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Probabilistic double hierarchy linguistic alternative queuing method for real economy development evaluation under the perspective of economic financialization

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ABSTRACT

With the development of science and technology, the new road of scientific economic and financial development has played a decisive role in supporting the financial undertaking. To accelerate the economic development, it is very important to increase the guiding role of financial undertaking in the real economy. Therefore, it is necessary to promote the development of the real economy under the perspective of economic financialization based on some actions. To judge the implementation effect of these actions, this paper develops a multiple criteria decision-making (MCDM) method to evaluate them. First, the decision-making matrices are established with the probabilistic double hierarchy linguistic term set in which the probabilities are added to all double hierarchy linguistic terms. Additionally, a weight-determining method is developed to obtain the weight vector of criteria, and we develop a MCDM method named the probabilistic double hierarchy linguistic alternative queuing method (PDHL-AQM), where the decision-making result is intuitive by a directed graph or a 0–1 precedence relationship matrix. Furthermore, we apply the PDHL-AQM to solve a practical MCDM problem involving the real economy development evaluation under the perspective of economic financialization. Finally, some comparative analyses are made to show the advantages and reasonableness of the PDHL-AQM.

1. Introduction

In recent years, China's economy is moving higher step by step. With the development of science and technology, the new scientific economic and financial pioneering path has played a decisive supporting role for the financial cause. To accelerate economic development, it is necessary to increase the guiding role of financial cause in

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the real economy (Li, 2020). To promote the development of the real economy under the perspective of economic financialization, some effective actions have been proposed: first, it is necessary to ensure that the financial development is a common progress with the real economy. The property of the financial industry is relatively special, and it does not have the property of value appreciation, because its essence is to control credit. Additionally, the financial industry needs to put forward reasonable management methods so that the financial market can play an effective role in promoting the real economy. Furthermore, scientific and reasonable allocation of financial resources can develop the real economy rapidly, but the development of the real economy certainly cannot come true without the innovative economic policies of the financial industry (Geng & Gao, 2020; Li, 2020).

To judge the implementation effect of these actions, it is a common practice to use some multiple criteria decision-making (MCDM) methods to evaluate them based on the evaluation information provided by some invited experts. In this process, two issues arise: one is how to express the evaluation information exactly, and the other one is the selection of the reasonable decision-making methods.

In order to solve the first issue, the fuzzy linguistic approach, defined by Zadeh (2012), is more appropriate considering that linguistic information can be obtained easily and used to reflect the real thoughts of people more correctly when experts propose their evaluation information. In recent years, based on the fuzzy linguistic approach, many linguistic representation models have been established including hesitant fuzzy linguistic term set (HFLTS) (Rodríguez et al., 2012), 2-tuple linguistic model (Herrera & Martínez, 2000; Wei et al., 2020), virtual linguistic term model (Xu, 2012; Xu & Wang, 2017), and type-2 fuzzy sets (Juang & Chen, 2013), etc. However, sometimes the linguistic representation models mentioned above cannot describe some complex linguistic terms accurately and comprehensively considering that people's cognition process and the decision-making information becomes more and more complex (Gou et al., 2020a). Therefore, based on the 2-tuple linguistic structure, Gou et al. (2017a) defined the concept of double hierarchy linguistic term set (DHLTS), which consists of two simple linguistic term sets (LTSs), the first hierarchy linguistic term set (LTS) is the main linguistic hierarchy and the second hierarchy LTS is the linguistic feature or detailed supplementary of each linguistic term in the first hierarchy LTS (Gou et al., 2017a). In recent years, many extensions of DHLTS have been developed such as double hierarchy hesitant fuzzy linguistic term set (Gou et al., 2017a), double hierarchy linguistic preference relation (Gou et al., 2020a), double hierarchy hesitant fuzzy linguistic preference relation (Gou et al., 2019), self-confident double hierarchy linguistic preference relation (Gou, Xu, Wang, et al., 2020), and linguistic preference orderings (Gou, Xu, & Zhou, 2020), etc.

Even though DHLTS can be used to express the complex linguistic information more correctly and completely, there still exist some new scenarios that need to be improved. First, it is unreasonable to give the same importance degree to each double hierarchy linguistic term (DHLT, the basic element of DHLTS) when aggregating the DHLTs provided by different experts. For example, when evaluating the price of a car, three experts with different importance degrees may use 'high', 'a little high', 'only a little low' to express their evaluations respectively, and then {high, a little

high, only a little low} can be used to represent the aggregated result if we use the existing linguistic representation models. However, the importance degrees of different experts have not been indicated. Secondly, one expert may consider himself to be '30% sure that the speed of the car is a little fast, 50% sure it is just right fast, and 20% sure it is much fast'. However, the representation {a little fast, just right fast, much fast} also neglects the importance degrees of these linguistic terms. To overcome these problems, Gou et al. (2020b) defined the concept of probabilistic double hierarchy linguistic term set (PDHLTS), and its basic element is called probabilistic double hierarchy linguistic element (PDHLE), which adds probabilities to all DHLTs to represent the importance degrees or belief degrees of the DHLTs for individual assessment, or the probabilistic distribution of collective DHLTs of all experts in group decision-making processes.

Additionally, for the second issue, the selection of the MCDM method is very important when solving practical MCDM problems under probabilistic double hierarchy linguistic environment. Generally, two kinds of MCDM methods are very common: The traditional MCDM solution methods (Fu & Liao, 2019; Gou et al., 2017a; 2020b; Liu et al., 2020; Wei & Gao, 2020; Wnuczak & Osiichuk, 2020) such as the TOPSIS method (Fu & Liao, 2019), the MULTIMOORA method (Gou et al., 2017a), the VIKOR method (Gou et al., 2020b), the TODIM method (Tian et al., 2020), etc.; and the outranking-based decision-making methods (Liu et al., 2018; 2019; Wang et al., 2020) such as the PROMETHEE (Liu et al., 2019), the DHHFL-LINMAP (Liu et al., 2018), the DHHFL-ORESTE (Wang et al., 2020), etc. Considering that these two kinds of MCDM methods can only obtain the decision-making results by some calculations, it is difficult to describe the decision-making results more intuitively. Therefore, this paper developed a new MCDM method, named by probabilistic double hierarchy linguistic alternatives queuing method (PDHL-AQM), which can be used to obtain the decision-making result intuitively by a directed graph or a 0–1 precedence relationship matrix (Gou et al., 2016; 2017b).

Based on the above motivations, the innovations and contributions of this paper are listed as follows:

1. Give a weight-determining method to obtain the weight vector of criteria, which is the important element in the process of decision-making using the PDHL-AQM.
2. Develop the PDHL-AQM to deal with MCDM problem, the decision-making result is intuitive by drawing the directed graph or establishing the 0–1 precedence relationship matrix.
3. Apply the PDHL-AQM method to solve a practical MCDM problem involving the real economy development evaluation under the perspective of economic financialization. Additionally, some comparative analyses are made to show the advantages and reasonableness of the PDHL-AQM.

The rest of this paper is organised as follows: Section 2 reviews some basic concepts related to PDHLTS and the precedence relationship between any two alternatives in MCDM problems. Section 3 proposes the weight-determining method and

the PDHL-AQM. Section 4 applies the PDHL-AQM to solve a practical MCDM problem involving the real economy development evaluation under the perspective of economic financialization, and some comparative analyses are made between the PDHL-AQM and some existing methods. Some conclusions are summarised in Section 5.

2. Preliminaries

In this section, we mainly introduce some concepts related to the PDHLTS, and the precedence relationship between any two alternatives in MCDM problems.

2.1. Probabilistic double hierarchy linguistic term set

Based on the 2-tuple linguistic structure (Herrera & Martínez, 2000), Gou et al. (2017a) proposed the concept of DHLTS. Let $S = \{s_t | t = -\tau, ..., -1, 0, 1, ..., \tau\}$ be the first hierarchy LTS, $O^t = \{o_k^t | k = -\varsigma, ..., -1, 0, 1, ..., \varsigma\}$ be the second hierarchy LTS of the linguistic term s_t in S. Then, the mathematical expression of DHLTS is

$$S_O = \left\{ s_{t<o_k^t>} \middle| t = -\tau, \ldots, -1, 0, 1, \ldots, \tau; k = -\varsigma, \ldots, -1, 0, 1, \ldots, \varsigma \right\} \quad (1)$$

where we call $s_{t<o_k^t>}$ double hierarchy linguistic term (DHLT). For convenience, the DHLTS can be expressed by a unified form $S_O = \{s_{t<o_k>} | t = -\tau, ..., -1, 0, 1, ..., \tau; k = -\varsigma, ..., -1, 0, 1, ..., \varsigma\}$. For more explanation of DHLTS, please refer to (Gou et al., 2017a; Gou & Xu, 2020).

