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Sensor fault detection and isolation for a class of uncertain nonlinear system using sliding mode observers

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ABSTRACT

Quick and timely fault detection is of great importance in control systems reliability. Undetected faulty sensors could result in irreparable damages. Although fault detection and isolation (FDI) methods in control systems have received much attention in the last decade, these techniques have not been applied for some classes of nonlinear systems yet. This paper deals with the issues of sensor fault detection and isolation for a class of Lipschitz uncertain nonlinear system. By introducing a coordinate transformation matrix for states and output, the original system is first divided into two subsystems. The first subsystem is affected by uncertainty and disturbance. The second subsystem just has sensor faults. The nonlinear term is separated to linear and pure nonlinear parts. For fault detection, two sliding mode observers (SMO) are designed for the two subsystems. The stability condition is obtained based on the Lyapunov approach. The necessary matrices and parameters are obtained by solving the linear matrix inequality (LMI) problem. Furthermore, two sliding mode observers are designed for fault isolation. Finally, the effectiveness of the proposed approach is illustrated by simulation examples.

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Fault detection; Lipschitz; nonlinear system; sliding mode observer; LMI

1. Introduction

With the development of industrial systems, the complexity of control systems has increased too, requiring high accuracy and speed. Real systems are subject to the disturbance in their inputs and outputs. Thus, it is important to include the disturbances in the control system modelling to obtain practical results, particularly in developing a fault detection method. The sensors play a crucial role in feedback control systems. If sensors provide incorrect information, tracking performance or regulation would be weak and may even cause system instability. In recent years, the study of fault detection to increase the reliability of systems has been of great interest to researchers [1–5]. Due to various nonlinearities in the systems, there is no general fault detection method for all nonlinear systems. The leader following tracking consensus problem for high order nonlinear dynamical multi-agent systems with switching topology and communication delay under noisy environments is investigated in [6]. One of the model-based FDI methods is the use of observers, which is one of the best ideas for fault detection and isolation. Observer-based fault detection is considered more than other fault detection methods [7–14]. In FDI applications, the observer is used to generate a residual. The residual is obtained from the difference between the actual output of the system and the estimated output of the observer. It is small in the absence of faults and after the occurrence of a fault, the residual increases and when exceeding a certain threshold, a fault is detected. The practical observers used in fault detection are adaptive observer, unknown input observer and sliding mode observer. There are many researches in actuator fault detection [15–18], but sensor FDI has been less studied than actuator FDI. Fault detection and isolation algorithm for attitude determination system of a satellite including a sun sensor was proposed in [19]. Sensor fault detection and isolation for a class of Lipschitz nonlinear systems with unstructured modelling uncertainty was developed by adaptive estimation approach in [20]. Although using the adaptive threshold is interesting but the capability to deal with the intermittent fault detection with non-zero initial states is not presented in [20]. To reduce fault detection time delay and for fault accommodation, an adaptive threshold with TS fuzzy system is implemented in [21]. Based on switched descriptor observer, sensor fault estimation and compensation for time delay switch systems were investigated in [22]. To detect incipient sensor faults, some researchers combined the sliding mode observer and lunberger observer [23]. The sliding mode observer is used for sensor fault estimation [24]. The present paper discusses sensor fault detection for a class of Lipschitz uncertain nonlinear system using the sliding mode observer. Firstly by introducing coordination transformation matrices for states and outputs, the

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original system is divided into two subsystems. One of them contains uncertainty and disturbance and the second one includes sensor faults only. Then, sensor faults in the second subsystem are formed as an actuator fault. The sliding mode observers are designed for fault detection and isolation. The sufficient condition of stability of the proposed FDI scheme is proved by the Lyapunov approach and solving LMI problem. Finally, if the estimated output error value is more than the threshold, the occurrence of sensor fault is detected.

In this paper, sensor fault detection and isolation of a special class of nonlinear systems, using a new technique to design new observers is considered. Two distinct motivations towards this research are, firstly there are many practical systems in use which fall in the class of nonlinear system investigated in this paper; and secondly, In spite of theoretical challenges to design sliding mode observer for FDI purpose, this technique provides high-performance results in terms of robustness against uncertainty and disturbances.

The rest of this paper is organized as follows: system modelling and mathematical preliminaries required for fault detection are described in Section 2. The observers design for sensor fault detection and the stability condition of error dynamics of the proposed observers based on the Lyapunov approach and solving LMI problem is presented in Section 3.1. The design of observers for fault isolation is discussed in Section 3.2. The simulation results with the proposed method on a Lipschitz nonlinear system in the presence of uncertainty and disturbance for an abrupt, an incipient and an intermittent fault are shown in Section 4, and some conclusions are drawn in Section 5.

2. Diagnosis structure

The class of nonlinear system is considered with the following form. It is assumed that only sensor fault occur in the system.

$$\dot{x} = Ax(t) + g(x)U(t) + M\eta(t) + Dd(t)$$
$$y(t) = Cx(t) + Ff_s \tag{1}$$

where $x \in R^n$, $U \in R^m$, $y \in R^p$, $f_s \in R^q$, $\eta \in R^r$, $d \in R^r$ denote respectively the vector of state variables, inputs, outputs, sensor faults, uncertainty and disturbance. $g(x)$ represents the nonlinear term, which can be rewritten as a combination of linear and pure nonlinear parts.

$$g(x) = g_l(x) + g_{nl}(x) \tag{2}$$

By substituting Equation (2) in Equation (1) and taking $B = g_l(x)$, Equation (3) is obtained.

$$\dot{x} = Ax(t) + BU(t) + g_{nl}(x)U(t) + M\eta(t) + Dd(t)$$
$$y(t) = Cx(t) + Ff_s \tag{3}$$

where $A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{p\times n}$, $M \in R^{n\times r}$, $D \in R^{n\times r}$ $F \in R^{p\times q}$, $p \geq q + r$ and F,M,D,C have full rank.

Assumption 2.1: rank(CM) = rank (M) and rank (CD) = rank (D).

Assumption 2.2: For every s with non-negative real part:

$$\text{rank}\begin{bmatrix} sI - A & M \\ C & 0 \end{bmatrix} = n + \text{rank}\,(M),$$
$$\text{rank}\begin{bmatrix} sI - A & D \\ C & 0 \end{bmatrix} = n + \text{rank}\,(D) \tag{4}$$

Assumption 2.3: The nonlinear term, $g_{nl}(x)$, is Lipschitz about the state x.

$$|g_{nl}(x) - g_{nl}(\hat{x})| \leq k_g|x - \hat{x}|, \quad \forall\, x, \hat{x} \in R^n \tag{5}$$

where k_g is the Lipschitz constant.

