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CODAS methods for multiple attribute group decision making with interval-valued bipolar uncertain linguistic information and their application to risk assessment of Chinese enterprises' overseas mergers and acquisitions

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ABSTRACT

Bipolar fuzzy set theory has been successfully applied in some areas, but there are situations in real life which can't be represented by bipolar fuzzy sets. However, all the existing approaches are unsuitable to describe the positive and negative membership degree an element to an uncertain linguistic label to have an interval value, which can reflect the decision maker's confidence level when they are making an evaluation. In order to overcome this limit, we propose the definition of interval-valued bipolar uncertain linguistic sets (IVBULSs) to solve this problem based on the bipolar fuzzy sets and uncertain linguistic information processing models. In this paper, we extend the traditional information aggregating operators to interval-valued bipolar uncertain linguistic sets (IVBULSs) and propose some IVBUL aggregating operators. Then, we extend the CODAS method to solve multiple attribute group decision making (MAGDM) issues with interval-valued bipolar uncertain linguistic numbers (IVBULNs) based on these operators. An example for risk assessment of Chinese enterprises' overseas mergers and acquisitions (M&As) is given to illustrate the proposed methodology.

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1. Introduction

K. T. Atanassov (1986)devised the concept of intuitionistic fuzzy sets (IFSs), which is a generalization of the definition of fuzzy set (Zadeh, 1965). The main merits of IFSs have membership degree and non-membership degree. The sum of these two pairs

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should not exceed 1. Over the last decades, the IFSs theory has been successfully applied to diverse domains. Later, the notion of IFSs is further generalized by allowing the membership and non-membership functions to assume interval values, thereby introducing the concept of interval-valued intuitionistic fuzzy sets (IVIFSs) (K. Atanassov & Gargov, 1989) and all kinds of corresponding applications. The bipolar fuzzy sets (BFSs) (Zhang, 1996) has appeared lately as a useful tool to depict fuzziness and uncertainty in MADM with the positive membership degree and the negative membership degree. But different from IFS, the value range of membership degree of the BFS is [−1, 1]. Naz and Shabir (2014) defined fuzzy bipolar soft sets and basic operations of union, intersection and complementation for fuzzy bipolar soft sets. Han et al. (2015) discussed the relationship between bipolar-valued fuzzy set and fuzzy set with its extensions. Sarwar and Akram (2017) presented various methods of construction of bipolar fuzzy competition graphs and designed an algorithm for computing the strength of competition among political candidates in a local government. Wei et al. (2017) investigated the MADM issues based on the aggregation operators with hesitant bipolar fuzzy information. X. R. Xu and Wei (2017)devised the MADM issues based on the aggregation operators with dual hesitant bipolar fuzzy information. Alghamdi et al. (2018) developed new methodologies in handling MCDM issues where the subjective data given by a DM are expressed with bipolar fuzzy information. Akram and Waseem (2018) presented novel applications of bipolar fuzzy graphs to decision making problems. Stanujkic et al. (2019) made a proposal for a new extension of the MULTIMOORA method to deal with bipolar fuzzy sets. Lu et al. (2017) presented some methods that apply bipolar 2-tuple linguistic operators to solving the bipolar 2-tuple linguistic MADM issues. Wei et al. (2018) investigated the MADM issues for risk evaluation of enterprise human capital investment with interval-valued bipolar 2-tuple linguistic numbers (IVB2TLNs). Gao et al. (2019) proposed the interval-valued bipolar uncertain linguistic sets (IVBULSs).

The CODAS method was proposed by Keshavarz Ghorabaee et al. (2016). It is a novel and stable method used to solve MADM problems with aid of deriving the Euclidean distance and Hamming distance to select the best alternative. Ghorabaee et al. (2018) developed the fuzzy CODAS method to select suppliers. Panchal et al. (2017) applied fuzzy CODAS to solve the maintenance decision problem. Karasan et al. (2019) proposed neutrosophic CODAS method to select wind energy plant locations. Yeni and Ozcelik (2019) defined the interval-valued intuitionistic fuzzy CODAS method for MAGDM. Peng and Li (2019) designed the hesitant fuzzy soft CODAS method for decision making.

This research has four main purposes. The first is to introduce the basic definition of IVBULSs and some basic operations between IVBULNs. The second lies in introducing some aggregation operators with interval-valued bipolar uncertain linguistic numbers (IVBULNs). The third is to establish some integrated ranking method for interval-valued bipolar uncertain linguistic MAGDM issues considering based on the CODAS method and the proposed aggregation operators. The final purpose is to demonstrate the application, practicality and effectiveness of the proposed MAGDM method using a case study about risk assessment of Chinese enterprises' overseas mergers and acquisitions (M&As). For the sake of clarity, the rest of such research is organized as follows. In the section 2, we introduce concepts of NFS, IVBULS and some basic operations of IVBULNs. In Section 3, we introduce some aggregation operators with IVBULNs. In Section 4, the CODAS method is developed based on proposed aggregation operators to solve MAGDM issues with IVBULNs. An illustrative example for risk assessment of Chinese enterprises' overseas mergers and acquisitions (M&As) is given in Section 5. Some conclusions are given in Section 6 to finish this paper.

2. Preliminaries

2.1. BFSs

The basic overview of BFSs (Zhang, 2006) is introduced.

Definition 1. (Zhang, 2006). Let X be a fix set. A BFS is an object having the form

$$B = \{\langle x, (\mu_B^+(x), \nu_B^-(x))\rangle | x \in X\} \tag{1}$$

where the positive membership degree function $\mu_B^+(x) : X \to [0,1]$ denotes the satisfaction degree of an element x to the property corresponding to a BFS B and the negative membership degree function $\nu_B^-(x) : X \to [-1,0]$ denotes satisfaction degree of an element x to some implicit counter property corresponding to a BFS B, respectively, and, for every $x \in X$.

Let $b = (\mu^+, \nu^-)$ be a bipolar fuzzy number (BFN).

Definition 2. (Gul, 2015). Some operations on BFNs are given.

$$(1) b_1 \oplus b_2 = \left(\mu_1^+ + \mu_2^+ - \mu_1^+\mu_2^+, -|\nu_1^-||\nu_2^-|\right);$$

$$(2) b_1 \otimes b_2 = \left(\mu_1^+\mu_2^+, \nu_1^- + \nu_2^- - \nu_1^-\nu_2^-\right);$$

$$(3) \lambda b = \left(1 - (1-\mu^+)^\lambda, -|\nu^-|^\lambda\right), \lambda>0;$$

$$(4) (b)^\lambda = \left((\mu^+)^\lambda, -1 + |1+\nu^-|^\lambda\right), \lambda>0;$$

$$(5) b^c = \left(1-\mu^+, |\nu^-| - 1\right);$$

2.2. The interval-valued bipolar uncertain linguistic set

Suppose that $S = \{s_i | i = 1, 2, \ldots, t\}$ is the linguistic term set with odd cardinality. Every label s_i expresses a possible value for a linguistic variable. It should satisfy the following characteristics (Herrera & Martinez, 2000):

(1) The linguistic term set is ordered: $s_i > s_j$, if $i>j$; (2) Max operator: $\max(s_i, s_j) = s_i$, if $s_i \geq s_j$; (3) Min operator: $\min(s_i, s_j) = s_i$, if $s_i \leq s_j$. For

example, S can be defined as

$$S = \{s_1 = extremelypoor, s_2 = verypoor, s_3 = poor, s_4 = medium,\\ s_5 = good, s_6 = verygood, s_7 = extremelygood\}$$

Let$\tilde{s} = [s_\alpha, s_\beta]$, where$s_\alpha, s_\beta \in S, s_\alpha$ and s_β are the lower and the upper limits, respectively. We call $\tilde{s}$ the uncertain linguistic variable (ULV). Let $\tilde{S}$ be the set of all the ULV sets (Z. S. Xu, 2004).

