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Robust fault estimation and fault-tolerant control for nonlinear Markov jump systems with time-delays

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ABSTRACT

The problem of the robust fault estimation and robust fault-tolerant control, for a class of nonlinear time-delayed Markov jump systems (MJSs) with both actuator and sensor faults, was studied. Firstly, by extending the system state, the actuator fault state vector, the sensor fault state vector and the original system state vector were extended to auxiliary state variables, and the original system was extended to a generalized system. Then, for the singular system, a generalized observer was designed to achieve the simultaneous estimation of its actuator faults, sensor faults and original system state. In addition, based on the observer estimation, a state feedback fault-tolerant controller was designed to make the closed-loop system stable and meet certain performance indicators. By solving linear matrix inequality, sufficient conditions for the existence of generalized observer and fault-tolerant controller were given. Finally, a numerical example and a practical example were given to demonstrate the effectiveness of the proposed approach.

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Markov jump systems; generalized observer; robust fault estimation; robust fault-tolerant control

1. Introduction

Markov jump systems (MJS) are systems consisting of several subsystems or modes that can be abstracted as time evolution and event-driven mechanisms that randomly switch from one mode to another at different times. During the operation, it will be affected by the external disturbance, human intervention and sudden changes in the external environment or damage to the internal components of the system. Because of this characteristic, MJS have been extensively studied by scholars in the field of control in the past few decades, and some achievements have been made.

In the actual control system, the faults of components, such as actuator and sensor faults inside the system, sudden changes in the external environment, the presence of external disturbances and network transmission delays, often lead to sudden changes in the structure and parameters of the system, reduce the stability performance of the system, and even lead to catastrophic accidents. In order to ensure that the system can still work normally in the presence of faults and external interference, the fault tolerance and anti-interference ability of the system are very important. Therefore, the stability analysis [1–5], fault detection and filtering [6–9], fault estimation and fault-tolerant control [10–14] of the system have attracted extensive attention of scholars, and have become a research hotspot in the field of control.

In [1], stochastic MJS with mixed time-varying delays and unknown partial transfer rate are discussed, and their exponential stability in the mean square is analysed. To obtain the less conservative stabilization condition, an appropriate weighting method is proposed in [2]. In [6], the H∞ filtering problem is studied for MJS with unknown transition probability. In [7], the H∞ filtering problem of discrete state MJS with time-varying delay is studied. A sliding surface is then constructed and a sliding mode controller is synthesized to ensure that the associated Markovian jump systems satisfy the reaching condition. Moreover, an observer-based sliding mode control problem is investigated in [9]. Although some attempts on fault estimation and fault-tolerant control for MJS have already been reported [14–18], the achieved results are only focused either on actuator faults [14] or on sensor faults [16]. Specially, the [15–18] have studied the problem of fault estimation and fault-tolerant control for a class of MJS with both actuator faults and sensor faults, but the upper bounds of faults and their derivatives need to be learned in advance. According to the above introduction, it is of great significance and practicality to carry out fault estimation and fault-tolerant control for MJS with actuator faults, sensor faults and external interference. In [19], the paper focuses on the fault-tolerant control (FTC) approach using fault estimation (FE) and fault compensation within the control system in which the design is achieved by integrating together the FE and FTC controller modules. In [20], the paper is concerned with the sliding mode control of uncertain nonlinear systems against actuator faults and external disturbances based on delta operator

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approach. At present, there are few studies on this issue at home and abroad, so it is worthwhile to discuss these issues in depth [21,22].

In this paper, a generalized observer design method is proposed for a class of nonlinear Markov jump systems with time-delays to estimate the original state of the system, actuator faults and sensor faults simultaneously. Based on the observer estimation, a design method of state feedback fault-tolerant controller is proposed to make the system stable and meet certain performance indexes in the case of simultaneous actuator and sensor fault and external disturbance, and the effectiveness of the proposed method is verified by numerical simulation.

The contributions of this paper are concluded as follows:

(1) In the case of uncertain state transition probability, the simultaneous estimation of actuator and sensor is given for a class of MJS with delay link and parameter uncertainty.
(2) This paper assumes that the state transition probability matrix has uncertainty in its estimated value, which is more practical than [15,16].
(3) In the design process of this paper, the information about actuator or sensor faults is not required to be determined in advance, thus leading to great advantages over the results in [22–24].

