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Impact of Returns Policies and Group-Buying On Channel Coordination

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by

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ABSTRACT

This dissertation investigates the role of two marketing practices—returns policies and group-buying services—in improving channel coordination. The first study (presented in Chapter Two) focuses on the interaction between two types of returns policies—returns of unwanted products from consumers to retailers and returns of unsold inventory from retailers to manufacturers. Even without the right to return unsold inventory to the manufacturer, the retailers may accept returns from consumers; by doing so, they benefit from a less price-sensitive market demand, an ability to screen for high-valuation consumers, and a competitive advantage (offering a returns policy makes a retailer more attractive to consumers). From the manufacturer's perspective, accepting returns may induce the retailers to order more stock, set lower prices, generate more sales, and therefore, improves the performance of the channel. However, under some conditions (e.g., when the marginal cost of stock-outs is relatively high), this study shows that this effect disappears and the manufacturer does not accept returns from the retailer in equilibrium. The second study (presented in Chapter Three) investigates the rationale for using group-buying services vis-a-vis the traditional posted-pricing mechanism. It focuses on the behavior of consumers and explores the role of heterogeneity in their valuation for the product and cost of purchasing via group-buying in the functioning of group-buying services as a price-discrimination device. Finally, the role of group-buying services in improving channel coordination under asymmetric information is studied in Chapter Four. This analysis shows that the availability of group-buying services provides an opportunity for the manufacturer to reduce the informational rents of the retailer arising from its private information about the market condition. Interestingly, the manufacturer can avoid paying these rents and regains the first-best profitability when asymmetry in information exists regarding the relative sizes of consumer segments. In other settings (e.g., when asymmetric information exists regarding consumers' price sensitivity),

leveraging the group-buying mechanism nevertheless allows the manufacturer to design a contract that requires lower rents and improves channel coordination to some extent.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1: INTRODUCTION

Marketing channels play an important role in every business, and many manufacturers rely on retailers and wholesalers to distribute their products to consumers. Retailers, for instance, offer a variety of services, such as providing guarantees, taking returns, offering financial services, etc., that add value to the consumers' experience. Furthermore, they sometimes help develop alternative options for consumers to purchase the product; these include on-line auctions, (e.g., Ebay), name-your-own-price (e.g., Priceline), and an option that has emerged relatively recently, i.e., group-buying services (e.g., Mobshop).

However, since the distribution channel is often comprised of independent members, each with its own decision variables and motivations, conflicts tend to arise among the channel's members. The importance of understanding how best to manage distribution channels has been the focus of much research attention in the marketing, economics, and management fields for a long time (e.g., see Spengler 1950, Jeuland and Shugan 1983, McGuire and Staelin 1986, Desiraju and Moorthy 1997, among others). This immense amount of effort has provided useful insights on many aspects of the distribution channel. In particular, researchers have proposed a variety of mechanisms to improve the functioning of the channel. This work on channel coordination includes contract design (Jeuland and Shugan 1983), implicit understanding from repeated interaction (Shugan 1985), franchising (Lal 1990), quantity discounts (Ingene and Parry 1995), performance requirements (Desiraju and Moorthy 1997), cooperative advertising (Corstjens and Lal 1989), and pull price promotions (Gerstner and Hess 1995), among others.

Recently, the advent of technological innovations—especially the Internet—creates numerous opportunities for firms to develop creative means to attract and better serve more customers. Many new distribution arrangements have emerged, and most interestingly, these have the potential to improve channel coordination and raise the overall performance of the entire system. For instance, manufacturers in various industries have added an on-line direct channel to the (traditional) indirect retail channel. In this context, Chiang, Chhajed and Hess (2003) show that such a dual-channel design induces the traditional retailers to lower their prices; consequently, the severity of the standard double-marginalization problem (Spengler 1950, Jeuland and Shugan 1983) is reduced and the channel's overall profitability is improved.

In a similar vein, this dissertation extends the literature on channel coordination by analyzing the role of two marketing practices—returns policies and group-buying services—in improving the performance of the distribution channel. An overview of each of these is given below.

The study of the first topic is motivated by the observation that though accepting returns from consumers is a common practice in the retail industry, not all manufacturers accept returns of unsold inventory from the retailers. The rationale for accepting returns by the manufacturers and the retailers (from their respective *customers*) has been explored in two non-overlapping streams of research—one deals with the returns from the retailers to the manufacturers (namely, manufacturer's returns and denoted *MR*) and the other with the returns from the consumers to the retailers (namely, consumer returns and denoted *CR*, hereafter).

Consider, for instance, the literature on manufacturer's returns. It has been shown that the manufacturer's returns policy can help share the risk with the retailers, safeguard the brand name, facilitate the distribution of new product information (Pasternack 1985, Kandel 1996, and Padmanabhan and Png 1995), and perhaps more interestingly, intensify retail competition—thereby, raising the manufacturer's profitability—when market demand is uncertain (Padmanabhan and Png 1997, 2004, Wang 2004). On the other hand, research on consumer returns focuses on the retailers' motivation for accepting returns—accepting returns from consumers allows the retailers to share the risk with consumers (therefore, raising their willingness-to-pay for the product; see Chu et al. 1998), screen for high-valuation consumers (Che 1996), and signal product quality (Moorthy and Srinivasan 1995).

Even though these two streams of work have laid the foundation for understanding many aspects of the returns policies, it is unclear how their interaction may impact the marketing decisions of the channel members. Accordingly, the second chapter of this dissertation addresses this gap in the literature and examines how a manufacturer's returns policy affects the retailers' prices, ordered stock levels, and consumer-returns policy; consequently, the analysis identifies conditions when *MR* and *CR* may be offered together and in isolation. Further, this study discusses the impact of other factors, such as the rate of returns from consumers and the cost of stock-outs to the retailers, on the above relationships.

The analysis of this interaction between *MR* and *CR* policies is conducted in a stylized setting where a manufacturer distributes its product to consumers through two competing retailers; both the manufacturer and the retailers may choose to accept returns from their respective customers. If *CR* is implemented, then it is helpful to distinguish between an initial demand and a net demand (i.e., net of returns from consumers) since they affect retail decisions in an important way.

First, consider the retailers' motivation for accepting returns from consumers in the absence of *MR*. By doing so, they can raise their prices due to consumers being less price-sensitive—consumers are less concerned about the price when having the right to return unwanted products. *CR* also allows the retailers to screen for high-valuation consumers. Moreover, offering *CR* makes a retailer more attractive to consumers (especially to those who are concerned about buying a mis-matched product) if the rival retailer does not accept returns; this allows the retailer to *steal* some market share from its competitor. However, accepting returns is costly. Interestingly, depending on their stocking and pricing strategies, the retailers will incur different types of cost when offering *CR*: (1) if they choose to order stock and set prices according to the initial demand, then they must bear the cost of unsold inventory (due to returns), and (2) if they base their stocking and pricing decisions on the net demand, then they will incur the cost of stock-outs[1].

Intuitively, if the marginal cost of stock-outs is negligible, then the retailers will accept returns from consumers—they can enjoy the above-mentioned benefits of the *CR* policies at no cost by ordering stock and setting prices according to the net demand. By contrast, if the marginal stock-out cost is not negligible, then the retailers must consider the trade-off between the benefit and cost of accepting consumer returns when making this decision.

Suppose that the marginal cost of stock-outs is not too high, then when offering *CR*, the retailers prefer bearing the cost of stock-outs to incurring the cost of excess inventory; consequently, they will order stock and set prices according to the net demand. Notice that the magnitude of stock-outs reflects the difference between the initial demand and the net demand, which increases as the rate of returns from consumers increases. Therefore, if consumers' rate of returns is low, then the cost of stock-outs will be low and can be recovered and the retailers will accept returns from consumers. In contrast, if this rate is high, implying a significant cost of accepting returns, then the wholesale price determines whether *CR* is offered or not: (1) if the wholesale price is low, then the retail prices will also be low; in this case, the retailers have little incentive to screen for high-valuation consumers, and therefore, *CR* will not be offered. (2) In contrast, if the wholesale price is high, then the retailers will be motivated to accept consumer returns since such policy allows them to screen for high-valuation consumers who can afford paying the high prices.

Finally, if the cost of stock-outs is relatively high, then the retailers will avoid stock-out problems by ordering stock according to the initial demand. When the product is always

[1]The stock level following this strategy is lower than the initial demand, resulting in stock-out problems (i.e., not all consumers, who are willing to purchase, can get the product initially). The retailers will then resell any returned items to those "rain-check" customers, and eventually, clear stock. Therefore, such retailers do not incur any costs due to excess inventory, and the only cost they must bear is that due to stock-outs.

available, the benefit of *CR* due to the above-mentioned competition effect is quite strong, and motivates the retailers to accept returns from consumers despite the cost of excess inventory.

Keeping the above in mind, now consider the manufacturer's perspective: For the manufacturer, accepting returns is costly since all excess inventory will be returned by the retailers. However, the manufacturer may nevertheless accept returns if such a policy raises its profitability. This possibility arises when the marginal cost of stock-outs is moderate. In this setting, by offering *MR*, the manufacturer induces the retailers to accept consumer returns and order stock according to the initial demand. By contrast, in the absence of *MR*, the retailers may or may not offer *CR* as discussed above; in both cases, they order less stock. (In particular, if *CR* is not offered, then sales are lower due to the demand being more price-sensitive; therefore, less stock is ordered. If consumer returns are accepted, then the level of ordered stock is lower due to: (1) the stocking strategy, which is based on the net demand, and (2) the motivation to reduce the cost of stock-outs[2].) This means that the manufacturer's returns policy can induce the retailers to order more stock. Higher stock level implies more intense competition between the retailers, lower prices, and higher profitability to the manufacturer. The trade-off between this benefit and the cost of excess inventory to the manufacturer (which is the production cost) determines whether *MR* will be offered—if the cost is relatively low (high), then the manufacturer will (not) accept returns from the retailers.

Most interestingly, the analysis shows that when the marginal cost of stock-outs is either negligible or relatively high, the benefit of accepting returns to the manufacturer completely disappears. In the latter setting, when the marginal cost of stock-outs is relatively high, the retailers will avoid this cost by ordering stock according to the initial demand if they choose

[2] When there is less stock to clear, the retailers can raise their prices; higher prices imply lower initial demand, and therefore, less consumers experiencing the problem of stock-outs (i.e., less stock-outs and lower stock-out cost).

to offer *CR*. Further, as discussed above, the retailers will accept consumer returns in this case. Here, even in the absence of *MR*, *CR* is still offered and stock is ordered according to the initial demand. In other words, *MR* neither changes the retailers' stocking (and pricing) strategy nor their consumer-returns policies. In fact, the nature of competition between the retailers remains unaffected by the manufacturer's returns policy; the manufacturer has nothing to gain from its returns policy.

Finally, consider the former setting, i.e, when the marginal cost of stock-outs is negligible. If *MR* is not offered, then the level of ordered stock is (solely) affected by the motivation to avoid excess inventory[3]—it is ordered according to the net demand. By offering *MR*, the manufacturer does induce the retailers to order more stock—they order stock based on the initial demand—but eventually, this increase in stock will be returned by consumers. In both cases, net sales are identical (and equal to the net demand). This means there is no benefit of offering *MR* when the marginal cost of stock-outs is negligible. The second chapter describes the structure of the market when *MR* and *CR* are (not) offered and provides a more detailed discussion on these issues.

The third and fourth chapters of this dissertation investigate the impact of an emerging practice of selling products to consumers via group-buying services. Recent research has shown that group-buying (which is a demand-aggregation mechanism) serves as a price-discrimination device that helps the firm extract more surplus when the market condition is uncertain (Anand and Aron 2003). The third chapter complements Anand and Aron (2003)'s work by focusing on disaggregate consumer behavior and investigating two dimensions of market heterogeneity that explain the rationale for using group-buying vis-a-vis the traditional posted-pricing mechanism. The analysis focuses on the behavior of consumers

[3]The latter force, i.e., the one due to the motivation to reduce the cost of stock-outs is not present in this case.

whose valuation for the product and cost of purchasing via group-buying[4] are heterogeneous. Consistent with Anand and Aron (2003)'s results (they do not consider disaggregate behavior), here too the analysis shows that group-buying allows for price discriminating higher-valuation consumers from those with lower valuation—consumers with high valuation will purchase via the traditional posted-pricing mechanism while low-valuation consumers purchase via group-buying, paying the lower group-buying price. However, consumer heterogeneity in both dimensions—valuation and cost of joining group-buying—is required for this price-discrimination mechanism to be profitable.

Intuitively, in the absence of heterogeneity in consumers' valuation—and thereby, willingness-to-pay, there is no benefit of market segmentation. Meanwhile, without sufficient heterogeneity in the cost of joining group-buying, it will be too costly (and even infeasible in some instances) for the firm to employ group-buying. For instance, if high-valuation consumers have too low a cost of joining group-buying, then the firm may need to reduce the posted price significantly to prevent them from purchasing the product via group-buying. Essentially, the firm has to compensate consumers' costs of joining group-buying. This means that the firm incurs significant costs when employing these services, and under some conditions, these costs may exceed any benefits arising from price-discrimination.

The fourth chapter builds on the previous one and investigates when a manufacturer may induce its retailer to offer group-buying services and characterizes the conditions under which channel coordination is improved. In practice, we notice that group-buying is employed by retailers in various industries and has triggered different reactions from manufacturers (The Wall Street Journal 2006). Further, the practice seems to arise where there is asymmetric information between the manufacturer and the retailer—the latter may be better informed about the market conditions than the former. For instance, the manufacturer may not

[4]In practice, consumers must spend extra time and efforts when purchasing via group-buying. Further, since these services are quite novel, some consumers may find it too costly to join group-buying, resulting in heterogeneity in terms of this cost among them.

know the precise distribution of high- and low-valuation consumers in the retailer's local market; in contrast, the retailer has a much clearer understanding of the distribution of consumer types (i.e., relative sizes of the consumer segments). Now, suppose that the size of the high-valuation segment is large, relative to that of the low-valuation segment. This market condition may allow the retailer to earn more profit by employing group-buying; however, the retailer may not reveal this to the manufacturer and can seek to leverage its informational advantage. Such asymmetry in information may also exist regarding other market parameters, such as price sensitivity of consumers in the different segments.

In general, under asymmetric information, the retailer earns rents (Myerson 1979), and from the manufacturer's perspective, this has a negative impact on the efficiency of the distribution channel. The analysis in Chapter Four shows that the availability of the group-buying mechanism provides an opportunity for the manufacturer to improve channel coordination under asymmetric information—that is, the manufacturer can reduce (and under some conditions, even avoid paying) the informational rents of the retailer arising from its private information about the market condition.

To illustrate the idea, Chapter Four analyzes a setting in which asymmetric information may exist between the manufacturer and its retailer—the retailer may be better informed about the market condition, i.e., the relative sizes of the consumer segments, and price sensitivities of consumers in different segments, than the manufacturer. Further, the market consists of two consumer segments—a high-valuation (H) and a low-valuation (L) segment; consumers in the H-segment are less price-sensitive than those in the L-segment—and the retailer has two pricing options in distributing the product, i.e, posted-pricing and group-buying. The relationship between these two parties is established by a contract, whose terms include a pricing mechanism, a wholesale price and a fixed fee.

When asymmetric information exists regarding the relative sizes of the high- and low-valuation segments, the result shows that the manufacturer avoids paying any informational rents and regains the first-best profitability (i.e., the profit it would earn under full information). Clearly, the profitability of posted-pricing and group-buying (and consequently, the retailer's choice between these two pricing mechanisms) depends on the state of the market[5]. Further, under a posted-pricing contract, the retailer's profitability is unaffected by the relative sizes of the consumer segments—no matter which segment (high or low) is relatively larger in size, the profitability of the posted-pricing contract remains the same and depends on the total demand of the two segments (which is fixed and known to the manufacturer). This allows the manufacturer to extract all the surplus from the retailer whenever the posted-pricing contract is chosen. As a result, the retailer is not doing any better by choosing the posted-pricing contract when it is not desired by the manufacturer.

On the other hand, the group-buying contract is more profitable when the size of the high-valuation segment is relatively large (and vice versa—see Footnote 5); consequently, it is desired by the manufacturer under this market condition and exhibits a (correspondingly) higher fixed fee. Here too, the retailer has no incentive to choose the group-buying contract when it is not desired by the manufacturer—if doing so, the retailer will incur a loss due to the high fixed fee exceeding the low gross-profit of the group-buying contract when the high-valuation segment is relatively small in size. In summary, the retailer always chooses the contract desired by the manufacturer in each market state, and reveals its private information about the market condition. As a result, it earns no informational rents.

Next, when asymmetric information exists regarding price sensitivities of consumers in the high- and low-valuation segments, the retailer does enjoy some informational rents. This is due to the fact that the profitability of both pricing mechanisms—posted-pricing and group-buying—is low (high) when price sensitivity of consumers happens to be high (low).

[5] It is more profitable to employ group-buying as a price-discrimination device when the state of the market is characterized by two consumer segments being more distinct from each other. For instance, if the size of the high segment is relatively large (compared to that of the low segment, making the two segments more distinct in this dimension from each other), then group-buying is more profitable (than posted-pricing), despite the cost associated with its employment.

Consequently, the full-information contract designed for the high (low) state would exhibit too low (high) a fixed fee. This motivates the retailer to choose the undesired contract when the market happens to be in the low state; by doing so, it pays a lower fixed fee (i.e., that of the contract designed for the high state) while earning a higher profit due to the market being in the low state. Nevertheless, these second-best contracts exhibit lower informational rents than those contracts that do no allow the manufacturer to specify the appropriate pricing mechanism. Notice that by choosing an appropriate pricing mechanism, the firm (retailer) earns higher profits; therefore, a contract of the former type (i.e., with a term on pricing mechanism) would allow the manufacturer to charge a higher fixed fee than that of the latter type. Further, the undesired contract of the former type clearly exhibits an *inappropriate* pricing mechanism, and thus, is less profitable to the retailer (compared to the undesired contract of the latter type, wherein the retailer is not obligated to any pricing mechanism). This higher fixed fee and lower profit reduce the retailer's incentive to choose the undesired contract; consequently, the retailer earns lower informational rents.

Interestingly, specifying the pricing mechanism could be considered as a type of performance requirement as in Desiraju and Moorthy (1997); that study examines the relative benefits of requirements on price - and/or effort - and not on group-buying. The analysis in Chapter Four shows that by leveraging group-buying, the manufacturer can reduce the negative impact of asymmetric information while improving channel coordination (to different extents, depending on the type of asymmetric information).

In summary, the three studies help develop a better understanding of the relationship between members of the distribution channel. Returns policies, which are commonly observed in the retail industry, have been a topic of academic interest for many years. The study in Chapter Two extends the literature on returns policies by investigating how a manufacturer's returns policy interacts with the retailers' returns policies and affects channel coordination. Chapters Three and Four are devoted to exploring the practice of group-buying due to the availability of the Internet, the emergence of e-commerce, and the expansion of social-networking in recent years. Relevant proofs of the mathematical results are provided in the Appendices.

CHAPTER 2: IMPACT OF RETURNS POLICIES ON CHANNEL COORDINATION

2.1 Introduction

Accepting returns from consumers is a common practice in the retail industry. On average, the rate of these returns is about 6-8% and can reach up to 10% in high season; in some product categories, the returns rate can be even higher, e.g., 20% for personal computers (The Wall Street Journal 1994, CEA 2005, Business Week 2007). In general, having the option to return purchases is valuable to consumers since it insures them against buying and keeping mismatched or defective products. For retailers, however, accepting returns tends to raise the cost of doing business. Nevertheless, almost all retailers offer consumers such an option.

In practice, manufacturers may accept returns[6] of unsold stock from retailers and this offsets at least some of the retailers' costs arising from consumer returns. Research has explored manufacturers' incentives to alleviate retailers' costs and several explanations have been put forth. For instance, the manufacturer's returns policy can help share the risk with the retailers, safeguard the brand name and facilitate the distribution of new product information (see e.g., Pasternack 1985, Kandel 1996, and Padmanabhan and Png 1995). An intriguing research finding in this context is that accepting returns from retailers can, under some conditions, raise the manufacturer's profitability.

More specifically, when demand is uncertain, Padmanabhan and Png (2004) show that when a manufacturer accepts returns, retail competition is intensified in the high demand state, leading to lower retail prices, higher sales and higher manufacturer profitability. When

[6] For expositional ease, we will refer to these as "manufacturer's returns", denoted *MR*, here and throughout the paper. In contrast, returns accepted by retailers are referred to as "consumer returns" and denoted *CR*.

there is no uncertainty about demand, though, retail competition is unaffected by the manufacturer's returns policy; consequently, in such a setting, the manufacturer has no incentive to accept returns from the retailers (Padmanabhan and Png 1997, 2004; Wang 2004). This stream of work, however, does not examine whether consumer returns affect the manufacturer's returns policy. Our goal is to extend this literature by exploring when and how consumer returns impact the manufacturer's optimal returns policy.

We investigate the strategic interaction between consumer returns and manufacturer's returns by addressing the following questions: (1) Given the fact that retailers may accept returns from consumers, how will the manufacturer's returns policy affect retail decisions, including retail pricing, stocking and consumer-returns policies? (2) Consequently, how will the manufacturer and the retailers make the decision on their respective returns policies? Specifically, (a) what are their respective benefit and cost associated with accepting returns? (b) how will retail competition be affected by the manufacturer's returns policy? (c) what are the factors that moderate the impact of the manufacturer's returns policy on the retailers' choice of consumer returns?

To highlight the strategic interaction between consumer returns and manufacturer's returns, we examine a setting in which a manufacturer distributes the product through two competing retailers. Each of these risk-neutral players decides whether to accept returns or not; and accepting returns from consumers affects the retail demand parameters[7]. Further, whenever the retailers accept returns from consumers, they will encounter two demand functions, including an initial-demand function and a net-demand function (i.e., net of returns from consumers). Using that demand structure, we determine the impact of the manufacturer's distribution strategy, i.e., the choice of the wholesale price and returns policy, on the retailers' choice of prices, stock levels, and consumer-returns policies.

[7]For example, when returns are accepted by retailers, consumers are willing to pay higher prices leading to a demand function that is less price-sensitive. In Appendix A, we develop an individual-level model that exhibits these features.

Suppose, for instance, that the manufacturer does not accept any returns from the retailers, the retailers must bear the cost associated with consumer returns, if any. Interestingly, depending on their pricing and stocking strategies, the retailers will incur different types of cost when accepting returns from consumers: (a) if the retailers choose to order stock and set prices according to the initial-demand function, then they will have enough stock to serve all consumers who are willing to purchase the product initially; since some of these consumers will return their purchase, resulting in some unsold inventory, the retailers must bear the cost of having excess inventory, and (b) if they base their stocking and pricing decisions on the net demand, then the stock they carry is lower than the initial demand, resulting in stock-outs problem (i.e., not all consumers, who are willing to purchase, can get the product initially); in this case, the retailers will incur the cost of such stock-outs[8]. Intuitively, if the cost due to stock-outs is greater than that due to excess inventory, the retailers will choose the former option.

In contrast, if the manufacturer accepts returns from the retailers, then excess inventory, if any, is taken care of by the manufacturer. Therefore, the retailers will choose to order stock and set prices according to the initial demand, i.e., the former option, when accepting returns from consumers. In other words, accepting consumer returns is costless to the retailers under the manufacturer's returns policy.

When making the decision on their returns policies, the retailers consider the trade-off between the above-mentioned costs and the benefit of accepting returns from consumers. Since consumers are less price-sensitive when they can return unwanted products, the retailers are able to charge a higher price and obtain a higher margin and profitability. Further, when a retailer accepts returns while it rival does not, the benefit from accepting consumer returns is enhanced since those consumers who are concerned about the risk of purchasing

[8]The retailers will resell any returned items to those "rain-check" consumers and eventually, clear their stock. Therefore, they do not incur any excess inventory; the only cost they must bear is that due to stock-outs. For detailed discussion, see the model section.

a mis-matched product are more likely choose the retailer who offers consumer returns. In other words, a returns policy makes the retail offer more attractive to consumers when not all retailers offer such a policy. However, this *attractiveness* is limited if the product may not be available (due to stock-outs) at the retailer who accepts returns—this is the case when this retailer chooses to order stock according to the net demand (i.e., the second option mentioned above).

Regarding the retailers' returns policies, we find that when the manufacturer accepts returns of unsold inventory, accepting consumer returns is costless to the retailers; they will accept returns from consumers. In contrast, without the right to return excess inventory to the manufacturer, the retailers' returns policies depend on other factors, including (1) the marginal cost of stock-outs, (2) the rate of returns from consumers, and (3) the wholesale price.

Specifically, if the marginal cost of stock-outs is negligible, the retailers will offer consumer returns; they will order stock as well as set the retail prices according to the net demand in this case. As mentioned above, the cost associated with accepting consumer returns when choosing this pricing option is the cost of stock-outs, which is negligible. In other words, the retailers are able to avoid the cost of consumer returns while benefiting from such policy. In the other extreme, if the marginal cost of stock-outs is high, all retailers will avoid stock-out problems by ordering stock according to the initial demand upon accepting returns from consumers. As pointed out earlier, when the product is always available, the *competitive* gain of accepting returns when the rival retailer does not (and at the same time, the *competitive* loss due to not accepting returns when the rival does) becomes significant, motivating the retailers to accept returns despite the cost of excess inventory. Finally, if the marginal cost of stock-outs is moderate, then the retailers will choose to bear this cost (instead of the cost of excess inventory) by ordering stock according to the net demand. Though consumer returns result in a less price-sensitive demand, the *competitive* gain from accepting returns is now limited due to the existing stock-outs problem. In equilibrium, if the rate of returns from consumers is low, then the cost of consumer returns is low[9]; the retailers accept returns from consumers. In contrast, if the rate of returns is significant, implying a significant cost of accepting returns, then the product's wholesale price determines whether consumer returns are accepted or not: (a) If the wholesale price is low, then the retail prices will also be low. In this case, the retailers have little incentive to screen for high-valuation consumers via a returns policy; they will choose to not accept returns from consumers. (b) In contrast, if the wholesale price is high, the retailers will be motivated to accept consumer returns since such policy allows them to screen for high-valuation consumers who can afford paying the high retail prices.

From the manufacturer's perspective, accepting returns is costly since all excess inventory will be returned by the retailers. However, the manufacturer still chooses to accept returns if such policy raises its profitability. This happens when the marginal cost of stock-outs is moderate. In this setting, by accepting returns of excess inventory, the manufacturer induces the retailers to offer *CR* as well as order stock and set prices according to the initial demand. In contrast, in the absence of the manufacturer's returns policy, the retailers may or may not offer *CR* (depending on the trade-off between the cost and benefit discussed above); in both cases, they will order less stock. If *CR* is not offered, then sales are lower due to higher price-sensitivity; consequently, stock is ordered less. In contrast, if consumer returns are accepted, the level of ordered stock is reduced as a result of two driving forces: (1) the motivation to avoid the cost of excess inventory—the retailers' stocking and pricing decisions are based on the net demand (instead of the higher initial demand)—and (2) the motivation to reduce the cost of stock-outs[10]. This means that the manufacturer's returns policy can

[9]Note that, in this case, the cost of accepting returns from consumers is the cost of stock-outs. When the rate of returns is low, the difference between the initial demand and the net demand is small; as a result, the level of stock-outs is low, implying a low level of stock-out cost.

[10]Recall that upon ordering stock according to the net demand, the retailers incur the cost of stock-outs. Most interestingly, given a marginal cost of stock-outs, the motivation to reduce the cost of stock-outs induces

induce the retailers to order more stock. Higher stock level implies more intense competition between the retailers, lower prices, and higher profitability to the manufacturer. We find that when the cost of stock-outs is moderate and the manufacturer's marginal cost of having excess inventory[11] is relatively low, the gain from accepting returns exceeds the associated cost and the manufacturer will be better off by offering the retailers the right to returns. Otherwise, if the marginal cost of taking back returns is high, the manufacturer will not accept any returns from the retailers.

Most interestingly, our analysis shows that under other conditions, i.e., when the marginal cost of stock-outs is either negligible or relatively high, the benefit of accepting returns to the manufacturer completely disappears and it will not accept returns at all. Consider the setting when the marginal cost of stock-outs is negligible. If *MR* is not offered, then the level of ordered stock is (solely) affected by the motivation to avoid excess inventory[12]—it is ordered according to the net demand. By offering *MR*, the manufacturer does induce the retailers to order more stock—they order stock based on the initial demand—but eventually, this increase in stock will be returned by consumers. In both cases, net sales are identical (and equal to the net demand). This means there is no benefit of offering *MR* when the marginal cost of stock-outs is negligible.

In the second setting, when the marginal cost of stock-outs is relatively high, the retailers will avoid this cost by ordering stock according to the initial demand if they choose to offer *CR*. Further, as discussed above, the retailers will accept consumer returns in this case. Here, even in the absence of *MR*, *CR* is still offered and stock is ordered according to the initial demand. In other words, *MR* neither changes the retailers' stocking (and pricing) strategy

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the retailers to order less stock: when there is less stock to clear, the retailers will raise the prices; higher prices imply lower initial demand, and consequently, less consumers experiencing the problem of stock-outs, i.e., less stock-outs and lower stock-out cost.

[11]For the manufacturer, this is the marginal cost of production.

[12]The latter force, i.e., the one due to the motivation to reduce the cost of stock-outs is not present in this case.

nor their consumer-returns policies. In fact, the nature of competition between the retailers remains unaffected by the manufacturer's returns policy; the manufacturer has nothing to gain from its returns policy.

The rest of this essay is organized as follows. In section §2.2, we provide a brief review of the literature on returns policies. §2.3 describes the basic model, including the retailers' demand functions under different combinations of returns policies. The optimal policies under different market conditions are characterized in §2.4. §2.5 explores the impact of risk-aversion on the part of consumers and presents some observations from numerical simulations. §2.6 concludes the paper. Details related to an individual-consumer-level model that can generate our demand functions, along with all other derivations, are presented in Appendix *A*.

2.2 Literature review

Here we discuss two streams of relevant research—the first deals with manufacturer's returns while the second with consumer returns. The literature on manufacturer's returns policies is quite extensive and examines the impact of a variety of factors. Marvel and Peck (1995), for instance, show that the decision to accept returns by manufacturers depends on the nature of demand uncertainty. Specifically, uncertainty over customer arrivals favors returns since returns compensate the retailers for holding risky inventory and result in optimal stock levels; in contrast, when the uncertainty is over consumer's valuation, a no-returns policy is recommended to avoid price distortion. Padmanabhan and Png (1995) provide a summary of the various explanations for the use of manufacturer's returns policies, including the need to (a) share the risk with the retailers when demand is uncertain, (b) safeguard the brand name, and (c) facilitate the distribution of new product information. More recently, Tsay (2002) shows how risk sensitivity affects the manufacturer's optimal returns policy.

Next, abstracting from the insurance role of manufacturer's returns policies, Pellegrini (1986) considers returns policies as an effective competitive tool for channel coordination when products are close substitutes and retailers are risk-neutral. The channel-coordination role of manufacturer's returns policies is also explored by Pasternack (1985), who explores how a partial credit for unsold stock can achieve channel coordination. Next, the manufacturer's returns policy can serve as a tool either to signal the quality of the new product when it is not observable by the retailers (as in Chu 1993) or to learn the demand for a new product (as in Sarvary and Padmanabhan 2001). Finally, Kandel (1996) provides arguments for offering returns based on the optimal allocation of responsibility for unsold inventory between the manufacturer and the retailers. His research discusses six factors that affect the choice of returns, including: (i) optimal inventory, (ii) capability to dispose of unsold stocks, (iii) risk-sharing, (iv) incentives to provide marketing efforts in terms of quality, service, and promotions, (v) beliefs about sales distribution–when there is asymmetric information between the manufacturer and the retailer, and (vi) costs of returns.

Perhaps the most relevant papers for our analysis are Padmanabhan and Png (1997, 2004) and Wang (2004). These papers raise the intriguing possibility that manufacturer returns may sometimes lead to more intense competition at the retail level. Wang (2004), for example, qualifies the results of Padmanabhan and Png (1997) to show that when demand is certain, there is no difference in stocking levels, retail prices, and profits between accepting and not accepting returns. Padmanabhan and Png (2004) identify conditions, under demand uncertainty, when a manufacturer's returns policy does intensify competition and raise profit.

In contrast to Padmanabhan and Png (1997, 2004) and Wang (2004), our paper considers a setting in which the retailers may choose to accept returns from consumers. In the presence of consumer returns, we show that even though the nature of competition remains Cournot-like, manufacturer's returns may nevertheless intensify retail competition under some conditions. In that sense, our analysis adds to this stream of literature.

We now turn to the second stream of literature, i.e., the one focusing on consumer returns policies. Che (1996) investigates the role of consumer returns policy in screening for high-valuation customers. He finds that when the retail cost is high and consumers are risk-averse, the retailer can protect its margin by selling only to high-valuation customers under a returns policy. Next, from a signaling perspective, Moorthy and Srinivasan (1995) (MS) argue that money-back guarantees, i.e., accepting returns from consumers, can credibly signal product quality. MS identify conditions under which money-back guarantees are necessary to signal quality; they also identify conditions under which these guarantees serve as a useful supplement to price in signaling quality.

Hess et al. (1996) investigate the role of a non-refundable charge in attenuating the moral hazard problem associated with accepting returns from consumers. That is the case when some consumers purchase the product with the intention of returning it after extracting some free value out of it. They find that the retailer is better off imposing a non-refundable charge when the trial value, the overall valuation, or the probability of consumer finding a matched product is high. Such a charge is also recommended when consumers' transaction cost or the salvage value of the returned product is low. Next, Chu et al. (1998) study the impact of consumers' opportunistic behavior, which results in abusive returns, on the firms' optimal compensation policy. By investigating the trade-off between an increase in consumers' willingness-to-pay and opportunistic behavior due to a returns policy, they find that a no-questions-asked refund policy is optimal; however, a full refund is not always recommended. Specifically, partial refund are more likely to be offered when (a) the probability of dissatisfaction is low, (b) usage rate during product trial is high, (c) consumers' cost of complaining is low, and (d) salvage value is low. Davis et al. (1995, 1998) and Yalabik et al. (2005) determine the optimal returns policy for the retailer in various settings. For instance, Davis et al. (1995) suggest using full-returns policies either when the transaction cost is low, the salvage value of the returned product is significant, or the probability of the match between consumers' expectation and the product's performance is small. Researchers in the operations management area are also interested in consumer returns with the focus on the impact of consumer returns on the design of the supply chain.

While this stream of research provides considerable insights, the focus is mainly on the risk-sharing role of consumer returns in a monopolistic environment. Our paper extends this literature by investigating the impact of consumer returns when retailers compete. We show, for instance, that even when consumers are risk-neutral, the retailers can benefit from accepting consumer returns. Overall, our paper contributes to the understanding of returns policies by investigating the strategic interaction between manufacturer's returns and consumer returns.

2.3 The model

We focus on a single upstream manufacturer, M, who sells its product through two risk-neutral competing downstream retailers, R_1 and R_2, who, in turn, distribute the product to consumers. The manufacturer offers both retailers the same terms, which include a uniform wholesale price, w, and a returns policy. As in Padmanabhan and Png (1997), here too, M offers to the retailers one of the following two options for returns: (a) full returns, denoted *MR*, and (b) no returns, denoted *NMR*. Under full returns, the manufacturer refunds the wholesale price to the retailers for any unsold quantity of the product; in contrast, under no returns, the retailers must bear the costs of any unsold items remaining in the ordered stock.

In the downstream market, R_1 and R_2 offer consumers the product at the retail prices, p_1 and p_2 respectively, along with their own returns policies. Analogous to the manufacturer's returns policy, we focus on the simple setting, in which each retailer offers to consumers one of two options: (a) accepting returns from consumers and giving them a full refund of the purchase price, denoted *CR*, or (b) not accepting any returns at all, denoted *NCR*. When accepting returns, retailer i ($i \in \{1, 2\}$) generates *initial* sales, $\hat{q}_i$, and receives $\rho\hat{q}_i$ in returns, where ρ is the rate of consumer returns. Consequently, the net sales volume, denoted q_i, equals $(1 - \rho)\hat{q}_i$. Meanwhile, when retailer i does not accept returns, all sales are final and the initial demand, $\hat{q}_i$, is identical to the net demand, q_i.

The sequence of events in this game unfolds in four stages (see Figure 2.1). In the first stage, the manufacturer sets the wholesale price, w, and decides on its returns policy, σ^M, where $\sigma^M \in \{MR, NMR\}$. In the second stage, the retailers choose, simultaneously and non-collusively, their consumer-returns strategies, denoted σ_i^R ($i = 1, 2$). Four possible combinations of consumer-returns policies can arise in this stage: (a) $\{NCR, NCR\}$ when no retailers accept consumer returns, (b) $\{NCR, CR\}$ when retailer R_1 does not accept returns from consumers while retailer R_2 does, (c) $\{CR, NCR\}$ when only retailer R_1 offers consumer returns, and (d) $\{CR, CR\}$ when both retailers accept consumer returns.

Next, in stage three, the retailers decide on the stock levels, s_i, that they will order given the manufacturer's distribution policy (i.e., σ^M and w) and the choice of consumer-returns policies (i.e., $\{\sigma_1^R, \sigma_2^R\}$) from stage two. In the final stage, R_1 and R_2 set their retail prices, p_1 and p_2 respectively, given the decisions made in all the previous stages. Subsequently, sales are realized and profits made.

When the manufacturer's returns policy is considered in conjunction with the retailers' consumer-returns policies, i.e., $\{\sigma^M, \sigma_1^R, \sigma_2^R\}$, there are eight possible combinations as depicted in Figure 2.1: (1) $\{NMR, NCR, NCR\}$, (2) $\{NMR, NCR, CR\}$, (3) $\{NMR, CR, NCR\}$, (4) $\{NMR, CR, CR\}$, (5) $\{MR, NCR, NCR\}$, (6) $\{MR, NCR, CR\}$, (7) $\{MR, CR, NCR\}$, and (8) $\{MR, CR, CR\}$. We use $\Pi^i_{j,k,l}$ to denote retailer i's profit when it employs consumer returns policy j, and the competing retailer and the manufacturer use returns policies k and l respectively. For expositional convenience, we will use a 1 to denote a full-returns policy and

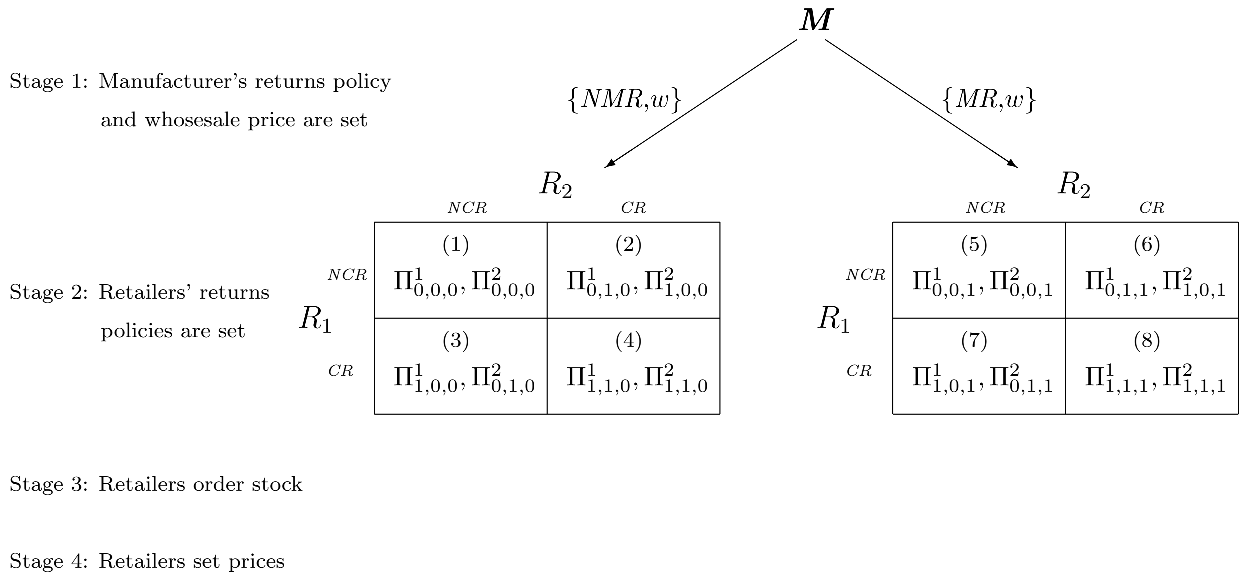


Figure 2.1: **The structure of the game.**

a 0 to denote a no-returns policy (i.e., $j, k, l \in \{0, 1\}$). With this notation, $\Pi^1_{1,0,1}$, for example, denotes R_1's profit when it offers CR, R_2 offers NCR and M offers MR. Finally, we use $\pi^M_{j,k,l}$ to denote M's profit, when R_1, R_2 and M use returns policies j, k, and l respectively.

Market demand depends on the retail prices, p_i $(i = 1, 2)$, and whether or not consumer returns are accepted. We focus on the following linear initial-demand structure:

$$\hat{q}_i = \alpha^i_{j,k} - \beta^i_{j,k}\, p_i + \gamma^i_{j,k}\, p_j, \tag{2.1}$$

where $j, k \in \{0, 1\}$ denote the returns policies of R_1 and R_2 respectively, and

$$q_i = \begin{cases} \hat{q}_i & \text{if} \quad \sigma^R_i = NCR, \\ (1-\rho)\hat{q}_i & \text{if} \quad \sigma^R_i = CR. \end{cases} \tag{2.2}$$

The above structure accommodates the impact of consumer returns on the various demand parameters (e.g., price sensitivity). In Appendix A, we develop one possible individual-consumer-level model that can result in such a (linear) demand structure for the eight scenarios examined here[13]. The specific demand functions that arise for the different combinations of consumer returns policies are given in Table 2.1.

Table 2.1: **Initial and net demand functions under different combinations of consumer returns policies**

$\{\sigma_1^R, \sigma_2^R\} = \{NCR, NCR\}$	$\{\sigma_1^R, \sigma_2^R\} = \{NCR, CR\}^a$	$\{\sigma_1^R, \sigma_2^R\} = \{CR, CR\}$
Initial-demand functions:		
$\hat{q}_i = 1 + 2\tau v - 3\tau p_i + \tau p_j$,	$\hat{q}_1 = 1 - \frac{\tau}{2} + 3\tau v + \mu - 3\tau p_1 + \frac{\tau}{2} p_2$,	$\hat{q}_i = 1 + \tau - 2\mu - \frac{3\tau}{2} p_i + \frac{\tau}{2} p_j$.
(where $i, j = 1, 2,\ i \neq j$).	$\hat{q}_2 = 1 + \frac{3\tau}{2} - \tau v - 3\mu - \frac{3\tau}{2} p_2 + \tau p_1$.	
Net-demand functions:		
$q_i = \hat{q}_i$.	$q_1 = \hat{q}_1$ and $q_2 = (1-\rho)\hat{q}_2$.	$q_i = (1-\rho)\hat{q}_i$.

Parameters^b:

$\rho \in [0, 1]$: rate of returns,

$v \in [\frac{1}{6}, \frac{1}{2}]$: certainty equivalent,

$\tau \in (0, +\infty)$: inverse of unit transportation cost, t,

$\mu \in [0, \bar{\mu}]$: aggregate costs of returns, where ($\bar{\mu} \overset{\text{def}}{=} \frac{1}{3} + \frac{\tau}{2} - \frac{\tau v}{3}$).

^{a}Demand functions when $\{\sigma_1^R, \sigma_2^R\} = \{CR, NCR\}$ are similar to those under $\{NCR, CR\}$ by symmetry.
^{b}See Appendix A for the description of these parameters.

We follow the literature (e.g., Che 1996, and Padmanabhan and Png 1997) in assuming that (a) returned items can be resold as long as there is demand[14], (b) unsold inventory (i.e., when demand is met) is worthless, (c) the manufacturer incurs only a constant marginal cost of production, c, and (d) both the manufacturer and the two retailers are risk-neutral.

[13] Developing the demand functions from first principles allows us to characterize the relationship between consumer returns and the various parameters more clearly; however, our qualitative insights will apply more generally.

[14] Assumption (a) is supported by the fact that 95% of returned products are in working condition; they are returned due to mis-match or change of mind (PC World 2008). Many products (e.g., house-hold items, electronics, etc.) are returned unopened and are put back on the shelf. The most common practice is reselling returned products as 'opened' items at discounted prices.

We now discuss the retailers' pricing strategies and specify their profit functions, along with that of the manufacturer, under each of the eight combinations of the returns policies. As noted in Wang (2004), the retailers set prices in stage four to clear the ordered stock whether returns are accepted by the manufacturer or not. Recall from (2.1) and (2.2) that when consumer returns are accepted, the retailers face an initial-demand function, $\hat{q}_i$ $(i = 1, 2)$, and a net-demand function, q_i $(i = 1, 2)$. Consequently, in our model, the retailers may choose one of the following two options, namely option A and option B, to set the retail prices in stage four: In option A, the retailer sets the price to clear stock according to the initial-demand function, $\hat{q}_i$, and in option B, the price is set so that stock is cleared according to the net-demand function, q_i.

Under option A, all consumers who are willing to purchase the product at the chosen retail prices will be served. This means that the initial demand is met completely; however, some customers return the product and the retailers end up with unsold inventory at the end of the selling period. Under *MR*, the manufacturer compensates the retailers for unsold inventory; in contrast, under *NMR*, the retailers must bear the cost of unsold inventory.

In contrast, under option B, prices are set to clear stock according to the net demand. Consequently, the retailers will generate more initial demand than the stock they carry. This means that some customers are not served initially due to stock-outs. Here, too, some of the initial buyers will return the product. In this case, as in Che (1996), the retailers are assumed to be reselling the returned items to other consumers; eventually, all consumers who are willing to purchase the product will be served and the stock will be cleared. However, the stock-out problem (i.e., some consumers cannot obtain the product immediately) can result in various negative reactions from consumers (see Campo et al. 2004, Fitzsimons 2000, among others). Here, to capture the negative impact of stock-outs, we assume that the retailers incur a constant marginal stock-out cost of η $(\eta \geq 0)$[15]. Intuitively, option B is

[15] The cost due to stock-outs is truly an opportunity cost. Past research has considered the adverse impact of stock-outs on both current and future profits of the firm (Fitzsimons 2000, Anderson et al. 2006).

meaningful only when the manufacturer does not offer *MR*. Furthermore, option B is likely to be preferred when the stock-out cost is relatively low[16] compared to the cost of having excess inventory (as in option A). Accordingly, our analysis focuses on two scenarios: (1) when the cost of stock-outs is relatively low and the retailers set their retail prices under *NMR* according to option B; and (2) when the stock-out cost is relatively high and the retailers set prices according to option A[17] when the manufacturer does not accept returns. Recall that when the manufacturer accepts returns, the retailers will choose option A to avoid the stock-out problem[18].

Clearly, the responsibility of unsold inventory affects the retailers' pricing decisions in stage four. Specifically, under *MR*, all unsold inventory can be returned to the manufacturer for refund; as a result, the retailers set their retail prices according to option A to avoid stock-outs when offering *CR*. Thus, under *MR*, retailer i's profit function is given by

$$\Pi^i_{j,k,1} = (1-\rho)^j \, \hat{q}_i \, (p_i - w) = q_i(p_i - w), \tag{2.3}$$

where $j = 1$ when retailer i offers *CR* and $j = 0$ otherwise; $k = 0, 1$, is the returns policy of the competing retailer.

A complete treatment of the stock-out problem would require, ideally, a dynamic model (at least a two-period model) which captures both the current and the future impact of stock-outs on the firm's demand and consequently, profit. In the context of our static model, the marginal cost due to stock-outs can be interpreted as the aggregate present-value measure of this cost. Such an interpretation allows us to focus on the optimal returns policies in a simpler analytical setting, without compromising too much on the qualitative insights.

[16]For instance, the stock-out cost for seasonal products is likely to be higher than that for commodity products. Further, different firms may consider stock-out costs differently. Discount retailers, such as Walmart, whose focus is on a low price strategy rather than on superior customer service, may perceive stock-out cost at a lower level compared to more upscale retailers.

[17]We abstract from endogenizing the retailers' choice between the two pricing options for analytical convenience. However, in the benchmark analysis of an integrated manufacturer, the choice between these two options is determined endogenously; the results show that indeed option A is chosen when the cost of stock-out is significant, and vice versa.

[18]Under zero marginal cost of stock-out, though both options are costless to the retailers when the manufacturer accepts returns, we assume that the retailers prefer option A to option B; this reflects the reality in the sense that option B is likely to require some additional effort to put the product back on the shelf (though this cost is not modeled explicitly in our framework).

The manufacturer's profit is:

$$\pi^M_{j,k,1} = \begin{cases} s_1(w-c)+s_2(w-c) & \text{if} \quad j=0 \text{ and } k=0 \\ s_1(1-\rho)w - s_1c + s_2(w-c) & \text{if} \quad j=1 \text{ and } k=0 \\ s_1(w-c)+s_2(1-\rho)w - s_2c & \text{if} \quad j=0 \text{ and } k=1 \\ s_1(1-\rho)w - s_1c + s_2(1-\rho)w - s_2c & \text{if} \quad j=1 \text{ and } k=1 \end{cases}$$
$$= s_1[(1-\rho)^j w - c] + s_2[(1-\rho)^k w - c], \tag{2.4}$$

where s_1, s_2 are the stock ordered by retailers R_1 and R_2 respectively, and j, k are their respective returns policies.

In contrast, under *NMR*, the retailers may choose either option A or B, depending on the level of stock-out cost. If pricing according to option A, R_1 and R_2 incur the cost of unsold inventory (but not the stock-out cost) and their profit functions are given by:

$$\Pi^{i,A}_{j,k,0} = \begin{cases} q_i p_i - q_i w & \text{if} \quad j=0 \\ \hat{q}_i p_i - \rho \hat{q}_i p_i - \hat{q}_i w & \text{if} \quad j=1 \end{cases}$$
$$= \hat{q}_i[(1-\rho)^j \, p_i - w], \tag{2.5}$$

where $j = 0, 1$, is the returns policy of retailer i and $k = 0, 1$, is the competing retailer's returns policy. Notice in (2.5) that the retailer is pricing to clear stock according to the initial demand (i.e., option A) and thus experiences excess inventory due to returns. Under option B, the retail prices are set to clear stock according to the net demand, i.e., $s_i = q_i$. Consequently, there are $\hat{q}_i - s_i = \hat{q}_i - q_i = \rho\hat{q}_i$ customers who are not served initially; this results in a stock-out cost equal to $\rho\hat{q}_i\eta$. Therefore, the profit function of retailer i is

$$\Pi^{i,B}_{j,k,0} \;=\; q_i\,(p_i - w) - j\,\rho\,\hat{q}_i\,\eta \;=\; q_i\,\Big[p_i - w - \frac{j\rho}{(1-\rho)^j}\,\eta\Big], \tag{2.6}$$

where j is the returns policy of retailer i and k is the returns policy of the competing retailer.

In either of the two cases, the manufacturer gets paid for all ordered stocks at the wholesale price, w, and its profit is:

$$\pi^M_{j,k,0} = (s_1 + s_2)(w - c), \tag{2.7}$$

where s_1, s_2 are the stocks ordered by retailers R_1 and R_2 respectively.

To highlight the interaction between consumer returns and manufacturer's returns, we focus on a setting where consumers are risk-neutral. Later we check for the robustness of the results when this assumption is relaxed. In addition, we assume that consumers do not incur any transaction cost of returning unwanted products and restrict the rate of returns, ρ, to be practically reasonable, i.e., $\rho \leq \frac{1}{2}$ [19].

To understand the manufacturer's preference for inducing consumer returns, we begin by analyzing a benchmark setting in which the manufacturer is the owner of the two retailers, R_1 and R_2 (see Appendix A for all the details). In this setting, the manufacturer does not have to tackle any incentive problems with the retailers. The optimal returns policies of such an integrated manufacturer are summarized in the following lemma.

Lemma 2.1 *When $\rho \leq \frac{1}{2}$, an integrated manufacturer will always choose to accept returns from consumers. However, its pricing strategy depends on the magnitude of stock-out cost, η, relative to the cost of production, c: If stock-outs are not too costly (i.e., $\eta < c$), then the integrated manufacturer sets the retail price to clear stock according to the net demand (i.e., option B). In contrast, if stock-out costs are such that $\eta \geq c$, then the retail price is set to clear stock according the initial demand (i.e., option A).*

[19] As mentioned earlier, the maximum rate of consumer returns is in the range of 10-20% (BusinessWeek, 2007).

By accepting returns from consumers, the integrated manufacturer enjoys a less price-sensitive demand. As a result, it can raise the retail price and obtain a higher profit. The gain due to accepting returns dominates the cost when the rate of returns is reasonable (i.e., $\rho \leq \frac{1}{2}$). In terms of pricing strategy, if the cost of stock-outs is relatively low, the integrated manufacturer prefers bearing this cost to incurring the cost of unsold inventory. Option B allows M to avoid unsold inventory by generating more demand, creating stock-outs and reselling returned products to the "rain-check" customers who could not get the product immediately. In contrast, if the cost of stock-outs is relatively high, the manufacturer will avoid the stock-out problem by choosing option A when setting the retail price. Equipped with this benchmark analysis, we now turn to our main model.

Here we examine three cases: (1) when the marginal cost of stock-outs, η, is negligible, i.e., $\eta = 0$, (2) when η is low, and (3) when η is high. Recall that the retailers' stocking and pricing decisions, i.e., the choice between option A and B, upon accepting returns from consumers depend on the level of the marginal cost of stock-outs. Specifically, in the first and second cases, the retailers set the retail prices in stage four according to option B to avoid the cost due to excess inventory when the manufacturer does not accept returns; in contrast, in the third case, they choose option A when setting the retail prices since stock-outs are too costly to bear. Focusing on each of the three cases allows us to derive an analytical solution regarding the interaction between consumer returns and manufacturer's returns as well as the impact of the remaining factor, i.e., the rate of consumer returns. Later, by comparing the equilibrium results of the three cases, we can highlight the moderating effect of the cost of stock-outs. To characterize the equilibrium in each case, we proceed as follows.

First, we solve stages four and three to determine the retailers' pricing and stocking strategies in each of the eight combinations of returns policies, noting that each combination exhibits a different demand function (cf. Table 2.1). After characterizing the retailers' pricing and stocking decisions, we investigate the choice of returns policies made by the retailers in stage two. Given a specific policy selected by M (i.e., $\{\sigma^M, w\}$), the retailers decide whether to accept consumer returns by comparing profits earned under different combinations of consumer-returns policies. Accordingly, we consider the following differences in profits of the retailers[20]:

$$\delta_1 = \Pi^2_{1,0,0} - \Pi^2_{0,0,0}, \tag{2.8}$$

$$\delta_2 = \Pi^1_{1,1,0} - \Pi^1_{0,1,0}, \tag{2.9}$$

$$\delta_3 = \Pi^2_{1,0,1} - \Pi^2_{0,0,1}, \text{ and} \tag{2.10}$$

$$\delta_4 = \Pi^1_{1,1,1} - \Pi^1_{0,1,1}. \tag{2.11}$$

The choice of consumer-returns policies under *NMR* is determined by the signs of δ_1 and δ_2[21]. Specifically, both retailers will offer consumer returns if $\Pi^2_{1,0,0} > \Pi^2_{0,0,0}$ and $\Pi^1_{1,1,0} > \Pi^1_{0,1,0}$, i.e., $\delta_1 > 0$ and $\delta_2 > 0$. No consumer returns are accepted by the retailers if $\delta_1 < 0$ and $\delta_2 < 0$. Other combinations of the retailers' returns policies, including asymmetric equilibria and multiple equilibria, may also arise under different combinations of the signs of δ_1 and δ_2[22]. In an analogous fashion, the equilibrium of the consumer-returns subgame under *MR* depends on the signs of δ_3 and δ_4.

Then, in the first stage of the game, anticipating the retailers' pricing, stocking, and consumer-returns strategies, we characterize the manufacturer's choice of wholesale price and returns policy. We now report the results for the three cases. When the cost of stock-outs, η, is low, i.e., in the first and second cases, the results, including the optimal prices, $p^i_{j,k,l}$, the optimal stock levels, $s^i_{j,k,l}$, the retailers' profits, $\Pi^i_{j,k,l}$, and the manufacturer's profit, $\pi^M_{j,k,l}$, are reported in Tables 2.3 and 2.4. When η is high (i.e., in the third case), the retailers

[20] To distinguish among the cases, when η is high (i.e., case 3), we use the notation $\hat{\delta}_1 = \hat{\Pi}^2_{1,0,0} - \Pi^2_{0,0,0}$ and $\hat{\delta}_2 = \hat{\Pi}^1_{1,1,0} - \hat{\Pi}^1_{0,1,0}$ (see Table 2.5 and Appendix *A*).

[21] Notice that by symmetry, $\Pi^1_{0,0,0} = \Pi^2_{0,0,0}$, $\Pi^1_{1,0,0} = \Pi^2_{1,0,0}$, and so on. Therefore, the signs of δ_1 and δ_2 are sufficient to determine the game's equilibrium.

[22] For instance, if $\delta_1 < 0$ and $\delta_2 > 0$, then both $\{NCR, NCR\}$ and $\{CR, CR\}$ will arise in equilibrium.

set the retail prices according to option A under *NMR*; those results are summarized in Table 2.5. Regularity conditions along with other details of the derivations are provided in Appendix A.

2.4 Results

Case 1: No stock-out cost

When there is no stock-out cost, the following proposition summarizes the properties of the equilibrium.

Proposition 2.1 *When consumers are risk-neutral and the retailers do not incur any stock-out cost (i.e., $\eta = 0$):*

(a) The retailers always choose to accept returns from consumers whether or not manufacturer's returns are offered. However, if the manufacturer accepts returns, then the retailers order stock and set prices according to the initial demand; otherwise, without the right to return excess inventory to the manufacturer, their stocking and pricing decisions are based on the net demand.

(b) The manufacturer is better off by not accepting returns from the retailers.

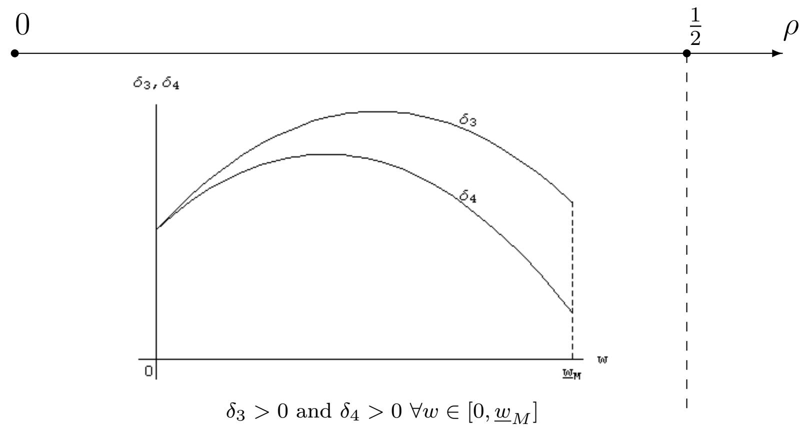
To understand the retailers' choice of consumer-returns policies, consider the following: By accepting returns, the retailers can charge a higher retail price due to the decrease in consumers' price sensitivity. Further, accepting returns also allows them to screen for high-valuation customers effectively. Most importantly, in this case, the retailers do not incur any cost associated with accepting returns from consumers. Under *MR*, this cost is compensated by the manufacturer; under *NMR*, by setting the retail prices to clear stock according to the net demand (i.e., option B), the retailers can effectively (and costlessly, since $\eta = 0$) clear all ordered stock by reselling returned items to those customers who cannot get the product initially due to stock-outs. Therefore, in the absence of stock-out cost, the retailers always benefit from offering consumer returns.

The manufacturer, however, is worse off when accepting returns from the retailers. Notice that by accepting returns, the manufacturer induces the retailers to set prices according to option A, which results in higher stock orders; otherwise, the retailers choose option B and order less stock. In both cases (i.e., under *NMR* and *MR*), the retailers' profits are determined by the net demand; further, in the absence of stock-out cost, they do not incur any cost except the wholesale price. This implies identical profit functions for the retailers, and hence identical retail prices in equilibrium under both *NMR* and *MR*[23]. Therefore, eventual sales are identical under both cases; any increase in the levels of stock when *MR* is offered will be eventually returned to the manufacturer. In other words, *MR* does not improve the net performance of the channel. By offering *MR*, the manufacturer induces option A; it is simply producing more and then taking back this additional production and incurring the cost of returns. Our result is in contrast with that of Wang (2004) and Padmanabhan and Png (2004), wherein the manufacturer is indifferent between *NMR* and *MR*. In their setting, since consumer returns are not considered, the retailers do not have any unsold inventory when demand is certain, and hence, there are no returns to the manufacturer.

<u>Case 2</u>: Low stock-out cost

In this setting, retail behavior is characterized in Lemma 2.2 and graphically illustrated in Figures 2.2, 2.3 and 2.4. (The specifications of the cut-off values, including ρ_1 and $\tilde{w}_1$, are provided in the appendix.) Notice from these figures that under *NMR*, δ_1 and δ_2 behave

[23]When $\eta = 0$, the retailers' profit functions under *NMR* and *MR* are identical (see equations 2.3 and 2.6). Therefore, the optimal retail prices are identical. From Tables 2.3 and 2.4, it can be seen that $p^i_{1,1,0} = p^i_{1,1,1} = \frac{3(1+\tau-2\mu)+4\tau w}{7\tau}$ when $\eta = 0$.



$\delta_3 > 0$ and $\delta_4 > 0$ $\forall w \in [0, \underline{w}_M]$

$\Rightarrow$ NE = {CR,CR} $\forall w \in [0, \underline{w}_M]$

$\Rightarrow$ NE = {CR,CR} $\forall w \in [0, \overline{w}_M]$

Figure 2.2: **Retailers' choice of consumer returns policies under *MR***

in three different patterns (i.e., pattern *I*, *II*, and *III*) depending on the magnitude of the returns rate, ρ. Here, for the purpose of illustration, we set $\eta = \frac{1}{4}$[24].