To understand the concept of DHLTS better, let $S = \{s_{-3} = none, s_{-2} = verylow, s_{-3} = low, s_{-3} = medium, s_{-3} = high, s_{-3} = veryhigh, s_{-3} = perfect\}$ be the first hierarchy LTS, and $O^1 = \{o_{-2} = farfrom, o_{-1} = alittle, o_0 = justright, o_1 = much, o_2 = verymuch\}$ be the second hierarchy LTS of s_1. Then, a Figure 1 can be drawn:
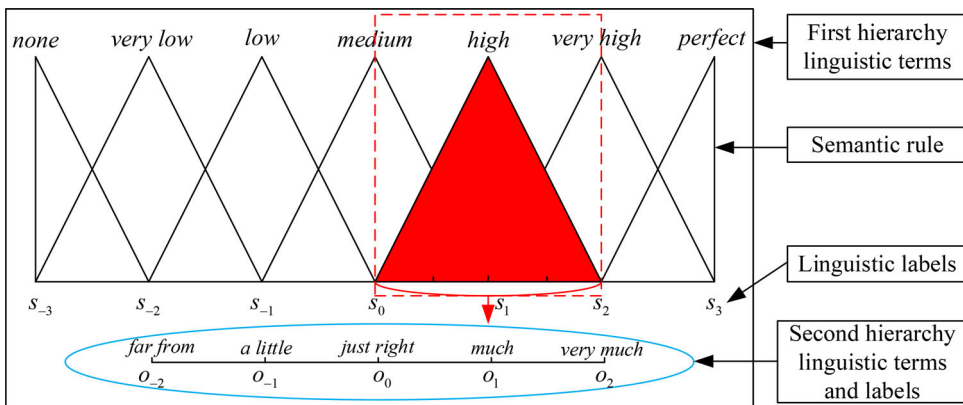

Figure 1. The second hierarchy LTS O^1 of the linguistic term s_1 in S. Source: The authors.

To make the calculations between two DHLTs, Gou et al. (2017a) developed two equivalent transformation functions f and f^{-1} based on the virtual DHLTS $\bar{S}_O = \{s_{t<o_k>}|t \in [-\tau, \tau]; k \in [-\varsigma, \varsigma]\}$:

Definition 1 (Gou et al., 2017a). Let $\bar{S}_O = \{s_{t<o_k>}|t \in [-\tau, \tau]; k \in [-\varsigma, \varsigma]\}$ be a VDHLTS. Then a numerical scale γ and the subscript (ϕ, φ) of any a DHLT $s_{\phi<o_\varphi>}$ that expresses the equivalent information to the numerical scale γ are transformed from one to another based on two functions f and f^{-1} :

$$f : [-\tau, \tau] \times [-\varsigma, \varsigma] \to [0, 1], f(s_{t<o_k>}) = \frac{t + (\tau + k)\varsigma}{2\varsigma\tau} = \gamma \tag{2}$$

$$f^{-1} : [0, 1] \to [-\tau, \tau] \times [-\varsigma, \varsigma],$$
$$f^{-1}(\gamma) = [2\tau\gamma - \tau]<o_{\varsigma(2\tau\gamma - \tau - [2\tau\gamma - \tau])}> = [2\tau\gamma - \tau] + 1<o_{\varsigma((2\tau\gamma - \tau - [2\tau\gamma - \tau]) - 1)}> \tag{3}$$

In the process of calculations, firstly, we can use f to transform DHLTs to the corresponding numerical scales. Then, the calculation between DHLTs can be made by making operations among numerical scales. Additionally, the calculation result can be transformed into the form of DHLT based on the anti-function f^{-1}. For example, let $S_O = \{s_{t<o_k>}|t = -4, ..., 4; k = -4, ..., 4\}$ be a DHLTS, $s_{2<o_1>}$ be a DHLT, $\frac{9}{16}$ be a real number. Then, we have $f(s_{2<o_1>}) = \frac{25}{32}$ and $f^{-1}\left(\frac{9}{16}\right) = s_{1<o_{-2}>}$.

In real decision-making processes, it is obvious that some experts may prefer to provide their own preferences for DHLTs. For example, one expert may consider himself to be '30% sure that the speed is a little fast, 50% sure it is just right high, and 20% sure it is much fast'. Additionally, the probabilities information is also common when we aggregate the preferences of some experts. For instance, in a group decision-making problem, 30% of a group of experts have the same opinion that the speed is 'a little fast', 50% of them think that it is 'just right fast', and 20% of them may access it is '*much fast*'. To deal with these kinds of evaluation information, Gou et al. (2020b) defined the concept of PDHLTS by combining the DHLTS and the probabilities information, and it can be shown as follows:

Definition 2 (Gou et al., 2020b). Let X be a fixed set, and S_O be a DHLTS. A PDHLTS on S_O can be defined by a mathematical form:

$$Z(p) = \{x_i|z(p)(x_i)|x_i \in X\} \tag{4}$$

where $z(p)(x_i)$ is a set of some values in S_O with probability information, denoting the possible membership degrees of the element $x_i \in X$ to the set $Z(p)$ as:

$$z(p)(x_i) = \{z^{(\sigma)}(p^{(\sigma)})|z^{(\sigma)} \in S_O, p^{(\sigma)} \geq 0, \sigma = 1, 2, ..., L, \sum_{\sigma=1}^{L} p^{(\sigma)} \leq 1\} \tag{5}$$

where $z^{(\sigma)}$ is the DHLT associated with the probability $p^{(\sigma)}$. We call $z^{(\sigma)}(p^{(\sigma)})$ a probabilistic double hierarchy linguistic term (PDHLT), and call $z(p)$ a PDHLE. L is the number of all PDHLTs in $z(p)$.

Then, Gou et al. (2020b) defined the score function and the variance of a PDHLE, and developed a comparison method between PDHLEs.

Definition 3 (Gou et al., 2020b). Let $z(p) = \{z^{(\sigma)}(p^{(\sigma)})|z^{(\sigma)} \in S_O, \sigma = 1, 2, ..., z(p)\}$ be a PDHLE. Then the score function and the variance of $z(p)$ are obtained respectively by

$$E(z(p)) = \sum_{\sigma=1}^{z(p)} f(z^{(\sigma)})p^{(\sigma)} / \sum_{\sigma=1}^{z(p)} p^{(\sigma)} \tag{6}$$

$$v(z(p)) = \sum_{\sigma=1}^{z(p)} ((f(z^{(\sigma)}) - E(z(p)))p^{(\sigma)})^2 / \sum_{\sigma=1}^{z(p)} p^{(\sigma)} \tag{7}$$

Let $z_1(p)$ and $z_2(p)$ be two PDHLEs, Then:

1. If $E(z_1(p)) > E(z_2(p))$, then $z_1(p) > z_2(p)$.
2. If $E(z_1(p)) = E(z_2(p))$, then
3. If $v(z_1(p)) > v(z_2(p))$, then $z_1(p) < z_2(p)$;
4. If $v(z_1(p)) = v(z_2(p))$, then $z_1(p) = z_2(p)$.

In this paper, the PDHLTs in a PDHLE are ranked in increasing order. Additionally, for a PDHLE $z(p)$, there exist three situations:

1. If $\sum_{\sigma=1}^{L} p^{(\sigma)} = 1$, then the probabilities of all possible DHLTs is complete;
2. If $\sum_{\sigma=1}^{L} p^{(\sigma)} < 1$, then the probabilities of all possible DHLTs is incomplete;
3. If $\sum_{\sigma=1}^{L} p^{(\sigma)} = 0$, then there is not any assessment provided by experts.

In fact, the probability information of a PDHLE $z(p)$ is usually incomplete, i.e. $\sum_{\sigma=1}^{L} p^{(\sigma)} < 1$. Then, it can be normalised by $\hat{z}(p) = \{\hat{z}^{(\sigma)}(\hat{p}^{(\sigma)})| \sum_{\sigma=1}^{L} \hat{p}^{(\sigma)} = 1\}$, where $\hat{p}^{(\sigma)} = p^{(\sigma)} / \sum_{\sigma=1}^{L} p^{(\sigma)}$ for all $\sigma = 1, 2, ..., z(p)$.

