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Examining Openness To Pedagogical Change Among Secondary Mathematics Teachers: Developing And Testing A Structural Model

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EXAMINING OPENNESS TO PEDAGOGICAL CHANGE AMONG SECONDARY MATHEMATICS TEACHERS: DEVELOPING AND TESTING A STRUCTURAL MODEL

by

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This dissertation, submitted by Cathy J. Williams in partial fulfillment of the requirements for the Degree of Doctor of Philosophy from the University of North Dakota, has been read by the Faculty Advisory Committee under whom the work has been done and is hereby approved.

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Title	Examining Openness to Pedagogical Change Among Secondary Mathematics Teachers: Developing and Testing A Structural Model
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ABSTRACT

Widespread adoption of the Common Core State Standards for Mathematics (NGA & CCSSO, 2010) and other career and college readiness standards have prompted changes in the pedagogical practices of secondary mathematics teachers in the United States. The purpose of this study was to examine factors contributing to a math teacher's willingness to alter pedagogical approaches. Key to the research was development of an instrument for measuring openness to change. The survey tool was created based on constructs drawn from the literature and was emailed to secondary mathematics teachers in the United States *(N* = 571). The instrument consisted of 65 questions pertaining to demographics, conception of mathematics, perceptions of learning mathematics, math mindset, teacher self-efficacy, professional identity, ambiguity tolerance, and attitude toward change. Exploratory and confirmatory factor analyses showed a six-factor structure to be effective for predicting openness to change. Structural equation modeling (SEM) techniques were used to test complexities among latent constructs and to support a theoretical model of correlations.

Results revealed significant differences along demographic lines on the openness-to-change scale, with females more open to adaptation than males, urban teachers more open than rural, and those without a math degree more open than math majors. Since high school teachers were much more likely than middle school teachers to hold a math degree—72.2 percent compared to 27.4 percent—this last result relates to the finding that middle school teachers are more change-ready than their high school counterparts. No significant

correlation was found between the change-scale score and age, experience, or years spent teaching mathematics.

The structural equation model tested in this study showed the six latent constructs combining in complex ways to explain math teacher willingness to alter teaching strategies. The structural equation model developed here serves to illuminate complex issues around math teacher change and provides a framework for diagnosing and remedying professional development challenges. The model suggests instructional change can be facilitated through attention to teachers' conception of mathematics, perceptions of learning mathematics, math mindset, self-efficacy, professional identity, and ambiguity tolerance.

CHAPTER I

INTRODUCTION

> Educational goals for students must reflect the importance of mathematical literacy. Toward this end, the K—12 standards articulate five general goals for all students: (1) that they learn to value mathematics, (2) that they become confident in their ability to do mathematics, (3) that they become mathematical problem solvers, (4) that they learn to communicate mathematically, and (5) that they learn to reason mathematically. These goals imply that students should be exposed to numerous and varied interrelated experiences that encourage them to value the mathematical enterprise, to develop mathematical habits of mind, and to understand and appreciate the role of mathematics in human affairs; that they should be encouraged to explore, to guess, and even to make and correct errors so that they gain confidence in their ability to solve complex problems; that they should read, write, and discuss mathematics; and that they should conjecture, test, and build arguments about a conjecture's validit*y*. (NCTM, 1989, p. 5)

By the time many U.S. students were first tested on the Common Core State Standards for Mathematics (CCSSM), it had been more than twenty-five years since the National Council of Teachers of Mathematics (NCTM) first suggested reforms very much like those now advanced by the Common Core. The 1989 NCTM *Curriculum and Evaluation Standards,* based upon the "assumption" that learning is an "active process," proposed a shift away from memorized procedures and rote practice toward deeper understanding, achieved through more student problem solving and student communication of strategies and logical arguments (NCTM 1989). These reforms have been slow to take root, and U.S. achievement data reflect this. While 4th- and 8th-grade students have shown growth on the Trends in International Math and Science Study (TIMSS) across the intervening years and have made "steady and significant" improvements on the National Assessment of Educational Progress

(NAEP), high school scores on these tests have remained stagnant by comparison (Dossey & McCrone, 2012). U.S high-school students rank nineteenth in the world on the Program for International Student Assessment (PISA) tests of problem solving, lagging far behind Singapore, Korea, Japan, and even neighboring Canada (OECD, 2012). Although some researchers are critical of international comparisons due to socioeconomic factors that disadvantage the U.S. (Carnoy & Rothstein, 2015, Rebell & Wolff, 2012), even when socioeconomic variables are accounted for, the U.S. lags behind top performing countries Canada, Finland and Korea, with only one-third of the gap closing (Carnoy & Rothstein, 2015). Petrilli and Wright (2016) use PISA data to point out that "poverty cannot explain away America's lackluster performance" (p. 1). The NAEP scores, of course, do not involve international comparisons. One possible explanation for sluggish achievement is that high school teachers in the U.S. have been slow to adopt reform shifts in instructional practice. Although it is hoped that widespread adoption of the CCSSM and accompanying changes in assessment will prompt long-awaited reform in secondary mathematics, unless mathematics education leaders address reasons behind teacher reluctance, professional development efforts are likely to be unsuccessful. It is one thing to say that teachers support the *Common Core State Standards* (O'Brien, 2014) and another thing altogether to say they are ready to shift pedagogical practices accordingly.

Statement of the Problem

It has long been understood that teacher practices are tied to teacher beliefs about teaching and learning (Stipek, Givvin, Salmon, & MacGyvers, 2001) and, in the case of math teachers, to beliefs about mathematics (Hoz & Wesman, 2008; Philipp, 2007; Chapman, 2002). Consequently, if we want to see a change in practice, we must first motivate a change

in beliefs. It is not enough to simply provide teachers with a new curriculum and hope for the best. Reform-based textbooks exist, but where teachers' beliefs do not align with the intentions of the curriculum, new books do not guarantee new reform-based instruction (Roehrig, 2005). It is also insufficient to simply offer professional development on reform-based strategies; teachers judge all new learning against currently held beliefs and tend to adapt learning to beliefs rather than altering beliefs to accommodate the new learning (Cohen & Ball, 1990).

> A shift to a problem solving approach to teaching requires deeper changes. It depends fundamentally on the teacher's system of beliefs, and in particular, on the teacher's conception of the nature of mathematics and mental models of teaching and learning mathematics. Teaching reforms cannot take place unless teachers' deeply held beliefs about mathematics and its teaching and learning change. (Ernest, 1989, p. 249)

In order to influence belief systems, we must first understand them, and in order to understand them, we must be able to assess them. This study provides a structural model for understanding pedagogical change, and through development of an instrument for measuring teacher beliefs, takes an essential next step toward instructional reform.

Theoretical Framework

Since the publication of the *NCTM Standards* in 1989, research and practice in mathematics education has been greatly influenced by the theory of constructivism. From a constructivist perspective, learning mathematics involves "incorporating new perceptions into an … existing cognitive structure" through "accommodation and assimilation," in such a way that conceptual frameworks are continually transformed (Schiro, 2008, p.108). This is very much in keeping with the student-centered "active learning" described in the 1989 *Standards*. Students are to actively construct meaning based on experience and interactions with phenomena—including expressions of the teacher's understanding—and make their own

sense. The Standards for Mathematical Practice (SMP) of the Common Core likewise promote construction of meaning by insisting on student reasoning, argument, and perseverance in problem solving (NGA & CCSSO, 2010).

Although constructivism is a theory of learning and not a theory of teaching (Simon, 1995), authors sometimes describe as "constructivist" a pedagogy that is consistent with the framework. Here is an example:

> The constructivist pedagogy … involve[s] the following characteristics:
>
> 1. Attention to the individual and respect for students' background and developing understandings of and beliefs about elements of the domain (This could also be described as student-centered);
>
> 2. Facilitation of group dialogue that explores an element of the domain with the purpose of leading to the creation and shared understanding of a topic;
>
> 3. Planned and often unplanned introduction of formal domain knowledge into the conversation through direct instruction, reference to text, exploration of a Web site, or some other means.
>
> 4. Provision of opportunities for students to determine, challenge, change or add to existing beliefs and understandings through engagement in tasks that are structured for this purpose; and
>
> 5. Development of students' meta-awareness of their own understandings and learning processes. (Richardson, 2003, p. 1626)

Traditional teaching of mathematics in the United States runs counter to constructivism in that it has tended toward the "transmission" of knowledge, with the teacher as expert in control of knowledge flow (Stipek et al., 2001). In a "typical American lesson," the teacher explains a rule or procedure, leads students through examples step by step, and only then assigns problems for homework (Stipek et al., 2001), problems that typically bear close resemblance to examples seen in class. Math reformists, who favor more constructivist approaches, object to this treatment of mathematics as a static collection of rules and

procedures. Such practice leaves little room for students to investigate, to conjecture, and to deeply process mathematical ideas.

It must be acknowledged that constructivism has come under attack in recent years (Hattie, 2009). Kirschner, Sweller, and Clark (2006), who define learning as "change in long-term memory" (p. 75), concluded constructivist approaches were less effective than direct instruction due to the demands on working memory. Under the broad umbrella of "minimally guided techniques," these researchers aggregated discovery learning, experiential learning, problem-based learning, inquiry learning, and constructivist learning, calling them "essentially pedagogically equivalent" (p. 75). The current study, however, does not equate these techniques (Hmelo-Silver, Duncan, & Chinn, 2007). Nor does it speak of the "straw man" version of constructivism (Liu & Matthews, 2005) often referenced by critics. There is no intended suggestion that students be left to "discover" mathematics without instruction from the teacher. On the contrary, the teacher is to carefully craft student experiences—including direct instruction—in such a way that students make sense and meaning of what is learned. In this way it is hoped students will do more than remember. The goal is also to build schemata for future acquisition and to build capacity to think theoretically about mathematical items. Dean and Kuhn (2006) have shown that direct instruction along is "neither a necessary nor sufficient condition for robust acquisition or for maintenance over time" (p. 384). Reform-based curricula with constructivist, student-centered approaches have been shown to be more effective than traditional classroom programs (Briars & Resnick, 2000; Riordan & Noyce, 2001; Schoenfeld, 2002).

Purpose of the Study

The question this study seeks to address is "How can we measure secondary math teachers' openness for instructional reform?" Currently no tool for doing so exists although Stipek and colleagues (2001) developed an instrument for exploring a related question among elementary teachers. Research has well established that beliefs impact teacher performance (Stipek et al., 2001) and suggests several factors in particular contribute to math teachers' willingness to re-imagine practice: teacher conceptions of mathematics (Sowder, 2007), teacher perceptions of how students learn math (Philipp, 2007), teacher perception of students' potential, or "math mindset" (Boaler, 2013; Dweck, 2006), teacher professional identity (Kelchtermanns, 2009), and teacher self-efficacy (Charalambous & Philippou, 2010). If newly proposed approaches are not in harmony with these components of a teacher's belief system, no matter the quality of the training, new strategies are likely to be ignored (Borko, Mayfield, Marion, Flexer, & Cumbo, 1997; Philipp, 2007). In a more general sense, one's ability to tolerate ambiguity also anticipates openness to change (Merenluoto & Lehtinen, 2004; Stein & Smith, 2011). All six of these constructs can be measured. What is required is a tool that addresses them all, the means to look inside the black box of teacher attitudes. With such a tool professional developers could—prior to asking teachers to reflect on new strategies—identify and assess the strength of potentially limiting perceptions. The aim of this study is to develop such an instrument and to test its validity.

Research Questions

1. How can math teacher openness to changing classroom practices be measured?
2. Which demographic measures predict openness to change?

3. How valid are certain constructs (math mindset, self-efficacy, constructivist perception of learning, dynamic conception of mathematics, professional identity, and ambiguity tolerance) for predicting a math teacher's receptivity to new practices?
4. How do these factors relate to one another as predictors of openness to reform?

Importance of the Study

The widespread adoption of the Common Core State Standards, which include eight standards for student habits of mind, has intensified a nationwide focus on teaching practices in U.S. math classrooms. In an effort to advance better instructional habits consistent with the Common Core, NCTM released *Principles to Action* (2014), outlining eight Mathematics Teaching Practices that support deeper, more student-centered learning. These teaching practices should, in effect, engage students in the Common Core mathematical practices. It is not realistic, however, to simply present these reform ideas to teachers and expect change to follow. We must be able to gauge teacher attitude toward reforms and willingness to embrace them.

This study illuminates relationships among factors impacting change. In order that professional development of math teachers be truly transformational, mathematics education leaders must deeply understand the complex associations among factors contributing to resistance. The structural model developed in this study brings together in graphic style much of the literature on math education reform. The model clarifies relationships among potentially limiting belief constructs and in doing suggests areas for math teacher training that go beyond subject matter and pedagogy.

Although much has been written about math education reform and obstacles to achieving it, a tool for measuring math teacher openness to change has not been available. This research validates such a tool, and in doing so provides districts the means to assess attitudes toward change. The survey instrument makes possible local identification of roadblocks to reform and allows leaders to gauge the relative strength of hindering beliefs. Administering the survey prior to teacher training may assist teacher educators in prioritizing attitudes to be addressed. In schools or districts where math teachers all score high in "openness to reform," trainers can potentially forego workshop sessions related to disposition and proceed directly to implementation of new pedagogies. In districts where results indicate limiting attitudes, use of the instrument will allow not only pre-training diagnostics and session roadmaps, but post-training feedback on the effectiveness of training; that is, had math-mindset training actually altered teacher mindsets? Had teachers become more comfortable with ambiguity? Had the general culture in the district started to shift?

This study also makes a contribution to the literature on math education reform in that it identifies demographic differences in openness to math reform. Differences exist along geographic, educational, and gender lines that are worthy of further exploration. The study found no support for widely held notions that age and experience are factors.

Effective professional development based on results of this study will include teacher reflection, the opportunity to become aware of and examine one's own deeply held beliefs. Without this metacognitive opportunity, a teacher's own scholarship goes undeveloped, as does "the scholarship of teaching in general" (Kelchtermans, 2009, p. 270). Finally, this study is significant in that provides further evidence that reform is a sensitive business; change must be implemented with caution due to important psychological factors at play.

Delimitations

The sample for this study was limited to secondary mathematics teachers working in the United States in the spring of 2015 and did not include participants from other countries, grade levels or disciplines. Participants were those who responded to an email or Twitter invitation to take an online survey. The sample was further limited to those teachers connected to math education leaders in the researcher's professional network. With the understanding that problems must be detected before they can be addressed, the study focused on identifying and measuring attitudinal impediments to math reform and did not deal directly with solutions to those impediments. Constructs analyzed in the study were those drawn from the literature on math reform. The study intentionally focused on change factors that were internal to the teacher and did not address important factors like adequate training, support, time and resources.

Assumptions

Typical of studies with large online surveys, this study assumed participants to be members of the target population. It also assumed participants were able to correctly interpret the questions and to answer them truthfully, expressing their own beliefs and opinions. Further, the study assumes that math teachers receptive to change will be more likely to pursue reform strategies. The researcher also believes it will continue to be important to foster in secondary students the Mathematical Practices outlined in the Common Core State Standards (or habits of mind very much in keeping with those practices should the standards themselves be rejected) and that it will therefore also continue to be important to develop

Mathematical Teaching Practices in teachers. More generally, since the study builds on a constructivist framework, it assumes that constructivist approaches to math instruction will continue to be supported by math education research. Since colleges of education have promoted constructivism for decades, it is likely to be supported well into the future.

Researcher's Background

The researcher has worked in the field of education for 34 years. In addition to teaching secondary mathematics and English for the same public school district for 23 years, she has taught university-level courses in English and education and for three years served as the curriculum director for the MATHCOUNTS program in Alexandria, VA. She is currently an instructional coach for the same district in which she taught for many years. As such, she is responsible for curriculum development and teacher professional development for grades 6-12 mathematics and English.

As curriculum director for MATHCOUNTS, she found herself in the Washington, DC area at the time NCTM's *Curriculum and Evaluation Standards* (1989) were first being released. It was her responsibility to know the *Standards* well and to develop alongside NCTM-appointed teachers classroom materials that reflected the intent of the *Standards*. When she returned to the Midwest from the DC area to take a math teaching position, she incorporated MATHCOUNTS materials into both classroom and after-school curricula and saw first-hand how well students learned math when given the opportunity to solve non-routine problems.

As a coach at the district level for the past six years, the researcher has led secondary math and English teachers in the development of Common Core-based curricula. She has also designed professional development sessions for teachers in both disciplines around the

incorporation of new pedagogies. In mathematics in particular, she has witnessed the reluctance of teachers to prioritize the Mathematical Practices Standards of the Common Core and a tendency to cling to the traditional lecture-followed-by-practice approach with which they themselves were taught.

Definitions

Common Core State Standards Mathematics: A set of K-12 academic standards developed for the National Governors Association and the Council of Chief States School Officers in an effort to define more coherent and rigorous guidelines for math education for any states choosing to adopt them (hence "Common"). "These Standards define what students should understand and be able to do in their study of mathematics" NGA & CCSSO, 2010, p. 4).

Standards for Mathematical Practice (SMP): A subset of the *Common Core State Standards* that define certain "processes and proficiencies" important to students' study of mathematics:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

These were derived from NCTM's Practice Standards and the National Research Council's strands of mathematical proficiency (NGA & CCSSO, 2010).

Mathematics Teaching Practices (MTP): A framework of eight practices for "strengthening the teaching and learning of mathematics," published by NCTM in *Principles to Action: Ensuring Mathematical Success for All* (2014).

Dynamic conception of mathematics: The extent to which a teacher thinks of mathematics as a "dynamic" science of inquiry as opposed to a "static" collection of rules and procedures (Hoz & Weisman, 2008).

Constructivist attitude toward learning mathematics: The extent to which a teacher values an "open"/exploratory/social/student-centered approach to teaching mathematics as opposed to a "closed" teacher-transmits-student-practices approach (Hoz & Weisman, 2008).

Mathematical mindset: The term *mindset* is used in the sense popularized by Carol Dweck (2006). The extent to which a teacher feels that math intelligence is malleable, a capacity that can be developed in any student.

Teacher self-efficacy: Teachers' "judgment of their capabilities to organize and execute courses of action required in order to obtain certain types of performances" (Bandura, 1986, p. 391). In this study, specifically the extent to which the teachers feel in possession of the means to impact student learning.

Professional identity: The magnitude of the teacher's identification with the role of math teacher (Adams, Hearn, Sturgis, & MacLeod, 2006). The extent to which a teacher values and feels pride in the role of math teacher and identifies with other math teachers.

Ambiguity tolerance: The level of comfort a teacher has with novelty, complexity, and uncertainty, the non-routine (Rasche, 2012; Budner, 1962).

Openness to change: The extent to which a teacher is willing to incorporate new pedagogy.

Reform: In the context of this paper, "reform" refers not to the current political teacher-accountability and high-stakes testing movements in the U.S. but to democratic changes in math pedagogy envisioned by the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), *Principles and Standards for School Mathematics* (NCTM, 2000), and more currently *Principals to Action Mathematics* (NCTM, 2014).

Measurement Model: A pictorial model of relationships among latent constructs and corresponding measurement items; used in confirmatory factor analysis.

Structural Model: A pictorial model representing theoretical covariance and regression relationships among constructs in a multivariate SEM study.

Summary

Why is it that mathematics instruction in today's secondary classrooms has changed so little since 1989 (Stigler & Hiebert, 2009)? NCTM, in *Principles to Action* (2014), is still citing "too much focus on learning procedures without any connection to meaning, understanding, or the applications that require these procedures" (p. 3). Given twenty-five years of literature inviting math teachers to make learning more interactive, student-centered, and conceptually grounded, how is that so little change has occurred? This study seeks to understand and measure the foundational beliefs that distinguish two types of teachers: those eager to embrace change in math pedagogy and those who are more reluctant. This study has been organized into five chapters. Chapter I provides an overview of the problem, along with a concise statement of purpose and the research questions. It also furnishes the theoretical context, significance, delimitations, and assumptions for the study. Chapter II provides a literature review to establish a rationale for measuring attitude toward change and for including each of the sub-constructs that comprise the instrument: math mindset, dynamic

conception of mathematics, constructivist attitude toward learning mathematics, professional identity, self-efficacy, and ambiguity tolerance. Chapter III is devoted to a description of the design and methodology of the study. Chapter IV explains the data results and analysis, and finally, Chapter V offers a summary, conclusions, discussion, recommendations, and reflection on the study.

CHAPTER II

LITERATURE REVIEW

In 2014 the National Council of Teachers of Mathematics released *Principles to Actions: Ensuring Mathematical Success for All*. One of its goals was to delineate for educators "productive and unproductive beliefs" regarding K-12 mathematics. NCTM called "unproductive" the belief that the teacher's role is to transmit mathematics, smoothing over problem solving for students by guiding them every step of the way. Also labeled "unproductive" was the idea that student work be based on routine procedures, memorization, and prescribed methods. Though careful not to brand these beliefs as "bad," NCTM classified them as likely to "hinder the effective implementation of effective instructional practices or to limit student access to important mathematics content and practices" (NCTM, 2014, p. 11). In line with more productive beliefs were these eight Mathematics Teaching Practices (MTP) promoted in *Principles to Action*:

- Establish mathematics goals to focus learning.
- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Pose purposeful questions.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.
- Elicit and use evidence of student thinking (NCTM, 2014, p. 10).

The MTP were intended as a framework to enable deeper student learning, the idea being that if teachers were to adopt these practices, learners would be better able to meet the eight

student Standards for Mathematical Practice (SMP) proposed in the Common Core document:

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

This study deals with challenges associated with moving math teachers toward adoption of the MTP so that students might be provided opportunities to experience the SMP. This chapter will summarize the literature on math teacher change with the aim of better understanding resistance to pedagogical reform.

Math Teacher Change

Unfortunately inertia is a powerful force in math education. Stigler and Hiebert (2009) point out that while reform movements come and go, "the substantive nature of what happens in the classroom stays pretty much the same" (p. 32). For these researchers stasis is a clear indication that teaching and teacher learning are cultural activities and therefore difficult to impact. In fact, they found that in the four-year period of major math reform efforts from 1995 to 1999, although many teachers claimed to be adjusting practices to the reforms, no changes were evident (p. 33).

In their video case studies associated with the Third International Mathematics and Science Study (TIMSS), Stigler and Hiebert (2009) drew cultural comparisons between U.S. math classrooms and math classrooms in countries that continue to achieve at higher rates. They concluded that no particular teaching strategy or model was responsible for higher

achievement. Not even the type of math problem given to students explained international differences. A close look at video cases showed the important difference to be the way in which teachers engaged students with the problems. In higher performing countries, teachers engaged students in "active struggle with core mathematics concepts and procedures" (Stigler & Hiebert, 2009, pp. 34-35), encouraging them to search for patterns in order to develop their own conceptual understanding. In contrast, U.S. teachers involved students in practicing arithmetic procedures and recalling information.

U.S. teachers not only tend to focus on the procedural but they often over-simplify processes for students. Colin Foster (2013), a British researcher, refers to this teacher tendency to "path-smooth" mathematical tasks for students as "reductionism":

> In a reductionist pedagogical paradigm, the subject is broken down into numerous tiny skills and pieces of knowledge, which are then taught separately and sequentially. The unstated assumption is that mastering these elements is equivalent to (but more manageable than) learning the original structure. Yet it is widely lamented that, when taught in this way, students often fail to see the purpose of these piecemeal bits of learning, quickly lose the various fragments and struggle to select appropriate ones and combine them when called on to solve more substantial problems. (p. 564)

Foster advocates for more complex tasks and holistic approaches that treat math learning as more than a "linear, unidirectional, ladder-like" endeavor (p. 576). Without these kinds of reforms, conversations like the following will persist in U.S. math classrooms:

> Student: I can do it when you're with me but I can't do it by myself!
> Teacher: Of course you can! You don't need me. I wasn't really doing anything—you did all the math!
> Student: I only know how to do it if you tell me what to do. (p. 575)

Researchers explain that if students are to remember and make use of the procedures they learn in math class, they must be "required to exert some intellectual effort in making sense of the procedures, perhaps wrestling with the question of why procedures work"

(Emerling, Hiebert & Gallimore, 2015, p. 50; Clark & Bjork, 2014; Hiebert & Grouws, 2007). There is wide support in the mathematics education research community for pedagogical reform in the direction of this deeper conceptualization (Boaler, 2002; Gresalfi & Cobb, 2011; Lampert, 2001; Stein, Silver, I Smith, 1998).

The literature on math teacher change suggests a number of reasons math teachers have been slow to move from "teaching as telling" toward implementation of the practices outlined in the NCTM standards (1989; 2000) and the more recent *Principles to Action* (NCTM, 2014). This study will propose six psychological constructs impacting a math teacher's openness to change (See Fig. 1). The rest of this chapter will be devoted to summarizing the research behind each construct.

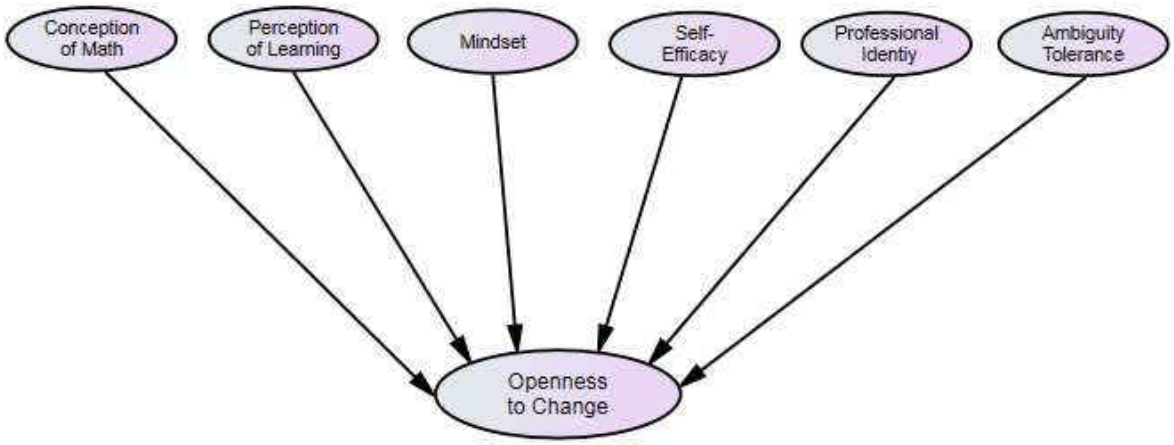


Figure 1. Simple Structural Model of Contributors to Change

Teacher Conceptions of Mathematics

Differing conceptual models of mathematics and mathematical understanding have fueled the "math wars" for decades now (Schoenfeld, 2004, p. 76), and some of the most consequential battles have been played out between teachers and staff developers. If all educators agreed on what mathematics *is*, reform would be more manageable. As it stands,

educators are divided regarding whether math is an object-to-be-transmitted or an experience-to-be-constructed (Chapman, 2002). That division has practical implications. In a three-year qualitative study of math teachers' perceptions of word problems, for example, Chapman (2002) found that those who held the object-to-be-transmitted view tended to use a traditional "show and tell approach," while those who held the experience-to-be-constructed view were apt to employ "student-centered," "inquiry-oriented" methods (p. 96). She suggests that professional development of math teachers must take into account the way in which they view math problems.