Lemma 2.2 *When consumers are risk-neutral and the marginal cost of stock-outs is low:*

(a) If the manufacturer accepts returns, then the retailers will accept returns from consumers. In addition, the retailers will set the retail prices to clear stock according to the initial demand (i.e., option A);

(b) If the manufacturer does not accept returns, then the retailers' consumer-returns policies depend on the rate of returns and the wholesale price: Under a low returns rate (i.e., $\rho < \rho_1$), the retailers accept consumer returns at all relevant wholesale prices. In contrast, when the rate of returns is high (i.e., $\rho > \rho_1$): (i) If $w < \tilde{w}_1$, the retailers will not accept returns, and (ii) If $w \geq \tilde{w}_1$, the retailers will accept returns from consumers. Regarding the retailers' pricing strategies, retail prices are set to clear net demand, i.e., according to option B, whenever consumer returns are accepted.

The intuition underlying this result can be explained as follows. Recall that accepting returns from consumers lowers price sensitivity which allows the retailers to charge higher

[24]In Appendix A, we show that when $\eta = \frac{1}{4}$, the retailers are indeed better off in equilibrium under option B than under option A.

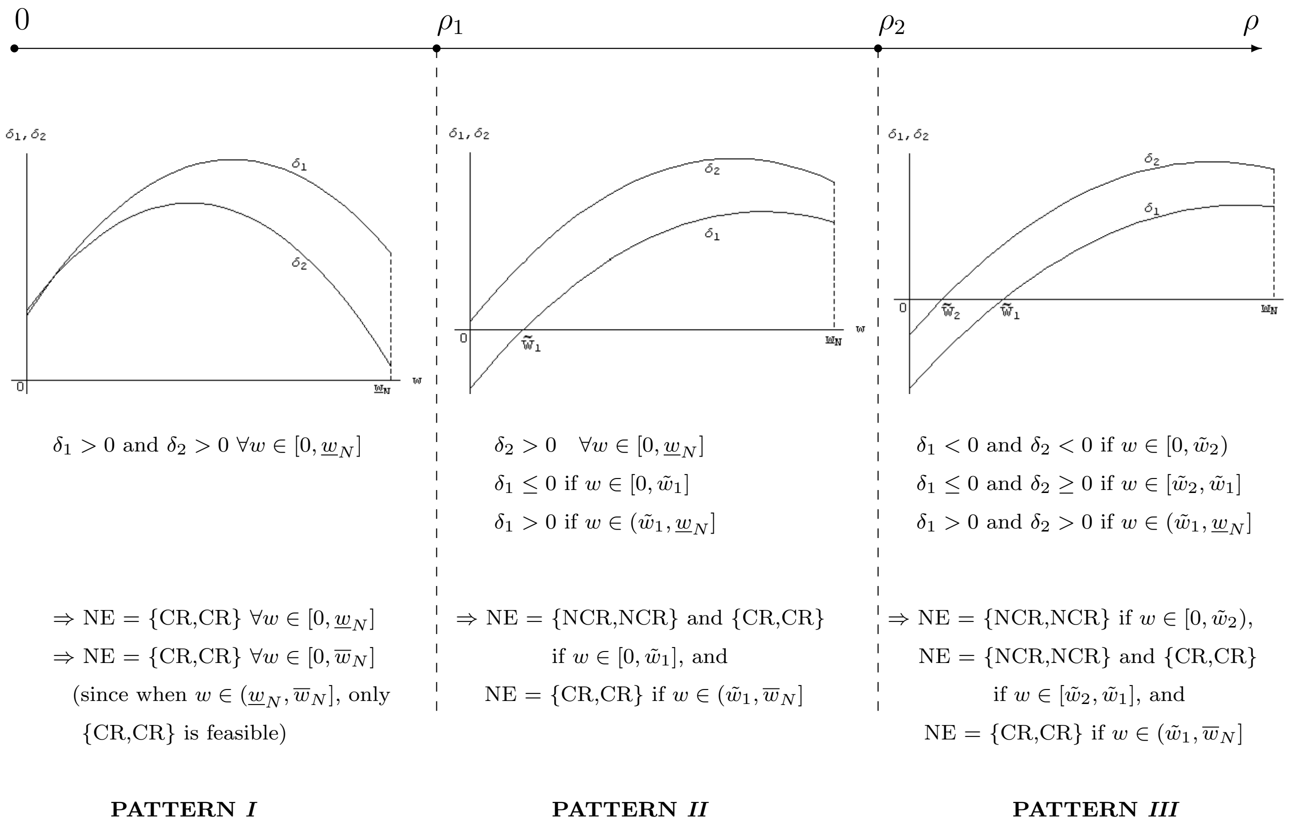


Figure 2.3: **Retailers' choice of consumer-returns policies under *NMR***

prices and earn higher profits. However, there is a cost associated with accepting returns from consumers. If the retailers set prices according to the initial demand (i.e., option A), then they incur the cost of unsold inventory due to returns from consumers; in contrast, if prices are set according to the net demand (i.e., option B), then they incur the cost of stock-outs instead. Under *MR*, though, the manufacturer refunds the wholesale price for any unsold stock, and therefore, accepting returns from consumers is costless to the retailers (given pricing option A). Therefore, the retailers always benefit from offering consumer returns under *MR*.

In contrast, when the manufacturer does not accept returns, the retailers are responsible for any unsold stock. In this case, they can avoid the cost of having excess inventory by ordering stock according to the net demand (i.e., choosing option B when setting the retail

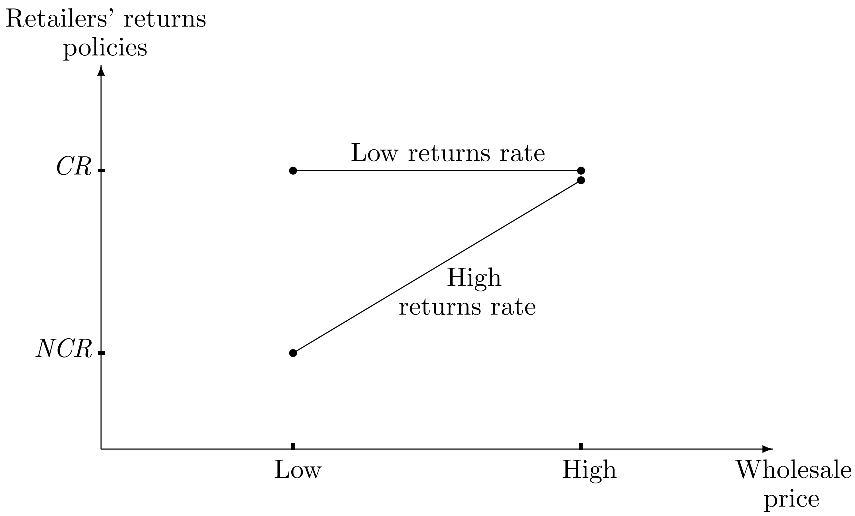


Figure 2.4: **Summary of retailers' returns policies under *NMR*.**

prices); however, they incur stock-out cost, which is increasing in the rate of returns (because net demand is lower with a higher returns rate[25]; see equations 2.1 and 2.2). When the rate of returns is relatively low, the cost associated with accepting consumer returns (i.e., the cost of stock-outs) is low, and the retailers are better off by accepting returns. When the returns rate is relatively high, this cost becomes significant and may deter the retailers from adopting a returns policy. In this latter setting, if the wholesale price were low, the retail prices will also be low and there is little incentive to accept returns from consumers. In contrast, if the wholesale price were higher, then retail prices will also be higher, and there is a greater incentive for the retailers to focus on the higher-valuation consumer segment. Accepting returns allows the retailers to screen for those consumers effectively. Consequently, accepting consumer returns when the wholesale price is high is beneficial despite the associated cost.

These results are broadly consistent with Che (1996). However, the findings of Che (1996) are based on the risk-sharing effect of consumer returns observed in a monopolist setting. In contrast, our results arise even in the absence of consumers' risk-aversion. In our setting, the motivation to accept consumer returns stems from the advantage of the less

[25] Recall that stock-out cost is proportional to the number of customers who are not served immediately due to the limited stock; see equation 2.6.

price-sensitive demand, which benefits the retailers in competing with the other retailers. In fact, with retail competition, adopting a no-returns policy while other retailers accept consumer returns is hurtful due to the higher own-price sensitivity and the lower cross-price sensitivity in the demand function (see Table 2.1 and Appendix *A*).

Anticipating the above-characterized behavior of the retailers, the manufacturer sets its distribution policy in the first stage. Notice that the manufacturer's profit function depends on the retailers' consumer-returns policies. The decision whether to accept returns from the retailers is made by comparing the profits earned under the two regimes, i.e., *NMR* and *MR*. We summarize the optimal wholesale prices under *NMR* and *MR* in Figure 2.5. Figure 2.6 depicts the manufacturer's distribution policy in equilibrium. The following proposition summarizes the findings. All derivations, along with the specifications of the cut-offs, are provided in Appendix *A*.

Proposition 2.2 *When consumers are risk-neutral and the marginal cost of stock-outs incurred by the retailers is low, the manufacturer's returns policy depends on the cost of production: If production cost is low (i.e., $c \leq c^*$), then the manufacturer accepts returns from the retailers; else, the manufacturer does not accept returns. The retailers, in equilibrium, will accept returns from consumers.*

When the cost of stock-outs is low, the retailers' pricing decision (i.e., option *A* versus option *B*) depends on the manufacturer's returns policy. Specifically, by offering *MR*, the manufacturer induces the retailers to choose option *A* and order stock based on the initial-demand function[26]; otherwise, under *NMR*, the retailers will choose option *B* and order stock based on the net demand. Intuitively, the level of ordered stock under option *B* is less than that under option *A*. This is due to (1) the net demand is lower than the initial

[26]Recall that setting the retail prices to clear stock according to the initial demand is equivalent to ordering enough stock to cover the initial demand. Analogously, setting prices to clear stock according to the net demand is equivalent to ordering stock enough to cover the net demand.

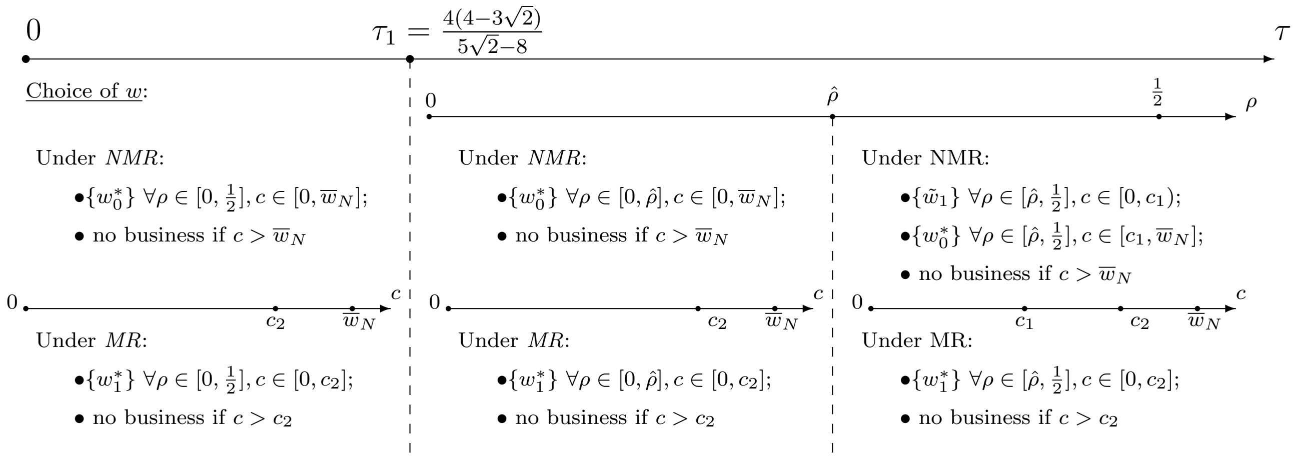


Figure 2.5: **Manufacturer's choice of wholesale price**

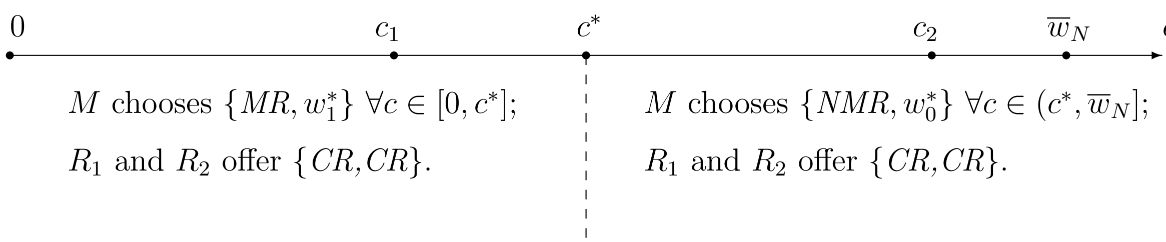


Figure 2.6: **Manufacturer's distribution policy in equilibrium**

demand, and (2) the retailers have an additional incentive to lower the level of ordered stock to reduce the cost of stock-outs, which equals $\eta\frac{\rho}{1-\rho}s$. Further, under option B, the cost of stock-outs is eventually incorporated in the retail prices[27], resulting in higher retail prices. Therefore, option A is more attractive (due to higher stock levels and lower retail prices) to the manufacturer than option B. In other words, the manufacturer has incentive to induce option A via accepting returns from retailers. However, that benefit is attenuated by the cost of accepting returns (which equals the cost of production). Hence, if this cost is low, the

[27]It can be seen that $p^i_{1,1,0} = p^i_{1,1,1} + \frac{4\rho}{7(1-\rho)}\eta$.

manufacturer will choose to accept returns; and when it is high, the manufacturer prefers a no-returns policy as outlined in the proposition.

Wang (2004) argues that in the framework of Padmanabhan and Png (1997), manufacturer's returns policy does not intensify retail competition when demand is certain since the retailers compete in the same manner under both *NMR* and *MR*, i.e., in a Cournot fashion. That argument may hold when consumer returns are not taken into consideration. However, when consumer returns are considered, even though manufacturer's returns policy does not shift the basis of competition from Cournot to Bertrand (Padmanabhan and Png 1997, Wang 2004), it does induce higher retail competition and alters the retailers' stocking and pricing strategies.

<u>Case 3</u>: High stock-out cost

When the cost of stock-outs is high, the retailers will avoid stock-out problem by choosing option A when setting the retail prices under *NMR*. Consequently, the optimal retail prices, the optimal levels of stock and the retailers' profits are different from those in the second case under the following combinations: $\{NMR, NCR, CR\}$, $\{NMR, CR, NCR\}$, and $\{NMR, CR, CR\}$. The results under the remaining combinations of returns policies are identical to those in the second case. Appendix A provides the detailed analysis of this case. We summarize the results, including the optimal prices, $\hat{p}^i_{j,k,0}$, stock levels, $\hat{s}^i_{j,k,0}$, and profits, $\hat{\Pi}^i_{j,k,0}$ in Table 2.5. The following proposition describes the characteristics of the equilibrium in this case.

Proposition 2.3 *When consumers are risk-neutral and the marginal cost of stock-outs incurred by the retailers is high, the retailers always order stock and set prices according to the initial demand. Further, they always accept returns from consumers. In contrast, the manufacturer is indifferent between accepting returns and not accepting returns from the retailers.*

Under high stock-out cost, the retailers set the retail prices to clear stock according to the initial demand (i.e., according to option A) under both *NMR* and *MR*. This means that in contrast to the case of low stock-out cost, the retailers are motivated to choose the pricing option desired by the manufacturer (which is option A) without additional incentive. In other words, *MR* does not provide any extra benefit to the manufacturer when the cost of stock-outs is high. In fact, *MR* neither shifts the nature of retail competition nor changes the retailers' pricing strategies—competition remains Cournot-like and prices are set according to option A. Though *MR* does increase the levels of stock and reduces the retail prices, all that gain is counter-balanced by the cost of returned product that the manufacturer has to cover. Consequently, the manufacturer is indifferent between accepting and not accepting returns.

Interestingly, this is reminiscent of Wang (2004) result in the absence of consumer returns. Note that in Wang (2004), when demand is certain, the retail prices and stock levels remain identical under *MR* and *NMR*. This means that accepting returns by the manufacturer has no impact on the channel's performance. In contrast, when consumer returns are considered, *MR* does improve the channel's performance due to the removal of the concern about unsold inventory. As noted above, though, the gain is counter-balanced by the loss due to the cost of returns.

2.5 Numerical simulation

This section aims to provide some robustness evidence of our findings when consumers are risk-averse. Recall that when consumers are assumed to be risk-neutral, their certainty equivalent is $v = \frac{1}{2}$. When consumers are risk-averse, the certainty equivalent decreases, i.e., $v < \frac{1}{2}$ (see Appendix A). Table 2.2 compares the equilibrium results when consumers are risk-averse ($v = 0.2$) to those when they are risk-neutral ($v = \frac{1}{2}$). The following variables (along with the respective notations) are reported in this table: (1) the manufacturer's choice of returns policy in equilibrium, σ^{M*}, (2) the optimal wholesale price under *NMR*, w^*_{NMR}, (3) the manufacturer's profit, π^{M*}_{NMR}, when choosing $\{NMR, w^*_{\text{NMR}}\}$, (4) the retailers' returns policies under *NMR*, $\sigma_i^R|_{\text{NMR}}$, (5) the retailers' choice of returns policies when the manufacturer sets the wholesale price at w^*_{NMR} and does not accept returns, $\sigma_i^{R*}|_{\text{NMR}}$, (6) the optimal wholesale price under *MR*, w^*_{MR}, (7) the manufacturer's profit, π^{M*}_{MR}, when choosing $\{MR, w^*_{\text{MR}}\}$, (8) the retailers' returns policies under *MR*, $\sigma_i^R|_{\text{MR}}$, and (9) the retailers' choice of returns policies when the manufacturer sets the wholesale price at w^*_{MR} and accepts returns, $\sigma_i^{R*}|_{\text{MR}}$.

We observe two possible patterns of the retailers' consumer returns policies under *NMR*, $\sigma_i^R|_{\text{NMR}}$ [28]: (Pattern *I*) the retailers choose to accept returns, i.e. $\{$*CR,CR*$\}$, for all relevant wholesale price, i.e. for all $w \in [c, w_2]$, and (Pattern *III*) the retailers's returns policies depend on the wholesale price, w; specifically, they choose (a) $\{$*NCR,NCR*$\}$ if $w \in [c, w_9)$, (b) either $\{$*NCR,NCR*$\}$ or $\{$*CR,CR*$\}$ if $w \in [w_9, w_8]$, and (c) $\{$*CR,CR*$\}$ if $w \in (w_8, w_2]$.

Observation 2.1 *As consumers become more risk-averse, accepting consumer returns can become more profitable to the retailers.*

Suppose the cost of production were low (in the table, $c = 0.1$), then the above observation is based on the fact that when the rate of returns is high ($\rho = \frac{1}{2}$), the pattern of consumer returns policies is changed from pattern *III* when consumers are risk-neutral ($v = \frac{1}{2}$) to pattern *I* when they are risk-averse ($v = 0.2$). (See Figure 2.3 for details on the difference between pattern *I* and *III*.) Intuitively, when consumers are risk-averse, the retailers need to share the risk with consumers. They can do so by either lowering the retail price or by offering consumer returns. As consumer risk-aversion increases, the retailers must compensate consumers more for the risk. Consequently, the no returns policy becomes less attractive. This observation is consistent with Che (1996).

[28] See Figure 2.3 for an illustration of the patterns.

Table 2.2: **Impact of consumers' risk aversion (v)**

ρ	c^a	v^b	σ^{M*}	w^*_{NMR}	Π^{M*}_{NMR}	$\sigma^R_i\|_{\text{NMR}}$	$\sigma^{R*}_i\|_{\text{NMR}}$	w^*_{MR}	Π^{M*}_{MR}	$\sigma^R_i\|_{\text{MR}}$	$\sigma^{R*}_i\|_{\text{MR}}$
0.01	0.1	$\frac{1}{2}$	***MR***	3.049	1.968	Pattern I	$\boldsymbol{\{CR, CR\}}$	3.051	**1.969**	Pattern I	$\boldsymbol{\{CR, CR\}}$
		0.2	***MR***	3.049	1.968	Pattern I	$\boldsymbol{\{CR, CR\}}$	3.051	**1.969**	Pattern I	$\boldsymbol{\{CR, CR\}}$
	0.3	$\frac{1}{2}$	***NMR***	3.149	**1.8364**	Pattern I	$\boldsymbol{\{CR, CR\}}$	3.152	1.8361	Pattern I	$\boldsymbol{\{CR, CR\}}$
		0.2	***NMR***	3.143	**1.756**	Pattern I	$\boldsymbol{\{CR, CR\}}$	3.158	1.754	Pattern I	$\boldsymbol{\{CR, CR\}}$
0.05	0.1	$\frac{1}{2}$	***MR***	3.043	1.881	Pattern I	$\boldsymbol{\{CR, CR\}}$	3.053	**1.886**	Pattern I	$\boldsymbol{\{CR, CR\}}$
		0.2	***MR***	3.043	1.881	Pattern I	$\boldsymbol{\{CR, CR\}}$	3.053	**1.886**	Pattern I	$\boldsymbol{\{CR, CR\}}$
	0.3	$\frac{1}{2}$	***NMR***	3.143	**1.756**	Pattern I	$\boldsymbol{\{CR, CR\}}$	3.158	1.754	Pattern I	$\boldsymbol{\{CR, CR\}}$
		0.2	***NMR***	3.143	**1.756**	Pattern I	$\boldsymbol{\{CR, CR\}}$	3.158	1.754	Pattern I	$\boldsymbol{\{CR, CR\}}$
0.5	0.1	$\frac{1}{2}$	***MR***	2.925	0.912	Pattern II	$\boldsymbol{\{CR, CR\}}$	3.1	**0.961**	Pattern I	$\boldsymbol{\{CR, CR\}}$
		0.2	***MR***	2.925	0.912	Pattern I	$\boldsymbol{\{CR, CR\}}$	3.1	**0.961**	Pattern I	$\boldsymbol{\{CR, CR\}}$
	0.3	$\frac{1}{2}$	***NMR***	3.025	**0.849**	Pattern I	$\boldsymbol{\{CR, CR\}}$	3.3	0.833	Pattern I	$\boldsymbol{\{CR, CR\}}$
		0.2	***NMR***	3.025	**0.849**	Pattern I	$\boldsymbol{\{CR, CR\}}$	3.3	0.833	Pattern I	$\boldsymbol{\{CR, CR\}}$

[a] Other parameters include $k = 0$, $\eta = \frac{1}{4}$, and $\tau = 0.2$.
[b] When $v = \frac{1}{2}$, consumers are risk-neutral; when $v < \frac{1}{2}$, consumers are risk-averse.

Observation 2.2 *Changes in consumer's level of risk-aversion does not affect the choice of manufacturer's returns policy or the choice of consumer returns policies in equilibrium.*

Given a reasonable rate of returns as in our analysis, i.e. $\rho \leq \frac{1}{2}$, the retailers are likely to accept returns from consumers even when consumers are risk-neutral. Raising the level of risk-aversion serves to only strengthen this result. When consumer returns are accepted under both *NMR* and *MR*, the choice of manufacturer's returns policy is based on the magnitude of production cost as noted in Proposition 2.2.

2.6 Conclusion and future research

This study extends the literature on returns policies by investigating the strategic interaction between the returns policies of a manufacturer and its retailers. We show that the manufacturer's returns policy can affect the intensity of retail competition even under Cournot-like conditions. Essentially, when consumer returns are accounted for, the manufacturer's returns policy shifts the responsibility of unsold inventory away from the retailers; consequently, they order more stock and price more competitively.

Further, we find that the manufacturer's returns policy affects the retailers' pricing strategies when the marginal cost of stock-outs is low. When unsold inventory can be returned to the manufacturer, the retailers order stock and set the retail prices to clear stock according to the initial demand. In contrast, when no returns are accepted and the cost of stock-outs is low, the retailers will price to clear stock according to the net demand to avoid bearing the cost of unsold inventory. This strategy results in lower levels of stock and higher retail prices. Such behavior creates an opportunity for the manufacturer to accept returns from retailers and provide them with appropriate incentives to improve channel profit. That opportunity does not arise when consumer returns are not accommodated in the analysis (see e.g., Padmanabhan and Png 1997, 2004; Wang 2004).

Interestingly, if the marginal cost of stock-outs is relatively high, our analysis reveals that the manufacturer's benefit of accepting returns is nullified by the cost of unsold inventory. Under such conditions, the manufacturer has no incentive to accept returns since that policy neither affects the nature of retail competition nor the pricing and stocking strategies of the retailers. Our analysis also provides insights about when retailers may accept returns from consumers. In addition to the role of risk-sharing and consumer screening (Che 1996), these policies can serve as a competitive tool at the retail level. In fact, retail profitability can be hurt under a no returns policy from a higher own-price-sensitive and lower cross-price-sensitive demand structure.

It is worth noting that our model does not consider competition at the manufacturer level. One may expect that competition provides additional incentives for the manufacturer to accept returns. In addition, we do not consider the salvage value of returned products and the cost of managing returns by either the manufacturer or the retailers. In practice, returned products after the selling period can be re-processed to be sold in secondary markets; this can help recover the cost of returns. To highlight the interaction between the returns policies of a manufacturer and its retailers, our analysis focused on a relatively streamlined model. We hope that our effort will spark further work in this important area of research.

Table 2.3: **Retailers' pricing and stocking strategies under *NMR* and low stock-out cost**

Combination 1: $\{\sigma^M, \sigma_1^R, \sigma_2^R\} = \{NMR, NCR, NCR\}$	**Combination 2**:[29] $\{\sigma^M, \sigma_1^R, \sigma_2^R\} = \{NMR, NCR, CR\}$	**Combination 4**: $\{\sigma^M, \sigma_1^R, \sigma_2^R\} = \{NMR, CR, CR\}$
$p^i_{0,0,0}$[30] $= \frac{3+6\tau v+8\tau w}{14\tau}$ $s^i_{0,0,0} = \frac{4(1+2\tau v-2\tau w)}{7}$ $\Pi^i_{0,0,0} = \frac{6(1+2\tau v-2\tau w)^2}{49\tau}$ $\pi^M_{0,0,0} = \frac{8(1+2\tau v-2\tau w)(w-c)}{7}$	$p^1_{0,1,0} = \frac{3[5-\tau(1-12v)+2\mu](1-\rho)+3\tau\rho\eta+37\tau(1-\rho)w}{70\tau(1-\rho)}$ $p^2_{1,0,0} = \frac{3[5+2\tau(3-v)-12\mu](1-\rho)+17\tau\rho\eta+23\tau(1-\rho)w}{35\tau(1-\rho)}$ $s^1_{0,1,0} = \frac{4\{[5-\tau(1-12v)+2\mu](1-\rho)+\tau\rho\eta-11\tau(1-\rho)w\}}{35(1-\rho)}$ $s^2_{1,0,0} = \frac{4\{[5+2\tau(3-v)-12\mu](1-\rho)-6\tau\rho\eta-4\tau(1-\rho)w\}}{35}$ $\Pi^1_{0,1,0} = \frac{6\{[5-\tau(1-12v)+2\mu](1-\rho)+\tau\rho\eta-11\tau(1-\rho)w\}^2}{1225\tau(1-\rho)^2}$ $\Pi^2_{1,0,0} = \frac{12\{[5+2\tau(3-v)-12\mu](1-\rho)-6\tau\rho\eta-4\tau(1-\rho)w\}^2}{1225\tau(1-\rho)}$ $\pi^M_{0,1,0} = (s^1_{0,1,0} + s^2_{1,0,0})(w-c)$	$p^i_{1,1,0} = \frac{3(1+\tau-2\mu)(1-\rho)+4\tau\rho\eta+4\tau(1-\rho)w}{7\tau(1-\rho)}$ $s^i_{1,1,0} = \frac{4[(1+\tau-2\mu)(1-\rho)-\tau\rho\eta-\tau(1-\rho)w]}{7}$ $\Pi^i_{1,1,0} = \frac{12[(1+\tau-2\mu)(1-\rho)-\tau\rho\eta-\tau(1-\rho)w]^2}{49\tau(1-\rho)}$ $\pi^M_{1,1,0} = \frac{8[(1+\tau-2\mu)(1-\rho)-\tau\rho\eta-\tau(1-\rho)w](w-c)}{7}$
Regularity conditions: (To ensure non-negative stocks) $w \le v + \frac{1}{2\tau} \overset{\text{def}}{=} w_1$	Regularity conditions: $w \le \min\{w_3, w_4\}$, where $w_3 \overset{\text{def}}{=} \frac{5-\tau(1-12v)+2\mu}{11\tau} + \frac{\rho\eta}{11(1-\rho)}$ $w_4 \overset{\text{def}}{=} \frac{5+2\tau(3-v)-12\mu}{4\tau} - \frac{3\rho\eta}{2(1-\rho)}$ $\eta < \frac{[5+2\tau(3-v)-12\mu](1-\rho)}{6\tau\rho} \overset{\text{def}}{=} \eta_2$ (to ensure $w_4 > 0$) Relabel: $w_3 \overset{\text{def}}{=} \underline{w}_N$	Regularity conditions: $w \le \frac{1+\tau-2\mu}{\tau} - \frac{\rho\eta}{1-\rho} \overset{\text{def}}{=} w_2$ $\eta < \frac{(1+\tau-2\mu)(1-\rho)}{\tau\rho} \overset{\text{def}}{=} \eta_1$ (to ensure $w_2 > 0$) Relabel: $w_2 \overset{\text{def}}{=} \overline{w}_N$
Demand functions: $q_i = 1 + 2\tau v - 3\tau p_i + \tau p_j$	Demand functions: R_1: $q_1 = 1 - \frac{\tau}{2} + 3\tau v + \mu - 3\tau p_1 + \frac{\tau}{2} p_2$ R_2: • Init. sales: $\hat{q}_2 = 1 + \frac{3\tau}{2} - \tau v - 3\mu - \frac{3\tau}{2} p_2 + \tau p_1$ • Net sales: $q_2 = (1-\rho)\hat{q}_2$	Demand functions: • Init. sales: $\hat{q}_i = 1 + \tau - 2\mu - \frac{3\tau}{2} p_i + \frac{\tau}{2} p_j$ • Net sales: $q_i = (1-\rho)\hat{q}_i$

Table 2.4: **Retailers' pricing and stocking strategies under *MR* and low stock-out cost**

Combination 5: $\{\sigma^M, \sigma_1^R, \sigma_2^R\}$ =$\{MR, NCR, NCR\}$	**Combination 6:**[31] $\{\sigma^M, \sigma_1^R, \sigma_2^R\}$ =$\{MR, NCR, CR\}$	**Combination 8:** $\{\sigma^M, \sigma_1^R, \sigma_2^R\}$ =$\{MR, CR, CR\}$
$p^i_{0,0,1} = \frac{3+6\tau v+8\tau w}{14\tau}$ $s^i_{0,0,1} = \frac{4(1+2\tau v-2\tau w)}{7}$ $\Pi^i_{0,0,1} = \frac{6(1+2\tau v-2\tau w)^2}{49\tau}$ $\pi^M_{0,0,1} = \frac{8(1+2\tau v-2\tau w)(w-c)}{7}$	$p^1_{0,1,1} = \frac{15-\tau(3-36v)+6\mu+37\tau w}{70\tau}$ $p^2_{1,0,1} = \frac{15+\tau(18-6v)-36\mu+23\tau w}{35\tau}$ $s^1_{0,1,1} = \frac{4[5-\tau(1-12v)+2\mu-11\tau w]}{35}$ $s^2_{1,0,1} = \frac{4[5+2\tau(3-v)-12\mu-4\tau w]}{35}$ $\Pi^1_{0,1,1} = \frac{6[5-\tau(1-12v)+2\mu-11\tau w]^2}{1225\tau}$ $\Pi^2_{1,0,1} = \frac{12(1-\rho)[5+2\tau(3-v)-12\mu-4\tau w]^2}{1225\tau}$ $\pi^M_{1,0,1} = [s^1_{0,1,1} + (1-\rho)s^2_{1,0,1}]w - (s^1_{0,1,1} + s^2_{1,0,1})c$	$p^i_{1,1,1} = \frac{3(1+\tau-2\mu)+4\tau w}{7\tau}$ $s^i_{1,1,1} = \frac{4(1+\tau-2\mu-\tau w)}{7}$ $\Pi^i_{1,1,1} = \frac{12(1-\rho)(1+\tau-2\mu-\tau w)^2}{49\tau}$ $\pi^M_{1,1,1} = \frac{8(1+\tau-2\mu-\tau w)[(1-\rho)w-c]}{7}$
<u>Regularity conditions</u>: (To ensure non-negative stocks) $w \leq v + \frac{1}{2\tau} \overset{\text{def}}{=} w_1$	<u>Regularity conditions</u>: $w \leq \min\{w_6, w_7\}$, where $w_6 \overset{\text{def}}{=} \frac{5-\tau(1-12v)+2\mu}{11\tau}$ $w_7 \overset{\text{def}}{=} \frac{5+2\tau(3-v)-12\mu}{4\tau}$ Relabel: $w_6 \overset{\text{def}}{=} \underline{w}_M$	<u>Regularity conditions</u>: $w \leq \frac{1+\tau-2\mu}{\tau} \overset{\text{def}}{=} w_5$ Relabel: $w_5 \overset{\text{def}}{=} \overline{w}_M$
<u>Demand functions</u>: $q_i = 1 + 2\tau v - 3\tau p_i + \tau p_j$	<u>Demand functions</u>: R_1: $q_1 = 1 - \frac{\tau}{2} + 3\tau v + \mu - 3\tau p_1 + \frac{\tau}{2}p_2$ R_2: • Init. sales: $\hat{q}_2 = 1 + \frac{3\tau}{2} - \tau v - 3\mu - \frac{3\tau}{2}p_2 + \tau p_1$ • Net sales: $q_2 = (1-\rho)\hat{q}_2$	<u>Demand functions</u>: • Init. sales: $\hat{q}_i = 1 + \tau - 2\mu - \frac{3\tau}{2}p_i + \frac{\tau}{2}p_j$ • Net sales: $q_i = (1-\rho)\hat{q}_i$

Table 2.5: **Retailers' pricing and stocking strategies under *NMR* and high η**

Combination 2:[a] $\{\sigma^M, \sigma_1^R, \sigma_2^R\} = \{NMR, NCR, CR\}$	**Combination 4:** $\{\sigma^M, \sigma_1^R, \sigma_2^R\} = \{NMR, CR, CR\}$
$\hat{p}^1_{0,1,0} = \frac{3[5-\tau(1-12v)+2\mu](1-\rho)+\tau(37-34\rho)w}{70\tau(1-\rho)}$	$\hat{p}^i_{1,1,0} = \frac{3(1+\tau-2\mu)(1-\rho)+4\tau w}{7\tau(1-\rho)}$
$\hat{p}^2_{1,0,0} = \frac{3[5+2\tau(3-v)-12\mu](1-\rho)+\tau(23-6\rho)w}{35\tau(1-\rho)}$	
$\hat{s}^1_{0,1,0} = \frac{4\{[5-\tau(1-12v)+2\mu](1-\rho)-\tau(44-48\rho)w\}}{35(1-\rho)}$	$\hat{s}^i_{1,1,0} = \frac{4[(1+\tau-2\mu)(1-\rho)-\tau w]}{7(1-\rho)}$
$\hat{s}^2_{1,0,0} = \frac{4\{[5+2\tau(3-v)-12\mu](1-\rho)-8\tau(2+\rho)w\}}{35(1-\rho)}$	
$\hat{\Pi}^1_{0,1,0} = \frac{6\{[5-\tau(1-12v)+2\mu](1-\rho)-\tau(11-12\rho)w\}^2}{1225\tau(1-\rho)^2}$	$\hat{\Pi}^i_{1,1,0} = \frac{12[(1+\tau-2\mu)(1-\rho)-\tau w]^2}{49\tau(1-\rho)}$
$\hat{\Pi}^2_{1,0,0} = \frac{12\{[5+2\tau(3-v)-12\mu](1-\rho)-2\tau(2+\rho)w\}^2}{1225\tau(1-\rho)}$	
$\hat{\pi}^M_{0,1,0} = (\hat{s}^1_{0,1,0} + \hat{s}^2_{1,0,0})(w-c)$	$\hat{\pi}^M_{1,1,0} = \frac{8[(1+\tau-2\mu)(1-\rho)-\tau w](w-c)}{7(1-\rho)}$
Regularity conditions: $w \leq \min\{\hat{w}_3, \hat{w}_4\}$, where $\hat{w}_3 \stackrel{\text{def}}{=} \frac{[5-\tau(1-12v)+2\mu](1-\rho)}{(11-12\rho)\tau}$, $\hat{w}_4 \stackrel{\text{def}}{=} \frac{[5+2\tau(3-v)-12\mu](1-\rho)}{2(2+\rho)\tau}$	Regularity conditions: $w \leq \frac{(1+\tau-2\mu)(1-\rho)}{\tau} \stackrel{\text{def}}{=} \hat{w}_2$

[a] By symmetry, the results of combination 3, i.e. $\{NMR, CR, NCR\}$, are similar to those of combination 2.

CHAPTER 3: OPTIMALITY OF GROUP-BUYING SERVICES

3.1 Introduction

With the advent of the Internet, electronic commerce has emerged. The business environment is changing rapidly and significantly—competition becomes more intense due to the lower barrier of entry and consumers are getting more powerful due to the explosion of social networks. These are both challenges and opportunities for firms in most industries. Firms have responded to (and taken advantage of) the changing business environment via creating numerous business models, such as on-line auction, direct on-line channels (Chiang, Chhajed and Hess 2003), third-party referral infomediaries (Chen et al. 2002), name-your-own-price channels (Fay 2004), and demand-aggregation mechanism (namely group-buying services; Anand and Aron 2003, and Kauffman and Wang 2001, 2002), among others. Most importantly, the employment of these innovative business models does have an impact on the strategic relationships between the parties of the distribution channel.

This chapter investigates the rationale—the benefits and costs—for using group-buying services, which can be seen as a response of the business community to the emerging trends of on-line shopping among consumers. This practice has a short but quite interesting history, especially in the B2C market. Upon its introduction in the late 1990s, the market has responded enthusiastically; it was predicted to be "the next big thing in e-commerce" (Adweek 2000). Many popular websites, such as Mercata.com, Mobshop.com, etc., were established and had attracted tens of thousands of participants. They were predicted to grow much faster than on-line auction services (such as Ebay.com; Business Week 2000). However, despite the rosy picture, many of them have failed after a few years of operation in the B2C market. But interestingly, the recent development of social networks has triggered a miracle revival of these services in China (they are called "tuangou" in Chinese, meaning team-buying; The Wall Street Journal 2006, 2008). Modified formats of group-buying services have also emerged recently in Europe and the US.

This phenomenon has attracted the interest of academic research, though modest. Anand and Aron (2003) provide an excellent survey of the various formats of group-buying, used in both B2B and B2C environments. A brief description of the group-buying concept is as follows: instead of a single posted price, the seller offers to customers a pricing schedule, which consists of many price tiers, depending on the total number of orders. For instance, using the example from Anand and Aron (2003), given the retail price of an electronic device being \$500, the unit price via group-buying will be \$480 if at least three units are demanded, \$450 if there is demand for five units or more, and so on. Most importantly, at the closing date of the sale, all consumers are paying the same price, irrespective of the price at the time they join group-buying.

Theoretical explanations of the rationale for using group-buying (versus posted-pricing) are offered by Anand and Aron (2003)—this mechanism serves as an effective price discrimination tool for firms to extract more market surplus when market demand is uncertain. They show that group-buying is only profitable if market demand is uncertain in a specific way: the demand curves in the high and low regimes must exhibit different levels of price sensitivity and intersect each other. While this work provides useful insights into the mechanism of group-buying services, a complete understanding of this phenomenon requires further research. Toward that goal, this essay explores alternative rationales for using group-buying services and provides a richer understanding of how this mechanism operates as a price-discrimination device.

Specifically, we complement the work of Anand and Aron (2003) by exploring other dimensions of market heterogeneity that determine the benefits and costs of group-buying services when they are used in addition to the traditional posted-pricing mechanism. In particular, we investigate consumer heterogeneity in their valuation for the product and the cost they incur when purchasing via different pricing mechanisms[29]. Further, note that in Anand and Aron (2003), the firm employs group-buying services as a replacement of the posted-pricing mechanism. In practice, however, retailers introduce the option of group-buying purchase in addition to the posted-pricing, i.e., both options are available to consumers for self-selection (The Wall Street Journal 2006).

We do so by developing an individual-level model in which consumers are heterogeneous in their valuations for the product as well as in their costs of purchasing it via group-buying. We find that even in the absence of market uncertainty, group-buying still serves as a price-discrimination device—consumers who have high valuation purchase the product in the traditional way, paying the high posted price while those who have low valuation purchase via group-buying and pay the low group-buying price.

However, it would require consumers' heterogeneity in both dimensions—valuation and cost of joining group-buying—to be profitable. Intuitively, in the absence of heterogeneity in consumers' valuation—and thereby, willingness-to-pay—, there is no benefit of market segmentation. Meanwhile, without sufficient heterogeneity in the cost of joining group-buying, it is too costly (and even infeasible in some instances) to employ this price-discrimination mechanism. For instance, if high-valuation consumers have too low the cost of joining group-buying, then the firm may need to significantly reduce the posted price to prevent them from purchasing the product via group-buying. Finally, the firm may have to compensate for consumers' cost of joining group-buying. This means that it must incur some cost when employing these services. Under some conditions, these costs may exceed the benefits from price discrimination. The analysis characterizes the conditions under which group-buying

[29]In practice, consumers must spend extra time and efforts when purchasing via group-buying. Further, since these services are quite novel, some consumers may find it too costly to join group-buying, resulting in heterogeneity in terms of this cost among them.

is (and is not) profitable to the firm. As an implication of our findings, one can explain the role of social-networking in the current development of group-buying services—the increasing popularity of social-networking strengthens the power of consumers and lowers their cost of joining various on-line communities. Group-buying services clearly benefit from this emerging trend.

The organization of this chapter is as follows. First, we provide a review of the related literature—group-buying and price discrimination—in §3.2. Next, §3.3 describes the structure of the market and develops the individual-level model of our analysis. Next, we summarize the results in §3.4 and discuss their implications in §3.5.

3.2 Literature review

Past research on group-buying, though modest, provides useful insights on consumers' behavior upon joining group-buying as well as how this pricing mechanism may benefit the firm. Kauffman and Wang (2001, 2002) initiate academic research on group-buying by investigating consumers' bidding behavior. They identify different effects that impact the evolution of group-buying bids. These include (a) a positive participation effect, i.e., the number of existing orders exhibits a positive effect on the placement of new orders, (b) a price-drop effect, that exists when the number of orders approaches the next tier, and (c) an ending effect, showing a significant increase in orders, placed by the end of the auction cycle.

At the aggregate level, Anand and Aron (2003) are the first to formalize the study of group-buying using an analytical model. They compare the performance of the group-buying mechanism to that of the traditional posted-pricing and identify the conditions, under which the former mechanism outperforms the latter. Their paper shows that group-buying is indeed a price-discrimination tool that allows the retailer (seller) to extract more surplus from the market, which exhibits demand heterogeneity. Specifically, demand heterogeneity is captured in this work by the uncertainty in the total demand of the entire market. It is the nature of this uncertainty that determines the profitability of group-buying services: only when uncertainty results in 'non-similar' demand curves in the high and low regimes (i.e., intersecting demand curves), employing group-buying is profitable.

Here, I extend the group-buying stream of research by exploring another dimension of market heterogeneity—consumers' valuation for the product—that explains the price-discrimination mechanism of group-buying. It is quite clear that this analysis, in its own nature, relates to the literature on price discrimination. The phenomenon of price discrimination has been identified a long time ago—in the first edition of Pigou's book, entitled "The Economics of Welfare", published in 1920. Later, it has attracted scholars in both economics and marketing, including, for instance, Phlips (1983), Spulber (1979), Tirole (1988), Narasimhan (1984), and others. Recently, two excellent review papers on price discrimination have been written (Armstrong 2006, Stole 2006), providing a complete picture of the economics of price discrimination.

Formally, price discrimination happens when "two varieties of a commodity are sold (by the same seller) to two buyers at different net prices" (Phlips 1983). The rationales for adopting price discrimination can be explained via the additional flexibility that the firm enjoys when marketing its product (Norman 1999); consequently, more surplus can be extracted from the market. The feasibility of price discrimination requires (a) some degree of market power (that allows the firm to charge a price not equal to the marginal cost), (b) ability to separate consumers into different segments, and (c) ability to prevent consumer arbitrage.

When the firm can observe all dimensions of consumers' heterogeneity and segment the market based on these characteristics, it may employ first-degree price discrimination by pricing each unit of the product precisely at the consumer's marginal willingness-to-pay (Spulber 1979). By doing so, the firm can extract the most surplus from consumers. (All surplus can be extracted in a monopolist setting.) However, first-degree price discrimination is not always preferred: Thisse and Vives (1988) show that the firms would collectively prefer uniform pricing strategies when the demand curve is highly elastic and the market is competitive.

In most instances, the firm only observes some of consumers' characteristics and therefore, cannot employ first-degree price discrimination. In this case, it may follow third-degree price discrimination. Obviously, this type of price discrimination does not allow the firm to extract all the surplus from consumers even under monopoly. In most cases, consumers are price discriminated based on their price elasticities and as a result, prices are higher in strong markets (i.e., with lower price elasticity) and lower in weak markets (i.e., with higher price elasticity; Robinson 1933, Borenstein 1985, and Holmes 1989). Most interestingly, in duopoly, firms may rank markets (as strong or weak) differently, resulting in 'best-response asymmetry'. In these settings, third-degree price discrimination may result in lower prices in all markets due to the fact that the 'market-stealing' effect (i.e., competition) is likely to dominate the 'rent-extraction' effect (Thisse and Vives 1988, and Stole 2006); consequently, firms may be worse off when price discriminating. Further, firms may leverage consumers' purchase history to price discriminate them based on their heterogeneous preferences for the product (e.g., see Fudenberg and Tirole 2000).

Finally, when the firm cannot directly separate consumers based on observable characteristics (i.e., first-degree as well as third-degree price discrimination is not feasible), it may rely on consumers' self-selection to segment the market. In this way, the firm follows second-degree price discrimination—it offers a schedule of prices to consumers for self-selection. This schedule of prices may be based on product quality (Mussa and Rosen 1978) or purchase volume, i.e, quantity (Maskin and Riley 1984). Other mechanisms, such as coupons (Narasimhan 1984), may also serve as a device for second-degree price discrimination. This is also the case when group-buying is offered.

By including an additional option—group-buying—for consumers to purchase the product, the retailer can indirectly price discriminate consumers in different market segments. They self-select the purchase mechanism (i.e., posted-pricing versus group-buying) based on their willingness-to-pay and cost of joining group-buying. Here, the price they pay is contingent on the purchase mechanism. Even in the setting, when each consumer demands only one unit of the product, this additional purchase option effectively creates a schedule of prices and allows the firm to extract more surplus from the market. In this perspective, this essay extends the literature on second-degree price discrimination, which has investigated pricing schedules based on quality (Mussa and Rosen 1978) and quantity (Maskin and Riley 1984).

3.3 The model

We consider a market served by a monopolist retailer, who has two options to set the price of its product: (a) It can set a single posted price, p^P, which is available to all customers; this is the traditional single posted-pricing strategy, denoted PP. (b) The retailer can follow a group-buying strategy (denoted GB), under which, in addition to the traditional posted-pricing, it allows consumers to purchase the product via group-buying. Here, we focus on a simple group-buying schedule, which consists of two price tiers, p_h and p_l ($p_h \geq p_l$). If the number of the group-buying orders, q, is less than a specific value, $\bar{q}$, then all group-buying consumers pay the high price, p_h. In contrast, if q exceeds $\bar{q}$, then they all pay the low price, p_l. In summary, under the GB strategy, the retailer offers to consumers a schedule of prices as follows:

$$
\begin{cases}
p^P \\
p^{GB} = \begin{cases} p_h & \text{if} \quad q < \bar{q}, \\ p_l & \text{if} \quad q \geq \bar{q}. \end{cases}
\end{cases}
\tag{3.1}
$$

On the demand side, the market consists of N consumers; each consumer is interested in purchasing at most one unit of the product and derives a valuation, v, for it. Without further loss of generality, we normalize the market size by setting $N = 1$. Further, we assume that a fraction, ϕ, of these consumers belongs to a high (H) segment and the remaining, $(1 - \phi)$, makes up the low (L) segment. In the L-segment, consumers' valuation for the product is uniformly distributed between 0 and 1, i.e., $v \sim U[0, 1]$. On the other hand, consumers in the H-segment have higher valuation on average; their valuation is uniformly distributed between v_0 and $v_0 + 1$, i.e., $v \sim U[v_0, v_0 + 1]$, where $v_0 \geq 0$. Figure 3.1 graphically illustrates the two market segments.

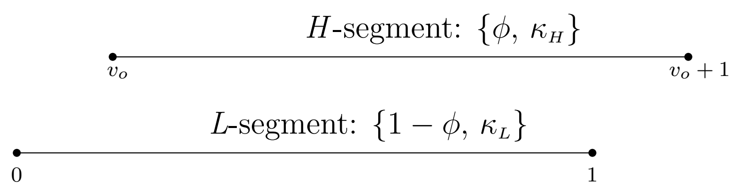


Figure 3.1: **The market structure**

Upon purchasing the product via the traditional posted-pricing mechanism, consumers pay p^P and receive the product immediately; they obtain a net utility: $v^P = v - p^P$. In contrast, if purchasing the product via group-buying (given that such option is available), then they must spend extra time and efforts in monitoring the auction, i.e., incur an extra cost, namely the group-buying cost. Regarding the distribution of this cost, we assume that consumers in the H-segment incur a high group-buying cost, κ_H, such that they never find it optimal to join group-buying; however, L-consumers have a lower group-buying cost, $\kappa_L < \kappa_H$, so that they may want to purchase the product via group-buying when available; this also requires $\kappa_L \leq 1$[30]. By doing so, L-consumers obtain a net utility, given by:

$$v^{G,L} = v - p^{GB} - \kappa_L. \tag{3.2}$$

Therefore, they will prefer the group-buying than the posted-pricing option if:

$$v^{G,L} \geq v^P \Leftrightarrow p^{GB} + \kappa_L \leq p^P. \tag{3.3}$$

Now, we derive the retailer's demand functions, $D^{j,i}$ (where $i \in \{H, L\}$ denotes the market segment, and j the retailer's pricing strategy—$j = P, G$ if the *PP* and *GB* strategies are employed, respectively). If following the *PP* strategy, then consumers who have $v^P \geq 0 \Leftrightarrow v \geq p^P$ will purchase the product. Thus, the demand functions in the H- and L-segments are given by:

$$D^{P,H} = \phi\,(1 + v_o - p^P), \text{ and} \tag{3.4}$$

$$D^{P,L} = (1 - \phi)(1 - p^P). \tag{3.5}$$

Together, total demand for the product under the *PP* strategy is:

$$D^P = D^{P,H} + D^{P,L} = 1 + \phi\, v_0 - p^P. \tag{3.6}$$

Next, consider the *GB* strategy. Note that since H-consumers always choose the posted-pricing option, the employment of the *GB* strategy implies that prices must be set so that L-consumers are motivated to join group-buying, i.e., according to (3.3). Given that (a)

[30] If $\kappa_L > 1$, then L-consumers, whose valuation for the product is $v \in [0, 1]$, will never join group-buying.

p^{GB} depends on q as described in (3.1), (b) the potential number of group-buying orders (at a given p^{GB}) is $(1-\phi)(1-p^{GB}-\kappa_L)$, and (c) consumers behave rationally based on this knowledge, the retailer has three options to set its pricing schedule as follows:

(A) Set p^P and $\bar{q}$ at high levels, i.e., $p^P \geq p_h + \kappa_L > p_l + \kappa_L$ and $\bar{q} > (1-\phi)(1-p_l-\kappa_L)$. In this case, knowing that the high price, p_h, will become effective due to high $\bar{q}$, only those L-consumers who have $v^{G,L}\big|_{p^{GB}=p_h} \geq 0$ will make the purchase (via group-buying). Demand in the L-segment is given by:

$$D^{G,L} = (1-\phi)(1-p_h-\kappa_L). \tag{3.7}$$

(B) Set high p^P and low $\bar{q}$, i.e., $p^P \geq p_h + \kappa_L > p_l + \kappa_L$ and $\bar{q} \leq (1-\phi)(1-p_l-\kappa_L)$. The low price is effective, inducing all consumers in the L-segment with $v^{G,L}\big|_{p^{GB}=p_l} \geq 0$ to purchase the product via group-buying. Therefore,

$$D^{G,L} = (1-\phi)(1-p_l-\kappa_L). \tag{3.8}$$

(C) Set p^P in the middle range, i.e., $p_h + \kappa_L > p^P \geq p_l + \kappa_L$, and $\bar{q}$ at a low level, i.e., $\bar{q} \leq (1-\phi)(1-p_l-\kappa_L)$. This schedule allows the low price, p_l, to prevail and generates the same demand as that in Option B (see equation 3.8).

In all cases, H-consumers always purchase the product at the posted price. Therefore, the demand function in the H-segment under the GB strategy, $D^{G,H}$, is identical to that under the PP strategy (see equation 3.4):

$$D^{G,H} = \phi\,(1+v_o-p^P). \tag{3.9}$$

The total demand under the GB strategy is $D^G = D^{G,H} + D^{G,L}$. These results are summarized in Table 3.1 and graphically illustrated in Figure 3.2.

Table 3.1: **Price options and demand functions under the GB strategy**

Pricing option	Specification	Demand functions
A	$p_l + \kappa_L < p_h + \kappa_L \leq p^P$ $\bar{q} > (1-\phi)(1-p_l-\kappa_L)$	$D^{G,H} = \phi\,(1+v_0-p^P)$ $D^{G,L} = (1-\phi)(1-p_h-\kappa_L)$
B	$p_l + \kappa_L < p_h + \kappa_L \leq p^P$ $\bar{q} \leq (1-\phi)(1-p_l-\kappa_L)$	$D^{G,H} = \phi\,(1+v_0-p^P)$ $D^{G,L} = (1-\phi)(1-p_l-\kappa_L)$
C	$p_l + \kappa_L \leq p^P < p_h + \kappa_L$ $\bar{q} \leq (1-\phi)(1-p_l-\kappa_L)$	$D^{G,H} = \phi\,(1+v_0-p^P)$ $D^{G,L} = (1-\phi)(1-p_l-\kappa_L)$

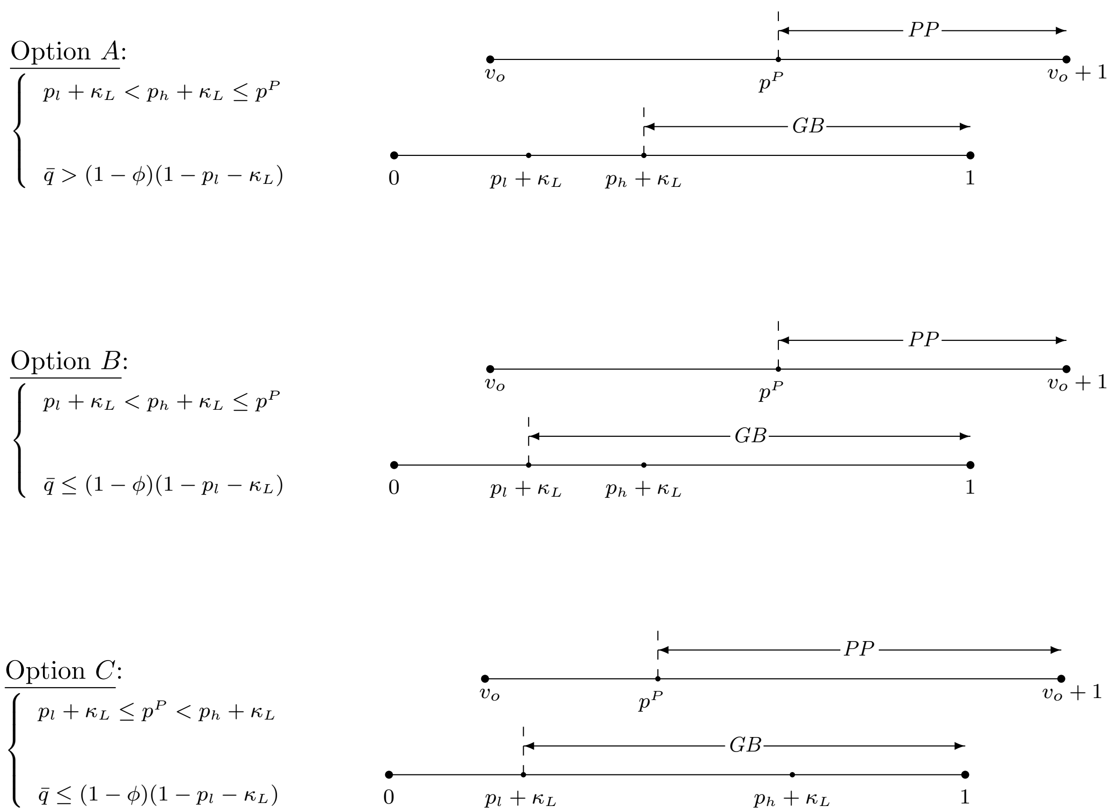


Figure 3.2: **Pricing options and demand structure under the GB strategy**

3.4 The retailer's choice of pricing strategy

The retailer decides whether to follow the PP or GB strategy by comparing the profitability of the two pricing strategies. Without loss of generality, we assume that the retailer incur zero production cost. In the following, we derive the optimal price(s) and the retailer's profits under each strategy. The choice of pricing strategy in equilibrium is discussed subsequently.

3.4.1 Profitability of the *PP* strategy

If employing the PP strategy, the retailer sets the single posted price, p^P, at the optimal level; it solves the following optimization problem:

$$\max_{p^P} \Pi^P \;=\; D^P\, p^P \;=\; (1+\phi\, v_0 - p^P)p^P. \tag{3.10}$$

This yields

$$p^{P^*} \;=\; \frac{1+\phi\, v_0}{2}, \text{ and} \tag{3.11}$$

$$\Pi^{P^*} \;=\; \frac{(1+\phi\, v_0)^2}{4}, \tag{3.12}$$

where the superscript * denotes the equilibrium solution in this case. Notes that the equilibrium posted price, p^{P^*}, increases as the H-segment becomes bigger (i.e., larger ϕ)—a bigger H-segment induces the retailer to charge a higher price to extract more surplus.

3.4.2 Prices and profitability of the *GB* strategy

Recall that the retailer has three options (i.e., Option A-C) to set its prices upon the employment of the GB strategy. In the following, we describe the retailer's optimization problem when choosing each of the three options along with the characterization of the solution. All the derivations are provided in Appendix B.

First, consider Option A. If choosing this option, the retailer generates demands in the H- and L-segments according to (3.9) and (3.7) respectively. The optimization problem is given by:

$$\max_{p^P,\, p_h,\, p_l,\, \bar{q}} \Pi^G \;=\; D^{G,H}\, p^P + D^{G,L}\, p_h \;=\; \phi\,(1 + v_0 - p^P)\, p^P + (1-\phi)(1 - p_h - \kappa_L)\, p_h \tag{3.13}$$

$$\text{s.t.} \quad \begin{cases} p^P \;\geq\; p_h + \kappa_L \;>\; p_l + \kappa_L \\ \bar{q} \;>\; (1-\phi)(1 - p_l - \kappa_L). \end{cases} \tag{3.14}$$

Notice that since p_l and $\bar{q}$ do not enter the profit function, the above problem is equivalent to:

$$\max_{p^P,\, p_h} \Pi^G \;=\; \phi\,(1 + v_0 - p^P)\, p^P + (1-\phi)(1 - p_h - \kappa_L)\, p_h \tag{3.15}$$

$$\text{s.t.} \quad p^P \;\geq\; p_h + \kappa_L, \tag{3.16}$$

given p_l and $\bar{q}$ taking any values that satisfy (3.14). The solution to this problem is then given by:

$$(a) \quad \begin{cases} p^{P^{**}} & = \frac{1+v_0}{2} \\ p_h^{**} & = \frac{1-\kappa_L}{2} \\ \Pi^{G^{**}} & = \phi\, \frac{(1+v_0)^2}{4} + (1-\phi)\, \frac{(1-\kappa_L)^2}{4}, \qquad \text{if } \kappa_L \;\leq\; v_0, \text{ and} \end{cases} \tag{3.17}$$

$$(b) \quad \begin{cases} \widehat{p}^P & = \frac{1+\phi\, v_0 + (1-\phi)\kappa_L}{2} \\ \widehat{p}_h & = \frac{1+\phi\, v_0 - (1+\phi)\kappa_L}{2} \\ \widehat{\Pi}^G & = \frac{[1-\phi\, v_0 - (1-\phi)\, \kappa_L]^2 + 4\,\phi\, v_0}{4}, \qquad \text{if } \kappa_L \;>\; v_0. \end{cases} \tag{3.18}$$

Next, if choosing Option B, then given the respective demand functions (i.e., equation 3.8 and 3.9), the retailer solves the following problem:

$$\max_{p^P,\,p_h,\,p_l,\,\bar{q}} \Pi^G \;=\; D^{G,H}\, p^P + D^{G,L}\, p_l \;=\; \phi\,(1+v_0-p^P)\,p^P + (1-\phi)(1-p_l-\kappa_L)\,p_l \tag{3.19}$$

$$\text{s.t.} \quad \begin{cases} p^P \;\geq\; p_h+\kappa_L \;>\; p_l+\kappa_L \\ \bar{q} \;\leq\; (1-\phi)(1-p_l-\kappa_L), \end{cases} \tag{3.20}$$

which is equivalent to:

$$\max_{p^P,\,p_l} \Pi^G \;=\; \phi\,(1+v_0-p^P)\,p^P + (1-\phi)(1-p_l-\kappa_L)\,p_l \tag{3.21}$$

$$\text{s.t.} \quad p^P \;>\; p_l+\kappa_L, \tag{3.22}$$

given that p_h and $\bar{q}$ are chosen according to (3.20). In Appendix B, we show that this option is feasible only when $\kappa_L \leq v_0$ and the solution is characterized by:

$$\begin{cases} p^{P^{**}} & = \frac{1+v_0}{2} \\ p_l^{**} & = \frac{1-\kappa_L}{2} \\ \Pi^{G^{**}} & = \phi\,\frac{(1+v_0)^2}{4} + (1-\phi)\,\frac{(1-\kappa_L)^2}{4}. \end{cases} \tag{3.23}$$

Finally, consider Option C. The optimization problem is then given by:

$$\max_{p^P,\,p_h,\,p_l,\,\bar{q}} \Pi^G \;=\; D^{G,H}\, p^P + D^{G,L}\, p_l \;=\; \phi\,(1+v_0-p^P)\,p^P + (1-\phi)(1-p_l-\kappa_L)\,p_l \tag{3.24}$$

$$\text{s.t.} \quad \begin{cases} p_h+\kappa_L \;>\; p^P \;\geq\; p_l+\kappa_L \\ \bar{q} \;\leq\; (1-\phi)(1-p_l-\kappa_L) \end{cases} \tag{3.25}$$

$$\Leftrightarrow \quad \max_{p^P,\, p_l} \Pi^G \;=\; \phi\,(1+v_0-p^P)\,p^P + (1-\phi)(1-p_l-\kappa_L)\,p_l \tag{3.26}$$

$$\text{s.t.} \quad p^P \;\geq\; p_l + \kappa_L, \tag{3.27}$$

given that p_h and $\bar{q}$ satisfy (3.25). The solution to this problem is as follows:

$$(a) \quad \begin{cases} p^{P^{**}} & = \frac{1+v_0}{2} \\ p_l^{**} & = \frac{1-\kappa_L}{2} \\ \Pi^{G^{**}} & = \phi\,\frac{(1+v_0)^2}{4} + (1-\phi)\,\frac{(1-\kappa_L)^2}{4}, \qquad \text{if } \kappa_L \;\leq\; v_0, \text{ and} \end{cases} \tag{3.28}$$

$$(b) \quad \begin{cases} \widehat{p}^P & = \frac{1+\phi\,v_0+(1-\phi)\kappa_L}{2} \\ \widehat{p}_l & = \frac{1+\phi\,v_0-(1+\phi)\kappa_L}{2} \\ \widehat{\Pi}^G & = \frac{[1-\phi\,v_0-(1-\phi)\,\kappa_L]^2+4\,\phi\,v_0}{4}, \qquad \text{if } \kappa_L \;>\; v_0. \end{cases} \tag{3.29}$$

First, notice that the result of Option B could be combined with that of Option C, which allows p_h to take any value in a wider range, i.e., $p_h > p_l^{**}$. Next, given a specific condition (i.e., κ_L), the retailer earns identical profit whether it sets the prices according to Option A, B, or C. For instance, if $\kappa_L \leq v_0$, while the effective group-buying price is p_h in Option A and p_l in Option C, both are set at an identical level, i.e., $p_h^{**} = p_l^{**}$. Given this observation, we summarize the characteristics of the pricing schedule (including the posted price, p^P, and the effective group-buying price, p^{GB}) and the profits of the GB strategy to the retailer in Lemma 3.1.

