2008-03-14

Similar but Different: The Complexities of Students' Mathematical Identities

Diane Skillicorn Hill
Brigham Young University - Provo

SIMILAR BUT DIFFERENT:

THE COMPLEXITIES OF STUDENTS'

MATHEMATICAL IDENTITIES

by

Diane Hill

A thesis submitted to the faculty of

Brigham Young University

in partial fulfillment of the requirements for the degree of

Master of Arts

Department of Mathematics Education

Brigham Young University

April 2008

BRIGHAM YOUNG UNIVERSITY

GRADUATE COMMITTEE APPROVAL

of a thesis submitted by

Diane Hill

This thesis has been read by each member of the following graduate committee and by majority vote has been found to be satisfactory.

Date	Keith Leatham, Chair
Date	Daniel Siebert
Date	Janet Walter
Date	Hope Gerson

BRIGHAM YOUNG UNIVERSITY

As chair of the candidate's graduate committee, I have read the thesis of Diane Hill in its final form and have found that (1) its format, citations, and bibliographical style are consistent and acceptable and fulfill university and department style requirements; (2) its illustrative materials including figures, tables, and charts are in place; and (3) the final manuscript is satisfactory to the graduate committee and is ready for submission to the university library.

Date	Keith Leatham Chair, Graduate Committee

Accepted for the Department

Date	Gerald Armstrong Chair, Department Mathematics Education

Accepted for the College

Date	Thomas W. Sederberg Associate Dean, College of Physical And Mathematical Sciences

ABSTRACT

SIMILAR BUT DIFFERENT:
THE COMPLEXITIES OF STUDENTS' MATHEMATICAL IDENTITIES

Diane Hill

Department of Mathematics Education

Master of Art

We, as a culture, tend to lump students into broad categories to describe their relationships with mathematics, such as 'good at math' or 'hates math.' This study focuses on five students each of whom could be considered 'good at math,' and shows how the beliefs that make up their mathematical identities are actually significantly different. The study examined eight beliefs that affect a student's motivation to do mathematics: confidence, anxiety, enjoyment of mathematics, skill level, usefulness of mathematics, what mathematics is, what it means to be good at mathematics, and how one learns mathematics. These five students' identities, which seemed to be very similar, were so intrinsically different that they could not be readily ranked or compared on a one-dimensional scale. Each student had a unique array of beliefs. For example, the students had strikingly different ideas about the definition of mathematics and how useful it is to the world and to the individual, they had varying amounts of confidence, different aspects

that cause anxiety, particular facets that they enjoy and different ways of showing enjoyment. Their commonly held beliefs also varied in specificity, conspicuousness, and importance. Recognizing that there are such differences among seemingly similar students may help teachers understand students better, and it is the first step in knowing how teachers can improve student's relationships with mathematics.

ACKNOWLEDGEMENTS

I would like to thank my family for their support and encouragement. My children, who thought it was so great when we sat down to do homework together, are hoping never to hear the word ‘thesis’ again. I am especially thankful for my husband, who loves me and believes that I can do anything.

I would also like to thank my advisor, Keith, who has spent many hours discussing ideas with me and encouraging me. He was so patient when life as a mother and grandmother distracted me from my thesis. He was always supportive of my need to take care of my family.

Table of Contents

Chapter 1: Rationale

Many students in the United States approach the mathematics classroom with predetermined ideas about their abilities to be successful. Some believe they are born "good at math" while others decide they are not "math people." There are even some education experts who believe in an innate ability with mathematics. For example, Barzun (1991) claimed that a student reaches a point in his education where he figures out if he is good at math or good at English. I have met many people who are dismissive of mathematics on the basis that they "cannot do math."

However, this is not a universal belief. For instance, in Japan, if a student does not do well in mathematics, the parents and teachers are more likely to blame lack of hard work than lack of innate abilities (Stigler & Hiebert, 1999). Furthermore, there is strong evidence that the part of the brain that enables us to use language is the same part that enables us to do mathematics (Devlin, 2000). That suggests that anyone that can talk can do mathematics.

Regardless, the beliefs that students have about mathematics seem to be causing problems for us as a country. We are facing a severe shortage of people who pursue careers in mathematics and other sciences. According to William Swanson (2005), CEO of Raytheon Company and member of a team working on a national initiative to study the lack of college graduates in mathematics and science, in China three-fourths of all bachelor's degrees are in math, science, and engineering fields, whereas, in the United States, only about one-third of bachelor's degrees are in those fields. Furthermore, universities are experiencing a drop in engineering graduates. Cal State University alone experienced a 31% drop in engineering graduates with a BS degree in the last decade (Swanson, 2005). Universities, such as the University of Wisconsin, are developing programs that include scholarships and innovative teaching to entice more students into mathematics and related fields (Carlson, 2004). In his 2006 State of the Union Address, President Bush named getting our children to study mathematics and science as one of the three things we need to do to compete with the rest of the world. It is important for us to figure out how students are thinking about mathematics before we can convince them to study it.

Many are tempted to make broad generalizations about the way people think about mathematics, or what is often referred to as their *mathematical identity*: some people believe they are good at math, some people do not. (A precise definition of mathematical identity will be discussed in chapter 2.) My niece, Kara, is a good example of how complicated a person's mathematical identity can be. She breezed through her mathematics courses, won state mathematics competitions, and finished calculus at 15. Then she declared she would never take another math class and deliberately chose a career that would not require additional math. When her major required 2 credits from the mathematics department, she looked for the "least mathematical" class she could find and signed up for a logic class. How can we characterize her mathematical identity? We would have to say she was good at math, but she avoided mathematics in the classroom. But could we say she hated math? Well, she said she makes a point to do arithmetic in her head instead of getting out a calculator, and she likes to talk to her young son about how some streets are parallel and some are perpendicular, and stop signs are hexagons etc. It sounds like she enjoys math. There must be other factors affecting how she thinks about mathematics.

The challenge to convince our children to study mathematics is not something we can accomplish until we know more about what they believe about mathematics. All of us have experiences with mathematics from a very young age, and all of those experiences combine to make up our mathematical identities. Understanding and better defining mathematical identity is the first step to understanding why students are not choosing mathematics. It is not sufficient to say that we should try to talk more people into studying mathematics. We need to know what the factors are behind their choice to study or not study mathematics.

Chapter 2: Theoretical Framework and Literature Review

Identity

It has been scarcely 50 years since Erikson, known as "identity's architect" (Friedman, 1999), presented his theories of the development of identity (Erikson, 1959, 1968). Since then, many have tried to build on Erikson's ideas (Cote & Levine, 2002; Hogg, Terry, & White, 1995; Marcia, 1966; Stryker & Burke, 2000). However, identity is still not well defined. It is generally considered to be one's sense of oneself. Bracher (2006) extends this to be "the sense of oneself as a force that matters in the world" (p. 6). Gee (2000) defines a person's identity as the "kind of person" they are. Sfard and Prusak (2005) define identity as "stories about persons" (p. 14). Cobb, Gresalfi and Hodge (in press) look at identity as being an association or affiliation with a group or person. Even Erikson is said to have felt that identity should be defined more precisely and that it could be viewed from different angles (Kroger, 2004). It is at times viewed as a structure or configuration, like a role or a category, (Hogg et al., 1995; Stryker & Burke, 2000) and at other times as a process, such as an interaction or positioning with others (van Langenhove & Harre, 1999).

Given so many different ideas about identity, it is surprising that words such as 'self' and 'identity' are often used in literature as if they were plainly understood. So, although I may believe that identity is one's sense of oneself, what I am thinking when I say that, may not be what another person is thinking when he says it. Cote and Levine (2002) have tried to define identity more precisely by including three aspects of identity: social identity, personal identity and ego identity (See figure 1.) Social identity refers to the groups we are members of. This could include nationality, gender, race, family, classroom or other group. Personal identity is how a person interacts with others, or his or her position in relation to others. This would include a person's behavior or his or her role within the group. For example, within a classroom, one student may be the one that challenges the teacher, whereas another student may quietly watch and listen. Ego identity refers to what is generally called the self—what a person thinks or believes about him or herself and about the world. Although these are described as different identities, they influence and are influenced by each other. One's social identity influences the kinds of interactions available; the kinds of interactions one participates in influences the beliefs one has about oneself; the beliefs one has influences the kinds of interactions one seeks out; the interactions one participates in influence the social settings one is affiliated with.

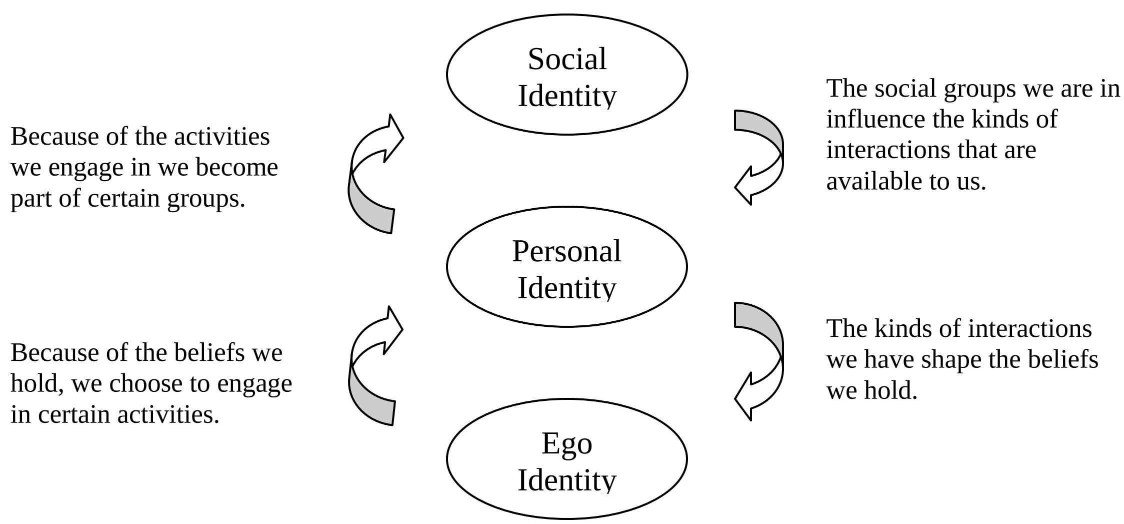

Figure 1. Three Levels of Identity (adapted from Cote & Levine, 2002).

The last few years researchers have begun to look at how identity can give us insight into problems in education (Bracher, 2006; Gee, 2000; Richards, 2006). By using identity as a lens to view what is happening in the classroom, we may gain insight into how to improve the classroom. Bracher (2006) claimed that learning, like all behavior, is motivated by the effort to maintain and enhance identity. In the mathematics classroom negative mathematical identities—especially persistent negative mathematical identities—have been a source of frustration for many teachers.

Mathematical identity is a person's identity with respect to mathematics. Psychologists and sociologists who study identity agree that the self, or the identity, is multifaceted, and they refer to those multiple components as identities (Hogg et al., 1995). Mathematical identity can be considered that part of a person's identity which pertains to mathematics. We may naively assume that we can easily observe a person's mathematical identity—we can see whether or not someone is able to do the assignments in class, or needs help with everything. However, mathematical identity can be looked at with the same aspects as Cote and Levine (2002) described for identity: social, personal, and ego identities (refer to figure 1).

To look at the social mathematical identity one needs to look at the groups or communities that the person is part of with relation to mathematics. This includes, of course, the mathematics classroom and other formal settings where mathematics is done; however, there are other social settings that must be included. Home, family, friends, race and nationality must be considered. Several people have done research on how the mathematical social identity affects the personal and ego identities of a student. Anderson and Gold (2006) found that among preschoolers, numeracy practices traveled between home and school. They stated that children may travel, as from home to school, but they "carry habits, attitudes and social identities along with them" (p. 278). Sfard and Prusak

(2005) compared two groups of students with different cultural experiences, in the same high school One was a group of native Israelis and the other a group of Russian immigrants to Israel. They found a difference in attitudes and practices when doing mathematics across the groups. Martin (2000) studied students with the social identity of being African-American. He looked for ways that being African-American influenced the kinds of mathematical opportunities available to the students.

Just as Cote and Levine (2002) described the interaction between social identity, personal identity, and ego identity, the same is found in mathematical identity. (Refer to figure 1.) Sfard and Prusak (2005) found that something about being in one or the other group (native Israeli or Russian immigrant) encouraged different experiences, interactions, and affiliations. For example, the native Israeli group tended to do the assignments more ritualistically, trying to please the teacher by the work. The immigrant group seemed to desire substantial learning for their own benefit, not to simply please the teacher. Based on the diagram (figure 1) I would infer that there was something about being in the different groups that led to different beliefs—possibly beliefs about the purpose of school or learning mathematics—that led the immigrant group to affiliate with mathematics differently. Affiliation is being closely associated with something or taking ownership of something. The immigrant group was choosing to make mathematics part of who they were, as suggested by the arrows going from ego to personal identity and from personal to social identity in figure 1. The native group tended to keep the mathematics at a distance, or did not affiliate with mathematics.

The mathematics classroom is also a place where researchers have looked at personal identity. Boaler and Greeno (2000) looked at two different kinds of mathematics classrooms: traditional lecture-based classes and discussion-based classes. The students they studied were all in advanced placement calculus, and so could all be considered successful at math. It turned out that the students who were in the discussion-based classes, who had different opportunities for interactions than the traditional classes, came away with more positive beliefs about mathematics than the students in the traditional classes. Cobb, Gresalfi, and Hodge (in press) also found that some classrooms offered experiences that encouraged affiliation with mathematics more than other classrooms. They particularly looked at the obligations students felt when in the classroom, both specifically-mathematical and in general.

The mathematical ego-identity is a little more illusive in the research than personal or social identities—it is hinted at, talked around and generally felt to be inherently understood. Instead of looking directly at the ego-identity, most of the research I have mentioned (Boaler & Greeno, 2000; Cobb et al., in press; Sfard & Prusak, 2005) looked at the behavior engaged in because of the ego-identity: engaging, interacting, affiliating. (Refer to the arrow going from the ego identity to the personal identity in figure 1.) This is a reasonable thing to do since we hope to encourage more students to affiliate with mathematics, but it skirts the issue of what kind of beliefs a person holds about him or herself with respect to mathematics—or the mathematical ego identity—that leads him or her to affiliate with mathematics.

Beliefs

In everyday language, the word 'belief' is often used interchangeably with words such as attitude, perception, opinion, disposition, or conviction. Because of the nature of these concepts, and the overlap between them, it is difficult to define and differentiate between them (Leder & Forgasz, 2002). I am using belief in a very broad sense that seems to encompass all of these. I am a using a definition espoused by Schoenfeld (1998) where he says, “beliefs are mental constructs that represent the codifications of people’s experiences and understandings (p. 19).” So, beliefs are the way we think about the things that happen to us and around us. This implies that we are always forming, changing or strengthening our beliefs. Schoenfeld goes on to say that people’s beliefs influence what they perceive in any situation or set of circumstances, what they consider to be possible or appropriate in those circumstances, the goals they might have in those circumstances, and the knowledge they might bring to bear in them. This is consistent with Cote and Levine’s (2002) view of the ego identity (figure 1) and how it shapes the experiences that a person engages in.

