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Thinking on the Brink: Facilitating Student Teachers' Learning Through In-the-Moment Interjections

Travis L. Lemon

A thesis submitted to the faculty of
Brigham Young University
in partial fulfillment of the requirements for the degree of

Master of Arts

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ABSTRACT

Thinking on the Brink: Facilitating Student Teachers' Learning

Through In-the-Moment Interjections

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Department of Mathematics Education

Master of Arts

In order to investigate ways pre-service student teachers (PSTs) might learn to teach with high-level tasks and effectively incorporate student thinking into their lessons a teaching experiment was designed and carried out by the cooperating teacher/researcher (CT). The intervention was for the CT to interject into the lessons of the PSTs during moments of opportunity. By interjecting a small question or comment during the lesson the CT hoped to support the learning of both the students of mathematics in the class and the PSTs. This in-the-moment interjecting was meant to enhance and underscore the situated learning of the PSTs within the context of actual practice. Essentially the PSTs learned how to manage and improve the discourse of the classroom in the moment of the discourse. This study utilized both an ongoing analysis of the data during collection in order to inform the instruction provided by the CT and a retrospective analysis of the data in order to develop an understanding of the developmental sequence through which PSTs progressed. The results suggest the interjections provided to the PSTs served multiple roles within the domains of mathematical development for the students of mathematics and pedagogical development for the PSTs. A classification of the interjections that occurred and the stages of development through which PSTs passed will be discussed. Implications from this work include increased attention to the groundwork leading up to the student teaching experience as well as an adjustment to the role of cooperating teacher to be more that of a teacher educator.

Keywords: student teaching, cooperating teacher, mathematical knowledge for teaching, teacher educator, learning to teach, situated cognition

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Brigham Young University

SIGNATURE PAGE

of a thesis submitted by

Travis L. Lemon

The thesis of Travis L. Lemon is acceptable in its final form including (1) its format, citations, and bibliographical style are consistent and acceptable and fulfill university and department style requirements; (2) its illustrative materials including figures, tables, and charts are in place; and (3) the final manuscript is satisfactory and ready for submission.

Date	Blake E. Peterson, Chair of Graduate Committee
Date	Keith R. Leatham
Date	Steven R. Williams
Date	Keith R. Leatham, Graduate Coordinator
Date	Thomas W. Sederberg, Associate Dean College of Physical and Mathematical Sciences

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INTRODUCTION

Tell me I forget, show me I remember, involve me and I understand. ~ Chinese Proverb.

Throughout my educational life as I have moved from student to teacher and teacher to mentor I have felt the truth of this proverb as my understanding has grown deeper by being both on the receiving and the giving end of involvement in educational experiences. The field of mathematics education has evolved and greatly enlarged in recent years. This is in large part due to the reform movement and publication of several standards documents by the National Council of Teachers of Mathematics (NCTM). The growth and evolution of the discipline of mathematics education has been fueled by the work of many researchers and teacher educators who have also adjusted their perspectives and understandings. The tone of this reform movement, as with the proverb quoted above, is one of involvement and understanding.

I feel as though my students and I have benefited greatly from the standards established by the National Council of Teachers of Mathematics (NCTM). The vision, established in *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and further clarified and articulated later in *Principles and Standards for School Mathematics* (NCTM, 2000), has at its heart that students will understand mathematics deeply by being involved in the construction of mathematical knowledge. As an undergraduate student receiving my pre-service training I was introduced to the first set of standards documents published by NCTM, *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and *Professional Standards for Teaching Mathematics* (NCTM, 1991), but only saw the tip of the iceberg when it came to the vision of reform. The learning-to-teach experiences in which I participated did little to overcome the thirteen years in which I had participated in an “apprenticeship of observation” (Lortie, 1975). Rather, as an undergraduate I was mostly told and sometimes shown what my

future classroom could possibly be like but rarely was I ever involved in the happenings of standards-based classrooms as a student. And, never did I have the opportunity to be involved in a standards-based classroom as the teacher. Coinciding with my first year of teaching, *Principles and Standards for School Mathematics* (NCTM, 2000) was published. The vision in this standards document resounded with my beliefs and goals for my students and spurred me to seek professional development opportunities in which I could increase my knowledge in order to better align my practice to the vision of the standards. It has been challenging to implement standards-based curricula and create an engaging learning environment. However, the efforts I have made have produced higher levels of student learning. I credit much of the success I have had as a teacher to the vision espoused in the NCTM standards, to which I was introduced to as an undergraduate, and have worked to realize in recent years.

The field of mathematics education places large amounts of attention on students being involved in a community of learners where knowledge is "social and shared" (Franke, Kazemi & Battey, 2007, p. 228). This emphasis reiterates the connection between involvement and learning. It is essential for students to think and reason, but ever more increasingly important for them to be involved in a learning community. This provides students with the opportunity to communicate and share their reasoning so that their collaborative inquiry can produce learning and understanding both individually and for the collective whole. The life-blood of a classroom environment, which fosters student learning in this way, is the quality of the discourse (Cobb, Wood & Yackel, 1993). It has been a challenge for me to learn how to create and maintain a learning environment that fosters high quality discourse and learning. And despite my experience and growth in this area, I find that I have to work continuously to maintain such an environment.

The focus of this work is on the training of pre-service student teachers (PSTs) as they seek to foster mathematical understanding through class discussion. The PSTs I have worked with in the past value student thinking; however, their abilities to successfully use it are in their infancy. I believe that with the proper learning experiences PSTs can gain greater understanding and skills that will allow them to produce rich mathematical discussions. I also believe it is my responsibility, as the cooperating teacher, to provide the best possible learning opportunities I can for PSTs, just as I do with my students of mathematics.

In order to develop possible instructional experiences that might foster such growth in the PSTs a teaching experiment, where I was the cooperating teacher/researcher (CT) and the PSTs were my students, was designed. Through this teaching experiment I inquired more deeply into my role as a cooperating teacher and also better defined some specific actions I might take (interjections and establishing norms) to foster the development of the PSTs.

Rationale

Within this review the reader will find a rationale for the main issues of this work, which include the research and standards calling for more teachers to employ discourse and utilize student thinking in the instruction they provide.

Discourse: What's the Big Deal?

The work of Vygotsky (1962, 1978, 1986, 1987) who's writings have made a profound impact on the research of learning and cognition, places emphasis on learning and growth through apprenticeship into social practices (Hicks, 1995). For Vygotsky, the central tool used to apprentice and assist the less experienced learner is the language of the more experienced expert. Central to Vygotsk's theory of learning is what he called the zone of proximal development (ZPD). The ZPD is the cognitive region that lies just beyond what the learner can

do alone. Anything that a child can learn with the assistance of another such as a teacher, peer or the instructional environment lies within the ZPD (Whilhelm, Baker & Hackett, 2001). As children repeatedly engage in activities carefully crafted by their teacher, which act as scaffolding for their learning, the children create understandings that facilitate full participation in a social world (Hicks, 1995).

The work of Vygotsky prompted a plethora of writing and research connected to socio-cultural and social constructivist learning theories. This work has helped educational researchers to better understand learning and cognition in general as well as in the field of mathematics education. Many mathematic education researchers (Cobb et al., 1993; Sfard, Forman & Kieran, 2001; Sfard, 2007; Ball, 2008; Schoenfeld, 2008; Hiebert et al., 1997; Goos, 2004) have built on the work of Vygotsky as they have theorized and studied the social and discursive practices of teaching and learning that take place in mathematics classrooms. As a result, discourse, the most essential tool of social interaction, has received a great deal of focus. Hiebert et al. (1997) suggested that it is through communication, which they further defined as "participating in social interaction, sharing thoughts with others and listening to others share their ideas" (p. 5), that understanding will develop. Franke et al. (2007) in their review of the most recent literature found "creating mathematical classroom discourse" to be one of the three central features of mathematics classroom practices that is associated with learning and understanding (p. 226). More strongly, Sfard (2007) has stated, "Learning mathematics is tantamount to modifying and extending one's discourse" (p. 565). Sfard added that one could view "discourse as the very object of learning" (p. 576). She further promoted the importance of an expert interlocutor that proactively seeks to assist the learners.

Standards: Discourse Focused

The National Council of Teachers of Mathematics has placed primacy on discourse in the classroom. The NCTM standards documents (NCTM, 1989, 1991, 2000, 2007) provided a vision of mathematics, which heavily utilizes discourse as a means to promote understanding and mathematical learning. The first of the standards documents *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) focused on communication as one of thirteen standards. This standards document advocated for students to take an active rather than a passive role as they "synthesize, critique, and summarize strategies, ideas, or conjectures that are the products of individual and group work" (NCTM, 1989, p. 67). This first standards document further suggested that mathematical reasoning takes place in collaboration rather than isolation. Additionally, it promoted the development of deep understanding as students "explain, conjecture, and defend [their] ideas orally and in writing" (NCTM, 1989, p. 79).

The *Principles and Standards for School Mathematics* (NCTM, 2000) further articulated the vision for the role of discourse in its opening paragraph by stating students should engage both "orally and in writing" (p. 3) as they communicate and work on worthwhile mathematical tasks. Communication in this second set of standards devotes greater attention to the role of discourse and classroom talk. This standards document, unlike the earlier, has a separate process standard addressing mathematical representations. Much of the discussion devoted to the use of symbols found alongside the standard for communication in *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) was moved to the representation standard in *Principles and Standards for School Mathematics* (2000). This change allowed for the communication standard to focus with greater clarity on the vision of quality discourse in mathematics classrooms.

As discussed above, the curriculum standards established by the National Council of Teachers of Mathematics in both *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989) and *Principles and Standards for School Mathematics* (NCTM, 2000) mentioned the role of communication and discourse. However, it was in the standards documents intended for teachers and focused on teaching that the major focus on discourse was found. In the *Professional Standards for Teaching Mathematics* (NCTM, 1991) three of the six standards contain the word "discourse" in the title of the standard, while the other three standards address discourse in a supportive manner. For example, Standard 1: Worthwhile Mathematical Tasks has, "promote communication about mathematics" as an attribute of such tasks (NCTM, 1991, p. 25). Likewise, Standard 5: Learning Environment and Standard 6: Analysis of Teaching and Learning, allude to the supportive role of discourse by suggesting that students should be able to raise questions, make conjectures and support their mathematical arguments and teachers should be able to describe and comment on this process.

Mathematics Teaching Today (NCTM, 2007) is the updated version of the first mathematics teaching standards document *Professional Standards for Teaching Mathematics* (NCTM, 1991). While *Mathematics Teaching Today* (NCTM, 2007) contains seven standards focused on the teaching and learning of mathematics as compared to the six in *Professional Standards for Teaching Mathematics* (NCTM, 1991) only one of the seven contains the word "discourse" in the title. This however should not be misinterpreted. The message of quality discourse or discussion that is "orchestrated" runs throughout six of the seven teaching and learning standards. It is of particular interest that "Learning Environment" takes a more central role in *Mathematics Teaching Today* (NCTM, 2007), in the previous teaching standards

document it followed the discourse standards, now learning environment precedes the discussion of discourse.

Nature of Quality Discourse in Mathematics Classrooms

The standards documents provide a vision of quality discourse in classrooms. The increased emphasis on student-to-student and student-to-teacher talk in the standards and other literature stands in sharp contrast to the more traditional one-way communication from teacher to student (NCTM, 2007; Franke et al., 2007). The traditional mode of instruction in classrooms across the United States revealed in the TIMSS research of 1995 and summarized later by Stigler & Hiebert (1999) as “learning terms and practicing procedures” (p. 27) has prevailed for more than a century (NCTM, 2007). This instructional style provides little room for students to discuss mathematics or collectively create meaning. Rather, the mathematical work under such a traditional model is more appropriately characterized as symbolic manipulation and memorization of procedures that are told or demonstrated by the teacher. There are many slight variations within the traditional discourse found in most classrooms. However, two prevailing patterns are discussed in the literature. The first is known as “Initiation-Reply-Evaluation” (IRE) where the teacher does both the initiating and the evaluating and the student seeks to reply with correct answers (Franke et al., 2007; Blanton, Berenson & Norwood; 2001). The second is “funneling” where the teacher asks questions, which move the students in a predetermined direction, or “funneling” them where the teacher wants them to go (Franke et al., 2007).

Some have misinterpreted the increased focus on discourse provided in the standards and other literature to mean simply that students should just be talking more. This, however, is a misinterpretation; the nature of the talk matters. If students are only sharing for the sake of sharing or writing for the sake of writing, there is no guarantee that their work will allow them to

make connections, enhance their ability to reason or build conceptual understanding (Simon, 1995). Students need to engage in a "robust mathematical discussion", which according to Mendez, Sherin & Louis (2007) is focused on at a least two dimensions. First, a mathematical dimension comprised of mathematical argumentation that utilizes representations, generalizations and justifications. Second, a discussion dimension comprised of engagement, intensity and concept building. As Stein et al. (2000) suggested communication between students and the teacher benefits students only in so far as it is in service of "doing mathematics."

Teachers Play and Important Role in Creating Quality Discourse

Teachers play a vital role in creating and maintaining quality discourse in the mathematics classroom (NCTM, 1991, 2007; Hiebert et al., 1997; Franke et al., 2007). There are many roles and actions a teacher can take. The teacher's orientation toward mathematics as either "calculational" or "conceptual" (Thompson et al., 1994), their beliefs, goals and knowledge (Schoenfeld, 1998), their ability to establish and maintain norms (Cobb et al., 1993) as well as the materials available to a teacher (Tarr et al., 2008) all effect the decisions a teacher will make and the discourse that will be created.

The number of variants and factors that influence any given teacher or classroom are vast for sure. Due to the scope of the teaching experiment conducted the focus of this work was on the following two areas of teacher influence: a) creating norms that foster a quality learning environment and support the growth of discourse, b) orchestrating discourse (student thinking) by making decisions, especially in-the-moment of instruction, which maintain the norms and foster mathematical reasoning, connections and deeper understanding. These two areas of focus have been chosen because of the dual roll that they played in this teaching experiment. They

were both the learning objectives for the PSTs and the means of intervention through which the CT sought to instruct the PSTs.

Norms: Learning Environment. A farmer carefully works to prepare the soil by plowing, tilling and then harrowing so that the seeds will have the best possible opportunity to sprout and grow. A builder will carefully excavate, level and pour concrete to insure a strong foundation on which a building may stand. Likewise, a teacher must work to prepare a learning environment and culture in which students are afforded the best opportunities to learn. In Mathematics Teaching Today (NCTM, 2007) the role of the teacher in creating a learning environment was further defined to include more than just the physical setting but "an intellectual environment in which serious engagement in mathematical thinking is the norm" (p. 40). It is the teacher's role to provide an atmosphere that values students' ideas, that allows time for students to grapple with significant mathematics and encourages learning as a collaborative practice in which students seek to clarify, justify and question the ideas being shared (NCTM, 2007).

In order for a teacher to establish and maintain such an environment, fertile to the growth of quality discourse, both a primary establishment and continual renegotiation of social and socio-mathematical norms must exist (Franke et al., 2007). In conjunction with the preparatory stages of classroom culture the teacher must continue to cultivate and nourish the learning of the students and their understanding of how to participate in a community of learners. This process of interaction and norm renegotiation occurs through the use of discourse and at the same time fosters discourse. It is through this cycle of norm maintenance and talking about mathematics, as well as talking about how to talk about mathematics, that essential elements of a high quality

mathematics learning environment (explaining, justifying, and collaborating) are mutually constructed by the teacher and the students (Cobb et al., 1993).

One might ask, “What norms should be established to promote growth and mathematical understanding?” This might be paralleled by the farmer asking, “Under what conditions or rules of cultivation will the crops best flourish?” In either case the answer is likely, “It depends!” The literature is full of examples of possible norms that could be established or that might emerge as a result of interaction between students and teacher (Franke et al., 2007; Stein et al., 2000). The intent of this review is not to address all such norms, but rather to emphasize their importance and the essential role of discourse in facilitating their establishment and maintaining their presence, such that high-levels of mathematical thinking can exist and be supported in mathematics classrooms.

Orchestrating Discourse: In-the-moment. The *Professional Standards for Teaching Mathematics* (NCTM, 1991) first introduced the teachers’ role in discourse as one of *orchestrating* classroom discussion by pulling together themes that have meaning just as one might put together chords in a piece of music to create a harmonic sound. *Professional Standards for Teaching Mathematics* also emphasized the importance of having an environment that is conducive to such discourse and provided three main suggestions as to what the teacher’s role should be: a) selection of worthwhile tasks that require thinking and reasoning, b) encouraging student participation through questioning and deciding which ideas should be pursued, c) monitoring and organizing students’ participation by maintaining an awareness of their thinking while seeking to involve all students. Within these three suggestions for teachers, student thinking and student ideas are referred to as the most important item for a teacher to have an awareness of as they look to nurture quality mathematical discourse.

In *Mathematics Teaching Today* (NCTM, 2007) the same vision for teacher involvement in orchestrating discourse was held. In addition to the three suggestions of how teachers can stimulate and manage classroom discourse from the previous standards document (NCTM, 1991) a focus on the use of a variety of representations and exploration of how various representations are alike and different was also provided.

The vision and suggestions provided by NCTM (1991, 2007) require teachers to be adept at making decisions in-the-moment in response to the discourse that unfolds and the student thinking that is revealed. Because of the uncertainty that is prevalent in varied student responses many teachers, both novice and veteran, may have misgivings about seeking to open up the mathematics to students in this way (Blanton et al., 2001; Borko & Mayfield, 1995; Franke et al., 2001).

Smith et al. (2009) suggested five practices that could assist teachers in orchestrating uncertain discussions: *anticipating, monitoring, selecting, sequencing and connecting*. These authors argue that a great deal of anxiety can be relieved if as many student responses as possible are *anticipated* ahead of time. The practice of *monitoring* also relies heavily on the creation of a list of anticipated responses as well as deciding what questions to ask to make student thinking more visible and explicit. The practice of *selecting* has to do with deciding which student work and thinking to focus on and discuss. This is greatly influenced by the goals a teacher has in mind and how each student's contribution will lead toward that goal. The practice of *sequencing* has to do with deciding how the student's ideas should be ordered so that they build toward the mathematical understanding desired. The teacher facilitates *connecting* as student solutions are connected to one another and to the main mathematical ideas of the lesson.

Similar to the framework provided by Smith et al. (2009), Franke et al. (2007) suggested the following principles could foster higher quality discourse: a) allowing for many forms of discourse that may support student learning, b) teachers playing a critical role in the classroom discourse, c) mathematical thinking should be made explicit by students, and d) participation by all is essential. Although these principles and others correlate with quality discourse, there seems to be very little research suggesting how one learns to implement these principles into their practice to orchestrate class discussions in an effective manner.

Learning to Orchestrate Discourse as a Pre-service Student Teacher (PST)

The literature pertaining to the instruction one might receive in learning to orchestrate quality classroom discourse either as a PST or a veteran teacher is scarce. Rather, the suggested method for learning to orchestrate discourse is that of learning by doing (Sherin, 2002). Franke et al. (2007) indicated that the work in the area of classroom discourse has only just begun. She also suggested that mathematical conversations in the classroom are the place where teachers can gain greater insight into student thinking and hone their skills. Franke et al. further promoted the solicitation and utilization of student thinking as a key reason for promoting discourse and orchestrating class discussions. She further suggested quality mathematical discourse can benefit the entire learning community, including the teacher. This work follows previous findings of (Franke et al., 2001), which provided evidence of a focus on student thinking as an essential component in facilitating teachers' learning that could be generative in nature. By generative the authors meant that it was understood and easily built upon. The professional development conducted by Franke et al. was done with Cognitively Guided Instruction (CGI) as a backdrop (Carpenter, et al., 1999). Teachers in the study had participated in professional development that focused on the utilization of student thinking in mathematics classrooms, according to CGI, four

years prior to the data collection. The observations and interviews conducted by Franke et al. revealed:

> that it is the engagement with student thinking that allowed teachers to develop understanding and connect ideas. As teachers engage with student thinking, they think about their daily work, about substance, content and process, and about their own students. They come to see that they can learn through working with their own students in their own classrooms; they receive continual feedback as children discuss their thinking (p. 685).