Then, Gao et al. (2019) proposed the definition and some operations of interval-valued bipolar uncertain linguistic sets (IVBULSs) based on the bipolar fuzzy sets (Zhang, 1996) and uncertain linguistic information processing models (Z. S. Xu, 2004).

Definition 3. (Gao et al., 2019)**.** An interval-valued bipolar uncertain linguistic sets (IVBULS)$\tilde{B}$ in$X$ is given

$$\tilde{B} = \left\{ \left[s^L_{\theta(x)}, s^R_{\theta(x)} \right], \left(\left[\mu^{L+}_B(x), \mu^{R+}_B(x) \right], \left[\nu^{L-}_B(x), \nu^{R-}_B(x) \right] \right), x \in X \right\} \tag{2}$$

where$\tilde{s}_{\theta(x)} \left(\tilde{s}_{\theta(x)} = \left[s^L_{\theta(x)}, s^R_{\theta(x)} \right] \right)$ $s^L_{\theta(x)}, s^R_{\theta(x)} \in S$, the positive membership degree function$\tilde{\mu}^+_B(x) \subset [0,1]$ which expresses the satisfaction degree of an element$x$ to uncertain linguistic variable $\tilde{s}_{\theta(x)}$ to the property corresponding to an IVBULS$\tilde{B}$ and the negative membership degree function $\tilde{\nu}^-_B(x) \subset [0,1]$ which denotes satisfaction degree of an element$x$ to uncertain linguistic variable $\tilde{s}_{\theta(x)}$ to some implicit counter property corresponding to an IVBULS$\tilde{B}$, respectively, and, for every $x \in X$.

If $s^L_{\theta(x)} = s^R_{\theta(x)}$, then the IVBULSs reduce to interval-valued bipolar linguistic sets; If $\mu^{L+}_B(x) = \mu^{R+}_B(x), \nu^{L-}_B(x) = \nu^{R-}_B(x)$, then the IVBULSs reduce to bipolar uncertain linguistic sets; If $s^L_{\theta(x)} = s^R_{\theta(x)}$ and$\mu^{L+}_B(x) = \mu^{R+}_B(x), \nu^{L-}_B(x) = \nu^{R-}_B(x)$, then the IVBULSs reduce to bipolar linguistic sets.

Let $\tilde{b} = \{\tilde{s}_\theta, (\tilde{\mu}^+, \tilde{\nu}^-)\} = \left\{ \left[s^L_\theta, s^R_\theta \right], \left(\left[\mu^{L+}, \mu^{R+} \right], \left[\nu^{L-}, \nu^{R-} \right] \right) \right\}$ be an interval-valued bipolar uncertain linguistic number (IVBULN).

Definition 4. (Gao et al., 2019). The expected values of $\tilde{b} = \left\{ [s_\alpha, s_\beta], \left(\left[\mu^{L+}, \mu^{R+} \right], \left[\nu^{L-}, \nu^{R-} \right] \right) \right\}$ is defined as

$$EV(\tilde{b}) = \frac{\alpha + \beta}{2t} \cdot \frac{\left(1 + \mu^{L+} + \nu^{L-}\right) + \left(1 + \mu^{R+} + \nu^{R-}\right)}{4} \tag{3}$$

Definition 5. (Gao et al., 2019)**.** Let $\tilde{b}_1 = \left\{ \left[s^L_{\theta_1}, s^R_{\theta_1} \right], \left(\left[\mu^{L+}_1, \mu^{R+}_1 \right], \left[\nu^{L-}_1, \nu^{R-}_1 \right] \right) \right\}$ and$\tilde{b}_2 = \left\{ \left[s^L_{\theta_2}, s^R_{\theta_2} \right], \left(\left[\mu^{L+}_2, \mu^{R+}_2 \right], \left[\nu^{L-}_2, \nu^{R-}_2 \right] \right) \right\}$ be two IVBULNs, the operations on IVBULNs are defined.

$$(1)\tilde{b}_1 \oplus \tilde{b}_2 = \left\{ \left[s^L_{\theta_1} \oplus s^L_{\theta_2}, s^R_{\theta_1} \oplus s^R_{\theta_2} \right], \begin{pmatrix} \left[\mu_1^{L+} + \mu_2^{L+} - \mu_1^{L+}\mu_2^{L+}, \mu_1^{R+} + \mu_2^{R+} - \mu_1^{R+}\mu_2^{R+} \right], \\ \left[-\left|\nu_1^{L-}\right|\left|\nu_2^{L-}\right|, -\left|\nu_1^{R-}\right|\left|\nu_2^{R-}\right| \right] \end{pmatrix} \right\}; \tag{4}$$

$$(2)\tilde{b}_1 \otimes \tilde{b}_2 = \left\{ \left[s^L_{\theta_1} \otimes s^L_{\theta_2}, s^R_{\theta_1} \otimes s^R_{\theta_2} \right], \begin{pmatrix} \left[\mu_1^{L+}\mu_2^{L+}, \mu_1^{R+}\mu_2^{R+} \right], \\ \left[\nu_1^{L-} + \nu_2^{L-} - \nu_1^{L-}\nu_2^{L-}, \nu_1^{R-} + \nu_2^{R-} - \nu_1^{R-}\nu_2^{R-} \right] \end{pmatrix} \right\}; \tag{5}$$

$$(3)\lambda\tilde{b}_1 = \left\{ \left[\lambda s^L_{\theta_1}, \lambda s^R_{\theta_1} \right], \begin{pmatrix} \left[1 - \left(1 - \mu^{L+}\right)^{\lambda}, 1 - \left(1 - \mu^{R+}\right)^{\lambda} \right], \\ \left[-\left|\nu^{L-}\right|^{\lambda}, -\left|\nu^{R-}\right|^{\lambda} \right] \end{pmatrix} \right\}, \lambda>0; \tag{6}$$

$$(4)\left(\tilde{b}_1\right)^{\lambda} = \left\{ \left[\left(s^L_{\theta_1}\right)^{\lambda}, \left(s^R_{\theta_1}\right)^{\lambda} \right], \begin{pmatrix} \left[\left(\mu^{L+}\right)^{\lambda}, \left(\mu^{R+}\right)^{\lambda} \right], \\ \left[-1 + \left|1 + \nu^{L-}\right|^{\lambda}, -1 + \left|1 + \nu^{R-}\right|^{\lambda} \right] \end{pmatrix} \right\}, \lambda>0. \tag{7}$$

