Using Student Input

In order to delve into the analysis it is important to know what it means for Ms. Cook to use student input. There are times in class discussion when it may appear that Ms. Cook is using student input because the student input initiates a way that Ms. Cook can talk about an idea. For example Ms. Cook may in response to student input decide "when and how to attach mathematical notation and language to students' ideas…. to provide information… to clarify an issue… to model…" (National Council of Teachers of Mathematics, 2007, p. 45). Ms. Cook had good reasons for why (clarify mathematical convention, address the student input without leaving ideas hanging, etc.) she decided to talk about the student input as the teacher and it may have been beneficial to the class discussion. Thus this decision to talk about student input is one way that Ms. Cook uses student input. When Ms. Cook talked about student input herself she used her own model of the mathematics to respond to the student input and she did not assist students in building their own model of the mathematics.

There are many different forms in which Ms. Cook used student input. For example in using student input Ms. Cook would decide to "listen carefully to students' ideas and decide what to pursue in depth from among the ideas that students generate during a discussion…encourage and accept the use of multiple representations…let students wrestle with a difficulty; and monitor students' participation in discussions and decide when and how to

encourage each student to participate" (National Council of Teachers of Mathematics, 2007, p. 45). These are all various forms of using student input. An important aspect of using student input is that the student input is built upon and becomes a part of the discussion. The input is not merely used as a springboard for the teacher to talk about an issue, but rather the student input is used to explore a mathematical idea further. In using student input the mathematics of the student input is taken and discussed further. As the mathematics was discussed Ms. Cook would either help the class reach a consensus about the mathematics or the discussion was postponed until later in the lesson.

To describe how Ms. Cook used student input I will describe three phases that were present when Ms. Cook used student input. The initial phase of using student input was to take the student input that was presented and build on the student's ideas. The second phase was to keep discussion of the idea going. The final phase of using student input was to end the discussion of the student input. More description of the three phases that encompass using student input will be given below. Although the three phases are separate, there are similarities between the phases and even some overlap.

The difference between the first phase of "initiating the conversation" and the second phase of "continuing the conversation" is initiating the conversation is in direct response to the mathematics that was presented. Or in other words after student input is presented Ms. Cook makes a move to use the student input, the move after the student input is presented is initiating the use of student input. On the other hand continuing the conversation of the mathematics is to continue to have students involved in the conversation of the mathematics instead of Ms. Cook herself taking over the discussion of the mathematics.

There are a few moves that Ms. Cook used to initiate and continue the discussion of student input. One move was Ms. Cook posed the question, if the input was a question, back to the class; or asked the student that presented student input, or other students, questions related to the mathematics of the student input. Ms. Cook taking the student input and posing it back to the class for their thoughts is what is termed a "reflective toss" (van Zee & Minstrell, 1997, p. 229), which was introduced in the methodology section. A reflective toss is when the teacher takes the student input and throws it back to the students to see what they think of the idea. Another move that Ms. Cook used was by asking the student that presented the student input clarifying questions. Clarifying questions have two different purposes. One purpose is to help Ms. Cook or other students to understand what the student that presented the input meant. Another purpose is to help the student that presented the input correct the work that they presented. A final way that Ms. Cook initiated or continued building on student ideas was by allowing students to discuss the mathematics of the student input without becoming involved in conversation. Students discussing an idea without Ms. Cook becoming involved in the discussion was a norm of Ms. Cook's class. Students knew that if Ms. Cook did not become involved in the conversation about the student input it was okay for them to continue their discussion of the input. In continuing the discussion of the ideas Ms. Cook did not necessarily have to do anything more than what she did to initially build on the ideas of the student input. For example, Ms. Cook would initiate the building by asking clarifying questions and continue to ask clarifying questions. Ms. Cook might also decide to use one way to initiate building on ideas and choose a different way to continue the discussion of the ideas. For example, Ms. Cook might decide to initiate the building on an idea by asking students what they think of the mathematics that was presented. To continue the discussion of ideas, Ms. Cook might decide to ask clarifying questions to students as they

respond with what they think about the student input instead of just taking over the conversation as the teacher.

Finally there is ending the discussion about the ideas that were presented in the student input. One way that Ms. Cook would end the discussion is by summarizing the discussion that had taken place about the student input. A similar way that Ms. Cook would end the discussion of the student input is by summarizing the discussion and then asking the student that presented the student input if her summary "made sense" or if that is what the student "meant." In both cases the student would usually respond with a yes or no. Therefore Ms. Cook didn't really know if the student understood what Ms. Cook had summarized, because the student never offered more evidence of understanding. A final way that Ms. Cook would end the discussion was by moving onto a new idea. Ms. Cook would either say, "It's time to move on," or postpone the discussion for a later time. In either case no conclusion was reached about the discussion of the mathematics and ideas were left hanging for students to think about. With all the ways that Ms. Cook chose to end the discussion she never followed up with students to see what they were thinking about the ideas associated with the student input. Ms. Cook never took the time to have students summarize or explain what had taken on in the discussion. She also did not ask students questions that were related to the mathematics that could test their understanding of the ideas that were discussed.

Variations of Uses of Student Input

There are varying degrees of complexity in Ms. Cook's uses of student input, which I have represented with what I call *brief use* of student input, *complex use* of student input, and *more complex* use of student input. A brief use of student input initiates the discussion of student input and then ends it, with no continuing moves. A complex use of student input includes all

phases of using student input. A more complex use of student input also includes all phases of using student input; but it contains many moves to continue the discussion.

Brief use of student input. The excerpt below is an example of a brief use of student input. Before the episode began students were working on an opener in which they were given that ABDC is a parallelogram. Students were asked to list all the properties of ABDC. They suggested the following properties: opposite sides are parallel, opposite angles are congruent, all the angles add to 360, there are 4 sides, exterior angles are 360, and the parallelogram is convex. Luke then presented the last property in the episode below:

Luke:	AD is equal to BC.
Ms. Cook:	So this segment is congruent to this segment [refers to the diagonals of the parallelogram]. Is that guaranteed?
Students:	No, yes
Ms. Cook:	So I have a question for you…. I want you to keep this thought in mind…. What is our definition of our parallelogram? Don't yell it out, just keep it in mind.

Following Ms. Cook's last statement she switched to questions about the homework. If this were the only time that the idea of the diagonals of a parallelogram being congruent were discussed this would have been a poor use of student input. However, the idea of the diagonals of a parallelogram being congruent was brought up two other times in the lesson. One time was during the launch of the task when a student asked, "How do we know that the diagonals of a parallelogram are congruent?" Ms. Cook again postponed the discussion of the idea at this time by saying the idea would be explored during the task. The idea of diagonals of a parallelogram being congruent was then discussed and disproved during the conclusion of the task.

The general moves that Ms. Cook made in the above excerpt were Ms. Cook initially took the statement, summarized the statement and asked the class a question that related to the statement, which would initiate using the student input. However, Ms. Cook did not do anything

to continue the discussion of the student input. Ms. Cook's reason for why she decided not to continue the discussion of whether or not the diagonals of a parallelogram were congruent align with one of Ms. Cook's goals for the lesson which was to have students list and prove all the properties of parallelograms. Thus Ms. Cook anticipated talking about whether diagonals were congruent as part of the task itself. During an interview when Ms. Cook was asked about this episode she said that since the idea would be explored later in class she merely wanted students to begin to think about how they could know that the diagonals were or were not congruent.

There was disagreement between the student answers, but Ms. Cook chose to end the discussion of the student input at that time. Ms. Cook left the idea without making it explicit that she wanted to come back to the idea later. However, she did ask students to think about the definition of a parallelogram in order to get them to start to realize what they have to work with to start proving whether or not the diagonals are congruent. Again Ms. Cook did not make it explicit to the students why she was asking them to think about the definition of a parallelogram.

If the idea of whether or not diagonals of a parallelogram are congruent had not been discussed later in the lesson then this would not be a good use of student input because the idea was left hanging and the students did not know the correct answer to the question. However, Ms. Cook did come back to the idea of whether or not the diagonals of parallelograms are congruent. The class took the time to walk through finding a counterexample for the diagonals of a parallelogram being congruent. One disadvantage that resulted from Ms. Cook's moves in the above episodes was Ms. Cook did not make it explicit to the class that they would discuss the idea at a later time in the class period. Instead during the opener students were left to wondering about whether or not the diagonals of a parallelogram are congruent.

There is more that can be added to the description of Ms. Cook's general moves made in the episode above by looking specifically at the mathematics that were presented and discussed. Luke essentially said that the diagonals of the parallelogram were congruent. Since the diagonals of rectangles and squares, which are both parallelograms, are congruent this might be a reason why Luke said the diagonals for the given parallelogram were congruent. Ms. Cook pointed to a figure on the board of a parallelogram that had no right angles and said, "This segment is congruent to this segment" (referring to the diagonals). Ms. Cook's pointing built on Luke's statement of saying AD and BC were congruent and gave students a visual representation of what Luke had claimed.

Ms. Cook then asked the students if it was always true that the diagonals of a parallelogram are congruent by saying "Is that guaranteed?" This move built on what Luke said by taking his statement from the parallelogram that the students were given and applying his statement to parallelograms in general and also moving students to thinking about whether or not this would always be true for parallelograms. As students responded to the question they gave mixed responses of yes and no. The students that said yes could have been thinking similarly to Luke–that all parallelograms have congruent diagonals. Although we don't know exactly what the students that responded by saying no could have been thinking, there are a few different things that we can infer that they might have been thinking. For example, these students could think that the diagonals of a parallelogram are sometimes congruent (rectangles and squares), or maybe that the diagonals of a parallelogram are never congruent. Ms. Cook did not ask for any explanation from students about their responses, thus it is impossible to know for sure what the students were thinking when they responded yes or when they responded no.

