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Automatika

Journal for Control, Measurement, Electronics, Computing and Communications

ISSN: 0005-1144 (Print) 1848-3380 (Online) Journal homepage: https://www.tandfonline.com/loi/taut20

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To cite this article: Cun-Fang Feng, Yan-Rong Tan, Ying-Hai Wang & Hai-Jun Yang (2017) Active backstepping control of combined projective synchronization among different nonlinear systems, Automatika, 58:3, 295-301, DOI: 10.1080/00051144.2018.1432466

To link to this article: https://doi.org/10.1080/00051144.2018.1432466

AUTOMATIKA, 2018
VOL. 58, NO. 3, 295–301
https://doi.org/10.1080/00051144.2018.1432466

Active backstepping control of combined projective synchronization among different nonlinear systems

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ABSTRACT
In this article, the authors have studied combination projective synchronization using active backstepping method. The main contribution of this effort is realization of the projective synchronization between two drive systems and one response system. We relax some limitations of previous work, where only combination complete synchronization has been investigated. According to Lyapunov stability theory and active backstepping design method, the corresponding controllers are designed to observe combination projective synchronization among three different classical chaotic systems, i.e. the Lorenz system, Rössler system and Chen system. The numerical simulation examples verify the effectiveness of the theoretical analysis. Combination projective synchronization has stronger anti-attack ability and anti-translated ability than the normal projective synchronization scheme realized by one drive and one response system in secure communication.

ARTICLE HISTORY
Received 13 March 2017
Accepted 13 January 2018

KEYWORDS
Combination projective synchronization; active backstepping design; Lyapunov stability theory

1. Introduction

Chaos systems are nonlinear dynamical systems that are highly sensitive to initial conditions. Modelling the dynamics of chaotic systems is a challenging problem with important real-world application, such as weather forecast [1–3], road traffic [4,5], stock market returns [6], etc. Chaotic systems are likely to lead completely different trajectories because of slight errors. Therefore, chaotic systems may require synchronization. In the late twentieth century, when the computational techniques became an important scientific tool, many scientists focused their efforts on developing deterministic methods to synchronize chaotic systems. Synchronization means two or more systems adjust each other to give rise to a common dynamical behaviour. As a key technique of secure communication, chaos synchronization has been extensively studied in recent decades and different notations have been proposed and studied, such as complete synchronization [7–9], generalized synchronization [10,11], phase synchronization [12,13], anti-phase synchronization [14–16] and projective synchronization [17–20].

Among them, the most preferred one for synchronization is projective synchronization. It has been successfully used to extend binary digital to M-nary digital for achieving fast communication. Projective synchronization is a recently discovered intriguing phenomenon which characterized by a scaling factor that the drive and the response systems synchronize proportionally. So far, the projective synchronization model of chaotic systems has mainly been limited to one drive system and one response system [18–20]. In secure communication, the typical approach is to transmit the information signal by means of one chaotic system.

However, since the transmitter is only one, this pattern is relatively easier to be attacked or decoded in the process of transmission. In order to ensure safer communication, combination synchronization, which has two drive systems (or three drive systems, or four drive systems) and one response system, has been proposed by Luo in 2011 [21]. This synchronization method has advantages over the usual drive-response synchronization within one drive system and one response system, such that it can provide greater security in secure communication. Because the transmitted signals can be split into two parts, each part is loaded in different drive systems or at different intervals. The signals in different intervals could be loaded in different drive systems. Thus, the transmitted signals can have stronger anti-attack ability and anti-translated capability than those transmitted by the usual transmission model. In the past three years, some researchers have been interested in the combination synchronization [21–27] for safer communication. In Ref. [22], combination synchronization among fractional-order chaotic systems was observed. Sun et al. investigated

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combination-combination synchronization between two drive systems and two response systems in a finite time [23]. Combination synchronization of chaotic systems is an open topic.