In contrast to Chapman, Schifter (1995) viewed conceptual differences as more continuous than dichotomous. In her work on reform's effect on attitudes, she described a progression of conceptions of math among teachers that ranged from highly rote and procedural to highly conceptual:

> 1. An ad hoc accumulation of facts, definitions, and computational routines;
>
> 2. Student centered activity but with little or no systematic inquiry into issues of mathematical structure and validity;
>
> 3. Student centered activity directed towards systematic inquiry into issues of mathematical structure and validity;
>
> 4. Systematic mathematical inquiry organized around investigations of big mathematical ideas. (p. 18)

At one end of the continuum were teachers focused on deriving answers, speaking "almost exclusively in the language of numbers and operations." At the other end were teachers concerned with the underlying framework of ideas and relationships, whose ultimate goal was coherent understanding (Thompson, Philipp, Thompson & Boyd, 1994).

Beswick (2012), crediting Ernest (1989), categorizes conceptions of math as "instrumentalist, Platonist, or problem solving" (p. 129-130). The instrumentalist sees math

as an amalgam of unrelated topics, each composed of a series of facts, skills and rules to be applied to practical purpose; the Platonist views it more as a connected static body of existing knowledge waiting to be discovered; and finally one with the problem solving view holds that math is a “dynamic and creative human invention” (p. 129-130).

The reality is that the mathematics practiced in classrooms is very different than the problem solving practice of mathematicians (Beswick, 2012). Math education leaders in this country have proposed to remedy this through introduction of standards of mathematical practice, and a new wave of math coaches have appeared on the scene to train teachers in these practices. Unfortunately when the beliefs of teachers and their trainers clash, "the teachers generally either ignore the new ideas or inappropriately assimilate them" (Borko et al., 1997, p. 270). In a qualitative study involving 14 third-grade teachers in math workshops, Borko et al. (1997) found teachers whose philosophies of math were not well aligned with NCTM reforms were not inclined to invest a lot of time in something that “didn’t need fixing” (p. 265). Some of these teachers, because they held beliefs that went unchallenged during the course of the workshop, continued to embrace strategies inconsistent with the goals of the training.

Likewise with reform curriculum materials: teachers will choose not to use them or use them improperly if the materials do not align with their conceptions (Philipp, 2007). In their qualitative study of seven elementary teachers, Remillard and Bryans (2004) found that a necessary condition of effective implementation of reform curriculum was a reform orientation on the part of the teacher.

What the reform movement has been hoping to nurture in teachers is an image of mathematics as a field of inquiry:

> We observe an object, or a relationship, or a phenomenon, and we ask: What properties must it have? How do we know? Do all objects that look like this have the same property? Just what does it mean to "look like this"? Are there different ways to understand this? With that mindset, simple objects or observations become the starting points for explorations, some of which become unexpectedly rich and interesting. (Schoenfeld, 2013, p. 18)

In a quantitative study of 176 Israeli high school math teachers, Hoz and Weizman (2008) use the terms "dynamic-changeable" and "static-stable" to distinguish between two diametrically opposed conceptions of mathematics. Teachers with a static conception see math as an infallible and immutable body of facts and rules held together by almost divine logic. Teachers with a dynamic view, on the other hand, conceive of math as a problem-driven creative endeavor, a social process of inquiry in which the question is as important as the answer and uncertainty and fallibility are assumed. The static conception holds math to be inherently more difficult than other disciplines, while the dynamic perspective holds that cognitive challenges are not unique to mathematics (p. 907).

Even among teachers who share a dynamic image of math, classroom lessons may reflect a traditional approach since beliefs alone do not account for the differences in teacher practices (Sztajn, 2003). Teachers point to other factors such as time, resources, and student behaviors to explain why they do not teach the way they believe they should (Raymond 1997). Elements as discernible as these, however, are easier to address than perceptions that lie beneath the surface. It is crucial that district leaders attend to beliefs in addition to pedagogy since teacher attitudes go a long way in determining *student* perceptions of math (Boaler & Greeno, 2000).

Teacher Perceptions of Mathematics Learning

Teachers' beliefs about math carry over to their beliefs about teaching math. Chapman's (2002) work linking math-as-object to show-and-tell approaches is consistent with earlier research on teachers' perceptions of knowledge. Teachers who believe that "knowing" amounts to representing something that exists outside the mind are likely to believe they can teach students simply by presenting information clearly and accurately (Cobb, Yackel, & Wood, 1992). Regrettably there are still many secondary mathematics teachers who would define "understanding math" as "the memorization and correct execution of standard algorithms" (Philipp 2007, p. 288). This is perhaps one of the biggest challenges of reform, changing notions of what it means to understand, particularly among high school teachers, who are more likely to have drawn their models of pedagogy from the university (Ball & Bass, 2002), where teaching is often "telling." It may be particularly difficult to steer teachers away from this view because it supports a teacher's sense of efficacy (Philipp 2007), perhaps especially in the math classroom:

> The conception of mathematics as a fixed set of facts and procedures restricts the content teachers must know; thus they can think that they have mastered the necessary content. The notion of teaching-as-telling provides a detailed but attainable model that teachers can hope to master. Telling students how to perform procedures also supports teachers' senses of efficacy, because the conventional nature of procedures is such that students cannot be expected to know them until the teacher shows them, and so the students' successes in mastering the procedures can be attributed to the teacher. (p. 281)

Hoz and Weizman (2008), in addition to defining static and dynamic conceptions of math held by teachers, describe two opposing attitudes toward teaching math: a "closed-strict" and an "open-tolerant." The "closed" attitude aligns with Freire's (1970) "banking model" and prioritizes math content over student development. The role of the teacher, as

authority, is to transmit knowledge to the passive student. The "open" attitude, on the other hand, aligns with constructivism and is driven by a concern for the student's development. The student is seen as capable of constructing meaning through problem solving opportunities, and math ability is viewed as malleable and acquirable. Learning is both personal and social. Hoz and Weizman (2008) found the most prevalent combination of math conception and teaching approach to be static-closed and the rarest to be dynamic-open (p. 910). In their study involving 176 Israeli math teachers, however, they found that a full 50% of teachers failed to adhere to a particular conception of math or the teaching of math, suggesting these attitudes are less polar than the researchers originally imagined.

Although teacher practices do not always strictly align with beliefs about math, it is interesting to note that practices are more likely to be consistent with math beliefs than with beliefs about *teaching and learning* (Raymond, 1997). Accordingly, in a structural equation model we would expect "perception of math learning" to be a particularly important construct predicting teacher practice. In fact, Gresalfi and Cobb (2011) cite ample research evidence that "improving practice involves reconceptualizing what it means to teach mathematics" (Cobb, McClain, Lamberg, & Dean, 2003; Chen & Ball, 1990; Franke, Carpenter, Levi, & Fennema, 2001; Kazemi & Franke, 2004; Schifter, 2001).

Mindset: Teacher Perceptions of Student Capabilities

A real concern on the part of many teachers resulting from the increased rigor of the CCSS is that not all students will be up to the challenge. The notion that some students cannot do the math is not new with the CCSS. "Sadly," says Rhona Weinstein (2002), "our system of education is largely built on beliefs and practices on the negative side—about differences in and limits to ability" (p. 1). Often teachers have lower expectations of low-

income and ELL students in particular and tend to use very basic procedural approaches with those students (Boaler, 2002). The more likely teachers are to treat students differently, the more likely students are to see themselves as the teacher does (Weinstein, 2002, p. 161).

Oddly, a student may be judged capable by one math teacher and incapable by another. Butler (2000) found that some teachers were more inclined to judge competency based on initial outcomes while others based judgments on subsequent achievement. This suggests the need to build consensus among teachers about what we are looking for when judging capability and to carefully define assessment criteria so that "what gets constructed as competent" ceases to vary (Gresalfi et al., 2009, p. 52).

Beyond a common description of competence lies the question of whether teachers view ability as fixed or malleable. Dweck's research in self-theory (2006) has shown that changing student "mindset" regarding this question can make all the difference in achievement. In studies, adolescent students who believed they could increase their intelligence—those with a growth/incremental mindset—earned significantly higher grades than peers with a more fixed/entity mindset (Blackwell, Trzesniewski, & Dweck, 2007; Dweck, 2006).

What impact does the teacher's mindset have on students? What happens when *the teacher* believes that a student is either born with math talent or is not? Particularly important to this study are Butler's (2000) findings that individuals with a fixed attitude toward the capability of others are likely to have a fixed attitude toward their own competence. Accordingly, a teacher with a limited view of a student's capacity might also have a limited view of her own ability to learn new instructional methods. Stipek et al. (2001) specifically suggest that an entity mindset may "undermine" math reform efforts. They reason that a

"shared focus on correctness" links a teacher's fixed attitude with traditional teaching practices. In their study involving 21 fourth- through sixth-grade teachers, they hypothesized that "the more teachers embraced traditional beliefs about mathematics teaching and learning and the less they embraced inquiry-oriented beliefs, the more they would espouse an entity theory of mathematics ability" (p. 215). Although results did not confirm a significant association between entity attitude and all traditional practices, they were able to establish an association between an entity mindset and a focus on "performance"—good grades and correct answers—over a more constructivist focus on understanding (p. 221).

Dweck (2006) suggests that teacher mindset does play a role in overall achievement; in countries that outperform the U.S., educators have a much less fixed mindset about student ability and believe that effort is the key to success. Anderson (2007) calls on teachers to deemphasize the "nature face" of math identity—the notion that math success is determined by innate predispositions out of the control of the student. Teachers should instead focus on engaging students in mathematical activities that foster a sense of competence.

Teacher Self-Efficacy

It was noted earlier that "teaching as telling"—a more traditional mode—was tied to teacher comfort and sense of self-efficacy (Philipp, 2007, p.281). More generally, teacher efficacy beliefs have been found to relate positively to attitude toward reforms and to openness to new instructional strategies and materials. As far back as Guskey (1988) it was observed that "teachers who express a high level of personal efficacy, who like teaching and feel confident about their teaching abilities, who are indeed highly effective in the classroom—these teachers also appear to be the most receptive to the implementation of new instructional practices" (p. 67). Teachers with low self-efficacy, on the other hand, are less

comfortable with change. Charalambous and Philippou (2010), studying 151 elementary math teachers five years into a district-mandated math curriculum reform, found that teachers with low self-efficacy had more concerns about the reform. “Teachers who were more comfortable with *pre-reform* [emphasis added] approaches tended to be more critical of the reform, exhibited more intense concerns about their capacity to manage the reform, and were more worried about its consequences on student learning” (p. 14).

Of specific application to the discussion of student-centered learning is the finding that "teachers with high efficacy beliefs were more open to student ideas" (Charalambous & Philippou, 2010, p. 3). This suggests that current calls for more student-centered, constructive approaches are likely to be better received by those who are already secure in the quality of their work. For others, it may be necessary to "reconceptualize their senses of efficacy” in order to be ready to accept math instructional reform (Philipp, 2007, p. 281). This of course is not easily accomplished, especially since the reforms themselves often erode teacher self-efficacy. Lasky (2005) points to the “guilt, frustration, and inefficacious vulnerability” teachers experience when a call for change characterizes them as “less effective” teachers (p. 911). This is exacerbated by the inherent “fishbowl” nature of the teaching (Kelchtermans, 2009; Lortie, 1975). It is, after all, a highly public act, observed by many, so it follows naturally that a teacher’s self-understanding is heavily “influenced by how others see him/her or what others say about him/her as a teacher” (Kelchtermans, 2009, p. 259). The social aspect of teaching revolves mostly around interaction with students, but the way teachers are viewed by peers also impacts self-understanding. When a teacher’s perspective is viewed by another as “outdated” the teacher becomes aware of that perspective

and its impact on the way others view him/her. “This awareness triggers…a critical examination” of the belief or attitude (p. 261). After all, a sense of belonging is important.

Professional Identity

Indeed another aspect of teacher self-understanding that has an impact on receptivity to change is strength of identity with the profession. This is due in part to what Lasky (2005) describes as an intricate intertwining of professional identity and self-worth. Nias (1989), too, notes that for teachers in particular overall self-image is very difficult to separate from professional self-image. For math teachers that image has two components, both a math and a teaching element. An Australian study of teachers who were identified by peers as strong math teachers found that “while the teachers saw themselves primarily as teachers, it was clear that they all had a strong mathematical sense of self and that their professional practice as mathematics teachers developed from both their pedagogical and discipline-based identities” (Grootenboer & Ballantyne, 2010, p. 226). It is essential to take into account this dual-natured construct since both aspects of identity are “foundational to teaching practice” (Grootenboer & Ballantyne, 2010).

Changing practice often involves professional-identity realignment. A math teacher may need to revise professional self-understanding in the light of math reforms, and such reshaping of self-image, as stated earlier, can be psychologically demanding. “When these deeply held beliefs are called into question—and the risk that this happens is always present—teachers feel that they themselves as a person are called into question” (Kelchtermans, 2009, p. 262).

Wenger (2010) explains that learning—a particular type of change, one that ought to occur during times of reform—necessarily shapes identity and may draw a teacher closer to

or further from her peers:

> Learning can be viewed as a process of realignment between socially defined competence and personal experience—whichever is leading the other. In both cases, each moment of learning is a claim to competence, which may or may not be embraced by the community. This process can cause identification as well as dis-identification with the community. In this sense, identification involves modulation: one can identify more or less with a community, the need to belong to it, and therefore the need to be accountable to its regime of competence. Creating an experience of knowledgeability (or lack of knowledgeability) involves a lot of identity work. Through this process of identification and the modulation of it, the practice, the community, and one's relationship with it become part of one's identity. Thus identity reflects a complex relationship between the social and the personal. Learning is a social becoming. (Wenger, 2010, p. 180)

It may be difficult, then, to predict whether a favorable attitude toward new practices will correspond to a stronger or weaker sense of professional identity. It may depend on the disposition of the community. Either way, Lasky (2005) tells us professional identity develops over time. She suggests the stronger the professional identity the less likely it will be eroded in times of institutional reform. The math reform movement "challenges most ways the majority of teachers have come to view themselves and their role in the teaching and learning process" (Gresalfi & Cobb, 2011, p. 273). A person with a strong connection to the role of "math teacher" will feel less threatened by change and so more open to it.

Ambiguity Tolerance

The final construct explored in this study, tolerance of ambiguity (TA), relates to a person's risk-taking attitude, which not only contributes significantly to a teacher's identity formation, but also relates directly to his/her feelings about change (Reio, 2005). McLain (2009) defines "ambiguity" as "a lack of information beyond risk or uncertainty, which requires an awareness of all possible outcomes" (p. 977). He explains that although ambiguity may hold attraction when a possible outcome is the improvement of a negative

state, often people experience stress in ambiguous situations. “A complex stimulus overwhelms the perceiver who must sift through a lot of information in order to understand the situation” (p. 977). Possible responses to novelty include avoidance, delay, and denial (Budner, 1962).

Although ambiguity tolerance is closely related to risk-taking attitude (RT), in this study the researcher strives to distinguish the two. On the other hand, due to reliance on items from Budner (1962), this study does not draw the fine line between TA and tolerance of uncertainty (TU) as described by Furnham and Marks (2013). In differentiating between RT and TA, the focus is on the probability of outcomes. In an ambiguous situation, the probabilities associated with outcomes are unknown and therefore cannot be evaluated. In a risk-taking situation, outcomes are known, and chances are taken accordingly (Furnham & Marks, 2013). In distinguishing between TA and TU, they focus on context, both locational and temporal. While TA is used “primarily in cognitive studies on decision-making, memory and perception,” TU is referenced in studies related to anxiety disorders (Furnham & Marks, 2013, p. 718). TA reflects a response to a current stimulus, with ambiguity an attribute of the stimulus itself, whereas TU is future-directed with the uncertainty an emotional response attributable to the individual (Furnham & Marks, 2013; Grenie, Barrette, & Ladouceur, 2005; Krohne, 1993). The three constructs are inter-related and the differences among them subtle. This study, in keeping with the Budner instrument, includes aspects of uncertainty in its measurement of ambiguity tolerance.

For two reasons the ambiguity tolerance variable is important to the study of math teacher change. For many teachers the mathematical practices of the CCSS suggest a dramatic shift in approach to instruction—to an as-yet unknown way of performing—and

Merenluoto and Lehtinen (2004) point out that conceptual change is only possible when a person is able to tolerate a high degree of ambiguity. When faced with a new knowledge system, the individual needs to trust that conflicts will be resolved or "avoidance behaviors" will result (p. 525). It is interesting to note that the secure attitude required to adapt to new practice is affected by the new practice itself. Because the ambiguity of educational reforms (*I don't know what this looks and feels like*) engenders uncertainty among teachers (*This makes me anxious*), capacity for novel behavior may be diminished at the very moment teachers need to rely on it (Reio, 2005). The implication is that one needs a fairly high tolerance for ambiguity before entering into a reorganization of practice.

Another reason the capacity to cope with ambiguity is relevant to this study is that a math teacher taking a more constructivist approach must be able to cope with the unknown of student responses. Since solutions strategies are not generally prescribed for inquiry-based tasks, teachers need to be able to think on their feet and let go of the need to know all answers ahead of time and to control the direction of thought (Stein & Smith, 2011). A teacher's willingness to take these risks impacts the way students learn math. Reio (2005) and Engel (2015), the latter speaking more in the language of risk-taking, suggest a teacher's attitude directly impacts student attitudes. When a teacher discourages uncertainty, students become less comfortable with ambiguity, and the discomfort eventually squelches natural curiosity (Engel, 2015), a requirement for deep engagement with mathematical ideas. The importance of openness to novelty extends beyond the math classroom. In a broader sense, risk-taking is a necessity for academic, social and professional success, for adaptation to an ever-changing world (Reio, 2005).

Tymula, Belmaker, Ruderman, Glimcher, and Levey (2013) found that the risk-taking behaviors of adolescents were attributable to their displaying a greater tolerance for ambiguity than adults. These findings suggest that perhaps there is a correlation between age and ambiguity tolerance. In a later study, however, these same researchers found "young, midlife, and older adults statistically indistinguishable in ambiguity attitude" (Tymula et al., 2012).

A Word on Career Stage

The career stage at which teachers find themselves during reform movements may play a role in willingness to grow and learn. Day and Sachs (2005) describe five phases of a teaching career: launching a career, stabilization, new challenges (experimentation), reaching a plateau, and the final phase. Beginning in the plateau phase, teachers stop striving and either enjoy or stagnate. It is particularly crucial, then, to identify perceptions among teachers in these last two phases that may predispose them toward unnecessary stagnation.

This is not to say that more experienced teachers are less open to change as some have asserted (Hargreaves, 2005). In fact, perhaps change is the very thing to ward off stagnation. Meister and Ahrens (2011) noted that plateauing occurs when teachers view their career as "void of new challenges" (p. 774). In Gusky's study (1988) there was no association between attitudinal constructs and years of experience. The current study confirms Gusky's findings.

Summary

This chapter has served to flesh out the issues surrounding math teacher change, focusing on six constructs identified by research as likely to impact a math teacher's openness to new practices. The next chapter explains how these constructs were

operationalized in the study in order to examine relationships among them. Chapter III describes the survey tool developed to measure constructs, the recruitment of survey participants, the procedures used to analyze survey data, and the exploratory and confirmatory techniques employed to develop and test a model of math teacher openness to change. Chapter IV relates results of the analyses, using statistical inference and a structural equation model to answer the research questions.

CHAPTER III

RESEARCH METHODS

The purpose of this study was to develop a means for identifying and measuring attitudes among secondary math teachers that impact their willingness to embrace instructional shifts. Another aim was to explain complex relationships among those inclinations. The study addressed four research questions:

1. How can math teacher openness to changing classroom practices be measured?
2. Which demographic measures predict openness to change?
3. How valid are certain constructs (math mindset, self-efficacy, constructivist perception of learning, dynamic conception of mathematics, professional identity, and ambiguity tolerance) for predicting a math teacher's receptivity to new practices?
4. How do these factors relate to one another as predictors of openness to change?

This study employed structural equation modeling (SEM) to analyze and represent the complex interrelations of six latent variables contributing to openness to change. The study was operationalized through the development of a survey instrument sent to secondary mathematics teachers around the U.S. Through a description of participants, measures, procedures and analysis, this chapter details how the research was carried out. It should be noted that a pilot study to develop the survey instrument was conducted in the fall of 2013 as

part of a final project for EFR 517: Advanced Research Methodologies. Results of that pilot study will also be discussed in this chapter.

Survey Methodology

A survey was used to gather data from secondary mathematics teachers. An advantage to the survey approach in the case of this study was the protection of participant anonymity, which may have helped to ensure honest responses (Rudestam & Newton, 2007). Also, the online delivery of the survey provided access to a large number of math teachers spread across a broad geographical region, another benefit typical of Internet surveys (Rudestam & Newton, 2007). This latter was important since a large sample size was crucial to the SEM analyses. Although answers to the first three research questions could have been addressed with a smaller sample drawn only from the rural state in which the research was first conducted, early analyses showed it would be necessary to reach out to other states in order to adequately answer the fourth question with a structural model (Kline, 2005). The calculations used to determine appropriate sample size are explained below.

Online surveys are known to present problems of both coverage and nonresponse bias. In this study, the sample was limited to teachers that could be reached through the researcher's professional network of mathematics education leaders. Although that network is far-reaching and enlisted participants from 45 states and D.C., coverage within states was not necessarily even. Additionally, as with many online surveys, this one was biased by an unknown rate of nonresponse. "Nonresponse error arises through the fact that not all people included in the sample are willing or able to complete the survey" (Couper, 2000, p. 473). It is possible that participants in this survey had special motivation, perhaps due to an interest

in the topic. To encourage others who may have been more reluctant, an incentive was provided in the form a chance to win one of seven $50 Amazon gift cards.

Finally, although asked to email the survey link to math teachers directly, one colleague in the researcher's professional network tweeted the link, inviting responses from "math teachers grades 7-12." There is no way of determining how many participants were solicited via Twitter this way, but since only 23% of adult Internet users were Twitter users as of 2014, some portion of the sample is biased accordingly and might also be expected to be biased relative to the age demographic (Duggan, Ellsion, Lampe, Lenhart, & Madden, 2015). That said, experience data for out-of-state respondents corresponds fairly well to national statistics for math teachers as shown in Tables 1 and 2 (IES, 2013, p.136). If

Table 1. Secondary Math Teachers by Years of Experience.

Years experience	Less than 3	3-9	10-20	Over 20
U.S. 2011-12				
(Grades 9-12)	12.0%	33.9%	35.0%	19.1%
Out-of-state	5.7%	19.2%	39.3%	35.8%
All participants	11.4%	18.2%	35.2%	35.2%

(IES, 2013, p. 136)

Table 2. Teachers by Age.

Age	Less than 30	30-39	40-49	50-59	60 or over
U.S. 2011-12					
(Grades K-12)	15.9%	19.3%	19.2%	20.5%	12.2%
Out-of-state	8.9%	24.8%	29.6%	23.5%	13.2%
All participants	14.9%	24.2%	27.8%	23.1%	10.0%

(IES, 2013, p. 136)

anything, the out-of-state data skews in the direction of experience, so Twitter did not have the effect of lowering overall experience levels.

The study employed a cross-sectional rather than a longitudinal design and thus yielded a snapshot of math teachers at a particular point in time. Although a longitudinal application of the survey tool could be used to track attitude changes across a period of reform, for the purposes of this study—which were to validate the instrument and the model—a cross-sectional design was appropriate.

The survey was administered using Qualtrics™ online data collection software. The full version of the survey can be found in Appendix A. Approval to conduct the survey was obtained from the Institutional Review Board (IRB) of the University of North Dakota (UND) (see Appendix B). For districts within the state of North Dakota, permission to contact teachers had been obtained in writing during the pilot, according to the policies of individual districts.

Pilot Study

In the pilot, conducted during the fall of 2013, the orginal survey tool was distributed to secondary mathematics teachers in North Dakota only. District officials signed letters of agreement that indicated there would be a follow-up survey within eighteen months.The online questionnaire was relayed to teachers through local administrators and regional coordinators, thus the number of participants solicited cannot be precisely determined. Given an approximate population of 600 and N at 186, the response rate was 31%, with 67 teachers from more rural areas responding and 119 from small cities. The sample was comprised of 106 females and 80 males, 71 middle-school teachers, 83 high-school teachers, and 28 who

taught at both levels. The sample reflected a range of teaching experience from a few months to 41 years. 84 participants had at least a master's degree.

The pilot instrument consisted of 22 items, only 13 of which survived exploratory factor analysis. The 13 items (see Appendix C) loaded strongly onto five normally distributed constructs, which explained 65.4% of the variance in responses to these items. The five constructs explored in the pilot correspond to five of the six to be explored in this study:

Table 3. Pilot Factor Loadings.

Varimax Rotation Component Matrix					
CcptMath	.82				
CcptMath	.84				
LrnMath1		.58			
LrnMath2		.72			
LrnMath3		.73			
LrnMath4		.73			
MndSet1			.75		
MndSet2			.72		
MndSet3			.83		
ProfImg1				.90	
ProfImg2				.91	
Effic1					.80
Effic2					.80

conception of mathematics, perceptions regarding learning math, math mindset, self-efficacy, and professional identity (not ambiguity tolerance). Exploratory analysis, however, left three of the five constructs with only two strongly loaded variables each (see Table 3).

Reliability analysis for the subscales in the pilot resulted in Cronbach's alpha ranging from .55 to .81, where ratings of .7 to .8 are considered satisfactory for comparing groups (Bland & Altman, 1997). Cronbach's for the 13-item "openness to change" total scale was .71 and would not increase with the deletion of any subscale item. The subscales correlated weakly but often significantly with one another (see Table 4), yet each of them correlated more strongly and significantly to the sum of the other four (see Table 5), substantiating consistency and the hypothesis that the five subscales combine well to measure openness to change.

When subscales were compared to two general questions about change—there was no change scale in the pilot—not all subscales correlated with "I am afraid to change the way I teach math," but *mindset* and *professional identity* did, with a fixed mindset and greater identity with profession corresponding to greater fear of change. The total change-openness scale correlated with the fear item ($r = .28$) at the $p < .001$ level. All subscales correlated significantly ($p = .01$) with the general item "Changes suggested by the Common Core make me want to leave teaching." The total change-openness scale correlated with this item ($r = .39$) at the $p < .001$ level. The medium-strength correlation between these two change items suggested that the tool, when strengthened, could be used to assess possible openness to change in math practices.

