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A Hilbert Space Geometric Representation of Shared Awareness and Joint Decision Making

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A HILBERT SPACE GEOMETRIC REPRESENTATION OF SHARED AWARENESS AND JOINT DECISION MAKING

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ABSTRACT

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Two people in the same situation may ascribe very different meanings to their experiences. They will form different awareness, reacting differently to shared information. Various factors can give rise to this behavior. These factors include, but are not limited to, prior knowledge, training, biases, cultural factors, social factors, team vs. individual context, time, resources, and technology. At the individual level, the differences in attaining separate actions by accessing shared information may not be considered as an anomaly from the perspective of rational decision-making. But for group behavior, reacting differently to the shared information can give rise to conflicts and deviations from an expected behavior, and are categorized as an anomaly or irrational behavior. The lack of proper recognition of the reasons for differences can even impede the shared action towards attaining a common objective. The manifestation of differences becomes noticeable in complex situations.

The shared awareness approaches that originate from available situational awareness models fail to recognize the reasons of an unexpected decision in these situations. One reason for this is that in complex situations, incompatible events can become dominant. Human information processing is sensitive to the compatibility of the events. This, and various other human psychological characteristics, require models to be developed that include comprehensive formalisms for both compatible and incompatible events in complex situations.

Quantum probability provides a geometrical probabilistic formalism to study the decision and the dynamic cognitive systems in complex situations. The event representation in Hilbert space provides the necessary foundation to represent an individual's knowledge of a situation. Hilbert space allows representing awareness as a superposition of indefinite states. These states form a complete N-dimensional Hilbert space. Within the space generated, events are represented as a subspace.

By using these characteristics of Hilbert space and quantum geometrical probabilities, this study introduces a representation of self and other-than-self in a situation. An area of awareness with the possibility of projection onto the same event allows representing shared awareness geometrically. This formalism provides a coherent explanation of shared awareness for both compatible and incompatible events. Also, by using the superposition principles, the dissertation introduces spooky action at a distance concept in studying shared awareness.

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CHAPTER 1

INTRODUCTION

1.1 A Narrative of the Problem

Are humans rational or irrational? Why and how do two people attain different actions, while accessing the same information? Seeking fathomable answers to these and similar questions has become the motivation to explore the human behavior further. Organizational studies, decision science, economics, and psychology are among the disciplines, which scrutinize the human behavior from various aspects. People categorize the accessed information with different heuristics and develop an understanding of the situations. To demonstrate the effect of contextuality and categorization the optical illusion in Figure **1** is a good example. Before showing the picture in Figure **1** to an individual, context-generating questions can be asked. For example,

- Which bird to do you see in this picture?
- Which mammal do you see in the picture?

These questions generate activates heuristics, which influence the answer of the individual drastically.

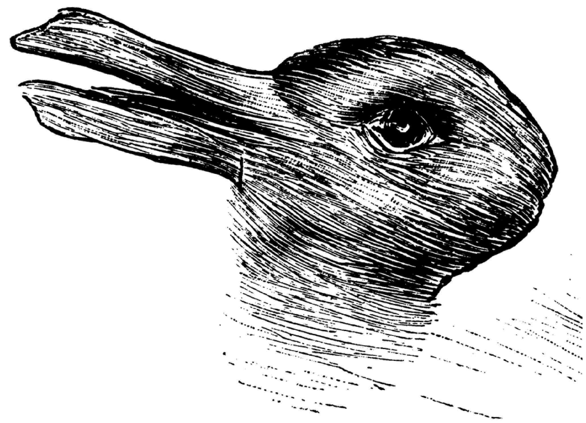

Figure 1 Effects of categorization and contextuality in understanding Rabbit–duck illusion (2017, April 5).

Another method to impose a context and bias the perception and the understanding of the individual could be to add a few of Easter eggs to the background. The people who live in a Christian culture will typically immediately recognize the picture as a rabbit rather than a duck. The Easter eggs form a context to those who know what Easter egg and rabbit mean in the context of Easter (Figure **2**).

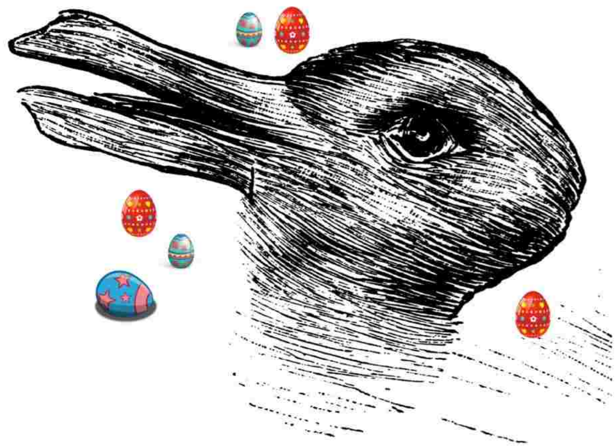

Figure 2 Incorporated context into the optical illusion Rabbit–duck illusion (2017, April 5).

Disposition, experience, framing, space, time and many other factors that are generated through cultural variances impact the understanding of a phenomenon. The ensuing perceptual variations mean that two people may access the same information, and act differently, or may view the same event but attend to different information. This is an indication of the impact of cognitive incongruences and heuristic effects. A relatively more scientific example is the Rorschach inkblot test, shown in Figure **3**.

Figure 3 One of the Rorschach inkblot test cards Rorschach test (2017, August 3).

The inkblot cards are shown to the individuals, and they are asked to describe their perceptions. The perception is high contextual and formed uniquely with the influence of the background, culture, frame, and many other factors.

An analysis of the Rorschach test, which is based on perceptual description, provides insight into an individual's psychology at that time. The perceptual description changes if a context is introduced before presenting the picture to an individual. This raises a concern for the reliability of the test. However, this is the interest in this dissertation. How can the mathematical model of this perception-biasing context be developed so that a shared awareness of phenomenon can be attained?

The heuristics that generate influences include, but are not limited to, representativeness, availability, anchoring, and adjustment heuristics. Widely-used modeling approaches that study human behavior fail to explain the so-called irrational behavior of people. However, irrationality may not be the explanation of behaviors that deviate from the classical prediction of rational human choice. It is true that utilizing traditional theories to explain seemingly irrational human result in *ad-hoc*, partial theories that coherency. A new approach, quantum cognition, provides an alternative explanation that does not require people to form probabilities and preferences in the way they would have to so as conform with the description of "classical rational agents". There is *a priori* in academia, which purports that social sciences are to study human behavior, and physical sciences are to explore the material world. This dichotomy can be because of preference; however, it has dominated academic studies. Social science has generated extensive scholarship in an attempt to explain human behavior. The classical assumption pervades these theories from top to bottom.

The classical assumptions limit the theoretical formalization of the information processing and decision making that constrain human social life. For example, the rules, norms, government, leadership position, and organizational requirements are all rooted in the human mind. If an alien comes from space, it cannot find a material embodiment of these. Besides, the notions such as intention, awareness at both individual and team level, are rooted in mind and communicated and shared with others. Thus, attempts to understand the seemingly irrational behavior should exceed the materialism paradigm, which is studied with classical physics. This problem emanates from the long and yet unsolved mind-body problem.

Social science is classified as a branch of natural science, which abides the causal closure of physics. Causal closure of physics introduces that everything in reality, including consciousness and social life, is composed of elementary objects. But there are two types of causal closure in the physics, which are classical and quantum. The classical one is a subset of the quantum one for specific situations. As a new approach that emerged 20 years ago, quantum cognition relies on the mathematical principles of quantum physics. It has broad implications for decision making, probability theory, and concept combinations. The mathematical foundation of this approach is important in two aspects. The first is the Hilbert space representation of the events.

The second is the geometrical projection of probabilities. Various seemingly irrational human behaviors are studied with these formalisms and provide an axiomatically coherent explanation to the anomalies, such as Kahneman-Tversky anomalies.

This research introduces rigorous mathematical principles used to understand the notion of awareness and shared awareness in complex situations. These two notions are discussed in a retrospective manner to reflect the evolution of these two notions so that the problem can become fathomable.

- Situation theory: Situation theory emerged as a mathematical framework to describe situations. It uses the set theory principles to describe and formalize the constraints of a situation. A significant contribution of situation theory is the developed ontological constructs of the information environment. Because of the limitations of the set theory principles, situation theory fails to provide a comprehensive approach to complex situations.
- Complex situations: Accessing the same information and taking different actions results in the recognition of the limits of situation theory. The limitations, in theory, constrain the predictions. Red cannot be red all the time. Hence, comprehensive and compatible theories are required to study complex situations.
- Awareness: One aspect of complex situations is the concept of awareness. The classical set theory does not allow developing a dynamic awareness construct. It limits the awareness with predefined sets.
- Situation awareness: Situation awareness models introduced substantial improvements to the information processing within a specific time and space. However, the state of the art

model fails to recognize the conjunction effects and other seemingly irrational behavior. This has become a major limitation in studying awareness with this model.

- Shared awareness: The notion of shared awareness is associated with situation awareness only at the phenomenon level in the framework of existing theories. This means that what is described as shared awareness is nothing but a measurement. The models fail to recognize the categorization of non-commuting (incompatible) events.
- Communication: Communication is the link between the interacting agents. Hence, communication models should demonstrate the truth and use meaning differences. How does the meaning become unique to the individual? The adaptive nature of the language, speech act concept should be articulated with the compatible theories, which eventually give rise to a model of shared awareness.
- Quantum cognition: Quantum cognition as an emerging discipline which provides a strong mathematical foundation for the listed challenges. The contextuality of quantum measurement provides the proper mathematical formalism to explain conjunction effects and other seemingly irrational human behavior.

1.2 Scope of the Research

The research presented in this dissertation is part of an ongoing research program conducted at Old Dominion University. The research program is being undertaken as a long-term initiative investigating management and engineering practices in complex situations. The program seeks to improve understanding of behavior in complex situations by making individual perspectives, and the associated awareness, and understanding the basic unit of analysis. This dissertation contributes to the program by introducing advanced mathematical formalisms needed to represent how individual awareness is formed, and how the awareness of multiple individuals may evolve to form a shared awareness. It captures the knowledge developed up to this point. The work presented focuses on the cognitive formation of individual or shared awareness about an event in a [complex] situation. The work places itself into a body of knowledge framed by work undertaken by seminal authors such as J. Busemayer, P. Gardenfors, D. Aerts, and A. Khrennikov.

The work presented is complete in the context of understanding how awareness is formed. It is, however, recognized that extensive research still needs to be conducted, for example, to elaborate on the time evolution of interactions, learning, and adaptive behavior.

This research adopts the terminology and lexicon of the quantum cognition paradigm, particularly as it applies to the formation of belief about an event and decision-making. The seminal text written by Busemeyer and Bruza (2012) is highly representative of the work and lexicon that is applied. It is recognized that many different approaches and paradigms are used to study decision making, complexity and the numerous, diverse fields that are associated with this research. While we recognize that differences in semantics and lexica generated by differing paradigms can greatly affect the degree to which individual work is understood, or even accepted, a full reconciliation of all of the fields is outside of the scope of the work conducted in this research. By time it would include other important theories.

1.3 Overview of the Dissertation

Thus far, a narrative of the problem has been presented. Next, in Chapter 2, an elaboration of the rationalist methodology that underlies the research is provided. Typically, the research methodology is presented after a detailed elaboration of literature, where the method is based on the concepts and conclusions that are derived from literature. No such clear delineation exists in rationalist research. Documentation of rationalist research will, in fact, seldom include an elaboration of the research method. This is partially due to the extremely non-linear, non-sequential progression of this type of work, from which little procedural detail can be recorded. What remains a requirement in rationalist work as it does in empirical studies is a basis on which the quality of the outcomes can judged. The methodology section provides a framing by which the conclusions of this dissertation can be evaluated.

Chapter 3 provides a literature review of this research. This section serves to provide a bounding of the research space based on the body of literature that it encompasses. It also provides detailed elaboration of pertinent topics, which form part of the results and theory derived in this dissertation. Situation theory and complex situation theory are discussed. The difference between these two approaches are articulated. The communication aspect of human interaction is presented and adaptive nature of the communication is related to the complex situation approach. A general theory of shared awareness is articulated as it fits into the complex situation approach. Chapter 4 presents the concept formulated from the literature. This concept establishes the basis for how shared awareness and decision making are addressed in this dissertation. It highlights how the concept addresses weaknesses found in other approaches and paradigms studying this problem. This chapter discusses quantum decision theory, Hilbert space, concept combination, and related mathematical formalisms. Chapter 5 presents the operationalization of the concept discussed in Chapter 4, which results in the Hilbert Space Geometric Representation of Shared Awareness and Joint Decision Making. The application of this representation to a situation, shared awareness, spooky action at a distance, and joint decision are provided. Finally, Chapter 6 presents conclusions, including a summary of the work, a discussion of the implications of this research, and possibilities for future research.

CHAPTER 2

METHODOLOGY

Conducting research is perceived as how knowledge generation is regulated with canons and methods. These methods and canons vary on the field of study, and the topic of interest. For example, in experimental particle physics, whether generating knowledge or filling a gap in the body of knowledge never or very rarely discussed. In physics, the theory is strong and foundationally well established. Consequently, there is a perception that whatever the experiment/observation is, there will be a correspondence in theory. The reason for this discernment could be a strong and mathematically well-outlined theory. However, there is another problem: the dominant empiricist perception in physics, especially younger generations who have given priority to becoming experts in techniques, rather than physics itself. Since the confidence and accuracy in the experiment are built in technique, at the individual level the robust learning of the theory is underestimated. Hence, an investigation in theory in empiricist paradigm is considered redundant in the short run.

The accumulation of knowledge requires maintaining a comprehensive view of the research methodology. It may give rise to various dualities. These dualities are akin to the egg-chicken contention, similar to the view that without theoretical physics there cannot be an experimental physics or vice versa.

The body of knowledge in both fields of engineering management and systems engineering is not foundationally well established; variances exist, and it is difficult to describe what the body of knowledge is. Generalized, context independent theoretical principles cannot be easily recognized. The rationalist and empiricist paradigms are deemed as convoluted and "*How can we* *gain knowledge?" i*s the primary concern for both rationalists and empiricists so much so that experts sustain disagreement on how to approach and perceive it. However, both rationalists and empiricists seek answers to respond to the questions that ensue from skepticism. Skepticism is one of the stimulating concept theories of knowledge. The desire to question what is sensed and conceptualization of it gives rise the discussion between empiricism and rationalism.

2.1 Objectivity and Subjectivity

Objectivity and subjectivity are the two terms that appear in any situation and are deemed as the reason for disagreements and conflicts. John Searle (YEAR) describes these two terms as systematically ambiguous between an epistemic and ontological sense. The epistemic aspect of the ambiguity is ascribed to the difference between types of knowledge claims. For example, the claim that Barack Obama became the 44^{th} President of the United States on January 20^{th}, 2009 is an epistemically objective claim. On the other hand, saying that Barack Obama was better than any other previous president is an epistemically subjective argument because this claim constitutes opinions. In the ontological sense, the mode's existence is the discussion, such as mountains, rivers, which are ontologically objective.

On the other hand, things like pain and the notion of beauty, are ontologically subjective, which varies with the participant and is depicted in Figure **4**.

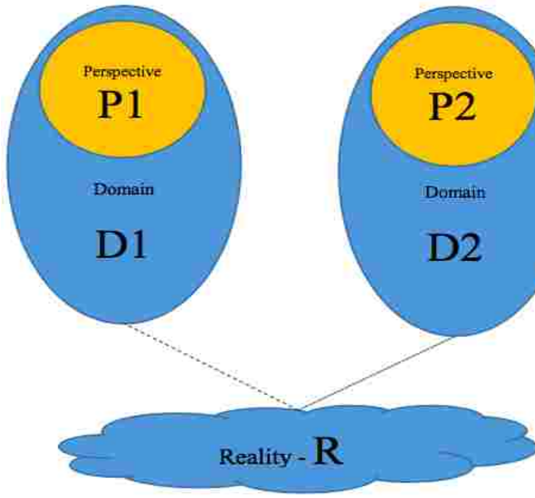

Figure 4 Two distinct domains of awareness that access the same reality (Canan & Soykan, 2016). The type of the knowledge claim that gives rise to epistemic subjectivity is recognized.

As an intellectual act, research is subject to this ambiguity. Unless properly recognized, this ambiguity can engender intellectual catastrophes.

2.2 Appearance and the Thing in Itself

Researchers strive to understand the objects, events, and relation(s) between them and other modes of existence through various methods. Understanding in a sense is between objectivity and subjectivity. These two terms have become an umbrella topic in the discussion of human understanding and knowledge. The motivation of this research emanates from how humans reach different understanding and take different actions after encountering the same object, cue, or word. Kant distinguishes "appearance" and "the thing in itself" in an attempt to prescind the objectivity and subjectivity. According to Kant, the human thought process includes the following faculties Kant, (2008 (Original 1781));

"Sensibility, learning and reason."

Rescher (1999) describes these faculties;

Sensibility, which conforms our sense perception of the object to the forms of sensibility namely space and time.

Understanding, which conforms our various individual judgments regarding objects to the categories of thought.

The reason, which conforms the collective totality of our judgments regarding objects to certain structural requirements of systemic unity. (p. #)

The relations between these faculties of human thought process are crucial in describing the objective and subjective knowledge claim. As a result, the conception of an object should include a general formalism, which describes the influence of objects on each other. Consider a situation that includes a conception process of an object through appearance (Kant, 2008 (Original 1781));

"We distinguish the mode in which we intuit them from the nature that belongs to them in themselves it is implied in this distinction that we place the later, consider in their nature, although we do not so intuit them." (p. #)

In this situation, suppose an orange is conceptualized. The characteristics that associated with appearances such as solidity, size, mass, motion and similar descriptions are objective, which means that are not constrained by the definition of the perceiver. The primary feature of the human thought process that maintains the conception of the appearance of an object is sensibility. Hence, this process is in the phenomenal category. When it comes to the taste, beauty, and color, which are considered subjective, the conceptualization can vary depending on the individual. The category that this process falls in is in the category of noumenon (Rescher, 1999);

> *"The concept of noumenon is a limiting concept, the function of which is to curb the pretensions of the sensibility." (p. #)*

John Locke in his work of "*An Essay on Human Understanding*" describes the same distinction between the perceiver dependent and independent qualities of an object as *primary and secondary qualities*. The discussion of this distinction has become a contentious issue since the rise of modern science. Locke (1690) says;

> *"… the Bulk, Figure, Number, Situation, and Motion, or Rest of their solid Parts; those are in them, whether we perceive them or no; and when they are of that size, that we can discover them, we have by these an Idea of the thing, as it is in itself, as is plain in artificial things. These I call primary Qualities.*
>
> *The Ideas of primary Qualities of Bodies, are Resemblances of them, and their Patterns do exist in the Bodies themselves; but the Ideas, produced in us by these Secondary Qualities, have no resemblance of them at all. There is nothing like our Ideas, existing in the Bodies themselves. They are in Bodies; we denominate from them, only a Power to produce those Sensations in us: And what is Sweet, Blue or Warm in Idea, is but the certain Bulk, Figure, and Motion of the insensible parts of the Bodies themselves, which we call so." (p. #)*

These arguments evoke the question of how can a human (researcher) chooses the thought process while developing a conception of an object. Human thought processes could not be confined in a selective box that functions as a one-way stimuli selector. When an encounter occurs, the human mind cannot be disentangled with that interacted stimulus. Also, a human cannot choose or limits the faculties that take place in the conceptualization process. Kant (2008 (Original 1781)) argues:

> *"Appearance can be nothing by itself, outside of our mode of representation. Unless the word appearance is recognized with a relation to something in itself that is an object of sensibility. There thus results in the concept of noumenon ... it is abstracted from everything that belongs to the form of sensible intuition". (p. #)*

This emphasizes the importance of both phenomenal and noumenal components on the human thought process in conceptualizing an object, a situation, or anything that is being encountered in research activity. In this respect, Kant defines the thing in itself as;

> *"the thing in itself is a creature of understanding." (p. #)*

Human understanding cannot operate only with sensibility, for instance, appearance. The functional role of the conception of the thing in itself is to maintain the entities of the situation within the limits of appearance.

> *"The concept of a noumenon is necessary, to prevent sensible intuition from being extended to things in themselves, and thus to limit the objective validity of sensible knowledge. The remaining things, to which it does not apply, are*

> *entitled noumena, to show that this knowledge cannot extend its domain over everything which the understanding thinks… The concept of noumenon is this a merely limiting concept, the function of which is to curb the pretension of To sensibility". (Author, YEAR, p. #)*

To maintain a complete and successful research, the desire for a comprehensive understanding of nature should be in the complementarity of the responses to the skepticism (shown in Figure **5**) approaches, which are rationalism and empiricism.

3.1 Rationalism vs. Empiricism

Regardless of the method, science is about making predictions. Freud and Einstein are two extreme examples of empricism and rationalism, repectively. Freud's prediction was that childhood experiences would have a heavy bearing on who the human would become. According to Popper (1959), Freud could make any data point work to service of this theory.

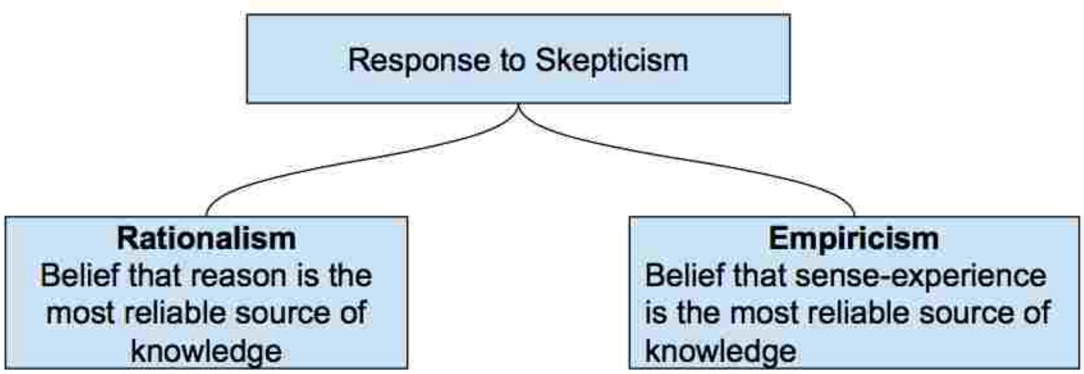

Figure 5 Knowledge generation categorized into two categories, rationalism and empiricism, in response to skepticism.

On the other hand, Einstein's prediction was different. Instead of looking backward and using past data to predict the present, he was looking ahead and predicting the future state of relations. This can be risky. However, it has the intuitive reasoning to impose not only the present reality but also a future one.

Another perspective in this end is Newton's argument regarding the derivation of laws (relations) (Einstein, 1922);

> *"Still believed that the basic concepts and laws of his system could be derived from experience." (p. #)*

Although this argument seems objective, the subjectivity in this argument has been suppressed. The subjectivity becomes noticeable in developing invariant theories. For example, grand unification theory is a challenging endeavor. It demonstrates how difficult it can be to come up with the unification of theories from different sources.

Newtown or Freud's beliefs on pure experience can be valid within the period in which they lived. For example, the success of Newtown's theory in his time was unarguably wonderful. However, this could be because of not being able to disseminate the findings all around the world, or any disproving argument did not reach to him. It is easy to find confirmation of a theory if one is looking for it. However, contradictions ensue from risky arguments, and every good scientific theory is prohibitive. The limitations of the Newtown's theory has been demonstrated by the latter scientists, and this does not diminish the fact that Newton achieved something extraordinary. However, it reveals an important fact about science (Popper, 1959);

> *"There can be no ultimate statements science: there can be no statements in science which cannot be tested, and therefore none which cannot in principle*

be refuted, by falsifying some of the conclusions which can be deduced from them." (p. #)

The traditional understanding of research method goes back to ancient Greeks. To look at the universe with a scientific eye is to observe with no preconceived notions. However, each human being has a preconceived notion of ideas with which we start. As discussed earlier, the human thought process cannot be segregated from these ideas. This would be even in conflict for the most extreme examples of empiricism, such as John Locke. On the other hand, pure abstract approaches cannot ameliorate the predictions affairs to better understand the universe (Einstein, 1922):

"Our experience up to date justifies us in feeling sure that in Nature is actualized the ideal of mathematical simplicity. It is my conviction that pure mathematical construct enables us to discover the concepts and the laws connecting them which gives us the way to the understanding of the phenomenon of Nature." (p. #)

3.2 Research Canons

A canon, by definition, is the rules, laws, and principles that provide a canvas for conducted research. Research canons are constructs of human intellect. Even the research canons themselves are subject to the various philosophical approaches. For example, a syncretic method can be a research methodology, and it includes pragmatism to attain a desired goal. Therefore, research canons are subjective, and the desired coherence should be attained in the plausibility of the arguments individually and, eventually, as a whole.

This research is closer to constructivism. The generic definition for constructivism comprises the interaction between experience and ideas. The experience gives rise to awareness, and thought processes fuse it with such as intuition, induction, or deduction.

The goal of this research to express the relations between the appearance and the thing in itself of the objects in a situation to describe the mathematical expression of shared awareness. The mathematical formalism in the context of this study aims to develop a framework of exploration of the relation between the object, observer, and situation. With a rationalist approach, this research aims to complement and enhance the empiricist endeavors.

CHAPTER 3

BACKGROUND OF THE STUDY

Awareness of the decision-making entity constrains the decisions in every situation. The state of the awareness of each entity is unique, and it includes the contributions of the generative process. The heuristics in human decision-making becomes active with the contributions of these characteristics. Various disciplines scrutinize different aspects of awareness formation and ensuing decisions under topics of situation theory, complex situations, situation awareness, shared awareness, and communication. As an interdisciplinary study, cognitive science fuses various approaches to form a cognitive representation of human cognitive functions that contribute decision-making. This chapter covers the contributions of these disciplines into the discussion of shared awareness.

3. 1 Situation Theory Perspective

The situation theory perspective (Barwise & Perry, 1983) initiative contributes the mathematical foundations of the understanding of situations. This initiative is based on set theory. An approach based on set theory fails to recognize the emerging constraints in a complex situation. An ensuing understanding from this awareness will be incomplete, limited in comprehension. This section articulates the type of abstraction and to demonstrate that in the complex situation it requires increasing the number of combinations, which makes it difficult to express in algorithms and mathematical models.