Lemma 3.1 *Upon the employment of the GB strategy,*

(a) if $\kappa_L \leq v_0$, then the optimal pricing schedule is characterized by:

$$\begin{cases} p^{P^{**}} & = \frac{1+v_0}{2} \\ p^{GB^{**}} & = \frac{1-\kappa_L}{2}; \end{cases} \tag{3.30}$$

the retailer then earns $\Pi^{G^{**}} = \phi \, \frac{(1+v_0)^2}{4} + (1-\phi) \, \frac{(1-\kappa_L)^2}{4}$.

(b) Otherwise, if $\kappa_L > v_0$, then the retailer sets its prices according to:

$$\begin{cases} \widehat{p}^{P} & = \frac{1+\phi \, v_0 + (1-\phi)\kappa_L}{2} \\ \widehat{p}^{GB} & = \frac{1+\phi \, v_0 - (1+\phi)\kappa_L}{2}, \end{cases} \tag{3.31}$$

and earns $\widehat{\Pi}^{G} = \frac{[1-\phi \, v_0 - (1-\phi) \, \kappa_L]^2 + 4 \, \phi \, v_0}{4}$.

This implies that the group-buying mechanism operates as a price-discrimination device: the firm has the posted price to target at consumers in the H-segment and the group-buying schedule to serve consumers in the L-segments; these prices are set at the optimal levels, reflecting the respective willingness-to-pay of consumers in the two segments. Most importantly, since the group-buying option is costly to consumers, the retailer has to (partially) compensate them for this cost; compensation is implemented via lowering the group-buying price. Further, if this cost becomes too big (i.e., exceeds v_0), then the *GB* strategy requires the prices in equilibrium being bound by the incentive constraints of L-consumers. (Otherwise, L-consumers will purchase via the posted-pricing option.)

3.4.3 Pricing strategy in equilibrium

Here, we compare the profitability of the PP and GB strategies (see Appendix L); the result is summarized in Proposition 3.1 and depicted graphically in Figure 3.3.

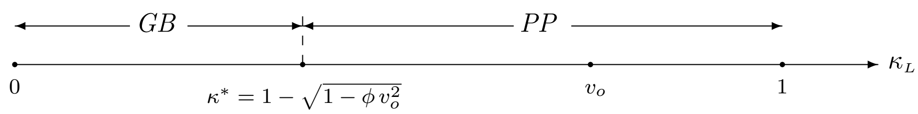


Figure 3.3: **Pricing strategy in equilibrium**

Proposition 3.1 *The retailer's pricing strategy depends on the cost that consumers in the L-segment incur upon joining group-buying,* κ_L*. If* κ_L *is not too big, i.e.,* $\kappa_L \leq \kappa^*$*, then the retailer will optimally introduce group-buying services in addition to the posted-pricing mechanism. In contrast, if* κ_L *is too big, i.e.,* $\kappa_L > \kappa^*$*, then it is better off by selling the product via the traditional posted-pricing mechanism.*

As discussed above, the group-buying mechanism serves as a price-discrimination device. It allows the retailers to extract more surplus from both the H- and L-segments via 'two' prices, i.e., the posted price and the group-buying price. However, it is costly to the retailer to employ this mechanism; this cost is proportional to κ_L (see Lemma 3.1). The retailer, therefore, must consider the trade-off between the benefit and the cost of price-discrimination when choosing between the two pricing strategies. If κ_L is sufficiently small, then the benefit of price-discrimination exceeds the cost and the retailer will choose the GB strategy in equilibrium. As κ_L increases, the cost of following the GB strategy increases; the retailer will be better off by employing the traditional PP strategy when κ_L exceeds a specific threshold, κ^*.

Additional investigation of κ^* reveals that as the size (ϕ) and (or) valuation (v_0) of the H-segment increase, then holding all else constant, it may become more profitable for the retailer to follow the GB rather than the PP strategy. This is due to the effect of these two factors on the profitability of price discriminating the two segments. Note that price-discrimination is essentially market segmentation and its profitability is positively driven by the difference in size (ϕ) and valuation (v_0) (which reflects willingness-to-pay) of consumers in these segments. Further, it can be seen that an increase in ϕ also results in a decrease in the cost of employing the group-buying mechanism (since as the size of the H-segment increases, the size of the L-segment decreases).

Finally, recall that the feasibility of the GB strategy requires heterogeneity in the cost of joining group-buying by consumers in the two segments (κ_H versus κ_L). Note that we made the assumption that κ_H is significantly large, making the group-buying mechanism unattractive to all consumers in the H-segment. It should be clear that this heterogeneity is the supporting condition, required by the feasibility of the GB strategy. In the absence of this heterogeneity, the employment of the GB strategy requires additional cost to separate the two segments. This leads to the following corollary:

Corollary 3.1 *In the absence of heterogeneity in the valuation and(or) group-buying cost among consumers, the retailer never finds the group-buying mechanism profitable.*

The explanation is straightforward. In the absence of heterogeneity in valuation between the two market segments, (i.e, $v_0 = 0$, for instance), there is obviously no benefit of segmentation. Second, if there is no heterogeneity in the group-buying cost, then segmentation would be more costly, and even unfeasible in most instances; consumers should be given sufficient incentives to choose the desired purchase mechanism. Here, heterogeneity in the cost of joining group-buying is necessary for this price-discrimination device to be operative.

3.5 Conclusion and future research

This chapter enriches our understanding of how group-buying operates and benefits the firm as a second-degree price-discrimination device. We show that the introduction of this mechanism allows the firm to price discriminate consumers who are heterogeneous in their valuation for the product, and therefore, extract more surplus from the market. However, the employment of this demand-aggregation mechanism is costly to the firm—it has to compensate consumers for the cost they incur when purchasing the product via group-buying. Finally, our analysis suggests that while consumers' heterogeneity in the cost of joining group-buying is required for group-buying to be feasible, heterogeneity in consumers' valuation for the product is needed for the profitability of this device.

Our results also helps understand the success/failure of group-buying services. When first introduced in the late 1990s, group-buying was a very new concept to consumers. For this and many other reasons, group-buying was perceived as an unattractive option; it seems to be too costly for consumers to join (The Wall Street Journal 2008). At that time, the employment of this demand-aggregation mechanism would require too much compensation and has proved to be unprofitable. When social-networking emerges and becomes popular, the concept of on-line social communities is familiar and so is that of on-line shopping communities, such as group-buying. It becomes less costly and hence, more profitable for retailers to employ.

In this study, the impact of group-buying on the profitability of the other channel member, i.e., the manufacturer, is not considered. This is the direction of investigation of the following chapter in this dissertation. Further, we do not consider competition in this analysis. While the benefit of group-buying services seems to be clear under monopoly, i.e., in our setting as well as in Anand and Aron (2003), it is not clear when the market is competitive and further research is needed to investigate the competitive reactions of channel members when group-buying is used. The use of group-buying may help the retailers to compete in the downstream market since they now have an additional distribution option to attract consumers who are price-sensitive. On the other hand, group-buying may be detrimental to manufacturers since it may raise the level of price competition among different brands and lower the margin and profitability to the manufacturers as suggested in the popular press (The Wall Street Journal 2006). This and other aspects of group-buying are interesting topics for future academic research.

CHAPTER 4: THE ROLE OF GROUP-BUYING IN CHANNEL COORDINATION

4.1 Introduction

The previous chapter and past research (Anand and Aron 2003) have provided the insights on the rationale for using group-buying services by retailers—this demand-aggregation mechanism serves as a price-discrimination device, allowing the retailers to extract more surplus from the market. However, it remains unclear about the impacts of this pricing mechanism on the profitability of other channel members (e.g., the manufacturer), and most importantly, the issue of channel coordination. For instance, given that group-buying offers significant discounts and attracts consumers in the price-sensitive segment, its employment should generate more sales, and thus, should be supported by the manufacturer. In practice, this is not always the case; we notice that retailers' engagement in this novel pricing mechanism has triggered different reactions from manufacturers. Some of them (e.g., Estee Lauder, Cartier, and BMW) even discourage their retailers from engaging in group-buying services (The Wall Street Journal 2006). Unfortunately, these *recommendations* are not always followed by the retailers. A few questions of interest to marketing managers then arise: How can we explain these conflicting behaviors of channel members—the manufacturer and the retailer? How does group-buying affect channel performance? How can the manufacturer leverage group-buying services to improve channel coordination? These questions motivate the third study in this dissertation.

Accordingly, this chapter explores how group-buying can help coordinate the behaviors of different members of the distribution channel. There are two central questions in this analysis: (1) When does the manufacturer have the incentive to induce its retailer(s) to offer group-buying services? (2) How may group-buying improve the efficiency of the distribution channel? The study addresses these two questions by analyzing a setting in which a manufacturer distributes its product to consumers through a retailer and asymmetric information may exist between these two parties—the retailer may be better informed about the market condition than the manufacturer. Further, the market consists of two consumer segments—a high- and a low-valuation segment; consumers in the high segment are less price-sensitive than those in the low segment—and the retailer may employ group-buying services, in addition to the conventional posted-pricing mechanism, in distributing the product to consumers. The relationship between these two parties is established via a (menu of) contract(s), whose terms include a pricing mechanism—posted-pricing or group-buying—, a wholesale price, and a fixed fee.

Regarding the type of asymmetric information, this study investigates three settings where asymmetric information may exist regarding: (1) the relative sizes of the two consumer segments, (2) price sensitivity of consumers in the high segment, and (3) price sensitivity of those in the low segment. In the first setting, while the size of the entire market is of common knowledge, the information about the size of the high segment, relative to that of the low segment, is unknown to the manufacturer. In the latter settings, the manufacturer is less informed about consumers' price sensitivities. More specifically, we assume that two realizations of the market condition, including a high and a low state regarding the focal market characteristics, may arise—when the market is in the high (low) state, the size of the high segment is relatively large (small) in the first setting and consumers' price sensitivity is high (low) in the latter two settings. This information is available to both parties. In addition, the manufacturer also has a prior knowledge of the probability that each of these market states may arise. However, only the retailer observes the actual realization observes the actual realization of the market state when asymmetry in information exists.

As in every principal-agent relationship (the manufacturer plays the role of the principal and the retailer is the agent), asymmetry in information may give rise to an adverse-selection problem. For instance, the manufacturer may not know the precise distribution of high- and low-valuation consumers in the retailer's local market (i.e., relative sizes of the consumer segments); in contrast, the retailer has a much clearer understanding of the distribution of consumer types. Now, suppose that the size of the high-valuation segment is large, relative to that of the low-valuation segment. This market condition may allow the retailer to earn more profit by employing group-buying; however, the retailer may not reveal this to the manufacturer and can seek to leverage its informational advantage. Such asymmetry in information may also exist regarding other market parameters, such as price sensitivity of consumers in the different segments.

In general, under asymmetric information, the retailer earns informational rents (Myerson 1979), and from the manufacturer's perspective, this has a negative impact on the efficiency of the distribution channel. The analysis shows that by including a term on pricing mechanism (i.e., leveraging the use of group-buying) in the contract, the manufacturer has an opportunity to better coordinate the distribution channel. This arrangement can help align the interests of the two parties (to different extents, depending on the type of asymmetric information), and therefore, reduces the informational rents of the retailer under asymmetric information.

Intuitively, the profitability of posted-pricing and group-buying to the retailer depends on the state of the market—for instance, given that group-buying serves as a price-discrimination device (which provides more benefits when the two consumer segments are more distinct), this pricing mechanism is more profitable when the market happens to be in: (1) the high state regarding the relative size of the high-valuation segment, (2) the low state regarding price sensitivity of consumers in the high-valuation segment, and (3) the high state regarding price sensitivity of those in the low-valuation segment (in these market conditions, the high- and low-valuation segments more distinct from each other in the respective dimensions). Therefore, the retailer's choice of pricing mechanism depends on the actual realization of the market condition. As a consequence, the pricing mechanism when included in the contract serves as a *sensor* of the market condition, which helps the manufacturer *detect* the state of the market via the retailer's choice of contract. In particular, the availability of group-buying provides additional flexibility to the retailer in setting prices; eventually, it could extract more surplus from the market by choosing the contract with an appropriate pricing mechanism. This means that the market itself provides some incentive (via this extra surplus) for the retailer to reveal its private information. When the retailer has less incentive to hide its private information (i.e., mis-inform the manufacturer about the market condition), the latter can reduce the informational rents of the former.

Most interestingly, the manufacturer can induce the retailer to reveal its private information without paying any informational rents when asymmetric information exists regarding the relative sizes of the two segments, and therefore, regains the first-best profitability (i.e., the profit it would earn under full information). Notice that the profitability of posted-pricing and group-buying (and consequently, the retailer's choice between these two pricing mechanisms) depends on the state of the market—for instance, it is more profitable to price discriminate (via group-buying) when the size of the high-valuation segment is large, relative to that of the low-valuation segment (since they are more distinct from each other). Further, under a posted-pricing contract, the retailer's profitability is unaffected by the relative sizes of the two consumer segments—no matter which segment (high- or low-valuation) is relatively larger in size, the profitability of the posted-pricing contract remains the same and depends on the total demand of the two segment (which is fixed and known to the manufacturer). This allows the manufacturer to extract all the surplus from the retailer whenever the posted-pricing contract is chosen. Consequently, the retailer is not doing any better by choosing the posted-pricing contract when it is not desired by the manufacturer. On the other hand, the group-buying contract is more profitable when the size of the high-valuation segment is relatively large (as discussed above); therefore, it is desired by the manufacturer under this market condition and exhibits a correspondingly higher fixed fee. Here too, the retailer has no incentive to choose this contract when it is not desired by the manufacturer—doing so implies paying a high fixed fee while earning a low gross-profit (due to the high-valuation segment being relatively small in size). In summary, the retailer always chooses the contract desired by the manufacturer in each market state, and reveals its private information about the market condition. As a consequence, it earns no informational rents.

Next, when asymmetric information exists regarding price sensitivities of consumers in the high- and low-valuation segments, the retailer does benefit from its private information and earn informational rents. This is due to the fact that the profitability of both the posted-pricing and group-buying mechanisms is low (high) when consumers' price sensitivity happens to be high (low). In this setting, the full-information contract designed for the high (low) state would exhibit too low (high) a fixed fee. This motivates the retailer to choose the undesired contract when the market is in the low state—by doing so, it pays a lower fixed fee (i.e., that of the contract designed for the high state) while earning a higher profit due to the market being in the low state. To prevent this from happening, the manufacturer has to pay some informational rents to the retailer. Nevertheless, these second-best contracts exhibit lower informational rents than those contracts that do not allow the manufacturer to specify the appropriate pricing mechanism. Notice that by choosing an appropriate pricing mechanism, the firm (retailer) earns higher profits; therefore, a contract of the former type (i.e., with a term on pricing mechanism) would allow the manufacturer to charge a higher fixed fee than that of the latter type. Further, the undesired contract of the former type clearly exhibits an *inappropriate* pricing mechanism, and thus, is less profitable to the retailer (compared to the undesired contract of the latter type, wherein the retailer is not obligated to any pricing mechanism). This higher fixed fee and lower profit reduce the retailer's incentive to choose the undesired contract; consequently, the retailer earns lower informational rents.

Interestingly, specifying the pricing mechanism could be considered as a type of performance requirement as in Desiraju and Moorthy (1997). From this perspective, this study extends their work (which examines the relative benefits of price - and/or effort - requirements) by showing that requirements on pricing mechanism—posted-pricing and group-buying—may also improve channel coordination by aligning the interests of the manufacturer and the retailer when asymmetry in information between the two parties exists.

Further, the findings in this chapter suggest that group-buying may be (more or less) profitable to the manufacturer. The manufacturer should not be too quick to (ineffectively) discourage the use of this demand-aggregation mechanism by its retailers, even if it appears to create a downward pressure on the wholesale price (which may negatively affect profit margin; The Wall Street Journal 2006). To the contrary, they could benefit from leveraging group-buying. By inducing group-buying, not only the profitability of the entire channel is improved but also the share of profit to the manufacturer may increase due to the reduction in informational rents of the retailers under asymmetric information.

This chapter is organized as follows. In §4.2, I review the related literature. §4.3 describes the model of the analysis. The characteristics of the full- and asymmetric-information (menu of) contracts in the first setting of asymmetric information (i.e., when asymmetric information may exist regarding the relative sizes of the two segments) are provided in §4.4 and 4.5. The extended analysis investigating the second and third setting of asymmetric information (i.e, regarding price sensitivities of consumers in the high and low segments) follows in §4.6. The chapter concludes in §4.7.

4.2 Literature review

Past research on group-buying, though modest, has provided useful insights on how this price-discovery mechanism may evolve as well as how it may benefit the firm. Kauffman and Wang (2001, 2002) initiate academic research on this business model by investigating consumers' bidding behavior. They identify different effects that impact the evolution of group-buying bids. These include (a) a positive participation effect, i.e., the number of existing orders exhibits a positive effect on the placement of new orders, (b) a price-drop effect, that exists when the number of orders approaches the next tier, and (c) an ending effect, showing a significant increase in orders, placed by the end of the auction cycle.

At the aggregate level, Anand and Aron (2003) are the first to formalize the study of group-buying using an analytical model. They compare the performance of the group-buying mechanism to that of the traditional posted-pricing and identify the conditions, under which the former mechanism outperforms the latter. Their paper shows that group-buying is indeed a price-discrimination tool that allows the retailer (seller) extracts more surplus from the market, which exhibits demand heterogeneity. Specifically, demand heterogeneity is captured in this work by the uncertainty in the total demand of the entire market. It is the nature of this uncertainty that determines when group-buying is profitable—only when uncertainty results in 'non-similar' demand curves in the high and low regimes (i.e., intersecting demand curves), the employment of group-buying is profitable.

This essay extends the group-buying stream of research in the following way. Given that group-buying could be used by the retailer as a price-discrimination device, it provides the insights on how the manufacturer can leverage this pricing mechanism to improve channel coordination when information about the market condition is asymmetric among the members of the distribution channel. In doing so, the essay integrates the streams of research on channel coordination, information economics and price discrimination. In the following, we briefly review each of these three relevant streams and discuss how our work fits in each of them.

The literature on channel coordination spans decades of research (e.g., see Spengler 1950, Jeuland and Shugan 1983, McGuire and Staelin 1986, Weng 1995, Desiraju and Moorthy 1997, Chiang, Chhajed and Hess 2003), and continues to be an important topic, particularly when the market is changing rapidly as a result of technology improvement. A variety of remedies to improve the performance of the distribution channel has been suggested, including contract design (Jeuland and Shugan 1983), implicit understanding from repeated interaction (Shugan 1985), franchising (Lal 1990), quantity discount (Ingene and Parry 1995), performance requirements (Desiraju and Moorthy 1997), cooperative advertising (Corstjens and Lal 1989), pull price promotion (Gerstner and Hess 1995), and direct channel (Chiang, Chhajed and Hess 2003) among others. Most recent research focuses on the employment of innovative business models to attract and better serve customers as well as to improve the profitability of the entire channel. For instance, Chiang, Chhajed, and Hess (2003) study the impact of the introduction of direct on-line channel by the manufacturer on the retailers' pricing decisions. They show that by adding a dual channel, the manufacturer induces the existing channel (i.e., the retailers) to expand sales via a more efficient pricing strategy. Another example of using communication innovation is Chen, Iyer, and Padmanabhan (2002). They investigate how Internet referral services, offered by a third party, affect competition among retailers and their profitability.

This essay is similar to, and distinctive from, these works in the following ways. It studies the role of group-buying, which is an emerging on-line business model, in better distributing products to consumers. The use of this pricing mechanism, in addition to the traditional posted-pricing, could be considered as an introduction of a new channel (compared to the direct channel). However, this mechanism is employed by the retailer, not the manufacturer as in direct channel. Nevertheless, we show that the manufacturer can leverage group-buying to improve the channel's performance (via a different mechanism though) and extract more surplus from the retailer—it helps reduce the informational rent due to the retailer when asymmetric information exists. In this manner, our work is related to the literature of information economics.

In particular, we apply the theory of adverse selection and countervailing incentives into the relationship among participants of the distribution channel. The pioneering work in this field is due to Akerlof (1970), Rothschild and Stiglitz (1976), and Guesnerie and Laffont (1984), Lewis and Sappington (1988), among others. For a survey of the various issues related to the incentives in the principal-agent relationships, see Sappington (1991). In addition, a complete exposition of the incentive theory is provided by Laffont and Martimort (2002).

In a channel context, the manufacturer serves the role of the principal, and the retailer is the agent. In most instances, the manufacturer is less informed about the state of the market than the retailer, who directly interacts with consumers. This asymmetric information gives rise to the adverse-selection problem upon the design and employment of the contract that regulates the relationship between these two parties. Specifically, the informational asymmetry provides an incentive for the retailer to overstate (or understate) the private information (i.e., the state of the market) in order to secure higher compensation (or better treatment) by the manufacturer. Consequently, in its optimal contract, the principal (i.e., the manufacturer) has to give up a specific amount of informational rent to the agent (i.e., the retailer) when the nature happens to be in its 'better' state (see, for example, Guesnerie and Laffont, 1984).

However, exceptions are applicable; we show that the nature of the asymmetric information determines whether the retailer will receive that informational rent or not. Specifically, in our framework, if information is asymmetric regarding consumers' price-sensitivity, then private information does benefit the retailer. In contrast, if asymmetric information exists regarding the relative sizes of the consumer segments, then the manufacturer can regain the first-best profitability by having a term on the pricing mechanism (i.e., inducing group-buying under some market conditions) in its contract with the retailer.

Finally, this paper relates to the price discrimination literature by its own nature. This phenomenon has been formally defined in the first edition of Pigou's book, "The Economics of Welfare", in 1920. Later, it has attracted scholars in both economics and marketing, including, for instance, Phlips (1983), Spulber (1979), Tirole (1988), Narasimhan (1984), and others. Recently, two excellent review papers on price discrimination have been written (Armstrong 2006 and Stole 2006). The availability of these two review papers provides a complete picture of the economics of price discrimination and allows us to discuss briefly the type of price discrimination, which is most relevant to group-buying, i.e., second-degree price discrimination.

Second-degree price discrimination arises when the firm cannot directly separate consumers into segments based on observable characteristics. Instead, it relies on consumers' self-selection to reveal their types and consequently, choose the 'right' price. To ensure proper self-selection, the firm needs to provide appropriate incentive to each type of consumers via its pricing schedule. It can do so by offering consumers a pricing schedule based on quality (Mussa and Rosen 1978) or quantity (Maskin and Riley 1984).

By having an additional option for consumers to purchase the product (i.e., via group-buying), the retailer can indirectly price discriminate consumers in different market segments. They self-select the purchase mechanism (i.e, group-buying versus posted-pricing) based on their willingness-to-pay and costs, incurred when joining group-buying. Even in the setting, when each consumer demands only one unit of the product (and therefore, price discrimination via quantity as in Maskin and Riley (1984) is not applicable), this additional purchase option can effectively create a schedule of prices and allows the firm to extract the most surplus out of each market segment. In this perspective, this essay extends the literature, adding a new tool, i.e., the pricing mechanism, to the toolbox of second-degree price discrimination (Mussa and Rosen 1978, and Maskin and Riley 1984).

In summary, our work intersects the above research streams and discovers a new device, the group-buying mechanism, that can improve the performance of the distribution channel. It provides the insights on the ability of the manufacturer to regain its first-best profitability under asymmetric information (using this demand-aggregation mechanism and possibly, other similar price-discrimination practices). Together with Anand and Aron (2003), this analysis enhances our understanding of the impacts of group-buying on the behaviors of the channel members.

4.3 The model

We focus on a market, which consists of two segments of consumers. Consumers in the high segment, denoted H, are less price-sensitive than those in the low segment, denoted L; their price sensitivities are β and γ respectively ($\beta < \gamma$) and the potential market sizes of the high and low segments are $\eta\,\alpha$ and $(1-\eta)\alpha$ respectively. The market is offered a product made by a monopolist manufacturer, M, at zero production cost[31] and distributed by a monopolist retailer, R. The relationship between the manufacturer and the retailer is established via a contract. This contract, denoted $\{j,\, w^j,\, F^j\}$, is designed by the manufacturer; its terms include: (a) a pricing mechanism, j, (b) a wholesale price, w^j, and (c) a fixed fee, F^j. The pricing mechanism, j, could be either posted-pricing, denoted P, or group-buying, denoted G. In general, the manufacturer could offer a menu of contracts and the retailer self-selects one from the menu.

Under the P contract, the retailer has a single posted price, p, to serve consumers in both segments of the market, who then make a 'take-it-or-leave-it' purchase decision. In contrast,

[31] Non-trivial cost of production does not provide any additional qualitative insight.

under the G contract, the retailer introduces a group-buying mechanism, in addition to the conventional posted-pricing, to sell the product.

Specifically, under the G contract, the retailer offers to consumers two purchase options: (a) purchase the product at the posted price[32], p^P, and (b) purchase the product via the group-buying mechanism and pay a price, p^{GB}, depending on the realization of the group-buying auction. We consider the simplest version of the group-buying pricing mechanism, which consists of two price tiers: a low price, p_1, and a high price, p_2. If the number of orders placed in the group-buying auction, q, is less than the cut-off level, $\bar{q}$, then the high price, p_2, is effective; otherwise, if the number of group-buying orders exceeds $\bar{q}$, then the low price, p_1, will be charged. Mathematically,

$$p^{GB} = \begin{cases} p_2 & \text{if} \quad q < \bar{q}, \\ p_1 & \text{if} \quad q \geq \bar{q}. \end{cases} \tag{4.1}$$

In contrast to the posted-pricing mechanism, consumers incur some intangible costs when purchasing the product via group-buying due to the extra time, efforts, and even emotion[33]. We capture the cost of joining group-buying, that consumers in the high and low segments incur, by κ_H and κ_L, respectively.

In this research, we focus on the overall effect of the group-buying mechanism on channel management via the construction of the contract between the manufacturer and the retailer, and abstract away from the stochastic aspect of the group-buying mechanism (see Kauffman and Wang 2001). Specifically, we employ an 'aggregate' treatment to the group-buying mechanism and focus on the effective price of the group-buying schedule. We provide here a brief description of this treatment and refer to Appendix C for the full exposition. Despite

[32]Notation p^P is introduced to denote the posted price under the G contract to distinguish from the posted price, p, of the P contract.

[33]Transaction time is extended by the duration of the group-buying auction. More efforts are spent to monitor the evolution of the auction. Emotional cost may arise when consumers feel disappointed if the effective group-buying price does not reach the (lower) tier they expect.

the fact that the group-buying pricing schedule includes multiple price tiers and either one may become effective from the consumers' perspective, this schedule is designed by the retailer in such a way, that one of the price tiers, either p_1 or p_2, is de facto the effective price and other parameters of the schedule, including the cut-off quantities and the other price(s), are set to support the realization of this price. Most importantly, the retailer is indifferent in choosing either the low or high price, p_1 or p_2, as the effective price. Therefore, the complex group-buying schedule could be treated, at the aggregate level, as a single effective price, p^{GB}, namely the group-buying price. The group-buying price will be set at the optimal level, along with the posted price, p^{P}, upon the employment of the G contract.

Even though the choice between the two purchase mechanisms (i.e., the conventional posted-pricing and the group-buying) is made by consumers, they are designed following a specific targeting strategy: the posted-pricing targets at consumers in the high segment while the group-buying targets at those in the low segment. To implement this targeting strategy, the posted price, p^{P}, and the group-buying price, p^{GB}, must be set to ensure proper incentives for each type of consumers to self-select the desired option; this imposes some constraints, which may or may not bind on the prices, p^{P} and p^{GB}. In Appendix C, we investigate the retailer's optimization problem and derive the conditions regarding the costs of joining group-buying (i.e., κ_L and κ_H), under which (a) the incentive-compatibility constraints on the part of consumers in both segments are not binding, (b) the incentive-compatibility constraint of consumers in the high segment binds on the prices, and (c) the incentive-compatibility constraint of low consumers is binding. In the subsequent analysis, we choose to focus on the first case, which requires the cost of joining group-buying being not too high for consumers in the low segment and significantly big for those in the high segment. These conditions, along with the derivation and the simplification of the retailer's optimization problem, are given in Appendix C.

Most importantly, there may exist asymmetric information between the manufacturer and the retailer about the condition of the market (i.e., η, β, and γ). This could happen due to the market being uncertain in these parameters. Consequently, there are three settings for investigation, in which asymmetric information may exist regarding: (a) the relative sizes of the two consumers segments, i.e., η, (b) price sensitivity of consumers in the high segment, and (c) price-sensitivity of those in the L-segment. We denote these three settings by the η-, β-, and γ-case respectively. Further, the analysis focuses on a simple structure, in which the state of the market, denoted i, could be either *high* (denoted h) or *low* (denoted l), i.e., $i \in \{h, l\}$. Compared to the low state, the high state is characterized by (a) a bigger relative size of the high segment (i.e., $\eta_h > \eta_l$) in the η-case, (b) a higher degree of price sensitivity of consumers in the high segment (i.e., $\beta_h > \beta_l$) in the β-case, and (c) a higher degree of price sensitivity of consumers in the low segment (i.e., $\gamma_h > \gamma_l$) in the γ-case. In all cases, the remaining parameters of the market (e.g., β and γ in the η-case) are fixed and known to all parties.

The realization of the market state is observed by the retailer when choosing the contract as well as when setting the retail prices. However, the manufacturer may or may not possess this information when designing the contract(s). If full information is available, the manufacturer will be able to determine the optimal contract, specific to the realized market state, and offers this first-best contract to the retailer. In contrast, under asymmetric information, the manufacturer does not observe the realization of the market condition (i.e., η_i in the η-case, etc.) when designing the contract(s); it will offer a menu of contracts and lets the retailer, who is better informed, choose one from the menu. As a matter of fact, asymmetric information must be taken into consideration in the design of this menu of contracts.

In this analysis, we investigate both the full- and asymmetric-information settings and determine the optimal (menu of) contracts that the manufacturer offers to the retailer as well as the retailer's choice of contracts. Details on the sequence of events under full and asymmetric information are provided in the following sections. Regarding the nature of asymmetric information, we focus on the η-case and investigate how the group-buying mechanism could be used in the contracts to recover the first-best outcome under asymmetric information. Then, in the extensions, the effects of asymmetric information in price sensitivities (i.e., β and γ) are investigated. In these settings, we show that having the group-buying mechanism, as an additional tool in the design of the contract(s), does not help the manufacturer recover the first-best outcome when information is asymmetric; it needs to reserve a specific amount of rent for the retailer, when the market is in the low state.

Finally, we assume a linear structure of the demand in both the H- and L-segments. Under the P contract with a single posted price, p, the demand functions in the H- and L-segments are given by:

$$D^{H,P}(p;\, \eta,\, \beta) = \eta\, \alpha - \beta\, p, \text{ and} \tag{4.2}$$

$$D^{L,P}(p;\, \eta,\, \gamma) = (1-\eta)\, \alpha - \gamma\, p, \text{ respectively;} \tag{4.3}$$

in contrast, under the G contract with a pricing schedule $\{p^P, p^{GB}\}$, demand functions[34] are:

$$D^{H,G}(p^P;\, \eta,\, \beta) = \eta\, \alpha - \beta\, p^P, \text{ and} \tag{4.4}$$

$$D^{L,G}(p^{GB};\, \eta,\, \gamma) = (1-\eta)\, \alpha - \gamma\, (p^{GB} + \kappa_L). \tag{4.5}$$

[34]Note that consumers in the low segment purchase the product via the group-buying mechanism. The total cost of purchasing the product to these consumers includes the group-buying price, p^{GB}, and the cost of joining group-buying, κ_L.

Given this demand structure, the retailer's profit functions, upon the employment of the P and G contracts, are given by:

$$\begin{aligned}\Pi^P(p, w, F; \eta, \beta, \gamma) &= [D^{H,P}(p; \cdot) + D^{L,P}(p; \cdot)](p - w) - F \\ &= \big[\alpha - (\beta + \gamma)p\big](p - w) - F, \text{ and} \qquad (4.6)\\ \Pi^G(p^P, p^{GB}, w, F; \eta, \beta, \gamma) &= D^{H,G}(p^P; \cdot)(p^P - w) + D^{L,G}(p^{GB}; \cdot)(p^{GB} - w) - F \\ &= (\eta\,\alpha - \beta\,p^P)(p^P - w) + \big[(1-\eta)\alpha - \gamma(p^{GB} + \kappa_L)\big](p^{GB} - w) - F, \qquad (4.7)\end{aligned}$$

respectively. Meanwhile, the profit functions of the manufacturer are:

$$\begin{aligned}\pi^P(w, F, p; \eta, \beta, \gamma) &= \big[D^{H,P}(p; \cdot) + D^{L,P}(p; \cdot)\big]w + F \\ &= \big[\alpha - (\beta + \gamma)p\big]w + F, \text{ and} \qquad (4.8)\end{aligned}$$

$$\begin{aligned}\pi^G(w, F, p^P, p^{GB}; \eta, \beta, \gamma) &= \big[D^{H,G}(p^P; \cdot) + D^{L,G}(p^{GB}; \cdot)\big]w + F \\ &= \big[\alpha - \beta\,p^P - \gamma(p^{GB} + \kappa_L)\big]w + F, \qquad (4.9)\end{aligned}$$

upon the employment of the P and G contracts.

4.4 The full-information contracts

Under full information, both the manufacturer and the retailer observe the realization of the parameters, characterizing the market, before making their respective decisions. Full information allows the manufacturer to design the optimal (i.e., first-best) contract, which could be either $\{P, w^P, F^P\}$ or $\{G, w^G, F^G\}$, depending on the state of the market.

The sequence of events, under full information, unfolds in five stages as follows. In the first stage, nature moves and the state of the market is realized. In the second stage, both M and R observe the realization of all parameters, that characterize the state of the market. Next, in stage three, M designs and offers to R the $\{j, w^j, F^j\}$ contract ($j \in \{P, G\}$), which requires R to employ pricing strategy j and pay a wholesale price of w^j and a fixed fee of F^j. In stage four, if R accepts the contract, then it sets the final retail prices according to the contract. Finally, in the last stage, demand is realized and profits made.

We derive the full-information G and P contracts in the most general setting, i.e., for any given market state, characterized by $\{\eta, \beta, \gamma\}$, in §4.4.1 and §4.4.2. Then, in §4.4.3, we compare the profitability of these two full-information contracts to determine the one that arises in the equilibrium of the η-case.

4.4.1 The full-information G contract

Under the G contract, the group-buying mechanism is introduced in addition to the conventional posted-pricing mechanism. Specifically, the retailer will offer a set of prices, which includes a posted price, p^P, and a group-buying schedule, p^{GB}. These prices must satisfy the incentive-compatibility constraints for consumers in the two segments to self-select the desired pricing mechanism as discussed above. A complete analysis of the retailer's optimization problem (see Appendix C) identifies the conditions, regarding the costs of joining group-buying (i.e., κ_H and κ_L), under which these incentive-compatibility constraints impact (i.e., bind or do not bind on) the retail prices. Here, we focus on the setting, in which the costs of joining group-buying, κ_H and κ_L, are such that the incentive-compatibility constraints are not binding on p^P and p^{GB}. This setting requires[35]:

$$\kappa_L \leq \frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma}, \text{ and} \tag{4.10}$$

$$\kappa_H \geq \frac{\kappa_L}{2} + \frac{1}{2}\Big[\frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma}\Big]. \tag{4.11}$$

In this setting, given the parameters of the market (i.e., η, β, and γ), the G contract, $\{G, w^G, F^G\}$, is designed by setting the wholesale price, w^G, and the fixed fee, F^G, so that the manufacturer's profit, $\pi^G(\cdot)$, is maximized. Assuming that the retailer's reservation profit equals to zero, it will accept the contract if its profit is non-negative, i.e., $\Pi^G(\cdot) \geq 0$. As a result, the G contract solves the following optimization problem:

$$\max_{\substack{w^G,\,F^G,\\ \mathring{p}^P,\,\mathring{p}^{GB}}} \pi^G(w^G, F^G, \mathring{p}^P, \mathring{p}^{GB}; \cdot) = \big[\alpha - \beta\,\mathring{p}^P - \gamma(\mathring{p}^{GB} + \kappa_L)\big] w^G + F^G, \tag{4.12}$$

subject to:

$$\begin{cases} \{\mathring{p}^P, \mathring{p}^{GB}\} = \underset{p^P,\,p^{GB}}{\operatorname{argmax}}\, \Pi^G(p^P, p^{GB}, w^G, F^G; \cdot) & = (\eta\,\alpha - \beta\,p^P)(p^P - w^G) + \big[(1-\eta)\alpha \\ & \quad - \gamma(p^{GB} + \kappa_L)\big](p^{GB} - w^G) - F^G, \\ \Pi^{G*} = \max_{p^P,\,p^{GB}} \Pi^G(p^P, p^{GB}, w^G, F^G; \cdot) \geq 0. & \end{cases} \tag{4.13}$$

We summarize the characteristics of the full-information G contract, given the market characterized by $\{\eta, \beta, \gamma\}$, in the following Lemma. The proof is provided in Appendix C.

Lemma 4.1 *Given that the cost of purchasing the product via the group-buying mechanism is sufficiently low for consumers in the low segment, i.e.,* $\kappa_L \leq \frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma}$*, and significantly high for those in the high segment, i.e,* $\kappa_H \geq \frac{\kappa_L}{2} + \frac{1}{2}\big[\frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma}\big]$*, the full-information G*

[35] Note that (4.10) requires $\frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma} > 0$. Otherwise, the incentive-compatibility constraint on the part of low consumers binds on the retail prices even if $\kappa_L = 0$ (see Appendix C).

contract is characterized by:

$$\begin{cases} w^{G*} &= 0, \\ F^{G*} &= \frac{\eta^2\,\alpha^2}{4\beta} + \frac{[(1-\eta)\alpha-\gamma\,\kappa_L]^2}{4\gamma}. \end{cases} \tag{4.14}$$

Under the employment of this contract, the manufacturer earns $\Pi^{G*} = F^{G*} = \frac{\eta^2\,\alpha^2}{4\beta} + \frac{[(1-\eta)\alpha-\gamma\,\kappa_L]^2}{4\gamma}$*, and the retailer sets the retail prices at:*

$$p^{P*} = \frac{\eta\,\alpha}{2\beta}, \text{ and} \tag{4.15}$$

$$p^{GB*} = \frac{(1-\eta)\alpha}{2\gamma} - \frac{\kappa_L}{2}. \tag{4.16}$$

Finally, the feasibility of this contract also requires:

$$(1-\eta)\alpha - \gamma\,\kappa_L \geq 0. \tag{4.17}$$

Upon the introduction of the group-buying mechanism under the G contract, the firm eventually has a schedule of 'two' prices: a posted price and a group-buying price. Consumers in the high segment will be interested in purchasing the product in the traditional way, paying the posted price. This is due to the fact that these consumers perceive the group-buying option to be too "costly" to join (i.e., high κ_H). In contrast, the group-buying mechanism is less "costly" for consumers in the low segment; therefore, they may have incentives to purchase via group-buying. Given that the costs of joining group-buying are sufficiently low for L-consumers and significantly high for those in the H-segment, the retailer will set the prices for high and low consumers, i.e., the posted price and the group-buying price, according to their respective willingness-to-pay[36]. Effectively, the retailer is now price discriminating the two segments via the group-buying mechanism. Notice that the retailer needs to further lower the group-buying price to compensate for the cost of joining group-buying incurred by low consumers, κ_L. Finally, since the wholesale price directly impacts the retail price, it is set at the production cost (i.e., $c = 0$) to eliminate the double-marginalization problem. In contrast, the fixed fee, which does not affect the retail price, is set at the highest level in order to extract all the surplus (i.e., profit) of the channel (see Moorthy 1987).

4.4.2 The full-information P contract

Upon the employment of the P contract, the retailer sells the product to all consumers via the traditional posted-pricing mechanism at a single posted price, p. The demand functions in the high and low segments are given by (4.2) and (4.3) respectively. Though the retailer, under some conditions, may choose to serve only one of the two market segments (and shuts down the other one), we focus on the setting, in which consumers in both segments are served[37]; this requires some conditions on the parameters of the market to hold. In the following, we describe the optimization problem, that the manufacturer faces when designing the P contract (i.e., $\{P, w^P, F^P\}$), given the market characterized by $\{\eta, \beta, \gamma\}$. The characteristics of this contract, along with the required conditions, are summarized in Lemma 4.2. All the derivations are provided in Appendix C.

[36]Willingness-to-pay at the aggregate level is proportional to the ratio of the market size and consumers' price sensitivity. For instance, given the same market size in the two segments (i.e., $\eta = \frac{1}{2}$), willingness-to-pay of consumers in the high segment is $\frac{\alpha}{4\beta}$, which is higher than that of consumers in the low segment, $\frac{\alpha}{4\gamma}$.

[37]i.e., $D^{H,P}(p, \cdot) \geq 0$ and $D^{L,P}(p, \cdot) \geq 0$.

To design the P contract, the manufacturer solves the following constrained optimization problem:

$$\max_{w^P, F^P, \mathring{p}} \pi^P(w^P, F^P, \mathring{p}; \cdot) = [\alpha - (\beta + \gamma)\mathring{p}]\, w^P + F^P \tag{4.18}$$

$$\text{s.t.} \quad \begin{cases} \mathring{p} = \underset{p}{\operatorname{argmax}}\ \Pi^P(p, w^P, F^P; \cdot) = \big[\alpha - (\beta + \gamma)p\big](p - w^P) - F^P, \\ \Pi^{P*} = \max_p \Pi^P(p, w^P, F^P; \cdot) \geq 0. \end{cases} \tag{4.19}$$

Lemma 4.2 *The full-information P contract, serving consumers in both the high and low segments, is characterized by:*

$$\begin{cases} w^{P*} = 0, \\ F^{P*} = \frac{\alpha^2}{4(\beta+\gamma)}. \end{cases} \tag{4.20}$$

Upon the employment of this contract, the manufacturer earns $\pi^{P*} = F^{P*} = \frac{\alpha^2}{4(\beta+\gamma)}$, and the retailer sets the retail price at $p^* = \frac{\alpha}{2(\beta+\gamma)}$. In addition, this contract is feasible under the following conditions:

$$(1 - \eta^2)\beta \geq \eta^2\,\gamma, \ \ \textit{and} \tag{4.21}$$

$$\eta(2 - \eta)\gamma \geq (1 - \eta)^2\,\beta. \tag{4.22}$$

Upon the employment of the P contract, the retailer has only a single price to serve both segments of the market. This price is set at the level, which reflects the 'average' willingness-to-pay of consumers in the two market segments, i.e., below the willingness-to-pay of consumers in the high segment and above that of consumers in the low segment.

Obviously, a single (posted) price does not allow the firm to extract all the surplus from the market. However, in contrast to the group-buying mechanism, consumers incur no extra cost (except the retail price), and therefore, no compensation is required. Finally, analogous to the G contract, the wholesale price is set at the production cost level and the fixed fee is set to extract the entire profit from the retailer.

4.4.3 The optimal full-information contract in equilibrium

Now, consider the η-case. For a given realization of the market state (i.e., η), the manufacturer with full information compares the profitability of the $\{P, w^P, F^P\}$ contract to that of the $\{G, w^G, F^G\}$ contract; the one with higher profitability is chosen in equilibrium. First, we investigate the optimal full-information contract in a benchmark setting, when the cost of joining group-buying is negligible for consumers in the low segment, i.e., $\kappa_L = 0$. (High consumers still incur a significantly high κ_H, as required by (4.11).) Next, the optimal contract under full information is determined for the case of non-trivial κ_L.

4.4.3.1 The benchmark: $\kappa_L = 0$

In this benchmark setting, consumers in the low segment do not incur any cost upon purchasing the product via group-buying (i.e., $\kappa_L = 0$). Lemma 4.3 characterizes the optimal contract, chosen by the manufacturer in this setting. The proof is given in Appendix *C*.

Lemma 4.3 *Given that both the* $\{P, w^P, F^P\}$ *and* $\{G, w^G, F^G\}$ *contracts are feasible*[38], *if the cost of purchasing the product via the group-buying mechanism is negligible for consumers in the low segment (i.e.,* $\kappa_L = 0$*) and significantly high for those in the high segment (i.e.,* $\kappa_H \geq$

[38]This requires the size of the high segment to be neither too small nor too big, i.e., $\frac{\beta}{\beta+\gamma} \leq \eta \leq \sqrt{\frac{\beta}{\beta+\gamma}}$. See Appendix *G* for more details.

$\frac{\alpha}{2\beta}$), *then the manufacturer always chooses the G contract, i.e.,* $\{G, w^G, F^G\}$, *in equilibrium.*

As discussed above, if choosing the $\{P, w^P, F^P\}$ contract, the single posted price is optimally set to reflect the 'average' willingness-to-pay of consumers in the high and low segments, and the firm cannot extract the most surplus out of each market segment. In contrast, the group-buying mechanism allows the firm to price discriminate between the two segments. Upon the employment of the $\{G, w^G, F^G\}$ contract, the retailer sets the two prices, p^P and p^{GB}, according to the willingness-to-pay of the two respective segments, and thus, extracts more surplus from the market. Most importantly, in this benchmark setting, since $\kappa_L = 0$, the firm can price discriminate without further lowering the group-buying price to compensate for κ_L. In other words, the group-buying mechanism serves as a price discrimination tool at no cost to the firm. Therefore, the G contract is always chosen by the manufacturer.

4.4.3.2 Non-trivial κ_L

Now, consider the setting in which consumers in the low segment incur a non-trivial cost of joining group-buying, (i.e., $\kappa_L > 0$); however, this cost should not be too high to satisfy condition 4.10. Based on Lemmas 4.1 and 4.2, we specify the ranges of η, in which both the $\{P, w^P, F^P\}$ and $\{G, w^G, F^G\}$ contracts could be employed. This result is summarized in the upper portion of Figure 4.1, where $\eta_1 \stackrel{\text{def}}{=} \frac{\beta(\alpha+\gamma\kappa_L)}{\alpha(\beta+\gamma)}$, $\eta_2 \stackrel{\text{def}}{=} 1 - \frac{\gamma\kappa_L}{\alpha}$, $\eta^* \stackrel{\text{def}}{=} \frac{\alpha\beta+\gamma[-\beta\kappa_L+\sqrt{\beta\kappa_L(2\alpha-\gamma\kappa_L)}]}{\alpha\beta+\gamma}$, $\kappa_1 \stackrel{\text{def}}{=} \frac{\alpha}{2\beta+\gamma}$, and $\kappa_2 \stackrel{\text{def}}{=} \frac{\alpha(\sqrt{\beta+\gamma}-\sqrt{\beta})}{\gamma\sqrt{\beta+\gamma}}$.

In these ranges of η, the manufacturer's choice of contract in equilibrium is determined by comparing the profits earned under the two contracts, i.e., $\{P, w^P, F^P\}$ and $\{G, w^G, F^G\}$. We summarize the result in Proposition 4.1 and provide the proof in Appendix C.

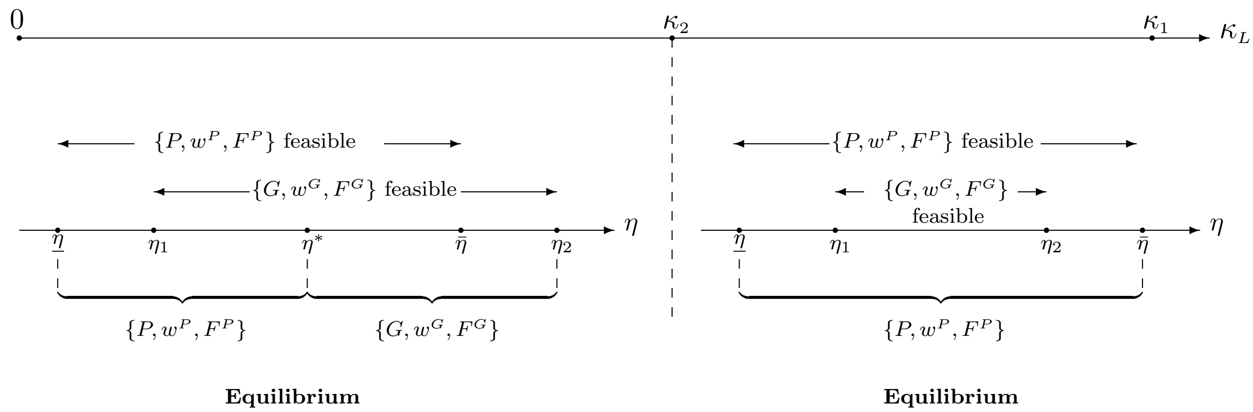


Figure 4.1: **Feasibility and choice of full-information contracts in equilibrium**

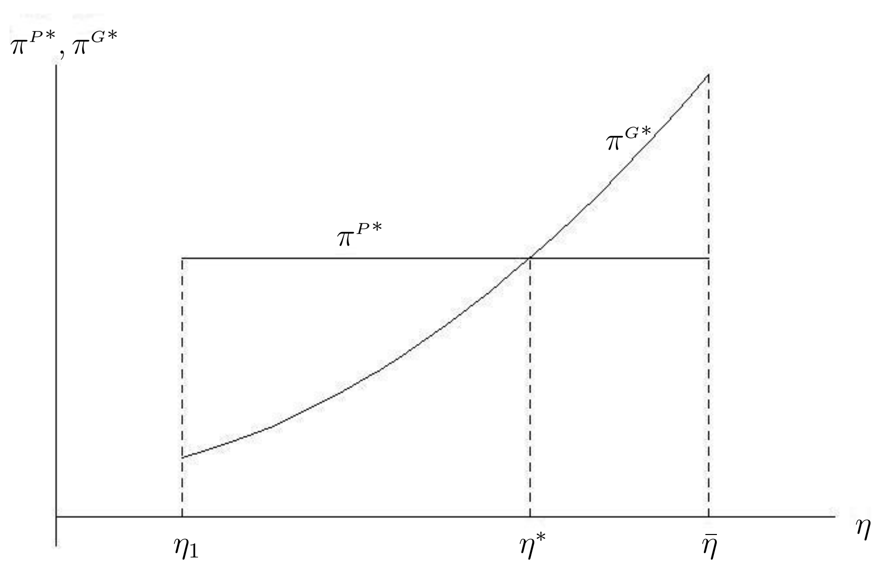


Figure 4.2: **Profitability of the full-information *G* and *P* contracts**

Proposition 4.1 *Given that the cost of joining group-buying is significantly high for consumers in the high segment, i.e.,* $\kappa_H \geq \frac{\kappa_L}{2} + \frac{\alpha}{2\beta}$,

(a) if the cost of joining group-buying is sufficiently low for consumers in the low segment, i.e., $\kappa_L \leq \kappa_2$, *then the manufacturer offers the* $\{P, w^P, F^P\}$ *contract to the retailer*

when the size of the H-segment is relatively small (i.e., $\eta < \eta^*$); otherwise, when the size of the H-segment is relatively big (i.e., $\eta \geq \eta^*$), the $\{G, w^G, F^G\}$ contract arises in equilibrium.

(b) *In contrast, if the cost of joining group-buying is not sufficiently low, i.e., $\kappa_2 < \kappa_L < \kappa_1$, then the manufacturer always chooses the $\{P, w^P, F^P\}$ contract whenever this contract is feasible (i.e., for all $\eta \in [\underline{\eta}, \bar{\eta}]$).*

Under full information, the manufacturer observes the realization of the market state (i.e., η) when designing the contracts. This allows M to design the first-best contract to offer to the retailer. As discussed above, the G contract helps the firm price discriminate the two market segments. Therefore, more surplus could be extracted under the G contract than under the P contract. However, under the G contract, since consumers in the low segment incur the cost of joining group-buying, κ_L, the group-buying price must be lowered further (i.e., below the willingness-to-pay of low consumers) to compensate for this κ_L; the G contract now becomes costly when κ_L is non-trivial. The manufacturer must consider the trade-off between the benefit and the cost of employing the G contract. If κ_L is high (i.e., $\kappa_2 < \kappa_L < \kappa_1$), then the cost of the G contract exceeds the benefit the firm could gain by price discriminating the two market segments; the G contract is thus dominated by the P contract.

It is only when κ_L is sufficiently low (i.e., $\kappa_L \leq \kappa_2$), the G contract may arise in equilibrium. In this case, the choice of contracts depends on the state of the market, i.e., η. Recall that under the P contract, the single posted price is set to reflect the 'average' willingness-to-pay of consumers in the two segments; in contrast, under the G contract, the firm has 'two' prices to serve the two segments.

If the size of the high segment is (relatively) big, i.e., $\eta > \eta^*$, then upon the employment of the P contract, the low segment, which is then (relatively) small in size, would 'contaminate' the single posted price: this price-sensitive segment imposes a downward pressure on the single posted price, while not providing too much of profit to the firm due to its small size. In contrast, under the G contract, the posted price is 'free' from this 'detrimental' downward pressure; it is set to reflect the willingness-to-pay of consumers in the high segment. (Those in the low segment now purchase via the group-buying mechanism and pay the group-buying price.) In other words, the benefit of price discrimination, upon the employment of the G contract, is high when the size of the H-(L-)segment is big (small). Further, as the size of the low segment becomes smaller, the cost of employing the G contract (due the need of compensating for κ_L) decreases. Together, the incremental benefit of the G contract, compared to the P contract, is sufficiently high when the high segment is relatively big; it exceeds the cost (due to κ_L) and hence, the G contract is dominating.

In contrast, when η is small (i.e., $\eta < \eta^*$), the above arguments hold in the opposite direction, i.e., the benefit of the G contract is now limited while its cost becomes significantly high. The G contract is dominated then.

4.5 The asymmetric-information contracts

Now, consider the asymmetric-information setting: the manufacturer does not observe the realization of the market state (i.e., η) when designing the contracts. However, it knows that the market could be in the high state, i.e., $\eta = \eta_h$, with probability ϕ, and in the low state, i.e., $\eta = \eta_l$ with probability $1 - \phi$. In contrast, the retailer, who is better informed than the manufacturer, observes the realization of the market state when making the decision to choose and accept the contracts, as well as when setting the retail prices.

In this analysis, we assume that $\kappa_L \leq \kappa_2$ and $\kappa_H \geq \frac{\kappa_L}{2} + \frac{\alpha}{2\beta}$ and let $\eta_h \in [\eta^*, \eta_2]$ and $\eta_l \in [\underline{\eta}, \eta^*)$[39]. Under asymmetric information, the manufacturer will offer a menu of

[39] These conditions ensure that the $\{G, w^G, F^G\}$ arises as a candidate of the optimal contract in equilibrium; see Proposition 4.1. Further, under these conditions, consumers in the high segment always purchase via the

contracts, $\{G, w^G, F^G\}$ and $\{P, w^P, F^P\}$, to the retailer for self-selection. Based on the result of Proposition 4.1, if the market is in the high state, then the G contract is more profitable; in contrast, if the market is in the low state, then the P contract is better. Therefore, the $\{G, w^G, F^G\}$ contract in the menu under asymmetric information is designed for the high state (i.e., η_h), and the $\{P, w^P, F^P\}$ contract is designed for the low state (i.e., η_l).

In the asymmetric-information setting, the sequence of events unfolds in five stages as follows. In the first stage, the manufacturer designs a menu of contracts, knowing that the market could be in the high state with probability ϕ and in the low state with probability $1-\phi$. In the second stage, nature moves and the state of the market is realized ($\eta_i \in \{\eta_h, \eta_l\}$). In the third stage, the retailer observes the realization of the market state and decides on which of the two contracts to accept, if any. In the forth stage, the retailer sets its retail prices according to the contract it accepts. Finally, in the last stage, demand is realized and profits made.

In the following, we describe the optimization problem that the manufacturer is facing when designing this menu of contracts. Note that this menu must ensure proper incentives for the retailer to participate (see the *IR-h* and *IR-l* constraints), as well as to choose the 'right' contract (i.e., $\{G, w^G, F^G\}$ when the market is in the high state and $\{P, w^P, F^P\}$ in the low state; see the *IC-h* and *IC-l* constraints). The characteristics of the optimal menu is summarized in Proposition 4.2. All the derivations are given in Appendix C.

The menu of contracts under asymmetric information regarding η solves the following constrained optimization problem:

---

traditional posted-pricing mechanism while those in the low segment will choose the group-buying option whenever this option is available (see Appendix C).

$$
\begin{aligned}
\max_{\substack{w^G, F^G, p^P_{h|h}, p^{GB}_{h|h} \\ w^P, F^P, p_{l|l}}} \pi = \; & \phi\, \pi^G_{h|h} \, + \, (1-\phi)\, \pi^P_{l|l} \\
= \; & \phi\, \pi^G(w^G, F^G, p^P_{h|h}, p^{GB}_{h|h}; \eta_h, \cdot) \, + \, (1-\phi)\, \pi^P(w^P, F^P, p_{l|l}; \eta_l, \cdot) \\
= \; & \phi \Big\{ \big[\alpha - \beta p^P_{h|h} - \gamma(p^{GB}_{h|h} + \kappa_L)\big] w^G + F^G \Big\} \\
& + \, (1-\phi) \Big\{ \big[\alpha - (\beta+\gamma) p_{l|l}\big] w^P + F^P \Big\},
\end{aligned} \tag{4.23}
$$

subject to:

$$
\textit{(FOC-G)}: \quad \{p^P_{h|h}, p^{GB}_{h|h}\} \, = \, \operatorname*{argmax}_{p^P,\, p^{GB}} \Pi^G_{h|h} \, = \, \Pi^G(p^P, p^{GB}, w^G, F^G; \eta_h, \cdot), \tag{4.24}
$$

$$
\textit{(FOC-P)}: \quad p_{l|l} \, = \, \operatorname*{argmax}_{p} \Pi^P_{l|l} \, = \, \Pi^P(p, w^P, F^P; \eta_l, \cdot), \tag{4.25}
$$

$$
\textit{(IR-h)}: \quad \Pi^{G\,*}_{h|h} \, = \, \max_{p^P,\, p^{GB}} \Pi^G_{h|h} \, \geq 0, \tag{4.26}
$$

$$
\textit{(IR-l)}: \quad \Pi^{P\,*}_{l|l} \, = \, \max_{p} \Pi^P_{l|l} \, \geq 0, \tag{4.27}
$$

$$
\textit{(IC-h)}: \quad \Pi^{G\,*}_{h|h} \, \geq \, \Pi^{P\,*}_{l|h} \, = \, \max_{p} \Pi^P_{l|h} = \Pi^P(p, w^P, F^P; \eta_h, \cdot), \tag{4.28}
$$

$$
\textit{(IC-l)}: \quad \Pi^{P\,*}_{l|l} \, \geq \, \Pi^{G\,*}_{h|l} \, = \, \max_{p^P,\, p^{GB}} \Pi^G_{h|l} = \Pi^G(p^P, p^{GB}, w^G, F^G; \eta_l, \cdot). \tag{4.29}
$$

Proposition 4.2 *Under asymmetric information, given that the cost of joining group-buying is significantly high for consumers in the high segment (i.e.,* $\kappa_H \geq \frac{\kappa_L}{2} + \frac{\alpha}{2\beta}$*) and sufficiently low for those in the low segment (i.e.,* $\kappa_L \leq \kappa_2$*), the manufacturer offers to the retailer a menu of two contracts,* $\{G, w^{G*}, F^{G*}\}$ *and* $\{P, w^{P*}, F^{P*}\}$*, where:*

$$
\begin{cases}
w^{G*} & = 0, \\
F^{G*} & = \frac{\eta_h^2 \alpha^2}{4\beta} + \frac{[(1-\eta_h)\alpha - \gamma \kappa_L]^2}{4\gamma}, \\
w^{P*} & = 0, \\
F^{P*} & = \frac{\alpha^2}{4(\beta+\gamma)}.
\end{cases} \tag{4.30}
$$

The former contract is chosen by the retailer when the market is in the high state, and the latter in the low state.

