A Protocol for Deciding on the Location of Production Unit Using Fuzzy Economic Variables

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ABSTRACT

In this paper, a fuzzy protocol for a decision-making problem on the choice of location for establishing a production unit from a set of possible locations is presented. Trapezoidal fuzzy numbers are used in the protocol. The ordering between fuzzy numbers is central to the protocol. A remarkable feature is that there is no case of failure, that is, there exists no case leading to indecision in this protocol. No defuzzification operation is used. A numerical illustration is given. The methodology used is a part of Fuzzy Analytical Hierarchical Methods (FAHM).

1. Introduction

In this paper, a problem of the choice of location for establishing production units of certain commodity is considered. The commodity is assumed to be sold in the local market of the particular location in which the production unit is situated. It is assumed that there is no connection between the local markets of these different locations as long as the particular commodity is concerned. Realistically, these locations may be situated in different countries and the commodity may be a food product with a short lifetime for use which prohibits its export to some other country. Again the commodity to be produced may be such that due to political reasons it may be excluded from being exported.

The producer of the commodity makes a survey through which he has some idea of the possible price of the commodity to be marketed and the cost associated with its production. He uses fuzzy economic variables to describe the price, the cost and the profit. The advantage the producer obtains by such use is that he can express his perceived possible uncertainties while estimating the price and the cost of production per unit commodity through fuzzy variables which would not be possible in case price and the cost of production are estimated using ordinary numbers. Thus in his process of decision-making he can now include the perceived uncertainties in his estimations.

Fuzzy sets were introduced by Zadeh [1] in 1965 to deal with uncertain situations which are non-probabilistic in their origin. The applications of fuzzy concepts are through possibilistic theories. Fuzzy concepts made quick headways in different branches of the theory and applications of mathematics. In the problems of decision-making the fuzzy concepts

have been used very extensively. Some recent works on fuzzy approaches in decision-making problem are in [2–11]. Specifically, he uses a Fuzzy Analytical Hierarchical Method which is abbreviated as a FAHM in the literatures. It is a fuzzy version of Analytical Hierarchical Method (AHM) [12] in the non-fuzzy, that is, crisp environment. The corresponding fuzzy versions of AHMs was discussed by Buckely [13]. Several Fuzzy Analytical Hierarchical Methods have been applied to decision-making problems in recent literatures [14–16]. The essence of this methodology is to create a hierarchy using fuzzy variables which is then applied to arrive at the decision. A speciality in this paper is that the decision-making protocol uses pure fuzzy concepts and does not rely on crisp concepts obtained through defuzzification for arriving at a decision.

In the following section, the technical concepts necessary for the purpose of the paper are described.

2. Technical Preliminaries

Let X be a universe of discourse. A fuzzy subset $\tilde{A}$ on X is defined by the membership function $\mu_{\tilde{A}}(x)$ which maps each element x in X to a real number in the interval [0,1] and is usually denoted as $\mu_{\tilde{A}} :\to [0,1]$ [1]. A fuzzy set is said to be normal if there exists at least one x for which $\mu_{\tilde{A}}(x) = 1$ [1]. A fuzzy set $\tilde{A}$ on X is convex if $\mu_{\tilde{A}}(\lambda x_1 + (1 - \lambda)x_2) \geq min(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2))$ for all $x_1, x_2 \varepsilon X$ and for $\lambda \varepsilon$ [0,1], where min denotes the minimum operator [1].

In this paper, we use the fuzzy numbers and fuzzy arithmetic in accordance with the function principle introduced by Chen [17] and considered in several works like [18–20]. The following concepts are described in Cheng [18].

A fuzzy number is a fuzzy subset of the real line which is both normal and convex. The membership function of a fuzzy number $\tilde{A}$ is usually represented as

$$\mu_{\tilde{A}(x)} = \mu_{\tilde{A}}^{l}(x), \quad x < b,$$
$$= 1, \quad b \leq x \leq c,$$
$$= \mu_{\tilde{A}}^{u}(x), \quad x > c,$$

where $\mu_{\tilde{A}}^{l}(x)$ is continuous from the right, strictly increasing for $x < b$ and there exists $a < b$ such that $\mu_{\tilde{A}}^{l}(x) = 0$ for $x < a$ and $\mu_{\tilde{A}}^{u}(x)$ is continuous from left, strictly decreasing for $x > c$ and there exists $d \geq c$ such that $\mu_{\tilde{A}}^{u}(x) = 0$ for $x \geq d$ [18].

