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Increasing Automaticity

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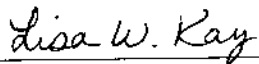
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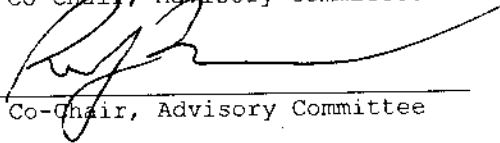
By

Sara Burns

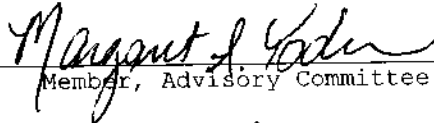
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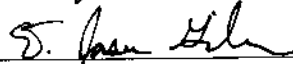
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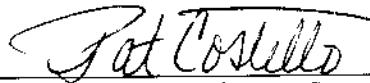
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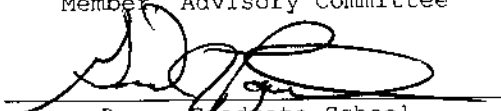
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Increasing Automaticity

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Submitted to the Faculty of the Graduate School of
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DEDICATION

This thesis is dedicated to my husband and two daughters,
Matt, Kylie, and Madison
for their support.

ACKNOWLEDGMENTS

I would like to thank two of my professors, Dr. Robert Thomas, and Dr. Cheryll Crowe, for their guidance and assistance throughout the last four years. I would also like to thank Dr. Patrick Costello, Dr. Ken Dutch, Dr. Lisa Kay, and Dr. Margaret Yoder for teaching me to be a good student. I would like to express my thanks to my husband, Matt, for his patience and for his support through the last few years when I did not foresee ever finishing my degree. Matt encouraged me and helped push me along the way to finish when I did not think I could work any harder. I would like to thank my daughters for allowing me to take time away from being a mom and allowing me to work on my homework. Finally, I would like to thank the members of my thesis committee and family members here in Richmond, Kentucky, especially my mom.

ABSTRACT

Automaticity is defined as the process of developing fluency in mathematics and the ability to answer a basic math problem routinely. Automaticity is one of the most important skills that a student can possess in mathematics. While there are many ways that an educator can implement strategies to increase automaticity in the classroom, the purpose of this study was to determine which of these methods of implementation is more effective: requiring students to practice automaticity three times per week or requiring students to practice automaticity five times per week.

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CHAPTER I

INTRODUCTION

According to National Numeracy, automaticity is the process of having fluency in mathematics and being able to routinely answer a basic math problem (National Numeracy, n.d.). In recent years, teachers have been more aware of the need for automaticity (computational fluency) and they are becoming more aware of the number of students who lack fluency in math (Pegg, Graham & Bellert, 2005, p. 4-49). Addition, subtraction, multiplication, and division skills are taught to students as early as kindergarten, but many high school students continue to struggle with fluency in the basic operations. Students need to be able to effectively master basic computations in mathematics before progressing to more general and abstract computations that require mathematical reasoning.

In the past, automaticity diagnostics have been used in schools to identify students' overall skills and to help increase the students' overall numeracy (Gersten & Chard, n.d.). Many students have not developed numeracy early on in school, which may have led to students' mathematical fluency declining (Jitendra & Sood, 2007, p. 145). Studies

have shown that when children cannot compute basic math problems with a quick response, the students will then not be able to carry out more difficult tasks in high school math classes.

Many students tend to rely on calculators because they are not comfortable with adding integers (or performing any math computation, for that matter). According to Caron, students benefit from additional practice, but not from additional testing (Caron, 2007, p. 278). Automaticity diagnostics and worksheets help to solve this problem because they are based on the premise that "with extended practice, specific skills can reach a level of proficiency where skill execution is rapid and accurate with little or no conscious monitoring" (Gersten & Chard, n.d.). Thus, the automaticity worksheets are used to decrease constant drilling of number skills.

In the 1970s, teachers began to notice that students had a lower fluency than what was expected and have been trying to overcome the challenges in the classroom ever since (Hung & Wang, 2010, p. 19). If a student cannot compute $5 + 7$ without the use of a calculator, it can be difficult for teachers to teach new content to students. It is imperative

for students to have fluency and numeracy in mathematics. Since improving automaticity has been shown to improve students' overall numeracy, teachers have begun to use automaticity worksheets to help students acquire the necessary skills to succeed (Gersten & Chard, n.d.).

At Eastern Kentucky University, Dr. Robert Thomas has been teaching students the benefits of automaticity diagnostics and worksheets. Dr. Thomas mentors students and helps pre-service math teachers learn how to help increase students' fluency in math by using the automaticity diagnostics and worksheets. Dr. Thomas modified the comprehensive test used by Gersten and Chard. The tests are to be given at the beginning, middle, and end of each year to test the students' fluency in basic mathematics skills. The comprehensive diagnostic test consists of addition, subtraction, multiplication, and division.

In Gersten & Chard's study, automaticity worksheets were used to increase students' fluency in math. The automaticity worksheets were given as a bell ringer each day, Monday through Friday. A bell ringer is a small group of questions that are given at the beginning of class to help students get on task. Results showed that the

students' overall fluency increased, but anecdotal data suggested that the students were bored, rather than enthusiastic, about the worksheets. To help the students become more engaged and interested in the automaticity worksheets, exit slips (consisting of a few problems to be completed in under a minute before the end of class) were given. The students' performance showed that they did not work to their full potential at the end of the period.

While practicing automaticity increases the students' overall numeracy and math computational fluency, teachers have a limited amount of time in the classroom. Therefore, it is crucial to produce results in student achievement effectively. The purpose of this study was to determine which method of implementation is more effective: requiring students to practice automaticity three times per week or requiring students to practice automaticity five times per week.

CHAPTER II

LITERATURE REVIEW

It is imperative for students to have the opportunity to develop number sense early in life to become more fluent with mathematics skills. In order for students to have number sense, they must be comfortable when working with numbers, be able to complete a mental math problem, and be able to compare numbers and numerals (Gersten & Chard, n.d.). Children begin to develop number sense at an early age and continue to build on the knowledge as new math skills are acquired and developed. Students need to be given the opportunity to develop a solid number sense. According to Jitendra and Sood, students currently have a low proficiency rate in mathematics (Jitendra & Sood, 2007, p. 145). Thus, teachers need to facilitate the improvement of numeracy in order for students to develop higher order mathematical thinking.

In 2011, Ivrendi conducted a study to determine what factors play a role in the development or lack of development of number sense in children (Ivrendi, 2011, p. 239). The study was conducted to determine how age, gender, parental income and education levels affect a child's

overall number sense. Only 71 of the 101 children studied were used in the data analysis because "a subsample of 30 children was randomly chosen for the reliability procedures of Assessing Number Sense and Head, Toes, Knees and Shoulders instruments" (Ivrendi, 2011, p. 239). The children were from a variety of economic levels and were evenly distributed among low, middle and high income families. Specific questions were asked to measure the child's ability to complete number production, counting, operations, and estimation.

In order to measure the children's self-regulation in the Ivrendi study, the instrument *Head, Toes, Knees and Shoulders* was utilized. *Head, Toes, Knees and Shoulders* is an instrument that measures children's memory, attention, and control; it asks students to do the opposite of what the teacher is telling students to do (i.e., if students are told to touch their heads, they would need to touch their toes) (Ponitz, et al., 2008, p. 141-158). The study took many variables into account to determine the most influential factor on the development of number sense (Ivrendi, 2011, p. 239). The study showed that self-regulation is the most influential factor in the

development of number sense. In order for someone to have self-regulation, they would need to have the ability to monitor and control their thoughts; thus, it is important for students to have self-regulation when working on mathematics. Students who stay focused in class and stay on topic benefit the most from learning in a classroom setting.

Teachers must help young students become adequately prepared to self-regulate in order to become fluent in number sense. By learning to self-regulate at an early age, students will be able to carry out higher order thinking later on in life. Therefore, when teachers determine how students are learning and obtaining fluency in math, they should also consider having the students learn about self-regulation to help enable each student to succeed in math, acquire computational fluency, and gain numeracy.

The Kentucky Core Academic Standards require first grade math teachers to incorporate activities that help build students' number sense. Upon leaving the first grade, students should "understand the order of the counting numbers and their relative magnitudes" (Kentucky Department of Education, 2010, p. 55). Number sense is also included

in the Quality Core Algebra I and Algebra II standards. Both Algebra I and Algebra II Quality Core standards have establishing number sense and operation skills as a foundational concept. In Algebra I the students are expected to:

Evaluate and simplify expressions requiring addition, subtraction, multiplication, and division with and without grouping symbols, translate real-world problems into expressions using variables to represent values, apply algebraic properties to simplify algebraic expressions, add and subtract polynomials, factor a monomial from a polynomial, multiply monomials, binomial, trinomials, and polynomials (Quality Core, 2011, p. 3).

In Algebra II, the Quality Core standards under establishing number sense and operation skills are the following: "identify complex numbers and write their conjugates, add, subtract, and multiply complex numbers, simplify quotients of complex numbers, perform operations on functions, including function composition, and determine domain and range for each of the given functions" (Quality Core, 2011, p. 3). Thus, Algebra I students must learn

basic mathematics before moving on to more complex mathematical problems in Algebra II.

As students develop number sense and continue to build upon fluency, teachers must make numeracy development a priority so that the students are able to recall mathematical facts automatically. This process has come to be called "automaticity" in mathematics and can be defined as the "phenomenon that a skill can be performed with minimal awareness of its use" (Axtell, et al., 2009, p. 527). Students need to be able to recall math facts when asked because students will need those basic skills to succeed in higher math.

According to Woodward, when a student does not have computational automaticity, that child has a higher cognitive load than other individuals who have automaticity when performing complex mathematical operations (Woodward, 2006, p. 269). A student whose cognitive load is too high has more information than the student can process. Math becomes more complex as students advance to higher grade levels. All students need to be able to do basic math computations (i.e., add, subtract, multiply, and divide) because each of the basic math computations is used in

everyday life. Being able to do numerical computations quickly will help students succeed as they develop their mathematical skills (Pegg, Graham & Bellert, 2005, p. 4-49).

Axtell demonstrated that students with automaticity tend to score higher on achievement tests that measure higher level skill development and retain more new knowledge after a period of time than those students without automaticity (Axtell, et al., 2009, p. 527). Students must not only be able to respond correctly, but students should also be fluent in math and have a short response time (Axtell, et al., 2009, p. 527).

Another advantage of being able to work fluently with basic math skills is that there is a lower chance of developing math anxiety (Axtell, et al., 2009, p. 527). Students tend to have math anxiety at some point in their schooling, but if teachers help students become more fluent with their mathematics, the students should not be as apprehensive about math. Students can become more confident in mathematical skills if a teacher helps them develop automaticity. Encouraging students to learn automaticity

will help them become fluent in math. Thus, students must practice automaticity in order to master automaticity.

Not all students will have automaticity with numbers; therefore, teachers may approach developing students' automaticity in different ways. Some may teach facts, some may drill math problems, and others will do both (Woodward, 2006, p. 269). Studies, such as that of Axtell, advise that constantly drilling students on math facts is not inevitably going to lead to students' achieving automaticity (Caron, 2007, p. 278). Teachers need to have students practice automaticity, but constant repetition may not be the best way to help students improve in automaticity.

One way to increase automaticity is Detect, Practice, and Repair. The procedure of Detect, Practice, and Repair has the same features that the automaticity worksheets have: "brief response times, many opportunities to respond, immediate feedback, and a self-management component in the form of self-graphing" (Axtell, et al., 2009, p. 529). A trial was conducted in 2009 to prove that Detect, Practice, and Repair would be beneficial to students learning automaticity. The trial included 36 middle school students

(13 in the control group and 23 in intervention). The procedure was as follows: students were given folders, the teacher read the instructions, and a metronome was set to 40 beats per minute. Then the students were given 80 seconds to complete the first page, which contained 48 division problems. The teacher displayed an answer board. Students found five problems that were incorrect. On the second page the students reviewed these five problems and the answer board was then removed. The teacher gave the students 60 seconds to complete the same problems that were rearranged on the final sheet, and in the end the students recorded their own progress (Axtell, et al., 2009, p. 527). The trial showed that the Detect, Practice, and Repair procedure increased the automaticity of students.

Hence, in order for all students to be able to succeed in mathematics, it is necessary for teachers to train students in automaticity. Studies have shown that it is vital to begin studying basic mathematics as early as kindergarten, and teachers should have their students begin practicing these skills. Thus, the various studies have also shown that in order for students to be more successful in math classes and to be able to solve higher order

thinking mathematical problems, students need to be able to quickly recall basic mathematical steps, which are developed through automaticity-type drills.