2. Problem statements

Consider a class of nonlinear time-delayed Markov jump systems with the following form:

$$\begin{cases} E\dot{x}(t) = A(r_t)x(t) + A_d(r_t)x(t-\tau) + B(r_t)u(t) \\ \qquad + D(r_t)\omega(t) + E(r_t)f_a(t) \\ \qquad + g(x(t), x(t-\tau)) \\ y(t) = C(r_t)x(t) + F(r_t)f_s(t) \\ x(t) = \varphi(t), t \in [-\tau, 0] \end{cases} \tag{1}$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the control input vector, $y(t) \in R^q$ is the control output vector, $w(t) \in R^p$ is the external disturbances input which belongs to $L_2[0,\infty)$, $L_2[0,\infty)$ represents the square integrable vector function space, $f_a(t) \in R^s$ and $f_s(t) \in R^w$ are unknown actuator faults and sensor faults, respectively. $\varphi(t)$ is the continuous initial state function on $[-\tau, 0]$, τ is the lag time constant of the system. x_0 and r_0 represent the initial state and the initial mode of the system, respectively. $g(x(t), x(t-\tau))$ represent the non-linear term of the system. The mode jumping process $\{r_t, t \geq 0\}$ is a Markov process with continuous time discrete state in the finite space $\Lambda = \{1, 2, \cdots, N\}$, and its state transition probability is as follows:

$$p(r_{t+h} = j | r_t = i) = \begin{cases} \pi_{ij}h + o(h), & i \neq j \\ 1 + \pi_{ij}h + o(h), & i = j \end{cases} \tag{2}$$

where $h > 0$, $\lim\limits_{h\to 0} = o(h)/h = 0$, π_{ij} represents the state transition probability from state i at time t to state j at time $t+h$. If $j \neq i$, then $\pi_{ij} > 0$, else $\pi_{ii} = -\sum_{j=1, j\neq i}^{N} \pi_{ij}$.$A(r_t), A_d(r_t), B(r_t), D(r_t), E(r_t), C(r_t)$ and $F(r_t)$ are constant matrices with appropriate dimensions.

Before system analysis, the following assumptions are made:

Assumption 1: *We assume that the actuator faults $f_a(t)$, sensor faults $f_s(t)$ and the external disturbance $\omega(t)$ are divisible and satisfy the following equations*

$$\begin{aligned} &f_a(t) \in L_2[0,\infty), \quad \dot{f}_a(t) \in L_2[0,\infty), \\ &f_s(t) \in L_2[0,\infty), \quad \dot{f}_s(t) \in L_2[0,\infty), \\ &\omega(t) \in L_2[0,\infty). \end{aligned}$$

Assumption 2: *The matrix N_i, L_i, T_i and Q_i with appropriate dimensions satisfy the following equation*

$$T_i\bar{A}_i - L_i\bar{C}_i - N_iT_iE = 0, T_iE + Q_i\bar{C}_i = I_{n+q+w} \tag{3}$$

Assumption 3: *The matrix B_i with appropriate dimensions satisfies $B_iB_i^T = I$.*

Definition 1 ([16]): The Markov jump system (1) with $u(t) = 0, \omega(t) = 0, f_a(t) = 0$ is said to be stochastically admissible if for all initial transitions x_0, initial modalities r_0 and all finite functions $\varphi(t)$ defined on $[-\tau, 0]$, the following inequality holds:

$$E\left\{\int_0^\infty ||x(t, \varphi(t), r_0)||^2 dt\right\} < \infty \tag{4}$$

where $E\{\cdot\}$ represents the mathematical expectation, and $||\cdot||$ represents the European norm of the vector.

Definition 2 ([22]): Given the scalar $\gamma > 0$, the Markov jump system (1) is said to be stochastically stable and meet γ disturbance attenuation, if there exists a constant $M(x_0, r_0)$ and $M(0, r_0) = 0$, such that the following inequality holds:

$$E\left\{\int_0^\infty [z^T(t)z(t)dt - \gamma^2\omega^T(t)\omega(t)]dt\right\} \\ \leq M(x_0, r_0) \tag{5}$$

Lemma 1 ([25]): *If the nonlinear term $g(x(t), x(t-\tau))$ of the system satisfies the Lipschitz condition, then there exists scalar ρ_1, ρ_2 satisfies the following formula:*

$$E\left\{\int_0^\infty [z^T(t)z(t)dt - \gamma^2\omega^T(t)\omega(t)]dt\right\} \\ \leq M(x_0, r_0) \tag{6}$$