After students responded Ms. Cook moved on to ask them to think about the definition of a parallelogram. She did not want the answer, but merely wanted students to think about the definition. During an interview Ms. Cook explained that she wanted students to start to think about the definition of a parallelogram, a quadrilateral with both pairs of opposite sides that are parallel, in order to get them to begin to think about what information they have to work with for a parallelogram and what they can prove given that information; however, Ms. Cook did not make her reason for students thinking about the definition of a parallelogram known to the class. There is no direct connection between the definition of a parallelogram and the idea that the diagonals of a parallelogram are not congruent; however, as students begin to explore what they can discover from just knowing that both pairs of opposite sides are congruent students will be able to prove many other properties of parallelograms. For example as students begin to work with just the information that opposite sides of a parallelogram are parallel they can then use alternate interior angles are congruent after drawing in the diagonals. Students can also use the reflexive property and then prove that the triangles that have been created by the transversals are congruent and thus that the opposite sides of a parallelogram are congruent. Students can continue to work with these properties to realize that in order to have the diagonals of a parallelogram be congruent then the angles of the parallelogram need to be right angles.

The idea of Luke's student input was initially built on by Ms. Cook summarizing with a picture of what Luke had presented. Luke's idea of diagonals being congruent in a parallelogram was built upon even more as Ms. Cook asked if the idea would hold for all parallelograms. However, at the time there was no consensus about whether or not diagonals of a parallelogram are always congruent. Students were unaware that the idea of whether or not diagonals of a parallelogram are congruent would be explored later in class, because Ms. Cook did not make it

known to the class that the idea would be explored in the task. Thus there were attempts to initially build on the student idea, but there was no conclusion for the students and the idea was left hanging. Ms. Cook never intended to address the proof of whether or not the diagonals of a parallelogram were congruent during the opener. It was an idea that she knew would be addressed during the task and conclusion of the task. Thus the question of why Ms. Cook used the opener in the first place remains. If she wanted students to not think about the proof and merely list the properties of parallelogram without proving them how would students know they were really properties that held for all parallelograms?

Complex use of student input. The next episode is an example of a complex use of student input. Before the episode began students were working on an opener in which they were given the figure found in Figure 3 and asked to draw the overlapping triangles ABC and DBC, and find the sides or angles that are in common. Right before Jack's question was asked a student presented that the segment BC was common in both triangles ABC and DBC.

Jack:	Well would ABC be a common angle?
Ms. Cook:	Good question, are you guys listening? Ok, Jack, ask your question again.
Jack:	Would angle ABC be a common angle?
Ms. Cook:	Ok, Tom what do you think?
Tom:	I think not because angle ABC is talking about the right angle or that angle right there [points to angle ABC] and the other angle…
Ms. Cook:	So are you meaning this angle right here [points to angle ABC], Tom?
Tom:	Yeah, and the other angle, the other triangle is using angle DBC, not A.
Ms. Cook:	So that angle is not the exact same angle of both triangles, right? Does that make sense Jack?
Jack:	Yes
Ms. Cook:	Ok, Melissa?
Melissa:	Can I do number 2?

To begin to build on the student input of the question of whether an angle is common Ms. Cook got the attention of the class and posed the question back to the class. This is one of the strategies

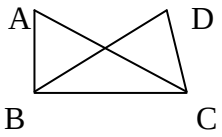

Figure 3

that was mentioned before that was termed a reflective toss. Thus, instead of Ms. Cook imposing her own mathematics she posed the question back to the class to have students discuss the mathematics of whether ABC would be a common angle. Ms. Cook then allowed another student, Tom, to answer the question. In the midst of Tom's response, instead of asking the students what they thought of what Tom said, like she did to initiate the conversation, Ms. Cook used a different method to continue the conversation about the mathematics. As Tom answered the question Ms. Cook chose to ask Tom a clarifying question ("So are you meaning this angle right here, Tom?"). In responding to Jack's initial question as well as to Ms. Cook's clarifying question Tom was doing his own mathematics, not Ms. Cook's mathematics. The reason that Tom was doing his own mathematics is he was not prompted or guided by Ms. Cook in a direction to do certain mathematics.

To end the discussion Ms. Cook summarized what she thought Tom meant by his answer. At this point, however, Ms. Cook imposed her model of the mathematics on the class instead of using Tom's model of the mathematics because she did not follow up with Tom to make sure that what she said was what Tom meant. Ms. Cook did follow up with Jack by asking whether he understood the answer, but Ms. Cook didn't really know if Jack understood because he did not offer any evidence that he understood aside from saying yes. A student may say yes thinking they understand or because they don't want to draw attention to themselves because they don't understand.

Although the moves that Ms. Cook made in this episode may look similar to the moves that she made in the previous episode in which the use of student input was brief, when the mathematics are examined closely there are big differences between this episode and the previous episode. The episode began with Jack asking if angle ABC is a common angle for triangles ABC and DBC. Initially one would think no, but then one might think, "Why would Jack think ABC is a common angle?" In the following excerpt Ms. Cook explained what she was thinking and what she did in response to Jack's question.

> When I was looking at this when he said ABC I knew he was referring to the whole angle, which part of it's in common and part of it's not. So I posed it back to the class because I wanted to see if they were thinking of the different parts. Like part of it is or were they just all thinking that it's included in it. So I kind of wanted feedback from them.

Ms. Cook initially talked about the whole angle ABC and knowing that part of it was "common" and part of it was not "common" in triangles ABC and DBC. If the intersection of line segments AC and DB were labeled E then angle EBC could be thought of as "common" in the sense that it makes at least a portion of an angle in each triangle, whereas angle EBA is not at all "common" to both triangles, as it lies exterior to triangle DBC. Ms. Cook explained that she posed the question back to the class to see if they were thinking about angle ABC as angles EBC and EBA or if they were thinking that angle ABC would be common in both triangles. So in other words she posed the question back to the class to assess what students were thinking about angle ABC.

Ms. Cook told Jack that his question was a good question. Ms. Cook supported this statement by getting the attention of the class and having Jack ask his question again. Ms. Cook then allowed Tom to attempt to answer Jack's question, which seems as though it would help Ms. Cook assess what Tom was thinking about ABC being common. Tom said that he didn't think that angle ABC was common, which built on Jack's question because Jack wanted to know

if angle ABC was a common angle. Tom then went on to explain why. Tom first began talking about angle ABC as a right angle, probably because it looked like a right angle, even though the information of ABC being a right angle was not given. Tom then changed his discussion of ABC being a right angle to "that angle right there [referring to angle ABC]." Tom began to talk about another angle, but Ms. Cook interrupted Tom to clarify what angle he was talking about. Ms. Cook explains below what she was thinking and why she asked her clarifying question.

> I asked the clarifying question about um…ok so we know part of it's in common, but is it part of the figure that we are looking for that's overlapping. I wanted them to think about it for a little while before we actually came to the conclusion of what it was whether it was common or not.

Ms. Cook explained that she asked the clarifying question in order to give students a chance to think about things before they came to a conclusion.

In the episode Ms. Cook did not say anything about part of angle ABC being common. She only let Tom answer Jack's question, clarified what angle Tom was talking about, and summarized what she thought Tom had said. She did not give students a chance to think about what part of angle ABC was common to both triangles, thus Ms. Cook was not able to assess what other students were thinking. Ms. Cook did not even pose any more questions to the class aside from Jack's original question. I don't think that the episode illustrated the question that Ms. Cook claimed to have asked in the excerpt above. The episode also does not reflect that she gave students a chance to think about the idea, because then Tom continued his explanation of why angle ABC was not common by saying that "the other triangle is using DBC not A." The term "using" is not clear. Did Tom mean that angle DBC is an angle of triangle DBC, but not an angle of triangle ABC or does he mean that the other triangle does not use A as a vertex and that is why ABC is not a common angle? After Tom's explanation Ms. Cook used different language

from Tom when she said the "angle is not the exact same angle of both triangles." Because Ms. Cook used different language we cannot tell for sure what Tom meant by "using." It is also not clear what Ms. Cook meant by the angle not being exactly the same for both triangles. To finish the episode Ms. Cook asked Jack if that made sense. Jack responds with a simple yes, but there is no evidence to support that he understood what Tom or Ms. Cook were trying to say. Asking closed questions to students is not a very effective way to assess students' thinking because they are not able to present an explanation for their answer or an explanation of their thinking.

When Ms. Cook was asked about this episode and how it related to her goals for the lesson Ms. Cook explained that it was a big part of the lesson, because when students "are looking at overlapping triangles that is the hardest thing for them to think about is what part is overlapping?" I would have thought that if this episode was such a big part of the lesson that more time would have been spent on making sure that students really understood what parts were overlapping. Ms. Cook never came back to the idea of overlapping parts or common parts of figures in her lesson. Since common parts of triangles are components of the triangles that both triangles completely share, Ms. Cook could have done more with what Tom had said. Although Ms. Cook seemed to know what Tom meant by a triangle "using" an angle she could have clarified what Tom meant to the class. Ms. Cook could have made sure that students really understood what parts were overlapping by focusing more on what it meant to be a common part of two triangles or even what it meant to not be a common part of two triangles. She could have asked another student to say what they thought Tom meant. Ms. Cook could have asked students for another part of the figure that had part in common in both triangles, but not the whole thing. Ms. Cook could have also asked students to list as many parts of the figure that were not

common to both triangles. Ms. Cook could have done more to explore the idea of common parts of figures.