Until now, combination synchronization mainly focuses on the combination complete synchronization [21], however, little attention has been paid to the combination projective synchronization. Combined projective synchronization becomes combined synchronization as scaling factor equals to 1. Compared to the normal projective synchronization scheme realized by one drive and one response system, combination projective synchronization has stronger anti-attack ability and anti-translated ability in secure communication. There are only a few papers on the combination projective synchronization [28,29]. On the other hand, the active control scheme [30–32] and backstepping design [33,34] have been widely recognized as two powerful design methods to control chaotic systems in recent years. The active control method is easier to manipulate so that it is used widely to control chaotic systems. Backstepping design represents a powerful and systematic technique that recursively interlaces the choice of the Lyapunov function. Consequently, in this paper, we use the active backstepping design [35–37] to achieve combination projective synchronization. It is a systematic design approach and consists of a recursive procedure by design a virtual control via Lyapunov stability theory.

Motivated by the above discussions, the aim of this paper is to study combination projective synchronization between two drive systems and one response system using active backstepping design. This paper is organized follows. In Section 2, we define the combination projective synchronization. In Section 3, we first introduce a brief description of the Lorenz system, Rössler system and Chen system. Second, we analyse the combination projective synchronization among Lorenz system, Rössler system and Chen system via the design of the active backstepping controllers based on the Lyapunov stability theory. Third, numerical simulations are given to confirm the validity of the proposed theoretical approach. Conclusions are drawn in the last section.

2. The definition of combination projective synchronization

The first drive system is given as follows:

$$\dot{X} = F(X) \tag{1}$$

and the second drive system is given by

$$\dot{Y} = G(Y) \tag{2}$$

The response system under the controller U is described by

$$\dot{Z} = H(Z) + U, \tag{3}$$

where $X = (x_1, x_2, \ldots, x_n)^T \in R^n$, $Y = (y_1, y_2, \ldots, y_n)^T \in R^n$ and $Z = (z_1, z_2, \ldots, z_n)^T \in R^n$ are the state variables of the systems (1)–(3), respectively; F, G, $H \in R^n$ are continuous nonlinear functions; $U = [u_1, u_2, \ldots, u_n]^T \in R^n$ is the controller to be designed.

Definition 2.1: For the drive systems described by Equations (1) and (2), we call they have realized combination projective synchronization with the response system (3) if there exists a non-zero constant α, such that the following condition:

$$\lim_{t\to\infty} \parallel e \parallel = \lim_{t\to\infty} \parallel X + Y - \alpha Z \parallel \to 0, \tag{4}$$

can be achieved, where $\parallel \cdot \parallel$ denotes the Euclidian norm of a vector. It implies that the error dynamical system between the drive systems and response system is globally asymptotically stable, and we call α a "scaling factor".

Remark 2.1: If $\alpha = 1$, then the combination projective synchronization problem discussed in this paper will be reduced to the combination complete synchronization.

3. Combination projective synchronization

3.1. System description

In this paper, we consider Lorenz system which is taken the following form:

$$\begin{cases} \dot{x}_1 = a_1(x_2 - x_1) \\ \dot{x}_2 = b_1 x_1 - x_2 - x_1 x_3 \\ \dot{x}_3 = x_1 x_2 - c_1 x_3, \end{cases} \tag{5}$$

where the system parameters are $a_1 = 10$, $b_1 = 28$ and $c_1 = \frac{8}{3}$ with which the system behaves chaotically; see Figure 1(a).

Consider the Rössler chaotic system described by the following differential equations:

$$\begin{cases} \dot{y}_1 = -y_2 - y_3 \\ \dot{y}_2 = y_1 + a_2 y_2 \\ \dot{y}_3 = -c_2 y_3 + b_2 + y_1 y_3, \end{cases} \tag{6}$$

which has a chaotic attractor as shown in Figure 1(b) when $a_2 = 0.2$, $b_2 = 0.2$, and $c_2 = 5.7$.

Consider the Chen system

$$\begin{cases} \dot{z}_1 = a_3(z_2 - z_1) \\ \dot{z}_2 = (c_3 - a_3) z_1 + c_3 z_2 - z_1 z_3 \\ \dot{z}_3 = z_1 z_2 - d z_3, \end{cases} \tag{7}$$

where $a_3 = 35$, $c_3 = 28$, and $d = 3$ with which the system behaves chaotically as shown in Figure 1(c).