Assumption 2.4: The vector d and η satisfy the following constraints:

$$\|d\| \leq \zeta, \quad \|\eta\| \leq \eta_0 \tag{6}$$

where ζ and η_0 are two known positive constants.

Basic steps of the proposed scheme are outlined in Figure 1.

The first step is to determine coordinate transformation matrices and applying them to the original system, resulting in the new system 1. Then the new system 1 is decomposed into two subsystems. The first subsystem contains uncertainty and disturbances, and the second one includes sensor faults only. Then, by augmenting first and second subsystems the new system 2 is obtained. Next step is to design sliding mode observers for fault detection and isolation. Moreover, observer error dynamics stability is achieved by the Lyapunov approach and solving LMI problem. The residual is calculated from the actual and estimated outputs. By evaluating the residual, a fault is detected and isolated.

Lemma 2.1: *There exist* $Z = Tx = \begin{pmatrix} Z_1 \\ Z_2 \end{pmatrix}$, $W = Sy = \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}$ *under Assumption* 2.1.

The coordinate transformation matrices T and S are elaborately determined through mathematical manipulation of the system equations so that they hold the following properties:

$$TAT^{-1} = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}, \quad TB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix},$$
$$TM = \begin{bmatrix} M_1 \\ 0 \end{bmatrix}, \quad TD = \begin{bmatrix} D_1 \\ 0 \end{bmatrix}, \quad SF = \begin{bmatrix} 0 \\ F_2 \end{bmatrix},$$
$$SCT^{-1} = \begin{bmatrix} C_1 & 0 \\ 0 & C_4 \end{bmatrix} \tag{7}$$

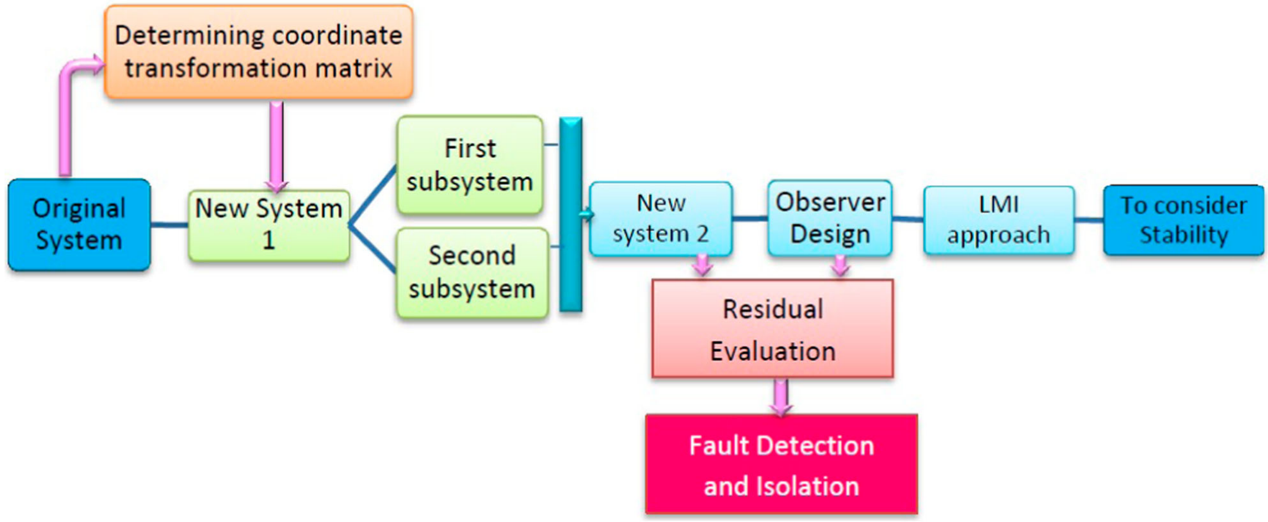

Figure 1. The structure of the proposed scheme for fault detection and isolation.

where:

$$T=\begin{pmatrix}T_1\\T_2\end{pmatrix}\in R^{n\times n},\quad S=\begin{pmatrix}S_1\\S_2\end{pmatrix}\in R^{p\times p},$$
$$T_1\in R^{r\times n},\quad S_1\in R^{r\times p},\quad Z_1\in R^{r}$$
$$W_1\in R^{r},\quad A_1\in R^{r\times r},\quad A_4\in R^{(n-r)\times(n-r)},$$
$$B_1\in R^{r\times m},\quad M_1\in R^{r\times r}, D_1\in R^{r\times r}$$
$$C_1\in R^{r\times r},\quad C_4\in R^{(p-r)\times(n-r)},\quad F_2\in R^{(p-r)\times q}.$$

3. Design of sliding mode observer

3.1. Fault detection

The coordinate transformation matrices T and S are introduced and system 1 is converted into the following two subsystems (8) and (9), where (8) accommodates uncertainties and disturbances but without any sensor faults and (9) includes sensor faults free of uncertainties or disturbances. The first subsystem is as follows:

$$\begin{aligned}\dot{Z}_1&=A_1Z_1+A_2Z_2+g_1(T^{-1}Z)U\\&\quad+B_1U+M_1\eta+D_1d\\W_1&=C_1Z_1\end{aligned}\tag{8}$$

And the second subsystem is as follows:

$$\begin{aligned}\dot{Z}_2&=A_3Z_1+A_4Z_2+B_{21}U+g_2(T^{-1}Z)U\\W_2&=C_4Z_2+F_2f_s\end{aligned}\tag{9}$$

where $g_1(T^{-1}Z)=T_1g_{nl}(T^{-1}Z)$ and $g_2(T^{-1}Z)=T_2g_{nl}(T^{-1}Z)$.

