AKCE International Journal of Graphs and Combinatorics

ISSN: 0972-8600 (Print) 2543-3474 (Online) Journal homepage: https://www.tandfonline.com/loi/uakc20

Graphs determined by signless Laplacian spectra

Ali Zeydi Abdian, Afshin Behmaram & Gholam Hossein Fath-Tabar

To cite this article: Ali Zeydi Abdian, Afshin Behmaram & Gholam Hossein Fath-Tabar (2020) Graphs determined by signless Laplacian spectra, AKCE International Journal of Graphs and Combinatorics, 17:1, 45-50, DOI: 10.1016/j.akcej.2018.06.009

To link to this article: https://doi.org/10.1016/j.akcej.2018.06.009

Published online: 01 Jun 2020.

AKCE INTERNATIONAL JOURNAL OF GRAPHS AND COMBINATORICS
2020, VOL. 17, NO. 1, 45–50
https://doi.org/10.1016/j.akcej.2018.06.009

Taylor & Francis
Taylor & Francis Group

AKCE International Journal of Graphs and Combinatorics

Graphs determined by signless Laplacian spectra

Ali Zeydi Abdian[a,*], Afshin Behmaram[b], Gholam Hossein Fath-Tabar[c]

[a] Department of the Mathematical Science, Lorestan University, Lorestan, Khorramabad, Iran
[b] Faculty of Mathematical Sciences, University of Tabriz, Tabriz, Iran
[c] Department of Pure Mathematics, Faculty of Mathematical Sciences, University of Kashan, Kashan 87317-53153, Iran

Received 7 January 2018; received in revised form 6 June 2018; accepted 14 June 2018

Abstract

In the past decades, graphs that are determined by their spectrum have received more attention, since they have been applied to several fields, such as randomized algorithms, combinatorial optimization problems and machine learning. An important part of spectral graph theory is devoted to determining whether given graphs or classes of graphs are determined by their spectra or not. So, finding and introducing any class of graphs which are determined by their spectra can be an interesting and important problem. A graph is said to be DQS if there is no other non-isomorphic graph with the same signless Laplacian spectrum. For a DQS graph G, we show that $G \cup rK_1 \cup sK_2$ is DQS under certain conditions, where r, s are natural numbers and K_1 and K_2 denote the complete graphs on one vertex and two vertices, respectively. Applying these results, some DQS graphs with independent edges and isolated vertices are obtained.

Keywords: Spectral characterization; Signless Laplacian spectrum; Cospectral graph

1. Introduction

Let $G = (V, E)$ be a simple graph with vertex set $V = V(G) = \{v_1, \ldots, v_n\}$ and edge set $E = E(G) = \{e_1, \ldots, e_m\}$. Denote by $d(v)$ the degree of vertex v. All graphs considered here are simple and undirected. All notions on graphs that are not defined here can be found in [1–5]. The join of two graphs G and H is a graph formed from disjoint copies of G and H by connecting each vertex of G to each vertex of H. We denote the join of two graphs G and H by $G \bigtriangledown H$. The complement of a graph G is denoted by $\overline{G}$.

Let $A(G)$ be the (0, 1)-adjacency matrix of graph G. The characteristic polynomial of G is $\det(\lambda I - A(G))$, and it is denoted by $P_G(\lambda)$. Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be the distinct eigenvalues of G with multiplicities $m_1, m_2, \ldots, m_n$, respectively. The multi-set of eigenvalues of $Q(G)$ is called the signless Laplacian spectrum of G. The matrices $L(G) = D(G) - A(G)$ and $Q(G) = SL(G) = D(G) + A(G)$ are called the Laplacian matrix and the signless

Peer review under responsibility of Kalasalingam University.

* Corresponding author.
E-mail addresses: abdian.al@fs.lu.ac.ir (A.Z. Abdian), behmaram@tabrizu.ac.ir (A. Behmaram), fathtabar@kashanu.ac.ir (G.H. Fath-Tabar).

https://doi.org/10.1016/j.akcej.2018.06.009

Please cite this article in press as: A.Z. Abdian, et al., Graphs determined by signless Laplacian spectra, AKCE International Journal of Graphs and Combinatorics (2018), https://doi.org/10.1016/j.akcej.2018.06.009.