Analyses of variance showed no significant difference in "math reform openness" by gender, highest degree attained, or school district type. A *t*-test revealed, however, a

difference in mean change openness between middle-school math teachers (53.0) and high-school math teachers (50.7) ($p < .01$). This finding would be confirmed in the larger study.

The research went beyond inferential statistical analyses during the pilot to an application of Structural Equation Modeling (SEM) in order to test hypotheses about causal

Table 4. Correlations of Subscale Constructs and Measures of Internal Consistency (Pilot).

Construct	Subscale Constructs	C1.	C2.	C3.	C4.	α
C1.	Conception of Mathematics					.60
C2.	Perceptions of Learning Math	.15*				.64
C3.	Math Mindset	.18*	.30**			.70
C4.	Professional Self-Image	-.18*	-.18*	-.13		.81
C5.	Self-Efficacy	.07	.14	.22**	-.19	.55

*Correlation is significant at the .05 level
**Correlation is significant at the .01 level

Table 5. Correlations of Subscales to Partial Sums (Pilot).

Subscale	Correlation to Total of Other Subscales
Conception of Mathematics	.222**
Perceptions of Learning Math	.334**
Math Mindset	.364**
Professional Self-Image (R)	.269**
Self-Efficacy	.251**

**Correlation is significant at the .01 level

relationships among the sub-constructs. Confirmatory factor analyses were conducted in AMOS using the maximum-likelihood estimation method (Myung, 2003).

Figure 21 of Appendix D shows the hypothesized measurement model used in the pilot for confirmatory factor analysis. It is over-identified, with 58 degrees of freedom, indicating there are enough known parameters to estimate the unknowns. Three covariance relationships were left out of the model due to low correlations indicated in the SPSS correlation matrix. The fit of the model to the data was quite good ($\chi 2$ = 59.744, df = 58, p = .412; SRMR = .0582; RMSEA = .013; CFI = .996; TLI = .994) and so suggested construct validity, except that some factor loadings—albeit significant—were weaker than they should have been (*Lrn2* at .45 and *MSet2* at .46). The structural equation model (See Figure 22, Appendix D) confirmed weak to medium-strength covariance between most construct pairs, but the constructs loaded onto "openness to change" at lower than desirable levels, the highest being professional identity at 0.27.

The pilot yielded several interesting findings. It made clear that current nationwide pressures toward instructional change have many math teachers in the state considering other careers (29.8% say, "Changes suggested by the Common Core make me want to leave teaching.") It also supported correlations suggested by the literature: change is difficult relative to strength of identity with the profession and for those with very fixed ideas about what math is and how students learn it. Mindset appeared to play a particularly important role.

Despite the strengths of the pilot, there were deficiencies that indicated a second study would be worthwhile. The quality of fit for both the measurement and structural models was high, and while there was some satisfaction in this, it has to be acknowledged that low correlations and loadings may have in fact explained these results. In evaluating the instrument in terms of validity, reliability, and item quality, the greatest concerns that

emerged were the set of subscales that failed to load more than two items: "conceptions of math," "professional self-image," and "math teaching self-efficacy."

The goal in the current study was thus to strengthen those subscales through the addition of items in order to boost the overall internal consistency of the scale and the reliability of the SEM model. Another key enhancement in the current study was the strengthening of the *change* scale. Since it represented the dependent variable, it was particularly important that it be valid and reliable. The first measurement model tested in the current study, using the enhanced survey tool, was very similar to the pilot model. The researcher is not aware of any competing models in the literature.

Dissertation Study

Participants

The population for this study was made up of part-time and full-time secondary mathematics teachers—at the outset from North Dakota exclusively but in the end from across the United States—whether working in public or private institutions. In early April of 2015, an email (see Appendix E) was sent to local administrators and regional coordinators in North Dakota, reminding them of their ongoing commitment to the research study and asking them to forward the new survey link to area math teachers. As in the pilot, the number of participants solicited could not be precisely determined, but given an approximate population of 600 math teachers in North Dakota and 244 responders, the response rate this time was a bit higher at 40.7%.

A priori determination of sample size is characteristic of structural equation modeling since the model itself dictates the number of participants required. There are various rules of thumb for minimum sample size and no clear agreement (Westland, 2010). A common rule

suggests 10 participants per item (Schreiber, Stage, King, Nora, & Barlow, 2006), yet another suggests either 200 or 20 times the number of parameters to be estimated, whichever is larger (Kline, 2005, p. 111, 178). With 55 items constituting the latent variables to be explored in this study, the need to increase the sample size became apparent.

Accordingly, the researcher decided to take advantage of membership in several national networks of math education leaders in order to reach out to teachers in other states. In May 2015, an IRB protocol change was approved (see Appendix B), and a letter (see Appendix E) was emailed to colleagues associated with the Council of Presidential Awardees in Mathematics, the Rutgers Institute of Discrete Mathematics, the Park City Mathematics Institute, and the National Council of Supervisors of Mathematics. Although it was late in the school year, the relayed emails resulted in an additional 465 math teachers logging onto the survey from May 29 to June 15.

Out of the combined 709 cases from within and outside North Dakota, 75 were cases in which the participant logged on but never actually began the survey; these were eliminated immediately. Because AMOS software for SEM is extremely sensitive to missing data, in order to address other gaps, rather than resort to imputation of data, the researcher performed listwise deletion on 8.4% of the remaining cases, where participants had begun the survey but had not progressed far in completing it. At that point there were 10 remaining cases with one or two items unanswered; those participants were eliminated as well. Of the 634 who took the survey, then, 10% were listwise deleted, resulting in a sample size of 571.

The sample was 34.7% male and 64.6% female, with one participant choosing "other" for gender and three choosing not to identify; this corresponded adequately to national math-teacher statistics of 42.7% male and 57.3% female (IES, 2013, pp. 134-136). Data on

ethnicity showed 91.6% of participants identified as White, compared to a national high school math statistic of 83.0% (IES, 2013, p. 139). This difference is no doubt due in large part to high participation from North Dakota (36%), whose population is 89.6% White compared to 77.7% nationally (U.S. Census Bureau, 2013). In terms of education, 61.1% of those surveyed held at least a masters degree, whereas among mathematics teachers nationally this figure was only 48.8% in 2012 (IES, 2013, p. 137). The difference is due at least in part to the high rate of master's degrees in North Dakota, which was heavily represented. 53.5% of study participants majored in math in college, with the figure at the national level at 64.5% among high school math teachers (IES, 2013, p. 139). The inclusion of middle school teachers, especially 6th grade, likely lowered the number. More high school mathematics teachers (58.0%) took the survey than middle school (39.6%). Figures 2 through 4 show participants' age and experience.

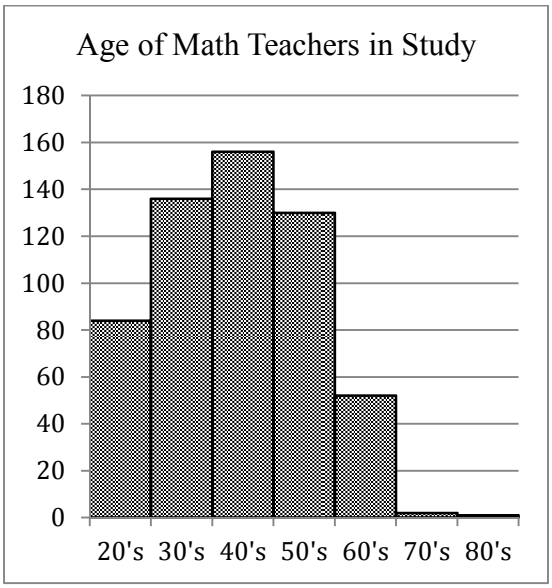


Figure 2. Histogram of Participants by Age. (Mode = 30; Median = 44)

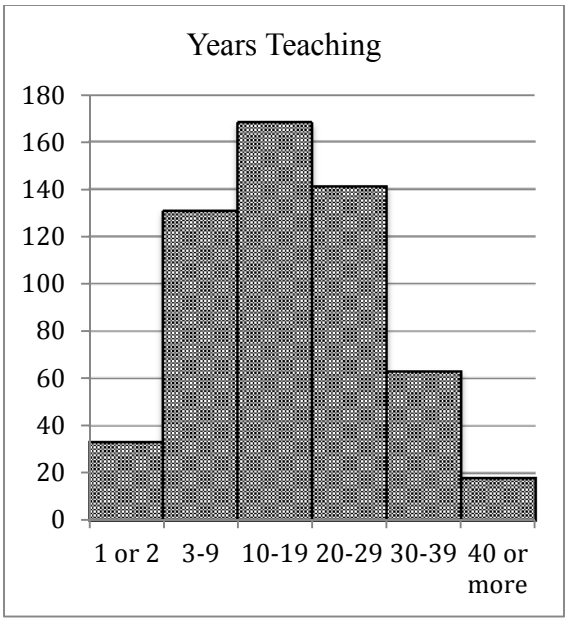


Figure 3. Histogram of Participants' Teaching Experience. (Mode = 3; Median = 16)

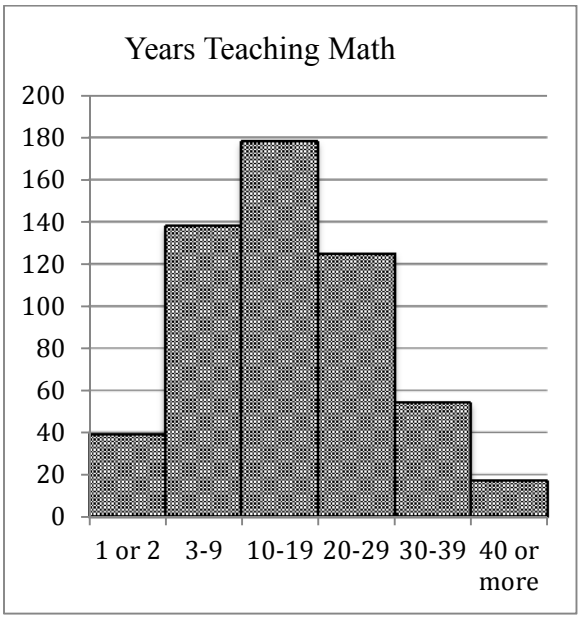


Figure 4. Histogram of Participants' Math Teaching Experience. (Mode = 10; Median = 15)

It is to be expected that these last two graphs look very much alike since 80.4% of teachers in the study had taught math throughout their years of teaching.

Again, given that participants were recruited most heavily and most directly from the state in which the research was done, it is not surprising to see in Table 6 that a high percentage of participants are from North Dakota. Nonetheless, participation from rural North Dakota is balanced by the 41% of particpants who teach in cities of population greater than 100,000. See Figure 5 for distribution by setting.

Table 6. Participants by State.

	Frequency	Percent		Frequency	Percent
AL	5	.9	NH	3	.5
AK	1	.2	NJ	12	2.1
AZ	5	.9	NM	2	.4
CA	35	6.1	NY	22	3.9
CT	3	.5	NC	3	.5
FL	4	.7	ND	207	36.3
GA	2	.4	OH	8	1.4
HI	2	.4	OK	5	.9
ID	38	6.7	OR	1	.2
IL	39	6.8	PA	8	1.4
IN	2	.4	RI	18	3.2
IA	2	.4	SC	3	.5
KY	3	.5	SD	1	.2
LA	1	.2	TN	1	.2
ME	4	.7	TX	7	1.2
MD	17	3.0	UT	1	.2
MA	11	1.9	VA	5	.9
MI	6	1.1	WA	8	1.4
MN	7	1.2	WV	2	.4
MS	3	.5	WI	33	5.8
MO	1	.2	WY	1	.2
NE	3	.5	DC	2	.4
NV	2	.4	Total	549	

No participants from AR, CO, DE, KS, MT, or VT

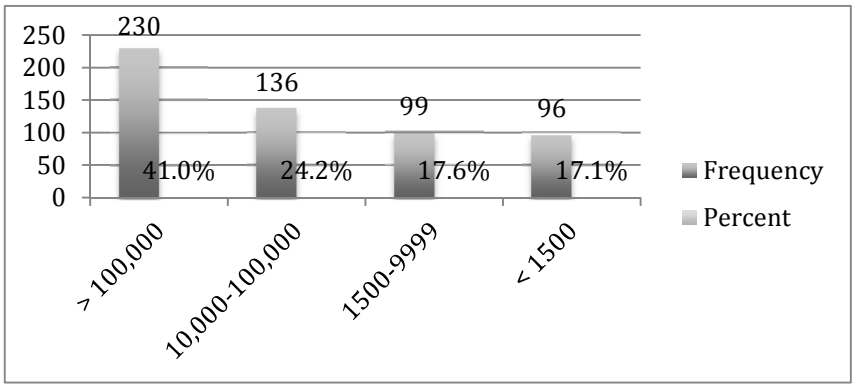


Figure 5. Number of Participants by Population of School Setting.

Procedures

Data collection took place during April, May and June of 2015. Secondary math teachers in North Dakota were invited in April via email to take a ten-minute survey. The email contained a link to an online survey and indicated a three-week window for completing it. After two weeks, the principal investigator asked North Dakota administrators to forward a reminder email to teachers (see Appendix E). The last week of May, after the survey had closed to North Dakota teachers, additional math teachers from outside North Dakota were invited by colleagues of the researcher to take the survey. Some received the invitation via email and some responded to a tweet on Twitter that read "Are u a 7-12 math teacher? Cathy Williams needs your help w/ dissertation task. She has ~10 min. anon survey: https://und.qualtrics.com/SE/?SID=SV_86ctP4jdsaTyldX … #**MTBoS** ." The survey closed to out-of-state participation on June 15. All survey results were strictly anonymous, with no information linking responses to individuals, schools, or districts.

Measures

In development of the instrument the constructs of interest were those identified by research. The survey tool (see Appendix A) consisted of ten demographics items and fifty-

five items measured on a six-point Likert-like scale (1=*strongly disagree*, 2=*disagree*, 3=*slightly disagree*, 4=*slightly agree*, 5=*agree*, 6=*strongly agree*). The items related to seven subscale constructs: conception of mathematics, perceptions of how students learn mathematics, mindset regarding mathematics ability, sense of professional identity, self-efficacy regarding the teaching of mathematics, ambiguity tolerance, and attitude toward change. Those that were related to conception of math, perceptions of learning math, math mindset, and self-efficacy drew heavily from items in the Stipek et al. study (2001) since that instrument was used to assess similar constructs among elementary math teachers.

In each subscale, roughly half of the items were worded positively and the other half negatively. Within all subscales, the challenge was to apply previously established scales to the context of teaching secondary mathematics. For this reason established scales were not used in their entirety and items were often adapted or supplemented. Specifically, items were slightly adapted for secondary mathematics teachers where necessary. For example, "Ability is something that remains relatively fixed throughout a person's life" became "Math ability is something that remains relatively fixed throughout a person's life."

In the conception of math scales, four items were drawn from Stipek and six other items were based on Hoz and Weizman (2008). An example item is "Math is mostly about finding the answer." The items were named "mth1-10" in SPSS but were later renamed (see below).

Among the perceptions-of-learning-math items, two were original, four items were based on Hoz and Weizman, one was from Stipek, and one item was based on NCTM's *Principles to Action* (2014). An example is "Math can be applied only after basic skills are mastered." The items were named "lrn1-8" in SPSS but were lated renamed (see below).

The math mindset subscale was based on Carol Dweck's (2006) work on entity theory. Six items are taken directly from the Stipek inventory and three are adapted for math from Dweck's own inventory For example, "There isn't much you can do about how much math ability you have." The items were named "mset1-9" in SPSS.

Three items from the self-efficacy subscale were adapted from a scale by Gibson and Dembo (1984). The other five items were taken from Stipek et al. An example is "I sometimes doubt my ability to teach math." The items were named "efc1-8" in.

Four of the professional identity items were based on a scale from Adams et al. (2006). An example is "I feel proud when I tell people I am a math teacher." The other two items in this scale were original. The items were named "pro1-6" in SPSS.

The ambiguity tolerance subscale consisted of six items drawn from the Multiple Stimulus Types Ambiguity Tolerance Scale-II as seen in McLain (2009) and two items adapted from the Rydell-Rosen Ambiguity Tolerance Scale (McDonald, 1970). An example item is "I try to avoid situations that are uncertain." The items were named "amb1-8" in SPSS. The change items were all original with this study, an example being, "I am quick to embrace new methods for teaching math." The items were named "chg1-8" in SPSS.

It should be noted here that early in the study, exploratory factor analysis revealed the math-conception and perceptions-of-learning-math items to be intermixed. They constituted two distinct factors, but not as the researcher had anticipated. Rather, they separated according to whether they had been phrased negatively or positively. The two constructs were realigned and renamed "dynamic" and "constructivist" in light of the Hoz and Weisman research (2008). The dynamic items measured the extent to which the teacher viewed math as a field of inquiry as opposed to a fixed body of procedures to be acquired for the purposes of

"answer-getting." Examples included "Math is a set of skills to be learned in sequence" and "Students who really understand math will have a solution quickly." The items were renamed "stat1-9" in SPSS.

The constructivist items measured the extent to which a teacher allowed students the opportunity to build their own understanding. Examples of this construct include, "In math, the questions are more important than the answers," and "Students need to construct their own understanding of a math concept." The items were renamed "cnstr1-7" in.

In the end, the seven subscale constructs were: math mindset, self-efficacy, constructivist perception of learning, dynamic conception of mathematics, professional identity, ambiguity tolerance, and attitude toward change. For each construct, items were summed in SPSS for purposes of correlating these subscales. Construct sums were named statSUM, cnstrSUM, msetSUM, proSUM, efcSUM, ambSUM, and chgSUM. The first six of these were then combined in an item called scaleSUM. When the model was simplified to include exactly four items per construct (see below), these new sums were labeled statSUM4, cnstrSUM4, msetSUM4, proSUM4, efcSUM4, ambSUM4, chgSUM4, and scaleSUM4.

The survey included ten demographic items, which were placed at the end of the survey. These included the individual demographics of gender, age, ethnicity, educational attainment, undergraduate major, areas of certification, grade level, years of experience in teaching, and years of experience teaching mathematics. One institutional demographic asked teachers to categorize the school district on a four-point scale from rural to urban.

Data Analysis

SPSS software was used for the early portions of the data analysis. Negatively worded items were reverse coded so that a high score on any item indicated openness to

change. To answer the first research question, "How can math teacher openness to changing practices be measured?" descriptive analyses were run in SPSS to test the distributional properties of each subscale item. SPSS exploratory factor analysis (EFA) was used to assess the unidimensionality of each proposed construct and to gauge the construct validity of scales. Since most subscales had not been established in their current forms in other studies, these analyses were particularly important. Principal axis factoring with oblique rotation was applied.

The internal reliability of each construct was tested in SPSS to confirm a Cronbach's Alpha of .70 or higher. The researcher later used AMOS software for confirmatory factor analysis. CFA is discussed in more detail later in this section.

To answer the second research question, "Which demographic measures associate with openness to reform?" *t*-tests were performed in SPSS as well as analyses of variance to determine whether demographic subgroups exhibited significant differences in the "change" variable and/or the scaleSUM variable. For example, the researcher examined the roles experience, education, and gender played in resistance to new approaches.

The third research question, "How valid are certain constructs for predicting a math teacher's receptivity to new research-based practices?" was addressed through regression analysis of subscale variables. Specifically, the researcher examined correlations between the "change" construct and each of the six independent variables (math mindset, self-efficacy, constructivist perception of learning, dynamic conception of mathematics, professional identity, and ambiguity tolerance) to determine which of the six associated most strongly with openness to change.

Finally, to answer the question, "How do these constructs relate to one another as predictors of openness to change?" the researcher used AMOS software and structural equation modeling (SEM) to test a proposed measurement model and a proposed structural model of relationships among constructs. SEM is a statistical methodology used to test structural theories regarding phenomena. SEM relates hypothesized theoretical models of relationships among variables to graphic depictions that clarify understanding of the proposed theory (Byrne, 2010). Because SEM is confirmatory in nature, it is important that the theoretical structures be grounded in research (Byrne, 2010). CFA assumes that measures "have been fully developed and their factor structures validated" (Byrne, 2010, p. 97).

In this study AMOS software for SEM was used both to confirm the measurement model (CFA) and to confirm the structural model. Within a SEM model there are both latent variables (not observed) and manifest (measured) variables. The measurement model consists of several manifest variables mapped onto each latent variable, and AMOS is used to confirm that the manifest items actually measure the latent construct. In the context of structural models, SEM also distinguishes between exogenous and endogenous variables. Exogenous latent variables—so called because their fluctuations are explained not by the model but by external influences—are the independent variables that predict changes in the endogenous or dependent variables (Byrne, 2010, p. 5). Both exogenous and endogenous may be latent variables (proposed constructs), operationalized through directly measured variables. In the structural model for this study, there are three exogenous variables theoretically predicting change in three endogenous variables, which in turn predict shifts in the endogenous variable "change." Each of these seven is determined by a number of measured items. Whether for CFA or to confirm a structural model, within AMOS software the researcher draws a

diagram of proposed relationships, and then the software generates goodness-of-fit indicators for that model, assessing how well correlations and covariances represented in the model are explained by its components. It does so through comparison of an estimated, model-based covariance matrix to the original sample covariance matrix (Fan, 1997).

Sample size is important to SEM. Since Kline's rule of thumb had suggested 20 participants per item, and since with a sample of 571 participation for this study fell short of the guideline for 55 items, the researcher elected to simplify the model through elimination of items (Kline, 2005). Although Hoyle (2012) makes clear that more complex models are often preferred with large samples due to replicability with other samples, he also states that simpler models can "provide better approximation to the population" (p. 226). In the case of this study the researcher determined 571 to be short of "large," based both on Kline's recommendation and the results of a test of SEM sample size at an online statistics calculator (Soper, 2015). The calculator suggested that for an anticipated effect size of 0.10, a desired statistical power of 0.80, and a 0.05 probability level, 28 items mapped onto seven latent variables would require a minimum sample size of 579 to detect effects—and 100 to test model structure. Accordingly, each subscale in this study was reduced to four items to achieve a simpler model. Chapter IV explains how these determinations were made.

The simple structural model first proposed appears in Figure 1. It shows six factors combining to predict the openness-to-change variable. When AMOS analyses revealed room for improvement in the model fit, the structural model was respecified. Structural changes were made to reflect both the research literature and tests of indirect effects performed in SPSS. The researcher postulated that since certain of the six constructs were more internal to the teacher and less concretely observable (self-efficacy, ambiguity tolerance, math mindset),

these were exogenous variables predicting other latent variables (professional identity, constructivist approach, static conception of math) on their way to predicting change. The respecified model in Figure 6 reflects this thinking. Figure 6, then, represents a competing model hypothesized to do a much better job of explaining how contributing factors predict change openness.

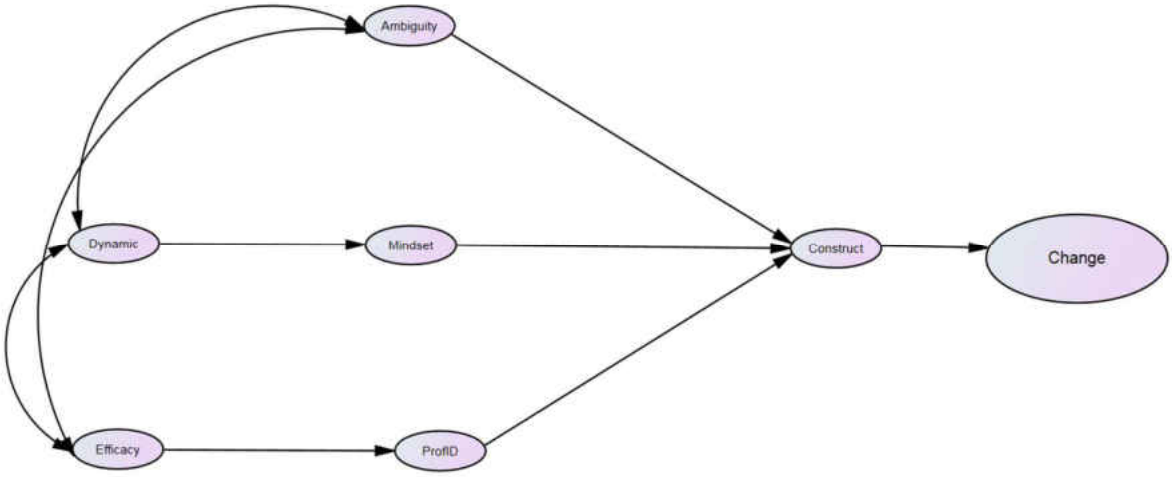


Figure 6. Complex Structural Model of Contributors to Change

In assessing goodness of fit for all models, care was taken to include at least one each of the incremental fit (CFI), parsimonious fit (RMSEA), and absolute fit (Chi-square and SRMR) indices to ensure comparison to both the independence and saturated models. The Comparative Fit Index (CFI) compares the hypothesized model with the null or independence model, which assumes all correlations among variables are zero. The CFI offers a good check against misspecification and recognizes values of .95 or higher as indicating goodness of fit (Hu & Bentler, 1999). The Root Mean Square Error of Approximation (RMSEA) test yields

a 90% confidence interval for its fit index and penalizes unnecessarily complex models. For the RMSEA, .06 or smaller is considered an indication of good fit (Hu & Bentler, 1999). The chi-square index, based on sound theory but of limited practical application here due to the effect of the large sample size, was used as a traditional first test, but the Standardized Root Mean Square Residual (SRMR) was employed as a preferred absolute-fit index, more likely to give meaningful output. The SRMR measures the mean absolute correlation residual or the average distance between predicted and observed correlations. With the SRMR, Kline (2005) recommends a reading of less than .10 for goodness of fit, while Hu and Bentler (1999) suggest .08 and Byrne (2010) favors .05.

Through the application of analyses described above, this is the first study that attempts to model the complexity of math teacher resistance to change. Validation of both the revised survey instrument and the measurement and structural models offers a clearer picture of obstacles to instructional reform and suggests a course of action for removing those impediments. Results are outlined in the next chapter.

CHAPTER IV

RESULTS

The dual purpose of this study was to validate a survey instrument for assessing math teacher openness to reform practices and to validate a structural model explaining factors contributing to that openness. The data analysis was accomplished in five phases. First, descriptive analyses of all variables were performed in order to understand characteristics of the sample (described in Chapter III) and to determine the suitability of variables for use in the model. Second, exploratory factor analyses (EFA) were performed along with tests of internal consistency in order to establish for each latent variable the validity and reliability of the scale. Third, correlation, regression, and means-difference tests were applied in order to explore relationships among variables. Fourth, confirmatory factor analysis (CFA) was employed to substantiate validity of the scales, verifying that survey items actually measured the latent constructs onto which they had been mapped. Finally, structural equation modeling (SEM) was used to test the fit of the hypothesized structural model to the relationships in the data. The final two phases involved five steps each. In the final phase there was at least one respecification of the model.

