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Financial position and performance in IFRS 17

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ABSTRACT

The general principles for determining the financial performance of a company is that revenue is earned as goods are delivered or services provided, and that expenses in the period are made up of the costs associated with this earned revenue. In the insurance industry, premium payments are typically made upfront, and can provide coverage for several years, or be paid many years before the coverage period starts. The associated costs are often not fully known until many years later. Hence, complexity arises both in determining how a premium paid should be earned over time, and in valuing the costs associated with this earned premium. IFRS 17 attempts to align the insurance industry with these general accounting principles. We bring this new accounting standard into the realm of actuarial science, through a mathematical interpretation of the regulatory texts, and by defining the algorithm for profit or loss in accordance with the new standard. Furthermore, we suggest a computationally efficient risk-based method of valuing a portfolio of insurance contracts and an allocation of this value to subportfolios. Finally, we demonstrate the practicability of these methods and the algorithm for profit or loss in a large-scale numerical example.

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1. Introduction

The accrual method for calculating profit or loss has for a long time been the standard method used when large companies prepare their financial statements. Hence, profit or loss is calculated as the difference between revenue and expenses, where revenue is earned as goods are delivered or services are provided, and expenses should match revenue, i.e. expenses are recognised in the period in which the associated revenue is recognised. This is different from the more basic cash method of calculating profit or loss, where revenue is recognised when cash is received and expenses when cash is paid. Naturally, the total profit or loss over the lifetime of a company is the same irrespective of method, but the method used will determine how profit or loss is allocated to different time periods. Since the main objective of financial statements of listed companies is to provide investors with the information they need to make informed economic decisions, the accrual method is the prescribed method in both International Financial Reporting Standards (IFRS) and Generally Accepted Accounting Practices (GAAP).

For some industries, the accrual method and the cash method for calculating profit or loss would give similar results, even in the short run. This is generally not the case in the insurance industry,

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due to the inherent nature of insurance products. In an insurance contract, an insurer agrees to compensate a policyholder in case of an insured event occurring. In exchange, the policyholder pays a premium in advance of the provision of coverage. When the contract is issued, the claim payments are uncertain in amount or timing, or both. The premium payment, the coverage period, and the payments due to claims in this coverage period can occur at different time points, sometimes many years apart. Hence, to recognise premium payments as revenue and claims payments as costs according to when they are paid would not accurately reflect the profit or loss in the period in question for insurance products, and would therefore be misleading.

For some types of insurance products, insurance coverage is typically purchased for one year at a time. Hence, to determine when the premium is earned is a fairly standard matter. Complexity arises when the total claims costs due to coverage provided during one reporting period only are fully determined several years later for some product lines. To faithfully represent the economics of this type of contract, the total claims cost somehow need to be measured and recognised in the period in which the premium is earned, i.e. as coverage is provided. For other types of insurance products, claims are generally paid as they are incurred, but on the other hand a premium payment often provides many years of coverage, and can be paid several years before the coverage period starts. Hence, the premium payment and any claims payments and other costs associated with the coverage that this premium provides need to be taken into account in order to derive a net liability value, i.e. an estimate of the profit or loss for the contract. This should be earned over the coverage period, since this is when services are provided. One also needs to take into account that when the coverage period is long, facts and circumstances might change in such a way that the net liability value changes compared to the estimated value at the time a contract is issued, and decide how this type of changes should affect profit or loss in the period, as well as the estimate of the remaining profit or loss for the contract.

As described in IASB (2017b), there is currently considerable variation in financial reporting practices in different jurisdictions for the insurance industry. The measurement of contracts and recognition of revenue also varies greatly compared to other industries, where accrual accounting as above is the general standard. The need for a consistent framework for accounting of insurance contracts has long been recognised, now resulting in IFRS 17, which provides a systematic way of recognising unearned profit over time.

1.1. Introducing IFRS 17

In the current European solvency regulatory framework, Solvency II, the concept of risk-based valuation of liabilities is one of the cornerstones in determining the solvency positions of insurance companies. In the coming years, risk-based valuation of liabilities will also need to be incorporated in how companies measure their financial position and performance, as the new international financial standard for insurance contracts, IFRS 17, comes into force.

The current accounting standard for insurance contracts, IFRS 4, is an interim standard, which sets some minimum requirements on the accounting policies in different jurisdictions, but apart from this allows considerable variation in financial reporting practices. The main objectives of IFRS 17 is thus to make accounting practices more consistent over different jurisdictions as well as making the financial statements of insurance companies more informative. It is stressed in IASB (2017a) that financial statements should ‘provide relevant information that faithfully represents . . . [insurance] contracts’ and should ‘reflect true underlying financial positions or performance arising from these insurance contracts’, in contrast to many current accounting practices that e.g. do not use current estimates of all cash flows, require no explicit risk measurement, or disregards the time value of money in the valuation (§ BC14 in IASB 2017b).

In Solvency II, market-consistent valuation is based on a cost-of-capital approach, and the formula and parameters used for the calculation are prescribed by the regulation (see [DIRECTIVE 2009/138/EC n.d](#) and [Commission Delegated Regulation \(EU\) 2015](#)). In IFRS 17, which

is principles based, there is much more freedom to choose what valuation technique to use, but the general principles are largely consistent with the ones in Solvency II. It is thus up to each company to choose the appropriate technique reflecting the true economics of its insurance contracts. Furthermore, in IFRS 17 the financial performance of a company will be linked to the valuation of its insurance liabilities, and this link is missing in Solvency II, which focuses solely on solvency.

The requirements in IFRS 17 raise questions regarding how to compute the risk adjustment for non-financial risk, defined as ‘the compensation that the entity requires for bearing the uncertainty about the amount and timing of the cash flows that arises from non-financial risk’ (§ 37 in IASB 2017a), which is part of the insurance liability value. Other considerations are how to calculate the confidence level that this risk adjustment corresponds to, as well as how to allocate the risk adjustment to a potentially large number of groups of contracts. With this new accounting standard for insurance contracts, the decisions made on these topics will influence, not only the financial position of the company at a certain date, but also the financial performance of the company for many years to come, since it directly affects the revenue streams presented by the company in its financial statements.

1.2. Our objective

Our objective is to present how to measure the financial performance of an insurer in accordance with IFRS 17 and demonstrate in terms of a realistic numerical example how the computational challenges may be handled efficiently. Throughout the paper we will disregard any costs not directly related to servicing contracts and we will only consider contracts that generate cash flows that are independent of financial asset values. In order to measure the financial performance of an insurer in accordance with IFRS 17, the following four components need to be determined:

Stochastic model. The policies written generate future payments to policyholders at random times and of random sizes. Groups of policies are therefore associated with stochastic cash flows and we must decide on a joint model for these stochastic cash flows.

Valuation method. In order to determine how much of the premium income can be considered as earned premium income resulting in profit or loss, the aggregate liability cash flow must be valued and revalued over time. We emphasise that it is the aggregate stochastic liability cash flow that is subject to capital requirements and possibly other capital costs for the insurer. Therefore we must decide on a method for valuation of aggregate stochastic cash flows.

Allocation method. Given that we assign a value to the aggregate liability cash flow, we must decide on how much of this value should be allocated to different groups of policies in order to determine profit or loss for these groups. Therefore, in addition to a risk-based valuation method we must decide on a risk-based allocation method.

Profit and loss algorithm. Given a joint stochastic model for liability cash flows, a valuation method for the aggregate liability cash flow, and an allocation method defining how the aggregate liability value should be split into contributions to this value for a partition of all policies into groups of policies, an accounting method defines profits and losses over time and across groups. We demonstrate that the IFRS 17 accounting standard defines an algorithm that, given a valuation method and an allocation method, defines profits and losses of groups of policies over time.

We emphasise that the four components above can be regarded as independent. Whereas IFRS 17 essentially defines the profit and loss algorithm, the stochastic model, the valuation method, and the allocation method can be chosen independently. That is, you may replace the valuation and allocation methods advocated here by your preferred choices (given a convincing motivation for doing so)! This is further highlighted in the organisation of the paper (see Section 1.3), which defines the profit and loss algorithm in terms of general liability values before defining the chosen valuation method and allocation method.

1.3. Organisation of the paper

This paper is organised as follows. Section 2 presents the liability cash flows and the different liability values that are key ingredients in the analysis of financial performance. Section 3 formalises the algorithm for financial performance of an insurer in accordance with IFRS 17. Section 4 presents valuation of aggregate insurance liability cash flows and allocation of these values to cash flows that are part of the aggregate cash flow. Section 4.1 presents multi-period cost-of-capital valuation, similar to what was developed in Möhr (2011) and Engstner et al. (2017) and presents explicit valuation and allocation formulas that hold exactly when cash flows and covariates representing the flow of information follow a Gaussian process. Section 5 presents a life-insurance example that demonstrates how the computational challenges that result from a large-scale implementation of an IFRS 17 financial performance analysis can be handled. Moreover, based on a standard mortality model with parameters estimated on data from the Human Mortality Database we illustrate properties of the valuation and allocation method and the IFRS 17 profit and loss algorithm for a life-insurance portfolio of realistic size. Palmborg et al. (2020) provide further details on the IFRS 17 profit and loss algorithm, computational details left out in the life-insurance example, and technical arguments on the allocation of an aggregate value to groups of contracts.

Throughout Sections 2 and 3, the presentation is in terms of general liability values, corresponding to the fulfilment cash flows in IFRS 17. The risk adjustment for non-financial risk is part of the liability value, and is made explicit in Section 4.

2. Liability cash flows and values

Consider an integer $i_0 < 0$, the set of time points $\mathcal{T} := \{i_0 - 1, i_0, \dots\}$ and the set of time periods $\mathcal{T}_+ := \{i_0, i_0 + 1, \dots\}$. Contracts issued during time period $t \in \mathcal{T}_+$ are issued in the time interval $(t - 1, t]$. Let current time be time 0. For $t \in \mathcal{T}$, let Gr_t be the set of groups that are part of the outstanding liability at time t . Each group of contracts is associated to an issuing period but there may be several groups associated to the same issuing period. For example, contracts issued during the same period might be grouped according to their product line or profitability profile. For each group g there exist $t_0 = t_0^{(g)}$ and $\tau = \tau^{(g)}$ such that $g \notin \text{Gr}_{t_0-1}$, $g \notin \text{Gr}_{t_0+\tau}$, and $g \in \text{Gr}_t$ for $t = t_0, t_0 + 1, \dots, t_0 + \tau - 1$. t_0 is the time of initial recognition of the group (see Remark 2.1), and τ is a positive integer chosen so that all contracts in the group are terminated after at most τ periods after t_0 . When a contract is terminated the insurance obligation is fulfilled, e.g. due to lapses, or the end of the decumulation phase for annuities. Whenever t_0 and τ appear without superscript (g) it is implicitly assumed that the group to which they correspond is clear from the context or refers to an arbitrary but fixed group.

Let $I_t^{(g)}$ denote the incremental net cash flow for group g in time period t , i.e. in the time interval $(t - 1, t]$. Let $P^{(t,g)}$ denote the net premium income during period t , adjusted for any other cash flows in this period, for contracts belonging to group g at time t , issued during this period. To avoid double counting, $I_t^{(g)}$ does not include net premium income from contracts belonging to group g that are issued in period t , but can include premium income for contracts belonging to group g that have been issued in previous periods, if this premium income is within the contract boundary (see Remark 2.2). If the premium income is not within the contract boundary it is viewed as a new contract, and hence is included in $P^{(t,g)}$ if this new contract belongs to group g . By definition, $I_{t_0}^{(g)} = 0$, since all cash flows in period t_0 will be included in $P^{(t_0,g)}$.

Let $X^{(t)}$ be the cash flow that corresponds to the outstanding liability as seen from time t (in runoff):

$$X^{(t)} := \sum_{g \in \text{Gr}_t} (I_{t+1}^{(g)}, \dots, I_{t_{\max}}^{(g)}), \quad t_{\max} := \max_{g \in \text{Gr}_t} (t_0^{(g)} + \tau^{(g)}).$$

Let $L^{(t)}$ denote the value of the outstanding liability cash flow as seen from time t , and let $L^{(t,g)}$ denote the liability value allocated to group g , i.e.

$$L^{(t)} = \sum_{g \in Gr_t} L^{(t,g)}.$$

If $g \notin Gr_t$, then $L^{(t,g)} = 0$. Note that $L^{(t,g)}$ is a part of the value $L^{(t)}$ of the sum $X^{(t)}$ that is allocated to one of its terms $X^{(t,g)} := (I_{t+1}^{(g)}, \dots, I_{t_{max}}^{(g)})$. Table 1 illustrates incremental liability cash flows and groups of contracts for the special case when $i_0 = -5$, $\tau = 5$ for all groups, and there is a single group per issuing period.

