Systems Science & Control Engineering
An Open Access Journal

ISSN: (Print) (Online) Journal homepage: https://www.tandfonline.com/loi/tssc20

# Event-triggered consensus control for general second-order multi-agent systems

Fan Liu & Zhijian Ji

SYSTEMS SCIENCE & CONTROL ENGINEERING: AN OPEN ACCESS JOURNAL
https://doi.org/10.1080/21642583.2020.1826003



# Event-triggered consensus control for general second-order multi-agent systems

Fan Liu and Zhijian Ji

Institute of Complexity Science, College of Automation, Qingdao University, Qingdao, People's Republic of China

**ABSTRACT**
The consensus problem of multi-agent systems with general second-order dynamics is studied. A distributed event-triggering strategy is proposed to reduce the communication frequency of agents and the update frequency of event-triggering controller. Under the fixed topology, a consensus protocol and event-triggering function are designed for each agent. The sufficient conditions of consensus are obtained by the stability theory, and the theoretical proof of excluding Zeno behaviour is presented. Finally, a simulation example is given to illustrate the effectiveness of the obtained results.

**ARTICLE HISTORY**
Received 23 June 2020
Accepted 16 September 2020

**KEYWORDS**
Consensus; multi-agent systems; general second-order; event-triggered

## 1. Introduction

In recent years, the cooperative control of multi-agent systems has become the research focus of many scholars (Ji et al., 2020; Ji & Yu, 2017; Lou et al., 2020; Qin & Yu, 2013; Qu et al., 2020; Zou et al., 2019, 2020). As an important basis of cooperative control, consensus is widely used in robot formation control, UAV task assignment, smart grid and so on. Therefore, great achievements have been obtained in the study of consensus problem (Liu et al., 2017; Richert & Cortés, 2013; Sakurama & Ahn, 2019; Sun et al., 2020, 2019; Zhang et al., 2019).

In the early research, in order to achieve consensus of multi-agent systems, it is usually assumed that agents exchange information continuously with their neighbours through the local network. However, in reality, the reserve energy of agents and the bandwidth of the network are limited. In order to ensure the stability of system performance and reduce the consumption of resources, a new control method needs to be designed. After leading into the event-triggered control strategy, the agent transmits information after completing specific tasks for the controller to update the received information, which effectively avoids continuous information exchange. Based on the event-triggered control, a centralized control strategy was proposed after the first-order multi-agent system was studied in Dimarogonas and Johansson (2009), and all agents share a triggering function. In Dimarogonas et al. (2012), a distributed control strategy was proposed to design a triggering function for each agent. Compared with Dimarogonas and Johansson (2009), the distributed control can reduce the communication frequency more effectively. In Xie et al. (2015), the consensus problem of the second-order multi-agent system under the directed graph is studied. The centralized and distributed control strategies were given respectively, and a new control protocol was designed. In Li et al. (2015), the consensus of second-order tracking control was studied under directed fixed topology and switching topology, and a new distributed event-triggered sampling scheme was proposed. The consensus problem of tracking control for two order multi-agent systems with nonlinear dynamics was studied in Zhao et al. (2018). In Cao et al. (2015), the consensus problem of second-order multi-agent systems was discussed by using sampling control and edge event driven control strategies. The mixed control strategy of periodic sampling and event driven was given in Liu and Ji (2017) and Cao et al. (2016). A fast convergence method for achieving consensus was studied in Qu et al. (2018). In Liang et al. (2019), the consensus problem of general linear systems based on the information of event-triggered state and event-triggered observer was studied. Hou et al. (2017) discussed the consensus conditions of general second-order multi-agent systems with communication delay.

According to the above work, at present, the research on the consensus problems of second-order systems is mostly focused on second-order integrators, while the results about general second-order systems are relatively few. And in the study of event-triggered control results, continuous communication still exists in event-triggered detection. Therefore, we further study the event-triggered consensus problems of general second-order systems, and propose a new event-triggered

**CONTACT** Zhijian Ji jizhijian@pku.org.cn

control scheme. The triggering conditions in the scheme only use the information received at the event-triggering instant, and Zeno behaviour is excluded. The sufficient conditions for the system state to achieve consensus are obtained.

The paper is organized as follows: Section 2 gives the definitions that will be used in this paper. Section 3 discusses the consensus conditions based on event-triggered control. The simulation example in Section 4 verifies the validity of the theoretical derivation. Finally, Section 5 presents the conclusion of this paper.

*Notations*: In this paper, the symbol $\mathbb{R}^n$ represents the $n$ dimensional real vectors space. The symbol $\mathbb{R}^{m\times n}$ represents the sets of $m \times n$ dimensional real matrix. $I_N$ denotes the $N$ dimensional identity matrix. $\mathbf{1}$ indicates an appropriate dimensional column vector composed of 1.

