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r-partite self-complementary and almost self-complementary r-uniform hypergraphs

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Abstract

A hypergraph H is said to be r-partite r-uniform if its vertex set V can be partitioned into non-empty sets $V_1, V_2, ..., V_r$ so that every edge in the edge set $E(H)$, consists of precisely one vertex from each set V_i, $i = 1, 2, \ldots, r$. It is denoted as $H^r(V_1, V_2, \ldots, V_r)$ or $H^r_{(n_1,n_2,\ldots,n_r)}$ if $|V_i| = n_i$ for $i = 1, 2, \ldots r$. In this paper we define r-partite self-complementary and almost self-complementary r-uniform hypergraph. We prove that, there exists an r-partite self-complementary r-uniform hypergraph $H^r(V_1, V_2, \ldots, V_r)$ where $|V_i| = n_i$ for $i = 1, 2, \ldots, r$ if and only if at least one of $n_1, n_2, \ldots, n_r$ is even. And we prove that, there exists an r-pasc $H^r(V_1, V_2, \ldots, V_r)$ where $|V_i| = n_i$ for $i = 1, 2, \ldots, r$ if and only if $n_1, n_2, \ldots, n_r$ are odd. Further, we analyze the cycle structure of complementing permutations of r-partite self-complementary r-uniform hypergraphs and r-partite almost self-complementary r-uniform hypergraphs.

Keywords: r-partite r-uniform hypergraph; r-partite self-complementary r-uniform hypergraph; r-partite almost self-complementary r-uniform hypergraph; Complementing permutation

1. Introduction

Let V be a finite set with n vertices. By $\binom{V}{k}$ we denote the set of all k-subsets of V. A k-uniform hypergraph is a pair $H = (V; E)$, where $E \subset \binom{V}{k}$. V is called a vertex set, and E an edge set of H. Two k-uniform hypergraphs $H = (V; E)$ and $H' = (V'; E')$ are isomorphic if there is a bijection $\sigma : V \to V'$ such that σ induces a bijection of E onto E'. If $H = (V; E)$ is isomorphic to $H' = (V; \binom{V}{k} - E)$, then H is called a self-complementary k-uniform hypergraph. Every permutation $\pi : V \to V$ which induces a bijection $\pi' : E \to \binom{V}{k} - E$ is called a self-complementing permutation.

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A. Symański, A. P. Wojda [1–3] and S. Gosselin [4], independently characterized n and k for which there exist k-uniform self-complementary hypergraphs of order n and gave the structure of corresponding complementing permutations.

A k-uniform hypergraph $H = (V; E)$ is called almost self-complementary if it is isomorphic with $H' = (V; \binom{V}{k} - E - \{e\})$ where e is an element of the set $\binom{V}{k}$. Almost self-complementary k-uniform hypergraph of order n may be called self-complementary in $K_n^k - e$. The almost self-complementary 2-uniform hypergraphs i.e. almost self-complementary graphs are introduced by Clapham in [5]. In [6], almost self-complementary 3-uniform hypergraphs are considered. And in [7], Wodja generalized corresponding results of [5] for $k = 2$ and of [6] for $k = 3$ for any $k \geq 2$.

A hypergraph H is said to be **r-partite r-uniform** [8] if its vertex set V can be partitioned into non-empty sets $V_1, V_2, \ldots, V_r$ so that every edge in the edge set $E(H)$, consists of precisely one vertex from each set V_i, $i = 1, 2, \ldots, r$. It is denoted as $H^r(V_1, V_2, \ldots, V_r)$ or $H^r_{(n_1,n_2,\ldots,n_r)}$ if $|V_i| = n_i$ for $i = 1, 2, \ldots, r$. An r-partite r-uniform hypergraph H with the vertex set $V = \bigcup_{i=1}^r V_i$, $V_i \cap V_j = \phi$, $\forall\, i \neq j$ and the edge set $E = \{e : e \subset V, |e| = r$ and $e \cap V_i \neq \phi$, for $i = 1, 2, \ldots, r\}$ is called **a complete r-partite r-uniform hypergraph**. It is denoted as $K^r(V_1, V_2, \ldots, V_r)$ or $K^r_{(n_1,n_2,\ldots,n_r)}$. We observe that, the total number of edges in $K^r_{(n_1,n_2,\ldots,n_r)}$ is $\prod_{i=1}^r n_i$. Given an r-partite r-uniform hypergraph $H = H^r(V_1, V_2, \ldots, V_r)$, we define its **$r$-partite complement** to be the r-partite r-uniform hypergraph $\bar{H} = \bar{H}^r(V_1, V_2, \ldots, V_r)$ where $V(\bar{H}) = V(H)$ and $E(\bar{H}) = E(K^r(V_1, V_2, \ldots, V_r)) - E(H)$.

We say $\bar{H}$ is the complement of H with respect to $K^r(V_1, V_2, \ldots, V_r)$. An r-partite r-uniform hypergraph $H = H^r(V_1, V_2, \ldots, V_r) = H^r(V)$ is said to be **self-complementary** if it is isomorphic to its r-partite complement $\bar{H} = \bar{H}^r(V_1, V_2, \ldots, V_r) = \bar{H}^r(V)$, that is there exists a bijection $\sigma : V \to V$ such that e is an edge in H if and only if $\sigma(e)$ is an edge in $\bar{H}$.

T. Gangopadhyay and S. P. Rao Hebbare [9] studied bi-partite self-complementary graphs, i.e. 2-partite self-complementary 2-uniform hypergraphs (r=2). They characterized the structural properties of bi-partite complementing permutations. In the present paper we study r-partite self-complementary r-uniform hypergraphs and r-partite almost self-complementary r-uniform hypergraphs.

In Section 2, we prove the existence of r-partite self-complementary r-uniform hypergraphs. In Section 3, we prove the existence of r-partite almost self-complementary r-uniform hypergraphs. In Sections 4 and 5 we analyze the cycle structure of complementing permutations of r-partite self-complementary r-uniform hypergraphs and the cycle structure of complementing permutations of r-partite almost self-complementary r-uniform hypergraphs respectively.

We use the shortform "r-psc" for r-partite self-complementary r-uniform hypergraph.

