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F index of graphs based on four new operations related to the strong product[☆]

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Abstract

For a molecular graph, the first Zagreb index of a graph is equal to the sum of squares of the vertex degrees of the graph and the forgotten topological index (F-index) of a graph is defined as the sum of cubes of the vertex degrees of the graph. These parameters have applications in chemistry and drug structures. In this paper, we study the F index of strong product of two connected graphs in which one of the graphs is obtained by using four new sums called F sums of graphs and the other is any connected graph.

Keywords: F index; Degree; Subdivision of graph; Total graph; Strong product

1. Introduction

Throughout this paper we consider only simple connected graphs, that is, connected graphs without loops and multiple edges. For a graph $G = (V, E)$ with vertex set $V = V(G)$ and edge set $E = E(G)$, the degree of a vertex v in G is the number of edges incident to v and is denoted by $d_G(v)$.

A graphical invariant is a number related to a graph which is structurally invariant. In chemical graph theory, these invariant numbers are also known as the topological indices. The first and second Zagreb indices of a graph are among the most studied vertex degree based topological indices. These indices were introduced by Gutman and Trinajestić [1], to study the structure dependency of the total π-electron energy on molecular structure, and this was elaborated on in [2]. Another vertex degree based topological index was defined in [1] where the Zagreb indices were introduced, and this index was not further studied until it was studied by Furtula and Gutman in the article [3]. A few basic properties of the forgotten topological index and the significant enhancement of physico-chemical applicability of the first Zagreb index are shown in [3].

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Also forgotten topological index of several widely chemical structures which often appear in drug molecular graphs were presented in [4]. The lower and upper bounds of forgotten topological index in terms of graph irregularity, Zagreb indices, graph size and maximum/minimum vertex degrees were given in [5].

For a (molecular) graph G, the first Zagreb index $M_1(G)$ and the second Zagreb index $M_2(G)$ are, respectively, defined as follows:

$$M_1 = M_1(G) = \sum_{v\in V(G)} d_G^2(v), \quad M_2 = M_2(G) = \sum_{uv\in E(G)} d_G(u)d_G(v).$$

Also, $M_1(G) = \sum_{uv\in E(G)}[d_G(u) + d_G(v)]$. For more details on these indices see the recent papers [6–13] and the references therein. The zeroth-order general Randić index is a more general case of the first Zagreb index [14,15] and see survey paper on Randić index [16].

In [17], exact expressions for the first and second Zagreb indices of graph operations containing the Cartesian product, composition, join, disjunction, and symmetric difference of graphs were presented. Also, exact expressions for the first and second Zagreb indices of graphs based on operations related to the Cartesian product and the lexicographic product were given in [18] and [19], respectively. The closed formulas for the F-index of four operations of graphs to Cartesian product were determined in [20].

In this work, we will study the F index of four new operations related to the strong product on graphs. For this purpose, we recall some operations on graphs in the following (see also [17–21]).

The strong product of two connected graphs G_1 and G_2, which is denoted by $G_1 \circ G_2$, is a graph such that the set of vertices is $V(G_1) \times V(G_2)$ and two vertices $u = (u_1, v_1)$ and $v = (u_2, v_2)$ are adjacent in $G_1 \circ G_2$ if and only if, either (i) $u_1 = u_2$ and v_1 is adjacent with v_2, or (ii) u_1 is adjacent with u_2 and $v_1 = v_2$, or (iii) u_1 is adjacent with u_2 and v_1 is adjacent with v_2.

It is easy to see that $d_{G_1\circ G_2}(u_1, v_1) = d_{G_1}(u_1) + d_{G_2}(v_1) + d_{G_1}(u_1)d_{G_2}(v_1)$.

For a connected graph G, there are four related graphs as follows:

(a) $S(G)$ is the graph obtained by inserting an additional vertex in each edge of G. Equivalently, each edge of G is replaced by a path of length 2.

(b) $R(G)$ is obtained from G by adding a new vertex corresponding to each edge of G, then joining each new vertex to the end vertices of the corresponding edge.

(c) $Q(G)$ is obtained from G by inserting a new vertex into each edge of G, then joining with edges those pairs of new vertices on adjacent edges of G.

(d) $T(G)$ has as its vertices the edges and vertices of G. Adjacency in $T(G)$ is defined as adjacency or incidence for the corresponding elements of G.

The graphs $S(G)$ and $T(G)$ are called the subdivision and total graph of G, respectively. For more details on these operations we refer the reader to [22]. The graphs $R(G)$ and $Q(G)$ are called the triangle parallel graph and the line superposition graph of G in [23], respectively.

Note that (i) $R(G)$ can be obtained by replacing each edge of G by a triangle, its vertex set is the union of $V(G)$ and $E(G)$, and its edge set is the union of the respective edge sets of G and $S(G)$; (ii) $Q(G)$ is a graph on the same vertex set as $S(G)$ whose edge set is the union of the edge sets of $S(G)$ and the line graph $L(G)$ of G.

Yarahmadi et al. [23] presented explicit formulas expressing the eccentric connectivity indices of $S(G)$, $R(G)$, $Q(G)$, $T(G)$ in terms of the eccentric connectivity index of the original graph G and some auxiliary invariants.

If G is P_3, then $S(P_3)$, $R(P_3)$, $Q(P_3)$ and $T(P_3)$ are shown in Fig. 1 (see [18,19]).

Based on the Cartesian product $G_1 \times G_2$ of two connected graphs G_1 and G_2 and the four types S, R, Q, T of graphs resulting from edge subdivision above, M. Eliasi and B. Taeri [21] introduced four new operations on these graphs.

The expression for the Wiener index $W(G_1 +_F G_2)$ of the F-sums of graph $G_1 +_F G_2$ in terms of $W(F(G_1))$ and $W(G_2)$ and the first and second Zagreb indices for the F-sums of graph were obtained in [21] and [18], respectively.

Also based on the strong product $G_1 \circ G_2$ of two connected graphs G_1 and G_2 and the four types S, R, Q, T of graphs resulting from edge subdivision, we also introduce four new operations on these graphs in the following:

Let $F \in \{S, R, Q, T\}$. The F-sum of G_1 and G_2, denoted by $G_1 \circ_F G_2$, is defined by $F(G_1) \circ G_2 - E^*$, where $E^* = \{(u, v_1)(u, v_2) \in E(F(G_1) \circ G_2) : u \in V(F(G_1)) - V(G_1), v_1v_2 \in E(G_2)\}$, i.e., $G_1 \circ_F G_2$ is a graph with the set of vertices $V(G_1 \circ_F G_2) = V(F(G_1)) \times V(G_2) = (V(G_1) \cup E(G_1)) \times V(G_2)$ and two vertices $u = (u_1, v_1)$ and $v = (u_2, v_2)$ of $G_1 \circ_F G_2$ are adjacent if and only if, either (i) $u_1 = u_2 \in V(G_1)$ and $v_1v_2 \in E(G_2)$, or (ii) $u_1u_2 \in E(F(G_1))$ and $v_1 = v_2 \in V(G_2)$, or (iii) $u_1u_2 \in E(F(G_1))$ and $v_1v_2 \in E(G_1)$.