## 2. Preliminaries

The information exchange topology network among $n$ agents can be depicted by an undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ with $N$ nodes. $\mathcal{V} = \{1, 2, \ldots, N\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of nodes and the set of edges, respectively. $\varepsilon_{i,j} \in \mathcal{E}$ means that nodes $i$ and $j$ are neighbours of each other, and node $i$ can receive information from node $j$. If there is a path between a point and all other points, then the undirected graph is connected. The associated adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is defined as $a_{ii} = 0$, $a_{ij} = 1$, if $\varepsilon_{i,j} \in \mathcal{E}$, and $a_{ij} = 0$ otherwise. The in-degree of node $i$ is called $d_i = \sum_{j=1}^N a_{ij}$, and the degree matrix is $D = \mathrm{diag}\{d_1, \ldots, d_N\}$. The Laplacian matrix $L$ of $\mathcal{G}$ is $L = D - \mathcal{A}$.

In this paper, we consider a multi-agent system composed of $N$ agents, each agent with general second-order linear dynamics is described as

$$\begin{cases} \dot{x}_i(t) = v_i(t) \\ \dot{v}_i(t) = ax_i(t) + bv_i(t) + u_i(t) \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}$, $v_i(t) \in \mathbb{R}$ and $u_i(t) \in \mathbb{R}$ represent the position, velocity and control input of the $i$th agent, respectively. $a$ and $b$ denote any real numbers. This paper will discuss the consensus problem of system (1), i.e. $\lim_{t\to\infty} |x_i - x_j| = 0$, $\lim_{t\to\infty} |v_i - v_j| = 0$, $i, j = 1, \ldots, N$, so that the system (1) can achieve consensus under event-triggered control mechanism and avoid Zeno behaviour.

**Assumption 2.1:** *The undirected topology $\mathcal{G}$ is connected.*

## 3. Main results

### 3.1. Event-triggered control

In this section, we propose a distributed event-triggered control strategy. $t_k^i$ denotes the $k$th event-triggering instant of agent $i$. The measurement error of agent $i$ is defined as $e_{xi}(t) = \tilde{x}_i(t) - x_i(t)$, $e_{vi}(t) = \tilde{v}_i(t) - v_i(t)$, where $\tilde{x}_i = x_i(t_k^i)$, $\tilde{v}_i = v_i(t_k^i)$, $t \in [t_k^i, t_{k+1}^i)$. The consensus protocol designed for each agent is

$$u_i(t) = -k_1 \sum_{j=1}^N a_{ij}(\tilde{x}_i(t) - \tilde{x}_j(t)) - k_2 \sum_{j=1}^N a_{ij}(\tilde{v}_i(t) - \tilde{v}_j(t)) \tag{2}$$

where $k_1$ and $k_2$ are nonnegative real numbers. Event-triggering instant sequence $\{t_k^i\}$ of agent $i$ is defined as:

$$t_{k+1}^i = \inf\{t > t_k^i | f_i(t) \geq 0\}$$

$\{t_k^i\}$ is determined by the following inequality:

$$f_i(t) = e_{xi}^2(t) + e_{vi}^2(t) - \frac{\eta}{\mu} \sum_{j=1}^N \frac{a_{ij}}{d_i}((\tilde{x}_i(t) - \tilde{x}_j(t))^2 + (\tilde{v}_i(t) - \tilde{v}_j(t))^2) - \frac{e^{-\nu t}}{d_i} \geq 0 \tag{3}$$

That is, $f_i(t)$ is the trigger function, where $\eta = \min\{\frac{k_1}{56}, \frac{k_2}{56}\}$, $\mu = \max\{k_1, k_2\}$ and $\nu$ are positive real number.

**Remark 3.1:** The event-triggering condition (4) in this article utilizes the latest event-triggered state values. The introduction of exponential term can exclude Zeno behaviour. However, the event-triggering conditions in the existing references generally utilized the real-time state values of each agent and its neighbours. Therefore, the event-triggering condition (3) effectively reduces the communication frequency.

To facilitate analysis, define $z = [z_x^T, z_v^T]^T$, $z_x = [z_{x1}^T, z_{x2}^T, \ldots, z_{xN}^T]^T$, $z_v = [z_{v1}^T, z_{v2}^T, \ldots, z_{vN}^T]^T$, $z_{xi} = x_i - \frac{1}{N}\sum_{j=1}^N x_j$, $z_{vi} = v_i - \frac{1}{N}\sum_{j=1}^N v_j$. The compact form of $z$ is $z = (I_2 \otimes M)y$, where $M = I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T$, $x = [x_1^T, x_2^T, \ldots, x_N^T]^T$, $v = [v_1^T, v_2^T, \ldots, v_N^T]^T$, $y = [x^T, v^T]^T$, We know that $z = 0$ if and only if $x_1 = x_2 = \ldots = x_N$, $v_1 = v_2 = \ldots = v_N$. Therefore, we can call vector $z$ the consensus error vector. Let $\tilde{y} = [\tilde{x}^T, \tilde{v}^T]^T$, $\tilde{x} = [\tilde{x}_1^T, \tilde{x}_2^T, \ldots, \tilde{x}_N^T]^T$, $\tilde{v} = [\tilde{v}_1^T, \tilde{v}_2^T, \ldots, \tilde{v}_N^T]^T$. From (1) and (2), we can get the error vector $z$ satisfying the following dynamics:

$$\dot{z}(t) = \begin{bmatrix} Mv(t) \\ M(ax(t) + bv(t) - k_1 L\tilde{x}(t) - k_2\tilde{v}(t)) \end{bmatrix} = \begin{bmatrix} 0 & I_N \\ aI_N & bI_N \end{bmatrix} z(t) - \begin{bmatrix} 0 & 0 \\ k_1 L & k_2 L \end{bmatrix} \tilde{y}(t) \tag{4}$$

**Lemma 3.1 (Olfati-Saber & Murray, 2004):** *For an undirected graph $\mathcal{G}$0 is an eigenvalue of $L$ if and only if $\mathcal{G}$ is connected and the minimum non-zero eigenvalue $\lambda_2(L)$ of $L$ is*

$$\lambda_2(L) = \min_{x\neq 0, \mathbf{1}^T x = 0} \frac{x^T L x}{x^T x}$$

**Lemma 3.2 (Nowzari & Cortes, 2016):** *Given any $x, y \in \mathbb{R}$ and $\epsilon \in \mathbb{R}$, $xy \le \frac{x^2}{2\epsilon} + \frac{\epsilon y^2}{2}$*

**Lemma 3.3 (Boyd et al., 1991):** *For linear matrix inequalities:*

$$\begin{bmatrix} R_1(x) & E(x) \\ E^T(x) & R_2(x) \end{bmatrix} \ge 0$$

*where $R_1(x) = R_1^T(x)$, $R_2(x) = R_2^T(x)$, The above inequality satisfies the following equivalent conditions:*

$$(1) R_2(x) > 0, R_1(x) - E^T(x) R_2^{-1}(x) E(x) \ge 0$$
$$(2) R_1(x) > 0, R_2(x) - E(x) R_1^{-1}(x) E^T(x) \ge 0$$

**Lemma 3.4 (Li & Duan, 2014):** *Consider a differential equation $\frac{\mathrm{d}u}{\mathrm{d}t} = f(t,u)$, $u(t_0) = u_0$, $t \ge t_0$, where $f(t,u)$ is continuous and satisfies the local Lipschitz condition in $u$. Let $[t_0, T)$ be the maximum existence interval of the solution $u(t)$, where $T$ can be infinite. If, for any $t \in [t_0, T)$, $v = v(t)$ satisfies*

$$\frac{\mathrm{d}s}{\mathrm{d}t} \le f(t,s), \quad s(t_0) \le u_0$$

*then $v(t) \le u(t), t \in [t_0, T)$.*

**Theorem 3.1:** *Consensus protocol (2) and event-triggering condition (3) enable multi-agent system (1) to achieve consensus if Assumption 2.1 holds and parameters a, b, $k_1$ and $k_2$ satisfy*

$$b \le \left(\frac{1}{4}k_2 - \frac{1}{8}k_1\right)\lambda_2(L) - 1 \tag{5}$$

$$a \le \left(\frac{1}{4}k_1 - \frac{1}{8}k_2\right)\lambda_2(L) - \frac{(1+a+b)^2}{\left(k_2 - \frac{1}{2}k_1\right)\lambda_2(L) - 4 - 4b} \tag{6}$$

$$\frac{1}{2} \le \frac{k_1}{k_2} \le 2 \tag{7}$$

***Proof:*** Consider a Lyapunov functions constructed as follows:

$$V(t) = \frac{1}{2} z^T(t) \begin{bmatrix} \delta L + I_N & I_N \\ I_N & I_N \end{bmatrix} z(t) \tag{8}$$

where $\delta = k_1 + k_2$, and because of $I_N > 0$, $\delta L + I_N - I_N \ge 0$, according to Lemma 3.3 , $V(t) \ge 0$. The derivative of $V_1$ along the trajectory of (4) is

$$\begin{aligned} \dot{V} &= z^T \begin{bmatrix} \delta L + I_N & I_N \\ I_N & I_N \end{bmatrix} \dot{z} \\ &= \begin{bmatrix} z_x^T + \delta z_x^T L + z_v^T & z_x^T + z_v^T \end{bmatrix} \\ &\quad \times \left( \begin{bmatrix} z_v \\ a z_x + b z_v \end{bmatrix} - \begin{bmatrix} 0 \\ k_1 L\tilde{x} + k_2 L\tilde{v} \end{bmatrix} \right) \\ &= z_x^T + \delta z_x^T L z_v + z_v^T z_x + a z_x^T z_v + b z_v^T z_x + b z_v^T z_v \\ &\quad - k_1 z_x^T L\tilde{x} - k_2 z_x^T L\tilde{v} - k_1 z_v^T L\tilde{x} - k_2 z_v^T L\tilde{v} \end{aligned} \tag{9}$$