Laplacian matrix of G, respectively, where $D(G)$ denotes the degree matrix. Note that $D(G)$ is diagonal. The multiset $\text{Spec}_Q(G) = \{[\lambda_1]^{m_1}, [\lambda_2]^{m_2}, \ldots, [\lambda_n]^{m_n}\}$ of eigenvalues of $Q(G)$ is called the signless Laplacian spectrum of G, where m_i denote the multiplicities of λ_i. The Laplacian spectrum is defined analogously.

For any bipartite graph, its Q-spectrum coincides with its L-spectrum. Two graphs are Q-cospectral (resp. L-cospectral, A-cospectral) if they have the same Q-spectrum (resp. L-spectrum, A-spectrum). A graph G is said to be DQS (resp. DLS, DAS) if there is no other non-isomorphic graph Q-cospectral (resp. L-cospectral, A-cospectral) with G. Van Dam and Haemers [6] conjectured that almost all graphs are determined by their spectra. Nevertheless, the set of graphs that are known to be determined by their spectra is too small. So, discovering infinite classes of graphs that are determined by their spectra can be an interesting problem. About the background of the question "Which graphs are determined by their spectrum?", we refer to [6]. It is interesting to construct new DQS (DLS) graphs from known DQS (DLS) graphs. For a DLS graph G, the join $G \cup rK_1$ is also DLS under some conditions [7]. Actually, a graph is DLS if and only if its complement is DLS. Hence we can obtain DLS graphs from known DLS graphs by adding independent edges.

Up to now, only some graphs with special structures are shown to be *determined by their spectra* (DS, for short) (see [1,8–30] and the references cited in them).

In this paper, we investigate signless Laplacian spectral characterization of graphs with independent edges and isolated vertices. For a DQS graph G, we show that $G \cup rK_1 \cup sK_2$ is DQS under certain conditions. Applying these results, some DQS graphs with independent edges and isolated vertices are obtained.

2. Some definitions and preliminaries

Some useful established results about the spectrum are presented in this section, will play an important role throughout this paper.

Lemma 2.1 (*[4,9,17]*). *For the adjacency matrix, the Laplacian matrix and the signless Laplacian matrix of a graph G, the following can be deduced from the spectrum:*

(1) The number of vertices.

(2) The number of edges.

(3) Whether G is regular.

For the Laplacian matrix, the following follows from the spectrum:

(4) The number of components.

For the signless Laplacian matrix, the following follow from the spectrum:

(5) The number of bipartite components.

(6) The sum of the squares of degrees of vertices.

Lemma 2.2 (*[17]*). *Let G be a graph with n vertices, m edges and t triangles and vertex degrees $d_1, d_2, \ldots, d_n$. Let $T_k = \sum_{i=1}^{n}(q_i(G))^k$, then*

$$T_0 = n,\ T_1 = \sum_{i=1}^{n} d_i = 2m,\ T_2 = 2m + \sum_{i=1}^{n} d_i^2 \text{ and } T_3 = 6t + 3\sum_{i=1}^{n} d_i^2 + \sum_{i=1}^{n} d_i^3.$$

For a graph G, let $P_L(G)$ and $P_Q(G)$ denote the product of all nonzero eigenvalues of L_G and Q_G, respectively. We assume that $P_L(G) = P_Q(G) = 1$ if G has no edges.

Lemma 2.3 (*[4]*). *For any connected bipartite graph G of order n, we have $P_Q(G) = P_L(G) = n\tau(G)$, where $\tau(G)$ is the number of spanning trees of G.*

For a connected graph G with n vertices and m edges, G is called unicyclic (resp. bicyclic) if $m = n$ (resp. $m = n + 1$). If G is a unicyclic graph that contains an odd (resp. even) cycle, then G is called odd unicyclic (resp. even unicyclic).