2.2. Precedence relationship between alternatives

In MCDM problems, it is very intuitive to show the decision-making result based on the directed graph and the 0–1 precedence relationship matrix (Gou et al., 2016; 2017b).

(1) The directed graph

In a directed graph drawn in MCDM, some small circles can be used to express the alternatives, called by alternative nodes. Then, the directed arc, drawn from the better alternative to the worse one, indicates the precedence relationship between two alternatives. Additionally, let A_i and A_r be two alternatives, if A_i is superior to A_r, denoted as $A_i \succ A_r$, then the directed arc between A_i and A_r can be drawn from A_i to A_r; if there is no different between A_i and A_r, denoted as $A_i \sim A_r$, then two directed arcs should be drawn: one is from A_i to A_r and the other one is from A_i to A_r. Specially, the directed arc can be omitted if two alternatives cannot be compared.
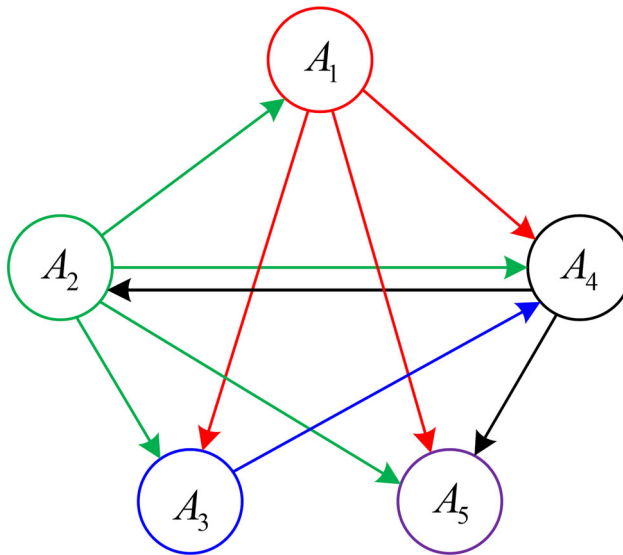
Figure 2. The directed graph of alternatives. Source: The authors.

For example, let $A = \{A_1, A_2, \ldots, A_5\}$ be the set of alternatives. A directed graph can be drawn as follows:

In Figure 2, some directed arcs can be explained: A_2 is superior to A_1, A_3, and A_5; there is no difference between A_2 and A_4; A_3 and A_5 cannot be compared.

(2) The 0–1 precedence relationship matrix

The directed graph can be transformed into the corresponding 0–1 precedence relationship matrix. Let $Q = (q_{ir})_{m \times m}$ be a 0–1 precedence relationship matrix, if $A_i \succ A_r$, then $q_{ir} = 1$ and $q_{ri} = 0$; if $A_i \sim A_r$, then $q_{ir} = q_{ri} = 1$; if both of them cannot be compared, then $q_{ir} = q_{ri} = 0$.

Then, the Figure 2 can be transformed into the 0–1 precedence relationship matrix Q below:

$$Q = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \end{array} \begin{array}{ccccc} A_1 & A_2 & A_3 & A_4 & A_5 \\ \begin{pmatrix} 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

In a directed graph, let $\Im_i$ be the number of directed arcs from A_i to other alternatives, and $\Re_i$ be the number of directed arcs which point to A_i. Then we obtain that $RV(A_i) = \Im_i - \Re_i$ is the ranking value of A_i. The larger the value of $RV(A_i)$, the between the alternative A_i.

Similarly, in the corresponding 0–1 precedence relationship matrix, the $\Im_i$ means the number of 1 and the $\Re_i$ means the number of 0 in each row of Q.

Finally, based on $RV(A_i)(i = 1, 2, \ldots, m)$, the rank of alternatives is obtained.

3. The PDHL-AQM for MCDM

In this section, we first propose the weight-determining method to obtain the weight vector of criteria. Then, the PDHL-AQM for MCDM is established.

First, a MCDM problem can be described as: Let $A = \{A_1, A_2, \ldots, A_m\}$ be a set of alternatives, $C = \{C_1, C_2, \ldots, C_n\}$ be a set of criteria, $DM^k = (dm_{ij}^k)_{m \times n}$ $(k = 1, 2, \ldots, K)$ be the decision-making matrices provided by experts $E = \{e^1, e^2, \ldots, e^K\}$ where dm_{ij}^k is a PDHLE, and $w = (w_1, w_2, \ldots, w_n)^T$ be the weight vector of experts.

3.1. Weight-determining methods

Based on the given decision-making matrices $DM^k = (dm_{ij}^k)_{m \times} n(k = 1, 2, \ldots, K)$ of all experts, first, it is necessary to aggregate them into an overall decision-making matrix $DM = (dm_{ij})_{m \times} n$ based on the following method:

We only summarise the linguistic terms located in the same location into the same set, and give the corresponding probability to each DHLT according to their frequencies.

For example, let

$$DM^1 = \begin{pmatrix} \{s_{0<o_2>}(1)\} & \{s_{-1<o_1>}(0.5), s_{0<o_3>}(0.5)\} & \{s_{0<o_2>}(1)\} & \{s_{0<o_2>}(1)\} \\ \{s_{0<o_2>}(1)\} & \{s_{0<o_2>}(1)\} & \{s_{1<o_2>}(0.4), s_{2<o_2>}(0.6)\} & \{s_{0<o_2>}(1)\} \\ \{s_{0<o_2>}(1)\} & \{s_{-2<o_1>}(0.4), s_{-2<o_2>}(0.6)\} & \{s_{0<o_2>}(1)\} & \{s_{0<o_2>}(1)\} \end{pmatrix}$$

$$DM^2 = \begin{pmatrix} \{s_{0<o_2>}(0.3), s_{1<o_1>}(0.7)\} & \{s_{-1<o_1>}(1)\} & \{s_{0<o_2>}(1)\} & \{s_{0<o_2>}(1)\} \\ \{s_{0<o_2>}(1)\} & \{s_{-1<o_1>}(0.2), s_{0<o_2>}(0.8)\} & \{s_{2<o_2>}(1)\} & \{s_{-1<o_3>}(0.6), s_{0<o_2>}(0.4)\} \\ \{s_{0<o_2>}(1)\} & \{s_{-2<o_2>}(1)\} & \{s_{0<o_2>}(0.4), s_{1<o_1>}(0.6)\} & \{s_{0<o_2>}(1)\} \end{pmatrix} \quad (1)$$

be two decision-making matrices given by experts e^1 and e^2, respectively, and the weight vector of them is $w = (0.4, 0.6)^T$. Then the overall decision-making matrix is obtained:

$$DM = \begin{pmatrix} \{s_{0<o_2>}(0.58), s_{1<o_1>}(0.42)\} & \{s_{-1<o_1>}(0.8), s_{0<o_3>}(0.2)\} & \{s_{0<o_2>}(1)\} & \{s_{0<o_2>}(1)\} \\ \{s_{0<o_2>}(1)\} & \{s_{-1<o_1>}(0.12), s_{0<o_2>}(0.88)\} & \{s_{1<o_2>}(0.16), s_{2<o_2>}(0.84)\} & \{s_{-1<o_3>}(0.36), s_{0<o_2>}(0.64)\} \\ \{s_{0<o_2>}(1)\} & \{s_{-2<o_1>}(0.16), s_{-2<o_2>}(0.84)\} & \{s_{0<o_2>}(0.64), s_{1<o_1>}(0.36)\} & \{s_{0<o_2>}(1)\} \end{pmatrix}$$

Then, an information entropy-based weight-determining method is developed:

Algorithm 1. The information entropy-based weight-determining method

Step 1. Calculate the expectation value of each element in DM based on Eq. (6), denoted by $E(dm_{ij})$. Then an adjusted decision-making matrix $E(DM) = (e(dm_{ij}))_{m \times n}$ is established.