This result implies that under asymmetric information regarding η, the manufacturer, by offering the menu of contracts as described above, can regain the first-best profitability. To understand the intuition underlying this interesting finding, consider the nature of market uncertainty and its impact on the profitability of the two contracts (i.e., the P and G contracts), as well as on the retailer's incentive of choosing the contracts. Whether the market is in the high or low state regarding η, the size of the entire market remains fixed. In other words, consumers just reallocate themselves from one segment to the other between the two states; the total demand of the market remains identical. Therefore, the P contract, which serves the entire market with a single price, provides the same profitability in both market states. Most importantly, having the information on the total market demand, the manufacturer can determine the fixed fee in the P contract (i.e., F^P) to extract all the surplus from the retailer whenever this contract is chosen. Regarding the G contract, as discussed above, it is more profitable when the market is in the high state (due to the higher benefit and the lower cost of price discrimination). Designed to be chosen in the high state, the fixed fee in the G contract (i.e., F^G) is set to extract all the gross profit that the retailer may earn upon the employment of the G contract in the high state.

It is the profitability of the two contracts in each market state that determines the retailer's incentive to choose either one of them. The above-mentioned pattern of profitability perfectly aligns the retailer's incentive to that of the manufacturer, and therefore, allows M to recover the first-best results even under asymmetric information. In particular, when the market is in the low state, the retailer has no incentive to choose the *undesired* G contract—choosing this contract in the low state results in negative profit to the retailer since the fixed fee, F^G, is higher than the gross profit (i.e., profit prior to paying the fixed fee) it would earn from this contract in the low state[40]. Most importantly, when the market is in the high state, the undesired contract, which is now the P contract, is not attractive to the retailer due to the fact that its profitability is identical to that when the market is in the low state and equals to the fixed fee it would pay to the manufacturer.

In summary, the term on the pricing mechanism in the contracts serves as a perfect *sensor* for the manufacturer to detect the state of the market (regarding the relative sizes of the two consumer segments) via the behaviors of the retailer. It helps eliminate the adverse incentive that the retailer would have under asymmetric information. In this setting, the private information regarding the relative sizes of the two consumer segment does not reward the retailer for its (possible) adverse selection of the pricing mechanism. This arrangement effectively improves channel coordination and increases the efficiency of the entire distribution system.

4.6 Extensions

4.6.1 Uncertainty-in-β case

Now, consider the setting when asymmetric information may exist regarding the price sensitivity of consumers in the high segment, i.e., β. First, we investigate the manufacturer's choice of full-information contracts—given the full-information G and P contracts, whose characteristics are given in Lemmas 4.1 and 4.2, the manufacturer will choose the optimal

[40] Note that the G contract is designed for the high state, and thus requires a high fixed cost. This fixed cost equals to the 'gross' profit of the retailer upon the employment of the G contract in the high state.

one, depending on the realized β that it observes. Next, we investigate the menu of contracts that M will offer to the retailer under asymmetric information regarding β. In this setting, though the manufacturer cannot recover the first-best outcome, it pays a lower informational rent when having a term on the pricing mechanism in the contract.

4.6.1.1 The optimal full-information contract in the uncertainty-in-β case

Based on the results of Lemmas 4.1 and 4.2, we specify the ranges of β, in which both the $\{G, w^G, F^G\}$ and $\{P, w^P, F^P\}$ are feasible. Then, we compare the profitability of the two contracts in these ranges of β to determine the optimal full-information contract. Proposition 4.3 describes the manufacturer's choice of full-information contracts in this β-case, given $\eta = \frac{1}{2}$. In Figure 4.3, where $\bar{\beta} \stackrel{\text{def}}{=} \frac{\gamma(2-\eta)\eta}{(1-\eta)^2}$, $\kappa_3 \stackrel{\text{def}}{=} \frac{(1-\eta)\alpha}{\gamma}$, $\kappa_4 \stackrel{\text{def}}{=} \frac{(2\eta-1)\alpha}{\gamma}$, and $\kappa_5 \stackrel{\text{def}}{=} \frac{\alpha[2(1-\eta)-\sqrt{2(1-2\eta^2)}]}{2\gamma}$, a complete description of the result is provided (i.e., when η takes on different values). All the proofs and the specification of β^* are provided in Appendix C.

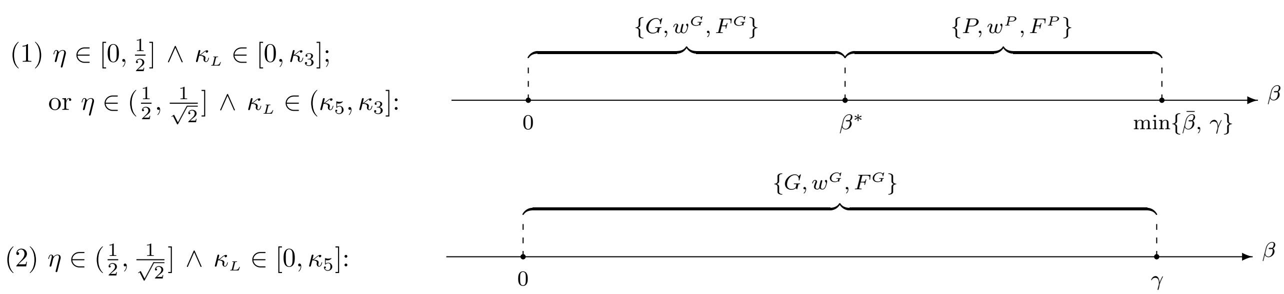


Figure 4.3: **The manufacturer's choice of full-information contracts in the β-case**

Proposition 4.3 *Given that κ_L is sufficiently low (i.e., $\kappa_L \leq \kappa_3$) and κ_H is significantly high, if the price sensitivity of high consumers is low, i.e., $\beta \leq \beta^*$, then the $\{G, w^G, F^G\}$ contract is chosen; in contrast, if β is high, i.e., $\beta > \beta^*$, then the $\{P, w^P, F^P\}$ contract arises in equilibrium.*

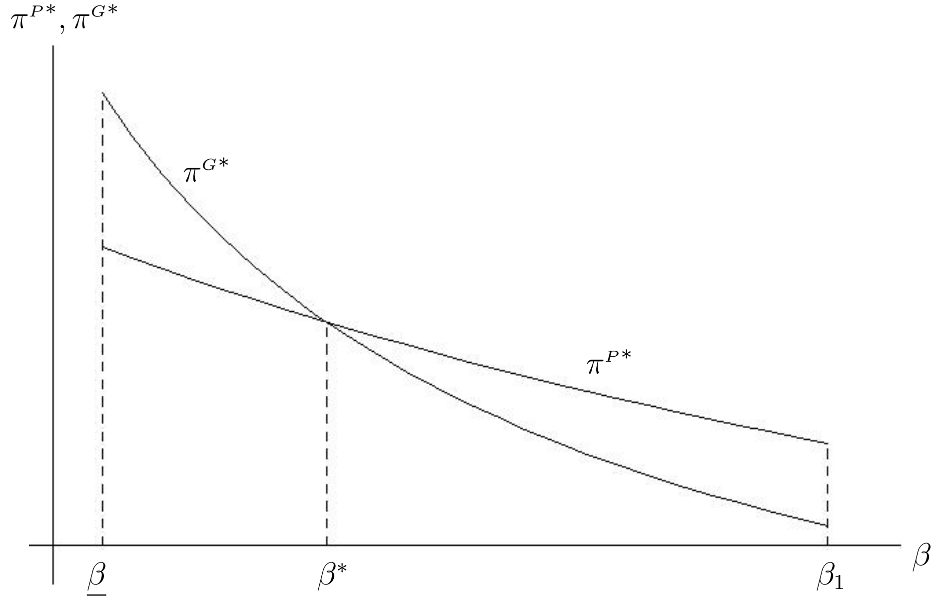


Figure 4.4: **Profitability of the full-information *G* and *P* contracts in the β-case**

The intuition of this result is quite straightforward. Notice that both contracts have higher profitability in the low state than in the high state due to a low β. However, the G contract is more profitable when the market is in the low state. Recall that the benefit of the G contract comes from its ability to price discriminate the two segments. This benefit increases as the two segments become more distinct with respect to their willingness-to-pay. This happens when the market is in the low state: the low price sensitivity of consumers in the high segments implies a high willingness-to-pay of these consumers, making the two segments significantly distinct. Therefore, the benefit of the G contract due to price discrimination is higher in the low state than in the high state; it exceeds the cost of compensating for κ_L upon the employment of this contract. In contrast, when the market is in the high state, the price sensitivity of consumers in the high segment is high and close to that of those in the low segment, making the two segments 'close' to each other. The benefit of price discrimination is limited; the cost now exceeds the benefit and the firm is better off by choosing the P contract.

4.6.1.2 The menus of contracts under asymmetric information regarding β

Now, we investigate the setting of asymmetric information regarding β. Though not observing the realization of β when designing the contracts, M knows that the high state (β_h) will realize with probability ϕ and the low state (β_l) with probability $1-\phi$. Here, we focus on the setting, described in the upper part of Figure 4.3, i.e., $\{\eta, \kappa_L\} \in [0, \frac{1}{2}] \times (0, \kappa_3] \cup (\frac{1}{2}, \frac{1}{\sqrt{2}}] \times (\kappa_5, \kappa_3]$. Further, we assume that $\beta_h \in (\beta^*, \min\{\bar{\beta}, \gamma\}]$ and $\beta_l \in (0, \beta^*]$. In this setting, the first-best contract (under full information) would be the P contract if the market is in the high state and the G contract in the low state.

In the following analysis, we show that the first-best outcome is not achievable under asymmetric information regarding β. However, the manufacturer can obtain the second-best outcome by offering a menu of P and G contracts. Specifically, it has three options to design the menu of contracts: (a) a menu of (only) P contracts (namely, the P-menu), (b) a menu of (only) G contracts (namely, the G-menu), and (c) a menu of P and G contracts, in which the former is designed for the high state and the latter for the low state. In the following, we describe the optimization problem that defines each of the three menus, and summarize the characteristics of each contract in Table 4.1. All derivations are provided in Appendix C.

(i) The menu of P contracts: $\{P, w_h^P, F_h^P\}$ (designed for the high state) and $\{P, w_l^P, F_l^P\}$ (designed for the low state). This menu solves the following problem:

$$\begin{aligned}\max_{\substack{w_h^P, F_h^P, \mathring{p}_{h|h}, \\ w_l^P, F_l^P, \mathring{p}_{l|l}}} \mathring{\pi}^P &= \phi\, \pi^P(w_h^P, F_h^P, \mathring{p}_{h|h}; \beta_h, \cdot) + (1-\phi)\, \pi^P(w_l^P, F_l^P, \mathring{p}_{l|l}; \beta_l, \cdot) \\ &= \phi \left\{ \left[\alpha - (\beta_h + \gamma)\mathring{p}_{h|h}\right] w_h^P + F_h^P \right\} + (1-\phi) \left\{ \left[\alpha - (\beta_l + \gamma)\mathring{p}_{l|l}\right] w_l^P + F_l^P \right\},\end{aligned} \tag{4.31}$$

subject to:

$$\textit{(FOC-h)}: \quad \mathring{p}_{h|h} \;=\; \underset{p}{\operatorname{argmax}}\; \mathring{\Pi}^{P}_{h|h} \;=\; \Pi^{P}(p, w^{P}_{h}, F^{P}_{h}; \beta_{h}, \cdot), \tag{4.32}$$

$$\textit{(FOC-l)}: \quad \mathring{p}_{l|l} \;=\; \underset{p}{\operatorname{argmax}}\; \mathring{\Pi}^{P}_{l|l} \;=\; \Pi^{P}(p, w^{P}_{l}, F^{P}_{l}; \beta_{l}, \cdot), \tag{4.33}$$

$$\textit{(IR-h)}: \quad \mathring{\Pi}^{P*}_{h|h} \;=\; \max_{p} \mathring{\Pi}^{P}_{h|h} \;\geq 0, \tag{4.34}$$

$$\textit{(IR-l)}: \quad \mathring{\Pi}^{P*}_{l|l} \;=\; \max_{p} \mathring{\Pi}^{P}_{l|l} \;\geq 0, \tag{4.35}$$

$$\textit{(IC-h)}: \quad \mathring{\Pi}^{P*}_{h|h} \;\geq\; \mathring{\Pi}^{P*}_{l|h} \;=\; \max_{p} \mathring{\Pi}^{P}_{l|h} = \Pi^{P}(p, w^{P}_{l}, F^{P}_{l}; \beta_{h}, \cdot), \text{ and} \tag{4.36}$$

$$\textit{(IC-l)}: \quad \mathring{\Pi}^{P*}_{l|l} \;\geq\; \mathring{\Pi}^{P*}_{h|l} \;=\; \max_{p} \mathring{\Pi}^{P}_{h|l} = \Pi^{P}(p, w^{P}_{l}, F^{P}_{l}; \beta_{l}, \cdot). \tag{4.37}$$

(ii) The menu of G contracts: $\{P, w^{G}_{h}, F^{G}_{h}\}$ and $\{P, w^{G}_{l}, F^{G}_{l}\}$. The optimal menu of G contracts then solves the following optimization problem:

$$\begin{aligned}
\max_{\substack{w^{G}_{h}, F^{G}_{h}, \mathring{p}^{P}_{h|h}, \mathring{p}^{GB}_{h|h} \\ w^{G}_{l}, F^{G}_{l}, \mathring{p}^{P}_{l|l}, \mathring{p}^{GB}_{l|l}}} \mathring{\pi}^{G} = \;& \phi\, \pi^{G}(w^{G}_{h}, F^{G}_{h}, \mathring{p}^{P}_{h|h}, \mathring{p}^{GB}_{h|h}; \beta_{h}, \cdot) \;+\; (1-\phi)\, \pi^{G}(w^{G}_{l}, F^{G}_{l}, \mathring{p}^{P}_{l|l}, \mathring{p}^{GB}_{l|l}; \beta_{l}, \cdot) \\
= \;& \phi \Big\{ \big[\alpha - \beta_{h}\, \mathring{p}^{P}_{h|h} - \gamma(\mathring{p}^{GB}_{h|h} + \kappa_{L})\big] w^{G}_{h} + F^{G}_{h} \Big\} + \\
& + (1-\phi) \Big\{ \big[\alpha - \beta_{l} \mathring{p}^{P}_{l|l} - \gamma(\mathring{p}^{GB}_{l|l} + \kappa_{L})\big] w^{G}_{l} + F^{G}_{l} \Big\},
\end{aligned} \tag{4.38}$$

subject to:

$$\textit{(FOC-h)}: \quad \{\mathring{p}^{P}_{h|h}, \mathring{p}^{GB}_{h|h}\} \;=\; \underset{p^{P}, p^{GB}}{\operatorname{argmax}}\; \mathring{\Pi}^{G}_{h|h} \;=\; \Pi^{G}(p^{P}, p^{GB}, w^{G}_{h}, F^{G}_{h}; \beta_{h}, \cdot), \tag{4.39}$$

$$\textit{(FOC-l)}: \quad \{\mathring{p}^{P}_{l|l}, \mathring{p}^{GB}_{l|l}\} \;=\; \underset{p^{P}, p^{GB}}{\operatorname{argmax}}\; \mathring{\Pi}^{G}_{l|l} \;=\; \Pi^{G}(p^{P}, p^{GB}, w^{G}_{l}, F^{G}_{l}; \beta_{l}, \cdot), \tag{4.40}$$

$$\textit{(IR-h)}: \quad \mathring{\Pi}^{G*}_{h|h} \;=\; \max_{p^{P}, p^{GB}} \mathring{\Pi}^{G}_{h|h} \;\geq 0, \tag{4.41}$$

$$\textit{(IR-l)}: \quad \mathring{\Pi}^{G*}_{l|l} \;=\; \max_{p^{P}, p^{GB}} \mathring{\Pi}^{G}_{l|l} \;\geq 0, \tag{4.42}$$

$$\textit{(IC-h)}: \quad \mathring{\Pi}^{G*}_{h|h} \;\geq\; \mathring{\Pi}^{G*}_{l|h} \;=\; \max_{p^P,\,p^{GB}} \mathring{\Pi}^{G}_{l|h} = \Pi^G(p^P, p^{GB}, w^G_l, F^G_l; \beta_h, \cdot), \text{ and} \tag{4.43}$$

$$\textit{(IC-l)}: \quad \mathring{\Pi}^{G*}_{l|l} \;\geq\; \mathring{\Pi}^{G*}_{h|l} \;=\; \max_{p^P,\,p^{GB}} \mathring{\Pi}^{G}_{h|l} = \Pi^G(p^P, p^{GB}, w^P_h, F^P_h; \beta_l, \cdot). \tag{4.44}$$

(iii) The menu of P and G contracts: $\{P, w^P, F^P\}$ (designed for the high state) and $\{G, w^G, F^G\}$ (designed for the low state). This menu solves the following optimization problem:

$$\begin{aligned}
\max_{\substack{w^P,\,F^P,\,p_{h|h},\\ w^G,\,F^G,\,p^P_{l|l},\,p^{GB}_{l|l}}} \quad \pi = {} & \phi\, \pi^P_{h|h} \;+\; (1-\phi)\, \pi^G_{l|l} \\
= {} & \phi\, \pi^P(w^P, F^P, p_{h|h}; \beta_h, \cdot) \;+\; (1-\phi)\, \pi^G(w^G, F^G, p^P_{l|l}, p^{GB}_{l|l}; \beta_l, \cdot) \\
= {} & \phi \Big\{ \big[\alpha - (\beta_h + \gamma) p_{h|h}\big] w^P + F^P \Big\} \\
& + (1-\phi) \Big\{ \big[\alpha - \beta_l\, p^P_{l|l} + \gamma (p^{GB}_{l|l} + \kappa_L)\big] w^G + F^G \Big\},
\end{aligned} \tag{4.45}$$

subject to:

$$\textit{(FOC-P)}: \quad p_{h|h} \;=\; \operatorname*{argmax}_{p}\, \Pi^P_{h|h} \;=\; \Pi^P(p, w^P, F^P; \beta_h, \cdot), \tag{4.46}$$

$$\textit{(FOC-G)}: \quad \{p^P_{l|l}, p^{GB}_{l|l}\} \;=\; \operatorname*{argmax}_{p^P,\,p^{GB}}\, \Pi^G_{l|l} \;=\; \Pi^G(p^P, p^{GB}, w^G, F^G; \beta_l, \cdot), \tag{4.47}$$

$$\textit{(IR-h)}: \quad {\Pi^P_{h|h}}^* \;=\; \max_{p} \Pi^P_{h|h} \;\geq 0, \tag{4.48}$$

$$\textit{(IR-l)}: \quad {\Pi^G_{l|l}}^* \;=\; \max_{p^P,\,p^{GB}} \Pi^G_{l|l} \;\geq 0, \tag{4.49}$$

$$\textit{(IC-h)}: \quad {\Pi^P_{h|h}}^* \;\geq\; {\Pi^G_{l|h}}^* \;=\; \max_{p^P,\,p^{GB}} \Pi^G_{l|h} = \Pi^G(p^P, p^{GB}, w^G, F^G; \beta_h, \cdot), \text{ and} \tag{4.50}$$

$$\textit{(IC-l)}: \quad {\Pi^G_{l|l}}^* \;\geq\; {\Pi^P_{h|l}}^* \;=\; \max_{p} \Pi^P_{h|l} = \Pi^P(p, w^P, F^P; \beta_l, \cdot). \tag{4.51}$$

By investigating the profitability of the three menus, it can be seen that the menu of P and G contracts outperforms the other two menus. First, if the market is in the high state,

Table 4.1: **Characteristics of the (menus of) contracts under asymmetric information regarding β**

		High state (β_h)	Low state (β_l)
Full information[a]	First-best contract: $\{j, w^j, F^j\}$ $(j \in \{P, G\})$	$w^{P*} = 0$ $F^{P*} = \frac{\alpha^2}{4(\beta_h+\gamma)}$	$w^{G*} = 0$ $F^{G*} = \frac{\eta^2\,\alpha^2}{4\beta_l} + \frac{[(1-\eta)\,\alpha-\gamma\,\kappa_L]^2}{4\gamma}$
Asymmetric information[b]	The menu of P contracts: $\{P, w_h^P, F_h^P\}$ & $\{P, w_l^P, F_l^P\}$	$w_h^{P*} = 0$ $F_h^{P*} = \frac{\alpha^2}{4(\beta_h+\gamma)}$	$w_l^{P*} = 0$ $F_l^{P*} = \frac{\alpha^2}{4(\beta_l+\gamma)} - \frac{(\beta_h-\beta_l)\alpha^2}{4(\beta_h+\gamma)(\beta_l+\gamma)}$
	The menu of G contracts: $\{G, w_h^G, F_h^G\}$ & $\{G, w_l^G, F_l^G\}$	$w_h^{G*} = 0$ $F_h^{G*} = \frac{\eta^2\,\alpha^2}{4\beta_h} + \frac{[(1-\eta)\,\alpha-\gamma\,\kappa_L]^2}{4\gamma}$	$w_l^{G*} = 0$ $F_l^{G*} = \frac{\eta^2\,\alpha^2}{4\beta_l} + \frac{[(1-\eta)\,\alpha-\gamma\,\kappa_L]^2}{4\gamma} - \frac{(\beta_h-\beta_l)\eta^2\,\alpha^2}{4\beta_h\,\beta_l}$
	The menu of P and G contracts: $\{P, w^P, F^P\}$ & $\{G, w^G, F^G\}$	$w^{P*} = 0$ $F^{P*} = \frac{\alpha^2}{4(\beta_h+\gamma)}$	$w^{G*} = 0$ $F^{G*} = \frac{\eta^2\,\alpha^2}{4\beta_l} + \frac{[(1-\eta)\,\alpha-\gamma\,\kappa_L]^2}{4\gamma} - \frac{(\beta_h-\beta_l)\alpha^2}{4(\beta_h+\gamma)(\beta_l+\gamma)}$

[a] see Proposition 4.3.
[b] where $c = 0$.

this menu gives the manufacturer the profit it would earn under full information with the P contract. Next, if the market is in the low state, the manufacturer can extract the higher surplus of the 'G-menu' while giving back to the retailer the smaller rent that would be due under the 'P-menu'[41]. Proposition 4.4 formalizes this result.

Proposition 4.4 *Under asymmetric information regarding β, the manufacturer cannot regain the first-best outcome with the menu of P and G contracts. However, this menu outperforms the menu of P contracts and the menu of G contracts: it allows the manufacturer to extract more surplus at a lower rent due to the retailer.*

First, under asymmetric information regarding β, the menu based on the first-best contracts could not be employed since the retailer will choose the undesired contract when the market is in the low state. To see this, recall the result of the full-information setting (see Figure 4.4): both contracts (i.e., the P and G contracts) are more profitable when the market is in the low state than in the high state. The P contract, designed for the high state,

[41] In Table 4.1, the last terms of F_l^{P*} and F_l^{G*} represent the rent due to asymmetric information in the P- and G-menus. It has been shown that $\frac{(\beta_h-\beta_l)\alpha^2}{4(\beta_h+\gamma)(\beta_l+\gamma)} < \frac{(\beta_h-\beta_l)\eta^2\,\alpha^2}{4\beta_h\,\beta_l}$ (see equation C.218).

includes a fixed fee, which is equal to the 'gross' profit of the retailer upon the employment of this contract in the high state; this fixed fee is smaller than the 'gross' profit that the retailer can earn when the market is in the low state. This gives the retailer an incentive to choose the P contract in the low state, in place of the desired G contract.

Therefore, under asymmetric information, the manufacturer must give up some information rent to the retailer in the low market state in order to mitigate the incentive of choosing the undesired contract. Compare the G- to the P-menu: (a) the fixed fees of the 'high' contracts in both menus are set to extract all the surpluses from the retailers in the high state, and (b) the G contract is more profitable than the P contract in the low state (due to its ability of price discriminating the two significantly 'distinct' segments of the market; see Proposition 4.3), and less profitable in the high state. This implies that when the market is in the low state, the retailer would earn more if choosing the undesired, 'high' G contract (i.e., $\{G, w_h^G, F_h^G\}$) in the G-menu than if choosing the 'high' P contract (i.e., $\{P, w_h^P, F_h^P\}$) in the P-menu. More incentives mean higher rent: the rent paid to the retailer in the G-menu is higher than that in the P-menu. However, the G-menu allows for higher surplus extraction in the low state, compared to the P-menu.

By having the menu of both the P and G contracts, the manufacturer can extract the most surplus while paying the lowest rent. The employment of the G contract in the low state and the P contract in the high state (as designed by this menu) allows for extracting the most surplus in both states, as shown in Proposition 4.3. Regarding the rent of this menu, it is the rent, required to prevent the retailer from choosing the undesired P contract, that needs to be paid. As discussed above, since the P contract is less profitable in the low state, this rent is lower than that required in the G-menu. In summary, the menu of P and G contracts combines the best of the two menus (i.e., the P- and G-menus). This is the second-best outcome that the manufacturer can achieve.

4.6.2 Uncertainty-in-γ case

Consider the γ-case. Analogous to the previous analysis, we begin by characterizing the first-best contracts under full information. Then, under asymmetric information, the optimal menu of contracts is investigated. Here, too, we show that the first-best outcome is unachievable: the manufacturer has to give up some rent to the retailer when the market is in the low state.

4.6.2.1 The optimal full-information contract in the uncertainty-in-γ case

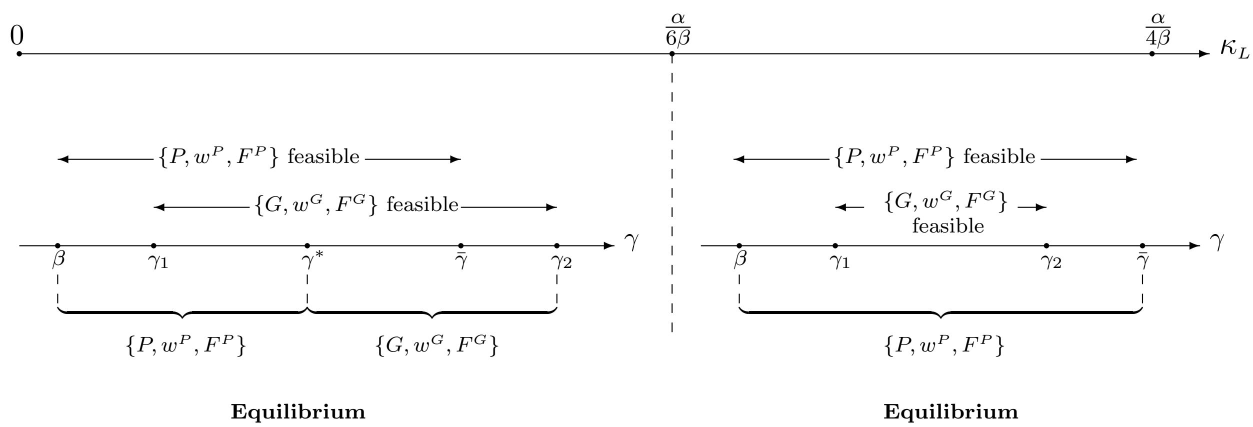


Figure 4.5: **Feasibility and the manufacturer's choice of full-information contracts when the market is uncertain in γ (given $\eta = \frac{1}{2}$)**

Under full information, the manufacturer chooses between the two full-information contracts, described in Lemmas 4.1 and 4.2, by comparing their respective profitabilities. To reduce the level of mathematical complexity, we focus on the setting of $\eta = \frac{1}{2}$. We report the result when κ_L is sufficiently low (i.e., $\kappa_L < \frac{\alpha}{6\beta}$) in Proposition 4.5. The complete result is presented in Figure 4.5. Figure 4.6 illustrates the profitability of the contracts when the market is uncertain in γ. The cut-offs, introduced hereby, include: (a) $\bar{\gamma} \stackrel{\text{def}}{=} 3\beta$, (b)

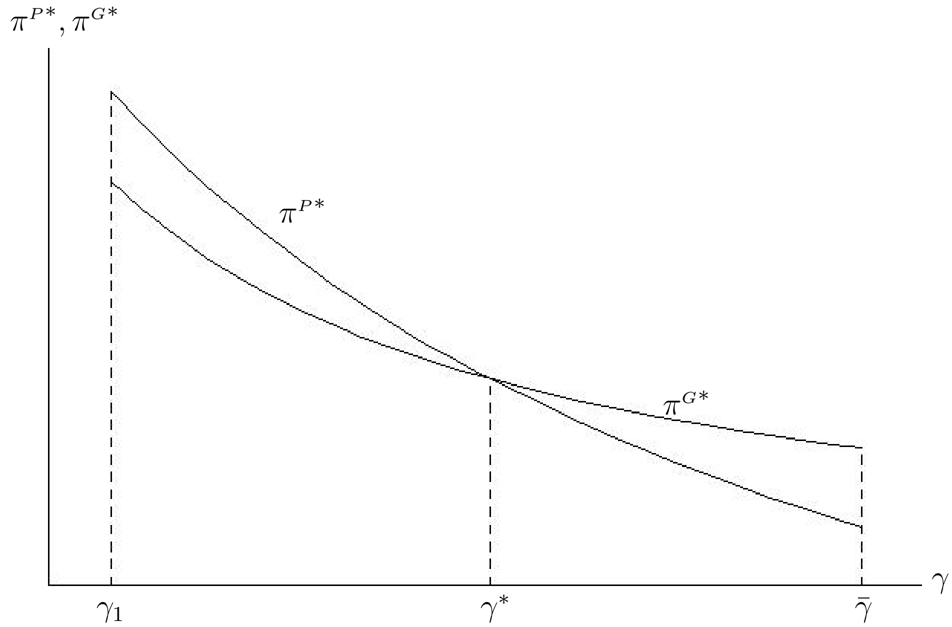


Figure 4.6: **Profitability of the full-information G and P contracts under uncertainty in γ (given $\eta = \frac{1}{2}$ and $\kappa_L \leq \frac{\alpha}{6\beta}$)**

$\gamma_1 \stackrel{\text{def}}{=} \frac{\alpha\beta}{\alpha - 2\beta\kappa_L}$, and (c) $\gamma_2 \stackrel{\text{def}}{=} \frac{\alpha}{2\kappa_L}$. Here, γ^* does not have a close-formed specification. However, its existence is shown in Appendix C, along with all the derivations.

Proposition 4.5 *Given that κ_L is sufficiently low (i.e., $\kappa_L < \frac{\alpha}{6\beta}$) and κ_H is significantly high, if the price sensitivity of consumers in the low segment is high, i.e., $\gamma > \gamma^*$, then the manufacturer offers the $\{G, w^G, F^G\}$ contract to the retailer. In contrast, if the price sensitivity of these consumers is low, i.e., $\gamma \leq \gamma^*$, then the $\{P, w^P, F^P\}$ contract is chosen in equilibrium.*

The logic that explains this result is similar to that of Proposition 4.3. The G contract allows the firm to price discriminate the two market segments. The benefit due to price discrimination increases as the two segments become more 'distinct' with respect to their willingness-to-pay, i.e., when the low segment becomes more price-sensitive. However, the

G contract is costly to employ due to the need of compensating consumers for κ_L. It is when the low segment is highly price-sensitive (i.e., γ is high), the benefit of the G contract exceeds its cost; it is then more profitable than the P contract. In contrast, when γ is low, the manufacturer cannot recover the cost of the G contract due to the limited benefit from price discrimination; the P contract is dominating the G contract.

4.6.2.2 The menus of contracts under asymmetric information regarding γ

This analysis investigates the setting of asymmetric information and uncertainty in γ. The manufacturer offers a menu of P and G contracts to the retailer. The P contract of the menu is designed for the low state and the G contract for the high state. Here, the result is similar to the setting of market uncertain in β: the menu of P and G contracts allows the manufacturer to extract more surplus at a lower rent, due to the retailer. Analogously, we describe the optimization problem of the manufacturer when designing (a) the menu of P contracts (namely, the P-menu), (b) the menu of G contracts (namely, the G-menu), and (c) the menu of P and G contracts. The characteristics of these menus are summarized in Table 4.2. We formalize the result in Proposition 4.6. The proof is given in Appendix C.

(i) The menu of P contracts: $\{P, w_h^P, F_h^P\}$ (designed for the high state) and $\{P, w_l^P, F_l^P\}$ (designed for the low state). This menu solves the following optimization problem:

$$\begin{aligned}\max_{\substack{w_h^P, F_h^P, \breve{p}_{h|h}, \\ w_l^P, F_l^P, \breve{p}_{l|l}}} \breve{\pi}^P &= \phi\, \pi^P(w_h^P, F_h^P, \breve{p}_{h|h}; \gamma_h, \cdot) + (1-\phi)\, \pi^P(w_l^P, F_l^P, \breve{p}_{l|l}; \gamma_l, \cdot) \\ &= \phi\left\{\left[\alpha - (\beta+\gamma_h)\breve{p}_{h|h}\right] w_h^P + F_h^P\right\} + (1-\phi)\left\{\left[\alpha - (\beta+\gamma_l)\breve{p}_{l|l}\right] w_l^P + F_l^P\right\},\end{aligned} \tag{4.52}$$

subject to:

$$\textit{(FOC-h)}: \quad \breve{p}_{h|h} \;=\; \underset{p}{\operatorname{argmax}}\; \breve{\Pi}^{P}_{h|h} \;=\; \Pi^{P}(p, w^{P}_{h}, F^{P}_{h}; \gamma_{h}, \cdot), \tag{4.53}$$

$$\textit{(FOC-l)}: \quad \breve{p}_{l|l} \;=\; \underset{p}{\operatorname{argmax}}\; \breve{\Pi}^{P}_{l|l} \;=\; \Pi^{P}(p, w^{P}_{l}, F^{P}_{l}; \gamma, \cdot), \tag{4.54}$$

$$\textit{(IR-h)}: \quad \breve{\Pi}^{P*}_{h|h} \;=\; \max_{p} \breve{\Pi}^{P}_{h|h} \;\geq 0, \tag{4.55}$$

$$\textit{(IR-l)}: \quad \breve{\Pi}^{P*}_{l|l} \;=\; \max_{p} \breve{\Pi}^{P}_{l|l} \;\geq 0, \tag{4.56}$$

$$\textit{(IC-h)}: \quad \breve{\Pi}^{P*}_{h|h} \;\geq\; \breve{\Pi}^{P*}_{l|h} \;=\; \max_{p} \breve{\Pi}^{P}_{l|h} = \Pi^{P}(p, w^{P}_{l}, F^{P}_{l}; \gamma_{h}, \cdot), \text{ and} \tag{4.57}$$

$$\textit{(IC-l)}: \quad \breve{\Pi}^{P*}_{l|l} \;\geq\; \breve{\Pi}^{P*}_{h|l} \;=\; \max_{p} \breve{\Pi}^{P}_{h|l} = \Pi^{P}(p, w^{P}_{l}, F^{P}_{l}; \gamma_{l}, \cdot). \tag{4.58}$$

(ii) The menu of G contracts: $\{P, w^{G}_{h}, F^{G}_{h}\}$ and $\{P, w^{G}_{l}, F^{G}_{l}\}$ (designed for the high and low state, respectively). This menu is the solution to:

$$\begin{aligned}
\max_{\substack{w^{G}_{h}, F^{G}_{h}, \breve{p}^{P}_{h|h}, \breve{p}^{GB}_{h|h} \\ w^{G}_{l}, F^{G}_{l}, \breve{p}^{P}_{l|l}, \breve{p}^{GB}_{l|l}}} \breve{\pi}^{G} =&\; \phi\, \pi^{G}(w^{G}_{h}, F^{G}_{h}, \breve{p}^{P}_{h|h}, \breve{p}^{GB}_{h|h}; \gamma_{h}, \cdot) \;+\; (1-\phi)\, \pi^{G}(w^{G}_{l}, F^{G}_{l}, \breve{p}^{P}_{l|l}, \breve{p}^{GB}_{l|l}; \gamma_{l}, \cdot) \\
=&\; \phi \Big\{ \big[\alpha - \beta\, \breve{p}^{P}_{h|h} - \gamma_{h}(\breve{p}^{GB}_{h|h} + \kappa_{L})\big] w^{G}_{h} + F^{G}_{h} \Big\} + \\
&+ (1-\phi) \Big\{ \big[\alpha - \beta \breve{p}^{P}_{l|l} - \gamma_{l}(\breve{p}^{GB}_{l|l} + \kappa_{L})\big] w^{G}_{l} + F^{G}_{l} \Big\},
\end{aligned} \tag{4.59}$$

subject to:

$$\textit{(FOC-h)}: \quad \{\breve{p}^{P}_{h|h}, \breve{p}^{GB}_{h|h}\} \;=\; \underset{p^{P}, p^{GB}}{\operatorname{argmax}}\; \breve{\Pi}^{G}_{h|h} \;=\; \Pi^{G}(p^{P}, p^{GB}, w^{G}_{h}, F^{G}_{h}; \gamma_{h}, \cdot), \tag{4.60}$$

$$\textit{(FOC-l)}: \quad \{\breve{p}^{P}_{l|l}, \breve{p}^{GB}_{l|l}\} \;=\; \underset{p^{P}, p^{GB}}{\operatorname{argmax}}\; \breve{\Pi}^{G}_{l|l} \;=\; \Pi^{G}(p^{P}, p^{GB}, w^{G}_{l}, F^{G}_{l}; \gamma_{l}, \cdot), \tag{4.61}$$

$$\textit{(IR-h)}: \quad \breve{\Pi}^{G*}_{h|h} \;=\; \max_{p^{P}, p^{GB}} \breve{\Pi}^{G}_{h|h} \;\geq 0, \tag{4.62}$$

$$\textit{(IR-l)}: \quad \breve{\Pi}^{G*}_{l|l} \;=\; \max_{p^{P}, p^{GB}} \breve{\Pi}^{G}_{l|l} \;\geq 0, \tag{4.63}$$

$$\textit{(IC-h)}: \quad \breve{\Pi}^{G*}_{h|h} \geq \breve{\Pi}^{G*}_{l|h} = \max_{p^P,\,p^{GB}} \breve{\Pi}^{G}_{l|h} = \Pi^G(p^P, p^{GB}, w^G_l, F^G_l; \gamma, \cdot), \text{ and} \tag{4.64}$$

$$\textit{(IC-l)}: \quad \breve{\Pi}^{G*}_{l|l} \geq \breve{\Pi}^{G*}_{h|l} = \max_{p^P,\,p^{GB}} \breve{\Pi}^{G}_{h|l} = \Pi^G(p^P, p^{GB}, w^P_h, F^P_h; \gamma_l, \cdot). \tag{4.65}$$

(iii) The menu of P and G contracts: $\{P, w^P, F^P\}$ (designed for the low state) and $\{G, w^G, F^G\}$ (designed for the high state)

$$\begin{aligned}
\max_{\substack{w^P,\,F^P,\,p_{l|l},\\ w^G,\,F^G,\,p^P_{h|h},\,p^{GB}_{h|h}}} \pi = {} & \phi\,\pi^G_{h|h} + (1-\phi)\,\pi^P_{l|l} \\
= {} & \phi\,\pi^G(w^G, F^G, p^P_{h|h}, p^{GB}_{h|h}; \gamma_h, \cdot) + (1-\phi)\,\pi^P(w^P, F^P, p_{l|l}, ; \gamma_l, \cdot) \\
= {} & \phi\Big\{\big[\alpha - \beta\, p^P_{h|h} - \gamma_h(p^{GB}_{h|h} + \kappa_L)\big] w^G + F^G\Big\} \\
& + (1-\phi)\Big\{\big[\alpha - (\beta + \gamma_l)p_{l|l}\big] w^P + F^P\Big\},
\end{aligned} \tag{4.66}$$

subject to:

$$\textit{(FOC-P)}: \quad p_{l|l} = \operatorname*{argmax}_{p} \Pi^P_{l|l} = \Pi^P(p, w^P, F^P; \gamma_l, \cdot), \tag{4.67}$$

$$\textit{(FOC-G)}: \quad \{p^P_{h|h}, p^{GB}_{h|h}\} = \operatorname*{argmax}_{p^P,\,p^{GB}} \Pi^G_{h|h} = \Pi^G(p^P, p^{GB}, w^G, F^G; \gamma_h, \cdot), \tag{4.68}$$

$$\textit{(IR-h)}: \quad {\Pi^G_{h|h}}^* = \max_{p^P,\,p^{GB}} \Pi^G_{h|h} \geq 0, \tag{4.69}$$

$$\textit{(IR-l)}: \quad {\Pi^P_{l|l}}^* = \max_{p} \Pi^P_{l|l} \geq 0, \tag{4.70}$$

$$\textit{(IC-h)}: \quad {\Pi^G_{h|h}}^* \geq {\Pi^P_{l|h}}^* = \max_{p} \Pi^P_{l|h} = \Pi^P(p, w^P, F^P; \gamma_h, \cdot), \text{ and} \tag{4.71}$$

$$\textit{(IC-l)}: \quad {\Pi^P_{l|l}}^* \geq {\Pi^G_{h|l}}^* = \max_{p^P,\,p^{GB}} \Pi^G_{h|l} = \Pi^G(p^P, p^{GB}, w^G, F^G; \gamma_l, \cdot). \tag{4.72}$$

Proposition 4.6 *Under asymmetric information regarding γ, the menu of P and G contracts does not help the manufacturer recover the first-best profitability. However, it provides an opportunity for the manufacturer to extract the most surplus at the lowest rent.*

Table 4.2: **Characteristics of the (menus of) contracts under asymmetric information regarding γ**

		High state (γ_h)	Low state (γ_l)
Full information[a]	First-best contract: $\{j, w^j, F^j\}$ ($j \in \{P, G\}$)	$w^{G*} = 0$ $F^{G*} = \frac{\eta^2\,\alpha^2}{4\beta} + \frac{[(1-\eta)\,\alpha - \gamma_h\,\kappa_L]^2}{4\gamma_h}$	$w^{P*} = 0$ $F^{P*} = \frac{\alpha^2}{4(\beta+\gamma_l)}$
Asymmetric information[b]	The menu of P contracts: $\{P, w_h^P, F_h^P\}$ & $\{P, w_l^P, F_l^P\}$	$w_h^{P*} = 0$ $F_h^{P*} = \frac{\alpha^2}{4(\beta+\gamma_h)}$	$w_l^{P*} = 0$ $F_l^{P*} = \frac{\alpha^2}{4(\beta+\gamma_l)} - \frac{(\gamma_h-\gamma_l)\alpha^2}{4(\beta+\gamma_h)(\beta+\gamma_l)}$
	The menu of G contracts: $\{G, w_h^G, F_h^G\}$ & $\{G, w_l^G, F_l^G\}$	$w_h^{G*} = \frac{(1-\phi)(\gamma_h-\gamma_l)\kappa_L}{\phi(\beta+\gamma_h)-(1-\phi)(\gamma_h-\gamma_l)}$ $F_h^{G*} = \frac{(\eta\,\alpha-\beta\,w_h^{G*})^2}{4\beta} + \frac{[(1-\eta)\alpha-\gamma_h(w_h^{G*}+\kappa_L)]^2}{4\gamma_h}$	$w_l^{G*} = 0$ $F_l^{G*} = \frac{\eta^2\,\alpha^2}{4\beta} + \frac{[(1-\eta)\,\alpha-\gamma_l\,\kappa_L]^2}{4\gamma_l} - \frac{\gamma_h-\gamma_l}{4}\left[\frac{(1-\eta)^2\alpha^2}{\gamma_h\,\gamma_l} - (w^{G*}+\kappa_L)^2\right]$
	The menu of P and G contracts: $\{P, w^P, F^P\}$ & $\{G, w^G, F^G\}$	$w^{G*} = \frac{(1-\phi)(\gamma_h-\gamma_l)\kappa_L}{\phi(\beta+\gamma_h)-(1-\phi)(\gamma_h-\gamma_l)}$ $F^{G*} = \frac{(\eta\,\alpha-\beta\,w_h^{G*})^2}{4\beta} + \frac{[(1-\eta)\alpha-\gamma_h(w_h^{G*}+\kappa_L)]^2}{4\gamma_h}$	$w^{P*} = 0$ $F^{P*} = \frac{\alpha^2}{4(\beta+\gamma_l)} - \frac{\gamma_h-\gamma_l}{4}\left[\frac{(1-\eta)^2\alpha^2}{\gamma_h\,\gamma_l} - (w^{G*}+\kappa_L)^2\right]$

[a] see Proposition 4.3.
[b] where $c = 0$.

Recall that (a) both the P and G contracts are more profitable in the low state than in the high state, and (b) the G contract is less profitable than the P contract in the low state. By the construction of the menu, the P contract is selected in the low state and the G in the high state. In other words, the menu provides the benefit of price discrimination in the high state and helps avoid the cost due to κ_L in the low state. However, it does require an amount of rent, paid to the retailer in the low state. This is the rent to mitigate the retailer's incentive to choose the undesired, G contract when the market is in the low state. Since the G contract is less profitable than the P contract in the low state, the rent is less than that of the P-menu. In summary, though the first-best is not recovered, the menu of P and G contracts gives the manufacturer the second-best outcome.

4.7 Conclusion and future research

In this chapter, we explore the role that group-buying services may play in improving channel coordination. Given that the retailer may introduce group-buying as an additional option for consumers to purchase the product, the manufacturer can take advantage of the retailer's *dual* pricing mechanism for better channel coordination. Here, the problem of channel coordination is one of asymmetric information whose existence requires, in general, the manufacturer to reserve some informational rent to the retailer due to its private information. Interestingly, by including pricing mechanism as a term in the contract and offering a menu of these contracts to the retailer for self-selection, the manufacturer is better off—it pays a lower rent (compared to the contract without the pricing term or one with a single pricing mechanism) and under some conditions, it needs not pay any rent at all.

Consider a menu of P and G contracts. Since one contract is more profitable in a specific market state and less profitable in the other (e.g., the P contract is more profitable when the market is in the high state regarding the price sensitivity of consumers in the L-segment), the pricing mechanism in the contract serves as a *sensor* of the market condition. In other words, the manufacturer can *detect* the state of the market when having this device in the contract. Most importantly, depending on the type of asymmetric information, the use of this device has different cost to the manufacturer.

Importantly, when asymmetric information exists regarding the relative sizes of the two segments, a term on pricing mechanism in the contract serves as a market detector at no cost to the manufacturer. In this setting, since the retailer earns identical profits in both market states—high and low—when employing the P contract, it has no incentive to deviate from the type of contract that the manufacturer desires. This results in an alignment of interests and allows the manufacturer to restore the outcome of the first-best setting (i.e., under full information) without paying any informational rent to the retailer.

However, when asymmetric information exists regarding price sensitivity of consumers in either segment—high and low—, the retailer will have incentive to deviate from the full-information contract that the manufacturer desires when the market is in the low state (in order to pay the low fixed fee of the high-state full-information contract). Informational rent is due to the retailer in this setting. But, since the employment of the optimal pricing mechanism raises the profitability, and consequently, the fixed fee of the high-state full-information contract, the retailer has less incentive to choose this contract (i.e., mis-inform the manufacturer about the market condition) when the market is in the low state; less incentives results in lower informational rent.

Overall, this study contributes to the literature on channel coordination and group-buying—it provides the insights on how group-buying may be used to improve channel coordination. A possible direction for future research on group-buying includes the exploration of other market characteristics, such as market fragmentation/consumer collective power and price transparency, that may impact the benefit of this mechanism. These are important aspects of the emerging on-line social/shopping communities. A systematic analysis is helpful in understanding the mechanism of this and other on-line business models as well as predicting the future of on-line shopping. Further, competition at both the upstream and downstream levels is not considered in our analysis. Since competition may enhance the benefit of this pricing mechanism to one party and raise the cost to the other party, future research in this direction is warranted.

APPENDIX A: PROOFS OF RESULTS IN CHAPTER 2

A.1 The demand structure

As noted in the model section, our focus is on a duopoly market served by two retailers, R_1 and R_2, with the same product supplied by a single manufacturer. The market consists of m consumers, each of whom desires at most one unit of the product offered by either retailer. Each consumer is characterized by two parameters: (a) a valuation for the product, and (b) a preference for shopping at a particular retailer.

The valuation for the product, V, is unknown to consumers ex-ante (i.e., at the time of purchasing), but is known ex-post (i.e., after they actually experience the product). Specifically, after using the product, consumers will be able to determine whether the product matches their needs or not. When the product is a match, consumers enjoy a high valuation, V_H; otherwise, the valuation is V_L $(0 \leq V_L < V_H)$. At the time of purchasing, though not knowing the actual valuation for the product, consumers have a prior belief that the product will (not) match their needs, and hence, will deliver a high (low) valuation of V_H (V_L) with probability ϕ (with probability $1 - \phi$); mathematically, $\text{prob}[V = V_H] \stackrel{\text{def}}{=} \phi$ and $\text{prob}[V = V_L] \stackrel{\text{def}}{=} 1 - \phi$, where $\phi \in [0, 1]$. It is important to note that the actual realization of V is independent from consumers' belief. For instance, when deciding to buy a computer, consumers may have a strong belief that the computer will be a match based on the observed product features, but may realize ex-post that this is not the case, e.g., due to the inconvenient keyboard design or the layout of the touch-pad. Further, we make the assumption that consumers are identical in terms of prior belief and risk-aversion (and therefore, identical in the certainty equivalent, v), but heterogeneous in their ex-post valuation, V.

When making the purchase decision, consumers do not know their ex-post valuation, and therefore, rely on their belief as well as level of risk aversion. In other words, they make the purchase decision based on the certainty equivalent, v, of the product. Specifically, if consumers are risk neutral, then $v = E[V] = \phi V_H + (1-\phi)V_L$; otherwise, if they are risk-averse, then $V_L \leq v < E[V]$ (see Che 1996). In contrast, after experiencing the product, consumers observe the realized valuation (either V_H or V_L), and make the decision to keep or return the product if a returns policy is available. The reasons for returning the product may include (1) dissatisfaction due to the mismatch between the product and the needs of consumers, i.e., a low ex-post valuation (V_L), and (2) opportunism (Chu et al. 1998). Denote the rate of returns from consumers by ρ. In this paper, ρ captures the fraction of consumers who either experience V_L or behave in an opportunistic manner per Chu et al. (1998) and return the product despite high valuation V_H[42].

Regarding consumers' preferences for the retailers, we assume that the m consumers are uniformly distributed on a Hotelling-type (1929) linear market, from $\underline{x}$ to $\bar{x}$ with a unit transportation cost of t while retailers R_1 and R_2 are located at a_1 and a_2 respectively ($\underline{x} < a_1 \leq \frac{\underline{x}+\bar{x}}{2} \leq a_2 < \bar{x}$). Further, the two retailers are assumed to be located symmetrically, i.e., $a_1 - \underline{x} = \bar{x} - a_2 \Leftrightarrow a_2 = \underline{x} + \bar{x} - a_1$. Given these market characteristics, we derive the demand functions under various combinations of consumer returns, $\{\sigma_1^R, \sigma_2^R\}$. Notice that by symmetry, the demand functions in the cases when only one retailer offers consumer returns (i.e., under $\{NCR,CR\}$ and $\{CR,NCR\}$), are identical.

A.1.1 Demand functions when no retailers accept consumer returns

Under a no-returns policy, consumers bear the entire risk associated with uncertain ex-post valuation. As a result, consumers' purchase decisions are based on the certainty equivalent,

[42] Note that in most businesses, it is the retailers' (and ultimately, the manufacturer's responsibility) to accept returns of defective products from consumers. In contrast, accepting returns from the two sources listed above is the choice of the retailers. Since we are interested in understanding the strategic reasons why the manufacturer and the retailers accept returns, our focus is on these sources of returns. This focus facilitates our treatment of ρ as a legitimate exogenous parameter. For a given level of product quality, the distribution of consumers who return the product due to a mismatch or opportunistic incentives is likely to be independent of firms' actions.

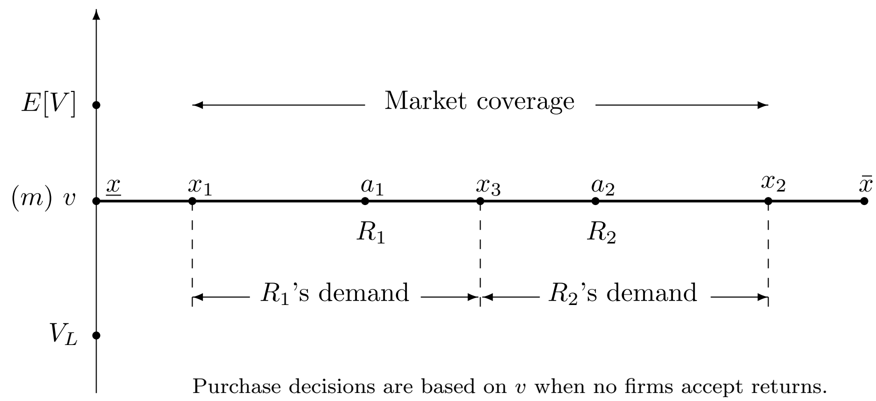


Figure A.1: **Market structure when no retailers accept consumer returns**

v. Figure A.1 depicts the market structure when neither retailer accepts returns, i.e., under $\{NCR,NCR\}$.

To derive the demand function of each retailer, we need to specify the locations of three critical consumers, including the two zero-utility consumers located at x_1 and x_2 and the indifferent consumer located at x_3. Consumers located between x_1 and x_3 will choose to buy from retailer R_1 while those located between x_3 and x_2 consider buying from retailer R_2. For a given set of retail prices $\{p_1, p_2\}$, the specifications of x_1, x_2, and x_3 are given below:

$$v - (a_1 - x_1)t - p_1 = 0 \quad \Leftrightarrow \quad x_1 = a_1 + \frac{p_1 - v}{t}, \tag{A.1}$$

$$v - (x_2 - a_2)t - p_2 = 0 \quad \Leftrightarrow \quad x_2 = a_2 - \frac{p_2 - v}{t}, \text{ and} \tag{A.2}$$

$$v - (x_3 - a_1)t - p_1 = v - (a_2 - x_3)t - p_2 \quad \Leftrightarrow \quad x_3 = \frac{a_1 + a_2}{2} - \frac{p_1 - p_2}{2t}. \tag{A.3}$$

Note that the above-mentioned market structure arises under the following conditions:

$$\begin{cases} \underline{x} & \leq x_1 \leq a_1 \\ a_2 & \leq x_2 \leq \bar{x} \\ a_1 & \leq x_3 \leq a_2 \end{cases} \Leftrightarrow \begin{cases} v - (a_1 - \underline{x})t & \leq p_1 \leq v \\ v - (\bar{x} - a_2)t & \leq p_2 \leq v \\ |p_1 - p_2| & \leq (a_2 - a_1)t. \end{cases} \tag{A.4}$$

Hereafter, we focus on the setting in which conditions A.4 are satisfied. As a result, the demand function of retailer R_1 has the following specification:

$$q_1 = m(x_3 - x_1) = \frac{mv}{t} + \frac{m(a_2 - a_1)}{2} - \frac{3m}{2t}p_1 + \frac{m}{2t}p_2. \tag{A.5}$$

Analogously, the demand function of retailer R_2 is:

$$q_2 = m(x_2 - x_3) = \frac{mv}{t} + \frac{m(a_2 - a_1)}{2} - \frac{3m}{2t}p_2 + \frac{m}{2t}p_1. \tag{A.6}$$

A.1.2 Demand functions when both retailers accept consumer returns

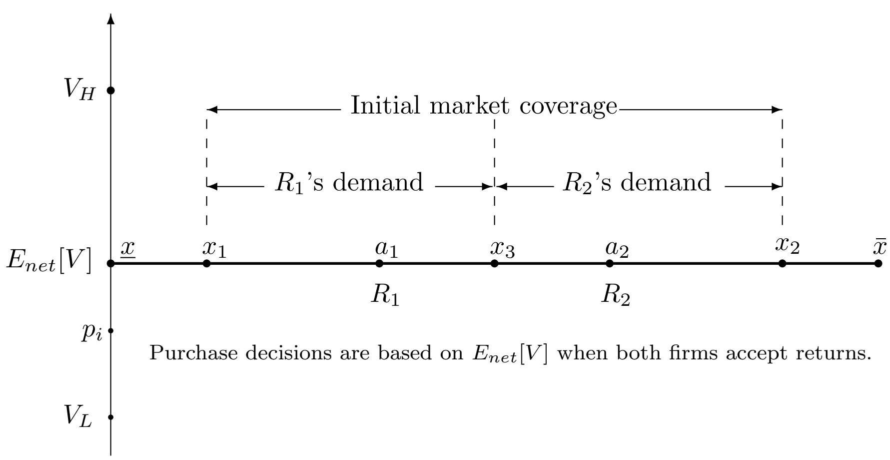


Figure A.2: **Market structure when both retailers accept consumer returns**

When returns are accepted, consumers have an option to return the product after purchase to get a full refund of the retail price. A returns policy releases consumers from the risk of buying a product with a low ex-post valuation (V_L). Despite a full refund, we assume that consumers will incur a cost of κ ($\kappa \geq 0$) to return the product. For instance, it costs consumers extra time and effort to bring the product back to the store[43]. At the time of purchasing, consumers believe that (1) with probability ϕ, the product will match their needs and deliver a high value, V_H, and hence, they will keep the product paying the retail price, p, and (2) with probability $1-\phi$, they will experience V_L and will return the product, incurring the cost of returns, κ. Based on this belief, consumers derive a net expected utility, $E_{net}[V] = \phi(V_H - p) + (1-\phi)(-\kappa)$ and make purchase decisions based on this expectation (see Figure A.2). After purchasing, a fraction ρ of consumers will return the product.

In this case, the initial demand functions are derived based on the locations of the three critical consumers, x_1, x_2 and x_3, whose specifications are given below:

$$\phi(V_H - p_1) + (1-\phi)(-\kappa) - t(a_1 - x_1) = 0 \Leftrightarrow x_1 = a_1 + \frac{(1-\phi)\kappa}{t} + \frac{\phi(p_1 - V_H)}{t}, \quad \text{(A.7)}$$

$$\phi(V_H - p_2) + (1-\phi)(-\kappa) - t(x_2 - a_2) = 0 \Leftrightarrow x_2 = a_2 - \frac{(1-\phi)\kappa}{t} - \frac{\phi(p_2 - V_H)}{t}, \quad \text{(A.8)}$$

$$\phi(V_H - p_1) + (1-\phi)(-\kappa) - t(x_3 - a_1) = \phi(V_H - p_2) + (1-\phi)(-\kappa) - t(a_2 - x_3)$$
$$\Leftrightarrow \quad x_3 = \frac{a_1 + a_2}{2} + \frac{\phi(p_2 - p_1)}{2t}. \quad \text{(A.9)}$$

Again, we assume that the conditions that ensure the above-mentioned market structure are satisfied:

$$\begin{cases} \underline{x} & \leq x_1 \leq a_1 \\ a_2 & \leq x_2 \leq \bar{x} \\ a_1 & \leq x_3 \leq a_2 \end{cases} \Leftrightarrow \begin{cases} V_H - \frac{(1-\phi)\kappa}{\phi} - \frac{(a_1-\underline{x})t}{\phi} & \leq p_1 \leq V_H - \frac{(1-\phi)\kappa}{\phi} \\ V_H - \frac{(1-\phi)\kappa}{\phi} - \frac{(a_1-\underline{x})t}{\phi} & \leq p_2 \leq V_H - \frac{(1-\phi)\kappa}{\phi} \\ |p_1 - p_2| & \leq \frac{(a_2-a_1)t}{\phi}. \end{cases} \quad \text{(A.10)}$$

[43]Note that in our setting, a positive κ limits the magnitude of opportunistic returns; in contrast, when $\kappa = 0$, such a limit will not arise.

As a result, retailer R_1's initial-demand function is:

$$\hat{q}_1 = m(x_3 - x_1) = \frac{m\phi V_H}{t} + \frac{m(a_2 - a_1)}{2} - \frac{m(1-\phi)\kappa}{t} - \frac{3m\phi}{2t}p_1 + \frac{m\phi}{2t}p_2. \tag{A.11}$$

Meanwhile, retailer R_2 faces initial demands according to the following function:

$$\hat{q}_2 = m(x_2 - x_3) = \frac{m\phi V_H}{t} + \frac{m(a_2 - a_1)}{2} - \frac{m(1-\phi)\kappa}{t} - \frac{3m\phi}{2t}p_2 + \frac{m\phi}{2t}p_1. \tag{A.12}$$

At the rate of returns equal to ρ, the retailers' net-demand functions are:

$$q_i = (1-\rho)\hat{q}_i, \text{ where } i = 1, 2. \tag{A.13}$$

A.1.3 Demand functions when one retailer accepts consumer returns

Consider the case when only R_2 accepts returns. (The case when only R_1 accepts returns is analogous by symmetry.) Having the option to return products, consumers who consider purchasing from R_2 make their decision based on the net expected utility, $E_{net}[V]$ (for definition, see the previous section). In contrast, those consumers who consider purchasing from R_1 will take the certainty equivalent, v, into account.