The outstanding liability cash flows can be partitioned into cash flows that correspond to insured events that have not yet occurred, and cash flows that correspond to insured events that have already occurred, including occurred claims that are not yet reported. The liability for remaining coverage, denoted by $L_{RC}^{(t,g)}$, is defined as the liability value for group g at time t allocated to the former, and the liability for incurred claims, denoted by $L_{IC}^{(t,g)}$, is defined as the liability value for group g at time t allocated to the latter. Hence,

$$L^{(t,g)} = L_{RC}^{(t,g)} + L_{IC}^{(t,g)}.$$

Let $L_{SP}^{(t,g)}$ denote the liability value for remaining coverage allocated to services provided between t and $t + 1$ for group g , and $L_{FS}^{(t,g)}$ the liability value allocated to future service (after $t + 1$) for group g , as seen from time t , hence

$$L_{RC}^{(t,g)} = L_{SP}^{(t,g)} + L_{FS}^{(t,g)}.$$

The liability values assigned to cash flows are computed by some method that takes the time value of money into account. It is common to handle this by considering discounting in terms of a money market account numeraire. From a modelling point of view, the parameters of the short rate model corresponding to the money market account should be such that the discount factors from the model agree with the given set of discount factors. In our context we need to consider liability values that

Table 1. Incremental liability cash flows illustrated for $i_0 = -5$, $\tau = 5$ for all groups, and a single group per issuing period. Since there is a single group per issuing period each group can be identified with the issuing period: $Gr_0 = \{-4; -3; -2; -1; 0\}$. The outstanding liability cash flow $X^{(0)}$ at time 0 corresponds to the vector of sums along the anti-diagonals of the cells coloured gray.

	1	2	3	4	5
-5	$I_{-4}^{(-5)}$	$I_{-3}^{(-5)}$	$I_{-2}^{(-5)}$	$I_{-1}^{(-5)}$	$I_0^{(-5)}$
-4	$I_{-3}^{(-4)}$	$I_{-2}^{(-4)}$	$I_{-1}^{(-4)}$	$I_0^{(-4)}$	$I_1^{(-4)}$
-3	$I_{-2}^{(-3)}$	$I_{-1}^{(-3)}$	$I_0^{(-3)}$	$I_1^{(-3)}$	$I_2^{(-3)}$
-2	$I_{-1}^{(-2)}$	$I_0^{(-2)}$	$I_1^{(-2)}$	$I_2^{(-2)}$	$I_3^{(-2)}$
-1	$I_0^{(-1)}$	$I_1^{(-1)}$	$I_2^{(-1)}$	$I_3^{(-1)}$	$I_4^{(-1)}$
0	$I_1^{(0)}$	$I_2^{(0)}$	$I_3^{(0)}$	$I_4^{(0)}$	$I_5^{(0)}$
1	$I_2^{(1)}$	$I_3^{(1)}$	$I_4^{(1)}$	$I_5^{(1)}$	$I_6^{(1)}$
⋮	⋯	⋯	⋯	⋯	⋯

are computed using discounting that depends on the discount factors that were realised at different times (the times of initial recognition of the various contract groups). To that end, let $L_{RC,t_0}^{(t,g)}$ denote the liability value for remaining coverage for group g at time t measured at the discount rates at time $t_0 \leq t$, where t_0 is the time of initial recognition of group g . This corresponds to setting the parameters of the short-rate model for the money market account numeraire to ensure consistency with discount factors at the time of initial recognition of a group of contracts. Similarly to $L_{RC,t_0}^{(t,g)}$, let $L_{FS,t_0}^{(t,g)}$ denote the liability value allocated to future service for group g as seen from time t measured at the discount rates at time t_0 . Furthermore, let $d_{t_0,t}$ denote the discount factor between t_0 and t , known at time t_0 . Note that $L_{RC,t_0}^{(t_0,g)} = L_{RC}^{(t_0,g)}$, $L_{FS,t_0}^{(t_0,g)} = L_{FS}^{(t_0,g)}$ and $d_{t_0,t_0} = 1$.

Remark 2.1 (Initial recognition): Initial recognition of a group of contracts is defined as the earliest of the following three dates: the beginning of the coverage period of the group; the date when the first payment from a policyholder in the group becomes due; and, for an onerous group, when the group becomes onerous (§ 25 in IASB 2017a). A group of contracts is onerous at initial recognition if the value of the outstanding liability cash flow for the group exceeds any cash flows for the group at that date (e.g. premium income received), resulting in a net outflow for the group (§ 47 in IASB 2017a). Hence, if the insurer has reason to believe that a contract group is onerous (due to e.g. pricing), this needs to be determined as the contracts are issued, by calculating the value of the outstanding liability cash flow at that date.

Remark 2.2 (Contract boundary): In IFRS 17 premiums are within the contract boundary if the insurer can force the policyholder to pay them, or if the insurer has to accept future premium payments from the policyholder, without being able to reprice the contract or change the benefit level to fully reflect the risks of the policyholder or the risks of the portfolio that the contract in question belongs to (§ 34 in IASB 2017a).

3. Formalising financial performance in IFRS 17

The key principles when measuring the financial performance of a company in accordance with IFRS 17 is that insurance contracts need to be divided into groups which should be valued as the sum of the fulfilment cash flows and the contractual service margin, the former being ‘a risk-adjusted present value of the future cash flows . . . consistent with observable market information’ and the latter ‘an amount representing the unearned profit in the group’ (§ IN6 in IASB 2017a). Hence, in contrast to Solvency II, IFRS 17 does not allow the company to recognise unearned future profit as a gain when the group of contracts is initially recognised. Instead the profit from the group will be recognised over the period that insurance coverage is provided, and as it is released from risk. However, the contractual service margin cannot be negative, i.e. if a group is or becomes loss-making, this loss will be recognised immediately.

The procedure for determining the financial position of a company can be summarised as follows:

- (a) Divide contracts into groups according to product line, profitability, and date issued (contracts in the same group have to be issued within one year of each other).
- (b) Valuation of the outstanding insurance liability as ‘estimates of future cash flows, . . . an adjustment to reflect the time value of money and the financial risks related to the future cash flows, . . . and a risk adjustment for non-financial risk’ (§ 32 in IASB 2017a).
- (c) Allocate the aggregate insurance liability value to each group of contracts.
- (d) Determine the contractual service margin, if any, as the amount that ensures that the initial recognition of a group of contracts does not result in any gain. Hence the sum of the insurance liability value allocated to the group, the contractual service margin, and any cash flows arising from the contracts at that date should be equal to zero (§ 38 in IASB 2017a).

- (e) In subsequent measurement, the contractual service margin at the end of the period, for each group, consists of the contractual service margin at the beginning of the period, adjusted for new contracts added to the group, if any; interest accreted on the contractual service margin in the period; the change in the liability value due to future service allocated to the group; and allocation of part of the contractual service margin to profit or loss in the period (§ 44 in IASB 2017a).

Note that the definition of the initial contractual service margin as the amount that makes the sum in step (d) equal to zero is only valid when the sum of the liability value and the net cash flows is non-negative. A negative net cash flow here corresponds to a net inflow (e.g. premium payments), and a positive net cash flow corresponds to a net outflow. This means that the contractual service margin cannot be negative. If the sum of the liability value and the net cash flows is positive, this amount will instead be the value of the loss component at initial recognition (§ 47 in IASB 2017a).

The profit or loss of a company consists of:

- (i) The part of the contractual service margin allocated to profit or loss in step (e) above, if any.
- (ii) Experience adjustments, i.e. the difference between ‘the estimate at the beginning of the period of the amounts expected to be incurred in the period and the actual amounts incurred in the period’ (Appendix A in IASB 2017a).
- (iii) The release from risk in the period, i.e. the change in the risk adjustment for non-financial risk due to services provided in the period.
- (iv) The loss component at initial recognition for onerous groups of contracts, or any increase or reversal of the loss component at subsequent measurement.
- (v) The change in the outstanding liability value and contractual service margin due to the effect of the time value of money and changes in the time value of money.

Note that step (e), (ii), (ii), and (v) are only valid in the case of a contractual service margin at the beginning of the period. If the contractual service margin at the beginning of the period is zero and there is instead a loss component, the development in the period will only give rise to a contractual service margin at the end of the period if the loss component is first fully reversed. When there is a loss component at the beginning of the period, a proportion of the estimate at the beginning of the period in step (ii), of the release from risk in step (iii), and of the change due to the time value of money in step (v), will instead adjust the loss component. Hence, in order to avoid double counting, only the remaining part of these amounts will be included in step (ii), (iii), and (v). Furthermore, note that the estimate at the beginning of the period in step (ii) together with the release from risk in step (iii) is the same as the liability value for remaining coverage allocated to services provided, $L_{SP}^{(t)}$.

Let $CSM^{(t,g)}$ denote the contractual service margin and $LC^{(t,g)}$ the loss component at time t for group g . The definition of the contractual service margin and the loss component as mathematical objects is as follows. We use the notation $x^+ := \max(x, 0)$ and $x^- := \max(-x, 0) = -\min(x, 0)$. The definition involves weights $W_t^{(g)}$ such that, for a given time t and group g , $W_t^{(g)}$ represents the proportion of the unearned profit at time t for group g that is allocated to profit or loss in periods $> t$. Therefore, $1 - W_t^{(g)}$ represents the proportion of the unearned profit at time t for group g that is allocated to profit or loss in period t . $W_t^{(g)}$ is a $[0, 1]$ -valued random variable known at time t . Similarly, the weight $U_t^{(g)}$ appearing in the definition is known at time t . $U_t^{(g)}$ represents an allocation of certain changes in the liability value in period t , between the loss component and the liability value excluding the loss component (see Remark 3.2).

Definition 3.1 (Algorithm for calculating CSM and LC): Consider a sequence of groups $(Gr_t)_{t \in \mathcal{T}}$ and sequences $(W_t^{(g)})_{t \in \mathcal{T}}$ and $(U_t^{(g)})_{t \in \mathcal{T}}$, where $W_t^{(g)}$ is $[0, 1]$ -valued and $W_t^{(g)} = 0$ for $g \notin Gr_t$. Fix $(t, g) \in \mathcal{T}_+ \times \cup_{s \in \mathcal{T}} Gr_s$.

If $g \notin \text{Gr}_{t-1}$, then set $\text{CSM}^{(t-1,g)} := 0$ and $\text{LC}^{(t-1,g)} := 0$.

If $g \in \text{Gr}_{t-1} \cup \text{Gr}_t$, $\text{CSM}^{(t-1,g)} \geq 0$ and $\text{LC}^{(t-1,g)} = 0$, then set

$$\Delta_1 := \frac{d_{t_0,t-1}}{d_{t_0,t}} \text{CSM}^{(t-1,g)} + \frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\text{FS},t_0}^{(t-1,g)} - L_{\text{RC},t_0}^{(t,g)} + P^{(t,g)} \quad (1)$$

and set $\text{CSM}^{(t,g)} := W_t^{(g)} \Delta_1^+$ and $\text{LC}^{(t,g)} := \Delta_1^-$.

If $g \in \text{Gr}_{t-1} \cup \text{Gr}_t$, $\text{CSM}^{(t-1,g)} = 0$ and $\text{LC}^{(t-1,g)} > 0$, then set

$$\Delta_2 := -\text{LC}^{(t-1,g)} - U_t^{(g)} \left(L_{\text{RC}}^{(t,g)} - L_{\text{RC}}^{(t-1,g)} + \frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\text{FS},t_0}^{(t-1,g)} - L_{\text{RC},t_0}^{(t,g)} \right), \quad (2)$$

$$\Delta_3 := -\Delta_2^- + \frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\text{FS},t_0}^{(t-1,g)} - L_{\text{RC},t_0}^{(t,g)} + P^{(t,g)} \quad (3)$$

and set $\text{CSM}^{(t,g)} := W_t^{(g)} \Delta_3^+$ and $\text{LC}^{(t,g)} := \Delta_3^-$.

The underlying principles for the development of the contractual service margin is that changes in the liability value only adjust the contractual service margin to the extent that these changes relate to future service, and that the contractual service margin is measured at discount rates locked in at initial recognition.

