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# Complement of the generalized total graph of fields

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**ABSTRACT**
Let $R$ be a commutative ring and $H$ be a multiplicative prime subset of $R$. The generalized total graph $GT_H(R)$ is the undirected simple graph with vertex set $R$ and two distinct vertices $x$ and $y$ are adjacent if $x+y \in H$. For a field $F$, $H=\{0\}$ is the only multiplicative prime subset of $F$ and the corresponding generalized total graph is denoted by $GT(F)$. In this paper, we investigate several graph theoretical properties of $\overline{GT(F)}$, where $\overline{GT(F)}$ is the complement of the generalized total graph of $F$. In particular, we characterize all the fields for which $\overline{GT(F)}$ is unicyclic, split, chordal, claw-free, perfect and pancyclic.

**KEYWORDS**
Generalized total graph; field; split; chordal; pancyclic; perfect

**1991 MATHEMATICS SUBJECT CLASSIFICATION**
Primary: 05C75; 05C25; Secondary:13A15; 13M05

## 1. Introduction

Let $R$ be a commutative ring with identity, $Z(R)$ be its set of all zero-divisors, $Z^*(R) = Z(R) \setminus \{0\}$ and $U(R)$ be the set of all units in $R$. Anderson and Livingston [3] introduced the *zero-divisor graph* of $R$, denoted by $\Gamma(R)$, as the (undirected) simple graph with vertex set $Z^*(R)$ and two distinct vertices $x, y \in Z^*(R)$ are adjacent if and only if $xy = 0$. Subsequently, Anderson and Badawi [1] introduced the concept of the *total graph* of a commutative ring. The *total graph* $T_\Gamma(R)$ of $R$ is the undirected graph with vertex set $R$ and for distinct $x, y \in R$ are adjacent if and only if $x + y \in Z(R)$. Tamizh Chelvam and Asir [4, 5, 15–19] have extensively studied about the total graph of commutative rings. Tamizh Chelvam and Balamurugan [22] studied about the complement of the generalized total graph of $\mathbb{Z}_n$. For a complete detail about total graphs, one can refer the survey [6, 14].

Recently, Anderson and Badawi [2] introduced the concept of the generalized total graph of a commutative ring $R$. A nonempty proper subset $H$ of $R$ to be a multiplicative prime subset of $R$ if the following two conditions hold: (i) $ab \in H$ for every $a \in H$ and $b \in R$; (ii) if $ab \in H$ for $a, b \in R$, then either $a \in H$ or $b \in H$. For a multiplicative prime subset $H$ of $R$, the *generalized total graph* $GT_H(R)$ of $R$ is the simple undirected graph with vertex set $R$ and two distinct vertices $x$ and $y$ are adjacent if and only if $x + y \in H$. For example, every prime ideal, union of prime ideals and $H = R \setminus U(R)$ are some of the multiplicative-prime subsets of $R$. One may note that total graphs cannot be studied for integral domains, where as the generalized total graph gives the scope to associate graph with even fields and integral domains. In a field $F$, $\{0\}$ is the only multiplicative prime subset of $F$. When $R$ is the field $F$ and $H = \{0\}$, we designate the graph as the generalized total graph of the field $F$ and denote the same by $GT(F)$. Tamizh Chelvam and Balamurugan [20, 21] have studied about the generalized total graph and its complement of commutative rings. Further they have studied domination properties of the generalized total graph of a field and its complement in [20]. In this paper, we study several other properties of the complement of the generalized total graph of fields.

Let $G = (V, E)$ be a graph with vertex set $V$ and edge set $E$. We say that $G$ is connected if there is a path between any two distinct vertices of $G$. The complement $\overline{G}$ of the graph $G$ is the simple graph with vertex set $V(G)$ and two distinct vertices $x$ and $y$ are adjacent in $\overline{G}$ if and only if they are not adjacent in $G$. For a vertex $v$ of a graph $G$, $deg(v)$ is the degree of the vertex $v$. $\delta(G)$ and $\Delta(G)$ denote the minimum and maximum degree of vertices in $G$ respectively. $K_n$ denotes the complete graph of order $n$ and $K_{m,n}$ denotes the complete bipartite graph. For the terms in graph theory which are not explicitly mentioned here, one can refer [8, 23], for the terms regarding algebra one can refer [13]. Note that if $R$ is finite, then $GT_{Z(R)}(R)$ is the unit graph [12].