Determining a person’s beliefs may be difficult. A person may not be able to articulate a belief—indeed, may have never tried to articulate it. For example, it is likely that most people have never tried to put into words what they believe that mathematics is. Furthermore, there may be vast differences between a professed belief and inferred beliefs, or what Schoenfeld (1998) calls “attributed” beliefs. Although researchers can never truly know what a person believes, they can “attribute” a belief to a person when he or she behaves in a way that is consistent with having that belief.

There are numerous beliefs a person may have which make up their mathematical identity. We may speak of a certain “belief,” but actually it makes little sense to talk about one belief by itself. Beliefs are interrelated. We look at systems of beliefs, which are groups of related beliefs. A person has a system of beliefs that makes sense to them, even if it does not make sense to someone else (Leatham, 2006). Often, looking at one belief by itself can be confusing or misleading. For example, in the research by Boaler and Greeno, (2000) they found that the majority of students in the traditional classes disliked mathematics, and the majority of students in the discussion-based classes liked mathematics. But when they looked at other beliefs about mathematics, they found that the two groups had very different beliefs about what is mathematics. When we study beliefs that make up a person's mathematical identity, we must consider a system of beliefs.

Defining a system of beliefs can be problematic. Systems overlap and vary from person to person. For example, a person may develop the belief within a music setting that they are not good at memorizing, which may turn out to be a belief important in other settings, including mathematics. One can, however, through experience and looking at previous research, come up with those beliefs that we can assume are related to mathematics. When other beliefs appear to be included—such as the ability to memorize—it can and should be added in for that individual.

The beliefs within the system differ in their importance to the individual. Beliefs vary along a central-peripheral dimension where the more central a belief the less likely it is to change (Rokeach, 1968). Centrality is neither determined by how long a belief is held, nor by the number of sources that substantiate it. It is determined by how many other beliefs are based on it (Bem, 1970). When people are faced with a contradiction among their beliefs, they will try to restore sense to the system by making as few changes to beliefs as possible. The more peripheral the belief, the more likely it is to be changed.

Motivational beliefs

One of my primary reasons for studying mathematical identity is to find out why relatively few students are choosing to pursue mathematics and how to motivate more students to study mathematics. Because of this, I wanted to look at motivational beliefs. I am using the framework from Wigfield and Eccles (2002) to organize my discussion of some motivational beliefs which I believe will be part of a person's mathematical identity. Wigfield and Eccles describe researchers in achievement motivation as those who are interested in why individuals engage in the variety of achievement-related behaviors that they do. They organize their discussion about achievement motivation around three questions students may ask themselves. I have phrased these questions pertaining to mathematics: Can I do the mathematics? Do I want to do mathematics? and What do I need to do to be successful at mathematics? Although I am not suggesting that the topics covered by these questions constitute a complete definition of mathematical ego-identity, I am suggesting that they are a vital part of how a person thinks about mathematics and, if a comprehensive definition were made, these topics should be included.

Some researchers have looked at a few of the same beliefs at which I am looking. For example, Martin (2000) considered beliefs about the importance of mathematics and opportunities in mathematical contexts, and motivations to learn mathematics. Boaler and Greeno (2000) found that students had different beliefs about how mathematics is learned and what it means to be good at mathematics and asked the participants about their enjoyment of mathematics. However, apparently no one has looked at several beliefs as a system to be looked at in relation to each other. It seems that research would benefit from, if not a comprehensive list, at least a working list of beliefs that are part of a person's mathematical identity. In the following sections of this chapter, I will consider some beliefs that may be included in that list. I have organized the list according to the achievement motivation work by Wigfield and Eccles (2002) as already mentioned.

Can I do the mathematics?

This question refers to a person's perceived competence (Wigfield & Eccles, 2002). I have included confidence and anxiety. These concepts have been studied extensively as related to mathematics.

The confidence, or belief in one's ability to perform a specific task, such as mathematics, is sometimes called self-efficacy (Bandura, 1993; Schunk & Pajares, 2002). One can talk about self-efficacy with respect to mathematics, or with respect to specific aspects of mathematics, such as algebra, or memorizing. However, one can also discuss confidence in a more general sense—an overall self-confidence. This is the way a person feels about himself in a general sense, not specific to any task (Schunk & Pajares, 2002). It is the person's self-concept. There is a relationship between a person's self-efficacy and self-concept, and there may also be confusion in determining whether it is self-efficacy or self-concept that one sees in another person.

Self-efficacy is an individual's personal belief in his or her own ability to achieve a goal. Self-efficacy has been found to have more impact on students' career choices, the classes that students take and how well the students do in those classes than other factors, such as socioeconomic status, parent beliefs, or actual grades

(Bandura, Barbaranelli, & Caprara, 1996). If we look at feelings of competency on a continuum with 'feeling competent at math' at one end and 'feeling incompetent at math' at the other, every person would fall somewhere on the continuum. Where a person falls on this continuum does not depend on how competent they actually are. It is possible to be over-confident and feel more competent than you actually are, which may be beneficial for a student; or to feel less competent than you really are, which is never beneficial for a student (Pajares & Miller, 1994). Forster, (2000) who studied mathematical identity, focused on self-efficacy and found this aspect to be very important in how a student succeeds. Much of the earlier research spoke of self-efficacy and confidence in the same way that recent research speaks of identity, making it important to include confidence in the beliefs we look at to describe a person's mathematical identity.

Mathematics anxiety is another belief that has been studied extensively. Some may argue that fear is a physiological response—sweaty palms, dry throat, racing heart beats, or even nausea (Perry, 2004)—and not a belief. However, it is a physiological response *caused* by a belief. I will treat anxiety as a belief. It is held by many students of mathematics. It turns out to be very widespread, and can be very debilitating. If we think of fear or anxiety on a continuum, the extreme anxiety would be at one end and security or peace (lack of fear or anxiety) would be at the other end. Every student would fit on the continuum at some point.

Anxiety is related to confidence, certainly a lot of anxiety could lead to a lack of confidence, but they are different and need to be considered independently, nonetheless, anxieties are still within the same system of beliefs. One may expect someone who is confident at mathematics to have little or no anxiety about doing mathematics; however, it is possible to be confident and still feel anxiety.

Do I want to do the mathematics?

This question refers to the reasons and purposes a person has for engaging or not engaging in an activity (Wigfield & Eccles, 2002). I will look at motivating factors such as enjoyment and beliefs about the usefulness of mathematics. A person may be able to do the math, and yet not want to do the math, either because they do not enjoy it or because they do not feel it would be useful to them.

The enjoyment a person feels when doing mathematics has an effect on whether they continue in mathematics. (I call this belief enjoyment while understanding that it is the other end of the spectrum, lack of enjoyment, that is most often brought to our attention. Nearly everyday I hear someone say, “I hate math.”) This emotion means almost nothing unless it is considered along with the belief of what mathematics is. A person who does not enjoy memorizing, and believes that mathematics is all about the memorizing, will probably dislike math. But if that same person believes mathematics is about thinking and problem solving instead of memorizing, he may very well like math. Enjoyment of mathematics has been looked at in light of making the classroom experience more ‘fun’ (Steward & Nardi, 2002); however, I am not considering enjoyment to be the same as entertainment. Entertainment allows the possibility of passivity—someone or something else entertains you. While you may enjoy being entertained, enjoyment is finding satisfaction or pleasure in something. It is possible to enjoy something that is difficult—to find satisfaction in doing something hard to do.

Beliefs about the usefulness of mathematics is another group of beliefs that make up a person’s mathematical identity. Even from early elementary school, students begin to form opinions about whether mathematics is useful and who it is useful for (Kloosterman, Raymond, & Emenaker, 1996). Perceived usefulness of mathematics for current needs as well as the future is an important factor in determining whether students choose to take mathematics classes (Reyes, 1984). Armstrong and Price (1982) found that both males and females ranked usefulness as the most important factor in their decisions to take more mathematics classes—even more important than whether they liked or disliked mathematics, or how good they perceived themselves to be at mathematics.

What do I need to do to be successful at mathematics?

Answering this question links motivation, cognition, and behavior (Wigfield & Eccles, 2002). It is not enough to want to do a task, students must consider what it will take to meet that goal. That means considering beliefs about what mathematics is, what it means to be good at mathematics, and how mathematics is learned.

As mentioned earlier, Boaler and Greeno (2000) interviewed students from two different kinds of classrooms—traditional and discussion-based. The students did not seem to be talking about the same mathematics. One group described mathematics as “procedures. Because you have to learn this, to learn this, to understand this (p. 180).” Another student from this same class said, “This is AP so it’s definitely going to be harder, but I feel as long as I can memorize the formulas and memorize the derivatives and things like that, then I should be pretty well off (p. 181).” Someone from the other style of classroom said of math, “I guess I’ve liked math overall because it's a lot better than English or social studies, just *because I don't like to memorize* just a bunch of stuff" (p. 182, italics added). 'Doing mathematics' means different things to different people. Some may describe it as conjecturing, questioning, proving, generalizing etc. But unfortunately many students in school believe that doing mathematics is simply following the rules set by the teacher, and knowing mathematics is remembering and applying the correct rule (Lampert, 1990).

Another similar group of beliefs would be beliefs about how a person learns mathematics. Some people believe that mathematics is learned only by practicing procedures. At one time drill and practice was the major means of instruction, especially for arithmetic, and is still part of many curricula (Resnick & Ford, 1981). Many, including the National Research Council (2001) insist that learning procedures is inadequate and students must have an opportunity to think mathematically by engaging in problem solving. As O'Brien (2007) has noted there are opponents of the reform movement that believe we should go "back to the basics" or back to drill and practice Suppose a person believed that mathematics should be learned by practicing procedures and he dutifully practices the procedures. If he still does not understand the concepts behind the practice, he may feel conflict within his system of beliefs. If his belief about learning mathematics by practicing procedures is more central than his belief in his ability to do mathematics, he might conclude that he just is not cut out for mathematics. If his belief about his ability to do mathematics is more central, then he may change his mind about how mathematics is learned.

Beliefs about how a person gains knowledge are demonstrated in the classroom. One may believe knowledge can be shared—as a teacher shares knowledge when she teaches by ‘telling’ or by demonstrating procedures (Smith, 1996). Or one may believe that knowledge must be constructed by each student (von Glasersfeld, 1983) by doing and discussing mathematics. Students develop beliefs about learning mathematics based on their experiences in the classrooms; however, student beliefs may not turn out to be what is expected. Schoenfeld (1988) found that students developed beliefs that were not anticipated because of the classroom settings. For instance, many students believed that if they could not solve a mathematics problem in just a few minutes, then they were not going to be able to solve it at all. This suggests that mathematics is something you either get or you do not get—you are either a ‘math person’ or you are not.

Summary

Mathematical identity is often referred to in the recent research in mathematics education; however, it is not well defined. Often, the research has focused on the students’ behavior that shows they are affiliating with mathematics (Cobb et al., in press), engaging with mathematics (Solomon, 2007), aligning themselves with mathematics (R. Anderson, 2007), or investing in mathematics (Martin, 2000). However, the students’ beliefs which lead to the behavior have not been studied in a direct manner as they relate to identity. We assume that students who choose to study mathematics have positive mathematical identities. Perhaps knowing more about the possible beliefs a student may have who chooses to study mathematics, we can eventually look to see what configurations of beliefs led to that positive mathematical identity.

Looking at beliefs about mathematics, or what Cote and Levine (2002) refer to as the ego-identity, is important to understanding mathematical identity. Mathematical identity is much more than simple broad categories. Educators and parents often group students too broadly, as do the students themselves. By looking at a person's beliefs we should be able to see a wide range of mathematical identities among people who are typically clustered together under the same label.

Research question:

How do students who could all be labeled as "good at mathematics" differ in the motivational beliefs that are included in their mathematical identities?

Chapter 3: Methodology

This is a case study of five students who society might characterize as having similar attitudes towards mathematics. They were chosen from an honors section of a first semester college calculus class at a large university. They all had a history of being good students and having done well in previous mathematics classes. I looked at the students' beliefs about mathematics to describe the mathematical identity for each of them. The class, part of a larger study, was videotaped daily. The five students who participated in my study worked together for most of the semester and their work in class was often captured on video. I chose these five students because I had videotape of their work together and because they knew each other well and could share their impressions about one another

The students were enrolled in a class with a discussion and problem solving format. Class was held three days a week for two hours each day from the beginning of September to the middle of December. Students worked in a collaborative setting on problems designed to elicit the building of critical mathematics in explorations. Tasks were intentionally problematic. It was expected that no one in the class would know immediately how to solve the problem, nor was there a single 'correct' solution. Groups presented their solutions to the entire class and it was discussed.

The students were guided by the assignments and the two teachers of the class to discover the main concepts of calculus. This may be similar to the sort of class that Cobb, Gresalfi and Hodge (in press) described as having the potential to change a person's mathematical identity; therefore, it is possible that simply being in such a class may have changed some of their beliefs. However, I have looked at the students at a time after the class has ended, and I am not trying to show a change in their identities. Having taken the class does become part of their histories and is an experience that they have in common—although it is not necessarily integrated into their past experiences in the same way. In some cases, where the student reports having a change in what he or she believes, I have considered possible explanations for those changes.

Data Collection

Written and video data were collected during the calculus class as part of the larger study. The data I have used from the larger study include a Mathematical Belief Questionnaire (see Appendix A), taken from Seaman, Szydlik, Szydlik, & Beam (2005), which asked the students to rate how much they agreed with particular statements about mathematics, and an open-response questionnaire on student perspectives about mathematics (see Appendix B). The students filled both of these out at the beginning of the semester and again at the end of the semester. I used information from these questionnaires to stimulate conversation during the interviews (see interview 3 below), and to triangulate the information that I gained from the interviews. For example, when a participant told me in the interview his or her definition of mathematics, I looked back at the questionnaires to see if what they said on the questionnaires was consistent with what they said in the interview.

From videotape taken during the class, I was able to see how the students behaved while doing mathematics. The behavior I saw on video led to certain lines of questioning. I also used clips from the videos during to refresh memories and stimulate conversation during the interviews (see interview 2 below).

Because I wanted to look at the students' beliefs about mathematics in depth, I felt that the information gleaned from the questionnaires and videos would not be complete without direct explanations by the students. Additional data specific to this study were collected in three separate interviews with each participant. The interviews each lasted about 50 minutes and were audio and video recorded and transcribed by me. During the interviews I tried to discover each person's beliefs about mathematics by having them do tasks, tell stories, relate memories, and describe feeling and thoughts. The specific material covered in each interview is described below.

Interview 1

In the first interview I wanted to focus on the individual students. I was interested in whether their experiences with family, friends or previous mathematics classes affected their present beliefs about mathematics (as in D. Anderson & Gold, 2006). I also wanted to encourage them to talk about what they thought "being good at math" means. There were two parts to the first interview. The protocol for interview 1 is found in Appendix C.

Background. I asked the subjects to describe their family, particularly concerning mathematics and college education of parents and siblings. I asked them to describe their own mathematical history, including favorite and least favorite classes and teachers, earliest memories, and attitudes toward mathematics.