2. Existence of r-partite self-complementary r-uniform hypergraphs

The concept of an r-partite self-complementary r-uniform hypergraph with partition $(V_1, V_2, \ldots, V_r)$ of vertex set V can be interpreted as a partitioning of the edge set of $K^r(V_1, V_2, \ldots, V_r)$ into two isomorphic factors. We note that a partitioning of the edge set of $K^r(V_1, V_2, \ldots, V_r)$ into two isomorphic factors is possible only if $K^r(V_1, V_2, \ldots, V_r)$ has an even number of edges i.e. $\prod_{i=1}^r n_i$ is even and this is true if and only if at least one of $n_1, n_2, \ldots, n_r$ is even. Conversely if we can construct an r-psc given that at least one n_i is even then we get a necessary and sufficient condition for existence of r-psc. Towards this we have the following theorem.

Theorem 2.1. *There exists an r-psc $H^r(V_1, V_2, \ldots, V_r)$ where $|V_i| = n_i$ for $i = 1, 2, \ldots, r$ if and only if at least one of $n_1, n_2, \ldots, n_r$ is even.*

Proof. Firstly we construct an r-psc $H^r(V_1, V_2, \ldots, V_r)$ given that at least one of $|V_i| = n_i$, $i = 1, 2, \ldots, r$ is even. Without loss of generality, let us suppose that n_1 is even. That is $n_1 = 2t$ for some positive integer t (say).

Let $V_i = \{u^i_1, u^i_2, \ldots, u^i_{n_i}\}$ for $i = 1, 2, \ldots, r$. Consider the complete $(r-1)$-partite $(r-1)$-uniform hypergraph, $K^{r-1}(V_2, V_3, \ldots, V_r) = K^{r-1}_{n_2,n_3,\ldots,n_r}$.

Consider the following partition of edge set of $K^r_{n_1,n_2,n_3,\ldots,n_r}$.

$E = \{e \cup \{u^1_j\} \mid e$ is an edge in $K^{r-1}_{n_2,n_3,\ldots,n_r}$ and $j = 1, 3, \ldots, 2t-1\}$

$\bar{E} = \{e \cup \{u^1_j\} \mid e$ is an edge in $K^{r-1}_{n_2,n_3,\ldots,n_r}$ and $j = 2, 4, \ldots, 2t\}$.

Let $H = H^r(V_1, V_2, \ldots, V_r)$ be the r-partite r-uniform hypergraph with edge set E. Fig. 1 gives a diagrammatic description of H. To prove that H is r-psc, we define a bijection $\sigma : V(H) \to V(H)$ as $\sigma = \prod_{i=1}^r (u^i_1\ u^i_2 \ldots u^i_{n_i})$. It can be easily checked that $H = H^r(V_1, V_2, \ldots, V_r)$ is self-complementary with σ as its complementing permutation. □

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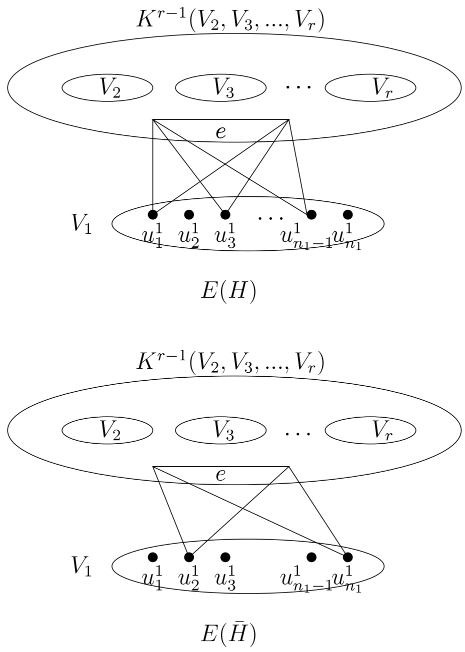

Fig. 1. Edge set of $H^r_{(n_1,n_2,\ldots,n_r)}$ and $\bar{H}^r_{(n_1,n_2,\ldots,n_r)}$.

It is clear that the partitioning of the edge set of $K^r(V_1, V_2, \ldots, V_r)$ into two isomorphic factors is not possible when $K^r(V_1, V_2, \ldots, V_r)$ has an odd number of edges. In the next section we define an r-partite almost self-complementary r-uniform hypergraph and give a condition on number of vertices for its existence.

3. Existence of r-partite almost self-complementary r-uniform hypergraphs

Definition 3.1 (*Almost Complete r-partite r-uniform Hypergraph*)**.** The hypergraph $\tilde{K}^r_{(n_1,n_2,\ldots,n_r)} = K^r_{(n_1,n_2,\ldots,n_r)} - e$ is called an almost complete r-partite r-uniform hypergraph where e is an edge in $K^r_{(n_1,n_2,\ldots,n_r)}$ called the deleted edge. Vertices of e will be called the special vertices.

Definition 3.2 (*Almost Self-complementary r-partite r-uniform Hypergraph*)**.** An r-partite r-uniform hypergraph $H(V_1, V_2, \ldots, V_r)$ such that $|V_i| = n_i$ for $i = 1, 2, \ldots, r$ is almost self-complementary if it is isomorphic with its complement $\bar{H}(V_1, V_2, \ldots, V_r)$ with respect to $\tilde{K}^r_{(n_1,n_2,\ldots,n_r)}$.

We use the shortform "r-pasc" for r-partite almost self-complementary r-uniform hypergraph.

A complete r-partite r-uniform hypergraph will have an odd number of edges if each of $n_1, n_2, \ldots, n_r$ is odd. In the next theorem we prove that this condition is sufficient as well for the existence of an r-pasc. The proof is constructive in nature. Since the special vertices are to be treated differently and since each set in the partition contains exactly one special vertex, we start with $V_i = V_i' \cup \{x_i\}$ such that V_i' contains even number of vertices for $i = 1, 2, \ldots, r$. To construct r-pasc, we consider the complete $r-1$-partite $r-1$-uniform hypergraph on $V_2, V_3, \ldots, V_r$ and then add each vertex of V_1 on each of these edges in such a way that we get the desired construction. The idea is the same as in the construction of Theorem 2.1.

