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On the radio number for corona of paths and cycles

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Abstract

Radio k-coloring of graphs is one of the variations of frequency assignment problem. For a simple connected graph G and a positive integer $k \leqslant diam(G)$, a radio k-coloring is an assignment f of positive integers (colors) to the vertices of G such that for every pair of distinct vertices u and v of G, the difference between their colors is at least $1 + k - d(u, v)$. The maximum color assigned by f is called its span, denoted by $rc_k(f)$. The radio k-chromatic number $rc_k(G)$ of G is $\min\{rc_k(f) \colon f$ is a radio k-coloring of $G\}$. If d is the diameter of G, then a radio d-coloring is referred as a radio coloring and the radio d-chromatic number as the radio number, denoted by $rn(G)$, of G. The corona $G \odot H$ of two graphs G and H is the graph obtained by taking one copy of G and $|V(G)|$ copies of H, and joining each and every vertex of the i^{th} copy of H with the i^{th} vertex of G by an edge. In this paper, for path P_n and cycle C_m, $m \geq 5$, we determine $rn(P_n \odot C_m)$ when n is even, and give an upper bound for the same when n is odd. Also, for $m \geq 4$, we determine the radio number of $P_n \odot P_m$ when n is even, and give both upper and lower bounds for $rn(P_n \odot P_m)$ when n is odd.

Keywords: Radio k-coloring; Radio k-chromatic number; Radio coloring; Radio number

1. Introduction

The problem of obtaining an assignment of frequencies to transmitters in some optimal manner is said to be Frequency Assignment Problem (FAP). Due to rapid growth of wireless networks and to the relatively scarce radio spectrum, the importance of FAP is growing significantly. One of the FAPs is the problem of assigning radio frequencies to transmitters at different locations without causing interference and reducing maximum frequency used. Hale [1] has modeled FAP as graph labeling problem as follows. Transmitters are represented by vertices of a graph and those vertices corresponding to very close transmitters are joined by edges. Maximum interference occurs among transmitters corresponding to adjacent vertices. Now, assigning frequencies to transmitters is same as assigning positive integers (colors) to vertices.

Motivated by channel assignment to radio stations, Chartrand et al. [2] have introduced radio k-coloring of graphs. For a simple connected graph G and an integer k, $1 \leqslant k \leqslant diam(G)$, a radio k-coloring of G is an assignment f of positive integers to the vertices of G such that $|f(u) - f(v)| \geqslant 1 + k - d(u, v)$ for all distinct vertices u and v of

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G. The maximum color assigned by f is called the span of f, denoted by $rc_k(f)$. The radio k-chromatic number $rc_k(G)$ of G is the minimum of spans over all radio k-colorings of G. A radio k-coloring f of G with span $rc_k(G)$ is referred as a minimal radio k-coloring of G. For some special values of k, there are some special names of radio k-colorings and as well as radio k-chromatic numbers in the literature. A radio 1-coloring is a proper coloring of G and $rc_1(G) = \chi(G)$. If $k = d$, the diameter of G, then a radio d-coloring is called a radio coloring and the radio d-chromatic number is said to be the radio number. Radio $(d-1)$-coloring and radio $(d-2)$-coloring are called antipodal coloring and nearly antipodal coloring respectively. The radio $(d-1)$-chromatic number and the radio $(d-2)$-chromatic number are called the antipodal number and the nearly antipodal number respectively.