Research Questions

Across the five phases of research, the study addressed four questions:

1. How can math teacher openness to changing classroom practices be measured?
2. Which demographic measures predict openness to change?

3. How valid are certain constructs (mindset, self-efficacy, constructivist perception of learning, dynamic conception of mathematics, professional identity, and ambiguity tolerance) for predicting a math teacher's receptivity to new practices?
4. How do these factors relate to one another as predictors of openness to change?

Phase I: Descriptive Analyses

In order to establish univariate normality of the data—particularly important to SEM analyses since many fitness tests assume normality—skewness and kurtosis checks were run in SPSS on all items corresponding to the seven scales. 38 of the original 55 items had skewness within one of zero and so could be classified as normally distributed (Lei & Lomax, 2005). All other items showed skewness absolute values between 1 and 2 and so were moderately nonnormal (Lei & Lomax, 2005). The skewness issues occurred primarily in the mindset, efficacy and professional identity scales. In the pared down model, four of the twenty-four exogenous items had slight skewness issues: mset9 (-1.31), efc2 (-1.26), pro3 (-1.37), and pro 4(-1.40). Had the sample been smaller, this would have presented a more serious concern. Lei and Lomax (2005) reported that "nonnormality conditions had no significant effect on the CFI" (p. 13) fit index with sample size of 500. This was especially true in applications of the maximum likelihood (ML) method of estimation used in the SEM portion of this study (Lei & Lomax, 2005; Shah & Goldstein, 2006).

Although there is no clear agreement on kurtosis guidelines (Kline, 2005), West et al. (1995) suggest a within-seven-of-zero range (i.e., more than -7 and less than 7). In this study, some items exhibited potentially problematic kurtosis. This was important to SEM and AMOS assumptions of multivariate normality, which cannot exist in the presence of too much univariate kurtosis (Byrne, 2010). Table 7 highlights three measurement items for

which kurtosis approached the limit for normality: mset7 (6.258), efic4 (5.260), and cnstr6 (6.324). Fortunately only one of these variables, efc4, would be used in the pared-down measurement and structural models later on; for the time being all three items were retained with the understanding they would require close monitoring. Table 8 demonstrates normality among the numeric demographic items. For purposes of *t*-tests, it is important to observe there are no skewness issues here (Byrne, 2010, p. 103).

It must be noted that for the most part the math teachers in the sample tended to express agreement with positively framed items and disagreement with items framed in the direction of resistance to change, and in that sense, the overall picture was brighter than anticipated. The mean response to a change item was 4.68, almost at the "agree" mark, suggesting the typical math teacher is not that adverse to change. With all negative items reversed coded, the range of responses was from 2.01 ("disagree") to 5.39 ("agree") with the mean and median response about half way between "slightly agree" and "agree" at 4.49 and 4.66 respectively. Of the original 47 independent items, only eight had mean responses in the "disagree" range (mset1, mset4R, dyn3R, dyn3R, dyn6R, dyn7R, dyn 8R, dyn9R, and amb7), the lowest corresponding to the very first item on the survey, mset1: "To be honest, you can't really change how much math talent you have." Of the six independent variables, mindset and dynamic conception of mathematics seemed at the outset important targets for teacher training, but overall the picture was brighter than anticipated.

Table 7. Descriptive Statistics on Items.

Item	N	Mean	StD	Skewness	Kurtosis
mset_1R	571	4.99	0.98	-1.16	1.30
mset_2R	571	3.86	1.41	0.01	-1.18
mset_3R	571	5.01	1.00	-1.15	1.12
mset_4R	571	2.95	1.21	0.46	-0.63
mset_5R	571	4.57	1.14	-0.91	0.43
mset_6	571	5.18	0.84	-1.35	2.92
mset_7	571	5.05	0.83	-1.76	6.26
mset_8R	571	4.02	1.22	-0.18	-0.75
mset_9R	571	5.03	0.88	-1.31	2.97
efc_1	571	5.10	0.84	-1.05	1.58
efc_2	571	5.12	0.94	-1.26	1.77
efc_3R	571	5.02	0.87	-1.26	2.56
efc_4	571	5.14	0.69	-1.21	5.26
efc_5R	571	3.94	1.21	-0.11	-1.14
efc_6	571	4.67	0.92	-0.79	1.03
efc_7R	571	4.60	1.01	-0.69	0.07
efc_8R	571	4.76	1.16	-0.87	-0.07
cnstr_1	571	4.24	1.18	-0.46	-0.37
cnstr_2	571	5.06	0.82	-1.18	3.23
cnstr_3	571	4.53	1.11	-0.57	-0.19
cnstr_4	571	4.34	1.00	-0.50	0.36
cnstr_5	571	4.65	1.04	-0.78	0.60
cnstr_6	571	5.36	0.73	-1.68	6.32
cnstr_7	571	4.80	0.92	-0.64	0.65
dyn_1R	571	4.43	1.27	-0.51	-0.78
dyn_2R	571	4.29	1.22	-0.51	-0.55
dyn_3R	571	3.39	1.18	0.21	-0.74
dyn_4	571	4.73	1.00	-0.66	0.07
dyn_5R	571	4.41	1.09	-0.45	-0.41
dyn_6R	571	3.64	1.37	0.00	-0.96
dyn_7R	571	2.78	1.16	0.64	-0.03
dyn_8R	571	3.57	1.29	0.13	-0.80
dyn_9R	571	3.67	1.20	-0.09	-0.58
pro_1	571	5.39	0.72	-1.10	1.23
pro_2R	571	4.89	1.28	-1.46	1.66
pro_3	571	5.33	0.80	-1.37	2.74

Table 7 cont.

Item	N	Mean	StD	Skewness	Kurtosis
pro_4	571	5.09	0.83	-1.40	3.95
pro_5	571	5.39	0.73	-1.13	1.27
pro_6R	571	5.08	1.04	-1.30	1.62
amb_1R	571	4.88	0.91	-0.90	1.07
amb_2	571	5.00	0.94	-1.05	1.35
amb_3	571	4.38	1.11	-0.56	-0.01
amb_4R	571	4.22	1.15	-0.51	-0.33
amb_5	571	4.14	1.05	-0.38	-0.17
amb_6R	571	3.83	1.24	-0.04	-0.94
amb_7	571	3.48	1.01	0.22	-0.24
amb_8R	571.00	3.96	1.04	-0.12	-0.61
Chg_1R	571.00	4.32	1.22	-0.32	-0.80
Chg_2	571	4.88	0.82	-0.99	2.72
Chg_3R	571	4.51	1.06	-0.63	0.26
Chg_4	571	4.32	1.05	-0.23	-0.29
Chg_5R	571	4.67	1.14	-0.85	0.29
Chg_6	571	4.87	0.87	-0.38	-0.25
Chg_7R	571	4.71	1.09	-0.70	-0.24
Chg_8	571	5.13	0.87	-0.91	0.91

Skewness and kurtosis issues
Bold items were those retained in final simplified model

Table 8. Descriptive Statistics on Key Demographics.

		Age	Years Teaching	Teaching Math
N	Valid	562	554	553
	Missing	9	17	18
Mean		43.52	17.02	16.03
Std. Deviation		12.01	10.97	10.84
Skewness		.142	.527	.670
Kurtosis		-.800	-.378	-.167

Phase II: Exploration of Constructs

Initial analyses of internal consistency relative to the more complex model showed scales corresponding to all seven constructs from the study to be reliable, with Cronbach's alpha coefficient ranging from 0.720 (mindset) to 0.850 (change). Subscale items were summed in SPSS, and correlations among subscale sums were analyzed. Table 9 shows significant but not strong correlations among all constructs, a good indication that excess collinearity would not be a problem and independent variables might be used well in combination to predict change. Hoyle (2012) suggests attention be given to correlations "at or near unity" (p. 266), but none of these are close to a value of one.

To assess the construct validity of the scales, exploratory factor analysis was performed on the 55 item scores. Principal axis factoring with oblimin (oblique) rotation was used since correlations among items were assumed (Preacher & MacCallum, 2003). Initial

Table 9. Subscale Correlations and Cronbach's Alpha.

Number	Subscale Construct	C1.	C2.	C3.	C4.	C5.	C6.	α
C1.	Mindset							.720
C2.	Efficacy	.267**						.754
C3.	Prof ID	.282**	.409**					.746
C4.	Static	.526**	.221**	.240**				.809
C5.	Construct	.386**	.242**	.371**	.503**			.756
C6.	Ambiguity	.451**	.357**	.335**	.560**	.503**		.797
C7.	Change	.321**	.266**	.444**	.303**	.443**	.393**	.850

**Correlation is significant at the .01 level; N = 571

factoring was based on eigenvalues, which identified eleven factors, but subsequent tests were based on seven factors as indicated by both the scree plot (see Figure 7) and research literature. Preacher and MacCallum (2003) suggest the scree plot is a better indicator than eigenvalues themselves. It provides a clear visual of relatively large changes in eigenvalues. Table 10 shows initial indications of the need to exclude seven items based on weak loadings: mset4, pro2, amb1, amb 2, cnstr3, stat4, and chg1. With these items removed, a second EFA model explained 40.7% of the variance among items, with Cronbach's staying about the same or improving so that the lowest was now .741 (mset, .824; efc, .754; pro, .815; stat, .801; cnstr, .741; amb, .772).

Discriminant validity of the subscales was established through comparison in each factor pair of the average AVE (average variance extracted) to the R^2 statistic corresponding to that pair's correlation (see Table 11). Discriminant validity indicates the within-construct correlations are greater than the between-construct correlations (Bollen & Lennox, 1991). For every construct pair, the average AVE far exceeded R^2, indicating the factors were indeed measuring distinct constructs. Collinearity diagnostics in SPSS confirmed there were no problems with multicollinearity. All variance inflation factors (VIF) were between 1.205 and 1.396, well under recommended maximums (O'Brien, 2007). The VIF is the inverse of tolerance, which is the percent of unshared variance between two variables (O'Brien, 2007). VIF readings between 1.205 and 1.396 indicate between 72% and 83% unshared variance between any two subscales, in other words, a good deal of discrimination among factors.

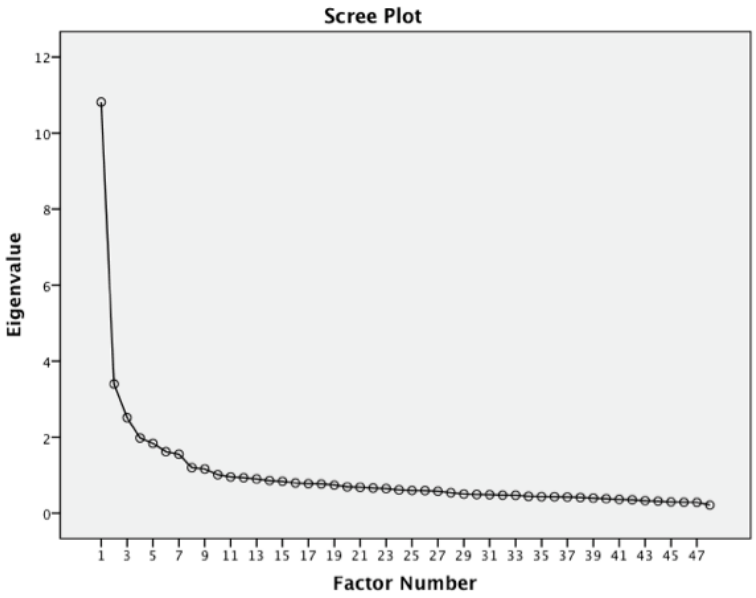


Figure 7. Scree Plot A.

At this point it was determined that initial findings merited further investigation. Of concern was the fact that some items were not loading strongly and would weaken any SEM models in AMOS. Parsimony was also a concern: a simpler model would be more effective in SEM. In fact, Hoyle (2012) describes several problems that arise in SEM with an overly complex model, among them difficulty in explaining and interpreting the model. Since the change scale was strong in terms of both internal consistency and content validity—no cross-loadings—it was removed from further EFA analysis so the researcher could focus on the "openness to change" scales, composed of the other 41 remaining items. The goal would be to pare down each subscale to approximately four items, taking care to preserve content validity, adequate factoring loadings and internal reliability of the constructs.

Table 10. Initial Factor Loadings.

Pattern Matrix[a]

	1	2	3	4	5	6	7
	construct	identity	efficacy	mindset	dynamic	ambig	change
cnstr_1	.439						
cnstr_2							
cnstr_4	.598						
cnstr_5	.453						
cnstr_6	.410						
cnstr_7	.681						
pro_1		.646					
pro_3		.670					
pro_4		.614					
pro_5		.805					
pro_6R		.586					
efc_1							
efc_2			.436				
efc_3R			.522				
efc_4			.493				
efc_5R			.568				
efc_6							
efc_7R			.414				
efc_8R			.662				
mset_1R				-.732			
mset_2R				-.528			
mset_3R				-.626			
mset_5R				-.656			
mset_6				-.424			
mset_7				-.481			
mset_8R				-.483			
mset_9R				-.701			
dyn_1R						.491	
dyn_2R						.549	
dyn_3R						.451	
dyn_5R						.431	
dyn_6R						.480	
dyn_7R						.531	
dyn_8R						.601	
dyn_9R						.403	
amb_3							-.495
amb_4R							-.475
amb_5							-.619
amb_6R							
amb_7							-.500
amb_8R							-.699
Chg_2					.517		
Chg_3R					.421		
Chg_4					.682		
Chg_5R					.497		
Chg_6					.598		
Chg_7R					.498		
Chg_8					.415		

Extraction Method: Principal Axis Factoring.
a. Rotation converged in 13 iterations.

Table 11. Discriminant Validity.

Statistics	Construct Pairs				
	mset/dyn	pro/efc	amb/cnstr	dyn/efc	dyn/amb
averAVE	0.765	0.778	0.624	0.313	0.640
R^2	0.176	0.126	0.160	0.018	0.138
	mset/pro	pro/cnstr	cnstr/dyn	efc/amb	
averAVE	0.907	0.752	0.610	0.649	
R^2	0.046	0.078	0.157	0.075	
	mset/cnstr	pro/dyn	cnstr/efc		
averAVE	0.748	0.768	0.619		
R^2	0.087	0.021	0.013		
	mset/amb	pro/amb			
averAVE	0.779	0.782			
R^2	0.110	0.067			
	mset/efc				
averAVE	0.774				
R^2	0.036				

Again, principal axis factoring with oblimin rotation was employed, this time with the specification that six factors be extracted since the change factor was not in play. The scree plot supported the extraction of six factors. EFA resulted in a model that explained 38.4% of variance in items, with four items failing to load: efc6, cnstr2, cnstr6, and amb6R. The process was repeated with these four items excluded in order to further refine the model. The resulting model, featured in Table 12, explains 39.8% of variance.

At this point the researcher chose for each subscale the four items loading most strongly, checking to make sure that Cronbach's alpha remained above .70. The researcher also carefully reexamined the text of each subset of surviving survey items to ensure that content validity had not been adversely affected. The only potential concern was the balance in efficacy between math efficacy and teaching efficacy, so that results would have to be considered accordingly. The resulting model is displayed in Table 13. It explains 45.2% of

Table 12. Factor Loadings B.

Pattern Matrix[a]						
	1	2	3	4	5	6
mset_1R			-.746			
mset_2R			-.522			
mset_3R			-.627			
mset_5R			-.642			
mset_6			-.420			
mset_7			-.486			
mset_8R			-.487			
mset_9R			-.698			
efc_1				.513		
efc_2				.519		
efc_3R				.495		
efc_4				.478		
efc_5R				.473		
efc_7R				.521		
efc_8R				.660		
cnstr_1						.408
cnstr_4						.650
cnstr_5						.466
cnstr_7						.717
dyn_1R	.499					
dyn_2R	.549					
dyn_3R	.469					
dyn_5R	.425					
dyn_6R	.506					
dyn_7R	.544					
dyn_8R	.610					
dyn_9R	.433					
pro_1		.651				
pro_3		.703				
pro_4		.613				
pro_5		.807				
pro_6R		.569				
amb_3					-.479	
amb_4R					-.477	
amb_5					-.681	
amb_7					-.525	
amb_8R					-.663	

Extraction Method: Principal Axis Factoring.
Rotation Method: Oblimin with Kaiser Normalization. 11 iterations.

Table 13. Factor Loadings C (Final) With Cronbach's Alpha.

	Pattern Matrix[a]						
	1	2	3	4	5	6	α
mset_1R	.774						
mset_3R	.654						.791
mset_5R	.664						
mset_9R	.669						
pro_1		.667					
pro_3		.727					.794
pro_4		.534					
pro_5		.843					
cnstr_1			-.383				
cnstr_4			-.725				.701
cnstr_5			-.439				
cnstr_7			-.763				
efc_1				.674			
efc_2				.644			.707
efc_7R				.625			
efc_8R				.503			
dyn_2R					.450		
dyn_6R					.580		.727
dyn_7R					.647		
dyn_8R					.675		
amb_3						-.453	
amb_5						-.660	.723
amb_7						-.569	
amb_8R						-.640	

Extraction Method: Principal Axis Factoring.

Rotation Method: Oblimin with Kaiser Normalization.

a. Rotation converged in 10 iterations.

variance among the 24 variables. The scree plots in Figures 8 and 9 shows six factors to be appropriate. The steepness of the first segment in the plot reflects mindset's accounting for 21% of total variance.

A new, simpler scale for the independent variable "change" was also created. Table 14 shows that the four items chosen, also according to loading and content, loaded strongly onto one variable, eigenvalues having suggested one was sufficient. The single-factor model explained 51.2% of variance among the four items.

For each of the other six constructs the researcher also confirmed unidimensionality by checking to see that the four items loaded well onto only one construct (Hair, Black, Babin, & Anderson, 2010). In each case, eigenvalues indicated exactly one factor, so convergent validity was confirmed.

Among the 28 variables in the new, more parsimonious model, it was noted that mset9, pro3, and pro4 scores displayed kurtosis readings of 2.96, 2.74, and 3.95 respectively. Although some classify these as indicators of severe nonnormality (Lei & Lomax, 2005), Byrne (2010) suggests only kurtosis with a value greater than 7.0 is a concern for

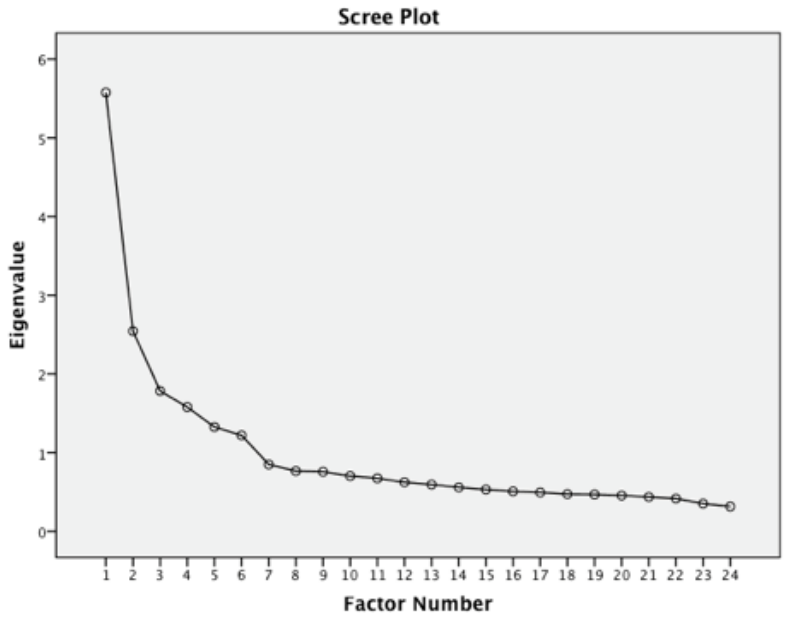


Figure 8. Scree Plot B.

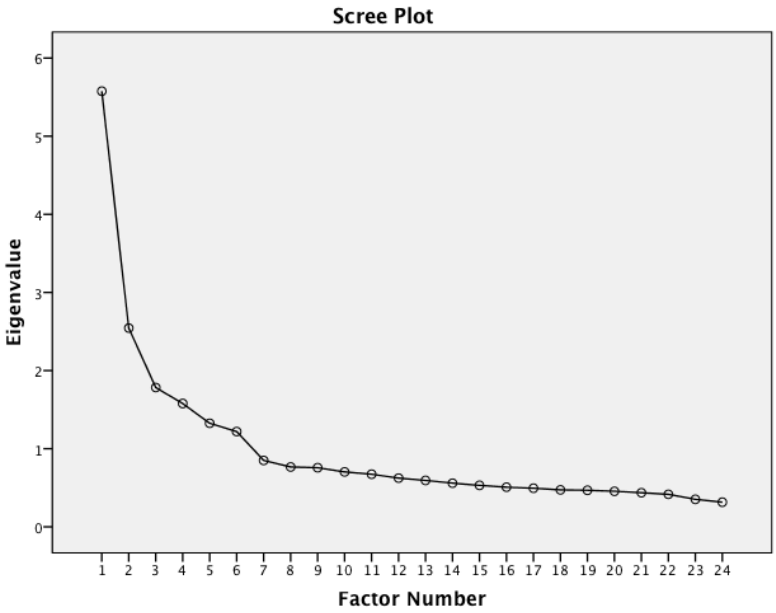


Figure 9. Scree Plot C.

SEM. Additionally, Lei and Lomax have found the maximum-likelihood method applied in this study to be highly robust to kurtosis, except where chi-square is concerned.

It was recognized that a few factor loadings were not as strong as they might be, in particular cnstr1 (-.383), constr5 (-.439), dyn2 (.450), and amb3 (-.453). The guideline for desired correlation is at least .50, with above .70 preferred (Hair et al., 2010). It would be important to confirm loadings using CFA in AMOS. Most EFA loadings were fairly strong, but SEM might demand stronger.

Table 14. Factor Loadings for Change Variable.

Factor Matrix[a]	
	1
Chg_2	.642
Chg_4	.820
Chg_5R	.592
Chg_6	.783

Extraction Method: Principal Axis Factoring.
a. 1 factors extracted. 8 iterations required.

Phase III: Testing Relationships

After development of the parsimonious model, the researcher used SPSS to combine the items for each construct into a subscale sum. Skewness and kurotis for each new sum were within one of zero, so these variables were all normally distributed.

Correlations

Pearson correlations were calculated to measure bivariate linear relationships among the constructs. Table 15 displays these correlations. As in the original model, correlations were all still significant at the .01 level but were slightly weaker than they had been for the full subscales. Worthy of note was the correlation between a growth mindset and a dynamic view of mathematics, $r(569) = .419$, $p = .000$. A possible explanation for this medium-strength association is that those who view math as a fixed body of difficult rules do not think everyone can learn math. A dynamic view also showed a medium-strength correlation with constructivist attitude, $r(569) = .396$, $p = .000$, suggesting that perhaps teachers who see math as a dynamic science are more likely to offer students the opportunity to interact dynamically with it. Both the willingness to allow this interaction, $r(569) = .400$, $p = .000$, and a dynamic view of math, $r(569) = .372$, $p = .000$, were associated with tolerance for ambiguity. As was the case in the full-subscale model, the three subscales correlating most strongly with change were constructivist attitude, $r(569) = .455$, $p = .000$, professional identity, $r(569) = .457$, $p = .000$, and ambiguity tolerance $r(569) = .474$, $p = .000$. In both models—as well as in the pilot—the lowest association with change corresponded to the efficacy variable. This may seem unusual, but the covariance structural model will provide evidence that the relationship between efficacy and change is complex.

Table 15. Subscale Correlations for Parsimonious Model.

Number	Subscale Construct	C1.	C2.	C3.	C4.	C5.	C6.	α
C1.	Mindset4							.791
C2.	Efficacy4	.190**						.707
C3.	Prof ID4	.215**	.355**					.794
C4.	Dynamic4	.419**	.135**	.145**				.727
C5.	Construct4	.295**	.113**	.280**	.396**			.701
C6.	Ambiguity4	.331**	.273**	.258**	.372**	.400**		.723
C7.	Change4	.322**	.176**	.425**	.326**	.443**	.406**	.791

Pearson correlations were also calculated to measure bivariate linear relationships among the dependent change variable and various demographic measures. The three continuous demographics were age, experience, and math experience. No significant correlation was observed between change and experience, $r(554) = .018$, $p = .674$. Likewise for math experience, there was virtually no association with change, $r(553) = .004$, $p = .924$). Nor was there association between change and age, $r(562) = .047$, $p = .263$. A scatterplot for age is illustrative (see Figure 10). Table 16 shows correlation coefficients for each construct with age and experience. These coefficients had been roughly the same in the larger model. Where correlations exist at all, they are weak. These findings run counter to widely held beliefs that older, more experience teachers are the ones likely to resist change. Whereas Tymula et al. (2013) had found differences in ambiguity tolerance to be statistically insignificant among age groups 21 to 25, 30 to 50, and 65 to 90, this study revealed a slight positive correlation between age and ambiguity tolerance.

Table 16 Construct Correlations with Age and Experience

	Age	Experience
Mindset	.079	.010
Professional Identity	.092*	.115**
Constructivist Attitude	-.057	-.026
Efficacy	.100*	.143**
Dynamic Math View	.126**	.126**
Ambiguity Tolerance	.084*	.037
Change Openness	.047	.018

**Correlation significant at 0.01 level (2-tailed).

*Correlation significant at 0.05 level (2-tailed).