Definition 6. Let $\tilde{b}_1 = \left\{ \left[s^L_{\theta_1}, s^R_{\theta_1} \right], \left(\left[\mu_1^{L+}, \mu_1^{R+} \right], \left[\nu_1^{L-}, \nu_1^{R-} \right] \right) \right\}$ and $\tilde{b}_2 = \left\{ \left[s^L_{\theta_2}, s^R_{\theta_2} \right], \left(\left[\mu_2^{L+}, \mu_2^{R+} \right], \left[\nu_2^{L-}, \nu_2^{R-} \right] \right) \right\}$ be two IVBULNs, then the Euclidean distance $ED(\tilde{b}_1, \tilde{b}_2)$ and Hamming distance$HD(\tilde{b}_1, \tilde{b}_2)$ between $\tilde{b}_1$ and $\tilde{b}_2$ is listed as follows:

$$ED\left(\tilde{b}_1, \tilde{b}_2\right) = \sqrt{\frac{\begin{pmatrix} \left(\frac{s^L_{\theta_1} - s^L_{\theta_2}}{t}\right)^2 + \left(\frac{s^R_{\theta_1} - s^R_{\theta_2}}{t}\right)^2 + \left(\mu_1^{L+} - \mu_2^{L+}\right)^2 \\ + \left(\mu_1^{R+} - \mu_2^{R+}\right)^2 + \left(\nu_1^{L+} - \nu_2^{L+}\right)^2 + \left(\nu_1^{R+} - \nu_2^{R+}\right)^2 \end{pmatrix}}{6}} \tag{8}$$

$$HD\left(\tilde{b}_1, \tilde{b}_2\right) = \frac{\begin{pmatrix} \left|\frac{s^L_{\theta_1} - s^L_{\theta_2}}{t}\right| + \left|\frac{s^R_{\theta_1} - s^R_{\theta_2}}{t}\right| + \left|\mu_1^{L+} - \mu_2^{L+}\right| \\ + \left|\mu_1^{R+} - \mu_2^{R+}\right| + \left|\nu_1^{L+} - \nu_2^{L+}\right| + \left|\nu_1^{R+} - \nu_2^{R+}\right| \end{pmatrix}}{6} \tag{9}$$

3. Some aggregation operators with IVBULNs

Let$\tilde{b}_j = \left\{ \left[s^L_{\theta_j}, s^R_{\theta_j} \right], \left(\left[\mu_j^{L+}, \mu_j^{R+} \right], \left[\nu_j^{L-}, \nu_j^{R-} \right] \right) \right\} (j = 1, 2, \ldots, n)$ be a set of IVBULNs. Gao et al. (2019) developed some IVBUL arithmetic aggregation operators.

Definition 7. The IVBUL weighted average (IVBULWA) operator is

$$\mathrm{IVBULWA}_{\omega}\left(\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_n\right) = \overset{n}{\underset{j=1}{\oplus}}\left(\omega_j \tilde{b}_j\right)$$

$$= \left\{ \left[\sum_{j=1}^{n} \omega_j s^L_{\theta_j}, \sum_{j=1}^{n} \omega_j s^R_{\theta_j}\right], \left(\begin{array}{c} \left[1 - \prod_{j=1}^{n}\left(1-\mu_j^{L+}\right)^{\omega_j}, 1 - \prod_{j=1}^{n}\left(1-\mu_j^{R+}\right)^{\omega_j}\right], \\ \left[-\prod_{j=1}^{n}\left|\nu_j^{L-}\right|^{\omega_j}, -\prod_{j=1}^{n}\left|\nu_j^{R-}\right|^{\omega_j}\right] \end{array} \right) \right\} \quad (10)$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ denotes the weight values of $\tilde{b}_j (j = 1, 2, \ldots, n)$, and $\omega_j > 0$, $\sum_{j=1}^{n} \omega_j = 1$.

Example 1. Let $\tilde{b}_1 = \{[s_5, s_6], ([0.2, 0.3], [-0.5, -0.4])\}$, $\tilde{b}_2 = \{[s_3, s_4], ([0.3, 0.4], [-0.6, -0.5])\}$, $\tilde{b}_3 = \{[s_4, s_5], ([0.4, 0.5], [-0.2, -0.1])\}$, $\tilde{b}_4 = \{[s_5, s_6], ([0.5, 0.6], [-0.2, -0.1])\}$ be four IVBULNs, $\omega = (0.2, 0.1, 0.3, 0.4)$ is the weight vector of $\tilde{b}_j (j = 1, 2, 3, 4)$. Then

$$\begin{aligned} &\mathrm{IVBULWA}_{\omega}\left(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \tilde{b}_4\right) = \overset{4}{\underset{j=1}{\oplus}}\left(\omega_j \tilde{b}_j\right) \\ &= \left\{ \left[\sum_{j=1}^{4} \omega_j s^L_{\theta_j}, \sum_{j=1}^{4} \omega_j s^R_{\theta_j}\right], \left(\begin{array}{c} \left[1 - \prod_{j=1}^{4}\left(1-\mu_j^{L+}\right)^{\omega_j}, 1 - \prod_{j=1}^{4}\left(1-\mu_j^{R+}\right)^{\omega_j}\right], \\ \left[-\prod_{j=1}^{4}\left|\nu_j^{L-}\right|^{\omega_j}, -\prod_{j=1}^{4}\left|\nu_j^{R-}\right|^{\omega_j}\right] \end{array} \right) \right\} \\ &= \left\{ \begin{array}{c} \left[\begin{array}{c} 0.2 \otimes s_5 \oplus 0.1 \otimes s_3 \oplus 0.3 \otimes s_4 \oplus 0.4 \otimes s_5, \\ 0.2 \otimes s_6 \oplus 0.1 \otimes s_4 \oplus 0.3 \otimes s_5 \oplus 0.4 \otimes s_6 \end{array} \right], \\ \left[\begin{array}{c} 1-(1-0.2)^{0.2} \times (1-0.3)^{0.1} \times (1-0.4)^{0.3} \times (1-0.5)^{0.4}, \\ 1-(1-0.3)^{0.2} \times (1-0.4)^{0.1} \times (1-0.5)^{0.3} \times (1-0.6)^{0.4} \end{array} \right], \\ \left[\begin{array}{c} -|-0.5|^{0.2} \times |-0.6|^{0.1} \times |-0.2|^{0.3} \times |-0.2|^{0.4}, \\ -|-0.4|^{0.2} \times |-0.5|^{0.1} \times |-0.1|^{0.3} \times |-0.1|^{0.4} \end{array} \right] \end{array} \right\} \\ &= \left\{[s_{4.5000}, s_{5.5000}], ([0.4000, 0.5019], [-0.2681, -0.1550])\right\} \end{aligned}$$

On the basis of the Applying the IVBUL arithmetic operators and geometric mean [59–60], Gao et al. (2019) proposed IVBUL geometric aggregation operators.

Definition 8. The IVBUL weighted geometric (IVBULWG) operator is given

$$\text{IVBULWG}_{\omega}\left(\tilde{b}_1, \tilde{b}_2, \ldots, \tilde{b}_n\right) = \underset{j=1}{\overset{n}{\otimes}}\left(\tilde{b}_j\right)^{\omega_j}$$
$$= \left\{\left[\underset{j=1}{\overset{n}{\otimes}}\left(s_{\theta_j}^{L}\right)^{\omega_j}, \underset{j=1}{\overset{n}{\otimes}}\left(s_{\theta_j}^{R}\right)^{\omega_j}\right], \left(\begin{array}{c}\left[\prod_{j=1}^{n}\left(\mu_j^{L+}\right)^{\omega_j}, \prod_{j=1}^{n}\left(\mu_j^{R+}\right)^{\omega_j}\right], \\ \left[-1+\prod_{j=1}^{n}\left(1+\nu_j^{L-}\right)^{\omega_j}, -1+\prod_{j=1}^{n}\left(1+\nu_j^{R-}\right)^{\omega_j}\right]\end{array}\right)\right\} \tag{11}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ is the weight values of $\tilde{b}_j (j = 1, 2, \ldots, n)$ with $0 \le \omega_j \le 1$, $\sum_{j=1}^{n} \omega_j = 1$.