If we see the literature of radio k-coloring for operation between graphs, it is studied only for Cartesian product of graphs. Even though, radio k-coloring is defined for $k \leqslant diam(G)$, some authors have studied it for $k \geqslant diam(G)$ as it is useful in finding radio k-chromatic number of larger graphs. Kchikech et al. [3] have given lower and upper bounds for $rc_k(P_n \Box P_n)$ when $k \geqslant 2n-3$. Also, they have given an upper bound for radio k-chromatic number of Cartesian product of two arbitrary graphs. Kim et al. [4] have determined the radio number of Cartesian product of path P_n and complete graph K_m as $\frac{mn^2-2n+4}{2}$ if n is even and $\frac{mn^2-2n+m+4}{2}$ if n is odd. Ajayi and Adefokun [5] have given bounds for the radio number for Cartesian product of path and star. Morris-Rivera [6] has determined that $rn(C_n \Box C_n)$ is $2p^3+4p^2-p$ if $n=2p$ and is $2p^3+4p^2+2p+1$ if $n=2p+1$. Saha and Panigrahi [7] found the exact value of radio number of $C_m \Box C_n$, toroidal grid, when at least one of m and n is even. Kola and Panigrahi [8] have given a lower bound for $rc_k(G)$ for an arbitrary graph G and using this lower bound, they have given a lower bound for radio k-chromatic number of prism graph $C_n \Box P_m$. Further, they proved this lower bound is exact for the radio number of $C_n \Box P_2$, when $n \equiv 1 \ mod\ 4$ and $n \equiv 2 \ mod\ 8$. The corona $G \odot H$ of two graphs G and H is the graph obtained by taking one copy of G and $|V(G)|$ copies of H, and joining each and every vertex of the ith copy of H with the ith vertex of G by an edge. It is easy to see that $G \odot H \ncong H \odot G$ if $G \ncong H$. Also, $diam(G \odot H) = diam(G) + 2$.

In this article, for $m \geq 5$, we determine the radio number for $P_n \odot C_m$ when n is even and we give lower and upper bounds for $rn(P_n \odot C_m)$ when n is odd. Also, for $m \geq 4$, we determine $rn(P_n \odot P_m)$ when n is even and give lower and upper bounds for the same when n is odd.

2. Results

We use the following definition and lemma to get the span of a radio coloring.

Definition 2.1. For a graph G of order n and a radio k-coloring f of G, let $x_1, x_2, x_3, \ldots, x_n$ be an ordering of vertices of G such that $f(x_i) \leq f(x_{i+1})$, $1 \leq i \leq n-1$. We define $\epsilon_i = f(x_i) - f(x_{i-1}) - (1 + k - d(x_i, x_{i-1}))$, $2 \leq i \leq n$.

Lemma 2.2. *For any radio k-coloring f of a graph G of order n,*

$$rc_k(f) = (n-1)(1+k) - \sum_{i=2}^{n} d(x_i, x_{i-1}) + \sum_{i=2}^{n} \epsilon_i + 1$$

where x_is are as given in Definition 2.1.

Proof.

$$\begin{aligned} f(x_n) - f(x_1) &= \sum_{i=2}^{n} [f(x_i) - f(x_{i-1})] \\ &= \sum_{i=2}^{n} [1 + k - d(x_i, x_{i-1}) + \epsilon_i] \\ &= (n-1)(1+k) - \sum_{i=2}^{n} d(x_i, x_{i-1}) + \sum_{i=2}^{n} \epsilon_i. \end{aligned}$$

Since $f(x_1) = 1$, $rc_k(f) = f(x_n) = (n-1)(1+k) - \sum_{i=2}^{n} d(x_i, x_{i-1}) + \sum_{i=2}^{n} \epsilon_i + 1$. ■

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To get a lower bound for the radio number of the graph under discourse, we use the lower bound technique for radio k-coloring given by Das et al. [9]. For a subset S of the vertex set of a graph G, let $N(S)$ be the set of all vertices of G adjacent to at least one vertex of S.

Theorem 2.3 ([9]). *If f is a radio k-coloring of a graph G, then*

$$rc_k(f) \geq |D_k| - 2p + 2\sum_{i=0}^{p} |L_i|(p-i) + \alpha + \beta,$$

where D_k and L_i's are defined as follows. If $k = 2p+1$, then $L_0 = V(C)$, where C is a maximal clique in G. If $k = 2p$, then $L_0 = \{v\}$, where v is a vertex of G. Recursively define $L_{i+1} = N(L_i)\backslash(L_0 \cup L_1 \cup \cdots \cup L_i)$ for $i = 0, 1, 2, \ldots, p-1$. Let $D_k = L_0 \cup L_1 \cup \cdots \cup L_p$. The minimum and the maximum colored vertices among the vertices of D_k are in L_α and L_β respectively.