Remark 3.1 (Change in the liability value relating to future service): The term $\frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\text{FS},t_0}^{(t-1,g)} - L_{\text{RC},t_0}^{(t,g)}$ appearing in (1), (2) and (3) in Definition 3.1 is the change in the liability value relating to future service in the period, measured at the discount rates determined at initial recognition of the contracts, and excluding any effect of the time value of money, since this effect is not seen as relating to future service in IFRS 17 (see § B97 in IASB 2017a). Note that $L_{\text{FS},t_0}^{(t-1,g)}$ is the liability value at $t-1$ (measured at the t_0 -discount rates) allocated to cash flows after time t , and unwinding the discount one period is captured by multiplying with $\frac{d_{t_0,t-1}}{d_{t_0,t}}$. Hence $\frac{d_{t_0,t-1}}{d_{t_0,t}} L_{\text{FS},t_0}^{(t-1,g)}$ is the liability value at time t for cash flows after t , measured at the t_0 -discount rates, but where all non-financial assumptions are based on the information at time $t-1$, while $L_{\text{RC},t_0}^{(t,g)}$ is the liability value at time t for cash flows after t , measured at the t_0 -discount rates, where all non-financial assumptions have been updated to include the information in period t .

In order for the definition of the contractual service margin $\text{CSM}^{(t,g)}$ and the loss component $\text{LC}^{(t,g)}$ to make economic sense, the weights $W_t^{(g)}$ and $U_t^{(g)}$ in Definition 3.1 must be suitably chosen. The weights $W_t^{(g)}$ correspond to quantities that are derived from so-called coverage units. Precise details are found in Section 3.1 below. For further details on the expressions in (1), (2), and (3), see Section A.1 in Palmborg et al. (2020).

Remark 3.2 (Weights $U_t^{(g)}$): The change in the liability value due to services provided and the change in the liability value due to the effect of the time value of money should be allocated between the loss component and the liability value excluding the loss component (§§ 50–51 in IASB 2017a). The weight $U_t^{(g)}$ in (2) in Definition 3.1 represents this allocation. In IASB (2017c) this weight has been set to $\text{LC}^{(t-1,g)} / L_{\text{RC}}^{(t-1,g)}$. Assuming that $P^{(t,g)} \geq 0$, that $L_{\text{RC}}^{(t,g)} \geq 0$, that $L_{\text{RC}}^{(t,g)} > 0$ implies that $L_{\text{FS}}^{(t,g)} \geq 0$, for all t , and disregarding any difference between liability values measured at current discount rates and measured at discount rates determined at t_0 , this choice of weight leads to the desirable properties that $\text{LC}^{(t,g)} = 0$ when $L_{\text{RC}}^{(t,g)} = 0$, that $\text{LC}^{(t-1,g)} / L_{\text{RC}}^{(t-1,g)} \in [0, 1]$, and thus ensures that the loss component satisfies the requirement $\text{LC}^{(t_0+\tau,g)} = 0$. We have used this suggested choice of weight in

the numerical examples in Section 5. However, we emphasise that it is possible to construct situations where this choice of weight can lead to strange behaviour for the loss component, see Section A.3 in Palmborg et al. (2020).

Remark 3.3 (Termination of CSM and LC): Definition 3.1, together with a suitable choice of weight $U_t^{(g)}$, ensures that the contractual service margin and the loss component equal zero at the end of the coverage period, i.e. that $\text{CSM}^{(t_0+\tau,g)} = 0$ and $\text{LC}^{(t_0+\tau,g)} = 0$. Note that $L_{\text{RC}}^{(t_0+\tau,g)} = L_{\text{FS},t_0}^{(t_0+\tau-1,g)} = L_{\text{RC},t_0}^{(t_0+\tau,g)} = 0$, and $P^{(t_0+\tau,g)} = 0$. Hence, if $\text{CSM}^{(t_0+\tau-1,g)} \geq 0$ and $\text{LC}^{(t_0+\tau-1,g)} = 0$, then $\Delta_1 = \frac{d_{t_0,t_0+\tau-1}}{d_{t_0,t_0+\tau}} \text{CSM}^{(t_0+\tau-1,g)} \geq 0$ in (1), hence $\text{LC}^{(t_0+\tau,g)} = 0$, and $\text{CSM}^{(t_0+\tau,g)} = 0$ since $W_{t_0+\tau}^{(g)} = 0$. If instead $\text{CSM}^{(t_0+\tau-1,g)} = 0$ and $\text{LC}^{(t_0+\tau-1,g)} > 0$, then $\Delta_2 \geq 0$ in (2), under the requirement that $U_{t_0+\tau}^{(g)} \geq \text{LC}^{(t_0+\tau-1,g)} / L_{\text{RC}}^{(t_0+\tau-1,g)}$, from which it follows that $\Delta_3 = 0$ in (3), thus $\text{LC}^{(t_0+\tau,g)} = \text{CSM}^{(t_0+\tau,g)} = 0$.

Remark 3.4 (Initial recognition): Note that $L_{\text{RC}}^{(t_0-1,g)} = \text{CSM}^{(t_0-1,g)} = \text{LC}^{(t_0-1,g)} = 0$, and $L_{\text{RC},t_0}^{(t_0,g)} = L_{\text{RC}}^{(t_0,g)}$. Thus, the somewhat odd discount factor d_{t_0,t_0-1} appearing in (1) at initial recognition can be defined arbitrarily, since it is multiplied by zero. Furthermore, since no unearned profit should be allocated to profit or loss at initial recognition of the group it is required that $1 - W_{t_0}^{(g)} = 0$. Thus, at initial recognition:

$$\begin{aligned}\text{CSM}^{(t_0,g)} &= (P^{(t_0,g)} - L_{\text{RC}}^{(t_0,g)})^+, \\ \text{LC}^{(t_0,g)} &= (L_{\text{RC}}^{(t_0,g)} - P^{(t_0,g)})^+.\end{aligned}$$

Profit or loss in reporting period t for group g , $\text{P\&L}_t^{(g)}$, is defined as follows in terms of the contractual service margin and loss component in Definition 3.1.

Definition 3.2 (Profit or loss): Profit or loss determined at time t for reporting period t and group g of contracts is given by

$$\text{P\&L}_t^{(g)} := L^{(t-1,g)} + \text{CSM}^{(t-1,g)} + P^{(t,g)} - (L^{(t,g)} + \text{CSM}^{(t,g)}) - I_t^{(g)}. \quad (4)$$

Remark 3.5 (Long time P&L): For a group of contracts initially recognised at time t_0 , where all claims are paid by time $t_0 + \tau$, the total profit or loss for the group over its lifetime is given by

$$\sum_{t=t_0}^{t_0+\tau} \text{P\&L}_t^{(g)} = \sum_{t=t_0}^{t_0+\tau} P^{(t,g)} - \sum_{t=t_0}^{t_0+\tau} I_t^{(g)}, \quad (5)$$

i.e. the net premium income minus the sum of the net liability cash flows for the group. This follows immediately from the definitions of the involved quantities, see Definition 3.1 and Remark 3.3. Any other proposed definition of $\text{P\&L}_t^{(g)}$ for which (5) does not hold would be seriously flawed.

Remark 3.6 (Loss component): Note that the loss component $\text{LC}^{(t,g)}$ does not appear explicitly in Definition 3.2. Instead, the loss component is part of the liability value, i.e. any changes in the loss component will be included in the difference $L^{(t-1,g)} - L^{(t,g)}$. This means that changes in the loss component in the period will directly affect profit or loss in the period. An increasing loss component will affect profit or loss negatively, and vice versa. Since the loss component is already taken into account in both the financial position and performance of the company through the computation of the liability value, one might wonder why it is necessary to track the loss component through the

algorithm in Definition 3.1. The reason is that the loss component is needed in order to determine if a currently onerous group gives rise to a contractual service margin at the end of the period. Here we get to one of the key principles in IFRS 17 regarding how unearned profit versus unearned losses should be treated. The definition of profit or loss in Definition 3.2 ensures that losses for groups of contracts are recognised immediately in the reporting period in which a previously profitable group becomes onerous, i.e. when $CSM^{(t-1,g)} > 0$ and $LC^{(t,g)} > 0$, or an onerous group becomes more onerous, i.e. when $LC^{(t,g)} > LC^{(t-1,g)}$. At the same time, the appearance of the change in the contractual service margin in Definition 3.2 together with its development in Definition 3.1 ensure that for a profitable group of contracts a proportion of the unearned profit is recognised later, due to appearance of the weight factor $W_t^{(g)}$.

Remark 3.7 (Liability for incurred claims): Note that it is the change in the total liability value, $L^{(t-1,g)} - L^{(t,g)}$ that appears in Definition 3.2. At the same time, the algorithm for calculating the contractual service margin in Definition 3.1 only depends on the liability for remaining coverage. Hence, the liability for remaining coverage affects profit or loss in period t both directly through the change in the liability value, $L_{RC}^{(t-1,g)} - L_{RC}^{(t,g)}$, and indirectly through the change in the contractual service margin, $CSM^{(t-1,g)} - CSM^{(t,g)}$, while the liability for incurred claims only affects profit or loss through the change in the liability value, $L_{IC}^{(t-1,g)} - L_{IC}^{(t,g)}$. This is a natural consequence of that the liability for incurred claims is the liability value allocated to cash flows corresponding to insured events that have already occurred, hence it relates to past or current service, and the contractual service margin should only be adjusted for changes in the liability value that relate to future service.

Remark 3.8 (Alternative definition of P&L): In IASB (2017a) profit or loss is defined in a way that is not obviously equivalent to Definition 3.2. We prove the equivalence in Proposition A.1 in Section A.2 in Palmborg et al. (2020) and choose the more intuitive expression for $P\&L_t^{(g)}$ in Definition 3.2.

3.1. Coverage units

The weights $W_t^{(g)}$ appearing in the definition of the contractual service margin are essential quantities since profit or loss is defined in terms of the contractual service margin. The weights are derived from so-called coverage units. The estimation of coverage units for the group should ensure that the release of the contractual service margin into profit or loss is in accordance with how services are provided. The number of coverage units in a group correspond to ‘the quantity of coverage provided by the contracts in the group, determined by considering for each contract the quantity of the benefits provided under a contract and its expected coverage duration’ (§ B119 in IASB 2017a). No further details are provided in IASB (2017a), hence it is largely up to the judgement of the insurer. Minimal guidance is provided in IASB (2017b), apart from that IASB rejected allocating the contractual service margin based on the pattern of expected cash flows (§ BC279 in IASB 2017b). Our interpretation of coverage units is as follows:

Let $B_s^{(g,k)}$ be the quantity of the benefits provided under the k th contract in group g in period s , and let $T_t^{(g,k)}$ be the time until the coverage period ends for the contract as seen from time t , taking non-negative integer values. We define the coverage units $CU_s^{(t,g,k)}$ in period s , as seen from time $t \leq s$, as

$$CU_s^{(t,g,k)} := B_s^{(g,k)} I_{\{T_t^{(g,k)} > s-t-1\}},$$

where I is the indicator function. $CU_t^{(t,g,k)}$ is known at time t . Note that for $T_t^{(g,k)} = 0$, $CU_t^{(t,g,k)} = B_t^{(g,k)}$ are the last coverage units provided under the contract, since $CU_t^{(t,g,k)}$ denotes the coverage units provided in $(t-1, t]$. The coverage unit $CU_s^{(t,g,k)}$ is the sum of the coverage units $CU_s^{(t,g,k)}$ over the index k .

$W_t^{(g)}$ is defined as the ratio of the expected remaining number of coverage units at time t over the lifetime of the group to the total number of coverage units for the group. The latter is the sum of the number of coverage units provided in reporting period t and the expected remaining number of coverage units at time t . We write

$$W_t^{(g)} := \frac{\mathbb{E}_t[\sum_{s=t+1}^{t_0+\tau} \text{CU}_s^{(t,g)}]}{\text{CU}_t^{(t,g)} + \mathbb{E}_t[\sum_{s=t+1}^{t_0+\tau} \text{CU}_s^{(t,g)}]},$$

where $\mathbb{E}_t[\cdot]$ denotes the expectation conditional on information up to time t . For convenience we let $\text{ERCU}^{(t,g)} := \mathbb{E}_t[\sum_{s=t+1}^{t_0+\tau} \text{CU}_s^{(t,g)}]$ and write

$$W_t^{(g)} := \frac{\text{ERCU}^{(t,g)}}{\text{CU}_t^{(t,g)} + \text{ERCU}^{(t,g)}}.$$

For one contract, omitting the indices g and k , note that if B_s is known at time t for $s = t + 1, \dots, t_0 + \tau$, then

$$\mathbb{E}_t \left[\sum_{s=t+1}^{t_0+\tau} \text{CU}_s^{(t)} \right] = \sum_{s=t+1}^{t_0+\tau} B_s \mathbb{E}_t[\mathbb{I}_{\{T_t > s-t-1\}}] = \sum_{s=t+1}^{t_0+\tau} B_s \mathbb{P}_t(T_t > s-t-1).$$

Furthermore, if $B_s := B$ for $s = t + u + 1, \dots, t_0 + \tau$ and zero otherwise, where B is known at time t and $0 \leq u \leq t_0 + \tau - t - 1$, then

$$\mathbb{E}_t \left[\sum_{s=t+1}^{t_0+\tau} \text{CU}_s^{(t)} \right] = B \sum_{s=u+1}^{t_0+\tau-t} \mathbb{P}_t(T_t > s-1).$$

Note that if $u = 0$, then the above expectation equals $B\mathbb{E}_t[T_t]$, i.e. the expected remaining coverage units at time t for the contract is the constant quantity of benefits times the expected coverage duration.