Lemma 2.4 (*[31]*). *For any graph G, $det(Q_G) = 4$ if and only if G is an odd unicyclic graph. If G is a non-bipartite connected graph and $|E(G)| > |V(G)|$, then $det(Q_G) > 16$, with equality if and only if G is a non-bipartite bicyclic graph with C_4 as its induced subgraph.*

Please cite this article in press as: A.Z. Abdian, et al., Graphs determined by signless Laplacian spectra, AKCE International Journal of Graphs and Combinatorics (2018), https://doi.org/10.1016/j.akcej.2018.06.009.

Lemma 2.5 ([32]). *Let H be a proper subgraph of a connected graph G. Then, $q_1(G) > q_1(H)$.*

3. Main results

We first investigate spectral characterizations of the union of a tree and several complete graphs K_1 and K_2.

Theorem 3.1. *Let T be a $DLS(DQS)$ tree of order n. Then $T \cup rK_1 \cup sK_2$ is DLS.*

$T \cup rK_1 \cup sK_2$ isDQS if$n$ is not divisible by 2 and $s = 1$.

Proof. Let G be any graph L-cospectral with $T \cup rK_1 \cup sK_2$. By Lemma 2.1, G has $n + r + 2s$ vertices, $n - 1 + s$ edges and $r + s + 1$ components. So each component of G is a tree. Suppose that $G = G_0 \cup G_1 \cup \cdots \cup G_{r+s}$, where G_i is a tree with n_i vertices and $n_0 \geq n_1 \geq \cdots \geq n_s \geq \cdots \geq n_{r+s} \geq 1$. Since G is L-cospectral with $T \cup rK_1 \cup sK_2$, by Lemma 2.3, we get $n_0 n_1 \cdots n_{r+s} = P_L(G) = n2^s$. We claim that $n_s = 2$. Suppose not and so $n_s \geq 3$. Therefore, $n_0 \geq n_1 \geq \cdots \geq n_s \geq 3$ and since $n_{s+1} \geq \cdots \geq n_{r+s} \geq 1$, one may deduce that $n2^s = n_0 n_1 \cdots n_{r+s} \geq 3^{s+1}$ or $n(\frac{2}{3})^s \geq 3$. Now if $s \longrightarrow \infty$, then $0 \geq 3$, a contradiction. Hence $n_s = 2$. By a similar argument one may show that $n_1 = n_2 = \cdots = n_{s-1} = 2$ and so $n_0 = n$ and $n_{s+1} = n_{s+2} = \cdots = n_{s+r} = 1$. Hence $G = G_0 \cup rK_1 \cup sK_2$. Since G and $T \cup rK_1 \cup sK_2$ are L-cospectral, G_0 and T are L-cospectral. Since T is DLS, we have $G_0 = T$, $G = T \cup rK_1 \cup sK_2$. Hence $T \cup rK_1 \cup sK_2$ is DLS. Let H be any graph Q-cospectral with $T \cup rK_1 \cup sK_2$. By Lemma 2.1, H has $n + r + 2s$ vertices, $n - 1 + s$ edges and $r + s + 1$ bipartite components. So one of the following holds:

(i) H has exactly $r + s + 1$ components, and each component of H is a tree.

(ii) H has $r + s + 1$ components which are trees, the other components of H are odd unicyclic.

If (i) holds, then H and $T \cup rK_1 \cup sK_2$ are both bipartite, so they are also L-cospectral. Since $T \cup rK_1 \cup sK_2$ is DLS, we have $H = T \cup rK_1 \cup sK_2$.

If (ii) holds, then by Lemma 2.4, $P_Q(H)$ is divisible by 4. Since T is a tree of order n, by Lemma 2.3, $P_Q(H) = n2^s$ is divisible by 4. Hence $T \cup rK_1 \cup sK_2$ is DQS when n is not divisible by 2 and $s = 1$. □

Remark 1. Some DLS trees are given in [33–38]. We can obtain DLS (DQS) graphs with independent edges and isolated vertices from Theorem 3.1.

