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Existence and Uniqueness of a Common Best Proximity Point on Fuzzy Metric Space

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ABSTRACT

In this paper, first we introduce the notion of proximally weakly compatible mappings and we extend the CLRg property (CLRg-Common limit in the range of g) to the case of non-self mappings. Then we prove the existence of proximally coincident point for this new class of mappings with the assumption of proximal CLRg property in fuzzy metric space. Further, we establish new common best proximity point result for proximally weakly compatible mappings in the setting of fuzzy metric space.

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Best proximity points; fixed points; weakly compatible mappings; fuzzy metric space

1. Introduction and Preliminaries

Fixed point theory is one of the most powerful and fruitful tools in nonlinear analysis and it has many applications in optimisation theory, economics, control theory and game theory. So, the researchers showed more interest to prove fixed point theorems for different kind of contractions in different domains and spaces. Later on, the study of notion of common fixed points of mappings satisfying certain contractive conditions gets attention by many researchers. In the sequel, in 1982, Sessa [1] proved the results on existence of common fixed points for weakly commuting pair of mappings in metric space. Further, Jungck [2] extended the concept weakly commuting pair of mappings to compatible mappings and obtained theorems for common fixed points in metric space. In 1996, Jungck [3] studied the notion of weakly compatible mappings to find fixed point results for set-valued non continuous mappings.

In case of non-self mappings, there is no assurance for the solution of fixed point equation $fx = x$. In this situation, one often tries to find an element x such that x is closest to fx . Such a point is called best proximity point. The best proximity point theorems possess the sufficient conditions that ensure the existence of approximate solutions for fixed point equations. For existence of best proximity point theorems in metric space, one can refer [4–7].

On the other hand, Zadeh [8] introduced the concept of fuzzy sets and studied some properties of fuzzy sets and its membership values. Later, George and Veeramani [9] introduced the fuzzy metric space and proved some basic results of metric spaces in the setting of fuzzy metric space. Fang [10] proved the theorems on common fixed point under

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ϕ -contraction for compatible and weakly compatible mappings in Menger probabilistic metric space. Xin-Qi Hu [11] gave common coupled fixed point theorems for mappings under ϕ -contractive conditions in fuzzy metric spaces and these results are generalisation of S. Sedghi et al. [12]. Later on, Aamri and Moutawakil [13] gave the concept of E. A property for mappings in metric space and derived results on existence of unique common fixed point for weakly compatible pairs. The notion of CLRg (Common limit in the range of g) property introduced by Sintunavarat and Kumam [14] to prove existence of common fixed point for weakly compatible mappings in fuzzy metric spaces in the sense of Kramosil and Michalek and in the sense of George and Veeramani. Then, M. Jain et al. [15] extended the concept of E. A property and (CLRg) property for coupled mappings and proved theorems on common fixed point for weakly compatible maps in fuzzy metric spaces, which are generalisation of the results in [11]. Recently, in [16], the authors derived new common fixed point theorems for weakly compatible mappings in fuzzy metric space using CLRg property and deduced some corollaries. These results extend the corresponding results in [15]. In the case of non-self mappings on fuzzy metric space, recently, the researchers desire to prove the existence and uniqueness of best proximity point for different kind of contractions. For more details, we refer [17–20].

In the light of above works, we are motivated to think that how one can get common fixed point for non-self weakly compatible mappings. In case of non-self mappings, one can identify that we cannot get common fixed point. At this moment, to find approximate common fixed point (called common best proximity point) we extend the notions weakly compatible mappings and CLRg property to the case of non-self mappings. So, in this research work, first we define the notions proximally weakly compatible and proximal CLRg property for non-self mappings in fuzzy metric space. Using this proximal CLRg property, we establish the existence of common best proximity point for proximally weakly compatible mappings in the setting of fuzzy metric space. The proposed results in this article generalise and extend some of results in [16].

First, we recall the following terminologies from the work of [9,11,21]:

Definition 1.1: [9] A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a continuous t -norm if it satisfies the following conditions:

- (i) $*$ is commutative, continuous and associative;
- (ii) $a * 1 = a$ for all $a \in [0, 1]$;
- (iii) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, for all $a, b, c, d \in [0, 1]$.