CHAPTER III

RESEARCH

In order to determine which method of implementation is more effective in increasing automaticity, research was conducted at Madison Southern High School in Berea, Kentucky. The two methods that were implemented were the following: (1) requiring students to practice automaticity three times per week and (2) requiring students to practice automaticity five times per week. The research using the automaticity diagnostics and worksheets took place in two Algebra I classes.

The Algebra I students entered high school in August 2013 with adequate pre-algebra skills (students were given a pre-test on prerequisite algebra skills on the first day of school). The students have been taught basic math skills in elementary and middle school. However, their skills needed to be assessed during the first week of school to help determine the overall growth of the students by the end of the study.

The students' overall automaticity ability determined which skills needed to be taught initially, starting with the first unit. The units were planned around the students'

skills, and the automaticity worksheets were given and discussed more in depth as time progressed. Not all students were comfortable with taking the timed worksheets, which meant students first needed to be trained on strategies in order to perform well. In a classroom, students learn from how a teacher teaches a topic. This concept is the same when dealing with learning new ways to perform various tasks in the classroom. One way a teacher can teach students how to handle timed tests is to allow the students to practice timed worksheets without being graded on performance. The researcher administered timed worksheets for practice to help the students be more comfortable with the timed tests.

The automaticity diagnostic test was administered on the third day of school (Appendix H). The diagnostic test was given to diagnose or assess all basic mathematics skills. After the students were tested over the addition, multiplication, subtraction, and division facts, the students recorded the overall results. Each student was given a sheet for recording time and overall scores on the diagnostics. The automaticity record sheet (Appendix E) has three rows, which include August, January, and May; but the

students altered the months to match those used in the study. The results indicated that many of the students took more than 10 minutes to complete the diagnostic test, and many students did not have all the correct answers. The data were used throughout the study to track the students' growth.

During the study, the automaticity worksheets were administered three days per week to the first period Algebra I class and five days per week to the fourth period Algebra I class. It was only possible for the first period Algebra I class to be given the worksheets three days per week because the students are in middle school and they are bused to the high school for class. They do not always attend five days per week; there were times during the study when the students from the middle school missed class. Therefore, the worksheets could not be given to the first period students every day due to the variations in their scheduling.

The automaticity worksheets (Appendix D) were given in both of the Algebra I classes at the beginning of class as a bell ringer and were graded when the given time elapsed. As instruction began, the students knew whether or not to

get an automaticity worksheet (first period only). If the students were not completing an automaticity worksheet, then they were working on a different bell ringer.

The worksheets begin with adding three. At the beginning of the study, the first worksheet (Appendix D) was distributed. Each operational treatment ranges from three through nine, except for multiplication, which ranges from two through nine. Each worksheet consists of either 49 or 56 questions of the same operation. All students began on the same remediation sheet (add three). A student can meet the benchmark and move on to the next worksheet if two or fewer problems are missed in two minutes or less. An online stopwatch was projected onto the whiteboard in order for the students to track the time used during the assessment. While the students graded the worksheets, they were observed to ensure no one was copying from another student or misstating the number of correct answers.

The directions for administering Dr. Thomas' revised worksheets are as follows:

- To begin, the teacher projects the timer or stopwatch set at 0 minutes (runs up to time limit).

- Once started, each student works on a remediation sheet for up to 5 minutes. Once completed, the student can grade the sheet (answers are at the top) using a colored pen or pencil. The student then records the elapsed time and number of problems missed or not completed. The teacher decides the method of recordkeeping.
- No time is allowed beyond 5 minutes. Students must stop working and grade their remediation sheets. Problems not completed are marked wrong.
- Each student progresses to the next remediation sheet when the amount of time required to complete the sheet drops to 2 minutes or less and 2 or less problems are incorrect. (Teacher discretion advised)
- The student works through the addition remediation sheets completely before moving to the multiplication, subtraction and division remediation sheets.

- Order of completion: Addition, Multiplication, Subtraction, Division.
- Once mastered, students then strive toward a maximum time of less than 1 minute per sheet. (Ultimate goal is automaticity) (Thomas, 2013¹).

When the students finished the automaticity bell ringer, they graded the papers using the answers at the top of the sheet. Each student graded his or her own paper using a colored pen or pencil to receive immediate feedback. At first, the answers were a crutch for some students (i.e., some students relied on the answers to finish the paper), but eventually the goal was for students not to have the urge to look at the answers (the students should be able to recall the answers "automatically"). The more time students spent on the automaticity worksheets, the more mathematics facts the students were able to recall.

After grading a worksheet, each student recorded the time and test grade (the number of correct answers out of the number of questions) on a tracking sheet (Appendix E).

¹ Complete directions for the automaticity protocol are in Appendix G.

There were no penalties for having to redo a worksheet because the worksheets were designed so students can move at their own pace. Once a student finished the addition automaticity bell ringers, that student moved on to multiplication, subtraction, and division.

Results from the diagnostics test were recorded using a separate Excel spreadsheet for each class. No student was singled out, and each student's name was changed to a number so that no individual could be identified. Tracking students was not difficult because the students were continuously tracking their work in a designated folder.

Each week, results were recorded to determine how the students progressed (or regressed). After each diagnostic test was given, the data were used to determine how many students improved and the amount of growth each student showed from the pre-test to the post-test.

Chapter IV

RESULTS

Throughout the study, students in two Algebra I classes at Madison Southern High School were assessed using the automaticity diagnostics. The overall diagnostic test (Appendix C) was given to both classes on August 16th, before the study began, to test the students' automaticity. School began on August 14th, but the study did not begin until the second week of school. Since the students were getting individually selected daily work (the automaticity worksheets), they could be considered to be the experimental units in the study.

Students' individual diagnostic pre-test scores and the amount of time the students used to complete the diagnostic test were recorded. The rate for a student was figured as follows: the number of correct answers divided by the amount of time a student used to complete the diagnostic test (the students were allowed up to twelve minutes). The data from the first period Algebra I class indicated that the mean score from the first diagnostic test was 98 out of 105 with a mean time of 7 minutes and 1 second (Appendix A). The data from the fourth period Algebra I class showed that the mean score from the first

diagnostic test was 91 out of 105 with a mean time of 10 minutes and 9 seconds (all students were allotted a maximum of 12 minutes) (Appendix B).

Each week the classes continued to complete the automaticity worksheets as bell ringers in order to give students practice and build their automaticity. Then on September 13th, the students were given the diagnostic test for the second time. The students had completed the automaticity worksheets for approximately four weeks. The first period students' mean score on the second diagnostic test was 103 out of 105 and the mean time used during the second diagnostic test was 6 minutes and 1 second.

The fourth period Algebra I class also completed the automaticity diagnostic test for the second time and increased their mean score and decreased their mean time. The average number of questions answered correctly in the fourth period on the second test was 97 out of 105, and the mean time used on the second diagnostic test was 9 minutes and 6 seconds. The ECU Transitions classes administer the diagnostic test three times during the duration of a school year, which is why the second diagnostic test was given,

even though results from the second test were not used in the overall analysis of growth for each class.

The diagnostic test was given a third time (post-test) on October 14th, and the data showed that first period's mean rate increased from 15.1 to 20.3 correct problems per minute to give a change of 5.2 correct problems per minute. The fourth period's mean rate increased from 9.6 to 12.7 correct problems per minute to give a change of 3.1 correct problems per minute (the change in mean rate included the first and the third diagnostic tests).

The Algebra I class rates were compared to the EKU Transitions values (the EKU Transitions document was used as a benchmark to compare with the study). For the students in the 2012-2013 EKU Transitions Program, the mean rate increased from 8.09 correct problems per minute in the fall of 2012 to 10.79 correct problems per minute in the spring of 2013 to give a change of 2.7 correct problems per minute (Thomas, 2013²). The rate increase is less than both of the Algebra I classes, but the mean score and mean time were both based on a total of 26,484 students. According to the EKU Transitions document in Appendix I, the document was

² The EKU Transitions table is located in Appendix F.

created "as a rough guide for evaluating the performance of students taking the automaticity pretest" (Thomas, 2013). In order to find a student's percentile ranking using the document in Appendix I, the corresponding grade level is listed as a column, and the percentile ranking is listed on the left at the beginning of the rows. The mean rate of the post-test was found in the corresponding grade level column of the ECU Transitions document.

First period included 27 students, and 26 students showed rate increases (one student did not take the pre-test); fourth period included 22 students, and 21 students showed increases. The student growth was computed by finding the difference in post-test rate and pre-test rate. The first period students increased their mean automaticity rate percentile by 22.5 percentage points and their median automaticity rate percentile by 23.2 percentage points. The fourth period students increased their mean automaticity rate percentile by 7.75 percentage points and their median automaticity rate percentile by 18.8 percentage points (Appendix J).

One factor that may have contributed to these performance differences was related to the initial student

abilities. The first period students were all accelerated middle school students who outscored the fourth period on the initial diagnostic by substantial margins: a mean rate of 15.1 correct problems per minute versus a mean rate of 9.6 correct problems per minute. The second factor that may have contributed to the performance difference was the abbreviated treatment duration. The treatment only lasted two months. The research tends to indicate that the brief treatment time per day is most effective over a long-term period.

The data for the two classes were compared using Minitab. The scores were copied into Minitab; the difference in scores was calculated for each student. Figure 1³ and Figure 2 show the growth in automaticity scores for the Algebra I students. The stacked dotplot indicates that fourth period students made greater gains from the pre-test to the post-test than first period students in raw scores. The boxplot in Figure 2 shows how the first period students' scores compared to the fourth period students' scores.

³ Figure 1, Appendix K.

results were not significant at the .05 level. The p value was .208, and the t statistic was -1.29. The t statistic and the corresponding large p value indicated there is not enough evidence to conclude that there is a difference in the means.

Figures 3 and 4⁴ show how the differences in rates of the first period students were much higher than the differences in rates of the fourth period students. Although the first period students did not have as much room for growth, the students in that section could still use the data to observe their improvements. In particular, since the rate includes the time taken to complete the diagnostic, they could see whether they were getting faster. Thus, all students, including students who began with a perfect score of 105 out of 105 on the diagnostic test, could see their improvements using the rate.

⁴ Figure 4, Appendix L.

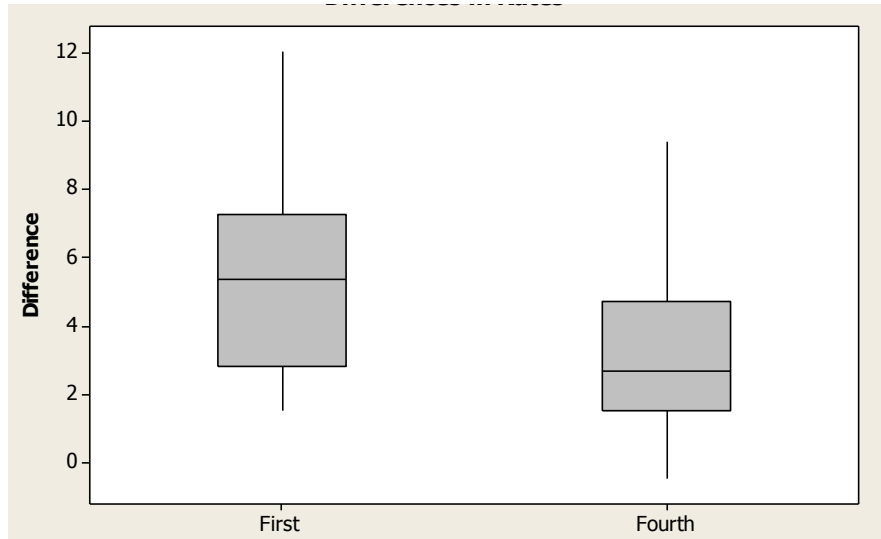


Figure 3. Differences in Rates Boxplot.