From $a_{ij} = a_{ji}$, $z_{xi} - z_{xj} = x_i - x_j = \tilde{x}_i - \tilde{x}_j - e_{xi} + e_{xj}$ and Lemma 3.2, $-z_x^T L\tilde{x}$ and $z_x^T L\tilde{v}$ in (6) can be amplified and transformed respectively.

$$\begin{aligned} & - z_x^T L\tilde{x} \\ &= -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(x_i - x_j)(\tilde{x}_i - \tilde{x}_j) \\ &= -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\tilde{x}_i - \tilde{x}_j)(\tilde{x}_i - \tilde{x}_j) \\ &\quad + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(e_{xi} - e_{xj})(\tilde{x}_i - \tilde{x}_j) \\ &\le -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\tilde{x}_i - \tilde{x}_j)(\tilde{x}_i - \tilde{x}_j) \\ &\quad + \frac{1}{8}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\tilde{x}_i - \tilde{x}_j)(\tilde{x}_i - \tilde{x}_j) \\ &\quad + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(e_{xi} - e_{xj})(e_{xi} - e_{xj}) \\ &\le -\frac{3}{8}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\tilde{x}_i - \tilde{x}_j)(\tilde{x}_i - \tilde{x}_j) + 2\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_{xi}^2 \\ &= -\frac{1}{8}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\tilde{x}_i - \tilde{x}_j)(\tilde{x}_i - \tilde{x}_j) + 2\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_{xi}^2 \\ &\quad - \frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(z_{xi} - z_{xj})(z_{xi} - z_{xj}) \\ &\quad - \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(z_{xi} - z_{xj})(e_{xi} - e_{xj}) \\ &\quad - \frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(e_{xi} - e_{xj})(e_{xi} - e_{xj}) \\ &\le -\frac{1}{8}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\tilde{x}_i - \tilde{x}_j)(\tilde{x}_i - \tilde{x}_j) + 2\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_{xi}^2 \\ &\quad - \frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(z_{xi} - z_{xj})(z_{xi} - z_{xj}) \\ &\quad + \frac{1}{8}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(z_{xi} - z_{xj})(z_{xi} - z_{xj}) \\ &\quad + \frac{1}{4}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(e_{xi} - e_{xj})(e_{xi} - e_{xj}) \end{aligned}$$

$$= -\frac{1}{8}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\tilde{x}_i - \tilde{x}_j)^2 + 3\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_{xi}^2 - \frac{1}{8}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(z_{xi} - z_{xj})^2 \quad (10)$$

$-z_x^T L\tilde{v}$ is transformed as follows:

$$\begin{aligned} -z_x^T L\tilde{v} &= -z_x^T Lz_v - z_x^T Le_v \\ &\le z_x^T Lz_v + \frac{1}{8}z_x^T Lz_x + 2e_v^T Le_v \\ &\le z_x^T Lz_v + \frac{1}{8}z_x^T Lz_x + 4\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_{vi}^2 \end{aligned} \quad (11)$$

Similarly, according to $a_{ij} = a_{ji}, z_{vi} - z_{vj} = v_i - v_j = \tilde{v}_i - \tilde{v}_j - e_{vi} + e_{xi}$ and Lemma 3.2, we can obtain

$$-z_v^T L\tilde{v} \le -\frac{1}{8}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\tilde{v}_i - \tilde{v}_j)^2 + 3\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_{vi}^2 - \frac{1}{8}\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(z_{vi} - z_{vj})^2 \quad (12)$$

$$-z_v^T L\tilde{x} \le -z_v^T Lz_x + \frac{1}{8}z_v^T Lz_v + 4\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_{xi}^2 \quad (13)$$

Substitute (10)–(13) into (9) to get

$$\begin{aligned} \dot{V} \le\ & z_x^T\left(aI_N - \left(\frac{1}{4}k_1 - \frac{1}{8}k_2\right)L\right)z_x \\ &+ z_x^T((1+a+b)I_N - (k_2 + k_1 - \delta)L)z_v \\ &+ z_v^T\left((1+b)I_N - \left(\frac{1}{4}k_2 - \frac{1}{8}k_1\right)L\right)z_v \\ &+ 7k_1\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_{xi}^2 + 7k_2\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}e_{vi}^2 \\ &- \frac{1}{8}k_1\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\tilde{x}_i - \tilde{x}_j)^2 - \frac{1}{8}k_2\sum_{i=1}^{N}\sum_{j=1}^{N} a_{ij}(\tilde{v}_i - \tilde{v}_j)^2 \end{aligned} \quad (14)$$