Definition 1.2: [9] A fuzzy metric space is an ordered triple $(X, M, *)$ such that X is a (nonempty) set, $*$ is a continuous t -norm and M is a fuzzy set on $X \times X \times [0, \infty)$ satisfying the following conditions, for all $x, y, z \in X, s, t > 0$:

- (i) $M(x, y, 0) = 0$;
- (ii) $M(x, y, t) > 0$;
- (iii) $M(x, y, t) = 1$ if and only if $x = y$;
- (iv) $M(x, y, t) = M(y, x, t)$;
- (v) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
- (vi) $M(x, y, \cdot) : (0, \infty) \rightarrow [0, 1]$ is continuous;

(vii) $M(x, y, \cdot) : [0, \infty) \rightarrow [0, 1]$ is left continuous.

In the Definition 1.2, if we assume the conditions (i), (iii), (iv), (v), (vii) then the triple $(X, M, *)$ is called a KM fuzzy metric space (in the sense of Kramosiland Michálek) and if we assume the conditions (ii), (iii), (iv), (v), (vi) then the triple $(X, M, *)$ is called a GV fuzzy metric space (in the sense of George and Veeramani).

For convenience, we denote $a^n = \underbrace{a * a * \dots * a}_{n \text{ times}}$, for all $n \in \mathbb{N}$.

Definition 1.3: [9] Let $(X, M, *)$ be a fuzzy metric space, then a sequence $\{x_n\}$ in X is said to be convergent to x if

$$\lim_{n \rightarrow \infty} M(x_n, x, t) = 1$$

for all $t > 0$.

Definition 1.4: [21] For each $a \in [0, 1]$, the sequence $\{*^n a\}_{n=1}^{\infty}$ is defined by $*^1 a = a$ and $*^n a = (*^{n-1} a) * a$. A t -norm $*$ is said to be of H -type if the sequence of functions $\{*^n a\}_{n=1}^{\infty}$ is equicontinuous at $a = 1$.

Definition 1.5: [11] Define $\Phi = \{\phi : \mathbb{R}^+ \rightarrow \mathbb{R}^+\}$, where $\mathbb{R}^+ = [0, \infty)$ and each $\phi \in \Phi$ satisfies the following condition:

($\phi - 1$) ϕ is nondecreasing;

($\phi - 2$) ϕ is upper semicontinuous from the right;

($\phi - 3$) $\sum_{n=0}^{\infty} \phi^n(t) < \infty$ for all $t > 0$, where $\phi^{n+1}(t) = \phi(\phi^n(t))$, $n \in \mathbb{N}$.

It is easy to prove that, if $\phi \in \Phi$, then $\phi(t) < t$ for all $t > 0$.

Here, we remind the notion of fuzzy distance in fuzzy metric space. Let A be a nonempty subsets of a fuzzy metric space $(X, M, *)$. The fuzzy distance of a point $x \in X$ from a nonempty set A for $t \geq 0$ is defined as

$$M(x, A, t) = \sup_{a \in A} M(x, a, t),$$

and the fuzzy distance between two nonempty sets A and B for $t \geq 0$ is defined as

$$M(A, B, t) = \sup\{M(a, b, t) : a \in A, b \in B\}.$$

Definition 1.6: [18] Let A and B be two nonempty subsets of a fuzzy metric space $(X, M, *)$. We define A_0 and B_0 as follows:

$$A_0 = \{x \in A : M(x, y, t) = M(A, B, t) \text{ for some } y \in B, \forall t \geq 0\},$$

$$B_0 = \{y \in B : M(x, y, t) = M(A, B, t) \text{ for some } x \in A, \forall t \geq 0\}.$$

We extend the Definition 1.1 in [16] in the setting of fuzzy metric space.

Definition 1.7: Let A and B be two nonempty subsets of a fuzzy metric space $(X, M, *)$ and let $f : A \rightarrow B$ and $g : A \rightarrow B$. We say the element $x \in A$ if proximally coincident point if

$$M(u, f(x), t) = M(A, B, t) = M(u, g(x), t),$$

for some $u \in A$ and for all $t > 0$.

