Fuzzy Information and Engineering

ISSN: 1616-8658 (Print) 1616-8666 (Online) Journal homepage: https://www.tandfonline.com/loi/tfie20

Iterative Solution Process for Multiple Objective Stochastic Linear Programming Problems Under Fuzzy Environment

Arindam Garai, Palash Mandal & Tapan Kumar Roy

FUZZY INFORMATION AND ENGINEERING
https://doi.org/10.1080/16168658.2020.1750871

Iterative Solution Process for Multiple Objective Stochastic Linear Programming Problems Under Fuzzy Environment

Arindam Garai [a], Palash Mandal[b] and Tapan Kumar Roy[b]

[a]Department of Mathematics, Sonarpur Mahavidyalaya, Kolkata, India; [b]Department of Mathematics, IIEST, Shibpur, India

ABSTRACT

This article presents one interactive algorithm, and thereby determines the Pareto optimal solution to multi-objective stochastic linear programming (MOSLP) problems in real-life oriented fuzzy environment. Among the various objective functions, there always exists one objective function, referred to as the main objective function in this article, to multi-objective models, whose optimal value is most vital to decision-makers. When the optimal value to *main* objective function meets the pre-determined aspiration level, and the corresponding values to other objective functions are satisfactory in nature, that Pareto optimal solution is acceptable to decision-makers. Again, in several existing interactive fuzzy optimisation methods to MOSLP models, all reference membership levels of expectations to objective functions are considered as a unity. However, this seems to be less rational that the expectation of each conflicting objective function simultaneously attains the individual goal. So, the present article proposes to employ the trade-off ratios of membership functions to analytically determine reference membership levels in a fuzzy environment. Numerical applications further illustrate this algorithm. Finally, conclusions are drawn.

KEYWORDS

Main objective function; trade-off ratio; chance-constrained programming; expectation model; fuzzy multi-objective optimisation; stochastic optimisation

1. Introduction

Multiple objective linear programming is the process of optimising simultaneously and systematically a collection of objective functions [1]. In crisp optimisation methods to solve multiple objective linear programming problems (MOLPP), usually, we are unable to employ preferences of decision-maker (DM) effectively. Therefore, on the basis of such imprecise information in real-life decision-making problems, usually, we use two approaches: fuzzy programming and stochastic programming.

By assuming that the DM has imprecise aspiration levels for each of the objective functions, mathematicians have proposed several methods in the literature for characterising Pareto optimal solutions to MOLPP in a fuzzy environment [2–4]. In 1970, Bellman and Zadeh introduced MOLPP in fuzzy environment for such decision-making problems [5]. In 1974, Tanaka, Okuda and Asai first formulated MOLPP with fuzzy parameters [6]. In fact, in 2015, Luhandjula has correctly observed that Tanaka et al. have helped raise the

CONTACT Arindam Garai fuzzy_arindam@yahoo.com

intellectual respectability of fuzzy optimisation [7,8]. As pointed out by Zimmermann in 1976 and later in 1978, various kinds of impreciseness can be categorised as fuzziness and consequently he developed one mathematical method to solve MOLPP in fuzzy environment [9]. In 1984 and 1986, interactive fuzzy optimisation method was first developed by Sakawa et al. [10]. In their article, DM is involved in solution steps. Subsequently, in real-life decision-making cases, interactive approaches have played an important role in deriving best-preferred compromise solutions to MOLPP. The main advantage of the interactive approach is that DM controls the search direction during the solution procedure; consequently, such solutions can usually achieve DM's aspiration levels.

Fuzzy optimisation methods were applied in solving real-world problems by Werners [11], Lai and Hwang [12], Gupta et al. [13], Kahraman et al. [14], Sahoo et al. [15], Zhou et al. [16], Wu et al. [17], Garg et al. [18], Wei et al. [19], Salehi et al. [20], Su et al. [21], Garai et al. [22,23] and several other mathematicians. We can find extensive studies of evolutionary computing approaches in fuzzy environment, e.g. Fernandeza et al. [24]. Fuzzy set theory also found applications in structural models; many researchers like Wang and Wang [25], Yeh et al. [26], Xu [27], Shih et al. [28] made distinctive implementations of fuzzy set theory. Fuzzy set theory has been used to yield decisions in multiple objective decision-making problems among multiple choices and multiple items by Yu et al. [29], Ebrahimnejad and Verdegay [30], Chakraborty et al. [31], Garai et al. [2], etc. Studies involving genetic algorithm have been studied by Martinez-Soto et al. [32], Jongsuebsuk et al. [33] and many others in recent past. Optimal design of fuzzy tracking controllers for autonomous mobile robots under perturbed torques has been developed by Melin et al. [34]. Emergency transportation planning in disaster relief supply chain management was elaborately discussed by Zheng and Ling [35]. A survey on nature-inspired optimisation algorithms with fuzzy logic was performed by Valdez et al. [36]. Also study of MOLPP involving imprecise objective functions, interactive techniques, KKT optimality conditions, etc. have been investigated by researchers in fuzzy environment in recent years [2]. E.g. an improved fuzzy self-adaptive learning multiple objective particle swarm optimisation algorithm has been developed for dynamic economic emission dispatch problem by applying interactive fuzzy optimisation method under stochastic environment by Bahman Bahmani-Firouzi et al. [37].

But in existing interactive fuzzy optimisation methods by Sakawa et al. [38], although DM may consider goals and tolerances of fuzzy objective functions, we may observe that DM cannot effectively specify any objective function as the main objective function. In other words, we may find that each of the objective functions is treated with equal importance in existing interactive solution method for solving MOLPP in fuzzy environment. Although DM can assign weights, etc. [39] to each of the fuzzy objective functions, such methods have several disadvantages.

We may further note that DM usually prefers the optimal solution in which optimal value of specified main objective function is most preferable and optimal values of other objective functions are acceptable to DM [3]. In other words, till date, mathematicians have used fuzziness of DM's aspirations to construct and solve optimisation models in fuzzy environment; but time is ripe to go beyond. In fuzzy environment, we need one iterative process in which most preferable Pareto optimal solutions to MOLPP may be generated based on specified main objective function [3].

Further, we may find that interaction in each step of decision process may satisfy plan of DM to actively participate in solution processes; but in long run, it may waste precious time and money of DM. In several recent cases, we may find that DM may not be available at each stage of decision-making activities. In 1979, Wierzbicki correctly raised one fundamental issue: 'what is more valuable: the perfection of a compromise, which is based on a never-perfect model, or the time of a top-rank decision maker?' [40]. If confronted with a multitude of questions (e.g. 'would you prefer this alternative to the other one?') at each and every stage of decision-making activity, DM may simply send the analyst back to where he/she belongs [40]! Consequently, it may be more rational to look for minimal yet efficient interaction with DM [3].

Further, we may observe that concept of reference membership levels, which first appeared in Sakawa et al. [41], can be viewed as one natural extension of the idea of reference point by Wierzbicki. In existing interactive fuzzy optimisation techniques, unity is taken as an initial reference membership level of each fuzzy objective function. But we better not expect each conflicting fuzzy objective function to attain the corresponding goal simultaneously [3]. Accordingly, we may propose that the initial reference membership level of each fuzzy objective function need to be based on analytically derived results.