Throughout this paper $F$ denotes a finite field. In this paper, we continue our study on graph theoretical properties of the complement $\overline{GT(F)}$. In Section 2, we study the graph theoretical properties namely pancyclic, unicyclic, split, claw-free and perfectness of $\overline{GT(F)}$. Also we obtain edge clique covering number of $\overline{GT(F)}$. In Section 3, we obtain a characterization for $\overline{GT(F)}$ to be planar and outerplanar.

## 2. Properties of $\overline{GT(F)}$

In this section, we prove that $\overline{GT(F)}$ is unicyclic, split, claw-free, perfect and pancyclic. Further we prove that when

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$\overline{GT(F)}$ is a path or bipartite or chordal. Also we discuss obtain the edge clique covering number of $\overline{GT(F)}$. We make use the following theorem, which gives the structure of the generalized total graph of a commutative ring.

**Theorem 2.1.** ([2, Theorem 2.2]) *Let $H$ be a prime ideal of a finite commutative ring $R$, and let $|H| = \lambda$ and $|\frac{R}{H}| = \mu$.*

(i) *If $2 \in H$, then $GT_H(R \setminus H)$ is the union of $\mu - 1$ disjoint $K_\lambda$'s;*
(ii) *If $2 \notin H$, then $GT_H(R \setminus H)$ is the union of $\frac{\mu-1}{2}$ disjoint $K_{\lambda,\lambda}$'s.*

Since $H = \{0\}$ is the only prime ideal in a field $F$, we have the following lemma which gives the structure for the generalized total graph $GT(F)$. Of course, the structure depends upon the characteristic of the field $F$. In fact if $char(F) = 2$, then $x + x = 0$ for every $x \in F$. When the $char(F) > 2$, for any $0 \neq x \in F, x \neq -x$ and $x + (-x) = 0$.

**Lemma 2.2.** *Let $F$ be a finite field. Then*

$$GT(F) = \begin{cases} \bigcup_{i=1}^{|F|} K_1 & \text{if } char(F) = 2; \\ K_1 \bigcup_{i=1}^{\frac{|F|-1}{2}} K_{1,1} & \text{if } char(F) > 2. \end{cases}$$

In view of the above Lemma 2.2, we have the following lemma for the complement $\overline{GT(F)}$.

**Lemma 2.3.** *Let $F$ be a finite field. Then the following are true:*

(i) *If $char(F) = 2$, then $\overline{GT(F)} = K_{|F|}$;*
(ii) *If $char(F) > 2$, then $\overline{GT(F)}$ is a connected bi-regular graph with $\Delta = |F| - 1$ and $\delta = |F| - 2$.*

Note that, a graph $G$ is said to be *unicyclic* if $G$ contains exactly one cycle.

**Theorem 2.4.** *Let $F$ be a finite field. Then the following hold:*

(i) *$\overline{GT(F)}$ is bipartite if and only if either $F \cong \mathbb{Z}_2$ or $F \cong \mathbb{Z}_3$;*
(ii) *$\overline{GT(F)}$ is neither a cycle nor an unicyclic graph.*

*Proof.* (i) If $F \cong \mathbb{Z}_2$ or $F \cong \mathbb{Z}_3$, then $\overline{GT(F)}$ is trivially a bipartite graph. Conversely assume that $\overline{GT(F)}$ is bipartite. If $char(F) = 2$ with $|F| \geq 4$, then $|F| = 2^n$ for some $n \in \mathbb{Z}^+$ and $n \geq 2$. By Lemma 2.3(i), $\overline{GT(F)}$ is a complete graph of order $\geq 4$. This implies $\overline{GT(F)}$ contains a $K_3$ as a subgraph and so $\overline{GT(F)}$ is not a bipartite graph. Hence $F \cong \mathbb{Z}_2$ when $char(F) = 2$.

Suppose $char(F) > 2$ with $|F| > 3$. Then $|F| \geq 5$. Let $S = \{0, x, y\}$ where $x, y \in F \setminus \{0\}, x \neq y$ and $x \neq -y$. Then the induced subgraph $< S >$ is $K_3$ and $K_3$ is a subgraph of $\overline{GT(F)}$. Therefore $\overline{GT(F)}$ is not bipartite. Hence $F \cong \mathbb{Z}_3$ when $char(F) > 2$.

(ii) Suppose $\overline{GT(F)}$ is a cycle. Then $|F| \geq 3$.

If $char(F) = 2$, by Lemma 2.3(i), $\overline{GT(F)}$ is a complete graph of order $\geq 4$, which is a contradiction.

If $char(F) > 2$, by Lemma 2.3(ii), $\overline{GT(F)}$ is bi-regular which is a contradiction.