Example 2. $\tilde{b}_1 = \{[s_5, s_6], ([0.2, 0.3], [-0.5, -0.4])\}$, $\tilde{b}_2 = \{[s_3, s_4], ([0.3, 0.4], [-0.6, -0.5])\}$, $\tilde{b}_3 = \{[s_4, s_5], ([0.4, 0.5], [-0.2, -0.1])\}$, $\tilde{b}_4 = \{[s_5, s_6], ([0.5, 0.6], [-0.2, -0.1])\}$ be four IVBULNs, $\omega = (0.2, 0.1, 0.3, 0.4)$ is the weight vector of$\tilde{b}_j (j = 1, 2, 3, 4)$. Then

$$\text{IVBULWG}_{\omega}\left(\tilde{b}_1, \tilde{b}_2, \tilde{b}_3, \tilde{b}_4\right) = \underset{j=1}{\overset{4}{\otimes}}\left(\tilde{b}_j\right)^{\omega_j}$$
$$= \left\{\left[\underset{j=1}{\overset{4}{\otimes}}\left(s_{\theta_j}^{L}\right)^{\omega_j}, \underset{j=1}{\overset{4}{\otimes}}\left(s_{\theta_j}^{R}\right)^{\omega_j}\right], \left(\begin{array}{c}\left[\prod_{j=1}^{4}\left(\mu_j^{L+}\right)^{\omega_j}, \prod_{j=1}^{4}\left(\mu_j^{R+}\right)^{\omega_j}\right], \\ \left[-1+\prod_{j=1}^{4}\left(1+\nu_j^{L-}\right)^{\omega_j}, -1+\prod_{j=1}^{4}\left(1+\nu_j^{R-}\right)^{\omega_j}\right]\end{array}\right)\right\}$$
$$= \left\{\begin{array}{c}\left[\begin{array}{c}(s_5)^{0.2} \otimes (s_3)^{0.1} \otimes (s_4)^{0.3} \otimes (s_5)^{0.4}, \\ (s_6)^{0.2} \otimes (s_4)^{0.1} \otimes (s_5)^{0.3} \otimes (s_6)^{0.4}\end{array}\right], \\ \left[0.2^{0.2} \times 0.3^{0.1} \times 0.4^{0.3} \times 0.5^{0.4}, 0.3^{0.2} \times 0.4^{0.1} \times 0.5^{0.3} \times 0.6^{0.4}\right], \\ \left[\begin{array}{c}-(1-0.5)^{0.2} \times (1-0.6)^{0.1} \times (1-0.2)^{0.3} \times (1-0.2)^{0.4}, \\ -(1-0.4)^{0.2} \times (1-0.5)^{0.1} \times (1-0.1)^{0.3} \times (1-0.1)^{0.4}\end{array}\right]\end{array}\right\}$$
$$= \left\{[s_{4.4434}, s_{5.4549}], ([0.3699, 0.4749], [-0.3205, -0.2175])\right\}$$

4. CODAS method for MAGDM with IVBULNs

Suppose that the following mathematical notations are represented the MAGDM problem with IVBULNs. Let$A = \{A_1, A_2, \ldots, A_m\}$ be alternatives sets and $G = \{G_1, G_2, \ldots, G_n\}$ be attributes sets. The $w = (w_1, w_2, \ldots, w_n)$ is the weight of attributes, where$0 \le w_j \le 1, \sum_{j=1}^{n} w_j = 1$, and a finite set of qualified experts $E = \{E_1, E_2, \ldots, E_q\}$, $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_q)$ is the weight of experts, where$0 \le \lambda_k \le 1$, $\sum_{k=1}^{q} \lambda_k = 1$. Suppose that $\tilde{R}^k = (\tilde{r}_{ij}^k)_{m \times n} = \left\{\left[(s_{\alpha_{ij}})^k, (s_{\beta_{ij}})^k\right], \left(\left[(\mu_{ij}^{L+})^k, (\mu_{ij}^{R+})^k\right], \left[(\nu_{ij}^{L-})^k, (\nu_{ij}^{R-})^k\right]\right)\right\}_{m \times n}$ is the

IVBUL group decision matrix, where $\left[(\mu_{ij}^{L+})^k, (\mu_{ij}^{R+})^k\right]$ and $\left[(\nu_{ij}^{L-})^k, (\nu_{ij}^{R-})^k\right]$ is the positive degree and negative degree which is given by the decision maker E_k for alternativeA_i under G_j, $\left[(\mu_{ij}^{L+})^k, (\mu_{ij}^{R+})^k\right] \subset [0,1]$, $\left[(\nu_{ij}^{L-})^k, (\nu_{ij}^{R-})^k\right] \subset [-1,0]$, $(s_{\alpha_{ij}})^k$, $(s_{\beta_{ij}})^k \in S, i = 1,2, \ldots, m,\ j = 1,2, \ldots, n, k = 1,2, \ldots, q$.

Then, an extended CODAS method is proposed to solve the MAGDM problems with IVBULNs. The calculating steps are involved as follows:

Step 1. Use the IVBULWA operator or IVBULWG operator to fuse matrix $\tilde{R}^k$ into the overall matrix$\tilde{R}$, $\tilde{R} = (\tilde{r}_{ij})_{m\times n} = \left\{[s_{\alpha_{ij}}, s_{\beta_{ij}}], \left(\left[\mu_{ij}^{L+}, \mu_{ij}^{R+}\right], \left[\nu_{ij}^{L-}, \nu_{ij}^{R-}\right]\right)\right\}_{m\times n}$.

$$\begin{aligned}\tilde{r}_{ij} &= \mathrm{IVBULWA}_\lambda\left(\tilde{r}_{ij}^1, \tilde{r}_{ij}^2, \ldots, \tilde{r}_{ij}^q\right)\\ &= \mathop{\oplus}_{k=1}^{q}\left(\lambda_k \tilde{r}_{ij}^k\right)\\ &= \left\{\left[\mathop{\oplus}_{k=1}^{q}\lambda_k (s_{\alpha_{ij}})^k, \mathop{\oplus}_{k=1}^{q}\lambda_k (s_{\beta_{ij}})^k\right], \left(\begin{array}{c}\left[1-\prod_{k=1}^{q}\left(1-\left(\mu_{ij}^{L+}\right)^k\right)^{\lambda_k}, 1-\prod_{k=1}^{q}\left(1-\left(\mu_{ij}^{R+}\right)^k\right)^{\lambda_k}\right],\\ \left[-\prod_{k=1}^{q}\left|\left(\nu_{ij}^{L-}\right)^k\right|^{\lambda_k}, -\prod_{k=1}^{q}\left|\left(\nu_{ij}^{R-}\right)^k\right|^{\lambda_k}\right]\end{array}\right)\right\}\end{aligned} \tag{12}$$

If the IVBULWG operator is chosen instead, we have

$$\begin{aligned}\tilde{r}_{ij} &= \mathrm{IVBULWG}_\lambda\left(\tilde{r}_{ij}^1, \tilde{r}_{ij}^2, \ldots, \tilde{r}_{ij}^q\right)\\ &= \mathop{\otimes}_{k=1}^{q}\left(\tilde{r}_{ij}^k\right)^{\lambda_k}\\ &= \left\{\left[\mathop{\otimes}_{k=1}^{q}\left((s_{\alpha_{ij}})^k\right)^{\lambda_k}, \mathop{\otimes}_{k=1}^{q}\left((s_{\beta_{ij}})^k\right)^{\lambda_k}\right], \left(\begin{array}{c}\left[\prod_{j=1}^{n}\left(\mu_{ij}^{L+}\right)^{\lambda_k}, \prod_{j=1}^{n}\left(\mu_{ij}^{R+}\right)^{\lambda_k}\right],\\ \left[-1+\prod_{j=1}^{n}\left(1+\left(\nu_{ij}^{L-}\right)^k\right)^{\lambda_k}, -1+\prod_{j=1}^{n}\left(1+\left(\nu_{ij}^{R-}\right)^k\right)^{\lambda_k}\right]\end{array}\right)\right\}\end{aligned} \tag{13}$$

Step 2. Compute the combined weight information for attributes.