Descriptive Words Task. I gave the students descriptive words and asked them to rate the words, from 0 to 5, according to how much the word describes someone who is good at mathematics (see Appendix C). I presented the words one at a time on a card. I asked the student to explain why he or she chose the response, and I encouraged them to give examples and experiences from their own lives. I was not so much interested in the numerical answer as in the explanation of why it was chosen. I started with 28 words. I found after the first interview that that was too many, it became rather boring. I cut the number of words back to 23. In some of the interviews I further adjusted the number of words to ensure the interview remained approximately 50 minutes.

Interview 2

The main emphasis in interview 2 was the interaction with the other members of the group. I was interested in how the participant felt when interacting with the group, and what he or she thought the others believed about mathematics. The interview protocol was different for each participant, but a sample interview protocol is found in Appendix D.

Watching and discussing video scenes from the calculus class*.* The five students in this study worked as a group and were often shown working on problems on the videotape of the class. I chose video clips showing different situations and asked the students to comment on what was happening in the video and how they felt about it. Some of the clips were the same for each student. For example, I showed each student a specific clip of the group talking about mathematics by personifying the equations, calling the derivative a “bad boy” in a fun-loving way. It was unclear from the video if all the students were comfortable with this kind of talk. I also showed each student two clips showing different reactions to new problems. In one clip the group was laughing and enjoying setting up a problem; in the other, the group responded with concern and doubt. I wanted to see if all the students felt the same about the problems, and if they could articulate what made the problems so different. I also showed each student a clip of him or herself presenting to the entire class, and discussing mathematics in the group. I wanted to see if those things caused anxiety. I asked them to describe what was happening and what they were thinking at the time if they could remember. I also asked if this situation was typical. I used this opportunity to gather information not only on the student being interviewed, but also on the other students in the group.

Continua. At the end of the last clip shown to the students, I paused the video on a screen where all 5 participants were shown. I asked them to tell me about the other students in the group. I presented the interviewee with four continua: (a) Anxiety to Peace, (b) Not Confident to Confident, (c) Enjoyment to Pain, and (d) Not Good at Mathematics to Good at Math. (See Appendix D.) Each extreme was placed at opposite ends of a line. Each continuum was on a separate piece of paper and printed landscape to give plenty of room for writing directly on the continuum. I had the interviewee place each of the other four students in the group one at a time on each of the four continua. I had them explain their reasoning as they did it. After they had placed the other students, I asked them to place themselves on the continua. Placing themselves was the most difficult, but having fellow students already on the continua forced the student to think about how he or she fit in relation to the others in the group.

I was interested in where the students placed one another on the continua, and I wondered if the teachers' points of view would be similar to the students' views on each other. I interviewed both of the teachers who taught the calculus class, and asked them to place these five students on the four continua. I structured the interview the same as this portion of the second interview was structured for the students. I had them place the students one at a time on the four continua, explaining their reasons as they did so. The calculus class ended 8 months before the interviews. It is possible that the responses would have been different immediately after the class.

Group Roles. Because we had just finished talking about the group and how all the students felt about certain aspects of mathematics, I asked each participant to describe the roles everyone played in the group when solving problems. I wanted to see how they positioned themselves in the group with respect to mathematics. (van Langenhove & Harre, 1999). For example, I was looking for statements about who was the leader, or the explainer, or the comic relief.

Interview 3

In interview 3 I wanted to look at the students' beliefs about the definition of mathematics and its usefulness. The protocol for interview 3 can be found in Appendix E.

Mathematical Categories Task. I gave each student 26 cards with an activity on each card and asked them to form categories in which to place the cards according to how the activities related to mathematics. (See Appendix E for a list of the activities.) They were told to make more than 1 and less than 26 categories, and then name the categories. Although I had asked the participants for their definition of mathematics, I also wanted to infer their definition of mathematics from their discussion of the activities and categories. I chose activities that varied in how they used math. Some of the activities used mathematics explicitly, like doing times tables and measuring a room, and some more implicitly, like arranging trophies from tallest to shortest or hanging a picture. I tried to find some activities that most people would say had no relationship to mathematics, like reading a book or writing an essay. I asked the participants to 'think out loud' as they sorted the cards. When they had the activities sorted into categories, I asked them to explain what the categories were and how each of the activities fit into that category.

Discussing the written surveys taken before and after the class. Many of the questions on the belief survey (found in Appendix A) needed additional explanation. For example, statement 11 says "Mathematicians are hired mainly to make precise measurements and calculations for scientists". A person may believe that mathematicians do make precise measurements and calculations, but not for scientists. I wanted to be clear about what part of the statement the student agreed or did not agree with, or why they agreed or did not agree with it. I read some of the statements to the students, and asked them to respond. We talked about why they thought as they did. If their response was different than on the original survey, we talked about what had changed. I was interested in having them talk about their beliefs, not so much the exact answer they chose on the survey. There was not time to talk about all of the statements, so before each interview I chose several statements that I thought would be interesting for that participant. I chose by looking at broad changes in answers from the beginning and end of the semester, where answers had stayed exactly the same, and topics that I wanted to talk to that person about more. I also asked them to re-answer some of the questions on the Student Perspective questionnaire, such as “What is mathematics?” and “What are the purposes of mathematics?”

Action/emotion statements. I gave the students statements in the form shown in Appendix E. I asked them to fill in the blank with an emotion evoked by the action. I asked them to try to give their first response, and then to try to articulate why they think they might have felt that way. I was disappointed by this task. Most of their responses were too general—like, “good.” There were only one or two responses that gave me any good information. It would have been better if I had given them a list of emotions and asked them to choose one from the list.

Analysis

All of the interviews were transcribed by me. I then took excerpts from the interviews and placed them in 8 categories: confidence, anxiety, enjoyment, mathematical skill, usefulness of mathematics, how mathematics is learned, what it means to be good at mathematics, and definition of mathematics. Some comments were placed in more than one category. I looked at not only what they said directly about a topic, but also whether or not what they said was consistent with the other things they said or did.

To look closely at the continua, I looked at both the order of the students on a given continuum and the placement on the continuum. To look at order I gave each name a number 1 through 5. For example, on the Confidence continuum, the person closest to the “not confident” end would be number 1. The next person along the continuum was number 2. The person closest to the “confident” end would be number 5. To show the placement on the continuum, I divided the continuum into 20 equally spaced segments. I assigned each name a number 1 through 20 according to where it was on the continuum. I placed the numbers on a spreadsheet. I looked at them pertaining to the person filling out the continuum and pertaining to topic. I looked at the places where the spreadsheet showed disagreement, for example, if everyone placed a person at the low end of a continuum, except for one person, I would look more closely at that person and see if I could figure out why. The inconsistencies showed potentially interesting things about the person being described or the person doing the describing. When I found an inconsistency I looked back at the other data to try to determine why the inconsistency existed. For instance, if one person ranked someone differently than everyone else, I looked at what that told me about the person being ranked, as well as the person who did the ranking.

After compiling the data according to topic, I looked at each person. I looked at how the person's beliefs fit together to make a person's mathematical identity. I tried to assess what each person believed about the 8 categories mentioned above, as well as their motivation for studying mathematics. I wrote a synopsis of what I found and how the beliefs fit together, which is in chapter 4. I sent each student his or her completed mathematical identity to verify that I had not misunderstood anything. I heard back from 3 of the students and made a couple of minor changes. Once I had a mathematical identity for each student, I looked across the students, to compare them in the different categories.

Chapter 4: Individual Mathematical Identities

I have synthesized information from all sources I had access to, to illustrate each student's mathematical identity. In each case I begin with a brief description of their backgrounds and how mathematics fits into their home and school history. I have covered background, confidence, anxiety, enjoyment, beliefs about usefulness of mathematics, how mathematics is learned, what it means to be good at math, and the definition of mathematics. The focus may be different for each student. I focused on what seemed important to the student—what they talked about the most—and what seemed unique about the student.

Bryce

Bryce[1] attended a large High School in California. He took mathematics classes through precalculus, getting mostly Bs because of his lack of effort. He felt that his lack of ability to do "thick, chewy algebra" came from his lack of motivation in high school and hindered his ability to do higher level math. Bryce is the youngest of six, and nearly everyone in his family has attended college. His father is an engineer, and, although Bryce assumed his father is good at math, he had "never seen him do it." Bryce lists his least favorite class in High School as math. The worst class was in 11th grade algebra 2. There was a substitute teacher every Friday. Bryce felt that nothing was learned, and the class was a waste of his time.

Although Bryce was not confident in his algebraic skills, he showed a lot of confidence in other areas. He had the confidence to put forth ideas in his group on how to approach a problem. In fact, he occasionally presented a solution idea so

[1] All participant's names are pseudonyms.

confidently that he talked the other group members into following his suggestion, even though he was wrong. Bryce described this:

> I think that is something that I have a problem with sometimes, because I think I know how to do something and I start doing it. And then like somebody's like, oh no you're not quite doing that right, and *I* try and argue it. And sometimes I convince them that I'm right. Even though I wasn't doing it right, I was able to convince people that I *was* doing it right.

It seemed that he had confidence in himself as a person—and it outweighed any lack of confidence in his mathematical ability. He had the confidence to stand up in front of the class and present his ideas, despite a lack of surety that the mathematics was correct. He often volunteered to present concepts to the class because he felt that it helped him learn it better.

Bryce had a fun-loving attitude towards life that spilled over into mathematics class. Although he did not really enjoy many aspects of mathematics, he was witty and playful to make it enjoyable. One might think that the laughter and making jokes about the problem was evidence that he was enjoying doing the math; however, Bryce reported that he did not enjoy doing the math, and joked around to make it "more bearable." Sometimes he joked around to cover up that he did not know what he was doing. However, it would not be true to say Bryce disliked all aspects of math. He placed himself in the middle of the Enjoyment Continuum when he is in a traditional classroom, and even higher when he is in a non-traditional setting, such as the calculus class in this study. He enjoyed learning about the "big concepts" and how they fit together. But his poor skills, or to use his words, "the whole math kicking me in the face," made doing mathematics not fun. It seemed that the skills that he failed to learn in junior high and high school hindered his enjoyment of mathematics on the college level.

Bryce defined mathematics very broadly: recognizing patterns and expanding on the patterns. He considered mathematics a way of looking at everything in the world; everything is either recognizing and using a pattern, or expanding on a known pattern. During the interviews he never used numbers as a test of whether something was mathematical; however, when he filled out the questionnaire at the end of the calculus class he defined mathematics as "the study of how numbers work." This seemed to fit with his idea of patterns. He did not look to see whether or not numbers are present, nor if operations are performed on numbers, but how the numbers fit into patterns.

He believed that when someone is good at mathematics they have mastered the ability to creatively expand on patterns and procedures. Bryce believed that anyone can learn to be good at creatively applying and using the principles of mathematics, but differentiated between what a normal person does when he is good at math, and what a truly brilliant person like Isaac Newton can do with math. Bryce believed that to learn mathematics a person needs to be patient and diligent. He learned mathematics by verbalizing and discussing the mathematical ideas, but acknowledged that some people are able to learn mathematics by themselves without social interaction.

Caleb

Caleb went to school in a large high school where he was on the advanced track for mathematics. He had calculus as a junior and statistics as a senior. Both of his parents were university professors in fields that do not involve mathematics. Education was important to them. He believed that no one in his family enjoys mathematics, except that his mother may have enjoyed it somewhat. When Caleb’s parents were unable to help him with mathematics problems, they arranged for him to get help in the math lab at the university. Although Caleb enjoyed mathematics at the time of the interviews, for most of high school mathematics was his least favorite subject. There were some years his grades were very low, including a quarter when he had to make-up a failing grade. Caleb attributed the low grades in part to problems outside the classroom, including family problems and illness. To a certain extent, some of his dislike of mathematics may have been caused by boredom. Caleb is very bright—some of his classmates said he was brilliant. The classes that he disliked were classes that did not challenge him. Caleb did not like calculus when he took it in high school because it was “just an easy A, basically, but you didn’t really learn anything.” When Caleb had statistics his senior year, he was challenged and interested; he enjoyed it so much he decided to pursue a career in actuarial science. Caleb has enjoyed the mathematics classes he has taken in college.

Caleb reported feeling some anxiety when he was introduced to a new concept, but once he had a chance to work with the concept and understand it, the anxiety went away. Tests did not cause Caleb to feel anxious. He enjoyed the challenge of tests, and found them exciting. Caleb had a lot of confidence in his ability to do math. When someone else in his group had a different answer than he did, Caleb began looking for the discrepancy in the other person's work because he felt that his own work is usually correct. And his work *was* typically correct. The members of the group in calculus class considered him to be the best mathematics student in their group, and maybe in the class. They looked to him for leadership in solving problems. However, the members of the group felt like there were times that he talked over their heads and did not explain the problems in a way they could understand.

Caleb believed that mathematics was figuring out "the way things work and the association between things," with the intent to use the information to understand, predict, recreate, or improve elements of life. Caleb thought that mathematics was implicitly useful and he believed that the mathematics classes he has taken and will take in college will be useful to him in his career. He planned to take some classes which are not required, because someone had recommended that they would be helpful to an actuarial scientist. He was motivated by the desire to understand the concepts in a deeper way than simply being able to do the problems. Caleb also believed that mathematics is found in most activities that people participate in, either consciously or subconsciously. That is, even though people are not consciously thinking about the math, it is still there.

Caleb believed that being good at mathematics means that you can understand the concepts and use them creatively, which is not necessarily demonstrated by good grades or high test scores. He felt that mathematics requires more creativity than other subjects where there is only one correct answer. Caleb compared doing a mathematics problem with mountain climbing. He explained that there may be one path that has been set, “it works every single time…it’s safe, it’s easy, you probably won’t get injuries or accidents, it’s been tested a lot,” but there may be other ways to “climb the mountain.” The alternative solutions may be more difficult, or they may be a way that no one has ever tried, but you can still solve the problem. The set routines and procedures that are often taught in a mathematics class are like the established path up the mountain.

Caleb believed that anyone can learn to be good at math, that it is not necessary to be gifted, but it does require persistence. He said one must be persistent in trying to understand the concepts and work through problems in his or her own way. If you are not persistent, he believed mathematics turns into “the rut of memorizing formulas.” Caleb contended that verbalizing the mathematical concepts can be helpful, but is not required in order to learn the mathematics. But he believed that everyone can benefit from learning by “discovery” [2]—in mathematics as well as other fields. In fact, he had a job in the upcoming semester as a teaching assistant in statistics and planned to use discovery methods as much as possible in his own teaching.

Debbie

Debbie grew up in a small town where she maintained a 4.0 grade point average. She took algebra 2 her junior year, which was the extent of the mathematics her high school offered; her senior year she had a college algebra class at the local

[2] When the students spoke about the format of the class they referred to it as “the guided opportunity to discover methods on your own” or, simply, “discovery.” The Belief Survey refers to “Discovery methods of teaching.” The teachers for this course described it as learning “in a collaborative setting designed to elicit the building of critical mathematics in explorations of carefully selected tasks.”

community college where she also received an A. She liked mathematics and felt like she was good at math. When she arrived at college, her feelings about mathematics changed. The fast pace and the difficulty of the tests caused Debbie a great deal of frustration and anxiety and resulted in poor grades. Mathematics was no longer enjoyable, but instead it was stressful. Debbie attributed many of the problems she had with mathematics to a traumatic experience she had as a youth. She had memorized a piano solo for a competition and during the performance she forgot part of it. Since that time she has had no confidence in her ability to memorize. Her mathematics classes in high school allowed students to use notes on the tests, thus eliminating the stress over memorizing formulas and such. In college, notes were not allowed on the tests. Debbie felt very anxious about whether she would remember what she needed for the tests.