Franke et al. further suggested that it was through listening to their students and closely examining their students' work and then sharing this with colleagues that teachers build principled teaching knowledge. Despite the success of many of the teachers that implemented CGI less than half were able to reach the highest level (4B on a 1 to 4 scale) of engagement with children's thinking. Some insight may be gained for working with PSTs from the work that has been done with practicing teachers, however, there has been an apparent void in the literature pertaining to the learning of discursive practices by PSTs.

The recent work of Peterson & Leatham (2009) has shed some light on possible difficulties that PSTs may encounter as they learn to teach in a discourse-oriented way (Baxter & Williams, 2010). Peterson & Leatham found three roadblocks for PSTs in their development of the ability to utilize student thinking and orchestrate discourse. First, PSTs had difficulty listening and understanding what students were saying. The PST would often not fully pay attention to what was being said because they were consumed in the act of keeping the class running. And if they did listen to the student's thought, often they did not understand what was being said because of either a lack of knowledge on the part of the PST or uniqueness in the strategy of the student. Second, PSTs experienced difficulty in recognizing teachable moments. Although a PST may have listened and understood what a student said they often would not recognize how that thinking could connect with the key mathematical concepts of the lesson and

build toward greater understanding for the entire class. The PSTs lacked the ability to find and see openings in the discourse that would lend themselves to mathematical development for the class. Finally, PSTs in some cases listened and understood what the student said, recognized it as a teachable moment and attempted to use the students ideas to build understanding for the class yet the PSTSs were still unsuccessful. This occurred for several reasons related mostly to the PSTs lack of knowledge, whether it was specialized content knowledge, pedagogical knowledge or content knowledge. As an implication of their work, Peterson & Leatham proposed that more work needs to be done to design learning activities from which PSTs can learn to overcome the roadblocks they face.

The work embodied here in is an attempt to do as Peterson & Leatham suggested. Acknowledging the call for productive discourse, by both researchers (Vygostky 1962, 1978, 1986, 1987; Cobb et al., 1993; Sfard, Forman & Kieran, 2001; Sfard, 2007; Ball, 2008; Schoenfeld, 2008; Hiebert et al., 1997; Goos, 2004) and reformers (NCTM, 1989, 1991, 2000, 2007) and also recognizing the lack of deliberate instructional interventions that might be employed to assist PSTs in learning to teach in a way that utilizes student thinking in a mathematically productive and discourse intensive manner I planned and carried out the teaching experiment described in this work.

LITERATURE REVIEW

In attempt to situate this work within the wider body of literature pertaining to the practicum or field experience of student teaching the reader will find a review of literature focused on: the purpose and expectations of the student teaching experience, the relationship between cooperating teachers and PSTs, and the deliberate actions or instruction that cooperating teachers might provide or PSTs might solicit.

Purpose and Expectations of the Student Teaching Experience

This section will focus on the perspectives of two main groups, the PSTs and the cooperating teachers, as they pertain to the student teaching experience. This review is done to help the reader better understand the contrasting perspective of the CT and the PSTs in this study, where deliberate norms and interventions were established by the CT.

PSTs Perceptions of the Purpose of Student Teaching.

Feiman-Nemser & Buchmann (1987) provided a glimpse as to what PSTs may view as the purpose of student teaching in their recount of two cases of student teaching. One of the PSTs in their study (Susan) "believed that 'actual concrete experience' was 'more valuable than all the reading and discussion and everything that can take place on a topic'" (p. 258). She felt that the student teaching experience was going to finally provide the practical experience she would need to learn to teach. Feiman-Nemser & Buchmann also stated that after Susan received a positive midterm evaluation she "seemed to lose interest in student teaching as a source of learning" (p. 261).

A second case provided by Feiman-Nemser & Buchmann (1987) focused on Molly and described her expectations for student teaching "'as a building on what I already know. I see it as a practice time, not as something totally new'" (p. 263). Molly described her view of student

teaching as “one more hoop to jump through . . . [like] working in a hotel kitchen before one becomes a chef” (p. 263). Molly hoped to have a chance to put it all together (what she had learned about teaching) and that doing so would be the true test of what she really new about teaching. In some ways Susan’s and Molly’s views of student teaching were like those of other student teachers studied by Calderhead (1988) in that they viewed the student teaching experience “like a driving test, in which there was a series of skills which they had to perform and on which they would be assessed. The object of the exercise was to pass and ‘after you’ve passed you can teach the way you really want to’” (p. 41).

The two PSTs studied by Feiman-Nemser & Buchmann (1987) were similar in some ways to those studied by Borko & Mayfield (1995). Borko & Mayfield found that all of the student teachers in their study “expressed the belief that a person learns to teach by doing—through experience, practice, and making mistakes” (p. 512). This common conception that learning to teach comes by doing it or by simply gaining experience is apparent in both the work of Feimann-Nemser & Buchmann and Borko & Mayfield. However, the work of Borko & Mayfield suggests that there was opportunity for PSTs to learn to teach from their cooperating teachers and university supervisors (although not fully taken advantage of). In addition to learning from experience, PSTs studied by Borko & Mayfield felt that observing others would help in developing their own teaching skills. These same PSTs also “hoped for some suggestions and feedback” (p. 515) from their cooperating teachers on lessons they taught, likely so that they could learn from the work they had done.

Some researchers have found that the preconceived ideas and images of PSTs concerning their views of teaching and what they can gain through student teaching are held tightly and not easily changed (Calderhead & Robson, 1991; Zeichner & Tabachnick, 1981). In fact, Zeichner,

Tabachnick & Densmore (1987) suggested that the images and beliefs held by PST are confirmed or more firmly entrenched after the student teaching experience. In contrast to Zeichner et al., others have suggested that the view taken towards student teaching, and teaching in general, by PSTs may be influenced by the cooperating teachers that they are placed with during their student teaching experience (Borko & Mayfield, 1995; Peterson & Williams, 2008). A more in depth look at the relationship and influence of cooperating teachers on PSTs will come later. Suffice it to say at this point that the teaching experiment I designed and carried was meant to provide the PSTs with more than an opportunity to gain experience but rather an opportunity to learn from deliberate actions taken by their cooperating teacher.

Cooperating Teachers Perceptions of the Purpose of Student Teaching

Borko & Mayfield (1995) found that cooperating teachers responded to a question about how one learns to teach with "the idea that one learns by teaching or by experience" (p. 507). Three of the twelve cooperating teachers in the study conducted by Borko & Mayfield talked about playing an active role in PSTs' learning however most of the others felt that they should not play an active role in the PSTs' learning. Cooperating teachers studied by Borko & Mayfield cited "experience" and "time" as the main mechanisms for learning to teach.

More recently, Leatham & Peterson (2010) conducted a survey of 45 secondary mathematics cooperating teachers in effort to discover their perceptions of the purpose of student teaching and their perceived role in accomplishing those purposes. Leatham & Peterson found eight categories of perceived purposes of student teaching: teacher interaction, real classroom, classroom management, student interaction, proving ground, affective development, enculturation, and generic. The categories that appeared most often were teacher interaction, real classroom and classroom management and led Leatham & Peterson to following conclusion,

"We believe that it is fair to say that, in general, for the cooperating teachers who mentor our student teachers, the primary purpose of student teaching is to interact with experienced teachers in real classrooms, and in so doing to learn how to successfully manage such classrooms" (p. 113). Like the participants of other studies focusing on the student teaching experience (Fiemann-Nemser & Buchmann, 1987; Borko & Mayfield,1995) the cooperating teachers surveyed by Leatham & Peterson placed great emphasis on experience, about half of the responses from cooperating teachers fit into the experience category.

None of the studies reviewed provided evidence of cooperating teachers perceiving the student teaching experience as one in which they, as the cooperating teachers, would provide clear learning goals and instruction for the PSTs in hopes of achieving those goals. On the contrary Feiman-Nemser & Buchmann (1987) and Borko & Mayfield (1995) both highlighted the missed opportunities for cooperating teachers to be teacher educators. Leatham & Peterson (2010) also highlighted the irony between sound pedagogical practice where, "we first articulate our learning outcomes and then design learning experiences that are likely to help students attain these outcomes" (p. 115), which cooperating teachers often employ with their students but not with their PSTs.

The Role of the Cooperating Teacher and their Relationship with PSTs

The literature suggests that the role of the cooperating teacher may in fact be the most influential on PSTs as they engage in their field practice experience (Borko & Mayfield, 1995; Calderhead, 1988; Feiman-Nemser, 1998; Feiman-Nemser & Buchmann, 1987). Calderhead (1988) and others (Borko & Mayfield, 1995) have documented the nature of cooperating teacher and PST relationships through case studies which have provided evidence that such relationships may be superficial or lacking in depth. Calderhead (1988), in his study of ten student teachers,

found that PSTs as well as the cooperating teachers focused mainly on management and organization. The PSTs would mimic their mentor's behavior not seeing the purpose underlying the action. For example, the PSTs would often circulate during instruction but they did not see an underlying reason for doing so. It was simply something they had observed their cooperating teachers do and they knew that their university supervisor expected them to circulate the room during instruction, so they did with out questioning or seeking any explicit insights as to what the goal of such action might be.

Borko & Mayfield (1995) in a similar vein found evidence that the relationship between four PSTs and their cooperating teachers were supportive in nature but not always productive. The PSTs "hoped for some suggestions and feedback, but they learned to be satisfied with very little" (p. 515). One reason Borko & Mayfield found that such disappointment occurred for PSTs was the belief held, by all involved, that learning to teach mainly comes through practice and experience. The cooperating teachers did not see any reason to provide feedback because what the PST really needed was more experience to work out the issues and find their way.

A similar issue identified by Borko & Mayfield (1995) as prohibitive to PST growth was the desire of both cooperating teacher and university supervisor to maximize comfort for the PSTs and eliminate as much risk for them as possible. Fernandez & Erbilgin (2009) found in an analysis of post-lesson conferences conducted by either the university supervisor or the cooperating teacher that cooperating teachers most often were evaluative in their comments about the PSTs lessons and almost always provided positive feedback to the PSTs. Leatham & Peterson (2010) also found that cooperating teachers desired for PSTs to "leave their student teaching experience as motivated, dedicated, and enthusiastic initiates of the teaching community" (p. 114). This desire likely promotes a cooperating teacher's role as one of support

provider rather than one of instructor or teacher educator. The eagerness to support the PSTs in their efforts and be positive in their interactions may inhibit an environment that would promote taking the risks necessary to learn to teach in different ways.

In addition to the roles of being supportive and evaluative as a cooperating teacher, Leatham & Peterson (2010) also found that cooperating teachers viewed their role in five main other ways. These included: provide experiences, model, facilitate reflection, share knowledge, and generic mentor. They further identified the cooperating teachers' "main roles as ones of providing a place to learn how to teach (provide experience), modeling effective teaching (model), and answering questions about teaching (facilitated reflection)" (p. 116).

Only one paper reviewed (Feiman-Nemser, 1998), which was not an investigative piece but rather a position paper, provided evidence of a cooperating teacher taking on the role of teacher educator. Feimann-Nemser demonstrated that this occurred as the cooperating teacher (Vivian) used a "think aloud" method for conveying important insights and information to the PSTs she worked with. In contrast to the case of Vivian, Feimann-Nemser like others (Borko & Mayfield, 1995; Leatham & Peterson, 2010) made the case that the usual role assumed by cooperating teachers and mentors is not one of teacher educator, even though the cooperating teacher's position within the student teaching experience is such that they are likely best able to facilitate student teaching as teacher education (Feiman-Nemser & Buchmann, 1987).

Possible Deliberate Actions or Instruction Cooperating Teachers Could Provide or PSTs Might Solicit

> For students to learn to engage with mathematics and with one another, and for classrooms to nurture such engagement, requires, of teachers, work too often left invisible. By making the effort to unpack the work involved for teachers and their pupils, such practices can be learned and such mathematical work can become more the norm (Ball, Lewis & Thames, 2008, p. 42).

Ball's remarks were part of an analysis of an episode of classroom instruction in which she was the instructor facilitating a discourse focused on the attributes of even and odd numbers. Within this same analysis Ball et al. also stated, "Exhorting teachers to engage students in mathematical reasoning is inadequate as a support for their practice. Parsing the work of teaching makes instructional practice visible, and hence potentially learnable" (p. 41).

Just as Ball et al. discussed the importance of "unpacking" the work of a teacher so that it is more "visible" other researchers have sought to find ways for cooperating teachers to make their practical knowledge visible for PSTs (Meijer, Zanting & Verloop, 2002; Timperley, 2001; Zanting et al., 1998). Zanting et al. provided the following three suggestions for cooperating teachers to make parts of their practical knowledge explicit: (a) make explicit their personal thoughts about teaching when discussing PSTs' lessons, (b) with PSTs present reflect on own lessons articulating the 'what', 'how', and 'why' of their teaching (c) jointly plan, give and analyze lessons with PSTs. In like manner, Timperley (2001) discussed the importance of improving the communication between cooperating teachers and PSTs. In her study, cooperating teachers were given training on how to better communicate with their PSTs. They specifically focused on what types of conversations to have and how to orchestrate these conversations so that they would provide greater benefit to the PST with whom they were engaging.

Meijer et al. (2002) also promoted an improved relationship between cooperating teacher and PST by studying ways that PSTs could elicit the practical knowledge of their cooperating teacher. This was done through stimulated recall interviews and concept mapping. In each of the two techniques the goal was to have the PSTs seek for similarities and differences in their conceptions of teaching as compared to the cooperating teachers so that they might further inquire and learn more about the practical knowledge utilized in teaching. Although these

researchers have sought to find ways to promote access of the mentor or expert teacher's knowledge to the novice PST, they did not specifically address the issue of orchestrating discourse in the classroom, but rather spoke of practical knowledge for teaching in a more general sense.

As described by Fernandez & Erbilgin (2009) cooperating teachers might work to employ "educative supervision" (Blanton et al., 2001) rather than "evaluative supervision" with their PSTs. Fernandez & Erbilgin (2009) stated that this can be done if the cooperating teacher uses "open-ended questioning related to observed classroom experiences [digs] at the student teachers' thinking, particularly related to mathematics pedagogy and mathematics, in order to help them learn from experiences in their own classroom. [And helps PSTs] connect ideas from their mathematics education program to their classroom practice" (p. 106). Frenandez & Erbilgin's conception of "educative supervision" comes from Blanton et al. (2001) who described how the university supervisor of a PST facilitated risk taking by encouraging a shift from a traditional paradigm of univocal discourse laden with IRE and funneling to a dialogic discourse. Blanton et al.'s work is an example of how a more experienced instructor might provide experiences for a PST within the ZPD of the PST.

The literature reviewed above provides some suggestions for improving the relationships between cooperating teachers and PSTs. The main point is that of improving the communication between PST and cooperating teacher. However, there is no mention of specific actions that a cooperating teacher might take in order to address specific learning goals with PSTs. The suggestions are very general in nature and do not provide well-defined actions or learning goals for the PSTs. The work of Blanton et al. (2001), which was built upon by Fernandez & Erbilgin (2009), does provide some specifics about helping a PST learn to more effectively create and use

quality classroom discourse. However, the focus of both Blanton et al. was on the role of the university supervisor and not the cooperating teacher.

The literature seems to suggest that setting norms between cooperating teachers and PSTs, to improve their relationship and communication, will be important in allowing for an increased focus on learning about teaching by the PSTs. However, it does not seem that there are any works in which a cooperating teacher seeks to study their own practices with PSTs, while implementing a specific intervention or "learning to teach" activity. In other words, this work seems unique in the sense that it has a cooperating teacher with learning objectives for his PSTs (learning to use student thinking and orchestrate discourse) and a "learning to teach" activity (interjections) through which he hopes to meet his objectives.

THEORETICAL FRAMEWORK

My Learning to Teach Experience

Like the PST described by Blanton et al. (2001) that shifted from having mostly univocal discourse laden with IRE and funneling to a more dialogic discourse, I (CT) have made a paradigm shift in my own teaching practice. This has come in large part as a result of implementing standards based curricula along with a standards-based learning environment. The transition has been difficult, yet rewarding. I was fortunate to be introduced to the vision of the NCTM standards documents during my pre-service training and also benefited by starting my career in a school district that choose to adopt a curricula full of worthwhile mathematical tasks.

Taking the risk of soliciting student thinking and building students' understanding through discourse was not easy for me although the vision was appealing. Increasing my ability to teach in a discourse-oriented way (Baxter & Williams, 2010) was difficult and didn't occur in isolation but rather through collaboration with colleagues. I sought out models of quality practice that I could observe, attended all the professional development I could and read case studies of other teachers (i.e. The Ron Kastleman case in Stien et al., 2000) in effort to improve my understanding of the vision of quality instruction called for in NCTM's standards (2000). I recall specific incidents in which I felt as though I was plunging into deep water without support or assistance. Some days I felt as though I was trudging forward and other days I rejoiced with colleagues in my successes. I wished for greater amounts of support and more experienced observers. It was very infrequent that I was able to have an instructional coach or colleague observe and give me feedback (I recall less than five times that this happened during my first five years).

Working with PSTs

It was not long after I began to make changes in my practice that I was asked to mentor a PST in my classroom. This PST embraced the standards-based curricula and reform style of instruction I had adopted. The difficulty of orchestrating whole class discussions however was ever prevalent and I did not at the time have any ideas as to how I might address the issue. Having taught myself from the literature I had been introduced to, the vision of the standards (NCTM, 2000), and through trial and error I was at a loss as to what specific action I might take beyond modeling for my PST. The following year I was fortunate to be involved with research attempting to make "visible" the issues faced by PSTs in developing the ability to use student thinking to orchestrate mathematical discussions (Peterson & Leatham, 2009).

Participating as a cooperating teacher in the research conducted by Peterson & Leathem (2009) caused me to look introspectively at what I do as a teacher and also at what I do as a mentor for PSTs. More specifically, I began to wonder what I could do, as the CT, to assist PSTs in developing their ability to orchestrate discourse in a whole class setting.

Change in Perspective: Teacher as Teacher Educator

Although student teaching naturally promotes interaction between PST and mentor teacher, I have felt a need to give more in terms of instruction. Having not received support in learning to teach as a new teacher I desired to improve the experience for those that I was asked to mentor. It is evident in the literature that PSTs are greatly influenced by their student teaching experience and their cooperating teacher (Peterson & Williams, 2008; Feinam-Nemser & Buchmann, 1987; Borko & Mayfield, 1995). As a cooperating teacher, I desired to influence the PSTs in the best possible way by having discussions with them and providing learning experiences from which they could benefit. My desire to be not just a mentor, that provides a

place for PSTs to experience teaching, but a teacher educator that provides "educative mentoring" (Feiman-Nemser, 1998) is consistent with Leatham & Peterson's (2010) call for mentor teachers to provide "learning-to-teach" activities. I believe that orchestrating discourse and using student thinking is an important part of a mathematics teacher's role and function. It was my intent to make the orchestration of a mathematical discussion that used student thinking a clear goal for the PSTs in this study and then to provide "learning-to-teach" activities that would foster growth toward this objective.

Cognitive Apprenticeship

Lave (1997) stated, "Apprenticeship forms of learning are likely to be based on assumptions that knowing, thinking, and understanding are generated in practice, in situations whose specific characteristics are part of practice as it unfolds" (p. 19). Thus, learning to orchestrate whole-class mathematical discourse might best be learned by engaging in the practice of orchestrating whole-class mathematical discourse. Lave further argued that learners or "apprentices" increase their knowledge and ability to think, act and interact properly when they do such things with people who do it well, as "legitimate, peripheral participants" (p. 19). The apprenticeship model discussed by Lave promotes understanding of the apprentice or learner in such a way that the apprentice becomes an "active agent" that may be more likely to act on their own behalf in terms of understanding the practice of a master practitioner.