Theorem 3.2. *Let G be a DQS odd unicyclic graph of order $n \geq 7$. Then $G \cup rK_1 \cup sK_2$ is DQS.*

Proof. Let H be any graph Q-cospectral with $G \cup rK_1 \cup sK_2$. By Lemma 2.4, $P_Q(H) = 4(2^s)$. By Lemma 2.1, H has $n + r + 2s$ vertices, $n + r$ edges and $r + s$ bipartite components. So one of the following holds:

(i) H has exactly $r + s$ components, and each component of H is a tree.

(ii) H has $r + s$ components which are trees, the other components of H are odd unicyclic.

If (i) holds, then we can let $H = H_1 \cup \cdots \cup H_{r+s}$, where H_i is a tree with n_i vertices and $n_1 \geq \cdots \geq n_{r+s} \geq 1$. Since $P_Q(H) = 4(2^s)$, by Lemma 2.3, we have $n_1 \cdots n_{r+s} = 4(2^s)$, $n_1 \leq 8$.

Since G contains a cycle, we have $q_1(H) = q_1(G) \geq 4$. Let $\Delta(H)$ be the maximum degree of H. If $\Delta(H) \leq 2$, then all components of H are paths, i.e., $q_1(H) < 4$, a contradiction. So $\Delta(H) > 3$. From $n_1 \leq 8$ and $n_1 \cdots n_{r+s} = 4(2^s) = 2^{(s+2)}$, we know that $H_1 = K_{1,7}$ (without loss of generality), $H_2 = \cdots = H_s = K_2$ and $H_{s+1} = \cdots = H_{r+s} = K_1$. Since $H = K_{1,7} \cup (s-1)K_2 \cup rK_1$ has $n + r + 2s$ vertices, we get $n = 6$, a contradiction to $n > 6$.

If (ii) holds, then we can let $H = U_1 \cup \cdots \cup Uc \cup H_1 \cup \cdots \cup H_r$, where U_i is odd unicyclic, H_i is a tree with n_i vertices. By Lemmas 2.3 and 2.4, $4(2^s) = P_Q(H) = 4^c n_1 \cdots n_r$. So $c = 1$, $H_1 = \cdots = H_s = K_2$ and $H_{s+1} = \cdots = H_{r+s} = K_1$. Since $H = U_1 \cup rK_1 \cup sK_2$ and $G \cup rK_1 \cup sK_2$ are Q-cospectral, U_1 and G are Q-cospectral. Since G is DQS, we have $U_1 = G$, $H = G \cup rK_1 \cup sK_2$. □

Remark 2. Some DQS unicyclic graphs are given in [39–44]. We can obtain DQS graphs with independent edges and isolated vertices from Theorem 3.2.

Theorem 3.3. *Let G be a non-bipartite DQS bicyclic graph with C_4 as its induced subgraph and $n \geq 5$. Then $G \cup rK_1 \cup sK_2$ is DQS.*

Please cite this article in press as: A.Z. Abdian, et al., Graphs determined by signless Laplacian spectra, AKCE International Journal of Graphs and Combinatorics (2018), https://doi.org/10.1016/j.akcej.2018.06.009.