Situation theory is a meta-theoretical construct that constitutes the cogitation of the circumvent condition of a problem. Rather than developing a higher understanding of specific phenomena from well-formulated perspectives, it focuses on an understanding of various bounding factors, which ultimately distort the practicality of the situations. Therefore, simple situations can become complex situations (Sousa-Poza, 2013) which require a dynamic replacement for understanding. The decisions that are made in a situation constrain the subsequent decision, action and a tangled world is generated.

4.1.1. Human Action and Social Structure

Understanding human action has long been studied and can be categorized as foundational structures and fundamental phenomenon. The former is associated with the normative approaches, such as normative sociology. Normative methodologies in human actions emerge via identification of social norms which are considered as constituting a common sense view of the world (Devlin, 1994). The latter is associated with ethnomethodology which considers human actions as fundamental and aims to explain how human action generates the collection of social norms and the common sense view of the world (Garfinkel, 1967). In the words of Garfinkel:

> *"Description of a society that it members, use and treat as known in common with other members, and with other members take for granted. Specifically, a description of the way decisions of meaning and fact are managed, how a body of factual knowledge of social structures is assembled in common sense situations of choice." (p. #)*

The ethnomethodologist view accentuates the human intervention to the social structure so that social norms and related structures emerge. This intervention is an interaction that changes the environment and the agents who are associated with the situation. This contextual interaction is not articulated in situation theory.

Rendering a situation comprehensible to all the agents in interaction cannot be expected to be enhanced and achieve the efficiency of the communication. It is true that speaker and listener share a knowledge and experience with the particular utterance. However, communication serves as a medium, through foundational structures and fundamental phenomena. The further ontological construct is necessary to develop a conceptual framework that provides a systemic perspective to the human actions.

4.1.2. Situation Theory

Identifying a measure of understanding and efficiency of communication is of paramount importance in decision sciences. The interaction of these two gives rise to a decision regarding a purposeful action. Situation theory endeavors to develop a conceptual framework to identify and scrutinize those interactions resulted a theory, which is concerned with various parameters such as human agents, real objects, information, processes such as refining the abstract structures ensued from and governed the behavior, and the actions of the human agents in social structures (Devlin, 1994). Therefore, situation theory can be perceived as limited part of reality, which constitutes a spatiotemporal and abstract extension of the mentioned parameters.

The conceptual framework of situation theory to study information in social phenomena results in a theoretical construct of the communication and human action (Devlin, 1994). The early ontological canvas of this approach can be seen in Figure 6.

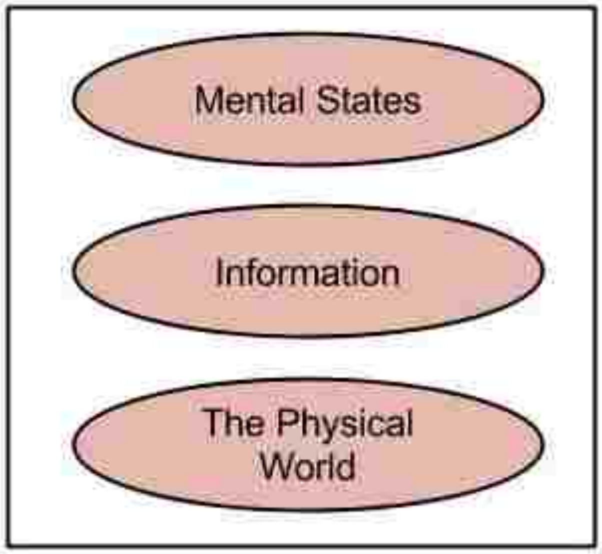

Figure 6 The Information Level (adapted from Devlin, 1994)). A theoretical formalism among the components of this construct is not introduced in this theory.

The distinct domains in Figure 6 emerge in studying cognition, communication, and human action in a comprehensive manner in situation theory. Here, situation theory introduces an intermediate level called "information domain" which is consistent with the available empirical evidence regarding cognition, communication, and human action (Devlin, 1994). Every human agent looks at phenomena via a grid, which is irrespective of the properties of the phenomena, and forms an ecological perception. The causality and relations among the phenomena in the physical world introduce a parameterization of information and mental states. This unique parameterization allows utilizing an interaction phenomenology with an adaptive behavior to study decision-making process.

The information level introduced in this theoretical formalism is important. This ontological construct supports the information environment, which includes the complex situation environment dimensions of the information era. However, the contribution of situation theory is at the material realm.

3. 2 Situation Theory and Information

An ontological classification of situation theory includes the following entities (Devlin, 1994):

- *Individuals: objects such as people, tables, etc. that agents can individuate*
- *Relations: uniformities individuated or discriminated by agents*
- *Spatial locations*
- *Temporal locations*
- *Situations: structured parts of the world discriminated by agents*
- *Types: higher order uniformities discriminated by agent, in other words, the nature of the immediate environment (situation type)*
- *Parameters: indeterminates that range over objects of various type. (p. #)*

With the framework of situation theory, human agents (as individuals) individuate reality, and human agents' behavior may vary on relations, spatial-temporal locations with different types and parameters. This is called a scheme on individuation.

3.2.1. Scheme of Individuation and Discrete Information

Each situation comprises information, which varies on the scheme of individuation. This information can be taken as discrete items as known infons (Devlin, 1994) which are articulated as:

"Infons are items of information. They are not things that in themselves are true or false. Rather a particular item of information may be true or false about a certain part of the world, "a situation." Infons may be combined to form what are known as compound infons: the

permissible combinatory operations consist of conjunction, disjunction, and bounded universal and existential quantification (by parameters)." (p. #)

There are various working descriptions of the items in information. A dynamic and comprehensive definition of the information item, such as infon, will ameliorate the conceptualization of the information item that mediates the interaction.

3.2.2. Infons, Information, and Constraints

Information items, such as utterances and cues, constitute meaning with the existence of an intelligent agent, a human. Information items become construable when they interact with humans. A solitary infon would be nothing but a material existence, which is subject to disappear. Thus, it is true that infons are always coupled with intelligent agents. This is the essential component of the communication. A text without a reader would mean nothing. Infons themselves cannot be classified as true or false; they are items of information and they need the situation to be true or false. Situation type and an infon constitute the entities of understanding, which can be acquired via different types of abstraction processes. These processes, however, include constraints (Devlin, 1994):

" the facilitators and inhibitors of information flow, are abstract links between types of situation. They may be natural laws, conventions, logical rules, linguistic rules, empirical, law-like correspondences. Their role in the information chain is quite well conveyed by the use of the words means". (p. #)

For example, for the following statement "vapor" means evaporation. Regarding entities of ontological construct of mental states, information, and the physical world, vapor is a type of a situation and evaporation is another type of situation. The infon mentioned above introduces a constraint that links two types. Vapor and evaporation are two distinct types of situations. The constraint here is a nomic (law-related) constraint. Constraints become effective with cogitation. There is another constraint that is manifested in the conceptual space and that is concept combination. Combined concepts evoke different construals for the same phenomena in different contexts.

Constraints emerge in the situations and functions by relating various regularities or uniformities across actual situations. For instance, in group environments, constraints are not developed because of the concurrent presence of situations. Rather, constraints become effective because of associative links established by the utterance of the infons, which can be represented in a conceptual space.

3.2.3. Constraints in Situation Theory

Infons, constraints, and type of situation can be represented with a classification introduced by Devlin (1991).

- Constraint 1, C_1: vapor means evaporation
- Constraint 2, C_2: the Tuesday after Labor Day is the first day of school

As described in the entity itemization earlier, types vary by the immediate environment. Here, the C_1 is a fact about the immediate environment, which is the world. The infon includes the types such as vapors and evaporation, location, and time. This nomic constraint is expressed by an infon that is supported by the appropriate situation, and it is a regularity in the world (Devlin, 1994):

> "*Any nomic constraint "C" will comprise a systemic, informational link between pairs of situations, situations of types, e.g. vapor and evaporate,*

respectively and as a systemic regularity in the world, this linkage will constitute a situation which supports constraint. This particular situation will include, in particular, the relevant causality between situations of type 1 and that situation of type 2 to which they are linked. A nomic constraint hold in the world because that is the world is; what makes a particular regularity a (nomic) constraint is the role it plays in guiding the flow of information. That is to say, in the case of nomic constraint, the distinction between the regularity (e.g. situation) and the constraint is essentially on of the abstraction and functionality." (p. #)

When an agent confronts a constraint, it can sense via observation that it is vapor. The observed instance of vapor is related to evaporation. After a certain number of repeated observations, the agent becomes aware of the constraint and can proceed in utilizing the constraint for any reason. The former, becoming aware of constraint, is called abstraction; the latter is called functionality. Subsequent observations result in the construction of a domain of awareness, which will be discussed later. The representation of the domain is in the conceptual space. The "*information link*" that Devlin indicated is phenomenological. It requires abstract mental representation.

C_2 is not a nomic relation as in the C_1 and does not manifest a ubiquitous regularity. This constraint is local, which are established based on common conventions of culture. Agents in different locations, in a different time interval, cannot even abstract and reach a functional move from the infon of type C_2.

Situations support different constraints and they become an action on human agents involvement, for instance, participant vs. observer. The behavior of an agent is probabilistic. The decision-making process is highly contextual, which is influenced by the associated constraints between the agent and the environment. The more an agent interacts with the environment, the more constraints are associated with the agent. The frequency of the interaction between two or more specific agents can generate a hierarchy of constraints. The hierarchy entails varying governance of the constraints. The manifestation of this governance becomes part of the contextuality of decision-making and influences the behavior of the agents. An example can be decision-making in a group. After several interactions in the group environment, the group identity becomes the dominant context. To maintain this identity, the individual considers the hierarchical constraints as normative constraints. The individual develops and sustains a sense of belonging by abiding these norms.

While describing the kinematics of motion, the constraints allow to formulate various parameters, and figures of measures. The constraint identification in human interaction will render the interaction formalization possible. Humans act under the influence of some cognitive/psychological parameters in the situation. The intent of one agent is transmitted with the illocutionary force of the exchange of infons. This whole construct is highly contextual and requires a theoretical approach that explains the influence of a context for every situation. For example, the following description of illocutionary act highlight the importance of information in human action:

> *"A re-specification of the concept of "illocutionary act" to exclude aprioristic efforts to isolate "propositional contents", and more fully to appreciate the socially situated availability of "what an utterance could be accomplishing " in situ, especially in respect of its properties of design,*

> *sequential implication, and turn-allocation relevance; in other words, its interactionally significant properties." (p. #)*

A situation includes human agents and the interaction with the environment. Every interaction, whether it be with the environment or another agent, generates learning. Considering the constraints in situation theory along with learning, the entities in a situation become connected (this will later be discussed further with the concept of entanglement). The ensuing condition in the situation can have various manifestations in the dimensions of the ontological canvas. The representation of the situation in the mental states is the dynamic adapted one, which is acquired through cognitive processes such as describing, defining, conceptualization, or nominalization. Later in the dissertation, this condition will be introduced as being aware of the situation. It is an indispensable human disposition to claim knowledge about phenomena while making decisions. It is challenging to abstract notions such as infons and constraints. For example, infons are not things in themselves; however, they are the building blocks of awareness in a situation. This indicates the importance of contextuality and interaction. The elements of interaction, which are infons and constraints, are highly contextual. This contextuality should have a compatible corresponding representation in the state of awareness. Especially with complex situations, the constraints require advanced formalism to comprehend the interactions and ensuing actions in the model. A set theory approach fails to recognize contextuality and hence limits the abstraction of complex social phenomena.

A popular notion in the management of an organization in these type circumstances is complexity. It is such popular that it becomes the "famous mistake" because of the reason that, everyone uses it without an agreed dictionary meaning. What makes a situation complex? What is complicated?

3.3 Complex Situations

Situations can be construed differently depending on the involving of the human agents. A situation may become complex, complicated, or simple according to the cognitive imposition of human agents. Situation acts as a constraint throughout any interaction with the environment, which is a generative process. Situation stipulates the establishment of awareness. The establishment of a cognizant entity demonstrates that the decision-making process a generative process, which is composed of a noumenon an (unbounded participation in life) and a phenomenon (a bounded observation of life) (Sousa-Poza, 2013). Within the cognitive dimension, this means that the situation is abstracted and has a corresponding domain that consists of perspectives. In this construct, being simple, or complex is a matter of comprehensibility (Brewer, 2010) or understanding of the situation while the constraints of the immediate environment affect the phenomena. A claim of true or false, right or wrong, will be subjective on many constraints.

3.3.1. A True-False Dichotomy or a Possible Fallibility: Pragmatic Idealism

A general non-academic perception of science is that discovery would invalidate the preceding in the same paradigm. For example, the discovery of the Standard Model in particle physics does not invalidate what earlier discoveries. Methodologically, they are similar. However, the scientists changed the grid and probing energy, and something new was observed. Changing the setting would result in something different. A true-false dichotomy within a situation is subject to constraints of the situation. As a result, claims/observations/results cannot be discerned as ultimate knowledge. This requires describing the influence of the context. The accessibility of claims surged with improvements of worldwide knowledge, such as Google, etc. Consequently, a knowledge claim is susceptible to objection/dispute, which renders any knowledge claim prone to fallibility in other reference frames.

3.3.2. Pragmatic Idealism

Idealism has become an umbrella term for any system of thought that perceives the object of knowledge as a mind dependent activity. Utilization of this principle, in the form of imposing human agents' ideal to situations, engenders unique cognitive constructs, which stimulates an understanding of the situation. The ensuing understanding can be more or less complex (no complexity = simplicity). Imposing the ideals to the situations is called process identification (Rescher, 1996) that is interactional. This process identification consists of a participant observer dyad, which constitutes identity-engendering factors such as cause-effect, activity-passivity, and action-interaction. Because of the uniqueness of each process, a spectrum of understanding with different modes of complexity emerges. Idealism warrants conceptualization, which pertains to pragmatism: both interpreting the situation and transpiring it into action. According to Rescher (1992a);

> *"Conceptual Idealism [states that] any fully adequate descriptive characterization of the nature of the physical ('material') reality must make reference to mental operations; some recourse to verbal characteristics or operations is required within the substantive content of an adequate account of what it is to be real." (p. #)*

A comprehensive cognitive construct becomes necessary to articulate a mental operation. Pragmatism, on the other hand, is easier to comprehend based on idealism. It can be perceived as “choose what is practical, ” and naïve, meaning it is consistent with the initial perspective of pragmatism in the context of this study. It does not aim to abandon a true-false dichotomy, rather it elucidates the rationale and appropriateness about the components of each situation such as belief evaluations, and actions can receive suitable, purpose-driven guidance (Rescher, 2000). The action that occurs with these constraints is called Pragmatic Idealism (PI) (Rescher, 1992a, 1992b, 1994).

Sousa-Poza et al. (2005) introduced another facet of Pragmatic Idealism with an actionable philosophy and understanding is transferred from experience and action. The distinguishing characteristic of the PI introduced in Canan and Sousa-Poza (2016a) and Sousa-Poza and Correa (2005) is the strong emphasis on fallibilism. The fundamental proposition in PI is that the perception of reality and design and management of a complex situation can be improved if the constraints in the situation are properly identified.

3.3.3. An Epistemic Component of Complexity

The foundational principle of the pragmatic idealism is that irrespective of the phenomenon, reality transcends human understanding. An intriguing example is the discovery of the standard model. Before the discovery, the physics community claimed that the atom was the smallest thing in the physical world. However, it turned out to be otherwise. Consequently, it can be said that understanding constitutes fallibility which induces complexity that can be introduced as (Sousa-Poza & Correa, 2005);

> *“… complexity is proportional to the probability of having/making an erroneous knowledge claim* $p(\varepsilon)$*.” (p. #)*

3.3.4. Situational Construct Model

The notion of fallibility can also emerge in any type situation in which there exists incongruence that constrains the interaction of agents. This induces complexity in a situational construct model that has the following components (Kovacic, 2007; Sousa-Poza, 2013);

- The Entity: a set of meta-attributes that constitute the elements of participant aspects of a situation,
- The Observer: the past, present, and future of an individual, mostly schema, and
- The Solution Form: composed by the teleological aspects of the situation associated with particulates.

This situational construct model relies on six (Figure 7) assessment parameters;

- Nature of the problem domain,
- Worldview or approach predisposition,
- Type of approach selected or required,
- Approach alignment,
- Problem framing, and
- Technical expertise.

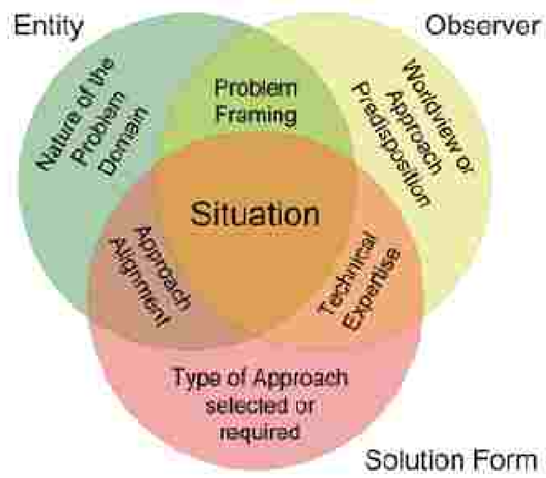

Figure 7 Situational Construct Model (Sousa-Poza, 2013)

This situational construct model is between the two ontological constructs used to study a complex situation within the scope of this work. This is particularly important when considering the situation construct as first-degree interaction (Sousa-Poza, 2013) this construct maps the RDP model (shown later in Figure 10) onto the Endsley situational awareness levels (Endsley, 1995).

3.4 Awareness

The dictionary definition of awareness from the Oxford English Dictionary is:

"having or showing realization, perception, or knowledge."

This definition, in fact, is acceptable. However, in management, decision-making studies notion of awareness manifests progressive and dynamic understanding, which forms the foundation of this work. The notion of awareness in pragmatic idealism framework will be discussed next/later.

3.4.1. Evolution of Awareness in Situation Theory

Situation theory sustains the natural ties of the abstract constructs to the reality; to understand dynamic-, transient-, and context-specific situations. Situation theory enables the involvement of an individual as a participant rather than a mere observer in understanding the reality (Sousa-Poza, 2013). The concomitant relation of the participant and observer dyad educes the following important principles:

- *"The incorporation of the participant and maintenance of the observer in the problems"* (Sousa-Poza, Kovacic & Keating, 2008)
- *"The practical nature of the problem"* (Sousa-Poza, 2013)
- *"The paradigm captured in Pragmatic Idealism, that leads to the present state of understanding of Situations Theory"* (Sousa-Poza & Correa, 2005)

These principles elicit the implementation of an ontological construct in the formation of awareness, which is known as the Representation of Reality (RDP) model.

3.4.2. Reality, Domain, Perspective (RDP) Model

The formation of a state of awareness is a generative process. A generative process constitutes epistemic processes such as inquiring, discovering, formulating, confirming, and communicating knowledge. Besides, there exist resources called "generative process-coordinated facts" which are investigated, discerned, and transmitted to induce the content of cognitive affairs. The process view of a human agent comprises distinct social aspects. The processual dispositions that render the human agent as an individual also require characterizing a human agent as a part of the social order of communicative interrelationships (Rescher, 1996). George H. Mead (1967) stresses that:

> *... the community is sustaining the role of communication among social being such that it is effectively impossible to study them sensibly isolation, abstracting from the interpersonal relations that shape virtually the whole spectrum of their activities. (p. #)*

A generative process (shown in Figure 8) constitutes three elements; observer, entity, and projected future action. Perception of complexity, in this regard as a feature of the situation, which ensues from the dissonance of cognitive representation of these three elements. The study of awareness requires including a generative process, which should include the construct of awareness of self and awareness of other than self (Figure 9).

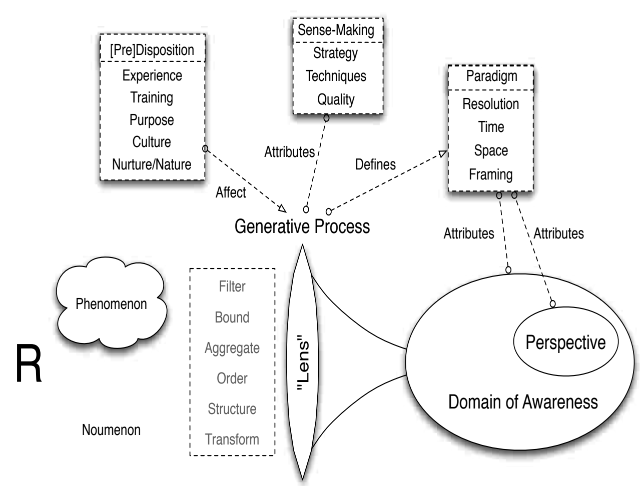

Figure 8 Generative Process: Components of the generative process in the construction of a domain of awareness (that which is comprehended) and perspective (that which is understood). Each generative process takes place as a continuous interaction within the environment. Due to the uniqueness of every individual, each generative process is unique.

The domain of awareness maintains an individual's ability to understand a problem. The degree of abstraction impacts the comprehensibility, which bounds the domain of awareness. Thus, the degree of complexity, as well as the effectiveness of domain of awareness can be represented by a nominal construct called abstraction distance (Figure 10) Pearl (1988, 2009).

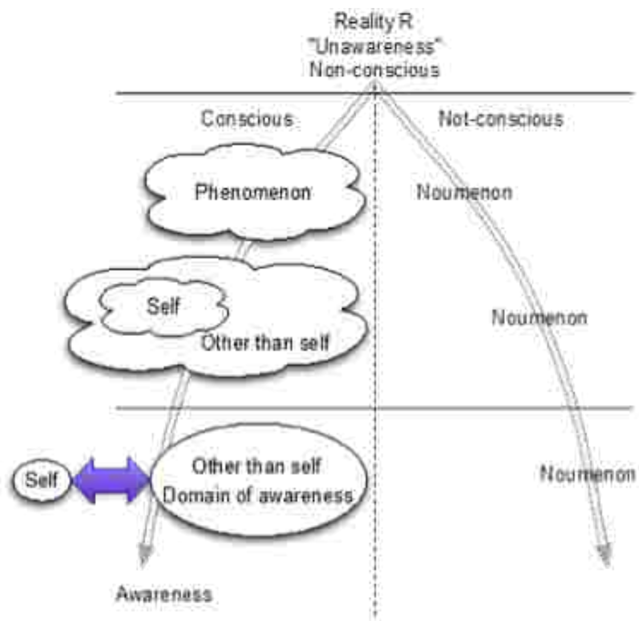

Figure 9 Formation of awareness of self and awareness of other than self.

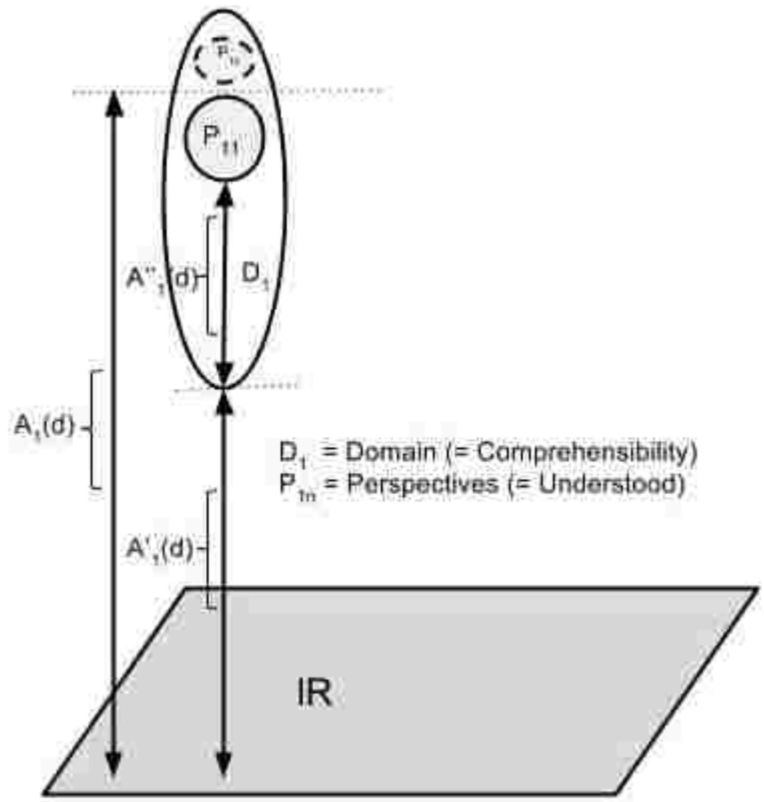

Figure 10 RDP model of a representation of abstraction on reality.

3.4.3. Comprehensibility, Understanding, and Complexity

The depiction of abstraction distance in Figure 10 includes the following items:

- IR - reality
- D_m - the domain of awareness (where m = 1 in Figure 10. Domain encompasses the comprehensibility.
- P_{mn} - perspective (where m = 1 in Figure 10 is formed in the domain of awareness. Perspective represents that what is understood of the problem/situation.
- $A_1(d)$ – the approximation distance between reality and what is understood. It is the uncertainty of the perspective. This is proportional to complexity.

- $A'_1(d)$ – the approximation distance between reality and comprehensibility. This is the aleatory uncertainty for any given perspective
- $A''_1(d)$ - the approximation distance between what is comprehensible and that which is understood. This may contain an element of epistemic uncertainty for a given perspective.

The domain is the essential component of the RDP construct in Figure 10. This ontological construct constitutes the formation of awareness and self-awareness. The assumptions, simplification, attributes, and strategies begin by intervening with the construct. The abstraction distance for the domain, $A'_m(d)$, impacts the ensued comprehensibility. For instance, a high comprehension ensues from a low abstraction $A'_m(d) \rightarrow 0$.