Brown, Collins & Duguid (1989) defined cognitive apprenticeship as the method in which students are enculturated "into authentic practices through activity and social interaction in a way similar to that evident – and evidently successful – in craft apprenticeship" (p. 37). Brown et al. further articulated this process as a progression through phases. The apprentice or learners first watch as the master or teacher provides "modeling" of the practice to be learned

through “authentic activity.” Then, in the second phase, the apprentice becomes active as the teacher supports or “scaffolds” (Hennessy, Deaney & Ruthven, 2005) their attempts at “doing” the task. This is often done through questioning or guided participation. The final phase is one of “empowerment”, which moves the apprentice into independent practice, while the teacher assumes the role of observer.

Traditionally student teaching allows for the PST to take part in the first and last phases of the cognitive apprenticeship model provided by Brown et al. PSTs begin by observing for a while and then the day comes at which point they are to assume the role of the teacher and practice independently. My personal experience followed this pattern. The amount of time if any spent on phase two, where the PST receives support from the mentor teacher varies and is mostly based upon the PST’s ability to properly manage student behavior. This focus on management during the second phase of the apprenticeship focuses the PST’s attention in the wrong direction (Feiman-Nemser & Buchmann, 1987). Rather than receiving classroom management supports, PSTs need to be engaged in “learning-to-teach” activities during phase two of the apprenticeship. This would allow them to be supported in their journey of becoming master teachers rather than masters of discipline.

Research Question

As one subscribing to socio-cultural views, which support the cognitive apprenticeship model, and also having the desire to better aid PSTs as they develop the skills and abilities needed to orchestrate whole-class conversations, in this study I will measure how specific interventions made by the CT (me) might possibly bridge the gap between the phases of observation and independent practice for PSTs. More specifically, this research seeks to find ways that a cooperating teacher might interject during the lessons of PSTs and how such

interjections may facilitate PST's access to knowledge and what might be learned from the PSTs responses to the interjections?

METHODS

To answer the research question a “teaching development experiment” (TDE) was conducted with a goal similar to that of Simon’s (2000) in that the researcher was seeking to “study development by fostering development as part of a continuous cycle of analysis and intervention” (p. 336). Simon further underscored the importance of TDE and other methodologies that are consistent with the “emergent perspective” (Cobb, 2000) in working to realize the reform of mathematics education and the “reinvention of mathematics teaching, [and] also the reinvention of teacher education” (p. 336). Furthermore, the selection of the TDE methodology allowed for the CT to serve as the researcher-teacher-educator and “work at the growing edge of [his] knowledge of how students (in this case PSTs) learn” (Simon, 2000, p. 340).

Specifically, the focus of the CT was on studying the development of the PSTs pedagogical practices for teaching mathematics, particularly their ability to use student thinking and orchestrate discourse, while providing interventions to foster their development. The pre-planned interventions came in the form of interjections during the lessons taught by the PSTs. Interjections, as referred to in this study, were comments or questions provided by the CT to the class as a whole or as side comments to the PSTs during the lessons taught by the PSTs. The intent of the interjections was two fold, maintain the opportunity for learning of mathematics by the students in the class and facilitate the opportunity for learning of the PSTs. The interjections will be described in much greater detail later.

The TDE methodology as described by Simon (2000) incorporates and builds upon the constructivist teaching experiment and whole-class teaching experiment methodologies as well as case study methodology. Cobb (2000) suggested that a teaching experiment methodology,

coupled with a focus on students' mathematical development, leads to consideration of the following three items: (a) social context of development, (b) activity and development of the teacher, (c) instructional sequences. The work done by the CT in this TDE focused intently on PSTs and their pedagogical development rather than students' mathematical development yet still has the following similar elements: (a) close attention was paid to the social context in which the student teaching took place and the CT proactively establishing social norms to foster the development of PSTs, (b) the cooperating teacher actively intervened (interjected) to provide instruction for the PSTs and this activity reciprocated in the development of the CT, and (c) a possible developmental sequence that cooperating teachers with similar goals can draw on in future years as they interact with PSTs was created.

Setting

University Program

The university's student teaching program places two PSTs with each cooperating teacher. Each of these pairs is also connected with another pair, which is located in another school, to form what is referred to as a cluster. The PSTs are arranged in clusters for the purpose of participating in focused observations and reflection meetings. The university program emphasizes student thinking in several ways. During the first five weeks of student teaching the PSTs are engaged in several learning-to-teach activities which include: focused observations, student interviews, daily journal writing, the writing of reflection papers, preparing a lesson with their partner and teaching the lesson while being observed by their university supervisor, cooperating teacher and peers followed by a peer reflection meeting. As described by Leatham & Peterson (2009) each of these activities places primacy on student thinking through observation, analysis and reflection.

The focused observations done by the PSTs include filling in an observation form (See Appendix A) for one lesson observed in the cooperating teacher's classroom and another form for a lesson observed in another teacher's classroom. The PSTs rotate through each of the following foci during the first five weeks of the student teaching experience: flow of lesson, mathematical discourse, students' mathematical experience, questions and answers, teachable moments. Upon completion of the two focused observations for the week, each PST is to synthesize the observations in a one- or two-page summary to be turned in to the university supervisor the following week.

The PSTs conduct interviews with students during weeks two, four and six of their experience. They are to select students that are achieving at an average level and ask them about their responses to questions from a recent assessment or homework assignment. The goal is to have the students share their thinking with the PST, not to provide the student with additional instruction. The PSTs are encouraged to listen closely to the students and pose additional questions or tasks that will assist them in gaining additional insight into students' understandings. After the interview the PST may provide additional instruction based upon the information gathered during the interview. After the interview is complete each PST writes a two- to three-page paper characterizing the understandings and misconceptions of the students that came to light as a result of the interview, and the recommendations they have for moving the student forward to greater understanding.

Daily journal writing is intended to serve as a constant reminder to the PSTs to focus on students' mathematical thinking and teachers' pedagogical practice. As they go about their daily activities they are to document their goals, happenings of the day, and insights gained into students' mathematical thinking. For example, one of the main prompts the PSTs were asked to

address required them to identify some student thinking they found interesting and describe how they might use that thinking during a lesson. The daily journal, although somewhat structured, requirement is more open than the other requirements for the PSTs. Entries in the daily journal can be focused on whatever issues or questions, pertaining to the student teaching experience, a PST desires. Its main purpose is to be a constant reminder to the PSTs that they should be recording and reflecting on the happenings of the day.

Beginning with week three, each of the PSTs prepares and teaches one lesson per week for the next three weeks. They are to collaborate with their partner to develop the lesson goals and corresponding tasks to help students reach those goals. Each of the PSTs in the pair at the host school teaches the same lesson to different class periods on the observation day. The other PST pair in the cluster along with the university supervisor and the cooperating teacher observe the lessons. The observing PSTs are given the charge to look closely at student thinking and how it develops throughout the lesson; one goal is to look at how students are experiencing mathematics during the lessons that day. After the lessons, a reflection meeting is held to allow for the sharing of ideas by the PSTs so they can self-reflect and also benefit from the input of their peers. The majority of the reflection meeting is spent on discussion of the development of the lesson and the mathematical thinking of the students. The PSTs do this while the university supervisor and the cooperating teacher reserve their comments for the end of the discussion.

The lessons taught by the PSTs during the first three weeks of student teaching provide each PST with the opportunity to participate in two reflection meetings a week, one at their school following the lesson they have taught, and one at the school of the other PST pair in the cluster. After participating in both reflection meetings, each PST writes a four- to six-page reflection paper about the experience. The purpose of the reflection paper is to have each PST

compare and contrast the mathematical thinking of the students and analyze lesson development across the lessons taught. The PSTs are to focus their reflection papers on the insights they gained into students' mathematical thinking by taking part in the planning, teaching, observation and reflection meetings. The PSTs are also asked as part of the reflection paper to propose possible improvements that could be made to the lessons and directions in which further learning may progress.

Following the first five weeks, each PST in the pair takes over half of the cooperating teacher's teaching load for the remaining nine weeks of the semester. The peer observations and reflection meetings are discontinued until the last week of the semester when one final observation and reflection meeting is held. During the final nine-weeks a university supervisor observes each PST on a weekly basis, and after each observation meets with the PSTs as a pair or individually to provide feedback and reflect on the lessons. The activities of the PSTs during the nine-week span of time are directed mainly by the cooperating teacher. The deliberate actions or interventions planned for this study spanned the entire fourteen weeks of the student teaching experience. However, the last nine weeks received the most intense focus.

School Site

The research takes place in a public junior high school (grades 7 to 9) of 1600 students located in a suburban area close in proximity to the university the PSTs attended. The classroom in which the PSTs are located can be characterized as a supportive environment in which PSTs are welcome and allowed freedom and opportunity to try new things and pursue ideas. The schedule of courses the PSTs assumed responsibility for included Algebra 1 and Geometry. The PSTs each taught two classes of Algebra and one class of Geometry, when they assumed their full teaching load. The average class size for these courses was thirty-six students.

Participants

Students

The students in this study, Bill and Melissa (pseudonyms), were pre-service student teachers (PSTs) from a large private university in the mountain west region of the United States. Prior to the student teaching experience, students are required to complete thirty-five credit hours of Mathematics courses, three credit hours of Statistics, thirteen credit hours of Mathematics Education courses and eleven credit hours of Secondary Education courses. The Mathematics Education Department seeks to train professional teachers so that they will be well-prepared and able to work at the secondary level. They seek to provide their graduates with the skills and abilities that will allow them to continue their development and focus on thinking about learners and effective ways of working with learners.

Bill could be described as being at the top of his class. He had performed exceptionally well in all of his undergraduate course work and had been mentored in research by one of the mathematics educators at the university. These experiences had prepared Bill well for the student teaching experience. Bill also expressed an interest in returning to study mathematics education in the near future and at some point becoming a mathematics educator and researcher.

Melissa came to the student teaching experience after several years of undergraduate work. She was married with three children at home and had taken some time off from school in order to be at home with her children. Unlike most of the PSTs coming from the same university Melissa's student teaching did not complete her undergraduate work and she did not graduate directly following the student teaching experience.

Researcher-Teacher-Educator

The author was the researcher-teacher-educator/cooperating teacher (CT) for this study. The CT had taught at the school site for nine years at the time of data collection. In attempts to align teaching and learning with the reforms advocated for in *The Principles and Standards for School Mathematics* (NCTM, 2000) the CT implemented a reformed curriculum, *Connected Mathematics Project* (CMP) (Lappan et al. 2006), as the main curriculum source in his classroom. Many other resources were also available and often drawn upon by the CT and his colleagues. Another focus of the CT was to promote discourse in the classroom as a means by which student thinking could be shared and built upon to develop understanding of mathematical concepts for all students. The CT's mode of instruction could be described as *discourse-oriented teaching* (Baxter & Williams, 2010). For these reasons the CT was chosen as a cooperating teacher when the university was developing their current PST program.

The role of the CT in this teaching experiment was to take part in generating and testing hypotheses as part of multiple iterations of the reflection-interaction cycle. In addition to providing support to the PSTs as most cooperating teacher do (see chapter on Literature Review) the CT observed almost all of the lessons taught by the PSTs and participated in conversations before and after the lessons to provide instructional assistance. Furthermore, the CT was a resource to the PSTs allowing the PSTs to take the lead but at the same time formed hypotheses about what might assist the PSTs in their development. The CT also developed personal knowledge through participation in the process and, through retrospective analysis, also created a model of the development of the learners (Simon, 2000).

Observer

A full-time faculty member of the Mathematics Education Department from the university that the PSTs attended fulfilled the role of observer. The role of observer complemented the role of the CT and amplified the analytical power of the research team. More specifically as Simon (2000) stated,

> The observer (a) represents a perspective from outside the teacher-student interaction, (b) calls attention to aspects of the data set that might be overlooked by the researcher-teacher, (c) introduces alternative explanations that broaden the analysis of the data, (d) challenges the researcher-teacher's interpretations and formulations, and (e) (by the nature of the collaboration) requires the researcher-teacher to articulate and communicate his or her ideas (and vice versa). (p. 341)

The observer often maintained his awareness of the happenings of the TDE and the work of the PSTs and CT by running the videotaping equipment used to gather some of the data.

Others

The focus of the teaching experiment was on the development and learning of the PSTs and their interaction with the CT. However, it should be acknowledged that others influenced this study. The observer, for example, had a direct impact on the work of the CT. Some others that potentially influenced this particular teaching experiment include: university supervisor, colleagues of the CT, other pre-service student teachers (OPSTs) from the cluster, the students receiving the mathematics instruction and colleagues of the observer. However, the influence of any of these other individuals, with exception of the university supervisor and the CT's advisor (who was also a colleague to the observer) from the university, was minimal when it came to setting the agenda for action within the teaching experiment.

Interventions (Deliberate Actions)

In accordance with Simon's (2000) description of teaching experiments, development was fostered through specific interventions. These interventions were selected based mainly on the experience of the CT in working with PSTs from previous years and the CT's desire to scaffold the learning to teach experience for PSTs and support opportunities for learning by both PSTs and students of mathematics in his class. The specific interventions that were used in working with the PSTs included both in-the-moment interjections during lessons taught by the PSTs and the establishment of norms between the PSTs and the CT. A rationale for each of the interventions is given below. It should be understood that the goal was to foster development in the PSTs. Therefore the interventions were developed and adjusted in response to the analysis of the data gathered, as an ongoing research cycle, throughout the fourteen-week student teaching experience (Cobb, 2000).

Interjections

Interjections were designed to be moments in which the CT would speak up and contribute in some way to the lesson of the PST during the course of instruction. Because of the complexity of mathematics classrooms, PSTs can often struggle with the development of high quality discourse (Blanton, Berenson & Norwood, 2001). Although PSTs may value student thinking and seek to build class discussions from student thinking, problems often arise (Leatham & Peterson, 2007; Peterson & Leatham, 2009). So, in order to foster the abilities of the PSTs to utilize student thinking as they orchestrated whole-class discussion the CT provided in-the-moment interjections during the course of the lessons taught by the PSTs. The intent of interjecting was to move the lesson forward mathematically and to assist the PST in moving the class in a productive direction. The CT hoped to provide the students in the class with a greater

opportunity of understanding the big mathematical ideas either related to or focused by the PSTs. The CT did not intend for the interjections to be moments in which he took over the lesson. Rather the interjections were meant to act as "scaffolds" (Hennessy, Deaney & Ruthven, 2005) or supports to assist the PST with the lesson and in learning how to teach.

The interjections also became markers within the contextualized moment of instruction. By interjecting, the CT flagged or marked a particular moment during the lesson in a way that allowed for it to be easily referred to later. Almost like the acronyms I am using as I write (PST, CT, TDE, etc.) allow me to refer back to a previously before mentioned item more easily. The interjections in some ways created an abbreviated reference to a particular moment in the lesson that could then be referred to later during post-lesson reflection. During post-lesson conversations, both formal and unscheduled, the PST that taught the lesson and the one who observed it along with the CT analyzed the happenings of the class and reflected on the interjections with the CT and discussed the purpose and intent of the interjection. Hypothetical learning opportunities were provided for the PSTs through such post-lesson conversations as well as during pre-lesson planning conferences (Simon, 1995).

Norms with PSTs

As an intervention and in order to facilitate the more prominent intervention of interjections, social norms were explicitly addressed with the PSTs. The first week of the student teaching experience, the PSTs and the CT had a discussion in which they explicitly discussed and clarified the norms for their interactions. From that point on the social norms were constantly renegotiated to meet the needs of the participants. The norms fit the description provided by Lo & Wheatly (1994), "The establishment of social norms are necessary for the smooth functioning of social interactions . . . [and such] norms are formed through negotiation, and are not fixed

rules" (p. 147). The norms were negotiated to allow for interjections into the lessons of the PSTs and to allow for other instruction that would support their learning. Just as a teacher would initiate and negotiate social norms to promote the learning of mathematics (Cobb et al., 1993), the CT insured that social norms were established to facilitate learning how to teach.

The following example is given to assist the reader in understanding the parallel between the norms established by the CT with his students and those established by the CT with the PSTs. At the beginning of each school year, the CT actively establishes norms with his students of mathematics to ensure that they are actively engaged in learning and also active members of a learning community in which they have a voice. This also lends itself to an understanding that the CT as the teacher is not the intellectual authority. For example, the CT specifically has a conversation with students about the importance of all members of the classroom contributing to the conversation just as members of a basketball team (or any group, band, dance, football, etc.) must contribute in order to produce the highest quality performance. The realization by students that the CT as their teacher is much like a coach and they as the members of the class are a team is empowering to them and helps them to be more engaged in the discourse of the classroom and contribute to the development of mathematics. In like manner the CT addressed his role with the PSTs in that he did not want them to view him as a threat or the ultimate authority. Rather that his role as the CT would be one of support and assistance with his goal being two fold: (1) supporting the learning opportunities for the students of mathematics in the class, (2) supporting the learning opportunities for the PSTs as they worked to bring about the first goal. Just as the CT established norms with his students of mathematics to change their perception of him to one of coach or facilitator so to the CT established norms with the PSTs to help them view the role of CT as that of guide or facilitator (teacher educator).

To be more explicit, it is important for the reader to realize that the CT was not seeking to support the PSTs in the sense of being generally supportive as much of the learning to teach literature suggests most cooperating teachers have been (Borko & Mayfield, 1995; Feiman-Nemser & Buchmann, 1987). Rather, the CT was seeking to implement the interjections and establish norms to provide learning opportunities for the PSTs. This of course can be easily seen as supportive, but as a specific supportive action of the learning goals the CT had identified for the PSTs (effectively using student thinking and orchestrating discourse). The CT made it clear to the PSTs that his desire was to support them and to support their opportunities for learning. He explicitly stated to the PSTs that they could call on him at anytime and made it especially clear even from the first lessons taught by the PSTs that this included calling on him while they were teaching.

Data Collection

Multiple data sources were pursued to provide for rich descriptions of the dynamic nature of the interactions between the PSTs and the CT. Having several data sources available helped promote accuracy in the findings as constant comparative analysis was realized. The following data was collected: (a) Video of one lesson taught by each PST each week, (b) Individual formal interviews conducted by the observer during weeks one, six and fourteen, (c) Audio recorded conversations between PSTs and the CT, (d) Journal entries of the PSTs and CT, (e) Field notes taken by the university supervisor and the CT, and (f) The required materials for student teaching established by the university program.

The lessons taught by the PSTs during the first five weeks of the student teaching experience and the subsequent reflection meetings were videoed. Following this initial five-week period, videoing of lessons continued on a weekly basis. The schedule for videoing followed the

pattern established during the first five weeks. The observer and several undergraduate research assistants took turns running the video equipment and recorded the classroom happenings for the entire class period. Unlike the first five weeks, there were no other student teachers present and no formal reflection meetings held during the videoing that took place from week six up until week fourteen.

The observer formally interviewed the PSTs during weeks one, six and fourteen of the student teaching experience. The interview questions are included as Appendices E, F and G. The "intensive interviewing" guidelines set forth by Charmaz (2006) in which the "interview fosters eliciting each participant's interpretation of his or her experience" (p. 25) were followed. The intent was for the PSTs to express themselves and make their thinking explicit. The interviews were not rigid but rather open-ended, allowing for the exploration of ideas and concerns brought forward by the PSTs.

The conversations between the PSTs and the CT were audio recorded. Not only were formally scheduled conversations audio recorded but the unscheduled conversations were recorded as well. Sometimes the most meaningful and significant conversations took place over lunch or on the way to the teacher's lounge. To capture as much of this data as possible, the CT carried a digital recorder with him and kept it activated when he and the PSTs were conversing. The goal was to gather as much data on the CT-PST interactions and conversations surrounding the lesson planning and reflection as possible. These conversations would be used to shed light on the response of the PSTs to the interjections and instruction provided by the CT.