Solving for the locations of the three critical consumers, x_1, x_2, and x_3, we get:

$$v - (a_1 - x_1)t - p_1 = 0 \Leftrightarrow x_1 = a_1 + \frac{p_1 - v}{t}, \tag{A.14}$$

$$\phi(V_H - p_2) + (1-\phi)(-\kappa) - t(x_2 - a_2) = 0 \Leftrightarrow x_2 = a_2 - \frac{(1-\phi)\kappa}{t} - \frac{\phi(p_2 - V_H)}{t}, \tag{A.15}$$

$$v - (x_3 - a_1)t - p_1 = \phi(V_H - p_2) + (1-\phi)(-\kappa) - t(a_2 - x_3)$$

$$\Leftrightarrow \quad x_3 = \frac{a_2 + a_1}{2} - \frac{\phi V_H - v}{2t} + \frac{(1-\phi)\kappa}{2t} + \frac{\phi p_2 - p_1}{2t}. \tag{A.16}$$

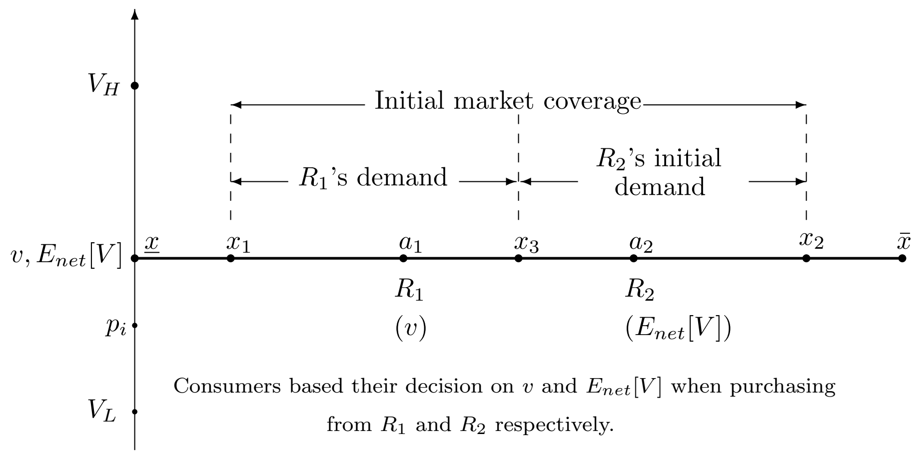


Figure A.3: **Market structure when only R_2 accepts consumer returns**

Analogously, this market structure requires:

$$\begin{cases} \underline{x} & \leq x_1 \leq a_1 \\ a_2 & \leq x_2 \leq \bar{x} \\ a_1 & \leq x_3 \leq a_2 \end{cases} \tag{A.17}$$

$$\Leftrightarrow \begin{cases} v - (a_1 - \underline{x})t & \leq p_1 \leq v \\ V_H - \frac{(1-\phi)\kappa}{\phi} - \frac{(a_1-\underline{x})t}{\phi} & \leq p_2 \leq V_H - \frac{(1-\phi)\kappa}{\phi} \\ (a_1 - a_2)t + (\phi V_H - v) + (1-\phi)\kappa & \leq \phi p_2 - p_1 \leq (a_2 - a_1)t + (\phi V_H - v) + (1-\phi)\kappa. \end{cases} \tag{A.18}$$

Consequently, R_1's demand function has the following specification:

$$q_1 = m(x_3 - x_1) = \frac{mv}{t} - \frac{m(\phi V_H - v)}{2t} + \frac{m(a_2 - a_1)}{2} + \frac{m(1-\phi)\kappa}{2t} - \frac{3m}{2t}p_1 + \frac{m\phi}{2t}p_2, \tag{A.19}$$

while R_2's initial-demand function, $\hat{q}_2$, and net-demand function, q_2, are given by:

$$\begin{aligned} \hat{q}_2 &= m(x_2 - x_3) \\ &= \frac{m\phi V_H}{t} + \frac{m(\phi V_H - v)}{2t} + \frac{m(a_2 - a_1)}{2} - \frac{3m(1-\phi)\kappa}{2t} - \frac{3m\phi}{2t}p_2 + \frac{m}{2t}p_1, \text{ and} \quad \text{(A.20)} \\ q_2 &= (1-\rho)\hat{q}_2, \text{ respectively.} \quad \text{(A.21)} \end{aligned}$$

A.1.4 Simplification of the demand functions

Without loss of generality, let $m = 2, V_H = 1, V_L = 0, a_1 = 0$, and $a_2 = 1$[44]. For further simplification, we assume that consumers are ex-ante completely unsure about their ex-post valuation of the product[45], i.e. $\phi = \frac{1}{2}$. In addition, we define $\tau \stackrel{\text{def}}{=} \frac{1}{t}$, $\mu \stackrel{\text{def}}{=} \frac{(1-\phi)\kappa}{t}$. With these simplifications, the demand functions can be rewritten as follows.

When no retailers offer consumer returns, i.e., under $\{NCR,NCR\}$, the demand functions are:

$$q_i = 1 + 2\tau v - 3\tau p_i + \tau p_j \quad (i, j = 1, 2 \text{ and } i \neq j). \quad \text{(A.22)}$$

When both retailers choose to offer consumer returns, i.e. under $\{CR,CR\}$, their initial sales are realized based on the initial-demand functions

$$\hat{q}_i = 1 + \tau - 2\mu - \frac{3\tau}{2}p_i + \frac{\tau}{2}p_j \quad (i, j = 1, 2 \text{ and } i \neq j). \quad \text{(A.23)}$$

[44]This requires: (1) $\underline{x} < 0$, (2) $\bar{x} > 1$, (3) $\underline{x} = 1 - \bar{x}$ (for symmetric location of the two retailers), and (4) $\underline{x}$ and $\bar{x}$ are such that the market is not fully covered (i.e., conditions stated by (A.4), (A.10), and (A.18) are satisfied).

[45]Relaxing this assumption does not change the insights of the analysis since a stronger belief (i.e., higher ϕ) is equivalent to a higher rate of returns, ρ and vice versa.

In the case where only retailer i offers consumer returns, its initial-demand function, $\hat{q}_i$, is:

$$\hat{q}_i = 1 + \frac{3\tau}{2} - \tau v - 3\mu - \frac{3\tau}{2}p_i + \tau p_j. \tag{A.24}$$

The demand function of retailer j, who does not accept returns from consumers, is

$$q_j = 1 - \frac{\tau}{2} + 3\tau v + \mu - 3\tau p_j + \frac{\tau}{2}p_i. \tag{A.25}$$

Next, we summarize the range of the parameters used in the model. We have $t \geq 0 \Leftrightarrow \tau \in (0, +\infty)$, and $V_L \leq v \leq E[V] \Leftrightarrow v \in [0, \frac{1}{2}]$. In addition, we need to ensure non-negative primary demand (i.e., the intercept) in each of the demand functions specified above. First, notice that the primary demand when no retailers accept returns is positive. Non-negativity of primary demand in (A.25) is guaranteed when $v \geq \frac{1}{6}$. Accordingly, we make the assumption that consumers are not extremely risk-averse so that $v \geq \frac{1}{6}$ and impose the following restraints on (A.23) and (A.24):

$$1 + \tau - 2\mu \geq 0 \Leftrightarrow \mu \leq \frac{1}{2} + \frac{\tau}{2}, \text{ and} \tag{A.26}$$

$$1 + \frac{3\tau}{2} - \tau v - 3\mu \geq 0 \Leftrightarrow \mu \leq \frac{1}{3} + \frac{\tau}{2} - \frac{\tau v}{3}. \tag{A.27}$$

These two conditions are satisfied when $0 \leq \mu \leq \bar{\mu}$, where $\bar{\mu} \stackrel{\text{def}}{=} \frac{1}{3} + \frac{\tau}{2} - \frac{\tau v}{3}$.

Table 2.1 summarizes this result—the demand functions of the retailers under various combinations of consumer returns policies and the range of the parameters.

A.2 The integrated-manufacturer model (Proof of Lemma 2.1)

Consider an integrated manufacturer (refer to as *IM*), who is the owner of the two retailers (or stores), R_1 and R_2. Let the demand functions of R_1 and R_2 be defined as in Table 2.1. The game unfolds in three stages. In the first stage, *IM* decides whether to accept consumer returns. Next, it chooses the production (stock) level, s, in the second stage, and then sets the retail prices, p_1 and p_2, in the third stage. We solve the game by employing backward induction.

A.2.1 The optimal price and stock level under *NCR*

From Table 2.1, the demand functions of the two retailers, R_1 and R_2, are:

$$q_1 = 1 + 2\tau v - 3\tau p_1 + \tau p_2, \text{ and} \tag{A.28}$$

$$q_2 = 1 + 2\tau v - 3\tau p_2 + \tau p_1 \text{ respectively.} \tag{A.29}$$

In the third stage, *IM* sets the retail prices to clear stock:

$$s = q_1 + q_2 = 2(1 + 2\tau v) - 2\tau p_1 - 2\tau p_2 \;\Leftrightarrow\; p_1 + p_2 = \frac{2(1 + 2\tau v) - s}{2\tau}. \tag{A.30}$$

Since *IM* may set different retail prices (i.e., $p_1 \neq p_2$), let $p_2 = p_1 + \epsilon$, where $\epsilon \geq 0$. The profit function of *IM* is then given by:

$$\pi^I \;=\; q_1 p_1 + q_2 p_2 - sc \;=\; \frac{\big[2(1 + 2\tau v) - 4\tau c\big]s - s^2 - 8\tau^2\epsilon^2}{4\tau}. \tag{A.31}$$

Notice that since $\frac{\partial \pi^I}{\partial \epsilon} = -4\tau\epsilon$ and $\frac{\partial^2 \pi^I}{\partial \epsilon^2} = -4\tau < 0$, *IM* is always better off by choosing $\epsilon = 0$; this means *IM* will set the same retail prices at the two retailers:

$$p_1 = p_2 = \frac{2(1+2\tau v) - s}{4\tau} \text{ (by (A.30))}. \tag{A.32}$$

Then, in the second stage, *IM* chooses the level of production (i.e., stock), s, so that its profit is maximized:

$$\max_s \pi^I = q_1 p_1 + q_2 p_2 - sc = s\left[\frac{2(1+2\tau v) - s}{4\tau} - c\right]. \tag{A.33}$$

Solving the first-order condition, we obtain the optimal stock level[46]:

$$s_0 = 1 + 2\tau v - 2\tau c. \tag{A.34}$$

Consequently, the optimal retail prices and profit of *IM* under *NCR* are as follows:

$$p_0 = \frac{1 + 2\tau v + 2\tau c}{4\tau}, \text{ and} \tag{A.35}$$

$$\pi_0^I = \frac{(1 + 2\tau c - 2\tau c)^2}{4\tau}. \tag{A.36}$$

Regularity requires the stock level to be non-negative, i.e., $s_0 \geq 0 \Leftrightarrow c \leq v + \frac{1}{2\tau} \overset{\text{def}}{=} \hat{c}_1$.

A.2.2 The optimal price and stock level under *CR*

Under *CR*, the initial- and net-demand functions are given by:

$$\hat{q}_i = 1 + \tau - 2\mu\frac{3\tau}{2}p_i + \frac{\tau}{2}p_j, \text{ and} \tag{A.37}$$

$$q_i = (1-\rho)\hat{q}_i, \tag{A.38}$$

[46]Note that the second-order condition holds in this case as well as in the subsequent analyses.

where $i, j = 1, 2$ and $i \neq j$.

IM has two options to set the retail prices in the third stage: (Option A) price to clear stock according to the initial demand, $\hat{q}_i$, and (Option B) price to clear stock according to the net demand, q_i. Choosing option A, *IM* will have enough stock to serve all customers who are willing to purchase the product at the chosen price. However, it will incur some unsold inventory due to returns at the end of the selling period. In contrast, under option B, *IM* will eventually clear all the ordered stock[47] but will incur the stock-out cost since the level of stock is not enough to cover the initial demand.

Consider pricing option A. At a given level of stock, s, *IM* sets the retail prices in the third stage as follows:

$$\begin{aligned} & s = \hat{q}_1 + \hat{q}_2 = 2(1+\tau-2\mu) - \tau(p_1+p_2) \\ \Leftrightarrow \quad & p_1 + p_2 = \frac{2(1+\tau-2\mu)-s}{\tau}. \end{aligned} \tag{A.39}$$

Analogous to the case under *NCR*, we can show that *IM* will set the same retail prices at the two retailers, i.e.,

$$p_1 = p_2 = \frac{2(1+\tau-2\mu)-s}{2\tau}. \tag{A.40}$$

At this price, it earns:

$$\begin{aligned} \pi_1^{I,A} &= (1-\rho)\hat{q}_1 p_1 + (1-\rho)\hat{q}_2 p_1 - sc \\ &= \frac{s[2(1+\tau-2k)(1-\rho)-(1-\rho)s-2\tau c]}{2\tau}. \end{aligned} \tag{A.41}$$

[47] By adopting the reselling policy (see Che 1996).

In contrast, under option B, IM sets the retail prices at:

$$\begin{aligned} s = q_1 + q_2 &= (1-\rho)[2(1+\tau-2k) - \tau(p_1+p_2)] \\ \Leftrightarrow \quad p_1 + p_2 &= \frac{2(1+\tau-2k) - \frac{s}{1-\rho}}{\tau}. \end{aligned} \tag{A.42}$$

Again, it can be shown that IM will optimally set the same retail prices:

$$p_1 = p_2 = \frac{2(1+\tau-2\mu) - \frac{s}{1-\rho}}{2\tau}. \tag{A.43}$$

Consequently, its profit function is given by:

$$\pi_1^{I,B} = (1-\rho)\hat{q}_1\, p_1 + (1-\rho)\hat{q}_2\, p_1 - \eta(\hat{q}_1 + \hat{q}_2 - s) - sc \tag{A.44}$$

$$\begin{aligned} &= (1-\rho)\hat{q}_1\, p_1 + (1-\rho)\hat{q}_2\, p_1 - \eta\frac{\rho}{1-\rho}s - sc \\ &= \frac{s[2(1+\tau-2k)(1-\rho) - s - 2\tau(1-\rho)c - 2\tau\rho\eta]}{2\tau(1-\rho)}. \end{aligned} \tag{A.45}$$

Upon comparing the profitability of the two options, IM will choose option A if and only if:

$$\begin{aligned} \pi_1^{I,A} \geq \pi_1^{I,B} \quad &\Leftrightarrow \quad \pi_1^{I,A} - \pi_1^{I,B} \geq 0 \\ &\Leftrightarrow \quad \frac{s\rho[s(2-\rho) + 2\tau\eta - 2(1+\tau-2k)(1-\rho)]}{2\tau(1-\rho)} \geq 0 \\ &\Leftrightarrow \quad s \geq \frac{2(1+\tau-2k)(1-\rho) - 2\tau\eta}{2-\rho} \overset{\text{def}}{=} \hat{s}. \end{aligned} \tag{A.46}$$

In summary, IM sets the retail price in the third stage as follows:

$$p_1 = p_2 = p = \begin{cases} \frac{2(1+\tau-2k)-s}{2\tau} & \text{if } s \geq \hat{s}, \\ \frac{2(1+\tau-2k)-\frac{s}{1-\rho}}{2\tau} & \text{if } s < \hat{s}. \end{cases} \tag{A.47}$$

Now, consider the second stage. *IM* sets s at the optimal level so that its profit is maximized, i.e.,

$$\max_s \pi^I = \begin{cases} \pi_1^{I,A} & \text{if } s \geq \hat{s}, \\ \pi_1^{I,B} & \text{if } s < \hat{s}. \end{cases} \tag{A.48}$$

Notice that $\pi_1^{I,A}$ is a concave quadratic function in s. Its maximum value,

$$\pi_1^{I,A*} = \frac{[(1+\tau-2k)(1-\rho)-\tau c]^2}{2\tau(1-\rho)}, \tag{A.49}$$

is obtained at

$$s_A^* = 1+\tau-2k-\frac{\tau}{1-\rho}c. \tag{A.50}$$

Regularity requires $s_A^* \geq 0 \Leftrightarrow c \leq \frac{(1+\tau-2k)(1-\rho)}{\tau} \stackrel{\text{def}}{=} \hat{c}_2$.

Similarly, $\pi_1^{I,B}$ is also a concave quadratic function in s, which obtains maximum at

$$s_B^* = (1+\tau-2k)(1-\rho)-\tau(1-\rho)c-\tau\rho\eta. \tag{A.51}$$

The maximum value of $\pi_1^{I,B}$ is

$$\pi_1^{I,B*} = \frac{[(1+\tau-2k)(1-\rho)-\tau(1-\rho)c-\tau\rho\eta]^2}{2\tau(1-\rho)}. \tag{A.52}$$

Here, regularity requires $s_B^* \geq 0 \Leftrightarrow \eta \leq \frac{(2-c)(1-\rho)}{\rho} \stackrel{\text{def}}{=} \bar{\eta}$.

Solving the optimization problem defined by (A.48)[48], we obtain the following result: (a) when $\eta < c$, *IM* chooses production level s_B^*, prices according to option B and earns profits $\pi_1^{I,B*}$, and (b) when $\eta \geq c$, *IM* is better off by setting the production level at s_A^* and pricing according to option A; it earns profit equal $\pi_1^{I,A*}$.

[48]The proof is available upon request.

A.2.3 *IM*'s choice of consumer returns

In the first stage, *IM* decides on its consumer-returns policy by comparing the profit earned under *NCR* to that under *CR*. For simplicity, we focus on the case of risk-neutral consumers and zero transaction cost for returning the product (i.e., $v = \frac{1}{2}$ and $\mu = 0$). In addition, we restrict the production cost, c, and stock-out cost, η, to be reasonable, i.e., $c \leq \bar{c} \overset{\text{def}}{=} \min\{\hat{c}_1, \hat{c}_2\}$ and $\eta \leq \min\{\bar{\eta}, \bar{c}\}$. Based on the previous results, we have two possibilities:

(1) When $0 \leq c < \eta$, *IM* chooses option A under *CR*. Therefore, it compares π_0^I to $\pi_1^{I,A*}$ when deciding whether to accept returns. Consider

$$\Delta_1 = \pi_1^{I,A*} - \pi_0^I = -\frac{\tau(1-2\rho)}{2(1-\rho)}c^2 + \frac{(1+\tau)^2(1-2\rho)}{4\tau}. \tag{A.53}$$

Notice that (a) Δ_1 is a concave quadratic function in $c \in [0, \eta]$, (b) $\Delta_1|_{c=0} = \frac{(1-2\rho)(1+\tau)^2}{4\tau} \geq 0$ since $\rho \leq \frac{1}{2}$, and (c) $\Delta_1|_{c=\eta} = \frac{(1-2\rho)[1+2\tau+(1-2\eta^2)\tau^2-\rho(1+\tau)^2]}{4\tau(1-\rho)} \geq 0$ (since $\rho \leq \frac{1}{2}$ and $\eta \leq \hat{c}_1$). This implies that $\Delta_1 \geq 0 \quad \forall c \in [0, \eta]$, i.e., *IM* is better off by choosing *CR*.

(2) When $\eta \leq c \leq \bar{c}$, *IM* sets prices according to option B and earns profit equal $\pi_1^{I,B*}$ under *CR*. Consider

$$\begin{aligned}\Delta_2 = \pi_{1,B}^{IM*} - \pi_0^{IM} = &-\frac{1}{2}\tau(1+\rho)c^2 + (1+\tau+\tau\eta)\rho c + \\ &\frac{(1+\tau)^2 + 2\rho^2(1+\tau+\tau\eta)^2 - (1+\tau)[3+\tau(3+4\eta)]\rho}{4\tau(1-\rho)}.\end{aligned} \tag{A.54}$$

Since (a) Δ_2 is a concave quadratic function in $c \in [\eta, \hat{c}_1] \subseteq [\eta, \bar{c}]$[49], (b) $\Delta_2|_{c=\eta} = \Delta_1|_{c=\eta} \geq 0$ (as shown above), and (c) $\Delta_2|_{c=\hat{c}_1} = \frac{[(1+\tau)(1-\rho)-2\tau\rho\eta]^2}{8\tau(1-\rho)} \geq 0$, we have $\Delta_2 \geq 0 \quad \forall c \in [\eta, \hat{c}_1]$, which implies $\Delta_2 \geq 0 \quad \forall c \in [\eta, \bar{c}]$. Consequently, *IM* also chooses to accept consumer returns when $\eta \leq c \leq \bar{c}$.

[49]More precisely, $\hat{c}_1 = \bar{c} = \min\{\hat{c}_1, \hat{c}_2\}$ since $\hat{c}_1 < \hat{c}_2$.

In summary, the integrated manufacturer will always choose to accept consumer returns. However, its pricing strategy depends on the magnitude of stock-out cost relative to the magnitude of production cost. Specifically, if stock-outs are less costly (i.e., $\eta < c$), then *IM* prefers incuring the cost of stock-outs to bearing the cost of unsold inventory; it sets the retail price to clear stock according to the net demand in this case. In contrast, if stock-outs are more costly (i.e., $\eta \geq c$), *IM* is in favor of having unsold inventory; it sets the retail price according to the initial demand. The statement of Lemma 2.1 follows. ■

A.3 Analysis with independent retailers

For expositional ease, we begin with the analysis of the case where the marginal cost of stock-outs, η, is low (i.e., case 2), followed by the analysis of cases 1 and 3. We solve the game in each case using backward induction: First, we solve for the retailers' optimal prices and stock levels in stages four and three. Then, we characterize the retailers' choice of consumer returns policy in the second stage. Finally, the manufacturer's distribution policy, including the optimal wholesale price and returns policy, is determined.

A.3.1 Proof of Lemma 2.2 and Proposition 2.2

A.3.1.1 The retailers' pricing and stocking strategies (Case 2, stages 4 and 3)

We start with the combinations of symmetric consumer returns under *NMR*, i.e., {*NMR,NCR, NCR*} and {*NMR,CR,CR*}. Then, we solve one of the two combinations with asymmetric consumer returns under *NMR*: {*NMR,NCR,CR*}. By symmetry, the results of the other asymmetric combination, {*NMR,CR,NCR*}, are similar to those of {*NMR,NCR,CR*}. Next, under *MR*, we investigate combinations {*MR,NCR,NCR*}, {*MR,CR,CR*}, and {*MR, NCR,CR*}. The results are summarized in Tables 2.3 and 2.4. Note: When η is relatively low, we focus on the setting where, under *NMR*, the retailers are willing to bear the stock-out cost (instead of the cost of unsold inventory), i.e., they set the retail prices in stage four to clear stock according to the net-demand functions when the manufacturer does not accept returns.

Combination 1: {*NMR,NCR,NCR*}

Under $\{NCR,NCR\}$, the net demand is identical to the initial demand, i.e., $q_i = \hat{q}_i = 1 + 2\tau v - 3\tau p_i + \tau p_j$, where $i, j = 1, 2$ and $i \neq j$ (see Table 2.1). The retailers set prices in stage four to clear stock and then, in stage three, they choose the stock levels that maximize their profits.

Stage 4: The retail prices, p_i $(i = 1, 2)$, are set to clear stock:

$$\begin{cases} s_1 &= 1 + 2\tau v - 3\tau p_1 + \tau p_2 \\ s_2 &= 1 + 2\tau v - 3\tau p_2 + \tau p_1 \end{cases} \Leftrightarrow \begin{cases} p_1 &= \frac{4+8\tau v-3s_1-s_2}{8\tau} \\ p_2 &= \frac{4+8\tau v-3s_2-s_1}{8\tau}. \end{cases} \tag{A.55}$$

Stage 3: The retailers determine the optimal level of stock by solving the following optimization problems:

$$\max_{s_1} \Pi_1 = s_1(p_1 - w), \text{ and} \tag{A.56}$$

$$\max_{s_2} \Pi_2 = s_2(p_2 - w). \tag{A.57}$$

Substituting for the retail prices, p_1 and p_2, from (A.55) and optimizing these profit functions simultaneously, we get the optimal stock levels, $s^1_{0,0,0}$ and $s^2_{0,0,0}$, that retailers R_1 and R_2 will order under combination 1:

$$s^1_{0,0,0} = s^2_{0,0,0} = \frac{4(1 + 2\tau v - 2\tau w)}{7}. \tag{A.58}$$

Consequently, the optimal retail prices, $p^i_{0,0,0}$ $(i = 1, 2)$, are

$$p^i_{0,0,0} = \frac{3 + 6\tau v + 8\tau w}{14\tau}. \tag{A.59}$$

At these optimal stock levels and retail prices, the retailers's profits are:

$$\Pi^i_{0,0,0} = s^i_{0,0,0}(p^i_{0,0,0} - w) = \frac{6(1 + 2\tau v - 2\tau w)^2}{49\tau}, \tag{A.60}$$

and the manufacturer gets

$$\pi^M_{0,0,0} = (s^1_{0,0,0} + s^2_{0,0,0})(w - c) = \frac{8(1 + 2\tau v - 2\tau w)(w - c)}{7}. \tag{A.61}$$

Regularity conditions: To ensure non-negative stock, i.e., $s^i_{0,0,0} \geq 0 \Leftrightarrow 1+2\tau v-2\tau w \geq 0$, the wholesale price, w, cannot exceed $w_1 \stackrel{\text{def}}{=} v + \frac{1}{2\tau}$.

Combination 4: {*NMR,CR,CR*}

In this case, both retailers accept consumer returns while the manufacturer does not accept any returns from the retailers. The initial-demand function, $\hat{q}_i$ $(i = 1, 2)$, and the corresponding net-demand function, q_i, are given by:

$$\hat{q}_i = 1 + \tau - 2\mu - \frac{3\tau}{2}p_i + \frac{\tau}{2}p_j, \quad (i, j = 1, 2; i \neq j), \text{ and} \tag{A.62}$$

$$q_i = (1 - \rho)\hat{q}_i, \text{ respectively.} \tag{A.63}$$

Stage 4: The retailers set the retail prices, p_i $(i = 1, 2)$, to clear stock according to the net-demand functions:

$$\begin{cases} s_1 &= (1-\rho)(1+\tau-2\mu-\frac{3\tau}{2}p_1+\frac{\tau}{2}p_2) \\ s_2 &= (1-\rho)(1+\tau-2\mu-\frac{3\tau}{2}p_2+\frac{\tau}{2}p_1) \end{cases} \Leftrightarrow \begin{cases} p_1 &= \frac{4(1+\tau-2\mu)(1-\rho)-3s_1-s_2}{4(1-\rho)\tau} \\ p_2 &= \frac{4(1+\tau-2\mu)(1-\rho)-3s_2-s_1}{4(1-\rho)\tau}. \end{cases} \tag{A.64}$$

Stage 3: The optimal levels of stock are determined by solving:

$$\max_{s_1} \Pi_1 = (1-\rho)(1+\tau-2\mu-\frac{3\tau}{2}p_1+\frac{\tau}{2}p_2)(p_1-w-\frac{\rho}{1-\rho}\eta)$$
$$= s_1(p_1-w-\frac{\rho}{1-\rho}\eta) \quad \text{(by (A.64)), and} \tag{A.65}$$

$$\max_{s_2} \Pi_2 = (1-\rho)(1+\tau-2\mu-\frac{3\tau}{2}p_2+\frac{\tau}{2}p_1)(p_2-w-\frac{\rho}{1-\rho}\eta)$$
$$= s_2(p_2-w-\frac{\rho}{1-\rho}\eta). \tag{A.66}$$

Solving the first-order conditions simultaneously with p_1 and p_2 given by (A.64), we obtain:

$$s^i_{1,1,0} = \frac{4[(1+\tau-2\mu)(1-\rho)-\tau(1-\rho)w-\tau\rho\eta]}{7} \quad (i=1,2). \tag{A.67}$$

At those stock levels, the resulting retail prices, $p^i_{1,1,0}$, the retailers' profits, $\Pi^i_{1,1,0}$, and the manufacturer's profit, $\pi^M_{1,1,0}$, are provided in Table 2.3.

Regularity conditions: Non-negativity of the stock levels requires $s^i_{1,1,0} \geq 0 \Leftrightarrow w \leq \frac{1+\tau-2\mu}{\tau} - \frac{\rho\eta}{1-\rho} \stackrel{\text{def}}{=} w_2$. Notice w_2 is positive when stock-out cost, η, is not too high, i.e., $\eta < \frac{(1+\tau-2\mu)(1-\rho)}{\tau\rho} \stackrel{\text{def}}{=} \eta_1$. This upper bound is consistent with the "low η" setting we examine here in case 2. Furthermore, as shown later, w_2 is the upper bound of the wholesale price under *NMR*. For expositional ease, we define $\overline{w}_N \stackrel{\text{def}}{=} w_2$.

Combination 2: {*NMR,NCR,CR*}

Under *NMR*, when only retailer R_2 accepts consumer returns, its initial-demand function is $\hat{q}_2 = 1+\frac{3\tau}{2}-\tau v-3\mu-\frac{3\tau}{2}p_2+\tau p_1$, and net sales are realized according to $q_2 = (1-\rho)\hat{q}_2$. Meanwhile, retailer R_1 has only one demand function, which is $q_1 = 1-\frac{\tau}{2}+3\tau v+\mu-3\tau p_1+\frac{\tau}{2}p_2$.

Stage 4: Retailer R_1 sets its retail price, p_1, to clear stock according to q_1, while its rival aims at clearing stock according to the net-demand function, q_2:

$$\begin{cases} s_1 &= 1 - \frac{\tau}{2} + 3\tau v + \mu - 3\tau p_1 + \frac{\tau}{2}p_2 \\ s_2 &= (1-\rho)(1 + \frac{3\tau}{2} - \tau v - 3\mu - \frac{3\tau}{2}p_2 + \tau p_1) \end{cases} \Leftrightarrow \begin{cases} p_1 &= \frac{4(1+2\tau v)(1-\rho)-3(1-\rho)s_1 - s_2}{8(1-\rho)\tau} \\ p_2 &= \frac{4(1+\tau-2\mu)(1-\rho)-3s_2-(1-\rho)s_1}{4(1-\rho)\tau}. \end{cases} \tag{A.68}$$

Stage 3: The retailers aim at maximizing (net) profits by choice of stocking levels, s_i $(i = 1, 2)$:

$$\max_{s_1} \Pi_1 = s_1(p_1 - w), \text{ and} \tag{A.69}$$

$$\max_{s_2} \Pi_2 = s_2(p_2 - w - \frac{\rho}{1-\rho}\eta). \tag{A.70}$$

The results, including the optimal stock levels, $s^i_{0,1,0}$ $(i = 1, 2)$, the optimal retail prices, $p^i_{0,1,0}$, and the profits of the retailers and the manufacturer, $\Pi^i_{0,1,0}$ and $\pi^M_{0,1,0}$, are reported in Table 2.3. Note that under *NMR*, the manufacturer gets paid for all the ordered stock; thus, it earns:

$$\pi^M_{0,1,0} = (s^1_{0,1,0} + s^2_{1,0,0})(w - c), \tag{A.71}$$

where $s^1_{0,1,0}$ and $s^2_{1,0,0}$ are the optimal stock levels ordered by R_1 and R_2 respectively.

Regularity conditions: Non-negative stock (i.e., $s^1_{0,1,0} \geq 0$ and $s^2_{1,0,0} \geq 0$) requires $w \leq \frac{5-\tau(1-12v)+2\mu}{11\tau} + \frac{\rho\eta}{11(1-\rho)} \overset{\text{def}}{=} w_3$ and $w \leq \frac{5+2\tau(3-v)-12\mu}{4\tau} - \frac{3\rho\eta}{2(1-\rho)} \overset{\text{def}}{=} w_4$ respectively. Together, we need $w \leq \min\{w_3, w_4\}$. In addition, $w_4 > 0$ if and only if $\eta < \frac{[5+2\tau(3-v)-12\mu](1-\rho)}{6\tau\rho} \overset{\text{def}}{=} \eta_2$. Further, we define $\underline{w}_N \overset{\text{def}}{=} w_3$ for expositional ease in the subsequent analysis.

Combination 5: {*MR,NCR,NCR*}

As noted in Wang (2004), the retailers set prices to clear stock in stage four when *MR* is offered. Then, in the third stage, they order stock so that profits are maximized as under $\{NMR,NCR,NCR\}$. Therefore, their optimal retail prices, $p^i_{0,0,1}$, stock levels, $s^i_{0,0,1}$, and profits, $\Pi^i_{0,0,1}$, are identical to those under $\{NMR,NCR,NCR\}$ (See Table 2.4).

Combination 8: $\{MR, CR, CR\}$

Under *MR*, the retailers set prices in stage four to clear stock according to the initial-demand function, $\hat{q}_i = 1 + \tau - 2\mu - \frac{3\tau}{2}p_i + \frac{\tau}{2}p_j$, where $i, j = 1, 2$ and $i \neq j$. Consequently, their net sales are given by $q_i = (1-\rho)\hat{q}_i$. Then in the third stage, they choose the levels of stock to order so that net profits are maximized.

Stage 4: The retailers clear stock according the initial demand by setting the retail prices as follows:

$$\begin{cases} s_1 &= 1 + \tau - 2\mu - \frac{3\tau}{2}p_1 + \frac{\tau}{2}p_2 \\ s_2 &= 1 + \tau - 2\mu - \frac{3\tau}{2}p_2 + \frac{\tau}{2}p_1 \end{cases} \Leftrightarrow \begin{cases} p_1 &= \frac{4(1+\tau-2\mu)-3s_1-s_2}{4\tau} \\ p_2 &= \frac{4(1+\tau-2\mu)-3s_2-s_1}{4\tau}. \end{cases} \tag{A.72}$$

Stage 3: They maximize net profits by choosing the optimal levels of stock to order:

$$\max_{s_1} \Pi_1 = (1-\rho)s_1(p_1 - w), \text{ and} \tag{A.73}$$

$$\max_{s_2} \Pi_2 = (1-\rho)s_2(p_2 - w), \tag{A.74}$$

where p_i $(i = 1, 2)$ are given by (A.72). This means the retailers will order stock according to:

$$s^i_{1,1,1} = \frac{4(1 + \tau - 2\mu - \tau w)}{7}, \tag{A.75}$$

and set the retail prices at:

$$p^i_{1,1,1} = \frac{3(1 + \tau - 2\mu) + 4\tau w}{7\tau}. \tag{A.76}$$

See Table 2.4 for the retailers' and the manufacturer's profits, $\Pi^i_{1,1,1}$ and $\pi^M_{1,1,1}$, under $\{MR, CR, CR\}$.

Regularity conditions: In this case, we need $w \leq \frac{1+\tau-2\mu}{\tau} \stackrel{\text{def}}{=} w_5$ for stock to be non-negative. In addition, w_5 is also the upper bound of the wholesale price under MR as shown later; we relabel $w_5 \stackrel{\text{def}}{=} \overline{w}_M$.

Combination 6: {*MR,NCR,CR*}

This is the case when only R_2 accepts consumer returns. Thus, R_2's initial- and net-demand functions are given by $\hat{q}_2 = 1+\frac{3\tau}{2}-\tau v-3\mu-\frac{3\tau}{2}p_2+\tau p_1$ and $q_2 = (1-\rho)\hat{q}_2$, respectively. Meanwhile, all of R_1's sales are final and equal to $q_1 = 1 - \frac{\tau}{2} + 3\tau v + \mu - 3\tau p_1 + \frac{\tau}{2}p_2$.

Stage 4: The retailers set prices to clear stock according to their (initial-) demand functions:

$$\begin{cases} s_1 &= 1 - \frac{\tau}{2} + 3\tau v + \mu - 3\tau p_1 + \frac{\tau}{2}p_2 \\ s_2 &= 1 + \frac{3\tau}{2} - \tau v - 3\mu - \frac{3\tau}{2}p_2 + \tau p_1 \end{cases} \Leftrightarrow \begin{cases} p_1 &= \frac{4(1+2\tau v)-3s_1-s_2}{8\tau} \\ p_2 &= \frac{4(1+\tau-2\mu)-3s_2-s_1}{4\tau}. \end{cases} \tag{A.77}$$

Stage 3: The retailers maximize their (net) profits by choosing the optimal levels of stock given the retail prices, p_1 and p_2, defined by (A.77), i.e.,

$$\max_{s_1} \Pi_1 = s_1(p_1 - w), \text{ and} \tag{A.78}$$

$$\max_{s_2} \Pi_2 = (1-\rho)s_2(p_2 - w) \tag{A.79}$$

The optimal stock levels are

$$s^1_{0,1,1} = \frac{4[5 - \tau(1-12v) + 2\mu - 11\tau w]}{35}, \text{ and} \tag{A.80}$$

$$s^2_{1,0,1} = \frac{4[5 + 2\tau(3-v) - 12\mu - 4\tau w]}{35}. \tag{A.81}$$

The complete results are provided in Table 2.4.

Regularity conditions: Stock levels, $s^1_{0,1,1}$ and $s^2_{1,0,1}$, are non-negative when $w \leq \min\{w_6, w_7\}$, where

$$w_6 \stackrel{\text{def}}{=} \frac{5 - \tau(1 - 12v) + 2\mu}{11\tau}, \text{ and} \tag{A.82}$$

$$w_7 \stackrel{\text{def}}{=} \frac{5 + 2\tau(3 - v) - 12}{4\tau}. \tag{A.83}$$

We also define $\underline{w}_M \stackrel{\text{def}}{=} w_6$.

A.3.1.2 The retailers' returns policies (Case 2, stage 2)

Under *NMR*:

The retailers' decision on consumer returns are made based on the behavior of the differences in their respective profits, i.e., δ_1 and δ_2 (see equations 2.8 and 2.9). Recall that we are focusing on the simple case with $v = \frac{1}{2}$, and $\mu = 0$. For the sake of mathematical tractability, we consider $\eta = \frac{1}{4}$. (This magnitude of η is considerably low compared to $w_1 = \frac{1}{2} + \frac{1}{2\tau}$. Later, the results obtained in equilibrium will be verified to ensure that pricing according to option B is indeed optimal for the retailers under *NMR*.) The remaining parameters are $\tau \in (0, +\infty)$ and $\rho \in [0, \frac{1}{2}]$.

First, we restate the regularity conditions that ensure feasibility of each of the four combinations of consumer returns (see Table 2.3). Specifically, combination 1, i.e., {*NMR,NCR, NCR*}, is feasible when $w \leq w_1 = \frac{1}{2} + \frac{1}{2\tau}$. Combination 2, i.e., {*NMR,NCR,CR*}, requires $w \leq \min\{w_3, w_4\}$, where $w_3 = \frac{5(1+\tau)}{11\tau} + \frac{\rho}{44(1-\rho)}$, and $w_4 = \frac{5(1+\tau)}{4\tau} - \frac{3\rho}{8(1-\rho)}$. (Note that $\eta < \eta_2$ when $\eta = \frac{1}{4}$, and $\rho \leq \frac{1}{2}$.) Finally, combination 4, i.e., {*NMR,CR,CR*}, is feasible when $w \leq w_2 = \frac{1+\tau}{\tau} - \frac{\rho}{4(1-\rho)}$. (The condition on η, i.e., $\eta < \eta_1$, can be shown to hold in this case.)

Before investigating the behavior of δ_1 and δ_2, we determine the ordering of the upper bounds of w in the four combinations. We have:

$$w_2 \geq w_1 \Leftrightarrow w_2 - w_1 = \frac{2(1+\tau) - (2+3\tau)\rho}{4\tau(1-\rho)} \geq 0 \Leftrightarrow \rho \leq \frac{2(1+\tau)}{2+3\tau}, \tag{A.84}$$

$$w_1 \geq w_3 \Leftrightarrow w_1 - w_3 = \frac{2(1+\tau) - (2+3\tau)\rho}{44\tau(1-\rho)} \geq 0 \Leftrightarrow \rho \leq \frac{2(1+\tau)}{2+3\tau}, \text{ and} \tag{A.85}$$

$$w_4 \geq w_2 \Leftrightarrow w_4 - w_2 = \frac{2(1+\tau) - (2+3\tau)\rho}{8\tau(1-\rho)} \geq 0 \Leftrightarrow \rho \leq \frac{2(1+\tau)}{2+3\tau}. \tag{A.86}$$

Notice that $\frac{2(1+\tau)}{2+3\tau} > \frac{1}{2}$. Therefore, when $\rho \leq \frac{1}{2} \Rightarrow \rho < \frac{2(1+\tau)}{2+3\tau}$, we have $w_3 < w_1 < w_2 < w_4$. Not all combinations are always feasible depending on the wholesale price, w. For instance, if the manufacturer sets the wholesale price, w, above w_3, combinations 2 and 3 exhibit negative stock, and cease to be feasible. Figure A.4 illustrates the feasibility of the four combinations under *NMR*. (Recall that w_3 and w_2 are relabeled as $\underline{w}_N$ and $\overline{w}_N$ respectively.)

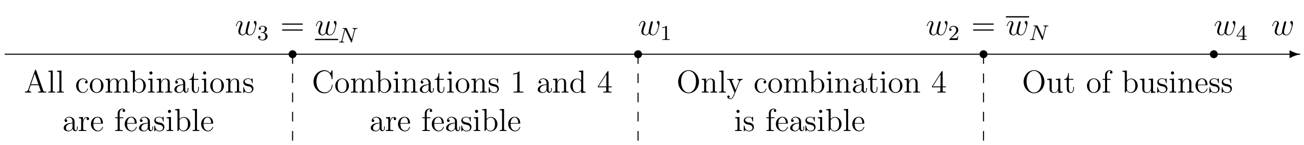


Figure A.4: **Feasibility of the four combinations of returns policies under *NMR***

First, consider $\delta_1 = \Pi^2_{1,0,0} - \Pi^2_{0,0,0}$. Given the parameters of this case, when $w \leq w_3$, both combinations 1 and 2 are feasible. We have:

$$\Pi^2_{0,0,0} = \frac{6(1+\tau-2\tau w)^2}{49\tau}, \text{ and} \tag{A.87}$$

$$\Pi^2_{1,0,0} = \frac{12[5(1+\tau)(1-\rho) - \frac{3}{2}\tau\rho - 4\tau(1-\rho)w]^2}{1225\tau(1-\rho)}. \tag{A.88}$$

This implies :

$$\delta_1 = \frac{3}{1225}\{-8\tau(17+8\rho)w^2 + 8[5(1+\tau)+2(10+13\tau)\rho]w + \\ + \frac{50(1+\tau)^2 - 30(1+\tau)(5+7\tau)\rho + (10+13\tau)^2\rho^2}{\tau(1-\rho)}\}. \tag{A.89}$$

Let A_1 be the term in the brackets on the RHS; δ_1 has the same behavior as A_1. Consider A_1, which is a concave quadratic function in $w \in [0, w_3]$. We have:

$$A_1|_{w=w3} = \frac{25(97-98\rho)[2(1+\tau)-(2+3\tau)\rho]^2}{242\tau(1-\rho)^2} > 0 \quad (\text{ since } \rho \leq \frac{1}{2}), \text{ and} \tag{A.90}$$

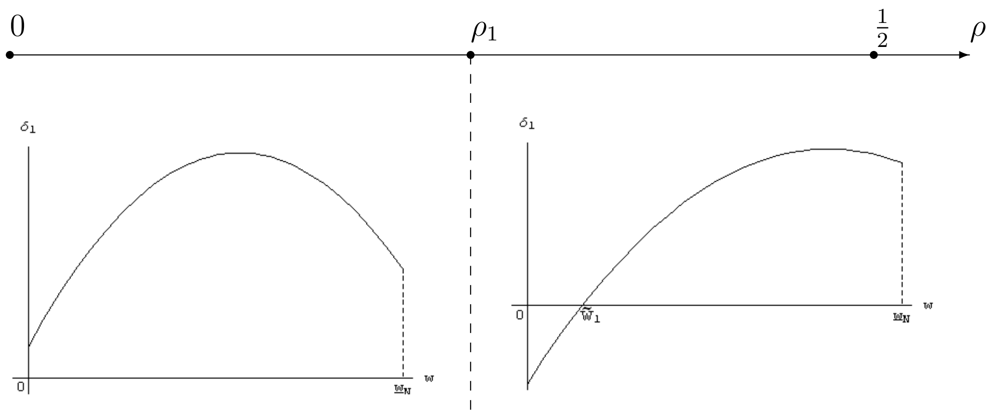
$$A_1|_{w=0} = \frac{(10+13\tau)^2\rho^2 - 30(1+\tau)(5+7\tau)\rho + 50(1+\tau)^2}{\tau(1-\rho)}. \tag{A.91}$$

Notice that $A_1|_{w=0}$ has the same sign as its numerator, which is a convex quadratic function in $\rho \in [0, \frac{1}{2}]$ with two solutions $\rho_1 = \frac{5[15+36\tau+21\tau^2-(1+\tau)\sqrt{25+110\tau+103\tau^2}]}{(10+13\tau)^2}$ and $\rho_1' = \frac{5[15+36\tau+21\tau^2+(1+\tau)\sqrt{25+110\tau+103\tau^2}]}{(10+13\tau)^2}$. It can be shown that $0 < \rho_1 < \frac{1}{2} < \rho_1'$. Therefore, when $0 \leq \rho \leq \rho_1$, the numerator of $A_1|_{w=0}$ is non-negative, which implies $A_1|_{w=0} \geq 0$. Consequently, when $0 \leq \rho \leq \rho_1$, we have $A_1 \geq 0 \Rightarrow \delta_1 \geq 0 \quad \forall w \in [0, w_3]$ (since $A_1|_{w=0} \geq 0$, $A_1|_{w=w_3} > 0$, and A_1 is a concave quadratic function of w). Otherwise, when $\rho_1 < \rho \leq \frac{1}{2}$, we have $A_1|_{w=0} < 0$. This means A_1 has one solution in $[0, w_3]$, which is $\tilde{w}_1 = \frac{5(1+\tau)+2(10+13\tau)\rho}{2\tau(17+8\rho)} - \frac{15[2(1+\tau)+(2+3\tau)\rho]}{2\tau(17+8\rho)\sqrt{2(1-\rho)}}$, and $A_1 < 0 \Rightarrow \delta_1 < 0$ when $0 \leq w < \tilde{w}_1$ and vice versa. Figure A.5 summarizes the behavior of δ_1.

Now, consider $\delta_2 = \Pi^1_{1,1,0} - \Pi^1_{0,1,0}$. From Table 2.3, we have:

$$\Pi^1_{0,1,0} = \frac{6[5(1+\tau)(1-\rho) + \frac{1}{4}\tau\rho - 11\tau(1-\rho)w]^2}{1225\tau(1-\rho)^2}, \text{ and} \tag{A.92}$$

$$\Pi^1_{1,1,0} = \frac{12[(1+\tau)(1-\rho) - \frac{1}{4}\tau\rho - \tau(1-\rho)w]^2}{49\tau(1-\rho)}. \tag{A.93}$$



$\delta_1 > 0 \quad \forall w \in [0, \underline{w}_N]$

$\delta_1 < 0 \quad$ if $0 \leq w < \tilde{w}_1$

$\delta_1 \geq 0 \quad$ if $\tilde{w}_1 \leq w \leq \underline{w}_N$

Figure A.5: **Behavior of δ_1**

Thus,

$$\begin{aligned}\delta_2 =& \frac{3}{1225}\{-2\tau(71+50\rho)w^2 + \frac{20(1+\tau)+(180+241\tau)\rho-50(4+5\tau)\rho^2}{1-\rho}w- \\ &- \frac{-400(1+\tau)^2+40(40+91\tau+52\tau^2)\rho-(2000+4840\tau+2889\tau^2)\rho^2+50(4+5\tau)^2\rho^3}{8\tau(1-\rho)^2}\}.\end{aligned} \tag{A.94}$$

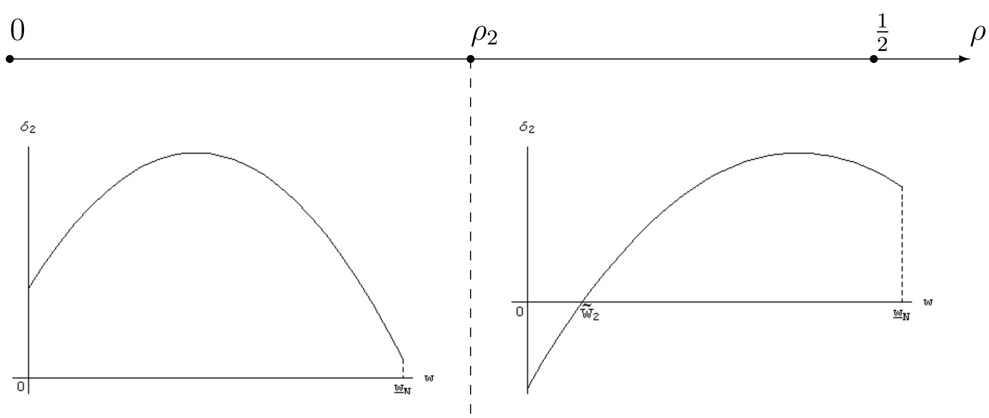
Denote the term in the bracket on the RHS by A_2, we can see that δ_2 behaves in the same manner as A_2, which is a concave quadratic function in $w \in [0, w_3]$. We have:

$$A_2|_{w=w_3} = \frac{900[2(1+\tau)-(2+3\tau)\rho]^2}{121\tau(1-\rho)} > 0 \quad \text{(by inspection), and} \tag{A.95}$$

$$A_2|_{w=0} = \frac{400(1+\tau)^2-40(40+91\tau+52\tau^2)\rho+(2000+4840\tau+2889\tau^2)\rho^2-50(4+5\tau)^2\rho^3}{8\tau(1-\rho)^2}. \tag{A.96}$$

Denote the numerator of $A_2|_{w=0}$ by N_2. Notice that $A_2|_{w=0}$ has the same sign as N_2, which is a cubic function in ρ with a negative third-order coefficient. Furthermore, it can be shown that (1) $N_2|_{\rho=0} = 400(1+\tau)^2 > 0$, (2) $N_2|_{\rho=\frac{1}{2}} = -6\tau(10+9\tau) < 0$, and (3) $\frac{\partial N_2}{\partial \rho} = -150(4+5\tau)^2\rho^2 + 2(20+27\tau)(100+107\tau)\rho - 40(1+\tau)(40+51\tau) < 0 \quad \forall \rho \in [0, \frac{1}{2}]$. This implies that N_2 is strictly decreasing in $\rho \in [0, \frac{1}{2}]$ and has one solution $\rho_2 \in [0, \frac{1}{2}]$, whose specification is available upon request.

In summary, when $0 \le \rho \le \rho_2$, we have: $A_2|_{w=0} \ge 0$ and $A_2|_{w=w_3} > 0 \Rightarrow A_2 \ge 0 \quad \forall w \in [0, w_3] \Rightarrow \delta_2 \ge 0 \quad \forall w \in [0, w_3]$. On the other hand, when $\rho_2 < \rho \le \frac{1}{2}$, we have $A_2|_{w=0} < 0$ and $A_2|_{w=w_3} > 0$. As a result, the concave quadratic function A_2 has one solution in $[0, w_3]$, which is $\tilde{w}_2 = \frac{20(1+\tau)+(180+241\tau)\rho-50(4+5\tau)\rho^2}{4\tau(1-\rho)(71+50\rho)} - \frac{15\sqrt{2(1-\rho)}[2(1+\tau)-(2+3\tau)\rho]}{\tau(1-\rho)(71+50\rho)}$. In this case, if $0 \le w < \tilde{w}_2$, then $A_2 < 0 \Rightarrow \delta_2 < 0$; otherwise, if $\tilde{w}_2 \le w \le w_3$, then $A_2 \ge 0 \Rightarrow \delta_2 \ge 0$. The behavior of δ_2 is depicted in Figure A.6.



$\delta_2 > 0 \quad \forall w \in [0, \underline{w}_N]$

$\delta_2 < 0 \quad$ if $0 \le w < \tilde{w}_2$

$\delta_2 \ge 0 \quad$ if $\tilde{w}_2 \le w \le \underline{w}_N$

Figure A.6: **Behavior of** δ_2

Given the behavior of δ_1 and δ_2, we can determine the retailers' consumer returns policies under *NMR*. First, it can be shown that $\rho_1 < \rho_2$ and $\tilde{w}_2 < \tilde{w}_1$ when $\rho \le \frac{1}{2}$. Therefore, when $0 \le \rho < \rho_1$, we have $\delta_1 > 0$ and $\delta_2 > 0\ \forall w \in [0, w_3]$. This implies that the equilibrium of the consumer returns subgame is $\{CR, CR\}$ under *NMR* when $\rho \in [0, \rho_1)$.

Next, when $\rho_1 \leq \rho < \rho_2$, we have $\delta_2 > 0 \; \forall w \in [0, w_3]$ while $\delta_1 \leq 0$ if $0 \leq w \leq \tilde{w}_1$ and $\delta_1 > 0$ if $\tilde{w}_1 < w \leq w_3$. Consequently, when $\rho \in [\rho_1, \rho_2)$: (a) if $0 \leq w \leq \tilde{w}_1$, then both {*CR,CR*} and {*NCR,NCR*} can arise in equilibrium , and (b) if $\tilde{w}_1 < w \leq w_3$, {CR,CR} is the only equilibrium.

Finally, when $\rho_2 \leq \rho \leq \frac{1}{2}$, we have: (a) $\delta_1 < 0$ and $\delta_2 < 0$ if $0 \leq w < \tilde{w}_2$, (b) $\delta_1 \leq 0$ and $\delta_2 \geq 0$ if $\tilde{w}_2 \leq w \leq \tilde{w}_1$, and (c) $\delta_1 > 0$ and $\delta_2 > 0$ if $\tilde{w}_1 < w \leq w_3$. Therefore, when $\rho \in [\rho_2, \frac{1}{2}]$, the equilibrium consumer-returns policies are (i) {*NCR,NCR*} when $0 \leq w < \tilde{w}_2$, (ii) {*NCR,NCR*} and {*CR,CR*} when $\tilde{w}_2 \leq w \leq \tilde{w}_1$, and (iii) {*CR,CR*} when $\tilde{w}_1 < w \leq w_3$. In addition, recall that since combination 4 remains feasible even when $w_3 < w \leq w_2$, {*CR,CR*} is also the equilibrium of the consumer returns subgame up to w_2. Figure 2.3 summarizes the retailers' consumer-returns policies when the manufacturer does not accept returns.

Under *MR*:

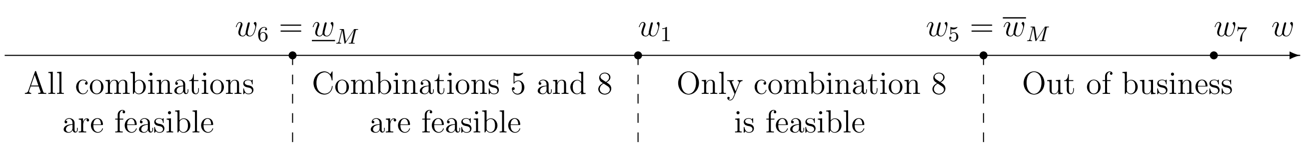


Figure A.7: **Feasibility of the four combinations of consumer returns under *MR***

Here, we determine the retailers' consumer-returns policies by considering δ_3 and δ_4, defined by (2.10) and (2.11). First, recall the regularity conditions of the four combinations under *MR*, which are: (a) $w \leq w_1 = \frac{1+\tau}{2\tau}$ in combination 5 (i.e., {*MR,NCR,NCR*}), (b) $w \leq \min\{w_6, w_7\}$ where $w_6 = \frac{14(1+\tau)}{31\tau}$ and $w_7 = \frac{14(1+\tau)}{11\tau}$ in combination 6 (i.e., {*MR,NCR,CR*}), and (c) $w \leq w_5 = \frac{1+\tau}{\tau}$ in combination 8 (i.e., {*MR,CR,CR*}). By inspection, we have

$w_6 < w_1 < w_5 < w_7$. Feasibility of the four combinations under MR is shown in Figure A.7. Again, recall that w_6 and w_5 are relabeled as $\underline{w}_M$ and $\overline{w}_M$ respectively.

When $w \leq w_6$, all combinations are feasible. First, consider δ_3. Since

$$\Pi^2_{0,0,1} = \frac{6(1+\tau-2\tau w)^2}{49\tau}, \text{ and} \tag{A.97}$$
$$\Pi^2_{1,0,1} = \frac{12(1-\rho)[5(1+\tau)-4\tau w]^2}{1225\tau}, \tag{A.98}$$

we have:

$$\begin{aligned}\delta_3 &= \Pi^2_{1,0,1} - \Pi^2_{0,0,1} \\ &= \frac{6}{1225\tau}\{-4\tau^2(17+8\rho)w^2 + 20\tau(1+\tau)(1+4\rho)w + 25(1+\tau)^2(1-2\rho)\}.\end{aligned} \tag{A.99}$$

Notice that (a) δ_3 is a concave quadratic function in $w \in [0, w_6]$, (b) $\delta_3|_{w=0} = \frac{6(1+\tau)^2(1-2\rho)}{49\tau} \geq 0$ when $\rho \leq \frac{1}{2}$, and (c) $\delta_3|_{w=w_6} = \frac{6(1+\tau)^2(97-98\rho)}{5929\tau} > 0$. This implies $\delta_3 \geq 0 \quad \forall w \in [0, w_6]$.

Next, consider δ_4. We have:

$$\Pi^1_{0,1,1} = \frac{6[5(1+\tau)-11\tau w]^2}{1225\tau}, \text{ and} \tag{A.100}$$
$$\Pi^1_{1,1,1} = \frac{12(1-\rho)(1+\tau-\tau w)^2}{49\tau}. \tag{A.101}$$

As a result,

$$\begin{aligned}\delta_4 &= \Pi^1_{1,1,1} - \Pi^1_{0,1,1} \\ &= \frac{6}{1225\tau}\{-\tau^2(71+50\rho)w^2 + 10\tau(1+\tau)(1+10\rho)w + 25(1+\tau)^2(1-2\rho)\},\end{aligned} \tag{A.102}$$

which is a concave quadratic function in $w \in [0, w_6]$. Since $\delta_4|_{w=0} = \frac{6(1+\tau)^2(1-2\rho)}{49\tau} \geq 0$ and $\delta_4|_{w=w_6} = \frac{432(1+\tau)^2(1-\rho)}{5929\tau} > 0$, we also have $\delta_4 \geq 0 \; \forall w \in [0, w_6]$.

In summary, we have $\delta_3 \geq 0$ and $\delta_4 \geq 0$. Therefore, the retailers will choose $\{CR, CR\}$ when the manufacturer chooses *MR*. (Note that as under *NMR*, $\{CR, CR\}$ arises in equilibrium under *MR* up to $w_5 = \overline{w}_M$.) Figure 2.4 summarizes the retailers' returns policies when the manufacturer accepts returns from the retailers. The statement of Lemma 2.2 then follows.

A.3.1.3 The manufacturer's distribution policy (Case 2, stage 1)

The optimal wholesale price under *NMR*:

Denote the profit function of the manufacturer under *NMR* by π_0^M. The wholesale price is set to maximize π_0^M. First, we derive the manufacturer's profit function, π_0^M, which depends on the retailers' returns policies (see Figure 2.3). Specifically, there are three possible cases:

(1) If $0 \leq \rho < \rho_1$, the retailers always choose to accept returns from consumers at all relevant wholesale prices, i.e., $w \in [c, \overline{w}_N]$. Thus, $\pi_0^M = \pi_{1,1,0}^M = \frac{8}{7}[1 + \tau - (1 + \frac{5\tau}{4})\rho - \tau(1 - \rho)w](w - c)$ (given the parameters of this case: $v = \frac{1}{2}, k = 0$, and $\eta = \frac{1}{4}$; see Table 2.3).

(2) If $\rho_1 \leq \rho < \rho_2$: (a) if $0 \leq w \leq \tilde{w}_1$, the retailers choose either $\{NCR, NCR\}$ or $\{CR, CR\}$, and (b) if $\tilde{w}_1 < w \leq \overline{w}_N$, they choose $\{CR, CR\}$. Consequently, if $c > \tilde{w}_1$, the manufacturer's profit function is $\pi_0^M = \pi_{1,1,0}^M \quad \forall w \in [c, \overline{w}_N]$. Otherwise, if $0 \leq c \leq \tilde{w}_1$, then the manufacturer earns (i) $\pi_{1,1,0}^M$ if $w \in (\tilde{w}_1, \overline{w}_N]$, and (ii) either $\pi_{0,0,0}^M$ or $\pi_{1,1,0}^M$ if $w \in [c, \tilde{w}_1]$. In this case, we consider the best scenario for the manufacturer. Since it can be shown that $\pi_{0,0,0}^M > \pi_{1,1,0}^M$ when $w \in [0, \tilde{w}_1]$[50], the best scenario for the manufacturer will be the case when the retailers choose $\{NCR, NCR\}$, i.e., $\pi_0^M = \pi_{0,0,0}^M$. In summary, the manufacturer's profit when $\rho_1 \leq \rho < \rho_2$ and $0 \leq c \leq \tilde{w}_1$ is given by:

$$\pi_0^M = \begin{cases} \pi_{0,0,0}^M = \frac{8}{7}(1 + \tau - 2\tau w)(w - c) & \text{if } c \leq w \leq \tilde{w}_1 \\ \pi_{1,1,0}^M = \frac{8}{7}[1 + \tau - (1 + \frac{5\tau}{4})\rho - \tau(1 - \rho)w](w - c) & \text{if } \tilde{w}_1 < w \leq \overline{w}_N. \end{cases} \tag{A.103}$$

[50] The proof is available upon request.

(Analogous to $\pi^M_{1,1,0}$, the expression of $\pi^M_{0,0,0}$ is obtained from Table 2 given the parameters of this case.)

(3) If $\rho_2 \leq \rho \leq \frac{1}{2}$, the retailers' choice of consumer returns is (a) $\{NCR,NCR\}$ if $0 \leq w < \tilde{w}_2$, (b) either $\{NCR,NCR\}$ or $\{CR,CR\}$ if $\tilde{w}_2 \leq w \leq \tilde{w}_1$, and (c) $\{CR,CR\}$ if $\tilde{w}_2 < w \leq \overline{w}_N$. As in the previous case, we consider the best scenario for the manufacturer when both $\{NCR,NCR\}$ and $\{CR,CR\}$ can arise. Here, we obtain an analogous structure for the manufacturer's profit function, i.e. (a) if $\tilde{w}_1 < c \leq \overline{w}_N$, then $\pi^M_0 = \pi^M_{1,1,0} \quad \forall w \in [c, \overline{w}_N]$, and (b) if $0 \leq c \leq \tilde{w}_1$, then

$$\pi^M_0 = \begin{cases} \pi^M_{0,0,0} & \text{if } c \leq w \leq \tilde{w}_1 \\ \pi^M_{1,1,0} & \text{if } \tilde{w}_1 < w \leq \overline{w}_N. \end{cases} \tag{A.104}$$

Having the manufacturer's profit function, we can determine the optimal wholesale price by solving the respective optimization problems.

Consider the first case (i.e., $0 \leq \rho < \rho_1$). The manufacturer's profit function is given by

$$\pi^M_0 = \pi^M_{1,1,0} = \frac{8}{7}[1 + \tau - (1 + \frac{5\tau}{4})\rho - \tau(1-\rho)w](w - c). \tag{A.105}$$

Solving the manufacturer's optimization problem, i.e,

$$\max_w \pi^M_0 = \pi^M_{1,1,0} = \frac{8}{7}[1 + \tau - (1 + \frac{5\tau}{4})\rho - \tau(1-\rho)w](w - c), \tag{A.106}$$

we obtain the optimal wholesale price:

$$w^*_0 = \frac{4(1+\tau) - (4+5\tau)\rho + 4\tau(1-\rho)c}{8\tau(1-\rho)}. \tag{A.107}$$

At this wholesale price, M earns:

$$\pi_0^{M*} = \frac{[4(1+\tau) - (4+5\tau)\rho - 4\tau(1-\rho)c]^2}{56\tau(1-\rho)}. \tag{A.108}$$

Notice that when $c \leq \overline{w}_N$, we always have $c \leq w_0^* \leq \overline{w}_N$, i.e. w_0^* is in the relevant range of w.