Proof. Let H be any graph Q-cospectral with $G \cup rK_1 \cup sK_2$. By Lemma 2.4, we have $P_Q(H) = 16(2^r)$. By Lemma 2.1, H has $n + r + 2s$ vertices, $n + 1 + s$ edges and $r + s$ bipartite components, where $n = |V(G)|$. So H has at least $r + s - 1$ components which are trees. Suppose that $H_1, H_2, \ldots, H_{r+s}$ are $r + s$ bipartite components of H, where $H_2, \ldots, H_r$ are trees. If H_1 contains an even cycle, then by Lemma 2.3, we have $P_Q(H) \geq P_Q(H_1) \geq 16$, and $P_Q(H) = 16(2^{s-1})$ if and only if $H = C_4 \cup (s - 1)K_2 \cup rK_1$. Since H has $n + r + 2s$ vertices, we get $n = 2$, a contradiction (G contains C_4). Hence $H_1, H_2, \ldots, H_{r+s}$ are trees. Since H has $n + r + 2s$ vertices, $n + 1 + r + 2s$ edges and $r + s$ bipartite components, H has a non-bipartite component H_0 which is a bicyclic graph. Lemma 2.4 implies that $P_Q(H) > P_Q(H_0) > 16$, and $P_Q(H) = 16(2^s)$ if and only if $H = H_0 \cup rK_1 \cup sK_2$ and H_0 contains C_4 as its induced subgraph. By $PQ(H) = 16(2^s)$, we have $H = H_0 \cup rK_1 \cup sK_2$. Since H and $G \cup rK_1 \cup sK_2$ are Q-cospectral, H_0 and G are Q-cospectral. Since G is DQS, we have $H_0 = G$, $H = G \cup rK_1 \cup sK_2$. Hence $G \cup rK_1 \cup sK_2$ is DQS. □

Remark 3. Some DQS bicyclic graphs are given in [45–48]. We can obtain DQS graphs with independent edges and isolated vertices from Theorem 3.3.

Theorem 3.4. *Let G be a DQS **connected** non-bipartite graph with $n \geq 3$ vertices. If H is Q-cospectral with $G \cup rK_1 \cup sK_2$, then H is a DQS graph.*

Proof. By Lemma 2.1, H has $n+r+2s$ vertices and at least $r+s$ bipartite components. We perform the mathematical induction on s.

H has $r + s$ components. Since H has at least $r + s$ bipartite components, each component of H is bipartite. Suppose that $H = H_1 \cup \cdots \cup H_{r+s}$, where H_i is a connected bipartite graph with n_i vertices, and $n_1 \geq \cdots \geq n_s \geq \cdots \geq n_{r+s} \geq 1$. Since H and $G \cup rK_1 \cup sK_2$ are Q-cospectral, by Lemma 2.1, G is a connected non-bipartite graph.

Let $s = 1$. For $n \geq 3$, $q_1(G) \geq 3$, since G has $K_{1,2}$ or K_3 as its subgraph. Obviously $\mathrm{Spec}_Q(H)$ has exactly $r + s$ eigenvalues that are zero. We show that if H is Q-cospectral with $G \cup rK_1 \cup K_2$, then H is a DQS graph. First we show that there is no connected graph Q-cospectral with $\mathrm{Spec}_Q(G') = \mathrm{Spec}_Q(G) \cup \left\{[2]^1\right\}$. In fact we prove that G' cannot have 2 as its eigenvalue. Obviously, $\mathrm{Spec}_Q(H) = \mathrm{Spec}_Q(G') \cup \left\{[0]^{r+1}\right\}$. But, in this case $|E(G')| = |E(G)| + 1$ and $|V(G')| = |V(G)| + 1$, which means that G' must be connected. Otherwise, G' contains 0 as its signless eigenvalues, a contradiction. Therefore, G is a proper subgraph of G' and so $q_1(G') \gneqq q_1(G) \geq 3$ (see Lemma 2.5), a contradiction. Therefore, G' cannot have 2 as its eigenvalue. By what was proved one can easily conclude that $\mathrm{Spec}_Q(H) = \mathrm{Spec}_Q(G) \cup \mathrm{Spec}_Q(K_2) \cup \mathrm{Spec}_Q(rK_1)$, since G is not a bipartite graph and so has not 0 as an its signless Laplacian eigenvalue. Therefore, $H = G \cup K_2 \cup rK_1$.

Now, let the theorem be true for s; that is, if $\mathrm{Spec}_Q(G_1) = \mathrm{Spec}_Q(G) \cup \mathrm{Spec}_Q(rK_1 \cup sK_2)$, then $G_1 = G \cup rK_1 \cup sK_2$. We show that it follows from $\mathrm{Spec}_Q(K) = \mathrm{Spec}_Q(G) \cup \mathrm{Spec}_Q(rK_1 \cup (s + 1)K_2)$ that $K = G \cup rK_1 \cup (s + 1)K_2$. Obviously, K has 2 vertices, one edge and one bipartite component more than G_1. So, we must have $K = G_1 \cup K_2$. □

Remark 4. In the following results graph G in $G \cup rK_1 \cup sK_2$ is a connected non-bipartite.