Since $\mu_{\tilde{A}}^{l}(x) : [a, b] \to [0, 1]$ is continuous and strictly increasing, the inverse function of $\mu_{\tilde{A}}^{l}(x)$ exists and is denoted by $\psi_{\tilde{A}}^{l}(x) : [0, 1] \to [a,b]$ which is continuous and strictly decreasing. Also since $\mu_{\tilde{A}}^{u}(x) : [c, d] \to [0, 1]$ is continuous and strictly decreasing, the inverse function of $\mu_{\tilde{A}}^{u}(x)$ exists and is denoted by $\psi_{\tilde{A}}^{u}(x) : [0, 1] \to [c, d]$ which is continuous and strictly increasing [17,18].

A fuzzy number reduces to a crisp number b (or c) if b = c, $\mu_{\tilde{A}}(x) = 0$ for $x < b$ and $\mu_{\tilde{A}}(x) = 0$ for $x > c$.

There are several types of fuzzy numbers dependent on the functions $\mu_{\tilde{A}}^{l}$ and $\mu_{\tilde{A}}^{u}$. One particular type is the triangular fuzzy number which is commonly referred to as TFN [17].

The fuzzy number $\tilde{A}$ is said to be a triangular fuzzy number if it is fully determined by the triplet (a, b, c) of crisp numbers such that $a < b < c$, with membership function otherwise,

$$\mu_{\tilde{A}(x)} = \frac{(x-a)}{b-a}, \quad a \leq x \leq b,$$

$$= \frac{(b-x)}{c-b}, \quad b \leq x \leq c,$$

$$= 0, \quad \text{otherwise,}$$

where a and c are the lower limit and upper limit respectively on the fuzzy number $\tilde{A}$. The TFN naturally corresponds to the real number b. Due to this natural correspondence, and the relative convenience of mathematically dealing with it, the TFN has been used in a large number of applications such as in [21–23].

A more general fuzzy number which is an extension of TFN is the trapezoidal fuzzy number.

The fuzzy number $\tilde{A}$ is said to be a trapezoidal fuzzy number if it is fully determined by a quadruplet (a,b,c,d) of crisp numbers such that $a < b < c < d$, with membership function,

$$\mu_{\tilde{A}(x)} = \frac{(x-a)}{b-a}, \quad a \leq x \leq b,$$

$$= 1, \quad b \leq x \leq c,$$

$$= \frac{(d-x)}{b-a}, \quad c \leq x \leq d,$$

$$= 0, \quad \text{otherwise,}$$

where a, b, c and d are the lower limit, lower mode, upper mode and upper limit respectively on the fuzzy number $\tilde{A}$ [18].

A trapezoidal fuzzy number is a generalization of TFN. When b $=$ c, the trapezoidal fuzzy number reduces to a triangular fuzzy number.

In order to rank trapezoidal fuzzy numbers, we use the distance index between the centroid point and original point given by Cheng [18].

The inverse functions of $\mu_{\tilde{A}}^l(x)$ and $\mu_{\tilde{A}}^u(x)$ for the case of trapezoidal fuzzy numbers described above are $\psi_{\tilde{A}}^l(x) = a + (b-a)y$ and $\psi_{\tilde{A}}^u(x) = d - (d-c)y$ [18].

The centroid point $(\tilde{x}_0, \tilde{y}_0)$ for fuzzy number $\tilde{A}$ is defined as

$$x_0(\tilde{A}) = \frac{\int_a^b (x\mu_{\tilde{A}}^l)\, dx + \int_b^c x\, dx + \int_c^d (x\mu_{\tilde{A}}^u)\, dx}{\int_a^b (\mu_{\tilde{A}}^l)\, dx + \int_b^c dx + \int_c^d (\mu_{\tilde{A}}^u)\, dx}, \quad y_0(\tilde{A}) = \frac{\int_0^1 (y\psi_{\tilde{A}}^l)\, dy + \int_0^1 (y\psi_{\tilde{A}}^u)\, dy}{\int_0^1 (\psi_{\tilde{A}}^l)\, dy + \int_0^1 (\psi_{\tilde{A}}^u)\, dy}. \quad [18]$$

Particularly in the case of the trapezoidal fuzzy number described above, for the purpose of ranking, we use distance index between the centroid point (x_0, y_0) and origin, that is, $R(\tilde{A}) = \sqrt{(x_0^2) + (y_0^2)}$ [18].