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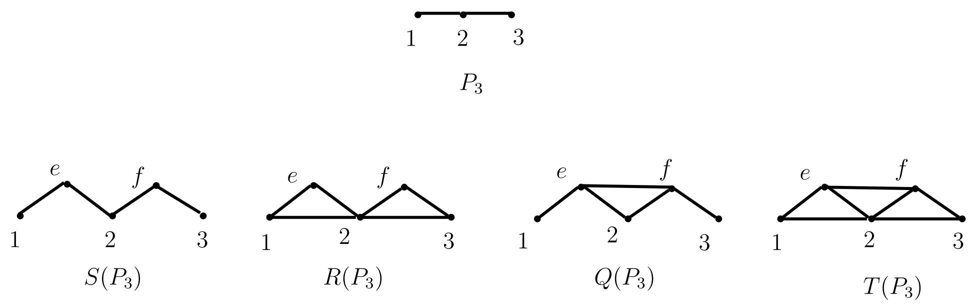

Fig. 1. P_3, $S(P_3)$, $R(P_3)$, $Q(P_3)$ *and* $T(P_3)$.

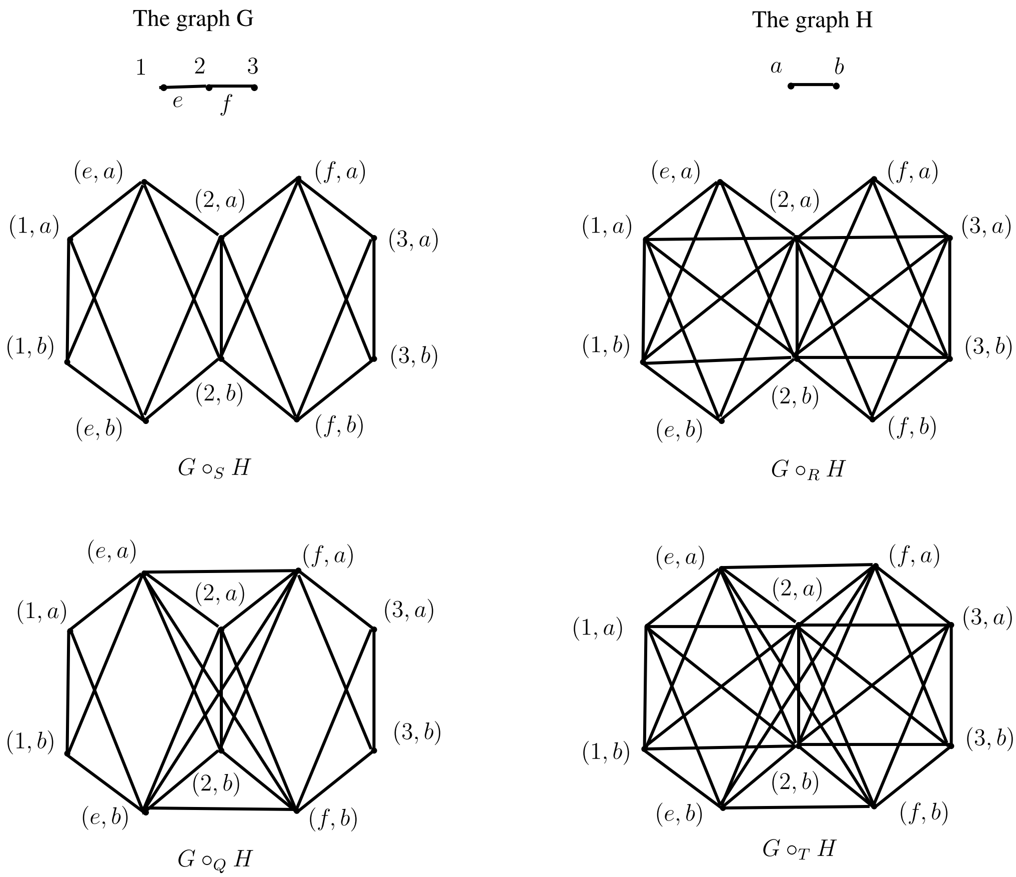

Fig. 2. Graphs G and H and $G \circ_F H$.

For any vertex $(x, y) \in V(G_1 \circ_F G_2)$, the degree of (x, y) in the F-strong product $G_1 \circ_F G_2$ is

$$d(x,y) = \begin{cases} d_{F(G_1)}(x) + d_{G_2}(y) + d_{F(G_1)}(x) d_{G_2}(y), & \text{if } x \in V(G_1); \\ d_{F(G_1)}(x) + d_{F(G_1)}(x) d_{G_2}(y), & \text{if } x \in V(F(G_1)) - V(G_1). \end{cases}$$

$P_3 \circ_S P_2$, $P_3 \circ_R P_2$, $P_3 \circ_Q P_2$ and $P_3 \circ_T P_2$ are shown in Fig. 2.

In this work, we will study the F index for the F-strong product of graphs.

2. The F index for F-strong product of graphs

Firstly, we will give the expression for the F index of $G_1 \circ_S G_2$ in terms of F index and Zagreb indices of graphs G_1 and G_2.

Theorem 1. *Let G_i be a connected graph with n_i vertices and e_i edges, $i = 1,2$. Then*

$$\begin{aligned} F(G_1 \circ_S G_2) =& [n_2 + 3M_1(G_2) + 6e_2]F(G_1) + [n_1 + 3M_1(G_1) + 14e_1]F(G_2) \\ &+ 6e_2 M_1(G_1) + 30e_1 M_1(G_2) + 6M_1(G_1)M_1(G_2) \\ &+ F(G_1)F(G_2) + 48e_1e_2 + 8n_2e_1. \end{aligned}$$

Proof. For any vertex $(x, y) \in V(G_1 \circ_S G_2)$, the degree $d(x, y)$ of (x, y) is

$$\begin{aligned} d(x, y) &= \begin{cases} d_{S(G_1)}(x) + d_{G_2}(y) + d_{S(G_1)}(x) d_{G_2}(y), & \text{if } x \in V(G_1); \\ d_{S(G_1)}(x) + d_{S(G_1)}(x) d_{G_2}(y), & \text{if } x \in V(S(G_1)) - V(G_1) \end{cases} \\ &= \begin{cases} d_{G_1}(x) + d_{G_2}(y) + d_{G_1}(x) d_{G_2}(y), & \text{if } x \in V(G_1); \\ 2 + 2d_{G_2}(y), & \text{if } x \in V(S(G_1)) - V(G_1). \end{cases} \end{aligned}$$