Example 1.8: Let $X = \mathbb{R}^2$ with usual metric $d((a_1, a_2), (b_1, b_2)) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$ and consider $A = \{(0, b) : 0 \leq b \leq 1\}$ and $B = \{(1, b) : 0 \leq b \leq 1\}$. And we define fuzzy metric on $X \times X \times [0, \infty)$ by $M(x, y, t) = (t/t + d(x, y))$. So we get $M(A, B, t) = (t/t + 1)$. Now we define $f : A \rightarrow B$ by $f(0, b) = (1, b^2)$ and $g : A \rightarrow B$ by $g(0, b) = (1, (b/2))$. Then clearly, we have

$$M((0, 1/4), f(0, 1/2), t) = M(A, B, t) = M((0, 1/4), g(0, 1/2), t).$$

So the point $(0, 1/2) \in A$ is proximally coincident of f and g .

Definition 1.9: [18] Let A and B be two nonempty subsets of a fuzzy metric space $(X, M, *)$ and let $f : A \rightarrow B$ and $g : A \rightarrow B$. We say the element $x \in A$ is common best proximity point if

$$M(x, f(x), t) = M(A, B, t) = M(x, g(x), t).$$

2. Main Results

In this section, first we define proximal CLRg property for non-self mappings which extends the definition (CLRg property) as in [16].

Definition 2.1: Let (X, M) be a fuzzy metric space under some continuous t -norm*. Two mappings $f, g : A \rightarrow B$ are said to have the proximal CLRg property if there exists a sequence $\{x_n\} \subseteq A$ and a point $z \in A$ with

$$M(u_n, fx_n, t) = M(A, B, t) = M(v_n, gx_n, t),$$

$$M(r, fz, t) = M(A, B, t) = M(s, gz, t)$$

such that

$$u_n \rightarrow s \text{ and } v_n \rightarrow s,$$

where $u_n, v_n, r, s \in A, \forall t \geq 0$.

Example 2.2: Let $X = \mathbb{R}^2$ with usual metric $d((a_1, a_2), (b_1, b_2)) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$ and consider $A = \{(-2, b) : 0 \leq b \leq 1\} \cup \{(1, b) : 0 \leq b \leq 1\}$ and $B = \{(-1, b) : 0 \leq b \leq 1\} \cup \{(0, b) : 0 \leq b \leq 1\}$. And we define fuzzy metric on $X \times X \times [0, \infty)$ by $M(x, y, t) = (t/t + d(x, y))$. So we get $M(A, B, t) = (t/(t + 1))$. Now we define $f : A \rightarrow B$ by

$$f(1, b) = \begin{cases} (-1, 1/2), & b = 0 \\ (0, b + 1/2), & 0 < b \leq \frac{1}{2} \\ (0, 1), & \frac{1}{2} < b \leq 1 \end{cases}, \quad f(-2, b) = \begin{cases} (-1, 0), & 0 \leq b < 1/2 \\ (-1, b - 1/2), & 1/2 \leq b \leq 1 \end{cases}$$

and $g : A \rightarrow B$ by

$$g(1, b) = \begin{cases} \left(0, \frac{1}{2} - \frac{3b}{4}\right), & 0 \leq b \leq 2/3 \\ (0, 1), & 2/3 < b \leq 1 \end{cases}, \quad g(-2, b) = (-1, 1 - b/2) \quad 0 \leq b \leq 1.$$

Now we consider the sequence $\{x_n\} = \{(1, 1/n)\}$ for $n \geq 2$ and $z = (1, 0)$. So we have $M((-2, 1/2), fz, t) = M(A, B, t) = M((1, 1/2), gz, t)$. Here $f(1, 1/n) = (0, 1/n + 1/2)$.

Therefore we obtain the sequence $\{u_n\} = (1, 1/n + 1/2)$ such that $M(u_n, fx_n, t) = M(A, B, t)$. Clearly $u_n \rightarrow (1, 1/2)$ as $n \rightarrow \infty$. And also $g(1, 1/n) = (0, 1/2 - 3/4n)$. Therefore we obtain the sequence $\{v_n\} = (1, 1/2 - 3/4n)$ such that $M(v_n, gx_n, t) = M(A, B, t)$. Clearly $v_n \rightarrow (1, 1/2)$ as $n \rightarrow \infty$.