Debbie was a very creative person and enjoyed music and art. She was accustomed to looking at the world according to creativity rather than mathematically. Once the mathematics was pointed out, Debbie found it beautiful, especially the way it can be adapted to so many things. Debbie realized that in her work as a geologist she will use many mathematical models; however, she knew that the computer will do the mathematics and it was enough for her just to know the mathematics is there.

Debbie defined mathematics as, “the use of numbers to really express how something works, the inner workings and the logic behind why it works.” This suggested that she believed that rather than being something to be solved or worked, mathematics is found in everything around us and can be discovered. She also believed that discovering mathematics in places where you may not expect it, such as geology, helps you "get the wheels turning" and think more creatively. Debbie believed that numbers and formulae are intrinsic in mathematics. She believed that you may arrive at the formula in different ways, or have different forms of the formula, but there is nearly always an applicable formula that can be discovered and applied.

Debbie believed that a key part of becoming good at mathematics is your character, which includes persistence and patience, as opposed to inherent talent in mathematics. She believed that people have different learning styles. She learned mathematics best by spending time in a social setting, discussing the problem. She felt that "discovery learning" worked well for her because it gave her more time with the concepts. She believed that explaining the problem to others can be a good gauge for you to see if you really understand the concepts. Debbie has found that you need to have a balance of being independent vs. dependent when doing math. She felt that one of her problems has been that she did not ask for help, or admit to others that she did not understand concepts. She thought she had to figure everything out for herself. However, she felt there is a danger when working with others in becoming too dependent on them to come up with ideas. She has been lulled into letting others come up with a plan, and then jumping in to help with the work.

When Debbie came to college her enjoyment and confidence suffered, but her overall mathematical identity remained positive. She placed herself below the others on the Mathematical Skills Continuum, but placed everyone, including herself, in the top half of the continuum. She said, "We're all good at math." In spite of several bad experiences in college mathematics classes, Debbie's belief about being good at mathematics was central for her and did not change.

Hannah

Hannah came from a very small farming community. Her graduating class had three students. She was very shy and soft spoken, especially as a freshman in a large university. She took the highest levels of mathematics offered at her school, which included pre-calculus, and excelled at it. She learned mathematics procedurally: either teaching herself from examples in the book, or having a teacher share time between more than one class. She enjoyed the calculations involved in doing mathematics—particularly fractions—and felt that her calculation skills were pretty good. She wondered at the time of the interview if she really enjoyed the mathematics or simply felt that it came easy to her. However, when it came to understanding the concepts behind the mathematics, Hannah's confidence was very low. She took one week of a college mathematics class on proofs and concluded "I don't know if I have the right way of thinking for math." There was a time in her calculus class when she appeared to be the only one in class understanding a concept. Hannah interpreted the experience negatively, saying, "it makes me think that I must not be getting it, if its that hard of a concept." Hannah compensated for her lack of understanding by hard work. She often did more than the required homework and put in more time than many of the other students. Others interpreted this extra diligence as dedication and enjoyment of mathematics, whereas she felt it was anxiety that motivated her to work hard and learn the mathematics.

Hannah believed that mathematics is solving problems: “this is what we have, figure out a way to make it work.” However, Hannah also believed that numbers play an important role. She believed that for mathematicians “the precise measurement is what it is all about…you have to find the right answer.” Hannah believed that mathematics is an important field on its own, and not merely a tool for the use of scientists. Mathematics is needed for many everyday activities such as cooking, reading a map, and keeping a planner. However, activities where the numbers are not explicit, like arranging trophies from tallest to shortest, were not considered mathematical to Hannah.

Hannah believed that mathematics is a serious subject, not to be taken lightly. Some students in the group joked around, personifying equations, for example, instead of calling a function f(x), they would call it Hector, and the derivative some other name. feeling that it made the mathematics more enjoyable. Hannah found the joking around to be a distraction and tried to ignore it. She felt that it interfered with her ability to learn the mathematics.

Hannah believed that to be good at mathematics a person needs to be brilliant, although people who are not brilliant can learn to be pretty good at mathematics if they are open to being taught, and work really hard. Consequently, she worked really hard and tried to listen and learn from what others in her group said, hoping they would teach her. Hannah believed that someone who is good at mathematics can learn mathematics by themselves without any social interaction; however, for someone like herself, social interaction, explaining or teaching someone else, is important in learning math. Hannah believed that learning by “discovery” is a valid alternative way to teach mathematics, but felt that teaching by "telling" is still better for those who are able to learn that way. Hannah planned on being a teacher with mathematics as her minor. She said she would use traditional "telling" as her first choice for teaching, and use "discovery" methods when a student does not get it using traditional methods.

Jack

Jack came from a large high school where he was on the advanced track for math. He took calculus as a junior and statistics as a senior. He earned all As. He liked doing mathematics and felt that he was pretty good at it. He has continued to excel in college courses. Being 'good at math' was important to Jack's self image. He reported feeling anxious before a test, not because he was afraid he would get a bad grade, but because he was afraid he would "do bad and just not be good at math." The desire to be "good at math" was a motivator for Jack.

Jack said for the most part he was confident about his work in mathematics, but felt like he did not always approach a problem the easiest way. He felt that he made the problems more complicated than they needed to be. Jack believed that learning mathematics in a group setting benefited him because he could talk about the problems and see when he was making a problem too complicated. When he was stuck on a problem, he would try to find someone he could talk to. However, he considered the need to talk about the mathematics a fault. He felt that someone who was really good at mathematics should be able to learn mathematics by themselves.

Jack enjoyed mathematics and was considering mathematics as a career; however, the only career he recognized in mathematics was teaching. He tried majoring in engineering—which he believed is related to mathematics, but not actually mathematics—but he did not enjoy the engineering classes. He said, “when you think of engineering there are all sorts of different fields you can go into. When you think of math, the only thing you think of is a mathematics teacher.” He was considering changing his major to mathematics education, even though the pay is low for teaching, because he enjoyed mathematics. Jack’s limited view of the career possibilities in mathematics was consistent with his view of the importance of mathematics. He considered mathematics a “sub-category of most things.” He believed that mathematics is not important in itself, but only as part of a science. Jack claimed, “I can’t think of anything now that would strike the average person as amazing or extraordinary, something that came from the mathematics field.” He believed that other than teaching, the job of the mathematician would be “crunching numbers and doing the calculations that other people don’t want to do or don’t know how to do.” Rather than being the queen of the sciences (Sartorius von Waltershausen, 1966), mathematics was the slave of the sciences.

Jack defined mathematics as “the study of numbers to do things”—and numbers played a very important role for him. Jack’s first test for mathematics in an activity was the use of numbers. Activities where he could think of a way numbers were involved, used mathematics. The numbers themselves were important, not performing operations on them: “Playing sudoku, that’s like basic math.” Driving a car involved mathematics if one was figuring out mileage or speed. His connections with mathematics were all very concrete numerical examples. However, because he could think of no way numbers could be involved with working on an electrical circuit or a lawyer arguing a case before a jury, he did not believe those used mathematics.

Jack also conceded that there is the geometrical aspect of mathematics. He called this category the symmetrical/geometrical group. He used symmetry to mean excellence in proportion of the parts to the whole, or the parts being arranged in their proper and pleasant arrangement. This included quilting, laying tiles, and arranging shapes. He is much more liberal with this aspect of mathematics; he even included cooking with a recipe—implying that tasting good and looking nice were comparable.

Chapter 5: Comparing and Contrasting the Mathematical Identities

By looking across the mathematical identities of the five students I can begin to answer my research question: how are they different? One may expect to find some variance in the amount of confidence or enjoyment, for example. But there are many instances where the essence of the beliefs seems to be different. Following is a cross-case analysis pointing out those differences in the beliefs about mathematics which seem to be more than just a difference in magnitude. The differences are organized here according to the achievement motivation questions discussed in chapter 2.

Can I do the math?

Recall that when students answer the question "Can I do the math?" they are considering beliefs about their own competency. There are two emotions that I have looked at to describe how competent a person feels: confidence and anxiety. The amount of confidence a person has is shown in how strongly one believes that one can do the math, while anxiety is the amount of fear one experiences when doing or thinking about doing math.

Confidence

When I interviewed the students, I talked to them about their confidence and about how they perceived their own skill level. I quickly found that there are not only varying levels of confidence, but different kinds of confidence, and different ways of determining someone else's level of confidence. All of the students in this study had enough self-efficacy to take an honors section of calculus and believe that they could succeed. If I were comparing them to students in remedial math, one would expect huge differences. However, the fact that they appear similar makes it more interesting that I found great differences in their confidences. When looking at a person's confidence, I have examined not only what the student told me herself or himself, but also what the student told me about others, what others said about the student, and whether the student acted as if he or she had confidence.

Caleb had the highest amount of self-efficacy to do mathematics in this group. He not only believed he has more confidence than the other students in the group, based on the Confidence Continuum, but the others, including the teachers, also placed him with more confidence than the others. Furthermore, Caleb acted as if he has self-efficacy for mathematics. When he and another member of the group came up with different answers, Caleb's initial response was to assume his own work was correct. He said, "generally I was pretty confident of my own answers, so generally I'd look at theirs first and then if theirs made sense then I'd look back at mine, see if mine made sense." This showed a great deal of confidence in his ability to do the mathematics. Caleb believed he could do the mathematics and that his answers were usually correct.

Hannah's confidence was much different. On the Confidence Continuum, Hannah placed herself at the very bottom of the scale, and the lowest in the group (see figure 2). She also behaved as though she has very little confidence. Recall that when it appeared that she was the only one understanding a concept, Hannah's response was to take that as evidence that she must not really understand it. She added, "I would never think that I knew something over anybody else, like that's just not how it is." However, on the Mathematics Skills Continuum Hannah placed herself on the top half of the scale and above both Debbie and Bryce (see figure 3). This clearly said she believed she was pretty good at mathematics. This appeared to be a contradiction. Why would someone who believed she is pretty good at mathematics talk and act as if she had very little ability?

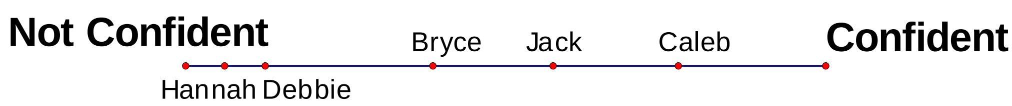

Figure 2: Hannah's Confidence Continuum**.**

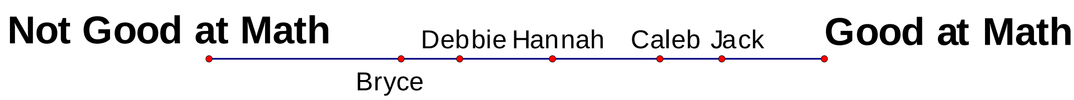

Figure 3. Hannah's Math Skills Continuum.

Hannah seemed to equate confidence with certainty. When I asked her to rate, on a scale from 1 to 5, how important confidence was for someone who was good at math, Hannah said she did not think it was important at all—she gave it a 1. She argued that the others were not really confident because they said things like “I think this is right, what do you think?” or making comments about not knowing how to begin a task. It seemed that Hannah felt someone who was confident would not need to ask for another’s advice. What some would see as healthy skepticism, Hannah saw as evidence of low confidence. Because Hannah saw the lack of certainty in herself even more than in others, she lacked confidence that she could do the math.

Bryce had confidence that is different from both Caleb and Hannah. Bryce placed himself lower than everyone else on the Mathematics Skills Continuum. He said that his poor algebra skills “crippled” him in the class. He also rated himself the lowest of the five students on the Confidence Continuum, saying that he made too many mistakes and got the wrong answer. However, Bryce acted as if he had confidence, and others saw confidence in him. He volunteered to make presentations to the class and even convinced his fellow group members to follow him when he was wrong. It appeared that Bryce was lacking in self-efficacy to do the mathematics, but had a great deal of confidence in himself as a person. Bryce's self-concept was strong enough that failures with mathematics problems did not deter him. He volunteered to make presentations to the class because he felt that "it makes more sense to you if you try and teach it."

Bryce's attitude toward certainty was quite different from Hannah's. Hannah kept the notion of uncertainty foremost in her mind, and it affected her behavior. She did not assert herself, even if she thought the others may be wrong. She felt that she was the little child being helped by the older brothers and sisters. Bryce, on the other hand, did not consider that his method may be wrong, until it was proven such. Although he readily admitted he was often wrong, his uncertainty had no affect on his behavior. Bryce behaved as if he were certain.

Anxiety

Another aspect which affected how competent a student felt was anxiety, which may be described as some amount of nervousness or fear. I expected to see a correlation between the confidence a person had and the amount of anxiety he felt. I saw that correlation in some people, but at other times I found the confidence and anxiety levels at odds. I found not only different levels of anxiety, but different kinds of anxiety. The anxieties may be pervasive or localized; the anxieties may be displayed in the student's behavior or remain internalized so that others do not see it. Many times anxieties can be related to previous experiences.

There were some similarities among the five students. On the Anxiety Continuum, no one placed him or herself with less anxiety than the other's placed them, and usually they placed themselves with more anxiety than the others saw in them. For example, Caleb showed very little anxiety, and everyone agreed that he and Jack had the least amount of anxiety. The others placed Caleb and Jack at the Peace end of the scale, sometimes placing Jack with less anxiety, and sometimes placing Caleb with less anxiety. Caleb also placed himself with the second lowest anxiety next to Jack; however, as shown in figure 4, Caleb separated himself from Jack by nearly half the continuum. He showed himself with much more anxiety than anyone else saw in him. Caleb said that he felt anxiety when he was faced with something new that he had never seen before. He said that when they learned about polar coordinates, in the calculus class he was in at the time of the interview, he found "it was really really new and I couldn't understand it. Like, I was very very anxious about that, but as we worked more into it, I got more confident that I would be able to learn it, I guess." The anxiety seemed to be very localized on new material. Caleb did not feel anxiety with tests, in fact, he responded that "no, I like the tests a lot," and he felt comfortable presenting in front of the class. Furthermore, Caleb seemed to keep his anxieties internalized. Other than an occasional comment about not knowing how to begin a problem (which is a comment made by all the students, and is expected by the design of the problems), Caleb did not do or say anything that would lead the others to believe that he was anxious. Figure 5 is an aggregate continuum showing where all of the students placed Caleb. All of the students and both of the teachers placed him in the top one-third of the continuum. They did not see that Caleb felt much anxiety about any of the math.

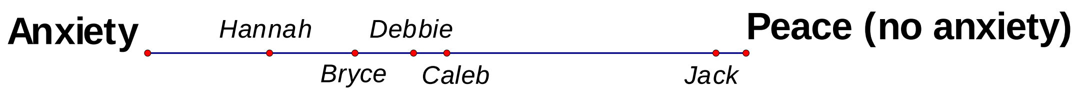

Figure 4. Caleb's Anxiety Continuum.