Perspective comprises two characteristics; one is on the domain of awareness represented by $A''_m(d)$ and the other represented by the total approximation distance $A_m(d)$. The overall complexity of the problem is represented by $A_m(d)$ which is a confluence of $A'_m(d)$ and $A''_m(d)$. Construction of $A_m(d) >> 0$ renders the situation complex. The impact of this high abstraction is that the comprehensibility of the perspective becomes very limited. Multiple constraints can influence and bound the understanding. Therefore, the earlier definition of complexity, which is the probability of making an erroneous knowledge claim, is sustained within this framework. In this construct, pragmatic idealism motivated scrutiny necessitates identifying the sources of uncertainty, whether $A'_m(d)$ or $A''_m(d)$ give rise to uncertainty. This is important because the uncertainty arising from the latter might reducible since the domain of awareness can comprise multiple perspectives (see Figure 10) "P_{1n}" with the established comprehensibility. On the other hand, if the former generates uncertainty; then it requires new assumptions and strategies to form a new domain of awareness.

3.4.4. Cognition, Schemata, and Knowledge

Obtaining, acquiring, organizing, and conveying knowledge constitute the essence of the human learning process. This generative process has cognitive and physical aspects. The construct of situation theory educes the fact that each person's perceiving and transpiring knowledge into action is distinguished and exclusive (Neisser, 1976) even though the stimuli or observed phenomenon would be same. Upon perception of an individual, stimuli become contingent to preexisting structures called *schemata*. A schema (Neisser, 1976);

> *"… is that portion of the entire perceptual cycle which is internal to the perceiver, modifiable by experience, and somehow specific to what is being perceived. The schema accepts information as it becomes available at sensory surfaces and is changed by that information; it directs movements and exploratory activities that make more information available by which it is further modified." (p. #)*

A noticeable aspect of the scheme is that it acts like a grid for information selection, processing, etc. A schema is not merely a plan or a format; rather it acts as a medium where stimuli and cognitive mechanism interact, which is inherently selective, is called perception. Perception can be conceived as the interaction between schema and available stimuli (Neisser, 1976). Receiving stimuli requires a perceptual system so that it can be transformed into meaning. Here, stimuli are not changed, a schema picks them up and they are altered and used and ultimately form the knowledge for sense making.

A schema implies the contextual and transience sensitivity of knowledge acquisition in the form of interaction with phenomenological elements. Knowledge generation is categorized into: empirical and rational. The former ensues from repeated observations, the later from coherence to provide justification (Dauer, 1974; Sousa-Poza, 2013; Sousa-Poza, Kovacic & Keating, 2008). Transpiring knowledge into action requires a transition from knowledge to understanding. This results in a temporal knowledge construct, which is pragmatic in nature and influences the sense-making and decision-making.

3.5 Situation Awareness (SA)

Situation awareness is an important theoretical attempt to understand human selectiveness and to model limited capacity attention. The development of a theoretical model of situation awareness includes of different paradigms. Situation awareness, in this regard, is an attempt to describe the process (Endsley, 1995).

The developed model for situation awareness by Endsley comprises the human factors that act as a constraint and become contingencies, as shown in Figure 11.

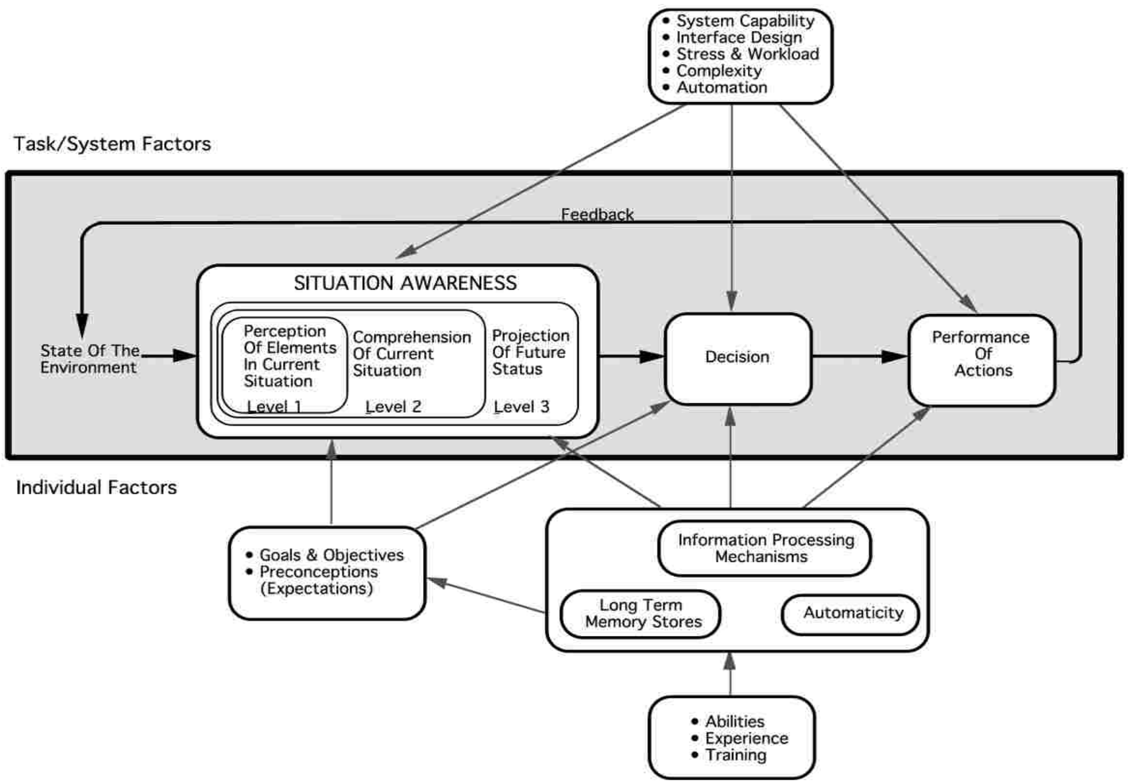

Figure 11 Model of Situation Awareness Representing a Dynamic Decision Making (Endsley, 1995)

SA, as a designated military concept, appeared when pilots recognized the necessity of gaining a comprehensive awareness about the enemy and themselves (Endsley, 1988). Subsequent SA studies resulted in a spectrum of models based on the practical needs of the application. There are two definition-based categories and two model-based categories.

3.5.1. Definition-Based SA Categories

The definition-based SA discussion distinguishes between SA as a state and SA as a process. The distinction between the approaches originates from the reference point for the SA formation. The state-based definition of SA is primarily concerned with the situation and the environment in which the human operates. The process-based definition is concerned with the human agent properties that are imposed on the situation and the environment.

3.5.1.1. State-Based Definitions of SA

Endsley (1995) defines SA as a state of knowledge. As such, it has to be separated from the processes that are used to acquire an SA. State approaches to SA are situation-based (Banbury & Tremblay, 2004). The situation-based classification maps perception in the environment to the human agents' cognition. *"State approaches limit the description of the process involved"* (Tremblay & Banbury, 2004, p. #) in achieving SA. Endsley's seminal model in this framework emerged to meet the practical needs of US fighter pilots. It is an iterative cycle where a stimulus from the environment is perceived. In this the framework, SA is defined as (Endsley, 1995):

> *"the perception of elements in the environment within a volume of time and space, the comprehension of their meaning, and the projection of their status*." *(p. #)*

The model includes three levels of situation awareness:

- Level 1 Situation Awareness: Perception of the elements in the environment,
- Level 2 Situation Awareness: Comprehension of current situation, and
- Level 3 Situation Awareness: Projection of future status.

Endsley' SA model (Figure 11) represents an information processing model. One manifestation of this is the separation of decision and performance of an action. The model does not provide a comprehensive cognition model of the SA. The face validity of this model, however, makes it very popular (Banbury & Tremblay, 2004).

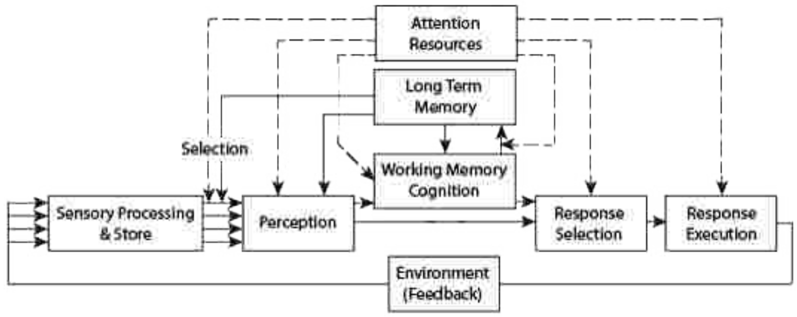

Figure 12 Human Information Processing (HIP) (Hollands, 1999).

3.5.1.2. Processed Based Definition of SA

Process views in the SA discussion are concerned with the operator-focused approach (Banbury & Tremblay, 2004). These take into account the properties of the human agent, such as cognitive abilities and perceptual processes, of which sensory processing is an example. The operator focused approach highlights the imposition of these abilities to the stimuli in the information processing framework (Figure 12) (Hollands, 1999). These processes are function-based and may include functions such as *"information extraction, information integration, mental picture formation, projecting and anticipation"* (Banbury & Tremblay, 2004, p. #). The Human Information Processing (HIP) model, shown in Figure 12, includes similar stages to the Endsley SA model (Figure 11). HIP and similar models articulate the psychological processes which start with a sensory input or the intuition of an operator while performing work-related tasks in the context of the human-system interaction.

The improvement in HIP-type psychological models in studying SA is the inclusion of a feedback loop that includes the environment. The inclusion of the environment advances the SA discussion beyond cybernetic competence and state of knowledge. In this framework, Smith and Hancock (1995) define SA as:

"adaptive externally directed consciousness." (p. #)

The inclusion of externally directed adaptation articulates the feedback loop that emphasizes the goal-oriented task preferences in achieving the objectives. The notion of externally directed adaptation is discussed with Smith and Hancock (1995) as:

"if the agent were to dictate private, incontestable (but dynamic) goals, SA would always be perfect because whatever perceived would that be the goal. However, boundaries of performance are often set by other or by nature or made by explicit ourselves at some previous point in time." (p. #)

This definition explicitly puts emphasis on the "externally directed." This indicates that the goal, which directs the SA, is a mere phenomenon, which is sensible. Both approaches complement each other in developing a concept of SA. The ensuing incompleteness in definition leads the discussion to another stage in the SA studies; that is modeling the SA.

There are two categories in SA models: descriptive and prescriptive. A descriptive model depicts how a process works in developing SA. The majority of the SA models are descriptive models (Pew & Mavor, 1998). The descriptive models include decision-making loops, the perception of stimuli, information processing, individual factors, attention, memory, and external factors. Hence, both process and state description, e.g. HIP and Endsley's model, are in this category. A prescriptive model steps forward and "recommends" courses of action. A prescriptive model sets the rules for how the processes in developing an SA work.

SA models and definitions emphasize the importance and impacts of the bounding factor in a situation. The "normative arbiter" (Smith & Hancock, 1995) as a source of constraint is important, yet an incomplete endeavor in developing an SA construct. The environments, in which SA is used, are various. For instance, a military environment constitutes not only normative but also descriptive as well prescriptive arbiters.

Decisions, for instance in a military context, must be acquired within a very narrow time space. This introduces incorporated contingencies, such as integrated meaning, individuals' understandings. The demand for a theory situation awareness emerged out of the discussions, and since then it is being conducted on different grounds. Some critiques of the theory (Endsley, 2015) can be listed as:

i. The three levels of situation awareness are linear;
ii. Endsley's model is data-driven information processing model;
iii. The product versus process distinction;
iv. The model of situation awareness is not cyclical or dynamic;
v. The situation awareness model fails to take into account meaning;
vi. Situation awareness is all contained in working memory; and
vii. The situation model only represents a Cartesian in the head view of the world and does not encompass the wider sociotechnical environment.

Improvements have occurred in the situation awareness model. However, a model cannot be an ultimate model and requires improvements to recognize the emerging challenges. Some SA models fail to include and preserve contextuality and the needs of cyber situation awareness constructs.

The definition and the three levels of the model require a further delineation of the components of situation awareness; because, what is being modeled, is not a mere information-processing model. For example, time and space are not formalized in the situation awareness constructs. The two elements are the needs of the operator to perceive and understand the stimuli in the immediate environment. Time becomes a constraint in situation awareness because of the transient nature of the stimuli or in a broader sense situation. The situation cannot be segregated from temporal aspects as well as spatial aspects. Situation awareness as a composition to all of these components maintains a cognitive existence, schema. Whenever it is invoked, it elicits as a construct of a mixture of all components.

As can be seen in Figure 11, decision-making and performance of actions are separated from the situation awareness construct. However, cogitation of a situation not only engenders the decision and action but also maintains an interaction, which begets new understanding. Therefore, everything that is required to maintain/operate a system is part of the situation awareness. This distinction appeared in the early stage of the discussions of the situation awareness. There is one group that perceives situation awareness as a productive awareness of the situation promptly. It is an engagement of stimuli via various processes such as analyzing, inferring, and determining the implication in the immediate environment. Another group perceives situation awareness as an understanding of datum that depends on the previously integrated knowledge (Adams, 1995).

3.5.2. Perception and Cognition

Situation awareness is engendered from imposing the human attributes to the incoming stimuli through various perceptual and encoding procedures as well as cognitive constructs (e.g. mental models, schema, etc.). This interaction-based approach is classified into: lower order processes and high-order processes. Lower order processes are perception and encoding; higher order processes are semantic and comprehension (Lichacz, 2001).

Knowledge as a cognitive element in the form of a mental state or a schema provides foundation of anticipation future action of agents. Thus, the incoming stimuli (e.g. events) transformed cognitively by the active schema of the perceiving individual. The proceeding information extraction, interpretations are directly biased. Neisser (1976) exemplify this as;

"The information picked up in vision is necessarily optical, consisting of patterns in the light over space and time. However, optical information can specify objects and events at various levels of abstraction and meaning. When we perceive a person's mood, we are not engaged in the same perceptual cycle as when we are attending to his lip movements. We develop a different set of anticipations; we pick up information that extends over a different span of time." (p. #)

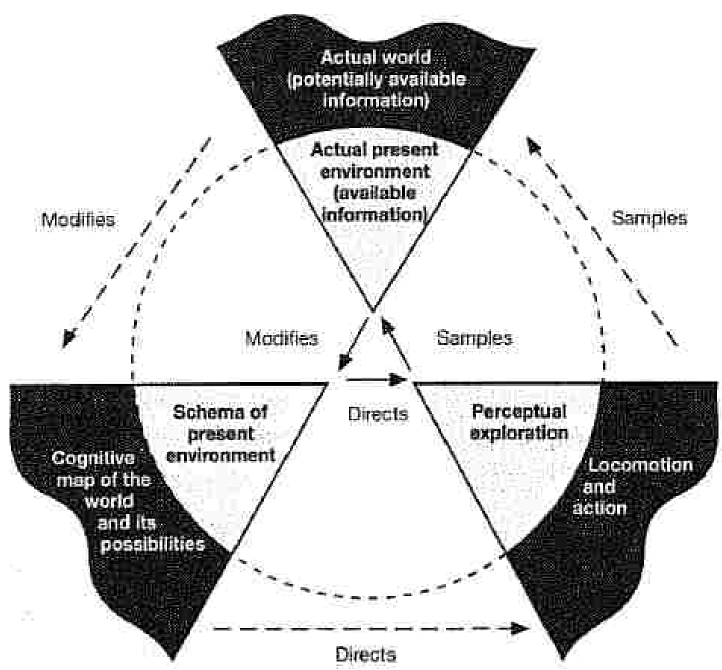

Figure 13 Perceptual Cycle by Neisser (1976)

The depiction of this process by Neisser (1976) can be seen in Figure 13. The major contribution of this model to the situation awareness discussion is that schema receive stimuli, then processes, modifies, and updates the schema. Then, it engenders a product as a state of the active schema, and a process as a state in perceptual cycle model (Woods, 1991). None of these models discuss the influence of context and mental state revision occurs due to the interaction.

3.5.2.1. What to Know

In a study conducted about Anti-Air Warfare (AAW) team in the AEGIS cruiser of the U.S. Navy revealed an interesting perception about decision-making and situation awareness. The Combat Information Center's main support to AAW is whether to engage with a threat or not. A closer look at this notion revealed a different picture as opposed to the general perception (Wolf, 1996);

"The engagement decision is a relatively simple one. Combat Information Center decision makers are primarily concerned with developing situation awareness, not with the determining which actions to take. The AAW team's primary task is to determine which set of contingencies exist; the procedures will then tell them what actions to take. The most important decisions were judgments about the nature of the situations, not the selections between alternative courses of action. Diagnosis requires that a decision maker perceive the need to adopt a hypothesis to explain observed events, generate one or more potential hypotheses, and evaluate them." (p. #)

In this regard, situation awareness comprises set of environmental conditions and system states where a participant interacts with them in the form of information extraction, information integration, mental picture formation, knowledge, projection, and anticipation. Thus, interaction renders the evolution of situation awareness as a proactive information seeking process, rather than passive receipt and storage process (Breton, 2004; Klein, 2000). Pro-active information seeking induces its problems in seeking what to know, accuracy vs. precision and the process is unique for each.

In addition to the process state discussion, Neisser's (1976) concept of schema introduces a convoluted process of situation awareness with decision-making. This precludes a procedural model of situation awareness model. This dyad is supported with the existing definition of the situation awareness, which mostly states that a match or correlation between an external world of stimuli and an internal world of mental representation (Liltzhoft, 2004). The state-process dichotomy of situation awareness entails other contingencies such accuracy vs. precision. If the situation awareness is a state, there should be a precise definition of knowledge that defines the particular state. On the other hand, if it is processed, then the process should be formulated. Accuracy and precision of a knowledge claim may not be the purported claim. Therefore, the state vs. the process and accuracy vs. the precision perception of situation awareness constructs may introduce irreversible problems. So, rather than producing a precise mental representation of outside world, or striving for accuracy on account of plausibility in situation awareness for action and goal achievement (Hollands, 1999). Therefore, the specification of a problem via knowledge, goals, and available information in the environment, and the actions of the human entails recognition of what must be known to achieve a goal. This approach requires a superposition construct in which the state of the system is indefinite until the measurement.

3.5.3. Situation Awareness and Adaptive Behavior

Situation awareness is an interactive process and includes adaptive behavior. The adaptive character of situation awareness initially appeared at the distinction between competence and

performance (Smith & Hancock, 1995) which is linked to the information processing problem and knowledge specification. The adaptive behavior in the situation awareness context provides the reasoning agent what to know to solve emerging problems to achieve the given goal.
The adaptive definition of situation awareness (Smith & Hancock, 1995) is;

> *"…situation awareness is adaptive, externally directed consciousness. Where consciousness to be that part of an agent's knowledge-generating behavior that is within the scope of intentional manipulation. Situation awareness generates purposeful behavior in specific task environment. The products of situation awareness are knowledge about and directed action within that environment. Situation awareness is more than performance, more fundamentally, it is the capacity to direct consciousness to generate competent performance given a particular situation as it unfolds." (p. #)*

Adaptive behavior is a process. It is a process which includes the impact of an external stimulus that eventually might elicit a pragmatic change in knowledge and the of the behavior agent to attain goals. The constituents of situations constraint the interaction and ensures that human agent's behavior and the goals match the available information in the situation (Hancock, 1995; Holland, 1992; Simon, 1982). Adaptive behavior is necessary for complex situations; however, too much of it may generate redundancy and unanticipated constraints (Canan & Sousa-Poza, 2016b). The disparities among the agents should be recognized within the situation so that the agent can have proper identification to develop solutions to attain the goals. This is where situation awareness becomes important because it requires the development of adaptive capability that meets the required specifications and criteria to attain a goal.

The discussed frameworks, which are the complex situation and pragmatic idealism, allow the evolution of situation awareness in an adaptive manner. However, this does not render situation awareness into something else. The construct of situation awareness is a dyad that has the components such as an environment or agents that form the situation together. Any stimuli, information, or a cue that conveys a meaning can act as a comparator. These stimuli can be normative or prescriptive. However, based on the interaction and constraints, the stimuli can be construed differently depending on the situation and the environment. At this confluence, the adaptation becomes an important attribute of the complex situations and pragmatic idealism (Canan, Sousa-Poza, & Kovacic, 2015).

3.5.4. Stimulus Flow and Situation Awareness

Descriptive situation awareness approaches perceive the stimuli (e.g. information) flow between stages before a decision. On the other hand, prescriptive approaches specify how stimuli interact to impact decision making, how the stimuli are being represented in the human agent domain (Baranski, 2004). Maintaining or having excellent situation awareness of the environment does not result in the conveyance of the knowledge. The fratricide incident that occurred in Iraq in 1993 (Snook, 2002) is a good example for this. The interconnectivity among entities was not working properly because of an unexpected constraint (Canan & Sousa-Poza, 2016b). In this regard, evaluating and describing the situation awareness on certain dynamics is essential. For example, military command and control requires a descriptively specified situation awareness so that prescriptive constructs can work (Baranski, 2004).

A framework for military command and control developed to study to situation awareness and decision-making called Critique, Explore, Compare and Adapt (CECA) model of C^2 can be seen in Figure 14. The CECA model, from the nominative perspective, is not a situation awareness model, but is rather a decision-making model. The detailed specifications reveal the fact that it includes a situational construct as well. There are similar points of CECA loop with the existing situation awareness model. The important component is the constructivist perspective of human perception. The constructivist perspective discerns how an interacting agent conveys the *a priori* knowledge coherently to the sensory systems.

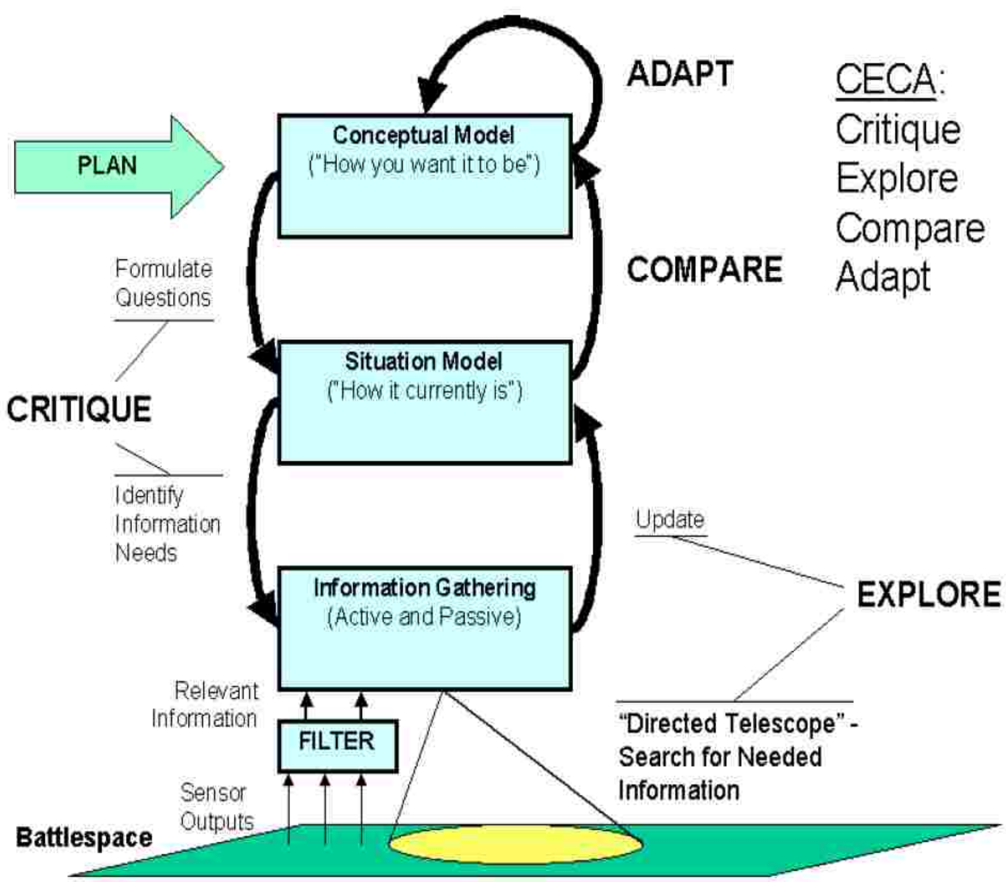

Figure 14 Critique, Explore, Compare, and Adapt (CECA) Loop (Bryant, 2003)

There is an ontology for mental events, and another for physical events. In the human context, this dichotomy appears as the agent and environment dyad for individual situation awareness. The term "situation awareness" demands a framework that spans mental and physical constraints, which mutually exist in situation awareness construct (Flach, 1995). For example, a participant-observer dyad can occur in any of these types of situations. This requires a consideration of the other part of the interaction which should be a coherent and plausible interpretation (Rock, 1985) to maintain a common understanding or group understanding. The CECA demonstrates the fact that human cannot fully fathom the experienced world. To complement this weakness, two concomitant models were prompted: a conceptual and a situation model. These two models provide a complementary fusion of the information from various sources so that a comprehensive understanding of the situation can occur.

For example, command and control, C2, is not an individual task. Rather, it consists of an intricate set of organizational procedures. A C2 model introduces the shared nature of the conceptual and situation models for the shared stimuli as well.

There are two key elements that affect the quality of C2 (Nordin, 1999);

> *"… are the commanders' vision, or conceptual model, and the degree to which that vision is shared among individuals and units who will contribute to accomplishing specified goals." (p. #)*

Is it enough to share the conceptual model? The unfolding events, for instance in the battlefield, emerge as a construct of the situation, conceptual, and gathered information. Therefore, shared awareness is critical to examine in the organizational and communal environment. As it can be seen in the CECA model, information in the form of a stimulus is subject to construal.

Consequently, rather than the massive information, the important factor in decision-making informativeness of the situation awareness and eventually how does shared awareness occur? When does it occur?

3.5.4.1. Situation Awareness and Cognitive Streaming

Cognitive streaming is about organizing information within a cognitive system. A key component in cognitive streaming is transitional probabilities. A stimulus perturbs the cognitive system, and received information is construed with the cognitive capabilities. An overlapping notion in situation awareness studies and the discussions in the cognitive behavioral science is that acquiring and interpreting information. Behavioral science perceives this as short-term memory and long-term memory interactions, and situation awareness community copes with this as a black box. The quantum cognition approach can provide answers to the intricate challenges of the black box.