For $v_1v_2 \in E(G_2)$ and $u_1u_2 \in E(S(G_1))$, if $u_1 \in V(G_1)$ and $u_2 \in V(S(G_1)) - V(G_1)$, i.e., u_2 is a new vertex inserted on an edge incident to u_1, then $(u_1, v_1)(u_2, v_2) \in E(G_1 \circ_S G_2)$ and $(u_1, v_2)(u_2, v_1) \in E(G_1 \circ_S G_2)$.

$$\begin{aligned} F(G_1 \circ_S G_2) &= \sum_{(u_1,v_1)(u_2,v_2) \in E(G_1 \circ_S G_2)} [d(u_1, v_1)^2 + d(u_2, v_2)^2] \\ &= \sum_{u_1 = u_2 \in V(G_1)} \sum_{v_1v_2 \in E(G_2)} [d(u_1, v_1)^2 + d(u_2, v_2)^2] \\ &\quad + \sum_{v_1 = v_2 \in V(G_2)} \sum_{u_1u_2 \in E(S(G_1))} [d(u_1, v_1)^2 + d(u_2, v_2)^2] \\ &\quad + \sum_{v_1v_2 \in E(G_2)} \sum_{u_1u_2 \in E(S(G_1))} [d(u_1, v_1)^2 + d(u_2, v_2)^2] \\ &= \sum\nolimits_1 + \sum\nolimits_2 + \sum\nolimits_3. \end{aligned}$$

Then

$$\begin{aligned} \sum\nolimits_1 &= \sum_{u_1 = u_2 \in V(G_1)} \sum_{v_1v_2 \in E(G_2)} [d(u_1, v_1)^2 + d(u_2, v_2)^2] \\ &= \sum_{u \in V(G_1)} \sum_{v_1v_2 \in E(G_2)} [(d(u) + d(v_1) + d(u)d(v_1))^2 + (d(u) + d(v_2) + d(u)d(v_2))^2] \\ &= \sum_{u \in V(G_1)} \sum_{v_1v_2 \in E(G_2)} [2d^2(u) + d^2(v_1) + d^2(v_2) + d^2(u)(d^2(v_1) + d^2(v_2)) \\ &+ 2d(u)(d(v_1) + d(v_2)) + 2d^2(u)(d(v_1) + d(v_2)) + 2d(u)(d^2(v_1) + d^2(v_2))] \\ &= \sum_{u \in V(G_1)} [e_2(2d^2(u)) + F(G_2) + d^2(u)F(G_2) + 2d(u)M_1(G_2) \\ &+ 2d^2(u)M_1(G_2) + 2d(u)F(G_2)] \\ &= 2e_2M_1(G_1) + n_1F(G_2) + M_1(G_1)F(G_2) + 2(2e_1)M_1(G_2) \\ &+ 2M_1(G_1)M_1(G_2) + 2(2e_1)F(G_2) \\ &= 2e_2M_1(G_1) + n_1F(G_2) + M_1(G_1)F(G_2) + 4e_1M_1(G_2) + 2M_1(G_1)M_1(G_2) + 4e_1F(G_2). \end{aligned}$$

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and

$$\sum_2 = \sum_{v_1=v_2\in V(G_2)} \sum_{u_1u_2\in E(S(G_1))} [d(u_1,v_1)^2 + d(u_2,v_2)^2]$$

(Without loss of generality, we assume that $u_1 \in V(G_1)$, $u_2 \in V(S(G_1)) - V(G_1)$.)

$$= \sum_{v\in V(G_2)} \sum_{u_1u_2\in E(S(G_1))} [(d(u_1)+d(v)+d(u_1)d(v))^2 + (d(u_2)+d(u_2)d(v))^2]$$

$$= \sum_{v\in V(G_2)} \sum_{u_1u_2\in E(S(G_1))} [d^2(u_1)+d^2(u_2)+d^2(v)(d^2(u_1)+d^2(u_2))+2d(v)(d^2(u_1)$$

$$+\ d^2(u_2)) + 2d(u_1)d(v) + 2d(u_1)d^2(v) + d^2(v)]$$

$$= \sum_{v\in V(G_2)} [F(S(G_1)) + d^2(v)F(S(G_1)) + 2d(v)F(S(G_1))$$

$$+\ 2d(v)M_1(G_1) + 2d^2(v)M_1(G_1) + 2e_1d^2(v)]$$

$$= n_2F(S(G_1)) + M_1(G_2)F(S(G_1)) + 4e_2F(S(G_1)) + 4e_2M_1(G_1)$$
$$+ 2M_1(G_1)M_1(G_2) + 2e_1M_1(G_2).$$

Note that $F(S(G_1)) = F(G_1) + 8e_1$, we have

$$\sum_2 = [n_2 + M_1(G_2) + 4e_2]F(G_1) + 2M_1(G_1)M_1(G_2) + 10e_1M_1(G_2) + 4e_2M_1(G_1) + 8e_1(n_2+4e_2).$$

$$\sum_3 = \sum_{v_1v_2\in E(G_2)} \sum_{u_1u_2\in E(S(G_1))} [d(u_1,v_1)^2 + d(u_2,v_2)^2]$$

$$= \sum_{v_1v_2\in E(G_2)} \sum_{\substack{u_1u_2\in E(S(G_1)) u_1\in V(G_1),\\ u_2\in V(S(G_1))-V(G_1)}} [(d(u_1)+d(v_1)+d(u_1)d(v_1))^2 + (d(u_2)+d(u_2)d(v_2))^2]$$

$$= \sum_{v_1v_2\in E(G_2)} \sum_{\substack{u_1u_2\in E(S(G_1)) u_1\in V(G_1),\\ u_2\in V(S(G_1))-V(G_1)}} [d^2(u_1)+d^2(u_2)+d^2(v_1)+d^2(u_1)d^2(v_1)+2d(u_1)d(v_1)$$

$$+ 2d^2(u_1)d(v_1) + 2d(u_1)d^2(v_1) + d^2(u_2)d^2(v_2) + 2d^2(u_2)d(v_2)]$$

$$= \sum_{v_1v_2\in E(G_2)} \sum_{\substack{u_1u_2\in E(S(G_1)) u_1\in V(G_1),\\ u_2\in V(S(G_1))-V(G_1)}} \big[[d^2(u_1)+d^2(u_2)] + [d^2(v_1)] + [d^2(u_1)d^2(v_1)]$$