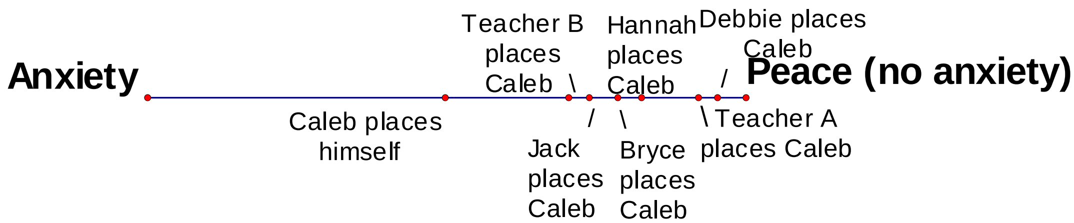

Figure 5. Caleb's Aggregate Anxiety Continuum.

It may be that Caleb's anxiety with new material stems from the fact that he has not always been successful at mathematics. During middle school and the first two years of high school Caleb struggled with mathematics. He attributed these problems to personal and family problems going on at the time and not to the difficulty of the mathematics. Although he has been successful and able to do very difficult problems, it may be that he worried about being back in a position where he did not understand.

Jack also had a very focused anxiety, although it was different than Caleb's anxiety. On the Anxiety Continuum, similar to Caleb, Jack places himself right in the middle of the continuum, and in the middle of the group, with Caleb and Hannah having less anxiety, and Bryce and Debbie having more anxiety (see figure 6). Everyone else places him with either the least anxiety, or second only to Caleb. However the cause of his anxiety is different. Jack explains what causes him to feel anxious:

> I get nervous or stuff like that before a test, not that I'm afraid of what's on the test but I'm afraid that I'll do bad on the test—there's a difference, I promise there's a difference—and not do bad and get a bad grade, but just do bad, and just not be good at mathematics.

Part of Jack's self-concept was that he was good at math; the anxiety came when he faced a situation where he might have found out that he had to change his self-concept. Bracher (2006) argues that people will go to great lengths to maintain their identities. Jack was not afraid of hard problems, or new problems, or presenting a solution in front of the class. His anxiety was focused on maintaining his self-concept of being good at math. We might wonder to what lengths Jack would go to protect his identity. Would he avoid situations where he may fail in order to maintain his sense of being "good at math?"

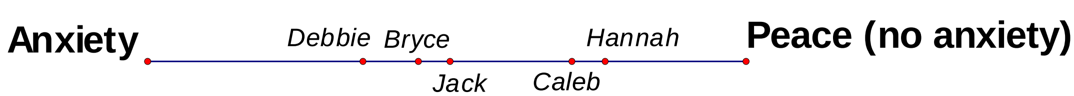

Figure 6. Jack's Anxiety Continuum.

Debbie's anxiety stemmed from the experience where she forgot part of a song during a recital. Even after many years, she still remembered how she felt that day and was convinced that she was not good at memorizing. Debbie reported feeling anxious and would freeze when she had to remember formulas or procedures, such as on tests. She said she would "try and remember everything and end up not remembering anything." Debbie attributed the problems she has had with mathematics classes to this inability to memorize. Unlike Caleb and Jack, who did not share their anxieties in class with their peers, Debbie was pretty open about hers.

Everyone, including herself, placed Debbie in the top half of the Anxiety Continuum, and nearly everyone believed her to be the one with either the highest anxiety or the second highest anxiety.

Hannah seemed to have a great deal of anxiety; however, it was not focused on any certain aspect of mathematics. One of the teachers in the calculus class described Hannah when she would come to office hours, saying, "the way she talked to me was, 'I don't understand this and I don't know what I'm going to do' kind of thing." Hannah placed herself at the very end of the 'anxiety' side of the Anxiety Continuum. She explained this:

> Interviewer: where would you be on the anxiety scale?
>
> Hannah: I think it's over here, I think that's just cause I feel that way about everything
>
> Interviewer: so you're anxious all the time…about other things besides math?
>
> Hannah: yeah, it's not just mathematics.

Hannah's anxiety was pervasive in all aspects of her life. This was understandable if we consider her background. Hannah came from a very small farming community. She graduated from High School in a class of three. When she took this calculus class she was a freshman at a large university where there were several times more people than in her entire home town. Anxiety was to be expected.

Hannah's anxiety was similar to Bryce's confidence. In both cases it was a global aspect of their respective identities which affected the local mathematical identity. In Bryce's case, a strong confidence in himself as a person gave him more confidence to do mathematics than he might otherwise have, or that might be warranted by his skill level. In Hannah's case, the anxiety she felt in all aspects of her life, negatively affected her ability to do mathematics—giving her more anxiety and less confidence than would be expected from her skill level.

Do I want to do the math?

As mentioned in chapter 2, when students answer the question "Do I want to do the math?" they are considering if it is worth the effort. I have looked at how much they enjoy studying mathematics, whether or not it is worth studying—or the usefulness of mathematics—and what motivates them to study the mathematics.

Enjoyment

The five students in this study worked together in the calculus class for most of a semester, and they seemed to have a lot of fun. They were often laughing and joking. I found that not only was there a great deal of variety in the amount of enjoyment, the students enjoy different aspects of the mathematics, and display their enjoyment in different ways.

It is no surprise that Jack enjoyed math, since being good at mathematics and liking mathematics were so important to his overall identity. He mentioned at least a dozen times during the interviews that he liked math. He placed himself at the top on the Enjoyment Continuum, and the one in the group that enjoyed it the most. Everyone else seemed to agree. The other students and the teachers all placed him as one of the top two students on the enjoyment scale. Jack has always enjoyed mathematics, even his earliest memories of grade school mathematics were happy, but interestingly, how much Jack enjoyed a class was associated with social aspects.

Jack's favorite mathematics class in high school was a class where the teacher organized extracurricular mathematics competitions, and Jack got to be a captain of one of the teams. His favorite college mathematics class was the calculus class in this study, which was a non-traditional, discussion based class where the students worked in groups. His least favorite class was his sophomore year in high school, when his family had just moved to a new city. He was one of three sophomores in a class of juniors and seniors. The social setting—not knowing others, being younger, feeling left out—made the class not as enjoyable as other mathematics classes.

It seemed that the way Jack showed his enjoyment of mathematics was by singing and making jokes about the mathematics. The group often made up names for equations. Rather than calling an equation f(x) or f'(x), the group would name them Hector or Alfred. Jack said,

> Personally I like being a little more loose with the terminology in math, and not so, I don't know, so…correct, like mathematical correctness of the problems. In fact when other guys would get up there from other tables and explain stuff, especially the ones that have really learned a lot about math, when they start explaining things with the correct terminology, I'd get kind of confused, and almost kind of bored. It makes it a little more fun.

It is interesting that Jack insisted that he liked math, but did not enjoy or feel comfortable using or hearing discourse from the mathematical community.

Bryce also liked to joke around during math; in fact, Bryce was most often the instigator. After watching a video clip of Bryce explaining a concept to the group using rather non-traditional language, I asked Hannah what that kind of talk said about Bryce. She said, “I think that he enjoys it—either that or he hates it so much that he has to find some way to make it bearable….” Although she said it facetiously (Hannah places Bryce as the person who enjoys mathematics the most), I think Hannah may have had more insight than she realized. The other students felt this same dichotomy. Two of the students placed him very high on the enjoyment continuum and two of them placed him in the ‘pain’ half of the continuum.

One of the teachers felt there was a difference between the joking Jack did and what Bryce did. She said that Jack joked around about the mathematics but was still engaged in the mathematics. Bryce was more likely to be off-task and pulling the group away from the mathematics. She felt that Bryce saw “his role in the group to be the comic relief.” This teacher believed that had Bryce really enjoyed the math, “he would have been more engaged in the math.” Bryce admitted that when he did not know what a question was asking, or what to do next, he would make jokes.

Bryce did not proclaim a great love for math, but he said he did not hate it either. He placed himself on the enjoyment end of the Enjoyment Continuum, especially when he was in a non-traditional setting such as the calculus class in this study. He said he enjoyed learning about concepts and how they fit together; he did not enjoy doing problems, particularly with “thick chewy algebra”. Bryce did not have a positive experience with mathematics in junior high and high school. Although he placed part of the blame on a teacher he did not get a long with, and another one that was gone every Friday, Bryce realized that the biggest problem was his own lack of effort. That brings us back to a reason Bryce did not enjoy mathematics more: he did not have the skills necessary to be successful. He admitted he enjoyed learning about the concepts, but felt like the algebra “kicks him in the face.”

Hannah came into this class liking math. Contrary to the rest of the people in her group (and nearly everyone I have ever met) Hannah’s favorite part of mathematics was fractions. She also liked solving simultaneous equations. It seemed that what Hannah enjoyed were the procedural aspects of mathematics. Hannah attended a very small school. Her last 2 years of high school she was the only one in her mathematics classes. She learned mathematics procedurally: either teaching herself from examples in the book, or having a teacher share time between more than one class. Once she came to the university, she began to question whether she really enjoyed math. One thing that interfered with Hannah’s enjoyment of mathematics was her anxiety level. I suspected there were several things in her life that she used to enjoy that were no longer enjoyable because of the pervasive anxiety.

One thing Hannah did not enjoy was the joking around with the mathematics. The next semester she was in the second semester class with the same format, and she was part of a different group. Once she saw how a different group worked, she realized “not having all that extra fluff helped me learn the mathematics better.” Hannah described times when the group in this study was working on a problem and the others would start joking around and she would “zone out” and “try not to listen to that part.” Yet still she maintained that the joking around showed that the others enjoyed the math, and she placed Bryce as the person who enjoyed mathematics more than the rest of the group. It seemed that Hannah felt deficient because she was not able to enjoy mathematics in that way. By the end of that semester Hannah said she did not enjoy mathematics, and she began to question whether she ever really enjoyed it. It was not until the next semester, after working with a different group that she began to enjoy mathematics again. When Hannah placed herself on the Enjoyment Continuum she asked if I wanted her to place herself where she would have been during the class in the study, or where she was at the time of the interview (which was after she had been working with the new group for some time). I had her mark both. Figure 7 shows what a sizable difference there was. It seems that Hannah’s beliefs about her enjoyment of mathematics were not central.

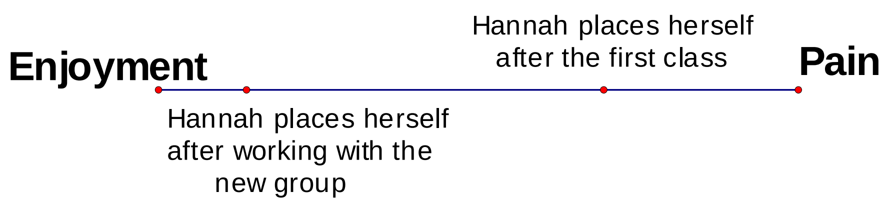

Figure 7. Hannah's placement of herself on the Enjoyment Continuum.

Usefulness of Mathematics

All five of these students came from families where both parents attended at least some university courses. They all believed that an education is important, and they all seemed to believe that studying mathematics was an important part of a good education. However, they differed in how they believe mathematics will be personally useful, and how useful mathematics is in and of itself.

Caleb believed that we use mathematics in nearly everything, although it is often used subconsciously. In the Mathematics Categories Task, Caleb included among his categories a ‘no math’ category, but placed only reading a book and hanging a picture on the wall in it. Furthermore, Caleb believed that the mathematics he learns will benefit him personally in his career. He was studying actuarial science which required a fair amount of math. He said, “I have to take 214, and I’ll probably take 343—it’s not really necessary, but I’ve been told that it will probably help….” Caleb planned to take more than the required mathematics classes, because he believed that they would be useful to him personally.

Debbie was studying Geology, which also required math. However, Debbie was not planning on taking any extra mathematics classes; she was grateful to be finished with the ones that were required. She did realize that mathematics is very ubiquitous. She said,

> Like with my work I realized, wow, how cool is that we have an equation that measures earthquakes, and the small earthquakes leading up to an earthquake. That’s pretty incredible to me. I think that mathematics is amazing. My mind couldn’t have come up with that. It’s just everything in general. Mathematics was used to create this building, mathematics was used to create cars. Yeah, I think it’s pretty incredible.

However, while Debbie wanted to understand the basics of the mathematics being used, she was happy to let the computer do the math.

It seemed that Debbie was unaccustomed to seeing mathematics in activities. When she did the Mathematical Categories Task, her categories were creative, organizational, practical/applicational and everyday things. But when I asked her about how they related to mathematics, it became clear that she was categorizing them according to creativity, not mathematics. As we talked about the specific activities, Debbie could usually come up with some relation to mathematics, but it was evident that other ways of looking at activities come more naturally. For example, she placed 'solving a logic puzzle' in the creative group, but could not come up with the kind of mathematics used. She said it "seemed creative to her." She also placed 'playing a video game' in the creative group, but when I asked about the math, she said she did not know, but it "wasn't like an everyday occurrence again to me." She seemed to be placing them according to the kind of activity she saw them as, and then thought of the mathematics they involved after the fact. I think that mathematics is much less personal for Debbie than it is for Caleb.

Jack's view of the usefulness of mathematics was very different from the others. He had a very negative view of mathematics, especially considering how important it was to him to be good at math. In contrast to Debbie, Jack claimed he "can't think of anything now that would strike the average person as amazing or extraordinary, something that came from the mathematics field." He added, "to me mathematics is more of a sub-category of most things. It's not really a—I don't know—it's not really a primary thing…." In contrast to both Caleb and Debbie, who thought that mathematics would be an integral part of their chosen careers, Jack was convinced that teaching is the only career available to a mathematician. When pressed for what else a mathematician would do, he said it would be "crunching numbers and doing the calculations that other people don't want to do or don't know how to do."

Motivation

The students were motivated to study mathematics by different things. For Caleb and Jack, enjoyment seemed on the surface to be the main motivation.

However, the deeper issues were different. For Caleb, he not only enjoyed it, he also wanted to learn mathematics because he felt it would be useful to him in his career. For Jack, he was motivated to study mathematics because it was important to his overall identity that he was good at math. Debbie was not motivated so much by her enjoyment of mathematics, but her love for Geology. The required mathematics courses were stops that had to be overcome along the path to geology. She was motivated to gain sufficient mathematical understanding to reach the geology goal.

Hannah's motivation came from what we usually consider a negative source: her anxiety. She attended the teacher's office hours more often than the others, and was more diligent about completing all the homework. Some of the others interpreted this as enjoyment or dedication. Jack said,

> You know, there are people who do their homework just to get As, but she did her homework and didn't complain about it, you know, she tried to understand it. Trying to understand something is more typical of someone who enjoys the subject or wants to learn more about the subject, rather than somebody who just tries to go through the motions and get through the subject.

Hannah explained the extra diligence as fear because she felt like she did not understand the concepts. She felt she had to work harder than everyone else.

Bryce's motivation was a little harder to pin down. He did not particularly enjoy the math. He had not chosen a major, so it was not something he needed to complete to reach some other goal. He did not seem to have any anxiety except in specific algebra or fraction problems—and then the anxiety seemed to be a deterrent not a motivator as in Hannah's case. Debbie suggested that grades were something that motivated Bryce. She said, "he's taking 112 again to just—he got a C in the class, I would be fine with a C. I would not enjoy it enough to take it again." Bryce also claimed the reason he retook calculus 112 was because he wanted to "try and improve that grade." However, I think there are some deeper reasons that motivated Bryce. Even though it was not a requirement to take even the first calculus class, Bryce considered taking the subsequent 113 class with the rest of his group.