Step 2. Obtain the normalised decision-making matrix $DM^N = (dm_{ij}^N)_{m \times n}$ by

$$P^a = \left(p_{ij}^a \right)_{m \times m} \tag{8}$$

Step 3. Calculate the information entropy IE_j of the $j-th$ criterion:

$$IE_j = \frac{2}{m(m-1)} \sum\nolimits_{i<r,\, i \neq r}^{m} |dm_{ij}^N - dm_{rj}^N| \tag{9}$$

Step 4. The larger the information entropy IE_j is, the greater the difference among alternatives will be. Therefore, the role of this criterion in alternative's comparisons will be larger. Then, the weight of each criterion can be obtained:

$$\omega_j = \frac{IE_j}{\sum_{j=1}^{n} IE_j} \tag{10}$$

3.2. The PDHL-AQM

Gou et al. (2016; 2017b) developed the original AQMs with hybrid fuzzy and ranking information first, and the hesitant fuzzy linguistic entropy and cross-entropy measures, respectively. The classical AQM mainly uses the directed graph or the 0–1 precedence relationship matrix to make decision. Therefore, A PDHL-AQM can be developed to deal with MCDM problems under probabilistic double hierarchy linguistic environment based on the directed graph or the 0–1 precedence relationship matrix.

Algorithm 2. The PDHL-AQM for MCDM

Input: The set of alternatives $A = \{A_1, A_2, \ldots, A_m\}$, the set of criteria $C = \{C_1, C_2, \ldots, C_n\}$, the decision-making matrices $DM^k = (dm_{ij}^k)_{m \times n}(k = 1, 2, \ldots, K)$ of experts $E = \{e^1, e^2, \ldots, e^K\}$, and the weight vector of experts $w = (w_1, w_2, \ldots, w_n)^T$.

Output: the rank of alternatives.

Step 1. Determine the weight vector of criteria $\omega = (\omega_1, \omega_2, \ldots, \omega_n)^T$ of criteria via Algorithm 1.

Step 2. Based on Definition 3, the pairwise comparison between any two alternatives with respect to each criterion is made, and then we can establish the directed graph or the 0–1 precedence relationship matrix with respect to each criterion. Specially, $(A_i \succ A_r)_j$ denotes that the alternative A_i is superior to A_r; $(A_i \prec A_r)_j$ denotes that the alternative A_i is inferior to A_r; and $(A_i \sim A_r)_j$ denotes that there is not any difference between A_i and A_r.

Step 3. Sum all weights of criteria with respect to $(A_i \succ A_r)_j$, we can obtain the overall pros weights $\omega(A_i \succ A_r) = \sum_{j \in (A_i \succ A_r)_j} \omega_j$ between any two alternatives A_i and A_r. Similarly, the overall cons weights $\omega(A_i \prec A_r)$, and the overall indifference weights $\omega(A_i \sim A_r)$ can be obtained.

Step 4. Obtain the overall indicated value with respect to the alternative pair (A_i, A_r) :

$$OI(A_i, A_r) = \frac{\omega(A_i \succ A_r) + \xi\omega(A_i \sim A_r)}{\omega(A_i \prec A_r) + \xi\omega(A_i \sim A_r)} \tag{11}$$

where the parameter $\xi \in [0, 1]$ indicates the important degree of $(A_i \sim A_r)$, and it can be given by decision makers directly.

Step 5. Obtain the relationship between any two alternatives:

$$\begin{cases} A_i \succ A_r, & \theta \leq OI(A_i, A_r) \\ A_i \sim A_r, & 1/\theta < OI(A_i, A_r) < \theta \\ A_i \prec A_r, & 0 < OI(A_i, A_r) < 1/\theta \end{cases} \tag{12}$$

where the parameter $\theta > 1$ is the given threshold value. Then, the ultima directed graph or the 0–1 precedence relationship matrix can be established.

Step 6. Calculate the synthetical value of each alternative A_i based on

$$RV(A_i) = \mathfrak{I}_i - \mathfrak{R}_i \tag{13}$$

where $\mathfrak{I}_i$ and $\mathfrak{R}_i$ are the number of 1 and 0 in the ultima 0–1 precedence relationship matrix, respectively.

Step 7. Rank all alternatives by ranking the synthetical values $RV(A_i)(i = 1, 2, \ldots, m)$ in decreasing order.

Step 8. End.

Then, a flow chart is drawn to show the proposed PDHL-AQM (Figure 3):

4. The application of the PDHL-AQM in real economy development evaluation

In this section, we apply the proposed PDHL-AQM to solve a practical MCDM problem involving the real economy development evaluation under the perspective of economic financialization. Additionally, some comparative analyses are made to show the advantages and reasonableness of the PDHL-AQM.

4.1. Background description

In recent years, China's financial development is gradually changing, and is constantly developing and making progress. Meanwhile, the rapid development of financial industry has greatly promoted the economic development of China. However, with the increasing growth of the finance industry, a network-type economic market in which the financial economy is separated from the real economy correspondingly
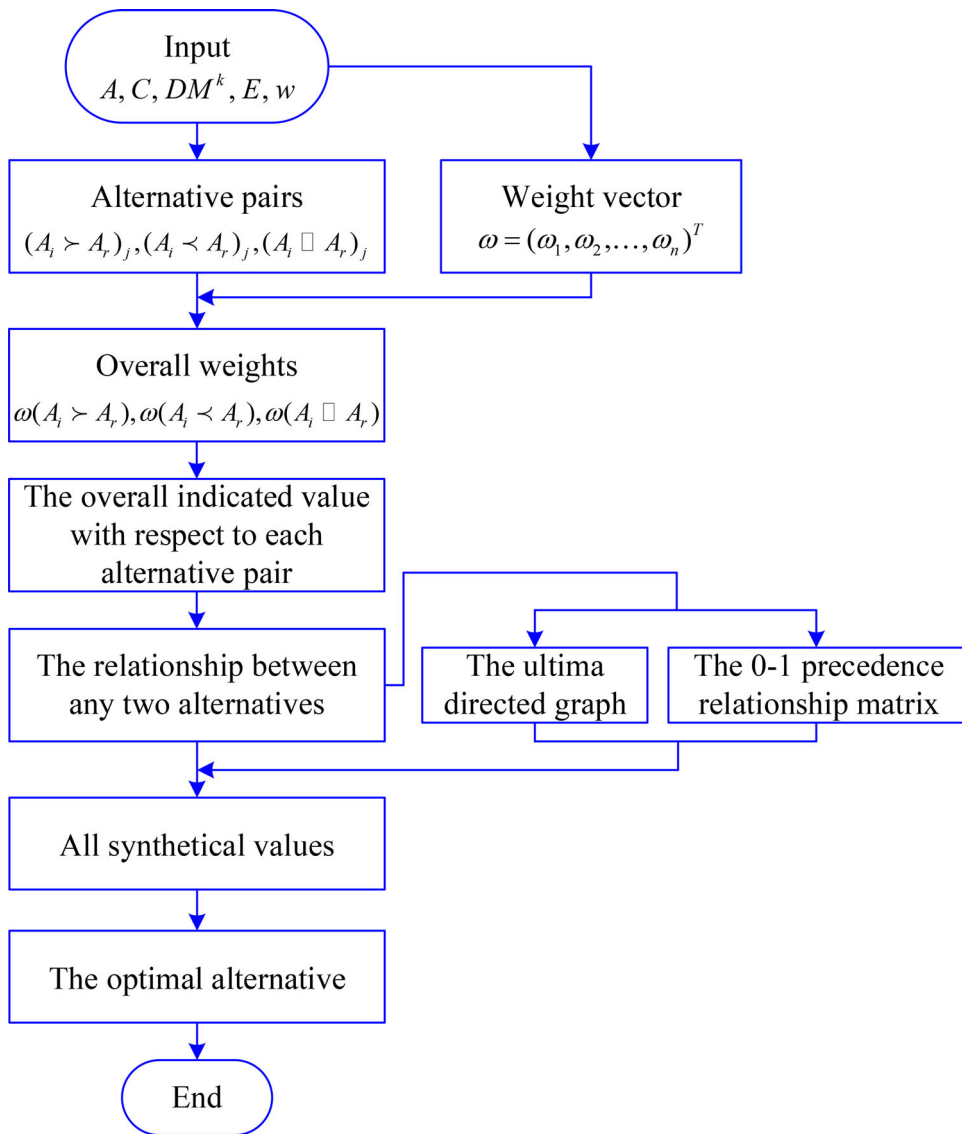
Figure 3. The flow chart of the PDHL-AQM. Source: The authors.

emerges, which leads to increasing risks of the real economy step by step. Therefore, this kind of economic market just deviates from the development route of our financial industry (Geng & Gao, 2020; Li, 2020). Then, one problem arises: How does finance enable the development of the real economy? Four proposals were put forward: (1) Formulating economic plans and providing market conditions for the development of a real economy; (2) Increasing the allocation of financial resources and reducing the inflow of funds from the real economy gradually; (3) Enhancing the value of finance in the market economy; (4) Innovating financial regulation methods and increasing the output of financial resources to the real economy (Li, 2020).