An important discussion in situation awareness is the vagueness of lower and higher order psychological processes. Rather than processes themselves, the recognition the interaction as a black box, and dealing only with inputs and outputs impedes the formalization of shared awareness. This generates compatibility problem and transmitting contextual influence. In this perspective, for instance, memory storage perceived as a continuum of behavior. On the contrary, cognitive stream approach suggests that (Jones, 2004)

> *"…it is one of a range of skilled behaviors with a key concept transitional probabilities which are defined as; the likelihood that certain types of event will occur following the occurrence of other events." (p. #)*

The anticipation is related with the transitional probabilities. For example, the anticipation of future states of an aircraft, and the inclusion of transitional probabilities allows the retrieval of information from memory by covering all aspects of past and future (Jones, 2004). The cognitive streaming predicts that (Tremblay, 2001);

> *… interference between tasks occurs when they draw upon the same mental (e.g. memory for order) rather than when their content is similar (e.g. both are spatial tasks). (p. #)*

Pragmatic idealism (Sousa-Poza & Correa, 2005) introduced complexity as the probability of making an erroneous knowledge claim. The probabilistic nature of human interaction with ecology, humans, machines, the organization is oversimplified by the nature of situation awareness and hence shared awareness. This oversimplification limits shared awareness in the material representation. However, the nature of the shared awareness requires advanced theories.

3.6 Shared Awareness

The discussion of shared situation awareness is vague. An important field to study shared awareness in an organizational setting is the military. To mitigate the fog and friction in the battlefield and to maintain an agile decision making in this environment, common operating picture or shared awareness is considered as *sine qua non*. Some argue that achieving a shared awareness emanates from common data (Maltz, 2010). However, accessing these common data is just the first perturbation of the process. Transforming data into a synchronized action requires a useful grasp of data via cognition (schema), and conceptual models. Consequently, the existence of shared data is not enough to attain a shared awareness because construing the information is unique to every individual.

Situation awareness is a continuum and it is a process. Any stimulus that is received by an individual is processed according to his/her predisposition (or culture). Stimuli are processed within the conditions of how an individual sees, hears, tastes, smell, feels, emotes, and thinks independently, in previous communities, and in the current community (or groups). These can be considered not only as complementary to the intelligence, aptitude, training, education, capabilities, instructions, technology, or anything that can be linked to doctrine organization, training, material, leadership, personnel, and facilities (DOTMLPF) (Maltz, 2010) but also mutually maintain formation of situation awareness. Hence, shared awareness requires further consideration in the context of the generative process.

3.6.1. The Nature of Shared Awareness

Human interaction with anything can have projections in cognitive, physical, and information dimensions of a situational construct. After a stimulus, e.g. utterance, is received in the physical form; it is construed in the cognitive domain. Moreover, then the received (incoming) meaning is transformed into an utterance that is ascribed to an understanding. Agents in the social environment and the organizational environment incessantly reason about the fellow soldier, the intent, and the given objective, which is the nature of the social interaction (Friedell, 1967) and form awareness. As discussed earlier, this awareness includes other than self, which is illustrated in Figure 9.

To be able to represent a proposition with the associated operator, the following representation of classical logic is used. To represent *"A thinks that B thinks x is true"* one can write:

$$(AB)x = A(Bx) \quad \text{(Eq. 1)}$$

This introduces an intricate operator relation, the “think” transpired into “action,” or to an “utterance.” Therefore, the formation of a common opinion on individual opinion includes all the constraints, which are associated with the involving operators. Thus,

$$(AB)x \overset{?}{\triangleq} A(Bx) \neq (A \cap B)x \quad \text{(Eq. 2)}$$

$$(BA)x \overset{?}{\triangleq} B(Ax) \neq (A \cap B)x \quad \text{(Eq. 3)}$$

$$(BA)x \overset{?}{\triangleq} (ABx) \quad \text{(Eq. 4)}$$

The problem emerges when these operators are not complementing each other. For example, in the case of $AB \neq BA$, this shared awareness approach fails in describing shared awareness. Shared awareness is part of the comprehensibility of the individuals, and, hence, it is part of situation awareness. The definition of shared awareness should not be described separately than situation awareness. Even though there are nominal alternatives to the shared awareness, they are just phenomena-level representations.

3.6.2. Notion of Sharing

There is an interesting etymology for the word share. In Old German, the word “Scare” means troop, the share of forced labor and in German “Schar” means troop, multitude, and band. The naive dictionary meaning suggests that the notion of sharing invokes goal driven common action. The dictionary meaning of the word “*share”* ("http://dictionary.reference.com"):

“Noun: A part or portion of something owned, allotted to, or contributed by a person or group.”

“Verb: to divide or apportion, to join with another or others in the use of something.”

The dictionary meaning does not convey meaning without the context. However, the notion of sharing includes a goal, something to share, and others who either participates or be part of the sharing process.

The juxtaposition of the words “share”, “situation”, and “awareness” with different combinations introduce conceptual and semantic issues. For instance, what does shared situation awareness stand for? Is it an awareness of a shared situation or shared awareness in a particular situation, or sharing of situational awareness? This discussion goes on, and changing word combinations does not ameliorate the concept of shared awareness. The discussion of situation awareness should avoid ascribing a meaning to the deterministic, sequential, or interconnectedness order of the steps of the situation. The individual, structural, and situational factors influence the formation of awareness. Consequently, different personalities prompt distinct awareness even for shared situations, and the same stimuli. None of these factors render the constructed awareness as some shared awareness situations are not merely because of being encountered and used simultaneously. All situation awareness constructs have manifested attributes within the context and willingness to share which gives rise to shared awareness, which is part of the cognitive schema or mental states.

3.6.3. Shared Situation Awareness: Is it a Proceeding Step in Situation Awareness?

Situation awareness descriptions/definitions incline to separate shared situation awareness from situation awareness as a construct. However, because every situation for an individual is constrained by a preceding one, a situational construct is stored in the schema. An individual situation awareness already constitutes a shared awareness or shareable awareness or even possibly a common understanding. When the right stimulus occurs, it prompts awareness so that sharing occurs as a situational construct. Thus, shared awareness is not a shared observation;

rather it is a separate construct, and the temporal aspect of it might be transient on individual situation awareness, yet it exists as a mental state.

3.6.4. Group vs. Shared Awareness

Every individual enters into a group environment with a unique awareness. In the group environment, every individual continues to develop awareness in the new context. However, the point is that an interaction commences at different levels between the interacting entities. The general expectation is that every individual develops individual situation awareness, shares each individual situational awareness with an additional requirement of being aware of the relevant action(s) of other group members and develop group shared situational awareness (Nofi, 2000; Weuve, 2000).

Regardless of the situation, individuals are there with their individual situation awareness. There are certain requirements to achieve shared situational awareness to fulfill a desired goal. This constrains the decision-making process in different aspects. For example, a training meeting or a workshop provides an environment for interaction among the individuals of an organization. These individuals may not know each other. However, these interactions improve the efficiency of the organization (Pentland, 2014) because once interacted, agents' mental states cannot be decomposed in meaning extraction.

3.6.4.1. Common Ground

Common ground is defined (Nofi, 2000) as:

"…is dynamic in nature and therefore is often a matter of explicit negotiation and communication. Common ground can fall apart and eventually can get lost; hence, it needs constant maintenance to keep the community, culture, and discipline alive." (p. #)

Reaching common ground is an important step in attaining shared knowledge, beliefs, and assumptions so that effective communication can be achieved with the establishment of a shared vocabulary to better the semantic agreement (Nofi, 2000). In this regard, communication is the most critical element in the development of situation awareness and in creating shared awareness. To understand each other, individuals should understand each other in the best way possible. Imparting a mental model to facilitate shared decision-making is a challenging process. The scope of this research includes a theoretical foundation for proper strategy development for common ground.

3.6.5. General Theory of Shared Awareness

The nature of human interaction causes the emergence of shared awareness. It is a perspective in the domain of awareness. It requires a perturbation to begin any necessary shift in cognitive dimension on the corresponding embodiments in the physical and/or information dimensions (Canan et al., 2015). The elucidation of this notion can engender more complexity rather than mitigating it. Shared awareness *"is proportional to the desire to share and the willingness to share by the entities to adapt from predispositions to establish a common disposition"* (Kovacic, 2013, p. #). In this context, since RDP support multiple perspectives, the desire to share or the willingness to share give rise to the necessary intrinsic or extrinsic stimulus to commence the adaptation. Consequently, shared awareness as a perspective can be perturbed and will be attained (Canan & Soykan, 2016).

Comprehensive adaptation is important, because attaining adaptation in the physical and information dimension (shown in Figure 15), in the form of illocution, does not result in common understanding or shared awareness. The solution is the adaptation in the corresponding cognitive elements (Canan et al., 2015). The disparity in cognitive dimension can be recognized

(Canan & Sousa-Poza, 2016a) by a feedback loop, supported by an RDP construct. Awareness as a cognitive entity feeds back to the reality in the form of decision and action. The cognitive disparities can only be observed in this type of extrinsic feedback loops. The same behavior, yet with different meanings, can occur and should be recognized properly. At this point, the salient contribution of the RDP in achieving an adaptation is that the adaptation is not a paradigm shift but rather a change in perspective, and it is a matter of awareness.

3.6.5.1. Categories of Shared Awareness

The notion of shared awareness is discussed under various topics. With the limitations of the classical approaches, it has become a conjectural construct. Limitations of classical theories, for example failing to recognize non-commutative events, results in a limited description of shared awareness. An interaction based general theory was introduced by Kovacic 2013);

Conditional Shared Awareness

Contextual Shared Awareness

Synthetic Shared Awareness

Synoptic Shared Awareness

These categories are based on the shared awareness definition:

1. *A state of shared comprehension established through adaptation resulting in a common context*

a. A state in which, conditional on the existence of a common disposition and the desire to share, a common comprehension is established.

b. A condition in which two entities with the common disposition and desire to share can attain a common comprehension of a situation.

(p. #)

3.6.6. Shared Awareness in Situation Theory Perspective

As discussed earlier, individuals manifest dualities. Heedlessness with dualities can have serious consequences, for example in recognizing the causes for paradoxes. These causes are discussed in the RDP construct discussion. Shared awareness, as a key element in this discussion, should be studied by including the influences from a noumenological and phenomenological component of the reality.

As elucidated in the RDP construct discussion, the establishment of a domain of awareness from noumenon (unbounded participation in life), and phenomenon (bounded observation of life).

3.6.6.1. The Information Environment

The information environment, Figure 15, is introduced in US JP-3-13 Information Operations publication (Command, 2012). It has three components: the cognitive dimension, the information dimension, and the physical dimension. The current discussion in shared awareness confines the notion of shared awareness into the fusion of information and physical dimension (illustrated with a box shown in Figure 15).

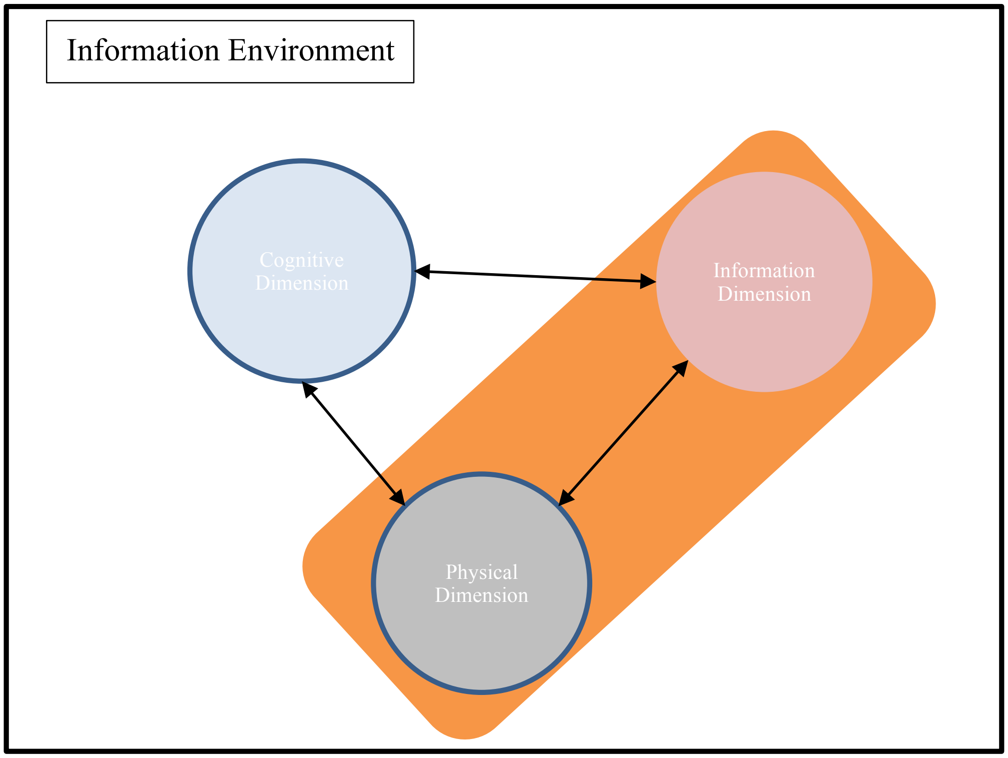

Figure 15 Information Environment and Constituent Dimensions from US JP-3-13 (Command, 2012)

This perception includes a common operating picture, shared data, interoperability, or shared information. However, as discussed in the "Understanding Information Age Warfare" the shared awareness is a state. The measurement of it is more complicated and should be distinct than the measurement of shared information (Alberts, Garstka, Hayes & Signori, 2001). The region illustrated with a rectangular box, in Figure 15, allows accessing the common data and shared information. The common data do not result in acquiring a shared situational awareness. Thus, measuring these phenomena within the information and physical dimensions does not result in measuring shared awareness. Shared situational understanding is not an end in itself (Maltz, 2010), rather it is an enabler for adaptability, synchronization, and mission command functions (e.g. disseminating intent). The shared awareness, introduced in "Understanding Information War Age" (Alberts et al., 2001), is based on the information environment components. Shared awareness, in this model, is projected on both to the cognitive dimension and information dimension. This is a significant improvement in situation awareness and shared awareness formalism endeavors. This model (Alberts et al., 2001) discusses two distinct sensing types: direct and indirect sensing mechanisms. Both sensing mechanisms in this construct are initiated in the physical dimension. Indirect sensing occurs when the object or event transforms into data and then feeds into the cognitive dimension. On the contrary, direct sensing results when the object or event directly feeds into the cognitive dimension. These are phenomenological components that contribute to the development of awareness.

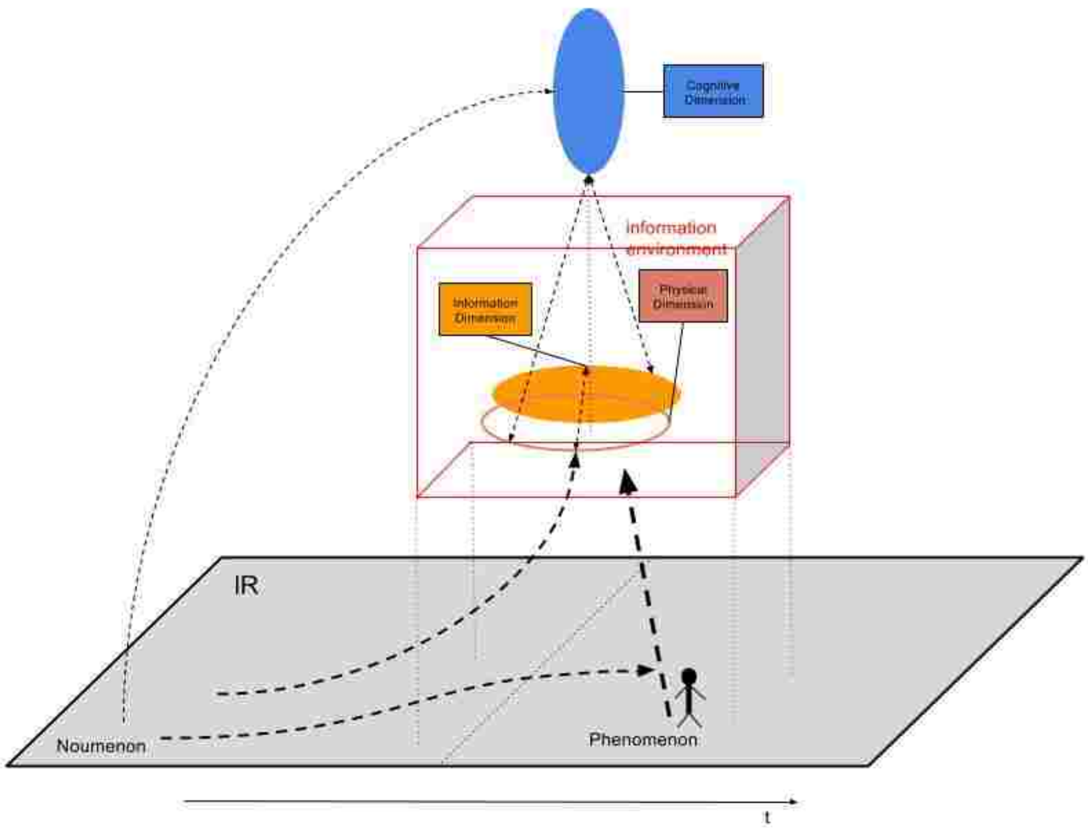

Figure 16 Improved information environment with the complex situation approach (The figure is taken from Canan and Sousa-Poza (2016a, 2016c)

The incorporation of RDP into the information environment framework results in the model shown in Figure 15. In this model, the noumenological contribution into the formation of a domain of awareness that is the cognitive dimension (Canan & Sousa-Poza, 2016a, 2016c). This contribution bifurcates to either through a phenomenon or directly to the cognitive dimension (Figure 15). This is elucidated as a generative process, in an earlier chapter, as a participant-observer dyad in the formation of a domain of awareness.

3.7 Communication

Communication is not a mere act of imparting information via language. It constitutes assertoric commitments, which can become fathomable with social interaction. The phenomena can be objective. However, the assertoric individual claims about the phenomena vary by the individual. To establish an understanding, further probing is necessary for individual and communal understanding to develop. Communication is facilitated through language, which allows imparting the mental models from sender to receiver through linguistic means.

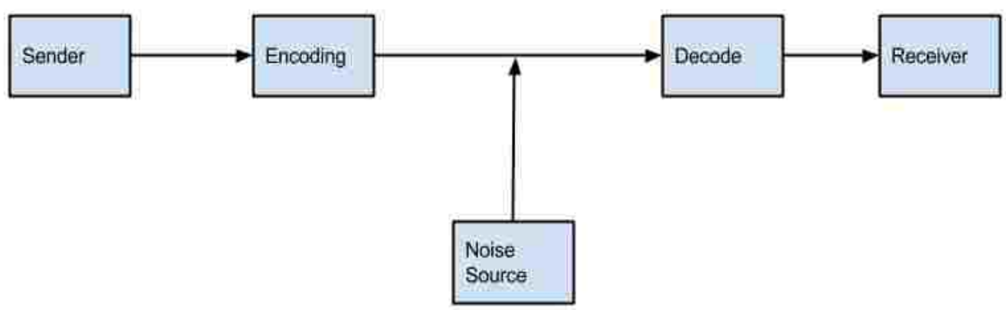

Figure 17 Shannon's Communication model. The sender encodes a message with the mental model and imparts to the receiver. The receiver decodes the message and establishes an understanding.

An abstract example of communication can be seen in Figure 17. A sender wants to convey an understanding to a receiver so that understanding can be established between sender and receiver in a situation. In certain situations, the meaning is conveyed for persuasion, or to make the receivers take action for an objective. This intricate process is conceptually well articulated in speech act theory. There are interlocutors that communicate with each other with locutions, which are words, phrases or utterances that constitute intrinsic meaning in the context used. The interlocutors perform an act of speaking or writing in which the intended action coded called illocution. The illocutionary force of an utterance is the ability of a locution to mobilize the target audience towards the intended action. The consequence of this process is the perlocution that is receiver takes action toward the desired objective Figure 18.

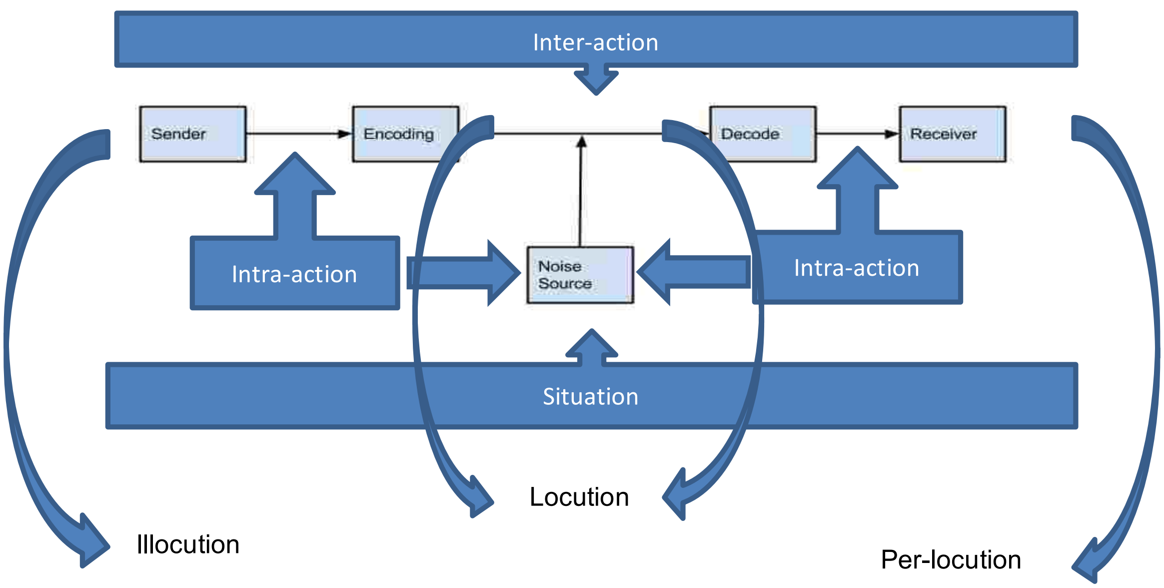

Figure 18 Speech act and its component projected onto the abstract communication model.

The meaning of information in communication is context dependent. A context can be intrinsic, in which a signifier is placed, or can be extrinsic causing a change in the paradigm in which the signifier and context are perceived. Regardless of being intrinsic or extrinsic with regard to cause, the difference generates discrepancy in meaning. There two types of discrepancies:

epistemic and ontological. The best possible decision is contingent on the individuals who perceive the phenomenon through these epistemic and ontological constructs. The nature of this construct elicits the existence of objectivity and subjectivity in both types of discrepancies. Attaining an objective understanding/awareness within an organization requires optimizing the objective and subjective individual claims to from an objective communal claim. At the confluence of these, communication becomes the probing process of other minds in establishing an understanding.

In a communication setting, agents aim to convey information in the most efficient way. Doing so creates a tacit agreement, a variety of operative assumptions (presuppositions) in the framework of a context of communication. The significance of these presuppositions is that they do not emerge from the context of the message, rather they arise from contextually-formed presuppositions, which are the individual (Rescher, 1998) and constrained by individual limits. Individual limits can be recognized by studying awareness with the generative process in situation theory. The generative process describes presupposition and situation theory renders the general governing principles of communication to function efficiently in different situations.

3.7.1. Communication and Awareness

Awareness is a cognitive construct. It cannot be confined to the material world. The individual interacts with the environment and develops a self and an other-than-self so that awareness is sustained. Communication becomes the necessary tool to understand others' minds. It maintains a flow of information among the minds so that new ideas emerge and decisions are made (Pentland, 2014). The facts of human life are discussed and studied within various disciplines with an inquiry of how human beings create social structures that are cooperative and productive. The information flow is important in understanding this inquiry for human beings who commune in social structures with assertoric commitments that may require further justification for decision-making.

3.7.2. Communication Models

Communication can be undertaken in many different ways. These include textbooks, public speeches, social media, telephone conversations, media, emails, radio, TV and so on. The models presented here constitute three levels of communication (Huseman, 1976),:

> *"intrapersonal communication public communication and interpersonal communication." (p. #)*

The first two communication types are not representing the communal aspects of communication. However, these two form the basis for the interpersonal communication, because it reflects some part of the generative process.

Communication models are a representation of certain aspects of events, structures, or systems made by using symbols (Chapanis, 1971). The salient examples of communication models such Laswell (1948), Shannon and Weaver (1949), Berlo (1960), Dance (1970), Barnlund (1970) discuss sender and receiver interactions with different approaches. They all agree that communication is a dynamic, continuous, circular, unrepeatable, irreversible, and complex process.

3.7.3. Systemic Model of Communication

Modeling communication as a system with the General System Theory principle provides a comprehensive view to the communication (Hall, 1956). The systemic approach to communication constitute the following axioms introduced by Watzlawick et al. (2011):

"i) The impossibility of not communicating; ii) Content and relationship in communication; iii) The punctuation of the sequence of events, and iv) Symmetrical and complementary interaction." (p. #)

This is an incomplete systemic approach because it does reflect the cognitive components of a system, which is part of humans. The cognitive dimension of humans into the system requires the inclusion of language.

3.7.3.1. Language as a Complex Adaptive System

In the human context, the characteristic of language is a social function. This manifestation has two folds: cognitive and physical functions which form a complex adaptive system (CAS). The system is composed of interacting agents. These agents render the system adaptive because agents' behavior is based on past interactions. The current and past interactions together prompt future possible behavior (Elis, 2007). The CAS framework renders language a dynamic, generative system rather than a static system of grammatical and syntactical principles. According to Elis (2007), this system has the following characteristics:

1. *The system consists of multiple agents,*
2. *The system is adaptive; past, present behavior involve forming future actions,*
3. *Competing factors affects the agents' behavior*
4. *The structure of language emerges from interrelated patterns of experience. (p. #)*

Communication in the human context can emerge at different points of social interaction. For example, joint actions, cooperative activities all require sharing human beings' intention, culture, with one another (Bratman, 1992, 1993; Herbert, 1996). The CAS approach permits studying language in human context in a way that the aggregated effect of many interacting constraints, including the structure of thought process, perceptual motor biases, cognitive limitations, and social-pragmatic factors (Chater, 2008) are taken into consideration.