$$+ [2d(u_1)d(v_1)] + [2d^2(u_1)d(v_1)] + [2d(u_1)d^2(v_1)] + [d^2(u_2)d^2(v_2)] + [2d^2(u_2)d(v_2)]\big]$$

$$= 2e_2F(G_1) + 16e_1e_2 + 2e_1F(G_2) + F(G_1)F(G_2) + 2M_1(G_1)M_1(G_2)$$

$$+\ 2F(G_1)M_1(G_2) + 2M_1(G_1)F(G_2) + 8e_1F(G_2) + 16e_1M_1(G_2).$$

Hence,

$$F(G_1 \circ_S G_2) = [n_2 + 3M_1(G_2) + 6e_2]F(G_1) + [n_1 + 3M_1(G_1) + 14e_1]F(G_2)$$
$$+\ 6e_2M_1(G_1) + 30e_1M_1(G_2) + 6M_1(G_1)M_1(G_2)$$
$$+\ F(G_1)F(G_2) + 48e_1e_2 + 8n_2e_1. \quad \blacksquare$$

Theorem 2. *Let G_i be a connected graph with n_i vertices and e_i edges, i = 1,2. Then*

$$F(G_1 \circ_R G_2) = 8[n_2 + 6e_2]F(G_1) + [n_1 + 20e_1]F(G_2) + 8F(G_1)F(G_2)$$
$$+\ 48e_1e_2 + 24e_2M_1(G_1) + 36e_1M_1(G_2) + 24M_1(G_1)M_1(G_2)$$
$$+\ 24F(G_1)M_1(G_2) + 12F(G_2)M_1(G_1) + 8n_2e_1.$$

Proof.

$$
\begin{aligned}
F(G_1 \circ_R G_2) &= \sum_{(u_1,v_1)(u_2,v_2)\in E(G_1\circ_R G_2)} [d(u_1,v_1)^2 + d(u_2,v_2)^2] \\
&= \sum_{u_1=u_2\in V(G_1)} \sum_{v_1v_2\in E(G_2)} [d(u_1,v_1)^2 + d(u_2,v_2)^2] \\
&\quad + \sum_{v_1=v_2\in V(G_2)} \sum_{u_1u_2\in E(R(G_1))} [d(u_1,v_1)^2 + d(u_2,v_2)^2] \\
&\quad + \sum_{v_1v_2\in E(G_2)} \sum_{u_1u_2\in E(R(G_1))} [d(u_1,v_1)^2 + d(u_2,v_2)^2] \\
&= \sum{}_1 + \sum{}_2 + \sum{}_3.
\end{aligned}
$$

Then

$$
\begin{aligned}
\sum{}_1 &= \sum_{u_1=u_2\in V(G_1)} \sum_{v_1v_2\in E(G_2)} [d(u_1,v_1)^2 + d(u_2,v_2)^2] \\
&= 8e_2M_1(G_1) + n_1F(G_2) + 4M_1(G_1)F(G_2) + 8e_1M_1(G_2) + 8M_1(G_1)M_1(G_2) + 8e_1F(G_2)
\end{aligned}
$$

since $d(u)$ in $R(G_1)$ is $2d(u)$ in G_1, i.e., $d_{R(G_1)}(u) = 2d_{G_1}(u)$.

$$
\begin{aligned}
\sum{}_2 &= \sum_{v_1=v_2\in V(G_2)} \sum_{u_1u_2\in E(R(G_1))} [d(u_1,v_1)^2 + d(u_2,v_2)^2] \\
&= \sum_{v_1=v_2\in V(G_2)} \sum_{\substack{u_1u_2\in E(R(G_1))\\ u_1,u_2\in V(G_1)}} [d(u_1,v_1)^2 + d(u_2,v_2)^2] \\
&\quad + \sum_{v_1=v_2\in V(G_2)} \sum_{\substack{u_1u_2\in E(R(G_1))\\ u_1\in V(G_1),u_2\in V(R(G_1))-V(G_1)}} [d(u_1,v_1)^2 + d(u_2,v_2)^2] \\
&= \sum_2{}' + \sum_2{}''
\end{aligned}
$$

$$
\begin{aligned}
\sum_2{}' &= \sum_{v_1=v_2\in V(G_2)} \sum_{\substack{u_1u_2\in E(R(G_1))\\ u_1,u_2\in V(G_1)}} [d(u_1,v_1)^2 + d(u_2,v_2)^2] \\
&= \sum_{v\in V(G_2)} \sum_{\substack{u_1u_2\in E(G_1)\\ u_1,u_2\in V(G_1)}} [(d(u_1)+d(v)+d(u_1)d(v))^2 + (d(u_2)+d(v)+d(u_2)d(v))^2] \\
&= \sum_{v\in V(G_2)} \sum_{\substack{u_1u_2\in E(G_1)\\ u_1,u_2\in V(G_1)}} [4(d^2(u_1)+d^2(u_2)) + 2d^2(v) + 4d^2(v)(d^2(u_1)+d^2(u_2)) \\
&\quad + 4d(v)(d(u_1)+d(u_2)) + 4d^2(v)(d(u_1)+d(u_2)) + 8d(v_1)(d^2(u_1)+d^2(u_2))] \\
&= 4n_2F(G_1) + 2e_1M_1(G_2) + 4M_1(G_2)F(G_1) + 8e_2M_1(G_1) + 4M_1(G_1)M_1(G_2) + 16e_2F(G_1),
\end{aligned}
$$

$$
\begin{aligned}
\sum_2{}'' &= \sum_{v_1=v_2\in V(G_2)} \sum_{\substack{u_1u_2\in E(R(G_1))\\ u_1\in V(G_1),u_2\in V(RG_1)-V(G_1)}} [d(u_1,v_1)^2 + d(u_2,v_2)^2] \\
&= \sum_{v\in V(G_2)} \sum_{\substack{u_1u_2\in E(G_1)\\ u_1\in V(G_1),u_2\in V(RG_1)-V(G_1)}} [(d(u_1)+d(v)+d(u_1)d(v))^2 + (d(u_2)+d(u_2)d(v))^2]
\end{aligned}
$$