As we know, the economic market is innovating step by step on the road of economy in China, and various policy systems are gradually improving. To promote the reform of economic market in the process of improvement, relevant departments will also adjust the economic market from the macroeconomic perspective. Suppose that four cities, denoted as $\{A_1, A_2, A_3, A_4\}$, want to investigate the relationship between finance and the real economy in each city according to the work done in the above four proposals, and then obtain the rank of these cities. Let these four proposals be the criteria, denoted as $\{C_1, C_2, C_3, C_4\}$, three experts $e^k (k = 1, 2, 3)$ with the same importance degree $w = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})^T$ are invited to evaluate these four cities with respect to these criteria based on the DHLTS S_O, where $S = \{s_{-4} = extremelybad, s_{-3} = verybad, s_{-2} = bad, s_{-1} = slightly \quad bad, s_0 = medium, s_1 = slightlygood, s_2 = good, s_3 = verygood, s_4 = extremelygood\}$ and $O = \{o_{-4} = farfrom, o_{-3} = scarcely, o_{-2} = onlyalittle, o_{-1} = alittle, o_0 = justright, o_1 = much, o_2 = verymuch, o_3 = extremelymuch, o_4 = entirely\}$. By combining the evaluations of experts, three decision-making metrices $DM^k = (dm_{ij}^k)_{4\times4}(k = 1, 2, 3)$ are established:

$$DM^1 = \begin{pmatrix} \{s_{-1<o_0>}(1)\} & \{s_{1<o_1>}(0.5), s_{2<o_{-1}>}(0.5)\} & \{s_{2<o_0>}(1)\} & \{s_{1<o_{-2}>}(1)\} \\ \{s_{1<o_{-1}>}(0.3), s_{2<o_1>}(0.7)\} & \{s_{3<o_{-1}>}(1)\} & \{s_{0<o_1>}(0.7), s_{1<o_{-1}>}(0.3)\} & \{s_{2<o_{-1}>}(1)\} \\ \{s_{2<o_1>}(1)\} & \{s_{0<o_{-1}>}(0.8), s_{1<o_2>}(0.2)\} & \{s_{2<o_2>}(1)\} & \{s_{-1<o_1>}(0.6), s_{0<o_2>}(0.4)\} \\ \{s_{1<o_{-1}>}(1)\} & \{s_{1<o_{-3}>}(1)\} & \{s_{-2<o_1>}(1)\} & \{s_{1<o_1>}(1)\} \end{pmatrix}$$

$$DM^2 = \begin{pmatrix} \{s_{-2<o_1>}(1)\} & \{s_{1<o_1>}(0.2), s_{2<o_{-1}>}(0.8)\} & \{s_{1<o_1>}(0.5), s_{2<o_0>}(0.5)\} & \{s_{0<o_1>}(0.4), s_{1<o_{-2}>}(0.6)\} \\ \{s_{2<o_1>}(1)\} & \{s_{3<o_{-1}>}(1)\} & \{s_{0<o_1>}(1)\} & \{s_{1<o_{-1}>}(1)\} \\ \{s_{1<o_{-1}>}(0.7), s_{2<o_1>}(0.3)\} & \{s_{1<o_2>}(1)\} & \{s_{1<o_2>}(1)\} & \{s_{-1<o_1>}(1)\} \\ \{s_{0<o_{-1}>}(0.5), s_{1<o_{-1}>}(0.5)\} & \{s_{0<o_1>}(0.8), s_{1<o_{-3}>}(0.2)\} & \{s_{-2<o_1>}(1)\} & \{s_{0<o_1>}(0.6), s_{1<o_1>}(0.4)\} \end{pmatrix}$$

$$DM^3 = \begin{pmatrix} \{s_{0<o_1>}(1)\} & \{s_{0<o_{-1}>}(1)\} & \{s_{1<o_1>}(1)\} & \{s_{1<o_{-2}>}(0.2), s_{2<o_1>}(0.8)\} \\ \{s_{0<o_1>}(0.5), s_{1<o_{-1}>}(0.5)\} & \{s_{1<o_2>}(0.3), s_{2<o_1>}(0.7)\} & \{s_{-1<o_2>}(1)\} & \{s_{2<o_{-1}>}(1)\} \\ \{s_{1<o_{-1}>}(1)\} & \{s_{0<o_{-1}>}(1)\} & \{s_{0<o_1>}(0.4), s_{1<o_2>}(0.6)\} & \{s_{0<o_2>}(1)\} \\ \{s_{0<o_{-1}>}(1)\} & \{s_{1<o_{-3}>}(0.6), s_{2<o_1>}(0.4)\} & \{s_{-1<o_2>}(1)\} & \{s_{-1<o_3>}(1)\} \end{pmatrix}$$

It is clear that this decision-making problem is a MCDM problem. Therefore, we apply the proposed PDHL-AQM to solve it.

Table 1. The relation between two alternatives with respect to each criterion.

	C_1	C_2	C_3	C_4
The relation between two alternatives	$(A_1 \prec A_2)_1$	$(A_1 \prec A_2)_2$	$(A_1 \succ A_2)_3$	
	$(A_1 \prec A_3)_1$	$(A_1 \succ A_3)_2$	$(A_1 \prec A_3)_3$	$(A_1 \succ A_3)_4$
	$(A_1 \prec A_4)_1$	$(A_1 \succ A_4)_2$	$(A_1 \succ A_4)_3$	$(A_1 \succ A_4)_4$
	$(A_2 \succ A_3)_1$	$(A_2 \succ A_3)_2$	$(A_2 \prec A_3)_3$	$(A_2 \succ A_3)_4$
	$(A_2 \succ A_4)_1$	$(A_2 \succ A_4)_2$	$(A_2 \succ A_4)_3$	$(A_2 \succ A_4)_4$
	$(A_3 \succ A_4)_1$	$(A_3 \prec A_4)_1$	$(A_3 \succ A_4)_3$	$(A_3 \prec A_4)_4$

Source: The authors.

4.2. Solving the MCDM problem by the PDHL-AQM

We can use the proposed PDHL-AQM method to solve this MCDM problem:

Step 1. First, the overall decision-making matrix is aggregated:

$DM =$

$$
\begin{pmatrix}
\left\{ s_{-2<o_1>}\left(\frac{1}{3}\right), \; s_{0<o_{-1}>}\left(\frac{1}{3}\right), & & & \\
s_{-1<o_0>}\left(\frac{1}{3}\right), \; s_{1<o_1>}\left(\frac{7}{30}\right), & \begin{array}{l} \left\{ s_{1<o_1>}\left(\frac{1}{2}\right), \right. \\ s_{2<o_0>}\left(\frac{1}{2}\right) \} \end{array} & \begin{array}{l} \left\{ s_{0<o_1>}\left(\frac{2}{15}\right), s_{1<o_{-2}>}\left(\frac{3}{5}\right), \right. \\ s_{2<o_1>}\left(\frac{4}{15}\right) \} \end{array} & \\
s_{0<o_1>}\left(\frac{1}{3}\right) \} \; s_{2<o_{-1}>}\left(\frac{13}{30}\right) \} & & & \\
\\
\left\{ s_{0<o_1>}\left(\frac{1}{6}\right), \; s_{1<o_2>}\left(\frac{1}{10}\right), & \left\{ s_{-1<o_2>}\left(\frac{1}{3}\right), \right. & & \\
s_{1<o_{-1}>}\left(\frac{4}{15}\right), \; s_{2<o_1>}\left(\frac{7}{30}\right), & s_{0<o_1>}\left(\frac{17}{30}\right), & \left\{ s_{1<o_{-1}>}\left(\frac{1}{3}\right), s_{2<o_{-1}>}\left(\frac{2}{3}\right) \right\} & \\
s_{2<o_1>}\left(\frac{17}{30}\right) \} \; s_{3<o_{-1}>}\left(\frac{2}{3}\right) \} & s_{1<o_{-1}>}\left(\frac{1}{10}\right) \} & & \\
\\
\left\{ s_{1<o_{-1}>}\left(\frac{17}{30}\right), \; s_{0<o_{-1}>}\left(\frac{3}{5}\right), & \begin{array}{l} \left\{ s_{0<o_1>}\left(\frac{2}{15}\right), \right. \\ s_{1<o_2>}\left(\frac{8}{15}\right), \\ s_{2<o_2>}\left(\frac{1}{3}\right) \} \end{array} & \left\{ s_{-1<o_1>}\left(\frac{8}{15}\right), s_{0<o_2>}\left(\frac{7}{15}\right) \right\} & \\
s_{2<o_1>}\left(\frac{13}{30}\right) \} \; s_{1<o_2>}\left(\frac{2}{5}\right) \} & & & \\
\\
\left\{ s_{0<o_{-1}>}\left(\frac{1}{2}\right), & \begin{array}{l} \left\{ s_{0<o_1>}\left(\frac{4}{15}\right), \right. \\ s_{1<o_{-3}>}\left(\frac{9}{15}\right), \end{array} & \left\{ s_{-2<o_1>}\left(\frac{2}{3}\right), \right. & \left\{ s_{-1<o_3>}\left(\frac{1}{3}\right), \right. \\
s_{1<o_{-1}>}\left(\frac{1}{2}\right) \} & s_{2<o_1>}\left(\frac{2}{15}\right) \} & s_{-1<o_2>}\left(\frac{1}{3}\right) \} & s_{0<o_1>}\left(\frac{1}{5}\right), s_{1<o_1>}\left(\frac{7}{15}\right) \}
\end{pmatrix}
$$