Theorem 3.3. *There exists an r-pasc $H^r(V_1, V_2, \ldots, V_r)$ where $|V_i| = n_i$ for $i = 1, 2, \ldots, r$ if and only if $n_1, n_2, \ldots, n_r$ are odd.*

Proof. We construct an r-pasc $H^r(V_1, V_2, \ldots, V_r)$ where $|V_i| = n_i$ for $i = 1, 2, \ldots, r$ when each n_i is odd. Let $n_i = 2t_i + 1$, for some positive integer t_i, for $i = 1, 2, \ldots, r$.

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Let $V_i' = \{u_1^i, u_2^i, \ldots, u_{2t_i}^i\}$ and $V_i = V_i' \cup \{x_i\}$, for $i = 1, 2, \ldots, r$.

Consider the complete $r-1$-partite $r-1$-uniform hypergraph $K^{r-1}_{(n_2,n_3,\ldots,n_r)}$ with edge set $E(K)$.

Consider the following partition of the edge set of $\tilde{K}^r_{(n_1,n_2,n_3,\ldots,n_r)}$, where $\{x_1, x_2, \ldots, x_r\}$ is the deleted edge.

$E_1 = \{e \cup \{u_i^1\} \mid e \in E(K),\ i = 1, 3, \ldots, 2t_1 - 1\}$,
$\bar{E}_1 = \{e \cup \{u_i^1\} \mid e \in E(K),\ \ i = 2, 4, \ldots, 2t_1\}$,
$E_{x_1} = \{\{x_1, u_{p_2}^2, u_{p_3}^3, \ldots, u_{p_r}^r\} \mid u_{p_i}^i \in V_i'$, and $i = 2, 3, \ldots, r$ and $p_2 = 1, 3, \ldots, 2t_2 - 1\}$,
$\bar{E}_{x_1} = \{\{x_1, u_{p_2}^2, u_{p_3}^3, \ldots, u_{p_r}^r\} \mid u_{p_i}^i \in V_i'$, and $i = 2, 3, \ldots, r$ and $p_2 = 2, 4, \ldots, 2t_2\}$.

Now we have to consider edges containing k special vertices along with x_1 for $k = 1, 2, \ldots, r-2$. For a given k, we have to consider all combinations of k special vertices $x_{j_1}, x_{j_2}, \ldots, x_{j_k}$ from $r-1$ special vertices and remaining $r-k-1$ vertices from $V_{s_1}', V_{s_2}', \ldots, V_{r-k-1}'$ where $j_1, j_2, \ldots, j_k, s_1, s_2, \ldots, s_{r-k-1}$ are distinct and belong to the set $\{2, 3, \ldots, r\}$.

Let $(r) = \{2, 3, \ldots, r\}$. For $k = 1, 2, \ldots, r-2$, let $J_k = \{j_1, j_2, \ldots, j_k\} \subset (r)$ and $J_k' = \{s_1, s_2, \ldots, s_{r-k-1}\} = (r) \setminus J_k$. Note that for each k, there are $\binom{r-1}{k}$ several k-subsets J_k. We consider all possible k-subsets.

For given k, we divide the edges containing k special vertices along with x_1 into two parts as follows.

Let,

$E_{x_1 x_{j_1} x_{j_2} \ldots x_{j_k}} = \{\{x_1, x_{j_1}, x_{j_2}, \ldots, x_{j_k}, u_{p_{s_1}}^{s_1}, u_{p_{s_2}}^{s_2}, \ldots, u_{p_{s_{r-k-1}}}^{s_{r-k-1}}\} \mid j_1, j_2, \ldots, j_k \in J_k,\ \ x_{j_m} \in V_{j_m},\ m = 1, 2, \ldots, k;\ s_1, \ldots, s_{r-k-1} \in J_k',\ u_{p_{s_i}}^{s_i} \in V_{s_i}',\ i = 1, 2, 3, \ldots, r-k-1$, and $p_{s_1} = 1, 3, 5, \ldots, 2t_{s_1} - 1\}$.
$\bar{E}_{x_1 j_1 j_2 \ldots j_k} = \{\{x_1, x_{j_1}, x_{j_2}, \ldots, x_{j_k}, u_{p_{s_1}}^{s_1}, u_{p_{s_2}}^{s_2}, \ldots, u_{p_{s_{r-k-1}}}^{s_{r-k-1}}\} \mid j_1, j_2, \ldots, j_k \in J_k,\ \ x_{j_m} \in V_{j_m},\ m = 1, 2, \ldots, k;\ s_1, \ldots, s_{r-k-1} \in J_k',\ u_{p_{s_i}}^{s_i} \in V_{s_i}',\ i = 1, 2, 3, \ldots, r-k-1$, and $p_{s_1} = 2, 4, 6, \ldots, 2t_{s_1}\}$.

Since $k \le r-2,\ r-k-1 \ge 1$. Hence the above partition is always a valid partition.

For given k let, $E_{x_1}^k = \bigcup_{J_k \subset (r)} E_{x_1 x_{j_1} x_{j_2} \ldots x_{j_k}}$ and $\bar{E}_{x_1}^k = \bigcup_{J_k \subset (r)} \bar{E}_{x_1 x_{j_1} x_{j_2} \ldots x_{j_k}}$.

Clearly, $E_1 \cup \bar{E}_1 \cup E_{x_1} \cup \bar{E}_{x_1} \cup (\bigcup_{k=1}^{r-2} E_{x_1}^k \cup \bar{E}_{x_1}^k)$ gives a partition of the edge set of $\tilde{K}^r(V_1, V_2, \ldots, V_r)$.

