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Interval Consistency Repairing Method for Double Hierarchy Hesitant Fuzzy Linguistic Preference Relation and Application in the Diagnosis of Lung Cancer

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ABSTRACT
Natural language is more in line with the real thoughts of people than crisp numbers considering that qualitative language information is more consistent with the expression habits of experts. Double hierarchy hesitant fuzzy linguistic preference relation (DHHFLPR) can be used to express complex linguistic preference information accurately because the pairwise comparison methods are more accurate than non-pairwise methods. Consistency reflects the rationalization of a preference relation and can be used to judge whether a preference relation is self-contradictory or not. In this paper, an interval consistency index of DHHFLPR is developed, which is consisted by the consistency indices of all double hierarchy linguistic preference relations associated with the DHHFLPR. Additionally, an average consistency index of DHHFLPR is given by calculating the average value of the consistency indices of all double hierarchy linguistic preference relations. Moreover, we develop a consistency checking and repairing method for DHHFLPR. Finally, we apply the proposed method into a practical group decision-making problem that is to identify the most critical factors in developing lung cancer, and some comparative analyses involving the connections and differences among the proposed consistency indices are analysed.

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1. Introduction

Considering the complexity of human cognition, sometimes qualitative language information is more consistent with the expression habits of experts than crisp numbers because we usually utilize natural languages to talk with others and express emotions or comments on something. Therefore, the concept of computing with word (CWW) (Zadeh, 2012) was proposed to deal with decision-making problems with linguistic information based on the fuzzy linguistic approach proposed by Zadeh (1975). Based on fuzzy linguistic approach, lots of linguistic representation models have been

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developed such as type-2 linguistic model (Zadeh, 1975), 2-tuple linguistic representation model (Herrera & Martínez, 2000; Wei & Gao, 2020), virtual linguistic term set (Xu & Wang, 2017), hesitant fuzzy linguistic term set (HFLTS) (Rodríguez et al., 2012) and probabilistic linguistic term set (Pang et al., 2016; Wei et al., 2020), etc. In practical decision-making processes, it is common that we need to deal with some complex linguistic terms such as "just right good" and "only a little high". To express the complex linguistic information accurately, Gou et al. (2017) introduced a new complex linguistic representation model named double hierarchy linguistic term set (DHLTS). Different from the traditional linguistic representation models, DHLTS consists of two hierarchy linguistic term sets, i.e., the first hierarchy linguistic term set and the second hierarchy linguistic term set. Specially, the second hierarchy linguistic term set is a linguistic feature or details supplementary of each linguistic term included in the first hierarchy linguistic term set. In addition, considering that complex uncertain linguistic information is common in decision-making processes, to represent this kind of linguistic information, Gou et al. (2017) extended the DHLTS to hesitant fuzzy environment and developed the concept of double hierarchy hesitant fuzzy linguistic term set (DHHFLTS). During the past several years, lots of research results about double hierarchy linguistic information have been developed such as linguistic preference ordering (Gou et al., 2020d), preference relations (Gou et al., 2018, 2019, 2020a, 2020b, 2020c), measure methodologies (Fu & Liao, 2019; Gou, Xu, et al., 2018) and decision-making methodologies (Krishankumar et al., 2019, Liu et al., 2019; Montserrat-Adell et al., 2019; Wang et al., 2020), etc.

Preference relations are more and more popular and have been utilized to model experts' preference information according to practical decision-making problems (Liao et al., 2018). In recent years, lots of preference relations have been proposed such as fuzzy preference relations (FPRs) (Alonso et al., 2008), hesitant fuzzy preference relations (HFPRs) (Liao et al., 2014), fuzzy linguistic preference relations (Alonso et al., 2009), hesitant fuzzy linguistic preference relations (HFLPRs) (Zhu & Xu, 2014), probabilistic linguistic preference relations (PLPRs) (Zhang et al., 2016) and linguistic preference relations with hedges (Wang et al., 2019), etc. Based on DHHFLTS, Gou et al. (2019) proposed the concept of double hierarchy hesitant fuzzy linguistic preference relation (DHHFLPR). As we know, the consistency issue of preference relation is very essential and important in decision-making processes, which reflects the rationalization of a preference relation since the lack of consistency may lead to inconsistent results. Therefore, the consistency checking and repairing approaches have been studied including additive consistency (Gou et al., 2019) and multiplicative consistency (Gou et al., 2020a). However, in the existing work, there exist some detected gaps in the consistency researches of the DHHFLPRs:

1. When introducing the concept of additive consistency index of DHHFLPRs, Gou et al. (2019) gave a normalization method by introducing the linguistic expected-value of double hierarchy hesitant fuzzy linguistic element (DHHFLE, i.e., the basic element of a DHHFLPR). Even though this normalization method reduces the dimension of all DHHFLEs and makes the calculation much simpler, the diversity of consistency index of original linguistic information is lost. In

addition, the multiplicative consistency index of DHHFLPRs is based on another normalization method by extending the short DHHFLEs and making all DHHFLEs have the same length (Gou et al., 2020a). However, the shortcoming of this method is that the original linguistic information is changed, so the accuracy of this consistency index will be greatly reduced.
2. When calculating the consistency index of a DHHFLPR, the necessary process mainly contains the normalization of DHHFLPRs, the calculation of additive consistent DHHFLPR or multiplicative consistent DHHFLPR, and the acquisition of consistency index by computing the distance between the normalized DHHFLPR and the additive consistent DHHFLPR or multiplicative consistent DHHFLPR. The calculation process is complex.
3. Generally, we can obtain only a result about the consistency index of a DHHFLPR, and the result is related to the parameter which is used to obtain the normalized DHHFLPR. Based on this result, we can only obtain partial result about the consistency index of DHHFLPR. To understand the consistency degree of DHHFLPR more comprehensively, it is necessary to develop a method to obtain all possible consistency indices associated with a DHHFLPR.

To overcome the previous shortcomings, in this paper, an interval consistency index (ICI) of a DHHFLPR is developed. The main contributions of this paper are listed as follows:

1. By dividing a DHHFLPR into some related double hierarchy linguistic preference relations (DHLPRs) (Gou et al., 2020b) and calculating the consistency indices of them, as well as proposing optimization-based models, we can obtain the worst consistency index (WCI) and the best consistency index (BCI) of a DHHFLPR and establish the ICI of this DHHFLPR by taking them as the lower and upper bounds, respectively. The ICI consists of all possible consistency indices of DHLPRs associated with the DHHFLPR. Additionally, a 0-1 linear programming is set up to obtain the optimum solutions of the given optimization-based models.
2. This paper gives a new concept of the average consistency index (ACI) of DHHFLPR by calculating the average value of the consistency indices of all DHLPRs associated with the DHHFLPR. Meanwhile, we find that the ACI and the normalized consistency indices (NCIs) obtained from (Gou et al., 2019, 2020a) are mostly equal. Therefore, the NCI can be regarded as an approximate reflection of the ACI.
3. Based on the ICI and ACI, we can check whether a DHHFLPR is of acceptable consistency or not. If not, we develop a consistency repairing method to improve the consistency. In this method, we can only adjust the related DHLPRs which have the smallest consistency indices, so the consistency repairing process will be much simpler.
4. We apply the proposed consistency checking and repairing methods into a practical group decision-making (GDM) method that is to identify the most critical factors associated with lung cancer.

5. The comparative analyses involving the connections and differences between these indices, ICI, NCIs and ACI, are analysed. Based on the discussions, it is better to utilize both the indices ICI and NCIs (or ACI) to reflect the consistency degrees of a DHHFLPR synthetically.