Then, the weight vector of criteria can be obtained based on Algorithm 1:

$$\omega = (0.2743, 0.1976, 0.3601, 0.1680)^T$$

Step 2. Calculate the relation between two alternatives with respect to each criterion, shown in Table 1.

Table 2. The overall pros weights, overall cons weights and overall indifference weights between two alternatives.

Overall pros weights	Weights	Overall cons weights	Weights	Overall indifference weights	Weights
$\omega(A_1 \succ A_2)$	0.3601	$\omega(A_1 \prec A_2)$	0.6399	$\omega(A_1 \sim A_2)$	0
$\omega(A_1 \succ A_3)$	0.3656	$\omega(A_1 \prec A_3)$	0.6344	$\omega(A_1 \sim A_3)$	0
$\omega(A_1 \succ A_4)$	0.7357	$\omega(A_1 \prec A_4)$	0.2743	$\omega(A_1 \sim A_4)$	0
$\omega(A_2 \succ A_3)$	0.6399	$\omega(A_2 \prec A_3)$	0.3601	$\omega(A_2 \sim A_3)$	0
$\omega(A_2 \succ A_4)$	1	$\omega(A_2 \prec A_4)$	0	$\omega(A_2 \sim A_4)$	0
$\omega(A_3 \succ A_4)$	0.6344	$\omega(A_3 \prec A_4)$	0.3656	$\omega(A_3 \sim A_4)$	0

Source: The authors.

Step 3. Obtain the overall pros weights $\omega(A_i \succ A_r)$, the overall cons weights $\omega(A_i \prec A_r)$, and the overall indifference weights $\omega(A_i \prec A_r)$ between any two alternatives A_i and A_r, shown in Table 2.

Step 4. Obtain the overall indicated value with respect to each alternative pair (A_i, A_r) :

$$OI(A_1, A_2) = 0.5627; OI(A_1, A_3) = 0.5763; OI(A_1, A_4) = 2.6821$$

$$OI(A_2, A_3) = 1.7770; OI(A_2, A_4) \to +\infty; OI(A_3, A_4) = 1.7352$$

Step 5. Let $\theta = 1.5$, then we can obtain the 0–1 precedence relationship matrix M^* and the ultima directed graph (Figure 4):

$$M^* = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} A_1 & A_2 & A_3 & A_4 \end{array} \\ \left(\begin{array}{cccc} 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{array} \right) \end{array}$$

Step 6. Calculate the synthetical values of all alternatives $A_i (i = 1, 2, 3, 4)$: $RV = \{2, 4, 3, 1\}$.

Step 7. The rank of all alternatives is obtained: $A_2 \succ A_3 \succ A_1 \succ A_4$.

Step 8. End.

4.3. Comparative analysis

Under probabilistic double hierarchy linguistic environment, only Gou et al. (2020b) developed a probabilistic double hierarchy linguistic VIKOR (PDHL-VIKOR). Therefore, we can solve the MCDM problem above by the PDHL-VIKOR method. Additionally, we can also solve this MCDM problem by deleting the second hierarchy linguistic terms of all decision-making matrices. Finally, some comparative analyses are set up.

(1) Solving the MCDM problem by the PDHL-VIKOR method

Based on the PDHL-VIKOR method (Gou et al., 2020b), the probabilistic double hierarchy linguistic group utility measure ($PDHLGU_i$), the probabilistic double hierarchy linguistic individual regret measure ($PDHLIR_i$), and the probabilistic double hierarchy linguistic compromise measure ($PDHLC_i$) for each alternative are obtained respectively and shown in Table 3.

Then, the ranks of alternatives are $A_2 \succ A_3 \succ A_4 \succ A_1$, $A_2 \succ A_3 \succ A_1 \succ A_4$, and $A_2 \succ A_3 \succ A_1 \succ A_4$ based on these three measures, respectively, and there is $PDHLC_3 - PDHLC_2 = 0.5526 - 0 > \frac{1}{3}$. Therefore, the optimal alternative is A_2.

(2) Solving the MCDM problem by deleting the second hierarchy linguistic terms of DM^k $(k = 1, 2, 3)$.

a. Using the AQM method. By deleting the second hierarchy linguistic terms of all decision-making matrices. Then, we can obtain the adjusted overall decision-making matrix DM' :

$$DM' =$$

$$
\begin{pmatrix}
\{s_{-2}\left(\frac{1}{3}\right), & \{s_0\left(\frac{1}{3}\right), & \{s_1\left(\frac{1}{2}\right), s_2\left(\frac{1}{2}\right)\} & \{s_0\left(\frac{2}{15}\right), \\
s_{-1}\left(\frac{1}{3}\right), s_0\left(\frac{1}{3}\right)\} & s_1\left(\frac{7}{30}\right), s_2\left(\frac{13}{30}\right)\} & & s_1\left(\frac{3}{5}\right), s_2\left(\frac{4}{15}\right)\} \\
\{s_0\left(\frac{1}{6}\right), & \{s_1\left(\frac{1}{10}\right), & \{s_{-1}\left(\frac{1}{3}\right), & \{s_1\left(\frac{1}{3}\right), s_2\left(\frac{2}{3}\right)\} \\
s_1\left(\frac{4}{15}\right), s_2\left(\frac{17}{30}\right)\} & s_2\left(\frac{7}{30}\right), s_3\left(\frac{2}{3}\right)\} & s_0\left(\frac{17}{30}\right), s_1\left(\frac{1}{10}\right)\} & \\
\{s_1\left(\frac{17}{30}\right), s_2\left(\frac{13}{30}\right)\} & \{s_0\left(\frac{3}{5}\right), s_1\left(\frac{2}{5}\right)\} & \{s_0\left(\frac{2}{15}\right), & \{s_{-1}\left(\frac{8}{15}\right), s_0\left(\frac{7}{15}\right)\} \\
& & s_1\left(\frac{8}{15}\right), s_2\left(\frac{1}{3}\right)\} & \\
\{s_0\left(\frac{1}{2}\right), s_1\left(\frac{1}{2}\right)\} & \{s_0\left(\frac{4}{15}\right), & \{s_{-2}\left(\frac{2}{3}\right), s_{-1}\left(\frac{1}{3}\right)\} & \{s_{-1}\left(\frac{1}{3}\right), \\
& s_1\left(\frac{9}{15}\right), s_2\left(\frac{2}{15}\right)\} & & s_0\left(\frac{1}{5}\right), s_1\left(\frac{7}{15}\right)\}
\end{pmatrix}
$$

Additionally, we can obtain the weight vector of criteria:

$$\omega' = (0.2440, 0.1755, 0.3551, 0.2254)^T$$

Then, the relation between two alternatives with respect to each criterion is shown in Table 4.