Let $H = H^r(V_1, V_2, \ldots, V_r)$ be the r-partite r-uniform hypergraph with edge set $E = E_1 \cup E_{x_1} \cup (\bigcup_{k=1}^{r-2} E_{x_1}^k)$. To prove H is r-pasc we define a bijection $\sigma : V(H) \to V(H)$ such that $\sigma = \prod_{i=1}^{k}((u_1^i\ u_2^i\ \ldots\ u_{n_i-1}^i)(x_i))$. It can be easily checked that $H = H^k(V_1, V_2, \ldots, V_k)$ is almost self-complementary with σ as its complementing permutation. □

4. Complementing permutations of r-partite self-complementary r-uniform hypergraphs

Let $V = \{V_1, V_2, \ldots, V_r\}$ be a partition of V. Any edge of $K^r(V_1, V_2, \ldots, V_r)$ is a r-subset of V containing exactly one vertex from each of the partitioned sets V_i, $i = 1, 2, \ldots, r$. Hence it is of the form $e = \{u_1, u_2, \ldots, u_r\}$ where $u_i \in V_i$, $i = 1, 2, \ldots, r$. If any r-subset of V contains more than one vertex from any one of the partitioned sets then we call it as an invalid edge. Hence any r-subset (r-tuple) of vertices in V is an invalid edge if and only if it contains at least two vertices from the same set of the partition. A permutation σ on V is said to a complementing permutation of an r-psc H, if $\sigma(e)$ is an edge in $E(\bar{H})$ whenever e is an edge in H. If σ is a complementing permutation then the corresponding mapping induced on the set of edges of $K^r_{(n_1,n_2,\ldots,n_r)}$ is denoted by σ'.

Definition 4.1. Let $V = \{V_1, V_2, \ldots, V_r\}$ be a partition of V. A permutation σ_{p_i} is said to be a pure permutation on the set V_i if it is a permutation on the set V_i that can be written as a product of disjoint cycles containing all the vertices of V_i.

Definition 4.2. Let $V = \{V_1, V_2, \ldots, V_r\}$ be a partition of V. A permutation σ_{m_j} is said to be a mixed permutation on any j sets of V, $2 \le j \le r$ if it can be written as a product of disjoint cycles where each cycle contains at least one vertex from each of the j sets.

First we characterize those permutations σ on V for which $\sigma(e)$ is an edge in $K^r_{(n_1,n_2,\ldots,n_r)}$ whenever e is an edge in $K^r_{(n_1,n_2,\ldots,n_r)}$. We call such permutation as a valid permutation.

Lemma 4.3. *Let $V = \{V_1, V_2, \ldots, V_r\}$ be a partition of V and σ be a valid permutation on V. If C is a cycle of σ containing two consecutive vertices from a single set of the partition say V_i for some i, $1 \le i \le r$ then σ must contain σ_{p_i} where σ_{p_i} is a pure permutation on V_i.*

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Proof. If C is a 2-cycle then we are done.

Let $C = (u_1\ u_2\ v_1\ v_2\ v_3\ \cdots\ v_j)$, $j \geq 1$ such that $u_1, u_2 \in V_i$.

Claim 1: $v_1, v_2, \ldots, v_j \in V_i$.

Proof of 1: Suppose not. That is for at least one $k, 1 \leq k \leq j$, $v_k \notin V_i$. Choose that k for which $v_k \notin V_i$ and $\sigma(v_k) \in V_i$. This is possible since $C = (u_1\ u_2\ v_1\ v_2\ \cdots\ v_j)$, $j \geq 1$ is a cycle of σ with $u_1 \in V_i$. Now u_1, v_k belong to different sets of the partition and hence any valid edge containing u_1 and v_k will give $\sigma(u_1)$ and $\sigma(v_k)$ both belonging to V_i, thus giving an invalid edge, a contradiction to σ is valid.

Claim 2: If there exists $u \in V_i$ not belonging to the cycle C then u belongs to C' where C' is another cycle of σ disjoint from C and contains vertices only from V_i.

Proof of claim 2: Let $C' = (u\ w_1\ w_2\ \cdots\ w_k)$ where at least one of the $w_1, w_2, \ldots, w_k$ (if any such exists) does not belong to V_i. Choose w_s such that $w_s \notin V_i$ and $\sigma(w_s) \in V_i$. Such a choice is possible since $u \in V_i$ and $C' = (u\ w_1\ w_2\ \cdots\ w_k)$ is a cycle of σ. Now any valid edge containing u_1, w_s from different sets will give an invalid edge containing $\sigma(u_1), \sigma(w_s)$ from some V_i, a contradiction to σ is valid.

Hence σ contains σ_{p_i}. □

Lemma 4.4. *Let $V = \{V_1, V_2, \ldots, V_r\}$ be a partition of V and σ be a valid permutation on V. If C is a cycle of σ containing vertices from $V_1, V_2, \ldots, V_t$, $t \geq 2$ then*

(i) $C = (u_1\ u_2\ \cdots u_t\ u_{t+1}\ u_{t+2}\ \cdots u_{2t} \cdots u_{(q-1)t+1} \cdots u_{qt})$ where $u_{it+j} \in V_j$ for all $i = 0, 1, \ldots, q-1$ and $j = 1, 2, \ldots, t$.

(ii) σ must contain σ_{m_t} where σ_{m_t} is a mixed permutation on $V_1, V_2, \ldots, V_t$.

Proof. Since C is a cycle containing vertices from $V_1, V_2, \ldots, V_t$, it must have length at least t and because of Lemma 4.3 no two consecutive vertices of C belong to the same set of the partition. The first t vertices of C must be one each from $V_1, V_2, \ldots, V_t$ in some order. If not that is suppose $C = (u_1\ u_2\ \cdots\ u_t \cdots)$ such that $u_i, u_j \in V_k$ with i and j are not consecutive and u_{i-1}, u_{j-1} belong to different V_i's for $i = 1, 2, \ldots, t$ then any valid edge containing u_{i-1} and u_{j-1} will give $\sigma(u_{i-1}) = u_i$ and $\sigma(u_{j-1}) = u_j$ both belonging to V_k, giving an invalid edge, a contradiction. Without loss of generality let $C = (u_1, u_2, \ldots, u_t, \ldots)$ where $u_j \in V_j$, $j = 1, 2, \ldots, t$.

Claim 1: $C = (u_1\ u_2\ \cdots\ u_t\ u_{t+1}\ u_{t+2} \cdots u_{2t} \cdots u_{(q-1)t+1} \cdots u_{qt})$, for $q \geq 1$ where $u_{it+j} \in V_j$ for all $i = 1, 2, \ldots, q-1$ and $j = 1, 2, \ldots, t$.