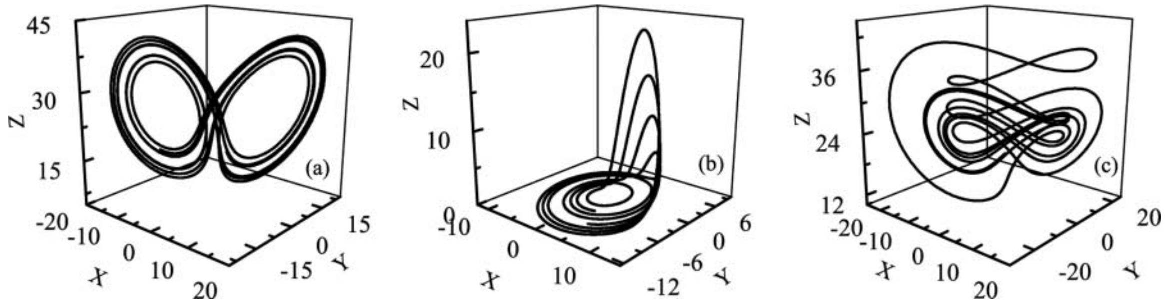

Figure 1. (a) Lorenz chaotic attractor, (b) Rössler chaotic attractor, (c) Chen chaotic attractor.

3.2. Design of the active backstepping controllers

In this section, we assume that the Lorenz system (5) and Rössler system (6) drive the Chen system (7). Thus, we rewrite the response system (7) under the controllers as follows:

$$\begin{cases} \dot{z}_1 = a_3(z_2 - z_1) + u_1 \\ \dot{z}_2 = (c_3 - a_3)z_1 + c_3 z_2 - z_1 z_3 + u_2 \\ \dot{z}_3 = z_1 z_2 - d z_3 + u_3, \end{cases} \tag{8}$$

where u_1, u_2 and u_3 in Equation (8) are the control functions to be designed for the purpose of the combination projective synchronization among systems (5), (6) and (8).

Define the error among systems (5), (6) and (8)

$$\begin{cases} e_1 = x_1 + y_1 - \alpha z_1 \\ e_2 = x_2 + y_2 - \alpha z_2 \\ e_3 = x_3 + y_3 - \alpha z_3, \end{cases} \tag{9}$$

then we obtain the error dynamical systems from Equation (9) as follows:

$$\begin{cases} \dot{e}_1 = \dot{x}_1 + \dot{y}_1 - \alpha \dot{z}_1 \\ \dot{e}_2 = \dot{x}_2 + \dot{y}_2 - \alpha \dot{z}_2 \\ \dot{e}_3 = \dot{x}_3 + \dot{y}_3 - \alpha \dot{z}_3 \end{cases} \tag{10}$$

Substituting Equations (5), (6) and (8) into Equation (10) yields

$$\begin{cases} \dot{e}_1 &= a_3 e_2 - a_3 e_1 + \phi_1 - \alpha u_1 \\ \dot{e}_2 &= c_3 e_1 - a_3 e_1 + c_3 e_2 - \dfrac{x_1}{\alpha} e_3 - \dfrac{y_1}{\alpha} e_3 - \dfrac{x_3}{\alpha} e_1 \\ & \quad - \dfrac{y_3}{\alpha} e_1 + \dfrac{e_1 e_3}{\alpha} + \phi_2 - \alpha u_2 \\ \dot{e}_3 &= \dfrac{x_1}{\alpha} e_2 + \dfrac{y_1}{\alpha} e_2 + \dfrac{x_2}{\alpha} e_1 + \dfrac{y_2}{\alpha} e_1 - \dfrac{e_1 e_2}{\alpha} - d e_3 + \phi_3 \\ & \quad - \alpha u_3, \end{cases} \tag{11}$$