On the other hand, in most real-life situations, it seems natural to consider that impreciseness should be expressed by a fusion of fuzziness and randomness rather than by either fuzziness or randomness [42]. To effectively handle DM's ambiguous judgements in MOLPP as well as the randomness of parameters involved in objective functions and/or constraints, Sakawa and his colleagues incorporated interactive fuzzy satisfying methods associated with deterministic problems into multiple objective stochastic programming problems [10,41]. In last few decades, many other scientists have introduced several stochastic programming models such as expectation optimisation, variance minimisation, probability maximisation, fractile criterion optimisation together with chance-constrained programming methods to derive satisficing solutions for DM from a set of Pareto optimal solutions. As Sakawa and other fellow researchers have described, in chance-constrained problems, for random data variations, a mathematical model can be formulated so that the violation of constraints is permitted up to specified probability levels [38].

In this article, we propose one iterative process to determine most preferable Pareto optimal solutions based on the specified main objective function and thereby using reference membership levels obtained analytically from trade-off ratios to multiple objective stochastic linear programing problems (MOSLPP) in fuzzy environment. If DM is not satisfied with this Pareto optimal solution, he/she may alter his/her aspiration levels and/or priority. Based on the updated information, another set of Pareto optimal solution may be yielded to MOSLPP in fuzzy environment. The iterative process may be continued until DM is satisfied with the latest Pareto optimal solutions. Here, for the sake of simplicity alone, we choose expectation optimisation model in a stochastic environment so that readers can easily digest the proposed theme of our manuscript without unnecessarily deviating into other details.

The rest of the article is organised as follows. In Section 2, we introduce a mathematical model of MOSLPP in fuzzy environment. In Section 3, we discuss the solution method to find preferable Pareto optimal solutions to MOSLPP based on specified main objective function in fuzzy environment. Related theorems are discussed here. In Section 4, we develop one iterative process to find preferable Pareto optimal solutions to MOSLPP based on specified main objective function in fuzzy environment. In Section 5, numerical examples are considered to further illustrate our proposed iterative process. Here, we compare results by the proposed process with several other existing solution techniques. In Section 6, conclusions are drawn and scopes for future research are given.

2. Expectation Model of Multiple Objective Chance-Constrained Programming in Fuzzy Environment

2.1. Formation of MOSLPP

Assuming that the coefficients in the objective functions and right-hand side constants of the constraints are random variables, we may formulate one general MOSLPP as follows:

$$\begin{aligned}&\text{minimize } (z_1(x)\ldots z_k(x))^{\mathrm{T}}\\&\text{subject to the constraints}\\&Ax \le \bar{b},\\&x \ge 0,\end{aligned} \tag{1}$$

Here x is an n-dimensional column vector of decision variables, A is an $m \times n$ coefficient matrix, $z_i(x), i = 1\ldots k$ are k conflicting objective functions with $z_i(x) = \bar{c}_i x, \forall i = 1\ldots k$, $\bar{c}_i, \forall i = 1\ldots k$ are n-dimensional random variable row vectors with finite means $E(\bar{c}_i)$ and $n \times n$ positive definite variance covariance matrices $V_i = [v^i_{jh}] = [\mathrm{Cov}\{c_{ij}, c_{ih}\}], \forall i = 1\ldots k$ and $\bar{b}$ is n-dimensional column vector whose elements are mutually independent random variables.

Since coefficients of model (1) are taken as random variables; we have to apply solution methods appropriate for stochastic events. Consequently, to deal with constraints associated with model (1), we may employ chance-constrained conditions that permit constraint violations up to specified probability limits. Suppose that a_q is the qth row vector of A and $\bar{b}_q$ is the qth element of $\bar{b}$. Here we assume that each random variable $\bar{b}_q$ has a continuous probability distribution function $F_q(r) = P(\bar{b}_q \le r)$. Hence $\forall q = 1\ldots m$, $\mathrm{P}(a_q x \le \bar{b}_q) \ge \beta_q$ implies that $a_q x \le F_q^{-1}(1-\beta_q)$. If we denote $X(\beta)$ as$X(\beta) = \{x | a_q x \le F_q{}^{-1}(1-\beta_q), q = 1\ldots m, x \ge 0\}$ and replace constraints in model (1) with chance-constrained conditions with satisfying probability levels $\beta_i, \forall i = 1\ldots k$, we may reformulate model (1) as chance-constrained problem as follows:

$$\begin{aligned}&\text{minimize } (z_1(x)\ldots z_k(x))^{\mathrm{T}}\\&\text{subject to the constraints}\\&x \in X(\beta)\end{aligned} \tag{2}$$

2.2. Expectation Model for MOSLPP

In expectation model, DM wishes to minimise expected values of objective functions subject to constraints in his/her attempt to deal with MOSLPP. Therefore by replacing objective functions $z_i(x) = \bar{c}_i x, \forall i = 1\ldots k$ with corresponding expectations, MOSLPP may be reformulated as follows:

$$\begin{aligned}&\text{minimize } (z_1^E(x)\ldots z_k^E(x))^{\mathrm{T}}\\&\text{subject to the constraints}\\&x\in X(\beta)\end{aligned}\tag{3}$$

where $z_i^E(x) = E[z_i(x)] = E[\bar{c}_i]x$ denotes expectation of $z_i(x) = \bar{c}_i x, \forall i = 1\ldots k$.

2.3. MOSLPP in Fuzzy Environment and Corresponding Existing Solution Techniques, Including Interactive Technique

It is well known that fundamental to MOSLPP (3) is the concept of Pareto optimal solutions. Considering the imprecise nature inherent in our judgements in MOSLPP (3), linguistic statements can be quantified by generating membership functions $\mu_i({z_i}^E(x)), i = 1\ldots k$ from goals and tolerances (supplied by DM) for expectations ${z_i}^E(x), i = 1\ldots k$ of corresponding fuzzy objective functions [38,43]. Consequently, in fuzzy environment, MOSLPP may be converted into optimisation problems having membership functions $\mu_i({z_i}^E(x)), i = 1\ldots k$ as follows:

$$\begin{aligned}&\max\min\{\mu_i({z_i}^E(x)), i = 1\ldots k\}\\&\text{subject to}\\&x\in X(\beta)\end{aligned}\tag{4}$$

We may note that the approach to model (4) is preferable only when fuzzy decision by Bellman and Zadeh [43] is a proper representation of fuzzy preferences of DM. However as Sakawa and other researchers have described several times, such situations seem to occur rarely in practice [38].

Consequently in existing interactive fuzzy optimisation techniques, initially, DM specifies aspiration levels of achievements for membership functions (i.e. reference membership levels $\hat{\mu}_1, \hat{\mu}_2\ldots\hat{\mu}_k$) of expectations ${z_i}^E(x), i = 1\ldots k$ of fuzzy objective functions $z_i(x), i = 1\ldots k$. Next for DM's reference membership levels $\hat{\mu} = (\hat{\mu}_1, \hat{\mu}_2\ldots\hat{\mu}_k)^{\mathrm{T}}$, corresponding M-Pareto optimal solutions nearest to $\hat{\mu} = (\hat{\mu}_1, \hat{\mu}_2\ldots\hat{\mu}_k)^{\mathrm{T}}$ in minimax sense or even better than that, if attainable at all, maybe obtained by solving following minimax problem:

$$\begin{aligned}&\min\max_{i=1\ldots k}\,(\hat{\mu}_i - \mu_i({z_i}^E(x)))\\&\text{subject to the constraints}\\&x\in X(\beta).\end{aligned}\tag{5}$$