Proof of claim 1: Suppose not. Let s and k be the smallest such that $u_{st+k} \notin V_k$. That is $u_{st+k} \in V_j$ for some $j \neq k$.

Case (i) Suppose $k = 1$. That is $u_{st+1} \notin V_1$. Then $u_{st+1} \in V_j$ for some $j \neq 1$. That is $u_{st+1} \in V_j$, $1 < j \leq t$. We have $u_{j-1} \in V_{j-1}$ and $u_{st} \in V_t$. Hence u_{j-1} and u_{st} belong to a valid edge but $\sigma(u_{j-1}) = u_j$ and $\sigma(u_{st}) = u_{st+1}$ both belong to V_j, a contradiction to σ is valid.

Case (ii) Suppose $k > 1$. $u_{st+k} \in V_j$, $j \geq 1$, $j \neq k$.

Suppose $j = 1$, that is $u_{st+k} \in V_1$. We have that $u_{st} \in V_t$ and $u_{st+(k-1)} \in V_{k-1}$. Thus u_{st} and $u_{st+(k-1)}$ belong to a valid edge whereas $\sigma(u_{st}) = u_{st+1}$ and $\sigma(u_{st+(k-1)}) = u_{st+k}$ both belong to V_1, a contradiction.

Suppose $j > 1$. We have $u_{j-1} \in V_{j-1}$ and $u_{st+(k-1)} \in V_{k-1}$. Thus u_{j-1} and $u_{st+(k-1)}$ belong to a valid edge but $\sigma(u_{j-1}) = u_j$ and $\sigma(u_{st+(k-1)}) = u_{st+k}$ both belong to V_j, which is a contradiction.

Hence length of C must be multiple of t.

Claim 2: Every cycle of σ containing the vertices from $V_1, V_2, \ldots, V_t, t \geq 2$ must be of the above type C with the same ordering of $V_1, V_2, \ldots, V_t$.

Proof of claim 2: Suppose σ contains a cycle $C'(\neq C)$ containing vertices from $V_1, V_2, \ldots, V_t$. Suppose for some $v_i \in V_i$ in C', $\sigma(v_i) \in V_k, k \neq i+1$ then for any edge e containing u_{k-1} in C and v_i, $\sigma(e)$ is an invalid edge, a contradiction.

Claim 3: All the vertices of $V_1, V_2, \ldots, V_t$ belong to cycles of type C.

Proof of claim 3: Suppose not. That is there is a cycle C'' in σ containing vertices from at least one of the sets $V_1, V_2, \ldots, V_t$ and vertices from $S = \{V_{t+1}, V_{t+2}, \ldots, V_r\}$. Without loss, let us suppose that C'' contain vertices from V_1 and S. Choose a vertex $u \in V_j$ where $V_j \in S$ from C'' such that $\sigma(u) \in V_1$. Clearly, $u \notin V_t$ and u_{qt} in C belongs to V_t. Any valid edge e containing u and u_{qt} gives $\sigma(e)$ to be invalid, a contradiction.

Hence σ must contain σ_{m_t}.

Further, $|V_1| = |V_2| = \cdots = |V_t| = q't$ for some $q' \geq 1$. □

From Lemmas 4.3 and 4.4 we immediately get the following theorem which characterizes all valid permutations.

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Theorem 4.5. *(i) Any valid permutation* σ *on* V *is of the form* $\sigma = \prod_{i=1}^{k}\sigma_{m_{t_i}} \prod_{j=1}^{s}\sigma_{p_j}$, *where* $\sigma_{m_{t_i}}$ *is a mixed permutation on* $V_{t_1+t_2+\cdots+t_{i-1}+1}$,
$V_{t_1+t_2+\cdots+t_{i-1}+2}, \dots, V_{t_1+t_2+\cdots+t_{i-1}+t_i}$ *for* $i = 1, 2, \dots, k$ *such that* $t_1 + t_2 + \cdots + t_k = t$ *and* $\sigma_{p_j}(V_{t+j}) = V_{t+j}$ *for* $j = 1, 2, \dots, s$ *and* $t + s = r$.
(ii) There cannot be a mixed permutation on $V_1, V_2, \dots, V_t$ *unless* $|V_1| = |V_2| = \cdots = |V_t| = qt$, *for some* $q \geq 1$.

The following remark gives a relation between the length of a cycle containing a particular edge in σ' and the lengths of cycles in σ containing the vertices of that edge where σ is a valid permutation.

Remark 4.6. Let $e' = \{u_1, u_2, \dots, u_r\}$ be any edge in $K^r_{n_1,n_2,\dots,n_r}$. Then the length of the cycle in σ' containing the edge e' is the least common multiple of the lengths of cycles in σ containing the vertices $u_1, u_2, \dots, u_r$ except for the edge which contains t vertices $u_{i_1}, u_{i_2}, \dots, u_{i_t}$ which belong to a cycle C of mixed permutation σ_m on t sets (out of $V_1, V_2, \dots, V_r$) of length qt such that $u_{i_{j+1}} = \sigma^q(u_{i_j})$ for $j = 1, 2, \dots, t$. For such an edge, length of the corresponding cycle in σ' depends on q instead of qt. Further σ will be a complementing permutation if and only if every cycle in the induced mapping σ' is of even length.

Following theorem gives the cycle structure of the complementing permutation of an r-psc.

Theorem 4.7. *A permutation* σ *is a complementing permutation of* r*-psc* $H^r(V_1, V_2, \dots, V_r)$ *if and only if following hold*
(i) σ *is valid, that is* $\sigma = \prod_{i=1}^{k}\sigma_{m_{t_i}} \prod_{j=1}^{s}\sigma_{p_j}$, *where* $\sigma_{m_{t_i}}$ *is a mixed permutation on* $V_{t_1+t_2+\cdots+t_{i-1}+1}, V_{t_1+t_2+\cdots+t_{i-1}+2}, \dots, V_{t_1+t_2+\cdots+t_{i-1}+t_i}$ *for* $i = 1, 2, \dots, k$ *such that* $t_1 + t_2 + \cdots + t_k = t$ *and* $\sigma_{p_j}(V_{t+j}) = V_{t+j}$ *for* $j = 1, 2, \dots, s$ *and* $t + s = r$.
(ii) either all the cycles in $\sigma_{m_{t_i}}$ *are of length even multiple of* t_i *for at least one* i, $i = 1, 2, \dots, k$ *or all the cycles in* σ_{p_j} *are of even length for at least one* j, $j = 1, 2, \dots, s$.