To do so, the rest of this paper is organized as follows: Section 2 reviews some concepts about the DHLTS, DHHFLTS and DHHFLPR. Section 3 proposes some consistency indices of DHLPRs and DHHFLPRs. Section 4 introduces the concepts of indices ICI and ACI of a DHHFLPR to evaluate the consistency degree of the DHHFLPR from different angles. Section 5 applies the proposed methods into a practical GDM method, and then makes some comparative analyses to obtain the connections and differences among the indices ICI, NCIs and ACI of DHHFLPRs. Finally, some concluding remarks and future research directions are summarized in Section 6.

2. Preliminaries

In this section, some concepts of DHLTS, DHHFLTS and DHHFLPR are reviewed.

Suppose that $S = \{s_t | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau\}$ and $O = \{o_k | k = -\varsigma, \ldots, -1, 0, 1, \ldots, \varsigma\}$ are the first hierarchy and the second hierarchy linguistic term set, respectively, and they are fully independent. A DHLTS, S_O, is in mathematical form of

$$S_O = \left\{ s_{t<o_k>} | t = -\tau, \ldots, -1, 0, 1, \ldots, \tau; k = -\varsigma, \ldots, -1, 0, 1, \ldots, \varsigma \right\} \tag{1}$$

We call $s_{t<o_k>}$ the double hierarchy linguistic term (DHLT), where o_k expresses the second hierarchy linguistic term when the first hierarchy linguistic term is s_t (Gou et al., 2017).

For a DHLTS as Eq. (1), considering that the linguistic labels of the first hierarchy linguistic term set are symmetrical distributions on both sides, and the semantics of linguistic terms on both sides are different and opposite. Therefore, the semantics of linguistic terms in the second hierarchy linguistic term sets should be also different. Let $S_O = \left\{ s_{t<o_k>} | t = -3, -2, -1, 0, 1, 2, 3; k = -3, -2, -1, 0, 1, 2, 3 \right\}$ be a DHLTS and $S = \{s_{-3} = none, s_{-2} = verylow, s_{-1} = low, s_0 = medium, s_1 = high, s_2 = veryhigh, s_3 = perfect\}$ and $O_3 = \{o_{-3} = farfrom, o_{-2} = onlyalittle, o_{-1} = alittle, o_0 = justright, o_1 = much, o_2 = verymuch, o_3 = entirely\}$ be the first hierarchy linguistic term set and the second hierarchy linguistic term set, respectively. Figure 1 shows the distributions of four parts of the second hierarchy linguistic term sets (Gou et al., 2017):

Remark 1. As depicted in Figure 1, the second hierarchy linguistic term sets can be classified into four main types according to the subscript t of the first hierarchy linguistic terms. When $t \geq 0$, the semantics of linguistic terms in the second hierarchy linguistic term set should be denoted as ascending order because of the meaning of the first hierarchy linguistic term set $S = \{s_t | t \geq 0\}$ is positive. On the contrary, the semantics of linguistic terms in the second hierarchy linguistic term set needs to be

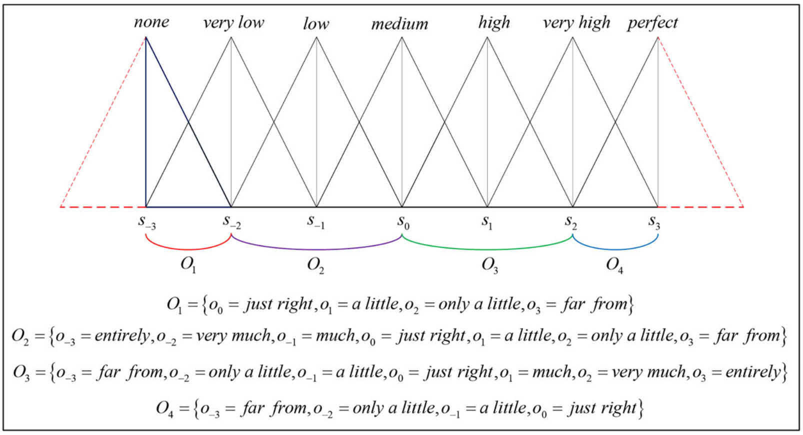

Figure 1. The distributions of four parts of the second hierarchy linguistic term sets.
Source: Gou et al. (2017).

selected with descending order when $t<0$. Specially, considering that both endpoint values s_τ and $s_{-\tau}$ only contain a half of area in Fig. 1, so Gou et al. (2017) only utilized $O_4 = \{o_k | k = -\varsigma, \ldots, -1, 0\}$ and $O_1 = \{o_k | k = 0, 1, \ldots, \varsigma\}$ to describe them, respectively. Specially, the second hierarchy linguistic term sets with respect to different first hierarchy linguistic term may be different. For convenience, we only utilize a uniform form $O = \{o_k | k = -\varsigma, \ldots, -1, 0, 1, \ldots, \varsigma\}$ to express all kinds of second hierarchy linguistic term sets.

Motivated by (Gou et al., 2017), an equivalent transformation function can be given to transform DHLTs into real numbers equivalently via extending the DHLTS S_O to a continuous DHLTS (CDHLTS) $\bar{S}_O = \left\{ s_{t<o_k>} \middle| t \in [-\tau, \tau]; k \in [-\varsigma, \varsigma] \right\}$.

Definition 1. (Gou et al., 2017). Let $\bar{S}_O$ be a continuous DHLTS, and $\gamma \in [0, 1]$ be a real number. Then, a function

$$f : [-\tau, \tau] \times [-\varsigma, \varsigma] \to [0, 1], f(s_{t<o_k>}) = \frac{t + (\tau + k)\varsigma}{2\varsigma\tau} = \gamma \tag{2}$$

is defined as a numerical scale of $\bar{S}_O$, and $f(s_{t<o_k>})$ is called the numerical index of the DHLT $s_{t<o_k>}$.

Then, Gou et al. (2017) extended S_O into hesitant fuzzy linguistic circumstance and developed the DHHFLTS: A DHHFLTS on X, H_{S_O}, is in mathematical form of $H_{S_O} = \{< x_i, h_{S_O}(x_i) > | x_i \in X\}$, where $h_{S_O}(x_i)$ is a set of some values in S_O, denoted as:

$$\begin{aligned} h_{S_O}(x_i) = \{ s_{\phi_l<o_{\varphi_l}>}(x_i) | s_{\phi_l<o_{\varphi_l}>} \in S_O; l = 1, 2, \ldots, L; \\ \phi_l \in \{-\tau, \ldots, -1, 0, 1, \ldots, \tau\}; \varphi_l \in \{-\varsigma, \ldots, -1, 0, 1, \ldots, \varsigma\}\} \end{aligned} \tag{3}$$

with L being the number of the DHLTs in $h_{S_O}(x_i)$ and $s_{\phi_l<o_{\varphi_l}>}(x_i)$ $(l = 1, 2, \ldots, L)$ in each $h_{S_O}(x_i)$ being the terms in S_O. $h_{S_O}(x_i)$ denotes the possible degree of the linguistic variable x_i to S_O. For convenience, we call $h_{S_O}(x_i)$ the DHHFLE.