For any two fuzzy numbers $\tilde{A}_1, \tilde{A}_2$ we have

(1) If $R(\tilde{A}_1) < R(\tilde{A}_2)$, then $\tilde{A}_1 < \tilde{A}_2$,
(2) If $R(\tilde{A}_1) = R(\tilde{A}_2)$, then $\tilde{A}_1 = \tilde{A}_2$,
(3) If $R(\tilde{A}_1) > R(\tilde{A}_2)$, then $\tilde{A}_1 > \tilde{A}_2$ [18].

The arithmetic operations required here for the trapezoidal fuzzy numbers are the following [18].

Let $\tilde{A}_1 = (a_{11}, a_{12}, a_{13}, a_{14})$ and $\tilde{A}_2 = (a_{21}, a_{22}, a_{23}, a_{24})$ be two trapezoidal fuzzy numbers. Then

(i) Addition
$$\tilde{A}_1 + \tilde{A}_2 = (a_{11} + a_{21}, a_{12} + a_{22}, a_{13} + a_{23}, a_{14} + a_{24}),$$
(ii) $-\tilde{A}_2 = (-a_{24}, -a_{23}, -a_{22}, -a_{21}),$
(iii) Substraction

$$\tilde{A}_1 - \tilde{A}_2 = (a_{11} - a_{24}, a_{12} - a_{23}, a_{13} - a_{22}, a_{14} - a_{21}) \ [18].$$

3. Deciding Location of Production Unit

The producer of certain commodity has the option of choosing his location of production in any one of the places out of n locations $L_1, L_2, \ldots$ and L_n. He estimates the possible price and cost of production for the commodity at each production unit [24,25]. It is assumed that the situation is that the commodity can only be sold locally. Since his estimation of possible price and cost of production necessarily involves uncertainties, he prefers to represent them through fuzzy numbers $\tilde{r}_i$ and $\tilde{c}_i$ respectively for the location L_i, $i = 1, 2, 3, \ldots n$. Particularly he uses trapezoidal fuzzy numbers. Thus mathematically the price and the production cost become fuzzy variables taking values in the set of trapezoidal fuzzy numbers.

He then executes the following protocol which is possible in view of the mathematics discussed in the previous section and is summarized in the following steps.

Step 1. The price (revenue per unit commodity) at the location L_i is estimated and represented by a trapezoidal fuzzy number $\tilde{r}_i$, $i = 1, 2, \ldots n$.

Step 2. The cost of production per unit commodity at the location L_i is estimated and represented by a trapezoidal fuzzy number $\tilde{c}_i$, $i = 1, 2, \ldots n$.

Step 3. The profit per unit commodity at the location L_i is represented by $\tilde{z}_i = \tilde{r}_i - \tilde{c}_i$, $i = 1, 2, \ldots n$, which is again a trapezoidal fuzzy number.

Step 4. The indices $R(\tilde{z}_1), \ldots, R(\tilde{z}_n)$ are calculated.

Step 5. The indices calculated in Step 4 are arranged in decreasing order $R(\tilde{z_{1N}}), \ldots, R(\tilde{z_{nN}})$.

Step 6. The fuzzy profits are arranged in the same order, that is, $\tilde{z_{1N}}, \ldots, \tilde{z_{nN}}$.

Step 7. The number $\tilde{z_{1N}}$ of the highest rank is chosen.

Step 8. The decision is the choice of the corresponding location of $\tilde{z_{1N}}$, that is, L_{1N}.

Step 9. If there are more than one highest rank $\tilde{z}_i$ s, then an arbitrary choice from them is made and then Step 8 is executed.

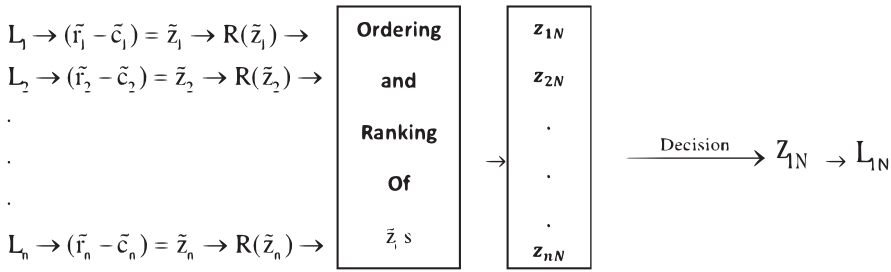The protocol is represented through Figure 1.