Next, we introduce a new class of non-self mappings, called proximally weakly compatible mappings in the setting of fuzzy metric space.

Definition 2.3: The mappings $f, g : A \rightarrow B$ are proximally weakly compatible if

$$M(u, fx, t) = M(A, B, t) = M(v, gx, t)$$

then $gu = fv$, for all $x, u, v \in A, \forall t \geq 0$.

Example 2.4: Let $X = \{(0, 1), (1, 0), (-1, 0), (0, -1)\}$ with usual metric $d((a_1, a_2), (b_1, b_2)) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$ and we consider $A = \{(-1, 0), (0, 1)\}, B = \{(0, -1), (1, 0)\}$. And we define fuzzy metric on $X \times X \times [0, \infty)$ by $M(x, y, t) = (t/(t + d(x, y)))$. So we get $M(A, B, t) = (t/(t + \sqrt{2}))$. Now define $f : A \rightarrow B$ by $f(a, b) = (b, a)$ and $g : A \rightarrow B$ by $g(a, b) = (-a, -b)$. Now we justify proximally weakly compatible of f and g , via following-possibilities:

$$\begin{cases} M((-1, 0), f(-1, 0), t) = \frac{t}{t + \sqrt{2}}, \\ M((0, 1), g(-1, 0), t) = \frac{t}{t + \sqrt{2}}, \end{cases}$$

then we get $g(-1, 0) = (1, 0) = f(0, 1)$ and

$$\begin{cases} M((0, 1), f(0, 1), t) = \frac{t}{t + \sqrt{2}}, \\ M((-1, 0), g(0, 1), t) = \frac{t}{t + \sqrt{2}}, \end{cases}$$

then we get $g(0, 1) = (0, -1) = f(-1, 0)$. Then f and g are proximally weakly compatible.

Remark 2.5: In the above definition, suppose we assume $A = B$, then clearly one can identify that $M(A, B, t) = 1$. This implies that $u = fx$ and $v = gx$. Then the Definition 2.3 reduces to weakly compatible mappings in [16].

The following existence theorem of coincident point and common fixed point for mappings using CLRg property were discussed in [16].

Lemma 2.6: [16] Let (X, M) be a fuzzy metric space under some continuous t -norm* and let $f, g : X \rightarrow X$ be mappings having the CLRg property, that is, there is a sequence $\{x_n\} \subseteq X$ and $z \in X$ such that $fx_n \rightarrow gz$ and $gx_n \rightarrow gz$. Assume that there exist $N \in \mathbb{N}$ and $\phi : (0, \infty) \rightarrow (0, \infty)$ such that $\phi(t) \leq t$ for all $t \in (0, \infty)$ and $M(fx, fy, \phi(t)) \geq *^N M(gx, gy, t)$ for all $x, y \in X$ and all $t > 0$. Then $fx = gz$, that is, f and g have a coincidence point.

Definition 2.7: [16] Define Φ' the family of all functions $\phi : (0, \infty) \rightarrow (0, \infty)$ such that the following properties are

$(\phi'_1) 0 < \phi(t)$ for all $t > 0$.

$(\phi'_2) \lim_{n \rightarrow \infty} \phi^n(t) = 0$ for all $t > 0$.

By condition (ϕ'_2) implies $\phi(t) < t$ for all $t > 0$. Clearly, $\Phi \subset \Phi'$.

Theorem 2.8: [16] Let $(X, M, *)$ be a fuzzy metric space such that $*$ is continuous t-norm of H-type and let $f, g : X \rightarrow X$ be weakly compatible mappings having the CLRg property. Assume that there exist $\phi \in \Phi$ and $N \in \mathbb{N}$ such that

$$M(fx, fy, \phi(t)) \geq *^N M(gx, gy, t)$$

for all $x, y \in X$ and all $t > 0$. Then f and g have a unique common fixed point.

First, we prove existence of proximally coincident point for mappings using CLRg property which improves the Lemma 2.6 and it helps to prove our main result.