Or equivalently if $v = \max\limits_{i=1\ldots k}(\hat{\mu}_i - \mu_i({z_i}^E(x)))$, we have:

$$\begin{aligned}&\text{minimize } v\\&\text{subject to the constraints}\\&\hat{\mu}_i - \mu_i({z_i}^E(x)) \le v, i = 1\ldots k,\ x\in X(\beta).\end{aligned}\tag{6}$$

3. Finding Pareto Optimal Solutions to MOSLPP in Fuzzy Environment Based on Specified Main Objective Function

We may observe in real-life decision-making problems that DM may like to suggest one specific objective function as a main objective function. And in the existing decision-making process, especially in the interactive process, DM's focus may lie essentially in the optimal value of the main objective function. And DM may get satisfaction only if the optimal value of the specified main objective function meets the aspiration level and values of other objective functions are satisfactory. But we may find that in existing interactive fuzzy optimisation methods to solve MOSLPP, each objective function is equally treated. As a result, DM cannot effectively specify any one objective function as a main objective function in existing interactive fuzzy optimisation methods to solve MOSLPP. Consequently, DM may not be able to find optimal values as per his/her specifications. In this article, we have taken these issues under consideration and have developed one iterative process, in which specified main objective function is prioritised over other objective functions to solve MOSLPP in fuzzy environment.

Another problem is to determine reference membership levels $\hat{\mu}_1, \hat{\mu}_2 \ldots \hat{\mu}_k$ of expectations $(z_i{}^E(x), i = 1 \ldots k)$ of fuzzy objective functions. We may note that DM usually specifies these reference membership levels $\hat{\mu}_1, \hat{\mu}_2 \ldots \hat{\mu}_k$ of expectations of fuzzy objective functions from past experiences, intuition, etc. In existing interactive fuzzy optimisation methods to solve MOSLPP, initial reference membership levels are taken as unity. But we should not always expect that each of such conflicting objective functions shall attain respective goals simultaneously. Hence in this article, we may propose to compute initial reference membership levels of expectations of fuzzy objective functions analytically from trade-off ratios.

Consequently, we may choose expectation of any one objective function, say $z_t{}^E(x), t \in \{1 \ldots k\}$ arbitrarily. Next using the chain rule, we may compute trade-off ratios π_{tj} between membership function $\mu_t(z_t{}^E(x))$ of $z_t{}^E(x)$ and membership functions $\mu_j(z_j{}^E(x)), j = 1 \ldots k, j \neq t$ of expectations of other objective functions $z_j{}^E(x)$, $j = 1 \ldots k, j \neq t$, one by one, as follows [10,41,42]:

$$\pi_{tj} = -\frac{\partial \mu_t(z_t{}^E(x))}{\partial \mu_j(z_j{}^E(x))} = -\frac{\partial \mu_t(z_t{}^E(x))}{\partial z_t{}^E(x)} \frac{\partial z_t{}^E(x)}{\partial z_j{}^E(x)} \left(\frac{\partial \mu_j(z_j{}^E(x))}{\partial z_j{}^E(x)} \right)^{-1}, \quad j = 1 \ldots k, j \neq t$$

Next, we may derive a set of numbers $\bar{\mu}_1 \ldots \bar{\mu}_k$ by using the following formula [38]:

$$\bar{\mu}_j = \left| \left(-\frac{\partial \mu_t(z_t{}^E(x))}{\partial \mu_j(z_j{}^E(x))} \right)^{-1} \right| \bar{\mu}_t, \quad j = 1 \ldots k, \;\; j \neq t$$

Next, we may take $\bar{\mu}_t = \xi$, any positive non-zero real number and $\hat{\mu} = \max\{\bar{\mu}_1 \ldots \bar{\mu}_k\}$. Clearly $\hat{\mu} \neq 0$.

Finally, the initial reference membership level $\hat{\mu}_i^{(0)}, i = 1 \ldots k$ of expectation of ith objective function may be determined by the following formula [38]:

$$\hat{\mu}_i^{(0)} = \bar{\mu}_i / \hat{\mu}, \quad \forall i = 1 \ldots k (\because \hat{\mu} \neq 0) \tag{7}$$

Finally, we may employ these reference membership levels of expectations of fuzzy objective functions to determine M-Pareto optimal solutions to generalised MOSLPP (2).

The relationship between M-Pareto optimal solution to model (6) and Pareto optimal solution to MOSLPP (1 maybe characterised by the following theorems)

Theorem 3.1: *If $x' \in X$ is a unique optimal solution to model (6), x' is Pareto optimal solution to MOSLPP (1).*

Proof: Let $x' \in X$ is a unique optimal solution to minimax problem (6) and v' be the corresponding optimal value of v. Without loss of generality, we may assume that all constraints of model (6) are active constraints. Therefore, we have

$$v' = \min\{|\hat{\mu}_i - \mu_i({z_i}^E(x'))|, \hat{\mu}_\alpha - \mu_\alpha({z_\alpha}^E(x'))\} \text{ or } |\hat{\mu}_i - \mu_i(z_i(x'))| = v' = \hat{\mu}_\alpha - \mu_\alpha(z_\alpha(x'))$$

If possible, suppose x' is not Pareto optimal solution to MOSLPP (1). Then $\exists$ at least one $x \in X$ such that ${z_m}^E(x)) \leq {z_m}^E(x'), \forall m = 1 \ldots k, m \neq n$ and ${z_n}^E(x) < {z_n}^E(x')$ for some n. Since $\forall i = 1 \ldots k$, $\mu_i({z_i}^E(x))$ is a strictly monotonic decreasing function for the expectation of minimising type of objective function ${z_i}^E(x)$, we have: $\mu_m({z_m}^E(x')) \leq \mu_m({z_m}^E(x))$ $\forall m = 1 \ldots k$, $m \neq n$ and $\mu_n({z_n}^E(x')) < \mu_n({z_n}^E(x))$. Hence $\forall j$, we have: $\hat{\mu}_j - \mu_j({z_j}^E(x)) \leq \hat{\mu}_j - \mu_j({z_j}^E(x')) = v'$ (since all constraints are active). Consequently, we may get: $\min\{|\hat{\mu}_i - \mu_i({z_i}^E(x))|, \hat{\mu}_\alpha - \mu_\alpha({z_\alpha}^E(x))\} \leq v'$, which is the desired contradiction to the fact that x' is a unique optimal solution to the minimax problem (6). Therefore, our assumption is wrong. Hence x' is Pareto optimal solution to MOSLPP (1) ■

Theorem 3.2: *If $x' \in X$ is a Pareto optimal solution of the MOSLPP (1), x' is an optimal solution to model (6) for some $\hat{\mu}_i$, $i = 1 \ldots k$.*

Proof: Let $x' \in X$ be Pareto optimal solution to MOSLPP (1); also assume that $\forall i = 1 \ldots k {z_i}^E(x')$ is the corresponding optimal value of expectation of objective function $z_i(x)$. Next, $\forall i = 1 \ldots k$, we may construct a membership function $\mu_i({z_i}^E(x))$ of expectation $({z_i}^E(x))$ of fuzzy objective function $z_i(x)$. Therefore $\forall i = 1 \ldots k$, we may choose reference membership levels $\hat{\mu}_i$ such that $\hat{\mu}_i - \mu_i({z_i}^E(x')) = v'$ and $v' \geq 0$. Consequently, all constraints of MOSLPP (1) become active constraints for $x' \in X$.