> I definitely thought about taking 113, just to kind of like get a broader, like expansion of what's going on in calculus. Because I know there are a lot more paths, because we just barely, you know, started getting to integrals, and what that's about, I know there are a lot more.

But Bryce knew he would "need to go and try to hone my skills a little better in 112" before attempting 113. I think that even though Bryce used jokes to cover up when he did not understand something, and it made it seem that he did not care about the math, his real motivation was to understand the mathematics.

What does it take to be successful at math?

When people answer the question "What will it take to be successful at math?" they are considering their beliefs about how people learn mathematics, what it means to be good at math, and how they define 'mathematics.'

How people learn mathematics.

These five students had all completed a non-traditional, discussion-based calculus class. It was clear from the interviews that the principles practiced in this class had an impact on how these students thought about learning mathematics. All of the students mentioned that they learned mathematics much better working in groups and discussing the math. Bryce encapsulated all of their feelings when he said, “if I understand something and I can explain it I understand it even better.” And everyone seemed to feel the same as Jack when he said he liked to have someone to “bounce ideas off.” However, the students had varied ideas about the importance of and the role of the groups.

Both Bryce and Debbie mentioned the importance of independence in learning mathematics. They felt there was a tendency to become too dependent on your group—especially when they had two dominant students like Caleb and Jack. Then when it was time for a test and the group was not there, they had trouble. Debbie brought up the point, however, that one could become too independent as well. She had had that problem in previous traditional classes. She had felt that she should be able to do the mathematics without asking for help. She was too independent. The group helped her overcome that problem, but tended to lead her to be too dependent. She felt there should be a balance when learning math.

Jack felt that working in a group helped him a lot and, most importantly, made it fun; however, he did not believe that it is necessary for learning math. When I asked him to rate how important being social was to being good at math, he said, ”point 5”. He felt “you can sit in the corner and learn how to do mathematics all by yourself.” Jack seemed to think that it was a fault in himself that he needed to be able to talk about the math. He said that “being social helps me to do better at math, but there are people out there who never talk to a soul, just listen to the teacher, and still be good at math." When I asked him if he thought they would be as good, he replied, "probably better."

The class also raised the issue of whether "discovery learning" is the best way to learn, or whether the more traditional method of "telling" is best. This class was the first experience any of the students had had with discovery learning, and they had differing reactions to it.

Caleb felt that discovery learning was the ideal way to learn. When I asked him if he could think of one thing that had affected his current attitude towards mathematics, he said, "probably the guided opportunity to discover methods on your own" like in the calculus class. He felt that most classes in other fields should be taught that way whenever possible. He felt that it would be a good way for anyone to learn. Caleb was planning on being a teaching assistant in a statistics class the semester after the interviews. He was planning to use discovery learning as much as possible when teaching the labs.

Hannah was not quite as enthusiastic. She felt that "everybody learns differently" and while discovery learning is great for some people, others learn perfectly well by the teacher telling them how to do the math. In fact, she believed that for people who "get it" by telling, telling would be the easiest method—for both the teacher and the student. She felt that not everyone needed to understand the math; they just needed to be able to do it.

For Hannah and Jack, learning mathematics procedurally seemed to be a relatively central belief. They were willing to admit that "discovery" learning could be beneficial, but not to the exclusion of a method that had been working extremely well for them for over 12 years. Both of these students had had great success in traditional classrooms. Recall that Caleb, on the other hand, had not always had success in the traditional classrooms. It is likely that he came into the class with beliefs similar to Jack and Hannah, because he asserted that taking this class did more to shape the way he thinks about math than any other event. However, Caleb's beliefs about leaning mathematics before the calculus class, whatever they were, were not strong enough to resist changing.

What it means to be good at mathematics

Hannah, as just mentioned, seemed to believe that the bottom line was the ability to do the math—"you have to find the right answer." She believed that "some people have amazing memories and memorize everything and do it that way." Other people "have to understand how it works." Hannah seemed to believe that some people were naturally good at mathematics and some were not. She said, "If you enjoy it and you're not good at it, then you need to be really diligent, but if you're just good at it, then you don't really need to be diligent." Hannah seemed to agree with Jack that while most people could learn to be pretty good at math, someone who was really good did not need to talk about the math, or try to understand the math, they were just able to do it.

Hannah seemed to think that discovery learning is a method that is for those who cannot understand the mathematics by traditional methods. She said of her experience learning by discovery that "this way I know different ways to do it. I could try, like if somebody isn't getting it telling them, then I can try it this way." It is interesting that Caleb feels so differently about this way of learning. The other students felt that Caleb was very intelligent. Bryce called him “freakin’ brilliant,” Debbie said he was “super smart,” and Hannah said he was the “dad” of the group. It seemed that according to Jack and Hannah, Caleb should be able to learn without discussion and without discovery learning; yet it was Caleb that believed that discovery learning was the best way for anyone to learn.

Contrary to Hannah, Caleb did not believe that the answer is the bottom line. He differentiated several times during the interviews between someone who was good at mathematics and someone who was good at taking mathematics tests. He talked about being “good at math in the sense that you can apply it and take the math to different situations, to new situations….” He added that being good at mathematics would mean that you are able to “get your own ideas.”

Summary

I began this study to see how students with similar mathematical identities differed in their beliefs. Bryce, Caleb, Debbie, Hannah and Jack have similar mathematical identities, in that they are choosing to engage in mathematics to some degree, either by choosing a mathematics-related career, or by choosing to study mathematics even though it is not required for a major. It is clear from this study that their mathematical identities are also quite different. A discussion of some of those differences is in Chapter 6.

Chapter 6: Conclusions and Implications

Unhealthy mathematical identities lead students to choose not to pursue professions in mathematics and mathematics-related fields. Studying mathematical identity may be a key to solving mathematics public image problems. However it is not a simple solution. Mathematical identities are much more complicated than being “good at math” or “liking math.” It is important for us to know more about the beliefs that make up a person’s mathematical identity to have hope of changing them.

Identity is not well-defined and is often talked about in seemingly contradictory terms. By using Cote and Levine’s (2002) model of identity we can see that there are different aspects of identity which include social settings, behavior, and beliefs. Most of the research in mathematics education has focused on behavior: either behavior that leads to certain beliefs, or behavior that is engaged in because of those beliefs. I have looked at the beliefs that are part of the mathematical identity.

I looked at five students who could all be considered “good at math” to determine the differences in their motivational beliefs: confidence, anxiety, enjoyment, ability, how mathematics is learned, what it means to be good at mathematics, usefulness of mathematics, and the definition of mathematics.

Conclusions

In comparing the beliefs of these five students I found, in answer to my research question, that their mathematical identities are intrinsically different. These identities could not be ranked on a one-dimensional scale as to which was better or healthier. Each person has a unique system of beliefs which makes sense to them. For example, the students had strikingly different ideas about what mathematics was and how useful it is, varying levels of confidence, different causes anxiety, particular facets that they enjoy and different ways of showing enjoyment. But not only is there a unique arrangement of beliefs for each person, the beliefs themselves are different. I will highlight the differences that were most salient to me: specificity, conspicuousness, changeability, and conformance.

Specificity

Beliefs varied in specificity from being very specific, or local to one aspect of mathematics, to being very global, or affect all aspects of a person's life. There is interplay between global beliefs and local beliefs specific to mathematics. Global beliefs, such as Bryce's confidence and Hannah's anxiety, affected the way the students behaved with respect to mathematics, and consequently had an affect on the local mathematical beliefs. In Bryce's case, he was more willing to try mathematical problems than his mathematical self-efficacy may have allowed. Research has been done comparing mathematics self-efficacy with academic self-concept (Lent, Brown, & Gore, 1997) which suggests that either one could be used to predict success; however, when dealing with performance deficits (like Bryce's algebra problems), the more global the feelings of competence (or incompetence) the less helpful in targeting problems. A strong positive self-confidence can mask the specific aspects that need remediation, such as algebra skills.

Hannah's anxiety kept her from participating in the way her skill level warranted. A negative global belief, such as high anxiety or low self-esteem, could cause students to engage in self-handicapping behaviors (Valentine, DuBois, &

Cooper, 2004) such as procrastination or, as in Hannah's case, unwillingness to share ideas and participate in the group.

Jack, on the other hand, had a local belief, being good at mathematics, that was an important part of his global identity. Protecting his global identity shaped his actions in mathematics. Bracher (2006) said that "learning, like all other behavior, is motivated by the effort to maintain and enhance identity" and because of that, "for learning to occur, either the process or the anticipated result of learning must provide support for identity in some way (p. 5)." It appeared that thus far, this has worked for good in Jack, he has studied hard, completed homework and taken tests in order to maintain that identity. However, one may wonder if there will be times when he will turn to avoidance of difficult subjects in order to maintain his identity of being good at mathematics.

Conspicuousness

Some beliefs were apparent, or clearly observable. For instance, the other students and the teachers were aware of Debbie's anxiety about memorizing and Caleb's confidence. However, some of the beliefs were only observable when examined closely. No one except Caleb seemed to be aware of Caleb's anxiety. No one except Bryce seemed to be aware of Bryce's motivation to study mathematics. This demonstrates how important it is to look closely at a person's beliefs. Although a belief seemed to be hidden, it was nonetheless important in understanding the individual.

This seems to be something different than Schoenfeld's (1998) differentiation between professed and attributed beliefs. These beliefs could be professed—if asked the right questions—and they could be attributed by an astute researcher—but again, only if presented with the right situations. In fact, it seemed as if in the classroom these beliefs were deliberately masked by the students. Whether consciously or unconsciously, Caleb chose not to talk about his anxiety and Bryce joked around and covered his desire to understand the mathematics. People choose to present themselves in certain ways. This is such a prominent dimension of human behavior that an entire subfield of social psychology is devoted to it called impression management or self-presentation (Bracher, 2006). It is likely that all students have some beliefs that are not apparent in the classroom—with varying degrees of importance to learning mathematics.

Changeability

Beliefs differ in their importance to a person. Central beliefs are vital to the belief system and are not easily changed, whereas peripheral beliefs are not as important and are more easily changed (Leatham, 2006). Debbie believed she was "good at math" and maintained that belief in spite of two years of a great deal of evidence to the contrary. That belief was central for Debbie. In contrast, Hannah started college believing she was good at math and received a B+ in her first Calculus class, but after a couple of weeks of a class on proofs she decided maybe she "wasn't cut out for math after all."

To understanding why Debbie's belief in her ability to do mathematics remained strong, we may gain insight by looking at a study done at Yale by Cohen (as described in Bem, 1970). After a riot on campus, where the police intervened very aggressively and, most of the student body believed, excessively brutally, Cohen and his assistants randomly chose students and asked them to write a strong forceful essay in favor of the police. They paid the students either $10, $5, $1, or 50 cents. After writing the essay, the student indicated his or her actual opinion on the incident. Interestingly, it was not the students who were paid the most that changed their minds. The less the individual was paid the more he persuaded him or herself. Writing the essay was not in line with the person's beliefs, and caused a dissonance. The person looks for a way to solve the dilemma. For the person who was paid very little, it was often resolved by changing their beliefs about the incident. For the person who was paid more money, the resolution was the money—they could 'explain' writing an essay about something that was not true by saying they did it for the money. The money was an escape hatch for a conflicting belief and action. Debbie also had a conflict between her belief that she was good at math and her performance in mathematics classes once she was in college. Her belief that, because of an earlier incident at a piano recital, she could not memorize well was her escape hatch. She did not have to change her belief in being good at math. She could make sense of her performance in a different way.

This is reminiscent of quasi-logical relationships (Leatham, 2006), or something that seems logical to the person holding the belief, but may not be logical for someone else. For example, in Debbie's case, she seemed to have linked the ideas that even though she is not good at memorizing, and math often requires memorization, as long as she does not have to memorize, she is good at math. This may not make sense to others, but this seems to have worked positively for Debbie. She has been able to maintain her identity of being good at mathematics and continue her studies toward her goal of being a geologist. Another student, whose quasi-logical relationship did not have a positive conclusion, may not benefit from the false belief as Debbie did.

At the same time that Debbie was keeping her identity strong and stable, Hannah was vacillating with her beliefs. She brought up cases where she had changed beliefs about enjoyment (enjoyed it in High School, did not enjoy it the first semester, enjoyed it again the second semester) and her ability (thought she was good at math, then decided she was not cut out for it), and with seemingly little perturbation. Rokeach (1968) had some insight into this. He said, “violation of any primitive beliefs supported by unanimous consensus may lead to serious disruption of beliefs about self-constancy or self-identity, and from this disruption other disturbances should follow, for example, disturbances in one’s feelings of competence and effectance (p. 7).” This would cause one to question the validity of many beliefs and cause major cognitive reorganization. Hannah showed signs of great upheaval among her self-beliefs. I did not gather data describing the changes in beliefs that Hannah experienced when she moved from a small farm-town, with a high school class of 3, to a large university with student body over 25,000, but it seems possible it was as that described by Rokeach. Hannah seems unique. There are not many people who could claim such a wide disparity between home and school; however, the disruption Rokeach mentioned could occur in other situations. In the schools there are many students who face “violation of primitive beliefs” in the form of divorce, death of a loved one, or other tragedies.

Conformance

Some beliefs are developed as a result of conformity to the group. Hannah changed her definition of how one enjoys mathematics based on the rest of the group. She came into the class thinking she enjoyed mathematics, but later wondered if she had ever really enjoyed it. Her quasi-logical relationship may have gone something like this:

a) Apparently, when people enjoy mathematics they joke around.

b) I am not comfortable joking around with the mathematics.

Therefore, I must not enjoy mathematics.

Hannah based her belief about whether she enjoyed mathematics on a deficit model: she assumed that in being different she was wrong.

Conformity is not always a bad thing. We rely on conformity for consensus, shared activities and social norms (Friend, Rafferty, & Bramel, 1990). However, we rely on independent thinking to discuss and explore the mathematical problems. We expect students to challenge each other, and to defend their ideas.

Implications for Teachers

One aspect for teachers to keep in mind is that the mathematical identity that one sees with a cursory glance is not necessarily the way it is. The interplay between global and local beliefs can disguise aspects of the mathematical identity. Additionally, some beliefs are not readily apparent. In a traditional classroom, there will probably not be enough interaction between student and teacher to allow the teacher to see the beliefs as they really are.

Behavior can also disguise a person's mathematical identity. Because of Bryce's joking around, neither the teachers nor the other students realized that he thought seriously about mathematics. Recall that Bryce had a very broad, healthy definition of mathematics and he expressed a desire to continue on with calculus even though he had not chosen a major and did not believe it would be required for graduation.

There are also implications for those who teach teachers. All five students in the study appreciated "discovery" learning; however, three of them included some qualifications, suggesting that although the class format was helpful for them if a person was "really good" at mathematics they would not need to discuss the mathematics. They implied it was a lower form of learning—something to be used if traditional methods did not work.