Proof. Suppose σ is a complementing permutation of an r-psc $H^r(V_1, V_2, \dots, V_r)$. Clearly, σ must be valid. From Theorem 4.5, we have that $\sigma = \prod_{i=1}^{k}\sigma_{m_{t_i}} \prod_{j=1}^{s}\sigma_{p_j}$. For convenience we denote $\sigma_{m_{t_i}}$ by σ_{m_i}.

Firstly we prove that for at least one i, $i = 1, 2, \dots, k$ either all the cycles in σ_{m_i} are of length even multiple of t_i or for at least one j, $j = 1, 2, \dots, s$ all cycles in σ_{p_j} are of even length.

Suppose not. That is for each i, $i = 1, 2, \dots, k$, σ_{m_i} contains at least one cycle of length odd multiple of t_i and for each j, $j = 1, 2, \dots, s$, σ_{p_j} contains at least one cycle of odd length. Let for each $i = 1, 2, \dots, k$, C_i be a cycle in σ_{m_i} of length odd multiple of t_i and for $j = 1, 2, \dots, s$, C'_j be a cycle in σ_{p_j} of odd length. Let length of C_i be $q_i t_i$ where q_i is odd for $i = 1, 2, \dots, k$ and length of C'_j be L'_j where L'_j is odd for $j = 1, 2, \dots, s$. Let $u_i \in C_i$, $i = 1, 2, \dots, k$ and $v_j \in C'_j$, $j = 1, 2, \dots, s$.

Consider the edge,

$$e' = \{u_1, \sigma^{q_1}(u_1), \sigma^{2q_1}(u_1), \dots, \sigma^{(t_1-1)q_1}(u_1), u_2, \sigma^{q_2}(u_2), \sigma^{2q_2}(u_2), \dots, \sigma^{(t_2-1)q_2}(u_2), \dots, u_k, \sigma^{q_k}(u_k), \sigma^{2q_k}(u_k), \dots, \sigma^{(t_k-1)q_k}(u_k), v_1, v_2, \dots, v_s\}$$

in $K^r_{(n_1,n_2,\dots,n_r)}$. The length of the cycle of σ' containing the edge e' is the least common multiple of $q_1, q_2, \dots, q_k, L'_1, L'_2, \dots, L'_s$ which is odd, a contradiction. Hence, either at least one q_i, $i = 1, 2, \dots, k$ is even or at least one L'_j, $j = 1, 2, \dots, s$ is even. Therefore, either for at least one i, $i = 1, 2, \dots, k$ all the cycles in σ_{m_i} are of length even multiple of t_i or for at least one j, $j = 1, 2, \dots, s$ all the cycles in σ_{p_j} are of even length. □

The following result proved by Gangopadhyay and S. P. Rao Hebbare [9], on the cycle structure of the complementing permutations of a bipartite self-complementary graph (2-partite self-complementary 2-uniform hypergraphs) follows from Theorem 4.7.

Corollary 4.8. *A permutation* σ *is a complementing permutation of bipartite self-complementary graph* $G(V_1, V_2)$ *if and only if either*
(i) $\sigma = \sigma_{p_1}\sigma_{p_2}$ *with all cycles in* σ_{p_1} *or* σ_{p_2} *are of even length or (ii)* $\sigma = \sigma_m$ *and every cycle of* σ_m *is of length a multiple of* 4 *and takes vertices alternately from* V_1 *and* V_2.

5. Complementing permutations of r-partite almost self-complementary r-uniform hypergraphs

Given an r-pasc H, let the edges of H be colored red and the remaining edges of $\tilde{K}^r_{(n_1,n_2,\ldots,n_r)}$ be colored green. Since the 2 factors are isomorphic, there is a permutation σ of the vertices of $\tilde{K}^r_{(n_1,n_2,\ldots,n_r)}$ that induces a mapping of the red edges onto the green edges. We consider σ as a permutation of the vertices of $K^r_{(n_1,n_2,\ldots,n_r)}$ and denote by σ' the corresponding mapping induced on the set of edges of $K^r_{(n_1,n_2,\ldots,n_r)}$. Thus σ' maps each red edge onto a green edge. However, the mapping σ' need not necessarily map each green edge onto a red edge. This would be so if σ' mapped e onto itself, but it may be that σ' maps e onto a red edge and some green edge onto e. Such a σ (which, for definiteness we shall always assume induces a mapping from red to green) will (as for r-psc) be called a complementing permutation. It will be useful to consider the cycles of the induced mapping σ'.

For the rest of the section we denote the deleted edge by $e = \{x_1, x_2, \ldots, x_r\}$ where $x_i \in V_i$ for $i = 1, 2, \ldots, r$ and call it as the missing edge. And the corresponding vertices $x_1, x_2, \ldots, x_r$ are called as the special vertices.

It is clear that the length of the cycle of σ' containing the edge $e = \{x_1, x_2, \ldots, x_r\}$ must be odd and all the other cycles of σ' must be of even length.

If σ is any permutation on V, for σ to be a complementing permutation it has to be valid and hence Theorem 4.5 holds. Remark 4.6 gives the relation between the lengths of cycles in σ and σ'. In addition an extra requirement that exactly one cycle of σ' containing the deleted edge is of odd length and all the other cycles of σ' are of even length changes the cycle structure of complementing permutation of r-pasc from that of r-psc. And we have the following theorem.