Since $x' \in X$ is Pareto optimal solution to MOSLPP (1),

$\exists$ no $y \in X$ such that $\forall i = 1 \ldots k, i \neq j, {z_i}^E(y) \leq {z_i}^E(x')$ and ${z_j}^E(y) < {z_j}^E(x')$, for some j

i.e. $\forall i = 1 \ldots k, i \neq j, \mu_i({z_i}^E(y)) \geq \mu_i({z_i}^E(x'))$ and $\mu_j({z_j}^E(y)) > \mu_j({z_j}^E(x'))$, for some j

[$\because \forall i = 1 \ldots k, \mu_i$ is monotonically decreasing function]

i.e. $\forall i = 1 \ldots k, i \neq j, \hat{\mu}_i - \mu_i({z_i}^E(y)) \leq \hat{\mu}_i - \mu_i({z_i}^E(x'))$ and

$\hat{\mu}_j - \mu_j({z_j}^E(y)) < \hat{\mu}_j - \mu_j({z_j}^E(x'))$, for some j

i.e. $\forall i = 1 \ldots k, \hat{\mu}_i - \mu_i({z_i}^E(y)) \leq \hat{\mu}_i - \mu_i({z_i}^E(x'))$

i.e. $\forall y \in X, \exists$ at least one $\alpha \in \{1 \ldots k\}$ such that $\hat{\mu}_\alpha - \mu_\alpha({z_\alpha}^E(y)) > \hat{\mu}_\alpha - \mu_\alpha({z_\alpha}^E(x')) = v'$

Hence it implies that x' is optimal solution to single objective optimisation model (6) for some $\hat{\mu}_i$, $i = 1 \ldots k$. Hence the result is proved.

For generating Pareto optimal solution using theorems 3.1 and 3.2, we have to verify uniqueness of this M-Pareto optimal solution. This can be performed by considering Pareto optimality test problem with decision variables $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}}$, $\varepsilon = (\varepsilon_1, \varepsilon_2 \ldots \varepsilon_k)^{\mathrm{T}}$ as follows [38,42]:

$$\begin{aligned}
&\text{maximize } \sum_{i=1}^{k} \varepsilon_i \\
&\text{subject to the constraints} \\
&\text{for minimizing type of objective functions: } {z_i}^E(x') - \varepsilon_i \geq {z_i}^E(x), i = 1 \ldots k, \\
&(\text{for maximizing type of objective functions: } {z_i}^E(x) - \varepsilon_i \geq {z_i}^E(x'), i = 1 \ldots k) \\
&x \in X(\beta).
\end{aligned} \tag{8}$$

■

Theorem 3.3: *Let $\bar{x}$ and $\bar{\varepsilon}$ are optimal solutions to model (8). Then:*

(1) *If $\bar{\varepsilon}_i = 0, i = 1 \ldots k$, x' is Pareto optimal solution to MOSLPP (1).*
(2) *If at least one $\bar{\varepsilon}_i > 0$, x' is not Pareto optimal solution of MOSLPP (1). Instead of x', $\bar{x}$ is Pareto optimal solution to MOSLPP (1).*

We may skip its proof as proof of the analogous theorem is already recorded in the literature.

4. Proposed Iterative Process to Find Pareto Optimal Solutions to Given MOSLPP in Fuzzy Environment

The above ideas can be further integrated into a general framework and an algorithm may be developed to find preferable Pareto optimal solutions to MOSLPP (1) based on specified main objective function in fuzzy environment. The steps of our proposed algorithm may be synthesised as follows

Step 1: Request DM to specify satisficing probability levels $(\beta_q, q = 1 \ldots m)$ for each constraint. Compute individual maximum and minimum of expectation of each objective function $(z_i(x), i = 1 \ldots k)$ under given constraints. If unbounded solution(s) is (are) found for expectations of objective function(s), we may assign suitable large number(s) as extremum(s). Based on the information, elicit goals and tolerances for the expectation of each fuzzy objective functions from DM.

Step 2: Using these goals and tolerances, construct membership functions $\mu_i({z_i}^E(x)), i = 1 \ldots k$ of expectations $({z_i}^E(x), i = 1 \ldots k)$ of fuzzy objective functions $(z_i(x), i = 1 \ldots k)$. If DM is not available or DM is unable to specify goal(s) and/or tolerance(s) for expectation(s) of fuzzy objective function(s), corresponding individual minimum and/or maximum value(s) or any suitable value(s) may be employed.

Step 3: Next, we have to determine initial reference membership levels $\hat{\mu}_i^{(0)}, i = 1 \ldots k$ for expectations of fuzzy objective functions analytically. First, expectation $({z_t}^E(x), t \in \{1 \ldots k\})$ of any one objective function may be chosen arbitrarily. Using chain rule, we may compute trade-off ratios π_{tj} between membership function $\mu_t({z_t}^E(x))$ of ${z_t}^E(x)$ and membership functions $\mu_j({z_j}^E(x))$, $j = 1 \ldots k, j \neq t$ of expectations $({z_j}^E(x), j = 1 \ldots k, j \neq t)$ of other objective functions one by one as follows [10,38,41,42]:

$$\pi_{tj} = -\frac{\partial \mu_t({z_t}^E(x))}{\partial \mu_j({z_j}^E(x))} = -\frac{\partial \mu_t({z_t}^E(x))}{\partial {z_t}^E(x)} \frac{\partial {z_t}^E(x)}{\partial {z_j}^E(x)} \left(\frac{\partial \mu_j({z_j}^E(x))}{\partial {z_j}^E(x)} \right)^{-1}, \quad j = 1 \ldots k,\ j \neq t$$

Step 4: Next, we may derive a set of numbers $\bar{\mu}_1, \bar{\mu}_2 \ldots \bar{\mu}_k$ by using the formula [10,38,41,42]

$$\bar{\mu}_j = \left| \left(-\frac{\partial \mu_t({z_t}^E(x))}{\partial \mu_j({z_j}^E(x))} \right)^{-1} \right| \bar{\mu}_t, j = 1 \ldots k, j \neq t$$

Here, we may choose $\bar{\mu}_t = \xi$ (any positive non-zero real number) and suppose that $\bar{\mu}_t = \xi$ $\hat{\mu} = \max\{\bar{\mu}_1, \bar{\mu}_2 \ldots \bar{\mu}_k\}$ (so, $\hat{\mu} \neq 0$). Finally, in fuzzy environment, initial reference membership levels $\hat{\mu}_i^{(0)}, i = 1 \ldots k$ of expectations of objective functions may be determined by the formula: $\hat{\mu}_i^{(0)} = {}^{\bar{\mu}_i}/_{\hat{\mu}}, \forall i = 1 \ldots k$ ($\because \hat{\mu} \neq 0$).

Step 5: Now we may request DM to specify the main objective function. Suppose that DM chooses $z_\alpha(x)$, for some $\alpha \in \{1 \ldots k\}$, as the main objective function. Next in fuzzy environment, by using initial reference membership levels of expectations of objective functions, following optimisation model may be solved.

$$\begin{aligned}
&\text{minimize } v \\
&\text{subject to the constraints} \\
&\hat{\mu}_\alpha^{(0)} - \mu_\alpha({z_\alpha}^E(x)) \leq v, \\
&|\hat{\mu}_i^{(0)} - \mu_i({z_i}^E(x))| \leq v, \;\; \forall i = 1 \ldots k, \; i \neq \alpha \\
&v \geq 0, x \in X.
\end{aligned}$$

Let $x^{(1)}, v^{(1)}$ are optimal values of x and v to the above model after the first iteration; also suppose that ${z_i}^E(x^{(1)}), \mu_i({z_i}^E(x^{(1)}))$ are corresponding optimal values of ${z_i}^E(x), \mu_i({z_i}^E(x)), \forall i = 1 \ldots k$, respectively. This completes the first iteration.

