AKCE International Journal of Graphs and Combinatorics

ISSN: 0972-8600 (Print) 2543-3474 (Online) Journal homepage: https://www.tandfonline.com/loi/uakc20

On the topological matrix and topological indices

Gülistan Kaya Gök

AKCE INTERNATIONAL JOURNAL OF GRAPHS AND COMBINATORICS
2020, VOL. 17, NO. 1, 252–258
https://doi.org/10.1016/j.akcej.2019.07.001

Taylor & Francis
Taylor & Francis Group

AKCE International Journal of Graphs and Combinatorics

On the topological matrix and topological indices

Gülistan Kaya Gök

Hakkari University, Department of Mathematics Education, Hakkari 30000, Turkey

Received 28 March 2019; received in revised form 24 June 2019; accepted 22 July 2019

Abstract

The topological matrix M of a graph G are indexed by $V(G)$. If $i, j \in L$ then the $(i, j) - entry$ of $M(G)$ is 1 and the $(i, j) - entry$ of $M(G)$ is 0 for otherwise. In this paper, an upper bound for this topological matrix is found and the weighted topological matrix is defined. Also, some inequalities for the topological energy and the weighted Laplacian topological energy and of $M(G)$ are obtained.

The Topological index $TI(G)$ of a graph G is qualified by $TI = |A + D|$. Some relations are found for basic mathematical operations of $TI(G)$ and some bounds are reported for some topological indices in this paper.

Keywords: Topological matrix; Topological indices; Bounds

1. Introduction

Let G be a simple connected graph on the vertex set $V(G)$ and the edge set $E(G)$. If v_i and v_j are adjacent, then we use the notation $v_i \sim v_j$. For $v_i \in V(G)$, the degree of v_i, denoted by d_i is the number of the vertices adjacent to v_i. Let $A(G)$ be adjacency matrix of G and $\lambda_1, \lambda_2, \ldots, \lambda_n$ be its eigenvalues (λ_1 is the greatest eigenvalue). The graph eigenvalues provide these well-known results: $\sum_{i=1}^{n} \lambda_i = 0$, $\sum_{i=1}^{n} \lambda_i^2 = 2m$, $\lambda_1 \geq \frac{2m}{n}$ [1,2]. In this paper, the reciprocals of the graph indices are examined under these basic concepts. The graph indices which is one of the topics involved in the studies of graph theory. An important part of these graph indices is topological indices that is used in chemical graph theory, particularly. Chemical graph theory can make topological representation of a molecule. The numerical values of this chemical structure are descriptors. A topological index is calculated from a graph representing a molecule and this index is also invariant number of graphs. Most recommended topological indices consists of vertex, edge, degree relationship. This, describes the atomic relationship in chemical graph theory. This means that, graph theory contributes organic compounds with calculations and creating diagrams. This paper indicates special bounds for some topological indices by the help of different mathematical formulas and series. These topological indices are described as follows:

Let A be an adjacency matrix and D be a graph distance matrix. The topological index defined by

$$TI = |A + D|$$

where $|A + D|$ denotes the determinant of the matrix addition.

Peer review under responsibility of Kalasalingam University.
E-mail address: gulistankayagok@hakkari.edu.tr.

https://doi.org/10.1016/j.akcej.2019.07.001

Please cite this article as: G. Kaya Gök, On the topological matrix and topological indices, AKCE International Journal of Graphs and Combinatorics (2019), https://doi.org/10.1016/j.akcej.2019.07.001.