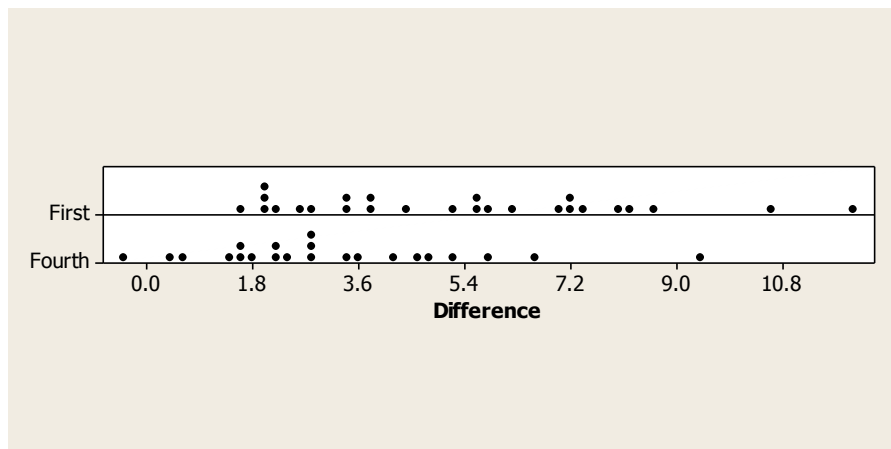


Figure 4. Differences in Rates Dotplot.

A two-sample t test was performed to compare the means of the differences in rate for the two classes (Appendix L). The t test results were significant at the .05 level. The p value was .005, and the t statistic was 2.96. The t statistic and the corresponding small p value indicated

there is enough evidence to conclude that there is a difference in the means.

In conclusion, virtually all students showed measureable increases in automaticity. The Algebra I classes and the ECU Transitions classes showed an increase in mean rate from using the automaticity worksheets and diagnostics. There was a difference in the raw scores in the two groups in the sample, but not a statistically significant one. However, using automaticity worksheets three days per week produced significantly more growth in rates.

Chapter V

LIMITATIONS

One of the limitations of the study was that there was a small sample of students. Since there were only two Algebra I courses available to participate in the study, the study could not include more students. There were 27 students in first period and 22 students in fourth period.

A second limitation of the study was that the sample was not randomly selected. Hence, the results could not be generalized to a larger population of interest.

Another limitation of the study was that randomization was not possible, which means that causation could not be established. The students could not be randomly assigned to the classes because the counselors determine class size and which math course students will be taking. Also, the treatments could not be randomly assigned to the two classes. The first period students were in middle school, and the schedules for high school and middle school are not always the same. The first period class did not always attend every day of the week. For example, they did not attend on the day the EXPLORE test was given at Foley

Middle School. This situation dictated how many days per week each class was able to do automaticity worksheets.

A fourth limitation of the study, related to the third limitation, was the differences in the students. For example, the first period Algebra I class consisted of eighth graders who tested into Algebra I and was conducted between 8:10 a.m. and 9:05 a.m. The students tested on a ninth-grade level on the MAP test (Measures of Academic Progress test) and were placed in the Algebra I class. The MAP test data are used to help teachers know what the student achievement levels are before entering the class. The fourth period Algebra I class included freshmen, and the class was conducted between 11:25 a.m. and 12:25 p.m. It is important to note that middle school students differ from high school students in important ways, which may have influenced the statistics.

A fifth limitation was that data from the ECU Transitions classes were used as a benchmark, but that data only included students who were administered the automaticity worksheets five days per week. There were no supplemental data to compare with the students who completed the worksheets three days per week.

REFERENCES

- Axtell, P. K., McCallum, R., Mee Bell, S., & Poncy, B. (2009). Developing math automaticity using a classwide fluency building procedure for middle school students: A preliminary study. *Psychology In The Schools, 46*(6), 526-538.
- Caron, T. A. (2007). Learning multiplication: The easy way. *Clearing House, 80*(6), 278-282.
- College Readiness Standards. (2012). ACT. Retrieved from <http://www.oprfhs.org/assessment-and-research/documents/ACTIdeasforProgress.pdf>
- Gersten, R., & Chard, D. J. (n.d.). Retrieved from <http://www.1donline.org/article/5838/>
- Hung, L., Wang, Z., (2010). Kindergarten children's number sense development through board games. *The International Journal of Learning, 17*(8), 19-32.
- Ivrendi, A., (2011). Influence of self-regulation on the development of children's number sense. *Early Childhood Education Journal I, 39*(4): 239-247.
- Jitendra, A. K, Sood, S., (2007). A comparative analysis of number sense instruction in reform-based and

traditional mathematics textbooks. *Journal Of Special Education*, 41(3), 145-157.

Kentucky Department of Education. (2010). *Kentucky Core Academic Standards*. Retrieved from <http://education.ky.gov/curriculum/docs/Documents/PO S%20with%20CCS%20for%20public%20review.pdf>

National Numeracy. (n.d.). Retrieved from <http://www.nationalnumeracy.org.uk/what-is-numeracy/index.html>

Pegg, J., Graham, L., & Bellert, A. (2005). The effect of improved automaticity of basic number skills on persistently low-achieving pupils. (P), 4.49-4.56. Retrieved from <http://www.emis.de/proceedings/PME29/PME29RRPapers/PME29Vol4PeggEtAl.pdf>

Ponitz, C., McClelland, M., Jewkes, A., Conner, C., Farris, C., & Morrison, F. (2008). Touch your toes! Developing a direct measure of behavioral regulation in early childhood. *Early Childhood Research Quarterly*, 23, 141-158.

Quality Core: Algebra II ACT Course Standards.(2011).

Quality Core. Retrieved from

<http://www.act.org/qualitycore/>

Thomas, Robert, (2013). Transitions data recap; Eastern Kentucky University, Department of Mathematics and Statistics.

Thomas, Robert, (2013). ECU transitions.

Woodward, J. (2006). Developing automaticity in multiplication facts: Integrating strategy instruction with timed practice drills. *Learning Disability Quarterly*, 29(4), 269-289.

APPENDIX A:
Algebra I 1st Period Data

APPENDIX B:
Algebra I 4th Period Data

APPENDIX C:
Automaticity Diagnostic Test

Time _____

Name _____

Teacher _____

Grade _____ Age _____

9	9	5	6	8	7	9
<u>+6</u>	<u>+12</u>	<u>+4</u>	<u>+5</u>	<u>+3</u>	<u>+4</u>	<u>+2</u>

7	7	12	9	9	7	4
<u>+7</u>	<u>+8</u>	<u>+4</u>	<u>+11</u>	<u>+6</u>	<u>+8</u>	<u>+7</u>

3	12	7	19	92	76	54
<u>+5</u>	<u>+9</u>	<u>+15</u>	<u>+8</u>	<u>+7</u>	<u>+12</u>	<u>+45</u>

17	11	16	19	9	15	49
<u>-8</u>	<u>-8</u>	<u>-9</u>	<u>-11</u>	<u>-6</u>	<u>-8</u>	<u>-37</u>

33	12	47	104	19	92	76
<u>-5</u>	<u>-9</u>	<u>-15</u>	<u>-43</u>	<u>-8</u>	<u>-7</u>	<u>-12</u>

5	5	5	5	5	5	5
<u>×11</u>	<u>×1</u>	<u>×5</u>	<u>×4</u>	<u>×12</u>	<u>×8</u>	<u>×2</u>

5	5	12	5	5	5	5
<u>×7</u>	<u>×9</u>	<u>×5</u>	<u>×10</u>	<u>×6</u>	<u>×3</u>	<u>×0</u>

Form A

$\frac{6}{\times 8}$ $\frac{6}{\times 1}$ $\frac{6}{\times 7}$ $\frac{8}{\times 6}$ $\frac{6}{\times 0}$ $\frac{6}{\times 4}$ $\frac{6}{\times 2}$

$\frac{6}{\times 5}$ $\frac{6}{\times 9}$ $\frac{6}{\times 6}$ $\frac{6}{\times 10}$ $\frac{6}{\times 12}$ $\frac{6}{\times 11}$ $\frac{6}{\times 3}$

$\frac{7}{\times 8}$ $\frac{7}{\times 1}$ $\frac{7}{\times 9}$ $\frac{7}{\times 3}$ $\frac{7}{\times 4}$ $\frac{7}{\times 12}$ $\frac{7}{\times 2}$

$\frac{7}{\times 5}$ $\frac{7}{\times 7}$ $\frac{7}{\times 6}$ $\frac{7}{\times 0}$ $\frac{7}{\times 10}$ $\frac{7}{\times 11}$ $\frac{7}{\times 6}$

$\frac{8}{\times 8}$ $\frac{8}{\times 1}$ $\frac{3}{\times 8}$ $\frac{8}{\times 3}$ $\frac{8}{\times 4}$ $\frac{8}{\times 9}$ $\frac{8}{\times 2}$

$\frac{8}{\times 5}$ $\frac{8}{\times 7}$ $\frac{12}{\times 6}$ $\frac{8}{\times 0}$ $\frac{10}{\times 8}$ $\frac{8}{\times 11}$ $\frac{8}{\times 6}$

$\frac{9}{\times 12}$ $\frac{9}{\times 4}$ $\frac{9}{\times 6}$ $\frac{9}{\times 9}$ $\frac{9}{\times 3}$ $\frac{9}{\times 0}$ $\frac{7}{\times 9}$

$\frac{3}{\times 9}$ $\frac{9}{\times 11}$ $\frac{9}{\times 2}$ $\frac{9}{\times 8}$ $\frac{9}{\times 1}$ $\frac{9}{\times 10}$ $\frac{5}{\times 9}$

APPENDIX D:
Automaticity Worksheets

Time _____

Name _____

$3+1=4$	$3+2=5$	$3+3=6$	$3+4=7$	$3+5=8$	$3+6=9$
$3+7=10$	$3+8=11$	$3+9=12$	$3+10=13$	$3+11=14$	$3+12=15$

3	3	3	8	3	7	4
<u>+8</u>	<u>+1</u>	<u>+7</u>	<u>+3</u>	<u>+3</u>	<u>+3</u>	<u>+3</u>

3	3	3	3	10	3	4
<u>+5</u>	<u>+9</u>	<u>+6</u>	<u>+10</u>	<u>+3</u>	<u>+11</u>	<u>+3</u>

3	3	12	3	3	8	3
<u>+4</u>	<u>+8</u>	<u>+3</u>	<u>+11</u>	<u>+5</u>	<u>+3</u>	<u>+7</u>

3	4	3	3	12	3	11
<u>+5</u>	<u>+3</u>	<u>+2</u>	<u>+4</u>	<u>+3</u>	<u>+11</u>	<u>+3</u>

3	3	3	3	3	9	3
<u>+9</u>	<u>+10</u>	<u>+6</u>	<u>+8</u>	<u>+5</u>	<u>+3</u>	<u>+11</u>

6	3	3	3	3	4	3
<u>+3</u>	<u>+12</u>	<u>+10</u>	<u>+11</u>	<u>+7</u>	<u>+3</u>	<u>+7</u>

4	3	6	3	3	11	3
<u>+3</u>	<u>+8</u>	<u>+3</u>	<u>+5</u>	<u>+12</u>	<u>+3</u>	<u>+2</u>

3	9	3	3	4	3	3
<u>+7</u>	<u>+3</u>	<u>+3</u>	<u>+6</u>	<u>+3</u>	<u>+11</u>	<u>+10</u>

Time _____

Name _____

$4+1=5$	$4+2=6$	$4+3=7$	$4+4=8$	$4+5=9$	$4+6=10$
$4+7=11$	$4+8=12$	$4+9=13$	$4+10=14$	$4+11=15$	$4+12=16$

4	4	4	8	4	7	4
<u>+8</u>	<u>+1</u>	<u>+7</u>	<u>+4</u>	<u>+4</u>	<u>+4</u>	<u>+2</u>

4	4	4	4	4	4	4
<u>+5</u>	<u>+9</u>	<u>+6</u>	<u>+10</u>	<u>+12</u>	<u>+11</u>	<u>+3</u>

4	4	12	4	4	8	4
<u>+4</u>	<u>+8</u>	<u>+4</u>	<u>+12</u>	<u>+5</u>	<u>+4</u>	<u>+7</u>

4	4	4	6	4	4	11
<u>+5</u>	<u>+3</u>	<u>+2</u>	<u>+4</u>	<u>+10</u>	<u>+11</u>	<u>+4</u>

4	4	4	4	4	9	4
<u>+9</u>	<u>+10</u>	<u>+6</u>	<u>+8</u>	<u>+5</u>	<u>+4</u>	<u>+11</u>

6	4	4	4	4	4	4
<u>+4</u>	<u>+12</u>	<u>+10</u>	<u>+11</u>	<u>+7</u>	<u>+4</u>	<u>+7</u>

4	4	4	4	4	4	4
<u>+3</u>	<u>+8</u>	<u>+6</u>	<u>+5</u>	<u>+12</u>	<u>+11</u>	<u>+2</u>