Furthermore, the 0–1 precedence relationship matrix M'^* and the ultima directed graph are obtained:
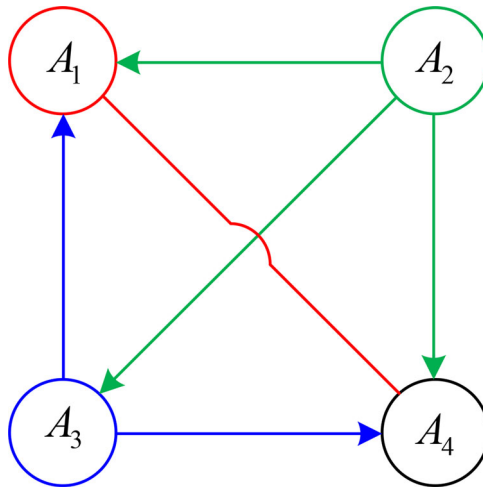
Figure 4. The ultima directed graph. Source: The authors.

Table 3. The calculation results of $PDHLGU_i$, $PDHLIR_i$ and $PDHLC_i$.

	$PDHLGU_i$	$PDHLIR_i$	$PDHLC_i$
A_1	0.6473	0.2743	0.8942
A_2	−0.6855	−0.0456	0
A_3	−0.0114	0.1976	0.5526
A_4	0.5083	0.3601	0.9479

Source: The authors.

$$
M'^{*} = \begin{matrix} & \begin{matrix} A_1 & A_2 & A_3 & A_4 \end{matrix} \\ \begin{matrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{matrix} & \begin{pmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \end{matrix}
$$

Then, the synthetical values of all alternatives is $RV = \{3, 3, 2, 2\}$, and the rank of alternatives is $A_2 \sim A_1 \succ A_4 \sim A_3$.b. Using some existing methods. In recent years, many MCDM methods were developed to deal with the probabilistic linguistic information. The decision-making results are summarised in Table 5.

(3) Discussion

The optimal decision-making result based on the PDHL-AQM and the PDHL-VIKOR method is the same. However, the PDHL-AQM is more intuitive considering we can observe the decision-making results clearly by the ultima directed graph and the 0–1 precedence relationship matrix. Additionally, the PDHL-VIKOR method is usually used to obtain the compromise solution. Therefore, it is common that the optimal solution is more than one alternative or no solution. On the contrary, we can always obtain the optimal solution using the proposed PDHL-AQM.

Furthermore, when solving the MCDM problem by deleting the second hierarchy linguistic terms of DM^k ($k = 1, 2, 3$), the rank of alternatives is $A_2 \sim A_1 \succ A_4 \sim A_3$,

Table 4. The relation between two alternatives with respect to each criterion.

The pair of alternatives	The relation between two alternatives			
(A_1, A_2)	$(A_1 \prec A_2)_1$	$(A_1 \prec A_2)_2$	$(A_1 \succ A_2)_3$	$(A_1 \prec A_2)_4$
(A_1, A_3)	$(A_1 \prec A_3)_1$	$(A_1 \succ A_3)_2$	$(A_1 \succ A_3)_3$	$(A_1 \succ A_3)_4$
(A_1, A_4)	$(A_1 \prec A_4)_1$	$(A_1 \succ A_4)_2$	$(A_1 \succ A_4)_3$	$(A_1 \succ A_4)_4$
(A_2, A_3)	$(A_2 \prec A_3)_1$	$(A_2 \succ A_3)_2$	$(A_2 \prec A_3)_3$	$(A_2 \succ A_3)_4$
(A_2, A_4)	$(A_2 \succ A_4)_1$	$(A_2 \succ A_4)_2$	$(A_2 \succ A_4)_3$	$(A_2 \succ A_4)_4$
(A_3, A_4)	$(A_3 \succ A_4)_1$	$(A_3 \prec A_4)_1$	$(A_3 \succ A_4)_3$	$(A_3 \prec A_4)_4$

Source: The authors.

Table 5. The decision-making results based on existing methods.

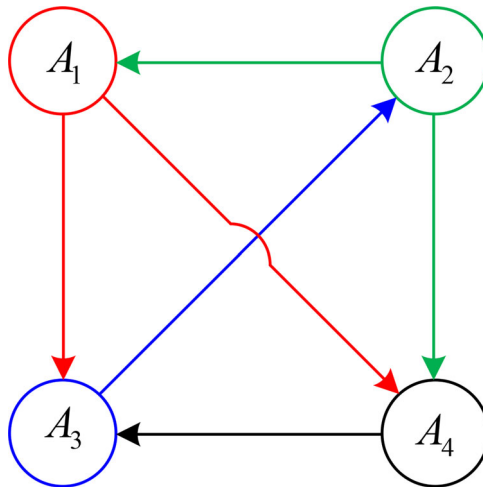| References	Methods	The decision-making results
Pang et al. (2016)	PL-TOPSIS	$A_2 \succ A_1 \succ A_3 \succ A_4$
Wu et al. (2018)	PL-MULTIMOORA	$A_2 \succ A_1 \succ A_3 \succ A_4$
Liao et al. (2017)	PL-LINMAP	$A_2 \succ A_1 \succ A_3 \succ A_4$
Pan et al. (2018)	PL-ELECTRE II	$A_2 \succ A_1 \succ A_3 \succ A_4$
Liu and Li (2018)	PL-PROMETHEE II	$A_2 \succ A_1 \succ A_3 \succ A_4$
Zhang et al. (2019e)	PL-TODIM	$A_2 \succ A_1 \succ A_3 \succ A_4$
Feng et al. (2019)	PL-QUALIFLEX	$A_2 \succ A_1 \succ A_3 \succ A_4$

Source: The authors.

Figure 5. The ultima directed graph about DM'. Source: The authors.

but it is unreasonable and illogical because we can find that $A_3 \succ A_2$ in Figure 5. Additionally, based on the existing methods, the rank of alternatives is $A_2 \succ A_1 \succ A_3 \succ A_4$, which differs from the proposed PDHL-AQM, but the optimal alternative is the same to the proposed PDHL-AQM. Therefore, the decision-making results are changed because the original information is changed by deleting the second hierarchy linguistic terms of all original decision-making matrices.

5. Conclusions and future research directions

This paper has developed a PDHL-AQM and used it to evaluate the real economy development under the perspective of economic financialization. First, a weight-determining method has been established to obtain the weight vector of criteria, and the

PDHL-AQM has been developed and the decision-making result is intuitive by drawing the directed graph or establishing the 0–1 precedence relationship matrix. Additionally, we have applied the proposed PDHL-AQM method to deal with a practical MCDM problem involving the real economy development evaluation under the perspective of economic financialization. Finally, some comparative analyses have been made to show the advantages and reasonableness of the PDHL-AQM.

By comparation, the PDHL-AQM is more intuitively considering we can observe the decision-making results clearly by the ultima directed graph and the 0–1 precedence relationship matrix, and we can always obtain the optimal solution using the proposed PDHL-AQM. Additionally, the PDHL-AQM is more logical because the PDHLTS will not lose the original information.

In the future, some interesting research directions will be considered. First, we will develop some novel decision-making methods such as probabilistic double hierarchy linguistic MULTIMOORA method, probabilistic double hierarchy linguistic LINMAP method, etc. Secondly, we will research some more popular economy problems such as Knowledge-based economy (Ježić, 2012), Post-transition economy (Wnuczak & Osiichuk, 2020), etc. Finally, the large-scale group decision-making is also the important research direction.

Disclosure statement

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References

Feng, X. Q., Liu, Q., & Wei, C. P. (2019). Probabilistic linguistic QUALIFLEX approach with possibility degree comparison. *Journal of Intelligent & Fuzzy Systems*, *36*(1), 719–730. https://doi.org/10.3233/JIFS-172112

Fu, Z. G., & Liao, H. C. (2019). Unbalanced double hierarchy linguistic term set: The TOPSIS method for multi-expert qualitative decision making involving green mine selection. *Information Fusion*, *51*, 271–286. https://doi.org/10.1016/j.inffus.2019.04.002

Geng, L. Z., & Gao, X. (2020). The development of finance and real economy from the perspective of economic financialization. *Money China*, *5*, 17.