More complex use of student input. The excerpt below is an example of a very complex use of student input. Before the episode the class had discussed that the exterior angles of a regular polygon sum to 360. Ms. Cook then asked what would happen if the polygon was not regular. Some students said that the exterior angles would still sum to 360 while other students said that the exterior angles would not sum to 360. The episode begins with Luke adding to the comments that students had made about whether or not the exterior angles of a non-regular polygon sum to 360. Luke is referring to the angles of a regular hexagon.

Luke:	So if you change one of the angles… you change it from 60 to 59 [an exterior angle of a regular hexagon] then the corresponding angle has to change to 61.
Ms. Cook:	So say I change this [points to an exterior angle of the regular hexagon she drew on the board] so it's no longer regular. Does the measure of this angle [points to the interior angle that corresponds with the exterior angle she pointed to before] change?
Students:	Yes.
Ms. Cook:	Is the measure of this angle [points to an exterior angle that is consecutive with the exterior angle that was changed above] going to change?
Students:	Yes.
Ms. Cook:	But are all of these going to change?
Tom:	No, not all of them.
Ms. Cook:	No not all of them. So would this [points to the interior angles of the hexagon with the changed angles] still add up to 720?
Students:	Yes.
Ms. Cook:	So this one wouldn't change [points to an interior angle of the hexagon], this one wouldn't change [points to another interior angle of the hexagon that is consecutive with the angle that she just pointed to and continues to point to the interior angles of the hexagon as she works her way around the hexagon], this one wouldn't change, and this one wouldn't change, but this one and this one would. So now these two [points to two consecutive interior angles of the regular hexagon], what's going to change if I change these two? These two still have to equal 120 together, right? So give me a measure?
Students:	140.

Ms. Cook: What would this [points to the angle that is consecutive to the angle that the students decided to change to 140] be?
Students: 100
Ms. Cook: Why?
Luke: Because it compensates.
Ms. Cook: But together didn't we say that these had to equal 120?
Students: No together they equal 240.
Ms. Cook: No, no because the two angles we had …
Luke: No because 120 and 120 is 240.
Ms. Cook: Oh yeah, sorry Luke. Thank you. So this is going to be 100. So now what's the measure of this angle? [Points to the exterior angle of the interior angle that measure was changed to 140]
Students: 40
Ms. Cook: And what's the measure of this angle? [Points to the exterior angle of the interior angle measure that was changed to 100]
Students: 80
Ms. Cook: Now what do they all add up to? [Referring to the exterior angles of the hexagon]
Students: 360
Ms. Cook: Why? We just changed them you didn't add them up. Claire?
Claire: Well they're still going to add up to 360 because there are still 6 straight angles.
Ms. Cook: Still 6 straight angles?
Claire: It's still six linear pairs so it's still going to add to 360.
Ms. Cook: So no matter how I change it's still going to add to 360.
Melissa: Yes, as long as it's' concave.
Ms. Cook: Convex polygon, we are restricting it to convex. Does it matter if it's [the hexagon] regular or not?
Students: No.
Ms. Cook: No, because if I change this side [points to one side of the regular hexagon] these two [referring to the angles that are at the ends of the side that she suggested to change] are still going to compensate in the same way, right?
Luke: If it's not regular you can find the whole, but not the individual.
Ms. Cook: Say that again Luke.
Luke: If you change it so it's not regular then everything is not the same measure so you can't find the individual angle measure you can only find the whole.
Ms. Cook: So if it's not a regular polygon like you weren't given these angles you were just given that it was a hexagon would you be able to find the measure of all these angles?
Students: No.
Ms. Cook: That's what Luke is saying you can't find each angle individually, but together you know that they add up to 360, right? Questions on that? I have a challenge for you then. Here's your challenge. You have a regular polygon and you know that one of the exterior angles has 36 degrees I want you to

find how many sides the polygon has. Does everyone have an answer on their paper? Ben.

Ben: Is it ten?

Ms. Cook: Tell us how you figured it out.

The episode began with Luke explaining that when you change one of the exterior angles of a regular hexagon from 60 to 59 then another angle needs to change. Ms. Cook decided to summarize what Luke had said by pointing to a drawing of a regular hexagon and explaining that if you change one of the exterior angles then the hexagon is no longer regular. This could have been an end to the discussion; however, to initiate the discussion of Luke's idea Ms. Cook then pointed to the other exterior angles of the hexagon individually and asked the students if the angles would change if the one that Luke suggested changed. The class decided that not all of the angles would change.

Ms. Cook then used many different moves to continue the discussion of Luke's idea of angles needing to compensate when one angle is changed. Ms. Cook first continued the discussion by asking the class if the interior angles of the hexagon would still sum to 720 if the exterior angles were changed. The class responded yes. Ms. Cook then chose to summarize what angles the students had said would and would not change if one angle was changed to 59. She then talked about changing two of the interior angles and asked students for a measure. She asked what the other angle would need to be. Ms. Cook was confused by student responses because she thought the two angles summed to 120 when in reality the angles summed to 240. The students helped Ms. Cook realize that the two angles summed to 240.

Ms. Cook continued the conversation by asking students for the measure of the exterior angles corresponding to the two angles that had changed. Ms. Cook asked the students what the exterior angles summed to and why. Claire explained that there were "6 straight angles" and that

is why the exterior angles summed to 360. Ms. Cook asked for clarification from Claire about what she meant by "straight angles." Claire explained that the straight angles were linear pairs. Although the discussion of the straight angles did not offer sufficient justification for why the angles sum to 360, Ms. Cook moved on and summarized that no matter how the hexagon's angles changed the exterior angles would still sum to 360. Melissa responded by saying yes as long as the hexagon was concave. Ms. Cook corrected Melissa and said the polygons needed to be convex polygons. She also asked students if the polygon needed to be regular. The students said no. Ms. Cook affirmed that no was the correct response and talked about the angles compensating if they were changed, as Luke had first presented.

Luke then made a comment about if the polygon was not regular you could find the whole, but not the individual. Ms. Cook asked Luke to repeat his statement. Luke expounded on his statement by explaining that if not all the interior angles are the same then you can't find the individual angles you can only find the measure of the sum of the angles. Ms. Cook rephrased Luke idea as a question to the class. The students responded no you could not find individual angles if the polygon was not regular. Ms. Cook then asked if the students had questions on Luke's idea of the angles compensating when they were changed.

Ms. Cook used many more moves to continue the conversation than she did in the previous two episodes that were presented. Ms. Cook summarized student responses, asked clarifying questions, and asked questions that were related to student input that was presented in association with the discussion that was taking place.

To end discussion of Luke's idea Ms. Cook went on to an idea that was related to Luke's but dealt more with knowing the measure of one exterior angle and being able to determine how

many sides the regular polygon had. Thus to the end the discussion of Luke's idea Ms. Cook challenged the students with a related mathematical task.

The moves that Ms. Cook made in this episode were similar to the moves that she made in the other two episodes that have been presented; however there were many more moves that she made to continue the discussion of student input than was presented in the other episodes. Also on closer analysis of the actual mathematics that occurred in this episode it became very apparent that the decisions that Ms. Cook made were much more complicated and involved as more students became involved in the conversation and there is an evolution of the discussion of Luke's student input. Below a description of analysis of the mathematics that was presented will be given.

The episode began by Luke stating that if you change one exterior angle of a regular hexagon from 60 to 59 then "the corresponding angle has to change to 61." Luke presented that one of the exterior angles would need to change because Ms. Cook wanted to know if the exterior angles of a polygon would still sum to 360 if the polygon was not regular. Thus Luke was presenting the change of one exterior angle from 60 to 59 in order to make the hexagon no longer a regular polygon. It is unclear what Luke meant by "the corresponding angle" needing to change to 61. It is true that one of the exterior angles will need to change if an exterior angle changes to 59, but there is not a specific angle that needs to change. Ms. Cook did not ask Luke for clarification about what he meant by the "corresponding angle" changing, even though when she was asked about this episode she explained that she was trying to figure out what Luke meant by his statement.

In order to figure out what Luke was saying Ms. Cook responded to Luke's comment by summarizing part of what Luke said by pointing to one of the exterior angles of the hexagon and

saying “So say I change this so it’s no longer regular.” Ms. Cook was saying that if you change one of the exterior angles then the hexagon is no longer regular. This built on Luke’s idea because Luke talked about changing one of the exterior angles from 60 to 59, thus Ms. Cook took Luke’s statement and made it visual for the class by pointing to a drawing of a regular hexagon. However Ms. Cook had no way of knowing what she said was what Luke meant because Ms. Cook never asked Luke if that was what he meant or went back to Luke for clarification of his idea.

Ms. Cook then led the students through a series of closed questions. First Ms. Cook pointed to two different angles of the hexagon and asked the class if those angles also changed. This move built on what Luke said because Ms. Cook took the idea of an angle changing and applied it to what other angles needed to change as well. Ms. Cook added to this even more by asking the students if all the angles were going to change. The students responded that not all the angles would change, which was the correct answer. There was no disagreement in the answer that was provided by students thus Ms. Cook affirmed that not all the angles changed and left the idea, because the students provided the expected response.

Ms. Cook continued the questioning by asking the students if the sum of the interior angles would still sum to 720. Ms. Cook might have asked this question to see what other properties of the hexagon the students thought would change as the hexagon was no longer a regular hexagon; however there was no explanation of why Ms. Cook chose to ask this question or no continued discussion of the idea of the interior angles still summing to 720.