where

$$\begin{cases} \phi_1 &= a_1 x_2 - a_1 x_1 - y_2 - y_3 - a_3 x_2 - a_3 y_2 + a_3 x_1 + a_3 y_1 \\ \phi_2 &= b_1 x_1 - x_2 - x_1 x_3 + y_1 + a_2 y_2 - c_3 x_1 - c_3 y_1 + a_3 x_1 \\ & \quad + a_3 y_1 - c_3 x_2 - c_3 y_2 + \dfrac{x_1 x_3}{\alpha} + \dfrac{x_1 y_3}{\alpha} + \dfrac{y_1 x_3}{\alpha} + \dfrac{y_1 y_3}{\alpha} \\ \phi_3 &= x_1 x_2 - c_1 x_3 - c_2 y_3 + b_2 + y_1 y_3 - \dfrac{x_1 x_2}{\alpha} - \dfrac{x_1 y_2}{\alpha} \\ & \quad - \dfrac{y_1 x_2}{\alpha} - \dfrac{y_1 y_2}{\alpha} + d x_3 + d y_3 \end{cases} \tag{12}$$

It is obvious that our object is to design proper controllers $u_i (i = 1, 2, 3)$ for stabilizing the error variables of system (11) at the origin. In this paper, we use active backstepping approach [35–37] which includes three steps.

Step 1. Let $v_1 = e_1$, then we obtain its derivative

$$\dot{v}_1 = \dot{e}_1 = a_3 e_2 - a_3 e_1 + \phi_1 - \alpha u_1, \tag{13}$$

where $e_2 = k_1(v_1)$ can be regarded as a virtual controller. For the design of $k_1(v_1)$ and u_1 to stabilize k_1-subsystem (13), consider the Lyapunov function $L_1 = \frac{1}{2} v_1^2$. The derivative of L_1 is

$$\dot{L}_1 = v_1 \dot{v}_1 = v_1 [a_3 k_1(v_1) - a_3 v_1 + \phi_1 - \alpha u_1]. \tag{14}$$

Then one can choose $\alpha u_1 = \phi_1 - a_3 v_1 + v_1$ and $k_1(v_1) = 0$, such that $\dot{L}_1 = -v_1^2 < 0$. It implies that the v_1-subsystem (13) is asymptotically stable. Since the virtual control function $k_1(v_1)$ is estimative, the error between e_2 and $k_1(v_1)$ is $v_2 = e_2 - k_1(v_1)$. Then we can obtain the following (v_1, v_2)-subsystem

$$\begin{cases} \dot{v}_1 &= a_3 v_2 - v_1 \\ \dot{v}_2 &= c_3 v_1 - a_3 v_1 + c_3 v_2 - \dfrac{x_1}{\alpha} e_3 - \dfrac{y_1}{\alpha} e_3 - \dfrac{x_3}{\alpha} v_1 \\ & \quad - \dfrac{y_3}{\alpha} v_1 + \dfrac{v_1}{\alpha} e_3 + \phi_2 - \alpha u_2. \end{cases} \tag{15}$$

Consider $e_3 = k_2(v_1, v_2)$ as a virtual controller to make system (15) asymptotically stable.

Step 2. In this step, in order to stabilize the (v_1, v_2)-subsystem (15), we can choose a Lyapunov function defined by $L_2 = L_1 + \frac{1}{2}v_2^2$. The time derivative of L_2 is

$$\begin{aligned}\dot{L}_2 &= \dot{L}_1 + \dot{v}_2 v_2 \\ &= a_3 v_1 v_2 - v_1^2 + v_2\Big[c_3 v_1 - a_3 v_1 + c_3 v_2 - \frac{x_1}{\alpha}k_2 \\ &\quad - \frac{y_1}{\alpha}k_2 - \frac{x_3}{\alpha}v_1 - \frac{y_3}{\alpha}v_1 + \frac{v_1}{\alpha}k_2 + \phi_2 - \alpha u_2\Big] \\ &= -v_1^2 + \Big(c_3 v_1 + c_3 v_2 - \frac{x_1}{\alpha}k_2 - \frac{y_1}{\alpha}k_2 - \frac{x_3}{\alpha}v_1 \\ &\quad - \frac{y_3}{\alpha}v_1 + \frac{v_1}{\alpha}k_2 + \phi_2 - \alpha u_2\Big)v_2 \end{aligned} \tag{16}$$