As a direct consequence of Theorem 2.3, we have the theorem below.

Theorem 2.4. *For any graph G and $1 \leq k \leq diam(G)$, we have*

$$rc_k(G) \geq \begin{cases} |D_k| - 2p + 2\sum_{i=0}^{p} |L_i|(p-i) & if\ k = 2p+1, \\ |D_k| - 2p + 2\sum_{i=0}^{p} |L_i|(p-i) + 1 & if\ k = 2p. \end{cases}$$

Following theorem gives an upper bound for the radio number of corona of path and cycle $P_n \odot C_m$. We refer the condition in the definition of radio k-coloring as radio k-coloring condition.

Theorem 2.5. *For $m \geq 5$,*

$$rn(P_n \odot C_m) \leq \begin{cases} (2m+2)p^2 + 2p & if\ n = 2p, \\ (2m+2)p^2 + (2m+4)p + m + 2 + (\frac{m}{2} - 1)p, & if\ n = 2p+1\ and\ m\ is\ even, \\ (2m+2)p^2 + (2m+4)p + m + 2 + (\frac{m-1}{2})p, & if\ n = 2p+1\ and\ m\ is\ odd. \end{cases}$$

Proof. To give an upper bound for the radio number, we define a radio coloring of $P_n \odot C_m$. Let $v_1v_2v_3 \ldots v_n$ be the path P_n and for $i = 1, 2, 3, \ldots, n$, let C_m^i be the copy of C_m in $P_n \odot C_m$ corresponding to the vertex v_i of P_n.

Case 1: Let $n = 2p$. To give a radio coloring, we first order the vertices of $P_n \odot C_m$ as follows. Let $x_1 = v_p$. We label the vertices of C_m^{n+1-i}, $i = 1, 2, 3, \ldots, p$, as $x_2, x_4, x_6, \ldots, x_{mn}$ starting from the vertices of C_m^n and once all vertices of C_m^n are labeled, we label the vertices of C_m^{n-1} and so on, in such a way that $d(x_i, x_{i+2}) > 1$ for $i = 2, 4, 6, \ldots, mn-2$. We label the vertices $v_n, v_{n-1}, v_{n-2}, \ldots, v_{p+1}$ as $x_{mn+2}, x_{mn+4}, x_{mn+6}, \ldots, x_{mn+n}$ respectively. Now, we label the vertices of C_m^{p+1-i}, $i = 1, 2, 3, \ldots, p$, as $x_3, x_5, x_7, \ldots, x_{mn+1}$ starting from the vertices of C_m^p and once all vertices of C_m^p are labeled, we label the vertices of C_m^{p-1} and so on, in such a way that $d(x_i, x_{i+2}) > 1$ for $i = 3, 5, 7, \ldots, mn-1$. Finally, we label the vertices $v_{p-1}, v_{p-2}, v_{p-3}, \ldots, v_1$ as $x_{mn+3}, x_{mn+5}, x_{mn+7}, \ldots, x_{mn+n-1}$ respectively.

Now, we define a coloring f by $f(x_1) = 1$ and for $i = 2, 3, 4, \ldots, mn+n$, $f(x_i) = f(x_{i-1}) + 1 + (2p+1) - d(x_i, x_{i-1})$. Next, we show that f is a radio coloring of $P_n \odot C_m$ with span $(2m+2)p^2 + 2p$. By definition of f, x_i satisfies radio coloring condition with x_{i+1}. Also it is easy to see that $f(x_i) - f(x_{i+3}) \geq 2p+1 = 1+(2p+1)-1 \geq 1+(2p+1) - d(x_i, x_{i+3})$. So, it remains to check the radio coloring condition for x_i and x_{i+2}. Suppose that x_i and x_{i+2} are on the same copy of C_m. Then by the ordering, $d(x_i, x_{i+2}) = 2$ and $d(x_i, x_{i+1}) = d(x_{i+1}, x_{i+2}) = p+2$. Therefore,

$$\begin{aligned} f(x_{i+2}) - f(x_i) &= f(x_{i+2}) - f(x_{i+1}) + f(x_{i+1}) - f(x_i) \\ &= p + p \\ &= 1 + (2p+1) - d(x_i, x_{i+2}). \end{aligned}$$