After the questions were asked and answered Ms. Cook summarized what angles of the hexagon changed when one exterior angle was changed to 61. Ms. Cook then built on Luke’s idea by asking students what would happen if two interior angles were changed; however it

might be unclear why Ms. Cook switched from talking about exterior angles of the hexagon to the interior angles of the hexagon. During an interview Ms. Cook stated it was "nice for him (Luke) to think about if we take one degree away from this one that one degree still has to be in the interior you have to add to the one up there." Thus from the interview Ms. Cook explained that she thought that Luke was talking about changing the measure of interior angles; however, there was no evidence from Luke's initial statement that showed that he was talking about interior angles as opposed to exterior angles.

Ms. Cook asked her students for a new measure for one of the interior angles that they had talked about changing. Instead of giving the measure of the other interior angle that changed Ms. Cook asked students what the angle measure would be. Instead of just accepting the angle measures that were given Ms. Cook asked students for an explanation of why the second angle had to be 100 if the first angle was 140. Luke explained that the angle "compensates" for the change in the other angle. Again there was no follow up to find out what Luke meant by "compensates." One can infer that if one of the angle increases by 20 then the other angle will need to "compensate" and decrease its measure by 20 degrees.

Then there was a debate between Ms. Cook and Luke about what the two angles sum to. Luke convinced Ms. Cook that the angles sum to 240 as opposed to 120. Ms. Cook then asked the class what the corresponding exterior angles changed to. This built on what Luke had originally presented about the exterior angles still summing to 360 if the hexagon was no longer regular, because Ms. Cook switched from talking about interior to exterior angles. Ms. Cook then asked the students if the exterior angles still summed to 360. The students responded that the angles still sum to 360. Ms. Cook then asked for an explanation.

Claire explained that the exterior angles summed to 360 because there were 6 straight angles. Ms. Cook asked for clarification of what Claire meant by straight angles. Claire changed from using straight angles to linear pairs. Claire said that there were 6 linear pairs and thus they summed to 360. However, 6 linear pairs would sum to 1080 not 360, thus it is unclear what Claire meant by 6 linear pairs summing to 360. The interior angles sum to 720 and 1080-720 is 360 making the exterior angles sum fixed. Ms. Cook took Claire explanation and did not ask for more clarification. Instead Ms. Cook stated that no matter how the angles changed they would still sum to 360, which was Luke's initial idea. Claire's idea was headed in the correct direction, thus it is interesting that Ms. Cook did not do more to explore her idea.

Melissa then stated that the exterior angles still sum to 360 as long as the polygon is concave. Melissa might have just mixed up the terms of concave and convex; however it is unclear what she meant by the polygon being concave. It is unclear because Ms. Cook merely corrects Melissa by stating that the polygons will be restricted merely to convex polygons. Ms. Cook then brings the conversation to what started the whole episode. Ms. Cook asked the students if "it matters" whether or not the polygon is regular. The students respond that it doesn't matter. Ms. Cook merely restated the students' response of no and explained by talking about the angles compensating. The word "compensating" is used again, but it is unclear whether or not the students know what compensating means.

Luke brings up a related idea by saying that you can find the "whole" but not the "individual" if the polygon is not regular. Ms. Cook asked Luke to restate what he had said. When Luke restated his statement he added more explanation. Luke explained that if you have a polygon that is not regular then not all the angles are the same, thus you cannot find the individual angles. It is unclear what Luke means by "all the angles are the same". Luke could be

referring to the interior or exterior angles of the hexagon. Luke's idea of not being able to find the "individual angles" is correct if the only given information is the number of sides of the polygon and the total sum of the angles; however, if more information is given then one might be able to find the individual angles. Luke went on to explain that you can only find the sum of the angles or the whole. Again this idea is correct depending upon how much information is given; it is not exactly clear what Luke meant by "whole." One can infer that Luke meant the sum of all the exterior angles, but there is no way to be certain because there is no more clarification from Luke. Ms. Cook then took Luke's statement and turned it to the class. The students agreed with Luke's comment. However, there is no evidence that the students really agreed with Luke's statement because nothing was offered from the class aside from their "no" response to Ms. Cook's question of if they would be able to find all the individual angles. To end the discussion of Luke's initial idea Ms. Cook asked if there were any questions relating to the sum of any convex polygon's exterior angles was 360. Since there were no questions Ms. Cook went on to ask the students if they could figure out the number of sides of the regular polygon if they were given the measure of one exterior angle.

Overall Luke's idea was explored more in depth than just confirming or denying the accuracy of his statement. However, Ms. Cook did a lot of leading students through the mathematics by closed questions. There was a conclusion of Luke's idea and there was some questioning of the students to see if they understood, even though the questions were closed questions.

Summary of uses of student input. Thus as was described in the previous pages there are varying degrees of how Ms. Cook uses student input. The uses that Ms. Cook implements in her classroom range from her merely talking about student input to her using very complex

moves and decisions to continue the conversation about the student input. Ms. Cook's variety in use of student input will be explored more with the connections between the different variables (student input, teacher response, time, and why) that were introduced in the framework chapter.

Intro of Discussion about Connections

As was mentioned in the framework chapter, four variables were looked at during data collection (see Figure 1). Initially these connections were analyzed separately in order to make sure that all of the connections were covered. During analysis I realized that there was overlap between the different connections. Thus connections 1, 2, and 3 will be talked about together and the variable of time (time constraints and lesson segments) will then be talked about with the connections related to time.

Episodes in which the student input is SI (student incomplete statements) will not be talked about further. There were only 28 episodes in which the student input was SI. Because episodes in which student input is SI did not happen very often it would be difficult to make any conclusions about SI compared to the amount of data collected for the other types of student input.

Connections 1, 2, and 3. Connections 1, 2, and 3 connect the variables of student input, teacher response and why. First a description will be given of how connections 1, 2, and 3 were initially analyzed. Then an overall discussion of connections 1, 2, and 3 will take place. In order to discuss the overall associations of connections 1, 2, and 3 this section will then be divided by the types of student input. For each type of student input there will be a discussion of the teacher response and why the teacher made that decision.

Connection 1 is the relationship between student input and teacher response. To find initial connections between the type of student input and the type of teacher response I took the

number of type of teacher responses and divided it by the total type of student input. For example, for SQ-NT I took the total number of SQ-NT and divided that number by the total number of SQ for all lessons that were observed. The results are in table 1. The order of the types of student input in table 1 was chosen because the types of student input range from the most interesting results to less interesting results.

Connection 2 is the relationship between the teacher response and why. In order to analyze this I took the interview transcript and coded each episode that Ms. Cook was asked about according to the decision (NT, TT, and RW) that she made in relation to the student input. I then tried to summarize Ms. Cook's reason for her decision in a few words.

Connection 3 is the relationship between the student input and why. Ms. Cook never explicitly said that the type of student input was the reason she made the decision she did. I also never asked her if the type of student input was the reason for her decision. Thus to analyze the relationship between student input and why I coded the interview transcript with the type of student input (SA, SQ, SS, SC, SI) and used the summary of Ms. Cook's reason why that I used in connection 2.

Student Answer (SA). The first category of student input SA had initially surprising results for what the teacher did in response to the student input. The largest response by Ms. Cook to SA was NT. Ms. Cook chose to NT almost two times as many times as she decided to RW or TT.

NT was surprising, because Ms. Cook is a big proponent of using student input. As I observed lessons I noticed a pattern of Ms. Cook eliciting an answer and then not talking about the answer. The first form of this pattern occurred when the class graded their homework.

Table 1

Relationship Between Type of Student Input and Teacher Response

Teacher Response	Type of Student Input	Not Talk		Teacher Talk		Run With	
		Number of episodes	Percentage	Number of episodes	Percentage	Number of episodes	Percentage
	Student Answers	385/701	54.9%	117/701	16.7%	199/701	28.4%
	Student Solutions	2/47	4.3%	4/47	8.5%	41/47	87.2%
	Student Questions	5/198	2.5%	106/198	53.5%	87/198	43.9%
	Student Comments	42/185	22.7%	40/185	21.6%	103/185	55.7%
	Student Incomplete Statements	9/28	32.1%	7/28	7%	12/28	42.9%

Ms. Cook said the number of the problem and the students responded with the answer. As I talked with Ms. Cook about this pattern of correcting the homework she responded, "Mostly because when I looked around they were nodding. They were okay with it." Ms. Cook chose to NT about the SA because there seemed to be agreement about the answer that was provided.

The next form of the pattern of eliciting an answer and then not talking about the answer was when input was presented to the class and Ms. Cook clarified or reviewed the input by using closed questions. An excerpt from one of the observed lessons in which Ms. Cook was helping to clarify which triangles in a figure were congruent helps illustrate this point.

Ms. Cook: Do you think these two triangles are congruent?
Students: No
Ms. Cook: What about these two?
Students: Yes
Ms. Cook: This one and this one?

Students: Yes
Ms. Cook: What about this one and this one?
Students: No

The excerpt illustrated closed questions that Ms. Cook asked and the responses (all correct) that the students provided that she did not go on to talk about. This relates to one of the reasons that Ms. Cook gave for her decisions in relation to SAs.

Ms. Cook said that one of the reasons she did what she did in relation to SA was she thought the SA was review of a topic that had already been covered in class. The following is an excerpt in which Ms. Cook was asked about why she NT in response to her asking students which properties from a list the students knew would be correct from what they discovered in the opener. The students responded that the first two properties were correct and then Ms. Cook didn't talk any more about the properties.

> Mostly because we had talked about it before when we proved triangles that we knew that if the lines were parallel then the opposite sides were going to be congruent. And I also knew that they would come back up when they proved the triangles were congruent. So when they proved that these two triangles were congruent to get the opposite angles they were also getting the sides. And they were pretty comfortable with it so I didn't think it needed to be brought back up again.