The reflective journal writing that took place during the first five weeks did not formally continue throughout the PSTs' experience. However, the CT continued to promote reflection and pressed the PSTs to rely upon their reflections of the day in preparation for future lessons. The

PSTs knew that upon completion of a lesson the CT would greet them with, “So, what do you think?” (as a way of pressing them to reflect) It did not take long before the PSTs began asking this question before the CT had a chance to verbalize it himself. The CT kept an informal journal in which he audio recorded his thoughts, insights and wonderings. The constant focus on reflection provided opportunity for what Dewey (1938) referred to as the sound psychology of “stop and think.” This facilitated, as explained further by Dewey, “a more comprehensive and coherent plan of activity” (p. 64).

The university supervisor visited the PSTs weekly and during these visits took extensive field notes on the lessons taught and the happenings of the classroom. The university supervisor then gave feedback to the PSTs in several different ways, either individually with each PST or collectively with both PSTs and the CT present. The CT did not interrupt the normal interactions between the PSTs and the university supervisor. The university supervisor audio recorded some of the conversations he had with PSTs. The lesson plans of the PSTs for the entire fourteen weeks were requested and a large number of Bill’s lesson plans were given to the CT.

Data Analysis

TDE methodology (Simon, 2000) utilizes data to investigate development and to inform interventions taken to foster development. The compound role played by the data “involves two important levels of data analysis: the ongoing analysis, which occurs during and between sessions with students [PSTs], and the retrospective analysis, which focuses on a sequence of sessions” (Simon, 2000, p. 341).

The first level of data analysis, informal ongoing data analysis, took place during the course of teaching and learning and was used to inform future instruction and interventions with the PSTs. Each week videoed lessons, field notes, conversations between PSTs and the CT were

reviewed in order to better understand the PSTs current level of thinking and development. It was on the daily evaluation of PSTs' thinking, especially as it applied to classroom discourse, that instructional actions for the following days were based (Simon, 2000). Insights and observations from colleagues both at the school and university levels were invaluable in enhancing the model of the PSTs thinking and development. It was during the ongoing data analysis of the TDE that the CT began to develop an understanding of the PSTs development (or lack there of) and sought to further that development by adjusting the pre-planned interventions and by providing additional learning opportunities.

The second level of data analysis, retrospective analysis, took place after completion of the fourteen-week student teaching experience and, although informed by the ongoing analysis, was done through constant comparative means and in the spirit of grounded theory. This was the case because the CT sought insight from the data and revisited the data of focus several times to build themes and understandings of the PSTs' progression and development (Charmaz, 2006). The CT made an initial pass through the video data carefully documenting the interjections that took place. The time at which each interjection took place during the lesson, the actual words spoken or action taken by the CT, and the context surrounding the interjection was recorded in order to give the CT a preliminary idea as to the nature and number of interjection within the video data.

Because of the significant number of the interjections (over a hundred in the video-data alone) the researcher at that point decided to target a few specific days and dig deeply into the data surrounding those days and the interjections that occurred. The intent of digging into the nature of the interjections more deeply for a few days was to allow for a finer-grained analysis and a more fully contextualized understanding of the nature of the interjections in order to

answer each part of the research question. By looking closely at the interjections for a few days a well-developed theory concerning the types of interjections could be developed to answer the first part of the research question (In what ways might a cooperating teacher interject?). A richer contextualization of the interjections also allowed for greater insight into the other parts of the research question. By looking at the post-lesson conversations between the CT and the PSTs insight into what PSTs might be learning, the second part of the research question (What knowledge was there opportunity for the PSTs to access because of the interjections?), was gained. And the PSTs response to the interjections both during their lessons and afterward during post-lesson conversations was also helpful in terms of answering the third part of the research question (How will PSTs respond to the interjections?) and also in facilitating one of the main purposes of a retrospective analysis by developing an "explanatory model" of PSTs' development (Simon, 2000).

Three days of video data were selected as the data of focus, the first lesson taught by the PSTs, the lessons taught during week six and the final lessons observed by the cluster. These data selections were made based upon the other data available at those points. For the first and last lessons selected there was an accompanying reflection meeting on the same day that was video recorded. For the second and third lessons selected there were interviews conducted on the same day by the observer that could also assist in providing more information about the experience of the PSTs on those days. By selecting lessons from the beginning, middle and end of the student teaching experience a cross time comparison of the PSTs progression from the beginning to the end of the student teaching experience could be realized and a more complete model of development created.

Upon selection of the videos to be fully analyzed, the reflection meetings corresponding to the first and last of these videoed lessons as well as the conversations on the three days of focus were transcribed. Additionally the three interviews which took place aligned nicely with the video data: one occurred at the beginning, one on the day of the second video and the final interview occurred on the same day as the final video. All interview data was also transcribed and analyzed. Unfortunately, do to technical error the first two interviews with Melissa were lost. So greater reliance on pre- and post-lesson conversations and the field notes of the CT was required in looking at Melissa's development. The CT then made a second pass across the videos within the data of focus once again identifying and documenting the interjections that took place and also categorizing the nature of the interjections.

The interjections were categorized by looking at the actual dialogue of the interjection and determining what opportunity for learning seemed to be made available based on the words used in the interjection and the context of the conversation leading up to the interjection. In a few cases, where the wording of the interjection was generic and the video may have not captured sufficient context, the intent and purpose of the CT in providing the interjection was relied upon in order to categorize the interjections. Two lenses for looking at the interjections were used to categorize the interjections, one focused on the opportunity for learning mathematics by the students in the class and the other focused on the opportunity for learning to teach by the PSTs. This was done because of the CT's initial goals for the TDE, to provide learning to teach opportunities for the PSTs and to maintain quality learning opportunities for the students of mathematics in the class, and also to answer the first two parts of the research question (In what might a cooperating teacher interject during the lessons of PSTs and how might such interjections facilitate PST's access to knowledge?).

Following the categorization based upon learning opportunities, a closer look was taken at the response and disposition of the PSTs to the interjections. This was done by looking at both their in the moment disposition towards the interjections as well as their comments about the interjections during post-lesson conversations. Through this analysis an explanatory model of the PST's pedagogical ability to teach mathematics in a discourse-oriented way was developed (Simon, 2000).

This second round of data analysis, retrospective analysis, progressed in stages and was guided by principle of constant comparison and in the spirit of grounded theory (Charmaz, 2006). The first videoed lesson, reflection meeting and interviews as well as conversations surrounding the first lessons taught by the PSTs were analyzed and a theory concerning the classification of the interjections and the PST's responses to the interjections was created. Then the second day of selected data was analyzed just as the first had been and the prior theory was measured against this data from the second day's data and revised. This refined theory was then measured against the final day's data and a final theory emerged. From the data and this analysis grew the categories for the different types of interjections that occurred (viewed in two ways in order to address the first two part of the research question) and the creation of a model of development based upon the PSTs responses to the interjections.

RESULTS

The intent of this chapter is to provide a summary and classification of the interjections that occurred in the data of focus and to present a model of development that the PSTs in this study moved through. The interjections, although spontaneous in nature and based on the happenings of the moment, can be classified in two main ways: *mathematical development* and *pedagogical development*. Melissa in her final reflection meeting commented on the dual nature of the interjections, “There are, kind of, two different interjections. One to help the mathematics be correct, the other one is to help the teacher to become better.” It is through these two lenses, mathematical development of the lesson for the students and pedagogical development of the PSTs, that a retrospective classification of the interjections took place. This is done in effort to answer the first two parts of the research question pertaining to the possible ways in which a CT might interject into the lessons of a PST and the knowledge that PSTs may have access to as a result of the interjections. Looking at the responses of the PSTs to the interjections, and then interpreting those responses to create an explanatory model of the PST’s abilities to teach in a discourse-oriented way addresses the third part of the research question.

Mathematical Development: Supporting the Mathematical Work of the Classroom

This section provides a description of the types of interjections that occurred as viewed through the lens of what opportunity for learning of mathematics the interjections seemed to promote. The focus is on the opportunity for learning that the interjections provided rather than the actual effect on learning or learning that was definitely created, since the actual effect was not precisely measured. The section is divided into two main subheadings: interjections spoken for the whole class (this includes the majority of interjections in the data), and interjections

spoken to individuals and small groups (this addresses the interaction the CT had with students in the class that can be viewed in the video data but for which there is no audio record).

Interjections Spoken for the Whole Class

Mathematical Connections and Distinctions. The first interjections that were verbalized during the first lessons taught, by both Bill and Melissa, B-1 and M-1[1] (see Figure 1), exemplified the type of interjections that were intended to point out a mathematical connection or distinction between ideas (For a detailed list of all interjections, see Appendices B and C). The first interjection in both cases dealt with creating a distinction between lines of symmetry and diagonals of polygons. This is clearly apparent from the words in the interjections themselves. To better understand the opportunity for mathematical learning within the interjections it can be helpful to understand the context in which the interjections occurred.

B-1.	"I-I'm wondering are the lines of symmetry the same as the diagonals always?"
M-1.	"I just have a question, are the…are the diagonals going to be lines of symmetry or vice versa?"

Figure 1. First interjections made during PSTs lessons.

The PSTs had prepared a task for the day (see Appendix D), which included a warm up activity or opener intended to help the students review the vocabulary of lines of symmetry and diagonals. The opener was in preparation for the larger task in which students would develop formulas for calculating both the number of lines of symmetry and the number of diagonals in regular polygons. During the opener, students were asked to draw lines of symmetry and diagonals for the given polygons. In Bill's class, students were asked to put their work from this

[1] The labeling of the interjections provides a letter that corresponds to the first letter of the PST's name and a number that indicates the order in which the interjection occurred within the data of focus. For example, B-1 is the first interjection recorded during Bill's lessons within the data of focus. A label of M-10 would indicate an interjection that occurred in Melissa's lesson and was the tenth of those occurring within the data focused on for this study.

task on the board and then explain. The student that put his work on the board for the parallelogram from 1c of the opener (see Figure 2) had drawn the diagonals along with a median even though the problem asked for lines of symmetry. As a result of this student's work being presented to the class, a discussion about why those lines (the diagonals drawn by the student) were not lines of symmetry occurred. The focus of the discussion was to show that the lines, which had been presented, were incorrect. An opportunity seemed to surface in this moment to help both the student that presented the misconception, as well as the other students in the class, to better understand and create a distinction between diagonals and lines of symmetry and when they may or may not be the same for a given polygon. Bill did take the discussion to the point of producing the correct answer. However, the CT saw an opportunity to further clarify and pursue a greater distinction and thus the mathematical distinction-making interjection, B-1.

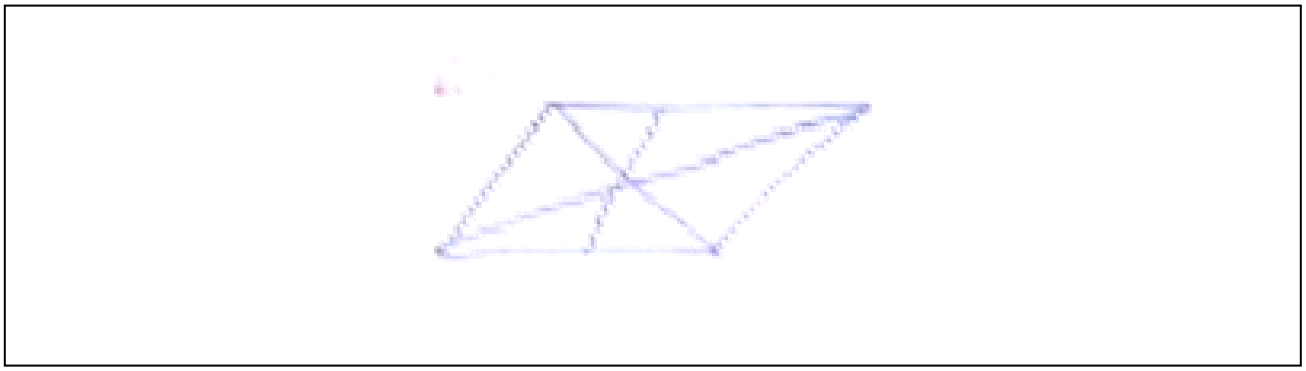

Figure 2. A student's attempt at drawing lines of symmetry for a parallelogram.

In Melissa's class on the same day a similar occurrence took place, involving interjection M-1, which focused on lines of symmetry and diagonals. Additionally, during Melissa's lesson two more mathematical connection-making interjections occurred, M-2 and M-4 (see Figure 3). Both of these interjections attempted to make mathematical connections by emphasizing the reasoning that was being repeated and applied in slightly different circumstances. For example, interjection M-2, occurred while Melissa was addressing students' questions from the previous

night's practice problems. The students were asked to classify quadrilaterals and were unsure about how to place a kite within a Venn diagram containing other quadrilaterals. During previous lessons, the classification of trapezoids had come up and the question of using an inclusive rather than an exclusive definition of trapezoid had been discussed. The determination to classify a parallelogram as a trapezoid or not, hinges on whether or not the definition of trapezoid allows for inclusion of quadrilaterals with "at least one" pair of parallel sides or exclusion by defining trapezoid as quadrilaterals with "exactly one" pair of parallel sides. Interjection M-2 was an attempt by the CT to point the class toward the discussion about trapezoids they had participated in previously and help them have the opportunity to see that a similar type of reasoning about the kite (Are kites inclusive of rhombi or not?) would facilitate its placement within the quadrilaterals. Interjection M-2 was a mathematical connection-making interjection because it was meant to direct students toward prior mathematical knowledge and repeat a previous line of reasoning.

M-2.	"I wondered if we could think about the trapezoid for a minute and maybe that would help us with the kite?"
M-4.	"Ms. Turner, you just, you show that they're touching the shape eight times. Is there some kind of connection between amount of touches and amount of them (lines of symmetry)?"

Figure 3. Connection making interjections during Melissa's first lesson.

Interjection M-4, which also occurred during the same lesson as M-2, was seeking to build mathematical understanding by pre-exposing students to a line of reasoning that was to be drawn upon later. This interjection, occurred during the first part of the mathematical task for the day (see Appendix D), focused on finding the number of lines of symmetry for a regular polygon. Melissa had just finished pointing out the way that lines of symmetry "touch" an even-

sided regular polygon is different then the way the lines of symmetry "touch" the sides of an odd-sided regular polygon (see Figure 4). The former has the lines of symmetry intersect at

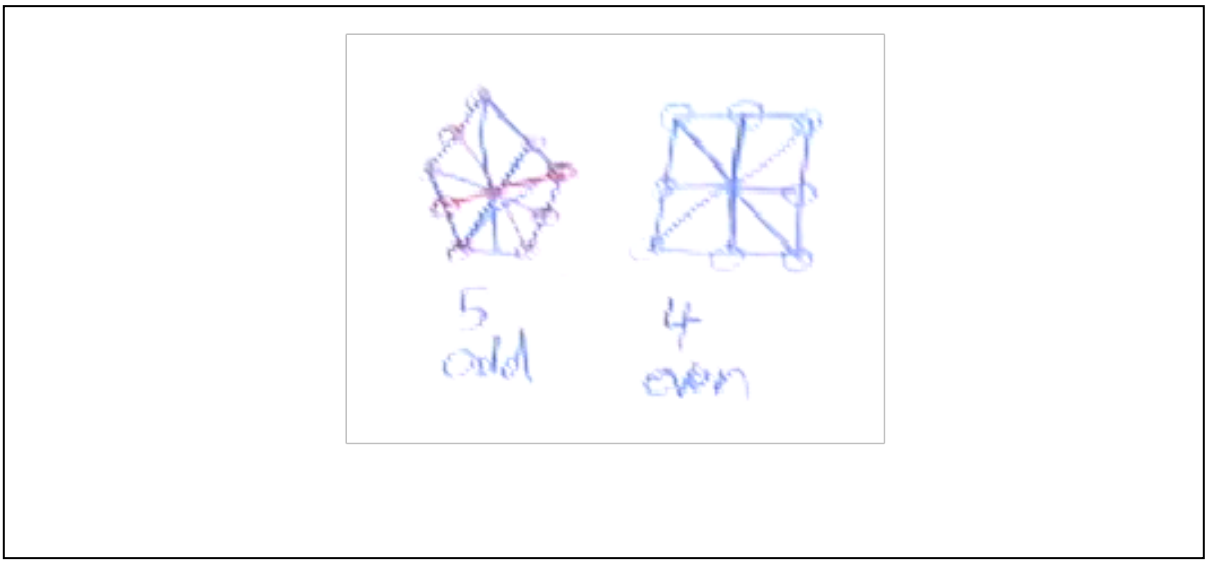

Figure 4. Melissa's demonstration of how lines of symmetry touch polygons.

either two vertices or two mid-points of sides and the later has the lines of symmetry always intersect the regular polygon at one vertex and one midpoint. This seemed to create an opening in the conversation for seeing that the lines of symmetry "touch" the polygon twice and yet for an *n*-gon there are *n* of them. This may also be thought of as an *n*-gon having half as many lines of symmetry as there are intersections or "touches" between the lines of symmetry and the regular polygon. This emphasizes that a counting of the intersections or the "touches" is a double counting.

The CT hoped interjection M-4, as a connection-making interjection, would lay the groundwork for the upcoming task to be done pertaining to diagonals. Diagonals, like lines of symmetry, intersect a polygon twice and the CT hoped to facilitate this connection (counting intersections of lines of symmetry with the polygon or intersections of diagonals with the

polygon results in a double counting of the segment) between the two ideas and scaffold the development of the formula for finding the number of diagonals in a convex polygon.

In moments such as those described above, the interjections provided by the CT came in the form of a question and were intended to focus the students on distinctions or connections that might help them increase their understanding of the concepts being learned. These interjections provided an opportunity for students that the PSTs did not provide and based on the analysis of post-lesson conversations would not have provided because they did not see the opportunity that the CT had noticed.

Encouraging Student Work. Prior to the PSTs' arrival, the CT had worked to link the development of the mathematics in the classroom directly to the contributions of the students in the class in a way similar to what could be described as discourse-oriented teaching (Baxter & Williams, 2010). The PSTs were given a similar expectation, of using student thinking, from the university's program for student teaching. The classroom climate was such that it promoted the expectation that students' work and student thinking be inseparable from the development of the mathematics being learned. The overarching purpose for the happenings of the classroom was the achievement of specified mathematical goals and the use of student thinking to help all of the students achieve those mathematical goals.

Despite the environment and expectations that had been given, the PSTs were not always sure how and when to involve students and build upon their work. Interjections M-5, M-8 and M-10 (see Figure 5) are examples of interjections that generally encouraged student contributions. Interjection M-5, which was made by the CT for the whole class to hear, is an example of a situation in which the students had not been given sufficient time to think and contribute and therefore the opportunity to develop the mathematics was inhibited.

M-5. "Maybe could we, since there were a few people who were gone, could we maybe have people get together for just a second and get some ratios down if they don't have them, give them just a second and then ask again for the ratios."

M-8. "Can we keep clarifying this? Each comment is making it more clear for me. Can someone else…"

M-10. "Jacob is going to do 14."

Figure 5. Interjections that encouraged student work.

Interjections M-8 and M-10 also encouraged student work because they promoted student sharing and engagement. Interjection M-10, although very specific in nature, "Jacob is going to do 14", came within the context of answering homework questions and the CT happened to be sitting next to Jacob and noticed that he had done number 14 on the homework. So, the CT encouraged Jacob to share his work with the class. Any student that might have been noticed to have number 14 from the homework could have equally likely been encouraged to share their work on the problem with the class. The specific strategy used by Jacob was not considered or brought to the awareness of the CT prior to this interjection. The CT was taking the opportunity to simply get the ball rolling with student volunteers to present their work from the practice problems that had been assigned.

The interjections that only encouraged student work promoted the participation of all students more generally. These interjections may appear more pedagogical in nature in that they appealed to the PST to allow all students an opportunity to think or to share. However, as mentioned it was through the sharing of student thinking that the mathematical goals for the lessons were realized. Unlike the mathematical-connection interjections these encouraging student work interjections came as suggestions or statements rather than questions and were more general rather than specific.