If the control function u_2 is chosen as $\alpha u_2 = c_3 v_1 + c_3 v_2 - \frac{x_3}{\alpha}v_1 - \frac{y_3}{\alpha}v_1 + \phi_2 + v_2$, and $k_2(v_1, v_2) = 0$, then $\dot{L}_2 = -v_1^2 - v_2^2 < 0$, which makes the (v_1, v_2)-subsystem (15) asymptotically stable. Let $v_3 = e_3$, one has the following (v_1, v_2, v_3)-subsystem:

$$\begin{cases} \dot{v}_1 = a_3 v_2 - v_1 \\ \dot{v}_2 = -a_3 v_1 - v_2 - \frac{x_1}{\alpha}v_3 - \frac{y_1}{\alpha}v_3 + \frac{v_1}{\alpha}v_3 \\ \dot{v}_3 = -d v_3 + \frac{x_1}{\alpha}v_2 + \frac{y_1}{\alpha}v_2 + \frac{x_2}{\alpha}v_1 + \frac{y_2}{\alpha}v_1 - \frac{v_1 v_2}{\alpha} \\ \qquad + \phi_3 - \alpha u_3. \end{cases} \tag{17}$$

Step 3. We can choose a Lyapunov function $L_3 = L_2 + \frac{1}{2}v_3^2$ in order to make the (v_1, v_2, v_3)-subsystem (17) stable.

The derivative of L_3 gives

$$\begin{aligned}\dot{L}_3 &= \dot{L}_2 + v_3\dot{v}_3 \\ &= -v_1^2 - v_2^2 + v_3\Big(\frac{x_2}{\alpha}v_1 + \frac{y_2}{\alpha}v_1 - d v_3 + \phi_3 - \alpha u_3\Big).\end{aligned} \tag{18}$$

We can choose $\alpha u_3 = \frac{x_2}{\alpha}v_1 + \frac{y_2}{\alpha}v_1 - d v_3 + \phi_3 + v_3$ so that $\dot{V}_3 = -v_1^2 - v_2^2 - v_3^2 < 0$, which imply the (v_1, v_2, v_3)-subsystem (17) asymptotically stable. By using the following properties: $v_1 = e_1$, $v_2 = e_2$, and $v_3 = e_3$, we know that $e_i (i = 1, 2, 3)$ go to zero as $t \to \infty$, which implies that the two drive systems (5) and (6) will achieve combination projective synchronization with the response system (8).

In what follows, we give numerical experiments to verify the effectiveness of our approach. The fourth-order Runge–Kutta algorithm is used in all of our simulations with time step being equal to 0.001. The initial values of the drive systems and the response system are given by $(x_{10}, x_{20}, x_{30}) = (-11.2, -8.4, 33.4)$, $(y_{10}, y_{20}, y_{30}) = (3, 5, 2)$ and $(z_{10}, z_{20}, z_{30}) = (10.5, 20, 38)$. The corresponding numerical results are shown in the following.

Figure 2 shows the combination projective synchronization among systems (5), (6), (8) with $\alpha = 2$.