Similar to the negation operation of the DHLT, the negation operation of the DHHFLE $h_{S_{O_{ij}}} = \left\{ h_{S_{O_{ij}}}^{(l)} \middle| l = 1, 2, \ldots, h_{S_{O_{ij}}} \right\}$ can be denoted by $Neg(h_{S_{O_{ij}}}) = \left\{ Neg(h_{S_{O_{ij}}}^{(l)}) \middle| \; l = 1, 2, \ldots, h_{S_{O_{ij}}} \right\}$ ($h_{S_{O_{ij}}}$ is the number of DHLTs in $h_{S_{O_{ij}}}$, $h_{S_{O_{ij}}}^{(l)}$ is the l–th DHLT in $h_{S_{O_{ij}}}$) . Then the concept of DHHFLPR is redefined as follows:

Definition 2. A DHHFLPR $\tilde{H}_{S_O}$ based on S_O is represented by a matrix $\tilde{H}_{S_O} = (h_{S_{O_{ij}}})_{m \times m}$, where $h_{S_{O_{ij}}} = \left\{ h_{S_{O_{ij}}}^{(l)} \middle| l = 1, 2, \ldots, h_{S_{O_{ij}}} \right\}$ is a DHHFLE, and $h_{S_{O_{ij}}}(i < j)$ satisfies $Neg(h_{S_{O_{ij}}}) = h_{S_{O_{ji}}}(i, j = 1, 2, \ldots, m)$ and $h_{S_{O_{ii}}} = \{ s_{0<o_0>} \}$.

3. Consistency indices of DHLPR and repairing method

In this section, the consistency indices of DHLPRs are developed based on distance measures at first. Additionally, a consistency repairing method for DHLPRs is proposed.

3.1. Consistency indices of DHLPR

Let $A = \{A_1, A_2, \ldots, A_m\} (m \geq 2)$ be a finite set of alternatives in a GDM problem. By making pairwise comparisons among alternatives under double hierarchy linguistic circumstance, and collecting the experts' preference information, the DHLPR of each expert is established and denoted by $R = (r_{ij})_{m \times m} \subset A \times A$, where the element r_{ij} expresses the preference degree of the alternative A_i to A_j.

Additionally, the negation operation of a DHLT $s_{t<o_k>}$ can be denoted by $Neg(s_{t<o_k>}) = s_{-t<o_{-k}>}$. Unlike the definition of DHLPR given in (Gou et al., 2020b), the concept of DHLPR can be redefined as follows:

Definition 3. A DHLPR R is presented by a matrix $R = (r_{ij})_{m \times m} \subset A \times A$, where $r_{ij} \in S_O$ $(i, j = 1, 2, \ldots, m)$ is a DHLT. r_{ij} satisfies $r_{ij} = Neg(r_{ji})$ and $r_{ii} = s_{0<o_0>}$.

Motivated by the additive transitivity of linguistic preferences (Alonso et al., 2008; Cabrerizo et al., 2010), an additive transitivity can be developed to characterize the consistency of DHLPR:

Definition 4. Let $R = (r_{ij})_{m \times m} \subset A \times A$ be a DHLPR on the basis of the DHLTS S_O. R is consistent if $f(r_{ij}) + f(r_{jk}) - f(r_{ik}) = \frac{1}{2}$ for $i, j, k = 1, 2, \ldots, m$.

Based on Definition 4, the consistency index of a DHLPR $R = (r_{ij})_{m \times m} \subset A \times A$ can be defined by utilizing Manhattan distance and Euclidean distance (Alonso et al., 2009; Dong et al., 2015; Gou et al., 2018a; Zhu & Xu, 2014), respectively.

Let $\sigma_{ijk} = f(r_{ij}) + f(r_{jk}) - f(r_{ik}) - \frac{1}{2}$. Then the consistency index of DHLPR based on the Manhattan distance can be defined as follows:

$$CI_M(R) = 1 - \frac{2}{3m(m-1)(m-2)} \sum_{i,j,k=1}^{m} |\sigma_{ijk}| \tag{4}$$

Similarly, the consistency index of DHLPR based on the Euclidean distance is established as follows:

$$CI_E(R) = 1 - \frac{2}{3}\sqrt{\frac{1}{m(m-1)(m-2)}\sum_{i,j,k=1}^{m}(\sigma_{ijk})^2} \tag{5}$$

Remark 2. Two key points need to be clarified:

1. Since $f(r_{ij}), f(r_{jk}), f(r_{ik}) \in [0,1]$, it is obvious that $\sigma_{ijk} - \frac{1}{2} \in \left[-\frac{3}{2}, \frac{3}{2}\right]$. Therefore, by adding a parameter $\frac{2}{3}$ to Eq. (4) and Eq. (5), the value of the consistency index can be limited to the interval $[0,1]$.
2. The larger the value of $CI(R)$ is, the more consistent the DHLPR R will be. If $CI(R) = 1$, then R is a consistent DHLPR.

For Eq. (4) and (5), considering the special structure of the DHLPR, there exist some repeating information when using $\sum_{i,j,k=1}^{m}|\sigma_{ijk}|$ and $\sum_{i,j,k=1}^{m}(\sigma_{ijk})^2$, which makes calculations more complicated. To avoid this shortcoming, we can only use the preference information in the upper triangle of the DHLPR to simplify Eq. (4) and Eq. (5), respectively:

$$CI_M(R) = 1 - \frac{2}{3\sum_{b=1}^{m-2} b(m-b-1)}\sum_{i<j<k}^{m}|\sigma_{ijk}| \tag{6}$$

$$CI_E(R) = 1 - \frac{2}{3}\sqrt{\frac{1}{\sum_{b=1}^{m-2} b(m-b-1)}\sum_{i<j<k}^{m}(\sigma_{ijk})^2} \tag{7}$$

3.2. Consistency repairing method for DHLPR

By Eq. (6) or (7), we can obtain the consistency index of a DHLPR. Then, based on the consistency thresholds provided by Gou et al. (2019) shown in Table 1, we can check whether a DHLPR is of acceptable consistency.

In Table 1, m is the number of the alternatives in a DHLPR, and T is the number of linguistic labels in the first hierarchy linguistic term set in a DHLTS.

Table 1. The values of consistency thresholds based on different m and T.

	$m=3$	$m=4$	$m=5$	$m=6$	$m=7$	$m=8$
$T=5$	0.8793	0.6970	0.6512	0.6226	0.6030	0.5888
$T=9$	0.8897	0.8485	0.8256	0.8113	0.8015	0.7944
$T=17$	0.9448	0.9242	0.9128	0.9056	0.9007	0.8972

Source: Gou et al. (2019).

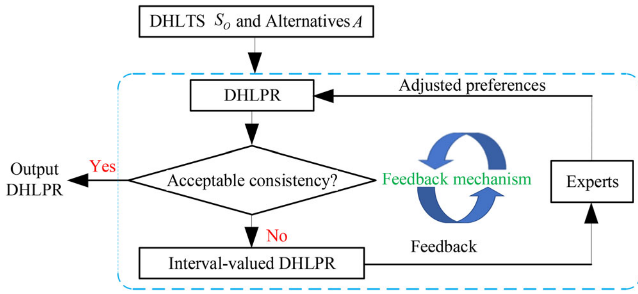

Figure 2. The consistency repairing flow of Algorithm 1.
Source: The Authors.

If the consistency index of a DHLPR is less than the corresponding consistency threshold, we can develop a consistency repairing method to improve the consistency index. Firstly, it is necessary to define a concept of additive consistent DHLPR.

Definition 5. Let $R = (r_{ij})_{m\times m} \subset A \times A$ be a DHLPR. If $\bar{r}_{ij} = \frac{1}{m}(\bigoplus_{\rho=1}^{m}(f(r_{i\rho}) + f(r_{i\rho}) - \frac{1}{2}))$ for all $i, j, \rho = 1, 2, \ldots, m; i \neq j$, then $\bar{R} = (\bar{r}_{ij})_{m\times m}$ is an additive consistent DHLPR.