Ms. Cook explained that the reason she responded the way that she did to the SA was because "we had talked about it before." When Ms. Cook said that the class had talked about it before she meant that the SA was review of a topic that they had covered in a previous class. Thus a reason to make a decision in relation to SA is the SA is review of a topic that has previously been covered.

The third way that the pattern of eliciting an answer and not talking about the answer occurred was when Ms. Cook was trying to elicit ideas from a student presenting work or correct

the work the student was presenting. An excerpt from one of the observed lessons below helps illustrate this point.

Ms. Cook: So what was your rise Mike?
Mike: 8
Ms. Cook: What was your run?
Mike: That one
Ms. Cook: So what was your slope?
Mike: 1
Ms. Cook: Do you need an x there to find the slope?
Mike: No (erases the one)

Ms. Cook led Mike through a series of closed questions to fix his work that he presented on the slope of a line. The responses that Mike provided were not discussed by Ms. Cook. Ms. Cook continued with her line of questioning to fix the work that Mike had presented.

The three patterns of SA-NT that were described above can help to describe times in which Ms. Cook might decide to TT or RW. One of the forms occurred when homework was graded. Ms. Cook would say the number to the problem the students would respond with the correct answer and Ms. Cook would move to the next problem. The next form was when Ms. Cook wanted to clarify or review the idea of the student input by asking students closed questions. The final form was when Ms. Cook wanted to help a student correct the work that he had presented. In all of these forms if students did not produce the SA that Ms. Cook had anticipated then Ms. Cook might decide to TT or RW. For example, when Ms. Cook was asked why she decided to TT or RW in association with particular SAs when the class was grading homework Ms. Cook said the following.

> Because there were two or three kids at least who still had questions about them. So I wanted to clarify those before we went on to the next group and mostly because they had to understand those to be able to do today's assignment.

Thus Ms. Cook explained that she decided to TT or RW because there were students that had questions about the SA. Ms. Cook thought it was important to address the SA in order for students to be able to do the task for the day.

Interestingly the four common reasons that Ms. Cook gave in relation to SA were the student input was review of a topic that had previously been covered, the student input aligned with the goal for the lesson, the student input had an idea that Ms. Cook wanted to clarify, and the student input was used to assess students. All four of these reasons do not relate to the reasons Ms. Cook gave for choosing to NT. Thus it is important to discuss when Ms. Cook might decide to TT or RW student input that are SA.

Student Solution (SS). The results of the teacher response to SS that was the largest was RW while TT and NT were about the same. The result of RW being the most often decision that Ms. Cook made in association with SS is not surprising, because Ms. Cook used student input to run her classroom. This idea again makes the results of the SA-NT so interesting. The reason that the SA-NT results are interesting is because Ms. Cook liked to use student input and did use student input. Thus one would think that no matter what the student input Ms. Cook would RW the input more than she would NT. An example of how Ms. Cook RW SS is in the excerpt below.

Ms. Cook: Okay, so I took a snapshot of Lisa's work. Just from looking at it, can you tell what Lisa did?
Sarah: She found the slope and the distance of all the lines.
Ms. Cook: So why would she care about finding the slope and the distance?
Sarah: For if all the sides are equal and what is parallel
Ms. Cook: So according to her she has a slope of zero over seven, zero over seven, zero over four and five over negative nine.
Students: What?
Ms. Cook: Mike
Mike: Wait so this is the first one, right? So AD can't exist right or can it? Aren't we trying to find the side measure not the diagonal measure?

Ms. Cook: So
Sarah: She found the diagonal measure
Ms. Cook: So she did find one of the diagonal slopes, right? Because this says its quadrilateral ABDC, right?
Mike: So Lisa, were you trying to find the diagonals or what?
Lisa: I didn't know it was DC

The above excerpt is only a portion of this episode of student input. The excerpt is meant to illustrate one example of how Ms. Cook would use student solutions to begin mathematical conversation with her students. Ms. Cook took the time when students were working on the task to take a snapshot of Lisa's work to start a conversation about what students did during the task. Lisa labeled her quadrilateral incorrectly, which led to an interesting class discussion. Ms. Cook did not closely analyze Lisa's work before taking a snapshot and did not know Lisa had labeled her quadrilateral incorrectly. Ms. Cook's decision illustrates that a teacher can use student solutions in class discussion without taking the time as the teacher before class discussion to analyze the student work, instead leaving the analysis of the student work for the students and the teacher during class discussion.

The SS-RW pattern was the most common type of response to SS. There were variations in how the SS was presented and how the RW occurred. In the excerpt above Ms. Cook presented a snapshot of the SS, sometimes the student would come to the board and present the SS or the student would verbalize the SS from their desk. Then Ms. Cook would respond by asking students what they thought about the work that was presented or having a back and forth conversation with the student that presented the work.

The reasons that Ms. Cook gave for why she chose to do what she did in association with student inputs that were SS also related to her reasons for RW. The three reasons that Ms. Cook gave in association with SS were the student input related to her goals for the lesson, the student

that was presenting the student input needed assistance in presenting their solution, and Ms. Cook wanted to make sure she understood the student input that was presented. Thus it is no surprise that the top decision that Ms. Cook made in response to SS was to RW.

There were times however that Ms. Cook chose to NT or TT about SS. One of the reasons that Ms. Cook gave for NT that might give us insight into why she would choose to NT about SS is Ms. Cook wanted to have a lot of different student input presented before she talked about the student input. A reason that Ms. Cook gave for TT that might help us understand why she would TT about a SS is Ms. Cook felt that there was an idea that was presented in the student input that needed to be clarified for either the student that presented the work or the other students in the class.

Student Question (SQ). The next category of student input is SQ. Ms. Cook chose more often to TT than the other decisions she could have made; however, her decisions to RW were very close to the same amount of her TT. The teacher response of TT to SQ however was initially a little surprising, but as I thought more about this idea and what Ms. Cook had said through interviews this result made sense. Below is an excerpt of an interview when Ms. Cook was asked why she TT about a student that presented that a polygon had to be a shape with an even number of sides.

> My first thought was why would he think it had to be an even number of sides because she had talked about a triangle and a quadrilateral, but I decided to talk to him. Ok let's make a conjecture then if it is an even number of sides what are we excluding? Triangles, pentagons, and heptagons. Then it was really easy to say oh ok it's not going to be an even number of sides. But I didn't want it to be hanging out there without addressing it because I wanted him to know that no it works for all polygons.

Thus the reason that TT made sense was Ms. Cook likes to address the ideas that students present so that ideas that are not correct are not "hanging out there." Even when Ms. Cook does not feel

that the input aligns with the goal of the lesson she can address the student input by TT. An episode of SQ-TT in the observed classroom can be described as a student asks a question and Ms. Cook responds to the question by merely answering the question without including others in the response to the student question.

An interesting result in relation to SQ was when Ms. Cook was asked why she made the decisions she did in relation to the SQ. Her reasons related to all three types of decisions a teacher can make. One reason Ms. Cook gave was she wanted either the SQ clarified so she understood or there was an idea in the SQ that needed to be clarified. Ms. Cook wanting the question clarified relates to Ms. Cook's reasons for RW. Ms. Cook wanting to clarify an idea related to her reasons for TT. Another reason that Ms. Cook gave was the SQ was review of a topic that had been covered in a previous class. Student input being review of a topic relates to Ms. Cook's reasons for deciding to TT. A final reason that Ms. Cook gave was the SQ was not aligned with her goals for the lesson. If student input is not aligned with Ms. Cook's goal this relates to Ms. Cook's reasons for NT. Thus two of the four reasons relate to TT, which is not surprising because that is what Ms. Cook decided to do the most in relation to SQ. One of the reasons relates to RW and one to NT, which will give us reasons for why Ms. Cook might make a different decision in relation to SQ other than TT.

Student Comment (SC). The largest result of Ms. Cook's response to SC was RW, which was not surprising because as was mentioned in an excerpt from Ms. Cook above, Ms. Cook loved to use student input. When a student shared a mathematical comment with the class Ms. Cook would RW the SC by either asking the students what they thought about the SC, or asking the students a question that related to the SC.

Ms. Cook gave reasons that were associated with her decisions in relation to SC that also support the data of RW being Ms. Cook's most common response and TT and NT occurring less often. One reason that Ms. Cook gave for her decisions in association with SC was the SC needed to be clarified. This also relates to Ms. Cook's reasons for TT and RW. Another reason that Ms. Cook gave for her decisions in association with SC was the SC was review. This also relates to Ms. Cook's reasons for TT and supports the percentage of Ms. Cook's decisions to TT about half of the time because two of Ms. Cook's four reasons for SC relate to TT. Another reason that Ms. Cook gave in association with SC was she wanted to postpone the discussion of the SC for another time. This reason relates to Ms. Cook's reasons for NT and supports the percent of NT being about a quarter of the time, because Ms. Cook gave one reason associated with NT out of the four reasons she gave. The final reason that Ms. Cook gave for her decisions in association with SC was the SC aligned with Ms. Cook's goals for her lesson. This reason also associates with Ms. Cook's reasons to RW. Thus the only decision that doesn't align with the decisions in association with SC is the RW, but the table only shows that the decision happened about a quarter of the time. Ms. Cook would RW SC about a quarter of the time because she felt that ideas in the SC needed to be addressed for one of the many reasons that have been given before.