Pointing to and Emphasizing Student Work. Interjections in this category pointed more specifically to one particular student's work or strategy in order to promote the mathematical development of the lesson, unlike interjections that only encouraged student thinking generally. Interjections B-2, B-3, B-9, B-13 and M-6 (see Figure 6) specifically identified a student, rather than simply appealing to the whole class. By focusing the attention of the class on one specific strategy, idea or question, the opportunity to progress toward the mathematical goals of the day was provided.

The interjections, which were pointing to a specific student's work and emphasizing it for the mathematical contribution that it made, at the same time encouraged student work. The tone of the interjections was such that when specific students were pointed to, they felt validated in their efforts, regardless of whether or not they were confident that their answer was right.

B-2.	CT, "I think Brady might have a really good idea." PST, "Oh yeah." Brady, "Okay." CT, "Would you be willing to share? That was cool."
B-3.	"Hmm, I'm wondering more about number four and number five. Are the ratios that Nya made the same kind of ratios that Cameron made?"
B-13.	"Can you repeat that Lauren? Say that again."
M-6.	"Before we go on there were some really big questions being asked back here. We don't need to necessarily answer them right now but I think maybe we should get them out there so as we go forward we keep thinking about them. Casey would you share the questions you had?"

Figure 6. Interjections that pointed to and emphasized one specific student's work.

For example, interjection M-6, which provided Casey with an opportunity to share her question with the class also led to a discussion about the role of ratios sometimes being that of a scale factor as well. This in turn lead to a clarification of the vocabulary of scale factor and ratio, followed by a rich discussion about internal ratios and corresponding ratios between similar triangles. The reciprocal relationship between scale factor and the corresponding ratios was also

made. So within this interjection, there was both an opportunity to encourage student work and also an opportunity to pursue the mathematical goal for the day.

Interjection B-13 (Can you repeat that Lauren? Say that again.) provided both an opportunity to point to and emphasize the contribution made by a student. Interjection B-13 also acted much like an exclamation point. Lauren had been involved in an episode of dialogue, B-9, which came previous to B-13, in which she had come to better understand the significance of corresponding sides when looking at ratios in similar triangles (a more detailed look at B-9 follows). Within just minutes of this previous clarification Lauren summarized her interpretation of a ratio as it pertained to similar triangles, “So, it can be between some sides in the same triangle and between two different triangles.” Which seemed to clarify the confusion that not only Lauren but many of the other students seemed to be having about the relationship between ratios, scale factors and corresponding sides of similar triangles. Because of the resolution that Lauren appeared to have, the CT interjected B-13, which provided an opportunity to point specifically to the mathematical clarification that she had achieved and to also emphasize that same clarification.

By interjecting questions that asked a student to repeat their comment or idea, emphasis was given to the idea and this underscored its importance and facilitated the development of the mathematical goals for the day. This type of interjection, in which specific student thinking was pointed to or emphasized, as with the other interjections, allowed the CT to provide opportunities to encourage student thinking and to move the students toward greater understanding of the mathematical goals for the lesson.

Pressing on Student Thinking (Clarifying). Interjections that provided an opportunity for mathematical development as a result of pressing on the student thinking that had already

been made available fit in this category. As specific students strategies and questions were presented to the class, a need arose for those strategies to be pressed upon in order to fully take advantage of the mathematical learning opportunities the student's work contained. Interjections that provided an opportunity to press for clarity came at times when students were at the front of the class sharing their ideas, like B-6, B-15 and M-12, and also when students asked a question from their seats, like B-9 (see Figure 7).

Interjection B-6 can be used to demonstrate the type of pressing that was done. Here Eliza was at the board explaining to the class how she was seeing the connection between a scale factor for similar triangles and the ratio of corresponding sides for the same set of similar triangles. To facilitate access for all students to the development of this mathematical idea, a pressing or clarifying interjection was used to provide a greater level of insight into the idea being shared. Once Eliza wrote the ratio as a fraction, the reciprocal relationship she was attempting to explain became much more obvious to the class.

B-6. "Eliza you are pointing at the ratio, can you go ahead and write the ratio as a fraction so we have the same format? Now talk more about that. What are you noticing? Maybe show us up on the top one as well."

B-9. "Somebody had a question up here, was it you Lauren?"
Lauren clarifies the question being asked, "I just asked was it on the same triangle or on different triangles? …Like long to short on the same triangle or two different triangles?"
CT, "So, would it make sense to have a long to short that involves more than one? Does that make sense Lauren?"

B-15. CT, "Could you write a similarity statement for use?" Student, "Sure."
CT, "Just to make it even more clear."
PST, "Can everyone follow along and make sure he is doing it?"

M-12. "Jacob can you, like outline the angle you are looking at and then outline the arc?"

Figure 7. Interjections that press on student thinking or clarify.

Interjection B-9, is an example of an interjection which allowed a student in the class, Lauren, to have the opportunity to come to greater understanding by simply pushing her to

clarify her question as it related to the work that had already been presented. This is the same Lauren that had her idea later underscored with interjection B-13 (Can you repeat that Lauren? Say that again.). The pressing that took place through the interjection of the CT seemingly facilitated Lauren's ability to sort out her ideas and later made a significant contribution to the development of the mathematics for the class. Interjections B-6 and B-9 show how interjections, which primarily pressed on student thinking at the same time, pointed to the student's thinking and emphasized it to the class as well as checked the understanding of that specific student. (A discussion of the multiple roles played by some interjections will be presented later.)

Not all interjections that press on students to clarify their thinking played as many roles as B-6 and B-9. Interjections B-7, B-11, B-12, M-14 (see Figure 8) are examples of interjections meant to underscore the importance of keeping track of mathematical ideas. These interjections did not press on any one student's thinking but provided an opportunity for all students to push their own thinking forward by carefully taking note and being precise in the their representations of the concepts. When ideas are expressed verbally or in writing they seem to gain clarity for the student sharing them or writing them down. The CT hoped that the interjections in Figure 8 would provide all students with the opportunity to clarify their own thoughts as they put them on paper with clear labeling.

Checks for Understanding. Interjection B-14 seemed to stand apart from the others in its role of developing mathematical understanding. Interjection B-14, "While Seth is doing that, how many of you know the value of x and x+2?" Rather than supporting development it instead primarily provided an opportunity to assess development. The majority of the time it was through observation that the PSTs and the CT gathered formative assessments (B-17, is one that was verbalized) of the students' progress. Interjection B-14, provided a quick showing of hands

for a problem that was to be presented just as class was ending and time did not warrant discussion.

B-7.	"When you have got them on your paper label them. Like label them this one is my ratio, small to big, this one is my ratio big to small, like label them so you know what is what. Otherwise, you know we can still have some confusion, so we need to keep it clear."
B-11.	"Once again, label them. Write down on your paper long to short and put down all the ones. And then, just like Mr. Lawrence has it up there, make yourself a couple of categories and put them all in there."
B-12.	"Write yourself a note somewhere and label what is happening. Write a ratio is … and give yourself an example or explanation and a proportion is. . . "
M-14.	"So, while we go through this you should be taking really good notes of what goes up. Some of you had some really good work some of you still need some. So, as we go through make sure that we get the ideas down on your paper."

Figure 8. Interjections that pressed students to clarify their work more generally.

Several of the interjections discussed above as pressing-on or clarifying student work interjections can also be viewed as interjections that provided an opportunity to check for understanding. Interjection B-6 pressed on Eliza's thinking and at the same time checked her thinking for deeper understanding of the mathematics.

Interrelatedness of Major Interjections Pertaining to Mathematical Development.

As mentioned previously, some interjections provided students with multiple opportunities and served multiple roles in the classroom. The categories above consider the context in which the interjections occurred and are an attempt to break down the complexity of the context in order to better view the primary role of the interjections as they supported the mathematical development taking place in the classroom. However, it is important to remember that an interjection's primary role or classification may not be its only one (see Figure 9). In fact, there are several interjections that encouraged student thinking and the development of mathematics from students' ideas, which could be viewed as having one primary role and

another secondary role. In Figure 9, a representation of the relationship between the different interjections, as it pertains to students' mathematical development, can be found. The purpose of the interjections was to create opportunities for the development of mathematical understanding and so this is the backdrop on which all of the interjections occurred. Within this primary purpose is found the secondary purpose of the interjections in creating opportunities for student involvement by encouraging their thinking. The other three types of interjections as discussed in the previous sections all fit within these two main purposes and are interrelated in nature.

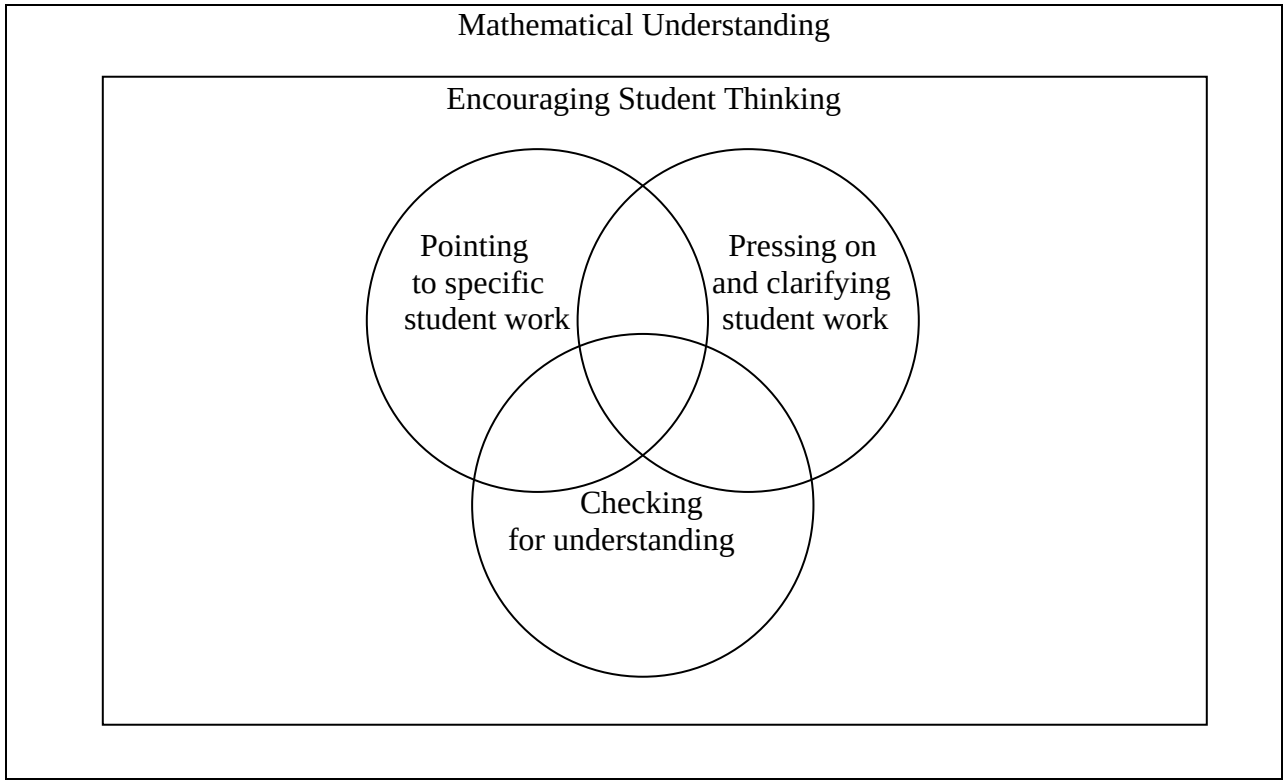

Figure 9. Relationship Between Interjections Pertaining to Mathematical Development.

Interjections B-2, B-3 and B-6 (see Figure 10) will be used to demonstrate this interrelatedness. Interjection B-2 shows how Brady's thinking was encouraged by being pointed to, (I think Brady might have a really good idea.) and then pressed upon (Would you be willing to share that was cool.). In like manner, interjection B-3 is primarily pointing to the work of two

students but at the same time is also checking to see if others in the class understand the work that was presented. Interjection B-6, discussed above involving Eliza, would be an example of a rich interjection because it points to specific work (Eliza's), it presses her thinking for clarity (Now talk more about that) and it checks for her understanding (What are you noticing?). These and several of the other interjections could be placed within the intersections of the three overlapping purposes of interjections in the center of Figure 9.

B-2.	CT, "I think Brady might have a really good idea." PST, "Oh yeah." Brady, "Okay." CT, "Would you be willing to share that was cool."
B-3.	"Hmm, I'm wondering more about number four and number five. Are the ratios that Nya made the same kind of ratios that Cameron made?"
B-6.	"Eliza you are pointing at the ratio, can you go ahead and write the ratio as a fraction so we have the same format? Now talk more about that. What are you noticing? Maybe show us up on the top one as well."

Figure 10. Sample interjections showing interrelatedness.

In contrast to those serving multiple purposes, some interjections seemed to serve just one purpose. Those that simply encouraged student work in general (see Figure 5) did not do much to serve any other purpose. Interjection B-13 (Can you repeat that Lauren? Say that again.) in like manner seemed to only point to Lauren's thinking and emphasize what she had said without any other purpose. Interjections B-7, B-11, B-12 and M-14 (see Figure 8) also were primarily meant to generally press students to clarify their thinking as they worked.

Interjections Spoken to Individuals or Small Groups

For this study, the interjections, which occurred so that all of the individuals in the classroom could hear them, took the primary focus. However, the CT was an active participant in all aspects of the lessons and happenings of the classrooms and interacted on an individual basis with both the students in the class and the PSTs during the lessons. The influence of these less

formal interjections on the mathematical development for the students in the class is discussed briefly below.

The nature of the classroom environment was one in which students in the class were frequently asked to engage in mathematical tasks which would scaffold their thinking toward the mathematical goal of the lesson. Students were also often given the opportunity to collaborate with one another on the tasks they were given. During this time of collaborative student work the PSTs, as well as the CT, observed students, provided them with clarifying questions and gave suggestions as to what students might attempt next. During this exploration phase of the lesson subtle and indirect interjections often took place. The CT would sometimes assist students in such a way that they would have a desirable approach to the task and a line of reasoning that would promote higher quality mathematical discussion. The efforts of the CT could be described as those referred to by Stein et. al (2009) as actions that maintained a high-level of cognitive demand for the task. In another sense, the actions of the CT might be described as messaging in nature. With an eye to the mathematical goal, the student thinking and the task at hand, the CT would scaffold a progression for the students. By encouraging students to listen to one another and share their thinking or by pointing to and pressing one of the students ideas the CT would utilize the task, which had been given, as a vehicle for the student thinking to point groups of students in the right direction.

The CT utilized many of the same techniques (see Figure 9) when interacting with small groups that he did when interjecting so that the whole class could hear him. These individual interactions with students, which were evidenced in the video data throughout the fourteen weeks, although not viewed as major interjections, had an impact upon the mathematical development of the class and the outcome of the lessons. Observing the CTs direct individual

interaction with students also became a learning to teach activity for Melissa as she shadowed the CT to observe the type of questioning and scaffolding he provided.

Pedagogical Development: Supporting the Learning of the Student Teachers

As a more experienced and more knowledgeable other (Dewey, 1938), the CT had a heightened sense of awareness to the critical learning moments for the students. It was during such teachable moments, when students in the class seemed primed and ready to make fragile ideas more solid, that several of the interjections took place. These openings, or teachable moments, that occurred within the discourse were not capitalized on by the PSTs because they either were unaware of, or unable to make the moves that needed to be made in-the-moment (Peterson & Leatham, 2009). Each of the sections that follow describes the interjections from the researcher's lens of *opportunity for pedagogical development.* In other words, the interjections are classified below according to their potential for providing learning opportunities to the PSTs. This classification directly correlates with the first two parts of the research question and cuts across the interjections in a different way than the previous lens of opportunity for mathematical development for the students in the class. Additional insight into the types of interjections is provided along with insight into the knowledge that PST's had the opportunity to access because of the interjections.

Learning to Clarify Expectations and Focus on the Mathematical Goal

As described above and shown in Figure 9, one of the major purposes and a backdrop for all of the interjections was to promote student understanding of the mathematical goals for the lessons. It became apparent, during the informal ongoing analysis of the data at the time of the TDE, that focusing on the mathematical goal was a struggle especially for Melissa. Interjections M-7, M-9 (see Figure 11) are evidence of the CT amending the requests of Melissa to the class

so that they would be more focused. The interjections in Figure 11 provided the opportunity for Melissa, primarily, and Bill, secondarily, to learn the importance of focusing their questions and requests of students on a clear mathematical goal.

Interjection M-7, is an example of an interjection that provided an opportunity to seek this type of clear mathematical focus. Melissa had just used the questions and work presented by Casey (M-6), which pertained to the relationship between scale factor and the ratios between corresponding sides of similar triangles. However, when directing students back to work she neglected to build upon Casey's work with a clear mathematical question and expectation. This lack of specificity was a concern to the CT and so he offered M-7 to help focus students on the mathematical connections within the task that had been given.

M-7.	"Could we maybe add to that, which ratios seem to be connected to scale factors?"
M-9.	"Ms. Turner, can we maybe list all of the ratios that are the same as that?"
M-13.	Other PST (Bill) jumps in and says, "Can we specify? An equation, we want to get to an equation." PST, "Yep, an equation, lets do that."

Figure 11. Interjections that focused on the mathematical goal.

Bill's interjection (M-13) into Melissa's lesson is another example of clarifying the expectation for students. The mathematical goal for the day as stated by Bill on his lesson plan shows that his goal did incorporate finding an equation to show the relationships between certain segments in a circle.

> Mathematical Goal from Bill's Lesson Plan:
> The students will see the relationships between segments, lengths of chords, secant and tangent lines. Specifically, the students will be able to identify similar or congruent triangles within the circles and use those triangles to set up equations to find unknown lengths.

Melissa, however, had not completely realized the importance this specificity would make in directing the work of the students. Bill had already taught the lesson and, just as the CT in many

previous occasions, felt there was a need for more than what was being provided, so he interjected. This interjection also provided clear evidence that the PSTs, in this case Bill especially, understood the importance of clarifying the mathematical tasks and goals for students.

Learning to Question

As mentioned previously, many of the interjections took on the form of a question. This allowed the PSTs to witness the modeling of questions. The questions that were interjected, and seemed to be the most mathematically productive, served two main purposes: 1) promoting connections and distinctions about mathematics (see Figures 1 and 3) and 2) promoting and pressing on the student thinking related to the mathematics (see Figure 7). It is learning how to formulate and use these mathematically productive questions by the PSTs on which we will now focus.

With many of the interjections coming in the form of a question, the PSTs noticed the importance of using quality questions that were clear, specific and pushed students to think more deeply about the mathematics. Bill, in his second interview (week six), stated the following:

> The questions I ask, and the questions we focus on as a class is really my role, to come up with good questions, to come up with topics that we need to discuss and really help further discussion. Also, knowing how to use that, how to kind of effectively use that question so that students can think about it and consider it and have a discussion about it.

And Melissa in her final interview (week fourteen) also acknowledged the importance of questions and that she had worked on learning to be a more effective questioner:

> But also, I am responsible for asking good questions, and so I made up just intriguing mathematical questions, enough that it's just exciting them to look for the math and be willing to…no, have the desire to work through the task and want to find out, versus, me again, pulling them to what they need to do, versus them wanting to find out and wanting to be, "Yeah, how would I solve that?" or "What is it going to be? I can't wait to solve it!", and so asking them questions along the way to lead them to that idea, that they are searching for.

A couple of interjections that exemplified how the opportunity of learning to question was made available through the intervention of the interjections and not just from the modeling of the CT, came during Bill's second lesson (week six). Bill actively pursued assistance in learning to question students. Two interjections initiated by Bill, B-5 and B-8 (see Figure 12), show direct requests from Bill to the CT for assistance with his questioning. This is evidence that in fact the interjections provided an opportunity for PSTs to learn how to question more effectively.