Theorem 5.1. *A permutation σ is a complementing permutation of r-pasc $H^r(V_1, V_2, \ldots, V_r)$ if and only if σ is valid, that is $\sigma = \prod_{i=1}^{k}\sigma_{m_{t_i}}\prod_{j=1}^{s}\sigma_{p_j}$ where each $\sigma_{m_{t_i}}$, $i = 1, 2, \ldots, k$ permutes vertices belonging to t_i number of sets of the partition $V_{t_1+t_2+\cdots+t_{i-1}+1}, V_{t_1+t_2+\cdots+t_{i-1}+2}, \ldots, V_{t_1+t_2+\cdots+t_{i-1}+t_i}$ and $\sigma_{p_j}(V_{t+j}) = V_{t+j}$ for $j = 1, 2, \ldots, s$ and $t + s = r$. Further, either*
(1) $\sigma = \prod_{i=1}^{k}\sigma_{m_{t_i}}\prod_{j=1}^{s}\sigma_{p_j}$ where $\prod_{j=1}^{s}\sigma_{p_j} = (x_{t+1})\cdots(x_{t+s})\prod_{\alpha}C_\alpha$, where each C_α is a cycle of even length containing vertices from a single set of the partition. And for $i = 1, 2, \ldots, k$, $\sigma_{m_{t_i}} = (x_{t_1+t_2+\cdots+t_{i-1}+1}\ x_{t_1+t_2+\cdots+t_{i-1}+2}\ \cdots\ x_{t_1+t_2+\cdots+t_{i-1}+t_i})\prod_{\beta}C_\beta$, where each C_β is a valid mixed cycle of length even multiple of t_i containing vertices from $V_{t_1+t_2+\cdots+t_{i-1}+1}, V_{t_1+t_2+\cdots+t_{i-1}+2}, \ldots, V_{t_1+t_2+\cdots+t_{i-1}+t_i}$.
or
(2) Among all the σ_{p_j}'s, $j = 1, 2, \ldots, s$, exactly one σ_{p_j} say σ_{p_1} is such that $\sigma_{p_1} = C\prod_{\gamma}C_\gamma$ where C is a cycle of odd length greater than 1 containing the vertex x_{t+1} and C_γ is a cycle of even length containing vertices from V_{t+1}. Hence $\sigma = (\prod_{i=1}^{k}\sigma_{m_{t_i}})(\prod_{j=2}^{s}\sigma_{p_j})\sigma_{p_1}$ where $\prod_{j=2}^{s}\sigma_{p_j} = (x_{t+2})\cdots(x_{t+s})\prod_{\alpha}C_\alpha$ where each C_α is a cycle of even length containing vertices from a single set of the partition. For each $i = 1, 2, \ldots, k$, $\sigma_{m_{t_i}}$ is as in (1).
or
(3) Among all the $\sigma_{m_{t_i}}$'s, $i = 1, 2, \ldots, k$, exactly one $\sigma_{m_{t_i}}$ say $\sigma_{m_{t_1}}$ is such that $\sigma_{m_{t_1}} = C\prod_{\delta}C_\delta$ where C is a valid mixed cycle on $V_1, V_2, \ldots, V_{t_1}$ having length odd multiple q_1 of t_1 ($q_1 > 1$), t_1 is even and C contains the special vertices $x_1, x_2, \ldots, x_{t_1}$ such that $\sigma^{q_1}(x_1) = x_2, \sigma^{q_1}(x_2) = x_3, \ldots, \sigma^{q_1}(x_{t_1-1}) = x_{t_1}, \sigma^{q_1}(x_{t_1}) = x_1$ and all other vertices from $V_1, V_2, \ldots, V_{t_1}$. Each C_δ is a valid mixed cycle on $V_1, V_2, \ldots, V_{t_1}$ of length even multiple of t_1. Hence $\sigma = \sigma_{m_{t_1}}(\prod_{i=2}^{k}\sigma_{m_{t_i}})(\prod_{j=1}^{s}\sigma_{p_j})$ where for $i = 2, 3, \ldots, k$, $\sigma_{m_{t_i}}$ is as in (1) *and $\prod_{j=1}^{s}\sigma_{p_j}$ is as in* (1).

Proof. Suppose σ is a complementing permutation of an r-pasc $H^r(V_1, V_2, \ldots, V_r)$. Clearly, σ is valid. From Theorem 4.5, we have that $\sigma = \prod_{i=1}^{k}\sigma_{m_{t_i}}\prod_{j=1}^{s}\sigma_{p_j}$ where σ_{m_1} permutes vertices belonging to $V_1, V_2, \ldots, V_{t_1}$, σ_{m_2} permutes vertices belonging to $V_{t_1+1}, V_{t_1+2}, \ldots, V_{t_1+t_2}$ and so on σ_{m_k} permutes vertices belonging to $V_{t_1+t_2+\cdots+t_{k-1}+1}$, $V_{t_1+t_2+\cdots+t_{k-1}+2}, \ldots, V_{t_1+t_2+\cdots+t_{k-1}+t_k}$ and $\sigma_{p_j}(V_{t+j}) = V_{t+j}$ for $j = 1, 2, \ldots, s$ and $t + s = r$. For convenience we denote $\sigma_{m_{t_i}}$ by σ_{m_i}.

Consider the deleted edge, $e = \{x_1, x_2, \ldots, x_{t_1}, x_{t_1+1}, \ldots, x_{t_1+t_2}, \ldots, x_{t_1+t_2+\cdots+t_{k-1}+1}, \ldots, x_{t_1+t_2+\cdots+t_k=t}, x_{t+1}, x_{t+2}, \ldots, x_{t+s=r}\}$. We must have the length of the cycle of σ' containing e to be odd.

First we prove that all the special vertices belonging to any particular mixed permutation must belong to the same cycle, that is for each $i = 1, 2, \ldots, k$, the special vertices $x_{t_1+t_2+\cdots+t_{i-1}+1}, x_{t_1+t_2+\cdots+t_{i-1}+2}, \ldots, x_{t_1+t_2+\cdots+t_{i-1}+t_i}$ belong to the same cycle of σ_{m_i}.