3.7.4. Pragmatics in Communication

Informative communication has at least two interlocutors: sender and receiver. There exists a tacit agreement in between these agents to maintain the recipient's acknowledgment that the sender has taken prudent measure to impart the truth.

Agents in communication aim to convey information in the most efficient way. Even though there is a tacit agreement, the interlocutors develop a variety of operative assumptions in the presupposition framework of the context of communication operations. The significance of this presupposition is that they do not emerge from the context of the message, rather they are developed from contextually-formed dispositions which are the consequences of a generative process (Rescher, 1998).

Construing received information and extracting knowledge of the interlocutor are among the essential stages in the process of communication. This essentiality ensues from the fact that once the message is imparted and the extraction (decoding) starts, the sender cannot intervene in this process. Therefore, the governing principle for verbal communication in an informative communication context is important. These can be listed as (Rescher, 1998):

- *The sender purports the claims to present the truth according to his/her understanding,*
- *The sender compiles the purported claims as accurately as he/she can omitting any misleading elements. (p. #)*

3.7.4.1. Context in Communication

The receiver (Figure 17) construes the received message and forms an understanding. The sender must, in turn, be careful in compiling the message to mitigate the possibility of misunderstanding. For example, consider an informative message in the form a text. It does convey a message in two ways; the substantiated meaning that words explicitly convey, and meaning that it may convey implicitly. Therefore, it can be said that a statement might have plural meaning. The process of distinguishing, reducing, or even eliminating the pluralism in the constructed meaning of a verbally compiled message is called interpretation (Rescher, 1998). The process of interpretation utilizes context as a reference point. The process of distinguishing, reducing, or even eliminating the pluralism in the constructed meaning of a verbally compiled message is called interpretation (Rescher, 1998). The process of interpretation utilizes context as a reference point. As Rescher says;

> *"in the process of interpretation context is not just important, it is everything." (p. #)*

The role of context is a principle of efficiency in communication.

3.7.4.2. Communication in Communal Setting

Humans have physical and cognitive needs. Knowledge, which can be converted into action, is the primary cognitive need. Acquiring and retaining knowledge is contingent on communally and cooperatively coordinated settings because one human life is too short to comprehend everything. Humans function in a larger community that operates in a spatial (social organizations)-temporal (cultural traditions) settings with communication, cooperation, and coordination (Rescher, 1998).

A viable and satisfactory communal existence requires that the people understand each other. From individuals' points of view, understanding is an endeavor to render the preceding and the proceedings of a rational person that is efficiently intelligible to the others in the communal existence. The communality of this rational procedure requires mechanisms such that cognitively-confined knowledge can be efficiently probed via communication.

The probing phenomenology is the key element in the progress of material science. In fact, social sciences utilize similar notions, however, named differently. In human interaction framework, presumptions emerge in a context where humans have questions and seek answers. The presumption is a thought instrument that is employed in circumstances where practical considerations are parameterizing and constraining human cognitive and communicative practices. For example, by changing presumptions, one can overcome a circumstance which lacks evidence for a conclusive claim that precludes reaching an answer. Rescher's (1998) arguments on presumptions are that a:

"[p]resumption affords useful cognitive and communicative resource. The obvious and evident advantage of presumption as an epistemic device is that it enables us vastly to extend the range of questions we can answer. It affords an instrument that enables us to extract a maximum of information from the communication situations. The presumption, in sum, is an ultimate pragmatic resource." (p. #)

Presumptions are transient. They become substantial in the framework of verbal communication and written communication. Presumptions are contextual and issue driven; a putative fact that is widely employed in communication (Rescher, 1998). Thus, the notion of presumption gains interest in logics, semantics, and epistemology.

Communication is a goal-oriented process, and the merits of it (e.g. efficiency) are sustained cognitively in a semantic domain on a mind-independent reality. The assumption of a mind-independent reality is essential for the invariance of diverse conceptions. The concept of mind-independent reality thinking is such that it provides a fixed independent reality thinking be such that it provides a fixed point, a stable center around which communication revolves around it (Rescher, 1998). However, this is very difficult to achieve.

Communication limits individual understanding, which is a contingency in making decision in complex environments. Individual limits can be recognized via awareness: A condition of having or showing realization, perception or knowledge (Kovacic, 2013).

3.7.5. Shared Awareness with Pragmatic Idealism Perspective

Pragmatic idealism as introduced in Sousa-Poza & Correa (2005) allows probabilistic representation of a knowledge claim or a perspective as can be seen in Figure 10. Multiple perspectives can exist with a certain probability in an ontological construct so that in the case of interaction (e.g. receiving stimuli), system states change and give rise to another understanding with new associated probability amplitudes. This ontological construct constitutes, real actuality, real probability, a real possibility, and a default state.

The depiction in Figure 19 along with the RDP model depicted in Figure 10 allows representing the potentialities state as a superposition of states in the domain of awareness. This ontology sustains the fallibilism, which is espoused the notion of complex situations. This indicates that (Sousa-Poza & Correa, 2005)

"as long as an alternative exists we cannot predict a specific outcome." (p. #)

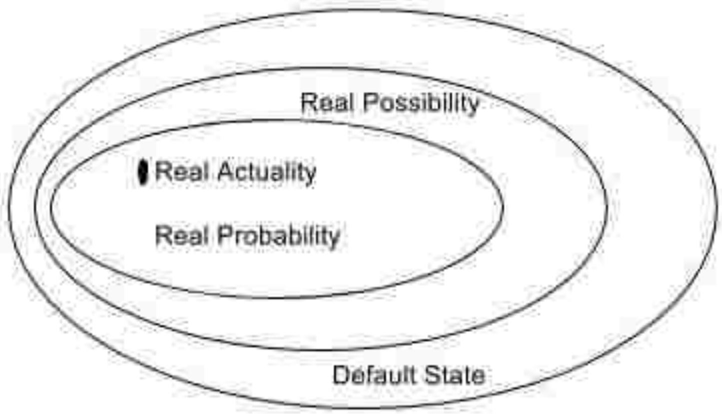

Figure 19 Ontology for Real Possibility, Real Probability, Real Possibility, and Default State. (Sousa-Poza & Correa, 2005)

3.8 Cognitive Science and Conceptual Space

Cognitive science is an interdisciplinary approach to studying thought, learning, reasoning, linguistics, and decision-making. There are two goals within the cognitive science: explanatory and constructive goals (Gardenfors, 2004). The former studies cognitive activities of humans and develop theories related to cognition. The second one is more engineering oriented. The aim is to develop artifacts such as decision aid tools, smart machines, and systems for complicated tasks so that human augmentation is advanced to attain difficult objectives. There is a common problem for both goals (Gardenfors, 2004):

> *"how the representations used by the cognitive systems are to be modeled in an appropriate way." (p. #)*

There are two dominant approaches to model representation. One approach, which is symbolism, describes the cognitive system akin to the Turing Machine and discerns cognition as computation. Another approach, associationism, is concerned with the associations among different kinds of information elements to represent cognitive phenomena.

> *"There are aspects of cognitive phenomena, however, for which neither symbolic representation or associationism appear to offer approved modeling tools." (p. #)*

Concept combination and concept learning are among the problematic processes for both approaches. Thus requires proper modeling approach so that the complex situation can be comprehensively represented in Figure 20.

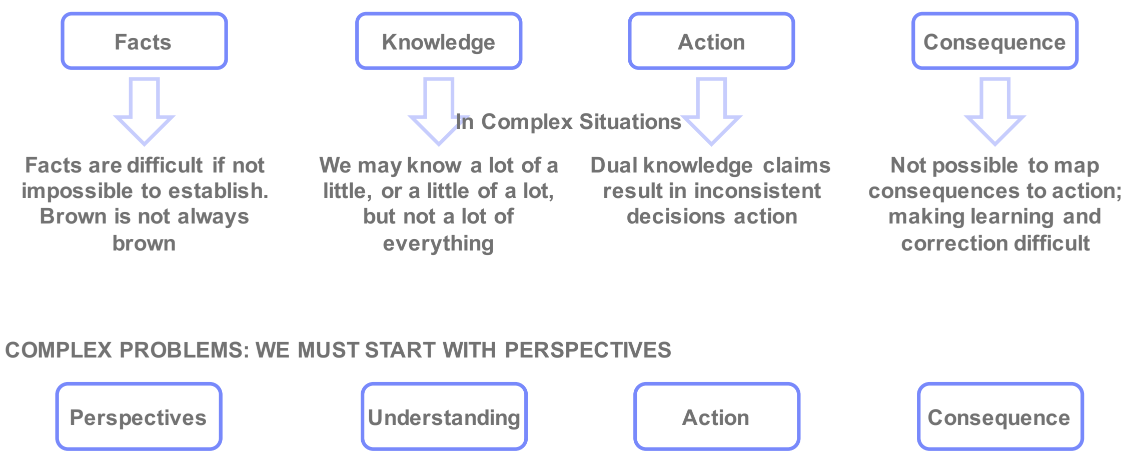

Figure 20 Simple Problems vs. Complex Problems

A phenomenon, or a fact, in reality, is not a single point. As introduced earlier, a domain of awareness (Figure 10) and constituent perspectives form meaning Pearl (1988, 2009). For example, brown is not brown all the time. The ascribed meaning is construed from a spectrum of possible interpretations (as addressed in Figure 20).

The developments in quantum cognition (Bruza & Cole, 2006; Busemeyer & Bruza, 2012; Bruza & Cole, 2006; Busemeyer & Bruza, 2012; Diederik Aerts & Czachor, 2004; Diederik Aerts, Sandro Sozzo, & Thomas Veloz, 2015a; Nelson & McEvoy, 2007), developments in concept theory (Diederik Aerts, Broekaert, Sozzo, & Veloz, 2013; Diederik Aerts & Gabora, 2004a, 2004b; Diederik Aerts & Sozzo, 2012) and conceptual space representation works (Gardenfors, 2004, 2013) demonstrate the necessity of a geometric representation for cognitive realm. Even though various groups have used geometric structures in cognitive studies, the conceptual form of the representation has been neglected.

There is no unique way of representing the phenomena in the cognitive domain. It is the same phenomena that can be represented at all levels with different levels of detail. All three levels complement each other. Consequently, situation awareness and decision-making in a complex situation can be more comprehensively formalized.

3.8.1. Symbolic Representation and Its Limitations

Representation and the process of information at the symbolic level are formed by symbol manipulation. Different cognitive states of the person are connected through logical and inferential relations (Gardenfors, 2004). There are two modeling approaches that describe the cognitive process at the symbolic level: logical inferences and syntactic parsing (Gardenfors, 2004):

> *When the symbols are used for modeling logical inferences, the expressions represent propositions, and they stand in various logical relations to each*

> *other. Information processing involves above all computations of logical consequences. (p. #)*

An agent in the cognitive realm is discerned akin to the logic machine that operates on well-formed expressions from some type of formal language. The symbolic representation maintains a functionalist philosophy of mind. As a result, the mind becomes a computation device. This computation device generates symbolic sentences based on the inputs from sensory sources. Based on the algorithms, verbal and non-verbal behaviors are generated. In this context, symbol manipulation becomes the program, which processes the inputs in symbolic form independent from what happens in the environment in that time.

Symbolic representation is the most classical form of representation used in the Artificial Intelligence (AI) community. The main objective of the AI community is to be able to describe the world and possible action with symbolic representation approach. Following this, smearing this representation or iterating with the powerful inference machine so that an AI, which would be capable of problem-solving, can be designed. However, the major limitation of the symbolic approach demonstrated itself as the combinatorial limitation.

Propositional representations in the symbolic representation limit the causal connections or dynamic interactions. The symbolic representation does not provide a natural way to separate different domains of information. Although various ways tried, the combinatorial limits impede the full application in this representation. In this paradigm, evolutionary emergence cannot be represented comprehensively. Overall, the symbolic approach is limited to comprehending the interaction in cognitive realm. The formation of semantic meaning, emergence, concept formation, and some other consequences of interactions require coherent representational approaches.

3.8.2. Associationism – Sub-conceptual Representation

The philosophical background of this approach goes back to John Locke (1690) and David Hume (1748). Both argue that the act of thinking establish associations among the constructed understanding of the world. This is akin to the notion of entangled states in quantum cognition because (Dellarosa, 1988):

> *"Events that co-occur in space or time become connected in mind. Events that share meaning or physical similarity become associated in mind. Activation of one unit activates others to which it is linked, the degree of activation depending on the strength of the association". (p. 29)*

The theory of concept (Diederik Aerts, Broekaert, Gabora, & Veloz, 2012; Diederik Aerts & Gabora, 2004a, 2004b; Diederik Aerts & Sozzo, 2012, 2013, 2014; Gabora, Rosch, & Aerts, 2008) supports the notion of interaction and entanglement. However, sub-conceptual space representation, or in the evolved form of connectionism, does not provide sufficient dimensional representation for the discussed characteristic of interactions. The connectionist approach is a network, and it is difficult to train these networks when they become complex. The complexity in the network impedes the generalizability of what is learned from one domain to another.

3.8.3. Conceptual Space Representation

The conceptual structure provides an epistemological framework to model various relations among experience, what is perceived, and imagined. The formalization of relations should be dynamic and adaptive because entities in the real world interact and change their perceived existence. The conceptual space representation recognizes dimensions of perception of a human. The geometric structure of representation provides a comprehensive possible possibility space, which supports the subjectivity in knowledge claims. For example, quantum probability theory uses space and subspaces rather than set theory.

The conceptual space approach provides a framework to study interrelation of concept. The primary concern is not about what is represented, rather how the representation is related to the other representation.

To define a domain in this framework Gardenfors (2004) discusses not only binary features but also dimensions. A dimension in this framework can be understood. For example, the time dimension can be represented for a human who perceives time as flowing from left (past) to the right (future) can be seen in Figure 21.

Figure 21 The Time Dimension (Gardenfors, 2004)

On the contrary, there are cultures that perceive time as circular. The representation of time concept of this culture should be compatible how it is perceived. Dimensions in this framework provide to assign properties to objects and to specify relations among the objects. A quality dimension in this construct signifies the qualities of the objects. Gardenfors (2004) characterizes dimensions into two: integral dimension and non-integral dimension. An integral dimension is defined as:

“one cannot assign an object a value on one dimension without giving it a value on the other.” (p. #)

A non-integral dimension is defined as:

“one can assign a value on one dimension without assigning a value to the other.” (p. #)

Integral Dimension	Non-integral Dimension
Hue / Brightness	Hue / Size

Figure 22 Examples of Integral and Non-integral Dimensions

Gallistel (1990) indicates that

“representing stimuli as points in descriptive space of modest dimensionality even implemented to the representation spectral composition. Even this is a source of confusion and misunderstanding in a scientific discussion of color. The colors like red, green, and blue do not have a straightforward translation into physical reality”. (p. #)

This resonates with the fact that brown is not brown all the time, a comprehensive dimensional representation requires to establish and support the efforts in understanding perceived reality. Gärdenfors (2004) defines a domain as:

“a set of integral dimensions that are separable from all other dimensions.”(p. #)

The conceptual space construct distinguishes concept and property. The property is an adjective, and it is defined a special region in the domain. The shape of this region is "convex." Consider two phenomena, p_1 and p_2 that belong to a concept. All the items in between p_1 and p_2 in conceptual space satisfy the criteria of being a convex region (Gabora et al., 2008; Gardenfors, 2004). This convex region in the domain defined by the integral dimension. For the color case, these dimensions are hue, chromaticness, and brightness.

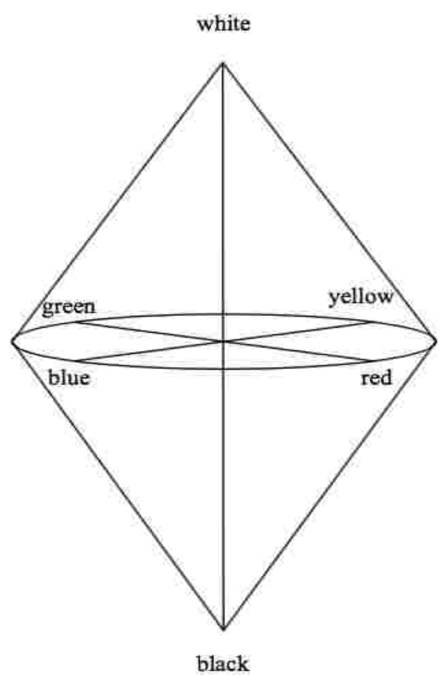

Figure 23 The spindle representation of color (Gardenfors, 2004).

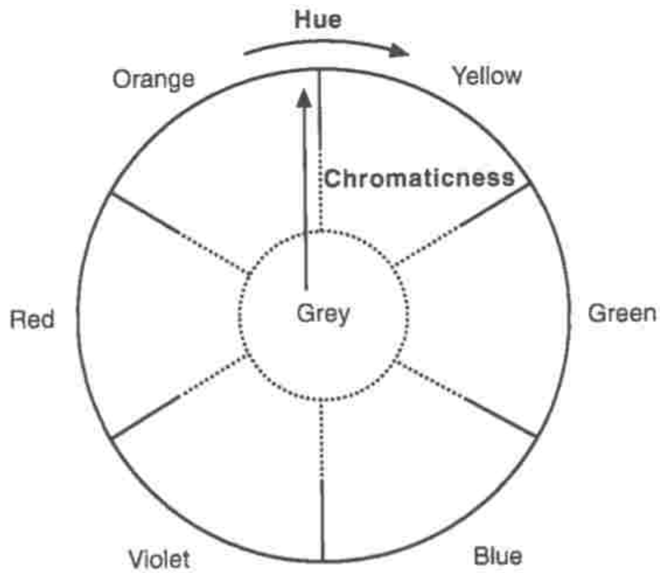

Figure 24 Color Circle (Gardenfors, 2004). The color circle is the vertical slice of the spindle in Figure 23.

Properties in this framework correspond to a single domain. However, the concepts can refer multiple domains. Thus, a concept can be a set of convex regions in a number of domains. Concepts in the conceptual space not only include the domains but also include the information regarding the different domains, and hence a causal relation among different domains can be extracted (Gabora et al., 2008).

The concept combination in this approach represented as the combination of these convex regions that correspond the concepts. Gabora, Rosch, and Aerts (2008) exemplifies this concept combination as;

"XY = region for some domain of modifier X replaces corresponding region for Y. This $modifier + modifier$ relationship is very language specific. For example, in RED BRICK, one replaces the original region for color for the concept BRICK with the corresponding region for RED." (p. #)

3.8.4 Ecological View

J.J. Gibson introduced an ecological perspective into the perception (Gibson, 1979). The introduced view includes an interaction approach. The developed perception of oneself and one's environment are tangled (Gibson, 1979; Hernes, 2007);

> *"The supposedly separate realms of the subjective and objective are only poles of attention." (p. #)*

The part of the generative process demonstrates its impact on this interaction approach. The micro and macro entities in the environment rely on each other to comprehend the possible complementarity through the functions so that action can emerge. Gibson (1979) introduced "affordance" to describe the function that is offered by the perceived other. This is supported by participant observer dyad because the world becomes meaningful with the reference points. The forms and extractable functions are concomitant as perceiving subject and perceived object (Gabora et al., 2008; Gibson, 1979; Hernes, 2007; Rescher, 1996, 1999).

The geometrical approach and ecological view both introduce important features. At the confluence of these thought provoking approaches, what becomes important are the concepts. The perceived objects are conceptualized, and the interaction with other concepts becomes possible with the concept representation of the perceived other (Gabora et al., 2008);

> *"it is only once objects in the world have been conceptualized that they are charged with the potential to dynamically interact in myriad ways with conceptions of other objects as well as with the goals, plans, schemas, desires, attitudes, fantasies, and so forth, that constitute human mental life. Moreover, it is through these interactions that their relations are discerned, and together*

they thereby come to function as an integrated internal model of the world, or worldview." (p. #)

Stimuli can originate from various sources, language, cues, etc., but how they are understood is based on the conceptual relations that the perceiving subject develops. This is important when discussed in the situation awareness and decision-making realms. This is because shared decision-making or shared goal attainment requires a comprehensive formalization so that the observer-participant dyad can be included in the discussion. Gregory Bateson (1973) discusses that:

"an ecological treatment of concepts opens up the possibility of making not just action but complex thought processes amenable to a more ecological approach." (p. #)

The domain of awareness and pragmatic idealism improvement to the notion of situation awareness allows studying shared awareness in complex situations with a dynamic formalism (Elanor Rosch, 1999a);

"it is the role of concepts to provide a bridge between what we think of like mind and what we think of as the world, and has articulated this position in terms of its implications for concepts. Concepts and categories do not represent the world in mind, as is generally assumed, but are a participating part of the mind-world whole. Therefore, they only occur as part of a web of meaning provided both by other concepts and by interrelated life activities.

> *This means that concepts and categories exist only in complex concrete situations."(p. #)*

The classical concept theories fail to include contextuality, concept combination, similarity, compatibility, and correlation. (A detailed discussion regarding failure can be found in Diederik (2016), Diederik et al. (2012), Diedrik and D'Hooghe (2009), Diederik, Gabora, and Sozzo (2013), Diederik and Sozzo (2012; 2013; 2014), Bordes, Glorot, Weston, and Bengio (2013), Bruza (2008), Bruza, Kitto, Nelson, and McEvoy (2009), Bruza, Kristy, McEvoy, and McEvoy (2008), Busemeyer and Bruza (2012), Busemeyer, Wang, and Lambert-Mogiliansky (2009), Carminati et al. (2015), Christia (2012), Conte (2008), Elio Conte (2008), Galli Carminati and Martin (2008), Gerhards and Schafer (2014), Goldstein (2009), Haven and Khrennikov (2012), Khrennikov (2010), Kirsty Kitto and Boschetti (2013a; 2013b), Kristy Kitto, Boschetti, and Bruza (2012); Litwin (2012), Nelson, Kitto, Galea, McEvoy, and Bruza (2013), Trueblood and Busemeyer (2012), Wang, Solloway, Shiffrin, and Busemeyer (2014), and Wendt (2015)).

The above-cited studies' contributions to the theory of concept is mathematical, which is the essential theoretical contribution. An ecological situation construct is difficult to illustrate with classical mathematics approaches. The mathematics of quantum theories, however, provides the framework to express the situational construct mathematically (Gabora et al., 2008);

> *"One point of similarity between quantum entities and concepts is that both differ from entities that can be described by classical physics, for which if a property is not actual then its negation is actual. If the property 'not green' is true of a particular ball, then the property 'green' is not true of that particular ball. However, for concepts, as in quantum mechanics, a property and its*

> *negation can both be potential. Thus for the concept BALL, if nothing is specified for the color, 'green' and 'not green' are both potentials. One could refer to this as a problem of nonclassical logic for concepts." (p. #)*

The notion of potential in the mathematics of quantum theory provides s sample space in reasoning. This is especially important in representing counter-factual reasoning for a situation. Another important contribution of the quantum theory to situation awareness studies is the observer effect and contextuality in the measurement (action);

> *"…much as properties of a quantum entity do not have definite values except in the context of a measurement, properties of a concept do not have definite applicabilities except in the context of a particular situation. In quantum mechanics, the states and properties of a quantum entity are affected in a systematic and mathematically well-modeled way by the measurement. Similarly, the context in which a concept is experienced colors inevitably how one experiences that concept. One could refer to this as an observer effect for concepts". (p. #)*

CHAPTER 4

CONCEPT

4.1 Quantum Cognition and Social Construction of Reality

Social science has generated extensive scholarship in an attempt to explain human behavior. The classical assumption pervades these theories from top to bottom. The attempts to explain seemingly irrational human behavior with these theories are all ad-hoc, partial, and do not form a coherent theory.

A comprehensive understanding of the existence of deviation of human behavior from the classical prediction of rational human choice (Bruza, Wang, & Busemeyer, 2015; Khrnnikov & Haven, 2009; Tversky & Kahneman, 1974) is required. For instance, order effects in human judgment, conjunction fallacies, have demonstrated that humans do not form probabilities and preferences in the way they would if humans were classical rational agents (Wendt, 2015).

The reason for the deviation from the predictions results from seeking the solution in a reduced domain. The rules, norms, government, leadership positions, and organizational requirements are all rooted in the human mind. Intentions and awareness at both the individual and team level are all rooted in mind. Thus, any attempt to understand the seemingly irrational behavior should go beyond the mere physical domain, which is studied with classical physics. This problem is also related to the long and yet unsolved mind-body problem. Seeking the answers in the domain of physicality reaches a dead end.

Social science is classified as a branch of natural science, which abides the causal closure of physics. Causal closure of physics introduces that everything in reality, including consciousness and social life, is composed of elementary objects. There two types of causal closure in the physics are classical and quantum. The classical closure is a subset of the quantum closure in specific situations.

The quantum domain demonstrates a significant contribution to the understanding of seemingly irrational behavior, such as non-commutative human behavior in interactions. In this domain, norms, rules, human conscious, words, cues, intelligent machines, and interaction (communication) are all represented as mind entities, which become quantum systems.

The mathematical foundation for quantum theory and the causal closure of quantum physics provide coherent and axiomatically consistent answers to the deviation from classical predictions. The representation of mind entities as a superposition of states, the implication of the quantum probability theory, the high contextuality of quantum decision theory, and interaction phenomenology are among the foundational principles that provide the mathematical formalism to study awareness in various contexts.