Choose $\frac{1}{2} \le \frac{k_1}{k_2} \le 2$. On the basis of Lemma 3.1, we can get that

$$\begin{aligned} \dot{V} \le\ & \left(a - \left(\frac{1}{4}k_1 - \frac{1}{8}k_2\right)\lambda_2(L)\right)z_x^T z_x + (1+a+b)z_x^T z_v \\ &- \left(1 + b - \left(\frac{1}{4}k_2 - \frac{1}{8}k_1\right)\lambda_2(L)\right)z_v^T z_v \\ &- 7\mu\sum_{i=1}^{N}(\sum_{j=1}^{N} a_{ij}(e_{xi}^2 + e_{vi}^2) \\ &- \frac{\eta}{\mu}\sum_{j=1}^{N} a_{ij}((\tilde{x}_i - \tilde{x}_j)^2 + (\tilde{v}_i - \tilde{v}_j)^2)) \\ \le\ & -\frac{1}{2}z^T\begin{bmatrix} \left(\left(\frac{1}{2}k_1 - \frac{1}{4}k_2\right)\lambda_2(L) - 2a\right)I_N & -(1+a+b)I_N \\ -((1+a+b)I_N & (\frac{1}{2}k_2 - \frac{1}{4}k_1)\lambda_2(L) - 2 - 2b)I_N \end{bmatrix} z - N\mu e^{\nu t} \end{aligned} \quad (15)$$

Let

$$\begin{bmatrix} \left(\left(\frac{1}{2}k_1 - \frac{1}{4}k_2\right)\lambda_2(L) - 2a\right)I_N & -(1+a+b)I_N \\ -((1+a+b)I_N & \left(\frac{1}{2}k_2 - \frac{1}{4}k_1\right)\lambda_2(L) - 2 - 2b\right)I_N \end{bmatrix} \ge 0$$

According to Lemma 3.3, if the above conditions are established, the conditions should be satisfied as follows:

$$b \le \left(\frac{1}{4}k_2 - \frac{1}{8}k_1\right)\lambda_2(L) - 1, a \le \left(\frac{1}{4}k_1 - \frac{1}{8}k_2\right)\lambda_2(L) - \frac{(1+a+b)^2}{\left(k_2 - \frac{1}{2}k_1\right)\lambda_2(L) - 4 - 4b}$$

Obviously, under the conditions of (5)–(7), $\dot{V}(t) \le 0$. The error system (4) is asymptotically stable, and the consensus of multi-agent system (1) can be achieved. ■

### 3.2. Event interval analysis

Next, the theorem of excluding Zeno behaviour is proposed.

**Theorem 3.2:** *The multi-agent system* (1) *does not exhibit Zeno behaviour under the consensus protocol* (2) *and the event-triggering condition* (3).

For agent $i$, the event-triggering function (3) shows that the event interval is the time of $e_{xi}^2 + e_{vi}^2$ from 0 to the threshold. From the definition of $e_{xi}$, $e_{vi}$, (1) and (3), we can get that the derivative of $e_{xi}^2 + e_{vi}^2$ with respect to $t \in [t_k^i, t_{k+1}^i)$ is as follows:

$$\begin{aligned} &\frac{\mathrm{d}(e_{xi}^2(t) + e_{vi}^2(t))}{\mathrm{d}t} \\ &= 2e_{xi}(t)\dot{e}_{xi}(t) + 2e_{vi}(t)\dot{e}_{vi}(t) \\ &= 2e_{xi}(t)(e_{vi}(t) - \tilde{v}_i(t)) + 2e_{vi}(t)(ae_{xi}(t) \\ &\quad + be_{vi}(t) - u_i(t) - a\tilde{x}_i(t) - b\tilde{v}_i(t)) \\ &\le (2+a)e_{xi}^2(t) + (2+a+b)e_{vi}^2(t) + \tilde{v}_i^2(t) \\ &\quad + (u_i(t) - a\tilde{v}_i(t) - b\tilde{v}_i(t))^2 \\ &\le (2+a+b)(e_{xi}^2(t) + e_{vi}^2(t)) + \tilde{v}_i^2(t) + (u_i(t) \end{aligned}$$

$$- a\tilde{v}_i(t) - b\tilde{v}_i(t))^2 \tag{16}$$

Let $c = 2 + a + b, \omega_i = \tilde{v}_i^2 + (u_i(t) - a\tilde{v}_i(t) - b\tilde{v}_i(t))^2$. It is known that $c$, $w_i$ are bounded positive real numbers. According to the above formula, we can get

$$\frac{\mathrm{d}(e_{xi}^2(t) + e_{vi}^2(t))}{\mathrm{d}t} \leq c(e_{xi}^2(t) + e_{vi}^2(t)) + \omega_i$$