The atom-bond connectivity index ABC is one of the popular degree-based topological indices in chemical graph theory [3] such that

$$ABC = ABC(G) = \sum_{v_i v_j \in E(G)} \sqrt{\frac{d_i + d_j - 2}{d_i d_j}}.$$

Randić index is a well known topological structure such that

$$R(G) = \sum_{v_i v_j \in E(G)} \frac{1}{\sqrt{d_i d_j}}.$$

It is referred that the articles [4] and [5] for the various properties on the Randić index. Another remarkable topological descriptor is the Harmonic index, characterized in [6] as

$$H(G) = \sum_{v_i v_j \in E(G)} \frac{2}{d_i + d_j}.$$

Recently [7], a graph invariant came into the focus of attention, defined as

$$F = F(G) = \sum_{i=1}^{n} d_i^3$$

which for historical reasons [8] was named forgotten topological index. F satisfies the identities

$$F(G) = \sum_{i \sim j} d_i^2 + d_j^2.$$

One of the aim of this paper is to define the weighted topological matrix of a graph and to provide upper bounds on the weighted topological matrix M_{ij} and the eigenvalues of the graph G. It is needed that the following results in order to effect these bounds:

The Topological matrix M_{ij} of a graph G is defined by

$$M_{ij}^{w} = \begin{cases} 1 & ; \quad i, j \in L \\ 0 & ; \quad \text{otherwise.} \end{cases} \tag{1.1}$$

where L is a link.

Let $\lambda_1(M)$ be greatest eigenvalue and $M = (m_{ij})$ be an nxn irreducible nonnegative matrix. Let $R_i(M) = \sum_{j=1}^{m} m_{ij}$ [9]. Then,

$$(min R_i(M) : 1 \leq i \leq n) \leq \lambda_1(M) \leq (max R_i(M) : 1 \leq i \leq n). \tag{1.2}$$

Let V be a vertex set, $v_i \in V$ and e_i be an average degree . If G is a simple, connected graph then [10]

$$\lambda_1(G) \leq max(\sqrt{e_i e_j} : 1 \leq i, j \leq n, v_i, v_j \in E) \tag{1.3}$$

The energy of a graph G is described as $E(G) = \sum_{i=1}^{n} |\lambda_i|$ where λ_i, $i = 1, 2, \ldots, n$ are the eigenvalues of graph G. The Laplacian energy of a graph G is qualified as $LE(G) = \sum_{i=1}^{n} |\mu_i - \frac{2m}{n}|$ where μ_i, $i = 1, 2, \ldots, n$ are the laplacian eigenvalues of graph G [2,11].

Lemma 1.1. *[11] Let $a_i, b_i \in R$ and a, b, A, B be real constants such that $i = 1, 2, \ldots, n$, $0 < a \leq a_i \leq A$ and $0 < b \leq b_i \leq B$. Then,*

$$|n \sum_{i=1}^{n} a_i b_i - \sum_{i=1}^{n} a_i \sum_{i=1}^{n} b_i| \leq \alpha(n)(A - a)(B - b)$$

where $\alpha(n) = n[\frac{n}{2}]([1 - \frac{1}{n}][\frac{n}{2}])$.

See [1,12–16] for details about graph theory and topological structures.

The scheme of the paper is as follows: In Section 1, a list of some previously known definitions and results are introduced. In Section 2, a new topological matrix and topological energy are defined for the weighted graphs. Also, some relations are established in terms of edges, vertices and degrees. In addition, different relationships are obtained for topological indices.

Please cite this article as: G. Kaya Gök, On the topological matrix and topological indices, AKCE International Journal of Graphs and Combinatorics (2019), https://doi.org/10.1016/j.akcej.2019.07.001.

2. Main results

2.1. On the topological matrix

In this section, the weighted Topological matrix and the weighted Topological energy are defined. Furthermore, a relationship is found between the weighted Topological energy and the eigenvalues of the weighted Topological matrix.

Definition 2.1. Let G be a simple, connected, edge weighted graph. The weighted Topological matrix $M^w = M^w(G)$ of G defined by,

$$M_{ij}^w = \begin{cases} w_{ij} & ; \quad i, j \in L \\ 0 & ; \quad \text{otherwise.} \end{cases} \tag{2.1}$$

where $w(u) = \sum_{i \sim u} t_i$, $w(v) = \sum_{j \sim v} c_j$.