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$$= \sum_{v\in V(G_2)} \sum_{\substack{u_1u_2\in E(G_1)\\ u_1\in V(G_1), u_2\in V(RG_1)-V(G_1)}} [4d^2(u_1) + d^2(v) + 4d^2(u_1)d^2(v) + 4d(u_1)d(v)$$
$$+ 4d(u_1)d^2(v) + 8d^2(u_1)d(v) + d^2(u_2) + d^2(u_2)d^2(v) + 2d^2(u_2)d(v)]$$
$$= 4n_2F(G_1) + 2e_1M_1(G_2) + 4M_1(G_2)F(G_1) + 8e_2M_1(G_1)$$
$$+ 4M_1(G_1)M_1(G_2) + 16e_2F(G_1) + 8n_2e_1 + 8e_1M_1(G_2) + 32e_1e_2,$$

$$\sum{}_2 = 8(n_2 + 4e_2)F(G_1) + 16e_2M_1(G_1) + 12e_1M_1(G_2) + 8M_1(G_2)F(G_1)$$
$$+ 8M_1(G_1)M_1(G_2) + 8n_2e_1 + 32e_1e_2.$$

$$\sum{}_3 = \sum_{v_1v_2\in E(G_2)} \sum_{u_1u_2\in E(R(G_1))} [d(u_1, v_1)^2 + d(u_2, v_2)^2]$$
$$= \sum_{v_1v_2\in E(G_2)} \sum_{\substack{u_1u_2\in E(S(G_1))\\ u_1,u_2\in V(G_1)}} [d(u_1, v_1)^2 + d(u_2, v_2)^2]$$
$$+ \sum_{v_1v_2\in E(G_2)} \sum_{\substack{u_1u_2\in E(S(G_1))\\ u_1\in V(G_1), u_2\in V(R(G_1))-V(G_1)}} [d(u_1, v_1)^2 + d(u_2, v_2)^2]$$
$$= \sum_3{}' + \sum_3{}''.$$

$$\sum_3{}' = \sum_{v_1v_2\in E(G_2)} \sum_{\substack{u_1u_2\in E(R(G_1))\\ u_1,u_2\in V(G_1)}} [d(u_1, v_1)^2 + d(u_2, v_2)^2]$$
$$= \sum_{v_1v_2\in E(G_2)} \sum_{\substack{u_1u_2\in E(R(G_1))\\ u_1,u_2\in V(G_1)}} [(d(u_1) + d(v_1) + d(u_1)d(v_1))^2 + (d(u_2) + d(v_2) + d(u_2)d(v_2))^2]$$
$$= \sum_{v_1v_2\in E(G_2)} \sum_{\substack{u_1u_2\in E(R(G_1))\\ u_1,u_2\in V(G_1)}} [d^2(u_1) + d^2(u_2) + d^2(v_1) + d^2(v_2) + d^2(u_1)d^2(v_1) + d^2(u_2)d^2(v_2)$$
$$+ 2d(u_1)d(v_1) + 2d(u_2)d(v_2) + 2d(u_1)d^2(v_1) + 2d(u_2)d^2(v_2) + 2d^2(u_1)d(v_1) + 2d^2(u_2)d(v_2)]$$
$$= \sum_{v_1v_2\in E(G_2)} \sum_{\substack{u_1u_2\in E(R(G_1))\\ u_1,u_2\in V(G_1)}} \big[4[d^2(u_1) + d^2(u_2)] + [d^2(v_1) + d^2(v_2)] + 4[d^2(u_1)d^2(v_1) + d^2(u_2)d^2(v_2)]$$
$$+ 4[d(u_1)d(v_1) + d(u_2)d(v_2)] + 4[d(u_1)d^2(v_1) + d(u_2)d^2(v_2)] + 8[d^2(u_1)d(v_1) + 2d^2(u_2)d(v_2)]\big]$$
$$= 8e_2F(G_1) + 2e_1F(G_2) + 4F(G_1)F(G_2) + 4M_1(G_1)M_1(G_2)$$
$$+ 4M_1(G_1)F(G_2) + 8F(G_1)M_1(G_2),$$

$$\sum_3{}'' = \sum_{v_1v_2\in E(G_2)} \sum_{\substack{u_1u_2\in E(R(G_1))\\ u_1\in V(G_1), u_2\in V(R(G_1))-V(G_1)}} [d(u_1, v_1)^2 + d(u_2, v_2)^2]$$
$$= \sum_{v_1v_2\in E(G_2)} \sum_{\substack{u_1u_2\in E(R(G_1))\\ u_1\in V(G_1), u_2\in V(R(G_1))-V(G_1)}} [(d(u_1) + d(v_1) + d(u_1)d(v_1))^2(d(u_2) + d(u_2)d(v_2))^2]$$

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$$= \sum_{v_1v_2 \in E(G_2)} \sum_{\substack{u_1u_2 \in E(R(G_1)) \\ u_1 \in V(G_1), u_2 \in V(R(G_1)) - V(G_1)}} [d^2(u_1) + d^2(u_2) + d^2(v_1) + d^2(u_1)d^2(v_1) + 2d(u_1)d(v_1)$$

$$+ 2d^2(u_1)d(v_1) + 2d(u_1)d^2(v_1) + d^2(u_2)d^2(v_2) + 2d^2(u_2)d(v_2)]$$

$$= \sum_{v_1v_2 \in E(G_2)} \sum_{\substack{u_1u_2 \in E(R(G_1)) \\ u_1 \in V(G_1), u_2 \in V(R(G_1)) - V(G_1)}} \Big[[d^2(u_1)] + [d^2(u_2)] + [d^2(u_1)d^2(v_1)] + [2d(u_1)d(v_1)]$$

$$+ [2d^2(u_1)d(v_1)] + [d^2(v_1)] + [2d(u_1)d^2(v_1)] + [d^2(u_2)d^2(v_2)] + [2d^2(u_2)d(v_2)] \Big]$$

$$= 8e_2F(G_1) + 16e_1e_2 + 4F(G_1)F(G_2) + 4M_1(G_1)M_1(G_2) + 8F(G_1)M_1(G_2)$$

$$+ \ 2e_1F(G_2) + 4M_1(G_1)F(G_2) + 8e_1F(G_2) + 16e_1M_1(G_2),$$

$$\sum\nolimits_3 = 16e_2F(G_1) + 12e_1F(G_2) + 8F(G_1)F(G_2) + 8M_1(G_1)M_1(G_2)$$

$$+ \ 16F(G_1)M_1(G_2) + 8F(G_2)M_1(G_1) + 16e_1M_1(G_2) + 16e_1e_2.$$

Hence,

$$F(G_1 \circ_R G_2) = 8[n_2 + 6e_2]F(G_1) + [n_1 + 20e_1]F(G_2) + 8F(G_1)F(G_2)$$

$$+ \ 48e_1e_2 + 24e_2M_1(G_1) + 36e_1M_1(G_2) + 24M_1(G_1)M_1(G_2)$$

$$+ \ 24F(G_1)M_1(G_2) + 12F(G_2)M_1(G_1) + 8n_2e_1. \quad \blacksquare$$

Theorem 3. *Let G_i be a connected graph with n_i vertices and e_i edges, $i = 1, 2$. Then*