This is troubling to the reform movement because both Hannah and Jack were planning to go into mathematics education and both expressed the intention to use "telling" as their main form of teaching, reserving "discovery" learning for those students who were not able to understand it with "telling". This is not a sentiment that has emerged in the literature. There have been studies showing that students liked the standards-based classes (Angier & Povey, 1999; Boaler & Greeno, 2000), and studies finding that many parents do not like them (Gellert, 2005; Remillard & Jackson, 2006). There have been teachers who believed that the standards-based teaching was only for the brightest students (Prawat & Jennings, 1997). But it is a strange idea that a student would find it the best way for him or herself, and still reject it as the best way for others to learn. This is something to look carefully at in our pre-service teachers. (Note: neither Hannah nor Jack had taken the courses required for mathematics education. Possibly learning more about teaching would help them understand the benefits of reform teaching.)

Implications for Research

This study has implications for research in mathematics education. There are ideas in methodology as well as theory that may be useful in future research.

Methodology.

Collecting information about beliefs can be challenging. There are many beliefs that a person does not articulate in the normal course of events, and when asked to articulate them may have trouble doing so. I found the continua very useful in helping the participants articulate their beliefs. At other times during the interviews, I used numerical scales to show agreement, which one could argue is similar to a continuum; however, the students often seemed uncomfortable choosing a number. It seemed too definite. I often had students wanting to choose something other than the numbers given, like "3.5" instead of either 3 or 4. Whereas, that is not an issue with a continuum. It is much less threatening to point to a spot on the continuum. Having placed themselves on the continua made it easier for them to talk about why in a more focused way. They had some kind of reference point to begin talking.

Continua are often used in research, but most commonly by the researcher as a way of synthesizing the data gathered. Many of the beliefs lend themselves very nicely to the format of a continuum, with opposite extremes at either end. Just as it helps the researcher see the big picture, it can help the participant see where he or she fits within the big picture. Because I was collecting data on what each participant thought about the other members of the group, the continua were convenient to help the participant place each member of the group in relation to each other. An added bonus came because I had them place the other students first. It is less personal and hence easier to pinpoint where someone else falls on the continua. In almost every case, after the student placed the others without any trouble, when I asked them to place themselves there was a disparaging comment like, “oh darn, I knew this was coming.” But after thinking through where they had been placed the others, it was easier to place themselves in relation to the others.

There are a few cases where continua have been used as a data collection technique, such as Osmo (2001) in the field of social work. However, in the field of mathematics education it appears to be mostly overlooked. As we continue to study mathematical identity, and the beliefs that are associated with it, it may be helpful to consider using continua to help make those beliefs more explicit.

Theory to consider for future research

Mathematical identity is a relatively new area of study within the field of mathematics education. We are just beginning to understand what is involved and how we may characterize a person’s mathematical identity. Going back to the model of identity by Cote and Levine (2002), most of the research has looked at the repercussions of the beliefs, or the behavior in the personal identity (see figure 1). The beliefs in the ego identity, which shape the behavior, are relatively overlooked. The beliefs are the vital link between the experience the teacher provides in the classroom and a person’s relationship with mathematics.

Much of the research on beliefs has focused on a small number of beliefs at a time, for example, beliefs about the nature of mathematics (Presmeg, 2002), self-efficacy (Bandura, 1993), anxiety (Perry, 2004), or how we learn mathematics (O'Brien, 2007). What this study suggests is that there is not a single, all-important belief that is necessary for a positive mathematical identity, it is the combination of beliefs that matters. Nor is it necessary for every belief to be positive. Consider Jack and his very negative beliefs about what mathematics is, and yet he was enthusiastically engaging in mathematics. It is a comfort to know that we do not have to 'fix' all of the beliefs.

Although we may tend to think of a mathematician as a certain sort of person, what this study tells us is that there is not a 'mathematician mold.' There are many configurations of beliefs that still produce a person who engages in, and affiliates with mathematics. What this means is that there is a wide variety of people who could decide to pursue careers in mathematics and sciences, and that is beneficial to the field. Someone like Debbie, who may not have the best skills and may not love mathematics, loves what mathematics can do.

Limitations and Future Research

This study is not without its limitations. I looked at five students in the same group in the same class. One thing I discovered as I met with Hannah was that the group she was working with made a big difference. She had a very different experience when she went on to the subsequent calculus class and worked with a different group of students. It was valuable to the study to have the input from members of a group who knew each other well, but it would be good to include more than one group. Something for future research may be to look at the kinds of beliefs a student holds which makes her struggle in one group and flourish in another.

One issue this study did not address is gender. Gender is, of course, part of person's identity, and shapes the kinds of interactions a person engages in, and hence influences the beliefs one holds. Gender differences have been studied at great length with regards to mathematics (e.g. Fennema & Sherman, 1977; Leedy, LaLonde, & Runk, 2003; Peterson & Fennema, 1985). Although gender was not the focus of this study, I observed differences between the males and females several times. One incident stands out in my mind. I had video of the group working on a problem and the Bryce, Caleb, and Jack were actively talking, laughing, and working on the problem, while Debbie and Hannah were watching. I showed this clip to all of the students and asked if this was typical, and why they thought it might be happening. They all thought it was not unusual, and Caleb attributed it to "male ego." There were also places in the data where beliefs seemed to divide down the gender lines. I chose not to follow up on that in this study. It would, however, be interesting in a larger study, with more participants, to see if we could characterize the differences in beliefs between male and female.

This study is just a small part of the struggle to comprehend mathematical identity. Personal beliefs make up just one part of mathematical identity, and the beliefs studied here are only a few of the possible beliefs a person may have. However, this is a good place to start. In this study I have tried to identify some beliefs a student may have. Future studies may try to identify combinations of beliefs that lead to healthy mathematical identities, or how we can modify the beliefs included here. A study that characterizes students' beliefs intermittently over a period of time may help us recognize the kinds of experiences that lead to healthy mathematical identities.

Mathematics is essential to our technological world. We need to have students who are strong in mathematics and who go on to choose professions in mathematics and science in order to compete in the world. Currently, many people do not consider mathematical fields as a viable option because of unhealthy mathematical identities. The beliefs they hold pertaining to mathematics limit their choices. We must continue to study mathematical identity to find ways to dispel inaccurate beliefs about mathematics.

Appendix A: Mathematical Beliefs Questionnaire

Mathematical Beliefs Questionnaire
Date________________________

Name____Honors Calculus Section___________________

Please circle the number which best describes your agreement with each statement.
1-Strongly disagree 2-Moderately disagree 3-Slightly disagree 4-Slightly agree 5-Moderately agree 6-Strongly agree

1. Solving a mathematics problem usually involves finding a rule or formula that applies.
   1 2 3 4 5 6
2. The field of math contains many of the finest and most elegant creations of the human mind.
   1 2 3 4 5 6
3. The main benefit from studying mathematics is developing the ability to follow directions.
   1 2 3 4 5 6
4. The laws and rules of mathematics severely limit the manner in which problems can be solved.
   1 2 3 4 5 6
5. Studying mathematics helps to develop the ability to think more creatively.
   1 2 3 4 5 6
6. The basic ingredient for success in mathematics is an inquiring nature.
   1 2 3 4 5 6
7. There are several different but appropriate ways to organize the basic ideas in mathematics.
   1 2 3 4 5 6
8. In mathematics there is usually just one proper way to do something.
   1 2 3 4 5 6
9. In mathematics, perhaps more than in other fields, one can find set routines and procedures.
   1 2 3 4 5 6
10. Math has so many applications because its models can be interpreted in so many ways.
    1 2 3 4 5 6
11. Mathematicians are hired mainly to make precise measurements and calculations for scientists.
    1 2 3 4 5 6
12. In mathematics, perhaps more than in other areas, one can display originality and ingenuity.
    1 2 3 4 5 6
13. There are several different but logically acceptable ways to define most terms in math.

1 2 3 4 5 6

14. Math is an organized body of knowledge which stresses the use of formulas to solve problems.

1 2 3 4 5 6

15. Trial-and-error and other seemingly haphazard methods are often necessary in mathematics.

1 2 3 4 5 6

16. Mathematics is a rigid discipline which functions strictly according to inescapable laws.

1 2 3 4 5 6

17. Many of the important functions of the mathematician are being taken over by the new computers.

1 2 3 4 5 6

18. Mathematics requires very much independent and original thinking.

1 2 3 4 5 6

19. There are often many different ways to solve a mathematics problem.

1 2 3 4 5 6

20. The language of math is so exact that there is no room for variety of expression.

1 2 3 4 5 6

21. The teacher should always work sample problems for students before making an assignment.

1 2 3 4 5 6

22. Teachers should make assignments on just that which has been thoroughly discussed in class.

1 2 3 4 5 6

23. Children should be encouraged to invent their own mathematical symbolism.

1 2 3 4 5 6

24. Each student should be encouraged to build on his own mathematical ideas, even if his attempts contain much trial and error.

1 2 3 4 5 6

25. Each student should feel free to use any method for solving a problem that suits him or her best.

1 2 3 4 5 6

26. Teachers should provide class time for students to experiment with their own mathematical ideas.

1 2 3 4 5 6

27. Discovery methods of teaching tend to frustrate many students who make too many errors before making any hoped for discovery.

1 2 3 4 5 6

28. Most exercises assigned to students should be applications of a particular rule or formula.

1 2 3 4 5 6

29. Teachers should spend most of each class period explaining how to work specific problems.

1 2 3 4 5 6

30. Teachers should frequently insist that pupils find individual methods for solving problems.

1 2 3 4 5 6

31. Discovery methods of teaching have limited value because students often get answers without knowing where they came from.

1 2 3 4 5 6

32. The teacher should provide models for problem solving and expect students to imitate them.

1 2 3 4 5 6

33. The average mathematics student, with a little guidance, should be able to discover the basic ideas of mathematics for herself or himself.

1 2 3 4 5 6

34. The teacher should consistently give assignments which require research and original thinking.

1 2 3 4 5 6

35. Teachers must get students to wonder and explore even beyond usual patterns of operation in math.

1 2 3 4 5 6

36. Teachers must frequently give students assignments which require creative or investigative work.

1 2 3 4 5 6

37. Students should be expected to use only those methods that their text or teacher uses.

1 2 3 4 5 6

38. Discovery-type lessons have very limited value when you consider the time they take up.

1 2 3 4 5 6

39. All students should be required to memorize the procedures that the text uses to solve problems.

1 2 3 4 5 6

40. Students of all abilities should learn better when taught by guided discovery methods.

1 2 3 4 5 6

Appendix B: Student Perspectives Questionnaire

(The questionnaire given at the end of the semester did not include the information about previous mathematics courses.)

Math 112H: Calculus I, Section 2
Student Introduction—Homework Assignment

Due Date: First Day of Class

Please help us to get to know you better by responding to the questions as completely as possible. One paragraph may be sufficient for some responses, while several paragraphs may be needed to provide the detail necessary to fully answer some questions. Expand the provided space as needed. Please submit your responses in the digital drop box in Blackboard.

Name:____

Please circle your current academic standing:
Freshman Sophomore Junior Senior

What is your declared (or expected) major?_____________________________

Exam (if you did not take an exam, please indicate)	**Score**
ACT	
SAT	
AP Calculus AB	
AP Calculus BC	
Math112H Pretest (required)	

Mathematics Background

High School Mathematics Courses	Grade Earned
University Mathematics Courses	Grade Earned

Perspectives on Mathematics Learning

1. List three necessary qualities of an excellent mathematics learner.

2. Which of the qualities you listed above, do you feel is your strongest? Please explain.

3. Which of the qualities you listed above, do you feel is your weakest? Please explain.

4. What does it mean to be a successful mathematics learner?

5. Describe an optimum classroom environment for learning mathematics. Why are these conditions optimum? What would be the practices within this environment?

Perspectives on Mathematics

6. What is mathematics?

7. What are the purposes of mathematics?

8. What do you like most about mathematics? Please explain.

9. What mathematics have you most enjoyed learning? Please be specific and explain why you find these particular topics engaging.

10. What do you find least appealing about mathematics? Why?

Perspectives on Mathematics Teaching

11. List three necessary qualities of an excellent mathematics teacher.

12. Please describe the teaching style of your best mathematics teacher.

Perspectives on Technology

13. What role does technology have in learning mathematics? Please explain.

14. What technologies have helped you learn mathematics? How?

Perspectives on Responsibilities

15. What do you feel are the responsibilities of a student in this course?

16. What do you feel are the responsibilities of a teacher in this course?

Appendix C: Protocol for Interview 1

I want to let you know that, since I will be asking some questions about your personal and family life, I will keep your identity anonymous in the published thesis. So I will be using a false name. Do you have one you'd like me to use, or should I just make one up?

Personal History

Where are you from?

    What is your favorite thing about your hometown?

What are you majoring in?

    Why did you choose that major?

    What career would you like to pursue?

What additional mathematics classes will you be taking?

What are your hobbies?

If you had tomorrow off, with no work and no school, what would you like to do?

Are you married? Kids?

    What does your spouse think about math?

What was your favorite subject in high school? Why?

What has been your favorite class you've taken in college?

    Why? (The subject? The teacher? The structure?)

What has been your least favorite class you've taken in college?

    Why?

What classes would you like to take, but probably won't because they won't fit in your schedule?

Family background

Brothers and sisters?

Are they older or younger?

Where do you fit?

Are you close in age?

Do your siblings like math?

What mathematics classes have they chosen to take beyond what is required?

Did you help your siblings/did your siblings help you?

Have your siblings gone to college?

What fields? Where they mathematics related?

Do you have similar interests with your siblings?

Tell me about your parents.

Did your parents go to college?

How much college?

What fields?

What do they do for work?

How far did your parents go in math?

Have your parents encouraged you to take mathematics classes?

How do your parents feel about math?

How do you know? Do they say that is how they feel?

Do your parents use mathematics in their work?

How?

Do your parents use mathematics at home?

How?

Did they help you with mathematics when you were in jr high/high school?

At what point were your parents no longer able to help you?

Do you think your parents like math?

Why do you think that?

How did your friends you hung out with in high school feel about math?

Did any of your friends from H.S. hate math?

How about your friends now?

Have any of your friends ever teased you about taking math?

When? What circumstances?

Who is your closest friend?

How does he/she feel about math?

Do you have friends in your mathematics classes?

Do you work together with your friends or other students in your classes?

Mathematics History

What is your earliest memory of doing mathematics at school?

Is it a good memory?

What mathematics classes did you take in High School?

What grades did you get?

What do you think was the biggest factor in getting the grades you did?

(hard work, natural ability, good teachers, etc)

What was your favorite mathematics class?

Why was it your favorite class?

What was your least favorite mathematics class?

Why?

Who was your favorite mathematics teacher?

What did he/she do different than the others?

Who was your least favorite mathematics teacher?

Why?

Are you taking a mathematics class now?

How are you doing in the class?

Describe your ideal mathematics class.

Describe your ideal mathematics teacher.

If you were to teach a mathematics class, who would you pattern your teaching style after?

Was there one event that you remember that has affected how you think about mathematics now?

Descriptive Words Task:

I will give you a word and I would like you to tell me to what extent you think this word describes someone who is good at math. Please rate it a ‘0’ if you think it does not describe someone good at mathematics at all, and a ‘5’ if you think it really describes someone good at math. I want you to give me your first impression.