Entropy is a conventional term from information theory which is also famous as the average (expected) amount of information (Ding & Shi, 2005) contained in each attribute. The larger the value of entropy in a specific attribute is, the smaller the differences in the ratings of alternatives with respect to this attribute. In turn, this means that this kind of attribute supplies less information and has a smaller weight.

1. Compute the normalized expected decision matrix$NE(\tilde{r}_{ij})_{m\times n}$ based on the expected decision matrix$E(\tilde{r}_{ij})_{m\times n}$, where:

$$NE(\tilde{r}_{ij}) = \frac{NE(\tilde{r}_{ij})}{\sum_{i=1}^{m} NE(\tilde{r}_{ij})}, j = 1, 2, \ldots, n, \tag{14}$$

2. Compute the vector of Shannon entropy$E = (E_1, E_2, \ldots, E_n)$, where:

$$E_j = -\frac{1}{\ln m} \sum_{i=1}^{m} NE(\tilde{r}_{ij}) \ln NE(\tilde{r}_{ij}) \tag{15}$$

   and $NE(\tilde{r}_{ij}) \ln NE(\tilde{r}_{ij})$ is defined as 0, if $NE(\tilde{r}_{ij}) = 0$.
3. Compute the vector of diversification degrees$H = (H_1, H_2, \ldots, H_n)$, where:

$$H_j = 1 - E_j, j = 1, 2, \ldots, n. \tag{16}$$

   The larger the degreeH_j, the more important the corresponding attributeG_j.
4. Compute the vector of objective weights$ow = (ow_1, ow_2, \ldots, ow_n)$ related attribute, where

$$w_j = \frac{H_j}{\sum_{j=1}^{n} H_j}, j = 1, 2, \ldots, n. \tag{17}$$

   where $ow_j \in [0, 1]$ and$\sum_{j=1}^{n} ow_j = 1$.
5. Determine the combined weights. Suppose that the subjective weight given by the DMs directly, is$sw = (sw_1, sw_2, \ldots, sw_n)$, where$sw_j \in [0, 1], j = 1, 2, \ldots, n, \sum_{j=1}^{n} sw_j = 1$. The objective weight is calculated by Eq.(17) directly, is $ow = (ow_1, ow_2, \ldots, ow_n)$, where$ow_j \in [0, 1], j = 1, 2, \ldots, n, \sum_{j=1}^{n} ow_j = 1$. Therefore, the combined weights of attributes $cw = (cw_1, cw_2, \ldots, cw_n)$ could be defined:

$$cw_j = \frac{ow_j * sw_j}{\sum_{j=1}^{n} ow_j * sw_j}, \tag{18}$$

where $cw_j \in [0, 1], j = 1, 2, \ldots, n, \sum_{j=1}^{n} cw_j = 1$.

Step 3. Obtain the interval-valued bipolar uncertain linguistic negative ideal solution (IVBULNIS):

$$IVBULNIS = (IVBULNIS_1, IVBULNIS_2, \ldots, IVBULNIS_n) \tag{19}$$

$$IVBULNIS_j = \left\{ [s_{\alpha_j}, s_{\beta_j}], \left(\left[\mu_j^{L+}, \mu_j^{R+} \right], \left[\nu_j^{L-}, \nu_j^{R-} \right] \right) \right\} \tag{20}$$

$$\left\{ [s_{\alpha_j}, s_{\beta_j}], \left(\left[\mu_j^{L+}, \mu_j^{R+} \right], \left[\nu_j^{L-}, \nu_j^{R-} \right] \right) \right\} = \min_i E \left\{ [s_{\alpha_{ij}}, s_{\beta_{ij}}], \left(\left[\mu_{ij}^{L+}, \mu_{ij}^{R+} \right], \left[\nu_{ij}^{L-}, \nu_{ij}^{R-} \right] \right) \right\} \tag{21}$$

Step 4. Derive the weighted Euclidean distance values$EDV(A_i, IVBULNIS)$ and Hamming distance values$HDV(A_i, IVBULNIS)$:

$$EDV(A_i, IVBULNIS) = \sum_{j=1}^{n} \omega_j ED(\tilde{r}_{ij}, IVBULNIS_j) = \sum_{j=1}^{n} \omega_j ED\left(\begin{array}{c} \left\{[s_{\alpha_{ij}}, s_{\beta_{ij}}], \left(\left[\mu_{ij}^{L+}, \mu_{ij}^{R+}\right], \left[\nu_{ij}^{L-}, \nu_{ij}^{R-}\right]\right)\right\}, \\ \left\{[s_{\alpha_j}, s_{\beta_j}], \left(\left[\mu_j^{L+}, \mu_j^{R+}\right], \left[\nu_j^{L-}, \nu_j^{R-}\right]\right)\right\} \end{array}\right) \tag{22}$$

$$HDV(A_i, IVBULNIS) = \sum_{j=1}^{n} \omega_j HD(\tilde{r}_{ij}, IVBULNIS_j) = \sum_{j=1}^{n} \omega_j HD\left(\begin{array}{c} \left\{[s_{\alpha_{ij}}, s_{\beta_{ij}}], \left(\left[\mu_{ij}^{L+}, \mu_{ij}^{R+}\right], \left[\nu_{ij}^{L-}, \nu_{ij}^{R-}\right]\right)\right\}, \\ \left\{[s_{\alpha_j}, s_{\beta_j}], \left(\left[\mu_j^{L+}, \mu_j^{R+}\right], \left[\nu_j^{L-}, \nu_j^{R-}\right]\right)\right\} \end{array}\right) \tag{23}$$

Step 5. Derive the interval-valued bipolar uncertain linguistic relative assessment matrix:

$$IVBULRAM = [IVBULRAM_{i\phi}]_{m\times m} \tag{24}$$

$$\begin{array}{c} IVBULRAM_{i\phi} \\ = \left(EDV(A_i, IVBULNIS) - EDV(A_\phi, IVBULNIS)\right) + \\ \left(\begin{array}{c} f\left(EDV(A_i, IVBULNIS) - EDV(A_\phi, IVBULNIS)\right) \times \\ \left(HDV(A_i, IVBULNIS) - HDV(A_\phi, IVBULNIS)\right) \end{array}\right) \end{array} \tag{25}$$

where $\phi = 1, 2, \cdots, m$ and a threshold function is given by Eq.(32):

$$f(x) = \begin{cases} 1 if |x| \geq \tau \\ 0 if |x| < \tau \end{cases}, \tag{26}$$

where τ is the threshold parameter value which is between 0.01 and 0.05. In this paper, $\tau = 0.02$ are always used to compute.

Step 6. Derive the interval-valued bipolar uncertain linguistic assessment score$IVBULAS_i$ by Eq. (33).

$$IVBULAS_i = \sum_{\phi=1}^{m} IVBULRAM_{i\phi}. \tag{27}$$

Step 7. Sort all the alternatives in accordance according to$IVBULAS_i$. The best alternative should have maximum value.