$$F(G_1 \circ_Q G_2) = [n_2 + 12e_2 + 9M_1(G_2)]F(G_1) + [n_1 + 6e_1 + 3M_1(G_1) + 6M_2(G_1)]F(G_2)$$

$$+ \ [n_2 + 4e_2 + M_1(G_2)][M_4(G_1) + 2M_2^1(G_1) + \sum_{u \in V(G_1)} d^2(u) \sum_{v \in N(u)} d(v)]$$

$$+ \ 6[e_2M_1(G_1) + e_1M_1(G_2) + 2(e_2 + M_1(G_2))M_2(G_1)]$$

$$+ \ 2[2F(G_1)F(G_2) + 3M_1(G_1)M_1(G_2)]$$

where $M_2^1(G_1) = \sum_{uv \in E(G_1)} (du+dv)dudv$

Proof.

$$F(G_1 \circ_Q G_2) = \sum_{(u_1,v_1)(u_2,v_2) \in E(G_1 \circ_Q G_2)} [d(u_1, v_1)^2 + d(u_2, v_2)^2]$$

$$= \sum_{u_1 = u_2 \in V(G_1)} \sum_{v_1v_2 \in E(G_2)} [d(u_1, v_1)^2 + d(u_2, v_2)^2]$$

$$+ \sum_{v_1 = v_2 \in V(G_2)} \sum_{u_1u_2 \in E(Q(G_1))} [d(u_1, v_1)^2 + d(u_2, v_2)^2]$$

$$+ \sum_{v_1v_2 \in E(G_2)} \sum_{u_1u_2 \in E(Q(G_1))} [d(u_1, v_1)^2 + d(u_2, v_2)^2]$$

$$= \sum\nolimits_1 + \sum\nolimits_2 + \sum\nolimits_3.$$

Then

$$\sum\nolimits_1 = \sum_{u_1 = u_2 \in V(G_1)} \sum_{v_1v_2 \in E(G_2)} [d(u_1, v_1)^2 + d(u_2, v_2)^2]$$

$$= 2e_2M_1(G_1) + n_1F(G_2) + M_1(G_1)F(G_2) + 4e_1M_1(G_2) + 2M_1(G_1)M_1(G_2) + 4e_1F(G_2)$$

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as this summation is same in $F(G_1 \circ_S G_2)$.

$$\sum 2 = \sum_{v_1=v_2 \in V(G_2)} \sum_{u_1u_2 \in E(Q(G_1))} [d(u_1,v_1)^2 + d(u_2,v_2)^2]$$

$$= \sum_{v_1=v_2 \in V(G_2)} \sum_{\substack{u_1u_2 \in E(Q(G_1)) \\ u_1 \in V(G_1), u_2 \in V((QG_1)-V(G_1))}} [d(u_1,v_1)^2 + d(u_2,v_2)^2]$$

$$+ \sum_{v_1=v_2 \in V(G_2)} \sum_{\substack{u_1u_2 \in E(Q(G_1)) \\ u_1,u_2 \in V(Q(G_1))-V(G_1)}} [d(u_1,v_1)^2 + d(u_2,v_2)^2]$$

$$= \sum_2' + \sum_2''.$$

$$\sum_2' = \sum_{v_1=v_2 \in V(G_2)} \sum_{\substack{u_1u_2 \in E(Q(G_1)) \\ u_1 \in V(G_1), u_2 \in V(QG_1)-V(G_1)}} [d(u_1,v_1)^2 + d(u_2,v_2)^2]$$

$$= \sum_{v \in V(G_2)} \sum_{\substack{u_1u_2 \in E(QG_1) \\ u_1 \in V(G_1), u_2 \in V(QG_1)-V(G_1)}} [(d(u_1) + d(v) + d(u_1)d(v))^2 + (d(u_2) + d(u_2)d(v))^2]$$

$$= \sum_{v \in V(G_2)} \sum_{\substack{u_1u_2 \in E(G_1) \\ u_1 \in V(G_1), u_2 \in V(QG_1)-V(G_1)}} [d^2(u_1) + d^2(v) + d^2(u_1)d^2(v) + 2d(u_1)d(v)$$

$$+ 2d(u_1)d^2(v) + 2d^2(u_1)d(v) + d^2(u_2) + d^2(u_2)d^2(v) + 2d^2(u_2)d(v)]$$

Consider

$$S_1 = \sum_{\substack{u_1u_2 \in E(QG_1) \\ u_1 \in V(G_1), u_2 \in V(QG_1)-V(G_1)}} d^2(u_1)$$

In S_1, $u_1 \in V(G_1)$ and d^2u_1 occurs du_1 times. Thus

$$S_1 = \sum_{\substack{u_1u_2 \in E(QG_1) \\ u_1 \in V(G_1), u_2 \in V(QG_1)-V(G_1)}} d^3(u_1) = F(G_1).$$

Let

$$S_2 = \sum_{\substack{u_1u_2 \in E(QG_1) \\ u_1 \in V(G_1), u_2 \in V(QG_1)-V(G_1)}} d^2(u_2)$$

as $u_2 = uv \in E(G_1)$, d^2u_2 occurs two times. Therefore

$$S_2 = 2 \sum_{u_2=uv \in V(QG_1)-V(G_1)} [du + dv]^2 = 2 \sum_{uv \in E(G_1)} [d^2u + d^2v + 2dudv]$$

$$= 2[F(G_1) + 2M_2(G_1)]$$

Hence

$$\sum_2' = n_2F(G_1) + 2e_1M_1(G_2) + M_1(G_2)F(G_1) + 4e_2M_1(G_1) + 4e_2F(G_1) + 2M_1(G_1)M_1(G_2)$$

$$+ 2n_2[F(G_1) + 2M_2(G_1)] + 2M_1(G_2)[F(G_1) + 2M_2(G_1)] + 8e_2[F(G_1) + 2M_2(G_1)]$$

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$$\sum_2'' = \sum_{v_1=v_2\in V(G_2)} \sum_{\substack{u_1u_2\in E(Q(G_1))\\ u_1,u_2\in V(QG_1)-V(G_1)}} [d(u_1,v_1)^2 + d(u_2,v_2)^2]$$

$$= \sum_{v\in V(G_2)} \sum_{\substack{u_1u_2\in E(QG_1)\\ u_1,u_2\in V(QG_1)-V(G_1)}} [(d(u_1)+d(u_1)d(v))^2 + (d(u_2)+d(u_2)d(v))^2]$$

$$= \sum_{v\in V(G_2)} \sum_{\substack{u_1u_2\in E(QG_1)\\ u_1,u_2\in V(QG_1)-V(G_1)}} \Big[[d^2(u_1)+d^2(u_2)] + d^2(v)[d^2(u_1)+d^2(u_2)]$$

$$+\ 2d(v)[d^2(u_1)+d^2(u_2)]\Big]$$

Consider

$$S_3 = \sum_{\substack{u_1u_2\in E(QG_1)\\ u_1,u_2\in V(QG_1)-V(G_1)}} [d^2(u_1)+d^2(u_2)]$$