Creative
Persistent
Organized
Diligent
Brilliant
Confident
Patient
Friendly
Teachable
Social
Open-minded
Introvert
Obedient
Verbal
Passionate
Light-hearted
Brave
Careful
Arrogant
Imaginative
Humble
Curious
Interested
Independent
Gifted
Focused
Motivated
Resourceful

(After each answer I may ask in what way it describes a mathematics person, an example, or what the participant thinks of when he or she sees that word.)

Appendix D: Sample Protocol for Interview 2

Sample Interview 2 protocol (for Bryce)

Video clip from 13Nov(2) 47:55 Starting new problem (Blob)	I'm going to show you 2 clips where you are starting new problems. Then we will compare/contrast them. As we are watching this video please comment on what you remember about what was going on and what you were thinking.
Video clip from 27Nov(1) 49:20 Starting a new problem (looking at emotions when starting a new problem, confidence, anxiety) (pause after 30 seconds)	As you watch this video please comment on what you remember about what was going on and what you were thinking. -How do you react to a new problem like this? -Is your reaction like the others? ("wow", "gee whiz"etc) -If you were given this problem alone, without your group, what would you think? In what ways would it be different from here, with the group? -Do you have confidence that you could do it alone?
49:50 Bryce starts decoding the problem	-Is this typical of you to jump in with ideas? -Do others sometimes do it? - Do you do it often? -Do ideas for solving it come to you right away? -I notice that Hannah and Debbie seem to be watching and listening to the others. What do you think they are thinking about while you are talking?
16oct(1) 40:21 Group presents- Bryce explains graphing derivatives	Please explain what is happening and what you are feeling at the time -are you comfortable presenting? -I think I heard that you volunteered to speak because you just learned this concept, is that correct? -is that typical for you? -Do you enjoy presenting? -Are you comfortable in front of a group in other

	situations? -Can you think of a time that you felt scared or nervous to present? -Can you think of a time that you felt excited to present? -
6Nov(1) 38 Bryce talking about the ticket line problem	Please tell what is going on and what you are thinking about during this clip. -This is an interesting way to talk about mathematics. What do you think it says about you? -
40 Hannah working to get caught up with everyone	-what happens when someone gets behind? -would you say that you are generally up with the others, or catching up to the others? -Are you generally helping others to catch up, or having someone help you catch up? -what happens when you are behind everyone else?
44:20 Differences in answers	-this is an example for Hannah, although I assume it has happened to you as well, when her answer does not match Jack's. -How do you resolve differences with the others? -What generally happens? -Is this typical of how differences are resolved? -Do you remember how this was resolved? -Are you generally confident about your answers?
29 Nov(2) 24:30 Hannah sits and watches silliness	Tell me what is going through your mind during this episode. -I see the three boys are having the conversation, and Hannah and Debbie seem to be watching. Is this typical behavior for the group? -What do you think they are thinking about when they are watching? -
28:49 Hannah corrects others	-Please describe what you think happened there. -Is this typical behavior for Hannah? -Are there others in the group who keep the group

	on track? -
Pause at 29	I would like to talk about the other members of the group.
For each: Caleb Debbie Jack Hannah Self (Bryce)	How do you think he/she feels about math? -on this pain/enjoyment continuum where do you think he/she fits? Why? -On this confident continuum where do you think he/she fits? Why? -On this continuum of 'good at math' where do you think he/she fits?
	On subsequent people compare with previous placements
	I would like you to think about the way your group works together. What roles do each of you play?

Continua used for Interview 2:

Math Skills Continuum

Not Good at Math ______________________________ **Good at Math**

Enjoyment Continuum

Enjoyment ______________________________ **Pain**

Confidence Continuum

Not Confident ______________________________ **Confident**

Anxiety Continuum

Anxiety ______________________________ **Peace (no anxiety)**

Appendix E: Protocol for Interview 3

'We are going to talk about what mathematics is and how useful it is. I have here 26 note cards. Each one has an activity on it. I want you to sort these cards into categories according to how the activity relates to mathematics. You get to make up the categories. You can have as many as you want—more than 2 and less than 26. When you have the categories, write down the categories on the sticky notes. Please think out loud as you are sorting the cards.'

Activities:

Arranging trophies from tallest to shortest
Sorting shapes
Cooking with a recipe
Memorizing times tables
Playing sudoku
Reading a book
Reading a map
Quilting
Playing a video game
Measuring your bedroom
Sending a text message
Solving a logic puzzle
Playing the drums
Driving a car
Drawing a map
Playing the piano
Working on an electrical circuit
Hanging a picture on the wall
Placing decorative tiles in your bathroom
Keeping a planner
A lawyer arguing a case before a jury
Writing a essay
Solving a scheduling problem
Playing solitaire
Doubling a recipe
Balancing your checkbook

(After they are finished sorting the activities into categories)

--for each category:

Tell me what this one is called.

What was the criteria for being in this category?

How does (each activity) fit in this category?

Discussing Belief Surveys:

I asked the students to explain answers on the Belief Questionnaires that they had filled out during the class (see Appendix A for a copy of the questionnaire).

Action/Emotion statements:

Please fill in the blank with how the following activities make you feel.

When I add a list of numbers without a calculator, I feel __________________.

When I first see a new mathematics problem, I feel ___________________.

When I can't answer a mathematics problem right away, I feel ___________________.

When I finish a big mathematics homework assignment, I feel ___________________.

When I help someone with a mathematics problem, I feel ___________________.

When I figure out a difficult mathematics problem, I feel __________________.

When I balance my checkbook, I feel _________________.

When I begin a mathematics test, I feel _________________.

When I present a solution to one of my mathematics problems in class, I feel ______.

When I get an answer wrong in class, I feel _________________.

When I make a mistake on an easy problem, I feel _________________.

When I disagree with an answer from someone else in my group, I feel _____.

When the teacher calls on me unexpectedly, I feel _________________.

When I work in a collaborative group on mathematics problems, I feel ___________.

When the members of my group get ahead of me, I feel _________________.

Bibliography

Anderson, D., & Gold, E. (2006). Home to school: Numeracy practices and mathematical identities. *Mathematical Thinking and Learning, 8*, 261-286.

Anderson, R. (2007). Being a mathematics learner: Four faces of identity. *The Mathematics Educator*, *17*, 7-14.

Angier, C., & Povey, H. (1999). One teacher and a class of school students: Their perception of the culture of their mathematics classroom and its construction. *Educational Review*, *51*, 147-160.

Armstrong, J., & Price, R. (1982). Correlates and predictors of women's mathematics participation. *Journal for Research in Mathematics Education, 13*, 99-109.

Bandura, A. (1993). Perceived self-efficacy in cognitive development and functioning. *Educational Psychologist, 28*, 117-148.

Bandura, A., Barbaranelli, C., & Caprara, G. V. (1996). Multifaceted impact of self-efficacy beliefs on academic functioning. *Child Development, 67*, 1206-1222.

Barzun, J. (1991). *Begin here: The forgotten conditions of teaching and learning* Chicago: University of Chicago.

Bem, D. (1970). *Beliefs, attitudes, and human affairs*. Belmont, California: Brooks/Cole Publishing Company.

Boaler, J., & Greeno, J. (2000). Identity, agency and knowing in mathematics worlds. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning* (pp. 171-201). Westport, CT: Ablex Publishing.

Bracher, M. (2006). *Radical pedagogy: Identity, generativity, and social transformation*. New York: Palgrave Macmillan.

Bush, G. W. (2006). State of the union address. Retrieved March 15, 2007, from http://www.whitehouse.gov/stateoftheunion/2006/print/index.html

Carlson, E. (2004). Facing shortage of U.S. Scientists, UW wants to boost math enrollment [Electronic Version]. Retrieved March 15, 2007 from http://wistechnology.com/article.php?id=1132.

Cobb, P., Gresalfi, M., & Hodge, L. L. (in press). An interpretive scheme for analyzing the identities that students develop in mathematics classrooms. *Journal of Research in Mathematics Education*.

Cote, J., & Levine, C. (2002). *Identity formation, agency, and culture: A social psychological synthesis*. Mahwah, New Jersey: Lawrence Erlbaum Associates.

Devlin, K. (2000). *The math gene: How mathematical thinking evolved and why numbers are like gossip*. New York: Basic books.

Erikson, E. (1959). *Identity and the life cycle: Selected papers*. New York: International Universities Press.

Erikson, E. (1968). *Identity, youth and crisis*. New York: W. W. Norton.

Fennema, E., & Sherman, J. (1977). Sex-related differences in mathematics achievement, spatial visualization and affective factors. *American Educational Research Journal*, *14*, 51-71.

Forster, P. (2000). Katie thought she couldn't do it but now she knows she can. *Educational Studies in Mathematics*, *43*, 225-242.

Friedman, L. (1999). *Identity's architect: A biography of Erik H. Erikson*. New York: Scribner.

Friend, R., Rafferty, Y., & Bramel, D. (1990). A puzzling misinterpretation of the Asch 'conformity' study. *European Journal of Social Psychology, 20*, 29-44.

Gee, J. (2000). Identity as an analytic lens for research in education. *Review of Research in Education, 25*, 99-125.

Gellert, U. (2005). Parents: Support or obstacle for curriculum innovation? *Journal of Curriculum Studies, 37*, 313-328.

Hogg, M. A., Terry, D. J., & White, K. M. (1995). A tale of two theories: A critical comparison of identity theory with social identity theory. *Social Psychology Quarterly, 58*, 255-269.

Kloosterman, P., Raymond, A. M., & Emenaker, C. (1996). Students' beliefs about mathematics: A three-year study. *The Elementary School Journal, 97*, 39-56.

Kroger, J. (2004). *Identity in adolescence: The balance between self and other*. New York: Routledge.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*, 29-63.

Leatham, K. (2006). Viewing mathematics teachers' beliefs as sensible systems. *Journal of Mathematics Teacher Education, 9*, 91-102.

Leder, G., & Forgasz, H. (2002). Measuring mathematical beliefs and their impact on the learning of mathematics: A new approach. In G. Leder, E. Pahkonen & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education?* Dordrecht, The Netherlands: Kluwer Academic Publishers.

Leedy, G., LaLonde, D., & Runk, K. (2003). Gender equity in mathematics: Beliefs of students, parents, and teachers. *School Science and Mathematics, 103*, 285-292.

Lent, R., Brown, S., & Gore, P. (1997). Discriminant and predictive validity of academic self-concept, academic self-efficacy, and mathematics-specific self-efficacy. *Journal of Counseling Psychology, 44*, 307-315.

Marcia, J. E. (1966). Development and validation of ego identity status. *Journal of Personality and Social Psychology, 3*, 551-558.

Martin, D. (2000). *Mathematics success and failure among African-American youth: The roles of sociohistorical context, community forces, school influence, and individual agency*. New Jersey: Lawrence Erlbaum Associates.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. In J. Kilpatrick, J. Swafford & B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: Nation Academy Press.

O'Brien, T. (2007). The old and the new. *Phi Delta Kappan, 88*, 664-668.

Osmo, R. (2001). A conceptual tool: Making social workers' assumptions explicit. *Social Work Education, 20*, 209-217.

Pajares, F., & Miller, M. D. (1994). Role of self-efficacy and self-concept beliefs in mathematical problem solving: A path analysis. *Journal of Educational Psychology, 86*, 193-203.

Perry, A. (2004). Decreasing math anxiety in college students. *College Student Journal, 38*, 321-324.

Peterson, P., & Fennema, E. (1985). Effective teaching, student engagement in classroom activities, and sex-related differences in learning mathematics. *American Educational Research Journal, 22*, 309-335.

Prawat, R. S., & Jennings, N. (1997). Students as context in mathematics reform: The story of two upper-elementary teachers. *The Elementary School Journal, 97*, 251-270.

Presmeg, N. (2002). Beliefs about the nature of mathematics in the bridging of everyday and school mathematical practices. In G. Leder, E. Pehkonen & G. Torner (Eds.), *Beliefs: A hidden variable in mathematics education?* Dordrecht, The Netherlands: Kluwer Academic Publishers.

Remillard, J. T., & Jackson, K. (2006). Old math, new math: Parents' experiences with standards-based reform. *Mathematical Thinking and Learning, 8*, 231-259.

Resnick, L. B., & Ford, W. W. (1981). *The psychology of mathematics for instruction*. Hillsdale, N.J.: Lawrence Erlbaum Associates.

Reyes, L. (1984). Affective variables and mathematic education. *Elementary School Journal, 84*, 558-581.

Richards, K. (2006). "Being the teacher": Identity and classroom conversation. *Applied Linguistics, 27*, 51-77.

Rokeach, M. (1968). *Beliefs attitudes and values: A theory of organization and change*. York, Pennsylvania: Jossey-Bass, Inc.

Sartorius von Waltershausen, W. (1966). *Gauss: A memorial* (H. W. Gauss, Trans.). Leipzig: S. Hirzel. (Original work published 1856).

Schoenfeld, A. (1988). When good teaching leads to bad results: The disasters of well-taught mathematics courses. *Educational Psychologist, 23*, 145-166.

Schoenfeld, A. (1998). Toward a theory of teaching-in-context. *Issues in Education, 4*, 1-95.

Schunk, D. H., & Pajares, F. (2002). The development of academic self-efficacy. In A. Wigfield & J. S. Eccles (Eds.), *Development of achievement motivation* (pp. 15-31). San Diego: Academic Press.

Seaman, C., Szydlik, J., Szydlik, S., & Beam, J. (2005). A comparison of preservice elementary teachers' beliefs about mathematics and teaching mathematics: 1968 and 1998. *School Science and Mathematics, 105*, 197-210.

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher, 34*(4), 14-22.

Smith, J. P. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education, 27*, 387-402.

Solomon, Y. (2007). Not belonging? What makes a functional learner identity in undergraduate mathematics? *Studies in Higher Education, 32*, 79-96.

Steward, S., & Nardi, E. (2002). I could be the best mathematician in the world...If I actually enjoyed it: Part 2. *Mathematics Teaching, 180*, 4-9.

Stigler, J., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. New York: Free Press.

Stryker, S., & Burke, P. (2000). The past, present, and future of an identity theory. *Social Psychology Quarterly, 63*, 284-297.

Swanson, R. (2005, June 11, 2005). Excerpts of remarks prepared for delivery to: California polytechnic state university commencement from http://web.archive.org/web/20051224142515/http://www.raytheon.com/newsroom/static/cms01_058576.pdf

Valentine, J., DuBois, D., & Cooper, H. (2004). The relation between self-beliefs and academic achievement: A meta-analytic review. *Educational Psychologist, 39*, 111-133.

van Langenhove, L., & Harre, R. (1999). Introducing positioning theory. In R. Harre & L. van Langenhove (Eds.), *Positioning theory* (pp. 14-31). Oxford: Blackwell Publishers Ltd.

von Glasersfeld, E. (1983). Learning as a constructive activity. In J. C. Bergeron & N. Herscovics (Eds.), *Proceedings of the fifth annual meeting of the north American chapter of the international group for the psychology of mathematics education* (Vol. 1, pp. 41-69). Montreal, Canada: Universile de Montreal.

Wigfield, A., & Eccles, J. S. (2002). Introduction. In A. Wigfield & J. S. Eccles (Eds.), *Development of achievement motivation* (pp. 1-11). San Diego: Academic Press.