5. Numerical example

Since the entering of the 21-st century, the world economic pattern has gone through some changes. These changes are mainly shown in the narrowing gap between the economic growth rates of developed countries and emerging countries and China remains the dominant guiding force for emerging economies. At the same time, since the adoption of the 2030 agenda for sustainable development in September 2015, the global economy has entered a new stage of development is entering a new stage of development. The "One Belt and One Road" designed by China in 2013 is in line with the 2030 agenda, demonstrating China's determination to promote global cooperation. With the deepening of the implementation process of the strategy of "going global", our country is providing more and more policy support for domestic enterprises. Under such a policy background, more and more Chinese enterprises go abroad to enhance their competitiveness through cross-border mergers and acquisitions and at the same time expand China's influence all over the world. Chinese enterprises have made some achievements through overseas mergers and acquisitions, but generally speaking, their mergers and acquisitions situation is not very optimistic. Although the scale of mergers and acquisitions is growing rapidly, the success rate of mergers and acquisitions is always at a low level. Overseas mergers and acquisitions are inherently risky because they involve two or more national themes and large amounts of capital. Furthermore, Chinese enterprises generally have a weak sense of control and are especially prone to ignore the integration in the later stage of mergers and acquisitions, and like to blindly follow the trend to embark on projects, leading to frequent failures of Chinese enterprises in overseas mergers and acquisitions. The risk assessment of Chinese enterprises' overseas mergers and acquisitions (M&As) could be regarded the corresponding MAGDM issues. To illustrate the method proposed above, we analyze a MAGDM with IVBULNs for risk assessment of Chinese enterprises' overseas mergers and acquisitions (M&As). Considering its own business development, a company wants to choose a Chinese enterprise for overseas mergers and acquisitions. There are five potential Chinese enterprises $A_i (i = 1, 2, 3, 4, 5)$ to assess. The experts have chosen four attributes for evaluation: ①G_1 is external environment risk; ②G_2 is financing risk; ③G_3 is operational integration risk; ④G_4 is value assessment risk. The five possible Chinese enterprises$A_i (i = 1, 2, 3, 4, 5)$ are gauged by using the IVBULNs in accordance with the four attributes (whose subjective weight vector$sw = (0.3, 0.2, 0.4, 0.1)^T$) by three experts (whose weighting vector$\lambda = (0.2, 0.5, 0.3)^T$) . The ratings are presented in the Tables 1–3.

Table 1. The IVBUL decision matrix by DM_1.

	G_1	G_2
A_1	$\{[S_6, S_7], ([0.4,0.5], [-0.4,-0.3])\}$	$\{[S_3, S_4], ([0.4,0.6],[-0.4,-0.2])\}$
A_2	$\{[S_5, S_6], ([0.6,0.7], [-0.3,-0.2])\}$	$\{[S_4, S_5], ([0.6,0.7],[-0.3,-0.2])\}$
A_3	$\{[S_2, S_4], ([0.3,0.6], [-0.4,-0.3])\}$	$\{[S_2, S_3], ([0.5,0.6],[-0.4,-0.3])\}$
A_4	$\{[S_4, S_5], ([0.7,0.8], [-0.2,-0.1])\}$	$\{[S_1, S_2], ([0.6,0.7],[-0.3,-0.1])\}$
A_5	$\{[S_3, S_4], ([0.3,0.4], [-0.3,-0.2])\}$	$\{[S_4, S_6], ([0.3,0.5],[-0.3,-0.1])\}$

	G_3	G_4
A_1	$\{[S_5, S_6], ([0.1,0.3],[-0.6,-0.5])\}$	$\{[S_6, S_7], ([0.3,0.4],[-0.5,-0.3])\}$
A_2	$\{[S_2, S_3], ([0.4,0.7],[-0.2,-0.1])\}$	$\{[S_2, S_3], ([0.5,0.6],[-0.3,-0.1])\}$
A_3	$\{[S_3, S_4], ([0.5,0.6],[-0.3,-0.1])\}$	$\{[S_4, S_6], ([0.4,0.5],[-0.4,-0.2])\}$
A_4	$\{[S_3, S_5], ([0.3,0.4],[-0.2,-0.1])\}$	$\{[S_4, S_5], ([0.3,0.7],[-0.2,-0.1])\}$
A_5	$\{[S_1, S_3], ([0.2,0.5],[-0.5,-0.4])\}$	$\{[S_3, S_4], ([0.3,0.4],[-0.6,-0.5])\}$

Source: The Authors.

Table 2. The IVBUL decision matrix by DM_2.

	G_1	G_2
A_1	$\{[S_5, S_6], ([0.3,0.4],[-0.5,-0.4])\}$	$\{[S_2, S_3], ([0.5,0.6],[-0.3,-0.1])\}$
A_2	$\{[S_4, S_5], ([0.3,0.6],[-0.4,-0.3])\}$	$\{[S_3, S_4], ([0.4,0.7],[-0.2,-0.1])\}$
A_3	$\{[S_1, S_3], ([0.2,0.5],[-0.5,-0.4])\}$	$\{[S_1, S_2], ([0.2,0.3],[-0.6,-0.4])\}$
A_4	$\{[S_3, S_4], ([0.4,0.5],[-0.5,-0.3])\}$	$\{[S_1, S_2], ([0.5,0.8],[-0.2,-0.1])\}$
A_5	$\{[S_2, S_3], ([0.5,0.6],[-0.4,-0.2])\}$	$\{[S_3, S_5], ([0.6,0.7],[-0.2,-0.1])\}$

	G_3	G_4
A_1	$\{[S_4, S_5], ([0.4,0.5],[-0.4,-0.3])\}$	$\{[S_5, S_6], ([0.4,0.6],[-0.4,-0.2])\}$
A_2	$\{[S_1, S_2], ([0.5,0.6],[-0.3,-0.2])\}$	$\{[S_1, S_2], ([0.6,0.7],[-0.3,-0.2])\}$
A_3	$\{[S_2, S_3], ([0.3,0.5],[-0.4,-0.3])\}$	$\{[S_3, S_5], ([0.1,0.3],[-0.6,-0.5])\}$
A_4	$\{[S_2, S_4], ([0.2,0.5],[-0.4,-0.3])\}$	$\{[S_3, S_4], ([0.4,0.7],[-0.2,-0.1])\}$
A_5	$\{[S_1, S_2], ([0.3,0.4],[-0.3,-0.1])\}$	$\{[S_2, S_3], ([0.6,0.7],[-0.3,-0.1])\}$

Source: The Authors.

Table 3. The IVBUL decision matrix by DM_3.

	G_1	G_2
A_1	$\{[S_2, S_4], ([0.2,0.5],[-0.4,-0.3])\}$	$\{[S_3, S_4], ([0.4,0.5],[-0.2,-0.1])\}$
A_2	$\{[S_5, S_6], ([0.2,0.7],[-0.3,-0.2])\}$	$\{[S_4, S_5], ([0.3,0.6],[-0.4,-0.2])\}$
A_3	$\{[S_2, S_4], ([0.1,0.6],[-0.4,-0.3])\}$	$\{[S_2, S_3], ([0.1,0.4],[-0.5,-0.3])\}$
A_4	$\{[S_4, S_5], ([0.3,0.6],[-0.4,-0.2])\}$	$\{[S_1, S_2], ([0.4,0.6],[-0.3,-0.2])\}$
A_5	$\{[S_3, S_4], ([0.4,0.7],[-0.3,-0.1])\}$	$\{[S_4, S_6], ([0.5,0.6],[-0.4,-0.3])\}$

	G_3	G_4
A_1	$\{[S_5, S_6], ([0.3,0.6],[-0.3,-0.2])\}$	$\{[S_2, S_3], ([0.3,0.7],[-0.3,-0.1])\}$
A_2	$\{[S_2, S_3], ([0.4,0.7],[-0.2,-0.1])\}$	$\{[S_2, S_3], ([0.5,0.8],[-0.2,-0.1])\}$
A_3	$\{[S_3, S_4], ([0.2,0.6],[-0.3,-0.2])\}$	$\{[S_4, S_6], ([0.2,0.4],[-0.5,-0.1])\}$
A_4	$\{[S_3, S_5], ([0.1,0.4],[-0.6,-0.3])\}$	$\{[S_4, S_5], ([0.3,0.7],[-0.2,-0.1])\}$
A_5	$\{[S_2, S_3], ([0.2,0.5],[-0.4,-0.3])\}$	$\{[S_3, S_4], ([0.5,0.6],[-0.4,-0.2])\}$

Source: The Authors.