$$L_1 \to (\tilde{r}_1 - \tilde{c}_1) = \tilde{z}_1 \to R(\tilde{z}_1) \to$$
$$L_2 \to (\tilde{r}_2 - \tilde{c}_2) = \tilde{z}_2 \to R(\tilde{z}_2) \to$$
.
.
.
$$L_n \to (\tilde{r}_n - \tilde{c}_n) = \tilde{z}_n \to R(\tilde{z}_n) \to$$

Ordering	z_{1N}
and	z_{2N}
Ranking	.
Of	.
$\tilde{z}_i$ s	z_{nN}

$\to$ $\xrightarrow{\text{Decision}}$ $Z_{1N} \to L_{1N}$

Figure 1. L_is are locations, $\tilde{r}_i$s are possible prices, $\tilde{c}_i$s are possible costs, $\tilde{z}_i$s are profits and $R(\tilde{z}_i)$s are indices.

4. An Illustration

The following is a numerical illustration of the above protocol described in Section 3.

Let there be four locations $L_i, i = 1, 2, 3, 4$.

Let $\tilde{r}_i$ and $\tilde{c}_i$, i= 1,2,3,4 be the price (revenue per unit commodity), and the cost of production per unit commodity respectively at these four locations be described respectively by the trapezoidal fuzzy numbers $\tilde{r}_1 = (25, 30, 35, 40)$, $\tilde{r}_2 = (35, 45, 55, 65)$, $\tilde{r}_3 = (40, 55, 70, 85)$, $\tilde{r}_4 = (44, 58, 72, 86)$ and $\tilde{c}_1 = (1, 5, 9, 13)$, $\tilde{c}_2 = (15, 20, 25, 30)$, $\tilde{c}_3 = (19, 25, 31, 37)$, $\tilde{c}_4 = (21, 28, 35, 42)$.

Then by the formula of the previous section $\tilde{z}_i = \tilde{r}_i - \tilde{c}_i$ we have, $\tilde{z}_1 = (12, 21, 30, 39)$, $\tilde{z}_2 = (5, 20, 35, 50)$, $\tilde{z}_3 = (3, 24, 45, 66)$, $\tilde{z}_4 = (2, 23, 44, 65)$ (Figures 2).

Now (12, 21, 30, 39) is given by the membership function

$$\mu_{\tilde{z}_1}(x) = \frac{(x - 12)}{9}, \quad 12 \le x \le 21,$$
$$= 1, \quad 21 \le x \le 30,$$
$$= \frac{(39 - x)}{9}, \quad 30 \le x \le 39.$$

The centroid of $\tilde{z}_1$ is

$$x_0(\tilde{z}_1) = \frac{\int_{12}^{21} \frac{x(x-12)}{9} \, dx + \int_{21}^{30} x \, dx + \int_{30}^{39} \frac{x(39-x)}{9} \, dx}{\int_{12}^{21} \frac{(x-12)}{9} \, dx + \int_{21}^{30} \, dx + \int_{30}^{39} \frac{(39-x)}{9} \, dx} = 25.5,$$

$$y_0(\tilde{z}_1) = \frac{\int_0^1 y(12 + 9y) \, dy + \int_0^1 y(39 - 9y) \, dy}{\int_0^1 (12 + 9y) \, dy + \int_0^1 (39 - 9y) \, dy} = 0.5.$$

Therefore the distance index between the centroid point (x_0, y_0) and the origin is $R(\tilde{z}_1) = \sqrt{(x_0^2) + (y_0^2)} = 25.50$.