In S_3, coefficient of d^2u = 2 $C^2_{d_{G_1}(u)}$ + $\sum_{v\in N(u)}$ d(v) – d(u)
= $d^2(u)$ – 2d(u) + $\sum_{v\in N(u)}$ d(v)
Therefore,

$$\sum_{u\in V(G_1)} d^2(u) = \sum_{u\in V(G_1)} [d^2(u) - 2d(u) + \sum_{v\in N(u)} d(v)]d^2(u)$$

$$= M_4(G_1) - 2F(G_1) + \sum_{u\in V(G_1)} d^2(u) \sum_{v\in N(u)} d(v)$$

For coefficient of dudv, let $u_1u_2 \in E(QG_1)$ with $u_1 = uv$ and $u_2 = wz$. As $u_1u_2 \in E(QG_1)$, we have either $v = w$ or z or $u = w$ or z. So uv is adjacent to all those vertices in G_1 which are adjacent to u and v. So number of such $dudv$ is $(du + dv - 2)$.

Therefore,

$$2\sum_{uv\in E(G_1)} dudv = 2\sum_{uv\in E(G_1)} (du+dv-2)dudv$$

$$= 2\sum_{uv\in E(G_1)} (du+dv)dudv - 4\sum_{uv\in E(G_1)} dudv$$

$$= 2M_2^1(G_1) - 4M_2(G_1)$$

So,

$$S_3 = M_4(G_1) - 2F(G_1) + \sum_{u\in V(G_1)} d^2(u) \sum_{v\in N(u)} d(v)$$

$$+\ 2M_2^1(G_1) - 4M_2(G_1)$$

$$So \sum_2'' = [n_2 + 4e_2 + M_1(G_2)][M_4(G_1) - 2F(G_1) + 2M_2^1(G_1) - 4M_2(G_1)]$$

$$+ \sum_{u\in V(G_1)} d^2(u) \sum_{v\in N(u)} d(v)$$

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$$\sum{}_2 = 2[e_1 + M_!(G_1)]M_1(G_2) + 4e_2M_1(G_1) + [n_2 + 4e_2 + M_1(G_2)][M_4(G_1) + F(G_1) + \sum_{u\in V(G_1)} d^2(u) \sum_{v\in N(u)} d(v) + 2M_2^1(G_1)]$$

$$\sum{}_3 = \sum_{v_1v_2\in E(G_2)} \sum_{u_1u_2\in E(Q(G_1))} [d(u_1,v_1)^2 + d(u_2,v_2)^2]$$
$$= \sum_{v_1v_2\in E(G_2)} \sum_{\substack{u_1u_2\in E(Q(G_1))\\ u_1\in V(G_1),u_2\in V(Q(G_1))-V(G_1)}} [d(u_1,v_1)^2 + d(u_2,v_2)^2] + \sum_{v_1v_2\in E(G_2)} \sum_{\substack{u_1u_2\in E(Q(G_1))\\ u_1,u_2\in V(Q(G_1))-V(G_1)}} [d(u_1,v_1)^2 + d(u_2,v_2)^2]$$
$$= \sum_3{}' + \sum_3{}''.$$

$$\sum_3{}' = \sum_{v_1v_2\in E(G_2)} \sum_{\substack{u_1u_2\in E(Q(G_1))\\ u_1\in V(G_1),u_2\in V(Q(G_1))-V(G_1)}} [d(u_1,v_1)^2 + d(u_2,v_2)^2]$$
$$= \sum_{v_1v_2\in E(G_2)} \sum_{\substack{u_1u_2\in E(Q(G_1))\\ u_1\in V(G_1),u_2\in V(Q(G_1))-V(G_1)}} [(d(u_1) + d(v_1) + d(u_1)d(v_1))^2 + (d(u_2) + d(u_2)d(v_2))^2]$$
$$= \sum_{v_1v_2\in E(G_2)} \sum_{\substack{u_1u_2\in E(Q(G_1))\\ u_1\in V(G_1),u_2\in V(Q(G_1))-V(G_1)}} [d^2(u_1) + d^2(u_1)d^2(v_1) + 2d(u_1)d(v_1) + 2d(u_1)d^2(v_1) + 2d^2(u_1)d(v_1) + d^2(v_1) + d^2(u_2) + d^2(u_2)d^2(v_2) + 2d^2(u_2)d(v_2)]$$
$$= 2e_2F(G_1) + F(G_1)F(G_2) + 2M_1(G_1)M_1(G_2) + 2M_1(G_1)F(G_2) + 2F(G_1)M_1(G_2) + 2e_1F(G_2) + [4e_2 + 2F(G_2) + 4M_1(G_2)][F(G_1) + 2M_2(G_1)]$$
$$= 6[e_2 + M_1(G_2)]F(G_1) + 3F(G_1)F(G_2) + 2M_1(G_1)M_1(G_2) + 2[e_1 + M_1(G_1) + 2M_2(G_1)]F(G_2) + 8M_2(G_1)[e_2 + M_1(G_2)],$$

$$\sum_3{}'' = \sum_{v_1v_2\in E(G_2)} \sum_{\substack{u_1u_2\in E(Q(G_1))\\ u_1,u_2\in V(Q(G_1))-V(G_1)}} [d(u_1,v_1)^2 + d(u_2,v_2)^2]$$
$$= \sum_{v_1v_2\in E(G_2)} \sum_{\substack{u_1u_2\in E(Q(G_1))\\ u_1,u_2\in V(Q(G_1))-V(G_1)}} [(d(u_1) + d(u_1)d(v_1))^2(d(u_2) + d(u_2)d(v_2))^2]$$
$$= \sum_{v_1v_2\in E(G_2)} \sum_{\substack{u_1u_2\in E(Q(G_1))\\ u_1,u_2\in V(Q(G_1))-V(G_1)}} \big[[d^2(u_1) + d^2(u_2)] + [d^2(u_1)d^2(v_1) + d^2(u_2)d^2(v_2)] + 2[d^2(u_1)d(v_1) + d^2(u_2)d(v_2)]\big]$$
$$= 2e_2[F(G_1) + 2M_2(G_1)] + F(G_2)[F(G_1) + 2M_2(G_1)] + 2M_1(G_2)[F(G_1) + 2M_2(G_1)],$$

$$\sum{}_3 = 8[e_2 + M_1(G_2)]F(G_1) + 4F(G_1)F(G_2) + 2M_1(G_1)M_1(G_2) + 2[e_1 + M_1(G_1) + 3M_2(G_1)]F(G_2) + 12[e_2 + M_1(G_2)]M_2(G_1).$$