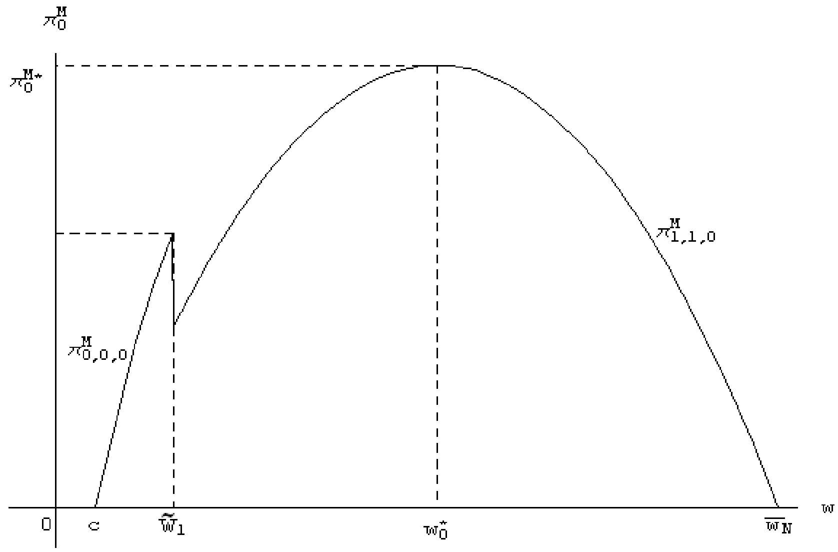


Figure A.8: **Manufacturer's profit when** $\rho_1 < \rho \leq \frac{1}{2}$ **and** $0 \leq c \leq \tilde{w}_1$**.**

In the second case (i.e., $\rho_1 \leq \rho < \rho_2$), we have two possibilities: (a) If $c > \tilde{w}_1$, then the manufacturer's profit function is the same as in the first case, i.e., $\pi_0^M = \pi_{1,1,0}^M$. Therefore, M will set the wholesale price at w_0^* to earn π_0^{M*}. (b) If $0 \leq c \leq \tilde{w}_1$, the manufacturer's profit function is defined by

$$\pi_0^M = \begin{cases} \pi_{0,0,0}^M = \frac{8}{7}(1+\tau-2\tau w)(w-c) & \text{if } c \leq w \leq \tilde{w}_1 \\ \pi_{1,1,0}^M = \frac{8}{7}[1+\tau-(1+\frac{5\tau}{4})\rho - \tau(1-\rho)w](w-c) & \text{if } \tilde{w}_1 < w \leq \overline{w}_N. \end{cases} \tag{A.109}$$

Notice that (i) both $\pi_{0,0,0}^M$ and $\pi_{1,1,0}^M$ are concave quadratic functions in w, (ii) $\pi_{0,0,0}^M$ obtains maximum at $\hat{w}_0^* = \frac{1+\tau+2\tau c}{4\tau}$, (iii) $\pi_{1,1,0}^M$ is maximized at $w = w_0^*$, and (iv) it can be shown that

$\hat{w}_0^* > \tilde{w}_1$ and $w_0^* > \tilde{w}_1$ in this case. This implies that π_0^M behaves in the manner depicted in Figure A.8.

Consequently, to determine the optimal wholesale price, the manufacturer compares π_0^{M*} to $\pi_{0,0,0}^M|_{w=\tilde{w}_1}$. It can be shown that $\pi_0^{M*} > \pi_{0,0,0}^M|_{w=\tilde{w}_1} \quad \forall \rho \in [\rho_1, \rho_2)$ and $c \in [0, \tilde{w}_1]$. Therefore, the manufacturer will set the wholesale price at w_0^* to obtain the optimal profit, π_0^{M*}, in this case.

In the third case, we have $\rho_2 \leq \rho \leq \frac{1}{2}$. The manufacturer's profit function exhibits the same structure as in the previous case, i.e., (a) if $\tilde{w}_1 < c \leq \overline{w}_N$, then $\pi_0^M = \pi_{1,1,0}^M \quad \forall w \in [c, \overline{w}_N]$, and (b) if $0 \leq c \leq \tilde{w}_1$, then

$$\pi_0^M = \begin{cases} \pi_{0,0,0}^M & \text{if } c \leq w \leq \tilde{w}_1 \\ \pi_{1,1,0}^M & \text{if } \tilde{w}_1 < w \leq \overline{w}_N. \end{cases} \tag{A.110}$$

Therefore, if $\tilde{w}_1 < c \leq \overline{w}_N$, the manufacturer sets the wholesale price at w_0^* to obtain profit equal to π_0^{M*} (see equations A.107 and A.108).

Otherwise, if $0 \leq c \leq \tilde{w}_1$, the manufacturer's choice of wholesale price is determined by comparing π_0^{M*} to $\pi_{0,0,0}^M|_{w=\tilde{w}_1}$[51]. Consider $\delta^M = \pi_0^{M*} - \pi_{0,0,0}^M|_{w=\tilde{w}_1}$, which is a convex quadratic function in c with two solutions c_1 and c_1' ($c_1 > c_1'$)[52]. Furthermore, it can be shown that (1) $c_1' < 0$, and (2) $0 \gtreqless c_1 < \tilde{w}_1$. Specifically, (a) if $\tau < \frac{4(4-3\sqrt{2})}{5\sqrt{2}-8} \stackrel{\text{def}}{=} \tau_1$, then $c_1 < 0 \quad \forall \rho \in [\rho_2, \frac{1}{2}]$, and (b) if $\tau \geq \tau_1$, then $\exists\, \hat{\rho} \in [\rho_2, \frac{1}{2}]$ such that when $\rho_2 \leq \rho < \hat{\rho}$, we have $c_1 < 0$ and otherwise, when $\hat{\rho} \leq \rho \leq \frac{1}{2}$, we have $c_1 \geq 0$.

This implies: (1) if $\tau < \tau_1$, then $c_1' < c_1 < 0 \quad \forall \rho \in [\rho_2, \frac{1}{2}]$; therefore, $\delta^M > 0 \quad \forall c \in [0, \tilde{w}_1]$, i.e., the manufacturer chooses w_0^* and earns profit equal π_0^{M*}. (2) If $\tau \geq \tau_1$, then (a) when $\rho \in [\rho_2, \hat{\rho})$, we have $c_1' < c_1 < 0$, which implies, again, $\delta^M > 0 \quad \forall c \in [0, \tilde{w}_1]$ and the optimal wholesale price is w_0^*, and (b) when $\rho \in [\hat{\rho}, \frac{1}{2}]$, we have $0 \leq c_1$; in this case, (i) if

[51] Again, it can be shown that $\hat{w}_0^* > \tilde{w}_1$ and $w_0^* > \tilde{w}_1$ in this case.

[52] The detailed analysis, along with specifications of δ^M, c_1, and c_1', is available upon request.

$0 \leq c < c_1$, then $\delta^M < 0$ and (ii) if $c_1 \leq c \leq \tilde{w}_1$, then $\delta^M \geq 0$. Therefore, the manufacturer's optimal wholesale price is $\tilde{w}_1$ if $0 \leq c < c_1$ and w_0^* if $c_1 \leq c \leq \tilde{w}_1$. See Figure 2.5 for a summary of the manufacturer's choice of wholesale price under *NMR*.

The optimal wholesale price under *MR*:

Under *MR*, the manufacturer's profit function is given by $\pi_1^M = \pi_{1,1,1}^M = \frac{8}{7}(1 + \tau - \tau w)[(1-\rho)w - c]$. Therefore, the optimal wholesale price is determined by solving the following optimization problem:

$$\max_w \pi_1^M = \frac{8}{7}(1 + \tau - \tau w)[(1-\rho)w - c]. \tag{A.111}$$

As a result, the optimal wholesale price under *MR* is:

$$w_1^* = \frac{1+\tau}{2\tau} + \frac{c}{2(1-\rho)}. \tag{A.112}$$

At this wholesale price, the manufacturer earns profit equal to:

$$\pi_1^{M*} = \frac{2[(1+\tau)(1-\rho) - \tau c]^2}{7\tau(1-\rho)}. \tag{A.113}$$

To ensure w_1^* being in the relevant range of w (i.e., $c \leq w_1^* \leq \overline{w}_M$), we need:

$$w_1^* \leq \overline{w}_M \quad \Leftrightarrow \quad c \leq \frac{(1+\tau)(1-\rho)}{\tau} \stackrel{\text{def}}{=} c_2, \text{ and} \tag{A.114}$$

$$c \leq w_1^* \quad \Leftrightarrow \quad c \leq \frac{(1+\tau)(1-\rho)}{\tau(1-2\rho)} \stackrel{\text{def}}{=} c_3. \tag{A.115}$$

Notice that $c_2 \leq \overline{w}_M \leq c_3$. This means (1) if $0 \leq c < c_2$, which implies $c < w_1^* < \overline{w}_M$, then the manufacturer sets the wholesale price at w_1^* and earns profits equal π_1^{M*}, and (2) if $c_2 \leq c \leq \overline{w}_M$, which implies $w_1^* \geq \overline{w}_M$, then the best M can do is to set the wholesale price at $\overline{w}_M$. In this case, the manufacturer earns zero profits: $\pi^M_{1,1,1}|_{w=\overline{w}_M} = 0$. (Note that when $w = \overline{w}_M$, the retailers order zero stock.) In other words, the manufacturer is out of business when $c \geq c_2$. In summary, under *MR*, the manufacturer sets its wholesale price at w_1^* and earns π_1^{M*} if $0 \leq c < c_2$. (See Figure 2.5.)

The manufacturer's returns policy:

Knowing the manufacturer's optimal wholesale prices and profits under *NMR* and *MR*, we investigate her returns policy by comparing the profits under the two regimes. First, consider the case when $\tau < \tau_1$. In this case, the manufacturer earns π_0^{M*} for all $c \in [0, \overline{w}_N]$ under *NMR*; in contrast, under *MR*, she earns π_1^{M*} if $0 \leq c < c_2$ and zero profit if $c \geq c_2$. Notice that $c_2 < \overline{w}_N$. There are two possibilities[53]: (1) $0 \leq c < c_2$, and (2) $c_2 \leq c \leq \overline{w}_N$.

(1) When $0 \leq c < c_2$, the manufacturer decides whether to accept returns from the retailers by comparing π_0^{M*} to π_1^{M*}. We have:

$$\pi_1^{M*} - \pi_0^{M*} = \frac{(1-4c)[8(1-\rho) + \tau(8-9\rho) - 4\tau(2-\rho)c]\rho}{56(1-\rho)} \tag{A.116}$$

Since the second term of the numerator can be shown to be positive when $c < c_2$, we have: (a) if $0 \leq c < \frac{1}{4}$, then $\pi_1^{M*} > \pi_0^{M*}$, i.e., the manufacturer will choose *MR*, and (b) if $\frac{1}{4} \leq c < c_2$, then $\pi_1^{M*} \leq \pi_0^{M*}$, i.e. the manufacturer will choose *NMR*. (Note that $c_2 > \frac{1}{4}$.)

(2) When $c_2 \leq c \leq \overline{w}_N$, the manufacturer gets zero profit under *MR* while she earns $\pi_0^{M*} \geq 0$ under *NMR*. Therefore, she will not accept returns from the retailers.

In summary, when $\tau < \tau_1$, the manufacturer chooses *MR* and sets the wholesale price at w_1^* if $0 \leq c < \frac{1}{4}$. Otherwise, if $\frac{1}{4} \leq c \leq \overline{w}_N$, she chooses *NMR* and sets the wholesale price at w_0^*. In both cases, the retailers always choose to offer consumer returns.

Next, we consider the case when $\tau \geq \tau_1$. If $0 \leq \rho < \hat{\rho}$, the manufacturer's profits are the same as when $\tau < \tau_1$. Therefore, the previous results hold if $0 \leq \rho < \hat{\rho}$.

[53] If $c > \overline{w}_N$, the manufacturer is out of business under both *NMR* and *MR*.

In contrast, if $\hat{\rho} \leq \rho \leq \frac{1}{2}$, then under *NMR*, the manufacturer earns $\pi^M_{0,0,0}|_{w=\tilde{w}_1}$ when $0 \leq c < c_1$ and π_0^{M*} when $c_1 \leq c \leq \overline{w}_N$. Meanwhile, under *MR*, M gets π_1^{M*} if $0 \leq c < c_2$ and zero profit if $c \geq c_2$. Since it can be shown that $c_1 < c_2$[54], we have three possibilities: (a) $0 \leq c < c_1$, (b) $c_1 \leq c < c_2$, and (c) $c_2 \leq c \leq \overline{w}_N$.

(a) When $0 \leq c < c_1$, the manufacturer earns $\pi^M_{0,0,0}|_{w=\tilde{w}_1}$ under *NMR* and π_1^{M*} under *MR*. Since it can be shown that $\pi_1^{M*} > \pi^M_{0,0,0}|_{w=\tilde{w}_1}$ $\forall\, c \in [0, c_1)$, the manufacturer will choose to offer *MR* and earns the optimal profit, π_1^{M*}, by setting the wholesale price at w_1^*.

(b) When $c_1 \leq c < c_2$, if choosing *NMR*, the manufacturer's profit is π_0^{M*}. On the other hand, M earns π_1^{M*} when offering *MR*. From the previous analysis, we have (i) if $c_1 \leq c < \frac{1}{4}$, the manufacturer is better off by choosing *MR* and setting the wholesale price at w_1^*, and (ii) if $\frac{1}{4} \leq c < c_2$, M will choose *NMR* and set the wholesale price at w_0^* to earn π_0^{M*}.

(c) When $c_2 \leq c \leq \overline{w}_N$, we have the same results as under $\tau < \tau_1$, i.e. the manufacturer's profit is zero under *MR* and π_0^{M*} under *NMR*. This implies that *NMR* is chosen and the manufacturer earns profit equal π_0^{M*} at the wholesale price of w_0^*.

Figure 2.6 summarizes the game's equilibrium.

Verifying that option B is optimal when $\{NMR, CR, CR\}$ arises in the equilibrium:

Consider the general linear demand functions: $\hat{q}_i = \alpha - \beta p_i + \gamma p_j$ $(i, j = 1, 2$ and $i \neq j)$ and $q_i = (1-\rho)\hat{q}_i$. Under *NMR*, if choosing option A, the retailers will set prices as follows:

$$\begin{cases} s_1 &= \alpha - \beta p_1 + \gamma p_2 \\ s_2 &= \alpha - \beta p_2 + \gamma p_1 \end{cases} \Leftrightarrow \begin{cases} p_1 &= \frac{\alpha(\beta+\gamma) - \beta s_1 - \gamma s_2}{\beta^2 - \gamma^2} \\ p_2 &= \frac{\alpha(\beta+\gamma) - \beta s_2 - \gamma s_1}{\beta^2 - \gamma^2}. \end{cases} \tag{A.117}$$

[54] In addition, it can also be shown that $c_1 < \frac{1}{4}$.

Here, the retailers incur the cost of unsold inventory because no returns are accepted by the manufacturer. Hence, their profits are equal to:

$$\Pi^{i,A}_{1,1,0} = (1-\rho)s_i p_i - s_i w, \tag{A.118}$$

where $i = 1, 2$ and p_i is defined by (A.117).

In contrast, if the retailers choose option B, their prices will be set as follows:

$$\begin{cases} s_1 &= (1-\rho)(\alpha - \beta p_1 + \gamma p_2) \\ s_2 &= (1-\rho)(\alpha - \beta p_2 + \gamma p_1) \end{cases} \Leftrightarrow \begin{cases} p_1 &= \frac{\alpha(\beta+\gamma)(1-\rho)-\beta s_1-\gamma s_2}{(\beta^2-\gamma^2)(1-\rho)} \\ p_2 &= \frac{\alpha(\beta+\gamma)(1-\rho)-\beta s_2-\gamma s_1}{(\beta^2-\gamma^2)(1-\rho)}, \end{cases} \tag{A.119}$$

and their profit functions are given by:

$$\Pi^{i,B}_{1,1,0} = s_i(p_i - w) - \frac{\rho s_i}{1-\rho}\eta, \tag{A.120}$$

where $i = 1, 2$ and p_i is defined by (A.119).

If $\{NMR, CR, CR\}$ arises in equilibrium, then by symmetry, each retailer orders the same level of stock; i.e., $s_1 = s_2 \stackrel{\text{def}}{=} s$. Then, option B is better than option A whenever:

$$\begin{aligned} & \Pi^{i,B}_{1,1,0} - \Pi^{i,B}_{1,1,0} \geq 0 \\ \Leftrightarrow \quad & \frac{s\rho[\alpha(1-\rho) - (\beta-\gamma)\eta - (2-\rho)s]}{(\beta-\gamma)(1-\rho)} \geq 0 \\ \Leftrightarrow \quad & \alpha(1-\rho) - (\beta-\gamma)\eta - (2-\rho)s \geq 0. \end{aligned} \tag{A.121}$$

To ensure that the retailers will choose option B when $\{NMR, CR, CR\}$ arises in equilibrium, we need to show that the levels of stock satisfy condition (A.121). From the previous analysis, when $\{NMR, CR, CR\}$ arises in equilibrium, the wholesale price is set at

$w_0^* = \frac{4(1+\tau)-(4+5\tau)\rho+4\tau(1-\rho)c}{8\tau(1-\rho)}$. At this wholesale price, the levels of stock ordered by the retailers are:

$$s = s^1_{1,1,0} = s^2_{1,1,0} = \frac{4[(1+\tau-2\mu)(1-\rho)-\tau(1-\rho)w-\tau\rho\eta]}{7} \tag{A.122}$$
$$= \frac{4(1+\tau-\tau c)(1-\rho)-\rho\tau}{14} \tag{A.123}$$

(Recall that $\eta = \frac{1}{4}$ and $\mu = 0$.) Substituting this stock level and the parameters of the demand function under $\{CR,CR\}$ from Table 2.1 into (A.121), we have:

$$\text{(A.121)} \quad \Leftrightarrow \quad \frac{2[(4c-5)\tau-4]\rho^2-4(1+6\tau c)\rho+12+(5+16c)\tau}{28} \geq 0 \tag{A.124}$$

Denote the expression in the numerator by A_3. We have: (1) A_3 is a concave quadratic function in $\rho \in [0, \frac{1}{2}]$ since $c \leq \overline{w}_N = \frac{1+\tau}{\tau} - \frac{\rho}{4(1-\rho)} \Rightarrow (4c-5)\tau - 4 \leq -\frac{\tau}{1-\rho} < 0$, (2) $A_3|_{\rho=0} = \frac{12+(5+16c)\tau}{28} > 0$, and (3) $A_3|_{\rho=\frac{1}{2}} = \frac{16+(5+12c)\tau}{56} > 0$. This implies $A_3 > 0 \quad \forall \rho \in [0, \frac{1}{2}]$, i.e., condition (A.121) is satisfied when $\{NMR,CR,CR\}$ arises in equilibrium. The statement of Proposition 2.2 follows. ■

A.3.2 Proof of Proposition 2.1

Here, we derive the solution of Case 1 (i.e., when $\eta = 0$), which is a special instance of Case 2. When the cost of stock-outs is negligible, based on the results from Tables 2.3 and 2.4, we obtain the differences in the retailers' profits (see equations 2.8-2.11) as follows:

$$\delta_1 = -\frac{24\tau(17+8\rho)}{1225}w^2 + \frac{24(1+\tau)(1+4\rho)}{245}w + \frac{6(1-2\rho)(1+\tau)^2}{49\tau}, \tag{A.125}$$
$$\delta_2 = -\frac{6\tau(71+50\rho)}{1225}w^2 + \frac{12(1+\tau)(1+10\rho)}{245}w + \frac{6(1-2\rho)(1+\tau)^2}{49\tau}, \tag{A.126}$$
$$\delta_3 = -\frac{24\tau(17+8\rho)}{1225}w^2 + \frac{24(1+\tau)(1+4\rho)}{245}w + \frac{6(1-2\rho)(1+\tau)^2}{49\tau}, \text{ and} \tag{A.127}$$

$$\delta_4 = -\frac{6\tau(71+50\rho)}{1225}w^2 + \frac{12(1+\tau)(1+10\rho)}{245}w + \frac{6(1-2\rho)(1+\tau)^2}{49\tau}. \tag{A.128}$$

To determine the retailers' choice of consumer-returns policy under *NMR*, we consider δ_1 and δ_2. Notice that δ_1 and δ_2 are concave quadratic functions in w. Recall that under *NMR*, the range of the wholesale price is $[c, \min\{w_1, w_2, w_3, w_4\}]$ (see Tables 2.3 and 2.4). Since $\delta_1|_{w=0} = \frac{6(1-2\rho)(1+\tau)^2}{49\tau} \geq 0$ (when $\rho \leq \frac{1}{2}$) and $\delta_1|_{w=w_1} = \frac{108(1-\rho)(1+\tau)^2}{1225\tau} > 0$, we have $\delta_1 \geq 0 \quad \forall w \in [0, w_1]$, which implies $\delta_1 \geq 0 \quad \forall w \in [c, \min\{w_1, w_2, w_3, w_4\}]$. Analogously, we have $\delta_2|_{w=0} = \frac{6(1-2\rho)(1+\tau)^2}{49\tau} \geq 0$ and $\delta_2|_{w=w_3} = \frac{432(1-\rho)(1+\tau)^2}{5929\tau} > 0$, which implies $\delta_2 \geq 0 \quad \forall w \in [c, \min\{w_1, w_2, w_3, w_4\}]$. As a result, under *NMR*, the retailers will choose to accept consumer returns at all relevant wholesale prices.

Under *MR*, too, the retailers will accept returns at all relevant wholesale prices. (Notice that $\delta_3 = \delta_1$ and $\delta_4 = \delta_2$.)

In the first stage, the manufacturer makes the decision on its distribution policy, including the returns policy and the wholesale price. Knowing that the retailers will always accept returns from consumers, the manufacturer compares profits under *NMR* and *MR*, which equal to $\pi^M_{1,1,0}$ and $\pi^M_{1,1,1}$ respectively, when making the decision whether to accept returns. Since $\pi^M_{1,1,0} - \pi^M_{1,1,1} = \frac{8\rho(1+\tau-\tau w)c}{7} \geq 0$ when $w \leq \frac{1+\tau}{\tau} = w_2$, M is better off by choosing *NMR*. The statement of Proposition 2.1 follows. ■

A.3.3 Proof of Proposition 2.3:

In this setting, η is so high that the retailers avoid stock-outs by pricing according to option A when accepting consumer returns under *NMR*. Specifically, under combinations 2 (i.e., $\{$*NMR,NCR,CR*$\}$), 3 (i.e., $\{$*NMR,CR,NCR*$\}$), and 4 (i.e., $\{$*NMR,CR,CR*$\}$), they price to clear stock according to the initial-demand function in stage four (instead of the net-demand function as in the case of low stock-out cost). Then, in the third stage, they aim at maximizing profits by choosing the optimal levels of stock. Note that under option A, the retailers do not incur stock-out cost but instead bear the cost of unsold inventory due to consumer returns. Under the remaining combinations, the retailers' pricing strategies are identical to those in Case 2. Consequently, the results under the remaining combinations, including combination 1 (i.e., $\{NMR,NCR,NCR\}$), combination 5 (i.e., $\{MR,NCR,NCR\}$), combination 6 (i.e., $\{MR,NCR,CR\}$), combination 7 (i.e., $\{MR,CR,NCR\}$), and combination 8 (i.e., $\{MR,CR,CR\}$), remain identical to those of Case 2.

A.3.3.1 The retailers' pricing and stocking strategies (Case 3, stages 4 and 3)

As noted above, the retailers set prices in stage four according to option A under three combinations 2, 3 and 4 when stock-out cost is high. Note that by symmetry, the results of combination 3 are similar to those of combination 2.

Combination 4: $\{NMR,CR,CR\}$

In stage four, the retailers set the retail prices as follow:

$$\begin{cases} s_1 &= 1+\tau-2\mu-\frac{3\tau}{2}p_1+\frac{\tau}{2}p_2 \\ s_2 &= 1+\tau-2\mu-\frac{3\tau}{2}p_2+\frac{\tau}{2}p_1 \end{cases} \Leftrightarrow \begin{cases} p_1 &= \frac{4(1+\tau-2\mu)-3s_1-s_2}{4\tau} \\ p_2 &= \frac{4(1+\tau-2\mu)-3s_2-s_1}{4\tau}. \end{cases} \tag{A.129}$$

Then, they choose the optimal levels of stock by solving the following optimization problems:

$$\begin{aligned} \max_{s_1} \Pi_1 &= (1-\rho)(1+\tau-2\mu-\frac{3\tau}{2}p_1+\frac{\tau}{2}p_2)p_1-s_1w \\ &= s_1[(1-\rho)p_1-w] \quad \text{(by (A.129)), and} \end{aligned} \tag{A.130}$$

$$\begin{aligned} \max_{s_2} \Pi_2 &= (1-\rho)(1+\tau-2\mu-\frac{3\tau}{2}p_2+\frac{\tau}{2}p_1)p_2-s_2w \\ &= s_2[(1-\rho)p_2-w]. \end{aligned} \tag{A.131}$$

Solving the first-order conditions simultaneously, we obtain the optimal stock levels:

$$\hat{s}^i_{1,1,0} = \frac{4[(1+\tau-2\mu)(1-\rho)-\tau w]}{7(1-\rho)} \tag{A.132}$$

The optimal prices, $\hat{p}^i_{1,1,0}$, profits of the retailers, $\hat{\Pi}^i_{1,1,0}$, and that of the manufacturer, $\hat{\pi}^M_{1,1,0}$, are given in Table 2.5. The optimal levels of stock are non-negative when $w \leq \frac{(1+\tau-2\mu)(1-\rho)}{\tau} \stackrel{\text{def}}{=} \hat{w}_2$.

Combination 2: {*NMR,NCR,CR*}

Here, only retailer R_2 accepts consumer returns. Therefore, in stage four, R_2 sets the retail price to clear stock with according to the initial-demand function, $\hat{q}_2 = 1 + \frac{3\tau}{2} - \tau v - 3\mu - \frac{3\tau}{2}p_2 + \tau p_1$. On the other hand, R_1 chooses the retail price to clear stock according to the demand function, $q_1 = 1 - \frac{\tau}{2} + 3\tau v + \mu - 3\tau p_1 + \frac{\tau}{2}p_2$. This means

$$\begin{cases} s_1 &= 1 - \frac{\tau}{2} + 3\tau v + \mu - 3\tau p_1 + \frac{\tau}{2}p_2 \\ s_2 &= 1 + \frac{3\tau}{2} - \tau v - 3\mu - \frac{3\tau}{2}p_2 + \tau p_1 \end{cases} \Leftrightarrow \begin{cases} p_1 &= \frac{4(1+2\tau v)-3s_1-s_2}{8\tau} \\ p_2 &= \frac{4(1+\tau-2\mu)-3s_2-s_1}{4\tau}. \end{cases} \tag{A.133}$$

Then, in the third stage, the retailers maximize their profits by choosing the optimal levels of stock:

$$\max_{s_1} \Pi_1 = s_1(p_1 - w), \text{ and} \tag{A.134}$$

$$\max_{s_2} \Pi_2 = s_2(1-\rho)p_2 - s_2 w. \tag{A.135}$$

The solution to this problem is provided in Table 2.5 along with the conditions of regularity.

A.3.3.2 The retailers' returns policies (Case 3, stage 2)

The retailers' pricing and stocking decisions as well as their profits under *MR* are identical to those in the setting where η is low. Therefore, the optimal consumer returns policies under *MR* remains unchanged (see Figure 2.2). Next, under *NMR*, we examine the following differences in profits:

$$\hat{\delta}_1 = \hat{\Pi}^2_{1,0,0} - \Pi^2_{0,0,0}, \tag{A.136}$$

$$\hat{\delta}_2 = \hat{\Pi}^1_{1,1,0} - \hat{\Pi}^1_{0,1,0}, \tag{A.137}$$

Again, recall that our focus is on the case of risk-neutral consumers (i.e., $v = \frac{1}{2}$), and zero transaction cost for returning the product (i.e., $\mu = 0$). The other relevant parameters are: $\tau \in (0, +\infty)$ and $\rho \in [0, \frac{1}{2}]$. Here, the regularity conditions under *NMR* require (a) $w \le w_1 = \frac{1}{2} + \frac{1}{2\tau}$ under combination 1, (b) $w \le \min\{\hat{w}_3, \hat{w}_4\}$, where $\hat{w}_3 = \frac{5(1+\tau)(1-\rho)}{(11-12\rho)\tau}$ and $\hat{w}_4 = \frac{5(1+\tau)(1-\rho)}{2(2+\rho)\tau}$, under combination 2, and (c) $w \le \hat{w}_2 = \frac{(1+\tau)(1-\rho)}{\tau}$ under combination 4. Again, it can be shown that $\hat{w}_3 \le w_1 \le \hat{w}_2 \le \hat{w}_4$ when $\rho \le \frac{1}{2}$. See Figure A.4 for feasibility of the four combinations under *NMR*, which shows that all four combinations are feasible only when $w \le \hat{w}_3$. (Note that to distinguish the analysis of Cases 2 and 3, we use the notation $\hat{w}_2, \hat{w}_3$, and $\hat{w}_4$ in place of w_2, w_3, and w_4 respectively.)

First, consider $\hat{\delta}_1$. We have

$$\hat{\delta}_1 = -\frac{24(17+\rho)(1-2\rho)\tau}{1225(1-\rho)} w^2 + \frac{24(1-2\rho)(1+\tau)}{245} w + \frac{6(1-2\rho)(1+\tau)^2}{49\tau}, \tag{A.138}$$

which is a concave quadratic function in $w \in [0, \hat{w}_3]$. It can be shown that (i) $\hat{\delta}_1|_{w=0} = \frac{6(1-2\rho)(1+\tau)^2}{49\tau} \ge 0$ and (ii) $\hat{\delta}_1|_{w=\hat{w}_3} = \frac{6(97-98\rho)(1-2\rho)^2(1+\tau)^2}{49(11-12\rho)^2\tau} \ge 0$. Therefore, $\hat{\delta}_1 \ge 0 \quad \forall w \in [0, \hat{w}_3]$.

Next, consider $\hat{\delta}_2$. Similarly, we have: (i) $\hat{\delta}_2$ is a concave quadratic function in w as

$$\hat{\delta}_2 = -\frac{6(1-2\rho)(71-72\rho)\tau}{1225(1-\rho)^2}w^2 + \frac{12(1-2\rho)(1+\tau)}{245(1-\rho)}w + \frac{6(1-2\rho)(1+\tau)^2}{49\tau}, \qquad \text{(A.139)}$$

(ii) $\hat{\delta}_2|_{w=0} = \frac{6(1-2\rho)(1+\tau)^2}{49\tau} \geq 0$, and (iii) $\hat{\delta}_2|_{w=\hat{w}_3} = \frac{432(1-\rho)(1-2\rho)^2(1+\tau)^2}{49(11-12\rho)^2\tau} \geq 0$. This implies that $\hat{\delta}_2 \geq 0 \quad \forall w \in [0, \hat{w}_3]$.

Since $\hat{\delta}_1 \geq 0$ and $\hat{\delta}_2 \geq 0 \quad \forall w \in [0, \hat{w}_3]$, the retailers always choose to accept consumer returns under *NMR*. In summary, the retailers accept consumer returns when stock-out cost is high, no matter of the returns policy of the manufacturer.

A.3.3.3 The manufacturer's distribution policy (Case 3, stage 1)

If the manufacturer does not accept returns from the retailers, its profit function is defined by $\hat{\pi}_0^M = \hat{\pi}_{1,1,0}^M = \frac{8[(1+\tau-2\mu)(1-\rho)-\tau w](w-c)}{7(1-\rho)}$. By optimization, M will set the wholesale price at $\hat{w}_0^* = \frac{c}{2} + \frac{(1-\rho)(1+\tau)}{2\tau}$ to earn $\hat{\pi}_0^{M*} = \frac{2[(1-\rho)(1+\tau)-\tau c]^2}{7(1-\rho)\tau}$. Notice that $\hat{w}_0^* \in [c, \hat{w}_2]$ as long as the cost of production satisfies $c \leq \hat{w}_2$.

In contrast, if the manufacturer chooses *MR*, then the profit function is given by $\hat{\pi}_1^M = \pi_{1,1,1}^M$ (see (A.111)). As shown in (A.111), here, the optimal wholesale price is w_1^* (see equation A.112). At this wholesale price, M's profit is equal to π_1^{M*} (see equation A.113 for the expression of π_1^{M*}).

Comparing $\hat{\pi}_0^{M*}$ with π_1^{M*}, it follows that the manufacturer is indifferent between accepting and not accepting returns from the retailers when stock-out cost is high. The statement of Proposition 2.3 follows. ■

APPENDIX B: PROOFS OF RESULTS IN CHAPTER 3

Proof of Proposition 3.1

B.1 Equilibrium prices and profits under the *GB* strategy

B.1.1 Option *A*

The retailer solves the constrained optimization problem, defined by (3.15) and (3.16), i.e.,

$$\max_{p^P,\, p_h} \Pi^G \;=\; \phi\,(1+v_0-p^P)\,p^P + (1-\phi)(1-p_h-\kappa_L)\,p_h \tag{B.1}$$

$$\text{s.t.} \quad p^P \;\geq\; p_h+\kappa_L. \tag{B.2}$$

The Lagrangian function associated with this problem is given by:

$$\mathscr{L} \;=\; \Pi^G + \lambda\,(p^P - p_h - \kappa_L), \tag{B.3}$$

which has the following first-order conditions:

$$\mathscr{L}_{p^P} \;=\; \phi\,(1+v_0-2p^P)+\lambda \;=\; 0, \tag{B.4}$$

$$\mathscr{L}_{p_h} \;=\; (1-\phi)(1-2p_h-\kappa_L)-\lambda \;=\; 0, \tag{B.5}$$

$$\mathscr{L}_{\lambda} \;=\; p^P - p_h - \kappa_L \;\geq\; 0, \text{ and} \tag{B.6}$$

$$\lambda\,\mathscr{L}_{\lambda} \;=\; 0. \tag{B.7}$$

If $\lambda = 0$, then by solving (B.4) and (B.5), we obtain:

$$p^{P^{**}} \;=\; \frac{1+v_0}{2}, \text{ and} \tag{B.8}$$

$$p_h^{**} \;=\; \frac{1-\kappa_L}{2}. \tag{B.9}$$

Note that the constraint is non-binding in this case; this requires:

$$p^{P^{**}} - p_h^{**} - \kappa_L \geq 0 \Leftrightarrow \frac{1}{2}(v_o - \kappa_L) \geq 0 \Leftrightarrow \kappa_L \leq v_0. \tag{B.10}$$

If $\lambda > 0$, then the constraint is binding. We solve the following system:

$$\begin{cases} p^P - p_h - \kappa_L = 0 \\ \mathscr{L}_{p^P} = 0 \\ \mathscr{L}_{p_h} = 0 \end{cases} \tag{B.11}$$

to obtain the corner solution to the problem:

$$\widehat{p}^P = \frac{1 + \phi\, v_0 + (1-\phi)\kappa_L}{2}, \tag{B.12}$$

$$\widehat{p}_h = \frac{1 + \phi\, v_0 - (1+\phi)\kappa_L}{2}, \text{ and} \tag{B.13}$$

$$\widehat{\lambda} = \phi\,(1-\phi)(\kappa_L - v_0). \tag{B.14}$$

This case arises when $\widehat{\lambda} > 0 \Leftrightarrow \kappa_L > v_0$.

B.1.2 Option B

When choosing Option B, the retailer solves the problem defined by (3.21) and (3.22):

$$\max_{p^P, p_l} \Pi^G = \phi\,(1 + v_0 - p^P)\, p^P + (1-\phi)(1 - p_l - \kappa_L)\, p_l \tag{B.15}$$

$$\text{s.t.} \quad p^P > p_l + \kappa_L. \tag{B.16}$$

The Lagrangian function and the first-order conditions are as follows:

$$\mathscr{L} = \Pi^G + \lambda\,(p^P - p_l - \kappa_L), \tag{B.17}$$

$$\mathscr{L}_{p^P} = \phi\,(1 + v_0 - 2p^P) + \lambda = 0, \tag{B.18}$$

$$\mathscr{L}_{p_l} = (1 - \phi)(1 - 2p_l - \kappa_L) - \lambda = 0, \tag{B.19}$$

$$\mathscr{L}_\lambda = p^P - p_l - \kappa_L > 0, \text{ and} \tag{B.20}$$

$$\lambda\,\mathscr{L}_\lambda = 0. \tag{B.21}$$

Notice that (B.20) and (B.21) imply $\lambda = 0$; given $\lambda = 0$, we solve (B.18) and (B.19) to obtain:

$$p^{P^{**}} = \frac{1 + v_0}{2}, \text{ and} \tag{B.22}$$

$$p_l^{**} = \frac{1 - \kappa_L}{2}. \tag{B.23}$$

Finally, (B.20) requires $p^{P^{**}} - p_l^{**} - \kappa_L > 0 \Leftrightarrow \kappa_L < v_0$.

B.1.3 Option *C*

Here, we solve the optimization problem defined by (3.26) and (3.27):

$$\max_{p^P,\,p_l} \Pi^G = \phi\,(1 + v_0 - p^P)\,p^P + (1 - \phi)(1 - p_l - \kappa_L)\,p_l \tag{B.24}$$

$$\text{s.t.} \quad p^P \geq p_l + \kappa_L. \tag{B.25}$$

Notice that this problem would be identical to the optimization problem in Option A upon replacing p_h for p_l; therefore, the solution (see (3.28) and (3.29)) could be derived analogously.

B.2 The retailer's pricing strategy in equilibrium

Here, we compare the profitability of the PP to that of the GB strategy (see Lemma 3.1).

First, consider $\kappa_L \leq v_0$. In this case, then the retailer earns $\Pi^{G^{**}} = \phi \frac{(1+v_0)^2}{4} + (1-\phi)\frac{(1-\kappa_L)^2}{4}$ upon the employment of the GB strategy. We have:

$$\Pi^{G^{**}} - \Pi^{P^*} = \frac{1}{4}(1-\phi)[\kappa_L{}^2 - 2\kappa_L + \phi\, v_0^2] \geq 0 \Leftrightarrow \kappa_L \leq 1 - \sqrt{1 - \phi\, v_0^2} \stackrel{\text{def}}{=} \kappa^*. \quad \text{(B.26)}$$

(It can be seen that $\kappa^* \in [0, v_0]\,\forall\, v_0 \in [0,1]$.) Therefore, the retailer will choose the GB strategy if $\kappa_L \leq \kappa^*$, and vice versa.

Next, if $\kappa_L > v_0$, then the profitability of the GB strategy is $\widehat{\Pi}^G = \frac{[1-\phi\, v_0 - (1-\phi)\,\kappa_L]^2 + 4\,\phi\, v_0}{4}$. We have:

$$\Pi^{P^*} - \widehat{\Pi}^G = \frac{1}{4}\,\kappa_L\,(1-\phi)[2(1-\phi\, v_0) - (1-\phi)\kappa_L]. \quad \text{(B.27)}$$

Recall that the group-buying price in this setting is $\widehat{p}^{GB} = \frac{1+\phi\, v_0 - (1+\phi)\kappa_L}{2}$; at this price, demand via group-buying is non-negative if:

$$D^{G,L} = (1-\phi)(1 - \widehat{p}^{GB} - \kappa_L) \geq 0 \Rightarrow (1-\phi)\kappa_L \leq 1 - \phi\, v_0, \quad \text{(B.28)}$$

which implies $\Pi^{P^*} - \widehat{\Pi}^G \geq 0$. Therefore, the retailer always chooses the PP strategy when $\kappa_L > v_0$. The statement of Proposition 3.1 follows. ■

APPENDIX C: PROOFS OF RESULTS IN CHAPTER 4

C.1 Derivation of the effective group-buying price

Upon the employment of the G contract, the retailer offers to consumers the following pricing schedule:

$$\begin{cases} p^P & \text{targets at } H \text{ consumers },\\ p^{GB} = \begin{cases} p_2 & \text{if} \quad q < \bar{q}, \\ p_1 & \text{if} \quad q \geq \bar{q} \end{cases} & \text{targets at } L \text{ consumers }. \end{cases} \tag{C.1}$$

Regarding the group-buying schedule, either p_1 or p_2 could realize, i.e., becomes effective. To implement its targeting strategy, the retailer must set the prices to ensure the following incentive compatibility constraints for consumers:

(1) H consumers always purchase the product using the traditional posted-pricing mechanism, paying p^P, when:

$$p^P \leq p_2 + \kappa_H \quad \text{if } q < \bar{q}, \text{ and} \tag{C.2}$$

$$p^P \leq p_1 + \kappa_H \quad \text{if } q \geq \bar{q}. \tag{C.3}$$

(2) For L consumers to join the group-buying mechanism, we need:

$$p_2 + \kappa_L \leq p^P \quad \text{if } q < \bar{q}, \text{ and} \tag{C.4}$$

$$p_1 + \kappa_L \leq p^P \quad \text{if } q \geq \bar{q}. \tag{C.5}$$

Given these constraints being satisfied, the demand functions upon the employment of the G contract are given by:

$$D^{H,G}(p^P; .) = \eta\,\alpha - \beta\, p^P, \text{ and} \tag{C.6}$$

$$D^{L,G}(p^{GB};.) = (1-\eta)\alpha - \gamma(p^{GB} + \kappa_L). \tag{C.7}$$

Upon the employment of the G contract, i.e., $\{G, w^G, F^G\}$, the cost of retailing includes the wholesale price, w^G, and the fixed fee, F^G; therefore, the retailer's optimization problem is as follows:

$$\begin{aligned}
\max_{p^P, p_2, p_1, \bar{q}} \Pi^G(.) &= D^{H,G}(p^P;.)(p^P - w^G) + D^{L,G}(p^{GB};.)(p^{GB} - w^G) - F^G \\
&= \begin{cases} (\eta\,\alpha - \beta\,p^P)(p^P - w^G) + [(1-\eta)\alpha - \gamma(p_2 + \kappa_L)](p_2 - w^G) - F^G & \text{if} \quad q < \bar{q}, \\ \\ (\eta\,\alpha - \beta\,p^P)(p^P - w^G) + [(1-\eta)\alpha - \gamma(p_1 + \kappa_L)](p_1 - w^G) - F^G & \text{if} \quad q \geq \bar{q}. \end{cases}
\end{aligned} \tag{C.8}$$

Investigating the behavior of Π^G, we notice that: (1) either p_2 or p_1 enters this profit function, (2) $\bar{q}$ affects the realization of p^{GB}, i.e., whether p_1 or p_2 arises as the effective group-buying price. In other words, assuming that consumers' utility is non-stochastic and their behavior is rational, the retailer can determine which price (i.e., p_1 or p_2) will become the effective group-buying price (at least, on average) by setting $\bar{q}$ at the appropriate level.

Abstracting away from the stochastic aspect of the group-buying mechanism, R has two options to set the optimal group-buying prices: (a) R can choose p_1 as the effective group-buying price and sets it at the optimal level, p_1^*, to maximize its profit, Π^G, given the two incentive compatibility constraints defined by (C.3) and (C.5) being satisfied, and (b) R chooses p_2 to be the effective price, which is set at the optimal level, p_2^*, taking (C.2) and (C.4) into consideration. The other parameters of the group-buying schedule (i.e., $\{p_2, \bar{q}\}$ in the former option and $\{p_1, \bar{q}\}$ in the latter) are set to support the realization of the chosen effective price in each of the two options. Specifically, in the first option, $p_2 > p_1^*$ and $\bar{q} \leq D^{L,G}(p_1^*;.)$, and in the second option, $p^P + \kappa_H \leq p_1 < p_2^*$ and $\bar{q} > D^{L,G}(p_1;.)$.

Notice that in equilibrium, the optimal effective group-buying prices, p_1^* and p_2^*, are identical (i.e., $p_1^* = p_2^*$), and so are the profits of the retailer in the two options: $\Pi^G(p^P, p_1^*, w^G, F^G; .) = \Pi^G(p^P, p_2^*, w^G, F^G; .)$. Therefore, the retailer is indifferent between the two options. Consequently, in the analysis of this paper, we simplify the treatment of the group-buying pricing with multiple price tiers to be one of a single effective price, p^{GB}; the non-effective group-buying price(s) and the cut-off level(s) of quantity are assumed to be set suitably as mentioned above.

In summary, upon the employment of the G contract, i.e., when both a posted price, p^P, and a group-buying pricing schedule, p^{GB}, are offered to target at consumers in the high and low segments respectively, the retailer's optimization problem can be simplified to:

$$\begin{aligned}
\max_{p^P, p^{GB}} \Pi^G(.) &= D^{H,G}(p^P; .)(p^P - w^G) + D^{L,G}(p^{GB}; .)(p^{GB} - w^G) - F^G \\
&= (\eta\,\alpha - \beta\,p^P)(p^P - w^G) + [(1-\eta)\alpha - \gamma(p^{GB} + \kappa_L)](p^{GB} - w^G) - F^G \qquad \text{(C.9)}
\end{aligned}$$

$$\text{s.t.} \quad \begin{cases} p^P \ \le p^{GB} + \kappa_H, \\ p^{GB} + \kappa_L \le p^P. \end{cases} \qquad \text{(C.10)}$$

C.2 The solution of the retailer's optimization problem upon the employment of the G contract

Consider the retailer's optimization problem when employing of the G contract, $\{G, w^G, F^G\}$:

$$\begin{aligned}
\max_{p^P, p^{GB}} \Pi^G(.) &= D^{H,G}(p^P; .)(p^P - w^G) + D^{L,G}(p^{GB}; .)(p^{GB} - w^G) - F^G \\
&= (\eta\,\alpha - \beta\,p^P)(p^P - w^G) + [(1-\eta)\alpha - \gamma(p^{GB} + \kappa_L)](p^{GB} - w^G) - F^G \qquad \text{(C.11)}
\end{aligned}$$

$$\text{s.t.} \quad \begin{cases} p^{GB} + \kappa_H - p^P \ \ge\ 0, & (IC_H) \\ p^P - p^{GB} - \kappa_L \ \ge\ 0. & (IC_L) \end{cases}$$

This constrained optimization problem has the following Lagrangian function:

$$\mathscr{L} = \Pi^{G}(.) + \lambda_{H}(p^{GB} - p^{P} + \kappa_{H}) + \lambda_{L}(p^{P} - p^{GB} - \kappa_{L}), \tag{C.12}$$

and the first-order conditions are:

$$\mathscr{L}_{p^P} = \eta_i\, \alpha - 2\beta_i\, p^P + \beta_i\, w^{GB} - \lambda_H + \lambda_L = 0, \tag{C.13}$$

$$\mathscr{L}_{p^{GB}} = (1-\eta_i)\, \alpha - 2\gamma_i\, p^{GB} - \gamma_i\, \kappa_L + \gamma_i\, w^{GB} + \lambda_H - \lambda_L = 0, \tag{C.14}$$

$$\mathscr{L}_{\lambda_H} = p^{GB} - p^P + \kappa_H \geq 0, \tag{C.15}$$

$$\mathscr{L}_{\lambda_L} = p^P - p^{GB} - \kappa_L \geq 0, \tag{C.16}$$

$$\lambda_H\, (p^{GB} - p^P + \kappa_H) = 0, \tag{C.17}$$

$$\lambda_L\, (p^P - p^{GB} - \kappa_L) = 0, \tag{C.18}$$

$$\lambda_H \geq 0, \tag{C.19}$$

$$\lambda_L \geq 0. \tag{C.20}$$

To solve this constrained optimization problem, we consider three cases: (a) $\lambda_H = \lambda_L = 0$, (b) $\lambda_H = 0$ and $\lambda_L > 0$, and (c) $\lambda_H > 0$ and $\lambda_L = 0$. (Notice that the case of $\lambda_H > 0$ and $\lambda_L > 0$ does not arise since $\kappa_H > \kappa_L$.)

First, if $\lambda_H = \lambda_L = 0$, then by solving equations (C.13) and (C.14), we obtain:

$$p^{P*} = \frac{\eta\, \alpha}{2\beta} + \frac{w^G}{2}, \text{ and} \tag{C.21}$$

$$p^{GB*} = \frac{(1-\eta)\, \alpha}{2\gamma} + \frac{w^G}{2} - \frac{\kappa_L}{2}. \tag{C.22}$$

Notice that this is the case of non-binding constraints, which arises under the following conditions:

$$(IC_{H})\&(IC_{L}) \Rightarrow \begin{cases} p^{GB^*} - p^{P^*} + \kappa_H \ \geq \ 0 \\ p^{P^*} - p^{GB^*} - \kappa_L \ \geq \ 0 \end{cases} \Leftrightarrow \begin{cases} \kappa_H \ \geq \ \frac{\kappa_L}{2} + \frac{1}{2}\left[\frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma}\right], \\ \kappa_L \ \leq \ \frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma}. \end{cases} \tag{C.23}$$

Next, if $\lambda_H = 0$ and $\lambda_L > 0$, which implies that (IC_L) is binding, we solve the following system:

$$\begin{cases} p^P - p^{GB} - \kappa_L \ = \ 0 \\ \mathscr{L}_{p^P}\big|_{\lambda_H=0} \ = \ 0 \\ \mathscr{L}_{p^{GB}}\big|_{\lambda_H=0} \ = \ 0, \end{cases} \tag{C.24}$$

and obtain:

$$\hat{p}^P = \frac{\alpha}{2(\beta+\gamma)} + \frac{w^G}{2} + \frac{\gamma\,\kappa_L}{2(\beta+\gamma)}, \tag{C.25}$$

$$\hat{p}^{GB} = \frac{\alpha}{2(\beta+\gamma)} + \frac{w^G}{2} - \frac{(2\beta+\gamma)\kappa_L}{2(\beta+\gamma)}, \tag{C.26}$$

$$\hat{\lambda}_L = \frac{\beta\,\gamma\,\kappa_L + \alpha[(1-\eta)\beta - \eta\,\gamma]}{\beta+\gamma}. \tag{C.27}$$

This means that the IC_L-binding case arises when

$$\hat{\lambda}_L > 0 \Leftrightarrow \kappa_L > \frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma}. \tag{C.28}$$

Finally, if $\lambda_H > 0$ and $\lambda_L = 0$, i.e., (IC_H) is binding, the first-order conditions become:

$$\begin{cases} p^{GB} - p^P + \kappa_H \ = \ 0 \\ \mathscr{L}_{p^P}\big|_{\lambda_L=0} \ = \ 0 \\ \mathscr{L}_{p^{GB}}\big|_{\lambda_L=0} \ = \ 0, \end{cases} \tag{C.29}$$

whose solution is as follows:

$$\tilde{p}^{P} = \frac{\alpha}{2(\beta+\gamma)} + \frac{w^{G}}{2} + \frac{\gamma\,(2\kappa_{H}-\kappa_{L})}{2(\beta+\gamma)}, \tag{C.30}$$

$$\tilde{p}^{GB} = \frac{\alpha}{2(\beta+\gamma)} + \frac{w^{G}}{2} - \frac{2\beta\,\kappa_{H}+\gamma\,\kappa_{L}}{2(\beta+\gamma)}, \tag{C.31}$$

$$\tilde{\lambda}_{H} = \frac{\beta\,\gamma\,(\kappa_{L}-2\kappa_{H}) - \alpha[(1-\eta)\beta-\eta\,\gamma]}{\beta+\gamma}. \tag{C.32}$$

The binding-IC_{H} case arises when:

$$\hat{\lambda}_{H} > 0 \;\Leftrightarrow\; \kappa_{H} < \frac{\kappa_{L}}{2} + \frac{1}{2}\Big[\frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma}\Big]. \tag{C.33}$$

In summary, the retailer's optimization problem upon the employment of the G contract is equivalent to:

(1) $$\max_{p^{P},\,p^{GB}} \Pi^{G}(.) = D^{H,G}(p^{P};.)(p^{P}-w^{G}) + D^{L,G}(p^{GB};.)(p^{GB}-w^{G}) - F^{G} \tag{C.34}$$

$$\text{if } \kappa_{L} \le \frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma} \text{ and } \kappa_{H} \ge \frac{\kappa_{L}}{2} + \frac{1}{2}\Big[\frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma}\Big], \tag{C.35}$$

(2) $$\max_{p^{P},\,p^{GB}} \Pi^{G}(.) \quad \text{s.t. } p^{P} - p^{GB} - \kappa_{L} = 0 \quad \text{if } \kappa_{L} > \frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma}, \tag{C.36}$$

(3) $$\max_{p^{P},\,p^{GB}} \Pi^{G}(.) \quad \text{s.t. } p^{GB} - p^{P} + \kappa_{H} = 0 \quad \text{if } \kappa_{H} < \frac{\kappa_{L}}{2} + \frac{1}{2}\Big[\frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma}\Big]. \tag{C.37}$$

In this paper, we focus on the first setting, when κ_{H} and κ_{L} satisfy (C.35).

C.3 The optimal full-information contracts in the η-case (Proofs of Lemmas 4.1-4.3 and Propositions 4.1)

C.3.1 Proof of Lemma 4.1:

Notice that the manufacturer needs to ensure the retailer just minimal incentive for participation; therefore, it will set the fixed fee, F^G, so that $\Pi^{G*} = 0$. Therefore, the constrained optimization problem, defined by (4.12)-(4.13), is rewritten as:

$$\max_{\substack{w^G,\, F^G,\\ \mathring{p}^P,\, \mathring{p}^{GB}}} \pi^G(\cdot) = \big\{(\eta\,\alpha - \beta\,\mathring{p}^P) + [(1-\eta)\alpha - \gamma(\mathring{p}^{GB} + \kappa^L)]\big\} w^G + F^G \tag{C.38}$$

$$\text{s.t.} \begin{cases} \{\mathring{p}^P, \mathring{p}^{GB}\} = \underset{p^P,\, p^{GB}}{\operatorname{argmax}}\; \Pi^G(p^P,\, p^{GB},\, w^G,\, F^G; \cdot) \\ \Pi^{G*} \; = 0. \end{cases} \tag{C.39}$$

$$\Leftrightarrow \max_{\substack{w^G,\, F^G,\\ \mathring{p}^P,\, \mathring{p}^{GB}}} \pi^G(\cdot) = \big\{(\eta\,\alpha - \beta\,\mathring{p}^P) + [(1-\eta)\alpha - \gamma(\mathring{p}^{GB} + \kappa^L)]\big\} w^G + F^G \tag{C.40}$$

$$\text{s.t.} \begin{cases} \frac{\partial \Pi^G(\cdot)}{\partial p^P}\Big|_{p^P = \mathring{p}^P} & = \; \eta\,\alpha - 2\beta\,\mathring{p}^P + \beta\,w^G = 0 \\ \frac{\partial \Pi^G(\cdot)}{\partial p^{GB}}\Big|_{p^{GB} = \mathring{p}^{GB}} & = \; (1-\eta)\alpha - \gamma(2\mathring{p}^{GB} - w^G + \kappa^L) = 0 \\ \Pi^{G*} & = \; \Pi^G(\mathring{p}^P,\, \mathring{p}^{GB},\, w^G,\, F^G; \cdot) = 0, \end{cases} \tag{C.41}$$

which has the following Lagrangian function:

$$\mathscr{L} = \pi^G(\cdot) + \lambda\, \frac{\partial \Pi^G(\cdot)}{\partial p^P}\Big|_{p^P = \mathring{p}^P} + \mu\, \frac{\partial \Pi^G(\cdot)}{\partial p^{GB}}\Big|_{p^{GB} = \mathring{p}^{GB}} + \delta\, \Pi^{G*}. \tag{C.42}$$

Solving the first-order conditions of this Lagrangian function:

$$\mathscr{L}_{w^G} = \big[\alpha - \beta\,\mathring{p}^P - \gamma(\mathring{p}^{GB} + \kappa^L)\big](1-\delta) + \gamma\,\lambda + \beta\,\mu \;=\; 0, \tag{C.43}$$

$$\mathscr{L}_{F^G} = 1 - \delta \;=\; 0, \tag{C.44}$$

$$\mathscr{L}_{\mathring{p}^P} = -\big[\eta\,\alpha - \beta(2\mathring{p}^P - w^G)\big]\delta - \beta\,\lambda - \beta\,w^G \;=\; 0, \tag{C.45}$$

$$\mathscr{L}_{\mathring{p}^{GB}} = \big[(1-\eta)\,\alpha - \gamma(2\mathring{p}^{GB} - w^{G} + \kappa^{L})\big]\delta - \gamma(2\mu + w^{G}) \;=\; 0, \tag{C.46}$$

$$\mathscr{L}_{\lambda} = \eta\,\alpha - \beta(2\mathring{p}^{P} - w^{G}) \;=\; 0, \tag{C.47}$$

$$\mathscr{L}_{\mu} = (1-\eta)\alpha - \gamma(2\mathring{p}^{GB} - w^{G} + \kappa^{L}) \;=\; 0, \tag{C.48}$$

$$\mathscr{L}_{\delta} = (\eta\,\alpha - \beta\,\mathring{p}^{P}) + \big[(1-\eta)\alpha - \gamma(\mathring{p}^{GB} - w^{G})\big] - F^{G} \;=\; 0, \tag{C.49}$$

we obtain the results given in Lemma 4.1. Here, the G contract, by nature, requires both the high and low segments to be served, i.e., $D^{H,G}(p^{P*};\cdot) \geq 0$ and $D^{L,G}(p^{GB*};\cdot) \geq 0$. Notice that $D^{H,G}(p^{P*};\cdot) = \frac{\eta\,\alpha}{2} > 0$ and

$$D^{L,G}(p^{GB*};\cdot) = \frac{(1-\eta)\alpha - \gamma\,\kappa^{L}}{2} \geq 0 \;\Leftrightarrow\; (1-\eta)\alpha - \gamma\,\kappa^{L} \geq 0. \tag{C.50}$$

The statement of Lemma 4.1 then follows. ■

C.3.2 Proof of Lemma 4.2:

Consider the P contract, $\{P, w^{P}, F^{P}\}$, under which consumers in both segments are served. This contract solves the constrained optimization problem, defined by (4.18)-(4.19). As under the G contract, here, too, the constraint regarding the retailer's profit is binding, i.e., $\Pi^{P*} = 0$. Consequently, this constrained optimization problem is rewritten as:

$$\max_{w^{P},\,F^{P},\,\mathring{p}} \pi^{P}(w^{P}, F^{P}, \mathring{p};\cdot) = \;[\alpha - (\beta+\gamma)\mathring{p}]\,w^{P} + F^{P} \tag{C.51}$$

$$\text{s.t.} \;\; \begin{cases} \mathring{p} \;=\; \underset{p}{\operatorname{argmax}}\; \Pi^{P}(p, w^{P}, F^{P};\cdot) \;\Leftrightarrow\; \frac{\partial \Pi^{P}(\cdot)}{\partial p}\Big|_{p=\mathring{p}} \;=\; \alpha - (\beta+\gamma)(2\mathring{p} - w^{P}) = 0, \\ \Pi^{P*} \;= \Pi^{P}(\mathring{p}, w^{P}, F^{P};\cdot) = 0. \end{cases} \tag{C.52}$$

Solving the first-order conditions of the Lagrangian function associated with this problem, i.e.,

$$\mathscr{L} = \pi^{P}(\cdot) + \lambda \frac{\partial \Pi^{P}(\cdot)}{\partial p}\Big|_{p=\mathring{p}} + \mu\, \Pi^{P*}, \tag{C.53}$$

we obtain the optimal full-information P contract as characterized by Lemma 4.2.

Here, to ensure both segments being served upon the employment of this contract, we need:

$$D^{H,P}(p^*;\cdot) \geq 0 \;\Leftrightarrow\; 2(\beta+\gamma)\eta - \beta \geq 0, \text{ and} \tag{C.54}$$

$$D^{L,P}(p^* : \cdot) \geq 0 \;\Leftrightarrow\; 2\beta + \gamma - 2(\beta+\gamma)\eta \geq 0. \tag{C.55}$$

Note that these are just the necessary conditions for the two segments being served under the P contract. Since the retailer may choose to serve only one of the two segments under some conditions, we derive the sufficient conditions for both segments being served.

First, consider the case when only the high segment is served. Denote the full-information P contract under which only consumers in the high segment are served by $\{P^{H}, w^{P}_{H}, F^{P}_{H}\}$ and the profit functions of the manufacturer and the retailer by $\pi^{P}_{H}(\cdot)$ and $\Pi^{P}_{H}(\cdot)$, respectively. This contract then solves the following optimization problem:

$$\max_{w^P_H, F^P_H, \mathring{p}_H} \pi^{P}_{H}(w^{P}_{H}, F^{P}_{H}, \mathring{p}_H; \eta, \beta) = D^{H,P}(\mathring{p}_H;\cdot) w^{P}_{H} + F^{P}_{H} = (\eta\,\alpha - \beta\,\mathring{p}_H)\, w^{P}_{H} + F^{P}_{H} \tag{C.56}$$

$$\text{s.t.} \begin{cases} \mathring{p}_H = \underset{p}{\operatorname{argmax}}\ \Pi^{P}_{H}(p, w^{P}_{H}, F^{P}_{H}; \eta, \beta) = D^{H,P}(p;\cdot)(p - w^{P}_{H}) - F^{P}_{H} \\ \Pi^{P*}_{H} = \Pi^{P}_{H}(\mathring{p}_H, w^{P}_{H}, F^{P}_{H};\cdot) \geq 0. \end{cases} \tag{C.57}$$

Here, too, the retailer's participation constraint is binding and the Lagrangian function is given by:

$$\mathscr{L} = \pi^{P}_{H}(\cdot) + \lambda \frac{\partial \Pi^{P}_{H}(\cdot)}{\partial p}\Big|_{p=\mathring{p}_H} + \mu\, \Pi^{P*}_{H}, \tag{C.58}$$

whose first-order conditions are solved to obtain the following results:

$$\begin{cases} w^{P*}_{H} & = \; 0, \\ F^{P*}_{H} & = \; \frac{\eta^2\, \alpha^2}{4\beta}, \\ \mathring{p}^{*}_{H} & = \; \frac{\eta\, \alpha}{2\beta}. \end{cases} \tag{C.59}$$

This contract would provide the manufacturer with a profit equal to $\pi^{P*}_{H} = F^{P*}_{H} = \frac{\eta^2\, \alpha^2}{4\beta}$. Therefore, it is dominated by the $\{P, w^{P}, F^{P}\}$ contract if

$$\pi^{P*} - \pi^{P*}_{H} \;\geq\; 0 \quad \Leftrightarrow \quad (1-\eta^2)\beta \;\geq\; \eta^2\, \gamma. \tag{C.60}$$

Now, consider the case when only consumers in the low segment are served. The full-information P contract serving only the low segment, denoted $\{P^{L}, w^{P}_{L}, F^{P}_{L}\}$, is determined by solving the following problem:

$$\max_{w^{P}_{L}, F^{P}_{L}, \mathring{p}_L} \; \pi^{P}_{L}(w^{P}_{L}, F^{P}_{L}, \mathring{p}_L; \eta, \gamma) = \; D^{L,P}(\mathring{p}_L; \cdot) w^{P}_{L} + F^{P}_{L} \;=\; \left[(1-\eta)\alpha - \gamma\, \mathring{p}_L\right] w^{P}_{L} + F^{P}_{L} \tag{C.61}$$

$$\text{s.t.} \; \begin{cases} \mathring{p}_L \;=\; \underset{p}{\operatorname{argmax}}\; \Pi^{P}_{L}(p, w^{P}_{L}, F^{P}_{L}; \eta, \beta) \;=\; D^{L,P}(p; \cdot)(p - w^{P}_{L}) - F^{P}_{L} \\ \Pi^{P*}_{L} \;=\; \Pi^{P}_{L}(\mathring{p}_L, w^{P}_{L}, F^{P}_{L}; \cdot) \geq 0. \end{cases} \tag{C.62}$$

Given that the retailer's participation constraint is binding, we solve the first-order conditions of the Lagrangian function associated with this problem:

$$\mathscr{L} = \pi^{P}_{L}(\cdot) + \lambda \frac{\partial \Pi^{P}_{L}(\cdot)}{\partial p}\Big|_{p=\mathring{p}_L} + \mu\, \Pi^{P*}_{L}, \tag{C.63}$$

to obtain:

$$\begin{cases} w_L^{P*} &= 0, \\ F_L^{P*} &= \frac{(1-\eta)^2\,\alpha^2}{4\gamma}, \\ \mathring{p}_L^{*} &= \frac{(1-\eta)\alpha}{2\gamma}. \end{cases} \tag{C.64}$$

Under this contract, the manufacturer earns $\pi_L^{P*} = F_L^{P*} = \frac{(1-\eta)^2\,\alpha^2}{4\gamma}$. As a result, this contract will not be chosen if:

$$\pi^{P*} - \pi_L^{P*} \geq 0 \quad \Leftrightarrow \quad \eta(2\eta-1)\gamma \geq (1-\eta)^2\,\beta. \tag{C.65}$$

Notice that (C.54) and (C.55) are satisfied whenever (C.65) and (C.60) hold, respectively. In summary, the full-information P contract, under which consumers in both segments are served, could be employed under the conditions characterized by (C.60) and (C.65). The statement of Lemma 4.2 then follows. ■

C.3.3 Proof of Lemma 4.3:

When $\kappa_L = 0$, from Lemmas 4.1 and 4.2, the manufacturer's profits upon the employment of the $\{G, w^G, F^G\}$ and $\{P, w^P, F^P\}$ contracts are:

$$\pi^{G*}\big|_{\kappa_L=0} = \frac{\eta^2\,\alpha^2}{4\beta} + \frac{(1-\eta)^2\,\alpha^2}{4\gamma}, \text{ and} \tag{C.66}$$

$$\pi^{P*} = \frac{\alpha^2}{4(\beta+\gamma)}, \text{ respectively.} \tag{C.67}$$

Since $\pi^{P*} - \pi^{G*}\big|_{\kappa_L=0} = -\frac{\alpha^2[(1-\eta)\beta-\eta\,\gamma]^2}{4\beta\,\gamma(\beta+\gamma)} < 0$, the manufacturer chooses the G contract in equilibrium.