Lemma 3.1: *The pair (A_4, C_4) is detectable if and only if Assumption 2.2 holds.*

A new state $Z_3=\int_0^t W_2(\tau)d\tau$ is defined so that

$$\dot{Z}_3=C_4Z_2+F_2f_s\tag{10}$$

Equations (9) *and* (10) *can be combined to form an augmented system as*

$$\begin{aligned}\begin{bmatrix}\dot{Z}_2\\\dot{Z}_3\end{bmatrix}&=\begin{bmatrix}A_4&0\\C_4&0\end{bmatrix}\begin{bmatrix}Z_2\\Z_3\end{bmatrix}+\begin{bmatrix}A_3\\0\end{bmatrix}Z_1+\begin{bmatrix}g_2(T^{-1}Z)\\0\end{bmatrix}U\\&\quad+\begin{bmatrix}B_2\\0\end{bmatrix}U+\begin{bmatrix}0\\F_2\end{bmatrix}f_s\\W_3&=Z_3\end{aligned}\tag{11}$$

System with Equation (11) *can be rewritten as*

$$\begin{aligned}\dot{Z}_0&=A_0Z_0+A_{01}Z_1+g(Z_0)U+B_0U+F_0f_s\\W_3&=C_0Z_0\end{aligned}\tag{12}$$

where:

$$A_{01}=\begin{bmatrix}A_3\\0\end{bmatrix}\in R^{(n+p-2r)\times r},$$
$$A_0=\begin{bmatrix}A_4&0\\C_4&0\end{bmatrix}\in R^{(n+p-2r)\times(n+p-2r)}$$
$$B_0=\begin{bmatrix}B_2\\0\end{bmatrix}\in R^{(n+p-2r)\times m},$$
$$C_0=\begin{bmatrix}0&I_{p-r}\end{bmatrix}\in R^{(p-r)\times(n+p-2r)}$$
$$Z_0=\begin{bmatrix}Z_2\\Z_3\end{bmatrix}\in R^{n+p-2r},\quad W_3\in R^{p-r},$$
$$F_0=\begin{bmatrix}0\\F_2\end{bmatrix}\in R^{(n+p-2r)\times q}$$
$$g(Z_0)=\begin{bmatrix}g_2(T^{-1}Z)\\0\end{bmatrix}$$

Accordingly, system (8) *can be rewritten as*

$$\dot{Z}_1 = A_1 Z_1 + \bar{A}_2 Z_0 + g_1(T^{-1}Z)U + B_1 U + M_1\eta + D_1 d$$
$$W_1 = C_1 Z_1 \tag{13}$$

where $\bar{A}_2 = [\, A_2 \;\; 0_{r\times(p-r)} \,]$.

Lemma 3.2: *The pair* (A_0, C_0) *is observable if Assumption* 2.2 *holds.*

Proof: From the Popov–Belevitch–Hautus (PBH) test, the pair (A_0, C_0) is observable if and only if:

$$\text{rank}\begin{bmatrix} sI - A_0 \\ C_0 \end{bmatrix} = \text{rank}\begin{bmatrix} sI - A_4 & 0 \\ -C_4 & sI \\ 0 & I \end{bmatrix} = n + p - 2r \tag{14}$$

If $s = 0$ then

$$\text{rank}\begin{bmatrix} sI - A_4 & 0 \\ -C_4 & sI \\ 0 & I \end{bmatrix} = \text{rank}\begin{bmatrix} -A_4 \\ -C_4 \end{bmatrix} + p - r$$

If Assumption 2.2 holds, it follows that (A_4, C_4) is observable and thus: rank $\begin{bmatrix} sI-A_4 \\ C_4 \end{bmatrix} = n - r$ when $s \neq 0$ and because (A_4, C_4) is observable then:

$$\begin{bmatrix} sI - A_4 & 0 \\ -C_4 & sI \\ 0 & I \end{bmatrix}\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = 0 \Rightarrow \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = 0$$

It means that the columns of $\begin{bmatrix} sI-A_4 & 0 \\ -C_4 & sI \\ 0 & I \end{bmatrix}$ are linearly independent and rank is $n + p - 2r$. This completes the proof. ■

It follows from Lemma 3.2 that there exists a matrix $L_0 \in R^{(n+p-2r)\times(p-r)}$ such that $A_0 - L_0 C_0$ is stable, and for any $Q_0 > 0$, the Lyapunov equation has a unique solution, $P_0 > 0$.

$$(A_0 - L_0 C_0)^T P_0 + P_0(A_0 - L_0 C_0) = -Q_0 \tag{15}$$

For subsystem (8) a sliding mode observer is designed like (16):

$$\dot{\hat{Z}}_1 = A_1\hat{Z}_1 + \bar{A}_2\hat{Z}_0 + g_1(T^{-1}\hat{Z})U + B_1 U + (A_1 - A_{1s})(C_1)^{-1}(W_1 - \hat{W}_1) + \nu_1$$
$$\hat{W}_1 = C_1\hat{Z}_1 \tag{16}$$

A_{1s} is a stable matrix. $\hat{Z}$ is defined as $\hat{Z} := col(C_1{}^{-1}W_1, \hat{Z}_2)$. The injection term ν_1 is defined by

$$\nu_1 = \begin{cases} k_1\dfrac{P_1(Z_1 - \hat{Z}_1)}{\|P_1(Z_1 - \hat{Z}_1)\|} & \text{if } Z_1 - \hat{Z}_1 \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

where $k_1 = \|M_1\|\eta_0 + \|D_1\|\zeta + \gamma$ and γ is a positive scalar. $P_1 \in R^{r\times r}$ is a symmetric positive definite matrix. For subsystem (12) following observer is designed:

$$\dot{\hat{Z}}_0 = A_0\hat{Z}_0 + A_{01}C_1^{-1}W_1 + \hat{g}_2(T^{-1}\hat{Z})U + B_0 U + L_0(W_3 - \hat{W}_3) + \nu_2$$
$$\hat{W}_3 = C_0\hat{Z}_0 \tag{18}$$

Where L_0 is the gain that will be defined and the injection term ν_2 is defined by

$$\nu_2 = \begin{cases} \dfrac{E_0(Z_3 - \hat{Z}_3)}{\|E_0(Z_3 - \hat{Z}_3)\|} & \text{if } Z_3 - \hat{Z}_3 \neq 0 \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

where $E_0 \in R^{q\times(p-r)}$.

3.1.1. Investigating the stability of observer error dynamics

If the state estimation errors are defined as $e_1 = Z_1 - \hat{Z}_1$ and $e_0 = Z_0 - \hat{Z}_0$, then the error dynamics with no sensor fault can be obtained as

$$\dot{e}_1(t) = A_{1s}e_1 + \bar{A}_2 e_0 + g_1(T^{-1}Z)U - g_1(T^{-1}\hat{Z})U + M_1\eta + D_1 d - \nu_1 \tag{20}$$

$$\dot{e}_0(t) = (A_0 - L_0C_0)e_0 + \bar{g}_2(T^{-1}Z)U - \bar{g}_2(T^{-1}\hat{Z})U - \nu_2 \tag{21}$$

Sufficient conditions for the existence of the proposed observers (16) and (18) are presented in the following theorem.