Again, (5, 20, 35, 50) is given by the membership function

$$\mu_{\tilde{z}_2}(x) = \frac{(x - 5)}{15}, \quad 5 \le x \le 20,$$
$$= 1, \quad 20 \le x \le 35,$$
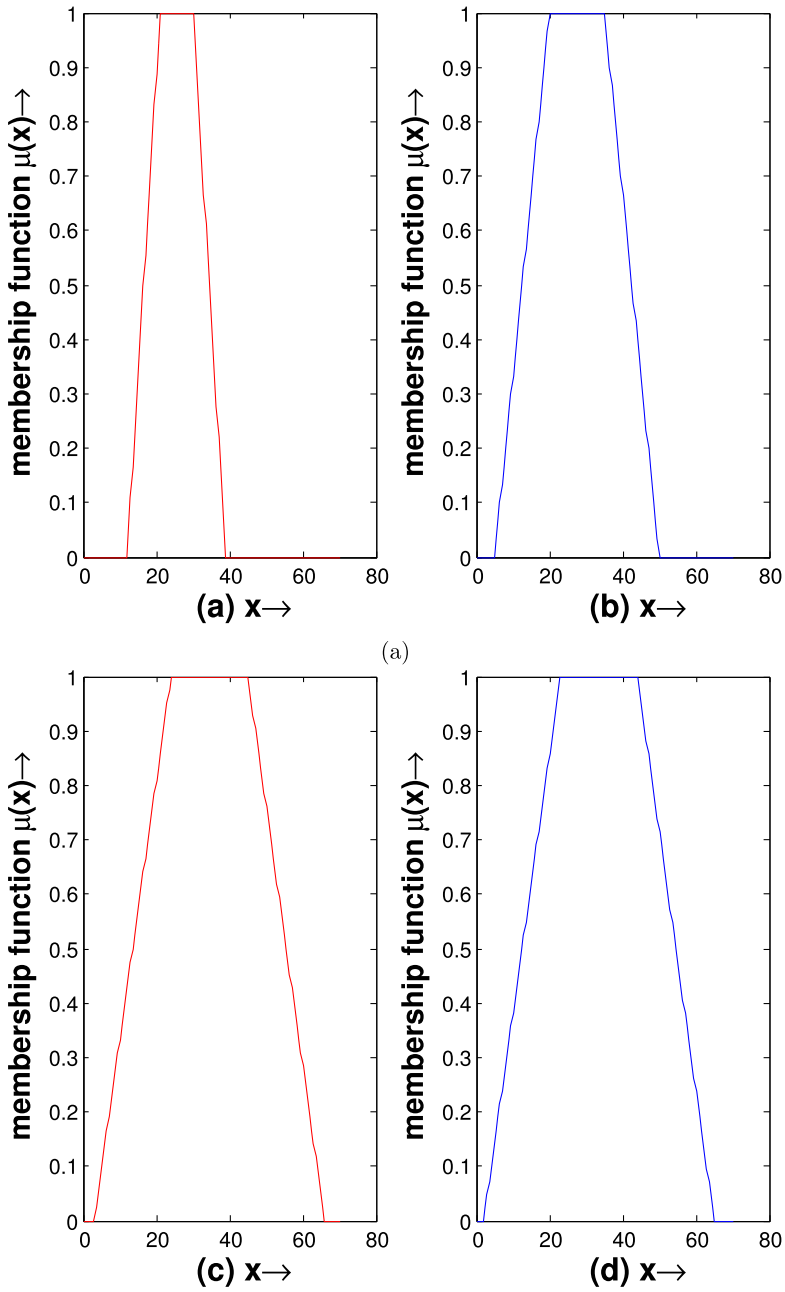$$= \frac{(50 - x)}{15}, \quad 35 \le x \le 50.$$

Figure 2. (a) Trapezoidal fuzzy number $\tilde{z}_1$, (b) trapezoidal fuzzy number $\tilde{z}_2$, (c) trapezoidal fuzzy number $\tilde{z}_3$ and (d) Trapezoidal fuzzy number $\tilde{z}_4$.
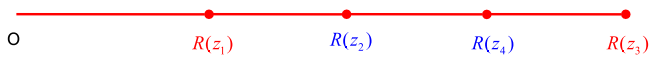

Figure 3. Indices of $\tilde{z}_1, \ldots, \tilde{z}_4$.

The centroid of $\tilde{z}_2$ is

$$x_0(\tilde{z}_2) = \frac{\int_5^{20} \frac{x(x-5)}{15}\,dx + \int_{20}^{35} x\,dx + \int_{35}^{50} \frac{x(50-x)}{15}\,dx}{\int_5^{20} \frac{(x-5)}{15}\,dx + \int_{20}^{35}\,dx + \int_{35}^{50} \frac{(50-x)}{15}\,dx} = 27.5,$$

$$y_0(\tilde{z}_2) = \frac{\int_0^1 y(5 + 15y)\,dy + \int_0^1 y(50 - 15y)\,dy}{\int_0^1 (5 + 15y)\,dy + \int_0^1 (50 - 15y)\,dy} = 0.5.$$

Therefore the distance index between the centroid point (x_0, y_0) and the origin is $R(\tilde{z}_2) = \sqrt{(x_0^2) + (y_0^2)} = 27.50.$