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Suppose that x_i and x_{i+2} are on different copies of C_m. Then $d(x_i, x_{i+2}) = 3$ and one of $d(x_i, x_{i+1})$ and $d(x_{i+1}, x_{i+2})$ is $p+2$ and the other is $p+1$. Therefore,

$$\begin{aligned} f(x_{i+2}) - f(x_i) &= f(x_{i+2}) - f(x_{i+1}) + f(x_{i+1}) - f(x_i) \\ &= 2(1 + (2p+1)) - d(x_i, x_{i+1}) - d(x_{i+1}, x_{i+2}) \\ &= 2p + 1 \\ &> 1 + (2p+1) - d(x_i, x_{i+2}). \end{aligned}$$

Suppose that x_i is on a copy of C_m and x_{i+2} is on P_n. Then $i = mn$ or $i = mn+1$. If $i = mn$, then $d(x_{mn}, x_{mn+2}) = p$, $d(x_{mn}, x_{mn+1}) = p+2$ and $d(x_{mn+1}, x_{mn+2}) = 2p$. Therefore,

$$\begin{aligned} f(x_{mn+2}) - f(x_{mn}) &= f(x_{mn+2}) - f(x_{mn+1}) + f(x_{mn+1}) - f(x_{mn}) \\ &= p + 2 \\ &= 1 + (2p+1) - d(x_{mn}, x_{mn+2}). \end{aligned}$$

If $i = mn+1$, then $d(x_{mn+1}, x_{mn+3}) = p - 1$, $d(x_{mn+1}, x_{mn+2}) = 2p$ and $d(x_{mn+2}, x_{mn+3}) = p+1$. Therefore,

$$\begin{aligned} f(x_{mn+3}) - f(x_{mn+1}) &= f(x_{mn+3}) - f(x_{mn+2}) + f(x_{mn+2}) - f(x_{mn+1}) \\ &= p + 3 \\ &= 1 + (2p+1) - d(x_{mn+1}, x_{mn+3}). \end{aligned}$$

Suppose that x_i is on path P_n and x_{i+2} is on a copy of C_m. Then $i = 1$, $d(x_1, x_3) = 1$, $d(x_1, x_2) = p+1$ and $d(x_2, x_3) = p+2$. Therefore,

$$\begin{aligned} f(x_3) - f(x_1) &= f(x_3) - f(x_2) + f(x_2) - f(x_1) \\ &= p + 1 + p \\ &= 1 + (2p+1) - d(x_1, x_3). \end{aligned}$$

Suppose both x_i and x_{i+2} are on P_n. Then $d(x_i, x_{i+2}) = 1$ and one of $d(x_i, x_{i+1})$ and $d(x_{i+1}, x_{i+2})$ is $p+1$ and the other is p. Therefore,

$$\begin{aligned} f(x_{i+2}) - f(x_i) &= f(x_{i+2}) - f(x_{i+1}) + f(x_{i+1}) - f(x_i) \\ &= 2(1 + (2p+1)) - d(x_i, x_{i+1}) - d(x_{i+1}, x_{i+2}) \\ &= 2p + 3 \\ &> 1 + (2p+1) - d(x_i, x_{i+2}). \end{aligned}$$