Then, we employ the PL-CODAS method to rank and select best Chinese enterprise.

Step 1. Use the IVBULWA operator to fuse matrix $\tilde{R}^k$ into the overall matrix $\tilde{R}$ (Table 4).

Step 2. Compute the combined weight values for attributes. Firstly, through Eq.(17), the objective weights of attributes is: $ow_1 = 0.2520, ow_2 = 0.2439, ow_3 =$

Table 4. The overall decision matrix by IVBULWA operator.

	G_1	G_2
A_1	$\{[S_{4.3}, S_{5.6}], ([0.2935,0.4523],[-0.4472,-0.3464])\}$	$\{[S_{2.5}, S_{3.5}], ([0.4523,0.5723],[-0.2814,-0.1149])\}$
A_2	$\{[S_{4.5}, S_{5.5}], ([0.3485,0.6536],[-0.3464,-0.2449])\}$	$\{[S_{3.5}, S_{4.5}], ([0.4205,0.6730],[-0.2670,-0.1414])\}$
A_3	$\{[S_{1.5}, S_{3.5}], ([0.1931,0.5528],[-0.4472,-0.3464])\}$	$\{[S_{1.5}, S_{2.5}], ([0.2456,0.4024],[-0.5238,-0.3464])\}$
A_4	$\{[S_{3.5}, S_{4.5}], ([0.4529,0.6107],[-0.3893,-0.2132])\}$	$\{[S_{1.3}, S_{2.3}], ([0.4949,0.7330],[-0.2449,-0.1231])\}$
A_5	$\{[S_{2.5}, S_{3.5}], ([0.4351,0.6021],[-0.3464,-0.1625])\}$	$\{[S_{3.5}, S_{5.5}], ([0.5217,0.6378],[-0.2670,-0.1390])\}$

	G_3	G_4
A_1	$\{[S_{4.5}, S_{5.5}], ([0.3185,0.4998],[-0.3979,-0.2942])\}$	$\{[S_{4.3}, S_{5.3}], ([0.3519,0.6021],[-0.3837,-0.1762])\}$
A_2	$\{[S_{1.5}, S_{2.5}], ([0.4523,0.6536],[-0.2449,-0.1414])\}$	$\{[S_{1.5}, S_{2.5}], ([0.5528,0.7186],[-0.2656,-0.1414])\}$
A_3	$\{[S_{2.5}, S_{3.5}], ([0.3188,0.5528],[-0.3464,-0.2132])\}$	$\{[S_{3.5}, S_{5.5}], ([0.1989,0.3751],[-0.5238,-0.2569])\}$
A_4	$\{[S_{2.5}, S_{4.5}], ([0.1931,0.4523],[-0.3933,-0.2408])\}$	$\{[S_{3.5}, S_{4.5}], ([0.3519,0.7000],[-0.2000,-0.1000])\}$
A_5	$\{[S_{1.3}, S_{2.5}], ([0.2517,0.4523],[-0.3622,-0.1835])\}$	$\{[S_{2.5}, S_{3.5}], ([0.5217,0.6243],[-0.3757,-0.1699])\}$

Source: The Authors.

Table 5. NIS of IVBULNs.

	G_1	G_2
NIS	$\{[S_{1.5}, S_{3.5}], ([0.1931,0.5528],[-0.4472,-0.3464])\}$	$\{[S_{1.3}, S_{2.3}], ([0.4949,0.7330],[-0.2449,-0.1231])\}$

	G_3	G_4
NIS	$\{[S_{1.3}, S_{2.5}], ([0.2517,0.4523],[-0.3622,-0.1835])\}$	$\{[S_{1.5}, S_{2.5}], ([0.5528,0.7186],[-0.2656,-0.1414])\}$

Source: The Authors.

Table 6. $EDV(A_i, IVBULNIS)$**and**$HDV(A_i, IVBULNIS)(i = 1, 2, 3, 4, 5)$.

Alternatives	$EDV(A_i, IVBULNIS)$	$HDV(A_i, IVBULNIS)$
A_1	0.2188	0.1627
A_2	0.1572	0.1247
A_3	0.1174	0.1025
A_4	0.1298	0.1028
A_5	0.0991	0.0758

Source: The Authors.

Table 7. IVBULRAM.

	A_1	A_2	A_3	A_4	A_5
A_1	0.0000	0.0997	0.1617	0.1490	0.2066
A_2	−0.0997	0.0000	0.0620	0.0492	0.1069
A_3	−0.1617	−0.0620	0.0000	−0.0124	0.0182
A_4	−0.1490	−0.0492	0.0124	0.0000	0.0576
A_5	−0.2066	−0.1069	−0.0182	−0.0576	0.0000

Source: The Authors.

$0.2505, ow_4 = 0.2536$; then, suppose that the subjective weights of attributes is:$sw_1 = 0.3, sw_2 = 0.2, sw_3 = 0.4, sw_4 = 0.1$; finally, through Eq.(18), combined weight values for attributes is: $cw_1 = 0.3025, cw_2 = 0.1951$, $cw_3 = 0.4009$, $cw_4 = 0.1015$.

Step 3. Obtain the NIS of IVBULNs (Table 5):

Step 4. Determine the$EDV(A_i, IVBULNIS)$ and$HDV(A_i, IVBULNIS)$ (Table 6):

Step 5. Compute the IVBULRAM (Table 7):

Step 6. Calculate the$IVBULAS_i(i = 1, 2, 3, 4, 5)$ by Eq. (27). The calculating results are derived in Table 8.

Table 8. $IVBULAS_i(i = 1, 2, 3, 4, 5)$.

Alternatives	A_1	A_2	A_3	A_4	A_5
IVBULAS	0.6170	0.1183	−0.2178	−0.1281	−0.3893

Source: The Authors.

Table 9. The overall decision matrix by IVBULWG operator.