4	4	4	4	4	4	4
<u>+7</u>	<u>+9</u>	<u>+4</u>	<u>+6</u>	<u>+3</u>	<u>+11</u>	<u>+10</u>

Time _____

Name _____

$5+1=6$	$5+2=7$	$5+3=8$	$5+4=9$	$5+5=10$	$5+6=11$	
$5+7=12$	$5+8=13$	$5+9=14$	$5+10=15$	$5+11=16$	$5+12=17$	

5	7	5	5	5	5	5
<u>+8</u>	<u>+1</u>	<u>+5</u>	<u>+4</u>	<u>+7</u>	<u>+9</u>	<u>+2</u>

7	5	5	5	5	5	5
<u>+5</u>	<u>+9</u>	<u>+6</u>	<u>+8</u>	<u>+12</u>	<u>+11</u>	<u>+3</u>

5	5	12	5	7	5	4
<u>+7</u>	<u>+8</u>	<u>+5</u>	<u>+12</u>	<u>+5</u>	<u>+8</u>	<u>+5</u>

7	5	5	6	5	5	11
<u>+5</u>	<u>+3</u>	<u>+2</u>	<u>+5</u>	<u>+8</u>	<u>+11</u>	<u>+5</u>

5	5	5	5	7	5	5
<u>+9</u>	<u>+8</u>	<u>+6</u>	<u>+1</u>	<u>+5</u>	<u>+9</u>	<u>+11</u>

6	5	5	5	5	5	4
<u>+5</u>	<u>+12</u>	<u>+8</u>	<u>+11</u>	<u>+7</u>	<u>+8</u>	<u>+5</u>

5	5	5	7	5	5	5
<u>+3</u>	<u>+8</u>	<u>+6</u>	<u>+5</u>	<u>+12</u>	<u>+9</u>	<u>+2</u>

5	5	5	5	5	5	5
<u>+5</u>	<u>+9</u>	<u>+7</u>	<u>+6</u>	<u>+3</u>	<u>+11</u>	<u>+8</u>

Time _____ Name _____

$6+1=7$	$6+2=8$	$6+3=9$	$6+4=10$	$6+5=11$	$6+6=12$	
$6+7=13$	$6+8=14$	$6+9=15$	$6+10=16$	$6+11=17$	$6+12=18$	

6	6	6	6	6	6	6
<u>+8</u>	<u>+1</u>	<u>+7</u>	<u>+4</u>	<u>+7</u>	<u>+9</u>	<u>+2</u>

6	6	6	6	7	6	6
<u>+5</u>	<u>+9</u>	<u>+6</u>	<u>+10</u>	<u>+12</u>	<u>+11</u>	<u>+3</u>

6	6	12	6	6	8	4
<u>+6</u>	<u>+8</u>	<u>+6</u>	<u>+12</u>	<u>+5</u>	<u>+6</u>	<u>+6</u>

6	6	6	6	6	6	11
<u>+5</u>	<u>+3</u>	<u>+2</u>	<u>+7</u>	<u>+10</u>	<u>+11</u>	<u>+6</u>

6	6	6	6	6	6	6
<u>+9</u>	<u>+10</u>	<u>+6</u>	<u>+1</u>	<u>+5</u>	<u>+9</u>	<u>+11</u>

6	6	6	6	6	6	4
<u>+7</u>	<u>+12</u>	<u>+10</u>	<u>+11</u>	<u>+6</u>	<u>+8</u>	<u>+6</u>

6	6	6	6	6	6	6
<u>+3</u>	<u>+8</u>	<u>+4</u>	<u>+5</u>	<u>+12</u>	<u>+9</u>	<u>+2</u>

6	6	7	6	6	6	6
<u>+6</u>	<u>+9</u>	<u>+6</u>	<u>+6</u>	<u>+3</u>	<u>+11</u>	<u>+10</u>

Time _____ Name _____

7+1=8 7+2=9 7+3=10 7+4=11 7+5=12 7+6=13
 7+7=14 7+8=15 7+9=16 7+10=17 7+11=18 7+12=19

7 7 7 7 7 7 7
+8 +1 +7 +4 +7 +9 +2

7 7 7 7 7 7 7
+5 +9 +6 +10 +12 +11 +3

7 7 12 7 7 7 4
+7 +8 +7 +12 +5 +8 +7

7 7 7 6 7 7 11
+5 +3 +2 +7 +10 +11 +7

7 7 7 7 7 7 7
+9 +10 +6 +1 +5 +9 +11

6 7 7 7 7 7 4
+7 +12 +10 +11 +7 +8 +7

7 7 7 7 7 7 7
+3 +8 +6 +5 +12 +9 +2

7 7 7 7 7 7 7
+7 +9 +7 +6 +3 +11 +10

Time _____

Name _____

$$\begin{array}{cccccc}
 8+1=9 & 8+2=10 & 8+3=11 & 8+4=12 & 8+5=13 & 8+6=14 \\
 8+7=15 & 8+8=16 & 8+9=17 & 8+10=18 & 8+11=19 & 8+12=20
 \end{array}$$

$$\begin{array}{cccccc}
 7 & 8 & 8 & 8 & 8 & 8 & 8 \\
 \underline{+8} & \underline{+1} & \underline{+7} & \underline{+4} & \underline{+8} & \underline{+9} & \underline{+2}
 \end{array}$$

$$\begin{array}{cccccc}
 8 & 8 & 8 & 8 & 8 & 8 & 8 \\
 \underline{+5} & \underline{+9} & \underline{+6} & \underline{+10} & \underline{+12} & \underline{+11} & \underline{+3}
 \end{array}$$

$$\begin{array}{cccccc}
 7 & 11 & 12 & 8 & 8 & 7 & 4 \\
 \underline{+8} & \underline{+8} & \underline{+8} & \underline{+12} & \underline{+5} & \underline{+8} & \underline{+8}
 \end{array}$$

$$\begin{array}{cccccc}
 8 & 8 & 8 & 6 & 8 & 8 & 11 \\
 \underline{+5} & \underline{+3} & \underline{+2} & \underline{+8} & \underline{+10} & \underline{+11} & \underline{+8}
 \end{array}$$

$$\begin{array}{cccccc}
 8 & 8 & 8 & 8 & 8 & 8 & 8 \\
 \underline{+9} & \underline{+10} & \underline{+6} & \underline{+1} & \underline{+5} & \underline{+9} & \underline{+11}
 \end{array}$$

$$\begin{array}{cccccc}
 6 & 8 & 8 & 8 & 8 & 7 & 4 \\
 \underline{+8} & \underline{+12} & \underline{+10} & \underline{+11} & \underline{+7} & \underline{+8} & \underline{+8}
 \end{array}$$

$$\begin{array}{cccccc}
 8 & 7 & 8 & 8 & 8 & 8 & 8 \\
 \underline{+3} & \underline{+8} & \underline{+6} & \underline{+5} & \underline{+12} & \underline{+9} & \underline{+2}
 \end{array}$$

$$\begin{array}{cccccc}
 8 & 8 & 8 & 8 & 8 & 8 & 8 \\
 \underline{+12} & \underline{+9} & \underline{+7} & \underline{+6} & \underline{+3} & \underline{+11} & \underline{+10}
 \end{array}$$

Time _____

Name _____

$9+1=10$ $9+2=11$ $9+3=12$ $9+4=13$ $9+5=14$ $9+6=15$
 $9+7=16$ $9+8=17$ $9+9=18$ $9+10=19$ $9+11=20$ $9+12=21$

9 9 9 9 9 7 9
+8 +1 +4 +5 +9 +9 +2

9 12 9 9 9 9 9
+5 +9 +6 +10 +3 +11 +12

9 9 11 9 9 9 9
+9 +8 +9 +12 +5 +8 +9

9 9 9 6 9 9 8
+5 +3 +2 +9 +10 +11 +9

7 9 9 9 9 7 9
+9 +10 +6 +1 +5 +9 +11

6 9 9 9 3 9 4
+9 +12 +10 +11 +9 +8 +9

9 9 9 9 9 7 9
+3 +8 +6 +5 +12 +9 +2

9 7 9 9 9 9 9
+9 +9 +4 +6 +3 +11 +10

Time _____

Name _____

$3+1=4$	$3+2=5$	$3+3=6$	$3+4=7$	$3+5=8$	$3+6=9$	
$3+7=10$	$3+8=11$	$3+9=12$	$3+10=13$	$3+11=14$	$3+12=15$	

11	14	12	4	8	5	13
<u>-3</u>	<u>-3</u>	<u>-3</u>	<u>-1</u>	<u>-3</u>	<u>-3</u>	<u>-3</u>

15	15	12	10	4	15	11
<u>-3</u>	<u>-12</u>	<u>-9</u>	<u>-7</u>	<u>-3</u>	<u>-3</u>	<u>-8</u>

12	7	13	14	4	10	4
<u>-3</u>	<u>-4</u>	<u>-10</u>	<u>-11</u>	<u>-3</u>	<u>-3</u>	<u>-3</u>

8	15	5	7	6	9	15
<u>-5</u>	<u>-3</u>	<u>-2</u>	<u>-3</u>	<u>-3</u>	<u>-6</u>	<u>-12</u>

14	11	14	12	4	8	15
<u>-11</u>	<u>-3</u>	<u>-3</u>	<u>-3</u>	<u>-1</u>	<u>-3</u>	<u>-12</u>

7	13	14	4	10	4	14
<u>-4</u>	<u>-10</u>	<u>-11</u>	<u>-3</u>	<u>-3</u>	<u>-3</u>	<u>-11</u>

13	9	11	8	15	5	7
<u>-10</u>	<u>-6</u>	<u>-8</u>	<u>-5</u>	<u>-3</u>	<u>-2</u>	<u>-3</u>

Time _____

Name _____

$$\begin{array}{cccccc}
 4+1=5 & 4+2=6 & 4+3=7 & 4+4=8 & 4+5=9 & 4+6=10 \\
 4+7=11 & 4+8=12 & 4+9=13 & 4+10=14 & 4+11=15 & 4+12=16
 \end{array}$$

$$\begin{array}{cccccc}
 12 & 9 & 16 & 8 & 16 & 15 & 11 \\
 \underline{-8} & \underline{-5} & \underline{-4} & \underline{-4} & \underline{-12} & \underline{-11} & \underline{-4}
 \end{array}$$

$$\begin{array}{cccccc}
 6 & 16 & 16 & 15 & 7 & 9 & 8 \\
 \underline{-4} & \underline{-4} & \underline{-12} & \underline{-4} & \underline{-3} & \underline{-4} & \underline{-4}
 \end{array}$$

$$\begin{array}{cccccc}
 13 & 7 & 12 & 14 & 5 & 13 & 5 \\
 \underline{-4} & \underline{-4} & \underline{-8} & \underline{-10} & \underline{-1} & \underline{-9} & \underline{-4}
 \end{array}$$

$$\begin{array}{cccccc}
 6 & 15 & 9 & 7 & 6 & 9 & 15 \\
 \underline{-2} & \underline{-11} & \underline{-4} & \underline{-3} & \underline{-4} & \underline{-5} & \underline{-4}
 \end{array}$$

$$\begin{array}{cccccc}
 15 & 11 & 14 & 12 & 16 & 8 & 16 \\
 \underline{-11} & \underline{-7} & \underline{-4} & \underline{-4} & \underline{-4} & \underline{-4} & \underline{-12}
 \end{array}$$

$$\begin{array}{cccccc}
 7 & 16 & 15 & 5 & 10 & 5 & 14 \\
 \underline{-4} & \underline{-12} & \underline{-11} & \underline{-1} & \underline{-4} & \underline{-4} & \underline{-10}
 \end{array}$$

$$\begin{array}{cccccc}
 14 & 9 & 11 & 8 & 15 & 5 & 7 \\
 \underline{-10} & \underline{-5} & \underline{-7} & \underline{-4} & \underline{-4} & \underline{-1} & \underline{-3}
 \end{array}$$

Time _____

Name _____

$5+1=6$	$5+2=7$	$5+3=8$	$5+4=9$	$5+5=10$	$5+6=11$
$5+7=12$	$5+8=13$	$5+9=14$	$5+10=15$	$5+11=16$	$5+12=17$

12	9	16	8	16	15	11
<u>-7</u>	<u>-5</u>	<u>-5</u>	<u>-3</u>	<u>-11</u>	<u>-10</u>	<u>-5</u>

6	11	17	15	7	9	9
<u>-5</u>	<u>-5</u>	<u>-12</u>	<u>-5</u>	<u>-2</u>	<u>-4</u>	<u>-5</u>