Theorem 3.1: *In the healthy system, the error dynamics* (20) *and* (21) *are asymptotically stable if there exist the matrices* $A_{1s} < 0$, $L_0 > 0$, $P_1 = P_1^T > 0$, $P_0 = P_0^T > 0$ *and the positive scalars* α_1, α_0 *such that*

$$\begin{bmatrix} \Pi_1 + \frac{1}{\alpha_1}P_1P_1 & P_1\bar{A}_2 \\ \bar{A}_2{}^T P_1 & \Pi_2 + \frac{1}{\alpha_0}P_0P_0 + \alpha I_{n+p-2r} \end{bmatrix} < 0 \tag{22}$$

where $\Pi_1 = A_{1s}{}^T P_1 + +P_1A_{1s}$, $\Pi_2 = (A_0 - L_0C_0)^T P_0 + P_0(A_0 - L_0C_0)$.

$$\alpha = \alpha_1 k_{g1}^2\|T^{-1}\|^2 + \alpha_0 k_{g2}^2\|T^{-1}\|^2$$

Proof: The Lyapunov function is chosen as follows:

$$V(e_1, e_0) = V_1(e_1) + V_0(e_0)$$
$$V_1(e_1) = e_1{}^T P_1 e_1,\; V_0(e_0) = e_0{}^T P_0 e_0$$

The time derivative of V_1 is given as

$$\dot{V}_1(e_1) = e_1^T(P_1A_{1s} + A_{1s}^T P_1)e_1 + 2e_1^T P_1\bar{A}_2 e_0 + 2e_1^T P_1 M_1\eta + 2e_1^T P_1 D_1 d + 2e_1^T P_1(g_1(T^{-1}Z) - g_1(T^{-1}\hat{Z}))U - 2e_1^T P_1\nu_1$$

Since the inequality $2X^TY \le (1/\alpha)X^TX + \alpha Y^TY$ is true for any positive scalar α, then:

$$\dot{V}_1(e_1) \le e_1^T(P_1A_{1s} + A_{1s}^TP_1)e_1 + 2e_1^TP_1\bar{A}_2e_0 + 2e_1^TP_1M_1\eta + 2e_1^TP_1D_1d + \frac{1}{\alpha}e_1^TP_1P_1^Te_1 + \alpha(g_1(T^{-1}Z) - g_1(T^{-1}\hat{Z}))^T \times (g_1(T^{-1}Z) - g_1(T^{-1}\hat{Z})) - 2e_1^TP_1\nu_1$$

With no sensor fault there is

$$Z - \hat{Z} = \begin{bmatrix} 0 \\ e_2 \end{bmatrix}$$

$$\|T^{-1}Z - T^{-1}\hat{Z}\| = \|T^{-1}e_2\| \le \|T^{-1}e_0\|$$

$$\|g_1(T^{-1}Z) - g_1(T^{-1}\hat{Z})\| \le k_{g1}\|T^{-1}\|\|e_0\|$$

$$\|g_2(T^{-1}Z) - g_2(T^{-1}\hat{Z})\| \le k_{g2}\|T^{-1}\|\|e_0\|$$

$$k_{g1} = \|T_1\|k_g, k_{g2} = \|T_2\|k_g$$

It can be obtained:

$$e_1^TP_1\nu_1 = k_1\|p_1e_1\|$$

$$\dot{V}_1(e_1) \le e_1^T\Pi_1e_1 + 2e_1^TP_1\bar{A}_2e_0 + \frac{1}{\alpha_1}e_1^TP_1P_1e_1 + \alpha_1k_{g1}^2\|T^{-1}\|^2\|e_0\|^2$$

In the same way as above the time derivative of V_0 is given as

$$\dot{V}_0(e_0) \le e_0^T\Pi_0e_0 + 2e_1^TP_1\bar{A}_2e_0 + \frac{1}{\alpha_0}e_0^TP_0P_0e_0 + \alpha_0k_{g2}^2\|T^{-1}\|^2\|e_0\|^2$$

Combining $\dot{V}_1$ and $\dot{V}_0$ yields:

$$\dot{V} = \dot{V}_1(e_1) + \dot{V}_0(e_0) \le \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}^T \wedge \begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$$

If there exist the matrices $A_{1s} < 0, L_0 > 0, P_1 = P_1^T > 0, P_0 = P_0^T > 0$ and the positive scalars α_1, α_0 such that inequality (22) is satisfied, then $\dot{V} < 0$ for any $e \ne 0$, where $e = [\begin{smallmatrix} e_1 \\ e_2 \end{smallmatrix}]$. This implies that the error dynamics are asymptotically stable. ■

By using the Schur complement approach the problem of finding matrices to stability inequality (22) can be transformed into the following LMI feasibility problem. The matrices X, y_0, $P_1 = P_1^T > 0$, $P_0 = P_0^T > 0$ and positive scalars α_1, α_0 exist such that:

$$\begin{bmatrix} X + X^T & P_1 & P_1\bar{A}_2 & 0 \\ P_1 & -\alpha_1 I & 0 & 0 \\ \bar{A}_2^TP_1 & 0 & \begin{matrix} \bar{A}_0^TP_0 + P_0A_0 \\ -C_0^Ty_0^T - y_0C_0 \\ +\alpha I \end{matrix} & P_0 \\ 0 & 0 & P_0 & -\alpha_0 I \end{bmatrix} < 0 \quad (23)$$

where $X = P_1A_{1s}$, $y_0 = P_0L_0$. When a sensor fault occurs at t_f, the error dynamics become as follows:

$$\dot{e}_1(t) = A_{1s}e_1 + \bar{A}_2e_0 + g_1(T^{-1}Z)U - g_1(T^{-1}\hat{Z})U + M_1\eta + D_1d - \nu_1 \quad (24)$$

$$\dot{e}_0(t) = (A_0 - L_0C_0)e_0 + \bar{g}_2(T^{-1}Z)U - \bar{g}_2(T^{-1}\hat{Z})U + F_0f_s - \nu_2 \quad (25)$$

By considering (25) it can be comprehended that e_0 is only affected by sensor faults f_s. The sensor fault distribution matrix is $F_0 = [\begin{smallmatrix} 0 \\ F_2 \end{smallmatrix}]$. Thus, the sensor fault affects the last $(p - r)$ components of e_0 namely, $e_{Z3} = Z_3 - \hat{Z}_3$. The residual for detecting sensor faults is chosen as: $\|e_{W3}\| = \|C_0e_0\| = \|e_{Z3}\|$. If $\|e_{W3}\| > \eta_0$ then there is a sensor fault, where η_0 is a specified threshold. The detection time, t_d, where $t_d \ge t_f$, is defined as the moment when $\|e_{W3}\|$ exceeds the threshold.