4.1.1. Probability Theories

Table 1 Comparison of some key axioms and theorems of the classical probability theory and quantum probability theory (Bruza et al., 2015)

Classical Probability Theory	Quantum Probability Theory
• Events are subsets of a universal set Ω. • The state of the cognitive system is represented by a function, p, defined on the subsets in Ω, and the probability of an event A is $p\ (A)$ • $p\ (A) > 1$, and $p\ (\Omega) = 1$ • If $(A \cap B) = \emptyset$, then $p\ (A \cup B) = p(A) + p(B)$. • The probability of event B given A equals $p(B\|A) = \frac{p(A\cap B)}{p(A)}$. • Law of total probability: $p(B) = p(A \cap B) + p(\sim A \cap B)$.	• Events are subspaces of a Hilbert space $\mathcal{H}$. Events, such as A and B, correspond to subspaces $\mathcal{H}_A$ and $\mathcal{H}_B$, respectively of $\mathcal{H}$. Associated with these subspaces are projectors $P_A$and P_B. • If their projectors are commutative, that is, $P_A P_B = P_B P_A$, then the events A and B are compatible. Otherwise, they are incompatible. • The state of the cognitive system is represented by a unit length vector S in the vector space, and the probability of event A equals $\left\|P_A \cdot S\right\|^2$. • $\left\|P_A \cdot S\right\|^2 \geq 0$ and $\left\|P_{\mathcal{H}} \cdot S\right\|^2 = 1$. • If $P_A P_B = 0$, then $\|(P_A + P_B) \cdot S\|^2 = \left\|P_A \cdot S\right\|^2 + \left\|P_B \cdot S\right\|^2$. • The probability of event B given A equals $\frac{\left\|P_B \cdot P_A \cdot S\right\|^2}{\left\|P_A \cdot S\right\|^2}$ • Violations of the law of total probability: $\left\|P_B \cdot S\right\|^2 \neq \left\|P_B \cdot P_A \cdot S\right\|^2 + \left\|P_B \cdot P_{\sim A} \cdot S\right\|^2$.

4.1.1.1. Classical Probability Theory

The foundation of probability theory in the existing modeling approaches is based on the classical probability theory. The axioms of the classical probability theory were introduced by Andrei N. Kolmogorov (1933). The theory is based on events belongs to subsets of the universal event set. For example, assume that A and B are two events in this subset representing the preferences of two individuals. These two events can be combined, A and B (conjunction), and are represented as a set intersection $A \cap B$ in the classical probability theory. The disjunction events are represented as $A \cup B$, which denotes A or B regarding the preference of the individual. According to Boolean logic, all these are considered events in an event space. The premise in this logic is that the events and preferences of individuals, are commutative, $A \cap \quad = B \cap A$ (Bruza et al., 2015). The classical probability introduces a probability function, which maps the events to the probabilities (Busemeyer & Bruza, 2012; Haven & Khrennikov, 2012).

4.1.1.2. Quantum Probability Theory

The events in quantum probability theory are defined as subspaces rather than sets (Nuemann, 1955). The set-theoretic mapping structure is replaced with a projective geometric structure of vector space (Bruza et al., 2015; Haven & Khrennikov, 2012; Khrennikov, 2009). Space is spanned by orthonormal unit basis vectors, and each basis vector corresponds to an elementary outcome (Busemeyer & Bruza, 2012) and provides the foundation for geometric projection of probabilities. Consequently, an event is a sub-space spanned by a subset of the basis vectors. The probability function of the classical approach has replaced the probability of an event defined by a projection of the even on the space that is spanned by the basis vector. Assume that a unit length state vector $|S\rangle$ in the Hilbert space. The probability of this event is represented by $p_v = ||P_{event}|S\rangle||^2$.

4.1.2. Hilbert Space

Hilbert space is an abstract vector space (D'Espagnat, 1999; Liboff, 1980; Nuemann, 1955), that is used in the mathematical representation of the quantum theory. It is a generalized form of Euclidian space, which can have infinite dimensions. As a vector space, Hilbert space is composed of abstract points $\mathbf{X}$. A point in $\mathbf{X}$ is represented by a "ket"$|X\rangle$, which is a vector (Busemeyer & Bruza, 2012). Any pair of vectors can be added, multiplied and generates another vector in the same space. The ket, $|X\rangle$ corresponds to a $N\times1$ matrix.

Hilbert space has several features that make it an ideal foundation to study complex macro phenomena. Hilbert space representation is applicable for cues, words, and contexts. Context is important in social and psychological settings, as is the case in quantum theory (Kirsty Kitto & Boschetti, 2013b). It is fundamental to extracting meaning from words or statements for the different states in which the word or message might be encountered.

4.1.2.1. Inner & Outer Product between the Vectors

Two important concepts in studying the vectors are an inner and outer product. The inner and outer product characterizes the relation between the operators and the vectors. An inner product allocates a scalar value to the interaction of two vectors in Hilbert space. This interaction is that a vector, $|X\rangle$, from Hilbert space operates on another vector $|Y\rangle$. The inner product is represented as:

$$\langle X|Y\rangle. \quad \text{(Eq. 5)}$$

The term $\langle X|$ is taken from the Hilbert space and is called “bra”. Bras belong to another vector space called dual space and the same algebra rules that is applicable to Hilbert space still valid. A bra can also be expressed as a matrix, which $1\times N$.

4.1.3. Representing Phenomena in Hilbert Space

The meaning ascribed to a phenomenon is studied with different representations. For example, messages, context, and cues are among the phenomena that are used in developing awareness and making a decision. These can be modeled at an extremely high level of resolution as would be representative of individual psychological, cognitive constructs, or at lower levels of resolution, representing sociological constructs such as norms, values, or culture.

Studies utilizing this framework have shown how understanding is generated through a complex interaction of words. A specific meaning of a word or message is rendered when individuals or groups interact. Meaning, in this case, is inherently contextual and not only compositional. This is analogous to the phenomenon of measurement observed in quantum physics (Wendt, 2015). Words are stored in the memory as nodes in a network of related concepts and connecting relations. The sets of context in a situation form a complete lattice in Hilbert space (Diederik Aerts & Sozzo, 2013). This formalism provides the necessary algebra for concept combination in Hilbert space (Diederik Aerts & Gabora, 2004a, 2004b). All these components constitute the meaning of a word and the meaning is influenced by the context.

The advantage of doing this in Hilbert space is that all the concepts are captured in the same space. In classical approaches, each concept would require an associated sample space to manifest the meaning. It is possible to convert messages into vectors in the Hilbert space and the influence of context on meaning for an individual or group, and ultimately its effects on their decisions and behaviors can be studied. Text, messaging and other content in social media or web pages can be monitored, converted into vectors.

Context is represented with a choice of basis. The superposition state for the word "W" which is about the cue "c" can be written as:

$$|W\rangle = a_0|0_c\rangle + a_1|1_c\rangle. \qquad \text{(Eq. 6)}$$

In this equation, the basis is taken from$\{|0_c\rangle, |1_c\rangle\}$, and the basis vector $|0_c\rangle$ represents the basis state, which is not re-called and $|1_c\rangle$ represents the basis vector, which is re-called Figure 25. With a different cue "cc", the superposition of the same word "W" can be generated by different choice of basis, $\{|0_{cc}\rangle, |1_{cc}\rangle\}$ Figure 25. The word "W" in relation to cue "cc" is then

$$|W\rangle = b_0|0_{cc}\rangle + b_1|1_{cc}\rangle. \qquad \text{(Eq. 7)}$$

The context is expandable with a basis in Hilbert space. The basis vector for cue, "ccc" is $\{x_1\rangle, \dots, x_n\rangle\}$ can be represented with the superposition of n potential vectors of ccc:

$$|ccc_n\rangle = q_1|x_1\rangle + q_2|xp_2\rangle + \cdots + c_n|q_n\rangle, \qquad \text{(Eq. 8)}$$

where $\sum_n |c_n|^2 = 1$ (Busemeyer & Bruza, 2012).

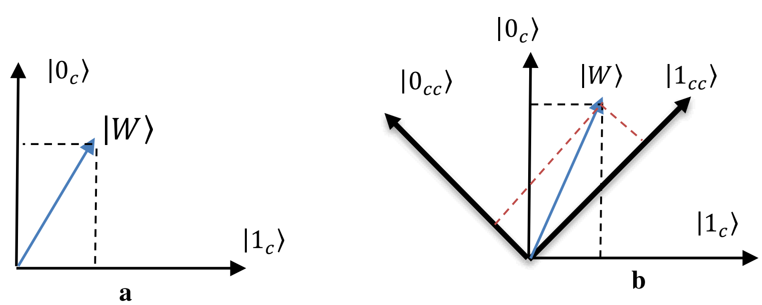

Figure 25 Geometric Representation of phenomena with basis vectors. The basis vectors represent context. The same phenomenon can be different in different contexts.

A word is stored cognitively with all possible concepts. Thus, in the case of interaction, quantum theory formalism renders the various meanings possible. For example, texts on social media or web pages can be transformed into vectors in Hilbert space. In the classical perspective, each concept should have an associated sample space to manifest meaning.

The derived equation can be expanded for concept combination in Hilbert space (Diederik Aerts & Gabora, 2004a, 2004b). The projection of the context results in a superposition of states. This formalism shows that concepts of meaning are inherently contextual, not compositional. The successive studies in this framework introduce the fact that words are stored in memory and not as isolated entities, but rather as nodes in a network of related words (Wendt, 2015). When presented as a stimulus, because of the existence of state vectors, the concepts in the superposition can be perturbed, and the decision can be projected to different subspaces.

Tensor products provide the required formalism to express the concept combination and context combination geometrically in Hilbert space (Diederik Aerts & Gabora, 2004a, 2004b).

In this approach, a cue word is represented with a very high-dimensional Hilbert space in the context of all of its associates. For example, the human awareness in superposition of ideas can be expressed as,

$$|Human\ Awareness\rangle = c_1|Idea_1\rangle + c_2|Idea_2\rangle + \cdots + c_N|Idea_n\rangle. \quad \text{(Eq. 9)}$$

Here, the coefficients are complex numbers, and square modulus of them give the relevance (probability) of a constituent idea to the situation and $\sum_n |c_n|^2 = 1$. To be more specific, the awareness vector of an individual for a democracy can be expressed as:

$$|Democracy\rangle = c_1|election\rangle + c_2|human\ rights\rangle + c_3|freedom\rangle$$
$$+ c_4|NATO\ hegemony\rangle + \cdots c_n|State_n\rangle. \quad \text{(Eq. 10)}$$

In this equation, the superposition includes orthogonal states.

4.1.4. Decision Making and Attitude Change

Decision-making, attitude change, and taking action are contextually constraining processes. The uncertainty in this formalism is ontological rather than epistemological. An agent that has acquired a good epistemology of a concept can form two distinct attitudes in two different contexts. The vector representation of this can be seen in Figure 26.

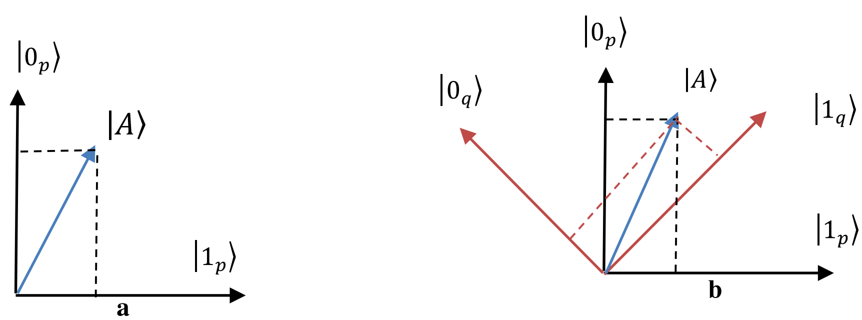

Figure 26 An agent decision represented in Hilbert Space. a) an agent in a context "p" b) The changing context to "q" for the same agent and projection changes.

Another contextual effect that can be seen in multi-agent decision making studies. Two agents with two distinct formed attitudes toward a social issue may act differently in different contexts. Epistemologically, both of the agents can have a well-established understanding of the situation; this can be seen in Figure 27. This is because of the context dependency of the processes. Context dependence is very high when two agents with same initial cognitive states can choose a different course of action if they are in different context (Kirsty Kitto & Boschetti, 2013a).

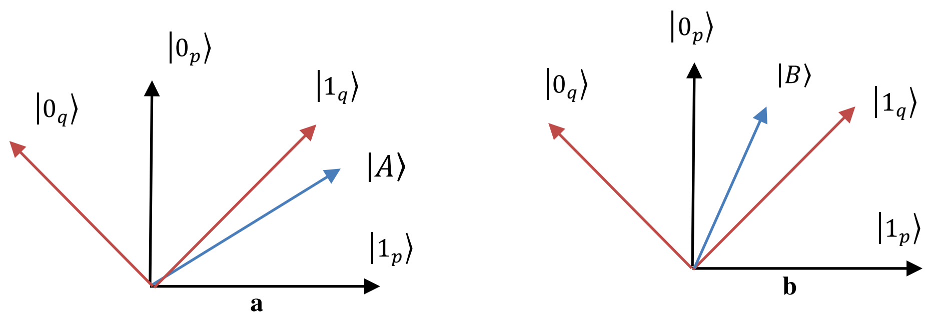

Figure 27 A set of agents all making a choice to act or not to act within a set of social context. Each has a different cognitive state, which can be measured on one of two different social groups or framing of the problem.

The most important part of this context formalism is the consideration of context itself. Here, the context itself become a part of the measurement and influences the outcome. The reference social context has a direct impact on the probability of a certain decision outcome. Therefore, being in the same society does not entail the attitude change nor does it make decision making identical for individuals.

A situation constitutes contexts that can influence the decision making agent and hence an attitude change can occur. The situations are categorized as micro and macro situations. A micro situation might arise from a wide range factors, such as external and internal factors. These factors are not limited to but include the socioeconomic status of an agent, educational background, race, beliefs. The aggregation of a micro situations can result in a macro situation. A macro situation represents, for instance, ideologies within a society.

In this approach, an agent's decision to act or not to act depends on both the current cognitive states $|A\rangle$ and the social context of the agent, which is comprised of macro and micro situations. The social context of an agent results from aggregating the attitudes of every other agent in the system. These two factors recursively interact in time, and both the cognitive states and the different framings of the issue will evolve in time.

4.1.5. Interaction Phenomenology

The wave function construct allows for a formulation of the interaction among different superposition states in Hilbert space. The available and presented information to the decision maker can be in two types i) already processed by an apparatus or another human, or ii) direct information. The human interacts with the information or the apparatus. The constituents of the superposition state can be annihilated, or the new ones can be created. The vector state that represents the agent becomes a new vector with new projections probabilities.

The interaction of two quantum superposition states in the form of wave mechanics can be written as

$$\Phi(r,t) = \sum_{n,m} a_m \, \psi_{A_m}(r,t). b_n \, \psi_{B_n}(r,t). \qquad \text{(Eq. 11)}$$

The interaction of a quantum superposition with an apparatus, which constitutes definite states represented with wave mechanics can be written as:

$$\Phi(r',t) = \sum_{n,m} a_m \, \psi_{A_m}(r_A,t)\phi_0(r_0,t). \qquad \text{(Eq. 12)}$$

The wave function formalism allows expressing the individual and impersonal behavior of individuals in the form of fields with the superposition of indefinite quantum states. For example,

$$\Psi(r,t) = \psi(r_i,t) + \phi\left(r_g,t\right), \qquad \text{(Eq. 13)}$$

where $\psi(r_i,t)$ represents the individual awareness field and $\phi\left(r_g,t\right)$ represents impersonal behavior (Martin, 2005).

4.1.6. Unitary Transformation Operators

There are two types of interaction; ecological perception interaction and agent-to-agent interaction. Describing an interaction between entities in the vector space allows using operators in multi-dimensional Hilbert Space. For example, a transformation operator U can be introduced to transform a vector from the existing basis to another orthonormal basis. This operator, U, is called unitary operator, which satisfies unitary property:

$$UU^{\dagger} = \mathbb{I}. \tag{Eq. 14}$$

A unitary operator transforms the basis $\{|\varphi_i\rangle, i = 1, \dots, N\}$ to another basis $\{|\phi_i\rangle, i = 1, \dots, N\}$;

$$|\phi_i\rangle = U|\varphi_i\rangle = \sum_m |\phi_m\rangle\langle\varphi_m|\varphi_i\rangle, \tag{Eq. 15}$$

where U is defined as

$$U = \sum_{m=1}^{N} |\phi_m\rangle\,\langle\varphi_m|,$$

$$U^{\dagger} = \sum_{m=1}^{N} |\varphi_m\rangle\,\langle\phi_m|. \tag{Eq. 16}$$

4.1.7. Contextuality

The contextuality of the measurement notion in quantum mechanics is introduced into the decision making and attitude change studies from various perspectives by Busemeyer and Bruza (2012), Trueblood and Busemeyer (2012), Kitto, Boschetti and Bruza (2012), Kitto (Kirsty Kitto & Boschetti, 2013a) , Aerts (Diederik Aerts & Liane Gabora, 2004a, 2004b), Conte, Khrennikov, Yuri, Todarello, Federici, and Zbilut (2008) and Khrennikov (2003a, 2003b). The contextuality of quantum decision theory becomes applicable with the Hilbert space. By projecting the individual cognitive state vector to basis vector of context (Trueblood & Busemeyer, 2012), the context in decision making is preserved. The discussed projection aspects of this geometric structure render the formalization of state vectors of adaptive behavior of agents in the environment.

The uncertainty in this formalism is ontological rather than epistemological. An agent that has acquired an understanding of a concept can form two distinct attitudes in two different contexts. The vector representation can be seen in Figure 28.

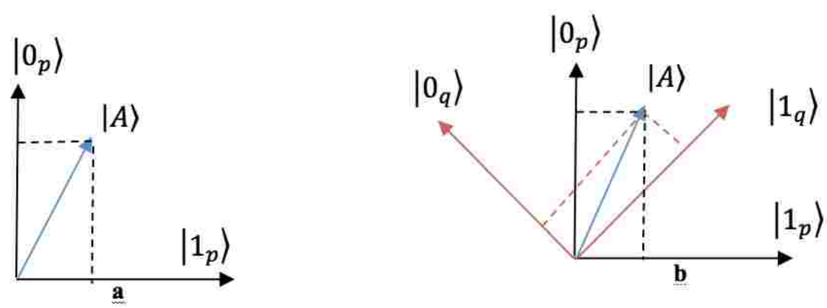

Figure 28: An agent decision represented in Hilbert Space. a) an agent in context "p" b) The changing context of the same agent and projections changes in another context "q." This Hilbert representation is simplified version of the two domain of awareness construct.

There are further contextual effects in decision-making. For example, in Figure 28b, an individual with a perspective A, can have two distinct decision in two separate contexts, *q,* and *p*. (Kirsty Kitto & Boschetti, 2013a). This represents paradoxes, in which two individuals with the same perspective can generate opposing, incompatible conclusions. As a converse to paradoxical outcomes, it is also possible to have two individuals with differing perspectives come to the same conclusion or action based on the effect of their individual contexts, or in other words, the enemy of my enemy is my friend.

Let's assume that a basic context $u \in X$ is represented by a projector $|u\rangle\langle u|$ where $|u\rangle$ is a unit vector and $B = \{|u\rangle \mid u \in X\}$. The ground state $\hat{p}$ of a concept is represented by unit vector $|x_{\hat{p}}\rangle$ superposition of the basis state B. $|x_{\hat{p}}\rangle = \sum_{u \in X} \alpha_u |u\rangle$ and $\alpha_u = \langle u|x_{\hat{p}}\rangle$. The α_u means that each of the basic states $u \in X$ is considered to have equal probability of being elicited (Diederik Aerts & Gabora, 2004b). The contexts are represented as projection operator, $P_{c_n} = \sum_{u \in B}|u\rangle\langle u|$. The calculation of the new superposition states with projection of context can be written as

$$|x_{p_n}\rangle = \frac{P_{c_n}|x_{\hat{p}}\rangle}{\sqrt{\langle x_{\hat{p}}|P_{c_n}|x_{\hat{p}}\rangle}} \Rightarrow \sum_{u \in B} \beta_{un}|u\rangle \qquad \text{(Eq. 17)}$$

Agent A has a state representation $|A\rangle$ for the cognitive states. The orthogonal basis sets for the context as can be seen in Figure 28. The basis is orthonormal because $|0_{q,p}\rangle$ represents the non-action while $|1_{q,p}\rangle$ represents action which cannot happen concurrently for the same agent. The probability of action in context "p" for $|A\rangle = a_{1p}|1_p\rangle + a_{0p}|0_p\rangle$ can be written as

$$\langle A|V_1|A\rangle = \langle A|1_p\rangle\langle 1_p|A\rangle = |a_{1p}|^2 \qquad \text{(Eq. 18)}$$

In the context "p," the higher probability is non-action, and in the context "q" it is taking action.

4.1.8. Interference Effects in Probability

Quantum probability theory replaces the sets in the classical approaches with vector space in Hilbert space. Quantum based outcomes (events) are defined geometrically in the Hilbert space. This transformation gives rise to a construct, which recognizes interference terms in probabilities. The interference term provides a possible explanation for human behaviors that are often classified as anomalies or as being irrational. The approach, however, still allows for classical probability theories, if appropriate, since they are, in essence, a simplified version of the quantum theory based isomorphism that is being used (Khrennikov, 2003a; Shimony, 2000).

One can write quantum theory-driven total probabilistic expression for decision process that constitutes different observables in Hilbert space as (Khrennikov, 2003b):

$$p_B(b_n) = \left(\sum_{j=1}^{n} a_j c_{nj}\right)^2 = \sum_{j=1}^{n} a_j^2\, c_{nj}^2 + 2\sum_{j\neq j\prime}\left[(a_{j\prime}, c_{nj})(a_j c_{nj\prime})\right] \qquad \text{(Eq. 19)}$$

In Eq. 19, there are two components. The first part of it is the classical probability expression, and the second part is the interference term that emerges in quantum mechanical expressions. By using Eq. 19 for observable sets in a cognitive system, one can get:

Observable A with the set $X = \{a_1,a_2\}$ and B with the set of $Y = \{b_1, b_2\}$ with the random variables a, and b. Then, a measurement of A over the elements of ξ cognitive systems that yields probability of:

$$p^a(x) = \frac{the\ number\ of\ results\ x}{the\ total\ number\ of\ elements}\ , x\ \epsilon\ X \qquad \text{(Eq. 20)}$$

Thus, $p^a(x)$ corresponds to obtain the result x in the ξ cognitive system. In this cognitive system one can perform observable A measurements for the elements of ensemble ξ_{b_i} ; $i = 1{,}2$ one can get

$$p^{a/b}(x/y) = \frac{the\ number\ of\ result\ x\ for\ ensemble\ \xi_y}{the\ total\ number\ of\ elements\ \ in\ \ \xi_y};\ x\ \epsilon\ X, y\ \epsilon\ Y \qquad \text{(Eq. 21)}$$

Eq. 21 can be inferred for $p^{a/b}(a_n/b_m)$ as the probability of answer a_n in the ξ cognitive system that already have chosen answer b_m. The classical probability framework entails the total number of probability as:

$$p^a(x) = p^b(b_1)p^{a/b}(x/b_1) + p^b(b_2)p^{a/b};\ x \epsilon X \quad \text{(Eq. 22)}$$

This indicates that classical expression neglects the interference terms because the probabilities are based on empirical frequencies. Consequently, any unprecedented behavior would be perceived irrational or abnormal. On the contrary, the quantum theoretical approach improves this probability expression and recognize the interference term.

$$p^a(x) = p^b(b_1)p^{a/b}(x/b_1) + p^b(b_2)p^{a/b} + 2\cos[\theta(x)]\sqrt{p^b(b_1)p^{a/b}(x/b_1)p^b(b_2)p^{a/b}(x/b_2)} \quad \text{(Eq. 23)}$$

In Eq. 23, $\theta(x)$ is the phase of the A interference between the state of mind in the cognitive system ξ and the ensemble ξ_y , $y \epsilon Y$ (Khrennikov, 2003b). Re-writing total probability, the interference total probability can be written:

$$p^a_\xi = \sum_{y \epsilon Y} p^b_\xi(y)p^{a/b}(x/y) + 2\cos[\theta_\xi(x)]\sqrt{\prod_{y \epsilon Y} p^b_\xi(y)\, p^{a/b}(x/y)} \quad \text{(Eq. 24)}$$

An advanced statement for doubt (uncertainty) that takes into account interference can be stated as:

$$D = A + B + 2\cos\theta\sqrt{AB} = \left|\sqrt{A} + e^{i\theta}\sqrt{B}\right|^2, A, B > 0, \quad \text{(Eq. 25)}$$

the probability can be written as:

$$p^a_\xi(x) = \left|\psi_\xi(x)\right|^2 \quad \text{(Eq. 26)}$$

where

$$\psi(x) \equiv \psi_{\xi}(x) = \sum_{y \in Y} \sqrt{p_{\xi}^{b}(y) p^{a/b}(x/y) e^{i\phi_{\xi}(x/y)}} \qquad \text{(Eq. 27)}$$

in Eq. 27, the phase $\phi_{\xi}(x/y)$ is defined as: $\phi_{\xi}(x/b_1) - \phi_{\xi}(x/b_2) = \theta_{\xi}(x)$

4.1.9. A Theory of Concepts

Hilbert space renders the representation of sets of contexts and properties of concepts possible. In the Hilbert space:

- States are represented as unit vectors or density operators,
- Contexts are orthogonal projections,
- Properties are orthogonal projections, and
- It is possible to model how context influence the state of a concept.

Modeling and representing context in decision-making is sin qua non. When developing a comprehensive situation awareness model, context is indispensable. The inadequate contribution of classical probability theory invokes non-classical probability model. For example, initial studies about the contradicting sentences of multi-sentence liar paradox can be represented in as an entangled state in Hilbert space (Diederik Aerts, Broekaert, & Smets, 1999). The interest of the quantum mechanics in mathematical modeling of contextual interaction appears in cognition (Diederik Aerts, Broekaerta, & Gaborab, 2011).