Time. Time was used as one of the variables because I thought it might play an important role in the reasons why Ms. Cook made her decisions. However, as I collected data, time constraints came up only once as a reason why Ms. Cook did what she did. From the pilot study there was a pattern of the teacher deciding to RW more during the opener and less as the class period went on. Thus I thought I might be able to find a similar pattern as I collected data. In this section there will be a description of the connections that time has with other variables and how I

ended up looking at the variable of time. Connections 4 and 5 connect the variables of the lesson segment, teacher response and why. A description of how connections 4 and 5 were analyzed will be given.

Connection 4. Connection number four is the connection between why and time (time constraints/lesson segments). I made sure to have a variety of episodes of student input that came in the lesson segments of the opener, homework questions, the launch of the task, and the conclusion of the task. The only reasons related to time constraints that Ms. Cook gave for why she decided to do what she did in response to student input was she ran out of time for the lesson, they would talk about the student input at a later time, and the student input had an idea that Ms. Cook wanted to address without taking the time to incorporate the idea into the class discussion. Thus Ms. Cook picks and chooses what student input she decides to RW and TT to make sure that what she does use focuses on the central goals of the lesson. Therefore I decided to look at the lesson segments (opener, homework question, etc.) and Ms. Cook's reasons for her decisions to see if there was a pattern. In looking at Ms. Cook's reasons for her decisions during the different lesson segments I also related those reasons to the reasons that Ms. Cook gave for her decisions in connection 2. In relating Ms. Cook's reasons between connections 2 and 4 there was no pattern between what she did and the lesson segment. The reasons NT, TT, and RW during the opener, homework questions, launch, and conclusion were all differing by a few decisions; therefore I chose not to report further on this connection.

Connection 5. Connection number five is the connection between Ms. Cook's response and time or the lesson segment. This connection was important because depending on the lesson segment Ms. Cook might decide to NT, TT, or RW. Thus the lesson segment was noted along

with the episode to become aware of patterns that might occur between the lesson segment and the type of episode of student input.

To figure out the frequency of the type of teacher response I took the number of the type of teacher responses and divided that number by the total number of episodes during that time in the lesson. For example, to find the frequency of NT during the conclusion, I took the number of NT and divided that number by the total number of episodes during the conclusion. The results can be found in Table 2.

In looking at the table the same conclusions that were made in connection number 4 can also be made in this connection. There is no real majority during any time in the lesson for the teacher response. All of the decisions occur less than half of the time and in many of the times during the lesson the decisions are almost about a third of the time. I decided to break down the results of table 2 even more. I took all of the lesson segments and divided them into the types of student input as well, which is illustrated in Table 3. More explanations will follow for Table 3.

Table 3 illustrates all of the episodes that occurred during the five observed lessons. The first section illustrates all of the episodes that happened during the lesson segment of the opener. The first column of numbers are ratios. The denominator of the ratio represents the total number of episodes of the specific type of student input that occurred during a specific lesson segment. The numerator of the ratio represents the specific type of student input the teacher decision that related to the student input. For example under NT and in the row of SQ the ratio 2/47means that of the 47 SQ that occurred during the opener Ms. Cook chose to NT twice. The second column of numbers are the ratios converted into percentages.

The results from the opener that are the most compelling deal with SQ and SS. Table 3 illustrates that Ms. Cook chose very rarely to NT about SQ. This means that Ms. Cook would

Table 2

Connection Between Teacher Response and Lesson Segment

Time							
Teacher response		NT		TT		RW	
		# of episodes	Percentage	# of episodes	Percentage	# of episodes	Percentage
	Opener	65/199	32.7%	52/199	26.1%	82/199	41.2%
	Homework Questions	139/366	38.0%	93/366	25.4%	134/366	36.6%
	Launch	96/214	44.9%	38/214	17.8%	80/214	37.4%
	Conclusion	145/377	38.5%	88/377	23.3%	144/377	38.2%

always address SQ even if it was her merely talking about the question herself. Ms. Cook choosing to address student questions most of the time also helps to illustrate how Ms. Cook values student input in her classroom, because she does not leave ideas hanging out with no discussion. Another interesting result of Table 3 is SS. Ms. Cook never chose to TT about SS during the opener. Thus when presented with a SS during the opener Ms. Cook would choose to NT or RW. This is an interesting result because it is not very often that we can say that something never happens.

The second section of Table 3 represents the episodes that occurred during the lesson segment of homework questions. The interesting results of the segment of homework questions just like the opener are associated with SQ and SS. Similar to the opener, during homework questions Ms. Cook decided to NT about SQ only twice. Thus the majority of the time Ms. Cook addressed SQ by either TT or RW. In association with SS during homework questions Ms. Cook always chose to RW. Ms. Cook never chose to NT or TT about SS. This is interesting because there are very few times in teaching that we can say that something always happens.

The third section of Table 3 represents episodes that occurred during the lesson segment of the launch of the task. Just like the opener and homework questions, the interesting results

Table 3

Connection Between Teacher Response, Lesson Segment, and Student Input

Teacher response	Time	Student Input	NT # of episodes		TT # of episodes		RW # of episodes	
	Opener	Student Question (SQ)	2/47	4.3%	24/47	51.1%	21/47	44.7%
		Student Comment (SC)	6/32	18.8%	6/32	18.8%	20/32	62.5%
		Student Solution (SS)	2/8	25.0%	0/8	0.0%	6/8	75.0%
		Student Answer (SA)	58/108	53.7%	18/108	16.7%	32/108	29.6%
	Homework Questions	SQ	2/70	2.9%	44/70	62.9%	24/70	34.3%
		SC	22/79	27.8%	17/79	21.5%	40/79	50.6%
		SS	0/8	0.0%	0/8	0.0%	8/8	100.0%
		SA	112/198	56.6%	30/198	15.1%	56/198	28.3%
	Launch	SQ	2/31	6.5%	12/31	38.7%	17/31	54.8%
		SC	5/26	19.2%	4/26	15.4%	17/26	65.4%
		SS	0/6	0.0%	0/6	0.0%	6/6	100.0%
		SA	87/148	58.8%	22/148	14.9%	39/148	26.4%
	Conclusion	SQ	1/50	2.0%	25/50	50.0%	24/50	48.0%
		SC	9/48	18.8%	13/48	27.1%	26/48	54.2%
		SS	0/25	0.0%	4/25	16.0%	21/25	84.0%
		SA	131/254	51.6%	46/254	18.1%	71/254	28.0%

deal with SQ and SS. The results of SQ during the launch follow the same results as during the opener and homework questions. Ms. Cook chose to almost never NT. That means that Ms. Cook would respond to SQ by either deciding to TT or RW. The other interesting result during the launch is for SS. Ms. Cook would decide to always RW. This is interesting because as a teacher trying to use student input you could know that during the launch you would always want to choose to RW in association with SS.

The last section of Table 3 represents the lesson segment of the conclusion of the task. Similar to the other lesson segments the most interesting results for the conclusion relate to SQ and SS. The results for SQ remain consistent with the results for SQ during the other lesson segments. Ms. Cook almost never to decided to NT about SQ. That means that Ms. Cook would either address SQ by deciding to TT or RW. The other interesting result for the conclusion was Ms. Cook never decided to NT about SS. Thus in association with any SS that was presented Ms. Cook either decided to TT or RW.

In summary, no matter when the lesson segment was Ms. Cook would most likely address SQ by either deciding to TT or RW. In regards to SS, during the opener Ms. Cook never decided to TT. During homework questions and the launch of the task Ms. Cook would always decide to RW. Finally during the conclusion of the task Ms. Cook would never decide to NT about SS.

Chapter 5: Conclusions/Discussion

First a summary and discussion of the variations of uses of student input will be given. Then, although time was one of the variables that was initially considered for data collection, during data analysis I realized it did not relate to the other variables as much as I thought it would. However, it is impossible to talk about time without talking about the other variables. After there is a discussion about time there will then be a description of how the type of student input, the teacher response, and the reason for why the decision was made were looked at together. These three variables were very closely related.

Variations of Uses of Student Input

As was described in the previous chapter there are variations in the way that Ms. Cook used student input. In all the variations of student input there was an initial building on of the student input that was presented and there was an end to the discussion of the student input. In some of the variations of student input there were moves to continue the conversation about the student input.

The first variation of use of student input was TT (teacher talk). In this variation Ms. Cook used the student input as a spring board to talk about a mathematical idea herself. Ms. Cook did not include students in the conversation about the mathematics. There were variations within TT. For example, the mathematics that Ms. Cook discussed may or may not be directly related to the mathematics of the student input that was presented; however, this was not an idea that was explored in depth through my study.

The next variation of use of student input was RW (run with) in which the RW was a *brief use* of student input. The *brief use* of student input included a student presenting student input, Ms. Cook making a move to initiate discussion about the mathematics of the student input,

and Ms. Cook making a move to end the discussion of the student input. A *brief use* of student input could be more beneficial than other *brief uses* to the class depending on how the use was left. If a *brief use* of student input was postponed for discussion later in the class or a later class period the use would be beneficial because although students might be interested in the mathematical ideas that relate to the student input they are told and therefore believe that there will be a discussion of the idea later so the students are not left wondering about the idea without the thought of there being a conclusion. On the other hand, if there was a *brief use* of student input in which the idea was left with no conclusion and no mention of coming back to the idea then the use would be a poor use of student input because some students might continue to think about the student input, think about the mathematics related to the student input, try to resolve the issue related to the student input, or come to their own conclusions and possibly have mathematical misconceptions. If students continue to think about the student input, the mathematics related to the input, or try to resolve the issue when the teacher is trying to move onto another idea it might be distracting to other students and may cause the teacher difficulties in moving onto another topic.