Suppose $\overline{GT(F)}$ is unicyclic. Then $|F| \geq 3$. If $char(F) = 2$, then $\overline{GT(F)}$ contains $K_4$ as a subgraph, which is not unicyclic. Suppose $char(F) > 2$. If $F \cong \mathbb{Z}_3$, then $\overline{GT_P(F)} = P_3$ which is not unicyclic. Suppose $|F| > 3$. Then $|F| \geq 5$. Let $S_1 = \{0, x, y\}$ where $x, y \in F \setminus \{0\}, x \neq y$ and $x \neq -y$. Let $S_2 = \{0, u, v\}$ where $u, v \in F \setminus S_1, u \neq v$ and $u \neq -v$. Then the induced subgraphs $< S_1 >$ and $< S_2 >$ are two different cycles of length 3 in $\overline{GT(F)}$ and so $\overline{GT(F)}$ is not unicyclic. □

Recall that, a *chordal graph* is a simple graph $G$ in which every cycle in $G$ of length four and greater has a *cycle chord*. Also, a *split graph* [9] is a graph in which the vertices can be partitioned into a clique and an independent set. The following characterization for split graphs is used to characterize when $\overline{GT(F)}$ is split.

**Theorem 2.5.** ([10, Theorem 6.3]) *Let $G$ be a connected graph. Then $G$ is a split graph if and only if $G$ contains no induced subgraph isomorphic to $2K_2$, $C_4$ and $C_5$.*

**Theorem 2.6.** *Let $F$ be a finite field. Then the following are equivalent:*

(i) *Either $char(F) = 2$ or $F \cong \mathbb{Z}_3$;*
(ii) *$\overline{GT(F)}$ is a split graph;*
(iii) *$\overline{GT(F)}$ is a chordal graph.*

*Proof.* 1 ⇒ 2. Assume that either $char(F) = 2$ or $F \cong \mathbb{Z}_3$. To prove that $\overline{GT(F)}$ is a split graph. If $char(F) = 2$, then by Lemma 2.3(i) and Theorem 2.5, $\overline{GT(F)}$ is a split graph. If $F \cong \mathbb{Z}_3$, then by Lemma 2.3(i), $\overline{GT(F)}$ is $P_3$ and so by Theorem 2.5, $\overline{GT(F)}$ is a split graph.

2 ⇒ 3. Note that every split graph is a chordal graph and proof is trivial.

3 ⇒ 1. Assume that $\overline{GT(F)}$ is a chordal graph. Suppose $char(F) > 2$ with $|F| > 3$. Then $|F| \geq 5$. Let $S = \{x, y, u, v\} \subset F$ where $y = -x$ and $v = -u$. Then $< S >= C_4$ is a chordless cycle in $\overline{GT(F)}$ and so $\overline{GT(F)}$ is not a chordal graph. Hence either $char(F) = 2$ or $F \cong \mathbb{Z}_3$. □

A graph $G$ is a *claw-free* if $G$ does not have the claw $K_{1,3}$ as the induced subgraph of $G$. Now we prove that $\overline{GT(F)}$ is claw-free.

**Theorem 2.7.** *Let $F$ be a finite field. Then $\overline{GT(F)}$ is a claw-free graph.*

*Proof.* If $char(F) = 2$, then $\overline{GT(F)}$ is complete and hence it is a claw-free graph. Suppose $char(F) > 2$. If $F \cong \mathbb{Z}_3$, then

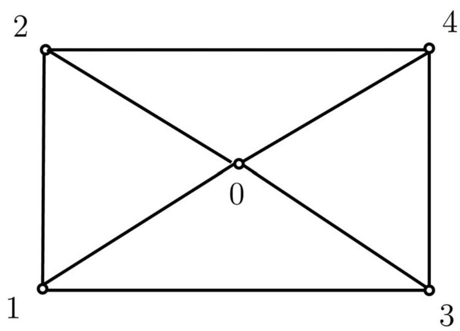


Figure 1. $\overline{GT_P(\mathbb{Z}_5)}$.

$\overline{GT(F)} = P_3$ which is claw free. When $|F| \geq 5$, consider a subset $S \subset V(\overline{GT(F)})$ with $|S| = 4$. By Lemma 2.3(ii), $deg_{<S>}(u) \geq 2$ for every $u \in S$. Hence $\overline{GT(F)}$ contains no vertex of degree 1 and so $\overline{GT(F)}$ is not a claw-free graph. □

A graph $G$ is *perfect* if and only if no induced subgraph of $G$ is an odd cycle of length at least five or the complement of one.