Hence,

$$\begin{aligned}F(G_1 \circ_Q G_2) =&[n_2 + 12e_2 + 9M_1(G_2)]F(G_1) + [n_1 + 6e_1 + 3M_1(G_1) + 6M_2(G_1)]F(G_2)\\&+ [n_2 + 4e_2 + M_1(G_2)][M_4(G_1) + 2M_2^1(G_1) + \sum_{u\in V(G_1)} d^2(u) \sum_{v\in N(u)} d(v)]\\&+ 6[e_2M_1(G_1) + e_1M_1(G_2) + 2(e_2 + M_1(G_2))M_2(G_1)]\\&+ 2[2F(G_1)F(G_2) + 3M_1(G_1)M_1(G_2)] \quad \blacksquare\end{aligned}$$

Combining Theorems 2 and 3 we get

Theorem 4.

$$\begin{aligned}F(G_1 \circ_T G_2) =& [8n_2 + 54e_2]F(G_1) + [n_1 + 12e_1]F(G_2) + 11F(G_1)F(G_2) + 24e_2M_1(G_1)\\&+ 12[e_1M_1(G_2) + e_2M_2(G_1)] + 24M_1(G_1)M_1(G_2) + 12M_1(G_1)F(G_2)\\&+ 30F(G_1)M_1(G_2) + 6M_2(G_1)[2M_1(G_2) + F(G_2)]\\&+ [n_2 + M_1(G_2) + 4e_2][M_4(G_1) + 2M_2^1(G_1) + \sum_{u\in V(G_1)} d^2(u) \sum_{v\in N(u)} d(v)]\end{aligned}$$

Example 1. Applying theorems above to the graphs $G_1 = P_m$ and $G_2 = P_n$, we have $n_1 = |V(G_1)| = m$, $n_2 = |V(G_2)| = n$, $e_1 = |E(G_1)| = m - 1$, $e_2 = |E(G_1)| = n - 1$, $M_1(G_1) = 4m - 6$, $M_1(G_2) = 4n - 6$, $M_2(G_1) = 4(m - 2)$, where $m > 2$, $M_2(G_2) = 4(n - 2)$, where $n > 2$, $M_4(G_1) = 16m - 30$, $M_2^1(G_1) = 16m - 36$, where $m \geq 3$, $\sum_{u\in V(G_1)} d^2(u) \sum_{v\in N(u)} d(v) = 16m - 36$, $F(G_1) = 8m - 14$ and $F(G_2) = 8n - 14$. And

$$F(P_m \circ_S P_n) = 728mn - 1078m - 990n + 1460, \text{ if } m \geq 2, n \geq 2;$$

$$F(P_m \circ_R P_n) = 2960mn - 4334m - 4680n + 6816, \text{ if } m \geq 2, n \geq 2;$$

$$F(P_m \circ_Q P_n) = 1952mn - 2758m - 3636n + 5056, \text{ if } m \geq 3, n \geq 2;$$

$$F(P_m \circ_T P_n) = 4184mn - 6014m - 7326n + 10412, \text{ if } m \geq 3, n \geq 2.$$

Example 2. Applying theorems above to the graphs $G_1 = C_m$ and $G_2 = C_n$, we have $n_1 = |V(G_1)| = m$, $n_2 = |V(G_2)| = n$, $e_1 = |E(G_1)| = m$, $e_2 = |E(G_1)| = n$, $M_1(G_1) = 4$ m, $M_1(G_2) = 4n$, $M_2(G_1) = 4$ m, $M_2(G_2) = 4n$, where $m, n \geq 3$, $M_4(G_1) = 16$ m, $M_2^1(G_1) = 16$ m, $\sum_{u\in V(G_1)} d^2(u) \sum_{v\in N(u)} d(v) = 16$ m, $F(G_1) = 8$ m, $F(G_2) = 8n$. And

$$F(C_m \circ_S C_n) = 728mn, \text{ if } m, n \geq 3;$$

$$F(C_m \circ_R C_n) = 2960mn, \text{ if } m, n \geq 3;$$

$$F(C_m \circ_Q C_n) = 1952mn, \text{ if } m, n \geq 3;$$

$$F(C_m \circ_T C_n) = 4184mn, \text{ if } m, n \geq 3.$$

Example 3. Applying theorems above to the graphs $G_1 = P_m$ and $G_2 = C_n$, we have $n_1 = |V(G_1)| = m$, $n_2 = |V(G_2)| = n$, $e_1 = |E(G_1)| = m - 1$, $e_2 = |E(G_1)| = n$, $M_1(G_1) = 4m - 6$, $M_1(G_2) = 4n$, $M_2(G_1) = 4m - 8$, $M_2(G_2) = 4n$, where $m \geq 2$, $n \geq 3$, $M_4(G_1) = 16m - 30$, $M_2^1(G_1) = 16m - 36$, where $m \geq 3$, $\sum_{u\in V(G_1)} d^2(u) \sum_{v\in N(u)} d(v) = 16m - 36$, $F(G_1) = 8m - 14$, $F(G_2) = 8n$. And

$$F(P_m \circ_S C_n) = 728mn - 990n, \text{ if } m \geq 2, n \geq 3;$$

$$F(P_m \circ_R C_n) = 2960mn - 4680n, \text{ if } m \geq 2, n \geq 3;$$

$$F(P_m \circ_Q C_n) = 1952mn - 3636n, \text{ if } m, n \geq 3;$$

$$F(P_m \circ_T C_n) = 4184mn - 7326n, \text{ if } m, n \geq 3.$$

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3. Summary and conclusions

In this work, we study the F index of graphs $G_1 \circ_F G_2$ based on operations related to the strong product, subdivision and total graph, and obtain the expressions for $F(G_1 \circ_S G_2)$, $F(G_1 \circ_R G_2)$, $F(G_1 \circ_Q G_2)$ and $F(G_1 \circ_T G_2)$ in terms of F indices of G_1, G_2, the first Zagreb indices of G_1, G_2, and the second Zagreb index of G_2, and use them to compute the F index of $P_m \circ_F P_n$, $C_m \circ_F C_n$ and $P_m \circ_F C_n$.

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