Also, $(3, 24, 45, 66)$ is given by the membership function

$$\mu_{\tilde{z}_3}(x) = \frac{(x - 3)}{21}, \quad 3 \le x \le 24,$$

$$= 1, \quad 24 \le x \le 45,$$

$$= \frac{(66 - x)}{21}, \quad 45 \le x \le 66.$$

The centroid of $\tilde{z}_3$ is

$$x_0(\tilde{z}_3) = \frac{\int_3^{24} \frac{x(x-3)}{21}\,dx + \int_{24}^{45} x\,dx + \int_{45}^{66} \frac{x(66-x)}{21}\,dx}{\int_3^{24} \frac{(x-3)}{21}\,dx + \int_{24}^{45}\,dx + \int_{45}^{66} \frac{(66-x)}{21}\,dx} = 34.5,$$

$$y_0(\tilde{z}_3) = \frac{\int_0^1 y(3 + 21y)\,dy + \int_0^1 y(66 - 21y)\,dy}{\int_0^1 (3 + 21y)\,dy + \int_0^1 (66 - 21y)\,dy} = 0.5.$$

Therefore the distance index between the centroid point (x_0, y_0) and the origin is $R(\tilde{z}_3) = \sqrt{(x_0^2) + (y_0^2)} = 34.50.$

Further, $(2, 23, 44, 65)$ is given by the membership function

$$\mu_{\tilde{z}_4}(x) = \frac{(x - 2)}{21}, \quad 2 \le x \le 23,$$

$$= 1, \quad 23 \le x \le 44,$$

$$= \frac{(65 - x)}{21}, \quad 44 \le x \le 65.$$

The centroid of $\tilde{z}_4$ is

$$x_0(\tilde{z}_4) = \frac{\int_2^{23} \frac{x(x-2)}{21}\,dx + \int_{23}^{44} x\,dx + \int_{44}^{65} \frac{x(65-x)}{21}\,dx}{\int_2^{23} \frac{(x-2)}{21}\,dx + \int_{23}^{44}\,dx + \int_{44}^{65} \frac{(65-x)}{21}\,dx} = 33.5,$$

$$y_0(\tilde{z}_4) = \frac{\int_0^1 y(2 + 21y)\,dy + \int_0^1 y(65 - 21y)\,dy}{\int_0^1 (2 + 21y)\,dy + \int_0^1 (65 - 21y)\,dy} = 0.5.$$

Therefore the distance index between the centroid point (x_0, y_0) and the origin is $R(\tilde{z}_4) = \sqrt{(x_0^2) + (y_0^2)} = 33.50.$

Since $R(\tilde{z}_3)$ has the highest value amongst $R(\tilde{z}_1)$, $R(\tilde{z}_2)$, $R(\tilde{z}_3)$, $R(\tilde{z}_4)$ (Figure 3), following the protocol of the previous section, the producer decides to choose the location L_3 for the setting up of his production unit.

Remark 4.1: Since a finite set of fuzzy numbers can always be ordered by the centroid method and the trapezoidal fuzzy numbers are closed under the operation of substraction, there is no failure case in the protocol. It is noteworthy that no defuzzification operation has been used here.

5. Conclusion

The specific finding of the present work is the postulation of a protocol to decide over a location of production unit in a fuzzy environment. The crux of the protocol is the ordering in the arithmetic of trapezoidal fuzzy numbers. The present work suggests the possibility of creating other decision-making schemes in the fuzzy environment for several other purposes as well. In one sense it can be viewed as an application of fuzzy arithmetic. The most important point to note here is that no defuzzification process has been executed as a part of it. This is, in addition to making the protocol free from deterministic considerations, also advantageous from the computational point of view. The defuzzification processes are often involved processes requiring a very large number of mathematical operations for their execution [26–28]. But it is not necessary in the present context which makes the scheme more economical in terms of computation. The scheme is illustrated with a simple numerical example in which hypothetical data are used. Real World applications with real data require large computational steps where the above mentioned advantage can be more evident. These applications can be in various fields of studies in economics like non-linear finance [29], supply chains [30,31], etc. where there are uncertainties in the economic environment. In fact, fuzzy AHP has already been used in such problems of which [14–16] are instances. The approach developed in this paper with suitable adaptations is supposed to be efficiently applicable to decision making problems from these areas as well.

Disclosure statement

No potential conflict of interest was reported by the authors.

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