Then, the consistency repairing method for a DHLPR is established as follows:

Algorithm 1. The consistency repairing method for a DHLPR

Step 1. Let $R^{(\mathbb{Z})} = ((r_{ij})_{m\times m})^{(\mathbb{Z})}$ ($\mathbb{Z} = 0, R^{(\mathbb{Z})}$ expresses the $\mathbb{Z}-$th power of R, indicating the number of iterations). Based on Definition 5, we can obtain the additive consistent DHLPR $\bar{R}^{(\mathbb{Z})} = ((\bar{r}_{ij})_{m\times m})^{(\mathbb{Z})}$.

Step 2. Obtain the $\overline{CI}$ based on Table 1.

Step 3. Calculate the consistency index $CI(R^{(\mathbb{Z})})$ based on Eq. (6) or (7). If $CI(R^{(\mathbb{Z})}) \geq \overline{CI}$, then go to Step 6; If not, then go to Step 4.

Step 4. Establish an interval-valued DHLPR $\tilde{R}^{(\mathbb{Z})} = ((\tilde{r}_{ij})_{m\times m})^{(\mathbb{Z})}$, where the element $(\tilde{r}_{ij})^{(\mathbb{Z})} = \left[\min\left\{(r_{ij})^{(\mathbb{Z})}, (\bar{r}_{ij})^{(\mathbb{Z})}\right\}, \max\left\{(r_{ij})^{(\mathbb{Z})}, (\bar{r}_{ij})^{(\mathbb{Z})}\right\}\right]$. Then feedback $\tilde{R}^{(\mathbb{Z})}$ to the corresponding expert and ask him/her to adjust the DHLPR based on $\tilde{R}^{(\mathbb{Z})}$.

Step 5. Receive the feedback adjusted preference information and obtain new DHLPR $R^{(\mathbb{Z}+1)} = ((r_{ij})_{m\times m})^{(\mathbb{Z}+1)}$. Let $\mathbb{Z} = \mathbb{Z} + 1$. Go back to Step 3.

Step 6. Let $*R = R^{(\mathbb{Z})}$, and output the adjusted DHLPR $*R$.

Figure 2 can be drawn to show the consistency repairing flow of Algorithm 1.

4. Interval consistency index of the DHHFLPR

When researching the additive consistency and multiplicative consistency of a DHHFLPR (Gou et al., 2019, 2020a), the scholars gave two kinds of synthesized NCIs of a DHHFLPR, but they are only approximate values. Therefore, in this section, based on a mixed 0-1 linear programming model, we shall develop an interval consistency index to get the range of the consistency indices of a DHHFLPR by collecting the consistency indices of all DHLPRs associated with the original DHHFLPR. Additionally, based on the consistency indices of all DHLPRs, an ACI of the DHHFLPR is developed.

4.1. A method to estimate the ICI of a DHHFLPR

Before introducing the ICI of a DHHFLPR, we need to obtain all DHLPRs divided from the DHHFLPR and calculate the consistency indices of these DHLPRs. Then, the ICI of the DHHFLPR can be established by collecting all consistency indices of DHLPRs. For a DHHFLPR $\tilde{H}_{S_O} = (h_{S_{O_{ij}}})_{m\times m}$, we can extract any one DHLT from each DHHFLE and then a DHLPR can be established as follows:

Definition 6. Let $\tilde{H}_{S_O} = (h_{S_{O_{ij}}})_{m\times m}$ be a DHHFLPR. $R = (r_{ij})_{m\times m}$ is a DHLPR associated with $\tilde{H}_{S_O}$, if $r_{ij} \in h_{S_{O_{ij}}}$, $r_{ij} = Neg(r_{ji})$ and $r_{ii} = s_{0<o_0>}$.

Additionally, we denote $\tilde{R}$ as the set of all DHLPRs associated with $\tilde{H}_{S_O}$. The number of all DHLPRs included in $\tilde{R}$ is denoted by N_R. Moreover, let $h_{S_{O_{ij}}}$ be the number of DHLTs of each DHHFLE included in the DHHFLPR $\tilde{H}_{S_O}$. Then, the number of the DHLPRs associated with the DHHFLPR is $N_R = \prod_{i=1}^{m-1}\prod_{j>i}^{m} h_{S_{O_{ij}}}$.

For example, suppose that

$$\tilde{H}_{S_O} = \begin{pmatrix} \{s_{0<o_0>}\} & \{s_{1<o_2>}\} & \{s_{1<o_0>}, s_{2<o_{-1}>}\} \\ \{s_{-1<o_{-2}>}\} & \{s_{0<o_0>}\} & \{s_{-1<o_2>}\} \\ \{s_{-1<o_0>}, s_{-2<o_1>}\} & \{s_{1<o_{-2}>}\} & \{s_{0<o_0>}\} \end{pmatrix}$$

is a DHHFLPR. Then, the related DHLPRs are

$$H^1_{S_O} = \begin{pmatrix} \{s_{0<o_0>}\} & \{s_{1<o_2>}\} & \{s_{1<o_0>}\} \\ \{s_{-1<o_{-2}>}\} & \{s_{0<o_0>}\} & \{s_{-1<o_2>}\} \\ \{s_{-1<o_0>}\} & \{s_{1<o_{-2}>}\} & \{s_{0<o_0>}\} \end{pmatrix} \text{and}$$

$$H^2_{S_O} = \begin{pmatrix} \{s_{0<o_0>}\} & \{s_{1<o_2>}\} & \{s_{2<o_{-1}>}\} \\ \{s_{-1<o_{-2}>}\} & \{s_{0<o_0>}\} & \{s_{-1<o_2>}\} \\ \{s_{-2<o_1>}\} & \{s_{1<o_{-2}>}\} & \{s_{0<o_0>}\} \end{pmatrix}$$

The ICI can be developed to estimate the consistency degree of a DHHFLPR by collecting all consistency indices of the related DHLPRs. The lower bound and upper bound of the ICI are the WCI and the BCI of the DHHFLPR $\tilde{H}_{S_O}$, respectively.

Definition 8. Let $\tilde{H}_{S_O} = (h_{S_{O_{ij}}})_{m\times m}$ be a DHHFLPR, and $R = (r_{ij})_{m\times m} \in \tilde{R}$ be the DHLPR associated with $\tilde{H}_{S_O}$. Then, the ICI of $\tilde{H}_{S_O}$ can be denoted as:

$$ICI(\tilde{H}_{S_O}) = \left[WCI(\tilde{H}_{S_O}), BCI(\tilde{H}_{S_O})\right] \tag{8}$$

where $WCI(\tilde{H}_{S_O}) = \min_{R\in\tilde{R}} CI(R)$ and $BCI(\tilde{H}_{S_O}) = \max_{R\in\tilde{R}} CI(R)$.