We still need a more precise definition of the quantity of benefits B_t and the time until the coverage period ends T_t . To that end, we use the opinions of the IFRS 17 Transition Resource Group (TRG) (IASB 2018) as guidance when making the following definitions. For the case when the coverage period starts at time t , or has started before time t , we want $\mathbb{E}_t[T_t]$ to be the expected remaining coverage period for the contract. Hence T_t should take into account lapses and cancellations, as well as deaths and other insured events to the extent that they affect the remaining coverage period (e.g. a contract that is terminated after the insured event occurs). This means that, depending on the type of contract under consideration and the contract terms, T_t can be fixed as seen from time t , or it can be a random variable.

To define the quantity of the benefits B_t is more complex, since there are many different product types and contract terms. The definition needs to be consistent with § B119 in IASB (2017a) which states that the allocation of the contractual service margin to profit or loss in each period should reflect the services provided under the group in that period. The services provided to a contract in one period is the coverage in the form of economic compensation if the insured event occurs in that period. Hence, as noted by the TRG in IASB (2018), the quantity of the benefits should be based on the claims payment in case of a valid claim, which is not the same as the amount that the insurer expects to pay out per contract. Some possible methods for determining B_t mentioned in IASB (2018) are using the maximum contractual cover for the contract in each time period, or using the benefit amount that the policyholder is expected to receive if the insured event occurs in each time period. The former method might be appropriate for some life-insurance contracts, where a fixed benefit payment is specified in the contract terms and paid out if the insured person is still alive at some future time point.

This is how we have defined B_t in the examples provided below and in Section 5. The latter method might be appropriate when no maximum contractual cover is specified. As an example, consider a contract which provides income when the policyholder is unable to work due to disability, and the benefit the policyholder receives depends on policyholder's salary before the insured event occurs. For this type of contract the quantity of the benefits for future reporting periods can be determined as the current salary adjusted for some measure of inflation. Another way to determine the quantity of the benefits for a group of contracts with no maximum contractual cover specified could be to use some simple model for the claim severity, i.e. the average payment per claim, based on historical data for similar contracts, and incorporating a trend component or inflation as needed.

There are also cases where a maximum contractual cover is specified, but using it to determine B_t might not be appropriate. E.g. consider a group of contracts consisting of some contracts with a very high maximum contractual cover, and others with a much lower maximum contractual cover, but where the historical claim severity is similar for both contract types. In this case, it might be more suitable to define the quantity of the benefits based on a model for the claim severity rather than the contractual maximum cover, since the very high maximum cover for some of the contracts has essentially no economic effect on the performance of the group. It is up to each insurer to judge how B_t should be defined in each specific case, to ensure that the number of coverage units in each period reflects the services provided.

When a contract is terminated ahead of time (due to e.g. lapse of the contract or death of the policyholder), the number of expected remaining coverage units that would have remained had the contract not been terminated needs to be derecognised from the group (§ 76 in IASB 2017a). Our interpretation of derecognition of coverage units is that the coverage units provided in the period should be adjusted in a similar manner. This ensures that part of the contractual service margin related to the derecognised contract is recognised in profit or loss for the period. Hence, if contract k in group g is derecognised in period t , we let $\text{ERCU}^{(t,g,k)} = 0$. The number of expected remaining coverage units that are derecognised is given by $\mathbb{E}_t[\sum_{s=t+1}^{t_0+\tau} B_s^{(g,k)} I_{\{\tilde{T}_t^{(g,k)} > s-t-1\}}]$, where $\tilde{T}_t^{(g,k)}$ denotes the time until the coverage period ends for the contract as seen from time t , had the contract not been terminated in period t . Hence, we let the coverage units provided in the period be given by $\text{CU}_t^{(t,g,k)} = \mathbb{E}_t[\sum_{s=t}^{t_0+\tau} B_s^{(g,k)} I_{\{\tilde{T}_t^{(g,k)} > s-t-1\}}]$. Thus the total number of coverage units for the contract is the same as it would have been had the contract not been terminated, but due to the derecognition of the contract the full number of coverage units are provided in the period, and none remain at the end of the period.

The number of coverage units provided in period t and the expected remaining number of coverage units at time t for some life-insurance contracts are defined below. We consider a group g of $N_{t-1}^{(g)}$ contracts at time $t-1$. For the k th contract of the group, let $\text{CU}_t^{(g,k)}$ and $\text{ERCU}^{(t,g,k)}$ denote the corresponding coverage units and expected remaining coverage units and set

$$\text{CU}_t^{(t,g)} := \sum_{k=1}^{N_{t-1}^{(g)}} \text{CU}_t^{(t,g,k)}, \quad \text{ERCU}^{(t,g)} := \sum_{k=1}^{N_{t-1}^{(g)}} \text{ERCU}^{(t,g,k)}.$$

In the three examples below, consider the k th insured person in group g , x periods old at time t , whose remaining lifetime is $T_{x,t}$.

Example 3.1 (Survival benefit): Consider an insurance contract with survival benefit B , paid if the insured person is still alive at time $u + t$. If the insured person is alive at time t , then

$$\text{CU}_t^{(t,g,k)} := 0, \quad \text{ERCU}^{(t,g,k)} := B \mathbb{P}_t(T_{x,t} > u)$$

If the insured person dies in period t , then

$$\text{CU}_t^{(t,g,k)} := B \mathbb{P}_t(T_{x,t} > u), \quad \text{ERCU}^{(t,g,k)} := 0$$

Example 3.2 (Life annuity I): Consider a life annuity during the decumulation phase, with benefit B paid out at the beginning of each period if the insured person is alive. If the insured person is alive at time t , then

$$CU_t^{(t,g,k)} := B, \quad ERCU^{(t,g,k)} := B \sum_{s=1}^{\infty} \mathbb{P}_t(T_{x,t} > s - 1)$$

If the insured person dies in period t , then

$$CU_t^{(t,g,k)} := B \sum_{s=0}^{\infty} \mathbb{P}_t(T_{x,t} > s - 1), \quad ERCU^{(t,g,k)} := 0$$

Example 3.3 (Life annuity II): Consider a life annuity during the accumulation phase, with benefit B paid out at the beginning of each period after time $u + t$ if the insured person is still alive. For this type of contract the total number of coverage units is $B \sum_{s=1}^{\infty} \mathbb{P}(T_{x,t} > u + s - 1)$: If the insured person is alive at time t , then

$$CU_t^{(g,k)} := 0, \quad ERCU^{(t,g,k)} := B \sum_{s=1}^{\infty} \mathbb{P}_t(T_{x,t} > u + s - 1)$$

If the insured person dies in period t , then

$$CU_t^{(g,k)} := B \sum_{s=1}^{\infty} \mathbb{P}_t(T_{x,t} > u + s - 1), \quad ERCU^{(t,g,k)} := 0$$

4. Valuation of liability cash flows and allocation to subgroups

In IFRS 17 the valuation of insurance liabilities should consist of the fulfilment cash flows and the contractual service margin. The fulfilment cash flows consist of estimates of future cash flows, and a risk adjustment for non-financial risk. The estimate of future cash flows is defined as ‘the expected value (ie the probability-weighted mean) of the full range of possible outcomes’, and it should be adjusted ‘to reflect the time value of money and the financial risks related to the future cash flows’ (§§ 32–33 in IASB 2017a). This is similar to the definition of the technical provisions in Solvency II, which consists of a best estimate and a risk margin (Article 77, §§ 1–2 in DIRECTIVE 2009/138/EC n.d). However, there is a difference between the valuation in the two regulations. In Solvency II, the risk margin ‘shall be such as to ensure that the value of the technical provisions is equivalent to the amount that insurance and reinsurance undertakings would be expected to require in order to take over and meet the insurance and reinsurance obligations’ (Article 77, § 3 in DIRECTIVE 2009/138/EC n.d). In IFRS 17, the risk adjustment for non-financial risk is defined as an adjustment ‘to reflect the compensation that the entity requires for bearing the uncertainty about the amount and timing of the cash flows that arises from non-financial risk’ (§ 37 in IASB 2017a), and it should reflect the fact that insurers ‘generally fulfil insurance contracts directly over time by providing services to policyholders, rather than by transferring the contracts to a third party’ (§ BC17 in IASB 2017b). Hence, in IFRS 17 it is the company’s own perspective that should be taken into account, rather than that of a reference undertaking. In most other aspects, the definition of the risk adjustment is consistent with the principles underlying the calculation of the risk margin. However, in Solvency II the method for calculating the risk margin is prescribed by the regulation, which states that it should be ‘calculated by determining the cost of providing an amount of eligible own funds equal to the Solvency Capital Requirement necessary to support the insurance and reinsurance obligations over the lifetime thereof’ (Article 77, § 5 in DIRECTIVE 2009/138/EC n.d), and furthermore it specifies a formula for the calculation as well as certain parameter values. This is in contrast to IFRS 17 where there is no specification of what

technique should be used as long as the risk adjustment has some general characteristics (see further § B91 in IASB 2017a).

The risk margin in Solvency II has been criticised for lacking a proper definition and theoretical foundation, which is addressed in e.g. Möhr (2011). Through the framework developed in Engstner et al. (2017), building on the setup in Möhr (2011), a multi-period cost-of-capital value is computed, and given certain assumptions explicit computations are possible. This cost-of-capital valuation is consistent with the requirements on the risk adjustment in IFRS 17, if taking the company's own perspective on the level of the cost-of-capital rate and risk aversion, and not that of a reference undertaking. We choose to present the cost-of-capital value V_0 , corresponding to the fulfilment cash flows in IFRS 17, without separating it into components. For the case when the explicit components of the liability value are required, the risk adjustment can be identified as the second term (the Σ -term) in (9). Similarly, the risk adjustment allocated to a specific group of contracts corresponds to the second term in (10).

4.1. Multi-period cost-of-capital valuation

We value an aggregate insurance liability by considering a hypothetical transfer of the liability to a subsidiary, whose sole purpose is to manage the run-off of the liability. Hence, the subsidiary is a financially separate entity, but is owned by the original entity, and thus has the same view on risk and the compensation required for bearing that risk as the parent entity. Note that this construction is in line with the hypothetical transfer to a reference undertaking in Solvency II. We use the term subsidiary to emphasise that the view on risk and compensation should be the company's own, not that of a third party.

We consider a money market account numeraire, and express values and cash flows via this numeraire. The evolution of the money market account is determined by a short-rate model that is assumed consistent with a given set of discount factors.

The liabilities considered throughout this paper are entirely non-replicable by financial instruments and therefore we do not need to consider any replicating portfolio that may be transferred along with the liability except for a numeraire position: an amount of cash invested fully in the money market account. The liability can be identified with a stochastic cash flow $X = (X_t)_{t=1}^\tau$, where τ corresponds to the time τ periods after the transfer has occurred when the run-off is complete. Time $t = 0$ here stands for the actual valuation time when the hypothetical transfer of the liabilities is considered. By considering a discrete-time liability cash flow we implicitly make the simplifying assumption that cash flows occur only at the preset finite set of times.

Along with the transfer of the liability, a cash amount V_0 is transferred to the subsidiary. V_0 is the amount that the parent entity would require as compensation for bearing the non-financial risks in the subsidiary. Consequently, V_0 is defined as the value of the liability cash flow X . In order to compute the compensation for bearing the risks in the subsidiary, and thus V_0 , capital costs throughout the run-off must be considered. Capital costs originate from the need for the subsidiary to hold buffer capital. This buffer capital is not necessarily the same as the regulatory required capital, instead it is the amount of capital that the risk averse owner requires the subsidiary to hold to ensure that it can fulfil the obligations to policyholders in most situations. What is meant by most situations, and thus the amount of buffer capital required, is based on the degree of risk aversion of the parent entity. On the other hand the owner of the subsidiary does not need to fulfil the obligations to the policyholders in all situations. More precisely, the owner of the subsidiary has limited liability which here means that if the required buffer capital turns out to be insufficient to meet the obligations to policyholders, then the subsidiary may be terminated at no further cost upon transferring the buffer capital to the policyholders.