Gou, X. J., Liao, H. C., Xu, Z. S., & Herrera, F. (2017a). Double hierarchy hesitant fuzzy linguistic term set and MULTIMOORA method: A case of study to evaluate the implementation status of haze controlling measures. *Information Fusion*, *38*, 22–34. https://doi.org/10.1016/j.inffus.2017.02.008

Gou, X. J., Liao, H. C., Xu, Z. S., & Herrera, F. (2020a). Consensus model handling minority opinions and non- cooperative behaviors in large-scale group decision-making under double hierarchy linguistic preference relations. *IEEE Transactions on Cybernetics*, *51*(1), 283–296. https://doi.org/10.1109/TCYB.2020.2985069

Gou, X. J., Liao, H. C., Xu, Z. S., & Herrera, F. (2020b). Probabilistic double hierarchy linguistic term set and its use for designing a VIKOR method for smart healthcare. *Journal of the Operational Research Society*. https://doi.org/10.1080/1331677X. 2020.1801485

Gou, X. J., Liao, H. C., Xu, Z. S., Min, R., & Herrera, F. (2019). Group decision making with double hierarchy hesitant fuzzy linguistic preference relations: Consistency based measures, index and repairing algorithms and decision model. *Information Sciences*, *489*, 93–112. https://doi.org/10.1016/j.ins.2019.03.037

Gou, X. J., & Xu, Z. S. (2020). Double hierarchy linguistic term set and its extensions: The state-of-the-art survey. *International Journal of Intelligent Systems*. https://doi.org/10.1002/int.22323

Gou, X. J., Xu, Z. S., & Liao, H. C. (2016). Alternative queuing method for multiple criteria decision making with hybrid fuzzy and ranking information. *Information Sciences*, *357*, 144–160. https://doi.org/10.1016/j.ins.2016.03.046

Gou, X. J., Xu, Z. S., & Liao, H. C. (2017b). Hesitant fuzzy linguistic entropy and cross-entropy measures and alternative queuing method for multiple criteria decision making. *Information Sciences*, *388–389*, 225–246. https://doi.org/10.1016/j.ins.2017.01.033

Gou, X. J., Xu, Z. S., Wang, X. X., & Liao, H. C. (2020). Managing consensus reaching process with self-confident double hierarchy linguistic preference relations in group decision making. *Fuzzy Optimization & Decision Making*. https://doi.org/10.1007/ s10700-020-09331-y

Gou, X. J., Xu, Z. S., & Zhou, W. (2020). Managing consensus by multiple stages optimization models with linguistic preference orderings and double hierarchy linguistic preferences. *Technological and Economic Development of Economy*, *26*(3), 642–674. https://doi.org/10. 3846/tede.2020.12736 https://doi.org/10.3846/tede.2020.12013

Herrera, F., & Martínez, L. (2000). A 2-tuple fuzzy linguistic representation model for computing with words. *IEEE Transactions on Fuzzy Systems*, *8*(6), 746–752. https://doi.org/10.1109/91.890332

Ježić, Z. (2012). Hypercomplex knowledge in a knowledge-based economy. *Economic Research-Ekonomska Istraživanja*, *25*(1), 132–144. https://doi.org/10.1080/1331677X.2012.11517500

Juang, C. F., & Chen, C. Y. (2013). Data-driven interval type-2 neural fuzzy system with high learning accuracy and improved model interpretability. *IEEE Transactions on Cybernetics*, *43*(6), 1781–1795. https://doi.org/10.1109/TSMCB.2012.2230253

Li, X. Y. (2020). Research on finance and real economy from the perspective of economic financialization. *Modern Business Trade Industry*, *31*, 134. https://doi.org/10.19311/j.cnki. 1672-3198.2020.31.060

Liao, H. C., Jiang, L. S., Xu, Z. S., Xu, J. P., & Herrera, F. (2017). A linear programming method for multiple criteria decision making with probabilistic linguistic information. *Information Sciences*, *415–416*, 341–355. https://doi.org/10.1016/j.ins.2017.06.035

Liu, N. N., He, Y., & Xu, Z. S. (2019). Evaluate public-private-partnership's Advancement using double hierarchy hesitant fuzzy linguistic PROMETHEE with subjective and objective information from stakeholder perspective. *Technological and Economic Development of Economy*, *25*(3), 386–420. https://doi.org/10.3846/tede.2019.7588

Liu, P. D., & Li, Y. (2018). The PROMETHEE II method based on probabilistic linguistic information and their application to decision making. *Informatica*, *29*(2), 303–320. https://doi.org/10.15388/Informatica.2018.169

Liu, X. D., Wang, Z. W., Zhang, S. T., & Liu, J. S. (2020). Probabilistic hesitant fuzzy multiple attribute decision-making based on regret theory for the evaluation of venture capital projects. *Economic Research-Ekonomska Istraživanja*, *33*(1), 672–697. https://doi.org/10.1080/1331677X.2019.1697327

Liu, S. H., Liu, X. Y., & Wang, X. L. (2018). Double hierarchy hesitant fuzzy linguistic LINMAP method for multiple attribute group decision making. In *2nd International Conference on Education Innovation and Economic Management* (pp. 265–272). DEStech. https://doi.org/10.12783/dtssehs/eiem2018/26943

Pan, L., Ren, P. J., & Xu, Z. S. (2018). Therapeutic schedule evaluation for Brain-Metastasized non-small cell lung cancer with a probabilistic linguistic ELECTRE II Method. *International*

Journal of Environmental Research and Public Health, *15*(9), 1799. https://doi.org/10.3390/ijerph15091799

Pang, Q., Wang, H., & Xu, Z. S. (2016). Probabilistic linguistic term sets in multi-attribute group decision making. *Information Sciences*, *369*, 128–143. https://doi.org/10.1016/j.ins.2016.06.021

Rodríguez, R. M., Martínez, L., & Herrera, F. (2012). Hesitant fuzzy linguistic terms sets for decision making. *IEEE Transactions on Fuzzy Systems*, *20*, 109–119. https://doi.org/10.1109/TFUZZ.2011. 2170076

Tian, X. L., Niu, M. L., Zhang, W. K., Li, L. H., & Herrera-Viedma, E. (2020). A novel TODIM based on prospect theory to select green supplier with Q-rung orthopair fuzzy set. *Technological and Economic Development of Economy*, 1–27. https://doi.org/10.3846/tede.2020.12736

Wang, X. D., Gou, X. J., & Xu, Z. S. (2020). Assessment of Traffic congestion with ORESTE method under double hierarchy hesitant fuzzy linguistic term set. *Applied Soft Computing*, *86*, 105864. https://doi.org/10.1016/j.asoc.2019.105864

Wei, G. W., & Gao, H. (2020). Pythagorean 2-tuple linguistic power aggregation operators in multiple attribute decision making. *Economic Research-Ekonomska Istraživanja*, *33*(1), 904–933. https://doi.org/10.1080/1331677X.2019.1670712

Wei, G., Lei, F., Lin, R., Wang, R., Wei, Y., Wu, J., & Wei, C. (2020). Algorithms for probabilistic uncertain linguistic multiple attribute group decision making based on the GRA and CRITIC method: application to location planning of electric vehicle charging stations. *Economic Research-Ekonomska Istraživanja*, *33*(1), 828–846. https://doi.org/10.1080/1331677X.2020.1734851

Wnuczak, P., & Osiichuk, D. (2020). The impact of management control systems and managers' educational background on the effectiveness of public cultural institutions in a post-transition economy. *Economic Research-Ekonomska Istraživanja*, *33*(1), 3166–3184. https://doi.org/10.1080/1331677X.2019.1691035

Wu, X. L., Liao, H. C., Xu, Z. S., Hafezalkotob, A., & Herrera, F. (2018). Probabilistic linguistic MULTIMOORA: Multi-criteria decision making method based on the probabilistic linguistic expectation function and the improved Borda rule. *IEEE Transactions on Fuzzy Systems*, *26*(6), 3688–3702. https://doi.org/10.1109/TFUZZ.2018.2843330

Xu, Z. S. (2012). *Linguistic decision making: theory and methods*. Springer.

Xu, Z. S., & Wang, H. (2017). On the syntax and semantics of virtual linguistic terms for information fusion in decision making. *Information Fusion*, *34*, 43–48. https://doi.org/10.1016/j.inffus.2016.06.002

Zadeh, L. A. (2012). *Computing with words: What is computing with words (CWW)?* Springer, pp. 1–40.

Zhang, Y. X., Xu, Z. S., & Liao, H. C. (2019e). Water security evaluation based on the TODIM method with probabilistic linguistic term sets. *Soft Computing*, *23*(15), 6215–6230. https://doi.org/10.1007/s00500-018-3276-9