Corollary 3.1. *The graph $K_n \cup rK_1 \cup sK_2$ is DQS.*

Proof. From [6] (Proposition 7), if $n = 1, 2$, then $K_n \cup rK_1 \cup sK_2$ is DQS. For $n \geq 3$, by Theorem 3.4 the result follows. □

In [49], Cámara and Haemers proved that a graph obtained from K_n by deleting a matching is DAS. In [50], it have been shown that this graph is also DQS.

Corollary 3.2. *Let G be the graph obtained from K_n by deleting a matching. Then $G \cup rK_1 \cup sK_2$ is DQS.*

Proof. From [6] (Proposition 7), if $n = 1, 2$, then $K_n \cup rK_1 \cup sK_2$ is DQS. For $n \geq 3$, by Theorem 3.4 the result follows. □

A regular graph is DQS if and only if it is DAS [6]. It is known that a k-regular graph of order n is DAS when $k = 0, 1, 2, n-1, n-2, n-3$ [17]. Hence a k-regular graph of order n is DQS when $k = 0, 1, 2, n-1, n-2, n-3$.

Please cite this article in press as: A.Z. Abdian, et al., Graphs determined by signless Laplacian spectra, AKCE International Journal of Graphs and Combinatorics (2018), https://doi.org/10.1016/j.akcej.2018.06.009.

Corollary 3.3. *Let G be a connected $(n-2)$-regular graph of order n. Then $G \cup rK_1 \cup sK_2$ is DQS.*

Corollary 3.4. *Let G be a connected $(n-3)$-regular graph of order n. Then $G \cup rK_1 \cup sK_2$ is DQS.*

Corollary 3.5. *Let G be a connected $(n-4)$-regular DAS graph. Then $G \cup rK_1 \cup sK_2$ is DQS.*

Remark 5. Some 3-regular DAS graphs are given in [6,51]. We can obtain DQS graphs with independent edges and isolated vertices and isolated vertices from Corollary 3.4.

Corollary 3.6. *Let F_n denote the friendship graph and G be Q-spectral with F_n, then $G \cup rK_1 \cup sK_2$ is DQS.*

Proof. It is well-known that F_n is DQS. By Theorem 3.4 the proof is completed. □