In order to make the arguments leading to the valuation of the liability cash flow in run-off precise, we consider the following mathematical setup. Let $(\mathcal{H}_t)_{t=0}^\tau$, with $\mathcal{H}_0 := \{\emptyset, \Omega\}$, be the run-off filtration representing the flow of information throughout the run-off of the liability. The cash flow X

is assumed adapted to $(\mathcal{H}_t)_{t=0}^{\tau}$ but this filtration may be larger than the filtration generated by X . For any $t = 1, \dots, \tau$, let V_t denote the value of the remaining liability cash flow at that time. Obviously, $V_{\tau} := 0$ since the run-off is complete at time τ . Let RC_t denote the required capital at time t .

At the time of the transfer of the liability from the insurer to the subsidiary, the subsidiary receives V_0 but is required to hold capital $RC_0 > V_0$. Hence, the parent entity injects $RC_0 - V_0$ in order to operate the subsidiary. At time 1 the policyholders demand X_1 and, given the information available at that time, the value of the remaining liability cash flow beyond time 1 is V_1 . Taking limited liability into account the payoff at time 1 for the owner of the subsidiary on the initial investment $RC_0 - V_0$ is therefore $(RC_0 - X_1 - V_1)^+$. However, the capital injection $RC_0 - V_0$ at time 0 would only be made available if the expected rate of return on this investment was sufficiently attractive:

$$\frac{\mathbb{E}[(RC_0 - X_1 - V_1)^+] - (RC_0 - V_0)}{RC_0 - V_0} = \eta_0 \quad (6)$$

for some cost-of-capital rate η_0 , which is determined by the parent entity, based on its required rate of return on capital invested in the subsidiary. The same reasoning holds at any time during the run-off, conditional on that the subsidiary has not been terminated at an earlier time. Therefore, the acceptability criterion at time t corresponding to (6) reads

$$V_t = RC_t - \frac{1}{1 + \eta_t} \mathbb{E}[(RC_t - X_{t+1} - V_{t+1})^+ | \mathcal{H}_t]. \quad (7)$$

If we were to assume, in line with the Solvency II regulatory framework, that at any time t the capital requirement is defined in terms of the conditional 99.5%-quantile of the $X_{t+1} + V_{t+1}$, conditional on \mathcal{H}_t , written

$$RC_t := \text{VaR}_{99.5\%}(X_{t+1} + V_{t+1} | \mathcal{H}_t), \quad (8)$$

then, upon replacing RC_t by the expression in (8) and noting that $V_{\tau} := 0$, we have arrived at a non-linear backward recursion for determining the value V_0 of the original liability. Notice that IFRS 17 does not prescribe the use of this specific risk measure.

Solving the backward recursion for $(V_t)_{t=0}^{\tau}$ numerically is in general a challenging task. However, under the strong assumption that the liability cash flow together with other variables generating the filtration can be described by a Gaussian process, and that the cost-of-capital rates $(\eta_t)_{t=0}^{\tau-1}$ are non-random, an explicit formula can be derived for V_0 :

$$V_0 = \mathbb{E}[R] + \sum_{t=0}^{\tau-1} c_t \left(\text{Var}(R | \mathcal{H}_t) - \text{Var}(R | \mathcal{H}_{t+1}) \right)^{1/2}, \quad (9)$$

where $R := \sum_{t=1}^{\tau} X_t$ and in case the risk measure is chosen as in (8):

$$c_t = \Phi^{-1}(0.995) - \frac{1}{1 + \eta_t} \left(0.995 \Phi^{-1}(0.995) + \varphi(\Phi^{-1}(0.995)) \right),$$

where Φ and φ denote the distribution and density function, respectively, of a standard normal random variable. If instead of $\text{VaR}_{99.5\%}(\cdot | \mathcal{H}_t)$ the risk measure Expected Shortfall at the level 99% is used, $\text{ES}_{99\%}(\cdot | \mathcal{H}_t)$,

$$c_t = \frac{\varphi(\Phi^{-1}(0.99))}{0.01} - \frac{1}{1 + \eta_t} \left(\frac{\varphi(\Phi^{-1}(0.99))}{0.01} \Phi \left(\frac{\varphi(\Phi^{-1}(0.99))}{0.01} \right) + \varphi \left(\frac{\varphi(\Phi^{-1}(0.99))}{0.01} \right) \right).$$

Notice again that IFRS 17 does not prescribe the use of this specific risk measure.

Since V_0 is non-random the expression (9) may look strange. However, due to the special property of conditional (co)variances for the multivariate normal distribution, the conditional variances in (9) are indeed non-random.

Remark 4.1 (Run-off filtration): We could take the filtration $(\mathcal{H}_t)_{t=0}^{\tau}$ to be the filtration generated by the cash flow $(X_t)_{t=1}^{\tau}$. However, when considering allocation of values to n groups it is more appropriate to consider the larger filtration generated by all the cash flows of the n groups, i.e. the filtration generated by $(X_t^{(k)})_{t=1}^{\tau}$, $k = 1, \dots, n$. In many cases, there may be relevant additional information available that can be expressed as the outcomes of a stochastic process. In any case we take $(\mathcal{H}_t)_{t=0}^{\tau}$ to be the filtration generated by a d -dimensional, where $d \geq n$, Gaussian process $(G_t)_{t=1}^{\tau}$ having $(X_t^{(k)})_{t=1}^{\tau}$, $k = 1, \dots, n$, as (a subset of its) marginal processes.

Remark 4.2 (Alternative valuation approaches): There are easily applied alternatives to the multi-period cost-of-capital valuation applied to the discounted outstanding liability R for which all the analysis in this paper holds with minor modification (and interpretation) of constants. The expression for V_0 in (9) corresponds to, under the aforementioned Gaussian model assumption, the solution to the backward recursion $V_t = \varphi_t(X_{t+1} + V_{t+1})$, $V_{\tau} := 0$, with

$$\varphi_t(Y) = \rho_t(Y) - \frac{1}{1 + \eta_t} \mathbb{E}[(\rho_t(Y) - Y)^+ | \mathcal{H}_t],$$

where ρ_t is a conditional version of VaR or ES given the information corresponding to \mathcal{H}_t (notice that here ρ_t is defined as operating on a loss variable rather than future net worth of a position, otherwise $\rho_t(Y)$ needs to be replaced by $\rho_t(-Y)$). For the expression for V_0 in (9) to hold, the only relevant mathematical properties of the map $\varphi_t : L^p(\mathcal{H}_{t+1}) \rightarrow L^p(\mathcal{H}_t)$ for $p \geq 1$ (where $L^p(\mathcal{H}_t)$ denotes the space of \mathcal{H}_t -measurable random variables Y with $\mathbb{E}[|Y|^p] < \infty$) are the following:

$$\begin{aligned} \text{if } \lambda \in L^p(\mathcal{H}_t), \quad & \text{then } \varphi_t(Y + \lambda) = \varphi_t(Y) + \lambda, \\ \text{if } \lambda \in [0, \infty), \quad & \text{then } \varphi_t(\lambda Y) = \lambda \varphi_t(Y), \\ \text{if } \mathbb{P}(Y \in A | \mathcal{H}_t) = \mathbb{P}(\tilde{Y} \in A | \mathcal{H}_t), \quad & \text{then } \varphi_t(Y) = \varphi_t(\tilde{Y}). \end{aligned}$$

Therefore, an application of the well-established standard deviation premium principle is straightforward since this corresponds to choosing $p = 2$ and

$$\varphi_t(Y) = \mathbb{E}[Y | \mathcal{H}_t] + \gamma_t \text{Var}(Y | \mathcal{H}_t)^{1/2},$$

where $(\gamma_t)_{t=0}^{\tau-1}$ is some sequence of non-negative constants. This choice of φ_t satisfies all requirements and gives the expression for V_0 in (9) with $c_t = \gamma_t$. There are many papers on multi-period valuation of insurance liabilities, many of which study a situation where the liability can be partly hedged by trading in financial assets. For further details, we refer to Barigou & Dhaene (2019), Delong et al. (2019a), Delong et al. (2019b), Engsner et al. (2020), Möhr (2011), Pelsser & Stadje (2014), Tsanakas et al. (2013) and the references therein.

Though IFRS 17 does not prescribe any specific technique for computing the risk adjustment, there is a requirement to disclose the confidence level that the result of the computation corresponds to (§ 119 in IASB 2017a). This requirement was included in order to supply users of financial statements with some means of comparing the risk adjustment between different companies, despite the fact of them potentially using different techniques for the calculation. This is not to say that IASB favours the confidence level technique over other techniques, but is instead a reflection of that the confidence level technique is the simplest to implement and that they did not want to burden companies with having to use more complex techniques for this benchmark in case the confidence level technique is deemed appropriate by the individual company (§§ BC215-BC217 in IASB 2017b).

From the Gaussian assumption follows immediately from (9) that

$$\mathbb{P}(R \leq V_0) = \Phi \left(\frac{\left(\sum_{t=0}^{\tau-1} c_t \left(\text{Var}(R | \mathcal{H}_t) - \text{Var}(R | \mathcal{H}_{t+1}) \right) \right)^{1/2}}{\text{Var}(R)^{1/2}} \right)$$

and if $\eta_t = \eta_0$ for all t then the expression simplifies further to

$$\mathbb{P}(R \leq V_0) = \Phi \left(c_0 \sum_{t=0}^{\tau-1} \left(\frac{\text{Var}(R | \mathcal{H}_t) - \text{Var}(R | \mathcal{H}_{t+1})}{\text{Var}(R)} \right)^{1/2} \right).$$

It is seen that the confidence level $\mathbb{P}(R \leq V_0)$ is an elementary function of η_0 and the variance profile, i.e. how the conditional variance of the outstanding liability cash flow decays to zero as the run-off progresses.

4.1.1. Allocations to contract groups

Since the risk adjustment in IFRS 17 should be based on the insurer's own view of risk, it can reflect diversification between different contract groups (§ B88 in IASB 2017a). Thus, many insurers would likely want to calculate the liability value for the company as a whole. However, the total value still needs to be allocated to a potentially large number of groups of contracts, since the measurement requirements in IFRS 17 need to be applied at this level.

A group of contracts in IFRS 17 is defined as follows: first the portfolio level of contracts has to be identified. A portfolio is a group of contracts 'subject to similar risks and managed together'. The next step is to subdivide each portfolio into a minimum of three groups depending on the estimated profitability of each contract at initial recognition. The first group consists of contracts that are onerous at initial recognition, the second group are contracts without any significant possibility of ever becoming onerous, and the third group consists of all other contracts. Finally, a group of contracts is only allowed to include contracts that are issued within one year of each other, hence for every future year newly issued contracts cannot be merged with previous groups of contracts, instead new groups will need to be formed. The group that an individual contract belongs to is decided when the contract is initially recognised and does not change subsequently – hence a group of onerous contracts can later on become profitable, and vice versa (see further §§ 14–24 in IASB 2017a).

A contract is onerous at initial recognition if the net of the fulfilment cash flows and any other cash flows arising from the contract is negative. For a group of contracts that is onerous, an immediate loss of this net value will have to be recognised in profit or loss. The allocation of the liability value to each group of contracts is thus important, since it can both influence if a group is onerous at initial recognition as well as if a group later on becomes onerous. For groups of contracts that are not onerous, the allocation of the liability value will instead influence the size of the contractual service margin (estimated future profits) for the group, which will be released into profit or loss over the lifetime of the group. Hence the allocation of the liability value to each group of contracts will directly affect the revenue stream for the company. This is a new regulatory concept compared to Solvency II, where the risk margin is allocated to lines of business (likely similar to the portfolio level of contracts in IFRS 17), but no further subdivision is needed, and no connection to any financial performance of the company is required.

IFRS 17 thus introduces a regulatory requirement on the allocation of risk being economically sound in order to ensure that the financial performance of the company reflects the true economic substance of the contracts. There is no other specific requirement in IFRS 17 on how the allocation should be made, only that the company 'is able to include the appropriate fulfilment cash flows in the measurement of the group . . . by allocating such estimates to the groups of contracts' (§ 24 in IASB 2017a). For individual companies it is likely important to have an allocation scheme that does not introduce unnecessary instability in the future revenue streams.