The weighted Topological eigenvalues $\rho_1^w, \rho_2^w, \ldots, \rho_n^w$ are the eigenvalues of its weighted Topological matrix M^w. These eigenvalues can be put in order such that $\rho_1^w \geq \rho_2^w \geq \cdots \geq \rho_n^w$.

Definition 2.2. Let G be a simple, connected weighted graph with n vertex. Let G be edge weighted graph and these weights be positive real numbers. The weighted Topological energy $ME = ME(G_w)$ of G_w is defined as follows:

$$ME = ME(G_w) = \sum_{i=1}^{n} |\rho_i^w| \tag{2.2}$$

where ρ_i^w is the eigenvalue of weighted Topological matrix M^w.

Definition 2.3. Let G be a simple, connected weighted graph. The weighted Laplacian Topological energy $LME = LME(G_w)$ of G_w is defined as follows:

$$ME = ME(G_w) = \sum_{i=1}^{n} |\rho_i^w - \frac{2m}{n}| \tag{2.3}$$

where ρ_i^w is the laplacian eigenvalue of weighted Topological matrix M^w.

Theorem 2.4. *If G is a simple, connected graph of M, then*

$$\lambda_1(G) \leq \frac{2e}{d_1}$$

where d_1 is the degree of G.

Proof. Let multiply the topological matrix with the diagonal matrix and the inverse of diagonal matrix. Let show this multiplication by $D(G)^{-1}M(G)D(G)$. Let consider an eigenvector of $D(G)^{-1}M(G)D(G)$ and this eigenvector be $X = (x_1, x_2, \ldots, x_n)^T$. Let one eigencomponent $x_i = 1$ and the other eigencomponent $0 < x_k \leq 1$ for every k. Let $x_j = max_k(x_k : v_i v_k \in E, i \sim k)$. It is known that

$(D(G)^{-1}M(G)D(G))X = \lambda_1(G)X$. It is implies that $\lambda_1(G)x_i = \sum_k (m_{1k}\frac{d_k}{d_1})x_k$. Since $(1, k) \in L$ then, $\lambda_1(G)x_i = \frac{1}{d_1}\sum_k (d_k)x_k$. It is well known that $\sum_{i=1}^n d_i = 2e$. Thus, $\lambda_1(G) \leq \frac{2e}{d_1}$. From (1.3), the inequality results that $\lambda_1(G) \leq \frac{2e}{d_1}$. $\square$

Corollary 2.5. *Let G has n vertices and m edges. Let $\bar{G}$ be the complement of a graph G. If G and $\bar{G}$ be connected non-singular graphs of M then,*

$$\lambda_1(G) - \lambda_1(\bar{G}) \leq \frac{(n-1)(2e - nd_1)}{(n-1-d_1)d_1}.$$

Please cite this article as: G. Kaya Gök, On the topological matrix and topological indices, AKCE International Journal of Graphs and Combinatorics (2019), https://doi.org/10.1016/j.akcej.2019.07.001.

Proof. Using Theorem 2.4,

$$\lambda_1(G)-\lambda_1(\bar{G}) \le \frac{2e}{d_1}-\frac{2\bar{e}}{n-1-d_1}.$$

Since, $2e+2\bar{e}=n(n-1)$, then

$$\lambda_1(G)-\lambda_1(\bar{G}) \le \frac{(n-1)(2e-nd_1)}{(n-1-d_1)d_1}. \quad \square$$

