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Research on RBF neural network model reference adaptive control system based on nonlinear U – model

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ABSTRACT

The overall objective of this study is to design the nonlinear U-model-based radial basis function neural network model reference adaptive control system, through research into a class of complex time-varying nonlinear plants. First, the ideal nonlinear plant is adopted as the reference model and transformed into the U-model representation. In the process, the authors establish the corresponding relationship between the degrees of the reference nonlinear model and the controlled nonlinear plants, and carry out research into the corresponding coefficient relationship between the reference nonlinear model and the controlled nonlinear plants. Also, the impact of the adjusting amplitude and tracking speed of the model on the system control accuracy is analyzed. Then, according to the learning error index of the neural network, the paper designs the adaptive algorithm of the radial basis function neural network, and trains the network by the error variety. With the weight coefficients and network parameters automatically updated and the adaptive controller adjusted, the output of controlled nonlinear plants can track the ideal output completely. The simulation results show that the model reference adaptive control system based on RBF neural network has better control effect than the nonlinear U-model adaptive control system based on the gradient descent method.

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RBF neural network; nonlinear U-model; model reference adaptive

1. Introduction

In the actual control system, the nonlinear characteristics are ubiquitous. Due to the inherent complexity of the nonlinear time-varying system, the establishment of a general-purpose and high-precision mathematical model has been the difficulty of research as well as the premise and foundation for the design of nonlinear control system. The early nonlinear models include Wiener model, bilinear system model, Hammerstein model, NARMAX model, nonlinear time series model, output affine model, NARX model and Lure model, etc., among which Wiener model and Hammerstein model are simple in structure, composed of the linear system and the nonlinear static gain in series. The nonlinear auto-regressive moving average model (NARMAX) has a certain degree of versatility with the ability to effectively describe any nonlinear system, and the U-model is evolved on the basis of it. The U-model represents a class of smoothing nonlinear discrete model; it has no loss of model accuracy so the modelling accuracy of nonlinear plant is improved [1].

In 2002, Professor Zhu [2] proposed the concept of U-model, and designed a class of known dynamic nonlinear systems with the U-model using the pole placement method, which yielded good control effect. This launched the preliminary study on the control method based on U-model and laid the foundation for the subsequent study of the U-model. On the basis of the pole placement control, Qiu et al. [3] compared and analyzed the U-model-based design method with the classical control design method, proving the superiority of the U-model in designing the control system. Chen et al. [4] modelled the complex CSTR chemical process simply with NARMAX modelling method and proposed a nonlinear pole placement controller based on U-model, which outdid the traditional PID control in response speed and control accuracy. Shafiq and Haseebiddon [5] put forward the U-model-based IMC control system with minimum mean square error, and introduced the learning rate parameters in the system inverse finding computational algorithm, which improved the stability of the algorithm but reduced the convergence speed. For complex nonlinear industrial process systems, Khan and Shafiq [6] presented the U-model-based adaptive inverse control method with minimum mean square error (LMS), and designed the outer ring by means of the linear control theory, which improved the interference suppression performance of the system. Combining the robustness of IMC and the approximation ability of nonlinear adaptive filter, Butt

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developed the adaptive system based on robust internal model control (IMC), hence the adaptive tracking for a class of stable nonlinear objects [7]. Hasan et al. [8] designed an adaptive controller based on the U-Model for the gas control equipment. After the comparison of the ARX and ARMAX models with the U model, it was demonstrated that the U-model was better in effect than the others. Xu et al. [9] presented the direct model reference adaptive control system which is modelled by nonlinear U-model, and introduced two compensation parameters. Thus the system output tracked the desired output through the adaptive adjustment of parameters. Du et al. [10] proposed the generalized predictive control algorithm based on the U model, which transformed the output of the controller into the solution of the polynomial and reduced the complexity of the nonlinear control system design. In [11], The U-model-based control system design method was extended to the non-minimum phase (NMP) dynamic system, in which the feedforward controller was used removed the NMP zero, and the system achieved the same output response as the desired MP device. Azhar et al. established the U-model to express the unknown MIMO bilinear system model, utilized the radial basis function neural network to identify the MIMO bilinear model online [12]. Also, the small gain theorem was adopted to analyze the real-time tracking and feedback effect of the system[13], and Newton–Raphson method was used to build up the learning feedforward controller (LFFC)[14]. In [15], the author selected the nonlinear U-model polynomial plant controller, which was designed by the method of linear polynomial control system, as the research target, and formed the U-block model framework, which made it possible to introduce the method of linear state space into the design of the nonlinear control system and offered a new direction for the design of nonlinear control system.

With favourable generalization ability and relatively simple network structure, neural network is not involved with complicated mathematical calculations and is characterized as the approximation to any nonlinear function with arbitrary precision. Therefore, it can be combined in many aspects of nonlinear research. With the degree of nonlinear U-model plant known, Chang et al. [16] identified the unknown parameters of nonlinear U-model plant by the least-squares algorithm and designed the radial basis function neural network (RBFNN) online controller. Shafiq et al. [17] combined the robustness of IMC with the ability of neural networks to identify arbitrary nonlinear functions, proposed the adaptive system based on IMC control for dynamic nonlinear plant and solved the control problems with a class of known nonlinear dynamic system. Butt et al. [18] put forward a universal and simple control law by combining the approximate ability of higher-order neural network with the control-oriented feature of the U-model. In [19], for nonlinear plant with stable structure and unknown parameters, which was identified and approximated by the high-order neural network, the adaptive internal model controller based on U-model was designed and applied into adaptive tracking of DC motor speed. Xu et al. [20] utilized the identification neural network to identify in real time the U-model time-varying parameters of nonlinear plant, and designed the self-tuning controller using Newton iteration algorithm, the output of which was adjusted by the results of online identification. This solved the control problems of the nonlinear system with unknown model.

In this paper, the design method of the RBF neural network model reference adaptive system based on nonlinear U-model is presented. The authors adopt the nonlinear U-model plant as the ideal reference model and establish the corresponding structure relationship between reference model and the controlled plants. Then, the study designs the adaptive algorithm based on the radial basis function neural network, the weights and parameters of which is adjusted through online error learning. Meanwhile, the control system is analyzed.

2. Nonlinear U-model

U-model can express a wide range of nonlinear discrete-time plant control models [2], whose control-oriented nonlinear model can be expressed as the polynomial with the $u(t-1)$.

$$y(t) = \sum_{j=0}^{M} \left[\prod_{l=1}^{n} \left(a_l + \sum_{i=1}^{m} y^l(t-i) \right) \right] \cdot \left[\prod_{l=1}^{n} \left(b_l + \sum_{i=2}^{m} u^l(t-i) \right) \right] u^j(t-1) \quad (1)$$

where, a_l and b_l the parameters of the model and are constants, M is the degree of the model input $u(t-1)$. $u(t-j)$ and $y(t-j)$ are input signals and output signals of the past time respectively. By simplifying Equation (1), we can get:

$$y(t) = \sum_{j=0}^{M} \alpha_j(t) u^j(t-1) \quad (2)$$

Where, the time-varying parameter $\alpha_j(k)$ is the function of the past inputs $u(t-j)$ and past outputs $y(t-j)$.

$$\alpha_j(t) = \left[\prod_{l=1}^{n} \left(a_l + \sum_{i=1}^{m} y^l(t-i) \right) \right] \cdot \left[\prod_{l=1}^{n} \left(b_l + \sum_{i=2}^{m} u^l(t-i) \right) \right]$$

$$= [a_1 + y(t-1) + y(t-2) + \cdots + y(t-m)]$$
$$[a_1 + y^2(t-1) + y^2(t-2) + \cdots + y^2(t-m)] \cdots$$
$$[a_n + y^n(t-1) + y^n(t-2) + \cdots + y^n(t-m)]$$
$$[b_1 + u(t-2) + u(t-3) + \cdots + u(t-m)]$$
$$[b_2 + u^2(t-2) + u^2(t-3) + \cdots + u^2(t-m)] \cdots$$
$$[b_n + u^n(t-2) + u^n(t-3) + \cdots + u^n(t-m)] \quad (3)$$

Take an example to illustrate the generality and simplicity of the nonlinear U-model, the expression of the nonlinear plant is:

$$y(t) = 0.1y(t-1)y(t-2) + 0.8u(t-2)u(t-1) - 0.3y(t-1)u^2(t-1) \quad (4)$$

According to Equation (2), we can get:

$$y(t) = \alpha_0(t) + \alpha_1(t)u(t-1) + \alpha_2(t)u^2(t-1) \quad (5)$$

Where, $\alpha_0(t) = 0.1y(t-1)y(t-2)$, $\alpha_1(t) = 0.8u(t-2)$, $\alpha_2(t) = 0.3y(t-1)$.

According to the above description of the U model, the expression of the nonlinear U model is very simple, and it can transform other models well, which is more general and plays an important role in solving the nonlinear modelling problem. Compared with other nonlinear models, U model has the following advantages:

(1) Compared with other nonlinear models, such as NARMAX model and Hammerstain model, the expression structure of the U model is simpler and more practical.
(2) The nonlinear U model can transform most nonlinear discrete time input-output models and the mapping is reversible.
(3) The U model is the polynomial structure of current input, and the mathematical expression is simple, which simplifies the complexity of the controller design.

3. The design of RBFNN model reference adaptive control system

3.1. The diagram of control system

Since the U-model represents a class of smooth nonlinear discrete models, the reference model can be expressed as the form of U-model:

$$y_m(k) = \sum_{j=0}^{n} \alpha_j(k)u^j(k-1) = \alpha_0(k) + \alpha_1(k)u(k-1) + \cdots + \alpha_n(k)u^n(k-1) \quad (6)$$

where, $u(k-1)$ is the input of the reference U-model, $y_m(k)$ is the output of the reference U-model, n is the degree of the reference U-model input $u(k-1)$, $\alpha_j(k)$ is the time-varying parameter of the reference U-model.

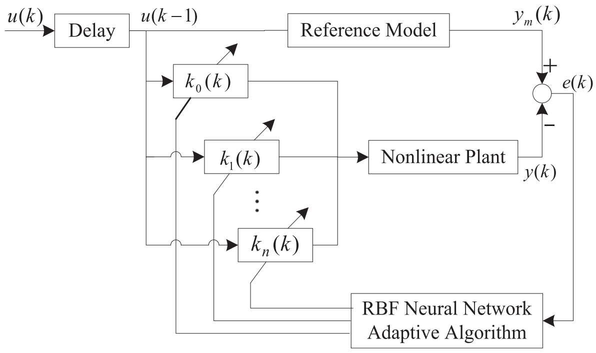

Figure 1. The diagram of model reference adaptive control system based on RBFNN.

After being sampled and held, the nonlinear plant can be conveniently transformed into the expression form of U-model. On the basis of typical model reference adaptive control, the adaptive mechanism based on RBF neural network is designed, and the output of the controlled plant can track the output of the reference model. The diagram of model reference adaptive control system based on RBF neural network is shown in Figure 1.

Where, $u(k-1)$ is the input of the reference model and nonlinear plant. $y_m(k)$ is the output of nonlinear reference U-model and $y(k)$ is the output of controlled nonlinear plant. $k_0(k), k_1(k), \cdots, k_n(k)$ are the compensation parameters of controller. The generalized error $e(k)$ is the deviation between the nonlinear reference U-model output and the controlled nonlinear plant output.

3.2. The analysis of corresponding relationship between controlled plant and reference model

For complex nonlinear plant, it can be discretized and expressed as the form of U-model:

$$y(k) = \sum_{j=0}^{m} \bar{\alpha}_j(k)u^j(k-1) = \bar{\alpha}_0(k) + \bar{\alpha}_1(k)u(k-1) + \cdots + \bar{\alpha}_m(k)u^m(k-1) \quad (7)$$

where, $u(k-1)$ is the input of controlled nonlinear plant, $y(k)$ is the output of controlled nonlinear plant, m is the degree of controlled nonlinear plant input $u(k-1)$, $\bar{\alpha}_j(k)$ is time-varying parameter of controlled nonlinear plant.

According to the diagram of model reference adaptive system, the corresponding relationship between the controlled plant and the reference model is analyzed in order to obtain the regulation rules of compensation parameters $k_0(k), k_1(k), \cdots, k_n(k)$.

(a) While $n > m$, the degree of the reference model is higher than the degree of the controlled plant model. According to the design requirements of the model reference adaptive algorithm, the parameters of the controlled plant are correspond to the parameters of the reference model one by one, let,

$$y(k) = \sum_{j=0}^{n} \tilde{\alpha}_j(k)u^j(k-1) = \tilde{\alpha}_0(k) + \tilde{\alpha}_1(k)u(k-1) + \cdots + \tilde{\alpha}_n(k)u^n(k-1)$$

Where

$$\begin{cases} \tilde{\alpha}_j(k) = \bar{\alpha}_j(k) & 0 \le j \le m \\ \tilde{\alpha}_j(k) = 0 & m < j \le n \end{cases}.$$

That is, the parameters of higher than the m degree term in the controlled plant model are set as zero, and the adaptive algorithm of the neural network is ineffective in adjusting the corresponding compensation parameters, which can cause a certain redundancy error to the system.

(b) While $n = m$,The degree of the reference model is equal to the degree of the controlled plant model, let,

$$y(k) = \sum_{j=0}^{n} \tilde{\alpha}_j(k)u^j(k-1) = \tilde{\alpha}_0(k) + \tilde{\alpha}_1(k)u(k-1) + \cdots + \tilde{\alpha}_n(k)u^n(k-1)$$

where $\tilde{\alpha}_j(k) = \bar{\alpha}_j(k), j = 0, 1, \cdots, n = m$.

The parameters of the controlled object model are one-to-one corresponding to the parameters of reference model, which are in line with the design requirements of the model reference adaptive algorithm. The neural network adaptive algorithm can effectively adjust all the compensation parameters. The system has the advantages of fast response, small adjustment range and relatively higher adjustment accurate.

(c) While $n < m$, the degree of the reference model is lower than that of the controlled plant model. According to the design requirements of the model reference adaptive algorithm, the parameters of the controlled plant are correspond to the parameters of reference model one by one, let,

$$y(k) = \sum_{j=0}^{n} \tilde{\alpha}_j(k)u^j(k-1) = \tilde{\alpha}_0(k) + \tilde{\alpha}_1(k)u(k-1) + \cdots + \tilde{\alpha}_n(k)u^n(k-1)$$

Where,

$$\begin{cases} \tilde{\alpha}_0(k) = \bar{\alpha}_0(k) + \bar{\alpha}_{n+1}(k)u^{n+1}(k-1) + \cdots \\ +\bar{\alpha}_m(k)u^m(k-1) \\ \tilde{\alpha}_j(k) = \bar{\alpha}_j(k), 1 \le j \le n \end{cases}.$$

That is, the polynomial part above the n degree in the controlled object model is incorporated into the zero-degree term. Although the system has no redundancy error, the adjustment of the compensation parameter $k_0(k)$ is relatively complicated and the adjustment speed is slowed down. The system adjustment precision is affected.

3.3. The design of control system

In the controller, compensation parameters are set as $k_0(k), k_1(k), \cdots, k_n(k)$, the time-varying parameters $\tilde{\alpha}_0(k), \tilde{\alpha}_1(k), \cdots, \tilde{\alpha}_n(k)$ is compensated by the $k_0(k), k_1(k), \cdots, k_n(k)$ respectively, then the output of the controlled plant is given as follows:

$$y(k) = \sum_{j=0}^{n} k_j(k)\tilde{\alpha}_j(k)u^j(k-1) = k_0(k)\tilde{\alpha}_0(k) + k_1(k)\tilde{\alpha}_1(k)u(k-1) + \cdots + k_n(k)\tilde{\alpha}_n(k)u^n(k-1) \quad (8)$$

The generalized error $e(k)$ of the system is defined:

$$e(k) = y_m(k) - y(k) \quad (9)$$

Substitute the (6) and (8) into (9), it can be available:

$$e(k) = \sum_{j=0}^{n} (\alpha_j(k) - k_j(k)\tilde{\alpha}_j(k))u^j(k-1) = \alpha_0(k) - k_0(k)\tilde{\alpha}_0(k) + (\alpha_1(k) - k_1(k)\tilde{\alpha}_1(k))u(k-1) + \cdots + (\alpha_n(k) - k_n(k)\tilde{\alpha}_n(k))u^n(k-1) \quad (10)$$

RBF neural network is used to train the compensation parameters, and the RBF neural network adaptive algorithm is designed to obtain a better control effect.

Select the network learning error index as follows:

$$E(k) = \frac{1}{2}e(k)^2 \quad (11)$$

The performance index function is minimized by adjusting the compensation parameters $k_0(k), k_1(k), \cdots, k_n(k)$.

Let $\boldsymbol{K}(k) = [k_0(k), k_1(k), \cdots, k_n(k)]^T$, the output of the RBF neural network is given as

$$\boldsymbol{K}(k) = \boldsymbol{WH} \quad (12)$$

where, $\boldsymbol{W}$ is the connection weight matrix between hidden layer and output layer of RBF neural network, $\boldsymbol{H} = [h_1, h_2, \cdots, h_s]^T$, s is the number of hidden layer nodes.

In the RBF network, $\boldsymbol{X} = [x_1, x_2, \cdots, x_r]^T$ is the input vector of the network, r is the number of the network inputs, $\boldsymbol{H} = [h_1, h_2, \cdots, h_s]^T$ is the radial basis vector of the RBF network, h_j is Gauss basis function:

$$h_j = \exp\left(-\frac{||\boldsymbol{X} - \boldsymbol{C}_j||^2}{2b_j^2}\right) \tag{13}$$

where, $i = 1, 2, \cdots, r; j = 1, 2, \cdots, s$. b_j is the basis width parameter of the node j, $b_j > 0$, $\boldsymbol{C}_j$ is the centre vector of the node j, $\boldsymbol{C}_j = [c_{j1}, c_{j2}, \cdots, c_{jr}]^T$, $\boldsymbol{B} = [b_1, b_2, \cdots, b_s]^T$.

Set the weight matrix as

$$\boldsymbol{W} = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1s} \\ w_{21} & w_{22} & \cdots & w_{2s} \\ \vdots & \vdots & \ddots & \vdots \\ w_{(n+1)1} & w_{(n+1)2} & \cdots & w_{(n+1)s} \end{bmatrix} \tag{14}$$

According to the gradient descent method and the chain rule, the learning algorithm of output layer neuron weights is given as:

$$\Delta w_{lj}(k) = -\eta \frac{\partial E(k)}{\partial w_{lj}} = -\eta \frac{\partial E(k)}{\partial y(k)} \frac{\partial y(k)}{\partial K(k)} \frac{\partial K(k)}{\partial w_{lj}} \tag{15}$$

Where,

$$\begin{cases} \Delta w_{1j}(k) = -\eta \frac{\partial E(k)}{\partial w_{1j}} = \eta e(k)\tilde{\alpha}_0(k)h_j \\ \Delta w_{2j}(k) = -\eta \frac{\partial E(k)}{\partial w_{lj}} = \eta e(k)\tilde{\alpha}_1(k)u(k-1)h_j \\ \cdots \\ \Delta w_{(n+1)j}(k) = -\eta \frac{\partial E(k)}{\partial w_{(n+1)j}} \\ = \eta e(k)\tilde{\alpha}_n(k)u^n(k-1)h_j \end{cases}$$

$$w_{lj}(k) = w_{lj}(k-1) + \Delta w_{lj}(k) + \beta[w_{lj}(k-1) - w_{lj}(k-2)] \tag{16}$$

where, $l = 1, 2, \cdots, n+1$, η is learning rate, β is momentum factor, $\eta \in [0, 1], \beta \in [0, 1]$.

The learning algorithms of basis width parameters and the central parameters are given as

$$\Delta b_j(k) = -\eta \frac{\partial E(k)}{\partial b_j} = \eta e(k) \frac{\partial y(k)}{\partial K(k)} \frac{\partial K(k)}{\partial b_j} = \eta e(k) \left[\sum_{k=0}^{n} \tilde{\alpha}_k(k) u^k(k-1) w_{(k+1)j}\right] h_j \frac{||x - c_{ij}||^2}{b_j^3} \tag{17}$$

$$b_j(k) = b_j(k-1) + \Delta b_j(k) + \beta[b_j(k-1) - b_j(k-2)] \tag{18}$$

$$\Delta c_{ij}(k) = -\eta \frac{\partial E(k)}{\partial c_{ij}} = \eta e(k) \frac{\partial y(k)}{\partial K(k)} \frac{\partial K(k)}{\partial c_{ij}} = \eta e(k)(\sum_{k=0}^{n} \tilde{\alpha}_k(k) u^k(k-1) w_{(k+1)j}) h_j \frac{x_i - c_{ij}}{b_j^2} \tag{19}$$

$$c_{ij}(k) = c_{ij}(k-1) + \Delta c_{ij}(k) + \beta[c_{ij}(k-1) - c_{ij}(k-2)] \tag{20}$$

Through the adjustment of neural network parameters and weights,change the compensate parameters of controller and the error of the system are close to zero, and adaptive tracking of the controlled nonlinear plant to the reference model is achieved.

3.4. The stability analysis

The main information source of model reference adaptive control law is the generalized error vector e. The complete asymptotic convergence of the model reference adaptive control system means that it can exist $\lim_{t\to\infty} e(t) = 0$ for any initial conditions. Therefore, the error equation is an important basis for the analysis and design of the model reference adaptive system. The error equation of the model reference adaptive control system based on the nonlinear U model is as follows:

$$e(k) = \sum_{j=0}^{n} (\alpha_j(k) - k_j(k)\tilde{\alpha}_j(k))u^j(k-1) = \alpha_0(k) - k_0(k)\tilde{\alpha}_0(k) + (\alpha_1(k) - k_1(k)\tilde{\alpha}_1(k))u(k-1) + \cdots + (\alpha_n(k) - k_n(k)\tilde{\alpha}_n(k))u^n(k-1) \tag{21}$$

According to the theory of the model reference adaptive control system, when the time tends to infinity, $\lim_{k\to\infty} \alpha_j(k) = \lim_{k\to\infty} k_j(k)\alpha_j'(k) j = 0, 1, \cdots, n$, there is, $\lim_{k\to\infty} e(k) = 0$, i.e. the error is convergence, the model reference adaptive control system based on the nonlinear U model is stable.

4. Simulation of model reference adaptive control system

In the adaptive control system based on nonlinear U model, the reference [9] directly uses the gradient descent method to design the adaptive law and adjust the compensation parameters. In this paper, the neural network adaptive law is used to adjust the compensation parameters, which obtain better control effects. The control effects of the two systems were compared through simulation experiments. The controller in the control system is composed of an adaptive RBF neural network, and the RBF neural network is used to calculate the output error of the controlled object and the ideal model, that is, to obtain the correction information. The adaptive algorithm adjusts online and real-time according to this information to ensure that the output of the actual object is consistent with the output of the ideal model.

In order to discuss the influence of the corresponding relationship between the reference model and the controlled plant model on the precision of the control system, three simulation examples are designed, and the fourth-degree, third-degree, and second-degree reference U-models are selected for simulation respectively.

Simulation 1: select the reference model and describe it as the structure of the fourth-degree U-model:

$$\begin{aligned} y_m(k) &= 0.529412u(k) - 0.1u(k-3)u(k-2)u(k) \\ &+ 0.529412u(k-2) - 0.823529y_m(k-1) \\ &- 0.294118y_m(k-2) + 1.058824u(k-1) \\ &+ 0.12u(k-3)u(k-2)u^2(k-1) \\ &+ 0.1u^3(k-1) - 0.12u^4(k-1) \end{aligned} \tag{22}$$

According to the corresponding relationship between the reference model and the controlled plant, three controlled nonlinear plants are selected and expressed as the form of the first-degree, fourth-degree and seventh-degree U-models respectively.

The first-degree controlled U-model plant:

$$y(k) = 0.5y(k-1) + 0.2y^2(k-1) + 0.2u(k-1) \tag{23}$$

While $n = 4$, comparison formula (6), it can be obtained:

$$\begin{aligned} y(k) &= \tilde{\alpha}_0(k) + \tilde{\alpha}_1(k)u(k-1) + \tilde{\alpha}_2(k)u^2(k-1) \\ &+ \tilde{\alpha}_3(k)u^3(k-1) + \tilde{\alpha}_4(k)u^4(k-1) \end{aligned} \tag{24}$$

Where $\tilde{\alpha}_0(k) = 0.5y(k-1) + 0.2y(k-1)y(k-1)$, $\tilde{\alpha}_1(k) = 0.2, \tilde{\alpha}_2(k) = 0, \tilde{\alpha}_3(k) = 0, \tilde{\alpha}_4(k) = 0$.

The fourth-degree controlled U-model plant:

$$\begin{aligned} y(k) &= 0.5y(k-1) + 0.2y^2(k-1) + 0.2u(k-1) \\ &- 0.7y(k-1)u^2(k-1) \\ &+ 0.5y(k-1)u^3(k-1) \\ &- 0.3y(k-1)u^4(k-1) \end{aligned} \tag{25}$$

While $n = 4$, comparison formula (6),it can be obtained:

$$\begin{aligned} y(k) &= \tilde{\alpha}_0(k) + \tilde{\alpha}_1(k)u(k-1) + \tilde{\alpha}_2(k)u^2(k-1) \\ &+ \tilde{\alpha}_3(k)u^3(k-1) + \tilde{\alpha}_4(k)u^4(k-1) \end{aligned}$$

Where $\tilde{\alpha}_0(k) = 0.5y(k-1) + 0.2y(k-1)y(k-1)$, $\tilde{\alpha}_1(k) = 0.2, \tilde{\alpha}_2(k) = -0.7y(k-1), \tilde{\alpha}_3(k) = 0.5y(k-1), \tilde{\alpha}_4(k) = -0.3y(k-1)$.

The seventh-degree controlled U-model plant:

$$\begin{aligned} y(k) &= 0.5y(k-1) + 0.2y^2(k-1) + 0.2u(k-1) \\ &- 0.7y(k-1)u^2(k-1) \\ &+ 0.5y(k-1)u^3(k-1) \\ &- 0.3y(k-1)u^4(k-1) \\ &+ 0.1y(k-1)u^5(k-1) \\ &+ 0.4y(k-1)u^6(k-1) \\ &+ 0.8y(k-1)u^7(k-1) \end{aligned} \tag{26}$$

While $n = 4$, comparison formula (6),it can be obtained:

$$\begin{aligned} y(k) &= \tilde{\alpha}_0(k) + \tilde{\alpha}_1(k)u(k-1) + \tilde{\alpha}_2(k)u^2(k-1) \\ &+ \tilde{\alpha}_3(k)u^3(k-1) + \tilde{\alpha}_4(k)u^4(k-1) \end{aligned}$$

Where,

$$\begin{aligned} \tilde{\alpha}_0(k) &= 0.5y(k-1) + 0.2y^2(k-1) \\ &+ 0.1y(k-1)u^5(k-1) \\ &+ 0.4y(k-1)u^6(k-1) \\ &+ 0.8y(k-1)u^7(k-1), \tilde{\alpha}_1(k) = 0.2, \end{aligned}$$

$$\begin{aligned} &\tilde{\alpha}_2(k) = -0.7y(k-1), \tilde{\alpha}_3(k) = 0.5y(k-1), \\ &\tilde{\alpha}_4(k) = -0.3y(k-1). \end{aligned}$$

The inputs of the RBF neural network are $e(k), y(k), y_m(k), e(k-1), e(k-2)$,the outputs of the RBF neural network are $k_0(k), k_1(k), k_2(k), k_3(k), k_4(k)$, the learning rate is $\eta = 0.01$, and the momentum factor is $\beta = 0.01$. The initial values of Gaussian function parameters are shown as:

$$b = [10, 10, 10, 10, 10, 10, 10]$$

$$c = \begin{bmatrix} -10 & -5 & -2 & 0 & 2 & 5 & 10 \\ -10 & -5 & -2 & 0 & 2 & 5 & 10 \\ -10 & -5 & -2 & 0 & 2 & 5 & 10 \end{bmatrix}^T$$

$$W = \begin{bmatrix} -0.5978 & -0.3135 & -0.3998 & 0.5745 & -0.2798 & -0.1967 & 0.2376 \\ -0.0416 & 1.6911 & 2.2545 & 2.2559 & 0.4914 & 0.6640 & -1.0074 \\ 0.2059 & -0.0059 & 0.3132 & 0.2008 & -0.3072 & 0.0390 & -0.2697 \\ 0.3078 & 0.0427 & -0.0490 & 0.1747 & -0.1722 & 0.3197 & 0.0959 \\ 0.2084 & 0.3584 & -0.2680 & 0.3834 & -0.4382 & 0.1227 & -0.1865 \end{bmatrix}$$

When the controlled plants are the first-degree, the fourth-degree and the seventh-degree U-models, the triangular, square and the sine wave signals are respectively selected as the input signals to simulate the system.

It can be seen from Figure 2, Figure 4, and Figure 6 that the outputs of the first-degree, fourth-degree, and seventh-degree controlled U-model plants can track the output of the fourth-degree reference U-model.

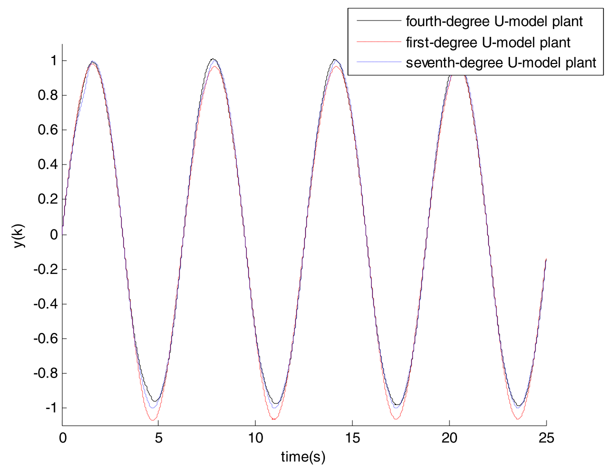

Figure 2. The system output curve of sine wave input.

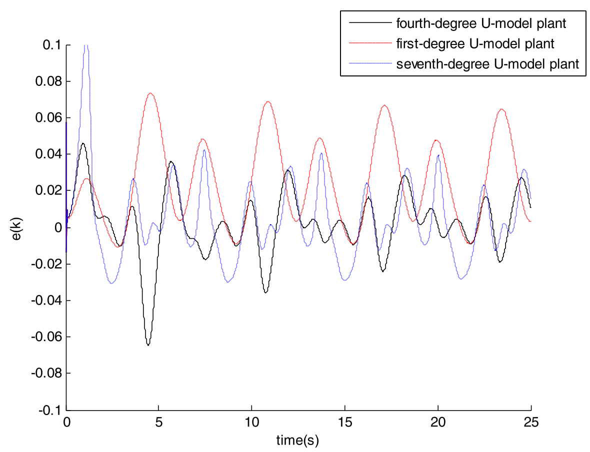

Figure 3. The system error curve of sine wave input.

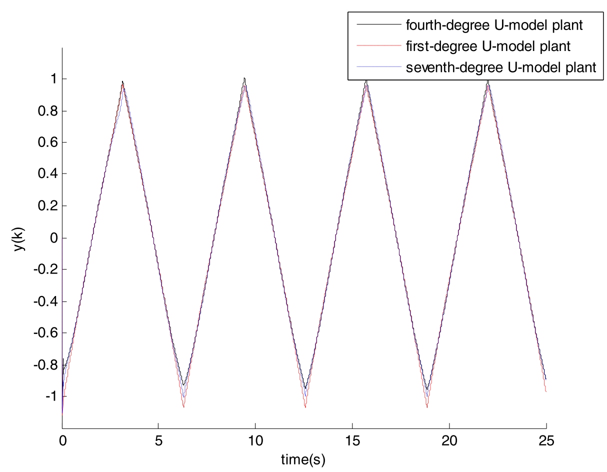

Figure 4. The system output curve of triangular wave input.

From Figure 3, Figure 5, Figure 7 can be obtained that, with the adaptation of the neural network adaptive algorithm, the system error is gradually reduced. When the controlled plant is the fourth-degree U-model, the system control effect is the best, the error adjustment speed is the fastest and the error is the smallest. When the controlled plant is the seventh-degree U-model, the system control effect is better, the error adjustment

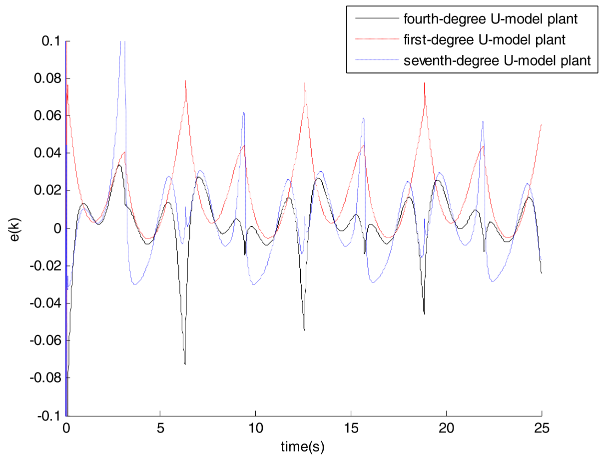

Figure 5. The system error curve of triangular wave input.

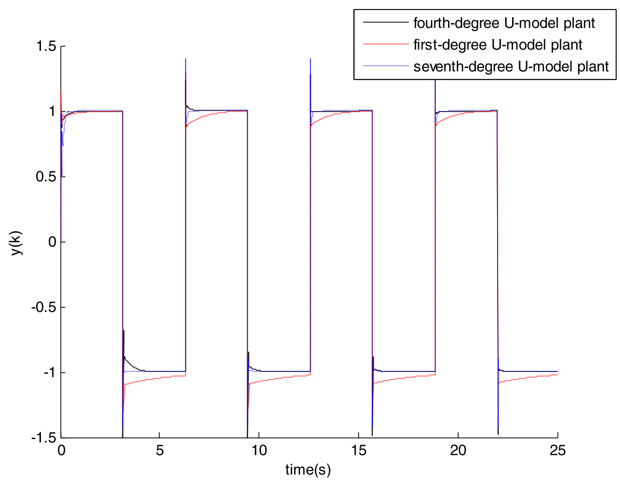

Figure 6. The system output curve of square wave input.

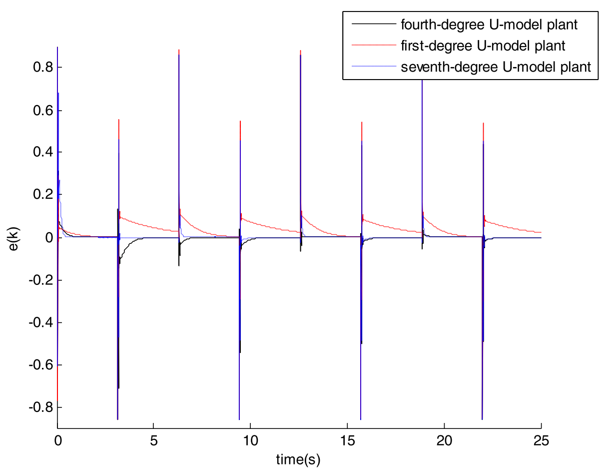

Figure 7. The system error curve of square wave input.

speed is faster and the error is smaller. When the controlled plant is the first-degree U-model, the system control effect is poor, the error adjustment speed is slow and the error is large.

Simulation 2: select the reference model and describe it as the structure of the third-degree U-model:

$$y_m(k) = 0.529412u(k) + 0.529412u(k-2) - 0.823529y_m(k-1) - 0.294118y_m(k-2) + 1.058824u(k-1) + 0.12u(k-2)u^2(k-1) - 0.12u^3(k-1) \quad (27)$$

According to the corresponding relationship between the reference model and the controlled plant, three controlled nonlinear plants are selected and expressed as the form of the first-degree, third-degree and fourth-degree U-models respectively.

The first-degree controlled U-model plant:

$$y(k) = 0.5y(k-1) + 0.2y^2(k-1) + 0.2u(k-1)$$

While $n = 3$, comparison formula (6),it can be obtained:

$$y(k) = \tilde{\alpha}_0(k) + \tilde{\alpha}_1(k)u(k-1) + \tilde{\alpha}_2(k)u^2(k-1) + \tilde{\alpha}_3(k)u^3(k-1) \quad (28)$$

where $\tilde{\alpha}_0(k) = 0.5y(k-1) + 0.2y(k-1)y(k-1)$, $\tilde{\alpha}_1(k) = 0.2, \tilde{\alpha}_2(k) = 0, \tilde{\alpha}_3(k) = 0$.

The third-degree controlled U-model plant:

$$y(k) = 0.5y(k-1) + 0.2y^2(k-1) + 0.2u(k-1) - 0.7y(k-1)u^2(k-1) + 0.5y(k-1)u^3(k-1) \quad (29)$$

While $n = 3$, comparison formula (6), it can be obtained:

$$y(k) = \tilde{\alpha}_0(k) + \tilde{\alpha}_1(k)u(k-1) + \tilde{\alpha}_2(k)u^2(k-1) + \tilde{\alpha}_3(k)u^3(k-1)$$

where $\tilde{\alpha}_0(k) = 0.5y(k-1) + 0.2y(k-1)y(k-1)$, $\tilde{\alpha}_1(k) = 0.2, \tilde{\alpha}_2(k) = -0.7y(k-1), \tilde{\alpha}_3(k) = 0.5y(k-1)$,

The fourth-degree controlled U-model plant:

$$y(k) = 0.5y(k-1) + 0.2y^2(k-1) + 0.2u(k-1) - 0.7y(k-1)u^2(k-1) + 0.5y(k-1)u^3(k-1) - 0.3y(k-1)u^4(k-1)$$

While $n = 3$, comparison formula (6),it can be obtained:

$$y(k) = \tilde{\alpha}_0(k) + \tilde{\alpha}_1(k)u(k-1) + \tilde{\alpha}_2(k)u^2(k-1) + \tilde{\alpha}_3(k)u^3(k-1)$$

Where, $\tilde{\alpha}_0(k) = 0.5y(k-1) + 0.2y^2(k-1) - 0.3y(k-1)u^4(k-1)$,

$$\tilde{\alpha}_1(k) = 0.2, \tilde{\alpha}_2(k) = -0.7y(k-1), \tilde{\alpha}_3(k) = 0.5y(k-1).$$

The inputs of the RBF neural network are $e(k), y(k), y_m(k), e(k-1)$, the outputs of the RBF neural network are $k_0(k), k_1(k), k_2(k), k_3(k)$, the learning rate is $\eta = 0.01$, and the momentum factor is $\beta = 0.01$. The initial values of Gaussian function parameters are shown as:

$$b = [10, 10, 10, 10, 10, 10, 10]$$

$$c = \begin{bmatrix} -10 & -5 & -2 & 0 & 2 & 5 & 10 \\ -10 & -5 & -2 & 0 & 2 & 5 & 10 \\ -10 & -5 & -2 & 0 & 2 & 5 & 10 \end{bmatrix}^T$$

$$W = \begin{bmatrix} -2.4586 & 2.9772 & -1.6776 & -2.2564 & -1.1290 & 4.3862 & 2.3587 \\ 4.9825 & -4.3001 & 6.8574 & 3.7188 & 0.7773 & -5.5144 & 2.1122 \\ 0.0165 & -0.0398 & 0.0135 & -0.0513 & 0.0407 & -0.0043 & 0.0406 \\ 1.1100 & 0.0414 & 0.1415 & -0.2090 & 0.8297 & -1.5834 & 1.6794 \end{bmatrix}$$

When the controlled plants are the first-degree, the third-degree and the fourth-degree U-models, the triangular, square and the sine wave signals are respectively selected as the input signals to simulate the system.

It can be seen from Figure 8, Figure 10, and Figure 12 that the outputs of the first-degree, third-degree, and fourth-degree controlled U-model plants can track the output of the third-degree reference U-model. From Figure 9, Figure 11, Figure 13 can be obtained that, with the adaptation of the neural network adaptive algorithm, the system error is gradually reduced. When the controlled plant is the third-degree U-model, the system control effect is the best, the error adjustment speed is the fastest and the error is the smallest. When the controlled plant is the fourth-degree U-model, the system control effect is better, the error adjustment speed is faster and the error is smaller. When the controlled plant is the first-degree U-model, the system control effect is poor, the error adjustment speed is slow and the error is large.

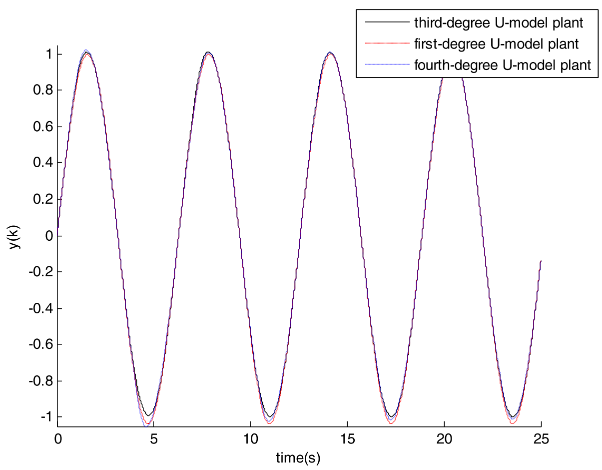

Figure 8. The system output curve of sine wave input.

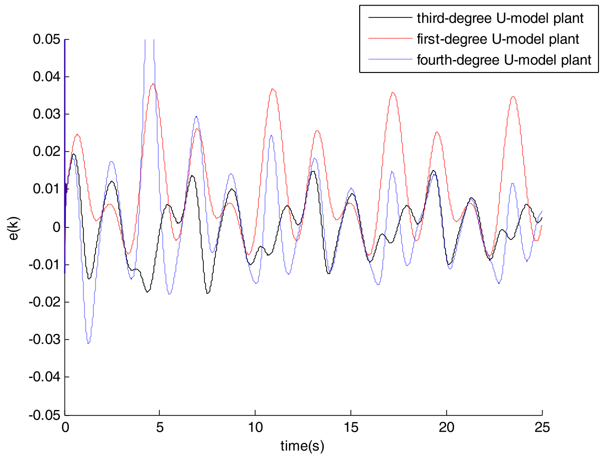

Figure 9. The system error curve of sine wave input.

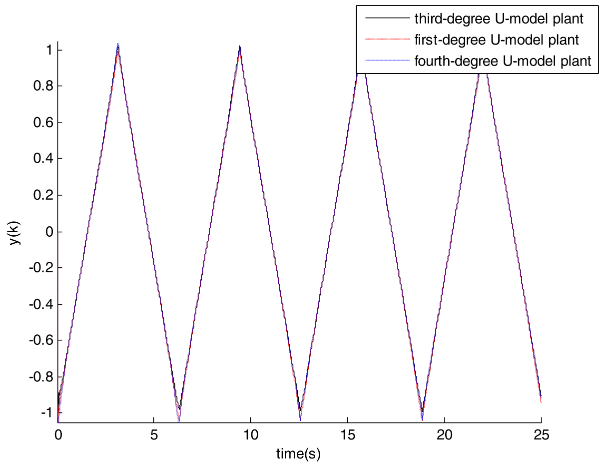

Figure 10. The system output curve of triangular wave input.

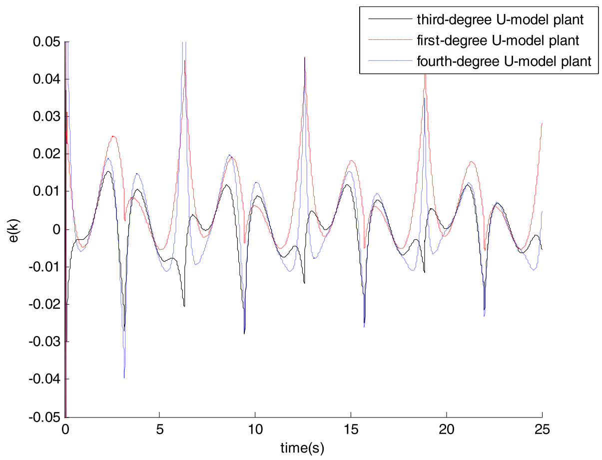

Figure 11. The system error curve of triangular wave input.

Simulation 3: select the reference model and describe it as the structure of the second-degree U-model:

$$y_m(k) = 0.529412u(k) - 0.12u(k-3)u(k-2)u(k) + 0.529412u(k-2) - 0.823529y_m(k-1) - 0.294118y_m(k-2) + 1.058824u(k-1) + 0.12u(k-2)u^2(k-1) \tag{30}$$

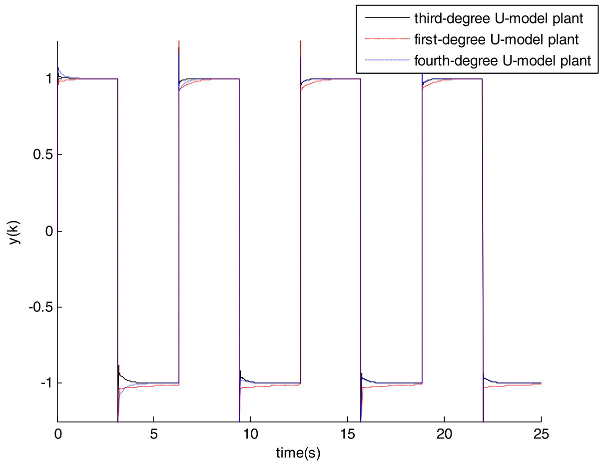

Figure 12. The system output curve of square wave input.

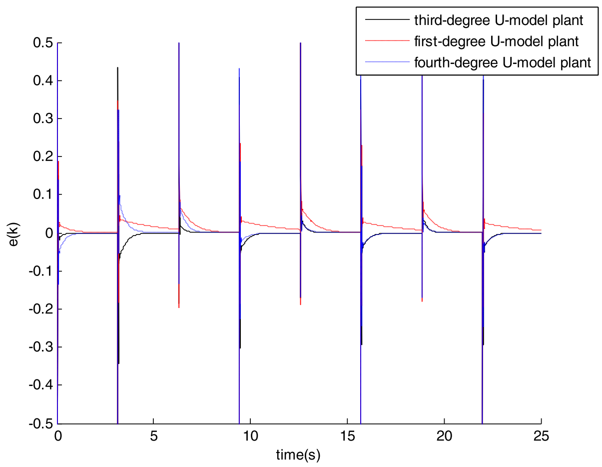

Figure 13. The system error curve of square wave input.

According to the corresponding relationship between the reference model and the controlled plant, three controlled nonlinear plants are selected and expressed as the form of the first-degree, second-degree and fourth-degree U-models respectively.

The first-degree controlled U-model plant:

$$y(k) = 0.5y(k-1) + 0.2y^2(k-1) + 0.2u(k-1)$$

While $n = 2$, comparison formula (6),it can be obtained:

$$y(k) = \tilde{\alpha}_0(k) + \tilde{\alpha}_1(k)u(k-1) + \tilde{\alpha}_2(k)u^2(k-1) \tag{31}$$

where $\tilde{\alpha}_0(k) = 0.5y(k-1) + 0.2y(k-1)y(k-1)$, $\tilde{\alpha}_1(k) = 0.2, \tilde{\alpha}_2(k) = 0$.

The second-degree controlled U-model plant:

$$y(k) = 0.5y(k-1) + 0.2y^2(k-1) + 0.2u(k-1) - 0.7y(k-1)u^2(k-1) \tag{32}$$

While $n = 2$, comparison formula (6),it can be obtained:

$$y(k) = \tilde{\alpha}_0(k) + \tilde{\alpha}_1(k)u(k-1) + \tilde{\alpha}_2(k)u^2(k-1)$$

where $\tilde{\alpha}_0(k) = 0.5y(k-1) + 0.2y(k-1)y(k-1)$, $\tilde{\alpha}_1(k) = 0.2, \tilde{\alpha}_2(k) = -0.7y(k-1)$.

The fourth-degree controlled U-model plant:

$$\begin{aligned} y(k) = {} & 0.5y(k-1) + 0.2y^2(k-1) + 0.2u(k-1) \\ & - 0.7y(k-1)u^2(k-1) \\ & + 0.5y(k-1)u^3(k-1) \\ & - 0.3y(k-1)u^4(k-1) \end{aligned}$$

While $n = 2$, comparison formula (6),it can be obtained:

$$y(k) = \tilde{\alpha}_0(k) + \tilde{\alpha}_1(k)u(k-1) + \tilde{\alpha}_2(k)u^2(k-1)$$

where,

$$\begin{aligned} \tilde{\alpha}_0(k) = {} & 0.5y(k-1) + 0.2y(k-1)y(k-1) \\ & + 0.5y(k-1)u^3(k-1) \\ & - 0.3y(k-1)u^4(k-1), \end{aligned}$$

$$\tilde{\alpha}_1(k) = 0.2, \tilde{\alpha}_2(k) = -0.7y(k-1).$$

The inputs of the RBF neural network are $e(k), y(k)$, $y_m(k)$, the outputs are of the RBF neural network $k_0(k), k_1(k), k_2(k)$, the learning rate is $\eta = 0.05$, and the momentum factor is $\beta = 0.01$. The initial values of Gaussian function parameters are shown as:

$$b = [10, 10, 10, 10, 10, 10, 10]$$

$$c = \begin{bmatrix} -10 & -5 & -2 & 0 & 2 & 5 & 10 \\ -10 & -5 & -2 & 0 & 2 & 5 & 10 \\ -10 & -5 & -2 & 0 & 2 & 5 & 10 \end{bmatrix}^T$$

$$W = \begin{bmatrix} 1.1533 & -1.5786 & -0.7232 & -0.5466 & 2.2968 & -0.9377 & 2.2644 \\ 0.7925 & 1.4870 & 2.1346 & -0.9937 & 2.2337 & 1.9277 & -2.5487 \\ 0.0366 & -0.1904 & 0.2495 & 0.0761 & 0.0641 & -0.0403 & 0.0227 \end{bmatrix}$$

When the controlled plants are the first-degree, the second-degree and the fourth-degree U-models, the triangular, square and the sine wave signals are respectively selected as the input signals to simulate the system.

It can be seen from Figure 14, Figure 16, and Figure 18 that the outputs of the first-degree, second-degree, and fourth-degree controlled U-model plants can track the output of the second-degree reference U-model. From Figure 15, Figure 17, Figure 19 can be obtained that, with the adaptation of the neural network adaptive algorithm, the system error is gradually reduced. When the controlled plant is the second-degree U-model, the system control effect is the best, the error adjustment speed is the fastest and the error is the smallest. When the controlled plant is the fourth-degree U-model, the system control effect is better, the error adjustment speed is faster and the error is smaller. When the controlled plant is the first-degree U-model, the system control effect is poor, the error adjustment speed is slow and the error is large.

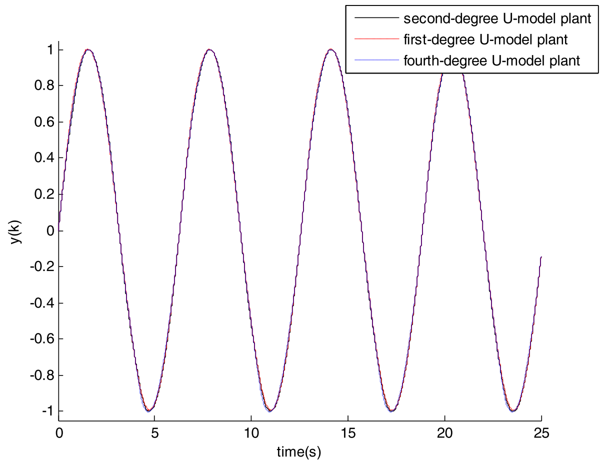

Figure 14. The system output curve of sine wave input.

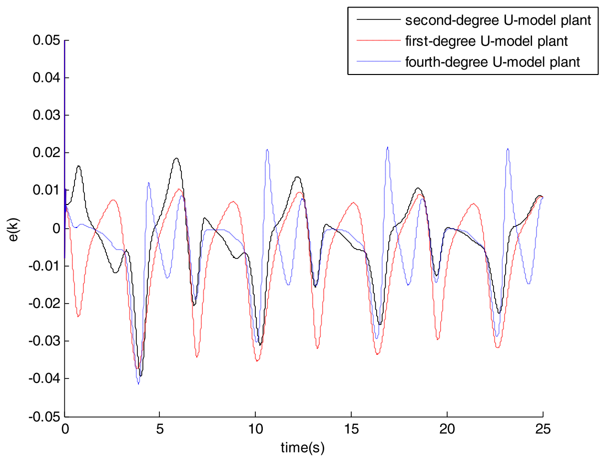

Figure 15. The system error curve of sine wave input.

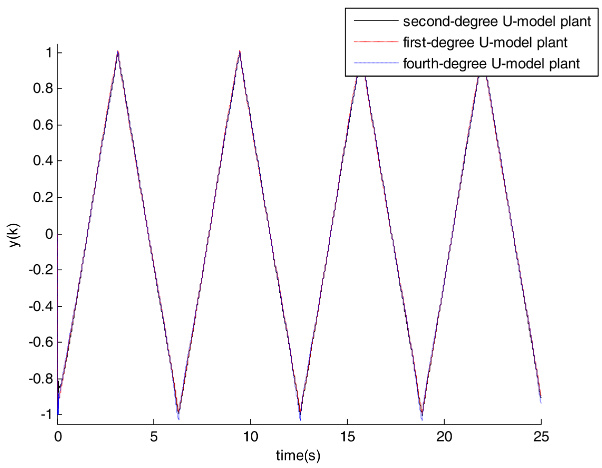

Figure 16. The system output curve of triangular wave input.

Through three simulation examples, it can be concluded that when the degree of the controlled plant model is the same as that of the reference model, the nonlinear plant has the best tracking effect to the reference model, the system adjustment accuracy is the

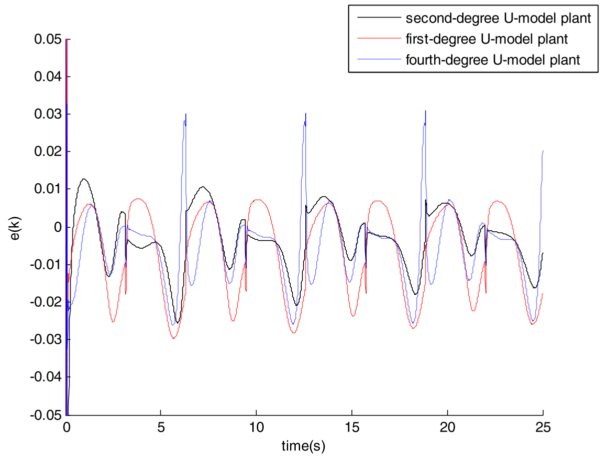

Figure 17. The system error curve of triangular wave input.

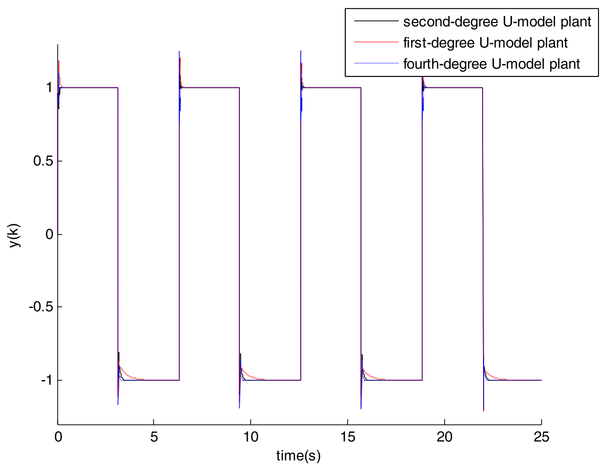

Figure 18. The system output curve of square wave input.

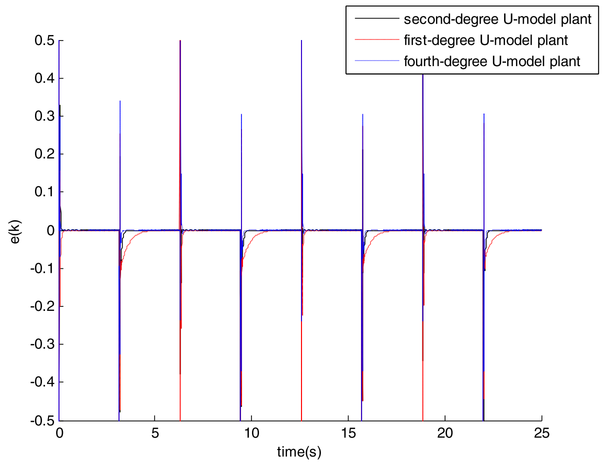

Figure 19. The system error curve of square wave input.

highest and the system error is the smallest; When the degree of the controlled plant model is higher than that of reference model, the adjustment of compensation parameter $k_0(k)$ is relatively complex, the nonlinear plant has the good tracking effect to the reference model, the system adjustment accuracy is high, the system error is small; When the degree of the controlled plant model is lower than that of reference model, the nonlinear plant has poor tracking effect to the reference model, the system adjustment accuracy is low, and the system error is large. Therefore, in order to achieve better tracking effect of the control system, the degree of reference model should be lower than or equal to the degree of controlled plant.

5. Conclusion

For complex time-varying nonlinear plants, the nonlinear U-model is established and a general design of RBF neural network model reference adaptive control system based on nonlinear U-model is proposed. The corresponding relationship between the reference model and the controlled plant influences the control accuracy of the system. The analysis shows that the system control accuracy is the best when the reference model and the nonlinear plant are in the same degree. This can serve as the guidance on the selection of reference model for different nonlinear plant model in adaptive control system design. Through the bridge of nonlinear U-model, the neural network algorithm is added into the classical model reference adaptive method to design the nonlinear system, which provides the theoretical basis for the nonlinear system design using the linear system method.

Disclosure statement

No potential conflict of interest was reported by the authors.

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References

[1] Xu F, Zhu Q, Zhao D, et al. U-model based design methods for nonlinear control systems a survey of the development in the 1st decade. Control Decis. 2013;28(7):961–977.
[2] Zhu Q, Guo LZ. A pole placement controller for nonlinear dynamic plants. Proc of IMechE, Part I: J Sys Control Eng. 2002;216(16):467–476.
[3] Qiu J, Zhu G, Zhu Q, et al. Comparative study on pole placement controller design with U-Model approach & classical approach. In: 2016 35th Control Conference (CCC). IEEE, Chengdu, China; 2016. p. 10436–10441.
[4] Guan X, Zhao D, Zhu Q. NARMAX modelling and U-model control design for continuous stirred tank reactor (CSTR). In 2016 35th Control Conference (CCC). IEEE, Chengdu, China; 2016. p. 1964–1969.
[5] Shafiq M, Haseebiddon M. U-model-based internal model control for non-linear dynamic plants. Proc of IMechE, Part I: J Sys Control Eng 2005; 219(6):449–458.
[6] Khan T, Shafiq M. A novel internal model control scheme for adaptive tracking of nonlinear dynamic plants. In: 2006 the 1st IEEE Conf on Industrial Electronics and Applications, Wuhan, China; 2006. p. 123–130.

[7] Butt NR, Shafiq M, Khan T. An adaptive root-solving controller for tracking of nonlinear dynamic plants. In Industrial Electronics and Control Applications, 2005. ICIECA 2005. International Conference on. IEEE, Ecuador; 2005. p. 144–149.
[8] Hasan E, Ibrahim RB, Ali SSA, et al. U-Model based adaptive control of gas process plant. Procedia Comput Sci. 2017;105:119–124.
[9] Xu F, Wang Z, Cheng Y, et al. Design of U-model enhanced adaptive control of nonlinear systems. In: Control Conference (CCC). IEEE, Chengdu, China; 2016. p. 5804–5809.
[10] Du W, Wu X, Zhu Q. Direct design of a U model-based generalized predictive controller for a class of non-linear(polynomial) dynamic plants. Proc of IMechE, Part I: J Syst Control Eng. 2012;226(1):27–42.
[11] Ji Q, Quanmin Z, Mokhtar N, et al. U-model enhanced control of nonminimum-phase dynamic systems. In Modelling, Identification and Control (ICMIC), 2016 8th International Conference on. IEEE, Algiers, Algeria; 2016. p. 648–653.
[12] Saad Azhar AS, Al-Sunni FM, Shafiq M. U-model based adaptive tracking scheme for unknown MIMO bilinear systems. In 2006 the 1st IEEE Conf on Industrial Electronics and Applications, Wuhan, China; 2006. p. 473–486.
[13] Saad Azhar AS, Al-Sunni FM, Shafiq M, et al. MIMO U-model based control: real-time tracking control and feedback analysis via small gain theorem. WSEAS Trans on Circ Syst. 2008;7(7):610–619.
[14] Saad Azhar AS, Al-Sunni FM, Shafiq M, et al. U-model based learning feedforward control of MIMO nonlinear systems. Electr Eng. 2010;91(8):405–415.
[15] Zhu Q, Zhao D, Zhang J. A general U-block model-based design procedure for nonlinear polynomial control systems. Int J Syst Sci 2015;47(14):3465–3475.
[16] Chang W, Wang W, Jia H. Radial basis functions neural network of vary learning rate based stochastic U-model. In: Electrical and Control Engineering (ICECE), 2011 International Conference on. IEEE, Yichang, China; 2011. p. 278–281.
[17] Butt NR, Shafiq M. Real-time adaptive tracking of DC motor speed using U-model based IMC. Autom Control Comput Sci. 2007;41(1):31–38.
[18] Butt NR, Shafiq M. Higher-order neural network based root-solving controller for adaptive tracking of stable nonlinear plants. In Engineering of Intelligent Systems, 2006 IEEE International Conference on. IEEE, Islamabad, Pakistan; 2006. p. 22–27.
[19] Shafiq M, Butt NR. Utilizing higher-order neural networks in U-model based controllers for stable nonlinear plants. Int J Control Autom. 2011;9(3):489–496.
[20] Xu F, Cheng Y, Ren H, et al. Research on adaptive neural network control system based on nonlinear U-model with time-varying delay. Math Probl Eng. 2014;2014(2014):1–7.