3.2. Fault isolation

When multiple sensor faults occur simultaneously the vector of sensor faults is denoted as $[f_{s1}^T, f_{s2}^T, \ldots, f_{sq}^T]^T$. For each possible $f_{si} \ne 0, i = 0, 1, 2, \ldots, q$; two observers are designed for two subsystems. The residuals obtained from the observers should only be sensitive to f_{si}. The following observer is designed for subsystem (13).

$$\dot{\hat{Z}}_{1i} = A_1\hat{Z}_{1i} + \bar{A}_2\hat{Z}_{0i} + g_1(T^{-1}\bar{Z}_i)U + B_1U + (A_1 - A_{1s})C_1^{-1}(W_{1i} - \hat{W}_{1i}) + \nu_{1i}$$
$$\hat{W}_{1i} = C_1\hat{Z}_{1i} \quad (26)$$

where $\hat{Z}_i$ denotes the estimated state, it is defined as $\hat{Z}_i := col(c_1^{-1}W_1, [I_{n-1}, 0]\hat{Z}_{0i})$ and $\hat{W}_i$ denotes the estimated output. The output error injection term ν_{1i} is defined as

$$\nu_{1i} = \begin{cases} (\|M_1\|\eta_0 + \|D_1\|\zeta + \gamma)\dfrac{P_1(Z_1 - \hat{Z}_{1i})}{\|P_1(Z_1 - \hat{Z}_{1i})\|} & \text{if } Z_1 - \hat{Z}_{1i} \ne 0 \\ 0 & \text{otherwise} \end{cases} \quad (27)$$

where γ is a positive scalar. The proposed observer for fault isolation has the following form for subsystem (12):

$$\dot{\hat{Z}}_{0i} = A_0\hat{Z}_{0i} + A_{01}C_1^{-1}W_{1i} + \bar{g}_2(T^{-1}\bar{Z}_i)U + B_0U + L_0(W_{3i} - \hat{W}_{3i}) + \bar{F}_{0i}\nu_{2i}$$
$$\hat{W}_{3i} = C_0\hat{Z}_{0i} \quad (28)$$

The output error injection term ν_{2i} is defined as

$$\nu_{2i} = \begin{cases} \gamma_3\dfrac{\bar{E}_{0i}(W_{3i} - \hat{W}_{3i})}{\|\bar{E}_{0i}(W_{3i} - \hat{W}_{3i})\|} & \text{if } e_{W_{3i}} = W_{3i} - \hat{W}_{3i} \ne 0 \\ 0 & \text{otherwise} \end{cases} \quad (29)$$

where $\bar{F}_{0i}$ represents the vector of all except f_{si} sensor faults, γ_3 is a positive scalar and E_{0i} is the i-th row of E_0 and $\bar{E}_{0i}$ consists all other rows. The error dynamics are expressed as

$$\dot{e}_{1i}(t) = A_{1s}e_{1i} + \bar{A}_2 e_{0i} + g_1(T^{-1}Z)U - g_1(T^{-1}\hat{Z})U + M_1\eta + D_1 d - v_{1i} \tag{30}$$

$$\dot{e}_{0i}(t) = (A_0 - L_0C_0)e_{0i} + \bar{g}_2(T^{-1}Z)U - \bar{g}_2(T^{-1}\hat{Z})U + F_{0i}f_{0i} - \bar{F}_{0i}(f_{si} - v_{2i}) \tag{31}$$

Multiple sensor faults can be isolated by comparing the residual $\|e_{W_{3i}}\|$, with a predefined threshold. If $\|e_{W_{3i}}\|$ exceeds the threshold, then it is concluded that $f_{si} \neq 0$. Considering the structure of F_0, the decision on which sensor is faulty is to be made.

4. Simulation

The effectiveness of the proposed sensor fault detection scheme is demonstrated by considering the following example. The state space of the system can be represented in the following form.

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 0.5 & 0\\ -1 & -1 & 0\\ 1 & -1 & -2\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\0\\1\end{bmatrix}U + \begin{bmatrix}0\\ -\sin(x_1)\\ 0\end{bmatrix}U + \begin{bmatrix}1\\0\\0\end{bmatrix}\eta + \begin{bmatrix}1\\0\\0\end{bmatrix}d$$

$$y = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix}f_s$$

$$\eta = 0.5\sin(t), \quad d = 0.2\cos(t),$$

$$U = 1.5\sin(t), \quad g(x) = \begin{bmatrix}0\\ -\sin(x_1)\\ 1\end{bmatrix}$$

The nonlinear term $g(x)$ has a Lipschitz constant. The terms η and d are added to the system equations to represent uncertainties and disturbances. Parameter uncertainty is assumed to change system matrix elements randomly. Sensor fault vector is $\begin{bmatrix}f_{s1}\\ f_{s2}\end{bmatrix}$ and the coordinate transformation matrices are obtained as $T = S = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$. In the new coordinate system, the system matrices become:

$$A = TAT^{-1} = \begin{bmatrix}0 & 0.5 & 0\\ -1 & -1 & 0\\ 1 & -1 & -2\end{bmatrix},$$

$$C = SCT^{-1} = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 1 & 1\end{bmatrix}, \quad B = TB = \begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$M = TM = \begin{bmatrix}1\\0\\0\end{bmatrix}, D = TD = \begin{bmatrix}1\\0\\0\end{bmatrix},$$

$$F = SF = \begin{bmatrix}0 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix}$$

By solving the LMI problem (22), following parameters are computed:

$$P_1 = 0.6567, \quad A_{1s} = -0.8938, \quad \alpha_0 = 1.1993,$$

$$E_0 = \begin{bmatrix}0.3852 & 0.0259\\ 0.0259 & 0.4318\end{bmatrix}$$

$$L_0 = \begin{bmatrix}0 & 0\\ 0 & 0\\ 1.7834 & -0.1048\\ -0.1414 & 1.5562\end{bmatrix}$$

To detect fault occurrence the norm of the output estimation error, $\|e_{W3}\|$, is selected as the residual. An adequate threshold level is decided based on practical considerations for system parameter uncertainty which is implemented by changing 10% in some randomly selected elements of system matrix. Performing many simulations confirmed that threshold level 0.02 provides satisfactory results in terms of fault detection sensitivity and false alarm rate. Furthermore, an upper band for initial transient errors is estimated by performing an extensive simulation study. To this end, randomly selected non-zero initial states are applied to the fault detection observer and the resulted transients are evaluated carefully. The initial transient error decays whiting 3.5 s. Therefore a time-varying threshold is chosen as Equation (32). Initial time-varying threshold facilitates fault detection during the transient period. Thus, a fault is detected whenever the residual goes over the threshold.

$$threshold = \begin{cases}\dfrac{1}{\frac{45}{3.5}t + 5} & t \leq 3.5\\ 0.02 & t > 3.5\end{cases} \tag{32}$$

4.1. Fault detection results

In this part, it is assumed that $f_{s2} = 0$ for all time. The residual value of fault-free system and after occurring the following abrupt fault is observed in Figure 2.

$$f_{s1}^{(abrupt)} = \begin{cases}0 & t < 12\\ 0.5 & t \geq 12\end{cases}$$

The residual value for non-zero initial states and without parameter uncertainty is shown in Figure 2(a). The trace of uncertainty is demonstrated in Figure 2(b) evidencing that threshold level 0.02 is a desirable choice. The result of non-zero initial states is shown in Figure 2(c). As observed, transient error falls below threshold level 0.02 before 3.5 s. The residual value in case of uncertainty and non-zero initial states is shown

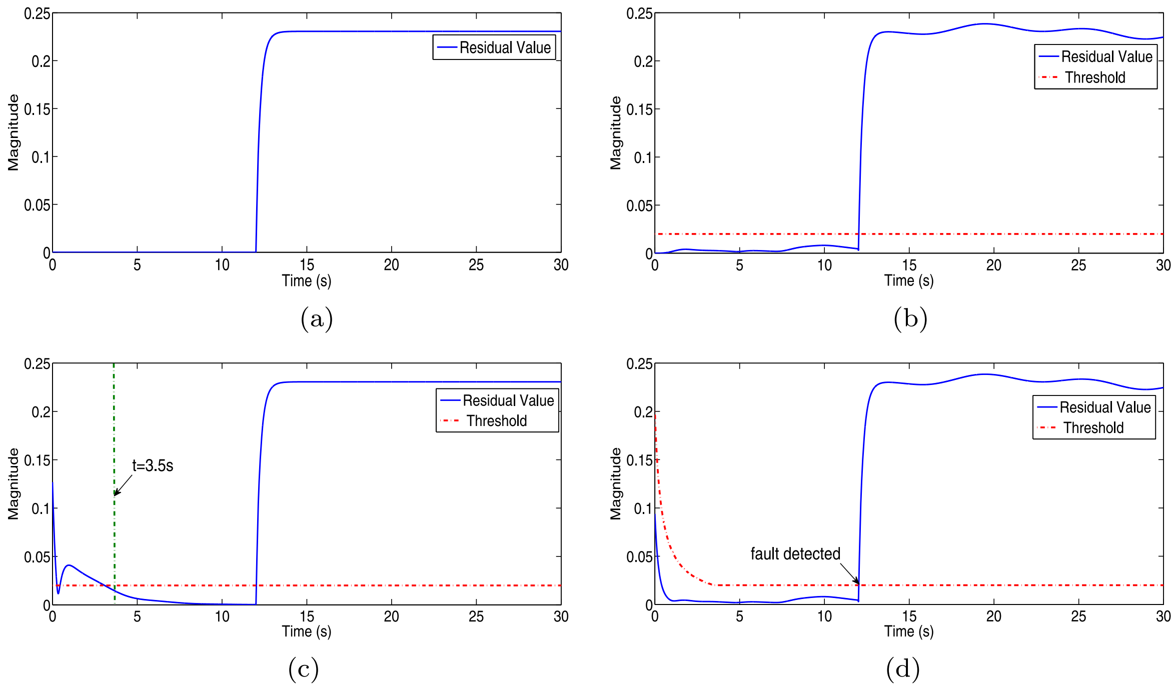

Figure 2. The influence of sensor abrupt fault on the residual. (a) Without uncertainty and zero initial states. (b) With uncertainty and zero initial states. (c) Without uncertainty and non-zero initial states. (d) With uncertainty and non-zero initial states.

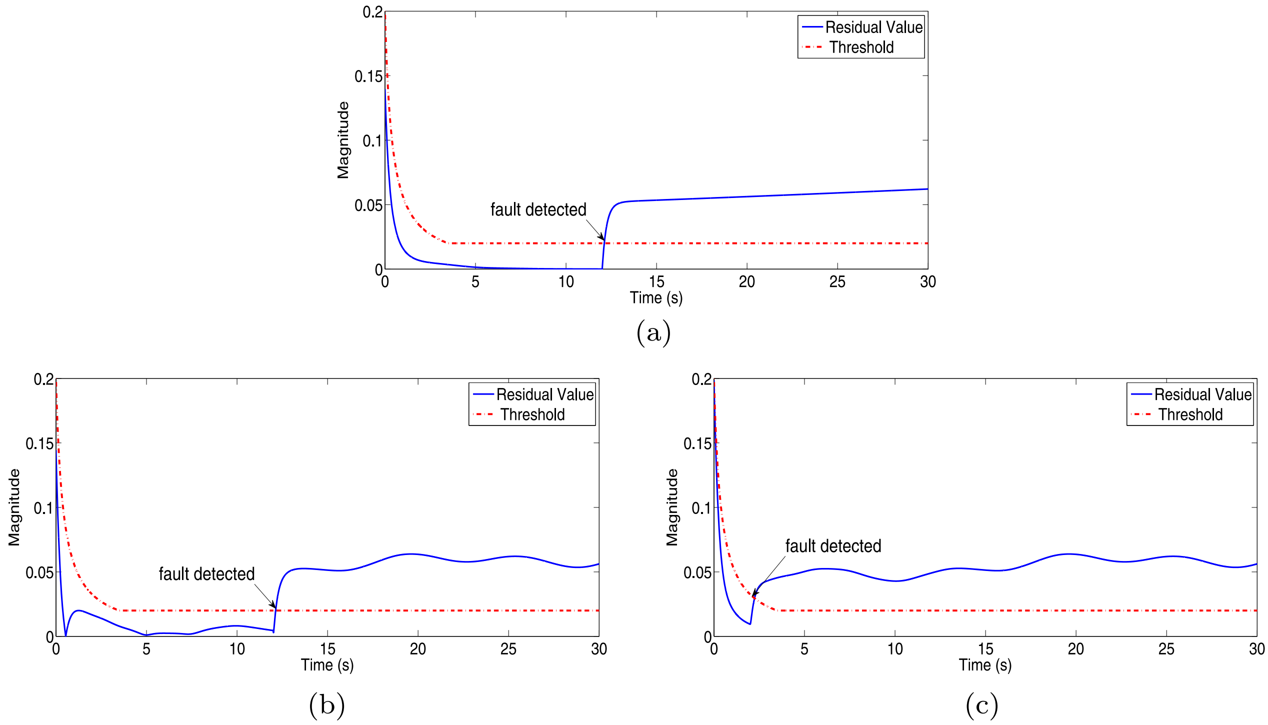

Figure 3. The incipient sensor fault detection. (a) Without uncertainty and non-zero initial states. (b) With uncertainty and non-zero initial states. (c) Fault occurs in transient period.

in Figure 2(d). The residual value exceeds the threshold at 12.02 s, therefore sensor fault is detected within 0.02 s.

Fault detection results for the following incipient fault with non-zero initial states but without parameter uncertainty and also with non-zero initial states and parameter uncertainty are shown in Figure 3(a,b), respectively. As observed in Figure 3(a) the residual value exceeds the threshold level at 12.11s, alarming a fault. The residual value exceeds threshold at 12.13 s in Figure 3(b) which reveals sensor fault. Figure 3(c) demonstrates the capability of the proposed scheme to

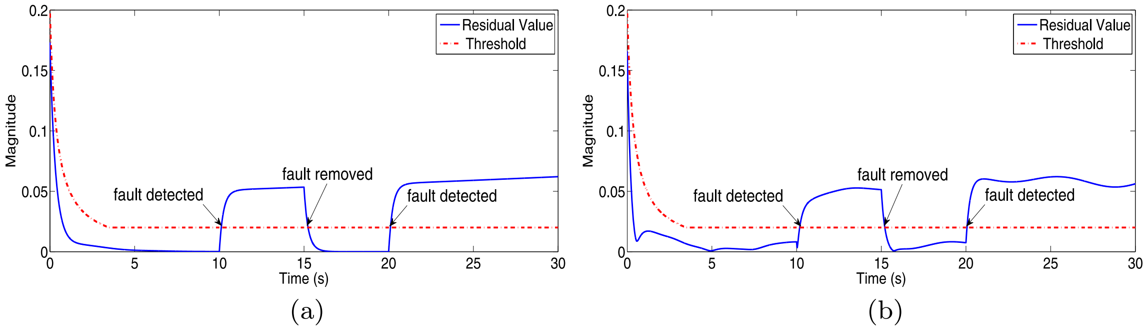

Figure 4. The intermittent sensor fault detection via the proposed scheme. (a) Without uncertainty and non-zero initial states. (b) With uncertainty and non-zero initial states.

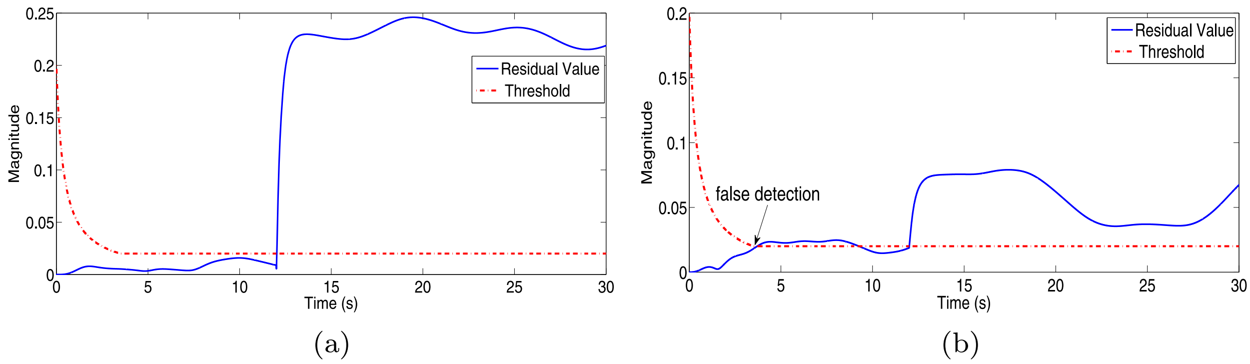

Figure 5. Simulation of large parameter uncertainty. (a) Abrupt fault. (b) Incipient fault.

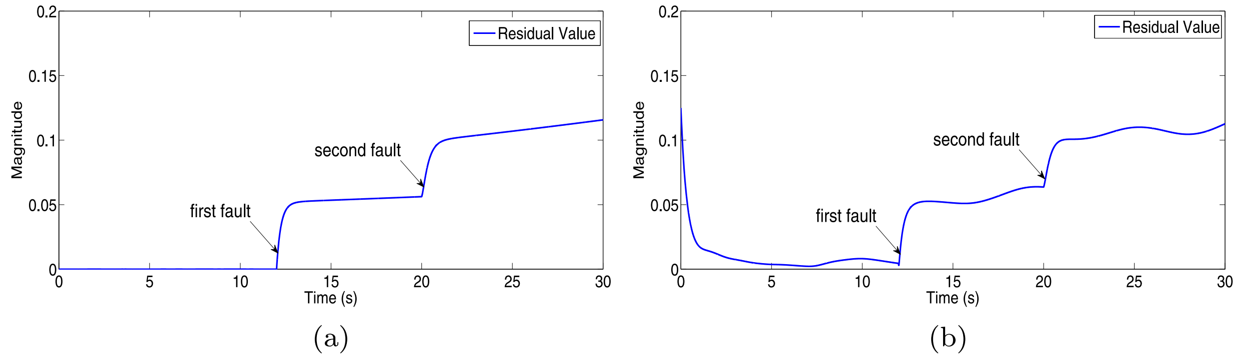

Figure 6. The residual in case of two successive sensors fault. (a) Without uncertainty and initial conditions. (b) With uncertainty and initial conditions.

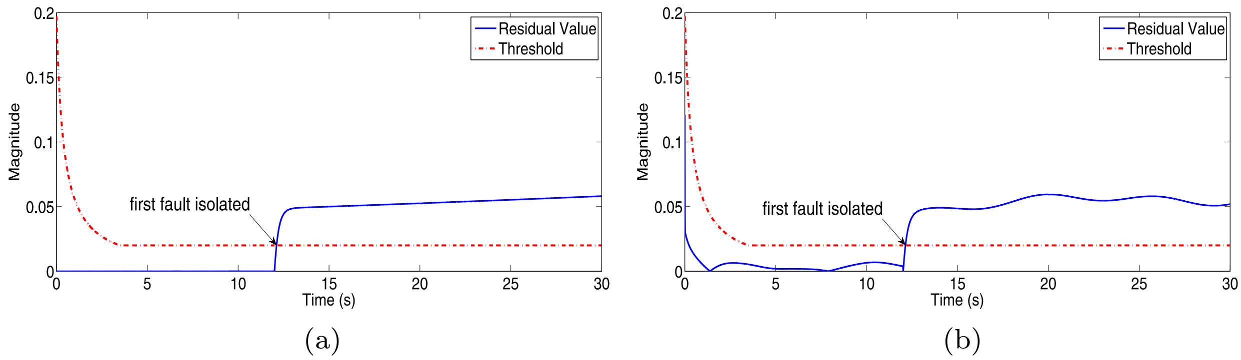

Figure 7. Isolation of first sensor fault, $fs1$. (a) Without uncertainty and initial conditions. (b) With uncertainty and initial conditions.

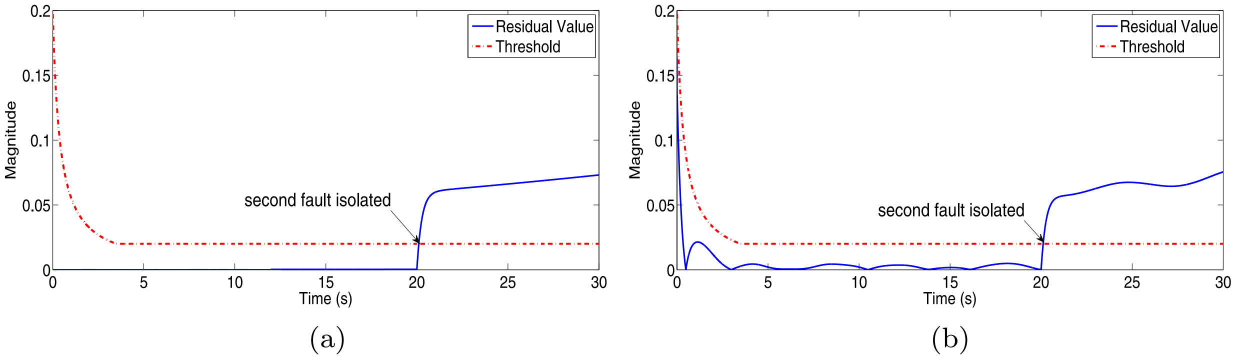

Figure 8. Isolation of second sensor fault, $fs2$. (a) Without uncertainty and initial conditions. (b) With uncertainty and initial conditions.

detect a sensor fault during the initial transient period, thanks to the adopted threshold function.

$$f_{s1}^{(incipient)} = \begin{cases} 0 & t < 12 \\ 0.1e^{0.01t} & t \geq 12 \end{cases}$$

The sensor fault detection via the proposed scheme for the following intermittent fault with non-zero initial states is demonstrated in Figure 4, where the effects of parameter uncertainty are seen in Figure 4(b). From Figure 4(a) the residual value exceeds the threshold at 10.11 s, thus a fault is detected. The residual remains larger than the threshold until 15.24 s. Then the residual falls below the threshold at 15.24 s and implies that the fault is removed. The residual exceeds the threshold at 20.10 s again, which implies that there is a fault after 20.10 s. From Figure 4(b) the residual first exceeds the threshold at 10.18 s and an incipient fault detected, then it falls below the threshold at 15.21 s indicating that the fault is removed. At 20.06 s a fault is detected as the residual exceeds the threshold.

$$f_{s1}^{(intermittent)} = \begin{cases} 0 & t < 10 \\ 0.1e^{0.01t} & 10 \leq t < 15 \\ 0 & 15 \leq t < 20 \\ 0.1e^{0.01t} & t \geq 20 \end{cases}$$

By more increasing, the level of parameter uncertainty, the probability of false detection would grow accordingly. The influence of 30% uncertainty in the randomly selected elements of the system matrix on abrupt and incipient fault detection are shown in Figure 5(a,b), respectively. As observed in Figure 5(b) false alarm is issued due to a large amount of parameter uncertainty.

4.2. Fault isolation results

In this part of simulations, two sensors fault are assumed as follows:

$$fs1 = \begin{cases} 0 & t < 12 \\ 0.1e^{0.01t} & t \geq 12 \end{cases},$$

$$fs2 = \begin{cases} 0 & t < 20 \\ 0.15e^{0.02t} & t \geq 20 \end{cases}$$

The fault detection capability of the proposed scheme without a trace of uncertainty and with zero initial states and also with a trace of uncertainty and non-zero initial states are demonstrated in Figure 6(a,b), respectively. After detecting a fault, the next step is to determine which sensor is faulty. The isolation result for f_{s1} and f_{s2} with zero initial states and without a trace of uncertainty are shown in Figure 7(a,b), respectively. In the same way, the isolation results for f_{s1} and f_{s2} with non-zero initial states and with a trace of uncertainty are shown in Figures 7(b) and 8(b), respectively. The residual generated by the observer is compared with the threshold obtained from Equation (32).

5. Conclusion

In this paper, a sensor fault detection and isolation method for a class of nonlinear systems was proposed. First, the original system was divided into two subsystems by applying coordinate transformation matrices. The first subsystem involved uncertainty and disturbance and the second subsystem only contained sensor faults. Then, the sensor fault was expressed as an actuator fault virtually and the new Sliding Mode Observer was designed for multiple sensors fault detection and isolation. The stability conditions were examined and the required parameters of the observers were obtained by solving the LMI problem. The performance of the proposed scheme is illustrated by a simulation study. To avoid false detection on initial transient, a time-varying threshold function is employed. The simulation results showed that the scheme detects and isolates faults correctly with good accuracy and speed without false alarm or missed detection, in the presence of a certain amount of uncertainty.

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