Another type of use of student input is a RW that was termed a *complex use* of student input. A *complex use* of student input takes the components of a *brief use* of student input and adds moves to continue the conversation of the student input that was presented. Ms. Cook first makes moves to initiate the conversation to build on the mathematics of the student input. Then there are moves made to continue the conversation of the mathematics. Finally Ms. Cook makes moves to end the discussion. In a *complex use* of student input the mathematics of the student input are taken and built on as students and Ms. Cook try to understand the mathematics that were presented in the student input.

The final type of use of student input is also a RW that was termed a *more complex use* of student input. A *more complex use* of student input takes the elements of a *complex use* of student input and adds more moves to continue the conversation about the mathematics. The more moves that are made to continue the conversation create an opportunity for more mathematics to be presented by students or the teacher. With more mathematics being present the teacher has a more difficult task in deciding what mathematics relate to the student input and how to approach what is presented.

Time

The interesting results of the variable of time relate to lesson segments and also to two specific types of student input, SQ (student questions) and SS (student solutions). Although Ms. Cook chose a few times to not use SQ by NT (not talk), for the most part no matter the lesson segment Ms. Cook addressed SQ by using the student input either by deciding to TT (teacher talk) or RW (run with). Thus Ms. Cook was interested in addressing the SQ through using the SQ no matter the time that it took from the lesson so that ideas were not left hanging for students to be confused by. In regards to SS, during the opener Ms. Cook never decided to TT. During homework questions and the launch of the task Ms. Cook always decided to RW. Finally during the conclusion of the task Ms. Cook never decided to use SS by NT.

Student Input, Teacher Response, and Why

The discussion of the variables of student input, teacher response, and why are organized by the different types of student input. The most interesting results came from the student input coded as SA (student answers). Ms. Cook decided to not use SA by NT (not talk) more often than she decided to use student input. This is because of a pattern that Ms. Cook used often in her class of asking closed ended questions and receiving a SA. If students presented Ms. Cook

with the SA she was anticipating or that was correct she moved onto another question or another idea. Ms. Cook would move on because the SA either answered the question that was asked or matched Ms. Cook's agenda of what she had planned for the class (McClain, 2002; Schoenfeld, 2010). If students did not present Ms. Cook with the SA she was anticipating, the SA was incorrect, or there seemed to be confusion, Ms. Cook would take the time to address the SA by using the student input either by deciding to TT (teacher talk) or RW (run with). In the situations in which the SA was not anticipated, not correct, or there was confusion Ms. Cook would need to make a decision without having time to reflect (Borko & Shavelson, 1990). Ms. Cook would address the SA because she felt that if the ideas were left they would confuse students.

SS (student solutions) followed the pattern that I thought would happen in all types of student input. A majority of the time Ms. Cook decided to use the SS by RW. Her reasons for her decisions in relation to SS also aligned with her reasons for RW. Thus if SS were presented Ms. Cook would RW the SS. RW student input was beneficial for her students because her students were able to communicate about mathematics and gain mathematical knowledge (Cobb, et al., 1992; Hiebert & Wearne, 1993; Lo, et al., 1994). Students were able to gain mathematical knowledge because as students explained their input they were able to solidify their own understanding (Chamberlin, 2005; Fraivillig, et al., 1999; Rosenthal, 1995; Wood, 1998). Students were also given an opportunity to bump up against ideas that didn't go with their thinking (Hiebert, 1992; Simon, 1995). A better explanation of Ms. Cook deciding to RW will be given at the end of the section.

SQ (student questions) was input that was usually addressed by Ms. Cook deciding to use the student input by either TT or RW. The reasons that Ms. Cook gave for her decisions in relation to SQ also related to all three of the decisions that a teacher can make. Thus it is

important to know when Ms. Cook would choose to not use SQ. SQ would be postponed (McClain, 2002) or not used if the SQ did not relate to Ms. Cook's goals for the lesson or she felt that the SQ would lead away from what the class was doing for the day (McClain, 2002; Schoenfeld, 2010). In relation to SQ Ms. Cook chose any of the three decisions that a teacher can make. Ms. Cook had good reasons for any of the decisions that she made in association with SQ.

In response to SC (student comments) Ms. Cook chose to RW about half of the time. The rest of the time was split between NT and TT. Ms. Cook would chose to NT about SC if she wanted to postpone that conversation of the SC for another time. Ms. Cook would chose to TT about SC when the SC was review of a topic that had previously been covered or if the SC had an idea that Ms. Cook felt needed to be clarified before moving on.

Thus overall Ms. Cook decided to use all student input no matter the type of input. The exception would be SA; Ms. Cook would choose to not use most SA. The rest of the time Ms. Cook would choose to use student input mostly by RW. The exception to when Ms. Cook chose to use student input by TT as opposed to RW is when the student input was a SQ.

Chapter 6: Implications

For Teachers

This study has many implications for teachers—for pre-service teachers, in-service teachers, and teacher educators. Pre-service and in-service teachers can learn a great deal from what an expert teacher does in her classroom. My study can help pre-service and in-service teachers know when and why they would decide to use student input. A majority of the time Ms. Cook would decide to use student input in some way, which is not the case for all teachers. Many teachers use the excuse of time for not addressing student input (Ball, 1993; Schoenfeld, 2008); however, Ms. Cook, an expert teacher, rarely used time as a reason for her decisions. Of the 48 episodes that Ms. Cook was asked about, she only mentioned time twice when discussing why she made the decisions she did in response to student input. One of the episodes in which she mentioned time she explained that it was the end of class and they did not have enough time to discuss the input they were discussing. Ms. Cook also said that she would discuss the input more during the next class period. The other time that Ms. Cook mentioned time was when a student presented input that Ms. Cook didn't want to take on at that particular time. She did not feel that the input related to the goals of her lesson, thus she postponed the discussion for a later time.

Many pre-service and even some in-service teachers seem to focus on time and getting to what they planned. Ball (1993) explained from her study in which she validated Sean numbers that time is something that must be considered when determining whether or not to validate an idea. Ball explained that she decided to explore Sean numbers in order to give students an opportunity to do mathematics like mathematicians. This reason is not a reason that most pre-service teachers would give for exploring an idea that did not align with their goals for the

lesson. In Schoenfeld's (2008) decision making model he listed one factor for decisions as the cost or time that it might take to explore an idea that was presented. From the results of Ball (1993) and Schoenfeld (2008) there is no question why a pre-service teacher may use time as a reason for not exploring an idea; however, from my study Ms. Cook illustrated that there are many more reasons that a teacher should focus on for using or not using student input.

Another idea that is closely related to this is when to address the input as the teacher or to let the input be discussed by the class. This is an issue that I struggled with as I was student teaching and learning to use student input. I felt that if I was going to use student input I needed to use everything that students said. I also felt that I needed more than one class member involved in the discussion. I did not think that I, as the teacher, could merely address student input by telling students things associated with their input. Other teachers also have an issue with knowing when to address student input themselves or letting the input be discussed by the class (Lobato, Clarke, & Ellis, 2005).

Lobato et al. (2005) described that one dilemma teachers face is when to tell students things and when not to. The authors explained that telling is not bad and that there are times when telling is required. Ms. Cook took the opportunity at times to tell students things or to not address all the student input that was presented. The times when Ms. Cook would tell students things I would term TT (teacher talk). The reasons that Ms. Cook gave for TT are the student input was review of an idea that had already been covered in a previous class and the student input had an idea that Ms. Cook wanted to address without taking the time to incorporate the idea into the class discussion.

In association with this idea of when to address student input the variable of time or the lesson segment is very useful. Ms. Cook would use all student inputs that were SQ (student

questions) no matter the lesson segment. She may have decided to TT or RW in response to the SQ, but she still used the SQ as an idea to build on and address. Although Ms. Cook always decided to use SQ it might not be a good idea for other teachers to always use SQ. There might have been times when Ms. Cook used a SQ in ways that did not productively build on the ideas. However, if a teacher is presented with a SQ and in the moment they are not too sure what to do with the SQ they can follow Ms. Cook's example and use the SQ either through TT or RW. There are variations in the use of student input and it would be beneficial to further explore what variations work better with SQ and if the lesson segment has an impact on the type of use as well.

In regards to SS (student solutions) Ms. Cook would do different things that either always happened or never happened during particular lesson segments. For example, during the opener Ms. Cook never decided to use SS by TT she either RW the SS or decided to not use the SS. Ms. Cook might have never been presented with a SS that she would have normally TT about during the lessons that I observed. Thus other teachers may find it useful when presented with SS to know that an expert teacher never TT about the SS, but they may also want to think about reasons why they would want to TT about a SS even if it is presented during the opener. During homework questions and the launch of the task Ms. Cook always decided to RW. Ms. Cook might have chosen SS to RW that did not end up being a productive use of building on student ideas. Although it would be useful to teachers to know that they can always anticipate RW SS during homework questions and the launch of the task, they may also want to consider how RW a particular SS would add to their lesson. It is a valuable decision to have in mind to RW a SS that a teacher is not too sure what to do with during homework questions or the launch; however, a teacher must also be open to other decisions that they could make. Finally during the

conclusion of the task Ms. Cook never decided to NT about SS. This is an important implication for teachers because during the conclusion of the task ideas are meant to come to a conclusion or consensus in order to not leave students confused. Thus when making decisions in regards to SS during the conclusion a teacher must think about whether the ideas associated with the SS are going to come to a conclusion or whether the teacher should let students think about the ideas until the next lesson or a time when the ideas can be discussed.