Regression

In this study a multiple linear regression was calculated to determine whether the change variable could be significantly predicted by the six independent variables: math mindset, efficacy, professional identity, dynamic math concept, constructivist approaches, and tolerance for ambiguity. Multiple linear regression is employed when several predictor

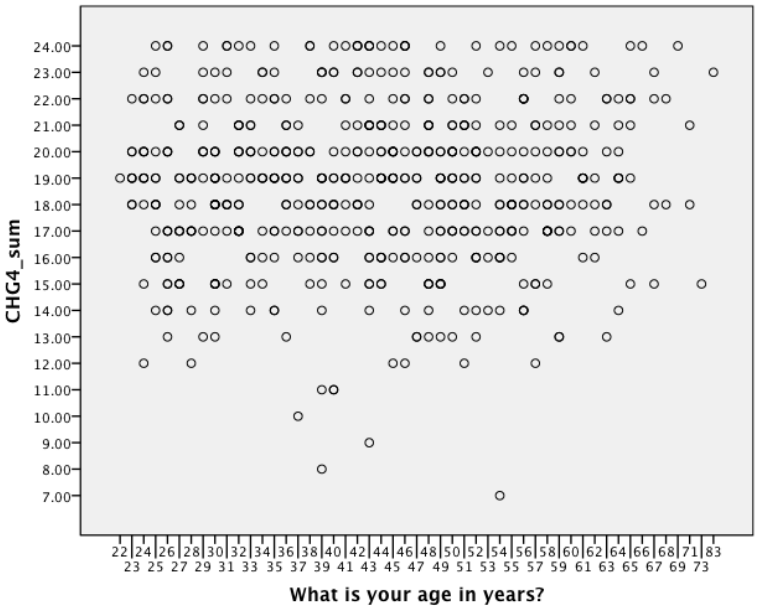


Figure 10. Scatterplot of Age vs. Change.

variables combine to "help obtain more accurate predictions" (Gravetter & Wallnau, 2013, p. 572). Often there is overlap in the independent variables, so each added predictor may not add much to the overall strength of the correlation. A significant regression equation was found in this study for the parsimonious model ($F(6, 564) = 66.947$, $p < .000$), with Pearson's R at .645 and R^2 at .416, meaning 41.6% of the variability in the change score was explained by a combination of the six other constructs. The survey results predicted change equal to .222 + .095(mset) – .012(efc) + .305(pro) + .102(dyn) + .203(cnstr) + .248(amb), with coefficients standardized. All but the efficacy construct were significant predictors of change. Regression analyses suggested perhaps the efficacy construct makes the instrument weaker.

Tests of Mean Differences

In the final, more parsimonious model used in this study, attitude toward change was measured with four items:

chg2: I try to adapt my instructional approaches to follow current best practices.

chg4: I am quick to embrace new methods for teaching math.

chg5R: Pressure to change my strategies makes me want to leave teaching.

chg6: I enjoy trying new ways of teaching mathematics.

This four-item scale had a Cronbach's alpha coefficient of .791, which would not increase with the removal of any item. These items loaded onto the change factor at .642, .820, .592, and .783, respectively. Preacher and MacCallum (2003) suggest no arbitrary cut-off be applied for loadings, so all of these were treated as acceptable going into SEM analysis. 51.2% of variance in the scale was explained through the items' association.

Independent sample *t*-tests were performed to test differences in mean change scores among demographic groups. All tests were conducted at the .95 confidence level, with each

4-item sum scale now having a score range of 0 to 24. The *t*-tests revealed interesting demographic differences in the means of the changeSUM variable. For example, females (M = 19.05, SD = 2.90) scored significantly higher on the change scale than males (M = 18.19, SD = 3.22; t = -3.24, p = .001), indicating a greater openness to new approaches. There was also a slight difference in the means of the changeSUM between whites and nonwhites, with nonwhites (M = 19.43, SD = 2.36) being more open to change than whites (M = 18.68, SD = 3.07; t = .053, p = .053). Middle school teachers were more open to change (M = 19.25, SD = 2.76) than high school teachers (M = 18.37, SD = 3.23; t = -3.476, p = .002) although it is interesting to note there was no significant difference between these two groups in the professional identity construct (t = .629, p = .529). It is also interesting to note that those holding degrees in mathematics were less open to change (M = 18.22, SD = 3.05) than those with other majors (M =19.32, SD = 3.01; t = -4.29, p = .000). Teachers in towns of population greater than 10,000 were slightly more ready to embrace change (M = 18.90, SD = 3.11) than their more rural counterparts (M = 18.42, SD = 2.98) (t = 1.76, p = .078). On the scaleSUM there was an even more significant difference between urban (M = 110.29, SD = 11.44) and rural (M = 105.38, SD = 11.50; t = 4.84, p = .000). Of particular interest to the researcher is the finding that math teachers in North Dakota were significantly less open to change (M = 17.89, SD = 2.86) than those working in other states (M = 19.21, SD = 3.08; t = -5.013, p = .000). This difference was seen in the scaleSUM as well, with North Dakota teachers (M = 102.73, SD = 9.44) trailing teachers in other states (M = 112.19, SD = 11.44); t = -10.491, p = .000). In fact, North Dakota trailed teachers from out of state on measures of all constructs.

Although each of the six categories mentioned above registered a *significant* difference in the change construct, it is important also to examine the relative magnitude of these differences. Table 17 below shows ethnicity, gender and grade level registering fairly small relative differences. Of these, grade level was of the greatest interest to the researcher since high school math scores had been observed to plateau in recent decades. High school teachers averaged half way between "slightly agree" and "agree" in response to change-scale questions, indicating they could indeed be less than eager to adopt new practices. Middle-school counterparts averaged closer to "agree" (recall that "neutral" was not an option in the Likert-like scale). In fact, middle-school teachers outscored high-school teachers on all four change items, with the largest (8% relative difference) and most significant difference (t = -3.83, $p < .0001$) coming on the item, "I am quick to embrace new methods for teaching mathematics." Here high school teachers averaged only a "slightly agree," with only 37.5% agreeing or strongly agreeing, compared to 50.9% of middle school teachers did (see Figure 11).

Table 17. Demographic Differences in Change Variable.

Demographic	Chg4 Mean (20=agree)	Mean Percent	Relative Difference
Nonwhite	19.43	80.96%	3.94%
White	18.68	77.83%	
Female	19.05	79.38%	4.62%
Male	18.19	75.79%	
MS	19.25	80.21%	4.68%
HS	18.37	76.54%	
Not major	19.32	80.50%	5.86%
Math Major	18.22	75.92%	

Other	19.21	80.04%	7.12%
ND	17.89	74.54%	
Urban	19.9	82.92%	7.72%
Rural	18.42	76.75%	

4=strongly disagree, 8 = disagree, 12 = slightly agree, 16 = slightly agree, 20 = agree, 24 = strongly agree

It must be pointed out that scale-sum scores (derived from the total of the six independent constructs) did not always align with the change-sum in categorical comparisons as they did for geographical location. For example, there was no significant difference between females and males on the scale-sum score. This is perhaps because females were slightly more likely to identify with the profession while males were more tolerant of ambiguity. There was also no scale-sum disadvantage for math majors, in part because they reported greater efficacy and identification with the profession. Strangely, high school teachers scored higher on the scale-sum (M =112.19, SD = 11.44) than middle school teachers *(*M =112.19, SD = 11.44; t = 2.223, p = .027), a directional misalignment with the change-sum results. This can be attributed to high school teachers scoring significantly higher on half of the subscales: efficacy, dynamic concept, and ambiguity tolerance.

Conflicting results for scale-sum and change-sum may seem strange in a study aiming to show connections between the two. SEM analysis will allow us to explore further the complex relationships among the seven constructs and come to a better understanding of this apparent incongruity. For now, one explanation is that, as noted earlier, the scale does not account for all of the variance in the change-sum. Another is that the subscale constructs do not work precisely in an additive manner. In other words, a teacher does not necessarily have to be strong in all six components to be open to change. A teacher could compensate for low

development in one area with high development in another. Thus, professional development could succeed along multiple paths, and could certainly be differentiated according to teacher needs. The next sections will deal with this idea in more detail.

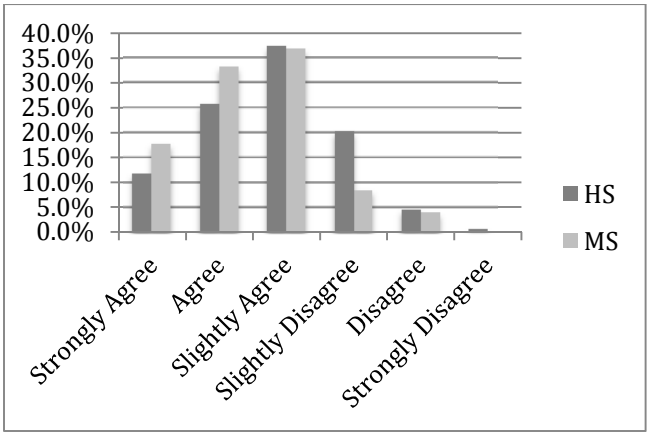


Figure 11. Bar Graph by Level: Embracing New Methods.

Phase IV: Confirmatory Factor Analysis

Before SEM can be employed to test relationships among latent constructs, CFA must be conducted to ensure the indicators measure the purported constructs. Without CFA, SEM would be essentially meaningless because constructs would lack validity (Crockett, 2012). The confirmatory factor analysis for this study was performed using structural equation modeling (SEM) techniques in AMOS software. Generally speaking, SEM approaches involve five steps: model specification, identification, estimation (ML), model assessment, and respecification (Bollen & Long, 1993).

Model specification, the proposal of a graphic model, is based on theory and research, which should suggest important latent constructs and the relationships among them (Crockett, 2012). For this study, research suggested six factors contributing to a teacher's openness to new strategies: math mindset, self-efficacy, strength of identity with the profession, dynamic conception of mathematics, constructivist attitude, and tolerance of ambiguity. Confirmatory

factor analysis (CFA) retested the construct validity of the latent variables. CFA was conducted on two distinct measurement models. The first measurement model represented the six independent 4-item constructs and the second included the (dependent) change as a seventh construct. The CFA models for math-teacher change are displayed in Figures 11 and 12 below.

Step two, model identification, involves certain statistical criteria that must be met in order to determine whether the model is capable of producing actual results (Crockett, 2012). Specifically, to be termed "identified" the model must yield a set of unique parameters as opposed to multiple possible assignments of values. The goal is a model that is "overidentified," meaning there are more known parameters than the number of parameters we hope to estimate. This allows for a number of degrees of freedom, which in turns allows for the possibility of a scientific rejection of the model (Byrne, 2010). In the case of the two measurement models in Figures 12 and 13, both were over-identified with 329 degrees of freedom in each (see Table 18). This large number is due to the 28 survey-item parameters and the many known associations among them.

Model estimation, the third step in SEM, occurs when software—in this case AMOS software—is used to determine whether the hypothesized model fits the data (Crockett, 2012). The researcher's goal is to find a model that minimizes the difference between the covariance matrix of the hypothesized model and the covariance matrix corresponding to the sample data (Byrne, 2010). In the drawing of the model, rectangles are used to represent observed

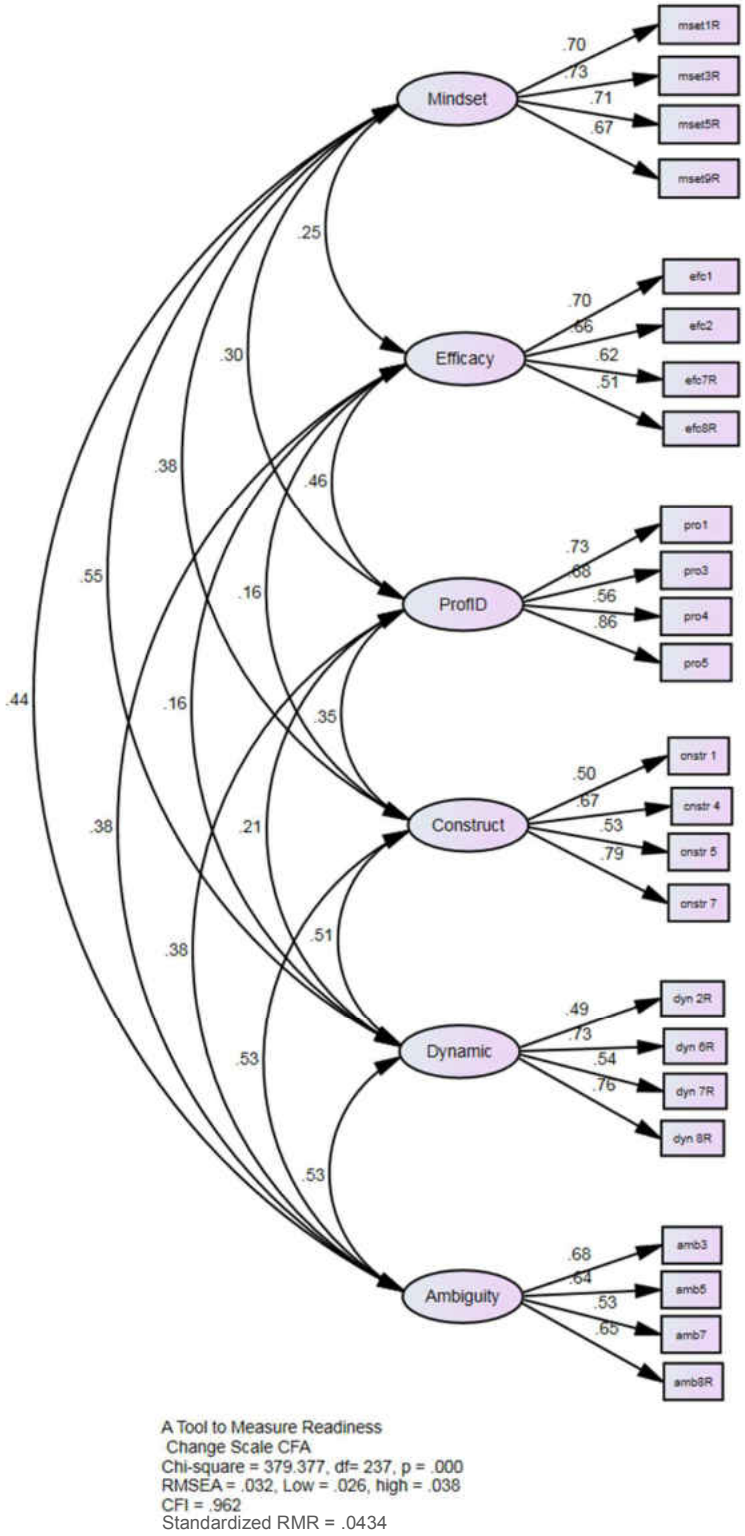


Figure 12. Six-Factor Measurement Model.

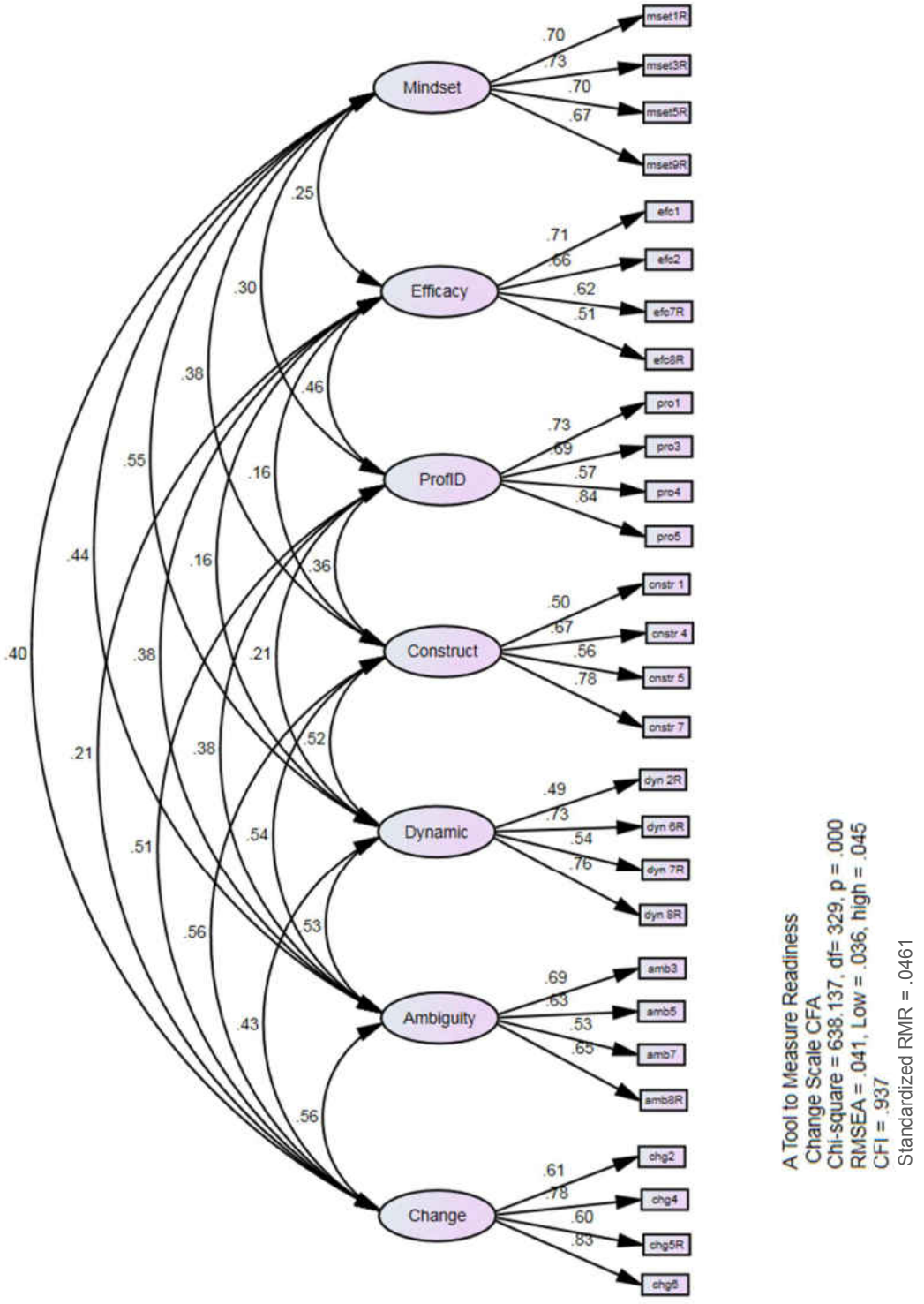


Figure 13. Seven-Factor Measurement Model.

Table 18. Identification of Measurement Models.

Seven-Factor Model		
Data points	= (28 x 29)/2	406
Factor loadings	21	
Latent variances	7	
Error variances	28	
Covariances	21	
Regressions	0	
Total estimates		77
Degrees of freedom	= 406 - 77	329

variables—in the case of this study, survey-item measures—and circles or ellipses are used to represent unobserved latent variables—in this case, theoretical constructs contributing to change-readiness. Single-headed arrows represent regression relationships, while double-headed arrows represent correlations or covariances. Readings along the arrows indicate standardized measures of association. Fitness indicators appear in small print beneath the diagram.

There are different methods for estimating these unknown parameters and, as described in the previous chapter, different tests of goodness of model fit. Functions for estimating parameters include the ordinary least squares method (OLS), the generalized least squares method (GLS), and the method used most often—including in this study—the maximum likelihood method (ML). The ML method of estimation relies on the probability distribution that makes the observed data "most likely" (Myung, 2003). This method is reliable and efficient with large samples. It estimates all parameters simultaneously and assumes multivariate normality (Crockett, 2012), though recall Lei and Lomax (2005) found it to be robust to kurtosis. Factors other than normality that affect fit and parameter estimates

are sample size (more than adequate in this study), model complexity (the reason for the pared-down model), and obviously misspecification of the model (Hoyle, 2012; Byrne, 2010; Fan, Thompson & Wang, 1999).

The fourth step in CFA or SEM is assessment of model fit. In each of the model diagrams in this chapter, indications of fit appear in small print below the diagram. For this study, the researcher used ML to make comparisons of the proposed model to both the independence and saturated models at opposite ends of a spectrum. The highly restrictive independence model is one with all correlations equal to zero, so we want distance from this model (CFI index expresses this). The unrestricted saturated model, on the other hand, computes exactly as many parameters as there are observed data points (no degrees of freedom; Byrne, 2010). We want to be close to this model (RMSEA and SRMR express this). Recall that desired values for the indices are CFI > .95, RMSEA < .05, SRMR < .05, and for χ^2 a low value that registers as nonsignificant, so our first measurement model fits the data well. This six-factor measurement model registered good fit on three indices, with CFI = .962, RMSEA = .032 (90% CI = .026, .038), SRMR = .043, and chi-square's indication of poor fit, not useful due to sample size effects, at $\chi^2(237) = 379.38$, $p < .001$. Lei and Lomax (2005) call chi-square the "least robust fit index," sensitive to both nonnormality and large sample size.

The second measurement model with all seven constructs fit the data fairly well also, with CFI = .937, RMSEA = .041 (90% CI = .033, .045), SRMR = .0461, and chi-square again affected by sample size: $\chi^2(329) = 638.14$, $p < .001$. Only the CFI failed to meet the standard, yet it was "close to .95" as Hu and Bentler recommend (1999). Although it is safe to disregard the chi-square readings, it is worth noting that Bollen (1989) recommended

division of chi-square by its degrees of freedom to reduce the effect of sample size. He suggested a quotient of 3.0 or less could indicate good fit. For the measurement models described here, the quotients, at 1.60 and 1.99 respectively, satisfy Bollen's standard. It should be noted, however, that a number of standardized residuals within the two measurement models were in the range |2.5| - |4.0|, suggesting some deviation from the sample covariance matrix (Hair, Black, Babin, & Anderson, 2010).

In the 6-construct measurement model, all paths were significant at $p \leq .001$ except for the covariance between efficacy and constructivist ($p = .006$) and between efficacy and dynamic ($p = .005$). When the change construct was added to the measurement model, all paths were significant at the level at $p \leq .001$ except the efficacy-constructivist covariance, which remained at $p = .006$. Critical ratios for loadings ranged from 8.8 to 13.8 in the first model and from 8.8 to 14.1 in the second, indicating statistically significant loadings.

The fifth step in CFA SEM analysis is respecification. However, since the measurement models were adequate to proceed toward design of a structural model, respecification was unnecessary at this point.

It should be noted that in both measurement models the convergent validity was found to be greater than had been indicated by SPSS analyses, with the four troublesome items loading more strongly so that three of them were now greater than .50 and dyn2 almost there at .49. Nonetheless, loadings greater than .70 would be preferred since an R^2 greater than .50 would indicate at least half of the variance in the item is explained by the latent construct (Hair, Black, Babin, & Anderson, 2010).

Phase V: Covariance Structural Modeling

Structural equation modeling involves the same five steps used in CFA but now applied to a structure that proposes regression relationships among the latent variables. In the case of this study, the first model specified, as in the pilot, was a fairly simple covariance model in which the six independent factors were regressed onto change (see Figure 14 with covariance arrows hidden). The model was over-identified with 329 degrees of freedom (See Table 19). Model estimation revealed fit to be fairly good, with CFI = .937, RMSEA = .041 (90% CI = .036, .045), SRMR = .0461, and chi-square's indication of poor fit not useful due to sample size effects: $\chi^2(329) = 638.137$, $p < .001$. All regression and covariance paths were significant at the $p \leq .001$ level. The model showed, however, rather poor loadings of the independent constructs onto change. Especially disappointing were nonsignificant regressions of the mindset ($p = .148$) and dynamic constructs onto change ($p = .452$). Efficacy regressed onto change at the $p = .036$ level. Perhaps the structure of associations was more complex than this model indicated. Moderate collinearity among constructs had suggested this might be the case.

Important to respecification of the model was examination of indirect effects among the latent constructs. The researcher needed to examine to what extent correlations among latent variables impacted their relationship with the independent variable change. Since efficacy had been deemed important by the literature but was contributing poorly to the regression model, the researcher theorized that perhaps it was due to overlap with the professional-identity construct. After all, Lasky (2005) had observed of teachers "Their sense of self-worth as a person was intricately intertwined with their professional identities" (p. 910).

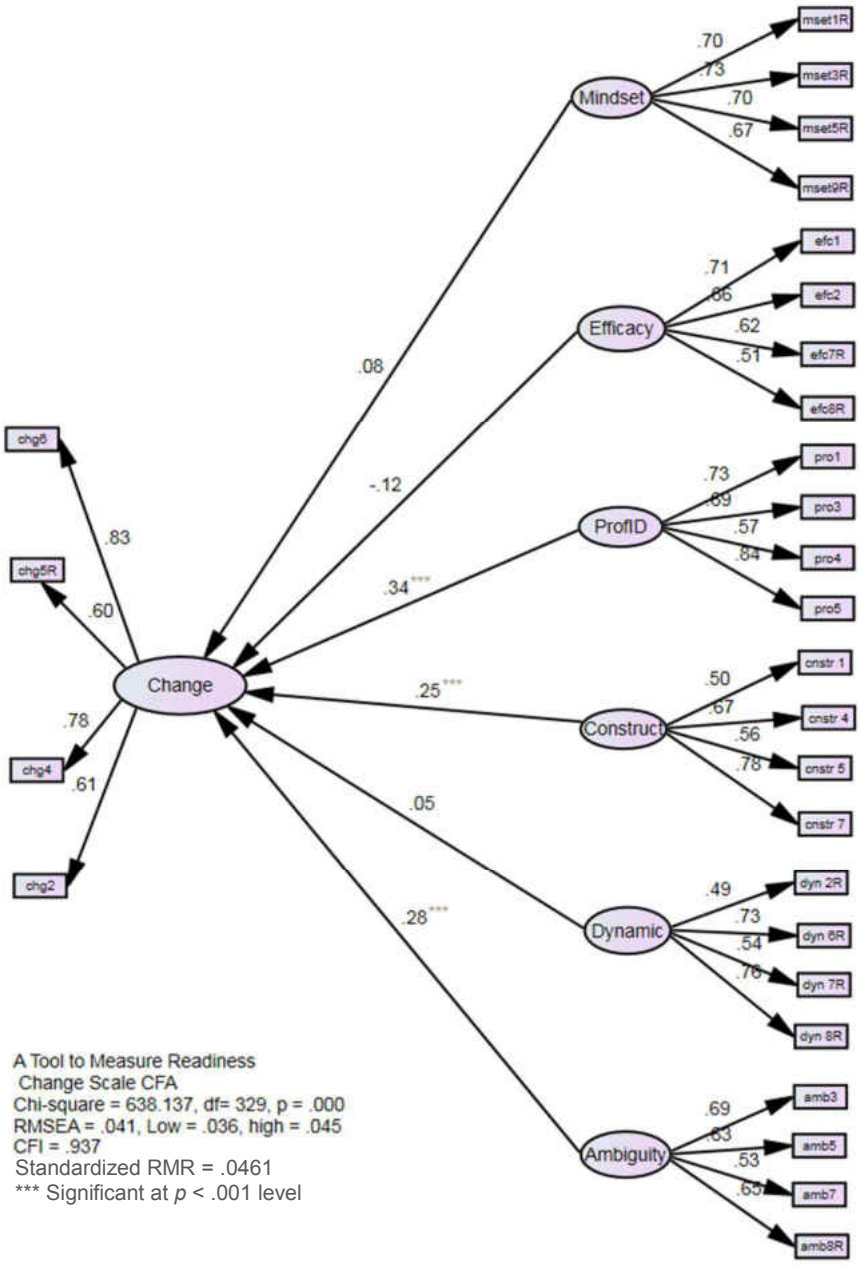


Figure 14. Simple Structural Model A.

Table 19. Identification of Structural Model A.