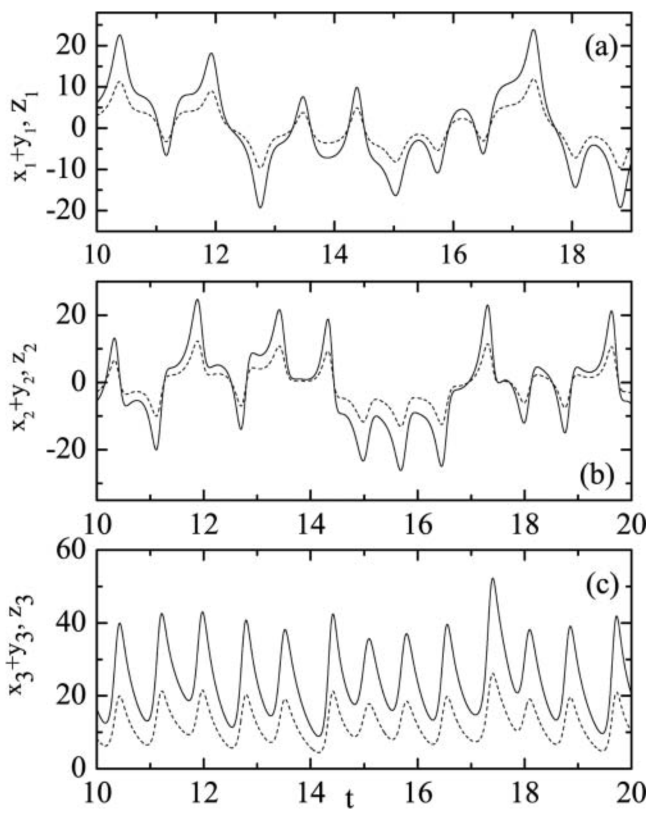

Figure 2. Combination projective synchronization between drive systems (5), (6) and response system (8) with $\alpha = 2$. (a) Time waveforms of the states $x_1 + y_1$ (solid) and z_1 (dashed), (b) time waveforms of the states $x_2 + y_2$ (solid) and z_2 (dashed), (c) time waveforms of the states $x_3 + y_3$ (solid) and z_3 (dashed).

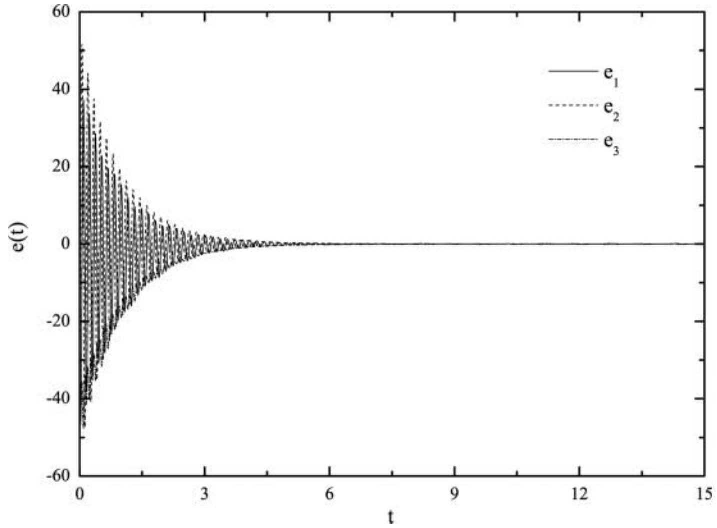

Figure 3. Time waveforms of the combination projective synchronization errors $e_i(t)(i = 1, 2, 3)$ between drive systems (5), (6) and response system (8) with $\alpha = 2$.

Figure 2(a–c) show the time waveforms of the states $x_1 + y_1$ and z_1, $x_2 + y_2$ and z_2, $x_3 + y_3$ and z_3, respectively. It can be easily seen that the phase angle between the synchronized trajectories is zero. Figure 3 displays the orbits of synchronization error $e_i(t)$, $i = 1, 2, 3$, as $t \to \infty$. From Figure 3, we can see that the error vector e converges to zero as time t goes to infinity. This shows that all the state variables achieve the combination projective synchronization.

The same results with $\alpha = -2$ are shown in Figures 4 and 5. Figure 4(a–c) show the time waveforms of the states $x_1 + y_1$ and z_1, $x_2 + y_2$ and z_2, $x_3 + y_3$ and z_3, respectively, where the phase angle between the synchronized trajectories is π. Furthermore, Figure 5