Consider n groups of contracts and write $R = \sum_{k=1}^n R_k$, where $R_k = \sum_{t=1}^{\tau} X_t^{(k)}$ is the sum of the (discounted) cash flow of group k throughout the run-off of the aggregate liability. The so-called Euler or gradient allocation of the value V_0 of R attributed to R_k is given by

$$\Lambda(R_k, R) = \mathbb{E}[R_k] + \sum_{t=0}^{\tau-1} c_t \frac{\text{Cov}(R_k, R \mid \mathcal{H}_t) - \text{Cov}(R_k, R \mid \mathcal{H}_{t+1})}{\left(\text{Var}(R \mid \mathcal{H}_t) - \text{Var}(R \mid \mathcal{H}_{t+1})\right)^{1/2}}. \quad (10)$$

The Euler allocation of the aggregate value to contract groups is sound in the sense that $V_0 = \Lambda(R, R)$ is fully allocated to the contract groups and, for each group, the value $\Lambda(R_k, R)$ allocated to the group does not exceed the corresponding stand-alone value $\Lambda(R_k, R_k)$ of the group's liability cash flow (obtained by replacing R by R_k in (9)):

$$\sum_{k=1}^n \Lambda(R_k, R) = \Lambda(R, R), \quad \Lambda(R_k, R) \leq \Lambda(R_k, R_k).$$

A proof of this statement is found in Appendix C in Palmberg et al. (2020).

5. Numerical insurance valuation and allocation to groups

We consider an insurance company issuing only contracts where claims payment is contingent on the policyholder being alive at certain future time points. We assume that the liability cash flow together with other variables generating the run-off filtration can be described by a Gaussian process, and that the cost-of-capital rate $\eta_t = \eta_0$ for $t = 0, 1, \dots, \tau$. Hence, the aggregate liability value $L^{(0)}$ is given by (9) with $c_t = c_0$ for $t = 0, 1, \dots, \tau$. Furthermore, the allocation of the value $L^{(0)}$ of R attributed to R_k (i.e. allocated to group k of contracts) is given by (10). For convenience we use a discount rate of zero across all reporting periods, i.e. we here disregard the effect of time value of money.

In order to calculate the liability value $L^{(0)}$ and allocate this value to the contract groups, we need estimates of the conditional variances and covariances in (9) and (10). Since the claims payment for all contracts are contingent on the policyholder still being alive, we need a model for the stochastic mortality rate. To this end, we estimate parameters of the Poisson log-bilinear model for mortality rates as proposed in Brouhns et al. (2002), where the Lee–Carter (Lee & Carter 1992) model for mortality rates is embedded in a Poisson regression model. For the estimation we use the R package StMoMo (Villegas et al. 2018), which defines the family of generalised age-period-cohort (GAPC) stochastic mortality models, of which the Poisson log-bilinear model in Brouhns et al. (2002) is a special case. The model assumes that the number of deaths $D_{x,t}$ in a population aged x years during period t satisfies

$$\mathcal{L}(D_{x,t} \mid E_{x,t}, \mu_{x,t}) = \text{Pois}(E_{x,t} \mu_{x,t}),$$

where $E_{x,t}$ is the so-called exposure to risk, and $\mu_{x,t}$ is the mortality rate at age x during period t . The mortality rate is modelled as

$$\log(\mu_{x,t}) = \alpha_x + \beta_x \kappa_t, \quad \kappa_t = \delta + \kappa_{t-1} + \xi_t,$$

where α_x measures the time-independent age effect, κ_t is the mortality trend, and β_x gives the sensitivity of the predictor at age x to variations in κ_t . The mortality trend κ_t is modelled as a Gaussian random walk with drift: δ is the drift parameter and (ξ_t) is an iid sequence with $\xi_t \sim N(0, \sigma_\kappa^2)$. The parameter constraints $\sum_t \kappa_t = 0$ and $\sum_x \beta_x = 1$, proposed in Lee & Carter (1992), are imposed to ensure model identification. The model parameters are estimated from data for Swedish males for year 1985 to 2018, for ages 0–90, from the Human Mortality Database (Human Mortality Database 2018) resulting in estimates $\hat{\delta} \approx -2.00$, $\hat{\sigma}_\kappa \approx 1.60$, and $(\hat{\alpha}_x)_{x=0}^{90}$, $(\hat{\beta}_x)_{x=0}^{90}$ and $(\hat{\kappa}_t)_{t=1985}^{2018}$ illustrated in Figure 1. We

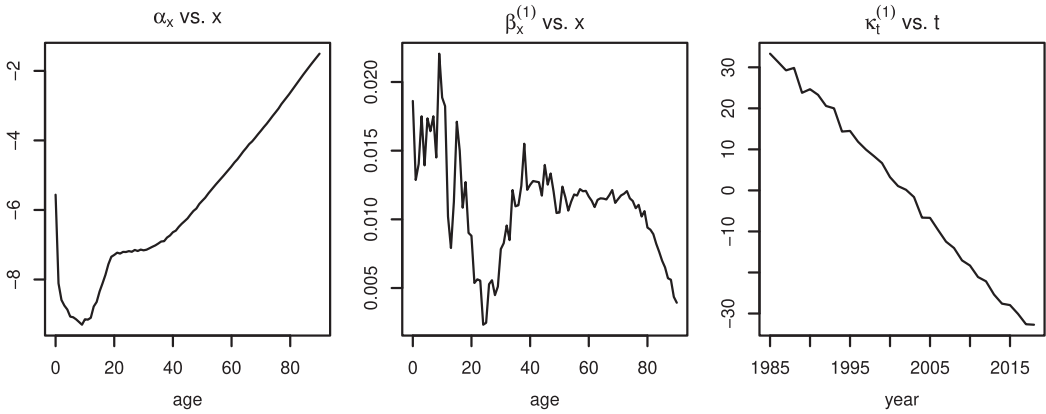


Figure 1. Parameter estimates for mortality model.

have chosen to exclude data for ages above 90 years in the estimation, due to data being sparse for these ages. Instead, we will use the estimated values $\hat{\alpha}_{90}$ and $\hat{\beta}_{90}$ for ages above 90 in our calculations. All estimated parameter values will be kept fixed for the future whole period modelled, i.e. no parameter risk is included.

5.1. Portfolio of life annuities

We consider an insurance company issuing only one type of contracts, a life annuity. If the insured person is still alive at age 65 years, a benefit of $B = 1$ is paid out in each reporting period until the period when the person dies, or the period when the insured person reaches age 100 years, whichever comes first. The annuity can be issued to individuals between the ages of 30 and 64 years. Denote the set of ages for which the benefit B is paid out by $PA := \{65, \dots, 100\}$.

Starting at time $t = -20$, new customers arrive according to a marked Poisson process, where the Poisson process has intensity $\lambda = 3000$, and the marks determine the age of the arriving customers according to a multinomial distribution. The parameters of the multinomial distribution are determined according to the proportion of different ages (between 30 and 64) of the general population of Sweden in year 2018 (age 30–40: 32.5%, age 41–50: 29.1%, age 51–64: 38.4%). Assuming that all contracts are profitable at the time of issue, we have one group of contracts per issuing year.

Let $N_{x,t}^{(g)}$ be the number of active contracts of x years old individuals belonging to group g at time t . We assume that deaths at time t of individuals conditional on the mortality rates at time $t-1$ are independent events. Hence, the number of active contracts at each time point t for groups of contracts issued before this time can be described as a nested binomial process as follows:

$$\mathcal{L}(N_{x,t}^{(g)} \mid N_{x-1,t-1}^{(g)}, \mu_{x-1,t-1}) = \text{Bin}(N_{x-1,t-1}^{(g)}, p_{x-1,t-1}),$$

with $p_{x-1,t-1} := \exp\{-\mu_{x-1,t-1}\}$.

We simulate one trajectory for the customer arrival process and one trajectory for the mortality trend $(\kappa_t)_{t=-19}^0$ over 20 years. At time $t = 0$ the outstanding liability consists of 20 groups of contracts, $\text{Gr}_0 = \{-19, \dots, 0\}$, and the liability cash flow $X := X^{(0)}$ takes the form

$$X = B \sum_{g \in \text{Gr}_0} \sum_{x \in PA} (N_{x,1}^{(g)}, \dots, N_{x,70}^{(g)}),$$

where $N_{x,t}^{(g)}$ denotes the number of x years old individuals at time t belonging to group g , under the assumption that no new contracts are issued after time 0. At time 0, the number $N_{x,0}^{(g)}$ of x years old

individuals belonging to group g is considered as a (non-random) model parameter $n_{x,0}^{(g)}$. Similarly, at time 0, the value κ_0 of the mortality trend is considered as a (non-random) model parameter. We denote by

$$X^{(g)} = B \sum_{x \in \text{PA}} \left(N_{x,1}^{(g)}, \dots, N_{x,70}^{(g)} \right), \quad g \in \text{Gr}_0,$$

the liability cash flows of the individual groups, and by

$$N_t^{(g)} = \sum_{x=30}^{100} N_{x,t}^{(g)}, \quad t \in \{1, \dots, 70\},$$

the total number of active contracts at time t in each group. We calculate the value $L^{(0)}$ of the outstanding liability according to (9), $L^{(0)} = V_0$, and allocate this value to the groups of contracts according to (10). The run-off filtration $(\mathcal{H}_t)_{t=0}^{70}$ is here taken to be the filtration generated by the cash flows of the 20 groups, and the total number of active contracts in each group:

$$\mathcal{H}_0 := \{\emptyset, \Omega\}, \quad \mathcal{H}_t := \sigma \left(X_t^{(g)}, N_t^{(g)}; g \in \text{Gr}_0 \right) \vee \mathcal{H}_{t-1}, \quad t = 1, \dots, 70.$$

In order to calculate the value of outstanding liability cash flows according to (9) we need to calculate $\mathbb{E}[R]$ and $\text{Var}(R \mid \mathcal{H}_t)$, where $R := \sum_{s=1}^{70} X_s$. Furthermore, to calculate the values allocated to contract groups according to (10) we need to calculate $\text{Cov}(R^{(g)}, R \mid \mathcal{H}_t)$, where $R^{(g)} := \sum_{s=1}^{70} X_s^{(g)}$. We calculate means and covariances based on the nested binomial process for the evolutions of active contracts over time, using the Lee–Carter model for the evolution of mortality rates. Given these means and covariances, the use of (9) and (10) means that we are assuming that

$$\left((X_t^{(g)}, N_t^{(g)}); g \in \text{Gr}_0, t \in \{1, \dots, 70\} \right)$$

is a random vector with a normal distribution. Although this is inconsistent with how the model for the cash flows and number of active contracts is defined, it is a reasonable approximation which enables efficient computation of all involved quantities.

The expectation $\mathbb{E}[R]$ of outstanding liability cash flows is given by

$$\begin{aligned} \mathbb{E}[R] &= B \sum_{s=1}^{70} \sum_{x \in \text{PA}} \mathbb{E}[N_{x,s}], \quad N_{x,t} := \sum_{g \in \text{Gr}_0} N_{x,t}^{(g)}, \\ \mathbb{E}[N_{x,t} \mid \mu] &= N_{x-t,0} \prod_{i=0}^{t-1} p_{x-t+i,i} = n_{x-t,0} \exp \left\{ - \sum_{i=0}^{t-1} \mu_{x-t+i,i} \right\}, \end{aligned}$$

where $n_{x-t,0} = \sum_{g \in \text{Gr}_0} n_{x-t,0}^{(g)}$. Taylor approximation yields

$$\mathbb{E}[N_{x,t} \mid \mu] \approx n_{x-t,0} \exp \left\{ - \sum_{i=0}^{t-1} \mathbb{E}[\mu_{x-t+i,i}] \right\} \left(1 - \sum_{i=0}^{t-1} (\mu_{x-t+i,i} - \mathbb{E}[\mu_{x-t+i,i}]) \right).$$

Hence,

$$\mathbb{E}[N_{x,s}] = \mathbb{E}[\mathbb{E}[N_{x,s} \mid \mu]] \approx n_{x-s,0} \exp \left\{ - \sum_{i=0}^{s-1} \mathbb{E}[\mu_{x-s+i,i}] \right\},$$

$$\mathbb{E}[\mu_{x,t}] = \exp \left\{ \alpha_x + \beta_x(t\delta + \kappa_0) + \frac{1}{2} \beta_x^2 t \sigma_\kappa^2 \right\}.$$

In order to calculate the variances $\text{Var}(R | \mathcal{H}_t)$ and covariances $\text{Cov}(R^{(g)}, R | \mathcal{H}_t)$ we first need to calculate the following covariances for the model at time 0:

$$\begin{aligned} \text{Cov}(X_t, X_s) &= B^2 \sum_{x,y \in \text{PA}} \text{Cov}(N_{x,t}, N_{y,s}), \\ \text{Cov}(X_t^{(g)}, X_s^{(h)}) &= B^2 \sum_{x,y \in \text{PA}} \text{Cov}(N_{x,t}^{(g)}, N_{y,s}^{(h)}), \\ \text{Cov}(X_t, X_s^{(g)}) &= B^2 \sum_{x,y \in \text{PA}} \sum_{h \in \text{Gr}_0} \text{Cov}(N_{x,t}^{(h)}, N_{y,s}^{(g)}). \end{aligned}$$

The calculation of the covariances above are found in Section B.1 in Palmberg et al. (2020).