Lemma 2.9: Let (X, M) be a fuzzy metric space under some continuous t-norm $*$ and let $f : A \rightarrow B$ and $g : A \rightarrow B$ be mappings having proximal CLRg property and there exists $\phi \in \Phi$ satisfying

$$M(u^f, v^f, \phi(t)) \geq *^N M(u^g, v^g, t)$$

provided

$$\begin{cases} M(u^f, f(x), t) = M(A, B, t) = M(u^g, g(x), t), \\ M(v^f, f(y), t) = M(A, B, t) = M(v^g, g(y), t), \end{cases}$$

for all $x, y, u^f, v^f, u^g, v^g \in A, \forall t > 0$. Then there exist $z \in A$ such that

$$M(u, fz, t) = M(A, B, t) = M(u, gz, t)$$

for some $u \in A$.

Proof: Since the pair (f, g) satisfies proximal CLRg property there exist a sequence $\{x_n\} \subseteq A$ and a point $z \in A$ with

$$M(u_n, fx_n, t) = M(A, B, t) = M(v_n, gx_n, t),$$

$$M(r, fz, t) = M(A, B, t) = M(s, gz, t)$$

such that

$$u_n \rightarrow s \text{ and } v_n \rightarrow s.$$

For all $n \in \mathbb{N}$, we have $M(u_n, r, \cdot)$ is non-decreasing function, then

$$\begin{aligned} M(u_n, r, t) &\geq M(u_n, r, \phi(t)) \\ &\geq *^N M(v_n, s, t). \end{aligned}$$

As $v_n \rightarrow s$, we have $M(v_n, s, t) \rightarrow 1$. Since $*$ is continuous then

$$\begin{aligned} \lim_{n \rightarrow \infty} M(u_n, r, t) &\geq \lim_{n \rightarrow \infty} *^N M(v_n, s, t) \\ &= *^N \left[\lim_{n \rightarrow \infty} M(v_n, s, t) \right] \\ &= *^N 1 = 1. \end{aligned}$$

Therefore, we obtain $u_n \rightarrow r$. By uniqueness of limit we get $r = s$. Then

$$M(r, fz, t) = M(A, B, t) = M(r, gz, t).$$

Now we prove the following existence theorem on common best proximity point for proximally weakly compatible mappings using proximal CLRg property. ■

Theorem 2.10: Let (X, M) be a fuzzy metric space under some continuous t -norm $*$ of H -type and let $f : A \rightarrow B$ and $g : A \rightarrow B$ be mappings having the proximal CLRg property with $f(A_0) \subseteq B_0$. Assume that there exist $\phi \in \Phi$ and $N \in \mathbb{N}$ satisfying

$$M(u^f, v^f, \phi(t)) \geq *^N M(u^g, v^g, t)$$

provided

$$\begin{cases} M(u^f, f(x), t) = M(A, B, t) = M(u^g, g(x), t), \\ M(v^f, f(y), t) = M(A, B, t) = M(v^g, g(y), t), \end{cases}$$

for all $x, y, u^f, v^f, u^g, v^g \in A, \forall t > 0$. Suppose the pair (f, g) is proximally weakly compatible, then there exists a unique $z \in A$ such that

$$M(z, fz, t) = M(A, B, t) = M(z, gz, t).$$

Proof: Since the pair (f, g) satisfies CLRg property there exist a sequence $\{x_n\} \subseteq A$ and a point $z \in A$ with

$$M(u_n, fx_n, t) = M(A, B, t) = M(v_n, gx_n, t),$$

$$M(r, fz, t) = M(A, B, t) = M(s, gz, t)$$

such that

$$u_n \rightarrow s \quad \text{and} \quad v_n \rightarrow s.$$

Then by Lemma 2.9, $r = s$. Therefore z is proximally coincident point of f and g . That is, $M(r, fz, t) = M(A, B, t) = M(r, gz, t)$. Since the pair (f, g) is proximally weakly compatible then $fr = gr$. Since $f(A_0) \subseteq B_0$ then there exists r' such that $M(r', fr, t) = M(A, B, t) = M(r', gr, t)$. Now we prove that $r' = r$. For, fix $\epsilon, t > 0$ arbitrary. As $*$ is of H -type, there exists $\eta \in (0, 1)$ such that if $a \in (1 - \eta, 1]$ then $*^m a > 1 - \epsilon$ for all $m \in \mathbb{N}$. We know that $\lim_{t \rightarrow \infty} M(r, z, t) = 1$, so there exists $t_0 > 0$ such that $M(r, z, t_0) > 1 - \eta$. Therefore, we