Step 6: Next, to determine the updated reference membership levels of expectations of objective functions after n iterations (n being natural number) in fuzzy environment, we may apply the method of bisection as follows [10,42]

$$\forall i = 1 \ldots k, \hat{\mu}_i^{(n)} = \begin{cases} (\mu_i({z_i}^E(x^{(n)})) + \hat{\mu}_i^{(n-1)})/2, & \text{if } \mu_i({z_i}^E(x^{(n)})) < \hat{\mu}_i^{(n-1)} \\ (1 + \mu_i({z_i}^E(x^{(n)})))/2, & \text{otherwise} \end{cases} \tag{9}$$

Step 7: Finally, employing these updated reference membership levels $\hat{\mu}_i^{(n)}$, we may solve the following optimisation model

$$\begin{aligned}
&\text{minimize } v \\
&\text{subject to the constraints} \\
&\hat{\mu}_\alpha^{(n)} - \mu_\alpha({z_\alpha}^E(x)) \leq v, \\
&|\hat{\mu}_i^{(n)} - \mu_i({z_i}^E(x))| \leq v, \forall i = 1 \ldots k, i \neq \alpha \\
&v \geq 0, x \in X.
\end{aligned}$$

Let $x^{(n+1)}, v^{(n+1)}$ are optimal values of x and v to the above model after $(n+1)$ iterations; also suppose ${z_i}^E(x^{(n+1)}), \mu_i({z_i}^E(x^{(n+1)}))$ are corresponding optimal values of ${z_i}^E(x), \mu_i({z_i}^E(x)), \forall i = 1 \ldots k$, respectively, after $(n+1)$ iterations.

Step 8: Proceeding in this way, we may stop when the iterative process converges or the main objective function $z_\alpha(x)$ has attained its goal. Suppose that the iteration stops after m steps and $x^{(m)}, z_j^E(x^{(m)}), \mu_j(z_j^E(x^{(m)}))$ are corresponding M-Pareto optimal values of $x, z_j^E(x), \mu_j(z_j^E(x)), j = 1 \ldots k$, respectively. To test uniqueness of the M-Pareto optimal solution, we may solve the following model [38,42]:

$$\text{maximize } \sum_{i=1}^{k} \varepsilon_i$$

subject to the constraints

for minimizing type of objective functions: $z_i^E(x^{(m)}) - \varepsilon_i \geq z_i^E(x),\ i = 1 \ldots k,$

(for maximizing type of objective functions: $z_i^E(x) - \varepsilon_i \geq z_i^E(x^{(m)}), i = 1 \ldots k,$)

$\varepsilon_i \geq 0, i = 1 \ldots k, x \in X.$

Let $\forall i = 1 \ldots k, \bar{\varepsilon}_i$ and $\bar{x}$ are optimal values of $\varepsilon_i$and x, respectively. If $\bar{\varepsilon}_i = 0, \forall i = 1 \ldots k$, solution after m iterations, i.e. the solution $x^{(m)}$ is Pareto optimal solution to the given MOSLPP. Otherwise, the solution $\bar{x}$ to the above model is Pareto optimal solution to given MOSLPP.

Step 9: If the DM is satisfied with the Pareto optimal values, stop. Otherwise request DM to update goal(s) or tolerance(s) or even main objective function and go to Step 2.

Finally, the iterative process is complete.

5. Numerical Examples

To illustrate our proposed iterative process further, we may modify the production planning problem, as given in the book titled *Linear and Multi-objective Programming with Fuzzy Stochastic Extensions* by *Sakawa et al.* [38] and consider the following three-objective stochastic linear programming problem in fuzzy environment.

$$\begin{aligned}
&\text{fuzzy maximize } z_1(x) = \bar{5}x_1 + \bar{5}x_2\\
&\text{fuzzy minimize } z_2(x) = \bar{5}x_1 + \bar{1}x_2\\
&\text{fuzzy maximize } z_3(x) = \bar{3}x_1 - \bar{8}x_2\\
&\text{subject to the constraints}\\
&5x_1 + 7x_2 \leq \overline{16.2},\ 9x_1 + x_2 \leq \overline{12.525}, -5x_1 + 3x_2 \leq \overline{3.85},\\
&x_1 \geq 0,\ x_2 \geq 0.
\end{aligned} \tag{10}$$

In this problem, we may consider that the coefficients of objective functions are represented as random variables with associated means as $E(\bar{5}) = 5; E(\bar{1}) = 1; E(\bar{3}) = 3; E(-\bar{8}) = -8$ and right-hand side constants of constraints are represented by normal random variables defined by $\overline{16.2} \sim N(16.2, 5^2); \overline{12.525} \sim N(12.525, 3^2), \overline{3.85} \sim N(3.85, 1^2)$. Also, suppose that DM specifies the satisficing probability levels as 0.8 for each constraint. Next, we may apply the proposed iterative process on the above model to yield preferable Pareto optimal solutions based on specified main objective function in fuzzy environment.

First, we may compute individual maximum values and individual minimum values of expectations $(z_i^E(x), i = 1, 2, 3)$ of objective functions under given constraints, as given in Table 1. Based on these individual maximum and minimum values, DM may be asked to

Table 1. Individual maximum and minimum values of expectations of objective functions.

Expectations ofobjective functions	Individual maximum values	Individual minimum values
${z_1}^E(x)$	10	0
${z_2}^E(x)$	6	0
${z_3}^E(x)$	3.3	−11.1

Table 2. Goals and tolerances to objective functions.

Expectations ofobjective functions	Goals	Tolerances
${z_1}^E(x)$	5	1.5
${z_2}^E(x)$	4	1
${z_3}^E(x)$	−2	2

specify goals and tolerances of expectations $({z_i}^E(x), i = 1, 2, 3)$ of objective functions in fuzzy environment, as given in Table 2.

Using these goals and tolerances of expectations of objective functions in fuzzy environment, we may construct corresponding membership functions $\mu_i({z_i}^E(x)), i = 1, 2, 3$ of expectations of objective functions in fuzzy environment as follows.

$$\mu_1({z_1}^E(x)) = \begin{cases} 1, & \text{if } {z_1}^E(x) \geq 5 \\ \frac{{z_1}^E(x)-3.5}{1.5}, & \text{if } 3.5 \leq {z_1}^E(x) \leq 5 \\ 0, & \text{if } {z_1}^E(x) \leq 3.5 \end{cases}$$

$$\mu_2({z_2}^E(x)) = \begin{cases} 1, & \text{if } {z_2}^E(x) \leq 4 \\ \frac{5-{z_2}^E(x)}{1}, & \text{if } 4 \leq {z_2}^E(x) \leq 5 \\ 0, & \text{if } {z_2}^E(x) \geq 5 \end{cases}$$

$$\mu_3({z_3}^E(x)) = \begin{cases} 1, & \text{if } {z_3}^E(x) \geq -2 \\ \frac{{z_3}^E(x)+4}{2}, & \text{if } -4 \leq {z_3}^E(x) \leq -2 \\ 0, & \text{if } {z_3}^E(x) \leq -4 \end{cases}$$