Consider a nonnegative function $\varphi : [0, \infty) \to \mathbb{R}_{\geq 0}$, which satisfies the following relation

$$\dot{\varphi} = c\varphi = w_i, \quad \varphi(0) = e_{xi}^2(t_k^i) + e_{vi}^2(t_k^i) = 0 \tag{17}$$

The solution of the above differential equation is $\varphi(t) = \frac{\omega_i}{c}(e^{ct} - 1)$. According to Lemma 3.4, we can obtain $e_{xi}^2(t) + e_{vi}^2(t) \leq \varphi(t - t_k^i)$. It can be seen from the trigger function that if

$$e_{xi}^2(t) + e_{vi}^2(t) \leq \frac{e^{-\nu t}}{d_i} \tag{18}$$

Then $f_i(t) \leq 0$. Therefore, the lower bound of the interval between event-triggering times $t_k^i$ and $t_{k+1}^i$ of agent $i$ can be determined by the time when $\varphi(t - t_k^i)$ increases from 0 to $\frac{e^{-\nu t}}{d_i}$. The lower bound $\tau_k^i = t_{k+1}^i - t_k^i$ can be obtained by the following equation:

$$\frac{\omega_i}{c}(e^{c\tau_k^i} - 1) = \frac{e^{-\nu(t_k^i + \tau_k^i)}}{d_i} \tag{19}$$

The solution of the above equation is

$$\tau_k^i = \frac{1}{c}\ln\left(1 + \frac{c}{w_i d_i} e^{-\nu(t_k^i + \tau_k^i)}\right) \tag{20}$$

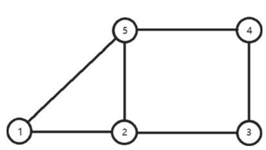


**Figure 1.** communication topology $\mathcal{G}$.

Assuming that Zeno behaviour occurs, there is a positive constant $t^*$ satisfying $\lim_{k\to\infty} t_k^i = t^*$. Let $\delta = \frac{1}{c}\ln(1 + \frac{c}{w_i d_i} e^{-\nu t^*})$, then $\tau_k^i \geq \delta$. According to $\lim_{k\to\infty} t_k^i = t^*$, there exist a positive integer $N^*$ for $\forall k \geq N^*$ to make $t^* - \delta < t_k^i \leq t^*$. Thus, $t^* < t_k^i + \tau_k^i \leq t_{k+1}^i$ when $k \geq N_0$. This is contradictory to $t_{k+1}^i \leq t^*, k \geq N_0$. Therefore, Zeno behaviour is strictly excluded.

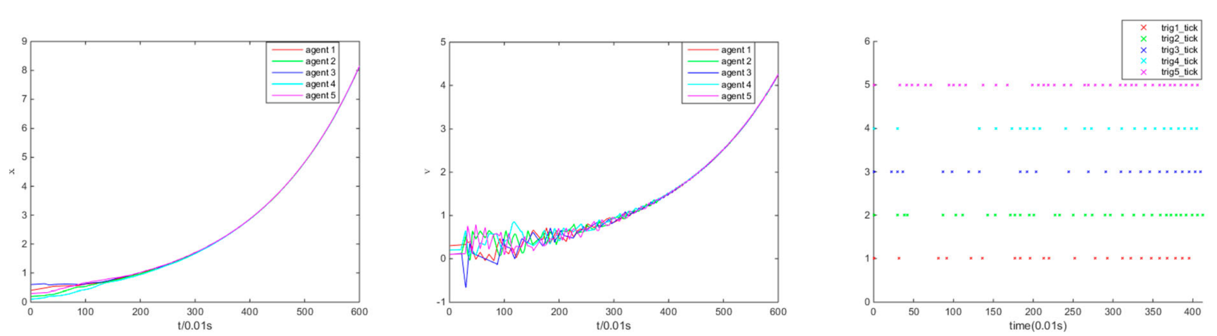
## 4. Simulation

In this section, the results are verified by simulation experiments. A multi-agent system consisting of 5 agents is considered. Its topology $\mathcal{G}$ is shown in Figure 1. The Laplacian matrix is

$$L = \begin{bmatrix} 2 & -1 & 0 & 0 & -1 \\ -1 & 3 & -1 & 0 & -1 \\ 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & -1 & 2 & -1 \\ -1 & -1 & 0 & -1 & 3 \end{bmatrix}$$