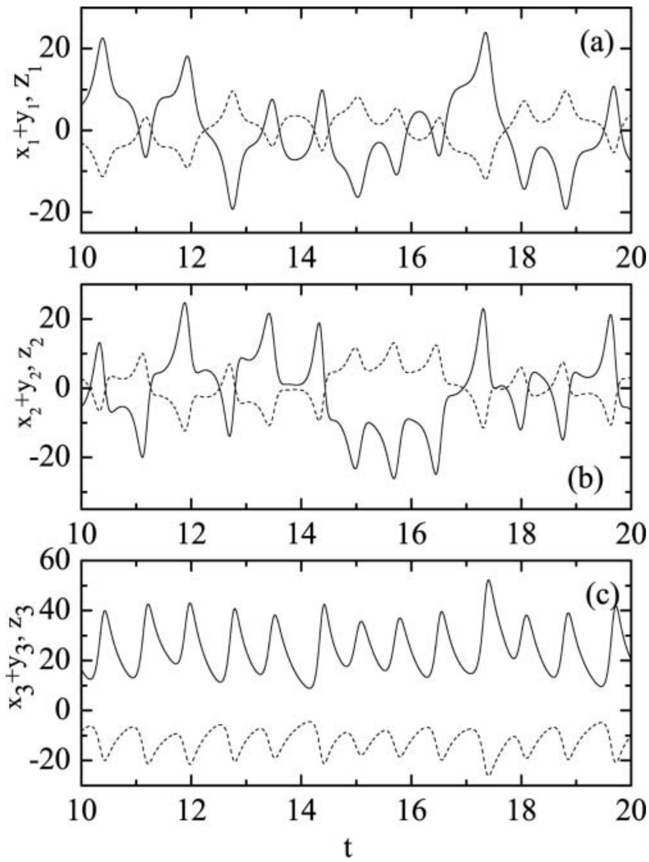

Figure 4. Combination projective synchronization between drive systems (5), (6) and response system (8) with $\alpha = -2$. (a) Time waveforms of the states $x_1 + y_1$ (solid) and z_1 (dashed), (b) time waveforms of the states $x_2 + y_2$ (solid) and z_2 (dashed), (c) time waveforms of the states $x_3 + y_3$ (solid) and z_3 (dashed).

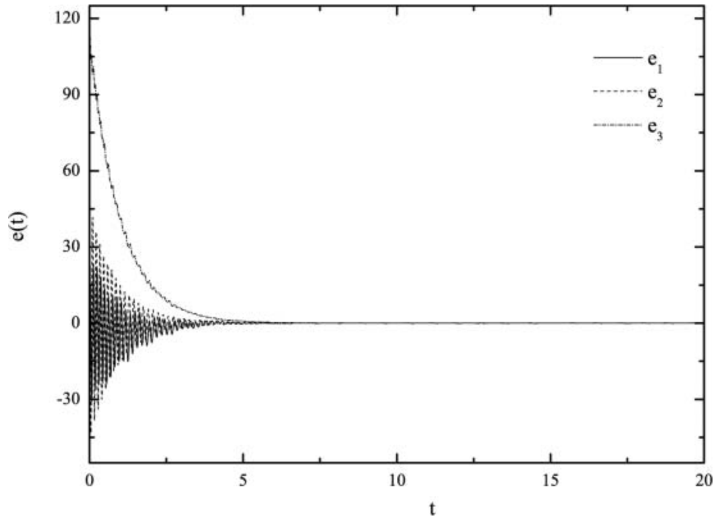

Figure 5. Time waveforms of the combination projective synchronization errors $e_i(t)(i = 1, 2, 3)$ between drive systems (5), (6) and response system (8) with $\alpha = -2$.

shows that the error vectors $e_i(t)$, $i = 1, 2, 3$, eventually converge to zero after the controllers are activated. It implies that the drive systems (5), (6) and response system (8) achieved the combination projective synchronization with $\alpha = -2$.

4. Conclusion

We have already analytically estimated and numerically simulated combination projective synchronization using an active backstepping design. The proposed control method is a systematic design method and contains a recursive procedure to make in-full range synchronization all state variables in a proportional state. Based on the Lyapunov stability theory, corresponding controllers to achieve combination projective synchronization are derived among three different classical chaotic systems: Lorenz system, Rössler system and Chen system. The numerical simulation results are conducted to illustrate the validity and feasibility of the theoretical analysis. Combination projective synchronization, including two drive systems, has stronger anti-attack ability and anti-translated ability than the projective synchronization in extending binary digital to M-nary digital for achieving fast communication.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

This work was supported by Key Projects of Educational Commission of Hubei Province [grant number D20161604].

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