Next, we may choose expectation of any one objective function, say ${z_1}^E(x)$, arbitrarily. By using chain rule, we may compute trade-off ratios between membership function $\mu_1({z_1}^E(x))$ of expectation of $({z_1}^E(x))$ and membership functions $\mu_j({z_j}^E(x)), j = 2, 3$ of expectations of other objective functions $({z_j}^E(x),\ j = 2, 3)$one by one as follows:

$$-\frac{\partial\mu_1({z_1}^E(x))}{\partial\mu_2({z_2}^E(x))} = -\frac{\partial\mu_1({z_1}^E(x))}{\partial {z_1}^E(x)}\frac{\partial {z_1}^E(x)}{\partial {z_2}^E(x)}\left(\frac{\partial\mu_2({z_2}^E(x))}{\partial {z_2}^E(x)}\right)^{-1} = \frac{110}{129};$$

$$-\frac{\partial\mu_1({z_1}^E(x))}{\partial\mu_3({z_3}^E(x))} = -\frac{\partial\mu_1({z_1}^E(x))}{\partial {z_1}^E(x)}\frac{\partial {z_1}^E(x)}{\partial {z_3}^E(x)}\left(\frac{\partial\mu_3({z_3}^E(x))}{\partial {z_3}^E(x)}\right)^{-1} = \frac{80}{129}$$

In order to determine initial reference membership levels $\hat{\mu}_i^{(0)}, i = 1, 2, 3$ of expectations of objective functions $z_i(x), i = 1, 2, 3$ in fuzzy environment, we may set $\bar{\mu}_1 = 1$. Thus, we may obtain: $\bar{\mu}_2 = 1.173$ and $\bar{\mu}_3 = 1.612$. And we may have $\hat{\mu} = 1.612$. Finally, using the formula: $\hat{\mu}_i^{(0)} = \bar{\mu}_i/\hat{\mu},\ i = 1, 2, 3$, initial reference membership levels of expectations of objective functions in fuzzy environment may be obtained as: $\hat{\mu}_1^{(0)} = 0.6202$, $\hat{\mu}_2^{(0)} = 0.7273$ and $\hat{\mu}_3^{(0)} = 1$.

Next suppose that DM specifies $z_2(x)$ as the main objective function. Proceeding as per our proposed iterative process, as discussed in Section 4, we may find M-Pareto optimal solution and consequently Pareto optimal solution to given MOSLPP as follows (¯denotes Pareto optimality):

$$\bar{x}_1 = 0.4623,\ \bar{x}_2 = 0.4235,\ z_1(\bar{x}) = 4.43,\ z_2(\bar{x}) = 2.73,\ z_3(\bar{x}) = -2.$$

Here we may observe that optimal value of expectation of the main objective function $z_2(x)$ is more preferable than corresponding aspiration level of DM and optimal values of expectations of all other objective functions $z_i(x), i = 1, 3$ in a proposed iterative process in fuzzy environment, are acceptable to DM (since each of them attained respective goals).

Comparison of Results

We may compare among optimal values obtained by the proposed iterative process and those obtained by other existing fuzzy optimisation methods in Table 3. Advantages of the proposed iterative process over several other existing solution methods are evident from Table 3. Here we may note that no two different Pareto optimal solutions can be similar or one Pareto optimal solution cannot be better over another. It is upon the DM to find out most preferable optimal solutions to MOSLPP in fuzzy environment. Moreover, completely different philosophies are ingrained in some of those existing techniques. Hence it may not be justifiable to compare elaborately.

Now, we may primarily focus on advantages of proposed iterative process over goal programming techniques. The goal programming method has weakness in at least three ways: 1. Because of the insistence on lexicographic optimality, some models may completely ignore lower priority objectives; 2. Some models disallow trade-off of small losses in high priority objectives for large gains in low priority objectives; 3. Some goal programming models ignore the non-constancy of the rates at which benefits from the objective attainments increase and they ignore non-constancy of the rates at which DM may trade-off attainments, etc. [44]. In our case, if we apply goal programming method having z_2 as the main objective function, corresponding optimal solution, as given in Table 3, shows that optimal expected values of z_1 and z_3 are very poor since optimal membership values of expectations are zero in both z_1 and z_3. Hence it may be reasonable to confess that such a solution maybe not acceptable to DM. Further, in the goal programming technique, usually, all objective functions are to be arranged in some order. Although we may consider only the main objective function as objective function and others as constraints, it may defeat the very purpose of goal programming. Moreover, in case of a large number of conflicting objective functions, it may not be feasible to rank all objective functions. But in a proposed iterative process, at one hand, we intrinsically employ trade-off rates among membership functions of expectations of fuzzy objective functions and on other hand, only one main objective function is necessary to run the process. It is said that goal programming's great virtue is that DM understand it easily; also, many of the alternate approaches are difficult to understand. This may be true in some instances, but should not be a guiding principle in the evolution of this field.

Table 3. Comparison of results.

	Pareto optimal solutions in						
	Existing techniques						
	Zimmermann's technique (Max-min operator)	Max-product operator	Sakawa's technique	Wu et al. technique [17]	Weighted sum approach	Goal programming	Proposed iterative process
Optimal values, ‾ denotes optimality	$\bar{x}_1 = 0.5455$, $\bar{x}_2 = 0.4545$, $z_1(\bar{x}) = 5$, $z_2(\bar{x}) = 3.18$, $z_3(\bar{x}) = -2$.	$\bar{x}_1 = 0.94$, $\bar{x}_2 = 1.04$, $z_1(\bar{x}) = 9.92$, $z_2(\bar{x}) = 5.76$, $z_3(\bar{x}) = -5.49$.	$\bar{x}_1 = 0.6583$, $\bar{x}_2 = 0.4263$, $z_1(\bar{x}) = 5.42$, $z_2(\bar{x}) = 3.72$, $z_3(\bar{x}) = -1.44$.	$\bar{x}_1 = 0.6583$, $\bar{x}_2 = 0.4263$, $z_1(\bar{x}) = 5.42$, $z_2(\bar{x}) = 3.72$, $z_3(\bar{x}) = -1.44$.	Weights: 0.25,0.5,0.25. $\bar{x}_1 = 0$, $\bar{x}_2 = 0$, $z_1(\bar{x}) = 0$, $z_2(\bar{x}) = 0$, $z_3(\bar{x}) = 0$.	$\bar{x}_1 = 0.145$, $\bar{x}_2 = 0.555$, $z_1(\bar{x}) = 3.5$, $z_2(\bar{x}) = 1.28$, $z_3(\bar{x}) = -4$.	$\bar{x}_1 = 0.4623$, $\bar{x}_2 = 0.4235$, $z_1(\bar{x}) = 4.43$, $z_2(\bar{x}) = 2.73$, $z_3(\bar{x}) = -2$.
Results	In the proposed iterative process, DM may effectively specify the main objective function; whereas in existing stochastic optimisation techniques under fuzzy environment, no such facility is available. And the optimal value of the main objective function $z_2(x)$ is more preferable to DM in the proposed iterative process than other existing techniques; moreover optimal values of other objective functions are also acceptable to DM (in our example, each objective function has attained its goal in the proposed iterative process).						
Remarks	Under fuzzy environment, the more effective and appropriate solution to MOSLPP may be obtained by applying our proposed iterative process than other existing fuzzy optimisation techniques. Ability to specify one main objective function may make the proposed iterative process more worthy as well as satisfactory to DM.						