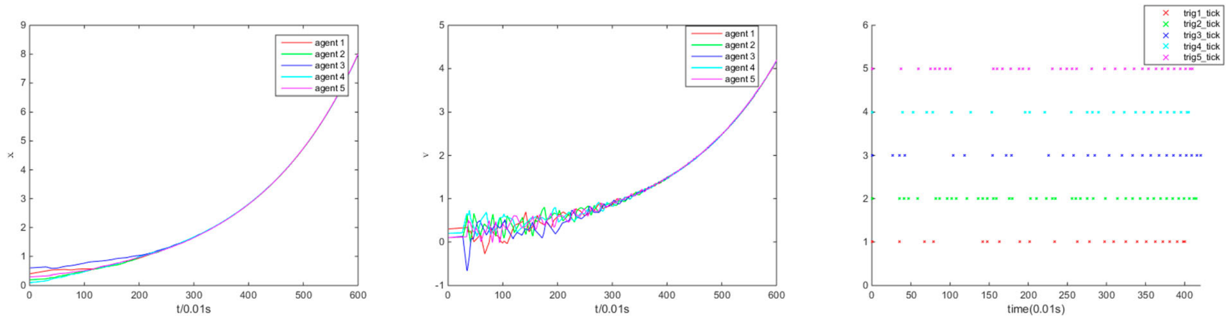
A multi-agent system (1), (2) based on event triggered control (3) is considered. Choose the parameters $k_1 = 7$, $k_2 = 8, a = 0.15, b = 0.1$. The initial state of the system is $x = [0.4, 0.2, 0.6, 0.1, 0.3]^T$, $v = [0.3, 0.1, 0.1, 0.1, 0.2, 0.1]^T$. Figures 2 and 3 show the simulation results based on event-triggering condition (3) and event-triggering conditions utilizing real-time state, respectively. In contrast, the communication frequency of Figure 2 is lower.

It can be seen from Figure 2 that under the event-triggered control, the position and velocity of each agent achieve consensus. And event-triggering time instants of each agent under the event-triggering condition (3) show that the proposed control strategy effectively reduces the communication frequency between agents and reduces the communication consumption of the system.




(a) Position trajectory. (b) Velocity trajectory. (c) Event-triggering time instants.

**Figure 2.** simulation results with event-triggering conditions (3). (a) Position trajectory. (b) Velocity trajectory. (c) Event-triggering time instants.




(a) Position trajectory. (b) Velocity trajectory. (c) Event-triggering time instants.

**Figure 3.** Simulation results of event-triggering conditions based on real-time state. (a) Position trajectory. (b) Velocity trajectory. (c) Event-triggering time instants.

## 5. Conclusions

In this paper, the consensus problem of general second-order multi-agent systems is considered. In order to reduce the communication consumption, the event trigger control protocol is designed so that each agent does not need to update the control input at any time, and its trigger condition update does not need to communicate with each neighbour at any time. Through theoretical analysis, it is proved that the system can achieve consistency under this control protocol. And the Zeno behaviour is strictly excluded. The correctness of theoretical analysis is verified by simulation examples. In the future work, we will focus on extending the results to unknown inputs problems and time-delay systems.

## Disclosure statement

No potential conflict of interest was reported by the author(s).

## Funding

This work was supported by the National Natural Science Foundation of China [grant number 61873136].

## References

Boyd, S., Ghaoui, L., Feron, E., & Balakrishnan, V. (1991). *Linear matrix inequality in systems and control theory*. SIAM.

Cao, M., Xiao, F., & Wang, L. (2015). Event-based second-order consensus control for multi-agent systems via synchronous periodic event detection. *IEEE Transactions on Automatic Control*, *60*(9), 2452–2457. https://doi.org/10.1109/TAC.2015.2390553

Cao, M., Xiao, F., & Wang, L. (2016). Second-order consensus in time-delayed networks based on periodic edge-event driven control. *Systems & Control Letters*, *96*(10), 37–44. https://doi.org/10.1016/j.sysconle.2016.07.001

Dimarogonas, D. V., Frazzoli, E., & Johansson, K. H. (2012). Distributed event-triggered control for multi-agent systems. *IEEE Transactions on Automatic Control*, *57*(5), 1291–1297. https://doi.org/10.1109/TAC.2011.2174666

Dimarogonas, D. V., & Johansson, K. H. (2009). Event-triggered control for multi-agent systems. IEEE, In Proceedings of the 48h IEEE Conference on Decision and Control (CDC) held jointly with 2009 28thChinese Control Conference (pp. 7131–7136), https://doi.org/10.1109/CDC.2009.539977

Hou, W., Fu, M., Zhang, H., & Wu, Z. (2017). Consensus conditions for general second-order multi-agent systems with communication delay. *Automatica*, *75*, 293–298. https://doi.org/10.1016/j.automatica.2016.09.042

Ji, Z., Lin, H., Cao, S., Qi, Q., & Ma, H. (2020). The complexity in complete graphic characterizations of multiagent controllability. *IEEE Transactions on Cybernetics*. https://doi.org/10.1109/TCYB.2020.2972403

Ji, Z., & Yu, H. (2017). A new perspective to graphical characterization of multiagent controllability. *IEEE Transactions on Cybernetics*, *47*(6), 1471–1483. https://doi.org/10.1109/TCYB.2016.2549034

Li, Z., & Duan, Z. (2014). *$H_\infty$ cooperative control of multi-agent systems: A consensus region approach*. CRC Press.