B-5.	"So Mr. Lemon, what would be a really good question to ask right here?" CT, "Maybe, I just heard a few people say, I'm confused now, maybe if some could kind of restate what the confusion is then we can have a good question to help clear it up."
B-8.	PST, "Is that a clear question Mr. Lemon?" Student, "Yes it is." Mr. Lemon asking the students, "Do you think it is?"

Figure 12. Bill's requests to receive support with questioning.

Although it was not verbalized and does not show up in Appendices B or C with the verbalized interjections Melissa requested specifically to shadow the CT during the exploration phase of the lesson. She had expressed uncertainty about what types of questions to ask students while they were working. However, she noticed the value of messaging or scaffolding student thinking in smaller groups so that it could benefit the whole class later. By shadowing the CT Melissa had the opportunity to see the direct interaction made with the students by the CT as a model from which she could learn questioning and scaffolding techniques.

Learning to Use Student Thinking

Several interjections provided an opportunity for the PSTs to learn how student thinking might be used effectively to develop mathematical ideas for the whole class. The interjections that encouraged (M-5, M-8, M-10), pointed to (B-2, B-3, B-9, B-13, M-6), or pressed on (B-6, B-9, B-15, M-12) student thinking provided the opportunity to develop mathematical ideas and

also modeled for the PSTs the moves that they could make in order to more effectively use student thinking.

Additionally, interjections B-16, M-15 and M-16 (see Figure 13) made as side comments between only the PST and the CT, while students worked on the task for the day, provided the PSTs with opportunities to reflect on the student thinking available to them and to make a plan

B-16.	"Have you thought about how to orchestrate the presentation, which methods to talk about first then second?"
M-15.	"Have you kind of thought about how to orchestrate what strategies to present first and so on?"
M-16.	CT: "How are you doing?" Melissa responds with the names of students that she feels have work that could be presented to help the whole class conversation. CT: "Okay my thought on these is that you are going to need to give a little bit more. People will have ideas about what will be congruent to what." Melissa: "Just move them through it." The CT shares the work of one group that could be helpful and then one of the observing PSTs adds her noticing of a student's work as well.

Figure 13. Interjections that facilitated learning to use student thinking.

for whole class discussion. Interjection M-16 exemplifies this reflection and planning on the part of Melissa. She was teaching the lesson in which, as discussed previously, Bill had interjected to help her clarify the goal of developing equations that would show the relationships between segments in circles. Melissa brought her ideas as to which students would be able to contribute to each part of the task forward and then, with the CT, she reflected on them. She then received some input as to the level of contribution that she might actually be able to get from the students as well as some additional ideas as to what other students might have to offer. Interjections like M-16 and those in Figure 13 allowed the PSTs opportunity to learn how they might effectively orchestrate student thinking in their lessons.

Learning to Maintain Norms

Many of the interjections served multiple roles and one that should not be neglected was that of maintaining the norms of the classroom. Some examples of this were the interjections that promoted the encouragement of student thinking (see Figure 5) and the use of student thinking (see Figures 6 and 7). These interjections had a primary role, which lent itself to the development of mathematics, and a secondary role, which maintained the norm of student engagement that had been established prior to the arrival of the PSTs. For example, interjection M-5 was provided to encourage student thinking and get the students involved.

> Interjection M-5:
> "Maybe could we, since there were a few people who were gone, could we maybe have people get together for just a second and get some ratios down if they don't have them, give them just a second and then ask again for the ratios."

At the same time, this interjection also maintained a norm of the classroom that students would be involved in discourse that is directed by the teacher.

Interjection B-6 is another example of an interjection that served both a primary role of pointing to and pressing on student thinking and a secondary role of maintaining a classroom norm. This interjection was intended to point to the idea that Eliza had about the reciprocal relationship she had noticed between a ratio involving corresponding sides and the scale factor between corresponding sides in similar triangles. It also provided other opportunities for learning because it pressed on Eliza to clarify her idea by asking her to say more about it. Another role served by this interjection was that of maintaining the norm in the classroom of presenting clear mathematical arguments, conjectures and justifications.

Considering the intent of student teaching being to allow the PSTs the opportunity to learn how to teach through the use of student thinking, and realizing the lack of experience of the PSTs, as well as being overwhelmed with so many things to attend to, it was important for the

CT to help maintain the norms of the class. By doing so, the CT provided an important opportunity to the PSTs by implicitly modeling an important part of teaching, that of maintaining norms that allow for a focus on mathematics and the thinking of students about the mathematics.

PSTs Response to the Interjections

The third part of the research question addressed the possible responses that the PSTs might have to the interjections. Their response was found by looking at some of the interjections themselves but was mainly based on the analysis of the interviews conducted with the PSTs as well as the post-lesson conversations the PSTs had with the CT. This section first addresses how the PSTs responded to the interjections by looking at how the PSTs perceived the effect of the interjections with regard to students' mathematical learning. The section then addresses the PSTs' responses to the interjections by looking at how they perceived the effect of the interjections on their own learning. Then in the next section the focus will shift to the explanatory model of the PSTs' development that was constructed by the CT through his observations during the ongoing analysis and through the retrospective analysis of the interjections, post-lesson conversations and interviews (Simon, 2000).

PSTs' Perceptions of the Effect of the Interjections on Student's Mathematical Learning

The intent of the CT from the outset of this study was to increase learning for both the students in the class and the PSTs. The data gathered contains evidence that the interjections did provide opportunities for the learning of mathematics by the students in the classes taught by the PSTs. This was evident as the interjections assisted the students in the classes to make *mathematical connections and distinctions.* As discussed previously, the very first interjections made by the CT provided the opportunity for students to create mathematical connections and mathematical distinctions. It was common for the CT to find openings in the discourse that

allowed for creating such opportunities of making connections, distinctions or sometimes extensions.

During the reflection meeting held after the first lesson taught by the PSTs one of the observing pre-service student teachers (OPSTs) stated the following about the effect of the interjections, B-1 and M-1,that occurred.

> OPST: It made sense why you asked it, I just was like, if that was planted that was perfect because the students were sitting there kind of like, kind of at the brink of, I kind of get it I don't kind of get it, and so you throw in something new and they were like what? So, it was good for them to think and come up with some things.

As evidenced in this OPST's comment the interjection seemed to come just as students seemed to be on the "brink" of understanding. This illustrates how the PSTs perceived that the interjections provided the students with opportunities to think and come up with some ideas that they may not have had otherwise.

In a similar manner the interjections categorized as: *encouraging student work, pointing to and emphasizing student work, pressing on student thinking (clarifying)* all were made to provide the opportunity for increased learning of the whole class and develop a shared understanding (Cobb, Wood & Yackel, 1993). The measure of the PSTs' perceptions of this increased thinking and involvement can be found in the comments made by the PSTs given during the three scheduled interviews, which occurred at the beginning (week 1), middle (week 6) and end (week 14) of the study. Bill stated the following in the first interview (week 1):

> I've found a lot, that often, students try to explain things that they can't really verbalize very well and often times they're thinking of a picture and Mr. Lemon does the great thing of asking them to go up to the board and draw the picture they're thinking of and that new image on the board creates other students kind of thinking, and what things that they say. So inviting them to the board also is something that helps kind of further the class discussion.

This response by Bill shows that he had noticed an increase in student thinking and involvement as a result of the CT's actions. Later, Bill reflected on the interjections during his third interview (week 14) and stated:

> He [the CT] noticed that students were seeing the relationship between the tangent of some angle and what ratio, I think it was like the tangent of 48 degrees and how that would relate to the side lengths and if we don't know one of the side lengths we can figure that out. I didn't go with it [the interjection] right away, I came back to it, but I felt like, looking back at that lesson, that I wish I'd gone there right then, because it would have been a smoother connection, because he saw it right then, and I thought, 'Well, we can come back to it in the lesson.' And just several experiences have taught me that when Mr. Lemon suggests something, it's usually a good idea to follow it. Those experiences have taught me that I usually just want to go with it, and notice after the fact, 'Oh yeah, that worked, instead of me saying, 'No, I think I want to stick to my plan,' which maybe is a good idea sometimes, but stick to my plan, and then realize looking back that 'oh, I should've done that'. That's just trusting his experience, and his understanding of teaching.

These comments by Bill demonstrate how he acknowledged the value of the interjections in terms of making things "work" in the classroom. This suggests that Bill perceived the interjections as worthwhile when it came to student learning. The comments of the OPST and of Bill were indicative of the value the PSTs saw in the interjections for promoting opportunities for student thinking. This also demonstrates a positive response to the interjections by the PSTs because of the common goal, which they shared with the CT, of increasing student learning through the use of student thinking.

The PSTs' Perceived Usefulness of the Interjections for Their Own Learning

The responses of the PSTs during the interviews revealed some important insights about how the PSTs felt when interjections occurred and the value of the interjections to the learning of the PSTs. Bill, during his second interview (week 6), stated the following:

> Most of all I really appreciated his interjections. I realize that he knows a lot more about teaching, he knows a lot more about this kind of teaching, than I do, he's been through it. So I really appreciate his interjections.

Melissa also expressed an appreciation for the interjections before the interviewer directly addressed interjections with her (week 14):

> Also during the lesson he's been willing to make suggestions as time's went on and willing to say….he's willing to say, "So Ms. Turner I wonder…" so I've appreciated his interjections, sometimes I wanted to pause for a second because I'm usually in the process of going somewhere and he's learned to wait for just a moment to see if I'll go there, and I've appreciated him waiting just to see if I will go there, but those interjections have been good.

Both PSTs appreciated the interjections and found them to be of benefit. They also suggested some reasons that the interjections may have been beneficial. One such reason of perceived benefit of the interjections was that of supporting the learning when the PST felt stuck. Both of the PSTs in one way or another said that they often found themselves stuck and thinking on their feet in front of the students and not knowing what to do, and then an interjection would come in and be very helpful. This, in essence, described the interjections as filling a gap between the instruction needed and the current level of skill of the PST. Bill described this gap in the following way:

> Even during the lesson, he would often interject and give some good comments, and thoughts, and things that have helped me. I still don't think I fully understand when he has said it, when he interjects, that I really understand where he's coming from. But I always identify that that was needed in that moment. And so I appreciate his interjections, but I'm not able to see it before he says it. But I think before, after, and during the lesson there's been a lot of support in helping me learn how to orchestrate discourse, which is probably part of the reason why I felt like I've grown so much in that area.

The significance of such gap-filling interjections for the students of mathematics in the class has already been discussed. It is the benefit to the PST upon which we now focus. The PSTs viewed the interjections as supports and positive contributions. They were willing to allow for them and appreciated them even if not understood at first. This suggests that one response of the PSTs to the interjections was that of confidence in their CT and a willingness to be instructed by him.

Another reason supplied by the PSTs for the effectiveness of the interjections was that the CT was experienced at having awareness of student thinking and as an active observer of the lesson was out among the students. The following comments addressed CT's awareness:

> Bill: I was focused on trying to see what they were doing and make sure I knew where they're at and it seemed like he was a lot more aware of what students were doing although he was doing less walking around. I mean I don't know. It felt like he was exerting less effort, but seeing more than I was. And so, because of that, he's often more aware of what students need right then than I am, when I'm teaching. And so, that awareness I think has really helped him to know what the class needs, and interjecting has been a way that he can communicate to me and to the whole class—a connection that we might need. And so, I really appreciated his interjections and in helping me to know more where the class is at, and also for the students to make the connections that they need to, you know we don't want to just leave them stranded so that I can learn by bad examples.
>
> Melissa: Because again, he's out among the students, generally, sitting right next to them, so he's hearing side comments, or something, that he knows maybe where they are, especially if I'm up trying to make a teaching move after not being with them for the past 5 or 10 minutes because I've just been up at the board.

This again shows that the PSTs acknowledged the value of student thinking and being aware of it. They both could see that a large number of the interjections created opportunities for using student thinking and therefore they implicitly gained an understanding of the importance of being aware of and using student thinking.

The impact of the interjections upon the PSTs likely allowed the CT to be more than a supportive mentor but to also act as a teacher educator to the PSTs. The CT had been seeking from the outset to provide enhanced learning opportunities for the PSTs and the interjections did effect the PSTs such that they viewed him in this light. Bill summarized his feeling about the CT's role in the following way:

> I really like the atmosphere that's in the class in that I'm learning how to teach and I often have questions about teaching that I maybe don't notice. And thinking about them as I'm teaching and trying to do the best that I can to finish the lesson, but still thinking about this and just as students when they have questions can raise their hands and ask the teacher, it's really helpful for me to ask Mr. Lemon, as my teacher in how to teach, a

> question that may go over the students' heads, but it will hit him and he'll know where I'm stuck and maybe give me a hint, or ask the class something and kind of get us through there. And so, I see it as me asking Mr. Lemon a question just as a student would ask their teacher a question about the mathematics, I'm asking about teaching.

The response of the PSTs to the interjections suggests that they valued them and also the CT for providing them to the extent that they were willing to allow the CT to fulfill the role of teacher educator with them.

In summary, the PSTs' response to the interjections was positive. They saw the opportunities for learning that the CT desired to provide as valuable because of their shared goal with the CT of soliciting and using student thinking effectively. The PSTs respected the CT's knowledge and awareness of students' mathematical thinking and became more open to accepting instruction and suggestions from the CT. The respect and confidence that the CT received from the PSTs likely allowed him to act more as a teacher educator for the PSTs than he might have been able to if they would not have been as open and receptive.

A Model of the Learning and Development of PSTs

Within this section the attention will turn to the development of the PSTs and the correlation between the interjections and their development. It was first through the ongoing analysis that a model of PSTs' development was considered in its preliminary form. Much discussion took place between the CT and the observer as well as the CT and his advisor at the university concerning the development of the PSTs and the role of the interventions. Some of the particular struggles and successes that the PSTs were having were acknowledged at that time. Then through retrospective analysis of the interjections within the video data as well as the analysis of the interviews and post-lesson conversations greater insight was sought about the thinking and development of the PSTs. The insights gained into the PSTs' perceived benefit of the interjections were presented in the last section. The retrospective analysis of the PSTs'

response to the interjections also provided for continued development of an explanatory model of the PSTs development (Simon, 2000).

Within the data there was evidence to support a progression of development by the PSTs. Within this section the main focus will be on the stages of development through which the PSTs moved during the study. The four stages of development that have been identified are: *Learning the Structure of a Discourse-Oriented Lesson and the Teacher's Role in It, Pursuing a Clear Goal, Desire to Bring the Students with You, Bringing the Students with You*. The first stage, *Learning the Structure of a Discourse-Oriented Lesson and the Teacher's Role in It,* was a stage of familiarization in which the PSTs came to better understand what was expected (realizing the importance of staying focused on a mathematical goal for example) and how to make it happen. The second stage, *Pursuing a Clear Goal,* occurred after the PSTs were more confident in their role and had taken ownership of the mathematical goal they had chosen. Stage three, *Desire to Bring the Students with You,* occurred when PSTs more fully realized the value of students' contributions and became more aware of the actions and work of the students. The fourth stage, *Bringing the Students with You,* occurred when the PSTs began to more carefully check for student understanding and, in turn, knew that they had helped students come to an understanding of the mathematics focused on for the day. A more in depth description of each stage of development follows.

Learning the Structure of a Discourse-Oriented Lesson and the Teacher's Role in It

The first phase of development the PSTs encountered was that of learning the structure of a discourse-oriented lesson and their role, as the teacher, within it. For anyone that has tried something new, this is a phase to which one can easily relate. For Melissa this phase was much more pronounced than it was for Bill. This may have been the case for many reasons, however

one possible explanation was that Bill had been engaged in undergraduate research under the direction of a Mathematics Educator, which had afforded him greater exposure to the role of a mathematics teacher. Melissa on the other hand was not as certain about her role and was more timid in her actions.

In order to better characterize this phase and describe the difficulties Melissa faced I will focus on the idea of having a "flow" to a lesson. Of course, this is just one of many items to attend to while learning to teach. Melissa found it very difficult to maintain a clear course or direction during her first several lessons. She would often jump from one topic to another or one student's thought to another's, uncertain about how much time or what to emphasize as she went. Melissa's first lesson had twice as many interjections as Bill's lesson did and three out of the four were to help make mathematical connections or distinctions. During the first several lessons Melissa taught, it became apparent that she had not internalized the mathematical goal for the lesson and was unsure about where she wanted students to be by the end of the class period. Her lack of preparation in planning caused her to miss the opportunities for creating mathematical connections, and disrupted a logical flow of ideas and building of concepts. Instead she was often caught following dead ends or unproductive leads that students in the class brought to her attention. In her conversations with the CT this was referred to as a "jerky" lesson and much of what was discussed early on had to do with eliminating the "jerks".

The difficulties Melissa had with staying focused on a specific learning goal while teaching her first few lessons, helped the CT to narrow his focus when working with her. In preparation for lessons the PSTs had prepared to teach, a discussion was often held about what students would walk away with as a consequence of the lesson. This focus of the CT with Melissa, along with the lesson planning templates provided by the university program prompted

the CT to create a lesson planning guide for the PSTs to use in thinking through their lesson plans ahead of time (See Appendix H). This lesson planning guide was meant to assist the PSTs in focusing on the mathematical goal of the lesson and to anticipate student thinking and begin to think through possible sequencing of student thinking and questions that might be beneficial in a way similar to that advocated by Smith et al. (2008). The focus of the CT and the university's program on learning goals was likely a contributing reason that this phase and the next *Pursuing a Clear Goal* were part of the PSTs development. As discussed by Leatham & Peterson (2010) sound pedagogical practice would reasonably begin with a well-articulated learning goal and then move toward facilitating the achievement of that goal.

Bill, in contrast to Melissa, was very familiar with the mathematical goal for each of the lessons and had often created an original task to help students build understanding toward the mathematical goal he was hoping to develop. One of the interjections, M-13, actually exemplifies the difference between the two PSTs. This interjection was not given by the CT but was provided by Bill during Melissa's lesson. He noticed the importance of having a clear goal and creating an expectation that the students reach that goal and interjected during Melissa's lesson to provide this clarity that he could see was lacking.

Interjections correlating to this phase of development most often were those that made mathematical connections or distinctions. This is understandably the case, since during this phase the PST's greatest area of need was knowing what mathematics to pursue and how to pursue it. It was also evident that a lack of preparation for any given lesson could cause the PST to revisit this stage of development. Melissa, in the third interview (week 14), provided this insight,

> It's definitely important and paramount that my preparation to understand the mathematics and the flow to lead up to that big idea, is there. I've noticed days when I haven't been as prepared, that obviously it didn't go as smoothly, but once that preparation's there, I'm knowing what to expect.

Although PSTs were soliciting student thinking during this phase and the CT was encouraging student thinking through interjections, the PSTs did not intently listen to students and as a consequence did not fully take advantage of opportunities to help students make connections.

It was the formulation and ownership of a clear mathematical goal for the lesson that seemed to allow for progression out of this phase and into the next phase of development for the PSTs. Bill likely started in the next phase, or at least arrived there very quickly. An indication of this came during one of Bill's interviews (week 6) where he stated the following:

> I think no matter what, like no matter where the lesson comes from, I feel like I really need to own the lesson and have ownership of it. I mean I need to know the ins and outs; I need to do the task myself—has really been important in lesson development.

In this response from Bill, it is evident that he understood the importance of taking ownership of the mathematical goal for which a task was intended. The PSTs realization that they needed to be well-prepared mathematically, knowing the content and possessing a clear understanding of the mathematical goal (taking ownership) that might be realized during the implementation of the task was a sign that the PSTs were moving out of this phase and into the next phase.

Pursuing a Clear Goal

When the PSTs took greater ownership of the mathematical goals they exhibited greater confidence in lesson planning. However, a couple of important elements of teaching were absent at this stage. The PSTs did not always consider the mathematical connections that might be made as they pursued their goal nor did they fully consider the role of the students and how to scaffold student thinking. Bill's early understanding of the importance of pursuing a clear mathematical goal may have been a result of his previous experiences and the development of both content knowledge and mathematical knowledge for teaching prior to the student teaching experience.