Six-Factor Model		
Data points	= (28 x 29)/2	406
Factor loadings	21	
Latent variances	7	
Error variances	28	
Covariances	15	
Regressions	6	
Total estimates		77
Degrees of freedom	= 406 - 77	329

Efficacy and professional identity indeed showed a weak to moderate correlation (r = .355) (Dancey & Reidy, 2004), but collinearity was not observed to be an issue. The variance inflation factor (VIF) between the two constructs, an indication of how much regression variance is increased due to collinearity, was only 1.144, far below rules of thumb suggesting 4 or sometimes 10 as an upper limit (O'Brien, 2007).

Regression tests were run to determine whether professional identity was mediating the effect of efficacy. Results showed that the regression of efficacy onto change indeed became insignificant when professional identity was taken into account. Note along the efficacy-change arrow in the first triangle in Figure 15 the reduction of R from .176 (p = .000) to .027 (p = .489) in light of the pro variable. This reflects full mediation by professional-identity (Baron & Kenny, 1986). Another way to illustrate this is to note that the percent of variance explained by the regression model in EFA does not change with the elimination of efficacy but drops from 41.6% to 34.1% without professional identity.

In addition to this case of full mediation, two cases of partial mediation also led to changes in the structural model. Dynamic math conception was found to be partially mediated by math mindset, meaning that math mindset still contributed significantly to

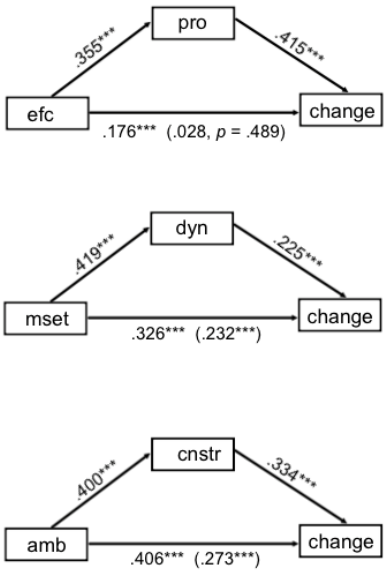


Figure 15. Indirect Effects

change with the addition of dynamic but contributed less. The same could be said of ambiguity tolerance and constructivist attitude: the first was partially mediated by the second. In these cases of partial mediation, the significance level was $p = .000$ for all loadings, but mediation effects caused substantial decreases in R values. Mindset's regression onto change falls from .326 to .232 and ambiguity's from .406 to .273.

Sobel tests were conducted to confirm the significance of the indirect effects illustrated in Figure 15. Results revealed that efficacy was indeed fully mediated by professional identity ($p = .000001$), dynamic conception was partially mediated by mindset ($p = 0$), and ambiguity tolerance was partially mediated by constructivism attitude ($p = 0$).

The Sobel test, however, assumes normality (Hayes, 2009), an issue in this study. Since the structural model indicated multivariate nonnormality, the bootstrap method was also applied to verify indirect effects. Bootstrapping involves creating a new sampling distribution through repeated selection of mini samples from the study's data set. This

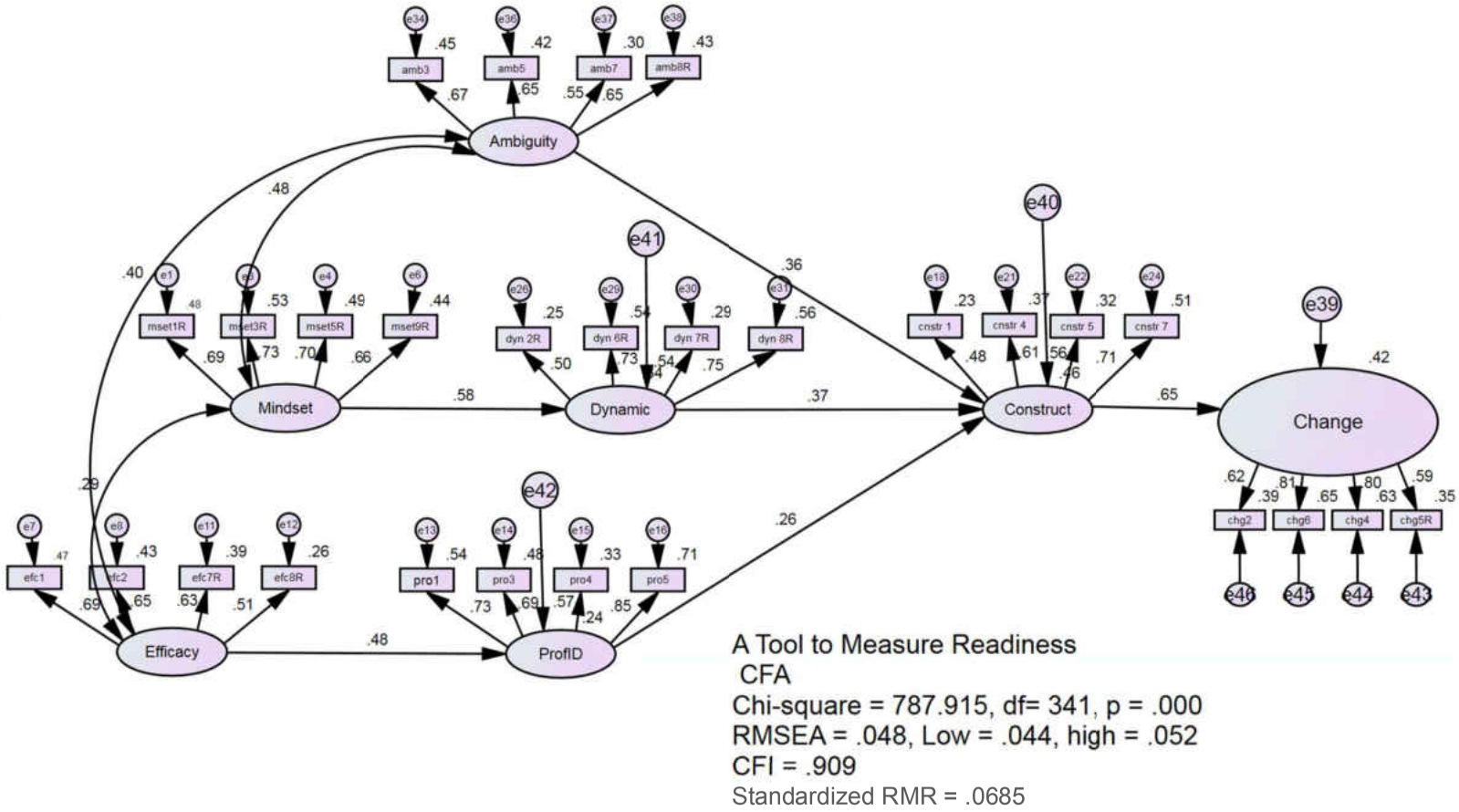


Figure 16. Complex Structural Model A.

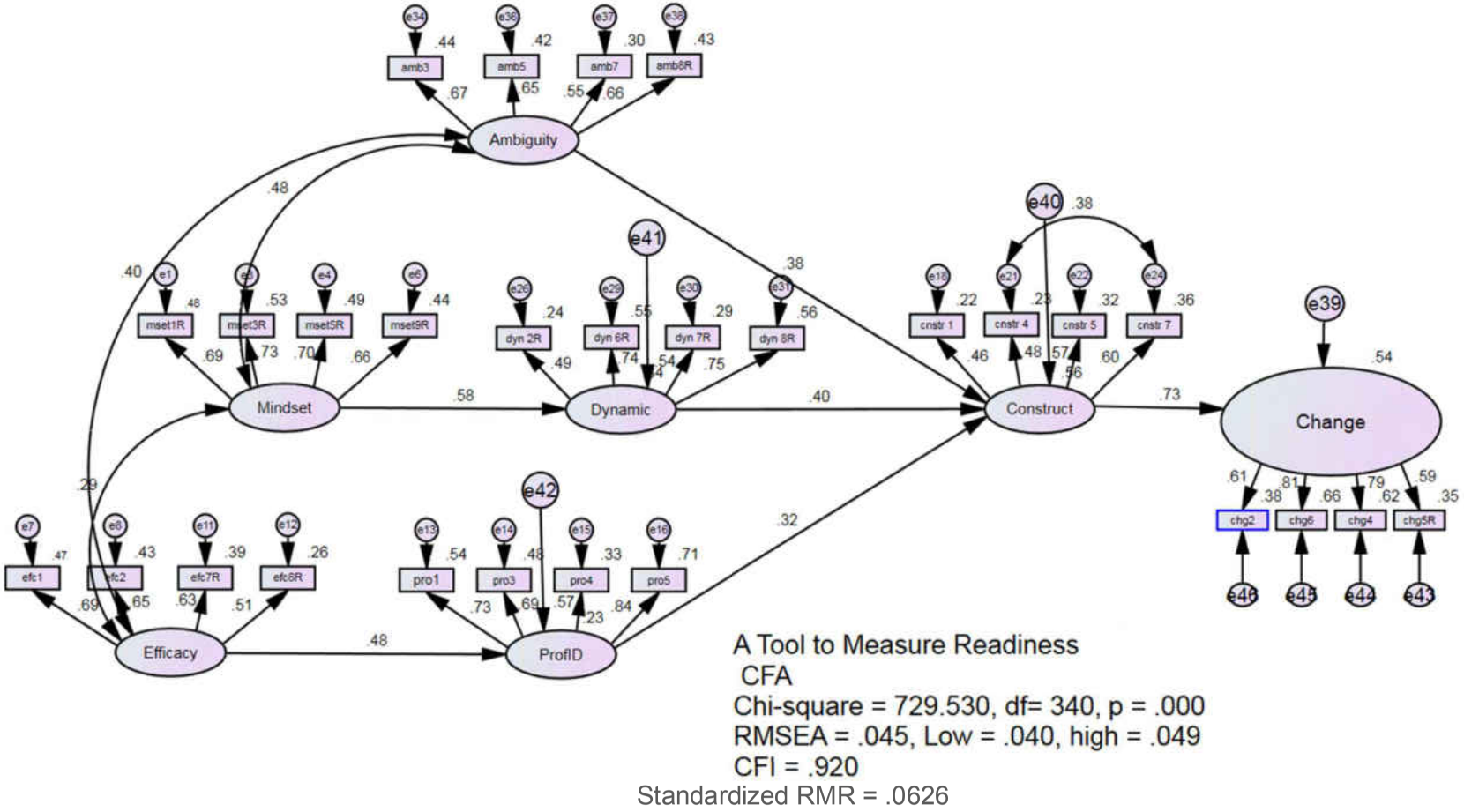


Figure 17. Complex Structural Model B: Modification.

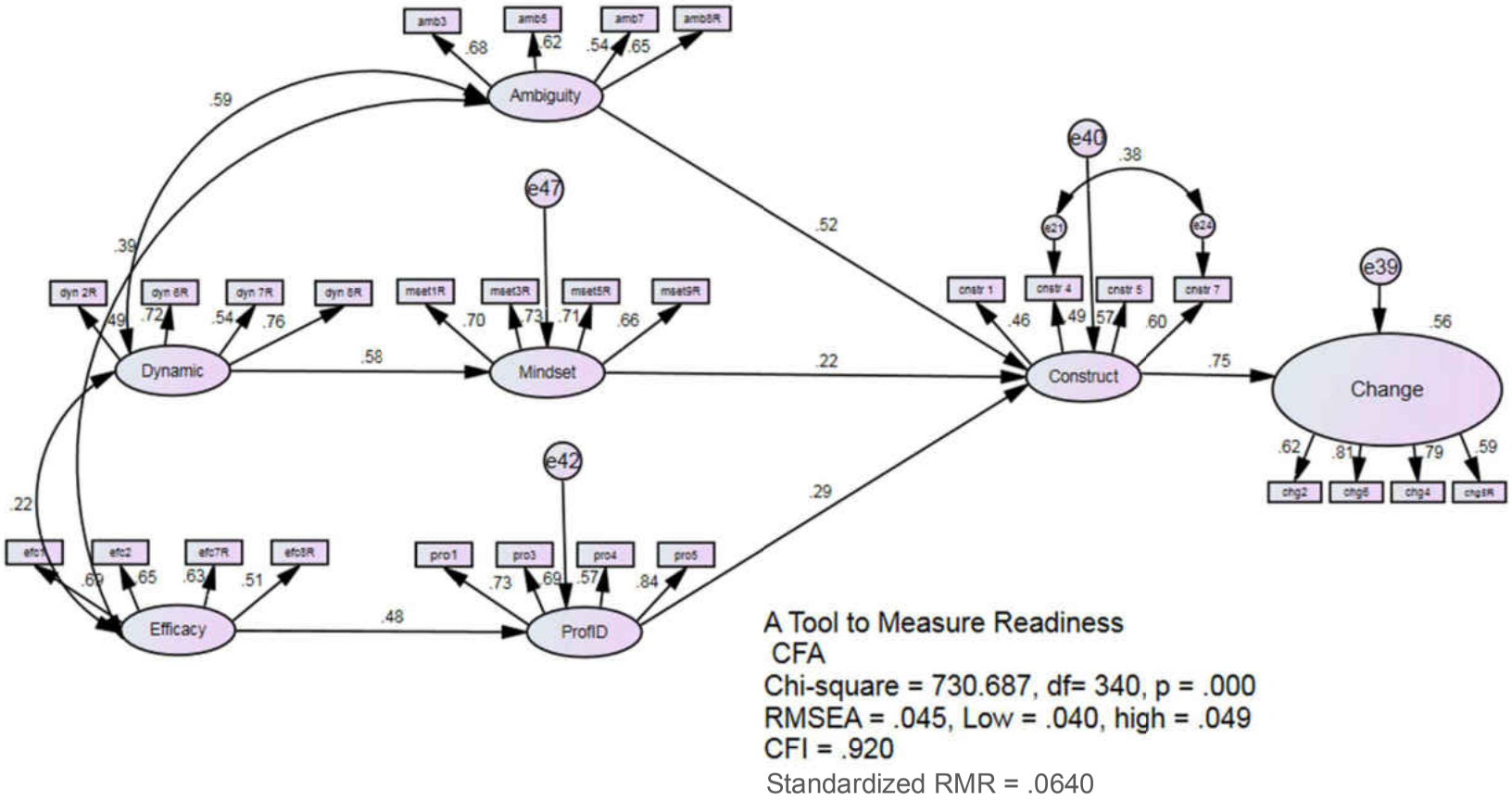


Figure 18. Complex Structural Model C: Restructured. (Final Model)

Table 20. Moderation Effects.

Coefficients[a]						
Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.
		B	Std. Error	Beta		
1	(Constant)	6.408	1.167		5.489	.000
	proSUM4	.543	.052	.428	10.397	.000
	efcSUM4	.038	.043	.036	.892	.373
	efic_pro_product	.024	.013	.070	1.778	.076

a. Dependent Variable: CHG4_sum

sampling is done with replacement. Based on the distribution of samples created, multiple estimates of a path coefficient are taken and an inference regarding indirect effect is drawn from these (Hayes, 2009). In this study the SPSS bootstrap analysis confirmed statistical significance for the patterns observed in the hypothesized model, 95% CI = [.11, .22].

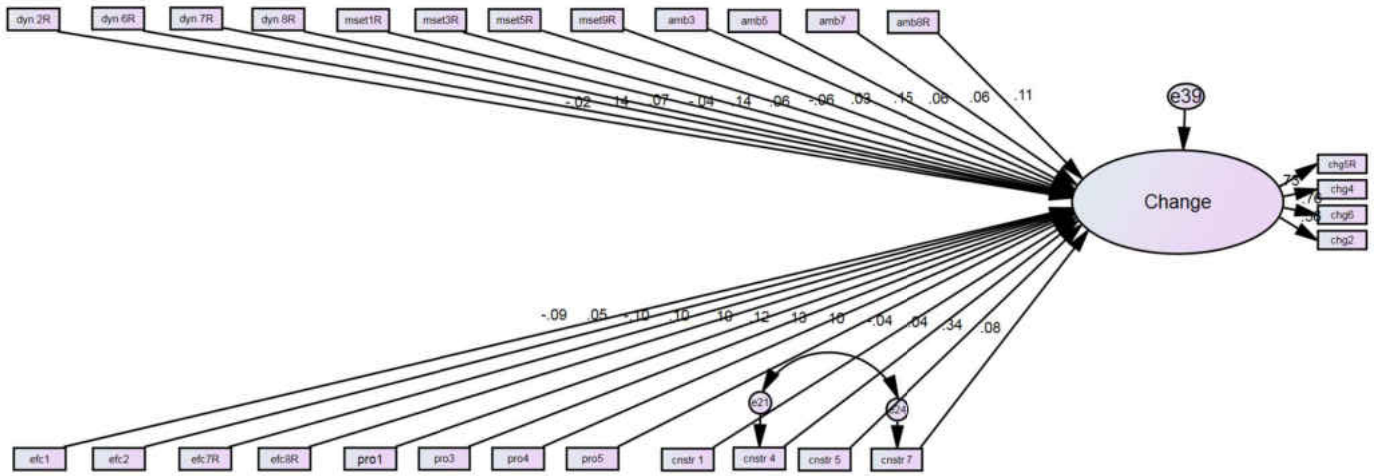
There are no doubt other indirect effects due to associations among constructs, but those discussed above were the relationships meriting exploration based on the literature. Figure 16 shows the more complex structural model drawn to reflect these effects. Table 20 shows the model in Figure 16 to be over-identified. In this model the loadings were much stronger than for the initial, simpler model, and the fit was still good, with CFI = .909, RMSEA = .048 (90% CI = .044, .052), SRMR = .0685, and chi-square's indication of poor fit still not useful due to sample size effects: $\chi^2(341) = 787.915$, $p < .001$.

Although less stringent guidelines for the fit of structural models suggested these goodness-of-fit readings were adequate, the researcher attempted to improve the fit through examination of modification indices. A modification index, provided within AMOS results as a user option, is an indication of the change in chi-square that would result if a correlation

were recognized by the model. The modification index for the association between error variables 21 and 24 was noted at 38.331,well beyond Joreskog and Sorbom's (1993) sufficiency guidelines for linking two variables in a structural model. Accordingly, significant improvement was achieved ($\chi^2(1) = 58.385$, $p < .0001$) through the recognition of covariance between errors 21 and 24, errors associated with two highly correlated items [$r(571) = .569$, $p = .000$] in the constructivist factor ("In math class, students need to develop their own solution strategies," and "Students need to construct their own understanding of a math concept"). Figures for the improved fit (see Figure 17) were CFI = .920, RMSEA = .045 (90% CI = .040, .049), SRMR = .0626, and $\chi^2(340) = 729.530$, $p < .001$, good indicators of fit for a structural model. For RMSEA and SRMR, readings $\leq .05$ are considered good and $\leq .08$ acceptable (Byrne, 2010; Hu & Bentler, 1999); CFI $\geq .95$ is good and $\geq .90$ acceptable (Byrne, 2010).

Figure 18 displays the final structural model and reflects one last change made for theoretical reasons. The researcher transposed the constructs mindset and dynamic conception of mathematics, judging it more likely that conception of math determines math mindset than other way around. The model was now in keeping with mediation findings. This model too was over-identified (see Table 21), and fit indices were almost unchanged from the previous model, with CFI = .920, RMSEA = .045 (90% CI = .040, .049), SRMR = .0640, and $\chi^2(340) = 730.687$, $p < .001$.

In order to further validate the structural model, the researcher compared it to a model in which the twenty-four survey items were regressed directly onto the change construct. As Figure 19 shows, the comparison model was a poor fit for the data, with CFI = .251, RMSEA = .136 (90% CI = .132, .139), SRMR = .2042, and $\chi^2(349) = 4009.178$. A test of chi-square



A Tool to Measure Readiness
CFA
Chi-square = 4009.178, df= 349, p = .000
RMSEA = .136, Low = .132, high = .139
CFI = .251
Standardized RMR = .2042

Figure 19. Structural Model with No Exogenous Factors.

Table 21. Structural Model C: Identification.

Six-Factor Model		
Data points	= (28 x 29)/2	406
Factor loadings	21	
Latent variances	7	
Error variances	32	
Covariances	4	
Regressions	6	
Total estimates		70
Degrees of freedom	= 406 - 77	336

differences revealed the model in Figure 18 to have a significantly better fit ($\chi^2(9) = 3278.491$, $p < .0001$).

In the final complex model, all regression weights were significant at the $p < .001$ level, indicating the loading problems registered by the simpler model had been addressed. The six regression relationships now had critical ratios ranging from 5.4 to 7.9, suggesting the model had something important to tell us.

The structural equation model in Figure 18 depicts the way openness to math reform develops along three pathways. For each path, the final precursor to change is a constructivist attitude toward classroom instruction. The upper path shows a prerequisite to constructivist methods is the capacity to tolerate ambiguity. The center path shows that a dynamic view of mathematics and a growth mindset regarding the learning of math also enable a constructivist perspective. Finally, the lower path indicates that a strong sense of self-efficacy, entailed by a strong identification with the math teaching profession, also makes possible the constructivist attitude necessary to make pedagogical changes called for in recent decades.

Summary

This chapter narrated the development through structural equation modeling of measurement models and a complex structural model for math-teacher openness to pedagogical change. Along the way, it addressed four research questions, explored through five phases of research: (1) descriptive analyses of items and subscales, (2) exploratory factor analysis and reliability testing of subscales, (3) tests of association and demographic difference in means, (4) confirmatory factor analysis of the measurement model through SEM, and (5) covariance structural modeling. The measurement models were shown to be sound, and a final complex structural model clarified relationships among factors contributing to teacher openness to change. The next chapter will discuss the meaning of these results and implications for professional development of mathematics teachers.

CHAPTER V:

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The purpose of this study was to examine factors contributing to math-teacher openness to adopting new instructional practices. The process was two-fold: first, to develop and validate a survey instrument for measuring openness to pedagogical change and, second, to build and validate a structural equation model for explaining factors contributing to openness. A pilot study applying exploratory factor analysis and dimension reduction tests had suggested the potential for certain theoretical constructs to combine to impact math teacher openness to change. In the pilot, however, the loading of survey items onto latent constructs was not sufficiently strong for meaningful application of SEM. The current study sought to build on the pilot by strengthening the survey instrument so that subsequent SEM investigations could yield more meaningful results.

This chapter will summarize the work of the previous chapters before delving into answers for each of the four research questions, addressing the meaning and implications of the research results. In closing, this chapter will discuss the limitations of the work and will make recommendations for further study.

Dissertation Summary

Chapter I discussed a decades-old call on the part of the National Council of Teachers of Mathematics for instructional practices that focus more on depth of student understanding than on transmission of information. In recent years, this call was taken up by the Common

Core State Standards (CCSS) movement and embodied in the Standards for Mathematical Practices (SMPs) (NGA & CCSSO, 2010). Although U.S. math teachers have embraced the content of the CCSS, there is no clear indication they are taking up the SMP. Chapter I suggested teacher belief systems might have to shift before practices truly begin to change. The researcher also discussed the importance to professional development of a tool for measuring openness to change and the importance to the research literature of a structural model explaining attitudes toward change. A constructivist theoretical framework was adopted as a foundation for the research.

Chapter II reviewed the literature relevant to math teacher change and provided a look at the specific shifts NCTM has proposed for mathematics pedagogy. The chapter examined through the relevant literature the six constructs this study proposed as contributors to change. First, varying conceptions of the nature of mathematics were discussed, with emphasis on the difference between dynamic and static views of the discipline (Hoz & Weisman, 2008). Then teacher conceptions of the way students learn math were examined, focusing on a contrast between active and passive experiences: teacher-transmitted versus student-constructed knowledge (Chapman, 2002; Hoz & Weisman, 2008; Philipp, 2007). Following that, Carol Dweck's (2006; 2000) mindset work illuminated the impact of teacher perceptions of students' capacity. Next, an exploration of the math-teacher efficacy literature made clear that teachers with a stronger sense of their own capacity were more able to allow opportunities for students to construct meaning (Charalambous and Philippou, 2010; Gresalfi & Cobb, Lasky, 2005). Closely tied to efficacy was the professional-identity literature, which suggested strong association with the identity "math teacher" as a factor that empowered math teachers to act with confidence during times of change (Kelchtermans, 2009; Lasky,

2005; Wenger, 2010). Finally, the literature on ambiguity tolerance explained how low tolerance inclines a teacher to control the flow of knowledge and leads to avoidance behaviors among teachers in times of change (Merenluoto and Lehtinen, 2004; Stein & Smith, 2011).

Chapter III described the research methods applied in this study. A sample of 571 math teachers from around the U.S. responded electronically to a survey of 55 Likert-like items and 10 demographic questions. Through exploratory factor analysis and dimension reduction, the 55 items were whittled down to 24 in order to create a parsimonious six-factor model to test in AMOS. Prior to SEM analyses, associations among constructs were explored and tests of demographic differences were performed. In AMOS software, structural equation modeling techniques were applied: first confirmatory factor analysis to test the validity the measurement models, then SEM techniques to test the fitness of possible structural models.

In Chapter IV the researcher shared results of the study. Along the way, math teacher age was shown to have less impact on change than expected, as were teaching experience and math teaching experience. Also, females were observed to outscore males on the change scale, just as middle-school teachers outscored high-school teachers. Of interest was the finding that math majors were less open to change than math teachers holding other degrees. Finally, urbanness of school setting was shown to predict change attitude, which perhaps explains the finding that North Dakota teachers are less ready for change than teachers in other states. These demographic findings were incidental to the development of two SEM models for math teacher change openness. A measurement model consisting of seven constructs was confirmed, as was a complex structural model that diagrammed associations among the seven constructs. Openness to pedagogical reform was shown to depend upon

complex interactions among mindset, self-efficacy, perception of learning, conception of mathematics, professional identity, and ambiguity tolerance. The next section reviews the results of the study in terms of the four research questions. Under each of the four, the meaning and implications of the research will be explored.

Research Questions

Question 1: How Can Math Teacher Openness to Changing Practices be Measured?

In earlier work, the principal investigator for this study had sought to determine what made some teachers eager to embrace recommended pedagogical reforms while others resisted at every turn. Combing the literature in search of a tool for measuring change attitudes, the researcher came up empty-handed. Experienced in developing training for math teachers, in this study she developed her own "change" items based on qualities she felt measured teacher inclination toward learning and applying new strategies. Those eight items comprised a construct of good internal consistency (α = .850). Only the chg2 item ("I try to adapt my instructional approaches to follow current best practices") veered from normality with skewness of 2.72, perhaps due to the ambiguity inherent in "try to." Even when pared down to four for SEM analyses, the scale held together well with alpha at .791.

Admittedly, other content experts were not invited to review the change items prior to sending the survey to participants. Nonetheless, the scale addressed key issues relative to adapting practice to match current research: Is it your intention? Is it your tendency? Are you comfortable with change? Do you have a positive disposition toward it? It makes sense that since change items incorporate terms such as "new," "adapt," and "change" itself, they do indeed measure attitude toward change:

chg2: I try to adapt my instructional approaches to follow current best practices.

chg4: I am quick to embrace new methods for teaching math.

chg5R: Pressure to change my strategies makes me want to leave teaching.

chg6: I enjoy trying new ways of teaching mathematics.