Li, H., Liao, X., Huang, T., & Zhu, W. (2015). Event-triggering sampling based leader-following consensus in second-order multi-agent systems. *IEEE Transactions on Automatic Control*, *60*(7), 1998–2003. https://doi.org/10.1109/TAC.2014.2365073

Liang, J., Liu, K., Ji, Z., & Wang, X. (2019). Event-triggered consensus control for linear multi-agent systems. *IEEE Access*, *7*, 144572–144579. https://doi.org/10.1109/Access.6287639

Liu, K., & Ji, Z. (2017). Consensus of multi-agent systems with time delay based on periodic sample and event hybrid control. *Neurocomputing*, *270*, 11–17. https://doi.org/10.1016/j.neucom.2016.12.106

Liu, K., Ji, Z., & Ren, W. (2017). Necessary and sufficient conditions for consensus of second-order multiagent systems under directed topologies without global gain dependency. *IEEE Transactions on Cybernetics*, *47*(8), 2089–2098. https://doi.org/10.1109/TCYB.2016.2616020

Lou, Y., Ji, Z., & Qu, J. (2020). New results of multi-agent controllability under equitable partitions. *IEEE Access*, *8*, 73523–73535. https://doi.org/10.1109/Access.6287639

Nowzari, C., & Cortes, J. (2016). Distributed event-triggered coordination for average consensus on weight-balanced digraphs. *Automatica*, *68*, 237–244. https://doi.org/10.1016/j.automatica.2016.01.069

Olfati-Saber, R., & R. M. Murray (2004). Consensus problems in networks of agents with switching topology and time-delays. *IEEE Transactions on Automatic Control*, *49*(9), 1520–1533. https://doi.org/10.1109/TAC.2004.834113

Qin, J., & Yu, C. (2013). Coordination of multiagents interacting under independent position and velocity topologies. *IEEE Transactions on Neural Networks and Learning Systems*, *24*(10), 1588–1597. https://doi.org/10.1109/TNNLS.2013.2261090

Qu, J., Ji, Z., Lin, C., & Yu, H. (2018). Fast consensus seeking on networks with antagonistic interactions. *Complexity*, *2018*(7831317), 1–15. https://doi.org/10.1155/2018/7831317.

Qu, J., Ji, Z., & Shi, Y. (2020). The graphical conditions for controllability of multi-agent systems under equitable partition. *IEEE Transactions on Cybernetics*.

Richert, D., & Cortés, J. (2013). Optimal leader allocation in UAV formation pairs ensuring cooperation. *Automatica*, *49*(11), 3189–3198. https://doi.org/10.1016/j.automatica.2013.07.030

Sakurama, K., & Ahn, H.-S. (2019). Multi-agent coordination over local indexes via clique-based distributed assignment. *Automatica*, *112*, 108670. https://doi.org/10.1016/j.automatica.2019.108670

Sun, Y., Ji, Z., & Liu, K. (2020). Event-based consensus for general linear multiagent systems under switching topologies. *Complexity*, 1–14. https://doi.org/10.1155/2020/5972749

Sun, Y., Ji, Z., Qi, Q., & Ma, H. (2019). Bipartite consensus of multi-agent systems with intermittent interaction. *IEEE Access*, *7*, 130300–130311. https://doi.org/10.1109/Access.6287639

Xie, D., Xu, S., Li, Z., & Zou, Y. (2015). Event-triggered consensus control for second-order multi-agent systems. *IET Control Theory & Applications*, *9*(5), 667–680. https://doi.org/10.1049/iet-cta.2014.0219

Zhang, X., Liu, K., & Ji, Z. (2019). Bipartite consensus for multi-agent systems with time-varying delays based on method of delay partitioning. *IEEE Access*, *7*, 29285–29294. https://doi.org/10.1109/ACCESS.2019.2902378

Zhao, M., Peng, C., He, W., & Song, Y. (2018). Event-triggered communication for leader-following consensus of second-order multiagent systems. *IEEE Transactions on Cybernetics*, *48*(6), 1888–1897. https://doi.org/10.1109/TCYB.2017.2716970

Zou, L., Wang, Z., Han, Q., & Zhou, D. (2019). Moving horizon estimation for networked time-delay systems under Round-Robin protocol. *IEEE Transactions on Automatic Control*, *64*(12), 5191–5198. https://doi.org/10.1109/TAC.9

Zou, L., Wang, Z., Hu, J., & Zhou, D. (2020). Moving horizon estimation with unknown inputs under dynamic quantization effects. *IEEE Transactions on Automatic Control*, 1. https://doi.org/10.1109/TAC.2020.2968975