**Theorem 2.8.** *Let F be a finite field. Then $\overline{GT(F)}$ is a perfect graph.*

*Proof.* By Lemma 2.2, $GT(F)$ have no induced subgraph that is an odd cycle of length at least 5. If $char(F) = 2$, then, by Lemma 2.4(i) $\overline{GT(F)}$ is complete and so $\overline{GT(F)}$ have no induced subgraph that is an odd cycle of length at least 5. Assume that $char(F) > 2$. If $F \cong \mathbb{Z}_3$, by Lemma 2.4(i) $\overline{GT(F)} = P_3$ which is not a cycle. If $|F| = 5$, the graph $\overline{GT(F)}(\mathbb{Z}_5)$ is given in Figure 1 and is perfect. When $|F| \geq 7$, consider a subset $S \subset V(\overline{GT(F)})$ with $|S| \geq 5$. By Lemma 2.3(ii), $deg_{<S>}(u) \geq 3$ for every $u \in S$. Hence $\overline{GT(F)}$ contains no vertex of degree 2 and so $\overline{GT(F)}$ is a perfect graph. □

A graph $G$ of order $m \geq 3$ is *pancyclic* ([7, Definition 6.3.1]) if $G$ contains cycles of all lengths from 3 to $m$. Also $G$ is called *vertex-pancyclic* if each vertex $v$ of $G$ belongs to a cycle of every length $\ell$ for $3 \leq \ell \leq m$.

**Theorem 2.9.** *Let F be a finite field. Then $\overline{GT(F)}$ is a pancyclic if and only if $|F| > 3$.*

*Proof.* Let $\overline{GT(F)}$ be a pancyclic graph. Note that $\overline{GT(\mathbb{Z}_2)} = K_2$ and $\overline{GT(\mathbb{Z}_3)} = P_3$. In both cases $\overline{GT(F)}$ is not a cycle, which is a contradiction to our assumption.

Conversely assume that $|F| > 3$. If $char(F) = 2$, then $\overline{GT(F)}$ is complete and so $\overline{GT(F)}$ is a pancyclic. If $char(F) > 2$, then $|F| \geq 5$. Let $F = \{0, x_1, ..., x_{\frac{|F|-1}{2}}, y_1, ..., y_{\frac{|F|-1}{2}}\}$ where each $x_i$ is the additive inverse of $y_i$ for $1 \leq i \leq \frac{|F|-1}{2}$. Note that $< \{0, x_1, ..., x_{\frac{|F|-1}{2}}\} > = < \{0, y_1, ..., y_{\frac{|F|-1}{2}}\} > = K_{\frac{|F|+1}{2}}$. Note that $P : 0 - x_1 - \cdots - x_{\frac{|F|-1}{2}} - y_1 - \cdots - y_{\frac{|F|-1}{2}} - 0$ is a spanning cycle in $\overline{GT(F)}$. By removing the vertices one by one from the set $S = \{y_{\frac{|F|-1}{2}}, y_{\frac{|F|-3}{2}}, ..., y_1, x_{\frac{|F|-1}{2}} \cdots, x_4, x_3\}$, we get cycles of lengths $|F| - 1, |F| - 2, ..., 4, 3$ as subgraphs in $\overline{GT(F)}$. From this, we get cycles of length from 3 to $F$ as subgraphs in $\overline{GT(F)}$. Hence $\overline{GT(F)}$ is pancyclic. □

**Corollary 2.10.** *Let F be a finite field. Then $\overline{GT(F)}$ is a vertex-pancyclic if and only if $|F| > 3$.*

Note that, an *edge clique cover* of a graph $G$ is a collection of cliques $L_1, L_2, ..., L_k$ such that $E(G) = \cup_{i=1}^{k} E(L_i)$. The minimum cardinality of an edge clique cover of $G$ is called *the edge-clique covering number* of $G$ and is denoted by $\theta_1(G)$.

The following lemma provides the clique number of $\overline{GT(F)}$ [20].

**Lemma 2.11.** ([20, Lemma 3.3]) *Let F be a finite field. Then*

$$\omega(\overline{GT(F)}) = \begin{cases} |F| & \text{if } char(F) = 2; \\ \frac{|F|+1}{2} & \text{if } char(F) > 2. \end{cases}$$

In the following lemma, we obtain the edge clique covering number of $\overline{GT(F)}$.