Question 2: Which Demographic Measures Predict Openness to Change?

Chapter IV reported significant differences in the change variable for six demographic categories: ethnicity, gender, college major, school level, state, and geographical setting. While the last two registered the most substantial differences, differences corresponding to school level were of also of interest to the researcher.

Teaching level. The introduction to this paper had proposed that stagnant high school math scores in the U.S. over the last several decades might be attributable to unwillingness among high school teachers to revise instructional practices. Results of this study confirm high school math teachers as indeed significantly less open to change than middle school teachers, but perhaps not to the degree anticipated. 64.7 % of high school teachers scored less than "agree" on the change scale while 53.1% of middle school teachers scored in that range. This difference is perhaps attributable to—for the high school teacher—a greater proximity to college lecture models (Ball & Bass, 2002) and a stronger focus on the content background necessary for college readiness. High school teachers are often concerned about preparing students for college admission tests and feel they must prioritize coverage of all rules and procedures. More worthy of note than the difference in these groups of teachers was the fact that 59.9% of *all* teachers scored 19 or lower on the change scale (where 4 = "strongly disagree," 4 = "disagree," 12 = "slightly disagree," 16 = "slightly agree," 20 = "agree," and 24 = "Strongly agree"), suggesting plenty of teachers at both levels bring some reluctance to reform efforts and could benefit from professional development around the need for change.

School leaders must be cautious, however, in interpreting all resistance in terms of readiness for change. The law of initiative fatigue must also be taken into consideration. This principle states that with each new initiative from the district, teachers will have less emotional energy to contribute toward change (Reeves, 2010). If change in math pedagogy is promoted in the context of many other reforms, it is perhaps overly optimistic to expect results.

Geography. The differences associated with geographic setting—with the more rural teachers less open to change—are perhaps of little surprise. After all, rural schools find themselves at a distinct disadvantage in recruiting qualified teachers due to low fiscal capacity, poorer working conditions and cultural isolation (Williams, 2003). Isolation in rural areas is not just cultural but also professional, with fewer opportunities for collaboration with peers and less access to high quality continuing education. Less peer interaction translates to less change since even "passive consumers" of professional learning are drawn into new practices through association with more change-inclined colleagues (Joyce & Showers, 1995). With nearly a third of America's teachers working in rural areas (Williams, 2003), resistance among this group can represent a serious roadblock to math education reform. It is particularly important that this group come to in-service training with an open attitude. Adequate training related to the six constructs outlined in this study will support them in doing so.

State. North Dakota is a good example of a state trying to meet the needs of rural teachers. Among North Dakota schools, 71.2% are classified as rural, accounting for 46% of its students. Only South Dakota has higher percentages (77.3% and 46.8%, respectively) (Williams, 2003). These data correspond to a national rate of 57% (NCES, 2013). Among participants in this study, a full third of ND teachers reported working in areas of population

less than 1500, where for out-of-staters this figure was only 7.6%. The rural nature of the North Dakota perhaps explains why its teachers scored lower on every construct in the study. The challenge in this state is to recruit a professional development team able to support learning among teachers spread across a wide geographic area. This can be particularly difficult since it involves coaches regularly traversing a 400-mile-wide state in which 71% of math teachers scored less than "agree" on the openness-to-change scale. Although North Dakota has eight education cooperatives dedicated to professional learning for teachers in eight regions in the state, only a few of these employ math specialists. This means that efforts must be carefully coordinated in order to ensure proper coaching for all math teachers in the state. In the past year, the Department of Public Instruction has formed a math leadership team comprised of math specialists from around the state. In beginning their work, this team would do well to pay attention to this study's findings about beliefs and attitudes, as would leadership groups from other rural areas.

Math degree. Results also showed math majors to be less inclined toward change than non-math majors. The researcher speculates this may be due in part to an association with teaching high school. As was stated in the previous chapter, among high school teachers 72.2% were math majors compared to 27.4% for middle school teachers. It is interesting to note that among math majors, the total score corresponding to the lower-path constructs in the structural model is significantly stronger ($p < .001$) while the upper-path total is weaker than for non-math majors ($p = .17$). This suggests that teachers who identify strongly with the subject matter will need more training in math mindset, more time to reflect on the nature of mathematics and what it means to learn it, and more support for ambiguity tolerance. Non-

majors on the other hand will need more opportunities to develop sense of efficacy and identification with the role "math teacher."

Gender. The structural model is not useful in explaining the greater openness to change among female participants. Females (M = 44.26) were significantly older than males (M = 42.14, t = -1.997, p = .046) and significantly higher in identification with the profession (M_f = 21.33, M_m = 20.94; t = -1.800, p = .07), but males were significantly more tolerant of ambiguity (M_f = 15.60, M_m = 16.66; t = 3.907, p = .000). Perhaps the change-attitude gender difference is attributable to a psychosocial construct not accounted for in the model. For example, there is literature to suggest that males tend to be overconfident, particularly when it comes to math (Bengtsson, Persson, & Willenhag, 2005; Jakobsson, Levin, & Kotsadam, 2013). Female rule-following behavior may also incline them to do as the best-practice literature suggests (Villalobos, 2009).

Ethnicity. In terms of ethnicity, the very low representation among groups other than White—with American Indians and Asians accounting for roughly 2% each of total participation and African Americans and Hispanics 1.4% each—makes it unrealistic to draw conclusions about differences. To have reflected well the ethnic proportions of teachers in this country, the last two groups alone would have to have been six times larger. Whites had a higher percentage of math majors (54.1%) than nonwhites (40.9%) and were a bit more likely to teach at the high school level (58.3% compared to 54.5%), so those associations may have contributed to lower change scores. Nonetheless, nonwhites were more urban with 45.5% working in a population center greater than 100,000 compared to 39.8% for whites.

Age and experience. Another demographic finding of very real interest was the fact that age is at best weakly associated with a reluctance to change. In application of the 4-item

scale for change, no significant association was found with age or experience. In fact, there was no significant difference in any independent construct scale along age lines. This suggests reformers need to avoid associating age with inertia and realize that even the youngest of teachers need to reflect on traditional beliefs and mindsets. It is not only experienced math teachers in U.S. classrooms who are clinging to outdated notions about the learning of mathematics.

Question 3: How Valid Are Certain Constructs for Predicting a Math Teacher's Openness to Changing Practices?

The study showed all six proposed constructs to be weak-to-moderate predictors of attitude toward change: math mindset, self-efficacy, constructivist perception of learning, dynamic conception of math, professional identity, and ambiguity tolerance, with the best predictor explaining 19.62% of variance in change. Combining the six factors in a multiple regression analysis, however, resulted in explanation of 41.6% of the variance in the change score. Discriminant validity was established for all constructs, and each scale exhibited internal reliability. In other words, all six of these constructs are worthy of attention when considering math teachers' openness to embracing reform pedagogy.

The best individual predictors of disposition toward change were constructivist attitude in the classroom, sense of professional identity, and tolerance for ambiguity, in that order. The constructivist findings are in keeping with the research of Draper (2002). It is likely that, given the historical context, a constructivist attitude was the best predictor of change-attitude because it measured a predisposition toward the kind of pedagogy advocated by the CCSS movement. A teacher who believes it is important for students to interact with mathematics and each other in a sense-making way is already philosophically aligned with

the Standards for Mathematical Practice (SMP) of the CCSS. The constructivist theoretical framework of this study is largely due to the constructivist underpinnings of the SMP. (The researcher speculates it would be unlikely, say, in another era, to find teachers with high constructivism scores eager to embrace the change advocated by a rigid back-to-basics movement.) The fact that constructivist attitude is an important contributor in this study emphasizes the need in CCSSM training to revisit the principles of constructivism.

Another finding that makes good sense—although it runs counter to pilot findings—is that identification with the profession is important to change. Teachers who see their work as "more than just a job" are likely to be motivated to invest in their practice. These teachers are apt to believe in continuous improvement, which naturally implies change. Perhaps these teachers are even more inclined to heed research on math education. It may very well be that when the National Council of Teachers of Mathematics says, "Let's move in this direction," those who identify most strongly with the profession are among the first to move. It would have been interesting to ask which participants were member of NCTM and to test association of membership with both professional identity and change. The findings related to professional identity are in keeping with the research of Gresalfi and Cobb (2011), who observed in math teachers motivation to change practice as they came to identify with reform teaching practices.

The finding that ambiguity tolerance is significantly linked to change-openness is in keeping with the work of Merenluoto and Lehtinen (2004). Their discussion of conceptual change makes clear that teachers need to be given opportunity to resolve conflicts in times of reform. Math teachers need to be well supported through ambiguity in order to feel confident

in implementing changes. This includes be given ample opportunity to reflect on those changes and the nature of any associated discomfort.

An unexpected finding in this study was that self-efficacy did not correlate more strongly with the change scale. Generally speaking, it would seem that self-efficacy gives one confidence for risk-taking in any career during times of adaptation. The low correlation is best explained through a discussion of the interaction of constructs.

Question 4: How Do Factors Relate to One Another as Predictors of Openness to Change?

Efficacy → Professional Identity → Construct Path. This study confirmed Lasky's (2005) claim that professional identity and self-efficacy are tightly interwoven. Although only moderately correlated ($R = .355$), the constructs exhibited indirect effects in their relation to change, such that efficacy was fully mediated by professional identity. In other words, although "professional identity" did not measure the same construct as "efficacy," it implied its presence and rendered it insignificant in terms of predicting change. The noncollinearity in the two constructs is illustrated by the fact that high school teachers (M =19.89, $SD = 2.78$) scored significantly higher than middle school ($M = 19.11$, $SD = 3.05$; $t = 3.143$, $p = .002$) on efficacy—perhaps due to the greater percentage of math majors—but virtually the same as middle school teachers on professional identity. Although they measure different constructs, it is difficult to talk about one factor without discussing the other. It would seem their intersection has to do with a sense of security. It is as if one construct entails a kind of motivation to change ("I can identify as this kind of math teacher") and the other the confidence to tackle the new approach ("I can do this"). Both provide a kind of defense against vulnerability in time of uncertainty. The

regression model from EFA tells us that motivation (proSUM) alone may suffice while confidence (efcSUM) alone will not. When we turn to the structural model, however, we see that the loadings onto constructivist attitude are a bit stronger when efficacy is included, even though the model fit is a bit stronger without it (see Figure 20). The researcher prefers to leave efficacy in the model due to its significant correlation with all other constructs, particularly professional identity and ambiguity tolerance, two of the best predictors of change. The negative association between efficacy and change described in Chapter IV, a surprising outcome, perhaps suggests that math teachers will low self-efficacy are just plain ready for guidance.

Clearly math education reform must involve the strengthening of a teacher's professional identity and the entailed sense of self-efficacy. Full implementation of professional learning communities (PLC's) formed around improving practice would help fortify a sense of belonging, but to truly strengthen a sense of professional identify it may first be necessary to address certain aspects of self-efficacy. PLC's could also be beneficial here, especially for bolstering the efficacy piece associated with pedagogical knowledge (efc8R: *I sometimes doubt my ability to teach math*). In PLC groups teachers plan and evaluate lessons together and so learn from one another. For the efficacy piece associated with content knowledge (efc1: *I think of myself as very good at math*; efc 2: *I am strong enough in math to teach it beyond the level at which I currently teach it*), PLC's may also be effective, but teachers with insufficient background will need professional development in math content, preferably accompanied by opportunities to experience the SMP. The need for content training among non-majors is supported by the finding that math majors scored significantly higher on efficacy items related to math background. Math majors did not

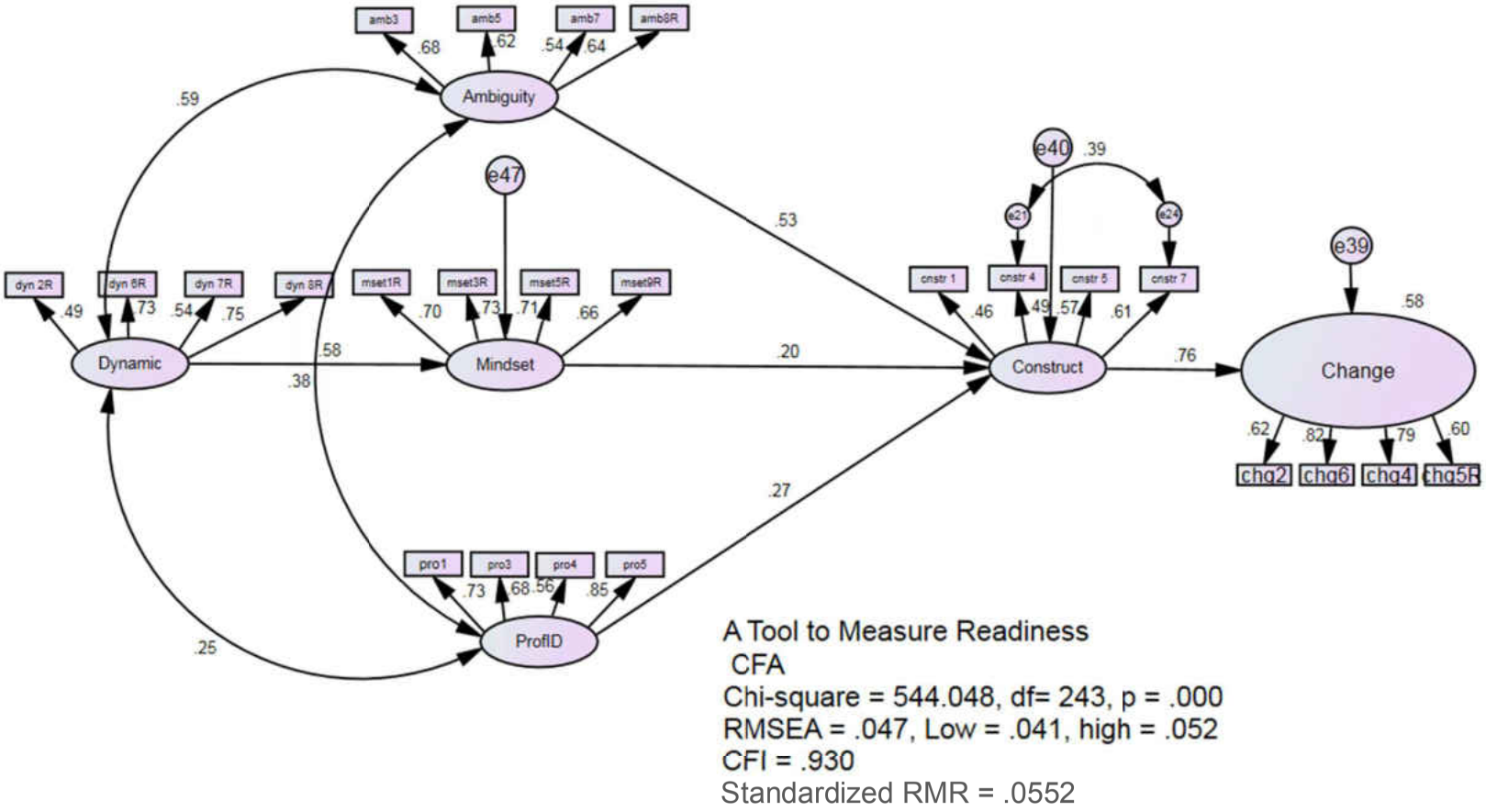


Figure 20. Complex Structural Model C Without Efficacy.

outscore their counterparts on any of the efficacy items related to effective teaching, however. In fact, on item efc6, not included in the parsimonious model (*No matter the students, I am able to help them improve their math skills*), non-majors (M =4.75, SD = .87) outscored math majors (M = 4.60, SD = .95) (t = -1.981, p = .048). In other words, the efficacy concerns to be addressed vary from teacher to teacher. Math majors may not need as much content training, but they will still need help implementing more effective instructional strategies. It may seem paradoxical that math majors were slightly less inclined toward change given their stronger efficacy, but recall the strength of their change orientation came from the lower path of the model: the efficacy and professional-identity constructs. Math majors scored lower in, and would require more professional development in, constructs in the upper two paths of the model: ambiguity tolerance, dynamic conception of math, and math mindset.

The regression of efficacy and professional identity onto constructivist attitude confirms the claim that math teachers secure in their capacity are more open to student-centered, inquiry-oriented approaches (Guskey, 1988; Charalambous and Philippou, 2010). Recall that Charalambous and Philippou (2010) had suggested efficacious teachers were more open to the ideas of students in general. Investing professional development time in efficacy- and identity-building activity might lead teachers toward these more constructivist attitudes, which in turn could create greater openness to current math reforms.

Dynamic → mindset → construct path. So how do constructs in the middle path lead to change-openness? Figure 15 showed math mindset mediating the impact of dynamic-math-view on change. In the literature, dynamic conception of mathematics and math mindset were linked transitively by their mutual association with constructivist attitude. The

literature had suggested teachers with a more static view of math—those who viewed math as an object or set of rules—were inclined toward traditional practices (Chapman, 2002; Hoz & Weisman, 2008). Likewise, those with a more fixed/entity mindset were observed to adopt transmission-style strategies (Stipek, 2001). The structural model in this study confirms these findings.

The model would fit the data equally well if mindset were regressed onto dynamic view. The structure in Figure 18 was preferred for theoretical reasons. Dynamic view of math is depicted as predicting mindset since one's view of math determines whom one believes can learn it, not the other way around. When a teacher conceives math as a science of inquiry and views a problem as accessible through multiple approaches, she is more inclined to a growth mindset, to believe every student can grow through mathematical experiences, making sense and meaning of underlying concepts through experimentation and productive struggle. The teacher who, on the other hand, views math as a fixed set of complex procedures to be drilled will naturally think it difficult for many students to acquire math. This latter teacher often fails to see that difficulty derives from lack of opportunity to understand underlying concepts.

It makes sense that a teacher with both a dynamic math view and a growth/incremental mindset would have a more open constructivist attitude toward learning. One who believes math is a science of inquiry is more likely to believe in the importance of questions (cnstr1: *In math, the questions are more important than the answers*) and to give students opportunity to share ideas (cnstr5: *It is really important to have students work in groups in math class*). And one who believes anyone can learn math is more likely to offer

social and contextual experiences where students can develop their own solutions (cnstr4) and construct their own understanding (cnstr7) through exposure to various pathways.

To shift math teachers along the middle track of the structural model, coaches must find a way to change perceptions of the discipline of mathematics. One way to accomplish this is through interaction with mathematicians in the field, who should be able to convey a picture of math as a living breathing science. Alternatively or additionally, coaches could provide teachers with opportunities to experience math as something other than a static body of rules by engaging them in non-routine problems for which clear solution paths are not immediately available. Teachers who have occasion to work collaboratively on such problems will learn quickly that multiple approaches are possible and that the best solutions do not always come from the colleagues with the strongest content background. These kinds of experiences will provide opportunities to reflect on and discuss what it means to construct mathematical meaning. As Linda Flowers (1994) explains:

> Problem solving [involves] intellectual moves that allow people to construct meaning—to interpret the situation; to organize, select, and connect information; to draw inferences, set goals, get the gist, … draw on past experiences, imagine options, and carry out intentions. (p. 24)

After time spent solving problems together, math teachers may benefit from discussing what they have learned and how they learned it.

The structural model in this study suggests math teachers will also benefit from studying Carol Dweck's work on mindset (2006; 2000). As they learn about "helpless" versus "mastery-oriented" patterns of behavior and fixed versus growth attitudes toward challenges (2002), they will not only see what is possible for students but what is possible for themselves. They will come to assess their own mindsets, and perhaps teachers with

previously fixed attitudes will begin to hunger for more than a "diet of easy [classroom] successes" (Dweck, 2000, p. 7).

Ambiguity → construct path. The top path of the structural model consists of one construct: ambiguity tolerance. Early analyses in this study had shown one of the strongest correlations among constructs to be between ambiguity tolerance and constructivist attitude (R = .400). The strength of this association was second only to that between mindset and dynamic math view (R = .419). The model confirms what research had suggested, that the capacity to navigate uncertainty makes it possible for teachers to take more constructivist approaches, to release to students some control over outcomes and learning (Smith & Stein, 2011). A teacher who tolerates ambiguity well will be open to student ideas and the organic unfolding of lessons. On the flip side, teachers with low tolerance for ambiguity will exhibit avoidance behaviors (Stein & Smith, 2011), cling to the familiar (Furnham & Marks, 2013), and stall forward movement (Budner, 1962). Since the ambiguity-intolerant tend toward more rigid, black-and-white views (Furnham & Marks, 2013), it is not unexpected to see also in the structural model a fairly strong association between ambiguity tolerance and dynamic view of math. A view of mathematics as a fixed set of known rules leaves little room for uncertainty.

In order to move teachers along the model's ambiguity → construct pathway, it may be more effective to reduce ambiguity than to try to build tolerance, but there are ways to do both. The Japanese lesson study approach, which has been spreading in the United States since the release of the first TIMSS Video Study (Doig & Groves, 2011), offers a means to decrease the "not knowing" a teacher experiences when adopting a more problem-based, student-centered approach. In lesson study, teachers in a PLC group carefully choose a study

problem and then anticipate together all the ways a student might solve or attack the problem. In lesson study done well, teachers plan together how they will respond to various solution strategies. Since teachers have the chance to "experiment with classroom practice and analyze it in detail" (Doig & Groves, 2011, p. 79), uncertainty about outcomes should naturally decrease.

Tolerance for the uncertainty can be enhanced by the presence of a math coach as lesson-study lessons move into the classroom. The coach's role would be to support improvement of teaching through modeling, co-teaching, and conferencing with the teacher about goals for the lesson (Mudzimiri, Burroughs, Luebeck, Sutton, & Yopp, 2014). Over time, with gradual release, the teacher should become more comfortable with student-centered approaches.

Construct → change. The structural model in this study indicates that if we address efficacy and identity, math-view and mindset, and ambiguity tolerance, these in turn will influence a teacher to adopt what Hoz and Weisman (2008) called "open" classroom practices (investigation of rich questions, social interaction, students actively constructing math knowledge). This does not mean, however, that teachers would not also benefit from direct training in constructivism and constructivism-based approaches. Teachers need opportunities to reflect on the theory that "we construct our knowledge of our world from our perceptions and experiences, which are themselves mediated through our previous knowledge" (Simon, 1995, p. 115). With an understanding of constructivist theory and its implications, teachers can learn to facilitate lessons more focused on student thinking.

The final causal relationship represented in the structural model is the strong regression of constructivist attitude onto change openness. Why are teachers with a more

open, constructivist conception of teaching mathematics also more open to change? Perhaps the operative word here is "open." The connection may have to do with a general underlying openness to experience and to a variety of ways of knowing. Teachers whose practices honor multiple pathways for student learning, who believe student knowledge is individually constructed through personal and social experience, perhaps also believe their own growth depends upon exposure to novel experiences. Put another way, perhaps teachers who believe students learn through exploration are also inclined to exploration themselves and therefore change. A general open-mindedness, functioning as a lurking variable regressed onto both factors, however, does not explain structural relationships depicted in the model.

The regression of constructivist attitude onto change may perhaps be better understood in a historical context. Constructivist perspectives have been shaping math reform in the U.S. since NCTM published its first set of *Standards* (Simon, 1995; NCTM, 1989), and the influence of those perspectives on NCTM's Practice Standards (NCTM, 2000) is reflected still in the SMP of the Common Core (NGA & CCSSO, 2010). In some sense, teachers with an open attitude toward constructivist-like practices are well on their way to fostering the Standards for Mathematical Practices. (Although no reference to the SMP were made in the survey, it is even possible teachers had them in mind when they read terms like "change," "best practices," and "new methods.") Classroom staples like small-group discussion and personal construction of solution strategies—referenced in the survey's constructivism items—are stepping-stones to Common Core practices of perseverance in problem solving, construction and critique of argument, and active use of structure and repeated reasoning. It is no surprise, then, that a constructivist attitude anticipates a willingness to move in this "new" direction.

In a last note on the structural model, it must be acknowledged that while it goes a long way to explain the variance in change-openness, clearly it leaves some elements unanalyzed. The researcher speculates there are additional psychological constructs at work, as well as outside contributing factors. For example, the Big Five personality trait of openness—different from any type of openness discussed here so far—includes an aspect of intellectual responsiveness (Kaufman et al., 2015), which may play a part. Recent work on curiosity, likely related to this "openness," suggests it may play a role, though Engel (2015) describes it as relatively stable from childhood to adulthood, with some individuals just more curious than others. Flexibility of thought is perhaps another contributor, as could be attraction to experimentation, preference for complexity, self-esteem, and perceived control over the change process. Even disposition toward mathematics and disposition toward students may be influences. Additionally, it is well established that math teachers must judge a reform to be worthwhile before they will embrace it (Cobb, Yackel, & Wood, 1992). Outside factors that no doubt play an important role are time to learn new practices, availability of adequate resources to implement them, and availability of support to sustain them. School/district culture and morale are also likely to be involved. After all, not all roadblocks to change are rooted in the teachers themselves. The goal in this study has not been to include all possible contributors to change, but rather to develop a model directly related to factors drawn from the literature on math pedagogy change.

Limitations

The results of the current study provide insight into the complex issue of instructional change in secondary mathematics. Nonetheless, the findings must be evaluated in the light of the study's limitations.

First, the study design was cross-sectional as opposed to longitudinal and so captured participant perceptions at a particular place and time. As was suggested in Chapter V just above, the historical backdrop of Common Core State Standards implementation may have had a particular impact in how U. S. math teachers interpreted items related to change. It may also have affected general disposition toward mathematics instruction, which in turn may have influenced the way teachers responded to certain items.

Second, the study was based on a sample of convenience (through the colleagues of the researcher) rather than on a random sample, so the possibility of sample bias exists, as do limitations to the generalizability of findings. More specifically the out-of-state sample was broader and less well defined due to less systematic recruiting procedures than used within North Dakota. Since out-of-state colleagues of the researcher forwarded the survey, it is impossible to determine to whom the survey was sent; it is unlikely it was distributed evenly within a state. It should also be noted that in multiple-regression analysis the six independent constructs combined to be a better predictor of change outside of North Dakota (R = .625, p = .000) than within (R = .512, p = .000), so careful consideration should be given to comparisons between these two geographically defined groups.

Additionally, the latent variables could not be objectively measured and so depended on the perceptions of participants and the extent to which they communicated beliefs honestly. Teaching methods were not observed, so the study was not able to draw connections between what teachers said they believed and the way beliefs manifested themselves in classroom practices.

No latent construct used in the study was measured with a well-established scale. The change scale, which functioned as the dependent variable, was wholly original to this study

and not validated through content experts, while other scales were adapted from established scales—in some cases from more than one scale—to fit the context of teaching mathematics.