13	7	12	15	8	13	16
<u>-5</u>	<u>-2</u>	<u>-7</u>	<u>-10</u>	<u>-3</u>	<u>-8</u>	<u>-11</u>

7	15	9	7	9	9	15
<u>-2</u>	<u>-10</u>	<u>-4</u>	<u>-5</u>	<u>-4</u>	<u>-5</u>	<u>-5</u>

8	11	17	12	16	8	9
<u>-5</u>	<u>-5</u>	<u>-5</u>	<u>-5</u>	<u>-5</u>	<u>-5</u>	<u>-5</u>

7	16	15	6	10	6	14
<u>-5</u>	<u>-11</u>	<u>-10</u>	<u>-1</u>	<u>-5</u>	<u>-5</u>	<u>-10</u>

17	9	11	8	15	6	7
<u>-12</u>	<u>-5</u>	<u>-6</u>	<u>-3</u>	<u>-5</u>	<u>-1</u>	<u>-2</u>

Time _____

Name _____

$6+1=7$	$6+2=8$	$6+3=9$	$6+4=10$	$6+5=11$	$6+6=12$
$6+7=13$	$6+8=14$	$6+9=15$	$6+10=16$	$6+11=17$	$6+12=18$

11	16	13	18	15	10	13
<u>-5</u>	<u>-6</u>	<u>-7</u>	<u>-6</u>	<u>-6</u>	<u>-6</u>	<u>-6</u>

15	15	12	17	13	16	20
<u>-6</u>	<u>-9</u>	<u>-6</u>	<u>-6</u>	<u>-6</u>	<u>-6</u>	<u>-6</u>

11	17	18	17	18	10	12
<u>-6</u>	<u>-11</u>	<u>-12</u>	<u>-11</u>	<u>-6</u>	<u>-4</u>	<u>-6</u>

15	15	14	16	15	11	18
<u>-6</u>	<u>-9</u>	<u>-8</u>	<u>-6</u>	<u>-9</u>	<u>-5</u>	<u>-12</u>

17	12	15	10	18	17	18
<u>-11</u>	<u>-6</u>	<u>-9</u>	<u>-4</u>	<u>-12</u>	<u>-6</u>	<u>-12</u>

15	14	12	11	16	14	16
<u>-9</u>	<u>-6</u>	<u>-6</u>	<u>-5</u>	<u>-10</u>	<u>-8</u>	<u>-10</u>

13	10	15	18	12	15	14
<u>-6</u>	<u>-6</u>	<u>-9</u>	<u>-12</u>	<u>-6</u>	<u>-6</u>	<u>-6</u>

Time _____

Name _____

$$\begin{array}{cccccc} 7+1=8 & 7+2=9 & 7+3=10 & 7+4=11 & 7+5=12 & 7+6=13 \\ 7+7=14 & 7+8=15 & 7+9=16 & 7+10=17 & 7+11=18 & 7+12=19 \end{array}$$

$$\begin{array}{cccccc} 11 & 16 & 19 & 18 & 15 & 10 & 13 \\ \underline{-4} & \underline{-9} & \underline{-7} & \underline{-7} & \underline{-7} & \underline{-7} & \underline{-6} \end{array}$$

$$\begin{array}{cccccc} 15 & 16 & 12 & 17 & 13 & 16 & 20 \\ \underline{-8} & \underline{-9} & \underline{-5} & \underline{-7} & \underline{-6} & \underline{-7} & \underline{-7} \end{array}$$

$$\begin{array}{cccccc} 11 & 18 & 19 & 18 & 18 & 10 & 12 \\ \underline{-7} & \underline{-11} & \underline{-12} & \underline{-11} & \underline{-7} & \underline{-3} & \underline{-7} \end{array}$$

$$\begin{array}{cccccc} 15 & 15 & 14 & 16 & 16 & 11 & 18 \\ \underline{-7} & \underline{-8} & \underline{-7} & \underline{-9} & \underline{-7} & \underline{-4} & \underline{-11} \end{array}$$

$$\begin{array}{cccccc} 19 & 12 & 16 & 10 & 19 & 17 & 18 \\ \underline{-12} & \underline{-7} & \underline{-9} & \underline{-3} & \underline{-12} & \underline{-7} & \underline{-11} \end{array}$$

$$\begin{array}{cccccc} 15 & 14 & 12 & 11 & 17 & 20 & 19 \\ \underline{-8} & \underline{-7} & \underline{-5} & \underline{-7} & \underline{-10} & \underline{-7} & \underline{-12} \end{array}$$

$$\begin{array}{cccccc} 13 & 10 & 15 & 18 & 12 & 19 & 14 \\ \underline{-6} & \underline{-7} & \underline{-8} & \underline{-11} & \underline{-7} & \underline{-12} & \underline{-7} \end{array}$$

Time _____

Name _____

$8+1=9$	$8+2=10$	$8+3=11$	$8+4=12$	$8+5=13$	$8+6=14$
$8+7=15$	$8+8=16$	$8+9=17$	$8+10=18$	$8+11=19$	$8+12=20$

11	16	20	19	15	10	13
<u>-3</u>	<u>-8</u>	<u>-8</u>	<u>-8</u>	<u>-7</u>	<u>-8</u>	<u>-5</u>

15	16	12	17	13	16	20
<u>-8</u>	<u>-8</u>	<u>-4</u>	<u>-9</u>	<u>-5</u>	<u>-8</u>	<u>-8</u>

11	18	20	19	18	10	12
<u>-3</u>	<u>-10</u>	<u>-12</u>	<u>-11</u>	<u>-8</u>	<u>-2</u>	<u>-8</u>

15	19	14	17	16	11	18
<u>-7</u>	<u>-8</u>	<u>-6</u>	<u>-8</u>	<u>-8</u>	<u>-3</u>	<u>-10</u>

20	19	17	10	20	17	12
<u>-12</u>	<u>-11</u>	<u>-9</u>	<u>-2</u>	<u>-12</u>	<u>-8</u>	<u>-8</u>

13	14	12	11	18	20	19
<u>-8</u>	<u>-8</u>	<u>-4</u>	<u>-8</u>	<u>-10</u>	<u>-8</u>	<u>-11</u>

13	10	15	18	14	20	14
<u>-8</u>	<u>-8</u>	<u>-8</u>	<u>-10</u>	<u>-8</u>	<u>-12</u>	<u>-6</u>

Time _____

Name _____

$9+1=10$	$9+2=11$	$9+3=12$	$9+4=13$	$9+5=14$	$9+6=15$	
$9+7=16$	$9+8=17$	$9+9=18$	$9+10=19$	$9+11=20$	$9+12=21$	

11	16	21	19	15	10	13
<u>-9</u>	<u>-7</u>	<u>-9</u>	<u>-9</u>	<u>-6</u>	<u>-9</u>	<u>-4</u>

15	16	12	17	13	16	20
<u>-9</u>	<u>-9</u>	<u>-3</u>	<u>-9</u>	<u>-4</u>	<u>-7</u>	<u>-9</u>

11	18	21	19	18	10	12
<u>-2</u>	<u>-9</u>	<u>-12</u>	<u>-10</u>	<u>-9</u>	<u>-1</u>	<u>-9</u>

15	19	14	17	16	11	18
<u>-6</u>	<u>-9</u>	<u>-5</u>	<u>-8</u>	<u>-9</u>	<u>-2</u>	<u>-9</u>

21	19	17	10	20	17	12
<u>-12</u>	<u>-10</u>	<u>-9</u>	<u>-1</u>	<u>-11</u>	<u>-8</u>	<u>-9</u>

13	14	12	11	18	21	20
<u>-9</u>	<u>-9</u>	<u>-3</u>	<u>-9</u>	<u>-9</u>	<u>-9</u>	<u>-11</u>

13	10	15	18	14	21	14
<u>-9</u>	<u>-9</u>	<u>-9</u>	<u>-9</u>	<u>-9</u>	<u>-12</u>	<u>-5</u>

Time _____

Name _____

$$\begin{array}{r} 2 \times 1 = 2 \\ 2 \times 7 = 14 \end{array}$$

$$\begin{array}{r} 2 \times 2 = 4 \\ 2 \times 8 = 16 \end{array}$$

$$\begin{array}{r} 2 \times 3 = 6 \\ 2 \times 9 = 18 \end{array}$$

$$\begin{array}{r} 2 \times 4 = 8 \\ 2 \times 10 = 20 \end{array}$$

$$\begin{array}{r} 2 \times 5 = 10 \\ 2 \times 11 = 22 \end{array}$$

$$\begin{array}{r} 2 \times 6 = 12 \\ 2 \times 12 = 24 \end{array}$$

$$\begin{array}{r} 2 \\ \times 5 \end{array}$$

$$\begin{array}{r} 2 \\ \times 1 \end{array}$$

$$\begin{array}{r} 2 \\ \times 9 \end{array}$$

$$\begin{array}{r} 2 \\ \times 12 \end{array}$$

$$\begin{array}{r} 2 \\ \times 11 \end{array}$$

$$\begin{array}{r} 2 \\ \times 7 \end{array}$$

$$\begin{array}{r} 2 \\ \times 8 \end{array}$$

$$\begin{array}{r} 2 \\ \times 7 \end{array}$$

$$\begin{array}{r} 2 \\ \times 6 \end{array}$$

$$\begin{array}{r} 2 \\ \times 3 \end{array}$$

$$\begin{array}{r} 2 \\ \times 10 \end{array}$$

$$\begin{array}{r} 2 \\ \times 4 \end{array}$$

$$\begin{array}{r} 2 \\ \times 5 \end{array}$$

$$\begin{array}{r} 2 \\ \times 2 \end{array}$$

$$\begin{array}{r} 2 \\ \times 8 \end{array}$$

$$\begin{array}{r} 2 \\ \times 11 \end{array}$$

$$\begin{array}{r} 6 \\ \times 2 \end{array}$$

$$\begin{array}{r} 8 \\ \times 2 \end{array}$$

$$\begin{array}{r} 2 \\ \times 12 \end{array}$$

$$\begin{array}{r} 2 \\ \times 1 \end{array}$$

$$\begin{array}{r} 2 \\ \times 9 \end{array}$$

$$\begin{array}{r} 2 \\ \times 4 \end{array}$$

$$\begin{array}{r} 2 \\ \times 2 \end{array}$$

$$\begin{array}{r} 11 \\ \times 2 \end{array}$$

$$\begin{array}{r} 2 \\ \times 7 \end{array}$$

$$\begin{array}{r} 9 \\ \times 2 \end{array}$$

$$\begin{array}{r} 2 \\ \times 6 \end{array}$$

$$\begin{array}{r} 2 \\ \times 3 \end{array}$$

$$\begin{array}{r} 2 \\ \times 9 \end{array}$$

$$\begin{array}{r} 2 \\ \times 10 \end{array}$$

$$\begin{array}{r} 2 \\ \times 5 \end{array}$$

$$\begin{array}{r} 2 \\ \times 11 \end{array}$$

$$\begin{array}{r} 2 \\ \times 12 \end{array}$$

$$\begin{array}{r} 2 \\ \times 8 \end{array}$$

$$\begin{array}{r} 2 \\ \times 4 \end{array}$$

$$\begin{array}{r} 2 \\ \times 3 \end{array}$$

$$\begin{array}{r} 2 \\ \times 1 \end{array}$$

$$\begin{array}{r} 3 \\ \times 2 \end{array}$$

$$\begin{array}{r} 10 \\ \times 2 \end{array}$$

$$\begin{array}{r} 2 \\ \times 6 \end{array}$$

$$\begin{array}{r} 2 \\ \times 2 \end{array}$$

$$\begin{array}{r} 2 \\ \times 10 \end{array}$$

$$\begin{array}{r} 2 \\ \times 4 \end{array}$$

$$\begin{array}{r} 2 \\ \times 10 \end{array}$$

$$\begin{array}{r} 3 \\ \times 2 \end{array}$$

$$\begin{array}{r} 4 \\ \times 2 \end{array}$$

$$\begin{array}{r} 2 \\ \times 8 \end{array}$$

$$\begin{array}{r} 2 \\ \times 7 \end{array}$$

$$\begin{array}{r} 2 \\ \times 5 \end{array}$$

$$\begin{array}{r} 2 \\ \times 6 \end{array}$$

$$\begin{array}{r} 2 \\ \times 2 \end{array}$$

$$\begin{array}{r} 2 \\ \times 9 \end{array}$$

$$\begin{array}{r} 2 \\ \times 11 \end{array}$$

$$\begin{array}{r} 2 \\ \times 12 \end{array}$$

$$\begin{array}{r} 2 \\ \times 3 \end{array}$$

$$\begin{array}{r} 2 \\ \times 1 \end{array}$$

Time _____

Name _____

$$\begin{array}{llllll}
 3 \times 1 = 3 & 3 \times 2 = 6 & 3 \times 3 = 9 & 3 \times 4 = 12 & 3 \times 5 = 15 & 3 \times 6 = 18 \\
 3 \times 7 = 21 & 3 \times 8 = 24 & 3 \times 9 = 27 & 3 \times 10 = 30 & 3 \times 11 = 33 & 3 \times 12 = 36
 \end{array}$$