Therefore f is a radio coloring of $P_n \odot C_m$. From the definition of f, we have $\sum_{i=2}^{mn+n} \epsilon_i = 0$. Since the sequence of distances $\{d(x_i, x_{i-1})\}_{i=2}^{mn+n}$ is such that the $2m$ terms $p+1, p+2, p+2, \ldots, p+2$ repeated p times, that is, up to $d(x_{mn+1}, x_{mn})$, $d(x_{mn+2}, x_{mn+1}) = 2p$ and an alternating sequence of $p+1$ and p from $d(x_{mn+3}, x_{mn+2})$ to $d(x_{mn+n}, x_{mn+n-1})$, we have

$$\begin{aligned} \sum_{i=2}^{mn+n} d(x_i, x_{i-1}) &= \sum_{i=2}^{mn+1} d(x_i, x_{i-1}) + d(x_{mn+2}, x_{mn+1}) + \sum_{i=mn+3}^{mn+n} d(x_i, x_{i-1}) \\ &= ((p+2)(2m-1)p + (p+1)p) + 2p + ((p+1)(p-1) + p(p-1)) \\ &= (2m+2)p^2 + 4pm - 1. \end{aligned}$$

Now, by Lemma 2.2, $rn(f) = (mn + n - 1)(2p + 1 + 1) - ((2m+2)p^2 + 4pm - 1) + 1 = (2m+2)p^2 + 2p$.

Case 2: Let $n = 2p + 1$ and m be even. As in Case 1, here also first we order the vertices of $P_n \odot C_m$. Let $x_1 = v_{p+1}$. We label $x_2, x_4, x_6, \ldots, x_{mn}$ as in Case 1 starting from the vertices of C_m^n ending after labeling $\frac{m}{2}$ vertices of C_m^{p+1}. We label the vertices $v_{p+1}, v_{p+2}, v_{p+3}, \ldots, v_n$ as $x_{mn+2}, x_{mn+4}, x_{mn+6}, \ldots, x_{mn+n-1}$ respectively. Now, we label $x_3, x_5, x_7, \ldots, x_{mn+1}$ as in Case 1 starting from the vertices of C_m^{p+1} ending after labeling all the vertices of C_m^1. Finally, we label the vertices $v_1, v_2, v_3, \ldots, v_p$ as $x_{mn+3}, x_{mn+5}, x_{mn+7}, \ldots, x_{mn+n}$ respectively.

Now, we define a coloring f by $f(x_1) = 1$ and for $i = 2, 3, 4, \ldots, mn + n$,

$$f(x_i) = \begin{cases} f(x_{i-1}) + 1 + (2p+2) - d(x_i, x_{i-1}) + 1, & \text{if } i \text{ is even and } d(x_i, x_{i-1}) = p+3, \\ f(x_{i-1}) + 1 + (2p+2) - d(x_i, x_{i-1}), & \text{otherwise.} \end{cases}$$

Checking the radio coloring condition for x_i and x_{i+2} is similar to the previous case except for the case that both x_i and x_{i+2} are on same copy of C_m. Then we have either $d(x_{i+1}, x_i) = d(x_{i+1}, x_{i+2}) = p+2$ or $d(x_{i+1}, x_i) = d(x_{i+1}, x_{i+2}) = p + 3$. As in the previous case, we can check the condition if $d(x_{i+1}, x_i) = d(x_{i+1}, x_{i+2}) = p + 2$. Suppose that $d(x_{i+1}, x_i) = d(x_{i+1}, x_{i+2}) = p+3$. Since only one of $i+1$ and $i+2$ is even, by the definition of f, we have either $f(x_{i+1}) = f(x_i)+1+(2p+2)-d(x_{i+1}, x_i)+1$ or $f(x_{i+2}) = f(x_{i+1})+1+(2p+2)-d(x_{i+2}, x_{i+1})+1$. Therefore,

$$\begin{aligned} f(x_{i+2}) - f(x_i) &= f(x_{i+2}) - f(x_{i+1}) + f(x_{i+1}) - f(x_i) \\ &= 2(1 + (2p+2)) - d(x_i, x_{i+1}) - d(x_{i+1}, x_{i+2}) + 1 \\ &= 2p + 1 \\ &= 1 + (2p+2) - d(x_i, x_{i+2}). \end{aligned}$$