Based on Definition 6, Eqs. (6) and (7), two models can be established to calculate $WCI(\tilde{H}_{S_O})$ and $BCI(\tilde{H}_{S_O})$, respectively, on the basis of Manhattan distance:

Model 1 (WCI) $\min_{R\in\tilde{R}} \quad 1 - \frac{2}{3\sum_{b=1}^{m-2} b(m-b-1)} \sum_{i<j<k}^{m} \left| f(r_{ij}) + f(r_{jk}) - f(r_{ik}) - \frac{1}{2} \right|$

$$s.t. \begin{cases} r_{ij} \in h_{S_{O_{ij}}} \\ r_{ij} = Neg(r_{ji}) \end{cases}$$

Model 2 (BCI) $\max_{R\in\tilde{R}} \quad 1 - \frac{2}{3\sum_{b=1}^{m-2} b(m-b-1)} \sum_{i<j<k}^{m} \left| f(r_{ij}) + f(r_{jk}) - f(r_{ik}) - \frac{1}{2} \right|$

$$s.t. \begin{cases} r_{ij} \in h_{S_{O_{ij}}} \\ r_{ij} = Neg(r_{ji}) \end{cases}$$

The above two models can be solved by a mixed 0-1 linear programming to obtain the corresponding optimum consistency solutions. The 0-1 variable can be denoted as:

$$\vartheta_{ij}^{(l)} = \begin{cases} 0 & if r_{ij} \neq h_{S_{O_{ij}}}^{(l)} \\ 1 & if r_{ij} = h_{S_{O_{ij}}}^{(l)} \end{cases}, \quad i,j = 1,2,\ldots,m; l = 1,2,\ldots,h_{S_{O_{ij}}} \text{and} \sum_{l=1}^{h_{S_{O_{ij}}}} \vartheta_{ij}^{(l)} = 1$$

We can utilize ϑ_{ij}^{l} to express the situation $r_{ij} \in h_{S_{O_{ij}}}$ equivalently. For instance, let $h_{S_{O_{13}}} = \left\{ h_{S_{O_{13}}}^{(1)}, h_{S_{O_{13}}}^{(2)}, h_{S_{O_{13}}}^{(3)} \right\} = \{ s_{-1<o_2>}, s_{0<o_1>}, s_{1<o_3>} \}$ be a DHHFLE. If $\left\{ \vartheta_{13}^{(1)}, \vartheta_{13}^{(2)}, \vartheta_{13}^{(3)} \right\} = \{1,0,0\}$, then $r_{13} = s_{-1<o_2>}$.

Theorem 1. Let $\tilde{H}_{S_O} = (h_{S_{O_{ij}}})_{m\times m}$ be a DHHFLPR, and $r_{ij} \in h_{S_{O_{ij}}}$. If $\vartheta_{ij}^{(l)} = \{0,1\}$ and $\sum_{l=1}^{h_{S_{O_{ij}}}} \vartheta_{ij}^{(l)} = 1$, then,

$$f(r_{ij}) = \sum_{l=1}^{h_{S_{O_{ij}}}} \left(\vartheta_{ij}^{(l)} \times f\left(h_{S_{O_{ij}}}^{(l)} \right) \right) \tag{9}$$

Proof. Without loss of generality, suppose that $r_{ij} = h_{S_{O_{ij}}}^{(k)} \in h_{S_{O_{ij}}}$. Because $\vartheta_{ij}^{(l)} = \{0,1\}$ and $\sum_{l=1}^{h_{S_{O_{ij}}}} \vartheta_{ij}^{(l)} = 1$, then

$$f(r_{ij}) = f\left(h_{S_{O_{ij}}}^{(k)}\right) = \vartheta_{ij}^{(k)} \times f\left(h_{S_{O_{ij}}}^{(k)}\right) + \sum_{l=1, l\neq k}^{h_{S_{O_{ij}}}} \left(\vartheta_{ij}^{(l)} \times f\left(h_{S_{O_{ij}}}^{(l)} \right) \right) = \sum_{l=1}^{h_{S_{O_{ij}}}} \left(\vartheta_{ij}^{(l)} \times f\left(h_{S_{O_{ij}}}^{(l)} \right) \right) \tag{10}$$

This completes the proof of Theorem 1. ■

Based on the 0–1 variable $\vartheta_{ij}^{(l)} = \begin{cases} 0 & if r_{ij} \neq h_{S_{O_{ij}}}^{(l)} \\ 1 & if r_{ij} = h_{S_{O_{ij}}}^{(l)} \end{cases} (i,j = 1,2,,m; l = 1,2,\ldots,h_{S_{O_{ij}}})$. Model 1 and Model 2 can be equivalently transformed into the next models:

$$\textbf{Model 3.} \quad \min_{R \in \tilde{R}} \quad 1 - \frac{2}{3\sum_{b=1}^{m-2} b(m-b-1)} \sum_{i<j<k}^{m} \left| q_{ij} + q_{jk} - q_{ik} - \frac{1}{2} \right|$$

$$s.t. \begin{cases} q_{ij} = \sum_{l=1}^{h_{S_{O_{ij}}}} (\vartheta_{ij}^{(l)} \times f(h_{S_{O_{ij}}}^{(l)})), \\ i,j = 1,2,\ldots,m q_{ij} = 1 - q_{ji}, \\ i,j = 1,2,\ldots,m \vartheta_{ij}^{(l)} = \{0,1\}, \\ i,j = 1,2,,m; l = 1,2,\ldots,h_{S_{O_{ij}}} \\ \sum_{l=1}^{h_{S_{O_{ij}}}} \vartheta_{ij}^{(l)} = 1, \quad i,j = 1,2,\ldots,m \end{cases}$$

$$\textbf{Model 4.} \quad \max_{R \in \tilde{R}} \quad 1 - \frac{2}{3\sum_{b=1}^{m-2} b(m-b-1)} \sum_{i<j<k}^{m} \left| q_{ij} + q_{jk} - q_{ik} - \frac{1}{2} \right|$$

$$s.t. \begin{cases} q_{ij} = \sum_{l=1}^{h_{S_{O_{ij}}}} (\vartheta_{ij}^{(l)} \times f(h_{S_{O_{ij}}}^{(l)})), \\ i,j = 1,2,\ldots,m q_{ij} = 1 - q_{ji}, \\ i,j = 1,2,\ldots,m \vartheta_{ij}^{(l)} = \{0,1\}, \\ i,j = 1,2,,m; l = 1,2,\ldots,h_{S_{O_{ij}}} \\ \sum_{l=1}^{h_{S_{O_{ij}}}} \vartheta_{ij}^{(l)} = 1, \quad i,j = 1,2,\ldots,m \end{cases}$$

Remark 3. (1) Based on Theorem 1, $f(r_{ij}) = \sum_{l=1}^{h_{S_{O_{ij}}}} (\vartheta_{ij}^{(l)} \times f(h_{S_{O_{ij}}}^{(l)})) = q_{ij}$. $q_{ij} = 1 - q_{ji}$ is equivalent to $r_{ij} = Neg(r_{ji})$. Additionally, the 0-1 variable $\vartheta_{ij}^{(l)}$ satisfies $\vartheta_{ij}^{(l)} = \{0,1\}$ and $\sum_{l=1}^{h_{S_{O_{ij}}}} \vartheta_{ij}^{(l)} = 1$. Therefore, Model 1 and Model 2 can be equivalently transformed into Model 3 and Model 4, respectively.

(2) Both Model 3 and Model 4 utilize the Manhattan distance, and we can calculate these two mixed 0-1 linear programming models and obtain the ICI of the DHHFLPR. Furthermore, we can also utilize the Euclidean distance to calculate the ICI by solving the corresponding mixed 0-1 linear programming models. Considering that the results of both Euclidean distance and Manhattan distance are similar, we only consider the result obtained by the Manhattan distance in remaining Sections.

4.2. The average consistency measure of the DHHFLPR

As discussed in Section 4.1, the ICI of the DHHFLPR is developed by calculating the consistency indices of all DHLPRs associated with the DHHFLPR. Based on the consistency indices of all DHLPRs, the ACI of a DHHFLPR can be defined as follows:

Table 2. The indices ACI and ICI of $\tilde{H}^{a}_{S_O}(a=1,2,3)$, respectively.