$$\begin{array}{lllllll}
 3 & 3 & 3 & 3 & 3 & 3 & 3 \\
 \times 11 & \times 1 & \times 2 & \times 12 & \times 7 & \times 9 & \times 4
 \end{array}$$

$$\begin{array}{lllllll}
 3 & 3 & 3 & 3 & 3 & 3 & 3 \\
 \times 6 & \times 9 & \times 3 & \times 10 & \times 7 & \times 8 & \times 5
 \end{array}$$

$$\begin{array}{lllllll}
 3 & 3 & 4 & 3 & 3 & 3 & 3 \\
 \times 4 & \times 12 & \times 3 & \times 9 & \times 5 & \times 1 & \times 2
 \end{array}$$

$$\begin{array}{lllllll}
 9 & 3 & 11 & 3 & 3 & 3 & 3 \\
 \times 3 & \times 8 & \times 3 & \times 11 & \times 6 & \times 10 & \times 3
 \end{array}$$

$$\begin{array}{lllllll}
 3 & 3 & 3 & 3 & 3 & 3 & 5 \\
 \times 7 & \times 2 & \times 9 & \times 5 & \times 4 & \times 11 & \times 3
 \end{array}$$

$$\begin{array}{lllllll}
 3 & 8 & 3 & 3 & 3 & 12 & 3 \\
 \times 11 & \times 3 & \times 1 & \times 10 & \times 3 & \times 3 & \times 8
 \end{array}$$

$$\begin{array}{lllllll}
 3 & 3 & 3 & 4 & 3 & 3 & 3 \\
 \times 4 & \times 5 & \times 12 & \times 3 & \times 5 & \times 7 & \times 6
 \end{array}$$

$$\begin{array}{lllllll}
 3 & 3 & 3 & 3 & 3 & 3 & 3 \\
 \times 6 & \times 2 & \times 8 & \times 11 & \times 9 & \times 1 & \times 3
 \end{array}$$

Time _____ Name _____

$4 \times 1 = 4$	$4 \times 2 = 8$	$4 \times 3 = 12$	$4 \times 4 = 16$	$4 \times 5 = 20$	$4 \times 6 = 24$
$4 \times 7 = 28$	$4 \times 8 = 32$	$4 \times 9 = 36$	$4 \times 10 = 40$	$4 \times 11 = 44$	$4 \times 12 = 48$

4	4	4	8	4	7	4
<u>$\times 8$</u>	<u>$\times 1$</u>	<u>$\times 7$</u>	<u>$\times 4$</u>	<u>$\times 4$</u>	<u>$\times 4$</u>	<u>$\times 2$</u>

4	4	4	4	4	4	4
<u>$\times 5$</u>	<u>$\times 9$</u>	<u>$\times 6$</u>	<u>$\times 10$</u>	<u>$\times 12$</u>	<u>$\times 11$</u>	<u>$\times 3$</u>

4	4	12	4	4	8	4
<u>$\times 4$</u>	<u>$\times 8$</u>	<u>$\times 4$</u>	<u>$\times 12$</u>	<u>$\times 5$</u>	<u>$\times 6$</u>	<u>$\times 7$</u>

4	4	4	6	4	4	11
<u>$\times 5$</u>	<u>$\times 3$</u>	<u>$\times 7$</u>	<u>$\times 4$</u>	<u>$\times 10$</u>	<u>$\times 11$</u>	<u>$\times 4$</u>

4	4	7	4	4	4	9
<u>$\times 9$</u>	<u>$\times 11$</u>	<u>$\times 4$</u>	<u>$\times 6$</u>	<u>$\times 8$</u>	<u>$\times 5$</u>	<u>$\times 4$</u>

6	4	6	4	4	4	4
<u>$\times 4$</u>	<u>$\times 12$</u>	<u>$\times 10$</u>	<u>$\times 11$</u>	<u>$\times 7$</u>	<u>$\times 1$</u>	<u>$\times 7$</u>

4	4	4	4	4	4	4
<u>$\times 3$</u>	<u>$\times 8$</u>	<u>$\times 6$</u>	<u>$\times 5$</u>	<u>$\times 12$</u>	<u>$\times 11$</u>	<u>$\times 5$</u>

4	4	4	4	4	4	4
<u>$\times 7$</u>	<u>$\times 2$</u>	<u>$\times 1$</u>	<u>$\times 9$</u>	<u>$\times 3$</u>	<u>$\times 11$</u>	<u>$\times 10$</u>

Time _____

Name _____

$8 \times 1 = 8$	$8 \times 2 = 16$	$8 \times 3 = 24$	$8 \times 4 = 32$	$8 \times 5 = 40$	$8 \times 6 = 48$	
$8 \times 7 = 56$	$8 \times 8 = 64$	$8 \times 9 = 72$	$8 \times 10 = 80$	$8 \times 11 = 88$	$8 \times 12 = 96$	

$\begin{array}{r} 8 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 1 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 9 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 9 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 2 \\ \hline \end{array}$
--	--	--	--	--	--	--

$\begin{array}{r} 8 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 11 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$
--	--	--	--	--	---	--

$\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 6 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 12 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 9 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 8 \\ \hline \end{array}$
--	--	--	---	--	--	--

$\begin{array}{r} 8 \\ \times 2 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 11 \\ \hline \end{array}$	$\begin{array}{r} 12 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 9 \\ \hline \end{array}$
--	--	--	--	---	---	--

$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 5 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 9 \\ \times 8 \\ \hline \end{array}$
--	--	--	--	--	--	--

$\begin{array}{r} 8 \\ \times 9 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 11 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 2 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 3 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 5 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 11 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$
--	---	--	--	--	---	--

$\begin{array}{r} 8 \\ \times 7 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 6 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 8 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 9 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 4 \\ \hline \end{array}$	$\begin{array}{r} 8 \\ \times 2 \\ \hline \end{array}$	$\begin{array}{r} 7 \\ \times 8 \\ \hline \end{array}$
--	--	--	--	--	--	--

Time _____

Name _____

$$\begin{array}{l} 3 \times 1 = 3 \quad 3 \times 2 = 6 \quad 3 \times 3 = 9 \quad 3 \times 4 = 12 \quad 3 \times 5 = 15 \quad 3 \times 6 = 18 \\ 3 \times 7 = 21 \quad 3 \times 8 = 24 \quad 3 \times 9 = 27 \quad 3 \times 10 = 30 \quad 3 \times 11 = 33 \quad 3 \times 12 = 36 \end{array}$$

$$3 \overline{)24} = 24 \div 3 = \frac{24}{3} = 8$$

[Write your answer to the right of the fraction]

$$\frac{33}{3} \quad \frac{9}{3} \quad \frac{12}{3} \quad \frac{36}{3} \quad \frac{15}{3} \quad \frac{24}{3} \quad \frac{30}{3}$$

$$\frac{18}{3} \quad \frac{21}{3} \quad \frac{3}{3} \quad \frac{6}{3} \quad \frac{27}{3} \quad \frac{36}{3} \quad \frac{33}{3}$$

$$\frac{30}{3} \quad \frac{12}{3} \quad \frac{9}{3} \quad \frac{21}{3} \quad \frac{30}{3} \quad \frac{6}{3} \quad \frac{24}{3}$$

$$\frac{36}{3} \quad \frac{15}{3} \quad \frac{27}{3} \quad \frac{24}{3} \quad \frac{30}{3} \quad \frac{9}{3} \quad \frac{18}{3}$$

$$\frac{21}{3} \quad \frac{24}{3} \quad \frac{33}{3} \quad \frac{6}{3} \quad \frac{27}{3} \quad \frac{36}{3} \quad \frac{12}{3}$$

$$\frac{30}{3} \quad \frac{12}{3} \quad \frac{21}{3} \quad \frac{15}{3} \quad \frac{18}{3} \quad \frac{27}{3} \quad \frac{24}{3}$$

$$\frac{27}{3} \quad \frac{9}{3} \quad \frac{36}{3} \quad \frac{3}{3} \quad \frac{6}{3} \quad \frac{15}{3} \quad \frac{18}{3}$$

$$\frac{3}{3} \quad \frac{18}{3} \quad \frac{33}{3} \quad \frac{24}{3} \quad \frac{12}{3} \quad \frac{21}{3} \quad \frac{27}{3}$$

Time _____

Name _____

$$\begin{array}{l} 4 \times 1 = 4 \\ 4 \times 7 = 28 \end{array}$$

$$\begin{array}{l} 4 \times 2 = 8 \\ 4 \times 8 = 32 \end{array}$$

$$\begin{array}{l} 4 \times 3 = 12 \\ 4 \times 9 = 36 \end{array}$$

$$\begin{array}{l} 4 \times 4 = 16 \\ 4 \times 10 = 40 \end{array}$$

$$\begin{array}{l} 4 \times 5 = 20 \\ 4 \times 11 = 44 \end{array}$$

$$\begin{array}{l} 4 \times 6 = 24 \\ 4 \times 12 = 48 \end{array}$$

$$4 \overline{)32} = 32 \div 4 = \frac{32}{4} = 8$$

[Write your answer to the right of the fraction]

$$\frac{44}{4}$$

$$\frac{8}{4}$$

$$\frac{12}{4}$$

$$\frac{36}{4}$$

$$\frac{16}{4}$$

$$\frac{24}{4}$$

$$\frac{40}{4}$$

$$\frac{20}{4}$$

$$\frac{32}{4}$$

$$\frac{4}{4}$$

$$\frac{24}{4}$$

$$\frac{28}{4}$$

$$\frac{48}{4}$$

$$\frac{44}{4}$$

$$\frac{36}{4}$$

$$\frac{24}{4}$$

$$\frac{20}{4}$$

$$\frac{48}{4}$$

$$\frac{32}{4}$$

$$\frac{16}{4}$$

$$\frac{20}{4}$$

$$\frac{40}{4}$$

$$\frac{36}{4}$$

$$\frac{28}{4}$$

$$\frac{24}{4}$$

$$\frac{32}{4}$$

$$\frac{12}{4}$$

$$\frac{4}{4}$$

$$\frac{12}{4}$$

$$\frac{24}{4}$$

$$\frac{44}{4}$$

$$\frac{40}{4}$$

$$\frac{20}{4}$$

$$\frac{36}{4}$$

$$\frac{24}{4}$$

$$\frac{44}{4}$$

$$\frac{20}{4}$$

$$\frac{32}{4}$$

$$\frac{40}{4}$$

$$\frac{28}{4}$$

$$\frac{48}{4}$$

$$\frac{32}{4}$$

$$\frac{20}{4}$$

$$\frac{8}{4}$$

$$\frac{36}{4}$$

$$\frac{32}{4}$$

$$\frac{32}{4}$$

$$\frac{24}{4}$$

$$\frac{12}{4}$$

$$\frac{32}{4}$$

$$\frac{28}{4}$$

$$\frac{48}{4}$$

$$\frac{40}{4}$$

$$\frac{4}{4}$$

$$\frac{8}{4}$$

$$\frac{44}{4}$$

Time _____

Name _____

$$\begin{array}{l} 5 \times 1 = 5 \quad 5 \times 2 = 10 \quad 5 \times 3 = 15 \quad 5 \times 4 = 20 \quad 5 \times 5 = 25 \quad 5 \times 6 = 30 \\ 5 \times 7 = 35 \quad 5 \times 8 = 40 \quad 5 \times 9 = 45 \quad 5 \times 10 = 50 \quad 5 \times 11 = 55 \quad 5 \times 12 = 60 \end{array}$$

$$5 \overline{)35} = 35 \div 5 = \frac{35}{5} = 7$$

[Write your answer to the right of the fraction]

$$\frac{20}{5} \quad \frac{10}{5} \quad \frac{40}{5} \quad \frac{55}{5} \quad \frac{15}{5} \quad \frac{60}{5} \quad \frac{50}{5}$$

$$\frac{45}{5} \quad \frac{25}{5} \quad \frac{5}{5} \quad \frac{60}{5} \quad \frac{50}{5} \quad \frac{30}{5} \quad \frac{35}{5}$$

$$\frac{10}{5} \quad \frac{30}{5} \quad \frac{40}{5} \quad \frac{35}{5} \quad \frac{50}{5} \quad \frac{15}{5} \quad \frac{55}{5}$$

$$\frac{50}{5} \quad \frac{25}{5} \quad \frac{60}{5} \quad \frac{30}{5} \quad \frac{5}{5} \quad \frac{45}{5} \quad \frac{60}{5}$$

$$\frac{5}{5} \quad \frac{40}{5} \quad \frac{30}{5} \quad \frac{55}{5} \quad \frac{25}{5} \quad \frac{10}{5} \quad \frac{50}{5}$$

$$\frac{60}{5} \quad \frac{15}{5} \quad \frac{45}{5} \quad \frac{25}{5} \quad \frac{60}{5} \quad \frac{45}{5} \quad \frac{40}{5}$$

$$\frac{10}{5} \quad \frac{30}{5} \quad \frac{50}{5} \quad \frac{5}{5} \quad \frac{60}{5} \quad \frac{55}{5} \quad \frac{15}{5}$$

$$\frac{55}{5} \quad \frac{60}{5} \quad \frac{60}{5} \quad \frac{40}{5} \quad \frac{45}{5} \quad \frac{25}{5} \quad \frac{10}{5}$$