Table 4. Pareto optimal solution to MOSLP model corresponding to the specified main objective functions.

Specified main objective functions	Pareto optimal solutions	Remarks
${z_1}^E(x)$	$\bar{x}_1 = 0.7485,\ \bar{x}_2 = 0.5305,$ ${z_1}^E(\bar{x}) = 6.39,\ {z_2}^E(\bar{x}) = 4.27, {z_3}^E(\bar{x}) = -2.$	More preferable optimal value for expectation of main objective function $z_1(x)$ than corresponding goal.
${z_2}^E(x)$	$\bar{x}_1 = 0.4623,\ \bar{x}_2 = 0.4235,$ ${z_1}^E(\bar{x}) = 4.43,\ {z_2}^E(\bar{x}) = 2.73, {z_3}^E(\bar{x}) = -2.$	More preferable optimal value for expectation of main objective function $z_2(x)$ than corresponding goal.
${z_3}^E(x)$	$\bar{x}_1 = 0.8467,\ \bar{x}_2 = 0.0394,$ ${z_1}^E(\bar{x}) = 4.43,\ {z_2}^E(\bar{x}) = 4.27, {z_3}^E(\bar{x}) = 2.23.$	More preferable optimal value for expectation of main objective function $z_3(x)$ than corresponding goal.

Again, suppose that DM specifies $z_1(x)$ as the main objective function. Applying the proposed iterative process with same goals and tolerances of expectations of objective functions, we may find Pareto optimal solutions to MOSLPP (10) in fuzzy environment as given in Table 4. Here, we may observe that optimal value of expectation $({z_1}^E(x))$ of the main objective function is more preferable than the corresponding goal; and optimal values of other objective functions are also acceptable to DM in a proposed iterative process (since both of these objective functions $z_i(x), i = 2, 3$ attain their respective goals).

Again, suppose that DM specifies $z_3(x)$ as main objective function. Applying the proposed iterative process with same goals and tolerances of expectations of objective functions, we may find Pareto optimal solutions to MOSLPP (10) in fuzzy environment as given in Table 4. Here we may observe that optimal value of expectation $({z_3}^E(x))$ of main objective function is more preferable than corresponding goal; and optimal values of other objective functions are also acceptable to DM in the proposed iterative process (since both of these objective functions $z_i(x), i = 1, 2$ attain their respective goals).

On the other hand, if the DM does not specify (or unable to specify) main objective function or DM is not available at all, we may generate Pareto optimal solutions by considering each objective function as main objective function one by one; it may help in finding most suitable Pareto optimal solution to given MOSLPP in fuzzy environment.

Conclusions

In this article, one general iterative process is developed to obtain preferable Pareto optimal solutions to MOSLPP based on specified main objective function in fuzzy environment. In several existing crisp and/or fuzzy optimisation methods to solve MOSLPP, no objective function can be effectively specified as the main objective function. Even if one can assign weights, goals, priorities, utility functions etc. to objective functions, those methods have major disadvantages (as were discussed in Section 5). But, in our proposed iterative process, we may obtain preferable Pareto optimal solutions corresponding to the specified main objective function. Moreover, we show through numerical examples in Section 5 that the optimal solutions obtained by our proposed iterative process are always Pareto optimal and may be more preferable to DM in comparison with solutions obtained by several other existing optimisation methods in fuzzy stochastic environment. Also, if DM alters main objective function, other sets of Pareto optimal solutions to MOSLPP may be quickly yielded based on the new main objective function. Further, the proposed iterative process may be applicable even if DM is unable to choose the main objective function or no DM is available at all.

Further, in existing fuzzy optimisation methods to solve MOSLPP, initial reference membership levels are arbitrarily set at unity. But we do not find it realistic to expect that expectations of each of the conflicting objective functions shall attain their respective goals simultaneously. Here, we have noted that trade-off rates among membership functions play one main role; consequently, in the proposed iterative process, we determine initial reference membership levels analytically by making use of trade-off ratios of membership functions of expectations of objective functions in fuzzy environment.

Future Research Directions

There are ample scopes for research involving proposed iterative process in future. We may extend it to other stochastic programming models. We may also employ various linear programming methods to solve final models obtained within this process. In future, we have scope to develop analogous iterative processes that can be applied to solve multiple objective stochastic non-linear optimisation models in fuzzy environment as well.

As Wierzbicki [40] said, we may note that real-life decision-making exercises may be looked upon as tools to handle complex problems of modern society. And such tools must be checked against real-life problems always. If there are complaints about efficiency of tools, an analyst must re-examine and redesign the model. When a new tool is found, analyst should be satisfied, but not to the extent of forgetting that he/she is constructing tools, which must again be checked in practice and further developed [40]. And proposed iterative process that is also interactive in nature may be one such optimal tool for DM to handle complex real-life problems under uncertainty.

This is especially important for us not to be influenced by only past successes and failures in explaining multi-objective methods to DM. If we bring into the classroom only what comes out of the boardroom, then the limitations of today's DM's will be imposed on tomorrow's.

Disclosure statement

No potential conflict of interest was reported by the author(s).

Notes on contributors

Arindam Garai, born and brought up at Burdwan. He is currently an Assistant Professor in the Department of Mathematics, Sonarpur Mahavidyalaya, Kolkata, India. He was awarded Ph.D. in Mathematics from IIEST, Howrah in 2018. The idea of T-set is his main contribution, till date. The other research interests include multi-objective optimization, inventory, and supply chain modelling in imprecise environments. He left his job at IBM to follow his passion towards teaching and research.

Palash Mandal, born and brought up at Kachukhali, Gosaba. He is currently an Assistant Teacher at Kalikapur Ramkamal Vidyapith (H.S.). He has been awarded the Ph.D. in Mathematics from IIEST, Howrah in 2018. He has published more than ten research articles in many reputed journals. His area of interest is optimization, fuzzy set theory, T-set and many more. He has around 5 years of teaching experience till date.

Dr. Tapan Kuma Roy, is currently a Professor in the Department of Mathematics, IIEST, Howrah, India. He completed the Doctor of Philosophy from the Vidyasagar University. He has published good number of articles on fuzzy and stochastic optimization, reliability optimization, inventory, supply chain in imprecise environments. Under his supervision, 23 students have been awarded Ph.Ds. He is much passionate about collecting rare books in regional languages.