From Lemma 4.2, the $\{P, w^P, F^P\}$ contract is feasible when:

$$(1-\eta^2)\beta \geq \eta^2\gamma \;\Leftrightarrow\; \eta \leq \sqrt{\frac{\beta}{\beta+\gamma}} \stackrel{\text{def}}{=} \bar{\beta}, \text{ and} \tag{C.68}$$

$$\eta(2\eta-1)\gamma \geq (1-\eta)^2\beta \;\Leftrightarrow\; \eta \geq 1-\sqrt{\frac{\gamma}{\beta+\gamma}} \stackrel{\text{def}}{=} \underline{\beta}. \tag{C.69}$$

Regarding the $\{G, w^G, F^G\}$ contract (see Lemma 4.1), given $\kappa_L = 0$, it is required that:

$$\frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma} \geq 0 \Leftrightarrow \eta \geq \frac{\beta}{\beta+\gamma} \tag{C.70}$$

$$\kappa_H \geq \frac{1}{2}\left[\frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma}\right] \tag{C.71}$$

Notice that since $1-\frac{\sqrt{\gamma}}{\sqrt{\beta+\gamma}} < \frac{\beta}{\beta+\gamma} < \sqrt{\frac{\beta}{\beta+\gamma}}$, from (C.68)-(C.70), we need: $\frac{\beta}{\beta+\gamma} \leq \eta \leq \sqrt{\frac{\beta}{\beta+\gamma}}$. Finally, if $\kappa_H \geq \frac{\alpha}{2\beta}$, then (C.71) always holds. The statement of Lemma 4.3 follows.■

C.3.4. Proof of Proposition 4.1:

Now, consider the case of non-trivial κ_L (i.e., $\kappa_L > 0$). From (C.68)-(C.69), the $\{P, w^P, F^P\}$ contract arises as a candidate for the optimal contract when $\eta \in [\underline{\eta}, \bar{\eta}]$. Meanwhile, the feasibility of the $\{G, w^G, F^G\}$ contract requires the following conditions (see Lemma 4.1):

$$(4.10)\&(4.17) \Leftrightarrow \eta_1 \stackrel{\text{def}}{=} \frac{\beta(\alpha+\gamma\,\kappa_L)}{\alpha(\beta+\gamma)} \leq \eta \leq 1-\frac{\gamma\,\kappa_L}{\alpha} \stackrel{\text{def}}{=} \eta_2. \tag{C.72}$$

Note that (C.72) requires $\kappa_L \leq \frac{\alpha}{2\beta+\gamma} \stackrel{\text{def}}{=} \kappa_1$. Further, analogous to the benchmark case, without loss of qualitative insights, we focus on the setting with significantly large κ_H (i.e., $\kappa_H \geq \frac{\kappa_L}{2} + \frac{\alpha}{2\beta}$), which warrants (4.11) to hold for all $\eta \in [0,1]$. Finally, it can be shown that (a) $\bar{\eta} > \eta_1 > \underline{\eta}$ for all $\kappa_L \leq \kappa_1$, and (b) $\eta_2 \geq \bar{\eta} \Leftrightarrow \kappa_L \leq \frac{\alpha(\sqrt{\beta+\gamma}-\sqrt{\beta})}{\gamma\sqrt{\beta+\gamma}} \stackrel{\text{def}}{=} \kappa_2$. (Notice that $\kappa_2 \in [0, \kappa_1]$.) Together, the conditions in η that are required for the $\{P, w^P, F^P\}$ and $\{G, w^G, FG\}$ contracts being feasible are summarized in Figure 4.1.

When both contracts are feasible (i.e., $\eta \in [\eta_1, \min\{\eta_2, \bar{\eta}\}]$), the manufacturer compares its profit earned under the $\{P, w^P, F^P\}$ contract to that under the $\{G, w^G, F^G\}$ contract:

$$\pi^{P*} - \pi^{G*} = -\frac{\alpha^2(\beta+\gamma)}{4\beta\,\gamma}\eta^2 + \frac{\alpha(\alpha-\gamma\,\kappa_L)}{2\gamma}\eta + \frac{1}{4}\big[2\alpha\,\kappa_L - \gamma\,\kappa_L^2 - \frac{\alpha^2\beta}{\gamma(\beta+\gamma)}\big] \stackrel{\text{def}}{=} \Delta. \quad \text{(C.73)}$$

Notice that (a) Δ is a concave, quadratic function in $\eta \in [\eta_1, \min\{\eta_2, \bar{\eta}\}]$, and (b) $\frac{\partial\Delta}{\partial\eta} = -\frac{\alpha\left[\beta\,\gamma\,\kappa_L - \alpha\,\beta + \alpha(\beta+\gamma)\eta\right]}{2\beta\,\gamma} \leq \frac{\partial\Delta}{\partial\eta}\Big|_{\eta=\eta_1} = -\alpha\,\kappa_L \leq 0$, i.e., Δ is decreasing in $\eta \in [\eta_1, \min\{\eta_2, \bar{\eta}\}]$.

If $\kappa_L \in (0, \kappa_2]$, then (a) $\min\{\eta_2, \bar{\eta}\} = \bar{\eta}$, (b) $\Delta\Big|_{\eta=\eta_1} = \frac{\gamma\,\kappa_L[2\alpha-(4\beta+\gamma)\kappa_L]}{4(\beta+\gamma)} \geq 0$[55], and (c) $\Delta\Big|_{\eta=\bar{\eta}} = -\frac{[\alpha(\sqrt{\beta+\gamma}-\sqrt{\beta})-\gamma\sqrt{\beta+\gamma}\kappa_L]^2}{4\gamma(\beta+\gamma)} \leq 0$. This implies that $\Delta = 0$ has one solution $\eta^* \in [\eta_1,\, \bar{\eta}]$, whose specification is given by:

$$\eta^* \stackrel{\text{def}}{=} \frac{\alpha\,\beta + \gamma[-\beta\,\kappa_L + \sqrt{\beta\,\kappa_L(2\alpha-\gamma\,\kappa_L)}]}{\alpha\beta+\gamma}. \quad \text{(C.74)}$$

In this case (i.e., $\kappa_L \in (0, \kappa_2]$), if $\eta \in [\eta_1, \eta^*]$, then $\Delta \geq 0 \Rightarrow \pi^{P*} \geq \pi^{G*}$, i.e., M chooses the $\{P, w^P, F^P\}$ contract. Otherwise, if $\eta \in (\eta^*, \bar{\eta}]$, then $\Delta < 0 \Rightarrow \pi^{P*} < \pi^{G*}$ and the $\{G, w^G, F^G\}$ contract arises.

If $\kappa_L \in (\kappa_2, \kappa_1]$, then (a) $\min\{\eta_2, \bar{\eta}\} = \eta_2$, (b) $\Delta\Big|_{\eta=\eta_1} \geq 0$ as shown above, and (c) $\Delta\Big|_{\eta=\eta_2} = -\frac{\gamma[\alpha^2-2\alpha(\beta+\gamma)\kappa_L+\gamma(\beta+\gamma)\kappa_L^2]}{4\beta(\beta+\gamma)} > 0$[56]. Therefore, $\Delta \geq 0 \Rightarrow \pi^{P*} \geq \pi^{G*}\ \forall\,\eta \in [\eta_1, \eta_2]$, i.e., M always chooses the $\{P, w^P, F^P\}$ contract in equilibrium. The statement of Proposition 4.1 then follows. ■

[55] since $2\alpha - (4\beta+\gamma)\kappa_L > 2\alpha - (4\beta+\gamma)\kappa_1 = \frac{\alpha\,\gamma}{2\beta+\gamma} > 0$.

[56] It can be seen that $\alpha^2 - 2\alpha(\beta+\gamma)\kappa_L + \gamma(\beta+\gamma)\kappa_L^2 < 0\ \forall\,\kappa_L \in (\kappa_2, \kappa_1]$ since (i) it is a convex, quadratic function in $\kappa_L \in (\kappa_2, \kappa_1]$, (ii) $\alpha^2 - 2\alpha(\beta+\gamma)\kappa_L + \gamma(\beta+\gamma)\kappa_L^2\Big|_{\kappa_L=\kappa_2} = 0$, and (iii) $\alpha^2 - 2\alpha(\beta+\gamma)\kappa_L + \gamma(\beta+\gamma)\kappa_L^2\Big|_{\kappa_L=\kappa_1} = -\frac{\alpha^2\,\beta\,\gamma}{(2\beta+\gamma)^2} < 0$.

C.4 The menu of contracts under asymmetric information regarding η (Proof of Proposition 4.2)

The optimal menu of contracts, under asymmetric information and uncertainty in η, solves the constrained optimization problem, defined by (4.23)-(4.29), i.e.,

$$\begin{aligned}
\max_{\substack{w^G,\,F^G,p^P_{h|h},p^{GB}_{h|h}\\ w^P,\,F^P,p_{l|l}}} \pi = \;& \phi\,\pi^G_{h|h} \;+\; (1-\phi)\,\pi^P_{l|l} \\
=\;& \phi\,\pi^G(w^G, F^G, p^P_{h|h}, p^{GB}_{h|h}; \eta_h, \cdot) \;+\; (1-\phi)\,\pi^P(w^P, F^P, p_{l|l}; \eta_l, \cdot) \\
=\;& \phi\Big\{\big[\alpha - \beta p^P_{h|h} - \gamma(p^{GB}_{h|h} + \kappa_L)\big]w^G + F^G\Big\} \\
& + (1-\phi)\Big\{\big[\alpha - (\beta+\gamma)p_{l|l}\big]w^P + F^P\Big\},
\end{aligned} \tag{C.75}$$

subject to:

$$(FOC\text{-}G): \quad \{p^P_{h|h}, p^{GB}_{h|h}\} \;=\; \underset{p^P,\,p^{GB}}{\operatorname{argmax}}\; \Pi^G_{h|h} \;=\; \Pi^G(p^P, p^{GB}, w^G, F^G; \eta_h, \cdot), \tag{C.76}$$

$$(FOC\text{-}P): \quad p_{l|l} \;=\; \underset{p}{\operatorname{argmax}}\; \Pi^P_{l|l} \;=\; \Pi^P(p, w^P, F^P; \eta_l, \cdot), \tag{C.77}$$

$$(IR\text{-}h): \quad \Pi^{G\,*}_{h|h} \;=\; \max_{p^P,\,p^{GB}} \Pi^G_{h|h} \;\geq 0, \tag{C.78}$$

$$(IR\text{-}l): \quad \Pi^{P\,*}_{l|l} \;=\; \max_{p} \Pi^P_{l|l} \;\geq 0, \tag{C.79}$$

$$(IC\text{-}h): \quad \Pi^{G\,*}_{h|h} \;\geq\; \Pi^{P\,*}_{l|h} \;=\; \max_{p} \Pi^P_{l|h} = \Pi^P(p, w^P, F^P; \eta_h, \cdot), \text{ and} \tag{C.80}$$

$$(IC\text{-}l): \quad \Pi^{P\,*}_{l|l} \;\geq\; \Pi^{G\,*}_{h|l} \;=\; \max_{p^P,\,p^{GB}} \Pi^G_{h|l} = \Pi^G(p^P, p^{GB}, w^G, F^G; \eta_l, \cdot). \tag{C.81}$$

Consider (C.76), where $\Pi^G(p^P, p^{GB}, w^G, F^G; \eta_h, \cdot) = (\eta_h\,\alpha - \beta\,p^P)(p^P - w^G) + \big[(1-\eta_h)\alpha - \gamma(p^{GB} + \kappa_L)\big](p^{GB} - w^G) - F^G$. By optimization, we obtain:

$$p^P_{h|h} = \frac{\eta_h\,\alpha}{2\beta} + \frac{w^G}{2}, \tag{C.82}$$

$$p^{GB}_{h|h} = \frac{(1-\eta_h)\alpha}{2\gamma} + \frac{w^G}{2} - \frac{\kappa_L}{2}, \text{ and} \tag{C.83}$$

$$\Pi^{G\,*}_{h|h} = \frac{(\eta_h\,\alpha - \beta\, w^G)^2}{4\beta} + \frac{\big[(1-\eta_h)\alpha - \gamma(w^G + \kappa_L)\big]^2}{4\gamma} - F^G. \tag{C.84}$$

Analogously, by optimizing $\Pi^G(p^P, p^{GB}, , w^G, F^G; \eta_l, \cdot) = (\eta_l\,\alpha - \beta\, p^P)(p^P - w^G) + \big[(1 - \eta_l)\alpha - \gamma(p^{GB} + \kappa_L)\big](p^{GB} - w^G) - F^G$ with respect to p^P and p^{GB}, we get:

$$p^P_{h|l} = \frac{\eta_l\,\alpha}{2\beta} + \frac{w^G}{2}, \tag{C.85}$$

$$p^{GB}_{h|l} = \frac{(1-\eta_l)\alpha}{2\gamma} + \frac{w^G}{2} - \frac{\kappa_L}{2}, \text{ and} \tag{C.86}$$

$$\Pi^{G\,*}_{h|l} = \frac{(\eta_l\,\alpha - \beta\, w^G)^2}{4\beta} + \frac{\big[(1-\eta_l)\alpha - \gamma(w^G + \kappa_L)\big]^2}{4\gamma} - F^G. \tag{C.87}$$

Next, we solve for $p_{l|l} = \underset{p}{\operatorname{argmax}}\, \Pi^P_{l|l} = \Pi^P(p, w^P, F^P; \eta_l, \cdot)$ and $p_{l|h} = \underset{p}{\operatorname{argmax}}\, \Pi^P_{l|h} = \Pi^P(p, w^P, F^P; \eta_h, \cdot)$[57], and obtain:

$$p_{l|l} = p_{l|h} = \frac{\alpha}{2(\beta+\gamma)} + \frac{w^P}{2}, \text{ and} \tag{C.88}$$

$$\Pi^{P\,*}_{l|l} = \Pi^{P\,*}_{l|h} = \frac{\big[\alpha - (\beta+\gamma)w^P\big]^2}{4(\beta+\gamma)} - F^P. \tag{C.89}$$

Further, since $\Pi^{P\,*}_{l|l} = \Pi^{P\,*}_{l|h}$, by (C.79) and (C.80), we have $\Pi^{G\,*}_{h|h} \geq \Pi^{P\,*}_{l|h} = \Pi^{P\,*}_{l|l} \geq 0$; this implies that (C.78) is redundant. Given these results, the above optimization problem is reduced to:

$$\max_{\substack{w^G, F^G,\\ w^P, F^P}} \pi = \phi \left\{ \big[\alpha - \beta p^P_{h|h} - \gamma(p^{GB}_{h|h} + \kappa_L)\big] w^G + F^G \right\} + (1-\phi) \left\{ \big[\alpha - (\beta+\gamma)p_{l|l}\big] w^P + F^P \right\}, \tag{C.90}$$

[57] Notice that $\Pi^P_{l|l} = \Pi^P_{l|h} = \Pi^P(p, w^P, F^P; \cdot) = \big[\alpha - (\beta+\gamma)p\big](p - w^P) - F^P$.

subject to:

$$\begin{cases} \Pi_{l|l}^{P\,*} \geq 0, \\ \Pi_{h|h}^{G\,*} \geq \Pi_{l|h}^{P\,*}, \\ \Pi_{l|l}^{P\,*} \geq \Pi_{h|l}^{G\,*}, \end{cases} \tag{C.91}$$

whose Lagrangian function is given by:

$$\mathscr{L} = \pi + \lambda\,\Pi_{l|l}^{P\,*} + \mu\,(\Pi_{h|h}^{G\,*} - \Pi_{l|h}^{P\,*}) + \delta\,(\Pi_{l|l}^{P\,*} - \Pi_{h|l}^{G\,*}). \tag{C.92}$$

The KKT first-order conditions of this problem are as follows:

$$\lambda \geq 0,\ \mu \geq 0,\ \delta \geq 0, \tag{C.93}$$

$$\mathscr{L}_{w^G} = \frac{1}{2}\big[\alpha - \gamma\,\kappa_L - (\beta+\gamma)w^G\big](\delta-\mu) + \frac{1}{2}\big[\alpha - \gamma\,\kappa_L - (\beta+\gamma)(2w^G - c)\big]\phi = 0 \tag{C.94}$$

$$\mathscr{L}_{F^G} = \delta - \mu + \phi = 0, \tag{C.95}$$

$$\mathscr{L}_{w^P} = \frac{1}{2}\big[\alpha - (\beta+\gamma)w^P\big](\mu-\delta-\lambda) + \frac{1}{2}\big[\alpha - (\beta+\gamma)(2w^P - c)\big](1-\phi) = 0, \tag{C.96}$$

$$\mathscr{L}_{F^P} = 1 - \phi - \delta - \lambda + \mu = 0, \tag{C.97}$$

$$\mathscr{L}_{\lambda} = \Pi_{l|l}^{P\,*} \geq 0 \tag{C.98}$$

$$\lambda\,\Pi_{l|l}^{P\,*} = 0 \tag{C.99}$$

$$\mathscr{L}_{\mu} = \Pi_{h|h}^{G\,*} - \Pi_{l|h}^{P\,*} \geq 0 \tag{C.100}$$

$$\mu\,(\Pi_{h|h}^{G\,*} - \Pi_{l|h}^{P\,*}) = 0 \tag{C.101}$$

$$\mathscr{L}_{\delta} = \Pi_{l|l}^{P\,*} - \Pi_{h|l}^{G\,*} \geq 0 \tag{C.102}$$

$$\delta\,(\Pi_{l|l}^{P\,*} - \Pi_{h|l}^{G\,*}) = 0 \tag{C.103}$$

By investigating the first-order conditions, we show that (a) $\lambda > 0$, (b) $\mu > 0$, and (c) $\delta = 0$. First, by (C.95) and (C.97), we have $\lambda = 1 > 0 \Rightarrow \Pi_{l|l}^{P\,*} = 0$. Next, if $\mu = 0$, then

$(C.95) \Rightarrow \delta = -\phi < 0$, which contradicts (C.93); therefore,

$$\mu > 0 \overset{\text{by (C.101)}}{\Rightarrow} \Pi^{G\;*}_{h|h} - \Pi^{P\;*}_{l|h} = 0. \tag{C.104}$$

Finally, suppose that $\delta > 0$, then by (C.103), we have $\Pi^{P\,*}_{l|l} - \Pi^{G\,*}_{h|l} = 0$. Together with (C.104), this implies:

$$(\Pi^{G\;*}_{h|h} - \Pi^{P\;*}_{l|h}) + (\Pi^{P\,*}_{l|l} - \Pi^{G\,*}_{h|l}) = 0, \tag{C.105}$$

$$\Leftrightarrow \frac{\alpha(\eta_h - \eta_l)\big[\alpha(\beta+\gamma)(\eta_h+\eta_l) - 2\beta(\alpha - \gamma\,\kappa_L)\big]}{4\beta\,\gamma} = 0, \tag{C.106}$$

which is a contradiction since $\eta_h > \eta_l$ and $\big[\alpha(\beta+\gamma)(\eta_h+\eta_l) - 2\beta(\alpha-\gamma\,\kappa_L)\big] > 0$[58]. Therefore, $\delta = 0$.

Given $\lambda = 1$, $\mu > 0$, and $\delta = 0$, we solve the following system of the first-order conditions:

$$\begin{cases} \mathscr{L}_{w^G}\big|_{\delta=0} = 0, \\ \mathscr{L}_{F^G}\big|_{\delta=0} = 0, \\ \mathscr{L}_{w^P}\big|_{\delta=0,\,\lambda=1} = 0, \\ \Pi^{P\,*}_{l|l} = 0, \\ \Pi^{G\;*}_{h|h} - \Pi^{P\;*}_{l|h} = 0, \end{cases} \tag{C.107}$$

to obtain the optimal menu of contracts:

$$w^{G*} = c, \tag{C.108}$$

$$F^{G*} = \frac{(\eta_h\,\alpha - \beta\,c)^2}{4\beta} + \frac{\big[(1-\eta_h)\alpha - \gamma(c+\kappa_L)\big]^2}{4\gamma}, \tag{C.109}$$

[58]The prices upon the employment of the G contract in the low state of the market, $p^P_{h|l}$ and $p^{GB}_{h|l}$, must be such that $p^P_{h|l} > p^{GB}_{h|l} \Leftrightarrow p^P_{h|l} - p^{GB}_{h|l} = \frac{\beta\,\gamma\,\kappa_L + \alpha[-\beta+\eta_l(\beta+\gamma)]}{2\beta\,\gamma} > 0 \Rightarrow \alpha(\beta+\gamma)\eta_l > \beta(\alpha - \gamma\,\kappa_L) \Rightarrow \alpha(\beta+\gamma)(\eta_h+\eta_l) > 2\alpha(\beta+\gamma)\eta_l > 2\beta(\alpha-\gamma\,\kappa_L) \Rightarrow \big[\alpha(\beta+\gamma)(\eta_h+\eta_l) - 2\beta(\alpha-\gamma\,\kappa_L)\big] > 0$.

$$w^{P*} = c, \tag{C.110}$$

$$F^{P*} = \frac{\big[\alpha - (\beta+\gamma)c\big]^2}{4(\beta+\gamma)}. \tag{C.111}$$

By substituting for $c = 0$, the statement of Proposition 4.2 then follows.■

C.5 The optimal full-information contracts in the β-case (Proof of Proposition 4.3)

Here, we investigate the manufacturer's choice of full-information contracts (i.e., $\{G, w^G, F^G\}$ vs. $\{P, w^P, F^P\}$) in equilibrium when the market is uncertain in β. Recall (from Lemma 4.1) the conditions that supports the $\{G, w^G, F^G\}$ contract[59]:

$$(1-\eta)\alpha - \gamma\,\kappa_L \;\geq\; 0 \;\Leftrightarrow\; \kappa_L \;\leq\; \frac{(1-\eta)\alpha}{\gamma} \stackrel{\text{def}}{=} \kappa_3, \text{ and} \tag{C.112}$$

$$\kappa_L \;\leq \frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma} \;\Leftrightarrow\; \beta \;\leq\; \frac{\eta\,\alpha\,\gamma}{(1-\eta)\alpha + \gamma\,\kappa_L} \stackrel{\text{def}}{=} \beta_1. \tag{C.113}$$

Meanwhile, the $\{P, w^P, F^P\}$ contract could be employed (see Lemma 4.2) when:

$$(1-\eta^2)\beta \;\geq\; \eta^2\,\gamma \;\Leftrightarrow\; \beta \;\geq\; \frac{\eta^2\,\gamma}{1-\eta^2} \stackrel{\text{def}}{=} \underline{\beta}, \text{ and} \tag{C.114}$$

$$\eta(2-\eta)\gamma \;\geq\; (1-\eta)^2\beta \;\Leftrightarrow\; \beta \;\leq\; \frac{\eta(2-\eta)\gamma}{(1-\eta)^2} \stackrel{\text{def}}{=} \bar{\beta}. \tag{C.115}$$

First, comparing β_1 to γ, we have: $\beta_1 - \gamma = -\frac{\gamma[(1-2\eta)\alpha + \gamma\,\kappa_L]}{(1-\eta)\alpha+\gamma\,\kappa_L}$. Therefore, if $\eta \leq \frac{1}{2}$, then $\beta_1 \;\leq\; \gamma \;\forall\,\kappa_L \;\in\; [0, \kappa_3]$. Otherwise, if $\eta > \frac{1}{2}$, then $\beta_1 \;\leq\; \gamma \;\Leftrightarrow\; \kappa_L \;\geq\; \frac{(2\eta-1)\alpha}{\gamma} \stackrel{\text{def}}{=} \kappa_4$. Notice that $\kappa_4 \;\leq\; \kappa_3 \;\Leftrightarrow\; \eta \;\leq\; \frac{2}{3}$.

Next, it can be shown that (a) $\underline{\beta} \leq \beta_1 < \bar{\beta}$ whenever $\kappa_L \leq \kappa_3$, (b) $\bar{\beta} \;\geq\; \gamma \;\Leftrightarrow\; \eta \;\geq\; \frac{2-\sqrt{2}}{2}$, and (c) $\underline{\beta} \;\geq\; \gamma \;\Leftrightarrow\; \eta \;\geq\; \frac{1}{\sqrt{2}}$.

[59]Here, we assume κ_H being significantly large so that $\kappa_H \;\geq\; \frac{\kappa_L}{2} + \frac{1}{2}[\frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma}]$ is satisfied.

Given $\beta < \gamma$ and the above relationships, we can determine the feasibility of the two contracts, $\{G, w^G, F^G\}$ and $\{P, w^P, F^P\}$, with respect to β. This result is summarized in Figure C.1.

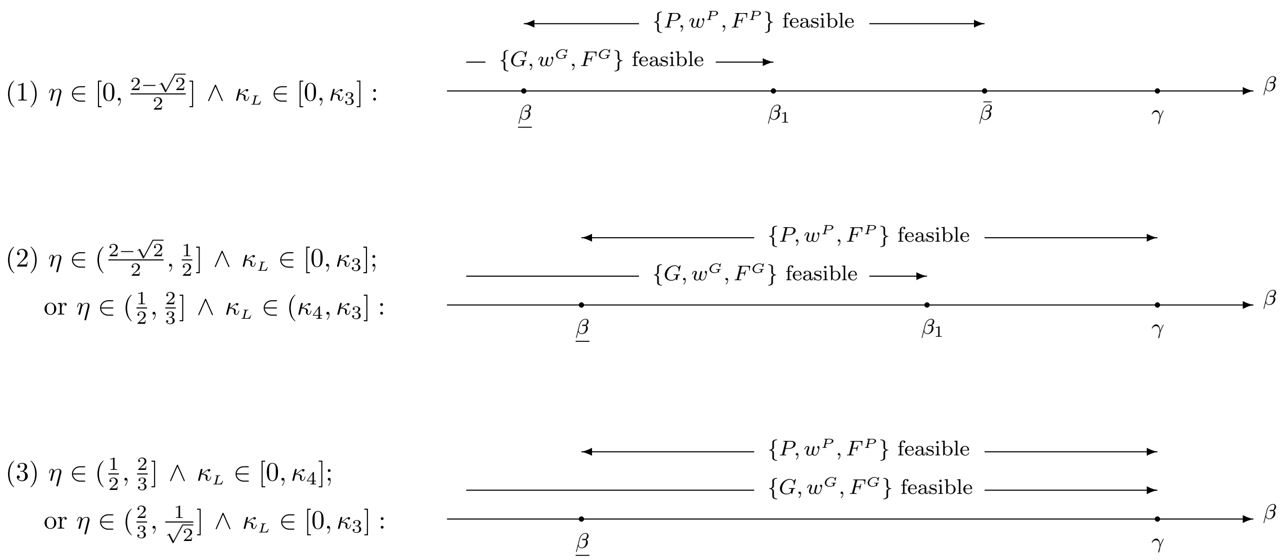


Figure C.1: **Feasibility of the full-information contracts when the market is uncertain in β**

Analogous to the setting of market uncertainty in η, we compare the profitability of the two contracts when both are feasible. Recall $\Delta = \pi^{P*} - \pi^{G*}$, which can be rewritten as follows:

$$\Delta = \frac{-[(1-\eta)\alpha - \gamma\,\kappa_L]^2\,\beta^2 - \gamma[\gamma^2\,\kappa_L^2 - 2(1-\eta)\alpha\,\gamma\,\kappa_L - 2(1-\eta)\eta\,\alpha^2]\beta - \eta^2\,\alpha^2\,\gamma^2}{4\beta\,\gamma\,(\beta+\gamma)}. \quad \text{(C.116)}$$

Notice that Δ has the same sign as the numerator of the RHS in (C.116), which is:

$$N_1 \stackrel{\text{def}}{=} -[(1-\eta)\alpha - \gamma\,\kappa_L]^2\,\beta^2 - \gamma[\gamma^2\,\kappa_L^2 - 2(1-\eta)\alpha\,\gamma\,\kappa_L - 2(1-\eta)\eta\,\alpha^2]\beta - \eta^2\,\alpha^2\,\gamma^2. \quad \text{(C.117)}$$

Consider the following cases: (a) $\eta \in [0, \frac{1}{2}]$ and $\kappa_L \in [0, \kappa_3]$, and (b) $\eta \in (\frac{1}{2}, \frac{2}{3}]$ and $\kappa_L \in [\kappa_4, \kappa_3]$. In these cases, both contracts are feasible when $\beta \in [\underline{\beta},\, \beta_1]$. Notice that $\frac{\partial N_1}{\partial \beta} = -2[(1-\eta)\alpha - \gamma\,\kappa_L]^2\,\beta - \gamma[\gamma^2\,\kappa_L^2 - 2(1-\eta)\alpha\,\gamma\,\kappa_L - 2(1-\eta)\eta\,\alpha^2]$ is decreasing in $\beta \in [\underline{\beta},\, \beta_1]$; therefore,

$$\frac{\partial N_1}{\partial \beta} \geq \frac{\partial N_1}{\partial \beta}\Big|_{\beta=\beta_1} = \frac{\gamma^2\,\kappa_L[-\gamma^2\,\kappa_L^2 + (1-3\eta)\alpha\,\gamma\,\kappa_L + 2(1-\eta)(1+2\eta)\alpha^2]}{(1-\eta)\alpha + \gamma\,\kappa_L} \geq 0 \quad \text{(C.118)}$$

(since the second term in the numerator of (C.118) is positive; it is a concave, quadratic function in $\kappa_L \in [0, \kappa_3]$, whose values, evaluated at $\kappa_L = 0$ and $\kappa_L = \kappa_3$, are $2(1-\eta)(1+2\eta)\alpha^2 > 0$ and $2(1-\eta^2)\alpha^2 > 0$, respectively). This implies that N_1 is increasing in $\beta \in [\underline{\beta}, \beta_1]$.

Since (a) N_1 is increasing in $\beta \in [\underline{\beta}, \beta_1]$, (b) $N_1\big|_{\beta=\underline{\beta}} = -\frac{\gamma^2[(1-\eta)\alpha-\gamma\,\kappa_L]^2\,\eta^2}{(1-\eta^2)^2} < 0$, and (c) $N_1\big|_{\beta=\beta_1} = -\frac{\eta\,\alpha\,\gamma^3\,\kappa_L[\gamma^2\,\kappa_L^2-(1-3\eta)\alpha\,\gamma\,\kappa_L-2(1-\eta)\alpha^2]}{[(1-\eta)\alpha+\gamma\,\kappa_L]^2} > 0$[60], there exists $\beta^* \in [\underline{\beta}, \beta_1]$, such that if $\beta < \beta^*$, then $N_1 < 0$, i.e., the $\{G, w^G, F^G\}$ contract is chosen. Otherwise, if $\beta \geq \beta^*$, then $N_1 \geq 0$, i.e., the $\{P, w^P, F^P\}$ contract arises in equilibrium. The specification of β^* is given by:

$$\beta^* = \frac{2\eta(1-\eta)\alpha^2 + 2(1-\eta)\alpha\,\gamma\,\kappa_L - \gamma^2\,\kappa_L^2}{2\big[(1-\eta)\alpha - \gamma\,\kappa_L\big]^2} - \frac{\sqrt{\gamma\,\kappa_L(2\alpha - \gamma\,\kappa_L)\big[4\eta(1-\eta)\alpha^2 + 2(1-2\eta)\alpha\,\gamma\,\kappa_L - \gamma^2\,\kappa_L^2\big]}}{2\big[(1-\eta)\alpha - \gamma\,\kappa_L\big]^2} \quad \text{(C.119)}$$

Consider the other cases, i.e., (a) $\eta \in (\frac{1}{2}, \frac{2}{3}]$ and $\kappa_L \in [0, \kappa_4]$, and (b) $\eta \in (\frac{2}{3}, \frac{1}{\sqrt{2}}]$ and $\kappa_L \in [0, \kappa_3]$. (Note that $\kappa_4 < \kappa_3$ in the former case and vice versa in the latter.) In these two cases, the two contracts are feasible simultaneously when $\beta \in [\underline{\beta},\, \gamma]$. Analogous to the previous case, we also have N_1 increasing in $\beta \in [\underline{\beta},\, \gamma]$ since $\frac{\partial N_1}{\partial \beta} \geq \frac{\partial N_1}{\partial \beta}\Big|_{\beta=\gamma} =$

[60]since the second term in the numerator of $N_1\big|_{\beta=\beta_1}$ is negative; it is a convex, quadratic function in $\kappa_L \in [0, \kappa_3]$, whose values, evaluated at $\kappa_L = 0$ and $\kappa_L = \kappa_3$, are $-2(1-\eta)\alpha^2 < 0$ and $-2(1-\eta)^2\alpha^2 < 0$, respectively.

$\gamma[-3\gamma^2\,\kappa_L^2 + 6(1-\eta)\alpha\,\gamma\,\kappa_L + 2(1-\eta)(2\eta-1)\alpha^2] > 0$[61]. Here, we have:

$$
\begin{aligned}
N_1\Big|_{\beta=\gamma} &= \gamma^2[-2\gamma^2\,\kappa_L^2 + 4(1-\eta)\alpha\,\gamma\,\kappa_L - (1-2\eta)^2\,\alpha^2] \geq 0 \\
\Leftrightarrow \kappa_L &\geq \frac{\alpha[2(1-\eta) - \sqrt{2(1-2\eta^2)}]}{2\gamma} \stackrel{\text{def}}{=} \kappa_5.
\end{aligned}
\tag{C.120}
$$

Notice that $\kappa_5 \in [0, \min\{\kappa_3,\, \kappa_4\}]$.

Therefore, if $\kappa_L < \kappa_5$, then (a) $N_1\big|_{\beta=\gamma} < 0$, and (b) $N_1\big|_{\beta=\underline{\beta}} < 0$ as shown above; this implies $N_1 < 0\ \forall \beta \in [\underline{\beta}, \gamma]$, and as a result, the $\{G, w^G, F^G\}$ contract is chosen in equilibrium. Otherwise, if $\kappa_L \geq \kappa_5$, then (a) $N_1\big|_{\beta=\gamma} \geq 0$, and (b) $N_1\big|_{\beta=\underline{\beta}} < 0$; therefore, the previous results are applicable, i.e., if $\beta < \beta^*$, then the $\{G, w^G, F^G\}$ contract is chosen; in contrast, if $\beta \geq \beta^*$, then the $\{P, w^P, F^P\}$ contract arises in equilibrium. The overall result is graphically summarized in Figure 4.3. The statement of Proposition 4.3 then follows. ■

C.6 The menu of contracts under asymmetric information regarding β (Proof of Proposition 4.4)

C.6.1 The menu of P contracts under asymmetric information regarding β

Consider the menu of P contracts (i.e., $\{P, w_h^P, F_h^P\}$ and $\{P, w_l^P, F_l^P\}$). This menu solves the problem, defined by (4.31)-(4.37)[62], i.e.,

$$
\begin{aligned}
\max_{\substack{w_h^P, F_h^P, \mathring{p}_{h|h}, \\ w_l^P, F_l^P, \mathring{p}_{l|l}}} \mathring{\pi}^P &= \phi\,\pi^P(w_h^P, F_h^P, \mathring{p}_{h|h}; \beta_h, \cdot) + (1-\phi)\,\pi^P(w_l^P, F_l^P, \mathring{p}_{l|l}; \beta_l, \cdot) \\
&= \phi\Big\{\big[\alpha - (\beta_h + \gamma)\mathring{p}_{h|h}\big](w_h^P - c) + F_h^P\Big\} \\
&\quad + (1-\phi)\Big\{\big[\alpha - (\beta_l + \gamma)\mathring{p}_{l|l}\big](w_l^P - c) + F_l^P\Big\},
\end{aligned}
\tag{C.121}
$$

[61] since the second term is positive for all $\kappa_L \in [0, \kappa_3]$; it is a concave, quadratic function in $\kappa_L \in [0, \kappa_3]$, whose values evaluated at $\kappa_L = 0$ and $\kappa_L = \kappa_3$ are $2(1-\eta)(2\eta-1) > 0$ and $(1-\eta^2)\alpha^2 > 0$, respectively.

[62] Here, too, we investigate the general case of $c \geq 0$.

subject to:

$$\mathring{p}_{h|h} \;=\; \underset{p}{\text{argmax}}\; \mathring{\Pi}^{P}_{h|h} \;=\; \Pi^{P}(p, w^{P}_{h}, F^{P}_{h}; \beta_{h}, \cdot) = \big[\alpha - (\beta_{h} + \gamma)p\big](p - w^{P}_{h}) - F^{P}_{h}, \tag{C.122}$$

$$\mathring{p}_{l|l} \;=\; \underset{p}{\text{argmax}}\; \mathring{\Pi}^{P}_{l|l} \;=\; \Pi^{P}(p, w^{P}_{l}, F^{P}_{l}; \beta_{l}, \cdot) = \big[\alpha - (\beta_{l} + \gamma)p\big](p - w^{P}_{l}) - F^{P}_{l}, \tag{C.123}$$

$$\mathring{\Pi}^{P*}_{h|h} \;=\; \max_{p} \mathring{\Pi}^{P}_{h|h} \;\geq 0, \tag{C.124}$$

$$\mathring{\Pi}^{P*}_{l|l} \;=\; \max_{p} \mathring{\Pi}^{P}_{l|l} \;\geq 0, \tag{C.125}$$

$$\mathring{\Pi}^{P*}_{h|h} \;\geq\; \mathring{\Pi}^{P*}_{l|h} \;=\; \max_{p} \mathring{\Pi}^{P}_{l|h} = \Pi^{P}(p, w^{P}_{l}, F^{P}_{l}; \beta_{h}, \cdot) = \big[\alpha - (\beta_{h} + \gamma)p\big](p - w^{P}_{l}) - F^{P}_{l}, \text{ and} \tag{C.126}$$

$$\mathring{\Pi}^{P*}_{l|l} \;\geq\; \mathring{\Pi}^{P*}_{h|l} \;=\; \max_{p} \mathring{\Pi}^{P}_{h|l} = \Pi^{P}(p, w^{P}_{h}, F^{P}_{h}; \beta_{l}, \cdot) = \big[\alpha - (\beta_{l} + \gamma)p\big](p - w^{P}_{h}) - F^{P}_{h}. \tag{C.127}$$

To simplify this problem, we solve for $\mathring{p}_{h|h}$ and $\mathring{p}_{l|l}$ and derive the specifications of $\mathring{\Pi}^{P*}_{h|h}$, $\mathring{\Pi}^{P*}_{l|l}$, $\mathring{\Pi}^{P*}_{l|h}$ and $\mathring{\Pi}^{P*}_{h|l}$. By optimizing the respective profit functions, we obtain:

$$\mathring{p}_{h|h} \;=\; \frac{\alpha}{2(\beta_{h} + \gamma)} + \frac{w^{P}_{h}}{2}, \tag{C.128}$$

$$\mathring{\Pi}^{P*}_{h|h} \;=\; \frac{[\alpha - (\beta_{h} + \gamma)w^{P}_{h}]^{2}}{4(\beta_{h} + \gamma)} - F^{P}_{h}, \tag{C.129}$$

$$\mathring{p}_{l|l} \;=\; \frac{\alpha}{2(\beta_{l} + \gamma)} + \frac{w^{P}_{l}}{2}, \tag{C.130}$$

$$\mathring{\Pi}^{P*}_{l|l} \;=\; \frac{[\alpha - (\beta_{l} + \gamma)w^{P}_{l}]^{2}}{4(\beta_{l} + \gamma)} - F^{P}_{l}, \tag{C.131}$$

$$\mathring{\Pi}^{P*}_{l|h} \;=\; \frac{[\alpha - (\beta_{h} + \gamma)w^{P}_{l}]^{2}}{4(\beta_{h} + \gamma)} - F^{P}_{l}, \text{ and} \tag{C.132}$$

$$\mathring{\Pi}^{P*}_{h|l} \;=\; \frac{[\alpha - (\beta_{l} + \gamma)w^{P}_{h}]^{2}}{4(\beta_{l} + \gamma)} - F^{P}_{h}. \tag{C.133}$$

Further, since $\mathring{\Pi}^{P*}_{h|l} - \mathring{\Pi}^{P*}_{h|h} \;=\; \frac{(\beta_{h}-\beta_{l})\left[\alpha^{2}-(\beta_{h}+\gamma)(\beta_{l}+\gamma){w^{P}_{h}}^{2}\right]}{4(\beta_{h}+\gamma)(\beta_{l}+\gamma)} \;>\; 0$[63], i.e., $\mathring{\Pi}^{P*}_{h|l} > \mathring{\Pi}^{P*}_{h|h}$, by (C.124) and (C.127), we have: $\mathring{\Pi}^{P*}_{l|l} \geq \mathring{\Pi}^{P*}_{h|l} > \mathring{\Pi}^{P*}_{h|h} \geq 0$, i.e., (C.125) is redundant.

[63]Upon the employment of the $\{P, w^{P}_{h}, F^{P}_{h}\}$, the retail price must be such that $\mathring{p}_{h|h} \geq w^{P}_{h} \;\Rightarrow\; w^{P}_{h} \leq \frac{\alpha}{\beta_{h}+\gamma} < \frac{\alpha}{\beta_{l}+\gamma} \;\Rightarrow\; \alpha^{2} - (\beta_{h} + \gamma)(\beta_{l} + \gamma){w^{P}_{h}}^{2} > 0$.

Given (C.128)-(C.133), the optimization problem in consideration simplifies to:

$$\max_{\substack{w_h^P, F_h^P, \\ w_l^P, F_l^P}} \mathring{\pi}^P = \phi\Big\{\big[\alpha - (\beta_h + \gamma)\mathring{p}_{h|h}\big](w_h^P - c) + F_h^P\Big\} + (1-\phi)\Big\{\big[\alpha - (\beta_l + \gamma)\mathring{p}_{l|l}\big](w_l^P - c) + F_l^P\Big\}$$

$$= \phi\Big\{\frac{1}{2}\big[\alpha - (\beta_h + \gamma)w_h^P\big](w_h^P - c) + F_h^P\Big\}$$
$$+ (1-\phi)\Big\{\frac{1}{2}\big[\alpha - (\beta_l + \gamma)w_l^P\big](w_l^P - c) + F_l^P\Big\} \tag{C.134}$$

$$\text{s.t.} \begin{cases} \mathring{\Pi}_{h|h}^{P*} \geq 0, \\ \mathring{\Pi}_{h|h}^{P*} \geq \mathring{\Pi}_{l|h}^{P*}, \text{ and} \\ \mathring{\Pi}_{l|l}^{P*} \geq \mathring{\Pi}_{h|l}^{P*}. \end{cases} \tag{C.135}$$

The Lagrangian function is then given by:

$$\mathscr{L} = \mathring{\pi}^P + \lambda\,\mathring{\Pi}_{h|h}^{P*} + \delta\,(\mathring{\Pi}_{h|h}^{P*} - \mathring{\Pi}_{l|h}^{P*}) + \mu\,(\mathring{\Pi}_{l|l}^{P*} - \mathring{\Pi}_{h|l}^{P*}). \tag{C.136}$$

The KKT first-order conditions of this problem are as follows:

$$\lambda \geq 0,\ \mu \geq 0,\ \delta \geq 0, \tag{C.137}$$

$$\mathscr{L}_{w_h^P} = \frac{1}{2}\Big\{\big[\alpha - (\beta_l + \gamma)w_h^P\big]\mu - \big[\alpha - (\beta_h + \gamma)w_h^P\big](\delta + \lambda) +$$
$$+ \big[\alpha - (\beta_h + \gamma)(2w_h^P - c)\big]\phi\Big\} = 0 \tag{C.138}$$

$$\mathscr{L}_{F_h^P} = \phi - \delta - \lambda + \mu = 0 \tag{C.139}$$

$$\mathscr{L}_{w_l^P} = \frac{1}{2}\Big\{-\big[\alpha - (\beta_l + \gamma)w_l^P\big]\mu + \big[\alpha - (\beta_h + \gamma)w_l^P\big]\delta +$$
$$+ \big[\alpha - (\beta_l + \gamma)(2w_l^P - c)\big](1-\phi)\Big\} = 0, \tag{C.140}$$

$$\mathscr{L}_{F_l^P} = 1 - \phi + \delta - \mu = 0, \tag{C.141}$$

$$\mathscr{L}_{\lambda} = \mathring{\Pi}_{h|h}^{P*} \geq 0, \tag{C.142}$$

$$\lambda\,\mathring{\Pi}_{h|h}^{P*} = 0, \tag{C.143}$$

$$\mathscr{L}_\delta = \mathring{\Pi}^{P*}_{h|h} - \mathring{\Pi}^{P*}_{l|h} \geq 0, \tag{C.144}$$

$$\delta\,(\mathring{\Pi}^{P*}_{h|h} - \mathring{\Pi}^{P*}_{l|h}) = 0, \tag{C.145}$$

$$\mathscr{L}_\mu = \mathring{\Pi}^{P*}_{l|l} - \mathring{\Pi}^{P*}_{h|l} \geq 0, \text{ and} \tag{C.146}$$

$$\mu\,(\mathring{\Pi}^{P*}_{l|l} - \mathring{\Pi}^{P*}_{h|l}) = 0 \tag{C.147}$$

Now, we show that (a) $\lambda > 0$, (b) $\mu > 0$, and (c) $\delta = 0$. First, from (C.139) and (C.141), we get: $\lambda = 1 > 0$. Next, if $\mu = 0$, then by (C.141), $\delta = -(1-\phi) < 0$, which contradicts (C.137); therefore,

$$\mu > 0 \overset{\text{by (C.147)}}{\Rightarrow} \mathring{\Pi}^{P*}_{l|l} - \mathring{\Pi}^{P*}_{h|l} = 0. \tag{C.148}$$

Finally, suppose that $\delta > 0$, then by (C.145), we have: $\mathring{\Pi}^{P*}_{h|h} - \mathring{\Pi}^{P*}_{l|h} = 0$. Together with (C.148), this implies:

$$(\mathring{\Pi}^{P*}_{h|h} - \mathring{\Pi}^{P*}_{l|h}) + (\mathring{\Pi}^{P*}_{l|l} - \mathring{\Pi}^{P*}_{h|l}) = 0 \;\Leftrightarrow\; \frac{(\beta_h - \beta_l)(w_h^{P2} - w_l^{P2})}{4} = 0 \;\Rightarrow\; w_h^P = w_l^P. \tag{C.149}$$

Let $w_h^P = w_l^P = w$; we solve the following system:

$$\begin{cases} \mathscr{L}_{w_h^P}\big|_{w_h^P = w} & = 0, \\ \mathscr{L}_{w_l^P}\big|_{w_l^P = w} & = 0, \\ \mathscr{L}_{F_l^P} & = 0, \end{cases} \tag{C.150}$$

for $\{\delta, \lambda, \mu\}$ and get $\delta = -\frac{(1-\phi)(\beta_l+\gamma)(w-c)}{w(\beta_h-\beta_l)} < 0$, which is a contradiction. Therefore, $\delta = 0$.

Given $\lambda = 1$, $\mu > 0$, and $\delta = 0$, we solve the following system of the first-order conditions:

$$
\begin{cases}
\mathscr{L}_{w_h^P}\big|_{\delta=0,\,\lambda=1} = 0, \\
\mathscr{L}_{w_l^P}\big|_{\delta=0} = 0, \\
\mathscr{L}_{F_l^P}\big|_{\delta=0} = 0, \\
\mathring{\Pi}^{P*}_{h|h} = 0, \\
\mathring{\Pi}^{P*}_{l|l} - \mathring{\Pi}^{P*}_{h|l} = 0,
\end{cases}
\tag{C.151}
$$

to obtain the optimal menu of P contracts as follows:

$$
w_h^{P*} = \frac{\phi(\beta_h+\gamma)c}{(2\phi-1)\beta_h + (1-\phi)\beta_l + \phi\,\gamma} \;=\; \frac{c}{1 - \frac{(1-\phi)(\beta_h-\beta_l)}{\phi(\beta_h+\gamma)}}, \tag{C.152}
$$

$$
F_h^{P*} = \frac{\left[\alpha - \frac{\phi(\beta_h+\gamma)^2 c}{(2\phi-1)\beta_h+(1-\phi)\beta_l+\phi\,\gamma}\right]^2}{4(\beta_h+\gamma)} \;=\; \frac{\left[\alpha - (\beta_h+\gamma)w_h^{P*}\right]^2}{4(\beta_h+\gamma)}, \tag{C.153}
$$

$$
w_l^{P*} = c, \tag{C.154}
$$

$$
\begin{aligned}
F_l^{P*} &= \frac{\left[\alpha - (\beta_l+\gamma)c\right]^2}{4(\beta_l+\gamma)} - \frac{1}{4}(\beta_h-\beta_l)\Big\{\frac{\alpha^2}{(\beta_h+\gamma)(\beta_l+\gamma)} - \frac{\phi^2(\beta_h+\gamma)^2c^2}{[(2\phi-1)\beta_h+(1-\phi)\beta_l+\phi\,\gamma]^2}\Big\} \\
&= \frac{\left[\alpha - (\beta_l+\gamma)c\right]^2}{4(\beta_l+\gamma)} - \frac{1}{4}(\beta_h-\beta_l)\Big[\frac{\alpha^2}{(\beta_h+\gamma)(\beta_l+\gamma)} - w_h^{P*2}\Big].
\end{aligned}
\tag{C.155}
$$

C.6.2 The menu of G contracts under asymmetric information regarding β

Consider the menu of G contracts, which solves the optimization problem, defined by (4.38).-(4.44), i.e.,

$$
\begin{aligned}
\max_{\substack{w_h^G, F_h^G, \mathring{p}^P_{h|h}, \mathring{p}^{GB}_{h|h} \\ w_l^G, F_l^G, \mathring{p}^P_{l|l}, \mathring{p}^{GB}_{l|l}}} \mathring{\pi}^G =\; & \phi\,\pi^G(w_h^G, F_h^G, \mathring{p}^P_{h|h}, \mathring{p}^{GB}_{h|h}; \beta_h, \cdot) \;+\; (1-\phi)\,\pi^G(w_l^G, F_l^G, \mathring{p}^P_{l|l}, \mathring{p}^{GB}_{l|l}; \beta_l, \cdot) \\
=\; & \phi\,\Big\{\big[\alpha - \beta_h\,\mathring{p}^P_{h|h} - \gamma(\mathring{p}^{GB}_{h|h} + \kappa_L)\big](w_h^G - c) + F_h^G\Big\} + \\
& + (1-\phi)\Big\{\big[\alpha - \beta_l \mathring{p}^P_{l|l} - \gamma(\mathring{p}^{GB}_{l|l} + \kappa_L)\big](w_l^G - c) + F_l^G\Big\},
\end{aligned}
\tag{C.156}
$$

subject to:

$$(FOC\text{-}h): \quad \{\mathring{p}^{P}_{h|h}, \mathring{p}^{GB}_{h|h}\} \; = \; \underset{p^P,\, p^{GB}}{\operatorname{argmax}} \; \mathring{\Pi}^{G}_{h|h} \; = \; \Pi^{G}(p^P, p^{GB}, w^G_h, F^G_h; \beta_h, \cdot), \tag{C.157}$$

$$(FOC\text{-}l): \quad \{\mathring{p}^{P}_{l|l}, \mathring{p}^{GB}_{l|l}\} \; = \; \underset{p^P,\, p^{GB}}{\operatorname{argmax}} \; \mathring{\Pi}^{G}_{l|l} \; = \; \Pi^{G}(p^P, p^{GB}, w^G_l, F^G_l; \beta_l, \cdot), \tag{C.158}$$

$$(IR\text{-}h): \quad \mathring{\Pi}^{G*}_{h|h} \; = \; \max_{p^P,\, p^{GB}} \mathring{\Pi}^{G}_{h|h} \; \geq 0, \tag{C.159}$$

$$(IR\text{-}l): \quad \mathring{\Pi}^{G*}_{l|l} \; = \; \max_{p^P,\, p^{GB}} \mathring{\Pi}^{G}_{l|l} \; \geq 0, \tag{C.160}$$

$$(IC\text{-}h): \quad \mathring{\Pi}^{G*}_{h|h} \; \geq \; \mathring{\Pi}^{G*}_{l|h} \; = \; \max_{p^P,\, p^{GB}} \mathring{\Pi}^{G}_{l|h} = \Pi^{G}(p^P, p^{GB}, w^G_l, F^G_l; \beta_h, \cdot), \text{ and} \tag{C.161}$$

$$(IC\text{-}l): \quad \mathring{\Pi}^{G*}_{l|l} \; \geq \; \mathring{\Pi}^{G*}_{h|l} \; = \; \max_{p^P,\, p^{GB}} \mathring{\Pi}^{G}_{h|l} = \Pi^{G}(p^P, p^{GB}, w^P_h, F^P_h; \beta_l, \cdot). \tag{C.162}$$

Analogous to the analysis of the menu of P contracts, we first derive the specification of $\mathring{p}^{P}_{h|h}, \mathring{p}^{GB}_{h|h}, \mathring{\Pi}^{G*}_{h|h}, \mathring{p}^{P}_{l|l}, \mathring{p}^{GB}_{l|l}, \mathring{\Pi}^{G*}_{l|l}, \mathring{\Pi}^{G*}_{l|h}$, and $\mathring{\Pi}^{G*}_{h|l}$, by optimizing the respective profit functions. The results are as follows:

$$\mathring{p}^{P}_{j|i} \; = \; \frac{\eta\,\alpha}{\beta_i} \; + \; \frac{w^G_j}{2}, \tag{C.163}$$

$$\mathring{p}^{GB}_{j|i} \; = \; \frac{(1-\eta)\alpha}{2\gamma} \; + \; \frac{w^G_j}{2} \; - \; \frac{\kappa_L}{2}, \tag{C.164}$$

$$\mathring{\Pi}^{G*}_{j|i} \; = \; \frac{(\eta\,\alpha - \beta_i\, w^G_j)^2}{4\beta_i} + \frac{\big[(1-\eta)\alpha - \gamma(w^G_j + \kappa_L)\big]^2}{4\gamma} - F^G_j, \text{ where } j, i \in \{h, l\}. \tag{C.165}$$

Here, too, the individual rationality condition given by (C.160) is redundant, given (C.159) and (C.162)[64].

[64]We have: $\mathring{\Pi}^{G*}_{h|l} - \mathring{\Pi}^{G*}_{h|h} \; = \; \frac{(\beta_h - \beta_l)(\eta^2\,\alpha^2 - \beta_h\,\beta_l\,w^{G\,2}_h)}{4\beta_h\,\beta_l} \; > \; 0 \; \Rightarrow \; \mathring{\Pi}^{G*}_{h|l} > \mathring{\Pi}^{G*}_{h|h}$. Then, by (C.159) and (C.162), it can be seen that $\mathring{\Pi}^{G*}_{l|l} \geq \mathring{\Pi}^{G*}_{h|l} > \mathring{\Pi}^{G*}_{h|h} \geq 0$.

Given these results, the above optimization problem simplifies to:

$$\max_{\substack{w_h^G,\,F_h^G,\\ w_l^G,\,F_l^G}} \mathring{\pi}^G = \phi\Big\{\big[\alpha - \beta_h\,\mathring{p}^P_{h|h} - \gamma(\mathring{p}^{GB}_{h|h} + \kappa_L)\big](w_h^G - c) + F_h^G\Big\} +$$

$$+ (1-\phi)\Big\{\big[\alpha - \beta_l\mathring{p}^P_{l|l} - \gamma(\mathring{p}^{GB}_{l|l} + \kappa_L)\big](w_l^G - c) + F_l^G\Big\}$$

$$= \phi\Big\{\frac{1}{2}\big[\alpha - \gamma\,\kappa_L - (\beta_h + \gamma)w_h^G\big](w_h^G - c) + F_h^G\Big\} +$$

$$+ (1-\phi)\Big\{\frac{1}{2}\big[\alpha - \gamma\,\kappa_L - (\beta_l + \gamma)w_l^G\big](w_l^G - c) + F_l^G\Big\} \tag{C.166}$$

$$\text{s.t.} \quad \begin{cases} \mathring{\Pi}^{G*}_{h|h} \geq 0, \\ \mathring{\Pi}^{G*}_{h|h} \geq \mathring{\Pi}^{G*}_{l|h}, \text{ and} \\ \mathring{\Pi}^{G*}_{l|l} \geq \mathring{\Pi}^{G*}_{h|l}, \end{cases} \tag{C.167}$$

whose Lagrangian function is given by:

$$\mathscr{L} = \mathring{\pi}^G + \lambda\,\mathring{\Pi}^{G*}_{h|h} + \delta\,(\mathring{\Pi}^{G*}_{h|h} - \mathring{\Pi}^{G*}_{l|h}) + \mu\,(\mathring{\Pi}^{G*}_{l|l} - \mathring{\Pi}^{G*}_{h|l}). \tag{C.168}$$

The KKT first-order conditions of this problem are as follows:

$$\lambda \geq 0,\ \mu \geq 0,\ \delta \geq 0, \tag{C.169}$$

$$\mathscr{L}_{w_h^G} = \frac{1}{2}\Big\{\big[\alpha - \gamma\,\kappa_L - (\beta_l + \gamma)w_h^G\big]\mu - \big[\alpha - \gamma\,\kappa_L - (\beta_h + \gamma)w_h^G\big](\delta + \lambda) +$$

$$+ \big[\alpha - \gamma\,\kappa_L - (\beta_h + \gamma)(2w_h^G - c)\big]\phi\Big\} = 0 \tag{C.170}$$

$$\mathscr{L}_{F_h^G} = \phi - \delta - \lambda + \mu = 0 \tag{C.171}$$

$$\mathscr{L}_{w_l^G} = \frac{1}{2}\Big\{ - \big[\alpha - \gamma\,\kappa_L - (\beta_l + \gamma)w_l^G\big]\mu + \big[\alpha - \gamma\,\kappa_L - (\beta_h + \gamma)w_l^G\big]\delta +$$

$$+ \big[\alpha - \gamma\,\kappa_L - (\beta_l + \gamma)(2w_l^G - c)\big](1-\phi)\Big\} = 0, \tag{C.172}$$

$$\mathscr{L}_{F_l^G} = 1 - \phi + \delta - \mu = 0, \tag{C.173}$$

$$\mathscr{L}_\lambda = \mathring{\Pi}^{G*}_{h|h} \geq 0, \tag{C.174}$$

$$\lambda \,\mathring{\Pi}^{G*}_{h|h} = 0, \tag{C.175}$$

$$\mathscr{L}_{\delta} = \mathring{\Pi}^{G*}_{h|h} - \mathring{\Pi}^{G*}_{l|h} \ \geq 0, \tag{C.176}$$

$$\delta\,(\mathring{\Pi}^{G*}_{h|h} - \mathring{\Pi}^{G*}_{l|h}) = 0, \tag{C.177}$$

$$\mathscr{L}_{\mu} = \mathring{\Pi}^{G*}_{l|l} - \mathring{\Pi}^{G*}_{h|l} \ \geq 0, \ \text{and} \tag{C.178}$$

$$\mu\,(\mathring{\Pi}^{G*}_{l|l} - \mathring{\Pi}^{G*}_{h|l}) = 0. \tag{C.179}$$

By (C.171) and (C.173), we have $\lambda = 1 > 0$. Further, if $\mu = 0$, then by (C.173), $\delta = -(1-\phi) < 0$, which contradicts (C.169). Thus,

$$\mu \ > \ 0 \ \overset{\text{by (C.179)}}{\Longrightarrow} \ \mathring{\Pi}^{G*}_{l|l} - \mathring{\Pi}^{G*}_{h|l} = 0. \tag{C.180}$$

Suppose $\delta > 0$; by (C.177), we must have $\mathring{\Pi}^{G*}_{h|h} - \mathring{\Pi}^{G*}_{l|h} = 0$. Together with (C.180), this implies:

$$(\mathring{\Pi}^{G*}_{h|h} - \mathring{\Pi}^{G*}_{l|h}) + (\mathring{\Pi}^{G*}_{l|l} - \mathring{\Pi}^{G*}_{h|l}) = 0 \ \Leftrightarrow \ \frac{(\beta_h - \beta_l)(w_h^{G2} - w_l^{G2})}{4} = 0 \ \Rightarrow \ w_h^G = w_l^G. \tag{C.181}$$

If $w_h^G = w_l^G = w$, then by solving the following system:

$$\begin{cases} \mathscr{L}_{w_h^G}\big|_{w_h^G = w} & = 0, \\ \mathscr{L}_{w_l^G}\big|_{w_l^G = w} & = 0, \\ \mathscr{L}_{F_l^G} & = 0 \end{cases} \tag{C.182}$$

for $\{\delta, \lambda, \mu\}$, we get $\delta = -\frac{(1-\phi)(\beta_l+\gamma)(w-c)}{w(\beta_h-\beta_l)} < 0$, which is a contradiction. Therefore, $\delta = 0$.