Suppose not. That is suppose for some i, $i = 1, 2, \ldots, k$, the special vertices $x_{t_1+t_2+\cdots+t_{i-1}+1}, x_{t_1+t_2+\cdots+t_{i-1}+2}, \ldots, x_{t_1+t_2+\cdots+t_{i-1}+t_i}$ do not belong to the same cycle of σ_{m_i}. That is at least one vertex among these vertices belongs to

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a different cycle. Without loss, suppose the vertices $x_{t_1+t_2+\cdots+t_{i-1}+1}, x_{t_1+t_2+\cdots+t_{i-1}+2}, \ldots, x_{t_1+t_2+\cdots+t_{i-1}+t_i-1}$ belong to a cycle C_1 and the vertex $x_{t_1+t_2+\cdots+t_{i-1}+t_i}$ belongs to a cycle C_2 of σ_{m_i} of lengths L_1 and L_2 respectively. Clearly, $L_1 = t_i q_1$ and $L_2 = t_i q_2$ for some positive integers q_1 and q_2. Note that both q_1 and q_2 must be odd. Since if either q_1 or q_2 is even then the cycle of σ' containing the edge e is of even length, a contradiction. Hence both q_1 and q_2 are odd. Further, since $L_1 = t_i q_1$ and $L_2 = t_i q_2$ there are vertices in C_1 and C_2 other than the special vertices which along with the remaining special vertices will form a valid edge and belong to a distinct cycle of σ' not containing e and at the same time having the same odd length as that of the cycle containing e, a contradiction. Hence, for each $i = 1, 2, \ldots, k$, the special vertices $x_{t_1+t_2+\cdots+t_{i-1}+1}, x_{t_1+t_2+\cdots+t_{i-1}+2}, \ldots, x_{t_1+t_2+\cdots+t_{i-1}+t_i}$ belong to the same cycle of σ_{m_i}. Moreover, the length of this cycle is $t_i q_i$, where q_i is odd.

Let C_{m_i} be the cycle in σ_{m_i} containing the special vertices $x_{t_1+t_2+\cdots+t_{i-1}+1}, x_{t_1+t_2+\cdots+t_{i-1}+2}, \ldots, x_{t_1+t_2+\cdots+t_{i-1}+t_i}$ and having length $L_i = t_i q_m^i$ respectively for $i = 1, 2, \ldots, k$, where each q_m^i is odd.

If C'_{m_i} is any other cycle in σ_{m_i} for any $i = 1, 2, \ldots, k$ having length $q t_i$ then q must be even otherwise we will get a cycle of σ' of odd length not containing the edge e.

Let C_{p_j} be the cycle in σ_{p_j} of length L'_j containing the special vertex $x_{t+j} \in V_{t+j}$, for each j, $j = 1, 2, \ldots, s$. Note that each L'_j, $j = 1, 2, \ldots, s$ is odd. If not then, the cycle of σ' containing the edge e will be of even length, a contradiction.

If C'_{p_j} is any other cycle in σ_{p_j} for any $j = 1, 2, \ldots, s$ then it must be of even length.

Observe that for any $1 \le i \le k$ and $1 \le j \le s$, at most one of q_m^i and L'_j can be greater than 1. If not then we will get a cycle of σ' of odd length not containing e, a contradiction. Further,
(1) If for some i, $i = 1, 2, \ldots, k$, $q_m^i > 1$ then t_i must be even with $\sigma^{q_m^i}(x_{t_1+t_2+\cdots t_{i-1}+l}) = x_{t_1+t_2+\cdots t_{i-1}+l+q_m^i \pmod{t_i}}$ and $q_m^n = 1$ for $n = 1, 2, \ldots, k$; $n \ne i$.
(2) If for some j, $j = 1, 2, \ldots, s$, $L'_j > 1$ then for any vertex $u \ne x_{t+j}$ in C_{p_j}, the cycle of σ' containing the special vertices other than x_{t+j} and u is of odd length which contains the edge e as well. And all the other cycles of σ' are of even length. □

The cycle structure of complementing permutations of bipartite ($r = 2$) almost self-complementary 2-hypergraphs (graphs) can be obtained from Theorem 5.1 as stated in the following corollary.

Corollary 5.2. *σ is a complementing permutation of 2-pasc (bipartite almost self-complementary graph) $H^2(V_1, V_2)$ if and only if σ is valid that is $\sigma = \sigma_m$ or $\sigma = \sigma_{p_1}\sigma_{p_2}$ where σ_m permutes vertices belonging to V_1, V_2 and σ_{p_1}, σ_{p_2} permute vertices belonging to V_1 and V_2 respectively. Further, either*
(i) $\sigma = \sigma_{p_1}\sigma_{p_2}$ where σ_{p_1} and σ_{p_2} have exactly one fixed special vertex x_1 and x_2 respectively, and all the other cycles of σ_{p_1} and σ_{p_2} are of even length.
or
(ii) $\sigma = \sigma_{p_1}\sigma_{p_2}$ where exactly one of σ_{p_1} and σ_{p_2} has exactly one cycle of odd length $L > 1$ containing the special vertex and the other has exactly one fixed special vertex. All the other cycles of σ_{p_1} and σ_{p_2} are of even length.
or
(iii) $\sigma = \sigma_m$ has a unique cycle of length $4h + 2$ containing the special vertices x_1, x_2 with $\sigma^{2h+1}(x_1) = x_2$, $h \ge 0$ and all the other cycles are of length a multiple of 4.

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References

[1] A. Szymański, A.P. Wojda, A note on k-uniform self-complementary hypergraphs of given order, Discuss. Math. Graph Theory 29 (2009) 199–202.
[2] A. Szymański, A.P. Wojda, Self-complementing permutations of k-uniform hypergraphs, Discrete Math. Theoret. Comput. Sci. 11 (2009) 117–124.
[3] A. Wojda, Self complementary hypergraphs, Discuss. Math. Graph Theory 26 (2006) 217–224.
[4] S. Gosselin, Constructing self-complementary uniform hypergraphs, Discrete Math. 310 (2010) 1366–1372.
[5] C.R.J. Clapham, Graphs self-Complementary in $K_n - e$, Discrete Math. 81 (1990) 229–235.
[6] L.N. Kamble, C.M. Deshpande, B.Y. Bam, Almost self-complementary 3-uniform hypergraphs, Discuss. Math. Graph Theory 37 (2017) 131–140.

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[7] A. Wojda, Almost self-complementary uniform hypergraphs, Discuss. Math. Graph Theory 38 (2018) 607–610.
[8] R. Diestel, Graph Theory, third ed., Springer-Verlag, 2005.
[9] T. Gangopadhyay, S.P. Rao Hebbare, Structural properties of r-partite complementing permutations, Tech. Report No. 19/77, I.S.I, Calcutta.

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