During this phase of development the PSTs possessed a clear understanding of the mathematical goal for the lesson, however, they often went in pursuit of this goal without full participation of the students in the room. Not only did the students in the room often find themselves somewhat bewildered but the CT also had a difficult time interjecting. The PSTs demonstrated a great amount of confidence during this phase, in fact they were often so sure of themselves that they did not wish for anyone to get in the way and mess things up. Bill's first lesson (week 3) and Melissa's second lesson (week 6) within the data of focus contain evidence of this phase of development.

The task for Bill's first lesson was one he had previously used during his practicum. Because of this he was very confident about the mathematical goal and how the lesson should flow. However, in many respects, this confidence did not allow for the openness that is needed in order to truly seek student participation and use student thinking. As evidenced in the dialogue following interjection B-1, Bill was not fully aware of the mathematical connections and distinctions that might need to be made. Bill's task sought to have students thinking and exploring the relationships between the number of sides of a polygon and the lines of symmetry as well as the diagonals. However, he was not certain about how to involve them in a whole-class conversation to summarize their work. He pressed forward with the lesson in such a manner that the CT could only interject twice, B-1 and B-2, in attempt to assist him.

The dialogue following interjection B-1, during the lesson, and the conversation during the reflection meeting, held following the lessons for the day, further demonstrate Bill's lack of understanding and openness to the interjection.

CT: "I-I'm wondering are the lines of symmetry the same as the diagonals always?"
Bill: "Oh", pause, "Are there thoughts? Bart."
Bart: "I don't think so because on the isosceles trapezoid it doesn't touch a point it just hits the segment."

Pause.

Bill: "Okay, thanks Bart. Okay, number two was about diagonals…"

This episode shows how Bill allowed for the interjection but did not exhibit a full understanding of its intent and did not seem open to furthering discussion about the distinction between lines of symmetry and diagonals, so he moved on to the next problem. It may be likely at this point that he also did not realize fully how to interpret and use the interjections (since it was the first time it had occurred). However, during the reflection meeting Bill expressed that he did lack an understanding of the distinctions and connections that might be made during this lesson when he stated:

> I didn't really exactly grasp the reason for his question. I mean I knew that maybe students were confused or that past students had been confused by this topic. Ah, so he wanted to bring it out, but I didn't understand that by comparing and contrasting you could better understand both, and so I kind of missed that, because I didn't really understand the purpose behind the question.

This lack of understanding on Bill's part as to connections and distinctions that might be made along with the eagerness to move on to the next item show how he was within the *Pursuing a Clear Goal* stage of development.

Melissa's second lesson also exemplifies this phase of development. Although the CT was able to interject more into this lesson, the interjections, M-5 through M-9 (see Figure 14), implicitly shared a common sentiment of 'slow down, listen to students, and press on their thinking.' The majority of the interjections during this lesson did seek to encourage more student thinking and engagement because Melissa was so eager to keep pushing forward.

This phase of development was an improvement over the first phase, *Learning the Role*, in that the PSTs possessed greater confidence because they knew where they were going as a result of establishing and taking personal ownership of a clear mathematical goal. However, PSTs were not fully open to student thinking, had not always considered the mathematical

M-5. "Maybe could we, since there were a few people who were gone, could we maybe have people get together for just a second and get some ratios down if they don't have them, give them just a second and then ask again for the ratios."

M-6. "Before we go on there were some really big questions being asked back here. We don't need to necessarily answer them right now but I think maybe we should get them out there so as we go forward we keep thinking about them. Casey would you share the questions you had?"

M-7. "Could we maybe add to that, which ratios seem to be connected to scale factors?"

M-8. "Can we keep clarifying this. Each comment is making it more clear for me. Can someone else…"

M-9. "Ms. Turner, can we maybe list all of the ratios that are the same as that?"

Figure 14. Interjections made during Melissa's second lesson.

connections and distinctions that might be made and were teaching in such a way that even the CT had difficulty finding openings in which he could interject.

Desire to Bring the Students with You

As the name of the phase suggests, this stage of development corresponded to an increased desire by the PSTs to involve students more fully in the lessons they taught. Although the classroom of the CT encouraged and used student thinking and the university program in which the PSTs were involved focused the PSTs on student thinking right from the start, it was apparent that a deeper level of focus on student thinking and understanding how to really use student thinking did not fully begin to occur until this phase. One would think that, of course, all teachers have a desire to bring students to an understanding of mathematics, but do they take the time to really listen to students and use the student's thinking to build understanding for the entire community of learners? Teachers in this phase exhibit openness to allowing for misconceptions to surface, student created work on the board, students responding to other students and time spent in sorting out multiple strategies or ideas. They may not have all of the

skills, abilities or tools to perfectly manage a classroom that encourages student thinking but they seek for the student involvement and input anyways.

There is evidence within the data that both of the PSTs demonstrated they had come to value student thinking at this deeper level during the student teaching experience.

Melissa stated in her final interview (week 14):

> Again, just the norms that it's okay to talk about it, but the quality comes from it being the student thinking rather than the teacher, that it's really centered around what's coming off their paper, what's coming out of their minds, and having them discuss it, and making their conjectures, and the teacher's role in that, obviously, is just to facilitate that discussion amongst the students, and clarify the times when we need to throw in convention and correct mathematics.

This statement came just after the interviewer asked if she had worked on or explicitly talked about how to promote quality discourse in the classroom with her CT. It shows that Melissa came to more fully see the importance of using student work. In the same interview Melissa also stated:

> After I had that day of reflecting on how I could be better, it's changed ever since that day, because there's a point, when you can make a conscious decision as the teacher that the students are the mathematicians and not just the teacher, and as soon as I gave that to them, the students, they were able to run with it, because they are fully capable, and given the opportunity to discuss it and having a task, obviously to promote that, just has been fantastic. So moving towards their thinking and their learning and having that be the main discussion of the classroom has been awesome. It's not easier because I'm talking less, but I notice as I am talking less, the class goes better. So that's what I've been working on, is not talking as much, in a sense.

She had come to see the benefit and possessed a desire to bring students to an understanding of the mathematical goal for the day.

In addition to the comments made by Melissa in her interview she also demonstrated a desire to learn how to better access and use student thinking by shadowing the CT. After realizing the importance of pursuing a clear mathematical goal, Melissa decided that she needed to be more open to student thinking that would contribute to the goal. This led her to seek to

know more about what to look for in student's work as they explored the task for the day. The CT and Melissa decided that shadowing during the explore phase of the lesson might be an effective way for her to gain insight into the types of questions she might use, as well as the things to look for on student's papers. This shadowing and the interjections made aside to Melissa (M-15 and M-16) are evidence that, by the end of student teaching, she had gained a desire to use student's thinking to move the students toward the goal for the day and she was seeking to find ways in which she could realize her desire.

Bill also expressed the desire to use student thinking in his second interview, which occurred after he taught the second lesson in the data of focus (week 6),

> Understanding what students are thinking to make sure they actually are there, and I'm not just superficially thinking that they're there. An awareness of where the students are at would really help me know. Because sometimes you can finish a lesson, and if you're only aware of what students are doing, which happens sometimes, at least for me, you know I thought it was a fine lesson but I'm not sure the students really got to where I hoped that they got. That would help me really determine whether or not that lesson was good or not, to really know what the students were thinking and what they were doing.

Here Bill talked about the importance of knowing what students were thinking and having an awareness of where the students are at, almost in the sense of assessment rather than using the student's thinking. However, it is important to realize the other aspects of Bill's interview on this day. He also discussed the importance of good questioning,

> The questions I ask, and the questions we focus on as a class is really my role, to come up with good questions, to come up with topics that we need to discuss and really help further discussion. Also, knowing how to use that, how to kind of effectively use that question so that students can think about it and consider it and have a discussion about it.

This comment, along with others during this same interview, about anticipating student responses and knowing when to have students come to the front of the room to use the board show that Bill was seeking not just to assess student thinking but to bring it out so that all students could have a discussion and move toward greater understanding.

In addition to Bill's interview, interjections B-5 and B-8 during Bill's second lesson, in which Bill sought for clarification from the CT, are evidence that he was seeking to find ways to better ask questions and needed help as he struggled to do so. The interjections during this phase of development are much more frequent as evidenced by Bill's second lesson. This is consistent with this phase of development because of the openness that was exhibited by PSTs during this phase. They were willing to allow for not just student input and thinking, but also were creating openings in the conversation through which the CT could interject and support their work. This phase of development lasted the longest and afforded the most growth to the PSTs because of their openness and willingness to learn.

Bringing the Students with You

Although the perfect lesson may be allusive for many teachers, because there are always aspects that might be improved upon, the PSTs made great strides during their student teaching experience toward having more and more lessons in which the students reached the mathematical goal for the day. These lessons can be described as ones in which the PSTs had a clear mathematical goal for the students and also had a well designed task for which they had anticipated a flow of both student thinking and a line of questioning to scaffold and press on that thinking. Additionally, their awareness of student understanding at the end of the lesson was another indicator that the PST had progressed to this phase of development.

The following portion of the transcript from Bill's final interview shows that he had experienced a good lesson during which he had brought the students to an understanding of the mathematical goal:

> Bill: The big idea was well intact, the goal, we noticed some relationships, and most of class time was spent going, supporting that big idea that students were able to explore, and then unpack, and then notice things, and be thinking about that big idea. Another thing was, there was, I think, enough

> scaffolding and support for the students, that they had the tools that they needed to notice and to learn and to make the connections that they needed to. There wasn't too much time spent in one activity, which can kind of, if it's too long on one activity, enthusiasm can kind of drop, and so I think there was enough variation in the activities, that the students could stay engaged in the different segments and parts of the lesson.

Interviewer: You may have already answered this in part, so just tell me, but what are some of the evidences you have that the lesson went well?

Bill: While we were summarizing, it felt like as I asked the question, a lot of students were really eager to respond and tell what thing they noticed, and the class atmosphere was very positive, and we all noticed these things, and it came out pretty quickly, which is one indicator to me, that if you ask a question and you have to wait a while, or a few seconds, then you notice it's not as relevant in their mind, a stronger connection could have been made. The timing and the unpacking was really where I began to see that students had made those connections.

Bill had in fact taught several quality lessons by the end of his student teaching experience. He had come to the student teaching experience well prepared and confident in the knowledge gained previously during his undergraduate work. Bill was able to add teaching skills and abilities to his prior knowledge, which helped him facilitate the learning of students and helped them to accompany Bill on his journey each day toward the mathematical goal.

In like manner, Melissa also taught lessons in which the students clearly had come to an understanding of the mathematical goal for the day. One indicator that Melissa had experienced this came as she described the importance of checking for students understanding in the following way:

> It's really those little moves that I need to be aware of, and I've talked lately with Mr. Lemon, about how much of my time is spent. So if I want to have that much time at the end to unpack, to clarify, to really check for student understanding, then I have to set aside that time, that no matter where we are in the exploration, that time is probably the most crucial.

Although Melissa had more struggles at the beginning of her student teaching experience she did gain an understanding of the importance of some essential components of a quality lesson. In her

final interview she made mention of the following: a clear mathematical goal, scaffolding student thinking, productive questioning, orchestrating a flow of ideas, and checking for student understanding.

The OPSTs during the final reflection meeting that was conducted during the final week of the student teaching experience also noticed a change in the nature of the interjections during the lessons of Bill and Melissa.

> OPST: I feel like before you were like, they were missing a big idea, let's interject so we can get that big idea out. Where these interjections were like, its just something small. Like, hey we can write it this way, instead of, hey can you get this idea out on the board. So I think definitely, I mean our teacher interjects all the time, and I am grateful for it. …
>
> Observer: So fine tuning versus overhaul.
>
> OPST: That is exactly what I am trying to get at.

This confirms the personal reflections of Bill and Melissa in showing that both of them by the end of the student teaching experience were now teaching lessons with clear goals in mind and were also able to move the students in the class toward those goals with minimal in-the-moment support or interjections from the CT.

DISCUSSION AND CONCLUSIONS

This chapter will address the conclusions to the research question: *In what ways might a cooperating teacher interject during the lessons of PSTs and how can such interjections facilitate the PSTs access to knowledge and what might be learned from the PSTs responses to the interjections?* The implications that might be drawn as a result of this research will also be made. Additionally, suggestions for possible future work that might be done will be provided.

Limitations

Before stating any conclusions or presenting a discussion concerning the results and analysis of this work it is important to establish the limitations of the findings. This study was deeply contextualized and included only one CT and two PSTs. The nature of the university's program for student teaching was also different from that commonly found at other universities. Because of these contextual factors the study does not provide results that can be fully generalized to any student teaching experience. Rather, the qualitative nature of the study allows for discussion about the nature of the student teaching experience that can occur.

It should also be noted that as suggested by Simon (2000) there may be issues with TDE because often times the researcher-teacher-educator is attempting to combine two difficulty processes "learning to conduct research and learning to teach" (p. 357). The CT in this case served the role of the researcher-teacher-educator and although he had worked with PSTs previously had not sought to provide in-the-moment interjections like he did here. Although, the CT had participated in other studies prior to this one this was the CT's first time to formally collect data on himself as a researcher. So, Simon's concerns are relevant with regard to this study. However, it is important to remember that this work as one following the TDE

methodology build directly upon the “emergent perspective” and thus can provide the opportunity to work at the growing edge of knowledge about how PSTs learn (Simon, 2000).

It should also be reiterated again that the model of development of the PSTs created by the CT was based on their response to the interjections but could have been influenced by many things. There is no claim that the interjections in and of themselves will cause PSTs to progress through the phases described. The CT himself given his goals and prior experience may have influenced the PSTs to progress in the way that they did more than anything else. However, it is reasonable to believe that the interjections were an effective learning to teach tool through which the CT was able to provide learning opportunities that otherwise may not have been available.

Findings

Ways of Interjecting

The interjections of the CT during the lessons of the PSTs, in the moment of instruction, and documented in the Results chapter provide a sample of what interjections may look like in a classroom. It is important to realize that the interjections that occurred during Bill and Melissa’s student teaching, three lessons of which were analyzed for this study, came within a culture of instructional improvement. This culture or environment was created and supported by the norms negotiated between the CT and the PSTs and the structure of student teaching, established by the university. Given different participants or a different structure and atmosphere the exact interjections might vary. However, with the intent that the interjections support both the learning of the students of mathematics and the PSTs, through the use of well-orchestrated discourse and student thinking, it is likely that interjections will focus on mathematical connections and distinctions as well as the thinking of students and bringing student thinking forward to be pressed upon and clarified.

Interjections Facilitating Access to Knowledge

It is beyond the scope of this work to argue the epistemological nature of the interjections or to what extent they may or may not have caused learning for the PSTs. The focus was instead upon finding evidence within the data to support a link between the interjections and PST development. Such a link and correlation does seem to be apparent. It should be understood that there is no claim that such a correlation implies that the development and learning of the PSTs was caused by the interjections of the CT. There were multiple things being experienced by the PSTs as they went through the student teaching process and it would be futile to attribute all of the growth and development of the PSTs to just one of those many things.

There is no evidence to suggest that the PSTs were hindered in their development because of the interjections. On the contrary, the PSTs appreciated the interjections and found value in their presence. The analysis suggests that PSTs did not have access to the opportunities for learning provided by the interjections at first but as time progressed the PSTs were able to more fully understand their intent, so well, in fact that, they began to request interjections in the moment of instruction. Since interjections occurred in the moment and were also discussed in greater depth after class, PSTs were provided with opportunities to learn from the interjections both in the moment and through reflective practice. In some ways the interjections acted as flags or sign posts for the PSTs and marked, in many instances, what the CT called “critical moments” in the class in terms of the opportunity for learning of the students. So even if the PST did not have access to the interjection in the moment of instruction, they still became aware of the moment and then later were given the opportunity to learn from the CT the importance of the moment and the possible teaching moves they might use in such moments. In many ways the interjections provided an opportunity of learning to teach for the PSTs as they engaged in the act

of teaching, just as questioning was often used by the PSTs to support and scaffold the learning of the students in their classes.

Implications

As described by Cobb (2000) a teaching experiment focused on student development leads to consideration of three areas, which for this study can be related to the following: (a) the social context in which the PSTs worked and the effort of the CT to establish and maintain norms to foster PSTs development, (b) the effort of the CT to actively intervene resulted in learning on the part of the CT, (c) a developmental sequence for PSTs was described such that it can inform the work of the CT and other cooperating teachers in their future work. A more in depth discussion of the implications for each of these items follows.

The Ground Work Matters

This work, as with other teaching experiments that focus on student development (Cobb, 2000), paid close attention to the social context in which the student teaching took place as the CT proactively established norms to foster the PSTs development. The findings here provide a reconfirmation of the work of many others (e.g. Cobb et al., 1993; Franke et al., 2007;Stein et al., 2000) that have gone before and found that the environment in which learning occurs matters. Cooperating teachers should look closely at the atmosphere and culture of their classrooms and the role they play in promoting norms of learning to teach for PSTs. The interjections maintained a norm and expectation of using student thinking. Although interjections may not be the only way a cooperating teacher can maintain this norm, this work implies that such norm maintenance should be an item of consideration for cooperating teachers. The norms negotiated between the CT and the PSTs were an important factor in what took place however, the structure of the student teaching experience was also important. For this reason consideration should be given to

the types of undergraduate preparatory experiences and coursework to be provided to teaching candidates. It was evident from the data that Bill's prior preparation played a role in his success as a PST. These two items (norms and undergraduate preparation) were important in creating an opportunity for the PSTs to learn and should be carefully considered and crafted for other PSTs as well.

Others considering the implementation of interjections with their PSTs need to carefully consider the establishment of norms between the cooperating teacher and the PSTs. It was the establishment of norms and a mutual understanding between the PSTs and the CT that the shared goal of all adults in the classroom was high levels of student learning through highly engaged students that contributed to the benefit provided by the interjections. Because the PSTs had respect for the CT and confidence in his experience and knowledge they were willing to accept the interjections and the instruction that he provided.

Cooperating Teacher as Teacher Educator

The role of a teacher is indisputably important when it comes to the learning of students. In this work, the CT established a clear learning goal for his PSTs and designed a tool (interjections) through which he planned to assist them in their learning. The CT found it important to narrow the learning goals on which the PSTs focused. The first couple of weeks proved to overwhelm the PSTs, especially Melissa, and after the CT focused the PSTs on a couple of specific items (i.e. articulation of a clear mathematical goal, asking quality questions) they were able to make more significant progress. This is similar in many ways to understanding the importance of a well-articulated learning goal for students of mathematics. As both Bill and Melissa took greater ownership of the learning goals for their students and were able to provide

greater focus on specific learning outcomes they to were able to see increases in student learning and understanding.

In many respects the CT provided what many have suggested a cooperating teacher acting as a teacher educator would provide (Fiemen-Nemser & Buchmann, 1987; Borko & Mayfield, 1995; Leatham & Peterson; 2010) and what others (Blanton et al., 2001) have suggested was needed in terms of narrowing the gap between university programs and the school experience for PSTs. For this set of PSTs the "double discontinuity" of which Felix Klein (Kilpatrick, 2009) spoke was likely diminished. The university program had been embraced by the CT because of his participation in the research that had lead to its development and the university in like manner had provided a strong vision of mathematical reasoning for the PSTs through the undergraduate program of study. In addition, the CT realized his role in helping the PSTs operate within their ZPD. The PSTs were given opportunity to learn how to teach while in the act of teaching and their learning was enhanced through the work of the CT.

The CT, as the teacher, also gained a great deal from the experience as Cobb (2000) suggested a teacher would during a teaching experiment. The CT came to recognize stages of development within the PSTs and to refine his instruction and interjections for them. One such refinement of instruction was the shadowing that Melissa did, after it was noticed that she was struggling with the collaborative (exploration) phase of lesson development. The CT also learned from his interaction with the observer and his advisor at the university the importance of establishing a clear, specific goal, instead of a general "learn how to teach" goal. Another activity the CT had the PSTs engage in was that of listing what they felt they had learned and what they felt they needed to still work on. This helped the CT focus his effort with the PSTs and provided insight that assisted the CT in developing learning experiences for the PSTs.