	$\tilde{H}^{1}_{S_O}$	$\tilde{H}^{2}_{S_O}$	$\tilde{H}^{3}_{S_O}$
ICI	[0.9063,0.9688]	[0.6563,0.9792]	[0.8125,0.9271]
ACI	0.9388	0.8279	0.8715

Definition 7. Let $\tilde{H}_{S_O}=(h_{S_{O_{ij}}})_{m\times m}$ be a DHHFLPR, $R^z=(r^z_{ij})_{m\times m}\in\tilde{R}$ $(z=1,2,\ldots,N_{\mathbb{R}})$ be the DHLPR associated with $\tilde{H}_{S_O}$ and N_R be the number of R in $\tilde{R}$. Then, the ACI of $\tilde{H}_{S_O}$ is denoted as:

$$ACI\left(\tilde{H}_{S_O}\right)=\frac{1}{N_R}\sum_{z=1}^{N_R}CI(R^z) \tag{11}$$

where $CI(R^z)=1-\frac{2}{3\sum_{b=1}^{m-2}b(m-b-1)}\sum_{i<j<k}^{m}\left|f(r^z_{ij})+f(r^z_{jk})-f(r^z_{ik})-\frac{1}{2}\right|$ and $N_R=\prod_{i=1}^{m-1}\prod_{j>i}^{m-1}(h_{S_{O_{ij}}})$.

The ACI of a DHHFLPR can be used to reflect the average level of the consistency index of the DHHFLPR from a statistical point of view. Meanwhile, it is comprehensive to show the overall situation of consistency index about a DHHFLPR based on the indices NCI, ICI, and ACI of the DHHFLPR.

Based on ACI and the consistency threshold shown in Table 2, we can check whether a DHHFLPR is of acceptable consistency. If not, an algorithm is developed to repairing its consistency.

Algorithm 2. The consistency checking and repairing method for a DHHFLPR

Step 1. Let $(\tilde{H}_{S_O})^{(\mathbb{Z})}=((h_{S_{O_{ij}}})_{m\times m})^{(\mathbb{Z})}$ be a DHHFLPR. $(R^z)^{(\mathbb{Z})}=((r^z_{ij})_{m\times m})^{(\mathbb{Z})}\in\tilde{R}$ $(z=1,2,\ldots,N_{\mathbb{R}})$ be the DHLPR associated with $\tilde{H}_{S_O}$.

Step 2. Calculate $\overline{CI}(\tilde{H}_{S_O})$ based on Table 1.

Step 3. Calculate the ICI and ACI of $(\tilde{H}_{S_O})^{(\mathbb{Z})}$. If $ACI((\tilde{H}_{S_O})^{(\mathbb{Z})})\geq\overline{CI}(\tilde{H}_{S_O})$, then go to Step 5. If not, go to Step 4.

Step 4. Obtain the DHLPR with the smallest consistency index, $CI((R^z)^{(\mathbb{Z})})=\min_{z=1,2,\ldots,N_{\mathbb{R}}}\{\tilde{R}\}$. Repair the consistency of $(R^z)^{(\mathbb{Z})}$ based on Algorithm 1. Go back to Step 2.

Step 5. Let $*\tilde{H}_{S_O}=(\tilde{H}_{S_O})^{(\mathbb{Z})}$, and output the adjusted DHHFLPR $*\tilde{H}_{S_O}$.

5. Case study

In the previous section, we defined the indices ICI and ACI of the DHHFLPR. Let the additive consistency index proposed in (Gou et al., 2019) be NCI and the multiplicative consistency index proposed in (Gou et al., 2020a) be NCI′. Firstly, we apply the proposed method to a practical GDM problem involving the identification of the most critical factors associated with lung cancer occurred. Then we make some comparative analyses about the connections and differences among these consistency indices ICI and ACI, NCI and NCI′ of the DHHFLPR.

5.1. The application in the diagnosis of lung cancer

Because the numbers of both new cases and dead cases of lung cancer rank the first among those of all malignancies, the lung cancer has become the major public health problem in China. As a country with the largest group of lung cancer patients in the world, China is facing new challenges regarding the early and accurate diagnoses and treatments for lung cancer against the increasing incidence of the lung cancer. In recent years, lots of researchers have investigated the risk factors in developing lung cancer (Christian et al., 2011; Goovaerts, 2010; Hosgood III et al., 2013; Lin et al., 2015; Lu et al., 2003; Marcus et al., 2015; Wood et al., 2000), etc. In order to effectively curb the incidence of lung cancer and improve people's quality of life, it is very important to identify the main causes in a region and manage them.

To improve people's health, since the fifth plenary session of the 18th CPC central committee, the state council and related ministries and commissions of China have issued a number of specific regulations since then, and the outline of the "healthy China 2030" plan has been released, which will help prevent and treat major diseases. According to the statistics, the medical cost caused by malignant tumors exceeds 220 billion yuan every year, and the number of new cases and deaths caused by lung cancer ranks the first. Therefore, lung cancer has become a major public health problem in our country. To reduce the incidence of lung cancer, finding the cause of the disease is an important work. At present, the risk factors in the development of lung cancer mainly include ecological environment (Lin et al., 2015; Lu et al., 2003), geographical location (Christian et al., 2011; Goovaerts, 2010; Hosgood III et al., 2013), smoking history (Marcus et al., 2015) and living and working environment (Wood et al., 2000), etc. Suppose that a city plans to investigate which one is the major factor that contributes to a city's lung cancer risk. The above four risk factors can be regarded as the alternatives $\{A_1, A_2, A_3, A_4\}$, and three experts $\{e^1, e^2, e^3\}$ are invited to evaluate these risk factors and provide their assessments. Let $S_O = \left\{s_{t<o_k>} | t = -4, \ldots, 4; \quad k = -4, \ldots, 4\right\}$ be a DHLTS with $S = \{s_{-4} = \mathit{extremelybad}, s_{-3} = \mathit{verybad}, s_{-2} = \mathit{bad}, s_{-1} = \mathit{slightlybad}, s_0 = \mathit{medium}, \quad s_1 = \mathit{slightlygood}, s_2 = \mathit{good}, \quad s_3 = \mathit{verygood}, s_4 = \mathit{extremelygood}\}$ and $O = \{o_{-4} = \mathit{farfrom}, o_{-3} = \mathit{scarcely}, o_{-2} = \mathit{only} \quad \mathit{alittle}, o_{-1} = \mathit{alittle}, o_0 = \mathit{justright}, o_1 = \mathit{much}, \quad o_2 = \mathit{verymuch}, o_3 = \mathit{extremelymuch}, o_4 = \mathit{entirely}\}$. Suppose that experts' assessments can be established by three DHHFLPRs $\tilde{H}^a_{S_O} (a = 1, 2, 3)$ shown as follows:

$$\tilde{H}^1_{S_O} = \begin{pmatrix} \{s_{0<o_0>}\} & \{s_{1<o_0>}\} & \{s_{2<o_2>}, s_{3<o_{-1}>}\} & \{s_{0<o_3>}\} \\ \{s_{-1<o_0>}\} & \{s_{0<o_0>}\} & \{s_{1<o_{-1}>}, s_{2<o_1>}\} & \{s_{-1<o_3>}, s_{0<o_2>}\} \\ \{s_{-2<o_{-2}>}, s_{-3<o_1>}\} & \{s_{-1<o_1>}, s_{-2<o_{-1}>}\} & \{s_{0<o_0>}\} & \{s_{-2<o_1>}, s_{-1<o_3>}\} \\ \{s_{0<o_{-3}>}\} & \{s_{1<o_{-3}>}, s_{0<o_{-2}>}\} & \{s_{2<o_{-1}>}, s_{1<o_{-3}>}\} & \{s_{0<o_0>}\} \end{pmatrix}$$