Hence f is a radio coloring. By the definition of f, we get $\sum_{i=2}^{mn+n} \epsilon_i = (\frac{m}{2} - 1)p$ and by the ordering of vertices, we have $\sum_{i=2}^{mn+n} d(x_i, x_{i-1}) = 2mp^2 + (6m+1)p + 2m + 1$. Now, by Lemma 2.2, we have

$$\begin{aligned} rn(f) &= (mn + n - 1)(2p + 2 + 1) - (2mp^2 + (6m+1)p + 2m + 1) + \left(\frac{m}{2} - 1\right) p + 1 \\ &= (2m+2)p^2 + (2m+4)p + m + 2 + \left(\frac{m}{2} - 1\right) p. \end{aligned}$$

Case 3: Let $n = 2p + 1$ and m be odd. First we order the vertices of $P_n \odot C_m$, similar to Case 2, with some modification. We label $x_2, x_4, x_6, \ldots, x_{mn-1}$ as in Case 2 starting from the vertices of C_m^n ending after labeling $\frac{m-1}{2}$ vertices of C_m^{p+1}. We label the vertices $v_n, v_{n-1}, v_{n-2}, \ldots, v_{p+1}$ as $x_{mn+1}, x_{mn+3}, x_{mn+5}, \ldots, x_{mn+n}$ respectively. Now, we label $x_1, x_3, x_5, \ldots, x_{mn}$, starting from the vertices of C_m^{p+1} ending after labeling all the vertices of C_m^1 as in Case 1. Finally, we label the vertices $v_p, v_{p-1}, v_{p-2}, \ldots, v_1$ as $x_{mn+2}, x_{mn+4}, x_{mn+6}, \ldots, x_{mn+n-1}$ respectively.

Now, we define a coloring f by $f(x_1) = 1$ and for $i = 2, 3, 4, \ldots, mn + n$,

$$f(x_i) = \begin{cases} f(x_{i-1}) + 1 + (2p+2) - d(x_i, x_{i-1}) + 1 & \text{if } i \text{ is even and } d(x_i, x_{i-1}) = p+3, \\ f(x_{i-1}) + 1 + (2p+2) - d(x_i, x_{i-1}) & \text{otherwise.} \end{cases}$$

As in Case 2, we can show that f is a radio coloring. Using Lemma 2.2, $rn(f) = (2m+2)p^2 + (2m+4)p + m + 2 + (\frac{m-1}{2})p$. ■

Example 2.6. The three cases of Theorem 2.5 are illustrated in Figs. 1–3.

Theorem 2.7. *If $n = 2p$ and $m \geq 5$, then $rn(P_n \odot C_m) = (2m+2)p^2 + 2p$.*

Proof. To show $rn(P_n \odot C_m) \geq (2m+2)p^2 + 2p$, we use Theorem 2.4. Let $v_1 v_2 v_3 \ldots v_n$ be the path P_n. We choose $L_0 = \{v_p, v_{p+1}\}$. Then we get, $|L_i| = 2m+2$, for $1 \leq i \leq p-1$, $|L_p| = 2m$ and $|D_{2p+1}| = |V(P_n \odot C_m)| = mn+n$. Now, by Theorem 2.4, we have

$$\begin{aligned} rn(G) &\geq mn + n - 2p + 2(2p) + 2\sum_{i=1}^{p-1}(2m+2)(p-i) + 2(2m)(0) \\ &= (2m+2)p^2 + 2p. \end{aligned}$$