Humans use concepts to develop an understanding of the world. Concepts provide a framework to classify and interpret the relation among the constituents of a situation. Concepts can be represented the various type of phenomena, which can be material or not. For example, take a mountain as a material example and pain as an abstract example. Concepts have been at the center of human understanding discussion. From the classical perspective, it can be identified with sufficient necessary properties. However, this approach fails to recognize in (Wittgenstein, 1953);

> *"… defining game such that frisbee, baseball, and roulette are classified as games, while wars debates and leisure walking are not. Because it is not possible to give a set of characteristics or rules that define concept". (p. #)*

The geometrical structure discussed earlier recognizes this issue and introduce the notion of the region, domain, to represent concepts. The idea of an organization of concepts around similar ones was introduced by Rosch (1978; 1983; 1999b). These old theories provide justification for the organization of concepts, however, do not directly address the combination problem. The famous example to challenge the theories in called the guppy effect (Diederik Aerts & Gabora, 2004a);

> *"the combination problem, already in the eighties, the so-called 'guppy effect' was identified, where guppy is not rated as a good example of 'pet,' nor of 'fish,' but it is rated as a good example of the combination 'pet-fish' … , also intuitively it is possible to understand the peculiarity: if (1) activation of 'pet' causes a small activation of guppy, and (2) activation of 'fish' causes a small activation of guppy, how is it that (3) activation of 'pet-fish' causes a large activation of guppy? Also, the explanation based theories, since they have not*

> *lent themselves to mathematical formulation, have not been able to model what happens when concepts combine." (p. #)*

The way that pet-fish type combined concepts are formalized in the Hilbert space representation. The state representation of concept permits expressing the combined concepts in the probability expression of quantum probability formalism.

4.1.10. Context, State, and Properties

Contextual uniqueness is described by the state representation of a concept. The context incites a change in the state of the concept so that the ascribed meaning changes. For example, the concept of *apple* in the context of dinner is a fruit. However, the same concept in a technology meeting would mean a computer. A theory in concept combination introduces solution to the so-called guppy effect because combination is identified through context, which is a different state (Diederik Aerts & Gabora, 2004a);

> *"A context can itself be a concept or aggregation of concepts, or it can be a goal or drive state, a previous lingering thought, feeling, or experience, or ones' physical surrounding. Since in this article we focus on the description of the combination of concepts the contexts that we consider are aggregations of concepts because it is this type of contexts that play a role in way concepts combine." (p. #)*

This is the contextual interaction of concepts. For example, the concept *apple* interacts with the concept *apple*. The ensuing combined concept "apple chip" is in a different state and it is constrained by context.

4.1.10.1. States

Two types of states are used in the quantum theory. A pure state is represented by a unit vector $|x\rangle \in \mathcal{H}^n$such that $\|x\| = 1$, where $\mathcal{H}^n$ represents the n dimensional Hilbert space. Another type of state, density state is represented by a density operator ρ, which is linear and self-adjoint on $\mathcal{H}^n$. The ρ satisfies the condition of $\rho_{ij} = \rho^*_{ji}$ for all i, j such that $1 \leq i, j \leq n$. The ρ operator is semi definite that means $\langle x|\rho|x\rangle \geq 0 \; \forall \; |x\rangle \in \mathcal{H}^n$. The sum of diagonal elements of this matrix, ρ, is $\sum_{i=1}^{n} \rho_{ii} = 1$. As introduced by (Diederik Aerts & Gabora, 2004a, 2004b) the concept pet and the situation described in guppy effect can be represented as states in n dimensional Hilbert space, $\mathcal{H}^n$:

$$p_1, p_2, \ldots, p_n \in \Psi \tag{Eq. 28}$$

of pet using unit vectors or density operators in Hilbert space $\mathcal{H}^n$.

4.1.10.2. Properties

Property in quantum theory represented by *means of a linear operator*, which is an orthogonal projection operator or an orthogonal operator. An orthogonal projection operator must satisfy $P^2 = P$, which can be expressed as $\sum_{j=1}^{n} P_{ij}P_{jk} = P_{ik}$.To represent a concept, there needs to be at least two orthogonal projection operator in the Hilbert space $\mathcal{H}^n$.

4.1.10.3. Contexts

Measurement in quantum theory is described by a linear operator represented by $n \times n$, M_{ij} that satisfies $M_{ij} = M^*_{ji}$. The representation of context in this approach, however, chosen to be represented because (Diederik Aerts & Gabora, 2004a, 2004b);

"the set of orthogonal projection operators that form the spectral decomposition of this self-adjoint operator, which is an equivalent representation. Note that we have been considering 'pieces of context' rather than total contexts, and a piece of context is represented by one of these projection operators. Hence, a (piece of) context e is represented by a projector P_e." (p. #)

A context e changes a state p (pure state) of the concept to state q as:

$$|x_q\rangle = \frac{P_e|x_p\rangle}{\sqrt{\langle x_p|P_e|x_p\rangle}} \quad \text{(Eq. 29)}$$

where $\mu(q, e, p) = \langle x_p|P_e|x_p\rangle$ is the probability of this change occurs.

In the case, p would be density state, the density operator is $\rho_p = \frac{P_e \rho_e P_e}{Tr \rho_p P_e}$ where $\mu(q, e, p) = Tr \rho_p P_e$ of the probability of change of the density operator.

The contextual change after interaction for the awareness vector for voting decision is illustrated in Figure 29.

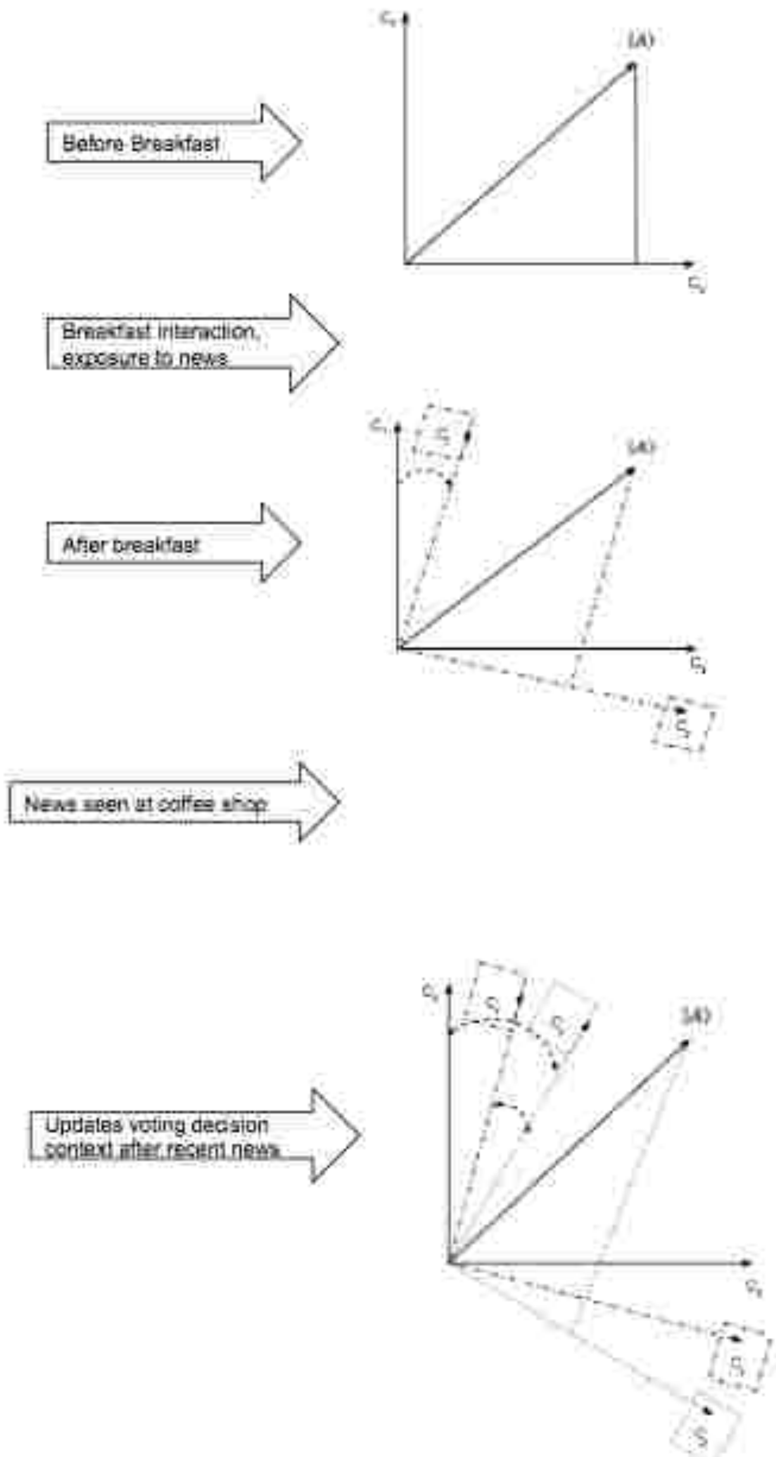

Figure 29 Depiction of contextual change for the voting decision of agent A. Axes C_1 represent the voting yes for candidate 1 and the other axes C_2 represents.

4.1.1.11. Concept Combination and Entanglement

Entanglement is manifested in the vector algebra. When the components of a system become entangled, they cannot be perceived as separate anymore. An entangled quantum system demonstrates an inter-component relation with the measurement settings (Bruza et al., 2009). An entangled quantum system and a correlated classical system are assumed to be the same. However, they are two distinct elements, for which the latter is a subset of the former.

The notion of entanglement discusses that how distinct and separated systems behave like one. Quantum entanglement is the existence of wave functions in the form (Haven & Khrennikov, 2012):

$$\psi(q_1, q_2) = \psi_1(q_1)\psi_2(q_2) + \psi_2(q_1)\psi_1(q_2). \tag{Eq. 30}$$

The tensor vector algebra that can be implemented in the Hilbert space provides a useful tool to study the conscious and unconscious interactions of various entities, as well as ensuing new superposition vector states that include possibilities of all types (explicit and implicit). This tool does not neglect the so-called classically hidden variables.

Different characteristics of individuals can be included in the Hilbert space representation. For example, Identity, Ego, Repressed, etc. Each of these sustains their own Hilbert space representation, e.g. H_{Ego} . The tensor product of all these forms a Hilbert space of an individual.

$$H_{Ego} \otimes H_{Identity} \otimes H_{Repressed} \ldots = H \tag{Eq. 31}$$

When an individual enters environments and his or her superposition state vector includes components from all of these Hilbert space. The H_{Ego} represented by the state vector $|CE\rangle$ and $H_{Repressed}$ represented by $|CR\rangle$.

The tensor products allow expressing the combination of two, for instance, as a superposition of the vectors in Hilbert Space.

$$H_{Ego} \otimes H_{Repressed} = |CE\rangle \otimes |CR\rangle \quad \text{(Eq. 31)}$$

where $|CE\rangle = a_0|CE0\rangle + a_1|CE1\rangle$ and $|CR\rangle = b_0|CR0\rangle + b_1|CR1\rangle$.

The tensor product outcome for this individual for this specific mood:

$$|CE\rangle \otimes |CR\rangle = a_0b_0|CE0\rangle|CR0\rangle + a_1b_0|CE1\rangle|CR0\rangle + a_0b_1|CE0\rangle|CR1\rangle + a_1b_1|CE1\rangle|CR1\rangle \quad \text{(Eq. 33)}$$

The tensor product and emerging entangled vector states contribute to the group model. The two fundamental group behavior principles are: 1) The conscious cooperation of the group members requires an unconscious emotional and phantasmatic communication between them; 2) The individuals in a group combine instantaneously and involuntarily according to the affective states called basic assumptions. Starting from and in contrast to the basic assumptions the group's work link to the reality can develop (Galli Carminati & Martin, 2008):

The Hilbert space tensor product allows expressing the quantum state of the group. The Hilbert space of the individuals: H_1, H_2, H_3, H_4, H_5. The tensor product of these individuals for a specific mood, T, can be expressed as:

$$H_{1T} \otimes H_{2T} \otimes H_{3T} \otimes H_{4T} \otimes H_{4T} = |CTH_1\rangle \otimes |CTH_2\rangle \otimes |CTH_3\rangle \otimes |CTH_4\rangle \otimes |CTH_5\rangle$$

$$= a_0b_0c_0d_0e_0|CTH_10\rangle|CTH_20\rangle|CTH_30\rangle|CTH_40\rangle|CTH_50\rangle + \cdots + a_1b_1c_1d_1e_1|CTH_21\rangle|CTH_31\rangle|CTH_41\rangle|CTH_51\rangle|CTH_51\rangle \quad \text{(Eq. 34)}$$

(Galli Carminati & Martin, 2008).

The notion of entanglement becomes important for modeling awareness. It provides transferring the contextual characteristics, the influence, to preserve the illocutionary force of the information, and to include a human aspect of the situation.

CHAPTER 5

RESULTS

5.1 Awareness and Shared Awareness

The formation of a domain takes place through the generative process. The generative process inculcates the mutual generation of self-awareness and awareness of other than self. The formation process is based on the situational construct; observer, entity, and solution form.

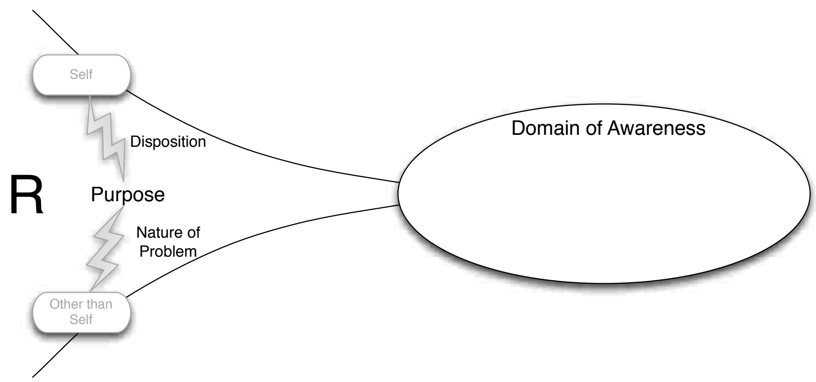

Figure 30 A domain of awareness is formed through the generative process. This domain of awareness is the result of the abstract representation in Figure 9

A Hilbert space vector formalism is employed to represent a phenomenon and to express a generative process interaction.

5.2 The Self and Other than Self

As represented in Figure 9 and Figure 30, an individual interacts with a phenomenon in reality. When the phenomenon observed an attribution is the cognitive domain takes place. To represent this an N-dimensional Hilbert space, $\mathcal{H}^N$, is used. By applying a Gram Schimd transformation (Strang, 1980) the independent vectors in the $\mathcal{H}^N$ can form an orthonormal basis:

$$\{|\varphi_i\rangle\,, i = 1, \dots, N\}. \qquad \text{(Eq. 35)}$$

Each basis vector, $|\varphi_i\rangle$, can be expressed in terms of projector operators:

$$\mathrm{P}_i = |\varphi_i\rangle\langle\varphi_i| \qquad \text{(Eq. 36)}$$

where

$$\sum_{i=1}^{N} \mathrm{P}_i = \mathrm{I}\,. \qquad \text{(Eq. 37)}$$

The basis vectors in this N-dimensional Hilbert space can represent the components of the generative process. Consequently, when an individual forms awareness through a generative process, it means describing the phenomenon with respect to basis vectors.

To simplify the representation assume a subset of two basis vectors:

$$\{|\varphi_i\rangle, |\varphi_j\rangle\}. \qquad \text{(Eq. 38)}$$

These two vectors span a space that forms a self-construct, which the phenomenon is perceived with the two components of the generative process. The projection of the basis vectors can be expressed as a linear combination:

$$\mathrm{P}_n = (\mathrm{P}_i + \mathrm{P}_j), \qquad \text{(Eq. 39)}$$

the vector representation is

$$P_n = |\varphi_i\rangle\langle\varphi_i| + |\varphi_j\rangle\langle\varphi_j| \qquad \text{(Eq. 40)}$$

By using the projection operator, the state representation of the phenomenon as self, in a space spanned by this N-dimensional space, can be written as:

$$\left|Ph_{Self}\right\rangle = \mathrm{I} \quad \cdot \left|Ph_{Self}\right\rangle = \sum_{i=1}^{N} \mathrm{P}_i \ \left|Ph_{Self}\right\rangle$$

$$= \sum_{i=1}^{N} |\varphi_i\rangle \langle\varphi_i|Ph_{self}\rangle \qquad \text{(Eq. 41)}$$

$$= \sum_{i=1}^{N} \langle\varphi_i|Ph_{self}\rangle|\varphi_i\rangle.$$

A ket is an object and a phenomenon is represented with the basis vector of the individual. The equation is represented by N different components of the generative process. The $N\times1$ matrix representation of this vector is:

$$\left|Ph_{Self}\right\rangle \to \begin{bmatrix} \langle\varphi_1|Ph_{self}\rangle \\ \langle\varphi_i|Ph_{self}\rangle \\ \langle\varphi_N|Ph_{self}\rangle \end{bmatrix} \qquad \text{(Eq. 42)}$$

The vector $\left|Ph_{Self}\right\rangle$ represent the awareness of self with the basis vector $\{|\varphi_i\rangle, i = 1, \dots, N\}$. Next, a generalized "other than self" is introduced. The other than self is accessible to the self. The construction of self and other than self is mutually a generative process, in other words, it is a reality bounded abstraction process. A phenomenon, which can be an action, a cue, an utterance etc., is abstracted to understand in terms of individual dispositions. As discussed in complex situation perspectives and RDP, two individuals can see the same information and develop different understandings. The construct of self is a partition of complete Hilbert space. The completeness theorem,

$$\sum_{i=1}^{N} \mathrm{P}_i = \mathrm{I} \qquad \text{(Eq. 43)}$$

renders the expansion of the vector representation to the complete orthonormal basis. The important distinction is that the partitions self and other than self can have different eigenvalues because of the orthogonality principle.

Assume that the possible understanding of a phenomenon $|Ph\rangle$, a system, for the basis vectors $\{|\varphi_i\rangle, |\varphi_j\rangle\}$ by using $\mathrm{P}_n = (\mathrm{P}_i + \mathrm{P}_j)$ projection operator:

$$|Ph_{Self}\rangle = \mathrm{P}_n|Ph\rangle. \qquad \text{(Eq. 44)}$$

The difference between self-vector and in the understanding of this phenomenon, e.g. system, can be expressed as:

$$\begin{aligned} |D\rangle &= |Ph\rangle - |Ph_{self}\rangle \\ &= |Ph\rangle - \mathrm{P}_n|Ph\rangle \\ &= (1 - \mathrm{P}_n)|Ph\rangle \\ &= (\mathbb{I} - \mathrm{P}_n)|Ph\rangle \end{aligned} \qquad \text{(Eq. 45)}$$

The orthogonality can be derived as:

$$\begin{aligned} \langle D|Ph_{Self}\rangle &= \langle Ph|(\mathbb{I} - \mathrm{P}_n)\mathrm{P}_n|Ph\rangle \\ &= \langle Ph|(\mathbb{I}\mathrm{P}_n - \mathrm{P}_n\mathrm{P}_n)|Ph\rangle \\ &= \langle Ph|\mathbb{I}\mathrm{P}_n|Ph\rangle - \langle Ph|\mathrm{P}_n\mathrm{P}_n|Ph\rangle \\ &\langle Ph|\mathrm{P}_n|Ph\rangle - \langle Ph|\mathrm{P}_n|Ph\rangle = 0 \end{aligned} \qquad \text{(Eq. 46)}$$

Therefore, “the other than self “can be expressed with a difference vector. The self and other than self are two orthogonal vectors span in the complete Hilbert space. The possible understandings are unique in these subspaces.

For example, consider two agents who aim to attain a shared awareness of a phenomenon, e.g. a cue as shown in Figure 31.

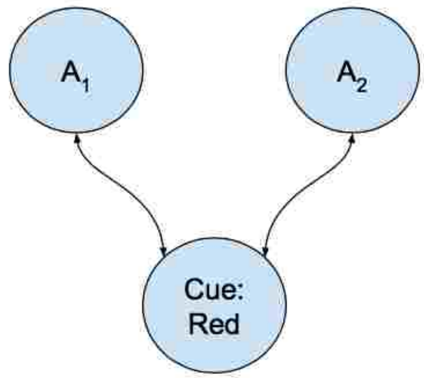

Figure 31 Two agents come across a cue. The goal of the agents is to attain a shared awareness. Each agent sees the cue, "red", but they interpret it differently.

The situation in Figure 31 constitutes, A_1, A_2 and the cue. Based on the introduced "other than self" expression, the A_1 develops awareness for this situation.

- The cue is represented with $|C\rangle$. The cue is accessible by both agents
- A_1 projects the vector $|C\rangle$ on to a subspace $\{|\varphi_j\rangle, |\varphi_k\rangle\}$ that is part of N dimensional space $\{|\varphi_i\rangle, i = 1, \dots N\}$ where $\sum_{i=1}^{N} P_i$

$$P_{A_1} = P_j + P_k \qquad \text{(Eq. 47)}$$
$$P_{A_1}|C\rangle = (P_j + P_k)|C\rangle$$

The "other than self" in this N-dimensional Hilbert space can be expressed as

$$|D\rangle = |C\rangle - P_{A_1}|C\rangle \qquad \text{(Eq. 48)}$$
$$= \left(\mathbb{I} - P_{A_1}\right)|C\rangle$$

5.3 Interaction Constraints of Shared Awareness

5.3.1 Interference and Order Effect

Shared awareness supports shared decision making. Human decision-making process demonstrates similarities to the quantum physics measurement process. The measurement process is a highly contextual process, which is a transition from potential to actual (Diederik Aerts, Arguëlles, et al., 2016). The concept of shared awareness should include individual attributes and the context. Suppose that are two agents with unique cognitive states, $|A_1\rangle$ and $|A_2\rangle$. As demonstrated earlier, even if these two agents are in the same context, the action might have different outcomes. Consequently, shared awareness and shared decision making is always contextual. Hence, conditional shared awareness category requires further elucidation.

Contextual sharing is defined (Kovacic, 2013) with commonality of the information, change in the situation, and spatial change. According the contextuality of interaction with the situation and the environment, besides the context, the order of interaction, the previous interaction, the cognitive state of the system, and compatibility of events constrain the shared awareness. Consequently, these two categories, conditional and contextual shared awareness, are considered as one category.

Considers the case for two possible decisions, which are decision X and Y. Depending on whether X and Y are compatible or not, attaining a shared decision towards a shared awareness of Y can be constrained by the event X based on the following scenarios:

- Decision Y and probability of Y is $P(Y) = ||P_Y|S\rangle||^2$
- Decision X or not-X ($\bar{X}$) then Y and the total probability is $P_{total} = ||P_Y P_X|S\rangle||^2 + ||P_Y P_{\bar{X}}|S\rangle||^2$.

This is important because the occurrence of an event (decision) results in the changes in the system, which is called state revision. Therefore, shared awareness model in attaining shared decision should exhibit a compatible description to reflect the state revision. State revision becomes a constraint for the non-compatible events. For example, the projection of event X and Y are $P_X$and P_Y. In the case of being compatible, these projections would give same probability. However, the non-compatible events do not give same result. Hence, the state revision should be reflected in the modeling with an effect called interference. Interference is $Int\ (XY) = P(Y) - P_{total}(Y)$ which can derived as:

$$
\begin{aligned}
P(Y) &= \|P_Y|S\rangle\|^2 = \|P_Y \mathrm{I}|S\rangle\|^2 \\
&= \|P_Y(P_X + P_{\bar{X}})|S\rangle\|^2 \\
&= \langle S|(P_X + P_{\bar{X}})P_Y P_Y (P_X + P_{\bar{X}})|S\rangle \\
&= \langle S|P_X P_Y P_X|S\rangle + \langle S|P_{\bar{X}} P_Y P_X|S\rangle + \langle S|P_X P_Y P_{\bar{X}}|S\rangle + \langle S|P_{\bar{X}} P_Y P_{\bar{X}}|S\rangle \\
&= \|P_Y P_X|S\rangle\|^2 + \|P_Y P_{\bar{X}}|S\rangle\|^2 + [\langle S|P_{\bar{X}} P_Y P_X|S\rangle + \langle S|P_X P_Y P_{\bar{X}}|S\rangle]
\end{aligned}
\quad \text{(Eq. 49)}
$$

The term $[\langle S|P_{\bar{X}} P_Y P_X|S\rangle + \langle S|P_X P_Y P_{\bar{X}}|S\rangle]$ corresponds to the interference term. If the interference term is decomposed:

$$
\begin{aligned}
&[\langle S|P_{\bar{X}} P_Y P_X|S\rangle + \langle S|P_X P_Y P_{\bar{X}}|S\rangle] = \\
&(\langle S|P_{\bar{X}} P_Y P_X|S\rangle + \langle S|P_X P_Y P_{\bar{X}}|S\rangle)(\langle S|P_{\bar{X}} P_Y P_X|S\rangle + \langle S|P_X P_Y P_{\bar{X}}|S\rangle)^* \\
&= \langle S|P_{\bar{X}} P_Y P_X|S\rangle\langle S|P_{\bar{X}} P_Y P_X|S\rangle^* + \langle S|P_{\bar{X}} P_Y P_X|S\rangle\langle S|P_X P_Y P_{\bar{X}}|S\rangle^* \\
&\quad + \langle S|P_X P_Y P_{\bar{X}}|S\rangle\langle S|P_{\bar{X}} P_Y P_X|S\rangle^* + \langle S|P_X P_Y P_{\bar{X}}|S\rangle\langle S|P_X P_Y P_{\bar{X}}|S\rangle^* \\
&= \langle S|P_{\bar{X}} P_Y P_X|S\rangle\langle S|P_X P_Y P_{\bar{X}}|S\rangle + \langle S|P_{\bar{X}} P_Y P_X|S\rangle\langle S|P_{\bar{X}} P_Y P_X|S\rangle + \langle S|P_X P_Y P_{\bar{X}}|S\rangle\langle S|P_X P_Y P_{\bar{X}}|S\rangle \\
&\quad + \langle S|P_X P_Y P_{\bar{X}}|S\rangle\langle S|P_{\bar{X}} P_Y P_X|S\rangle \\
&= 2 \cdot |\langle S|P_{\bar{X}} P_Y P_X|S\rangle| \cdot \cos\theta
\end{aligned}
\quad \text{(Eq. 50)}
$$

The interference term can be zero, positive or negative. If the events are compatible, this means that events can be expressed with a shared basis φ;

- Event (decision) X spans in $\varphi_X \subseteq \varphi$
- Event (decision) Y spans in $\varphi_Y \subseteq \varphi$

Consequently, the conjunction of events is not affected by the order of projection, $P_X P_Y = P_Y P_X$. However, concepts become entangled because of the interaction. The superposition of system state should include the states of conjunction $A \cap B$ and the disjunction $A \cup B$. On the contrary, non-commutative events (decisions) can result in conflict rather than sharing, depending on the event order.