Theorem 2.6. *Let G be a simple, connected graph with m edges and $ME(G)$ be a Topological energy of G then,*

$$|ME(G)+ME(G-e)| \le \sqrt{4m+4\sqrt{m(m-\delta)}}.$$

Proof. Let λ_i be $i-th$ eigenvalue of $M(G)$. Define $\lambda_i' = \lambda_i(G-e)$. It is known that $ME(G)=\sum_{i=1}^{n}|\lambda_i|$. Since $\sum_{i=1}^{n}(\lambda_i)^2 = 2m$ and $\sum_{i=1}^{n}(\lambda_i')^2 = 2(m-\delta)$ then,

$$\sum_{i=2}^{n}(|\lambda_i|+|\lambda_i'|)^2 \le 2m-(\lambda_1)^2+2m-2\delta-(\lambda_i)^2 \tag{2.4}$$

$$+2\sqrt{(2m-(\lambda_1)^2)(2m-2\delta-(\lambda_i)^2)} \tag{2.5}$$

$$\le 4m-\frac{4m^2}{n^2}+\frac{4m^2-4m\delta-4\delta^2}{n^2-2n+1}+2\sqrt{(2m-\frac{4m^2}{n^2})(2m-2\delta-\frac{4m^2}{n^2})}. \tag{2.6}$$

This means that,

$$ME(G)+ME(G-e) \le |\lambda_1|+|\lambda_1'|+\sum_{i=2}^{n}|\lambda_i|+\sum_{i=2}^{n}|\lambda_i'| \tag{2.7}$$

$$\le |\lambda_1|+|\lambda_1'| \tag{2.8}$$

$$+\sqrt{4m-\frac{4m^2}{n^2}+\frac{4m^2-4m\delta-4\delta^2}{n^2-2n+1}+2\sqrt{(2m-\frac{4m^2}{n^2})(2m-2\delta-\frac{4m^2}{n^2})}}. \tag{2.9}$$

Since $|\lambda_1| \ge \frac{2m}{n}$ and $|\lambda_1'| \ge \frac{2(m-\delta)}{n-1}$, the inequality turns into

$$ME(G)+ME(G-e) \le \frac{2m}{n}+2\frac{(m-\delta)}{n-1} \tag{2.10}$$

$$+\sqrt{4m-\frac{4m^2}{n^2}+\frac{4m^2-4m\delta-4\delta^2}{n^2-2n+1}+2\sqrt{(2m-\frac{4m^2}{n^2})(2m-2\delta-\frac{4m^2}{n^2})}}. \tag{2.11}$$

It is constructed a sequence $G_n (n \ge 2)$ of graphs to complete the argument such that $|ME(G_n)+ME(G_n-e)| \longrightarrow \sqrt{4m+4\sqrt{m(m-\delta)}}$. This completes argument. $\square$

Theorem 2.7. *Let G_w be a weighted graph with n vertices and m edges. Let $\rho_1^w \ge \rho_2^w \ge \cdots \ge \rho_{k-1}^w \ge \rho_k^w (k \le n)$ be k non-zero laplacian eigenvalues of the weighted Topological matrix and $MLE(G_w)$ be a Topological Laplacian energy of G_w then,*

$$|MLE(G_w)| \ge \sqrt{(\frac{2m(n-1)}{n})^2 k-\alpha(k)|\rho_1^w-\rho_k^w|^2}.$$

where $\alpha(k)=k[\frac{k}{2}]([1-\frac{1}{k}][\frac{k}{2}])$ while $[x]$ denotes integer part of a real number x.

Proof.

If $MLE(G_w)$ is a weighted Topological Laplacian energy of G_w then, $MLE(G_w)=\sum_{i=1}^{k}|\rho_i^w-\frac{2m}{n}|$ and

$$\sum_{i=1}^{k}|\rho_i^w-\frac{2m}{n}|^2=\sum_{i=1}^{n}|\rho_i^w-\frac{2m}{n}|^2=\sum_{i=1}^{n}(\rho_i^w-\frac{2m}{n})^2. \tag{2.12}$$

Seeing that,

$$\sum_{i=1}^{n}(\rho_i^w - \frac{2m}{n})^2 = \sum_{i=1}^{n}((\rho_i^w)^2 - \frac{4m}{n}\rho_i^w + \frac{4m^2}{n^2}) \tag{2.13}$$

$$=(2m)^2 - \frac{8m^2}{n} + \frac{4m^2}{n^2}. \tag{2.14}$$

It requires that,

$$\sum_{i=1}^{k}|\rho_i^w - \frac{2m}{n}|^2 = 4m^2 - \frac{8m^2}{n} + \frac{4m^2}{n^2}. \tag{2.15}$$