Therefore, $rn(P_n \odot C_m) \geq (2m+2)p^2 + 2p$. ■

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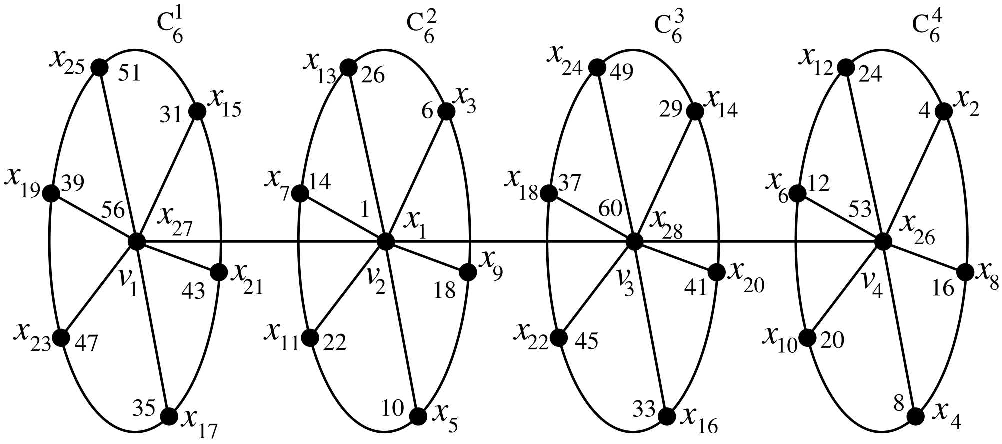

Fig. 1. The ordering vertices of $P_4 \odot C_6$ and the radio coloring of $P_4 \odot C_6$ given in Case 1 of Theorem 2.5.

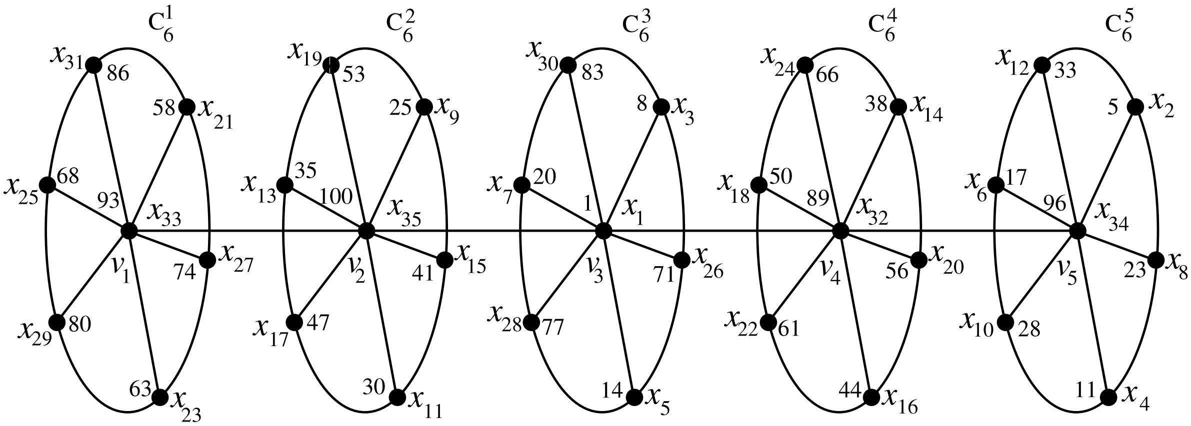

Fig. 2. The ordering vertices of $P_5 \odot C_6$ and the radio coloring of $P_5 \odot C_6$ given in Case 2 of Theorem 2.5.