$$\tilde{H}^2_{S_O} = \begin{pmatrix} \{s_{0<o_0>}\} & \{s_{2<o_1>}, s_3, s_{4<o_0>}\} & \{s_{0<o_2>}, s_1, s_{2<o_3>}\} & \{s_{1<o_1>}\} \\ \{s_{-2<o_{-1}>}, s_{-3}, s_{-4<o_0>}\} & \{s_{0<o_0>}\} & \{s_{0<o_2>}, s_1, s_{2<o_{-1}>}\} & \{s_{-1<o_{-1}>}, s_0, s_{1<o_2>}\} \\ \{s_{0<o_{-2}>}, s_{-1}, s_{-2<o_{-3}>}\} & \{s_{0<o_{-2}>}, s_{-1}, s_{-2<o_1>}\} & \{s_{0<o_0>}\} & \{s_{-3<o_{-1}>}, s_{-2}, s_{-1<o_1>}\} \\ \{s_{-1<o_{-1}>}\} & \{s_{1<o_1>}, s_0, s_{-1<o_{-2}>}\} & \{s_{3<o_1>}, s_2, s_{1<o_{-1}>}\} & \{s_{0<o_0>}\} \end{pmatrix}$$

$$\tilde{H}^3_{S_O} = \begin{pmatrix} \{s_{0<o_0>}\} & \{s_{3<o_1>}\} & \{s_{1<o_2>}\} & \{s_{1<o_2>}\} \\ \{s_{-3<o_{-1}>}\} & \{s_{0<o_0>}\} & \{s_{-2<o_1>}, s_{-1}, s_{0<o_{-2}>}\} & \{s_{-1<o_{-1}>}, s_0, s_{1<o_2>}\} \\ \{s_{-1<o_{-2}>}\} & \{s_{2<o_{-1}>}, s_1, s_{0<o_2>}\} & \{s_{0<o_0>}\} & \{s_{-2<o_3>}\} \\ \{s_{-1<o_{-2}>}\} & \{s_{1<o_1>}, s_0, s_{-1<o_{-2}>}\} & \{s_{2<o_{-3}>}\} & \{s_{0<o_0>}\} \end{pmatrix}$$

The above decision-making problem is a GDM problem. Next, we can calculate the indices ICI and ACI of these three DHHFLPRs $\tilde{H}^a_{S_O}(a = 1, 2, 3)$, and check whether all of them are of acceptable consistencies and repair the DHHFLPR with unacceptable consistency based on Algorithm 2. Finally, we can obtain the optimal alternative.

Firstly, the indices ICI and ACI of these three DHHFLPRs $\tilde{H}^a_{S_O}(a = 1, 2, 3)$ are shown in Table 2.

Then, we can use the ACI to check whether a DHHFLPR is of acceptable consistency. Based on the results shown in Table 2, there is $ACI(\tilde{H}^2_{S_O}) \leq \overline{CI}$. Next, we can utilize Algorithm 2 to repair $\tilde{H}^2_{S_O}$.

Step 1. Let $(\tilde{H}^2_{S_O})^{(0)} = \tilde{H}^2_{S_O}$. Find the DHLPRs $(R^{2,k*})^{(0)} = ((r^{2,k*}_{ij})_{4\times 4})^{(0)}$ $(k = 1, 2, 3)$ which are associated with $\tilde{H}^{2(0)}_{S_O}$ respectively and have the smallest consistency index 0.6563:

$$(R^{2,1*})^{(0)} = \begin{pmatrix} \{s_{0<o_0>}\} & \{s_{4<o_0>}\} & \{s_{0<o_2>}\} & \{s_{1<o_1>}\} \\ \{s_{-4<o_0>}\} & \{s_{0<o_0>}\} & \{s_{0<o_2>}\} & \{s_{1<o_2>}\} \\ \{s_{0<o_{-2}>}\} & \{s_{0<o_{-2}>}\} & \{s_{0<o_0>}\} & \{s_{-3<o_{-1}>}\} \\ \{s_{-1<o_{-1}>}\} & \{s_{-1<o_{-2}>}\} & \{s_{3<o_1>}\} & \{s_{0<o_0>}\} \end{pmatrix}$$

$$(R^{2,2*})^{(0)} = \begin{pmatrix} \{s_{0<o_0>}\} & \{s_{4<o_0>}\} & \{s_{0<o_2>}\} & \{s_{1<o_1>}\} \\ \{s_{-4<o_0>}\} & \{s_{0<o_0>}\} & \{s_1\} & \{s_{1<o_2>}\} \\ \{s_{0<o_{-2}>}\} & \{s_{-1}\} & \{s_{0<o_0>}\} & \{s_{-3<o_{-1}>}\} \\ \{s_{-1<o_{-1}>}\} & \{s_{-1<o_{-2}>}\} & \{s_{3<o_1>}\} & \{s_{0<o_0>}\} \end{pmatrix}$$

$$(R^{2,3*})^{(0)} = \begin{pmatrix} \{s_{0<o_0>}\} & \{s_{4<o_0>}\} & \{s_{0<o_2>}\} & \{s_{1<o_1>}\} \\ \{s_{-4<o_0>}\} & \{s_{0<o_0>}\} & \{s_{2<o_{-1}>}\} & \{s_{1<o_2>}\} \\ \{s_{0<o_{-2}>}\} & \{s_{-2<o_1>}\} & \{s_{0<o_0>}\} & \{s_{-3<o_{-1}>}\} \\ \{s_{-1<o_{-1}>}\} & \{s_{-1<o_{-2}>}\} & \{s_{3<o_1>}\} & \{s_{0<o_0>}\} \end{pmatrix}$$

Step 2. Calculate the additive consistent DHLPRs $(\bar{R}^{2,k*})^{(0)} = ((\bar{r}^{2,k*}_{ij})_{4\times 4})^{(0)}$ $(k = 1, 2, 3)$ respectively, and obtain three interval-valued DHHFLPRs which are established by the DHLPRs and the correspondingly additive consistent DHLPRs. Based on the suggestions, expert e^2 provides the new preferences shown as follows:

$$\left(\tilde{H}^2_{S_O}\right)^{(1)} = \begin{pmatrix} \{s_{0<o_0>}\} & \{s_{2<o_1>}, s_3, s_{4<o_0>}\} & \{s_1, s_{2<o_1>}, s_{2<o_3>}\} & \{s_{1<o_1>}\} \\ \{s_{-2<o_{-1}>}, s_{-3}, s_{-4<o_0>}\} & \{s_{0<o_0>}\} & \{s_{0<o_2>}, s_1, s_{2<o_{-1}>}\} & \{s_{-1<o_{-1}>}, s_{-1<o_2>}, s_0\} \\ \{s_{-1}, s_{-2<o_{-1}>}, s_{-2<o_{-3}>}\} & \{s_{0<o_{-2}>}, s_{-1}, s_{-2<o_1>}\} & \{s_{0<o_0>}\} & \{s_{-2}, s_{-2<o_2>}, s_{-1<o_1>}\} \\ \{s_{-1<o_{-1}>}\} & \{s_{1<o_1>}, s_{1<o_{-2}>}, s_0\} & \{s_2, s_{2<o_{-2}>}, s_{1<o_{-1}>}\} & \{s_{0<o_0>}\} \end{pmatrix}$$

Table 3. The indices ACI and ICI of $\tilde{H}^{2(1)}_{S_O}$, respectively.

	$(\tilde{H}^2_{S_O})^{(1)}$
ICI	[0.8333,1]
ACI	0.9205

Source: The Authors.