Just as the teacher educators that the PSTs had taken courses from during their undergraduate work had sought to produce learning that would prepare them to teach, so too the CT hoped to provide learning-to-teach experiences (through the interjections) for the PSTs. The results of this study imply that the role played by the CT can and should be similar to the work of a teacher educator. If it is, then likely both cooperating teachers and PSTs will benefit.

Allow for Visibility and Parsing of Teaching

Interjections into the lessons of the PSTs provided a way for the teachers involved to identify important aspects of practice and address them either in-the-moment of instruction or later through reflection. The act of interjecting resulted in making the work of teaching more "visible" for the PSTs, as Ball et al. (2008) has suggested it needs to be for progress to occur. Just as the PSTs were able to move their students and themselves forward more effectively once they had a clear mathematical goal in mind, so too was the CT able to move himself and the PSTs forward in the work of learning to teach once he focused his efforts on clear teaching goals. Because the PSTs were adult learners and had a strongly vested interest in learning to teach the CT allowed for them to chose what they wished to focus on learning. The interjections did not necessarily change because of the PSTs' specific learning goals, the intent of the interjections to provide learning opportunities for students in the classes as well as the PSTs was constant throughout. However, the PSTs did feel more at ease and were grateful for the focus that the CT developed with them, rather than attempting to learn all of the components of discourse-oriented teaching at once. The more specific focus of the PSTs reduced their feelings of being overwhelmed and facilitated their learning. This "parsing" as advocated for by Ball et al. (2008) assisted in making the work of teaching more "visible" for the PSTs. The PSTs appreciated the interjections and also mentioned that they had the opportunity to identify weaknesses in their

abilities to teach and were given the autonomy to choose where they wanted to focus their efforts.

This study also implies that the student teaching experience can be about learning to teach rather than about learning classroom management. Clear goals and expectations as well as supported opportunities to reach those goals can produce development and learning in PSTs. It is likely that the closer PSTs are to actual practice when they receive instruction and support in learning to teach, the more apt they can be to learn and develop quality teaching practices. The interjections studied here are one way by which the work of teaching might be "parsed" and made more visible for PSTs while they are in the act of teaching. By providing the interjections the CT was able to focus on the development of the PSTs and assist them in that development. This work, like that of other teaching experiments (Cobb, 2000), produced a sequence of development for the PSTs from which the CT and other cooperating teachers will be able to draw as additional work is done to assist PSTs in learning-to-teach.

The model of development that was created by the CT for the PSTs in this study may also serve to benefit the work of other researchers and teacher educators as they look into PST development. The phases or stages of progression that emerged from the responses of the PSTs to the interjections may inform the work of others that seek to provide the PSTs they work with opportunities to learn to teach through the use of students' thinking and in a discourse-oriented way.

Future Work

The CT identified four stages of development through which the PSTs in this study progressed. Continued refinement of these stages, as well as the addition of items that may have not been considered, could be produced through additional iterations of this study. Researchers

may use the developmental stages found here to inform their work and instructional design for PSTs. For example, the CT noticed it was important for the PSTs to take ownership of the mathematical goal they had for the students. Bill possessed this understanding from the start and Melissa struggled with it. In future work with PSTs, the CT will focus on what it means to have a 'clear mathematical goal' for a lesson more explicitly so that PSTs, like Melissa, can progress more quickly. Additionally, the CT noticed the PSTs seemed to progress the most during the third stage, *Desire to Bring Students with You*, this seemed to be due to their realization that they were not as open to student contributions as they had hoped to be and also a realization that good questioning techniques, which they acknowledged they lacked, allowed for an intense focus on student responses (pointing to them, pressing on them) and seemed to produce higher levels of student engagement and learning. In future work the CT hopes to address these issues and better instruct the PSTs by helping them to see more explicitly some of the roadblocks they might face (Peterson & Leatham, 2009), in using student thinking. One such possible approach might be to read some cases with the PSTs or to do some deliberate reading of transcripts and coding from other classroom conversations (Scherrer & Stein, 2010) in order to create an awareness of critical moments within a lesson and how to formulate questions that will take advantage of those moments.

The CT only analyzed three of the lessons taught by each of the PSTs along with conversations and interviews connected to those lessons. Future iterations could paint a more complete picture of PST development by analyzing a larger portion of the data set collected. The CT feels that there is still more to be learned from the data he gathered and with more time and resources would dig more deeply into the lessons and conversations of the PSTs.

The progression in development had by the PSTs in this study is encouraging and it would be significant to expand such work to more cooperating teachers. Interjecting is likely just one means by which the knowledge of more experienced instructors can be made visible for PSTs. It would be encouraging for other researchers to conduct similar exploratory studies to find additional ways to dissect the act of teaching. If more such methods for focusing our efforts and identifying the critical areas of need were made available to mentors and instructional coaches of mathematics, it is likely that the field could enlarge what is known about knowledge for teaching and learning to teach.

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Appendix A: Sample Observation Record form used by the PSTs

Date day/sub		Teacher		Per:
Math Topic		Observation Focus	**The Flow of the Lesson**	
Time	Teacher's approach (Tasks, Questions, Approaches, blackboard, materials)	Student's reaction (Comments, Responses, Thinking, Notes)		
Remarks and opinion of observation				

Appendix B: Interjections into the lessons taught by Bill

Spoken openly during lessons taught by Bill

Jan.

B-1. "I-I'm wondering are the lines of symmetry the same as the diagonals always?"

B-2. CT, "I think Brady might have a really good idea." PST, "Oh yeah." Brady, "Okay." CT, "Would you be willing to share that was cool."

Feb.

B-3. "Hmm, I'm wondering more about number four and number five. Are the ratios that Nya made the same kind of ratios that Cameron made?"

B-4. "So, we can keep going but I am curious is, if we multiply by that ratio, which direction it will take us, lets keep talking but I am curious more about that, keep going."

B-5. "So Mr. Lemon, what would be a really good question to ask right here?"
CT, "Maybe, I just heard a few people say, I'm confused now, maybe if some could kind of restate what the confusion is then we can have a good question to help clear it up."

B-6. "Eliza you are pointing at the ratio, can you go ahead and write the ratio as a fraction so we have the same format? Now talk more about that. What are you noticing? Maybe show us up on the top one as well."

B-7. "When you have got them on your paper label them. Like label them this one is my ratio, small to big, this one is my ratio big to small, like label them so you know what is what. Otherwise, you know we can still have some confusion, so we need to keep it clear."

B-8. PST, "Is that a clear question Mr. Lemon?" Student, "Yes it is."
Mr. Lemon asking the students, "Do you think it is?"

B-9. "Somebody had a question up here, was it you Lauren?"
Lauren clarifies the question being asked, "I just asked was it on the same triangle or on different triangles? …Like long to short on the same triangle or two different triangles?"
CT, "So, would it make sense to have a long to short that involves more than one? Does that make sense Lauren?"

B-10. "I'm wondering if there is confusion between a big to small ratio versus a big to small scale factor?"

B-11. "Once again, label them. Write down on your paper long to short and put down all the ones. And then, just like Mr. Lawrence has it up there, make yourself a couple of categories and put them all in there."

B-12. “Write yourself a note somewhere and label what is happening. Write a ratio is … and give yourself an example or explanation and a proportion is. . . ”

B-13. “Can you repeat that Lauren? Say that again.”

B-14. “While Seth is doing that, how many of you know the value of x and x+2?”

April

B-15. CT, “Could you write a similarity statement for use?” Student, “Sure.”
CT, “Just to make it even more clear.”
PST, “Can everyone follow along and make sure he is doing it.”

Spoken only to Bill while students worked

Jan.

B-16. “Have you thought about how to orchestrate the presentation, which methods to talk about first then second?”

April

B-17. “I am noticing those that are most successful have started marking what they know.” PST, “yeah.” CT, “You might just..” PST, “I just tell them.” CT, “that is important.”

Appendix C: Interjections into the lessons taught by Melissa

Spoken openly during lessons taught by Melissa

Jan.

M-1. "I just have a question, are the…are the diagonals going to be lines of symmetry or vice versa?"

M-2. "I wondered if we could think about the trapezoid for a minute and maybe that would help us with the kite?"

M-3. "Can they say that again?"

M-4. "Ms. Turner, you just, you show that they're touching the shape eight times. Is there some kind of connection between amount of touches and amount of them (lines of symmetry)?"

Feb.

M-5. "Maybe could we, since there were a few people who were gone, could we maybe have people get together for just a second and get some ratios down if they don't have them, give them just a second and then ask again for the ratios."

M-6. "Before we go on there were some really big questions being asked back here. We don't need to necessarily answer them right now but I think maybe we should get them out there so as we go forward we keep thinking about them. Casey would you share the questions you had?"

M-7. "Could we maybe add to that, which ratios seem to be connected to scale factors?"

M-8. "Can we keep clarifying this. Each comment is making it more clear for me. Can someone else…"

M-9. "Ms. Turner, can we maybe list all of the ratios that are the same as that?"

April

M-10. "Jacob is going to do 14."

M-11. CT, "Shouldn't that be 2a?" PST, "Oh sorry thank you, you're right. Because the angle is half."
CT, "So if the arc was 'a' then the angle would be half 'a'."

M-12. "Jacob can you, like outline the angle you are looking at and then outline the arc?"

M-13. Other PST (Bill) jumps in and says, "Can we specify? An equation, we want to get to an equation." PST, "Yep, an equation, lets do that."

M-14. “So, while we go through this you should be taking really good notes of what goes up. Some of you had some really good work some of you still need some. So, as we go through make sure that we get the ideas down on your paper.”

Spoken only to Melissa while students work

Jan.

M-15 “Have you kind of thought about how to orchestrate what strategies to present first and so on?”

April

M-16 “How are you doing?” Melissa responds with the names of students that she feels have work that could be presented to help the whole class conversation. “Okay my thought on these is that you are going to need to give a little bit more. People will have ideas about what will be congruent to what.” Melissa, “Just move them through it.” The cooperating teacher shares the work of one group that could be helpful and then one of the observing PSTs adds her noticing of a student’s work as well.

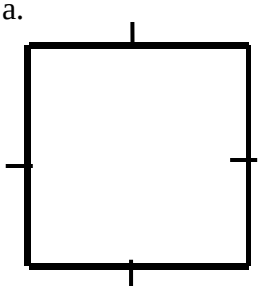
Appendix D: Task Sheets for Polygon Lesson on January 23rd

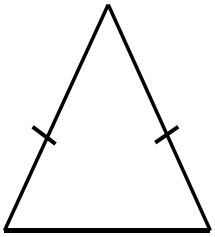
Name_____ Period_____

Symmetry and Diagonals Opener

1. In the following triangles and quadrilaterals, identify all the lines of symmetry:

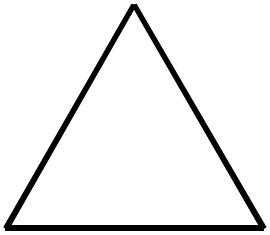
a. b. c.

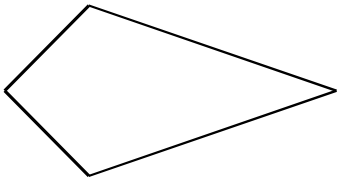


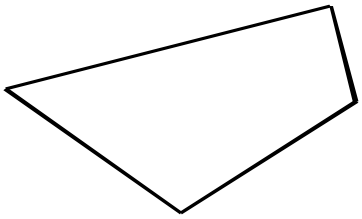


2. In the following triangles and quadrilaterals, identify all the diagonals:

a. b. c.







Name: _____
Class: _____

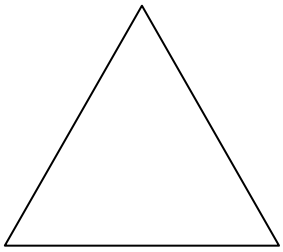
Regular Polygons:
Symmetry and Diagonals

A regular polygon is _____

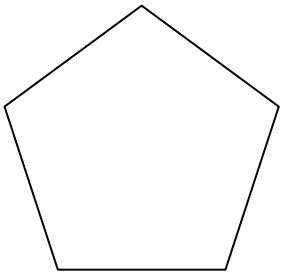
What is a regular triangle? _____

What is a regular quadrilateral? _____

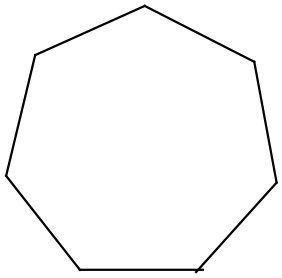
Find the lines of symmetry and the number of lines of symmetry on these regular polygons.



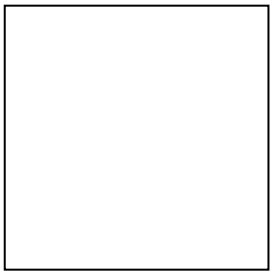
3 sides



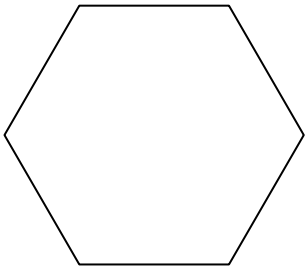
5 sides



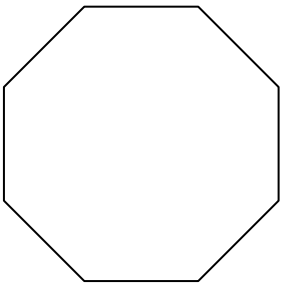
7 sides



4 sides



6 sides



8 sides

Given an “n-gon” (a regular polygon with n sides), how many lines of symmetry will it have?

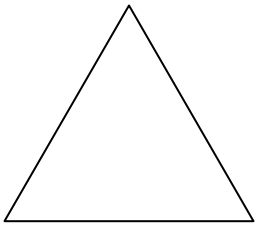
Why does this pattern hold?

_____

_____

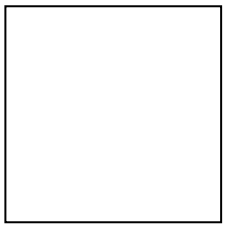
_____

Our next task is to find the number of diagonals an “n-gon” will have.



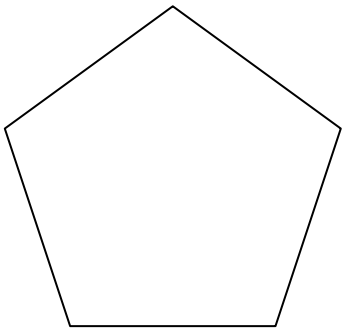
3 sides

Number of Diagonals:



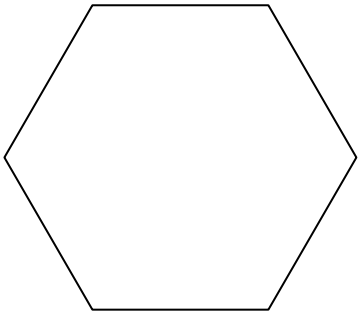
4 sides

Number of Diagonals:



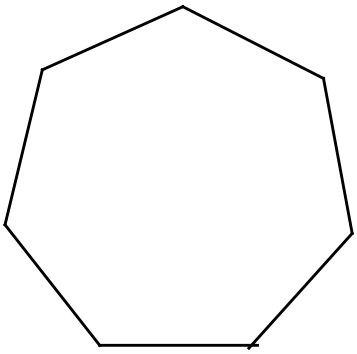
5 sides

Number of Diagonals:



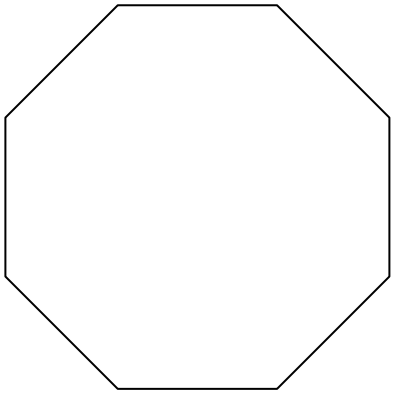
6 sides

Number of Diagonals:



7 sides

Number of Diagonals:



8 sides

Number of Diagonals:

How many diagonals will an “n-gon” have?

Explain why this formula holds:

Appendix E: First Interview Protocol for PSTs

(A flexible interviewing technique will be used. These questions will be the base on which the interviews will be established.)

1. What do you envision as a good lesson? What components will it have? How will you measure the effectiveness of the lesson?

2. What helps you be successful in learning and understanding something new for yourself?

3. What do you believe is necessary to create a quality class discussion in a math classroom?

4. What role can or should you play as the teacher to assist in the development of big ideas and concepts during a lesson? What role can or should you play as the teacher to facilitate quality discourse among students during a lesson? What connection, if any, do you see between development of big ideas during a lesson and the development of discourse?

5. What specific skills or abilities would you like to work on to help you improve your teaching? Why?

Appendix F: Second Interview Protocol for PSTs

1. Now that you have observed and taught a few lessons, what do you envision as a good lesson? What components will it have? How will you measure the effectiveness of the lesson? How if at all has your vision of a good lesson evolved over the last seven weeks?

2. What do you believe is necessary to create a quality class discussion in a math classroom? What evidence or examples of quality class discussion have you seen?

3. How do you view your role as the teacher in facilitating lesson development? How do you view your role as the teacher in facilitating quality discourse in the classroom? What successes and struggles have you had in facilitating discourse?

4. What would you define as a teachable moment or critical point in a lesson? Why?

5. What specific skills or abilities would you like to work on to help you improve your teaching? Why?

Appendix G: Third Interview Protocol for PSTs

1. Of the lessons you have taught over the past weeks can you describe one that you felt was a good lesson? What components did it have? How do you know that it was an effective lesson? What did the students do and what did you do that made it effective? What do you need to do as a teacher to create an effective lesson? Describe all components you can.

2. What do you believe is necessary to create a quality class discussion in a math classroom? What evidence or examples of quality class discussions have you seen? What have you or your cooperating teacher done to promote quality discourse?

3. How do you view your role as the teacher in facilitating lesson development? How do you view your role as the teacher in facilitating quality discourse in the classroom? What successes and struggles have you had in facilitating discourse?

4. What specific skills and abilities have you developed during your student teaching experience? What specific things has your cooperating teacher done to assist you in that growth? As you begin your career in teaching what specific skill would you like to work on to help you improve your teaching? Why? Do you feel like you have a vision of what quality mathematics instruction should be?

5. What would you define as a teachable moment or critical point in a lesson? Why? Can you think of a time you recognized a teachable moment and handled it well? Please

describe what happened? Can you think of a time when you recognized a teachable moment but did not handle it well? Please describe what happened.

6. Can you identify something specifically that your cooperating teacher did to help you learn your role as a teacher? What? Do you feel like your cooperating teacher has assisted you in improving your ability to orchestrate discourse? How? What feedback do you have for your cooperating teacher? Is there something you really liked? What? What things could he improve upon to greater benefit student teachers in the future?

7. During your experience your cooperating teacher has been interjecting during the class discussions. Tell me your opinion of the interjections. What impact, if any, has this had on the lessons? What impact, if any, has this had on you learning to teach? In what ways, if any, has your experience with these interjections differed if the interjections were "Travis initiated" or "you initiated"--that is, if you invited Travis to interject or if he raised his hand and requested to interject. To what extent did you feel obligated to try to do what Travis suggested in his interjections?

Appendix H: Lesson planning template created by CT

Lesson Planning Discussion Guide
Goal for the Lesson:

Getting Started: warm-up HW?'s review pre-teach mini-lesson

TASK:
Launch: (Schema) Hook:

Context:

Expectations:

Questions:

Resources:

Explore: Grouping arrangement: individual pairs fours whole-class
Anticipated Student work:

Questions:

Looking Ahead to Summary:

Summary: Orchestrating Discussion
Student Work: means of display: overhead board poster

Order of student presentation:

Questions: