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Advertising investment under switching costs

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ABSTRACT
Switching costs are exceedingly important in service industries including platform firms and IT firms, and, have a strong effect on firms' advertising investments and market structure. This study examines the effects of switching costs on advertising using a two-stage discrete-time dynamic duopoly model. Firstly, we argue that switching costs reduce firms' advertising investments. Secondly, both brand stealing effects and brand expansion effects of advertising promote firms' competition regarding pricing and advertising investments. Finally, firms with high prices invest more in advertising thank others. Because of switching costs, firms compete in terms of both price and advertising investment. This article captures the relationship between switching costs and advertising investments in detail. The managerial policy is that firms to determine advertising investment should consider switching costs.

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1. Introduction

Switching costs are very popular in practice and arise from many economic activities (Li et al., 2018; Nagengast et al., 2014; Otrodi et al., 2019; Villas-Boas, 2015). For example, many cellular phone carriers charge very high cancellation fees for canceling a contract, which yield switching costs (Chuah et al., 2017). Abdullah et al. (2019) further listed an example of switching costs in supply chain field.

Interestingly, Luo et al. (2014) found that durable goods rely less on advertising than non-durable goods. In general, durable goods face higher switching costs than non-durable goods. This interesting result reflects that switching costs influence advertising investment. This motivates this research to capture the relationship between switching costs and advertising investment.

This paper aims to examine the effects of switching costs on firms' advertising investments in theory. In this paper, we discuss the effects of switching costs on advertising investments. The effects of switching costs on advertising are discussed

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using a two-stage duopoly model. Because of switching costs, two firms compete in terms of both price and advertising investment.

The contributions of this article lie in two aspects: In theory, this article highlights the effects of switching costs on advertising investment. This adds the literature about the relationship between switching costs and advertisement. As we known, this is the first paper to consider the relationship between switching costs and advertisement.

In application, this article supports robust theory for firm's advertising investment. This paper suggests that when firms make decision about advertising investment, switching costs should be taken into account. Based on the conclusions of this article, firms can make optimal decision about advertising investment when switching costs affect the market.

This paper is organized as follows. A model is established in the Section 3. A two-stage duopoly model with switching costs is introduced. When advertisement is considered, the technique of endogenous brand advertisement in Baye and Morgan (2009) is employed. Conclusions are presented in Section 4. Concluding remarks are made in the final section.

2. Literature review

Literature includes switching costs and advertising investment. Then, the related literature is briefly remarked and the knowledge gap is described.

2.1. Switching costs

Much research about switching costs appears in recent years. Burnham et al. (2003), Klemperer (1995) and Farrell and Klemperer (2007) discussed switching costs in their interesting survey papers. Klemperer (1995) systematically provided many interesting examples and describes switching costs in microeconomics, industrial organization and international trade. Burnham et al. (2003) identified three types of switching costs: procedural switching costs, financial switching costs and relational switching costs. Whitten et al. (2010), and Lee et al. (2011) discussed the effects of switching costs on strategy choice.

In theory, much literature focuses on the relationship between price and switching costs (Blut et al., 2015; Nie, 2018; Nie et al., 2018a; Zhang et al., 2015), innovation and switching costs (Nie et al., 2018b; Yang et al., 2018), outputs and switching costs (Nie et al., 2019; Nie & Wang, 2019) and so on (Nie et al., 2021; Xiao et al., 2020). Almost all previous papers before 2009 argue that switching costs result in price increases. Recently, Dubé et al. (2009) challenged this idea with a numerical simulation. Cabral (2009) argued, using a mathematical model, that small switching costs reduce price. Doganoglu (2010) further proved this conclusion with uncertain demand. At the same time, Viard (2007) also found these phenomena using data on 800-number portability. Rhodes (2014) further presented the conditions that switching costs result in price increases. Ramadan et al. (2019) recently identified effects of switching costs about Amazon on U.S. market. Amaldoss and He (2019) captured the relationship between switching costs and price. Koo et al. (2020) described the effects of switching costs on loyalty of hotel.

In empirical research, researchers pay attention to the relationship between the switching costs and special goods. Many empirical studies provide evidence for the strong effects of switching costs on credit cards (Chen, Chen, et al., 2020; Chen, Wang, et al., 2020; Stango, 2002), cigarettes (Elzinga & Mills, 1999) and computer software (Larkin, 2004). All of these empirical studies support the importance of switching costs. Anderson and Simester (2013) examined the effects of product standards, customer learning, and switching costs on advertisement.

Recently, some research develops the relationship between switching costs and firms' behavior (Fabra & García, 2015). Switching costs have many effects on firms' strategies and there exists a large literature in this field. Morita and Waldman (2010) discussed the effects of switching costs on firms' maintaining service. Wang and Wen (1998) addressed the effects of switching costs on strategic invasion. Chen (1997) developed a theory of switching costs. In fact, switching costs affect the strategies of many firms. Biglaiser et al. (2013) argued the switching costs deter the entrance under dynamic situation. Doganoglu and Grzybowski (2013) showed that the switching costs reduce the demand. Wang and Nie (2020) argued that free shopping shuttle bus strategies promote switching costs. Nie et al. (2018b) recently identified the relationship between switching costs and innovative investment. Switching costs improve innovation under symmetric cases, while switching costs have no effects on innovation under asymmetric situation (Nie et al., 2018b).

2.2. Advertising investment

This work is closely related to Baye and Morgan (2009) studied of endogenous brand advertisement. The research about advertisement is briefly introduced. Bagwell (2007) surveyed the topic of advertisement. Anderson and Renault (2006) explored a theory concerning a monopoly firm's choice of advertising content and the information disclosed to consumers. There are many branches in the study of advertisement. Anderson and Renault (2009) developed a significant comparative advertisement theory on the basis of some interesting economic phenomena. Baye and Morgan (2009) developed a theory of the relationship between advertising and pricing. In their interesting paper, Baye and Morgan discuss endogenous brand advertisement. Chen and Wen (2013) recently developed the theory of vertical cooperative advertisement based on a dual-brand model with a single manufacturer and a single retailer.

2.3. Knowledge gap

The existed literature focuses on both switching costs and advertisement investment. Switching costs have impacts on price, trade and industrial structures. Advertising investments are determined the properties of goods, competitions and so on. Rare papers highlight the effects of switching costs on advertising investment except Anderson and Simester (2013) with field experiments. By large-scale randomized field experiments, Anderson and Simester (2013) found that switching costs deters advertising investment. Moreover, no literature focuses on the acting mechanism between switching costs and advertising investment.

Based on the interesting conclusions of Anderson and Simester (2013), it is important to capture the mechanism that switching costs affect advertisement. To illustrate the mechanism of switching costs affecting advertisement, it is necessary to establish theory to capture it. This article fills in these gaps and captures the operating mechanism of switching costs affecting advertisement. By establishing the operating mechanism, this article supports robust decision in theory. Furthermore, this article supports model for further research literature about switching costs and advertising investment.

3. Model

Here, a model is established to capture the relationship between switching costs and advertisement investment. A duopoly model with advertisement and switching costs is established. This study highlights the effects of switching costs on advertisement. When advertisement is addressed, this paper refers to the model of Baye and Morgan (2009), in which brand advertisement is endogenous.

Notations are presented as follows. $F = \{1, 2\}$ represents two firms. The products of these two firms are functionally identical. There are two stages. $s \in [0, S]$ represents the switching costs, which are uniformly distributed with the density function $f(s) = \frac{1}{S}$. $S>0$ is a constant. The expected value of switching costs is $E(s) = \frac{S}{2}$. p_t^i is the price of firm i at stage t for $t = 1, 2$ and $i = 1, 2$.A^i is the advertisement investment for firm i and $i = 1, 2$. We denote $A = (A^1, A^2)$ and $p_t = (p_t^1, p_t^2)$.

The market size is N. At the first stage, the two firms have the same brand awareness and launch advertising investment competitions. At the second stage, the two firms establish their brand, and there are two types of consumers. The first type of consumer($N_1 \leq N$) is loyal to one of the two firms, and the second type of consumer($N_2 = N - N_1$) views the sellers as identical. This loyalty is established after the first stage. Each consumer buys a unit product at each stage. This paper addresses switching costs based on brand loyalty. u_0 is a constant, and the consumer discount factor is 1. β^i stands for the brand value of products produced by firm i and is determined by advertisement investment. The utility value for a consumer with $\beta^i \in [0, 1]$ purchasing from firm i for $i = 1, 2,$ is

$$u(p, \beta^i) = u^1(p_1^i) + u^2(p_2^i, \beta^i) = \beta^i + 2u_0 - \sum_{t=1}^{2} p_t^i \tag{1}$$

where $u^1(p_1^i) = u_0 - p_1^i$ and $u^2(p_2^i, \beta^i) = \beta^i + u_0 - p_2^i$. Moreover, $\beta^i = \alpha_1 \frac{A^i}{A^1 + A^2} + \alpha_2 A^i$, where the term α_1 captures potential brand stealing effects of brand advertisement and the term α_2 captures brand expansion effects, which is similar to that of Baye and Morgan (2009). The first type of consumer changes from the product of firm i to firm j with $i \neq j$ at stage 2, if and only if the following relation holds:

$$\beta^i - p^i - s \le \beta^j - p_2^j. \tag{2}$$

(2) implies that this consumer changes his product if and only if the utility value increases by doing so. $q_t = (q_t^1, q_t^2)$ represents the output quantity of the two firms at stage $t = 1, 2$. The cost incurred by firm i to produce a unit of product at each stage is assumed to be zero. For $i = 1, 2,$ the profit value of firm i is

$$V^i = p_1^i q_1^i(p_1) - A^i + p_2^i q_1^i(p_2, s, q_1, A). \tag{3}$$

For convenience, we must digress from other important factors discussed in the literature, such as transportation cost and holdup. The timing of this game is as follows. In the first stage, the two firms simultaneously set prices. Consumers decide to buy products from one firm and have no information about the two firms' prices and advertising in the second stage. Then, the two firms launch advertising campaigns to establish their brands. In the second stage, the two firms establish their brands and simultaneously set prices. Because of switching costs, consumers decide to buy products from one firm.

4. Discussion

4.1. Main remarks

The model in the above section is addressed here. We first outline the demand function on the basis of (1, 2) and the advertisement input. The demand at the first stage is outlined as follows:

$$q_1^1 = \begin{cases} N & p_1^1 < p_1^2 \\ \frac{1}{2}N & p_1^1 = p_1^2 \\ 0 & p_1^1 > p_1^2 \end{cases}, q_1^2 = \begin{cases} N & p_1^1 > p_1^2 \\ \frac{1}{2}N & p_1^1 = p_1^2 \\ 0 & p_1^1 < p_1^2 \end{cases} \tag{4}$$

We further note that there are no new consumers entering into this market at the second stage. To simplify the model, we always assume that $|p_2^1 - p_2^2| \le S$ and $\left|(\beta^1 - p_2^1) - (\beta^2 - p_2^2)\right| \le S$.Otherwise, one type of market is fully occupied by one firm at the second stage. This case is neglected under this assumption. Without a loss of generality, we assume $p_2^1 < p_2^2$. In the second stage, because of the effects of switching costs, we immediately have the following demand function:

$$q_2^1 = q_1^1 + \frac{N_2 q_1^2}{N}\frac{p_2^2 - p_2^1}{S} + \frac{N_1}{N} sign\{(\beta^1 - p_2^1) - (\beta^2 - p_2^2)\} \max\left\{ q_1^2 \frac{(\beta^1 - p_2^1) - (\beta^2 - p_2^2)}{S}, q_1^1 \frac{(\beta^2 - p_2^2) - (\beta^1 - p_2^1)}{S} \right\}, \tag{5}$$

$$q_2^2 = q_1^2 + \frac{N_2 q_1^2}{N}\frac{p_2^1 - p_2^2}{S} + \frac{N_1}{N} sign\{(\beta^2 - p_2^2) - (\beta^1 - p_2^1)\}\max\left\{q_1^1 \frac{(\beta^2 - p_2^2) - (\beta^1 - p_2^1)}{S}, q_1^2 \frac{(\beta^1 - p_2^1) - (\beta^2 - p_2^2)}{S}\right\}, \tag{6}$$

where $sign(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x<0 \end{cases}$ is a signal function.

In (5), the sum of the first term and the second term on the right represents the demand in the first stage. The third term on the right represents the number of consumers of the second type switching their products. The fourth term illustrates customers of the first type who switched to another product. The model is analyzed using backward induction. For $i, j = 1, 2$ and $i \neq j$, we have

$$V^i = p_1^i q_1^i + p_2^i\{q_1^i + \frac{N_2 q_2^i}{N}\frac{p_2^j - p_2^i}{S} - A^i + \frac{N_1}{N} sign\{(\beta^i - p_2^i) - (\beta^j - p_2^j)\}\max\{q_1^j \frac{(\beta^i - p_2^i) - (\beta^j - p_2^j)}{S}, q_1^i \frac{(\beta^j - p_2^j) - (\beta^i - p_2^i)}{S}\}\}. \tag{7}$$

4.2. The second stage

We discuss two cases of the second stage in this section. One is $(\beta^1 - p_2^1) - (\beta^2 - p_2^2) < 0$ and the other is $(\beta^1 - p_2^1) - (\beta^2 - p_2^2) \geq 0$. $\beta^2 - \beta^1 = \alpha_1 \frac{A^2 - A^1}{A^1 + A^2} + \alpha_2(A^2 - A^1) = \alpha_1\left(1 - \frac{2A^1}{A^1 + A^2}\right) + \alpha_2(A^2 - A^1) = \alpha_1\left(-1 + \frac{2A^2}{A^1 + A^2}\right) + \alpha_2(A^2 - A^1)$.

Case 1. $(\beta^1 - p_2^1) - (\beta^2 - p_2^2) < 0$.

In this case, the first firm dominates in the two types of markets, and we have

$$V^1 = p_1^1 q_1^1 + p_2^1\left[q_1^1 + \frac{N_2 q_1^2}{N}\frac{p_2^2 - p_2^1}{S} - \frac{N_1 q_1^1}{N}\left(\frac{p_2^1 - p_2^2}{S} + \frac{\beta^2 - \beta^1}{S}\right)\right] - A^1, \tag{8}$$

$$V^2 = p_1^2 q_1^2 + p_2^2\left[q_1^2 - \frac{N_2 q_1^2}{N}\frac{p_2^2 - p_2^1}{S} + \frac{N_1 q_1^1}{N}\left(\frac{p_2^1 - p_2^2}{S} + \frac{\beta^2 - \beta^1}{S}\right)\right] - A^2. \tag{9}$$

Consider $Max_{p_2^1, A_2^1} V^1$ and $Max_{p_2^2, A_2^2} V^2$. V^i is concave in p_2^i for $i = 1, 2$.Thus, $\frac{\partial V^1}{\partial A^1} = \frac{p_2^1 N_1 q_1^1}{NS}\left[\frac{2A^2\alpha_1}{(A^1 + A^2)^2} + \alpha_2\right] - 1$ and $\frac{\partial^2 V^1}{\partial (A^1)^2} = -\frac{4p_2^1 N_1 q_1^1 \alpha_1}{NS}\frac{A^2}{(A^1 + A^2)^3}$. For (9), we have $\frac{\partial V^2}{\partial A^2} = \frac{p_2^2 N_1 q_1^1}{NS}\left[\frac{2A^1\alpha_1}{(A^1 + A^2)^2} + \alpha_2\right] - 1$ and $\frac{\partial^2 V^2}{\partial (A^2)^2} = -\frac{4p_2^2 N_1 q_1^1 \alpha_1}{NS}\frac{A^1}{(A^1 + A^2)^3}$. V^1 is therefore concave in A^1, and V^2 is concave in A^2. The equilibrium is uniquely determined by its first-order optimal conditions, which are outlined as follows:

$$\frac{\partial V^1}{\partial p_2^1} = f_1 = q_1^1 - 2p_2^1\frac{N_1 q_1^1 + N_2 q_1^2}{NS} + p_2^2\frac{N_1 q_1^1 + N_2 q_1^2}{NS} - \frac{N_1 q_1^1}{N}\frac{\beta^2 - \beta^1}{S} = 0, \tag{10}$$

$$\frac{\partial V^2}{\partial p_2^2} = f_2 = q_1^2 - 2p_2^2\frac{N_1q_1^1 + N_2q_1^2}{NS} + p_2^1\frac{N_1q_1^1 + N_2q_1^2}{NS} + \frac{N_1q_1^1}{N}\frac{\beta^2 - \beta^1}{S} = 0, \tag{11}$$

$$\frac{\partial V^1}{\partial A_2^1} = f_3 = \frac{p_2^1N_1q_1^1}{NS}\left[\frac{2A^2\alpha_1}{(A^1 + A^2)^2} + \alpha_2\right] - 1 = 0, \tag{12}$$

$$\frac{\partial V^2}{\partial A_2^2} = f_4 = \frac{p_2^2N_1q_1^1}{NS}\left[\frac{2A^1\alpha_1}{(A^1 + A^2)^2} + \alpha_2\right] - 1 = 0. \tag{13}$$

(10) and (11) jointly indicate

$$p_2^1 = \frac{SN}{3(N_1q_1^1 + N_2q_1^2)} + \frac{Sq_1^1}{3(N_1q_1^1 + N_2q_1^2)} - \frac{q_1^1N_1(\beta^2 - \beta^1)}{3N(N_1q_1^1 + N_2q_1^2)}, \tag{14}$$

$$p_2^2 = \frac{SN}{3(N_1q_1^1 + N_2q_1^2)} + \frac{Sq_1^2}{3(N_1q_1^1 + N_2q_1^2)} + \frac{q_1^1N_1(\beta^2 - \beta^1)}{3N(N_1q_1^1 + N_2q_1^2)}. \tag{15}$$

It is difficult to obtain the explicit solution to the system of equations in (10)–(13). According to the implicit function theorem, we have the following conclusion.

Proposition 1. *If $p_2^1 < p_2^2$ and $(\beta^1 - p_2^1) - (\beta^2 - p_2^2) < 0$ at equilibrium, we have $\frac{\partial p_2^i}{\partial S} > 0$, $\frac{\partial p_2^i}{\partial \alpha_1} < 0$, $\frac{\partial p_2^i}{\partial \alpha_2} < 0$, $\frac{\partial A^i}{\partial S} < 0$, $\frac{\partial A^i}{\partial \alpha_1} > 0$, $\frac{\partial A^i}{\partial \alpha_2} > 0$, $\frac{\partial p_2^i}{\partial N_1} < 0$ and $\frac{\partial A^i}{\partial N_1} > 0$.*

Proof. See Appendix. ■

Remark s. Greater switching costs yield a higher price at the final stage by virtue of the above conclusion. $\frac{\partial p_2^i}{\partial N_1} < 0$ and $\frac{\partial A^i}{\partial N_1} > 0$ illustrate that a larger market size of the first type stimulates more advertising investment and improves price competition. Both the potential brand stealing effects and the brand expansion effects of advertising improve firms' competition regarding pricing and advertising investments.

From Proposition 1, we have that switching costs deter advertising investments. This is consistent with the reality. For example, according to their annual report, Rinhe Pharmacy Co. Ltd, a famous medicine producer in Center China, reduced its advertisement in recently years because of the higher switching costs of its products.[1]

Case 2. $(\beta^1 - p_2^1) - (\beta^2 - p_2^2) \geq 0$.

Under $(\beta^1 - p_2^1) - (\beta^2 - p_2^2) \geq 0$, the first firm focuses on the first type of market, and the second focuses on the second type. The profit values are restated as follows: $V^1 = p_1^1q_1^1 + p_2^1[q_1^1 + \frac{N_2q_1^1}{N}\frac{p_2^2 - p_2^1}{S} + \frac{N_1q_1^2}{N}\left(\frac{p_2^2 - p_2^1}{S} + \frac{\beta^1 - \beta^2}{S}\right)] - A^1$, and $V^2 = p_1^2 + p_2^2[q_1^2 - \frac{N_2q_1^2}{N}\frac{p_2^2 - p_2^1}{S} - \frac{N_1q_1^2}{N}\left(\frac{p_2^2 - p_2^1}{S} + \frac{\beta^1 - \beta^2}{S}\right)] - A^2$.

If the equilibrium solution is an interior point or $(\beta^1 - p_2^1) - (\beta^2 - p_2^2) > 0$, which is similar to the above analysis, we reach the same conclusions as Proposition 1, because the first-order optimal conditions are very similar to (10)–(13). The analysis for this case is very similar to the one for the previous case, so we may omit it here.

We now discuss $(\beta^1 - p_2^1) - (\beta^2 - p_2^2) = 0$. Profit values are rewritten as

$$\begin{aligned} \underset{p_2^1, A_2^1}{Max}\, V^1 &= p_1^1 q_1^1 + p_2^1\left[q_1^1 + \frac{N_2 q_1^2}{N}\frac{p_2^2 - p_2^1}{S}\right] - A^1 \\ S.T. \quad & (\beta^1 - p_2^1) - (\beta^2 - p_2^2) = 0 \end{aligned}, \tag{16}$$

$$\begin{aligned} \underset{p_2^2, A_2^2}{Max}\, V^2 &= p_1^2 q_1^2 + p_2^2\left[q_1^2 - \frac{N_2 q_1^2}{N}\frac{p_2^2 - p_2^1}{S}\right] - A^2 \\ S.T. \quad & (\beta^1 - p_2^1) - (\beta^2 - p_2^2) = 0 \end{aligned}. \tag{17}$$

For (16) and (17), on the basis of the corresponding Lagrangian function, the first-order optimal conditions are

$$f_5 = q_1^1 - 2p_2^1 \frac{q_1^2 N_2}{NS} + p_2^2 \frac{q_1^2 N_2}{NS} - \lambda_1 = 0, \tag{18}$$

$$f_6 = q_1^2 - 2p_2^2 \frac{q_1^2 N_2}{NS} + p_2^1 \frac{q_1^2 N_2}{NS} + \lambda_2 = 0, \tag{19}$$

$$f_7 = \lambda_1\left[\frac{2A^2\alpha_1}{(A^1 + A^2)^2} + \alpha_2\right] - 1 = 0, \tag{20}$$

$$f_8 = \lambda_2\left[\frac{2A^1\alpha_1}{(A^1 + A^2)^2} + \alpha_2\right] - 1 = 0, \tag{21}$$

$$f_9 = (\beta^1 - p_2^1) - (\beta^2 - p_2^2) = 0, \tag{22}$$

where $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$ are the Lagrangian multipliers of (16) and (17), respectively. (20) and (21) imply that $\lambda_1 > 0$ and $\lambda_2 > 0$. On the basis of (18)–(22), we have the following conclusions.

Proposition 2. *For $(\beta^1 - p_2^1) - (\beta^2 - p_2^2) = 0$, at equilibrium we have the following relations: $\frac{\partial p_2^1}{\partial S} > 0, \frac{\partial p_2^2}{\partial S} > 0$, $\frac{\partial p_2^1}{\partial N_2} < 0$ and $\frac{\partial p_2^2}{\partial N_2} < 0$.*

Proof. See Appendix. ■

Remark s. The relationship between price at the second stage and parameters is addressed for the case where $(\beta^1 - p_2^1) - (\beta^2 - p_2^2) = 0$. $\frac{\partial p_2^1}{\partial S} > 0$ and $\frac{\partial p_2^2}{\partial S} > 0$ imply that higher expected switching costs yield higher prices. Switching costs therefore increase prices. $\frac{\partial p_2^1}{\partial N_2} < 0$ and $\frac{\partial p_2^2}{\partial N_2} < 0$ illustrate that firms compete more intensely if the second type of market is larger. We further note that brand stealing effects and brand expansion effects have no impact on price if $(\beta^1 - p_2^1) - (\beta^2 - p_2^2) = 0$.

Equilibrium is achieved at the second stage, and the first stage is addressed in the next subsection.

4.3. The first stage

The first stage is addressed based on (4). We have $p_1^1 = p_1^2$ and $q_1^1 = q_1^2$ if two firms enter into this market. This paper focuses on just these two firms in this market.

If the two firms cooperatively price, $p_1^1 = p_1^2 = u_0$ such that the firms achieve maximum benefit and all consumers enter into this market. In this case, the two firms establish a Cartel, and switching costs have no effect on prices in the first stage. If the two firms freely compete in the first stage, then they price under marginal cost, and $p_1^1 = p_1^2$.

If $(\beta^1 - p_2^1) - (\beta^2 - p_2^2) < 0$, $q_1^1 = q_1^2$, (12) and (13) imply that $A^2 > A^1$. $q_1^1 = q_1^2$ and $(\beta^1 - p_2^1) - (\beta^2 - p_2^2) \geq 0$ also indicate that $A^2 > A^1$. This effect is summarized as follows.

Proposition 3. *If $p_2^1 < p_2^2$, we have $A^2 > A^1$.*

Remark s. The above conclusions illustrate that firms without a price advantage invest more in advertisement. Firms without price advantage try to improve the market share and advertisement is an efficient way to improve market share. This result is consistent with the large-scale randomized field experiment results of Anderson and Simester (2013) and many economic phenomena. This article supports robust theory for Anderson and Simester (2013).

The managerial policy is that firms with higher price invest more advertising than other firms to maintain market share. Actually, many firms with big brands expend much to maintain the market share or brand advantages.

5. Conclusions

This study develops a theory of the effect of switching costs on advertising. We argue that switching costs reduce competition both regarding pricing and advertising investment. Under switching cost, all producers reduce advertisement investment. Moreover, firms with higher prices are more likely to invest in advertising.

This article initially captures the impacts of switching costs on advertisement, which amplifies the literature of both switching costs and advertisement. The acting mechanism of switching costs affecting advertising investment is captured. The interaction of switching costs and advertisement is added to the literature of switching costs in this paper. As a byproduct, many industries with durable goods are analyzed and this article amplifies the literature of industrial economics.

The managerial implication lies in two aspects: On one hand, when firms determine advertising investment, the properties of productions or switching costs should be considered. On the other hand, to improve competition, government should establish standards to reduce switching costs.

There are some limitations about this article which are our further researching issues. In fact, advertising investment may act to deter other firms from entering into this industry if the first type of market is large enough. This seems more difficult. This study employs linear functions to simplify the problem. The model in this study can be extended to general situations. Moreover, this work uses a special type of advertisement and it is interesting when applied to other types of advertisements.

Note

1. http://www.renheyaoye.com/cn/gsjs.jsp

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Appendix

Proof of Proposition 1 Since it is difficult to obtain an explicit solution for the above system of Equations (10)–(13), the implicit function theorem is employed to show this conclusion. We denote the following Jacobi matrix for the systems of equations in (10)–(13):

$$J=\begin{bmatrix} \frac{\partial f_1}{\partial p_2^1} & \frac{\partial f_1}{\partial p_2^2} & \frac{\partial f_1}{\partial A^1} & \frac{\partial f_1}{\partial A^2} \\ \frac{\partial f_2}{\partial p_2^1} & \frac{\partial f_2}{\partial p_2^2} & \frac{\partial f_2}{\partial A^1} & \frac{\partial f_2}{\partial A^2} \\ \frac{\partial f_3}{\partial p_2^1} & \frac{\partial f_3}{\partial p_2^2} & \frac{\partial f_3}{\partial A^1} & \frac{\partial f_3}{\partial A^2} \\ \frac{\partial f_4}{\partial p_2^1} & \frac{\partial f_4}{\partial p_2^2} & \frac{\partial f_4}{\partial A^1} & \frac{\partial f_4}{\partial A^2} \end{bmatrix}=$$

$$\begin{bmatrix} -2\frac{N_1q_1^1+N_2q_1^2}{NS} & \frac{N_1q_1^1+N_2q_1^2}{NS} & \frac{N_1q_1^1}{NS}\left[\frac{2A^2\alpha_1}{(A^1+A^2)^2}+\alpha_2\right] & -\frac{N_1q_1^1}{NS}\left[\frac{2A^1\alpha_1}{(A^1+A^2)^2}+\alpha_2\right] \\ \frac{N_1q_1^1+N_2q_1^2}{NS} & -2\frac{N_1q_1^1+N_2q_1^2}{NS} & -\frac{N_1q_1^1}{NS}\left[\frac{2A^2\alpha_1}{(A^1+A^2)^2}+\alpha_2\right] & \frac{N_1q_1^1}{NS}\left[\frac{2A^1\alpha_1}{(A^1+A^2)^2}+\alpha_2\right] \\ \frac{N_1q_1^1}{NS}\left[\frac{2A^2\alpha_1}{(A^1+A^2)^2}+\alpha_2\right] & 0 & -\frac{4p_2^1N_1q_1^1\alpha_1}{NS}\frac{A^2}{(A^1+A^2)^3} & \frac{\partial f_3}{\partial A^2} \\ 0 & \frac{N_1q_1^1}{NS}\left[\frac{2A^1\alpha_1}{(A^1+A^2)^2}+\alpha_2\right] & \frac{\partial f_4}{\partial A^1} & -\frac{4p_2^2N_1q_1^1\alpha_1}{NS}\frac{A^1}{(A^1+A^2)^3} \end{bmatrix}$$

In the above equation, we have the relations $\frac{\partial f_3}{\partial A^2}=\frac{p_2^1N_1q_1^1}{NS}[\frac{2\alpha_1}{(A^1+A^2)^2}-\frac{4A^2\alpha_1}{(A^1+A^2)^3}]$ and $\frac{\partial f_4}{\partial A^1}=\frac{p_2^2N_1q_1^1}{NS}[\frac{2\alpha_1}{(A^1+A^2)^2}-\frac{4A^1\alpha_1}{(A^1+A^2)^3}]$. Since the two profit functions are concave, we have $\det J>0$. The relationship between p_2^i, A^i, and S, α_1, α_2 is addressed., By simple calculation, we have $\frac{\partial f_1}{\partial S}=2p_2^1\frac{N_1q_1^1+N_2q_1^2}{NS^2}-p_2^2\frac{N_1q_1^1+N_2q_1^2}{NS^2}+\frac{N_1q_1^1}{N}\frac{\beta^2-\beta^1}{S^2}$, $\frac{\partial f_2}{\partial S}=2p_2^2\frac{N_1q_1^1+N_2q_1^2}{NS^2}-p_2^1\frac{N_1q_1^1+N_2q_1^2}{NS^2}-\frac{N_1q_1^2}{N}\frac{\beta^2-\beta^1}{S^2}$, $\frac{\partial f_3}{\partial S}=-\frac{p_2^1N_1q_1^1}{NS^2}[\frac{2A^2\alpha_1}{(A^1+A^2)^2}+\alpha_2]$ and $\frac{\partial f_4}{\partial S}=-\frac{p_2^2N_1q_1^1}{NS^2}[\frac{2A^1\alpha_1}{(A^1+A^2)^2}+\alpha_2]$.$\frac{\partial f_1}{\partial \alpha_1}=-N_1q_1^1\frac{A^2-A^1}{SN(A^1+A^2)}$, $\frac{\partial f_2}{\partial \alpha_1}=q_1^1N_1\frac{A^2-A^1}{SN(A^1+A^2)}$, $\frac{\partial f_3}{\partial \alpha_1}=\frac{2p_2^1N_1q_1^1}{NS}\frac{A^2}{(A^1+A^2)^2}$ and $\frac{\partial f_4}{\partial \alpha_1}=\frac{2p_2^2N_1q_1^1}{NS}\frac{A^1}{(A^1+A^2)^2}$. $\frac{\partial f_1}{\partial \alpha_2}=-q_1^1N_1\frac{A^2-A^1}{NS}$, $\frac{\partial f_2}{\partial \alpha_2}=q_1^1N_1\frac{A^2-A^1}{NS}$, $\frac{\partial f_3}{\partial \alpha_2}=\frac{p_2^1N_1q_1^1}{NS}$ and $\frac{\partial f_4}{\partial \alpha_2}=\frac{p_2^2N_1q_1^1}{NS}$. By the implicit function theorem, the unique functions $p_2^1(S,N_2,\alpha_1,\alpha_2), p_2^2(S,N_2,\alpha_1,\alpha_2), A^1(S,N_2,\alpha_1,\alpha_2)$ and $A^2(S,N_2,\alpha_1,\alpha_2)$ exist, and are all differentiable. (14) and (15) imply that $\frac{\partial p_2^i}{\partial S}>0$. (12) and (13) indicate that $\frac{\partial p_2^i}{\partial \alpha_1}<0, \frac{\partial p_2^i}{\partial \alpha_2}<0, \frac{\partial A^i}{\partial S}<0, \frac{\partial A^i}{\partial \alpha_1}>0$ and $\frac{\partial A^i}{\partial \alpha_2}>0$.(14) and (15) also yield $\frac{\partial p_2^i}{\partial N_1}<0$ and $\frac{\partial A^i}{\partial N_1}>0$.

The conclusion is obtained and the proof is complete. ■

Proof of Proposition 2 We utilize the implicit function theorem to show this conclusion. We denote the following Jacobi matrix for the systems of equations in (18)–(21):

$$\bar{J}=\begin{bmatrix} \frac{\partial f_5}{\partial p_2^1} & \frac{\partial f_5}{\partial p_2^2} & \frac{\partial f_5}{\partial A^1} & \frac{\partial f_5}{\partial A^2} \\ \frac{\partial f_6}{\partial p_2^1} & \frac{\partial f_6}{\partial p_2^2} & \frac{\partial f_6}{\partial A^1} & \frac{\partial f_6}{\partial A^2} \\ \frac{\partial f_7}{\partial p_2^1} & \frac{\partial f_7}{\partial p_2^2} & \frac{\partial f_7}{\partial A^1} & \frac{\partial f_7}{\partial A^2} \\ \frac{\partial f_8}{\partial p_2^1} & \frac{\partial f_8}{\partial p_2^2} & \frac{\partial f_8}{\partial A^1} & \frac{\partial f_8}{\partial A^2} \end{bmatrix}=\begin{bmatrix} -2\frac{q_1^2N_2}{NS} & \frac{q_1^2N_2}{NS} & 0 & 0 \\ \frac{q_1^2N_2}{NS} & -2\frac{q_1^2N_2}{NS} & 0 & 0 \\ 0 & 0 & -\frac{4\lambda_1\alpha_1A^2}{(A^1+A^2)^3} & \frac{\partial f_7}{\partial A^2} \\ 0 & 0 & \frac{\partial f_8}{\partial A^1} & -\frac{4\lambda_2\alpha_1A^1}{(A^1+A^2)^3} \end{bmatrix}$$

in the above equation, we have the relation $\frac{\partial f_7}{\partial A^2} = \frac{2\lambda_1\alpha_1}{(A^1+A^2)^2} - \frac{4\lambda_1 A^2\alpha_1}{(A^1+A^2)^3}$ and $\frac{\partial f_8}{\partial A^1} = \frac{2\lambda_2\alpha_1}{(A^1+A^2)^2} - \frac{4\lambda_2 A^1\alpha_1}{(A^1+A^2)^3}$. We know that $\det\bar{J}>0$. Furthermore, $\frac{\partial f_5}{\partial S} = \frac{q_1^2 N_2}{NS^2}(2p_2^1 - p_2^2)$, $\frac{\partial f_6}{\partial S} = \frac{q_1^2 N_2}{NS^2}(2p_2^2 - p_2^1)$, $\frac{\partial f_7}{\partial S} = 0$, $\frac{\partial f_8}{\partial S} = 0$, $\frac{\partial f_5}{\partial N_2} = -\frac{q_1^2}{NS}(2p_2^1 - p_2^2)$, $\frac{\partial f_6}{\partial N_2} = -\frac{q_1^2}{NS}(2p_2^2 - p_2^1)$, $\frac{\partial f_7}{\partial N_2} = 0$ and $\frac{\partial f_8}{\partial N_2} = 0$. By the implicit function theorem, the unique functions $p_2^1(S, N_2)$ and $p_2^2(S, N_2)$ exist, are differentiable, and satisfy the following relation:

$$\frac{\partial p_2^1}{\partial S} = -\frac{\begin{vmatrix} \frac{\partial f_5}{\partial S} & \frac{\partial f_5}{\partial p_2^2} & \frac{\partial f_5}{\partial A^1} & \frac{\partial f_5}{\partial A^2} \\ \frac{\partial f_6}{\partial S} & \frac{\partial f_6}{\partial p_2^2} & \frac{\partial f_6}{\partial A^1} & \frac{\partial f_6}{\partial A^2} \\ \frac{\partial f_7}{\partial S} & \frac{\partial f_7}{\partial p_2^2} & \frac{\partial f_7}{\partial A^1} & \frac{\partial f_7}{\partial A^2} \\ \frac{\partial f_8}{\partial S} & \frac{\partial f_8}{\partial p_2^2} & \frac{\partial f_8}{\partial A^1} & \frac{\partial f_8}{\partial A^2} \end{vmatrix}}{\det\bar{J}} > 0, \quad \frac{\partial p_2^2}{\partial S} = -\frac{\begin{vmatrix} \frac{\partial f_5}{\partial p_2^1} & \frac{\partial f_5}{\partial S} & \frac{\partial f_5}{\partial A^1} & \frac{\partial f_5}{\partial A^2} \\ \frac{\partial f_6}{\partial p_2^1} & \frac{\partial f_6}{\partial S} & \frac{\partial f_6}{\partial A^1} & \frac{\partial f_6}{\partial A^2} \\ \frac{\partial f_7}{\partial p_2^1} & \frac{\partial f_7}{\partial S} & \frac{\partial f_7}{\partial A^1} & \frac{\partial f_7}{\partial A^2} \\ \frac{\partial f_8}{\partial p_2^1} & \frac{\partial f_8}{\partial S} & \frac{\partial f_8}{\partial A^1} & \frac{\partial f_8}{\partial A^2} \end{vmatrix}}{\det\bar{J}} > 0,$$

$$\frac{\partial p_2^1}{\partial N_2} = -\frac{\begin{vmatrix} \frac{\partial f_5}{\partial N_2} & \frac{\partial f_5}{\partial p_2^2} & \frac{\partial f_5}{\partial A^1} & \frac{\partial f_5}{\partial A^2} \\ \frac{\partial f_6}{\partial N_2} & \frac{\partial f_6}{\partial p_2^2} & \frac{\partial f_6}{\partial A^1} & \frac{\partial f_6}{\partial A^2} \\ \frac{\partial f_7}{\partial N_2} & \frac{\partial f_7}{\partial p_2^2} & \frac{\partial f_7}{\partial A^1} & \frac{\partial f_7}{\partial A^2} \\ \frac{\partial f_8}{\partial N_2} & \frac{\partial f_8}{\partial p_2^2} & \frac{\partial f_8}{\partial A^1} & \frac{\partial f_8}{\partial A^2} \end{vmatrix}}{\det\bar{J}} < 0, \quad \frac{\partial p_2^2}{\partial N_2} = -\frac{\begin{vmatrix} \frac{\partial f_5}{\partial p_2^1} & \frac{\partial f_5}{\partial N_2} & \frac{\partial f_5}{\partial A^1} & \frac{\partial f_5}{\partial A^2} \\ \frac{\partial f_6}{\partial p_2^1} & \frac{\partial f_6}{\partial N_2} & \frac{\partial f_6}{\partial A^1} & \frac{\partial f_6}{\partial A^2} \\ \frac{\partial f_7}{\partial p_2^1} & \frac{\partial f_7}{\partial N_2} & \frac{\partial f_7}{\partial A^1} & \frac{\partial f_7}{\partial A^2} \\ \frac{\partial f_8}{\partial p_2^1} & \frac{\partial f_8}{\partial N_2} & \frac{\partial f_8}{\partial A^1} & \frac{\partial f_8}{\partial A^2} \end{vmatrix}}{\det\bar{J}} < 0.$$

The conclusion is obtained and the proof is complete. ■