Given $\lambda = 1$, $\mu > 0$, and $\delta = 0$, the optimal menu of G contracts is the solution to the following system of the first-order conditions:

$$
\begin{cases}
\mathscr{L}_{w_h^G}\big|_{\delta=0,\,\lambda=1} = 0, \\
\mathscr{L}_{w_l^G}\big|_{\delta=0} = 0, \\
\mathscr{L}_{F_l^G}\big|_{\delta=0} = 0, \\
\mathring{\Pi}_{h|h}^{G*} = 0, \\
\mathring{\Pi}_{l|l}^{G*} - \mathring{\Pi}_{h|l}^{G*} = 0.
\end{cases}
\tag{C.183}
$$

This menu is characterized by:

$$
w_h^{G*} = \frac{\phi(\beta_h + \gamma)c}{(2\phi - 1)\beta_h + (1-\phi)\beta_l + \phi\,\gamma} = \frac{c}{1 - \frac{(1-\phi)(\beta_h - \beta_l)}{\phi(\beta_h + \gamma)}}, \tag{C.184}
$$

$$
F_h^{G*} = \frac{\left[\eta\,\alpha - \beta_h\, w_h^{G*}\right]^2}{4\beta_h} + \frac{\left[(1-\eta)\alpha - \gamma(w_h^{G*} + \kappa_L)\right]^2}{4\gamma}, \tag{C.185}
$$

$$
w_l^{G*} = c, \tag{C.186}
$$

$$
F_l^{G*} = \frac{\left[\eta\,\alpha - \beta_l\, c\right]^2}{4\beta_l} + \frac{\left[(1-\eta)\alpha - \gamma(c + \kappa_L)\right]^2}{4\gamma} - \frac{1}{4}(\beta_h - \beta_l)\Big[\frac{\eta^2\,\alpha^2}{\beta_h\,\beta_l} - w_h^{G*2}\Big]. \tag{C.187}
$$

Notice that $w_h^{G*} = w_h^{P*}$.

C.6.3 The menu of P and G contracts under asymmetric information regarding β

Recall the optimization problem that defines the optimal menu of P and G contracts (i.e., $\{P, w^P, F^P\}$ and $\{G, w^G, F^G\}$) under asymmetric information regarding β:

$$
\begin{aligned}
\max_{\substack{w^P,\,F^P,\,p_{h|h},\\ w^G,\,F^G,\,p_{l|l}^P,\,p_{l|l}^{GB}}} \pi = \;& \phi\,\pi_{h|h}^P + (1-\phi)\,\pi_{l|l}^G = \phi\,\pi^P(w^P, F^P, p_{h|h}; \beta_h, \cdot) + (1-\phi)\,\pi^G(w^G, F^G, p_{l|l}^P, p_{l|l}^{GB}; \beta_l, \cdot) \\
= \;& \phi\Big\{\big[\alpha - (\beta_h + \gamma)p_{h|h}\big](w^P - c) + F^P\Big\} \\
& + (1-\phi)\Big\{\big[\alpha - \beta_l\, p_{l|l}^P + \gamma(p_{l|l}^{GB} + \kappa_L)\big](w^G - c) + F^G\Big\},
\end{aligned}
\tag{C.188}
$$

subject to:

$$\textit{(FOC-P)}: \quad p_{h|h} \;=\; \underset{p}{\text{argmax}}\; \Pi^{P}_{h|h} \;=\; \Pi^{P}(p, w^{P}, F^{P}; \beta_h, \cdot), \tag{C.189}$$

$$\textit{(FOC-G)}: \quad \{p^{P}_{l|l}, p^{GB}_{l|l}\} \;=\; \underset{p^{P},\, p^{GB}}{\text{argmax}}\; \Pi^{G}_{l|l} \;=\; \Pi^{G}(p^{P}, p^{GB}, w^{G}, F^{G}; \beta_l, \cdot), \tag{C.190}$$

$$\textit{(IR-h)}: \quad {\Pi^{P}_{h|h}}^{*} \;=\; \max_{p} \Pi^{P}_{h|h} \;\geq 0, \tag{C.191}$$

$$\textit{(IR-l)}: \quad \Pi^{G\,*}_{l|l} \;=\; \max_{p^{P},\, p^{GB}} \Pi^{G}_{l|l} \;\geq 0, \tag{C.192}$$

$$\textit{(IC-h)}: \quad {\Pi^{P}_{h|h}}^{*} \;\geq\; {\Pi^{G}_{l|h}}^{*} \;=\; \max_{p^{P},\, p^{GB}} \Pi^{G}_{l|h} = \Pi^{G}(p^{P}, p^{GB}, w^{G}, F^{G}; \beta_h, \cdot), \text{ and} \tag{C.193}$$

$$\textit{(IC-l)}: \quad \Pi^{G\,*}_{l|l} \;\geq\; {\Pi^{P}_{h|l}}^{*} \;=\; \max_{p} \Pi^{P}_{h|l} = \Pi^{P}(p, w^{P}, F^{P}; \beta_l, \cdot). \tag{C.194}$$

Here, too, we first derive the specifications of $p_{h|i}, {\Pi^{P}_{h|i}}^{*}, p^{P}_{l|i}, p^{GB}_{l|i}, \Pi^{G\,*}_{l|i}$ $(i \in \{h, l\})$, by optimizing the respective profit functions; the results are as follows:

$$p_{h|i} = \frac{\alpha}{2(\beta_i + \gamma)} + \frac{w^{P}}{2}, \tag{C.195}$$

$${\Pi^{P}_{h|i}}^{*} = \frac{\left[\alpha - (\beta_i + \gamma) w^{P}\right]^2}{4(\beta_i + \gamma)} - F^{P}, \tag{C.196}$$

$$p^{P}_{l|i} = \frac{\eta\, \alpha}{2\beta_i} + \frac{w^{G}}{2}, \tag{C.197}$$

$$p^{GB}_{l|i} = \frac{(1-\eta)\alpha}{2\gamma} + \frac{w^{G}}{2} - \frac{\kappa_L}{2}, \text{ and} \tag{C.198}$$

$$\Pi^{G\,*}_{l|i} = \frac{(\eta\, \alpha - \beta_i\, w^{G})^2}{4\beta_i} + \frac{\left[(1-\eta)\alpha - \gamma(w^{G} + \kappa_L)\right]^2}{4\gamma} - F^{G}. \tag{C.199}$$

Next, it can be seen that (C.192) is redundant, given (C.191) and (C.194)[65]. Therefore, the above optimization problem simplifies to:

[65]Specifically, since ${\Pi^{P}_{h|l}}^{*} - {\Pi^{P}_{h|h}}^{*} = \frac{(\beta_h - \beta_l)\left[\alpha^2 - (\beta_h + \gamma)(\beta_l + \gamma) {w^{P}}^2\right]}{4(\beta_h + \gamma)(\beta_l + \gamma)} > 0 \;\Rightarrow\; {\Pi^{P}_{h|l}}^{*} > {\Pi^{P}_{h|h}}^{*}$, then by (C.191) and (C.194), we have: $\Pi^{G\,*}_{l|l} \;\geq\; {\Pi^{P}_{h|l}}^{*} \;>\; {\Pi^{P}_{h|h}}^{*} \;\geq\; 0$.

$$
\begin{aligned}
\max_{\substack{w^P,\,F^P,\\ w^G,\,F^G}} \pi =\ & \phi\Big\{\big[\alpha-(\beta_h+\gamma)p_{h|h}\big](w^P-c)+F^P\Big\}\\
&+\ (1-\phi)\Big\{\big[\alpha-\beta_l\,p^P_{l|l}+\gamma(p^{GB}_{l|l}+\kappa_L)\big](w^G-c)+F^G\Big\}\\
=\ & \phi\Big\{\frac{1}{2}\big[\alpha-(\beta_h+\gamma)w^P\big](w^P-c)+F^P\Big\}\\
&+(1-\phi)\Big\{\frac{1}{2}\big[\alpha-\gamma\,\kappa_L-(\beta_l+\gamma)w^G\big](w^G-c)+F^G\Big\},
\end{aligned}
\tag{C.200}
$$

$$
\text{s.t.}\quad
\begin{cases}
{\Pi^P_{h|h}}^* \ \geq 0,\\
{\Pi^P_{h|h}}^* \ \geq\ {\Pi^G_{l|h}}^*,\ \text{and}\\
{\Pi^G_{l|l}}^* \ \geq\ {\Pi^P_{h|l}}^*.
\end{cases}
\tag{C.201}
$$

The Lagrangian function of this problem is:

$$
\mathscr{L}=\pi+\lambda\,{\Pi^P_{h|h}}^*+\delta\,({\Pi^P_{h|h}}^*-{\Pi^G_{l|h}}^*)+\mu\,({\Pi^G_{l|l}}^*-{\Pi^P_{h|l}}^*),
\tag{C.202}
$$

which has the following KKT first-order conditions:

$$
\lambda\ \geq\ 0,\ \mu\ \geq\ 0,\ \delta\ \geq\ 0,
\tag{C.203}
$$

$$
\begin{aligned}
\mathscr{L}_{w^P}=\ & \frac{1}{2}\Big\{\big[\alpha-(\beta_l+\gamma)w^P\big]\mu-\big[\alpha-(\beta_h+\gamma)w^P\big](\delta+\lambda)+\\
&+\big[\alpha-(\beta_h+\gamma)(2w^P-c)\big]\phi\Big\}=0
\end{aligned}
\tag{C.204}
$$

$$
\mathscr{L}_{F^P}=\phi-\delta-\lambda+\mu=0
\tag{C.205}
$$

$$
\begin{aligned}
\mathscr{L}_{w^G}=\ & \frac{1}{2}\Big\{-\big[\alpha-\gamma\,\kappa_L-(\beta_l+\gamma)w^G\big]\mu+\big[\alpha-\gamma\,\kappa_L-(\beta_h+\gamma)w^G\big]\delta+\\
&+\big[\alpha-\gamma\,\kappa_L-(\beta_l+\gamma)(2w^G-c)\big](1-\phi)\Big\}=0,
\end{aligned}
\tag{C.206}
$$

$$
\mathscr{L}_{F^G}=1-\phi+\delta-\mu=0,
\tag{C.207}
$$

$$\mathscr{L}_\lambda = \Pi^{P\;*}_{h|h} \geq 0, \tag{C.208}$$

$$\lambda\, \Pi^{P\;*}_{h|h} = 0, \tag{C.209}$$

$$\mathscr{L}_\delta = \Pi^{P\;*}_{h|h} - \Pi^{G\;*}_{l|h} \geq 0, \tag{C.210}$$

$$\delta\,(\Pi^{P\;*}_{h|h} - \Pi^{G\;*}_{l|h}) = 0, \tag{C.211}$$

$$\mathscr{L}_\mu = \Pi^{G\,*}_{l|l} - \Pi^{P\;*}_{h|l} \geq 0, \text{ and} \tag{C.212}$$

$$\mu\,(\Pi^{G\,*}_{l|l} - \Pi^{P\;*}_{h|l}) = 0. \tag{C.213}$$

First, from (C.205) and (C.207), we get: $\lambda = 1 > 0$. Next, if $\mu = 0$, then (C.207) implies that $\delta = -(1-\phi) < 0$, which contradicts (C.203); hence,

$$\mu \;>\; 0 \;\overset{\text{by (C.213)}}{\Longrightarrow}\; \Pi^{G\,*}_{l|l} - \Pi^{P\;*}_{h|l} = 0. \tag{C.214}$$

Analogous to the previous analysis, we show that $\delta = 0$ by contradiction. Specifically, assume that $\delta > 0$, then by (C.211), we have:

$$\Pi^{P\;*}_{h|h} - \Pi^{G\;*}_{l|h} = 0. \tag{C.215}$$

Together, (C.214) and (C.215) imply that:

$$(\Pi^{G\,*}_{l|l} - \Pi^{P\;*}_{h|l}) + (\Pi^{P\;*}_{h|h} - \Pi^{G\;*}_{l|h}) = 0 \;\Rightarrow\; w^{P\,2} - w^{G\,2} \;=\; \frac{\alpha^2}{(\beta_h+\gamma)(\beta_l+\gamma)} - \frac{\eta^2\,\alpha^2}{\beta_h\,\beta_l}. \tag{C.216}$$

Recall one of the conditions, required upon the employment of the G contract (Lemma 1):

$$\kappa_L \leq \frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma} \;\Leftrightarrow\; \beta \leq \frac{\eta\,\alpha\,\gamma}{(1-\eta)\alpha + \gamma\,\kappa_L} \;\Rightarrow\; \beta < \frac{\eta\,\gamma}{(1-\eta)} \;\Leftrightarrow\; \eta > \frac{\beta}{\beta+\gamma} \tag{C.217}$$

$$\Rightarrow \eta > \frac{\beta_h}{\beta_h+\gamma} > \frac{\beta_l}{\beta_l+\gamma} \;\Rightarrow\; \frac{\alpha^2}{(\beta_h+\gamma)(\beta_l+\gamma)} - \frac{\eta^2\,\alpha^2}{\beta_h\,\beta_l} < 0 \;\Rightarrow\; w^P < w^G. \tag{C.218}$$

However, given $\lambda = 1$, we solve the following system:

$$\begin{cases} \mathscr{L}_{w^P}\big|_{\lambda=1} & = 0, \\ \mathscr{L}_{w^G} & = 0, \\ \mathscr{L}_{F^G} & = 0, \end{cases} \tag{C.219}$$

for $\{w^P, w^G, \mu\}$ and obtain

$$w^P = \frac{\phi(\beta_h + \gamma)c}{\phi\,\gamma - \beta_h(1+\delta-2\phi) + \beta_l(1+\delta-\phi)}, \text{ and} \tag{C.220}$$

$$w^G = \frac{(1-\phi)(\beta_l + \gamma)c}{(1-\phi)\gamma + \beta_h\,\delta + \beta_l(1-\phi-\delta)}. \tag{C.221}$$

This implies $w^P - w^G = \frac{c(\beta_h - \beta_l)\left[\gamma\,\delta + \gamma(1-\phi)^2 + \beta_l(1-\phi+\delta)(1-\phi) + \beta_h\,\delta\,\phi\right]}{\left[\phi\,\gamma - \beta_h(1+\delta-2\phi) + \beta_l(1+\delta-\phi)\right]\left[(1-\phi)\gamma + \beta_h\,\delta + \beta_l(1-\phi-\delta)\right]} > 0$ (by inspection), which contradicts (C.218). Therefore, $\delta = 0$.

Given $\lambda = 1$, $\mu > 0$, and $\delta = 0$, the first-order conditions become:

$$\begin{cases} \mathscr{L}_{w^P}\big|_{\delta=0,\,\lambda=1} = 0, \\ \mathscr{L}_{w^G}\big|_{\delta=0} = 0, \\ \mathscr{L}_{F^G}\big|_{\delta=0} = 0, \\ {\Pi^P_{h|h}}^* = 0, \\ {\Pi^G_{l|l}}^* - {\Pi^P_{h|l}}^* = 0, \end{cases} \tag{C.222}$$

whose solution characterizes the optimal menu of P and G contracts:

$$w^{G*} = c, \tag{C.223}$$

$$F^{G*} = \frac{(\eta\,\alpha - \beta_l\,c)^2}{4\beta_l} + \frac{\big[(1-\eta)\alpha - \gamma(c+\kappa_L)\big]^2}{4\gamma} - \frac{1}{4}(\beta_h - \beta_l)\Big[\frac{\alpha^2}{(\beta_h+\gamma)(\beta_l+\gamma)} - w^{P*2}\Big] \tag{C.224}$$

$$w^{P*} = \frac{\phi(\beta_h + \gamma)c}{(1-\phi)\beta_l + (2\phi - 1)\beta_h + \phi\,\gamma} = \frac{c}{1 - \frac{(1-\phi)(\beta_h - \beta_l)}{\phi(\beta_h + \gamma)}}, \tag{C.225}$$

$$F^{P*} = \frac{\left[\alpha - (\beta_h + \gamma)w^{P*}\right]^2}{4(\beta_h + \gamma)}. \tag{C.226}$$

Given $c = 0$, the characteristics of the three menus, i.e., the menu of P contracts, the menu of G contracts, and the menu of P and G contracts, are summarized in Table 4.1. The statement of Proposition 4.4 then follows.■

C.7 The optimal full-information contracts in the γ-case (Proof of Proposition 4.5)

Consider the setting of market uncertainty in γ. Further, recall that for analytical simplicity, we focus on the case of $\eta = \frac{1}{2}$. In this setting, the conditions that support the $\{P, w^P, F^P\}$ contract (Lemma 4.2), include: (a) $(1-\eta^2)\beta \geq \eta^2\,\gamma \;\Leftrightarrow\; \gamma \leq 3\beta \overset{\text{def}}{=} \bar{\gamma}$, and (b) $\eta(2-\eta)\gamma \geq (1-\eta)^2\beta$, which is true for all $\gamma > \beta$, given $\eta = \frac{1}{2}$. Regarding the $\{G, w^G, F^G\}$ contract, we assume that κ_H is significantly high so that (4.11) is satisfied. The other two conditions, i.e. (4.10) and (4.17), imply that the $\{G, w^G, F^G\}$ contract arises as a candidate for the optimal full-information contract in equilibrium if:

$$\gamma_1 \overset{\text{def}}{=} \frac{\alpha\,\beta}{\alpha - 2\beta\,\kappa_L} \leq \gamma \leq \frac{\alpha}{2\kappa_L} \overset{\text{def}}{=} \gamma_2, \text{ and} \tag{C.227}$$

$$\kappa_L \leq \frac{\alpha}{4\beta}. \tag{C.228}$$

Notice that (a) $\gamma_1 > \beta$, and (b) $\gamma_2 \leq \bar{\gamma} \;\Leftrightarrow\; \kappa_L \leq \frac{\alpha}{6\beta}$. Together, we can determine the feasibility of the two contracts in γ; this result is provided in the upper portion of Figure 4.5.

Now, we compare the profitability of these two contracts when both are feasible, i.e., (a) $\gamma \in \{\gamma_1, \bar{\gamma}\}$ if $\kappa_L \leq \frac{\alpha}{6\beta}$, and (b) $\gamma \in \{\gamma_1, \gamma_2\}$ if $\frac{\alpha}{6\beta} < \kappa_L \leq \frac{\alpha}{4\beta}$. Recall that $\Delta = \pi^{P*} - \pi^{G*}$ has the same sign as N_1 (see equations C.116 and C.117), which can be rewritten as a function in γ as follows:

$$N_1 = -\beta\,\kappa_L^2\,\gamma^3 - \frac{1}{4}(\alpha - 2\beta\,\kappa_L)^2\,\gamma^2 + \frac{1}{2}\alpha\,\beta(\alpha + 2\beta\,\kappa_L)\gamma - \frac{\alpha^2\,\beta^2}{2}. \tag{C.229}$$

Consider the first case, i.e., when $\kappa_L \in [0, \frac{\alpha}{6\beta}]$ and $\gamma \in [\gamma_1, \bar{\gamma}]$. Notice that N_1 is a cubic function in $\gamma \in [\gamma_1, \bar{\gamma}]$ with a negative third-order coefficient. Further, it can be shown that (a) $\frac{\partial N_1}{\partial \gamma}\big|_{\gamma=\gamma_1} = \frac{\alpha\,\beta^2\,\kappa_L(2\alpha^2 - 11\alpha\,\beta\,\kappa_L + 8\beta^2\,\kappa_L^2)}{(\alpha - 2\beta\,\kappa_L)^2} > 0\ \forall\,\kappa_L \in [0, \frac{\alpha}{6\beta}]$, (b) $\frac{\partial N_1}{\partial \gamma}\big|_{\gamma=\bar{\gamma}} = -\beta\big[(\alpha - \frac{7}{2}\beta\,\kappa_L)^2 + \frac{83\beta^2\,\kappa_L^2}{4}\big] < 0$, (c) $N_1\big|_{\gamma=\gamma_1} = \frac{\alpha^2\,\beta^3\,\kappa_L(2\alpha^2 - 9\alpha\,\beta\,\kappa_L + 8\beta^2\,\kappa_L^2)}{(\alpha - 2\beta\,\kappa_L)^3} > 0\ \forall\,\kappa_L \in [0, \frac{\alpha}{6\beta}]$, and (d) $N_1\big|_{\gamma=\bar{\gamma}} = -\beta^2(\alpha - 6\beta\,\kappa_L)^2 < 0$. Therefore, there exists a solution[66], γ^*, of N_1 in $[\gamma_1, \bar{\gamma}]$ such that (a) if $\gamma \le \gamma^*$, then $N_1 \ge 0$, i.e., M will choose the $\{P, w^P, F^P\}$ contract, and (b) if $\gamma > \gamma^*$, then $N_1 < 0$, i.e., the $\{G, w^G, F^G\}$ contract will be chosen.

Next, consider the other case, when $\kappa_L \in (\frac{\alpha}{6\beta}, \frac{\alpha}{4\beta}]$ and $\gamma \in [\gamma_1, \gamma_2]$. Notice that (a) N_1 can be rewritten as a concave, quadratic function in $\kappa_L \in (\frac{\alpha}{6\beta}, \frac{\alpha}{4\beta}]$[67], (b) $N_1\big|_{\kappa_L=\frac{\alpha}{6\beta}} = -\frac{\alpha^2(9\beta^3 - 24\beta^2\,\gamma + 4\beta\,\gamma^2 + \gamma^3)}{36\beta} > 0\ \forall\gamma \in [\beta, \bar{\gamma}]$[68], and (c) $N_1\big|_{\kappa_L=\frac{\alpha}{4\beta}} = -\frac{\alpha^2(4\beta^3 - 12\beta^2\,\gamma + \beta\,\gamma^2 + \gamma^3)}{16\beta} > 0\ \forall\gamma \in [\beta, \bar{\gamma}]$. This implies that $N_1 > 0\ \forall\kappa_L \in (\frac{\alpha}{6\beta}, \frac{\alpha}{4\beta}]$, i.e., M always chooses the $\{P, w^P, F^P\}$ contract when $\kappa_L \in (\frac{\alpha}{6\beta}, \frac{\alpha}{4\beta}]$.

A summary of this result is provided in Figure 4.5. The statement Proposition 4.5 then follow. ■

[66]The complex specification of the cubic function does not allow for a closed-form solution of γ^*.
[67]$N_1 = -\beta\,\gamma^2\,(\beta + \gamma)\kappa_L^2 + \alpha\,\beta\,\gamma(\beta + \gamma)\kappa_L - \alpha^2(\frac{\gamma}{2} - \frac{\beta}{2})^2$.
[68]Note that in this case, we have $\gamma_2 < \bar{\gamma}$.

C.8 The menu of contracts under asymmetric information regarding γ (Proof of Proposition 4.6)

C.8.1 The menu of P contracts under asymmetric information regarding γ

Consider the menu of P contracts (i.e., $\{P, w_h^P, F_h^P\}$ and $\{P, w_l^P, F_l^P\}$), which is the solution to the the problem, defined by (4.52)-(4.58), i.e.,

$$\begin{aligned}
\max_{\substack{w_h^P, F_h^P, \breve{p}_{h|h}, \\ w_l^P, F_l^P, \breve{p}_{l|l}}} \breve{\pi}^P &= \phi\, \pi^P(w_h^P, F_h^P, \breve{p}_{h|h}; \gamma_h, \cdot) + (1-\phi)\, \pi^P(w_l^P, F_l^P, \breve{p}_{l|l}; \gamma_l, \cdot) \\
&= \phi \Big\{ \big[\alpha - (\beta + \gamma_h)\breve{p}_{h|h}\big] w_h^P + F_h^P \Big\} \\
&\quad + (1-\phi)\Big\{ \big[\alpha - (\beta + \gamma_l)\breve{p}_{l|l}\big] w_l^P + F_l^P \Big\}, \qquad \text{(C.230)}
\end{aligned}$$

subject to:

$$\breve{p}_{h|h} = \underset{p}{\operatorname{argmax}}\, \breve{\Pi}_{h|h}^P = \Pi^P(p, w_h^P, F_h^P; \gamma_h, \cdot) = \big[\alpha - (\beta + \gamma_h)p\big](p - w_h^P) - F_h^P, \qquad \text{(C.231)}$$

$$\breve{p}_{l|l} = \underset{p}{\operatorname{argmax}}\, \breve{\Pi}_{l|l}^P = \Pi^P(p, w_l^P, F_l^P; \gamma, \cdot) = \big[\alpha - (\beta_l + \gamma)p\big](p - w_l^P) - F_l^P, \qquad \text{(C.232)}$$

$$\breve{\Pi}_{h|h}^{P*} = \max_p \breve{\Pi}_{h|h}^P \geq 0, \qquad \text{(C.233)}$$

$$\breve{\Pi}_{l|l}^{P*} = \max_p \breve{\Pi}_{l|l}^P \geq 0, \qquad \text{(C.234)}$$

$$\breve{\Pi}_{h|h}^{P*} \geq \breve{\Pi}_{l|h}^{P*} = \max_p \breve{\Pi}_{l|h}^P = \Pi^P(p, w_l^P, F_l^P; \gamma_h, \cdot) = \big[\alpha - (\beta + \gamma_h)p\big](p - w_l^P) - F_l^P, \text{ and} \qquad \text{(C.235)}$$

$$\breve{\Pi}_{l|l}^{P*} \geq \breve{\Pi}_{h|l}^{P*} = \max_p \breve{\Pi}_{h|l}^P = \Pi^P(p, w_l^P, F_l^P; \gamma_l, \cdot) = \big[\alpha - (\beta + \gamma_l)p\big](p - w_h^P) - F_h^P. \qquad \text{(C.236)}$$

We follow the same procedure to solve this problem. First, we derive the specification of $\breve{p}_{h|h}$, $\breve{p}_{l|l}$, $\breve{\Pi}_{h|h}^{P*}$, $\breve{\Pi}_{l|l}^{P*}$, $\breve{\Pi}_{l|h}^{P*}$ and $\breve{\Pi}_{h|l}^{P*}$ by optimizing the respective profit functions. The results are as follows:

$$\breve{p}_{j|i} = \frac{\alpha}{2(\beta+\gamma_i)} + \frac{w_j^P}{2}, \text{ and} \tag{C.237}$$

$$\breve{\Pi}_{j|i}^{P*} = \frac{[\alpha - (\beta+\gamma_i)w_j^P]^2}{4(\beta+\gamma_i)} - F_j^P. \tag{C.238}$$

Here, since $\breve{\Pi}_{h|l}^{P*} - \breve{\Pi}_{h|h}^{P*} = \frac{(\gamma_h-\gamma_l)\left[\alpha^2-(\beta+\gamma_h)(\beta+\gamma_l)w_h^{P\,2}\right]}{4(\beta+\gamma_h)(\beta+\gamma_l)} > 0$[69] $\Rightarrow \breve{\Pi}_{h|l}^{P*} > \breve{\Pi}_{h|h}^{P*}$, by (C.233) and (C.236), we have: $\breve{\Pi}_{l|l}^{P*} \geq \breve{\Pi}_{h|l}^{P*} > \breve{\Pi}_{h|h}^{P*} \geq 0$, i.e., (C.234) is redundant.

Given (C.237)-(C.238), the optimization problem is simplified to:

$$\begin{aligned}
\max_{\substack{w_h^P, F_h^P,\\ w_l^P, F_l^P}} \breve{\pi}^P &= \phi\left\{\left[\alpha - (\beta+\gamma_h)\breve{p}_{h|h}\right](w_h^P - c) + F_h^P\right\} + (1-\phi)\left\{\left[\alpha - (\beta+\gamma_l\breve{p}_{l|l}\right](w_l^P - c) + F_l^P\right\} \\
&= \phi\left\{\frac{1}{2}\left[\alpha - (\beta+\gamma_h)w_h^P\right](w_h^P - c) + F_h^P\right\} \\
&\quad + (1-\phi)\left\{\frac{1}{2}\left[\alpha - (\beta+\gamma_l)w_l^P\right](w_l^P - c) + F_l^P\right\}
\end{aligned} \tag{C.239}$$

$$\text{s.t.} \begin{cases} \breve{\Pi}_{h|h}^{P*} \geq 0, \\ \breve{\Pi}_{h|h}^{P*} \geq \breve{\Pi}_{l|h}^{P*}, \text{ and} \\ \breve{\Pi}_{l|l}^{P*} \geq \breve{\Pi}_{h|l}^{P*}. \end{cases} \tag{C.240}$$

The Lagrangian function of this problem is as follows:

$$\mathscr{L} = \breve{\pi}^P + \lambda\,\breve{\Pi}_{h|h}^{P*} + \delta\,(\breve{\Pi}_{h|h}^{P*} - \breve{\Pi}_{l|h}^{P*}) + \mu\,(\breve{\Pi}_{l|l}^{P*} - \breve{\Pi}_{h|l}^{P*}). \tag{C.241}$$

Next, we derive the KKT first-order conditions of this Lagrangian function:

$$\lambda \geq 0,\ \mu \geq 0,\ \delta \geq 0, \tag{C.242}$$

[69] Due to the condition on the retail price: $\breve{p}_{h|h} \geq w_h^P \Rightarrow w_h^P \leq \frac{\alpha}{\beta+\gamma_h} < \frac{\alpha}{\beta+\gamma_l} \Rightarrow \alpha^2 - (\beta+\gamma_h)(\beta+\gamma_l)w_h^{P\,2} > 0$.

$$\mathscr{L}_{w_h^P} = \frac{1}{2}\Big\{\big[\alpha - (\beta+\gamma_l)w_h^P\big]\mu - \big[\alpha - (\beta+\gamma_h)w_h^P\big](\delta+\lambda) + \\ + \big[\alpha - (\beta+\gamma_h)(2w_h^P - c)\big]\phi\Big\} = 0 \tag{C.243}$$

$$\mathscr{L}_{F_h^P} = \phi - \delta - \lambda + \mu = 0 \tag{C.244}$$

$$\mathscr{L}_{w_l^P} = \frac{1}{2}\Big\{ - \big[\alpha - (\beta+\gamma_l)w_l^P\big]\mu + \big[\alpha - (\beta+\gamma_h)w_l^P\big]\delta + \\ + \big[\alpha - (\beta+\gamma_l)(2w_l^P - c)\big](1-\phi)\Big\} = 0, \tag{C.245}$$

$$\mathscr{L}_{F_l^P} = 1 - \phi + \delta - \mu = 0, \tag{C.246}$$

$$\mathscr{L}_\lambda = \breve{\Pi}_{h|h}^{P^*} \geq 0, \tag{C.247}$$

$$\lambda\,\breve{\Pi}_{h|h}^{P^*} = 0, \tag{C.248}$$

$$\mathscr{L}_\delta = \breve{\Pi}_{h|h}^{P^*} - \breve{\Pi}_{l|h}^{P^*} \geq 0, \tag{C.249}$$

$$\delta\,(\breve{\Pi}_{h|h}^{P^*} - \breve{\Pi}_{l|h}^{P^*}) = 0, \tag{C.250}$$

$$\mathscr{L}_\mu = \breve{\Pi}_{l|l}^{P^*} - \breve{\Pi}_{h|l}^{P^*} \geq 0, \text{ and} \tag{C.251}$$

$$\mu\,(\breve{\Pi}_{l|l}^{P^*} - \breve{\Pi}_{h|l}^{P^*}) = 0 \tag{C.252}$$

In the following, we show that (a) $\lambda > 0$, (b) $\mu > 0$, and (c) $\delta = 0$. First, we have $\lambda = 1 > 0$ based on (C.244) and (C.246). Next, if $\mu = 0$, then by (C.246), $\delta = -(1-\phi) < 0$, which contradicts (C.242); therefore,

$$\mu > 0 \overset{\text{by (C.252)}}{\Rightarrow} \breve{\Pi}_{l|l}^{P^*} - \breve{\Pi}_{h|l}^{P^*} = 0. \tag{C.253}$$

Finally, suppose that $\delta > 0$, then by (C.250), we have: $\breve{\Pi}_{h|h}^{P^*} - \breve{\Pi}_{l|h}^{P^*} = 0$. Together with (C.253), this implies:

$$(\breve{\Pi}_{h|h}^{P^*} - \breve{\Pi}_{l|h}^{P^*}) + (\breve{\Pi}_{l|l}^{P^*} - \breve{\Pi}_{h|l}^{P^*}) = 0 \;\Leftrightarrow\; \frac{(\gamma_h - \gamma_l)(w_h^{P2} - w_l^{P2})}{4} = 0 \;\Rightarrow\; w_h^P = w_l^P. \tag{C.254}$$

Let $w_h^P = w_l^P = w$; by solving the following system:

$$\begin{cases} \mathscr{L}_{w_h^P}\big|_{w_h^P=w} &= 0, \\ \mathscr{L}_{w_l^P}\big|_{w_l^P=w} &= 0, \\ \mathscr{L}_{F_l^P} &= 0, \end{cases} \tag{C.255}$$

for $\{\delta, \lambda, \mu\}$, we get $\delta = -\frac{(1-\phi)(\beta+\gamma_l)(w-c)}{w(\gamma_h-\gamma_l)} < 0$, which contradicts the assumption that $\delta > 0$. Therefore, $\delta = 0$.

Given $\lambda = 1$, $\mu > 0$, and $\delta = 0$, the first-order conditions now become:

$$\begin{cases} \mathscr{L}_{w_h^P}\big|_{\delta=0,\,\lambda=1} = 0, \\ \mathscr{L}_{w_l^P}\big|_{\delta=0} = 0, \\ \mathscr{L}_{F_l^P}\big|_{\delta=0} = 0, \\ \breve{\Pi}_{h|h}^{P*} = 0, \\ \breve{\Pi}_{l|l}^{P*} - \breve{\Pi}_{h|l}^{P*} = 0. \end{cases} \tag{C.256}$$

The solution to this system of equations is given by:

$$w_h^{P*} = \frac{c}{1 - \frac{(1-\phi)(\gamma_h-\gamma_l)}{\phi(\beta+\gamma_h)}}, \tag{C.257}$$

$$F_h^{P*} = \frac{\left[\alpha - (\beta+\gamma_h)w_h^{P*}\right]^2}{4(\beta+\gamma_h)}, \tag{C.258}$$

$$w_l^{P*} = c, \text{ and} \tag{C.259}$$

$$F_l^{P*} = \frac{\left[\alpha - (\beta+\gamma_l)c\right]^2}{4(\beta+\gamma_l)} - \frac{1}{4}(\gamma_h - \gamma_l)\Big[\frac{\alpha^2}{(\beta+\gamma_h)(\beta+\gamma_l)} - w_h^{P*2}\Big]. \tag{C.260}$$

C.8.2 The menu of G contracts under asymmetric information regarding γ

Consider the menu of G contracts under market uncertainty in γ; this menu solves the optimization problem, defined by (4.59).-(4.65), i.e.,

$$\max_{\substack{w_h^G, F_h^G, \breve{p}_{h|h}^P, \breve{p}_{h|h}^{GB} \\ w_l^G, F_l^G, \breve{p}_{l|l}^P, \breve{p}_{l|l}^{GB}}} \breve{\pi}^G = \phi\, \pi^G(w_h^G, F_h^G, \breve{p}_{h|h}^P, \breve{p}_{h|h}^{GB}; \gamma_h, \cdot) + (1-\phi)\, \pi^G(w_l^G, F_l^G, \breve{p}_{l|l}^P, \breve{p}_{l|l}^{GB}; \gamma_l, \cdot)$$

$$= \phi \Big\{ \big[\alpha - \beta\, \breve{p}_{h|h}^P - \gamma_h(\breve{p}_{h|h}^{GB} + \kappa_L)\big] w_h^G + F_h^G \Big\} +$$

$$+ (1-\phi) \Big\{ \big[\alpha - \beta \breve{p}_{l|l}^P - \gamma_l(\breve{p}_{l|l}^{GB} + \kappa_L)\big] w_l^G + F_l^G \Big\}, \tag{C.261}$$

subject to:

$$\textit{(FOC-h)}: \quad \{\breve{p}_{h|h}^P, \breve{p}_{h|h}^{GB}\} = \underset{p^P,\, p^{GB}}{\operatorname{argmax}}\ \breve{\Pi}_{h|h}^G = \Pi^G(p^P, p^{GB}, w_h^G, F_h^G; \gamma_h, \cdot), \tag{C.262}$$

$$\textit{(FOC-l)}: \quad \{\breve{p}_{l|l}^P, \breve{p}_{l|l}^{GB}\} = \underset{p^P,\, p^{GB}}{\operatorname{argmax}}\ \breve{\Pi}_{l|l}^G = \Pi^G(p^P, p^{GB}, w_l^G, F_l^G; \gamma_l, \cdot), \tag{C.263}$$

$$\textit{(IR-h)}: \quad \breve{\Pi}_{h|h}^{G*} = \max_{p^P,\, p^{GB}} \breve{\Pi}_{h|h}^G \geq 0, \tag{C.264}$$

$$\textit{(IR-l)}: \quad \breve{\Pi}_{l|l}^{G*} = \max_{p^P,\, p^{GB}} \breve{\Pi}_{l|l}^G \geq 0, \tag{C.265}$$

$$\textit{(IC-h)}: \quad \breve{\Pi}_{h|h}^{G*} \geq \breve{\Pi}_{l|h}^{G*} = \max_{p^P,\, p^{GB}} \breve{\Pi}_{l|h}^G = \Pi^G(p^P, p^{GB}, w_l^G, F_l^G; \gamma, \cdot), \text{ and} \tag{C.266}$$

$$\textit{(IC-l)}: \quad \breve{\Pi}_{l|l}^{G*} \geq \breve{\Pi}_{h|l}^{G*} = \max_{p^P,\, p^{GB}} \breve{\Pi}_{h|l}^G = \Pi^G(p^P, p^{GB}, w_h^P, F_h^P; \gamma_l, \cdot). \tag{C.267}$$

First, we derive the specification of $\breve{p}_{j|i}^P, \breve{p}_{j|i}^{GB}$, and $\breve{\Pi}_{j|i}^{G*}$ $(i, j \in \{h, l\})$ by optimizing the respective profit functions:

$$\breve{p}_{j|i}^P = \frac{\eta\, \alpha}{\beta} + \frac{w_j^G}{2}, \tag{C.268}$$

$$\breve{p}^{GB}_{j|i} = \frac{(1-\eta)\alpha}{2\gamma_i} + \frac{w^G_j}{2} - \frac{\kappa_L}{2}, \text{ and} \tag{C.269}$$

$$\breve{\Pi}^{G*}_{j|i} = \frac{(\eta\,\alpha - \beta\, w^G_j)^2}{4\beta} + \frac{\big[(1-\eta)\alpha - \gamma_i(w^G_j + \kappa_L)\big]^2}{4\gamma_i} - F^G_j. \tag{C.270}$$

Next, given (C.264) and (C.267), it can be seen that condition (C.265) is redundant[70]. Consequently, the above optimization problem simplifies to:

$$\begin{aligned}
\max_{\substack{w^G_h, F^G_h,\\ w^G_l, F^G_l}} \breve{\pi}^G =&\ \phi\Big\{\big[\alpha - \beta\,\breve{p}^P_{h|h} - \gamma_h(\breve{p}^{GB}_{h|h} + \kappa_L)\big](w^G_h - c) + F^G_h\Big\} + \\
&+ (1-\phi)\Big\{\big[\alpha - \beta\breve{p}^P_{l|l} - \gamma_l(\breve{p}^{GB}_{l|l} + \kappa_L)\big](w^G_l - c) + F^G_l\Big\} \\
=&\ \phi\Big\{\frac{1}{2}\big[\alpha - \gamma_h\,\kappa_L - (\beta+\gamma_h)w^G_h\big](w^G_h - c) + F^G_h\Big\} + \\
&+ (1-\phi)\Big\{\frac{1}{2}\big[\alpha - \gamma_l\,\kappa_L - (\beta+\gamma_l)w^G_l\big](w^G_l - c) + F^G_l\Big\}
\end{aligned} \tag{C.271}$$

$$\text{s.t.} \quad \begin{cases} \breve{\Pi}^{G*}_{h|h} \geq 0, \\ \breve{\Pi}^{G*}_{h|h} \geq \breve{\Pi}^{G*}_{l|h}, \text{ and} \\ \breve{\Pi}^{G*}_{l|l} \geq \breve{\Pi}^{G*}_{h|l}, \end{cases} \tag{C.272}$$

whose Lagrangian function is given by:

$$\mathscr{L} = \breve{\pi}^G + \lambda\,\breve{\Pi}^{G*}_{h|h} + \delta\,(\breve{\Pi}^{G*}_{h|h} - \breve{\Pi}^{G*}_{l|h}) + \mu\,(\breve{\Pi}^{G*}_{l|l} - \breve{\Pi}^{G*}_{h|l}). \tag{C.273}$$

The KKT first-order conditions of this problem are as follows:

$$\lambda \geq 0,\ \mu \geq 0,\ \delta \geq 0, \tag{C.274}$$

$$\begin{aligned}
\mathscr{L}_{w^G_h} = \frac{1}{2}\Big\{&\big[\alpha - \gamma_l\,\kappa_L - (\beta+\gamma_l)w^G_h\big]\mu - \big[\alpha - \gamma_h\,\kappa_L - (\beta+\gamma_h)w^G_h\big](\delta+\lambda) + \\
&+ \big[\alpha - \gamma_h\,\kappa_L - (\beta+\gamma_h)(2w^G_h - c)\big]\phi\Big\} = 0
\end{aligned} \tag{C.275}$$

[70]Notice that $\breve{\Pi}^{G*}_{h|l} - \breve{\Pi}^{G*}_{h|h} = \frac{(\gamma_h-\gamma_l)\left[(1-\eta)^2\,\alpha^2 - \gamma_h\,\gamma_l\,(w^G_h+\kappa_L)^2\right]}{4\gamma_h\,\gamma_l} > 0 \Rightarrow \breve{\Pi}^{G*}_{h|l} > \breve{\Pi}^{G*}_{h|h}$. Therefore, by (C.264) and (C.267), we have: $\breve{\Pi}^{G*}_{l|l} \geq \breve{\Pi}^{G*}_{h|l} > \breve{\Pi}^{G*}_{h|h} \geq 0$.

$$\mathscr{L}_{F_h^G} = \phi - \delta - \lambda + \mu = 0 \tag{C.276}$$

$$\mathscr{L}_{w_l^G} = \frac{1}{2}\Big\{ - \big[\alpha - \gamma_l\,\kappa_L - (\beta + \gamma_l)w_l^G\big]\mu \,+\, \big[\alpha - \gamma_h\,\kappa_L - (\beta + \gamma_h)w_l^G\big]\delta + \\ +\, \big[\alpha - \gamma_l\,\kappa_L - (\beta + \gamma_l)(2w_l^G - c)\big](1 - \phi)\Big\} = 0, \tag{C.277}$$

$$\mathscr{L}_{F_l^G} = 1 - \phi + \delta - \mu = 0, \tag{C.278}$$

$$\mathscr{L}_{\lambda} = \breve{\Pi}_{h|h}^{G^*} \geq 0, \tag{C.279}$$

$$\lambda\,\breve{\Pi}_{h|h}^{G^*} = 0, \tag{C.280}$$

$$\mathscr{L}_{\delta} = \breve{\Pi}_{h|h}^{G^*} - \breve{\Pi}_{l|h}^{G^*} \geq 0, \tag{C.281}$$

$$\delta\,(\breve{\Pi}_{h|h}^{G^*} - \breve{\Pi}_{l|h}^{G^*}) = 0, \tag{C.282}$$

$$\mathscr{L}_{\mu} = \breve{\Pi}_{l|l}^{G^*} - \breve{\Pi}_{h|l}^{G^*} \geq 0, \text{ and} \tag{C.283}$$

$$\mu\,(\breve{\Pi}_{l|l}^{G^*} - \breve{\Pi}_{h|l}^{G^*}) = 0. \tag{C.284}$$

By (C.276) and (C.278), we have $\lambda = 1 > 0$. Further, if $\mu = 0$, then by (C.278), $\delta = -(1 - \phi) < 0$, which contradicts (C.274). Thus,

$$\mu \,>\, 0 \;\;\overset{\text{by (C.284)}}{\Longrightarrow}\;\; \mathring{\Pi}_{l|l}^{G^*} - \mathring{\Pi}_{h|l}^{G^*} = 0. \tag{C.285}$$

Now, we show that $\delta = 0$ by contradiction. Suppose $\delta > 0$; by (C.282), we must have $\breve{\Pi}_{h|h}^{G^*} - \breve{\Pi}_{l|h}^{G^*} = 0$. Together with (C.285), this implies:

$$(\breve{\Pi}_{h|h}^{G^*} - \breve{\Pi}_{l|h}^{G^*}) + (\breve{\Pi}_{l|l}^{G^*} - \breve{\Pi}_{h|l}^{G^*}) = 0 \;\Leftrightarrow\; \frac{(\gamma_h - \gamma_l)(w_h^G - w_l^G)(w_h^G + w_l^G + 2\kappa_L)}{4} = 0 \;\Rightarrow\; w_h^G = w_l^G. \tag{C.286}$$

Given $w_h^G = w_l^G = w$, by solving the following system:

$$
\begin{cases}
\mathscr{L}_{w_h^G}\big|_{w_h^G=w} & = 0, \\
\mathscr{L}_{w_l^G}\big|_{w_l^G=w} & = 0, \\
\mathscr{L}_{F_l^G} & = 0
\end{cases}
\tag{C.287}
$$

for $\{\delta, \lambda, \mu\}$, we get $\delta = -\frac{(1-\phi)(\beta+\gamma_l)(w-c)}{(\gamma_h-\gamma)(w+\kappa_L)} < 0$, which is a contradiction. Therefore, $\delta = 0$.

In summary, we have $\lambda = 1$, $\mu > 0$, and $\delta = 0$. By solving the following system of equations (derived from the first-order conditions):

$$
\begin{cases}
\mathscr{L}_{w_h^G}\big|_{\delta=0,\,\lambda=1} & = 0 \\
\mathscr{L}_{w_l^G}\big|_{\delta=0} & = 0 \\
\mathscr{L}_{F_l^G}\big|_{\delta=0} & = 0 \\
\breve{\Pi}_{h|h}^{G*} & = 0 \\
\breve{\Pi}_{l|l}^{G*} - \breve{\Pi}_{h|l}^{G*} & = 0,
\end{cases}
\tag{C.288}
$$

we obtain the optimal menu of G contracts as follows:

$$
w_h^{G*} = \frac{c}{1 - \frac{(1-\phi)(\gamma_h-\gamma_l)}{\phi(\beta+\gamma_h)}} - \frac{\kappa_L}{1 - \frac{\phi(\beta+\gamma_h)}{(1-\phi)(\gamma_h-\gamma_l)}}, \tag{C.289}
$$

$$
F_h^{G*} = \frac{\big[\eta\,\alpha - \beta\,w_h^{G*}\big]^2}{4\beta} + \frac{\big[(1-\eta)\alpha - \gamma_h(w_h^{G*} + \kappa_L)\big]^2}{4\gamma_h}, \tag{C.290}
$$

$$
w_l^{G*} = c, \tag{C.291}
$$

$$
F_l^{G*} = \frac{\big[\eta\,\alpha - \beta\,c\big]^2}{4\beta} + \frac{\big[(1-\eta)\alpha - \gamma_l(c + \kappa_L)\big]^2}{4\gamma_l} - \frac{1}{4}(\gamma_h - \gamma_l)\Big[\frac{(1-\eta)^2\,\alpha^2}{\gamma_h\,\gamma_l} - (w_h^{G*} + \kappa_L)^2\Big]. \tag{C.292}
$$

C.8.3 The menu of P and G contracts under asymmetric information regarding γ

Here, we solve the optimization problem that defines the optimal menu of P and G contracts under asymmetric information and uncertainty in γ (i.e., $\{P, w^P, F^P\}$ and $\{G, w^G, F^G\}$, designed for the low and high state respectively):

$$\begin{aligned}
\max_{\substack{w^P, F^P, p_{l|l}, \\ w^G, F^G, p^P_{h|h}, p^{GB}_{h|h}}} \quad \pi = \; & \phi\, \pi^G_{h|h} \;+\; (1-\phi)\, \pi^P_{l|l} \\
= \; & \phi\, \pi^G(w^G, F^G, p^P_{h|h}, p^{GB}_{h|h}; \gamma_h, \cdot) \;+\; (1-\phi)\, \pi^P(w^P, F^P, p_{l|l}, ; \gamma_l, \cdot) \\
= \; & \phi \Big\{ \big[\alpha - \beta\, p^P_{h|h} - \gamma_h(p^{GB}_{h|h} + \kappa_L)\big] w^G + F^G \Big\} \\
& + (1-\phi) \Big\{ \big[\alpha - (\beta + \gamma_l) p_{l|l}\big] w^P + F^P \Big\},
\end{aligned} \tag{C.293}$$

subject to:

$$\textit{(FOC-P)}: \quad p_{l|l} \;=\; \operatorname*{argmax}_{p}\, \Pi^P_{l|l} \;=\; \Pi^P(p, w^P, F^P; \gamma_l, \cdot), \tag{C.294}$$

$$\textit{(FOC-G)}: \quad \{p^P_{h|h}, p^{GB}_{h|h}\} \;=\; \operatorname*{argmax}_{p^P, p^{GB}}\, \Pi^G_{h|h} \;=\; \Pi^G(p^P, p^{GB}, w^G, F^G; \gamma_h, \cdot), \tag{C.295}$$

$$\textit{(IR-h)}: \quad {\Pi^G_{h|h}}^* \;=\; \max_{p^P, p^{GB}} \Pi^G_{h|h} \;\geq 0, \tag{C.296}$$

$$\textit{(IR-l)}: \quad {\Pi^P_{l|l}}^* \;=\; \max_{p} \Pi^P_{l|l} \;\geq 0, \tag{C.297}$$

$$\textit{(IC-h)}: \quad {\Pi^G_{h|h}}^* \;\geq\; {\Pi^P_{l|h}}^* \;=\; \max_{p} \Pi^P_{l|h} = \Pi^P(p, w^P, F^P; \gamma_h, \cdot), \text{ and} \tag{C.298}$$

$$\textit{(IC-l)}: \quad {\Pi^P_{l|l}}^* \;\geq\; {\Pi^G_{h|l}}^* \;=\; \max_{p^P, p^{GB}} \Pi^G_{h|l} = \Pi^G(p^P, p^{GB}, w^G, F^G; \gamma_l, \cdot). \tag{C.299}$$

To simplify this problem, we derive the specification of $p_{l|i}, {\Pi^P_{l|i}}^*, p^P_{h|i}, p^{GB}_{h|i}$, and ${\Pi^G_{h|i}}^*$ ($i \in \{h, l\}$) by optimizing the respective profit functions. The results are provided below:

$$p_{l|i} = \frac{\alpha}{2(\beta+\gamma_i)} + \frac{w^P}{2}, \tag{C.300}$$

$$\Pi_{l|i}^{P\,*} = \frac{\big[\alpha - (\beta+\gamma_i)w^P\big]^2}{4(\beta+\gamma_i)} - F^P, \tag{C.301}$$

$$p_{h|i}^P = \frac{\eta\,\alpha}{2\beta} + \frac{w^G}{2}, \tag{C.302}$$

$$p_{h|i}^{GB} = \frac{(1-\eta)\alpha}{2\gamma_i} + \frac{w^G}{2} - \frac{\kappa_L}{2}, \text{ and} \tag{C.303}$$

$$\Pi_{h|i}^{G\,*} = \frac{(\eta\,\alpha - \beta\,w^G)^2}{4\beta} + \frac{\big[(1-\eta)\alpha - \gamma_i(w^G+\kappa_L)\big]^2}{4\gamma_i} - F^G. \tag{C.304}$$

Here, too, it can be seen that (C.297) is redundant, given (C.296) and (C.299)[71]. This result, together with (C.308)-(C.310), allows for a simplification of the above optimization problem as follows:

$$\begin{aligned}
\max_{\substack{w^P,F^P,\\ w^G,F^G}} \pi = \;& \phi\Big\{\big[\alpha - \beta\,p_{h|h}^P - \gamma_h(p_{h|h}^{GB}+\kappa_L)\big]w^G + F^G\Big\} \\
&+ (1-\phi)\Big\{\big[\alpha - (\beta+\gamma_l)p_{l|l}\big]w^P + F^P\Big\} \\
=\;& \phi\Big\{\frac{1}{2}\big[\alpha - \gamma_h\,\kappa_L - (\beta+\gamma_h)w^G\big](w^G - c) + F^G\Big\} \\
&+ (1-\phi)\Big\{\frac{1}{2}\big[\alpha - (\beta+\gamma_l)w^P\big](w^P - c) + F^P\Big\},
\end{aligned} \tag{C.305}$$

$$\text{s.t.} \begin{cases} \Pi_{h|h}^{G\,*} \ge 0, \\ \Pi_{h|h}^{G\,*} \ge \Pi_{l|h}^{P\,*}, \text{ and} \\ \Pi_{l|l}^{P\,*} \ge \Pi_{h|l}^{G\,*}. \end{cases} \tag{C.306}$$

[71]Since $\Pi_{h|l}^{G\,*} - \Pi_{h|h}^{G\,*} = \frac{(\gamma_h-\gamma_l)\big[(1-\eta)^2\,\alpha^2 - \gamma_h\,\gamma_l(w^P+\kappa_L)^2\big]}{4\gamma_h\,\gamma_l} > 0 \Rightarrow \Pi_{h|l}^{G\,*} > \Pi_{h|h}^{G\,*}$, by (C.296) and (C.299), we have: $\Pi_{l|l}^{P\,*} \ge \Pi_{h|l}^{G\,*} > \Pi_{h|h}^{G\,*} \ge 0$.

The Lagrangian function of this problem is:

$$\mathscr{L} = \pi + \lambda\,\Pi^{G\;*}_{h|h} + \delta\,(\Pi^{G\;*}_{h|h} - \Pi^{P\;*}_{l|h}) + \mu\,(\Pi^{P\,*}_{l|l} - \Pi^{G\;*}_{h|l}), \tag{C.307}$$

which has the following KKT first-order conditions:

$$\lambda \,\geq\, 0,\; \mu \,\geq\, 0,\; \delta \,\geq\, 0, \tag{C.308}$$

$$\begin{aligned}\mathscr{L}_{w^P} = \frac{1}{2}\Big\{ &- \big[\alpha - (\beta + \gamma_l)w^P\big]\mu + \big[\alpha - (\beta + \gamma_h)w^P\big]\delta + \\ &+ \big[\alpha - (\beta + \gamma_l)(2w^P - c)\big](1 - \phi)\Big\} = 0\end{aligned} \tag{C.309}$$

$$\mathscr{L}_{F^P} = 1 - \phi + \delta - \mu = 0 \tag{C.310}$$

$$\begin{aligned}\mathscr{L}_{w^G} = \frac{1}{2}\Big\{ &\big[\alpha - \gamma_l\,\kappa_L - (\beta + \gamma_l)w^G\big]\mu - \big[\alpha - \gamma_h\,\kappa_L - (\beta + \gamma_h)w^G\big](\delta + \lambda) + \\ &+ \big[\alpha - \gamma_h\,\kappa_L - (\beta + \gamma_h)(2w^G - c)\big]\phi\Big\} = 0,\end{aligned} \tag{C.311}$$

$$\mathscr{L}_{F^G} = \phi - \delta - \lambda + \mu = 0, \tag{C.312}$$

$$\mathscr{L}_{\lambda} = \Pi^{G\;*}_{h|h} \,\geq 0, \tag{C.313}$$

$$\lambda\,\Pi^{G\;*}_{h|h} = 0, \tag{C.314}$$

$$\mathscr{L}_{\delta} = \Pi^{G\;*}_{h|h} - \Pi^{P\;*}_{l|h} \,\geq 0, \tag{C.315}$$

$$\delta\,(\Pi^{G\;*}_{h|h} - \Pi^{P\;*}_{l|h}) = 0, \tag{C.316}$$

$$\mathscr{L}_{\mu} = \Pi^{P\,*}_{l|l} - \Pi^{G\;*}_{h|l} \,\geq 0, \text{ and} \tag{C.317}$$

$$\mu\,(\Pi^{P\,*}_{l|l} - \Pi^{G\;*}_{h|l}) = 0. \tag{C.318}$$

First, from (C.310) and (C.312), we get: $\lambda = 1 > 0$. Next, if $\mu = 0$, then (C.310) implies that $\delta = -(1 - \phi) < 0$, which contradicts (C.308); hence,

$$\mu \,>\, 0 \;\;\overset{\text{by (C.318)}}{\Longrightarrow}\;\; \Pi^{P\,*}_{l|l} - \Pi^{G\;*}_{h|l} = 0. \tag{C.319}$$

Now, we show that $\delta = 0$, also by contradiction. Assume that $\delta > 0$, then by (C.316), we have:

$$\Pi^{G}_{h|h}{}^{*} - \Pi^{P}_{l|h}{}^{*} = 0. \tag{C.320}$$

Together, (C.319) and (C.320) give us the following relationship:

$$(\Pi^{P}_{l|l}{}^{*} - \Pi^{G}_{h|l}{}^{*}) + (\Pi^{G}_{h|h}{}^{*} - \Pi^{P}_{l|h}{}^{*}) = 0 \;\Rightarrow\; {w^{P}}^{2} - (w^{G} + \kappa_L)^2 \;=\; \frac{\alpha^2}{(\beta+\gamma_h)(\beta+\gamma_l)} - \frac{(1-\eta)^2\,\alpha^2}{\gamma_h\,\gamma_l}. \tag{C.321}$$

Recall one of the conditions, required upon the employment of the G contract (Lemma 1):

$$\kappa_L \leq \frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma} \;\Rightarrow\; \frac{\eta\,\alpha}{\beta} - \frac{(1-\eta)\alpha}{\gamma} > 0 (\text{ since } \kappa_L > 0) \;\Leftrightarrow\; 1-\eta < \frac{\gamma}{\beta+\gamma} \tag{C.322}$$

$$\Rightarrow 1-\eta < \frac{\gamma_l}{\beta+\gamma_l} < \frac{\gamma_h}{\beta+\gamma_h} \;\Rightarrow\; \frac{\alpha^2}{(\beta+\gamma_h)(\beta+\gamma_l)} - \frac{(1-\eta)^2\,\alpha^2}{\gamma_h\,\gamma_l} > 0$$

$$\Rightarrow w^{P} > w^{G} + \kappa_L. \tag{C.323}$$

However, given $\lambda = 1$, we solve the following system:

$$\begin{cases} \mathscr{L}_{w^G}\big|_{\lambda=1} & = 0 \\ \mathscr{L}_{w^P} & = 0 \\ \mathscr{L}_{F^P} & = 0 \end{cases} \tag{C.324}$$

for $\{w^P, w^G, \mu\}$ and obtain

$$w^{P} = \frac{(1-\phi)(\beta+\gamma_l)c}{(1-\phi)\,\beta + \gamma_h\,\delta + \gamma_l(1-\phi-\delta)}, \text{ and} \tag{C.325}$$

$$w^{G}=\frac{(1-\phi+\delta)(\gamma_h-\gamma_l)\kappa_L+\phi(\beta+\gamma_h)c}{\phi\,\beta+\gamma_h(2\phi-1+\delta)+\gamma_l(1-\phi+\delta)}. \tag{C.326}$$

Denote the denominators in the RHS of (C.325) and (C.326) by D_1 and D_2 respectively and notice that these terms must be positive for the prices to be positive. From (C.325) and (C.326), we have: $w^P-(w^G+\kappa_L) = -\frac{c(\gamma_h-\gamma_l)[\beta\,\delta+\beta\,(1-\phi)^2+\gamma_h\,\delta\,\phi+\gamma_l(1-\phi+\delta)(1-\phi)]}{D_1\,D_2}-\frac{\phi(\beta+\gamma_h)\kappa_L}{D_2}<0$ (by inspection), which contradicts (C.323). Therefore, $\delta=0$.

Given $\lambda=1$, $\mu>0$, and $\delta=0$, we solve the first-order conditions, i.e.,

$$\begin{cases} \mathscr{L}_{w^P}\big|_{\delta=0}=0, \\ \mathscr{L}_{F^P}\big|_{\delta=0}=0, \\ \mathscr{L}_{w^G}\big|_{\delta=0,\,\lambda=1}=0, \\ \Pi^{G\,*}_{h|h}=0, \\ \Pi^{P\,*}_{l|l}-\Pi^{G\,*}_{h|l}=0, \end{cases} \tag{C.327}$$

and obtain the optimal menu of P and G contracts as follows:

$$w^{P*}=c, \tag{C.328}$$

$$F^{P*}=\frac{\big[\alpha-(\beta+\gamma_l)c\big]^2}{4(\beta+\gamma_l)}-\frac{1}{4}(\gamma_h-\gamma_l)\Big[\frac{(1-\eta)^2\,\alpha}{\gamma_h\,\gamma_l}-(w^{G*}+\kappa_L)^2\Big] \tag{C.329}$$

$$w^{G*}=\frac{c}{1-\frac{(1-\phi)(\gamma_h-\gamma_l)}{\phi(\beta+\gamma_h)}}-\frac{\kappa_L}{1-\frac{\phi(\beta+\gamma_h)}{(1-\phi)(\gamma_h-\gamma_l)}}, \tag{C.330}$$

$$F^{G*}=\frac{(\eta\,\alpha-\beta\,w^{G*})^2}{4\beta}+\frac{\big[(1-\eta)\alpha-\gamma_h(w^{G*}+\kappa_L)\big]^2}{4\gamma_h}. \tag{C.331}$$

We summarize the characteristics of the three menus, i.e., the menu of P contracts, the menu of G contracts, and the menu of P and G contracts, when $c=0$ in Table 4.2. The statement of Proposition 4.6 then follows.■

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