Setting $a_i = |\rho_i^w - \frac{2m}{n}|$, $b_i = |\rho_i^w - \frac{2m}{n}|$, $a = b = |\rho_k^w - \frac{2m}{n}|$ and $A = B = |\rho_1^w - \frac{2m}{n}|$, $i = 1, 2, \ldots, k$. Lemma 1.1 becomes

$$|k\sum_{i=1}^{k}|\rho_i^w - \frac{2m}{n}|^2 - (\sum_{i=1}^{k}|\rho_i^w - \frac{2m}{n}|)^2| \le \alpha(k)(|\rho_1^w - \frac{2m}{n}| - |\rho_k^w - \frac{2m}{n}|)^2. \tag{2.16}$$

Thus, the inequality transforms into

$$(4m^2 - \frac{8m^2}{n} + \frac{4m^2}{n^2})k - (MLE(G_w))^2 \le \alpha(k)|\rho_1^w - \rho_k^w|^2. \tag{2.17}$$

Hence,

$$|MLE(G_w)| \ge \sqrt{(\frac{2m(n-1)}{n})^2 k - \alpha(k)|\rho_1^w - \rho_k^w|^2}. \quad \square$$

2.2. *Some notes on the topological indices*

In this section, some results are given for Topological indices of graph operations. These results are obtained in terms of Topological indices and in fact, the bounds are tight. Also, upper bounds on the some topological indices are determined involving just the forgotten topological index, the Randić index, the ABC index, the Harmonic index and the edges.

Theorem 2.8. *Let G and H be two simple graph and E_G, E_H be edge sets, V_G, V_H be vertex sets, respectively. Then,*

(1) $TI(G+H) = (-1)^{V^+-1}(V^+-1)2^{V^+}$.
(2) $TI(G\cup H) = (-1)^{V^\cup-1}(V^\cup-1)2^{V^\cup}$.
(3) $TI(G\cap H) = (-1)^{V^\cap-1}(V^\cap-1)2^{V^\cap}$.

where $|V_G + V_H| = V^+$, $|V_G \cup V_H| = V^\cup$ and $|V_G \cap V_H| = V^\cap$.

Proof. (1) Let E_G and E_H be edge sets, V_G, V_H be vertex sets, $V = V_G \cup V_H$, $E = E_G \cup E_H \cup \{(u_G, u_H) \mid u_G \in V(G), u_H \in V(H)\}$. It is obvious from the definition,

$$TI(G+H) = (-1)^{V^+-1}(V^+-1)2^{V^+}.$$

(2) Let denote $G = G\times H$ edge sets. To define the product $G\times H$ of two graphs think any two points $u = (u_G, u_H)$ and $v = (v_G, v_H)$ and $u, v \in V = V_G \times V_H$. Hence, u and v are adjacent in $G\times H$ whenever [$u_G = v_G$ and u_H adjacent to v_H] or [$u_H = v_H$ and u_G adjacent to v_G]. Then,

$$TI(G\cup H) = (-1)^{V^\cup-1}(V^\cup-1)2^{V^\cup}.$$

(3) If two graphs G and H have at least one common vertex then their intersection will be a graph such that $V(G\cap H) = V(G)\cap V(H)$ and $E(G\cap H) = E(G)\cap E(H)$. Hence,

$$TI(G\cap H) = (-1)^{V^\cap-1}(V^\cap-1)2^{V^\cap}. \quad \square$$

Please cite this article as: G. Kaya Gök, On the topological matrix and topological indices, AKCE International Journal of Graphs and Combinatorics (2019), https://doi.org/10.1016/j.akcej.2019.07.001.