have that $*^m M(r, z, t_0) > 1 - \epsilon$ for all $m \in \mathbb{N}$. We note that, $M(r', r, \phi(t_0)) \geq *^N M(r', r, t_0)$. Similarly, we can obtain

$$\begin{aligned} M(r', r, \phi^2(t_0)) &\geq *^N M(r', r, \phi(t_0)) \\ &\geq *^{N^2} M(r', r, t_0). \end{aligned}$$

In general, we have $M(r', r, \phi^k(t_0)) \geq *^{N^k} M(r', r, t_0)$ for all $k \in \mathbb{N}$.

As $\phi \in \Phi'$, then $\phi^k(t_0) \rightarrow 0$. Also, as $t > 0$, there is $k_0 \in \mathbb{N}$ such that $\phi^{k_0}(t_0) < t$. It follows,

$$M(r', r, t) \geq M(r', r, \phi^{k_0}(t_0)) \geq *^{N^{k_0}} M(r', r, t_0) > 1 - \epsilon.$$

Since $\epsilon, t > 0$ are arbitrary, we deduce that $M(r', r, t) = 1$ for all $t > 0$. Then $r' = r$. Hence we get $M(r, fr, t) = M(A, B, t) = M(r, gr, t)$. In the same manner, we can prove the uniqueness of common best proximity point.

The following example illustrates the above theorem. ■

Example 2.11: Let $X = \mathbb{R}^2$ with usual metric $d((a_1, a_2), (b_1, b_2)) = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$ and consider $A = \{(0, b) : 0 < b < \infty\}$ and $B = \{(1, b) : 0 < b < \infty\}$. And we define fuzzy metric on $X \times X \times [0, \infty)$ by $M(x, y, t) = (t/t + d(x, y))$. Then (X, M) is a fuzzy metric space under $*(a, b) = \min\{a, b\}$. One can identify $M(A, B, t) = (t/t + 1)$. Now we define $f : A \rightarrow B$ by

$$f(0, b) = \begin{cases} (1, 1), & 0 < b \leq 1 \\ (1, b), & 1 < b < 3, \\ (1, 4) & 3 \leq b \end{cases}$$

and $g : A \rightarrow B$ by

$$g(0, b) = \begin{cases} (1, 7), & 0 < b \leq 1 \\ (1, 10 - 3b), & 1 < b < 3. \\ (1, 1) & 3 \leq b \end{cases}$$

Now we consider the sequence $\{x_n\} = \{(0, (5n - 4)/2n)\}$ for $n \geq 1$ and $z = (0, 5/2)$. So we get $M((0, 5/2), fz, t) = M(A, B, t) = M((0, 5/2), gz, t)$. Here we have $f(0, ((5n - 4)/2n)) = (1, ((5n - 4)/2n))$. Therefore we obtain the sequence $\{u_n\} = (0, ((5n - 4)/2n))$ such that $M(u_n, fx_n, t) = M(A, B, t)$. Clearly $u_n \rightarrow (0, 5/2)$ as $n \rightarrow \infty$. And also $g(0, ((5n - 4)/2n)) = (1, 10 - ((15n - 12)/2n))$. Therefore we obtain the sequence $\{v_n\} = (0, 10 - ((15n - 12)/2n))$ such that $M(v_n, gx_n, t) = M(A, B, t)$. Clearly $v_n \rightarrow (0, 5/2)$ as $n \rightarrow \infty$. It shows f and g have proximal CLRG property. By assuming $\phi(t) = (t/4)$ for all $t > 0$, one can easily verify that f and g agree the proximal contractive condition (1) for any $N \in \mathbb{N}$. Also we can observe f and g are proximally weakly compatible. Then f and g have a unique common best proximity point $(0, 5/2)$ in A .

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