The background of this study involves communication. Communication and language are important because of the interactive nature of human decision-making. The representation of the utterances and cues in Hilbert space allow developing models of contextual interaction.

Suppose that agents of the situation in Figure 31 initiate communication from A_2 to A_1 to better the shared awareness and eventually the shared decision (Figure 32).

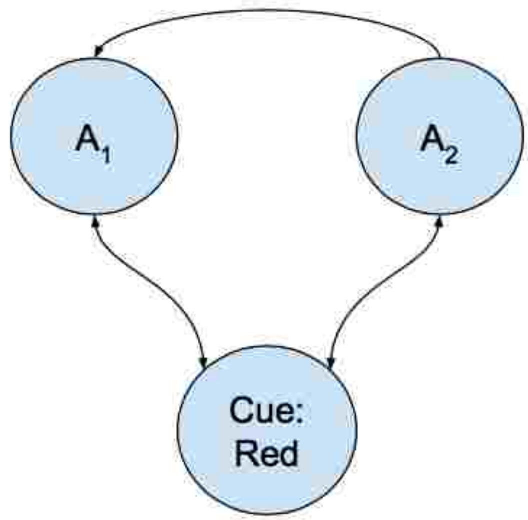

Figure 32 Communicating agents attain a shared decision on a simultaneously accessible phenomenon

In this case, agents are interlocutors, and A_2 has the intent to initiate a verbal exchange. The illocutionary force in the locution generates an impetus to either alter understanding or to elucidate the context of the locution of the utterance. In the context of Hilbert space representation, the locution from A_2 to A_1 is formed within the context of A_2, however, is construed in the context of A_1.

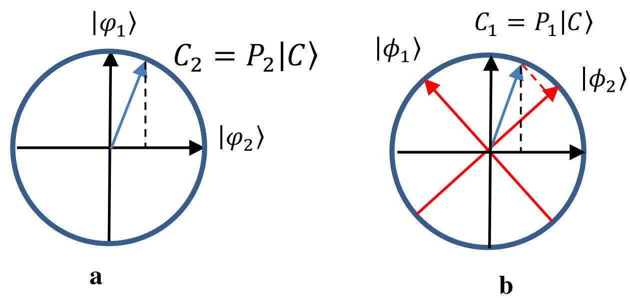

Figure 33 Representation of the utterance of a phenomenon. The formation of the utterance is in the context of A_2. However, it is construed in the context of A_1

The same vector $|C\rangle$ is expressed with two different subspace bases. For the A_1 the basis is

$$|C_1\rangle = a_1|\phi_1\rangle + a_2|\phi_2\rangle.$$

$$P_1 = |\phi_1\rangle\langle\phi_1| + |\phi_2\rangle\langle\phi_2| \quad \text{(Eq. 51)}$$

The basis of the A_2 is

$$|C_2\rangle = b_1|\varphi_1\rangle + b_2|\varphi_2\rangle.$$

$$P_2 = |\varphi_1\rangle\langle\varphi_1| + |\varphi_2\rangle\langle\varphi_2| \quad \text{(Eq. 52)}$$

The interference effect can occur in communication in many different ways. The orders of the locution utterance and the order of actions all constrain the construing process because of the high contextuality. Consequently, a model of human behavior or decision aid tools should include the effect of interference in attaining shared awareness. As introduced earlier, interference can have a negative or positive influence. As a result, a certain order of communication can prompt unexpected outcomes. The effect of communication can be expressed in two ways. By using the transformation operators, one can write the new other than self as:

$U_{12}A_1 = U_{12}P_1|C\rangle$

$$U_{12} = |\varphi_1\rangle\langle\phi_1| + |\varphi_2\rangle\langle\phi_2| \qquad \text{(Eq. 53)}$$

When the P_1on the state vector of the cue $|C\rangle$:

$$P_1|C\rangle = a_1|\phi_1\rangle + a_2|\phi_2\rangle \qquad \text{(Eq. 54)}$$

After receiving a stimulus from the agent A_2, the agent A_1 demonstrate a desire to change the context of the understanding according to the agent A_2:

$$\begin{aligned} U_{12}|C_1\rangle &= (|\varphi_1\rangle\langle\phi_1| + |\varphi_2\rangle\langle\phi_2|)(a_1|\phi_1\rangle + a_2|\phi_2\rangle) \\ &= \langle\phi_1|a_1|\phi_1\rangle|\varphi_1\rangle + \langle\phi_1|a_2|\phi_2\rangle|\varphi_1\rangle + \langle\phi_2|a_1|\phi_1\rangle|\varphi_1\rangle + \langle\phi_2|a_2|\phi_2\rangle|\varphi_2\rangle \end{aligned} \qquad \text{(Eq. 55)}$$

Because of the orthogonality, the second and third terms give zero inner product. Hence:

$$U_{12}P_1|C\rangle = |C_{12}\rangle = \underbrace{a_1'}_{\substack{new \\ amplitute}} |\varphi_1\rangle + \underbrace{a_2'}_{\substack{new \\ amplitute}} |\varphi_2\rangle. \qquad \text{(Eq. 56)}$$

This equation is illustrated in Figure 34. Even though a transformation occurred as depicted in Figure 34c, the projections are still different. This requires an additional projection on $|C_{12}\rangle$.

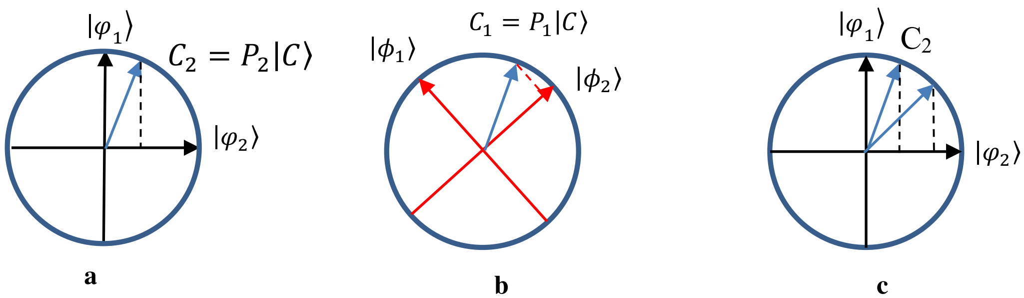

Figure 34 Impacts of communication the context (a basis that spans in the subspace of N-dimensional Hilbert Space). The angle between the vector $|\boldsymbol{C}\rangle$ and A_1 basis changes and the context become more aligned. Even though both contexts become based on the same source, the projections of the vector are different. (c) represents the transformed basis. However, because of the different amplitudes there is still disagreement.

The difference between two vectors in Figure 34c, can be represented as;

$$|D\rangle = |C_2\rangle - |C_{12}\rangle. \qquad \text{(Eq. 57)}$$

the difference between two vectors in Hilbert space can be expressed as:

$$d = \left||C_2\rangle - |C_{12}\rangle\right| \qquad \text{(Eq. 58)}$$

By using the inner product, one can have

$$d = (\langle C_2| - \langle C_{12}|, |C_2\rangle - |C_{12}\rangle)$$

$$d = \sqrt{\langle C_2|C_2\rangle - \langle C_2|C_{12}\rangle - \langle C_{12}|C_2\rangle + \langle C_{12}|C_{12}\rangle} \qquad \text{(Eq. 59)}$$

Since C_{12} and C_2 are not orthogonal vectors, the inner product is different than zero.

Assume that both vector are aligned with different basis vectors:

$$|C_2\rangle = |\varphi_2\rangle$$

$$|C_{12}\rangle = |\varphi_1\rangle. \qquad \text{(Eq. 60)}$$

In this case the distance between two vector;

$$d = \sqrt{\underbrace{\langle C_2|C_2\rangle}_{1} - \underbrace{\langle C_2|C_{12}\rangle}_{0} - \underbrace{\langle C_{12}|C_2\rangle}_{0} + \underbrace{\langle C_{12}|C_{12}\rangle}_{1}} \qquad \text{(Eq. 61)}$$

then,

$$d = \sqrt{2}. \qquad \text{(Eq. 62)}$$

This means two individual having a non-degenerate superposition vectors, will have higher disagreement then:

$$d' = \sqrt{\underbrace{\langle C_2|C_2\rangle}_{1} - \underbrace{\langle C_2|C_{12}\rangle}_{\neq 0} - \underbrace{\langle C_{12}|C_2\rangle}_{\neq 0} + \underbrace{\langle C_{12}|C_{12}\rangle}_{1}}, \qquad \text{(Eq. 63)}$$

$$0 < d' < \sqrt{2}.$$

This situation can be illustrated as in Figure 34c.

To further explore the shared awareness in this representation in a situation where C_{12} and C_2 have different likely event outputs, as in Figure 34c. One of the agents (or both) willingly changes the awareness vector, as described in Figure 34.

- This can result in complete overlap with of both agents, or
- One of the agents (or both of the agents) shifts its vectors in a way that the likely outcome projection will be the same as in Figure 35.

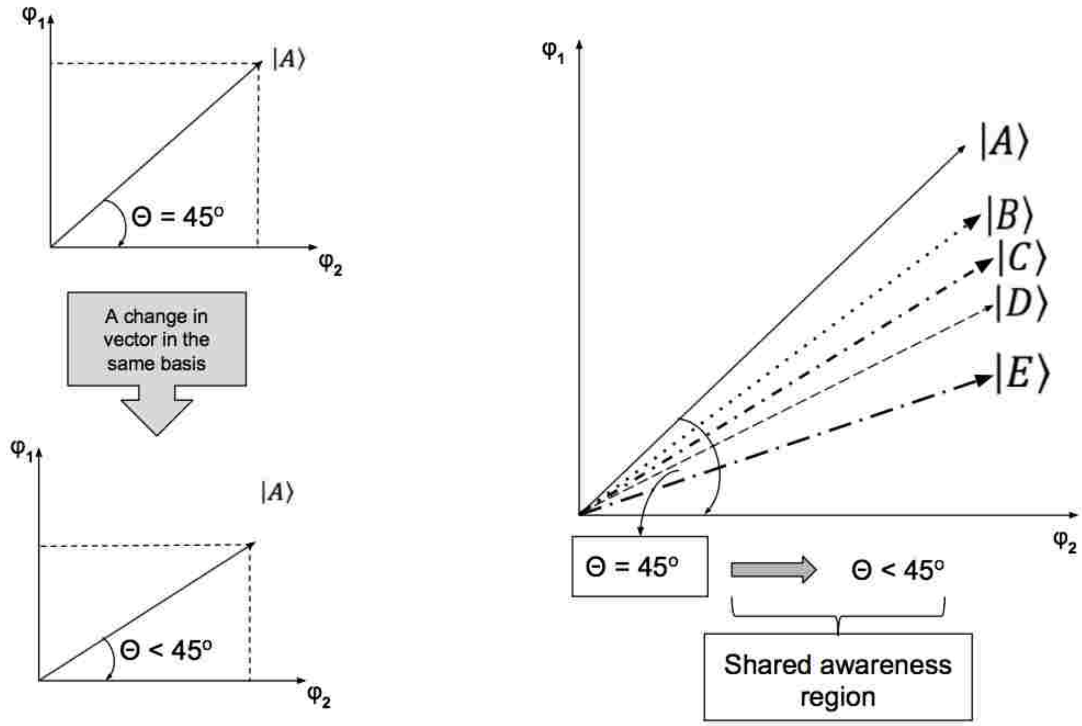

Figure 35 Geometrical representation of shared awareness. Since the outcome is represented by the projection of the state vector to the basis, the projection is on the desired outcome and sharing is attained.

Consider a group of five agents. Agents B, C, D and E have a projection on φ_1. Agent A has an equal projection for two of the basis. This group has a desire for the outcome φ_1, and after communication with other agents, A recognizes that the group members are different and that a group action is desired. Shared awareness can be acquired based on the goal attainment objective. Agent A does not need to agree (overlap with others), yet can acquire shared awareness. The desire to goal attainment can result in a move toward to the φ_2 bases. When the angle between the vector A and φ_2 becomes less than 45^0, the agent A will be in the region where the projection of the action is φ_2. Therefore, a model based on this geometrical projection approach will allow modeling shared awareness in terms of area of sharing rather than limiting cases of symbolism or associationism. Rather than a symbolic or associationist sharing awareness representation, an area of awareness can be sufficient to attain a shared awareness to acquire a shared decision making.

Any further strategy can be built on this vector difference to ameliorate the shared awareness. In this simple situation, there are two types stimuli, the cue from the object and the locution imparted from the agent A_2 to A_1. In the first case, an awareness of the situation is developed, and after the interaction, the developed awareness is updated.

The RDP construct, introduced earlier in Section 3.4.2 recognizes this fact with a domain of awareness that supports multiple perspectives. A desire to fit in the group, a desire to act in an organization, or a desire to adapt the norms of a new society can be expressed in this type of change. The change can also be at the micro and/or macro level. Micro level changes and the desire to share are related to the generative process of the individual, economic status, immigration status, and education status. Macro-level changes are governed by ideologies and organizational cultures.

5.3.2 Combined Concept

As introduced earlier in Section 4.1.9, Theory Concept, concept combination can be modeled in a contextual way other than the traditional cognitive representations. This is considered a contextual combination. Consequently, in describing a situation, and in the case of contextual interaction, attaining shared awareness recognizes proper contextual modeling.

Concept combination constrains human reasoning in developing awareness in two ways:

- Already combined concepts, or
- Emergent concepts.

The influence of the interference through concept combination is different than the order effects in the communication. To demonstrate this effect (Diederik Aerts, Sandro Sozzo, & Tomas Veloz, 2015b) introduced two types of reasoning:

> *"human reasoning is a specifically structured for a superposition of two processes: a logical reasoning and emergent reasoning. Logical reasoning combine entities, and emergent reasoning is about the formation of combined cognitive entities as newly emerging entities."(p. #)*

The discussion of these two reasons requires the use of a special Hilbert space type called Fock space. Fock space is direct sum of tensor product of concepts of single Hilbert space. This construct recognizes a specific state for interference term, which can represent either conjunction $X \cap Y$ or disjunction $X \cup Y$.

The superposition of the state system in this situation can be expressed as:

$$|X \ \cap \ Y\rangle = x|X\rangle + \ y|Y\rangle + \frac{xy}{\sqrt{2}}(|X\rangle + \ |Y\rangle) \qquad \text{(Eq. 64)}$$

The $|X\rangle + \ |Y\rangle$ is entangled state and is not decomposable. This is a result of interaction between the entities. The projection of $|X \ \cap \ Y\rangle$ on to the Fock space provides the associated weighting factor of each state to a event. The outcome of the projection (Diederik Aerts et al., 2015b);

$$\gamma\mu(X)\mu(Y) + \alpha(\frac{\mu(X) + \mu(Y)}{2} + \Re\langle X|\mu|Y\rangle) \qquad \text{(Eq. 65)}$$

The term $\Re\langle X|\mu|Y\rangle$ represents the interference term. The solution for this equation is attainable only:

$$\langle X|\mu|Y\rangle \begin{cases} \sqrt{1-\mu(X)}\sqrt{1-\mu(Y)}\cos\theta & if \ (\mu(X) + \mu(Y)) > 1 \\ \sqrt{\mu(X)}\sqrt{\mu(Y)} \ \cos\theta & if \ (\mu(X) + \mu(Y)) < 1 \end{cases} \qquad \text{(Eq. 66)}$$

This result demonstrates that two separate entities can attain a shared decision by ascribing an identical meaning to the action.

5.3.3 Entanglement and Spooky Action at a Distance

Representing a phenomenon is the primary constraint in cognitive science and mathematical modeling. The discussed three levels of representation complement each other. The geometrical representation, introduced by Gardenfors (2004), demonstrates that a geometric perspective of concepts is attainable. In the quantum theory, the geometric perspective in Hilbert space, shown in Figure 25, allows expressing the concepts and associates that contribute to the meaning of the concept by forming a superposition of basis vectors. This superposition vector includes all the possible meaning of a phenomenon, e.g. a word, in the context of all associates. For example,

$$|KFC\ Sandwich\rangle = c_1|chicken\rangle + c_2|cheese\rangle + c_3|hungry\rangle + c_4|lettuce\rangle + c_5|cheap\rangle + c_6|one - dollar\ bil\rangle + \cdots c_n|State_n\rangle \quad \text{(Eq. 67)}$$

The word sandwich constitutes these entire possible contexts. The meaning of democracy can be activated by not only targeting "sandwich" but also the associated context (Nelson & McEvoy, 2007);

> *"the activation at a distance rule assumes that the target is, in quantum terms, entangled with its associates because of learning and practicing the language in the world. Associative entanglement causes the studied target word to simultaneously activate its associated structure". (p. #)*

Shared awareness in this context is interesting. Sometime ago, KFC broadcasted a commercial for a “99” cent sandwich. The sandwich has all the regular ingredients in the picture. However, there is a small one-dollar bill picture in the lettuce part of the sandwich.

The awareness for cheap food notion is activated through this incorporated image, which generates concept combination through an unintentional interaction. The associates *hungry* and *cheap* can generate shared awareness to purchase KFC sandwich.

Studying interaction between a commander and subordinate can better the notion of entanglement in attaining shared awareness.

The concept activation is important to attain a change in the context basis vectors as depicted in Figure 34. The activation of specific for a concept can be achieved by activating the associated in the superposition. The emergent situation in attaining shared awareness can be modeled in this way.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.1 Summary

This study presented a retrospective overview of the approaches to attain shared awareness and introduced a geometric representation of shared awareness in cognitive dimension. The difference between simple and complex situations is articulated by providing an approach base comparison. Situation theory uses the principles of set theory, which fails to recognize the causes of paradoxes in complex situations. Mathematical modeling of simple situations uses the principles of situation theory. From the information and communication perspective, situation theory introduces a foundational ontological construct. However, because of the incomplete theoretical foundation, the introduced construct fails to understand the constraints of a complex situation, the relations among it, and relations among the entities of the situation. The transition to complex situations with the same principles that are used in the situation theory provides an incomplete picture. A complex situation approach introduces a fallibility component in order to recognize the sources of a true-false dichotomy. This fallibility is part of the complexity definition, which is later discussed as Pragmatic Idealism. Projecting the ontological construct of the situation theory environment on to the pragmatic idealism conceptual model, the cognitive dimension of the information environment is re-conceptualized. The mathematical foundation of the relations among the three dimensions of the information environment is introduced. Objectivity and subjectivity are discussed in this complex situation construct in two folds: epistemic and ontological. A conceptual model, the sense of subjectivity and objectivity, is discussed regarding reality domain and perspective.

In this conceptual model, a domain of awareness is introduced. Awareness of the entities in the situation is described regarding this abstract construct. The ensuing domain of awareness supports multiple perspectives and understanding become contingent on the comprehensibility of this domain. Consequently, a model of comprehensibility, understanding, and complexity become attainable with reality domain and perspective abstraction. A brief overview of the generative process is provided, and the influence of various components of the generative process is discussed. At the confluence of a situation and awareness, a theoretical model of human selectiveness and limited capacity attention become noticeable, which is called a situation awareness model. A retrospective and comparative discussion of situation awareness models provides the evolution of situation awareness models. In understanding shared awareness, a comparison between process and state base models becomes important. The interaction between stimulus and the phases of situation awareness model requires improvements to characterize and express shared awareness. The impact of stimulus to the cognitive system is articulated to demonstrate the influence of communication as an interaction in attaining shared awareness. Shared awareness is studied in the situation awareness community. Existing approaches to shared awareness are limited and confined to the information and physical dimensions. Consequently, discussions of shared awareness are limited at the substance level and the cognitive aspect of shared awareness is oversimplified or even neglected. The ensuing understanding of the shared awareness is limited at the consequence level, and mental models are not accessible through classical approaches. Shared action or shared decisions are measured and described as shared awareness. However, measured phenomena are the consequence of the shared awareness regardless of the complexity of the situation. Hence, neglected cognitive representation of shared awareness results in incomplete shared awareness construct. One indication is that there are numerous attempts to name the shared awareness to describe the emerging shared phenomenon. The failure of the existing approaches in modeling non-compatible events is demonstrated, and an alternative method is discussed.

Communication is an essential tool to extract knowledge and understanding of others. The discussed concept of pragmatic idealism incorporates the notion complexity to the communication model by coupling the pragmatic idealism to all interlocutors. The speech act perspective of communication model, illocutionary force, illocution, and per-locution can be expressed with introduced mathematical models. Communication is important to enhance the quality of awareness models of humans. This becomes paramount important in human augmentation and designing the hybrid team models.

Modeling human behavior, understanding, and language is the subject of the cognitive science. Two goals of cognitive science, explanatory and constructive goals, are discussed regarding two representations: symbolism and associationism. The limitations of these two representations are articulated about complex situations. A third representation approach, conceptual space representation, however, provides a thought-provoking representation alternative. The similarities and complementarities between the concept of a domain in a conceptual space and the domain of awareness are discussed. Another contribution of the conceptual space representation is the concept combination. An analysis of concept combination in ecological view of concepts is articulated in this geometrical representation.

Since the set theory based classical approaches fail to provide a comprehensive explanation, as a new approach, the advantages and contribution of quantum cognition are articulated. The mind dependent construction of reality and the superposition of indefinite states are used to provide an explanation to a long time debated anomalies. A comparison between classical and quantum probability theory is provided to illustrate differences between two approaches. The Hilbert space is discussed. The significance of this abstract vector space is given briefly. The vector algebra of ket and bra is articulated. A Hilbert space representation of phenomena, attitude and decision-making, context and interaction phenomenology is described. Measurement in quantum mechanics is contextual. Projection operators describe this contextuality and social examples of these are articulated.

Another important contribution of quantum probabilities to the understanding of the complex social phenomenon is to be able to express interference effects. The interference effect and its expression in vector algebra are discussed.

A theory of concept, entanglement, states, context, and tensor products of vectors in Hilbert space is discussed. In this theory of concept, concept combination and entanglement is detailed with an interaction aspect.

The dissertation introduced a Hilbert space representation of a situation. This is important to describe the self and other than self in the situation. Describing the situation in N-dimensional Hilbert space renders this construct possible. An important aspect of shared awareness is the stimulus. The interaction among the entities in the situation can be of different types. Human-ecology and human-to-human interactions are among the types of the interactions. This dissertation is concerned with human-to-human interaction and its impetus in attaining shared awareness. A shared phenomenon is exemplified, and a state of awareness of each entity in this situation is iterated. Shared awareness model must be comprehensive enough to be able to distinguish conjunction and disjunction effects.

The developed theoretical construct demonstrates the implementation of unitary transformation after initial projection of the understanding of the other. The ensuing transformation provides a framework to quantify the difference between two or more domains.

An illustrated construct of shared awareness is group action introduced as a geometrical representation of shared awareness. The introduced formalism defines an awareness region in the Hilbert space. The significance of this desired region is the projectability of individual awareness on the desired objectives.

Concept combination is a feature that constrains the notion of shared awareness. Existing situation models and shared awareness model cannot provide a representation of conjunction and disjunction events with a dynamic feature. This concept combination allows studying emergence, holistic views, and systems thinking with a cognitive representation. The introduced interaction phenomenology in Hilbert space describes entanglement with interaction. Entanglement is crucial in studying shared awareness because entangled states cannot be decomposed. The ensuing superposition of state after an interaction includes entangled states. This superposition can include various meaning activation process, which may result in a classically unprecedented meaning of awareness. This is discussed with spooky action at a distance.

6.2 Future Research

Mind dependent construction of a situation is necessary for numerous paradigms. Especially in developing a cyber situation, the introduced Hilbert space construct can have profound implications. Cyber situation awareness in cyber and information operations can be modeled in this approach. The design of hybrid team requires a contextual sensitive shared mental models. A machine with a contextual sensitivity can become the necessary component of the decision aid tools. Promising developments in artificial intelligence renders the rebel agents and explainable

AI (XAI) possible. The contextuality of quantum mathematical principles can enhance the state of the art machines. Designing agile teams with these reasoning machines should include contextuality and conceptual algorithm. This could render the rebel agent and XAI manageable by adjusting the context.

Human augmentation in information environment with ontological improvement in cognitive dimension is an important step in the possible future research. This will better cyberspace dependent interoperability in organizations.

Next generation organization theory requires the inclusion of this augmentation because shared awareness among the heterogeneous members of a hybrid team is indispensable.

The representation of contextuality in this approach can enhance the modeling endeavors in risk studies. For example, representation of heuristics such as availability, anchoring, and representativeness in Hilbert space will improve the probability space of the model. Incorporating a geometric probability in risk will replace the event set with the event space. Information operation's target audience modeling will be significantly improved with the incorporation of quantum principles. For example, framing that is employed in mess media, social media, and influence of these phenomena in faction formation can be the subject of the future research. Hilbert space formalism provides the foundation for these type of modeling of social dynamics. In this regard, conflict resolution can also be studied with this approach. The introduced concept of RDP and domain of awareness recognizes the causes of subjectivity. For example, two cultures may share the phenomenon; however, construing can give two conflicting meanings. If this is not recognized properly, the solution cannot be established.

The existence of Google, social media, and various other platforms renders the information accessibility quicker and easier than ever before. The ensuing improved accessibility can diminish the accuracy of the information, which can include misinformation and disinformation. To identify the possible consequences of misinformation and disinformation all three dimensions, information, cognitive, and physical should be invariantly modeled. Hilbert space and Fock space constructs can improve the implication space with a compatible model. Misinformation and disinformation can generate aporia (~ impasse), which can be difficult to overcome with traditional reasoning methods. Negation and abduction become the necessary reasoning to resolve the aporia. Abductive reasoning can be overcome by developing counterfactual and as a result, the shared awareness can ameliorate, and conflict can be mitigated.

The conjunction and disjunction effects may constrain cyber security of critical infrastructure. Critical infrastructures rely on artificial intelligence on machine learning. This introduces cyber security issues. Algorithms that ignore conjunction and interference effects may become vulnerable to cyber-attack.

Humans rely on information. Information has infinite potential because meaning is arbitrary. To improve the benefits of this potential, compatible theories should be developed.

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