References

[1] X. Liu, L. Pengli, Signless Laplacian spectral characterization of some joins, Electron. J. Linear Algebra 30 (1) (2015) 30.
[2] R.B. Bapat, Graphs and Matrices, Springer-Verlag, New York, 2010.
[3] N.L. Biggs, Algebraic Graph Theory, Cambridge University press, Cambridge, 1933.
[4] D. Cvetković, P. Rowlinson, S. Simić, An introduction to the theory of graph spectra, in: London Mathematical Society Student Teyts, vol. 75, Cambridge University Press, Cambridge, 2010.
[5] D.B. West, Introduction to Graph Theory, Prentice hall, Upper Saddle River, 2001.
[6] E.R. Van Dam, W.H. Haemers, Which graphs are determined by their spectrum? Linear Algebra. Appl. 373 (2003) 241–272.
[7] X. Lizhen, C. He, On the signless Laplacian spectral determination of the join of regular graphs., Discrete Math. Algorithms Appl. 6 (04) (2014) 1450050.
[8] A.Z. Abdian, Lowell W. Beineke, A. Behmaram On the spectral determinations of the connected multicone graphs $K_r \bigtriangledown sK_t$, arXiv preprint arXiv:1806.02625.
[9] A.Z. Abdian, S.M. Mirafzal, On new classes of multicone graph determined by their spectrums, Alg. Struc. Appl. 2 (2015) 23–34.
[10] A.Z. Abdian, Graphs which are determined by their spectrum, Konuralp J. Math. 4 (2016) 34–41.
[11] A.Z. Abdian, Two classes of multicone graphs determined by their spectra, J. Math. Ext. 10 (2016) 111–121.
[12] A.Z. Abdian, Graphs cospectral with multicone graphs $K_w \bigtriangledown L(P)$, TWMS. J. App and Eng. Math. 7 (2017) 181–187.
[13] A.Z. Abdian, The spectral determination of the multicone graphs $K_w \bigtriangledown P$, 2017. arXiv preprint arXiv:1703.08728.
[14] A.Z. Abdian, S.M. Mirafzal, The spectral characterizations of the connected multicone graphs $K_w \bigtriangledown LHS$ and $K_w \bigtriangledown LGQ(3,9)$, Discrete Math. Algorithms Appl. 10 (2018) 1850019.
[15] A.Z. Abdian, S.M. Mirafzal, The spectral determinations of the connected multicone graphs $K_w \bigtriangledown mP_{17}$ and $K_w \bigtriangledown mS$, Czechoslovak Math. J. (2018) 1–14. http://dx.doi.org/10.21136/CMJ.2018.0098-17.
[16] A.Z. Abdian, The spectral determinations of the multicone graphs $K_w \bigtriangledown mC_n$, arXiv preprint arXiv:1703.08728.
[17] C. Bu, J. Zhou, Signless Laplacian spectral characterization of the cones over some regular graphs, Linear Algebra Appl. 436 (2012) 3634–3641.
[18] D. Cvetković, P. Rowlinson, S. Simić, Signless Laplacians of finite graphs, Linear Algebra Appl. 423 (1) (2007) 155–171.
[19] D. Cvetković, S. Simić, Towards a spectral theory of graphs based on the signless Laplacian, II, Linear Algebra Appl. 432 (2010) 2257–2272.
[20] K.C. Das, On conjectures involving second largest signless Laplacian eigenvalue of graphs, Linear Algebra Appl. 43 (2010) 2.
[21] W.H. Haemers, X.G. Liu, Y.P. Zhang, Spectral characterizations of lollipop graphs, Linear Algebra Appl. 428 (2008) 2415–2423 3018–3029.
[22] R. Merris, Laplacian matrices of graphs: A survey, Linear Algebra Appl. 197 (1994) 143–176.
[23] S.M. Mirafzal, A.Z. Abdian, Spectral characterization of new classes of multicone graphs, Stud. Univ. Babeş-Bolyai Math 62 (3) (2017) 275–286.
[24] S.M. Mirafzal, A.Z. Abdian, The spectral determinations of some classes of multicone graphs, J. Discrete Math. Sci. Cryptogr. 21 (1) (2018) 179–189.
[25] R. Sharafdini, A.Z. Abdian, Signless Laplacian determinations of some graphs with independent edges, Carpathian Mathematical Publication (2018) in press.
[26] W. Yi, F. Yizheng, T. Yingying, On graphs with three distinct Laplacian eigenvalues, Appl. Math. J. Chinese Univ. Ser. A 22 (2007) 478–484.
[27] J. Wang, H. Zhao, Q. Huang, Spectral charactrization of multicone graphs, Czechoslovak Math. J. 62 (2012) 117–126.
[28] HS.H. Günthard, H. Primas, Zusammenhang von Graph theory und Mo-Theorie von Molekeln mit Systemen konjugierter Bindungen, Helv. Chim. Acta 39 (1925) 1645–1653.
[29] M.H. Liu, B.L. Liu, Some results on the Laplacian spectrum, J. Comput. Appl. Math. 59 (2010) 3612–3616.
[30] J. Zhou, C. Bu, Laplacian spectral characterization of some graphs obtained by product operation, Discrete Math. 312 (2012) 1591–1595.
[31] M. Mirzakhah, D. Kiani, The sun graph is determined by its signless Laplacian spectrum, Electron. J. Linear Algebra. 20 (2010) 610–620.
[32] M. Chen, B. Zhou, On the signless Laplacian spectral radius of cacti, Croat. Chem. Acta 89 (4) (2016) 1–6.
[33] G. Aalipour, S. Akbari, N. Shajari, Laplacian spectral characterization of two families of trees, Linear Multilinear Algebra 62 (2014) 965–977.
[34] R. Boulet, The centipede is determined by its Laplacian spectrum, C. R. Acad. Sci., Paris I 346 (2008) 711–716.
[35] C. Bu, J. Zhou, H. Li, Spectral determination of some chemical graphs, Filomat 26 (2012) 1123–1131.