Given the covariances above we form the high-dimensional covariance matrix $\Sigma^{(t)}$, for the model at time 0, of the random vector

$$\left(\sum_{s=t+1}^{70} X_s, \left(\sum_{s=t+1}^{70} X_s^{(g)} \right)_{g \in \text{Gr}_0}, \left((X_s^{(g)})_{s=1}^t, (N_s^{(g)})_{s=1}^t \right)_{g \in \text{Gr}_0} \right) \quad (11)$$

and notice that

$$\begin{aligned} \text{Var}(R | \mathcal{H}_t) &= \text{Var} \left(\sum_{s=t+1}^{70} X_s \mid \left((X_s^{(g)})_{s=1}^t, (N_s^{(g)})_{s=1}^t \right)_{g \in \text{Gr}_0} \right), \\ \text{Cov}(R^{(g)}, R | \mathcal{H}_t) &= \text{Cov} \left(\sum_{s=t+1}^{70} X_s^{(g)}, \sum_{s=t+1}^{70} X_s \mid \left((X_s^{(g)})_{s=1}^t, (N_s^{(g)})_{s=1}^t \right)_{g \in \text{Gr}_0} \right). \end{aligned}$$

We emphasise at this point that due to the assumed multivariate normality of the random vector in (11), these conditional variances and covariances are indeed non-random as seen from time 0. To calculate the conditional variances and covariances needed, we use properties of the multivariate normal distribution. For a multivariate normal vector $Z \sim N_n(\mu, \Sigma)$, write

$$\mu = \begin{pmatrix} \mu_{1:m} \\ \mu_{m+1:n} \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \Sigma_{1:m,1:m} & \Sigma_{1:m,m+1:n} \\ \Sigma_{m+1:n,1:m} & \Sigma_{m+1:n,m+1:n} \end{pmatrix}$$

for $m \in \{1, \dots, n-1\}$. Then the conditional distribution of (Z_1, \dots, Z_m) given (Z_{m+1}, \dots, Z_n) is multivariate normal with mean vector

$$\mu_{1:m|m+1:n} = \mu_{1:m} + \Sigma_{1:m,m+1:n} \Sigma_{m+1:n,m+1:n}^{-1} (Z_{m+1:n} - \mu_{m+1:n}),$$

assuming that the inverse $\Sigma_{m+1:n,m+1:n}^{-1}$ exists, and covariance matrix

$$\Sigma_{1:m|m+1:n} = \Sigma_{1:m,1:m} - \Sigma_{1:m,m+1:n} \Sigma_{m+1:n,m+1:n}^{-1} \Sigma_{m+1:n,1:m}.$$

Hence, the conditional distribution of $(\sum_{s=t+1}^{70} X_s, (\sum_{s=t+1}^{70} X_s^{(g)})_{g \in \text{Gr}_0})$ given $((X_s^{(g)})_{s=1}^t, (N_s^{(g)})_{s=1}^t)_{g \in \text{Gr}_0}$ is multivariate normal with covariance matrix

$$\Sigma_{1:m|m+1:n}^{(t)} = \Sigma_{1:m,1:m}^{(t)} - \Sigma_{1:m,m+1:n}^{(t)} (\Sigma_{m+1:n,m+1:n}^{(t)})^{-1} \Sigma_{m+1:n,1:m}^{(t)}$$

where $\Sigma^{(t)}$ is the covariance matrix for the random vector in (11).

The covariance structure of the liability cash flows $(X_t)_{t=1}^{70}$ seen from time $t = 0$ is shown in Figure 2. The left figure shows the terms $(\text{Var}(R | \mathcal{H}_t) - \text{Var}(R | \mathcal{H}_{t+1}))^{1/2}, t = 0, \dots, 69$, in the formula for the outstanding liability cash flow at time $t = 0$. The right figure shows the correlation matrix for the liability cash flows $(X_t)_{t=1}^{70}$ seen from time $t = 0$.

The allocation to contract groups is demonstrated in Figure 3. The upper-left figure corresponds to a 10% cost-of-capital rate, with parameters for the mortality model estimated from data for Swedish males, as described previously. Hence in this example, we have used $\sigma_\kappa = 1.60$ as the volatility for the mortality trend. From this figure it is clear that the liability value is just marginally larger than the expected value of the liability cash flows. However, we would expect a larger difference in a real world setting for a number of reasons. Firstly, in this example the only risk modelled is longevity risk, since we assume that the only expenses for the insurer are payments to policyholders. In reality, the insurer would also have administrative expenses, and thus be exposed to the risk that these expenses increase more than expected over the lifetime of the portfolio. Furthermore, we have only modelled contracts with a single premium paid upfront, and no right for the policyholder to terminate the contract ahead of time. With periodic premium payments or a surrender value being paid out if the contract is terminated by the policyholder, there would be uncertainty associated with future fees drawn by the insurer, giving rise to lapse risk. Both expense risk and lapse risk should be included in the valuation, as per the definition of non-financial risk in IASB (2017a).

Secondly, in this example we have estimated the parameters for the mortality model based on data for the whole population of Swedish males. When valuing the outstanding liability of an insurer, the mortality model would need to be adjusted to capture the mortality experience of the insurer, i.e. to a much smaller population. How to go about adjusting the mortality model based on data for a large population to the subset consisting of only insured persons, and further, to the even smaller population of an individual insurance company, is a question in its own right. It is however reasonable to assume that the variability in data for an insurer’s portfolio would be higher than in data for the population as a whole.

To conclude, the risk that an individual insurer is exposed to would generally be higher than what is captured by our example. In order to illustrate this effect in a simple manner without complicating our model, we have adjusted the volatility parameter σ_κ in the mortality trend, the results of which can be seen in Figure 3, where we also show the effect of changing the cost-of-capital rate from 10% to 20%. We emphasise that this is simply a way to illustrate the effect of overall higher volatility in the cash

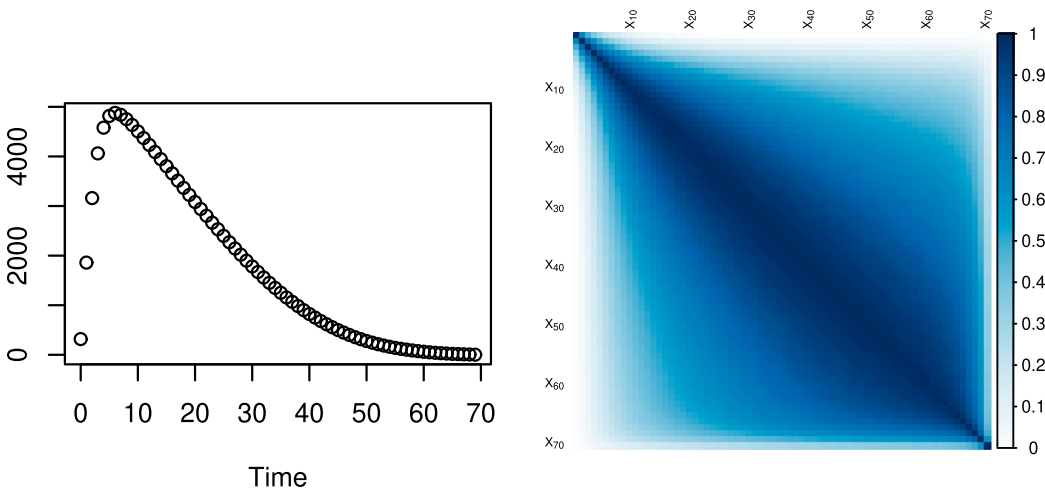


Figure 2. The left figure shows the terms $(\text{Var}(R | \mathcal{H}_t) - \text{Var}(R | \mathcal{H}_{t+1}))^{1/2}, t = 0, \dots, 69$, in the formula for the outstanding liability cash flow at time $t = 0$. The right figure shows the correlation matrix for the outstanding liability cash flow at time $t = 0$.

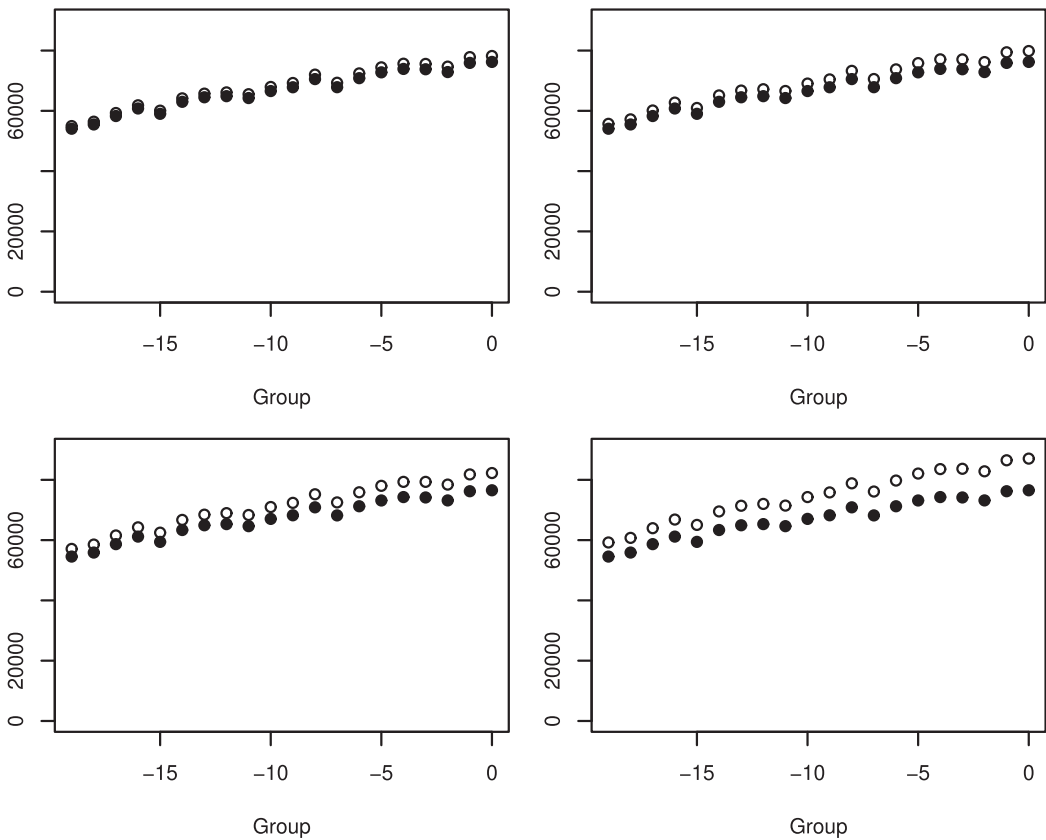


Figure 3. The figures show the allocation $\Lambda(R^{(g)}, R)$, $g = -19, \dots, 0$, of the liability value (empty circles) and the expected value of the outstanding liability cash flows (filled circles) at time $t = 0$ to the 20 groups of contracts with the following parameters: 10% cost-of-capital rate (left), 20% cost-of-capital rate (right), $\sigma_{\kappa} = 1.60$ (top), and $\sigma_{\kappa} = 4.81$ (bottom).

flow model than what can be captured in our simple setting, and should not be seen as a suitable way of adjusting the mortality model for an insurer. In fact, one can argue that the model for the mortality trend estimated for the whole population should be valid for smaller subsets of that population, at least long-term. Furthermore, even if the insurer has reason to believe that its particular portfolio consists of a certain subset of the population that would not see the same longevity improvements as the population as a whole, this is likely difficult to ascertain statistically for a portfolio of this size. One way of including the unsystematic mortality risk due to the size of the individual insurer's portfolio is given in Jarner & Møller (2015). In order to derive an alternative to the longevity risk calculations in the standard formula in Solvency II, they assume that the mortality rates of each insurer is proportional to the mortality rates for the whole insurance industry, with a mortality trend estimated based on data for the Danish population. However, the proportionality constant is unknown and has to be determined based on the insurer's mortality experience over a certain time period. The unsystematic risk is due to the fact that this proportionality constant needs to be reestimated every year.