Although using student input is important it is also important to know the times when it would be beneficial to not use student input. My study can help pre-service and in-service teachers know that they do not have to use every student input that is presented, because there is more student input presented than can be addressed (Ball, 1993; Leikin & Dinur, 2007; Sherin, 2002; van Zee & Minstrell, 1997).

Ms. Cook's reasons for not using student input (Not Talk) were the student input was not aligned with her goals for her lesson, she wanted many different student inputs presented before she discussed them, and she wanted to postpone the discussion for another time. The reasons that Ms. Cook presented can help pre-service teachers prepare for times when they may not want to use student input.

The above implications are also useful for teacher educators. If it is beneficial for pre-service and in-service teachers to know when and when not to use student input then it is also important for teacher educators to know, because teacher educators are the people that will teach and help the teachers. It is important for teacher educators to also know about the different variations of using student input so that they can help teachers to understand that there are different choices that they can make in relation to using student input.

For Students

My study also has implications for students that would be in a classroom where the teacher uses student input. The first implication is that if students are a part of a classroom that uses student input students would be respected as mathematical thinkers (Ball, 1993). The students would be respected as mathematical thinkers because their input will be heard by the teacher and other students in the classroom. For example when a SQ was posed Ms. Cook more often than not addressed the student input by deciding to RW the question or deciding to address the question herself by deciding to TT. Thus students in the classroom would know that their ideas were respected by Ms. Cook because she would not disregard the input that they presented. Ball (1993) described that when teachers validate their student's ideas the students feel respected and are more willing to share ideas.

Another implication is when a teacher uses student input students are able to listen to other students and are able to learn more about mathematics (Cobb, et al., 1992; Hiebert & Wearne, 1993; Lo, et al., 1994). Often during class discussion Ms. Cook would let students discuss ideas without her being involved in the discussion. Ms. Cook expressed the following in relation to students presenting work and discussing it.

> If students start to listen to what each other are saying then they'll get a lot more ideas than just my way, because I might just think about it one way. But they are approaching it from a lot of different ways and they start to see, "Oh there are a lot of different ways that I can think about math and it just doesn't have to be Ms. Cook's way." And besides that they bring up some really good ideas that I would never ever think about for approaching it.

As Ms. Cook said above when students present their mathematical work they are able to better understand the mathematics and students are also given the opportunity to listen to other students. As students listen to each other they are able to realize that there is more than one way

to look at a problem. Thus many of the implications for students are related to students being able to better understand and express mathematical ideas.

For Research

My study also provides implications for future research. The variation in uses of student input are a great way to define "using student input;" however, more research could be done on when and why a particular variation of a use of student input would be beneficial. Since there are so many variations within using student input just telling teachers that they should use student input is not enough. For example, more could be done to research when it is more beneficial for a teacher to TT (teacher talk) or *briefly use* a particular type of student input.

The way in which student input and teacher responses were categorized was a useful organizational tool. The spread sheet that was presented in figure two and the spreadsheets that represent my data found in Appendix D helped me to focus my observations as I collected data. As I filled in the spreadsheets after each observation I was able to know for the next observation exactly what types of student input and what types of teacher responses I needed to discuss with Ms. Cook during the interviews. Thus during my lesson observations I was more focused, because the types of student input and teacher responses narrowed as I discussed more of them with Ms. Cook. The categorization would be recommended for others looking to do research in student input and teacher responses.

The cyclical way in which data was collected and analyzed was also a useful tool that provided information that was needed in order to observe another classroom (Strauss & Corbin, 1994). Lesson observations were held with a post-lesson interview with Ms. Cook. The data were looked at following the collection in order to gain insight into what happened in the observation and what could be improved for the next observation. The cyclical collection and

analysis of data helped to ensure that there were enough types of student input and teacher responses during different times of the lesson in order to answer my research questions. The cyclical data collection and analysis might prove useful for other researchers.

Another implication of my study for research is the interview and observation being on the same day. This assisted Ms. Cook to have the lesson and her decisions fresh on her mind (Leikin & Dinur, 2007; Lyle, 2003; Westerman, 1991). It also allowed me to have a good glimpse into the decisions she made. Thus conducting the observation and interview on the same day would help other researchers that are interested in knowing why a teacher does certain things.

The research that was presented was of one teacher during a portion of the school year with only 5 lessons that were observed. Thus one limitation of the study that could be improved with future research was looking at one expert teacher as opposed to being able to compare more than one expert teacher. Having more expert teachers included in a study could help to see if other expert teachers use student input in the same way, at the same time, or for the same reasons.

Another limitation of the study that can be improved with future research was the lessons that were observed were only during a small portion of the school year. In observing different times during the school year others would be able to determine if there were different times during the year that the teacher uses student input differently.

A final limitation of my study that can be improved deals with the questions that were asked to Ms. Cook. I feel that if I would have directly asked Ms. Cook about the different types of student input and if they had an effect on her decisions I would have had even more insight into her decisions. Other questions that might have been beneficial to ask Ms. Cook would be in

relation to time. If I would have asked at the end of the interview if Ms. Cook felt that any of her decisions that she made during class discussion either related to the time of the lesson or the time left in class I feel that time would have been a bigger variable to analyze. Although my study creates a more detailed view of an expert teacher using student input there is still more research that can be done in this area.

Conclusion

Through my study I was able to gain better insight into how an expert teacher makes decisions. In categorizing the different types of student input I found that most of the time Ms. Cook would decide to use student input. This will help me as well as other teachers in leading class discussion, because we will have the reassurance that time is not as big of a factor that we might have thought and taking the time to discuss student input is beneficial. Although the study added to the model of how to lead class discussion by showing how an expert teacher makes decisions during class discussion more research should be done in this area to continue to make a better model of how to conduct class discussion.

Appendix A

In conducting a pilot study I viewed video of a previously recorded expert junior high mathematics teacher. The two lessons that were observed were of two Algebra lessons from two different school years. First video of the reflection meetings was viewed to see what the goals of the lessons were. The goals for both lessons related to students being able to identify slope and y-intercept from different representations. Next video of the lessons were watched and the episodes of student input were identified and then coded into the categories of not talking about (NT), teacher talking about (TT), and running with student input (RW) (the categories will be explained more later in the methodology chapter). Finally the reflection meeting was watched for each lesson to see if any of the questions I had were asked or answered.

Appendix B

FIELD NOTES

Date	
Teacher	
Lesson Goals/ Other Goals	

Time of Episode: _____ Elapsed time: _____ Episode 1
Description of Student input:
_____
_____Preliminary Code _____
Description of Episode:
_____
_____Preliminary Code _____
Thoughts or Questions:
_____
_____
_____

Time of Episode: _____ Elapsed time: _____ Episode 2
Description of Student input:
_____
_____Preliminary Code _____
Description of Episode:
_____
_____Preliminary Code _____
Thoughts or Questions:
_____
_____
_____

Time of Episode: _____ Elapsed time: _____ Episode 3
Description of Student input:
_____
_____Preliminary Code _____
Description of Episode:
_____
_____Preliminary Code _____
Thoughts or Questions:
_____
_____

Appendix C

Interview Protocols NT

I noticed that in regards to this student's input that you didn't say much or use the input very much in your lesson today.

Can you explain to me a little bit about what you were thinking about what this student said and what you decided to do as a result?

How did this student's input relate to the goal of your lesson?

Was this student's input something that you had anticipated?

Interview Protocols TT

I noticed that in relation to this student's input you decided to talk about the student's response. Can you tell me more about what you thought of what the student said and what you decided to do as a result?

I noticed that you didn't incorporate other students in the discussion of this student's idea. Can you tell me more about that?

How does this student's input relate to the goal of your lesson?

Was this student's input something that you had anticipated?

Interview Protocols RW

I noticed that in regards to this student's input you incorporated what they said to hold a class discussion about the topic.

Can you tell me more about what you thought about what the student said and what you decided as a result?

I noticed that you incorporated other students in the discussion.

Can you tell me more about why you incorporated other students in the conversation?

How does this student's input relate to the goal of your lesson?

Was this student's input something that you had anticipated?

Appendix D

Interviews Table

Episode of S.I.	Category of S.I.	NT	TT	RW
	SQ	2,	2, 2, 3,	1, 1, 1, 2, 2, 2, 2, 4,
	SC	2, 2, 3,	1, 2, 2, 3, 4,	1, 2, 3, 3, 4,
	SS	4,		1, 1, 2, 2, 2, 3, 3, 3, 4, 4
	SA	1, 3, 4, 4,	1, 2, 2,	1, 1, 2, 4, 4, 4, 4
	SI	4		

Black = 11/2/10
Blue = 11/29/10

Red= 12/3/10
Green=12/13/10
Purple = 12/15/10

1 = opener
2 = homework questions
3 = launch of task
4 = conclusion of task

S.I. = student input

SQ = student question

SC = student comment

SS = student solution

SA = student answer

SI = student incomplete statement

Lessons Table

	Category of S.I.			
Episode of S.I.		**NT**	**TT**	**RW**
	SQ	1, 1, 2, 2, 3, 3, 4,	1,1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4,	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4,4, 4, 4,
	SC	1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 4,	1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4,	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4,
	SS	1, 1,	4, 4, 4, 4,	1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4,

	1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1 , 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3,3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4,		1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4,
SA	4, 4, 4, 4, 4, 4, 4, 4,	4, 4, 4, 4,	4, 4,

	4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4,		
SI	1, 2, 2, 2, 3, 4, 4, 4, 4, 4, 4,	1, 1, 2, 2, 3, 4, 4,	1, 1, 2, 2, 2, 2, 2, 2, 3, 4, 4, 4,

1 = opener
2 = homework questions
3 = launch of task
4 = conclusion of task

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