Please cite this article in press as: A.Z. Abdian, et al., Graphs determined by signless Laplacian spectra, AKCE International Journal of Graphs and Combinatorics (2018), https://doi.org/10.1016/j.akcej.2018.06.009.

[36] P.L. Lu, X.D. Zhang, Y. Zhang, Determination of double quasi-star tree from its Laplacian spectrum, J. Shanghai Univ. 14 (3) (2010) 163–166.
[37] X.L. Shen, Y.P. Hou, Some trees are determined by their Laplacian spectra, J. Nat. Sci. Hunan Norm. Univ. 29 (1) (2006) 21–24 (in Chinese).
[38] Z. Stanić, On determination of caterpillars with four terminal vertices by their Laplacian spectrum, Linear Algebra Appl. 431 (2009) 2035–2048.
[39] C. Bu, J. Zhou, H. Li, W. Wang, Spectral characterizations of the corona of a cycle and two isolated vertices, Graphs Combin. 30 (2014) 1123–1133.
[40] M.H. Liu, Some graphs determined by their (signless) Laplacian spectra, Czechoslovak Math. J. 62 (137) (2012) 1117–1134.
[41] M.H. Liu, H.Y. Shan, K.C. Das, Some graphs determined by their (signless) Laplacian spectra, Linear Algebra Appl. 449 (2014) 154–165.
[42] X. Liu, S. Wang, Y. Zhang, X. Yong, On the spectral characterization of some unicyclic graphs, Discrete Math. 311 (2011) 2317–2336.
[43] J.F. Wang, F. Belardo, Q. Zhang, Signless Laplacian spectral characterization of line graphs of T-shape trees, Linear Multilinear Algebra 62 (2014) 1529–1545.
[44] Y. Zhang, X. Liu, B. Zhang, X. Yong, The lollipop graph is determined by its Q-spectrum, Discrete Math. 309 (2009) 3364–3369.
[45] G. Guo, G. Wang, On the (signless) Laplacian spectral characterization of the line graphs of lollipop graphs, Linear Algebra Appl. 438 (2013) 4595–4605.
[46] X. Liu, S. Zhou, Spectral characterizations of propeller graphs, Electron. J. Linear Algebra. 27 (2014) 19–38.
[47] J.F. Wang, Q.X. Huang, F. Belardo, E.M. Li Marzi, On the spectral characterizations of 1-graphs, Discrete Math. 310 (2010) 1845–1855.
[48] J.F. Wang, Q.X. Huang, F. Belardo, E.M. Li Marzi, Spectral characterizations of dumbbell graphs, Electron. J. Combin. 17 (2010).
[49] M. Cámara, W.H. Haemers, Spectral characterizations of almost complete graphs, Discrete Appl. Math. 176 (2014) 19–23.
[50] S. Huang, J. Zhou, C. Bu, Signless Laplacian spectral characterization of graphs with isolated vertices, Filomat 30 (14) (2017).
[51] F.J. Liu, Q.X. Huang, H.J. Lai, Note on the spectral characterization of some cubic graphs with maximum number of triangles, Linear Algebra Appl. 438 (2013) 1393–1397.

Please cite this article in press as: A.Z. Abdian, et al., Graphs determined by signless Laplacian spectra, AKCE International Journal of Graphs and Combinatorics (2018), https://doi.org/10.1016/j.akcej.2018.06.009.