Finally, in the paring down of the model, an attempt was made to maintain a balance in the efficacy construct between math efficacy and teaching efficacy. Since the factor played a small role in the overall model, one wonders if future investigation might examine more closely the content of this variable. Perhaps to focus on teaching efficacy exclusively would yield different results.

Future Research

As implied above, replication of this study with a random stratified sample of secondary math teachers from across geographic regions of the country would be beneficial. It would also be of value to extend the research by linking beliefs to practices as Stipek et al. (2001) did in their study of elementary mathematics teachers. In particular, it would be interesting to see whether teachers with high change-openness scores did indeed implement newly introduced strategies more quickly than their peers and whether, in a more general sense, beliefs aligned with practices.

In addition to further research suggested by the limitations of this study, it would be interesting to expand the structural model to include a construct for uncertainty tolerance, described by Furnham and Marks (2013) as distinct from ambiguity tolerance. Then not only could we examine attitudes toward the ambiguity inherent in certain changes but also the anxiety teachers experience in response. A risk-taking construct might be added as well, and yet at some point the number of constructs results in a model that is too complex. Another avenue might be to undertake separately a close examination of the three constructs TA, TU, and RT and the personality trait intellectual openness, all relative to teacher change.

Future studies should also include qualitative research to explain this study's results in greater detail. Interviews with secondary math teachers would bring a dimension of understanding not possible without participant voices. A purposeful sampling of participants with high or low scores on the change scale—or for that matter, on any of the constructs—would add significant insight to the findings of this study.

Finally, an experimental study could be undertaken which tracked scores on the change scale before, during and after implementation of an extensive professional development program. Ideally the program would include elements like those described earlier in this chapter: lesson study in PLC's, collaborative problem-solving, reflection on mindset, constructivism, and the nature of mathematics.

Conclusion

The purpose of this study was to examine factors contributing to a math teacher's openness to changing instructional practices. Central to the research was the development and validation of two tools: an instrument for measuring openness to change and a structural equation model for explaining how various factors contribute to change-openness.

The results of the study indicated that six factors contribute to openness to new practice: a growth math-mindset, a strong sense of self-efficacy, a dynamic view of mathematics, a constructivist perception of learning math, a strong sense of professional identity, and a high tolerance for ambiguity. The structural model showed these factors to have complex inter-relationships. Other findings were that urban math teachers were found to be more open to change than their rural counterparts, and middle school teachers were found to be slightly more ready than high school teachers. No association was found between attitude toward change and age or experience.

This study offers the first tool for assessing math teacher receptivity toward instructional reform. Additionally, the structural equation model developed in this work is the first to illuminate complex issues around math teacher change. It provides a framework for analyzing, diagnosing and remedying professional development challenges and offers rich ground for further research in motivation and math teacher education.

APPENDICES

Appendix A
Survey

The purpose of the current survey is to ask teachers about their thoughts and experiences regarding the teaching of mathematics. Please try to answer the questions honestly. Your identity will be unknown to me and responses will be kept anonymous. If there are significant findings from this study, the results may be published in a research journal, but all references to place will be anonymized; that is, there will be no references to names of persons, schools, or districts.

Thanks for your participation. If you complete all questions, you will be able to link to a prize drawing for $50 Amazon gift cards at the end of the survey.

Cathy Williams

For each of the following statements, click the column that indicates your level of agreement: 1 (strongly disagree) to 6 (strongly agree) [Underlined items were retained in parsimonious model.]

Conceptions of Mathematics (Dynamic)
(Later: Constructivist Attitude or Dynamic View of Mathematics**)*
1. Mathematics involves mostly facts and procedures that have to be learned.
2. In math, you can be creative and discover things on your own.
3. There is usually one best way to solve a math problem.
4. Students who really understand math will have a solution quickly. **
5. In math, the questions are more important than the answers. *
6. Mathematics is a science of inquiry and exploration.
7. Math is characterized by certainty.
8. Mathematics is continually growing, changing and being revised.
9. Math is more about ideas than numbers.
10. Math is mostly about finding the answer.

Perceptions of Learning Math (Construct)
(Later: Constructivist Attitude or Dynamic View of Mathematics**)*
11. Math can be applied only after basic skills are mastered. **
12. In math class, students need to develop their own solution strategies. *
13. It is really important to have students work in groups in math class. *
14. Learning math requires receiving clear explanations. **
15. When learning math, students benefit from making mistakes.
16. Math is a set of skills to be learned in sequence. **
17. Students need to construct their own understanding of a math concept. *
18. The best way to understand math is to do lots of problems.

Mindset

19. To be honest, you can't really change how much math talent you have.
20. Some people just have a knack for math and some just don't.
21. Math ability is something that remains relatively fixed throughout a person's life.
22. All of my students would be good at math if they worked hard at it.
23. I can improve my math skills but I can't change my basic math ability.
24. No matter who you are, you can learn math.
25. It's possible to change even your basic level of math intelligence.
26. In math there will always be some students who simply won't "get it".
27. There isn't much you can do about how much math ability you have.

Professional Identity (ProfID)

27. Being a member of the math teaching profession is important to me.
28. Teaching math is just what I do to earn a living.
29. I feel proud when I tell people I am a math teacher.
30. I can identify positively with other math teachers.
31. Teaching mathematics means more to me than a job.
32. I don't really feel like a member of the math teaching profession.

Self-Efficacy (Efficacy)

33. I think of myself as very good at math.
34. I am strong enough in math to teach it beyond the level at which I currently teach it.
35. When I teach math, I often find it difficult to interpret students' wrong answers.
36. I am good at communicating math material to students.
37. I don't always know what to do to help my students learn math better.
38. No matter the students, I am able to help them to improve their math skills.
39. When my answer to a math problem doesn't match another math teacher's answer, I usually assume my answer is wrong.
40. I sometimes doubt my ability to teach math.

Ambiguity Tolerance (Ambiguity)

41. I try to avoid problems that don't have one best solution.
42. It is more fun to tackle a complicated problem than one that is simple to solve.
43. I like to fool around with new ideas even if they turn out to be a waste of time.
44. A problem has little attraction for me if I don't think it has a solution.
45. I tolerate ambiguous situations well.
46. What we are used to is always preferable to what is unfamiliar.
47. I generally prefer novelty over familiarity.
48. I try to avoid situations that are uncertain.

Attitude Toward Changing Practice (Change)

49. I prefer to teach math the way it was taught to me.
50. I try to adapt my instructional approaches to follow current best practices.
51. I don't want to change the way I teach math.

52. I am quick to embrace new methods for teaching math.
53. Pressure to change my strategies makes me want to leave teaching.
54. I enjoy trying new ways of teaching math.
55. I am afraid to change the way I teach math.
56. Gaining new knowledge about teaching math is invigorating.

Demographics

56. Gender	What is your gender: 1) Male 2) Female 3) Other 4) Choose not to identify
57. Age	What is your age in years? (textbox)
58. Ethnicity	1) White/Caucasian 2) African American/Black 3) American Indian 4) Asian American/Asian 5) Mexican American/Chicano 6) Puerto Rican American 7) Other (please specify)
59. Education	What is your highest level of education? 1) BA/BS 2) MA/MS 3) PhD
60. Degree	Do you have a degree in mathematics (that is, a full major in math)? 1) Yes 2) No (Please specify your undergraduate major field.) (textbox)
61. Certification	In what areas other than mathematics are you certified to teach? (text box)
62. Experience	How many years of teaching experience do you have? (textbox)
63. Math Experience	How many years of math teaching experience do you have? (textbox)
64. Setting	In what setting do you teach? 1) In a city whose population is greater than 10,000 2) In a large town with population greater than 1500 but less than 10,000 3) In a rural, small-town, or consolidated district (population is less than 1500)

65. At what grade level do you teach mathematics? (Check all that apply.)

Level
1) 6th
2) 7th
3) 8th
4) 9th
5) 10th
6) 11th
7) 12th

Thank you so much for completing the survey. If you would like to be entered in the drawing for one of five $50 gift cards, please click on the link below, which will submit your responses and then take you to a separate contest site. No one will be able to connect your name to the survey you have taken here.

https://und.qualtrics.com/SE/?SID=SV_8k17LVBZpUZgIzr

If you DO NOT wish to enter the drawing, click on the arrow below to submit your responses.

Prize Drawing Survey

If you would like to be entered in the drawing for one of five $50 Amazon gift cards, please enter your name, email address, and phone number below. The phone number will only be used if you win a prize and we cannot reach you by email.

What is your name? Please include first and last name. (text box)

What is your email address? (text box)

What is your phone number? (text box)

Thank you and good luck! Be sure to click the arrow to the right to submit your name to the drawing.

Appendix B
IRB Approval

UNIVERSITY OF UND NORTH DAKOTA

INSTITUTIONAL REVIEW BOARD
c/o RESEARCH DEVELOPMENT AND COMPLIANCE
DIVISION OF RESEARCH
TWAMLEY HALL ROOM 106
264 CENTENNIAL DRIVE STOP 7134
GRAND FORKS ND 58202-7134
(701) 777-4279
FAX (701) 777-6708

October 10, 2013

Cathy Williams
815 28th Avenue South
Grand Forks, ND 58201

Dear Ms. Williams:

We are pleased to inform you that your project titled, "Openness to Reform Among High School Mathematics Teachers" (IRB-201310-118) has been reviewed and approved by the University of North Dakota Institutional Review Board (IRB). The expiration date of this approval is December 24, 2013.

As principal investigator for a study involving human participants, you assume certain responsibilities to the University of North Dakota and the UND IRB. Specifically, any adverse events or departures from the protocol that occur must be reported to the IRB immediately. It is your obligation to inform the IRB in writing if you would like to change aspects of your approved project, prior to implementing such changes.

When your research, including data analysis, is completed, you must submit a Research Project Termination form to the IRB office so your file can be closed. A Termination Form has been enclosed and is also available on the IRB website.

If you have any questions or concerns, please feel free to call me at (701) 777-4279 or e-mail michelle.bowles@research.und.edu.

Sincerely,

Michelle L. Bowles, M.P.A., CIP
IRB Coordinator

MLB/jle

Enclosures

REPORT OF ACTION: EXEMPT/EXPEDITED REVIEW

University of North Dakota Institutional Review Board

Date: 10/10/2013 **Project Number:** IRB-201310-118

Principal Investigator: Williams, Cathy

Department: Educational Foundations and Research

Project Title: Openness to Reform Among High School Mathematics Teachers

The above referenced project was reviewed by a designated member for the University's Institutional Review Board on 10/10/2013 and the following action was taken:

☐ Project approved. **Expedited Review** Category No. _____
Next scheduled review must be before: _____

☐ **Copies of the attached consent form with the IRB approval stamp dated _____ must be used in obtaining consent for this study.**

☒ Project approved. **Exempt Review** Category No. 2
This approval is valid until DEC 24 2013 as long as approved procedures are followed. No periodic review scheduled unless so stated in the Remarks Section.

☐ **Copies of the attached consent form with the IRB approval stamp dated N/A must be used in obtaining consent for this study.**

☐ Minor modifications required. The required corrections/additions must be submitted to RDC for review and approval. **This study may NOT be started UNTIL final IRB approval has been received.**

☐ Project approval **deferred. This study may not be started until final IRB approval has been received.**
(See Remarks Section for further information.)

☐ Disapproved claim of exemption. This project requires Expedited or Full Board review. The Human Subjects Review Form must be filled out and submitted to the IRB for review.

☐ Proposed project is not human subjects research as defined under Federal regulations 45 CFR 46 or 21 CFR 50 and does not require IRB review.

☐ Not Research ☐ Not Human Subject

PLEASE NOTE: Requested revisions for student proposals MUST include adviser's signature. All revisions MUST be highlighted and submitted to the IRB within 90 days of the above review date.

☒ Education Requirements Completed. (Project cannot be started until IRB education requirements are met.)

Research may begin at all sites listed on the IRB application form, except for Fargo Public Schools and GNWEC. Once letters of approval from Fargo and GNWEC have been received, they must be sent to the UND IRB office prior to beginning any research at those sites.

cc: Cheryl Hunter, Ph.D.

Michelle L Bartles 10/10/2013
Signature of Designated IRB Member Date
UND's Institutional Review Board

If the proposed project (clinical medical) is to be part of a research activity funded by a Federal Agency, a special assurance statement or a completed 310 Form may be required. Contact RDC to obtain the required documents.

(Revised 10/2006)

UNIVERSITY OF NORTH DAKOTA.

UND.edu

Institutional Review Board
Twamley Hall, Room 106
264 Centennial Drive Stop 7134
Grand Forks, ND 58202-7134
Phone: 701.777.4279
Fax: 701.777.6708

May 28, 2015

Principal Investigator:	Cathy Williams
Project Title:	Openness to Reform Among High School Mathematics Teachers
IRB Project Number:	IRB-201310-118
Project Review Level:	Exempt 2
Date of IRB Approval:	05/27/2015
Expiration Date of This Approval:	10/09/2016

The Protocol Change Form and all included documentation for the above-referenced project have been reviewed and approved via the procedures of the University of North Dakota Institutional Review Board.

You have approval for this project through the above-listed expiration date. When this research is completed, please submit a termination form to the IRB.

The forms to assist you in filing your project termination, adverse event/unanticipated problem, protocol change, etc. may be accessed on the IRB website: http://und.edu/research/resources/human-subjects/

Sincerely,

Michelle L. Bowles, M.P.A., CIP
IRB Coordinator

MLB/jle

Cc: Robert Stupnisky, Ph.D.

Appendix C
Pilot Items

Openness to Mathematics Education Reform,
Mean, and Standard Deviation (strongly disagree = 1, strongly agree = 6)

Survey Questions	Agree or Strongly Agree	*M*	*SD*
Conceptions of Mathematics			
1. Mathematics involves mostly facts and procedures that have to be learned. (R)	35.8	3.8	1.2
2. Students who really understand math will have a solution quickly. (R)	44.6	4.1	1.1
Perceptions of Learning Mathematics			
3. Doing mathematics is a creative process.	66.3	4.7	.9
4. It is really important to have students work in groups in math class.	58.0	4.5	1.0
5. Students need to construct their own understanding of a math concept.	81.0	5.3	.7
6. When learning math, students benefit from making mistakes.	62.6	4.7	.8
Math Mindset			
7. Not everyone can learn to do math. (R)	76.1	5.0	1.2
8. To be honest, you can't really change how much math talent you have. (R)	85.0	5.1	.8
9. In math classes in school, there will (NOT) always be some students who simply won't "get it." (R)	44.7	4.1	1.3
Professional Self-Image			
10. I feel proud when I tell people I am a math teacher.	89.8	1.7	.7
11. Teaching mathematics is more than a job to me.	88.8	1.6	.7
Math Teaching Self-Efficacy			
12. I have enough training to deal with almost any learning problem.	32.4	3.6	1.3
13. I am an expert in how students learn mathematics.	19.7	3.3	1.3

Appendix D
Pilot Models

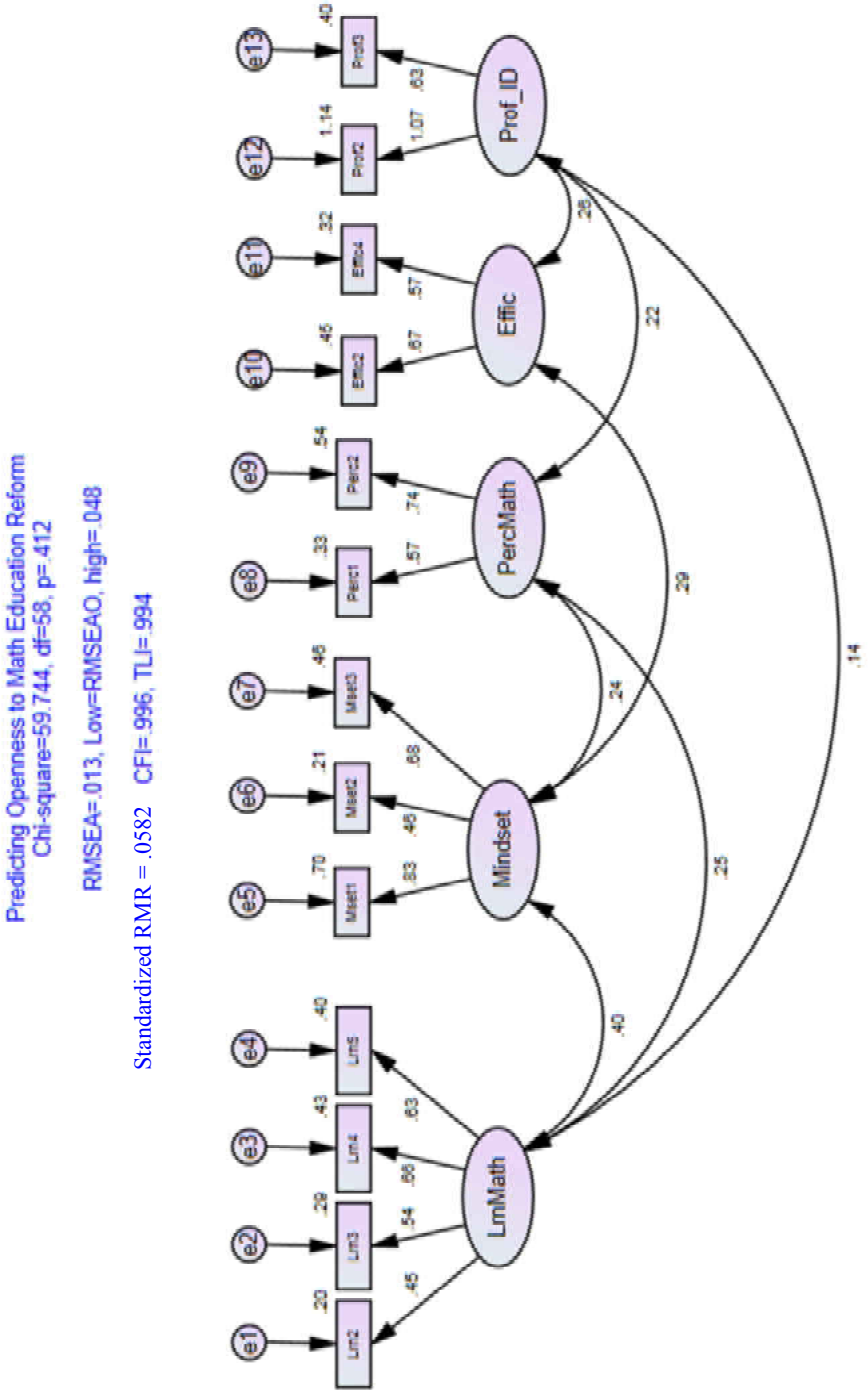


Figure 21. Pilot Measurement Model.

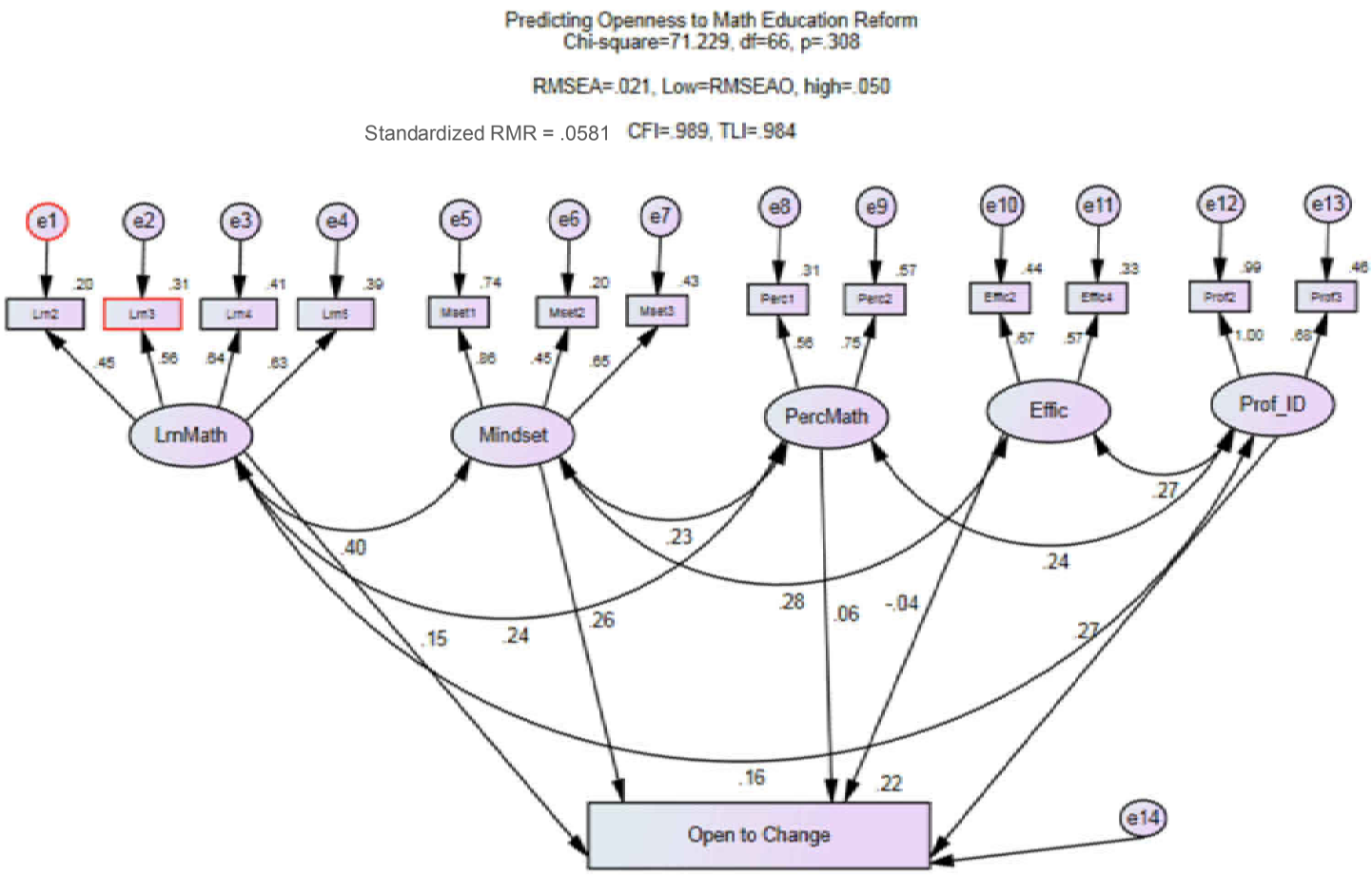


Figure 22. Pilot Structural Model.

Appendix E
Recruitment Letters

Dear Colleague,

You may recall that in late October of 2013 your district/region agreed to participate in research related my dissertation work at UND. At that time you forwarded a survey to math teachers for me, with the understanding there would be a follow-up survey in 18 months. This email contains a link to that follow-up survey.

Thank you so much for agreeing to forward the email below. Please send it at your earliest convenience to any teacher who teaches math in any grade from 6 through 12. If you are so inclined, you might add a note (in place of this blue one) encouraging them to help out a colleague with her research. Also, I would really appreciate it if you could copy me on the email you send.

I am truly grateful for your support! By way of a small thank you, I will send you a report of the findings in January.

Thanks again, so much,

Cathy Williams

SUBJECT LINE: Win $50 Gift Card -- Math Teacher Survey Link

Dear North Dakota math teacher,

My name is Cathy Williams. I am a former math teacher, currently employed as an instructional coach in Grand Forks, and I am working on my dissertation at the University of North Dakota. In the era of Common Core State Standards, I am interested to learn how math teachers view their math teaching experience and hope to gain your help in understanding this through a short survey. The anonymous survey contains 64 agreement-scale questions and should take approximately 10 minutes to complete. The survey will be available until May 7.

Here are 3 reasons to participate:
1) To have your name entered into 5 drawings for $50 Amazon gift cards.
2) To have your district receive a summary of the findings.
3) To contribute to the research on math teacher education.

Here is the link to the survey: https://und.qualtrics.com/SE/?SID=SV_dhtc5TGarN2AI4d

I would be so grateful for your input! Thank you for helping me with this assignment.

Cathy Williams
University of North Dakota

Hi, _____,

In October 2013, you signed a letter agreeing to allow _____ math teachers to participate in my dissertation survey. I sent the survey to you two weeks ago in hopes that you would forward it to all 6-12 math teachers. In case you haven't had the opportunity to do so, I am sending this reminder. If you could send them email below and copy me on the mailing, I would be tremendously grateful for your support! Let me know if you are not the person to whom I should be addressing this request. Have a great weekend! ~ Cathy Williams

SUBJECT LINE: Win $50 Gift Card -- Math Teacher Survey Link

Dear North Dakota math teacher,

My name is Cathy Williams. I am a former math teacher, currently employed as an instructional coach in Grand Forks, and I am working on my dissertation at the University of North Dakota. In the era of Common Core State Standards, I am interested to learn how math teachers view their math teaching experience and hope to gain your help in understanding this through a short survey. The anonymous survey contains 64 agreement-scale questions and should take approximately 10 minutes to complete. The survey will be available until May 7.

Here are 3 reasons to participate:
1) To have your name entered into 5 drawings for $50 Amazon gift cards.
2) To have your district receive a summary of the findings.
3) To contribute to the research on math teacher education.

Here is the link to the survey: https://und.qualtrics.com/SE/?SID=SV_dhtc5TGarN2AI4d

I would be so grateful for your input! Thank you for helping me with this assignment.

Cathy Williams
University of North Dakota

Hi, colleagues,

I am in the midst of my dissertation work and find I need to increase my sample size. I wonder if you could do me a favor and send this short note and link to any mathematics teachers of grades 7-12 students in your area. I know the school year is drawing to a close, so I would be so grateful if you could send it at your earliest convenience.

Thank you so much. I hope you all have a relaxing summer. ~ Cathy Williams

SUBJECT: Math Teacher Survey - Win $50 Gift Card

Dear math teacher,

My name is Cathy Williams. I am a former math teacher currently at work on my dissertation at the University of North Dakota, and I am wondering if you would do me the huge favor of taking a survey. In the era of Common Core State Standards, I am interested to learn how math teachers view their math teaching experience. The anonymous survey contains 64 agreement -scale questions and should take approximately 10 minutes to complete. The survey will be available until June 15. Here are 3 reasons to participate:

1) To have your name entered into 2 drawings for $50 Visa gift cards.
2) To have your district receive a summary of the findings.
3) To contribute to the research on math teacher education.

Here is the link to the survey
https://und.qualtrics.com/SE/?SID=SV_86ctP4jdsaTyldX

I would be so grateful for your input! Thank you for helping me with this assignment.

Cathy Williams
Graduate Student
University of North Dakota

Friday, June 12, 2015

Thanks to those of you who were able to forward my survey to teachers. For those of you who have not, you still have a few more days. I appreciate your support!

Cathy Williams

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