**Theorem 2.12.** *Let F be a field. Then*

$$\theta_1(\overline{GT(F)}) = \begin{cases} 1 & \text{if } char(F) = 2; \\ 2 & \text{if } F \cong \mathbb{Z}_3; \\ 2 + \frac{|F|-1}{2} & \text{otherwise}. \end{cases}$$

*Proof.* If $char(F) = 2$, then $\overline{GT(F)}$ is complete and so $\theta_1(\overline{GT(F)}) = 1$. If $F = \mathbb{Z}_3$, then $\overline{GT(F)} = P_3$ and so $\theta_1(\overline{GT(F)}) = 2$. Assume that $char(F) > 2$ and $|F| > 3$. List the elements of $F$ as $F = \{0, x_1, ..., x_{\frac{|F|-1}{2}}, y_1, ..., y_{\frac{|F|-1}{2}}\}$ where $x_i$ is the additive inverse $y_i$ for all $1 \leq i \leq \frac{|F|-1}{2}$. Let $S = \{0, x_1, ..., x_{\frac{|F|-1}{2}}\}$, $T = \{0, y_1, ..., y_{\frac{|F|-1}{2}}\}$ and $S_i = (S \setminus \{x_i\}) \cup \{y_i\}$ for $1 \leq i \leq \frac{|F|-1}{2}$. Then $< S > = < T > = < S_i > = K_{\frac{|F|+1}{2}}$ in $\overline{GT(F)}$. By Lemma 2.11, $< S >$, $< T >$ and $< S_i >$ are cliques in $\overline{GT(F)}$ for all $1 \leq i \leq \frac{|F|-1}{2}$.

Also $\cup_{i=1}^{\frac{|F|-1}{2}} E(< S_i >) \cup E(< S >) \cup E(< T >) = E(\overline{GT(F)})$ and so $\theta_1(\overline{GT(F)}) = 2 + \frac{|F|-1}{2}$. □

## 3. When $\overline{GT(F)}$ is planar or outerplanar

In this section, we discuss about planarity and outerplanarity of $\overline{GT(F)}$. The following two results are used for characterization of planar and outerplanar nature of $\overline{GT(F)}$.

**Theorem 3.1.** [8, Theorem 9.7] *A graph is planar if and only if it does not contain a subdivision of $K_5$ or $K_{3,3}$.*

**Theorem 3.2.** [11, Theorem 11.10] *A graph G is outerplanar if and only if it does not contain a subdivision of $K_4$ or $K_{2,3}$.*

**Theorem 3.3.** *Let F be a finite field. Then the following hold:*

(i) $\overline{GT(F)}$ *is planar if and only if* $|F| \leq 5$;
(ii) $\overline{GT(F)}$ *is outer planar if and only if* $|F| \leq 3$.

*Proof.* (i) Assume that $|F| \leq 5$. If $F$ is either $\mathbb{Z}_2$, or $\mathbb{Z}_3$ then the graph $\overline{GT(F)}$ is either $K_2$, or $P_3$. If $F \cong \mathbb{F}_4$, then $\overline{GT(F)}$ is $K_4$ and so $\overline{GT(F)}$ is planar. If $F \cong \mathbb{Z}_5$, then the planar embedding of $\overline{GT(\mathbb{Z}_5)}$ is given in the above Figure 1.

Conversely, assume that $\overline{GT(F)}$ is planar. Let us consider $|F| \geq 7$. If $char(F) = 2$, then $|F| \geq 8$. By Lemma 2.4(i), $\overline{GT(F)}$ is complete which is not planar.

Consider $char(F) > 2$. Let $F = \{0, x_1, ..., x_{\frac{|F|-1}{2}}, y_1, ..., y_{\frac{|F|-1}{2}}\}$ where $x_i$ is the additive inverse $y_i$ for all $1 \leq i \leq \frac{|F|-1}{2}$. Since $|F| \geq 7$, we can choose a set $S = \{0, x_1, x_2, x_3, y_1, y_2\}$. Then $< S >$ contains a subdivision of $K_{3,3}$ in $\overline{GT(F)}$ and so $\overline{GT(F)}$ is not planar.

(ii) If $F$ is either $\mathbb{Z}_2$ or $\mathbb{Z}_3$, the proof is trivial.

Conversely, if $F \cong \mathbb{F}_4$ then $\overline{GT(F)} = K_4$ which is not outerplanar. For $F \cong \mathbb{Z}_5$, from the Figure 1, $\overline{GT(F)}$ contains a subdivision of $K_{2,3}$ and so $\overline{GT(F)}$ is not outerplanar. For $|F| \geq 7$, the proof follows as in (i) above. □

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