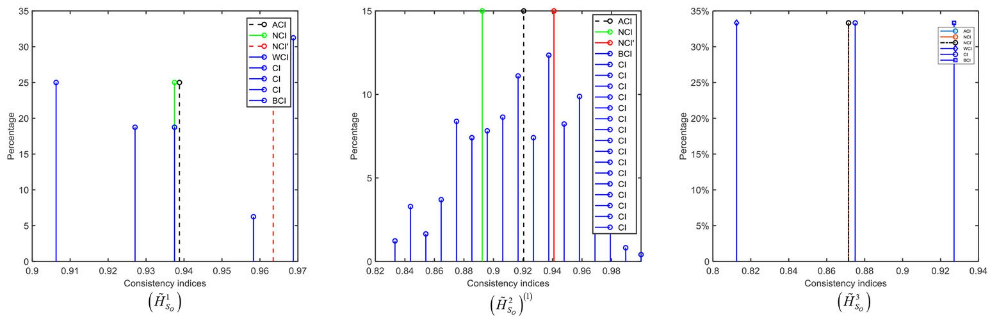

Figure 3. The consistency distributions of all DHLPRs associated with $\tilde{H}^a_{S_O}(a = 1, 2, 3)$, respectively.
Source: The Authors.

Step 3. Calculate the indices ICI and ACI of DHHFLPR $(\tilde{H}^2_{S_O})^{(1)}$, the results are shown in Table 3.

Obviously, $ACI((\tilde{H}^2_{S_O})^{(1)}) > \overline{CI}$. Then based on the synthetical value of alternative (Gou et al., 2019) shown as follows:

$$SV(A_i) = \frac{1}{3}\sum_{a=1}^{3}\frac{1}{4}\sum_{j=1}^{4}\frac{1}{h^a_{ij}}\sum_{l=1}^{h^a_{ij}} f\left(h^{a(l)}_{ij}\right) \tag{12}$$

we obtain all synthetical values of alternatives: $SV = \{0.6766, 0.4353, 0.3824, 0.5056\}$. Therefore, in this city, the ecological environment is the main factor in developing the lung cancer.

Based on the decision-making result, some suggestions about the main factor in developing the lung cancer can be provided. The city should pay close attention to the ecological environment. In addition, the remaining factors should also be considered because the synthetical value of them are not very low. Therefore, the city needs to pay some degrees of attention to these factors according to this ranking: ecological environment $\succ$ geographical location $\succ$ smoking history $\succ$ living and working environment.

5.2. Comparative analysis

Next, based on (Gou et al., 2019) and (Gou et al., 2020a), we can calculate the indices NCI and NCI′) of these three DHHFLPRs, and the indices ICI, NCI (NCI′) and ACI of these three DHHFLPRs $\tilde{H}^a_{S_O}(a = 1, 2, 3)$ are shown in Figure 3.

In Figure 3, the x-axis with blue dotted lines shows the consistency indices of all DHLPRs associated with the DHHFLPRs $\tilde{H}^a_{S_O}(a = 1, 2, 3)$, respectively. The x-axis with black, green, and red dotted lines shows the indices ACI, NCI and NCI′ of the

Table 4. The indices NCI (NCI′), ACI and ICI of $\tilde{H}^a_{S_O}(a = 1, 2, 3)$, respectively.

	$\tilde{H}^1_{S_O}$	$(\tilde{H}^2_{S_O})^{(1)}$	$\tilde{H}^3_{S_O}$
ICI	[0.9063,0.9688]	[0.8333,1]	[0.8125,0.9271]
NCI	0.9375	0.8924	0.8715
NCI′	0.9635	0.9410	0.8715
ACI	0.9388	0.9205	0.8715

Source: The Authors.

DHHFLPRs $\tilde{H}^a_{S_O}(a = 1, 2, 3)$, respectively. The y-axis shows the percentages of consistency indices of the DHLPRs associated with the DHHFLPRs $\tilde{H}^a_{S_O}(a = 1, 2, 3)$, respectively. Additionally, the results of indices ICI, ACI and NCI (NCI′) of the DHHFLPRs $\tilde{H}^a_{S_O}(a = 1, 2, 3)$ are summarized in Table 4.

Based on Figure 3 and Table 4, some comparative analyses can be made. Firstly, the connections of these consistency indices are summarized as follows:

1. The index ICI contains all possible consistency indices of each DHHFLPR, and the indices NCI (NCI′) and ACI are obtained by different methods for calculating average values. The values of the indices NCI (or NCI′) and ACI are included in the index ICI of the corresponding DHHFLPR. Additionally, the indices NCI presented in (Gou et al., 2019, 2020a) are also included in the index ICI of the corresponding DHHFLPR.
2. Even though the normalization methods are different, the indices NCI and NCI′ of each DHHFLPR are close or the same. Specially, if the number of all DHHFLEs included in a DHHFLPR is equal to or less than 2, then the indices NCI and NCI′ values must be the same because it is not necessary to add any novel DHLT.
3. The indices NCI and NCI′ values are very close to those of the index ACI because all of them are based on average operations, which means that we can utilize the normalization approaches to approximately measure the average consistency of the DHHFLPR.

Then, the differences among the indices ICI, ACI, and NCI (NCI′) of the DHHFLPR can be analyzed and summarized as follows:

1. As we discussed above, the values of the indices ACI and the NCI (NCI′) of each DHHFLPR are almost the same because their calculation methods are similar. Therefore, we can obtain that $ACI(\tilde{H}_{S_O}) \approx NCI(\tilde{H}_{S_O}) \approx NCI'(\tilde{H}_{S_O})$. However, there exist some differences on the calculations of the indices ACI and the index NCI (NCI′): the ACI is to calculate the average value of all DHLPRs associated with the DHHFLPR (See Eq. (11)), and the index NCI (NCI′) is to calculate the average value of all DHLPRs obtained by the corresponding positions of DHLTs (See (Gou et al., 2019, 2020a)).
2. There exist obvious different consistency reflections between the indices ACI, NCI (NCI′) and the ICI of each DHHFLPR. The main reason is that the index ICI is the comprehensive representation of all consistency indices of a DHHFLPR, and the index ICI provides the lower and upper bounds of the

consistency indices of a DHHFLPR. However, the indices ACI and the NCI (NCI′) only represent the average values, which are very close to the overall consistency degrees of a DHHFLPR. Therefore, the relation among them is $ACI(\tilde{H}_{S_O})\approx NCI(\tilde{H}_{S_O})\approx NCI'(\tilde{H}_{S_O}) \in ICI(\tilde{H}_{S_O})\left[WCI(\tilde{H}_{S_O}), BCI(\tilde{H}_{S_O})\right]$.

6. Conclusions and future research directions

In this paper, we developed an ICI of a DHHFLPR, which consists of all possible consistency indices of the DHHFLPR, and proposed a simpler consistency measure to obtain the consistency index of a DHHFLPR. Then, a new concept of ACI of the DHHFLPR was given by calculating the average value of all DHLPRs associated with the DHHFLPR. Based on the ACI and the obtained DHLPRs, we developed a simpler consistency repairing method to improve the DHHFLPR is of unacceptable consistency. Finally, we applied the proposed consistency checking and repairing methods into a practical GDM method that is to identify the most critical factors associated with lung cancer, and some comparative analyses involving the indices ICI, NCI and ACI was analyzed to understand these proposed indices more clearly.

As future work, some other consistency calculation methods, the consensus reaching methods under some special circumstances, and the applications in large-scale GDM will be investigated to perfect the consistency theory of DHHFLPR.

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