Theorem 2.9. *Let G and H be two simple graphs. If G and H are isomorf then topological indices of G and H are equal.*

Proof. Let V_G and V_H be vertex sets of G and H, respectively. Let E_G and E_H be edge sets of G and H, respectively. Since G and H are isomorf graphs then $V_G = V_H$ and $E_G = E_H$. Thus, the adjacency matrix and the distance matrix of these graphs are equal that is ; $A(G) = A(H)$ and $D(G) = D(H)$. It is seen that $TI(G) = TI(H)$. □

Furthermore, some formulas for Topological index of popular graphs are obtained, directly. Let K_n, P_n, C_n and S_n show the n-vertex complete graph, path, cycle and star graph, respectively.

Corollary 2.10.
Let G be a simple connected graph with n vertices. Then,
(1) For $2 \le n \le 6$,

$$TI(K_n) = TI(C_n) = TI(S_n).$$

(2) For $2 \le n \le 3$,

$$TI(K_n) = TI(C_n) = TI(S_n) = TI(P_n).$$

(3) For $n \ge 2$,

$$TI(K_n) = TI(S_n) = (-1)^{n-1}(n-1)2^n.$$

Theorem 2.11. *Let G be a connected graph with m edges, then*

$$4H_{-1}(G) - F(G) - 2R^2_{-1}(G) \le 2\sqrt{m(F(G) + 2R^2_{-1}(G))}.$$

Proof. Minkowski inequality gives

$$\Big(\sum_{v_iv_j \in E(G)} (d_i + d_j + 1)^2\Big)^{\frac{1}{2}} \le \Big(\sum_{v_iv_j \in E(G)} (d_i + d_j)^2\Big)^{\frac{1}{2}} + \Big(\sum_{v_iv_j \in E(G)} 1\Big)^{\frac{1}{2}}.$$

In the Bernoulli inequality, $(1+x)^\alpha$ is greater than or equal to $1+\alpha x$ for $x \ge -1$. It gives $(1+d_i+d_j)^2 \ge 1+2(d_i+d_j)$. Hence, $\sum_{v_iv_j \in E(G)}(1 + 2(d_i + d_j)) = m + 2\sum_{v_iv_j \in E(G)}(d_i + d_j)$. It follows that,

$$\Big(m + 2\sum_{v_iv_j \in E(G)} (d_i + d_j)\Big)^{\frac{1}{2}} \le \Big(\sum_{v_iv_j \in E(G)} (d_i + d_j)^2\Big)^{\frac{1}{2}} + \sqrt{m}.$$

From the above inequality,

$$m + 2\sum_{v_iv_j \in E(G)} (d_i + d_j) \le \sum_{v_iv_j \in E(G)} (d_i + d_j)^2 + m + 2\sqrt{m\sum_{v_iv_j \in E(G)} (d_i + d_j)^2}.$$

It is known that, $\sum_{v_iv_j \in E(G)}(d_i^2 + d_j^2) = \sum_{v_iv_j \in E(G)}((d_i + d_j)^2 - 2d_id_j)$. By expanding the terms under summation,

$$\sum_{v_iv_j \in E(G)} (d_i + d_j)^2 = F(G) + 2R^2_{-1}(G).$$

It is concluded that

$$2\sum_{v_iv_j \in E(G)} (d_i + d_j) \le F(G) + 2R^2_{-1}(G) + 2\sqrt{m(F(G) + 2R^2_{-1}(G))}.$$

Hence,

$$4H_{-1}(G) - F(G) - 2R^2_{-1}(G) \le 2\sqrt{m(F(G) + 2R^2_{-1}(G))}. \quad \square$$

Please cite this article as: G. Kaya Gök, On the topological matrix and topological indices, AKCE International Journal of Graphs and Combinatorics (2019), https://doi.org/10.1016/j.akcej.2019.07.001.