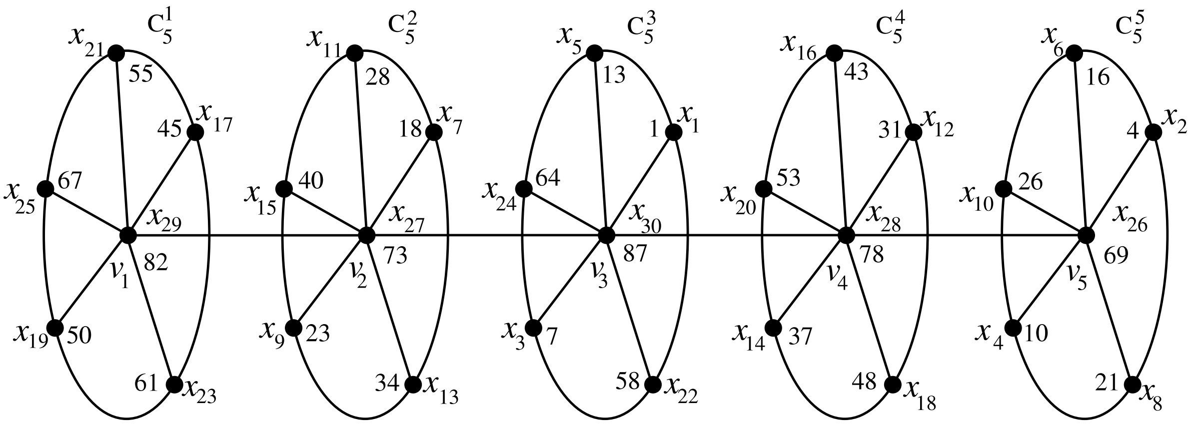

Fig. 3. The ordering vertices of $P_5 \odot C_5$ and the radio coloring of $P_5 \odot C_5$ given in Case 3 of Theorem 2.5.

Theorem 2.8. *If $n = 2p + 1$ and $m \geq 5$, then $rn(P_n \odot C_m) \geq (2m + 2)p^2 + (2m + 4)p + m + 2$.*

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Proof. Let $v_1v_2v_3 \ldots v_n$ be the path P_n. By choosing $L_0 = \{v_{p+1}\}$ and using Theorem 2.4, we get the required lower bound. ■

In the remaining part of the paper, we determine the radio number of $P_n \odot P_m$ when n is even. For n odd, we give upper and lower bounds for the same. It is easy to see that $P_n \odot P_m$ is a subgraph of $P_n \odot C_m$.

Theorem 2.9. *If $n = 2p$ and $m \geq 4$, then $rn(P_n \odot P_m) = (2m+2)p^2 + 2p$.*

Proof. Since $P_n \odot P_m$ is a subgraph of $P_n \odot C_m$, by Theorem 2.5, we have $rn(P_n \odot P_m) \leq (2m+2)p^2 + 2p$ for $m \geq 5$. For $m = 4$, we do exactly same as in Case 1 of Theorem 2.5 and get $rn(P_n \odot P_4) \leq 10p^2 + 2p$. Now, to get the lower bound for $rn(P_n \odot P_m)$, we choose L_0 same as in the proof of Theorem 2.7 and we get $rn(P_n \odot P_m) \geq (2m+2)p^2 + 2p$. ■

Following theorem gives upper and lower bounds for $rn(P_n \odot P_m)$ when n is odd.

Theorem 2.10. *If $n = 2p + 1$ and $m \geq 4$, then*

$$rn(P_n \odot P_m) \leq \begin{cases} (2m+2)p^2 + (2m+4)p + m + 2 + (\frac{m}{2} - 1)p & if\ m\ is\ even, \\ (2m+2)p^2 + (2m+4)p + m + 2 + (\frac{m-1}{2})p & if\ m\ is\ odd, \end{cases}$$

and $rn(P_n \odot P_m) \geq (2m+2)p^2 + (2m+4)p + m + 2$.

Proof. Since $P_n \odot P_m$ is a subgraph of $P_n \odot C_m$, by Theorem 2.5, we have $rn(P_n \odot P_m) \leq (2m+2)p^2 + (2m+4)p + m + 2 + (\frac{m}{2} - 1)p$ if $m \geq 6$ is even and $rn(P_n \odot P_m) \leq (2m+2)p^2 + (2m+4)p + m + 2 + (\frac{m-1}{2})p$ if $m \geq 5$ is odd. For $m = 4$, we do exactly same as in Case 2 of Theorem 2.5 and get $rn(P_n \odot P_4) \leq 10p^2 + 13p + 6$. Now, to get the lower bound for $rn(P_n \odot P_m)$, we choose L_0 same as in the proof of Theorem 2.8 and obtain $rn(P_n \odot P_m) \geq (2m+2)p^2 + (2m+4)p + m + 2$. ■

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