ORCID

Arindam Garai http://orcid.org/0000-0003-2004-2489

References

[1] Zimmermann HJ. Applications of fuzzy set theory to mathematical programming. Inf Sci (Ny). 1985;36:29–58.
[2] Garai A, Mandal P, Roy TK. Multipollutant air quality management strategies: T-sets based optimization method under imprecise environment. Int J Fuzzy Syst. 2017;19(6):1927–1939.
[3] Garai A, Mandal P, Roy TK. Interactive intuitionistic fuzzy method in multi-objective optimization, Int. J. Fuzzy Comput. Model. 2016;2(1):14–26.
[4] Jimenez M, Bilbao A. Pareto-optimal solutions in fuzzy multi-objective linear programming. Fuzzy Sets Syst. 2009;150:2714–2721.
[5] Bellman RE, Zadeh LA. Decision-making in a fuzzy environment. Manage Sci. 1970;17:141–164.
[6] Tanaka H, Okuda T, Asai K. On fuzzy mathematical programming. J Cybern. 1974;3:37–46.
[7] Luhandjula MK. Fuzzy optimization: Milestones and perspectives. Fuzzy Sets Syst. 2015;274: 4–11.
[8] Luhandjula MK. Fuzzy optimization: An appraisal. Fuzzy Sets Syst. 1989;30(3):257–282.
[9] Zimmermann HJ. Fuzzy programming and linear programming with several objective functions. Fuzzy Sets Syst. 1978;1(1):45–55.
[10] Sakawa M, Yano H. Interactive decision-making for multiple nonlinear programming using augmented minimax problems. Fuzzy Sets Syst. 1986;20(1):31–43.
[11] Werners B. An interactive fuzzy programming system. Fuzzy Sets Syst. 1987;23:131–147.
[12] Lai YJ, Hwang CL. Fuzzy multiple objective decision-making: methods and applications. New York: Springer Verlag; 1994. (Lecture notes in economics and mathematical systems; vol. 404)
[13] Gupta AP, Harboe R, Tabucanon MT. Fuzzy multiple-criteria decision-making for crop area planning in Narmada river basin. Agric Syst. 2000;63:1–18.
[14] Kahraman C, Ertay T, Büyüközkan G. A fuzzy optimization model for QFD planning process using analytic network approach. Eur J Oper Res. 2006;171(2):390–411.
[15] Sahoo NC, Panda SK, Dash PK. A current modulation scheme for direct torque control of switched reluctance motor using fuzzy logic. Mechatronics (Oxf). 2000;10(3):353–370.
[16] Zhou S, Lam J, Zheng WX. Control design for fuzzy Systems based on relaxed non-quadratic stability and $H\infty$ performance conditions. IEEE Trans Fuzzy Syst. 2007;15(2):188–199.
[17] Wu Y-K, Liu C-C, Lur Y-Y. Pareto-optimal solution for multiple objective linear programming problems with fuzzy goals. Fuzzy Optimum Decision-Making. 2015;14:43–55.
[18] Garg H, Rani M, Sharma SP, et al. Intuitionistic fuzzy optimization method for solving multi-objective reliability optimization problems in interval environment. Expert Syst Appl. 2014;41(7):3157–3167.
[19] Wei G, Wang H, Zhao X, et al. Hesitant triangular fuzzy information aggregation in multiple attribute decision-making. J. Intell Fuzzy Syst: Appl Eng Techn. 2014;26(3):1201–1209.
[20] Salehi S, Selamat A, Mashiinchi MR, et al. The synergistic combination of particle swarm optimization and fuzzy sets to design granular classifier. Knowl Based Syst. 2015;76:200–218.
[21] Su S-F, Chen M-C. Enhanced fuzzy Systems for type 2 fuzzy and their Application in dynamic System Identification). 16th world Congress of the International fuzzy Systems Association.. Atlantis Press (2015)
[22] Garai A, Mandal P, Roy TK. Intuitionistic fuzzy T-sets based optimization method for production-distribution planning in supply chain management. OPSEARCH. 2016;53(4):950–975.

[23] Garai A, Mandal P, Roy TK. Intuitionistic fuzzy T-sets based solution method for multiple objective linear programming problems under imprecise environment. Notes Intuit Fuzzy Sets. 2015;21(4):104–123.
[24] Fernandeza A, Lopezb V, Jesusa M, et al. Revisiting evolutionary fuzzy systems: Taxonomy, applications, new trends and challenges. Knowl Based Syst. 2015;80:109–121.
[25] Wang GY, Wang WQ. Fuzzy optimum design of structure. Eng Optim. 1985;8:291–300.
[26] Yeh YC, Hsu DS. Structural optimization with fuzzy parameters. Comput Struct. 1990;37(6): 917–924.
[27] Xu C. Fuzzy optimization of structures by the two-phase method. Comput Struct. 1989;1(4): 575–580.
[28] Shih CJ, Lee HW. Level-cut approaches of first and Second Kind for unique solution design in fuzzy engineering optimization problems. Tamkang J Sci Eng. 2004;7(3):189–198.
[29] Yu L, Wang S, Lai KK. An intelligent-agent-based fuzzy group decision-making model for financial multicriteria decision support: The case of credit scoring. Eur J Oper Res. 2009;195(3): 942–959.
[30] Ebrahimnejad A, Verdegay JL. A novel approach for sensitivity analysis in linear programs with trapezoidal fuzzy numbers. J Intell Fuzzy Syst. 2014;27(1):173–185.
[31] Chakraborty D, Jana DK, Roy TK. A new approach to solve multi-objective multi-choice multi-item Atanassov's intuitionistic fuzzy transportation problem using chance operator. J Intell Fuzzy Syst. 2015;28(2):843–865.
[32] Martinez-Soto R, Castillo O, Castro J. Genetic algorithm optimization for type-2 non-singleton fuzzy logic controllers. Recent Adv Hybrid Approaches Design Intell Syst Series Stud Comput Intell. 2014;547:3–18.
[33] Jongsuebsuk P, Wattanapongsakorn N, Charnsripinyo C. Network intrusion detection with fuzzy genetic algorithm for unknown attacks. The International Conference on Information Networking 2013 (ICOIN). 2013: 1–5. DOI:10.1109/ICOIN.2013.6496342.
[34] Melin P, Astudillo L, Castillo O, et al. Optimal design of type-2 and type-1 fuzzy tracking controllers for autonomous mobile robots under perturbed torques using a new chemical optimization paradigm. Expert Syst Appl. 2013;40(8):3185–3195.
[35] Zheng Y-J, Ling H-F. Emergency transportation planning in disaster relief supply chain management: a cooperative fuzzy optimization approach. Soft Comput. 2013;17(7):1301–1314.
[36] Valdez F, Melin P, Castillo O. A survey on nature-inspired optimization algorithms with fuzzy logic for dynamic parameter adaptation. Expert Syst Appl. 2014;41:6459–6466. DOI:10.1016/j.eswa.2014.04.015.
[37] Bahmani BF, Farjah E, Azizipanah R. An efficient scenario-based and fuzzy self-adaptive learning particle swarm optimization approach for dynamic economic emission dispatch considering load and wind power uncertainties. Energy Volume. 2013;50(1):232–244.
[38] Sakawa M, Yano H, Nishizaki I. Linear and multi-objective programming with fuzzy stochastic extensions. New York: Springer Science Business Media; 2013.
[39] Guu S-M, Wu Y-K. Weighted coefficients in two-phase approach for solving the multiple objective programming problems. Fuzzy Sets Syst. 1997;85:45–48.
[40] Wierzbicki AP. The Use of Reference Objectives in Multiobjective Optimization. Multiple Criteria Decision Making Theory and Application. Lecture Notes in Economics and Mathematical Systems book series (LNE). Vol. 177, pp 468–486. ISBN: 978-3-642-48782-8. DOI:10.1007/978-3-642-48782-8_32.
[41] Sakawa M, Yano H, Yumine T. An interactive fuzzy satisficing method for multi-objective linear-programming problems and its application. IEEE Trans Syst Man and Cybernetics. 1987;17:654–661.
[42] Sakawa M, Matsui T. An interactive fuzzy satisficing method for multiobjective stochastic integer programming with simple recourse. Appl Math (Irvine). 2012;3:1245–1251.
[43] Dempe S, Ruziye A. On the calculation of a membership function for the solution of a fuzzy linear optimization problem. Fuzzy Sets Syst. 2012;188(1):58–67.
[44] Zeleny M.: Technical note: the pros and cons of goal programming. Comput Ops Res. 1981;8(4):357–359.