Time _____

Name _____

$$6 \times 1 = 6$$
$$6 \times 7 = 42$$

$$6 \times 2 = 12$$
$$6 \times 8 = 48$$

$$6 \times 3 = 18$$
$$6 \times 9 = 54$$

$$6 \times 4 = 24$$
$$6 \times 10 = 60$$

$$6 \times 5 = 30$$
$$6 \times 11 = 66$$

$$6 \times 6 = 36$$
$$6 \times 12 = 72$$

$$6 \overline{)54} = 54 \div 6 = \frac{54}{6} = 9$$

[Write your answer to the right of the fraction]

$$\frac{72}{6}$$

$$\frac{42}{6}$$

$$\frac{12}{6}$$

$$\frac{48}{6}$$

$$\frac{54}{6}$$

$$\frac{24}{6}$$

$$\frac{30}{6}$$

$$\frac{18}{6}$$

$$\frac{60}{6}$$

$$\frac{6}{6}$$

$$\frac{54}{6}$$

$$\frac{72}{6}$$

$$\frac{36}{6}$$

$$\frac{66}{6}$$

$$\frac{72}{6}$$

$$\frac{30}{6}$$

$$\frac{66}{6}$$

$$\frac{60}{6}$$

$$\frac{42}{6}$$

$$\frac{48}{6}$$

$$\frac{12}{6}$$

$$\frac{54}{6}$$

$$\frac{36}{6}$$

$$\frac{12}{6}$$

$$\frac{24}{6}$$

$$\frac{6}{6}$$

$$\frac{72}{6}$$

$$\frac{18}{6}$$

$$\frac{42}{6}$$

$$\frac{18}{6}$$

$$\frac{30}{6}$$

$$\frac{6}{6}$$

$$\frac{48}{6}$$

$$\frac{12}{6}$$

$$\frac{54}{6}$$

$$\frac{66}{6}$$

$$\frac{48}{6}$$

$$\frac{60}{6}$$

$$\frac{36}{6}$$

$$\frac{72}{6}$$

$$\frac{24}{6}$$

$$\frac{42}{6}$$

$$\frac{36}{6}$$

$$\frac{54}{6}$$

$$\frac{12}{6}$$

$$\frac{30}{6}$$

$$\frac{6}{6}$$

$$\frac{18}{6}$$

$$\frac{60}{6}$$

$$\frac{24}{6}$$

$$\frac{72}{6}$$

$$\frac{42}{6}$$

$$\frac{48}{6}$$

$$\frac{36}{6}$$

$$\frac{54}{6}$$

$$\frac{66}{6}$$

Time _____

Name _____

$$\begin{array}{l} 7 \times 1 = 7 \quad 7 \times 2 = 14 \quad 7 \times 3 = 21 \quad 7 \times 4 = 28 \quad 7 \times 5 = 35 \quad 7 \times 6 = 42 \\ 7 \times 7 = 49 \quad 7 \times 8 = 56 \quad 7 \times 9 = 63 \quad 7 \times 10 = 70 \quad 7 \times 11 = 77 \quad 7 \times 12 = 84 \end{array}$$

$$7 \overline{)63} = 63 \div 7 = \frac{63}{7} = 9$$

[Write your answer to the right of the fraction]

$$\frac{84}{7} \quad \frac{49}{7} \quad \frac{35}{7} \quad \frac{7}{7} \quad \frac{21}{7} \quad \frac{56}{7} \quad \frac{63}{7}$$

$$\frac{42}{7} \quad \frac{63}{7} \quad \frac{56}{7} \quad \frac{28}{7} \quad \frac{14}{7} \quad \frac{70}{7} \quad \frac{77}{7}$$

$$\frac{28}{7} \quad \frac{7}{7} \quad \frac{21}{7} \quad \frac{77}{7} \quad \frac{63}{7} \quad \frac{56}{7} \quad \frac{84}{7}$$

$$\frac{84}{7} \quad \frac{56}{7} \quad \frac{35}{7} \quad \frac{14}{7} \quad \frac{70}{7} \quad \frac{21}{7} \quad \frac{7}{7}$$

$$\frac{70}{7} \quad \frac{21}{7} \quad \frac{63}{7} \quad \frac{77}{7} \quad \frac{84}{7} \quad \frac{56}{7} \quad \frac{28}{7}$$

$$\frac{84}{7} \quad \frac{7}{7} \quad \frac{14}{7} \quad \frac{56}{7} \quad \frac{35}{7} \quad \frac{63}{7} \quad \frac{70}{7}$$

$$\frac{35}{7} \quad \frac{56}{7} \quad \frac{84}{7} \quad \frac{63}{7} \quad \frac{77}{7} \quad \frac{35}{7} \quad \frac{14}{7}$$

$$\frac{63}{7} \quad \frac{28}{7} \quad \frac{21}{7} \quad \frac{84}{7} \quad \frac{7}{7} \quad \frac{84}{7} \quad \frac{56}{7}$$

Time _____

Name _____

$$\begin{array}{l} 8 \times 1 = 8 \quad 8 \times 2 = 16 \quad 8 \times 3 = 24 \quad 8 \times 4 = 32 \quad 8 \times 5 = 40 \quad 8 \times 6 = 48 \\ 8 \times 7 = 56 \quad 8 \times 8 = 64 \quad 8 \times 9 = 72 \quad 8 \times 10 = 80 \quad 8 \times 11 = 88 \quad 8 \times 12 = 96 \end{array}$$

$$8 \overline{)72} = 72 \div 8 = \frac{72}{8} = 9$$

[Write your answer to the right of the fraction]

$$\frac{64}{8} \quad \frac{8}{8} \quad \frac{56}{8} \quad \frac{72}{8} \quad \frac{16}{8} \quad \frac{24}{8} \quad \frac{40}{8}$$

$$\frac{80}{8} \quad \frac{32}{8} \quad \frac{96}{8} \quad \frac{24}{8} \quad \frac{56}{8} \quad \frac{48}{8} \quad \frac{88}{8}$$

$$\frac{40}{8} \quad \frac{72}{8} \quad \frac{24}{8} \quad \frac{64}{8} \quad \frac{8}{8} \quad \frac{96}{8} \quad \frac{16}{8}$$

$$\frac{56}{8} \quad \frac{88}{8} \quad \frac{96}{8} \quad \frac{48}{8} \quad \frac{32}{8} \quad \frac{80}{8} \quad \frac{72}{8}$$

$$\frac{16}{8} \quad \frac{64}{8} \quad \frac{40}{8} \quad \frac{88}{8} \quad \frac{56}{8} \quad \frac{24}{8} \quad \frac{48}{8}$$

$$\frac{8}{8} \quad \frac{56}{8} \quad \frac{72}{8} \quad \frac{80}{8} \quad \frac{32}{8} \quad \frac{96}{8} \quad \frac{88}{8}$$

$$\frac{24}{8} \quad \frac{96}{8} \quad \frac{48}{8} \quad \frac{40}{8} \quad \frac{64}{8} \quad \frac{56}{8} \quad \frac{72}{8}$$

$$\frac{64}{8} \quad \frac{32}{8} \quad \frac{16}{8} \quad \frac{88}{8} \quad \frac{72}{8} \quad \frac{80}{8} \quad \frac{96}{8}$$

Time _____

Name _____

$$9 \times 1 = 9$$

$$9 \times 7 = 63$$

$$9 \times 2 = 18$$

$$9 \times 8 = 72$$

$$9 \times 3 = 27$$

$$9 \times 9 = 81$$

$$9 \times 4 = 36$$

$$9 \times 10 = 90$$

$$9 \times 5 = 45$$

$$9 \times 11 = 99$$

$$9 \times 6 = 54$$

$$9 \times 12 = 108$$

$$9 \overline{)63} = 63 \div 9 = \frac{63}{9} = 7$$

[Write your answer to the right of the fraction]

$$\frac{54}{9}$$

$$\frac{36}{9}$$

$$\frac{72}{9}$$

$$\frac{63}{9}$$

$$\frac{18}{9}$$

$$\frac{45}{9}$$

$$\frac{90}{9}$$

$$\frac{108}{9}$$

$$\frac{81}{9}$$

$$\frac{9}{9}$$

$$\frac{54}{9}$$

$$\frac{27}{9}$$

$$\frac{63}{9}$$

$$\frac{99}{9}$$

$$\frac{90}{9}$$

$$\frac{45}{9}$$

$$\frac{99}{9}$$

$$\frac{81}{9}$$

$$\frac{72}{9}$$

$$\frac{18}{9}$$

$$\frac{36}{9}$$

$$\frac{54}{9}$$

$$\frac{63}{9}$$

$$\frac{54}{9}$$

$$\frac{108}{9}$$

$$\frac{36}{9}$$

$$\frac{81}{9}$$

$$\frac{9}{9}$$

$$\frac{72}{9}$$

$$\frac{99}{9}$$

$$\frac{90}{9}$$

$$\frac{18}{9}$$

$$\frac{45}{9}$$

$$\frac{54}{9}$$

$$\frac{108}{9}$$

$$\frac{81}{9}$$

$$\frac{36}{9}$$

$$\frac{9}{9}$$

$$\frac{99}{9}$$

$$\frac{90}{9}$$

$$\frac{45}{9}$$

$$\frac{27}{9}$$

$$\frac{108}{9}$$

$$\frac{63}{9}$$

$$\frac{54}{9}$$

$$\frac{72}{9}$$

$$\frac{18}{9}$$

$$\frac{36}{9}$$

$$\frac{81}{9}$$

$$\frac{72}{9}$$

$$\frac{81}{9}$$

$$\frac{9}{9}$$

$$\frac{99}{9}$$

$$\frac{63}{9}$$

$$\frac{108}{9}$$

$$\frac{54}{9}$$

APPENDIX E:
Automaticity Recording Device

Name: _____ period: _____

Add 3							
Add 4							
Add 5							
Add 6							
Add 7							
Add 8							
Add 9							
Sub 3							
Sub 4							
Sub 5							
Sub 6							
Sub 7							
Sub 8							
Sub 9							
Mult 2							
Mult 3							
Mult 4							
Mult 5							
Mult 6							
Mult 7							
Mult 8							
Mult 9							
Div 3							
Div 4							
Div 5							
Div 6							
Div 7							
Div 8							
Div 9							
Test Score Aug.							
Test Score <input type="checkbox"/> Sept.							
Test Score <input type="checkbox"/> Oct.							

APPENDIX F:
EKU Transitions Table

Fall 2012			Winter 2012			Spring 2013		
Percentile	Automatichy Number Missed	Automatichy Time	Percentile	Automatichy Number Missed	Automatichy Time	Percentile	Automatichy Number Missed	Automatichy Time
49	12	10	48	7	8.24	51	4	7.43
48.24	20.96	11.00	47.90	12.23	11.03	49.98	13.22	8.90
Median Fall 2012:			Median Winter 2012:			Median Spring 2013:		
Mean Fall 2012:			Mean Winter 2012:			Mean Spring 2013:		

n = 26484

% of students who improved:

66.72%
FALL/WINTER DIFFERENCE

% of students who improved:

64.91%
WINTER/SRING DIFFERENCE

Source(s): Thomas, Robert, (2013). Transitions recap; Eastern Kentucky University, Department

APPENDIX G:

EKU Suggested High School Readiness Transition Remediation
Regimen Protocol Guidelines

Suggested High School Readiness Transition Remediation Regimen Protocol Guidelines
Middle School/High School (Grades 6 to 12)

The remediation regimen for students is done at the beginning of class during the “warm up” or “bell ringer” time. The total remediation exercise can take 6 – 7 minutes.