Theorem 2.12. *Let G be a connected regular graph with m edges, then*

$$\frac{ABC(G)}{R(G)} \leq \sqrt{2(H_{-1}(G) - m)}.$$

Proof. By the definition of ABC index,

$$\sum_{v_i v_j \in E(G)} \sqrt{\frac{d_i + d_j - 2}{d_i d_j}} \geq \sqrt{\sum_{v_i v_j \in E(G)} \frac{d_i + d_j - 2}{d_i d_j}}.$$

It follows that,

$$ABC^2(G) \geq \frac{\sum_{v_i v_j \in E(G)} d_i + d_j - 2}{\sum_{v_i v_j \in E(G) d_i d_j}}.$$

Indeed, it is seen that

$$ABC^2(G) \geq \frac{2H_{-1}(G) - 2m}{R^{-2}(G)}.$$

Hence,

$$\frac{ABC(G)}{R(G)} \geq \sqrt{2(H_{-1}(G) - m)}. \quad \square$$

3. Conclusion

Topological indices is the corner stone of the theories in this paper. Therefore, new inequalities and relations are obtained for topological structures throughout this paper. Firstly, the weighted graphs are examined and some special bounds are formed. In continuation, to gain a more intuitive understanding of the topological index of composite graphs and original graphs, some relations are found. Lastly, some relationships are stated between different topological indices.

Acknowledgments

The author would like thank for the valuable suggestions of referees.

References

[1] R.B. Bapat, Graphs and Matrices, Indian Statistical Institute, New Delhi 110016, India, 2010.
[2] K.C. Das, S.A. Mojalal, On energy and Laplacian energy of graphs, Electron. J. Linear Algebra 31 (2016) 167–186.
[3] I. Gutman, Degree-based topological indices, Croat. Chem. Acta 86 (2013) 351–361.
[4] I. Gutman, B. Furtula (Eds.), Recent Results in the Theory of RandiĆ Index, Univ., Kragujevac, 2008.
[5] X. Li, Y. Shi, A survey on the Randić index, MATCH Commun. Math. Comput. Chem. 59 (2008) 127–156.
[6] S. Fajtlowicz, On conjectures of Graffiti–II, Congr. Number 60 (1987) 187–197.
[7] B. Furtula, I. Gutman, A forgotten topological index, J. Math. Chem. 53 (2015) 1184–1190.
[8] I. Gutman, On the origin of two degree-based topological indices, Bull. Acad. Serb. Sci. Arts 146 (2014) 39–52.
[9] R.A. Horn, C.R. Johnson, Matrix Analysis, Cambridge University Press, New York, 1985.
[10] K.C. Das, P. Kumar, Some new bounds on the spectral radius of graphs, Discrete Math. 281 (2004) 149–161.
[11] I.Z. Milovanović, .E.I. Milovanović, A. Zakić, A short note on graph energy, MATCH Commun. Math. Comput. Chem. 72 (2014) 179–182.
[12] A. Aslam, S. Ahmad, M.A. Binyamin, W. Gao, Calculating topological indices of certain OTIS interconnection networks, Open Chem. 17 (2019) 220–228.
[13] A. Aslam, M.F. Nadeem, Z. Zahid, S. Zafar, W. Gao, Computing certain topological indices of the line graphs of subdivision graphs of some rooted product graphs, Mathematics 7 (5) (2019) 393.
[14] S. Büyükköse, G. Kaya Gök, Graf Teoriye Giris, Nobel Akademik Yayıncılık Egitim Danısmanlık Tic.Ltd.Sti, Ankara, 2018.
[15] W. Gao, Z. Iqbal, M. Ishaq, A. Aslam, R. Sarfraz, Topological aspects of dendrimers via distance based descriptors, IEEE Access 7 (1) (2019) 35619–35630.
[16] C. Vasudev, Graph Theory with Applications, Vol. 1, New Age International Publishers, New Delhi/ Indian, 2006, pp. 4–5, 21, 56–57.

Please cite this article as: G. Kaya Gök, On the topological matrix and topological indices, AKCE International Journal of Graphs and Combinatorics (2019), https://doi.org/10.1016/j.akcej.2019.07.001.