	G_1	G_2
A_1	$\{[S_{3.9393}, S_{5.4792}]$, ([0.2814,0.4472],[−0.4523,−0.3519])$\}$	$\{[S_{2.4495}, S_{3.4641}]$, ([0.4472,0.5681],[−0.2935,−0.1210])$\}$
A_2	$\{[S_{4.4721}, S_{5.4772}]$, ([0.3051,0.6481],[−0.3519,−0.2517])$\}$	$\{[S_{3.4641}, S_{4.4721}]$, ([0.3979,0.6684],[−0.2855,−0.1515])$\}$
A_3	$\{[S_{1.4142}, S_{3.4641}]$, ([0.1762,0.5477],[−0.4523,−0.3519])$\}$	$\{[S_{1.4142}, S_{2.4495}]$, ([0.1951,0.3757],[−0.5362,−0.3519])$\}$
A_4	$\{[S_{3.4641}, S_{4.4721}]$, ([0.4104,0.5802],[−0.4198,−0.2338])$\}$	$\{[S_{1.2311}, S_{2.2587}]$, ([0.4850,0.7145],[−0.2517,−0.1312])$\}$
A_5	$\{[S_{2.4495}, S_{3.4641}]$, ([0.4222,0.5795,[−0.3519,−0.1712])$\}$	$\{[S_{3.4641}, S_{5.4772}]$, ([0.4945,0.6249],[−0.2855,−0.1654])$\}$

	G_3	G_4
A_1	$\{[S_{4.4721}, S_{5.4772}]$, ([0.2781,0.4768],[−0.4205,−0.3188])$\}$	$\{[S_{3.9393}, S_{5.0261}]$, ([0.3464,0.5795],[−0.3941,−0.1931])$\}$
A_2	$\{[S_{1.4142}, S_{2.4495}]$, ([0.4472,0.6481],[−0.2517,−0.1515])$\}$	$\{[S_{1.4142}, S_{2.4495}]$, ([0.5477,0.7065],[−0.2714,−0.1515])$\}$
A_3	$\{[S_{2.4495}, S_{3.4641}]$, ([0.2942,0.5477],[−0.3519,−0.2338])$\}$	$\{[S_{3.4641}, S_{5.4772}]$, ([0.1625,0.3622],[−0.5362,−0.3448])$\}$
A_4	$\{[S_{2.4495}, S_{4.4721}]$, ([0.1762,0.4472],[−0.4373,−0.2639])$\}$	$\{[S_{3.4641}, S_{4.4721}]$, ([0.3464,0.7000],[−0.2000,−0.1000])$\}$
A_5	$\{[S_{1.2311}, S_{2.4495}]$, ([0.2449,0.4472],[−0.3751,−0.2304])$\}$	$\{[S_{2.4495}, S_{3.4641}]$, ([0.4945,0.5976],[−0.4024,−0.2276])$\}$

Source: The Authors.

Step 7. According to$IVBULAS_i(i = 1, 2, 3, 4, 5)$, we can sort all the Chinese enterprises. Obviously, the order is$A_1 > A_2 > A_4 > A_3 > A_5$. That's to say,$A_1$ is the optimal alternative.

If the IVBULWG operator is applied instead, the problem can be solved in a similar way.

Step 1. Use the IVBULWG operator to fuse matrix$\tilde{R}^k$ into the overall matrix$\tilde{R}$ (Table 9).
Step 2. Compute the combined weight values for attributes. Firstly, through Eq.(17), the objective weights of attributes is: $ow_1 = 0.2522, ow_2 = 0.2433, ow_3 = 0.2508, ow_4 = 0.2537$; then, suppose that the subjective weights of attributes is:$sw_1 = 0.3, sw_2 = 0.2, sw_3 = 0.4, sw_4 = 0.1$; finally, through Eq.(18), combined weight values for attributes is:$cw_1 = 0.3027, cw_2 = 0.1946, cw_3 = 0.4012, cw_4 = 0.1015$.
Step 3. Obtain the NIS of IVBULNs (Table 10):
Step 4. Determine the$EDV(A_i, IVBULNIS)$ and$HDV(A_i, IVBULNIS)$ (Table 11):
Step 5. Compute the IVBULRAM (Table 12):
Step 6. Calculate the$IVBULAS_i(i = 1, 2, 3, 4, 5)$ by Eq.(27). The calculating results are derived in Table 13.
Step 7. According to$IVBULAS_i(i = 1, 2, 3, 4, 5)$, we can sort all the Chinese enterprises. Obviously, the order is$A_1 > A_2 > A_4 > A_3 > A_5$. That's to say,$A_1$ is the optimal alternative.

From the above analysis, it is easily seen that although the overall rating values of the alternatives are completely same by using two operators, respectively, and, the most desirable Chinese enterprise isA_1.

Lu et al. (2017) defined the concept of bipolar 2-tuple linguistic sets. Wei et al. (2018) proposed some arithmetic and geometric aggregation operators with interval-

Table 10. NIS of IVBULNs.

	G_1	G_2
NIS	$\{[S_{1.4142}, S_{3.4641}], ([0.1762,0.5477],[-0.4523,-0.3519])\}$	$\{[S_{1.2311}, S_{2.2587}], ([0.4850,0.7145],[-0.2517,-0.1312])\}$
	G_3	G_4
NIS	$\{[S_{1.2311}, S_{2.4495}], ([0.2449,0.4472],[-0.3751,-0.2304])\}$	$\{[S_{1.4142}, S_{2.4495}], ([0.5477,0.7065],[-0.2714,-0.1515])\}$

Source: The Authors.

Table 11. $EDV(A_i, IVBULNIS)$**and**$HDV(A_i, IVBULNIS)(i = 1,2,3,4,5)$.

Alternatives	$EDV(A_i, IVBULNIS)$	$HDV(A_i, IVBULNIS)$
A_1	0.2123	0.1557
A_2	0.1593	0.1271
A_3	0.1205	0.1040
A_4	0.1289	0.1005
A_5	0.1008	0.0775

Source: The Authors.

Table 12. IVBULRAM.

	A_1	A_2	A_3	A_4	A_5
A_1	0.0000	0.0816	0.1435	0.1386	0.1897
A_2	−0.0816	0.0000	0.0619	0.0570	0.1081
A_3	−0.1435	−0.0619	0.0000	−0.0084	0.0197
A_4	−0.1386	−0.0570	0.0084	0.0000	0.0510
A_5	−0.1897	−0.1081	−0.0197	−0.0510	0.0000

Source: The Authors.

Table 13. $IVBULAS_i(i = 1,2,3,4,5)$.

Alternatives	A_1	A_2	A_3	A_4	A_5
$IVBULAS$	0.5534	0.1454	−0.1941	−0.1362	−0.3684

Source: The Authors.

valued bipolar 2-tuple linguistic sets (IVB2TLSs) and analyzed the characteristics of these operators and presented methods that apply these operators to solve the MADM problems with IVB2TLNs for the risk evaluation of enterprise human capital investment. But all these methods can't deal with the MADM with IVBULNs. Thus, the proposed methods in this paper are more effective than the existing methods. The existing methods are some special cases of the proposed methods in this paper.

6. Conclusion

The CODAS is a quite useful and efficient ranking method to tackle the complicated MAGDM issues, which a number of attributes are employed to assess a limited lot of alternatives by some experts. In this paper, we expand the CODAS method to the MAGDM with IVBULNs on the basis of two kinds of distances measures and proposed aggregating operators. Compared to existing methods, the prime advantages of the proposed method are that not only tackle the IVBULNs but also has strong ability to differentiate the optimal alternative. Then, the definition, comparative method and distance of IVBULNs are briefly reviewed. Motivated by the traditional aggregation operators, we develop two aggregation operators for aggregating IVBULNs and further analyze their prominent properties. Moreover, the CODAS method is extended to tackle MAGDM issues with IVBULNs which can fully consider Euclidean distances and Hamming distances. Finally, a practical case for risk assessment of Chinese enterprises' overseas mergers and acquisitions is designed to show the developed approach. The main contributions of this study is three folds: (1) the classical CODAS method is expanded to IVBULNs; (2) the CODAS (PL-CODAS) method for MAGDM issues with IVBULNs is designed, which supplies DMs with effective way for tackling MAGDM; (3) a case study about risk assessment of Chinese enterprises' overseas mergers and acquisitions is given to demonstrate the application, practicality and effectiveness of the proposed MAGDM issues.

For our future studies, we shall continue to extend our proposed models to other fuzzy environments and other applicable domains.

Disclouser statement

The disclosure statement has been inserted. Please correct if this is inaccurate.

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