5.2. Profit or loss for a portfolio of survival benefits

To illustrate the development of profit or loss and the contractual service margin in accordance with IFRS 17, we consider an insurance company that at time $t = 0$ issues 1000 insurance contracts to policyholders aged 50 years. The insurer make a single benefit payment $B = 1$ to each policyholder

who survives until age 70. All policyholders pay the same single premium P at time $t = 0$, and no new contracts are issued after this time, hence the liability consists of a single group of contracts, and the time of initial recognition is $t_0 = 0$. We simulate the financial performance of the company until all contracts are terminated, which happens either when the policyholder dies, or when the benefit is paid at time $\tau = 20$, whichever comes first.

The financial performance is calculated through the algorithm for calculating the contractual service margin and loss component (Definition 3.1) and profit or loss as in Definition 3.2. Since we only have one group of contracts, we drop the superscript (g) in the following. For this simple example, we note that $L_{\text{SP}}^{(t)} = 0$, $t = 0, \dots, \tau - 2$, since service is only provided at time τ (i.e. in period τ). Furthermore, since payments are made immediately as claims are incurred (when an insured person survives until age 70), $L_{\text{IC}}^{(t)} = 0$. Hence, $L^{(t)} = L_{\text{RC}}^{(t)} = L_{\text{FS}}^{(t)}$, $t = 0, \dots, \tau - 2$, and $L^{(\tau-1)} = L_{\text{RC}}^{(\tau-1)} = L_{\text{SP}}^{(\tau-1)}$. We also note that $L^{(-1)} = L^{(\tau)} = 0$.

We simulate five independent trajectories $(\kappa_t^{(i)})_{t=1}^{\tau}$, $i = 1, \dots, 5$, for the mortality trend, and, as in the previous example, we let the number of active contracts remaining at each time point t be described as a nested binomial process. For the i th mortality trend trajectory and at each time point t , the value $L^{(i,t)}$ of the outstanding liability is calculated according to (9) using the run-off filtration $(\mathcal{H}_s^{(i,t)})_{s=0}^{\tau-t}$ that applies to a hypothetical run-off starting at time t and finishing at time τ . Hence

$$L^{(i,t)} = \mathbb{E}[R^{(i,t)} \mid \mathcal{H}_0^{(i,t)}] + c_0 \sum_{s=0}^{\tau-1-t} \left(\text{Var}(R^{(i,t)} \mid \mathcal{H}_s^{(i,t)}) - \text{Var}(R^{(i,t)} \mid \mathcal{H}_{s+1}^{(i,t)}) \right)^{1/2}.$$

We let $N_s^{(i,t)}$ denote the number of active contracts at time $s \geq t$ for a run-off starting at time t , given a development of the mortality trend up to time t according to $(\kappa_s^{(i)})_{s=1}^t$. Seen from time t , $\kappa_t^{(i)}$ and $N_t^{(i,t)}$ are (non-random) model parameters. Since here we do not consider any new contracts, and the only cash flows to the policyholders are at $\tau = 20$, we obtain $R^{(i,t)} = N_{\tau}^{(i,t)}$. Furthermore, the run-off filtration $(\mathcal{H}_s^{(i,t)})_{s=0}^{\tau-t}$ only depends on the development of the number $N_{t+s}^{(i,t)}$ of contracts that are active at time $t + s$:

$$\mathcal{H}_0^{(i,t)} = \{\emptyset, \Omega\}, \quad \mathcal{H}_s^{(i,t)} = \sigma(N_{t+s}^{(i,t)}) \vee \mathcal{H}_{s-1}^{(i,t)}, \quad s = 1, \dots, \tau - t.$$

In order to use the algorithm for CSM and LC in Definition 3.1, we also need to calculate the weights $(U_t)_{t=1}^{\tau}$ and $(W_t)_{t=1}^{\tau}$. We choose $U_t^{(g)}$ according to IASB (2017c), see also Remark 3.2. $(W_t)_{t=1}^{\tau}$ are determined based on the coverage units provided in each period, and the expected remaining coverage units (see Section 3.1). Since all contracts considered in this example are survival benefits, the coverage units provided and remaining for each time point can be determined as in Example 1 in Section 3.1, hence if the k th insured person is alive at time t , then

$$\text{CU}_t^{(t,k)} := 0, \quad \text{ERCUCU}_t^{(t,k)} := B\mathbb{P}_t(T_{x,t} > 70 - x)$$

and if the k th insured person dies in period t , then

$$\text{CU}_t^{(t,k)} := B\mathbb{P}_t(T_{x,t} > 70 - x), \quad \text{ERCUCU}_t^{(t,k)} := 0,$$

where $T_{x,t}$ is the remaining lifetime of a randomly chosen individual who is x periods old at time t , and

$$\text{CU}_t^{(t)} := \sum_{k=1}^{N_{t-1}} \text{CU}_t^{(t,k)}, \quad \text{ERCUCU}_t^{(t)} := \sum_{k=1}^{N_t} \text{ERCUCU}_t^{(t,k)}$$

Assuming that the i th trajectory for the mortality trend corresponds to the observable information, the subscript t in \mathbb{P}_t means that the probability is calculated with respect to the model with parameters

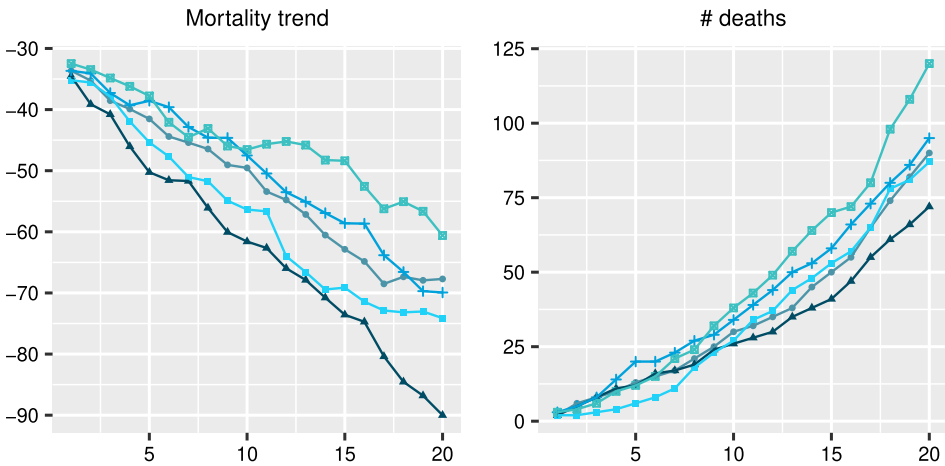


Figure 4. The left figure shows the trajectories for the mortality trend $\kappa_t^{(i)}$, $t = 1, \dots, \tau$, $i = 1, \dots, 5$. The right figure shows the trajectories for the accumulated number of deaths in the portfolio consisting of 1000 contracts.

(α_x) , (β_x) , δ , σ_κ and $\kappa_0^{(i,t)} := \kappa_t^{(i)}$. To calculate the probability at time t of an individual aged x at time t surviving until time $t + 70 - x$, $\mathbb{P}_t(T_{x,t} > 70 - x)$, we use the same approximations as when calculating the liability value.

$$\begin{aligned} \mathbb{P}_t(T_{x,t} > 70 - x) &= \mathbb{E}_t \left[\mathbb{P}_t \left(T_{x,t} > 70 - x \mid \mu^{(i,t)} \right) \right] = \mathbb{E}_t \left[\exp \left\{ - \sum_{s=0}^{69-x} \mu_{x+s,s}^{(i,t)} \right\} \right] \\ &\approx \exp \left\{ - \sum_{s=0}^{69-x} \mathbb{E}_t [\mu_{x+s,s}^{(i,t)}] \right\} \\ &= \exp \left\{ - \sum_{s=0}^{69-x} \exp \left\{ \alpha_{x+s} + \beta_{x+s}(s\delta + \kappa_0^{(i,t)}) + \frac{1}{2} \beta_{x+s}^2 s \sigma_\kappa^2 \right\} \right\}. \end{aligned}$$

The trajectories for the mortality trend $\kappa_t^{(i)}$, $t = 1 \dots, \tau$ are demonstrated in Figure 4, together with the trajectories for the accumulated number of deaths. The profit or loss, the contractual service margin and the loss component are shown in Figure 5. Note the different patterns of profit or loss depending on the size of the initial premium P compared to the initial liability value $L^{(0)}$. The initial premium is determined according to $nP = (1 + m)L^{(0)}$, where m is the margin added to the initial liability value. The figures on the left demonstrate profit or loss, contractual service margin and loss component when $m = 0$, i.e. no margin is added to the initial liability value when determining the premium. Note that the development of the contractual service margin and loss component is closely related to the trajectory for the mortality trend. Furthermore, the profit or loss for trajectories where a CSM is built up is either zero or slightly above zero, until time $\tau = 20$ when any remaining CSM is released into profit or loss. For trajectories where we instead have a positive LC, profit or loss can be both positive and negative, and tends to fluctuate around zero. These patterns are even clearer in the middle figures ($m = 0.1$) and left figures ($m = -0.1$). When the contract group is very profitable it will give rise to a positive CSM during the whole period, hence the insurer will be protected against losses since the CSM acts as a buffer against any adverse developments (as seen from the insurer's perspective) over the lifetime of the contracts. When the contract group is loss-making from its recognition, this loss has to be realised immediately, and any consecutive changes will directly affect profit or loss, leading to higher volatility in the financial performance.

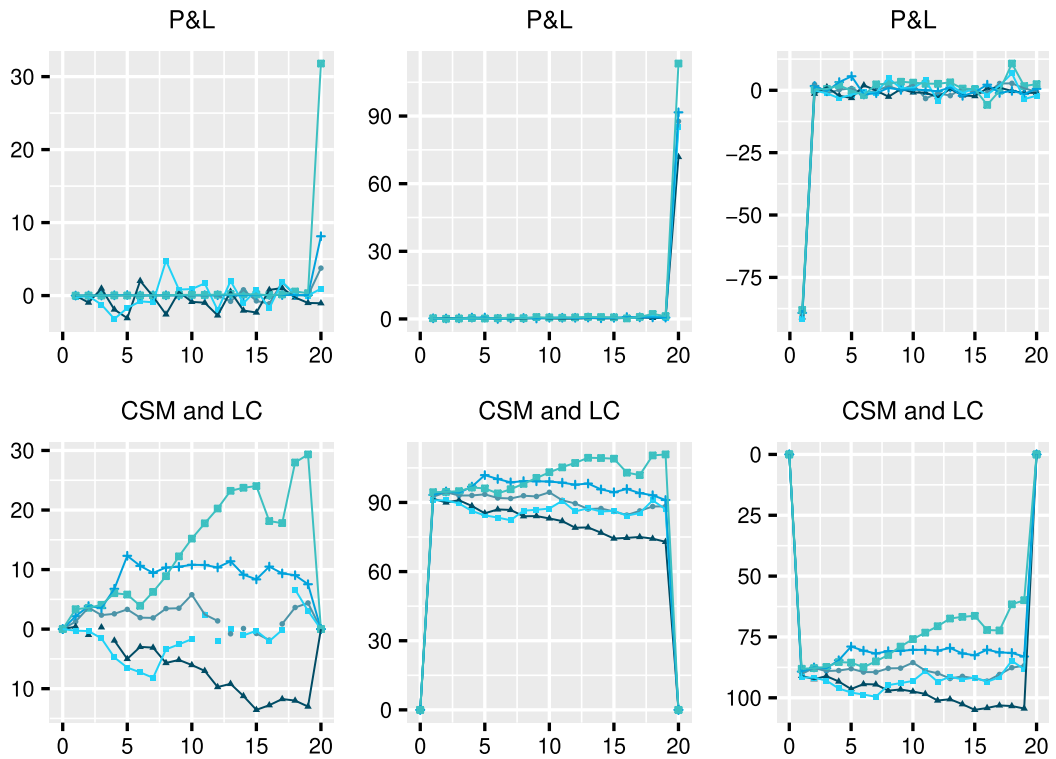


Figure 5. P&L (top), CSM and LC (bottom) for a portfolio of 1000 policyholders, with margin 0 (left), margin 10% (middle), and margin -10% (right).

Disclosure statement

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