The teacher gives students 30 seconds to procure the appropriate remediation sheet. Suggested organizational methods are given as follows:

- To begin, the teacher projects the timer or stopwatch set at 0 minutes (runs up to time limit).
- Once started, each student works on a remediation sheet for up to 5 minutes. Once completed, the student can grade the sheet (answers are at the top) using a colored pen or pencil. The student then records the elapsed time and number of problems missed or not completed. The teacher decides the method of record keeping.
- No time is allowed beyond 5 minutes. Students must stop working and grade their remediation sheets. Problems not completed are marked wrong.
- Each student progresses to the next remediation sheet when the amount of time required to complete the sheet drops to 2 minutes or less and 2 or less problems are incorrect. (Teacher discretion advised)
- The student works through the addition remediation sheets completely before moving to the multiplication, subtraction and division remediation sheets.
- Order of completion: Addition, Multiplication, Subtraction, Division

- Once mastered, students then strive toward a maximum time of less than 1 minute per sheet. (Ultimate goal is automaticity.)

These exercises are supplementary to regular mathematics instruction. Teachers are cautioned to avoid “drill and kill” when implementing these regimens. Daily drill and repetition are encouraged, but not to the exclusion of regular arithmetic and mathematics instruction. A little bit each day or at regular intervals show the best promise for increasing fluency.

Teachers are encouraged to conduct weekly interim assessments with timed sheets of their own design. Suggestion: The sheet could have 10 to 15 addition or multiplication problems, whichever is appropriate.

Students are allowed 1 to 1½ minutes to complete the assessment. Students then grade their own assessment using a colored pen and record the results as directed by teachers.

Teachers can monitor each student’s scores and measure the results against self-reported progress on the daily timed worksheets, making individual adjustments for students. (For example: If a student misses problems with plus 7’s on the interim assessment, the teacher requires the student to re-visit the plus 7’s sheets during the daily bell-ringer exercises.)

Note: The 105 question diagnostic assessment is only to be administered in August, January and May.

Getting started

- Many teachers start all students on the same remediation sheet for the first exercise.
- The students that complete the worksheet in the targeted time with 2 or less errors move on to the next remediation sheet for the next exercise.
- In the beginning, teachers may want to allow students with advanced computational fluency skills to work through more than 1 sheet per exercise.
- Within 2 weeks, students are working at different levels and on different remediation sheets (differentiated learning).

Teachers are asked to remind students that they are competing not with each other, but with themselves.

URL for Online Stopwatch: <http://www.online-stopwatch.com/full-screen-stopwatch/>

Reminders

These exercises are **supplementary to regular mathematics instruction**. Teachers are cautioned to avoid “drill and kill” when implementing these regimens. Daily drill and repetition are encouraged, but not to the exclusion of regular arithmetic and mathematics instruction. A little bit each day or at regular intervals show the best promise for increasing fluency.

Encouraging parents to help in this process is recommended, but the same cautionary measures hold. An overemphasis on these protocols can cause students to become bored, frustrated, discouraged or turned off.

Related fun activities are also encouraged.

Recommendations

- Teachers adjust targeted times as appropriate.
- Teachers supplement the fluency practice with oral activities and flash cards.
- Flash cards should only have 1 problem per card with the answer on the back.
- Students should be shown the answer after each query right or wrong (immediate feedback).

Additional Notes

Keep track of students that are not progressing through the regimen. This lack of progress may be an indication of other, more serious mathematical or learning difficulties or deficiencies.

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APPENDIX H:
Automaticity Diagnostic Test Instructions

Automaticity Diagnostic Test Instructions

Please read the following instructions to your students:

This assessment is a mental math test that measures your automaticity skills which are basic math skills. This test is not for a grade but is very important.

Directions:

- You may not use a calculator.
- You may not ask your teacher for help.
- When you are finished you must record your completion time in the upper left corner of the test and then turn your paper face down.
- No talking is allowed until everyone is finished. You may read a book until time is up.
- Remember there is a time limit posted on the board.
- Time is an important factor in the overall assessment of this test.
- Quickness/speed is most important.
- If you do not know an answer, you can guess but do not stop to count on your fingers, etc.

Maximum Times for Automaticity Diagnostic Test

Grade	Maximum Time (in minutes)
2	10
3	20
4	15
5	15
6	15
7	15
8	15
9-12	12

APPENDIX I:
Automaticity Scores by Grade/Percentile

Automaticity Scores by Grade/Percentile

	1	2	3	4	5	6	7	8	9	10	11	12
	17	2155	2553	2145	2337	2728	2733	2941	1906	505	409	373
10	5.60	3.90	1.50	2.00	3.07	4.47	5.20	5.86	7.23	7.05	6.50	7.58
20	5.66	4.83	1.86	2.80	4.17	5.67	6.48	7.53	8.96	8.85	7.97	10.21
30	5.70	6.08	2.25	3.40	4.98	6.74	7.89	9.04	10.86	10.95	9.30	11.85
40	5.80	6.80	2.65	3.88	5.60	7.84	9.13	10.53	12.30	12.45	10.80	13.41
50	5.80	7.40	3.02	4.35	6.33	9.00	10.47	12.06	14.11	14.32	11.95	15.45
60	5.98	7.80	3.41	4.92	7.21	10.20	12.12	13.88	16.20	16.02	13.43	17.25
70	6.12	8.30	3.85	5.49	8.29	11.67	14.18	16.26	18.94	18.53	15.60	19.87
80	6.34	9.20	4.45	6.25	10.12	14.14	16.86	19.09	22.13	21.50	18.35	23.52
90	6.82	11.74	5.72	8.00	12.43	18.14	21.72	24.00	27.42	25.43	22.99	28.31

You may use this table as a rough guide for evaluating the performance of students taking the automaticity pretest. Look for the student's score in the corresponding grade level column. The number at the left end of the row is the percentile ranking for that score based on the Fall 2011 pretest from all participating counties.

As rough rules of thumb:

- A student with a raw score at/above (grade level)+12 is on pace to 70th percentile
- A student with a raw score at/above 1.5*(grade level)+6 is on pace to 80th percentile
- A student with a raw score at/above 2*(grade level)+4 is on pace to 90th percentile

You can make your own determination of the cutoff percentile level beyond which intervention is not deemed to be necessary.

	1	2	3	4	5	6	7	8	9	10	11	12
	17	2155	2553	2145	2337	2728	2733	2941	1906	505	409	373
10	5.60	3.90	1.50	2.00	3.07	4.47	5.20	5.86	7.23	7.05	6.50	7.58
20	5.66	4.83	1.86	2.80	4.17	5.67	6.48	7.53	8.96	8.85	7.97	10.21
30	5.70	6.08	2.25	3.40	4.98	6.74	7.89	9.04	10.86	10.95	9.30	11.85
40	5.80	6.80	2.65	3.88	5.60	7.84	9.13	10.53	12.30	12.45	10.80	13.41
50	5.80	7.40	3.02	4.35	6.33	9.00	10.47	12.06	14.11	14.32	11.95	15.45
60	5.98	7.80	3.41	4.92	7.21	10.20	12.12	13.88	16.20	16.02	13.43	17.25
70	6.12	8.30	3.85	5.49	8.29	11.67	14.18	16.26	18.94	18.53	15.60	19.87
80	6.34	9.20	4.45	6.25	10.12	14.14	16.86	19.09	22.13	21.50	18.35	23.52
90	6.82	11.74	5.72	8.00	12.43	18.14	21.72	24.00	27.42	25.43	22.99	28.31

APPENDIX J:

Automaticity Percentile Table

1st Period												
Date	Time	Score	# Correct	Time	Date	% ile	Score	# Correct	Time	Date	% ile	Ratio Increase
16-Aug	6.84	97.69	15.59	5.77	13-Sep	55.00	103.12	19.57	5.34	14-Oct	72.50	5.34
Mean				5.56		54.00	104.00	18.53	5.14		68.50	5.14
Median												
4th Period												
Date	Time	Score	# Correct	Time	Date	% ile	Score	# Correct	Time	Date	% ile	Ratio Increase
16-Aug	10.04	91.27	9.72	8.97	13-Sep	37.50	96.70	12.00	8.46	14-Oct	39.50	3.29
Mean				8.83		21.00	100.50	10.78	8.34		29.80	2.77
Median												

APPENDIX K:
Growth in Automaticity Scores

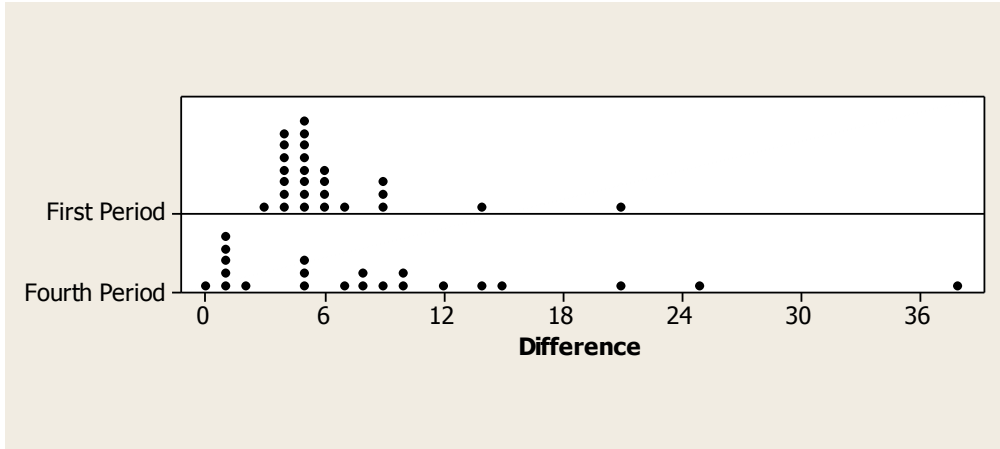


Figure 1. Growth in Automaticity Scores Dotplot.

Two-Sample T-Test and CI: First Period, Fourth Period

Two-sample T for First Period vs Fourth Period

	N	Mean	StDev	SE Mean
First Period	26	6.31	3.79	0.74
Fourth Period	22	9.05	9.30	2.0

Difference = μ (First Period) - μ (Fourth Period)
 Estimate for difference: -2.74
 95% CI for difference: (-7.09, 1.62)
 T-Test of difference = 0 (vs not =): T-Value = -1.29 P-Value = 0.208 DF = 26

APPENDIX L:
Differences in Rates

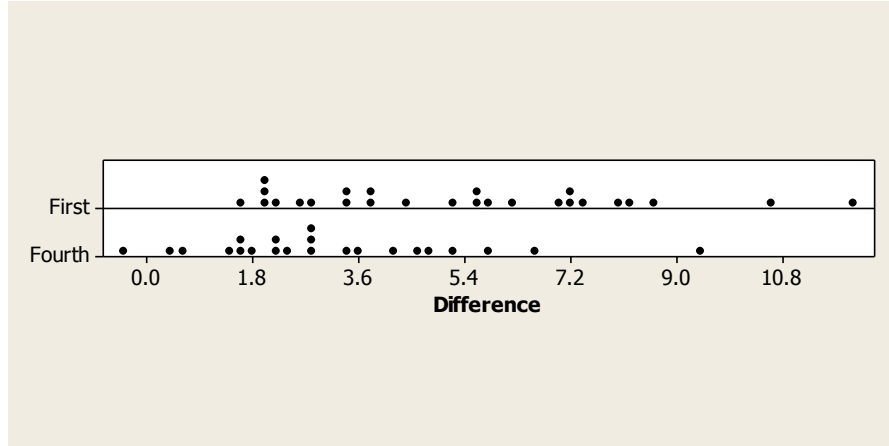


Figure 4. Differences in Rates Dotplot.

Two-Sample T-Test and CI: 1st Difference, 4th Difference

Two-sample T for 1st Difference vs 4th Difference

	N	Mean	StDev	SE Mean
1st Difference	26	5.34	2.81	0.55
4th Difference	22	3.16	2.28	0.49

Difference = μ (1st Difference) - μ (4th Difference)

Estimate for difference: 2.176

95% CI for difference: (0.697, 3.655)

T-Test of difference = 0 (vs not =): T-Value = 2.96

P-Value = 0.005 DF = 45