Lemma 2 ([26]): *Given the matrix X and Y with appropriate dimension, if there exist a scalar $\varepsilon > 0$ and vector*

$x \in R^n, y \in R^n$ *such that*

$$2x^TXYy \le \varepsilon^{-1}x^TX^TXx + \varepsilon y^TY^TYy \tag{7}$$

Define a new state variable $\bar{x}(t) = \begin{bmatrix} x(t) \\ f_a(t) \\ f_s(t) \end{bmatrix} \in R^{n+q+w}$, *then system (1) can be equivalently transformed into*

$$\begin{cases} E\dot{\bar{x}}(t) = \bar{A}_i\bar{x}(t) + \bar{A}_{di}x(t-\tau) + \bar{B}_iu(t) \\ \qquad + \bar{D}_i\omega(t) + \bar{g}(x(t), x(t-\tau)) \\ y(t) = \bar{C}_i\bar{x}(t) \end{cases} \tag{8}$$

where

$$E = \begin{bmatrix} I_n & 0 & 0 \end{bmatrix}, \bar{A}_i = \begin{bmatrix} A_i & E_i & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\bar{A}_{di} = \begin{bmatrix} A_{di} \\ 0 \\ 0 \end{bmatrix}, \quad \bar{B}_i = \begin{bmatrix} B_i \\ 0 \\ 0 \end{bmatrix}, \quad \bar{D}_i = \begin{bmatrix} D_i \\ 0 \\ 0 \end{bmatrix},$$

$$\bar{g}(x(t), x(t-\tau)) = \begin{bmatrix} g(x(t), x(t-\tau)) \\ 0 \\ 0 \end{bmatrix},$$

$$\bar{C}_i = \begin{bmatrix} C_i & 0 & F_i \end{bmatrix}.$$

The system (8) is a generalized system with state variables including the original system state, actuator faults and sensor faults. If an effective observer is designed for the system (8), then the original system state, actuator faults and sensor faults can be estimated simultaneously.

3. Main results

3.1. Robust observer design

For the system (8), we propose a generalized observer that can simultaneously estimate the original system state, actuator and sensor faults. The observer design is as follows:

$$\begin{cases} \dot{z}(t) = N_iz(t) + L_iy(t) + T_i\bar{A}_{di}\hat{x}(t-\tau) \\ \qquad + T_i\bar{B}_iu(t) + T_i\bar{g}(\hat{x}(t), \hat{x}(t-\tau)) \\ \hat{\bar{x}}(t) = z(t) + Q_iy(t) \end{cases} \tag{9}$$

where $z(t) \in R^{n+q+w}$ is the observer intermediate variable, $\hat{x}(t)$ and $\hat{x}(t-\tau)$ are the estimated state of $x(t)$ and time-delayed state of $x(t-\tau)$, respectively, $\hat{\bar{x}}(t)$ is the estimated state of $\bar{x}(t)$, N_i, L_i, T_i and Q_i are matrices to be solved with appropriate dimensions.

Definition $e(t) = \bar{x}(t) - \hat{\bar{x}}(t)$, then from the formula (7), formula (9) and assumption 2, we can obtain the following formula:

$$\begin{aligned} e(t) &= \bar{x}(t) - \hat{\bar{x}}(t) = \bar{x}(t) - z(t) - Q_i\bar{C}_i\bar{x}(t) \\ &= (I_{n+q+w} - Q_i\bar{C}_i)\bar{x}(t) - z(t) \\ &= T_iE\bar{x}(t) - z(t) \end{aligned} \tag{10}$$

then we can obtain the following error dynamic system:

$$\begin{aligned} \dot{e}(t) &= T_iE\dot{\bar{x}}(t) - \dot{z}(t) \\ &= T_i\bar{A}_i\bar{x}(t) + T_i\bar{A}_{di}x(t-\tau) \\ &\quad + T_i\bar{B}_iu(t) + T_i\bar{D}_i\omega(t) \\ &\quad + T_i\bar{g}(x(t), x(t-\tau)) - N_iz(t) - L_iy(t) \\ &\quad - T_i\bar{A}_{di}\hat{x}(t-\tau) - T_i\bar{B}_iu(t) + N_iT_iE\bar{x}(t) \\ &\quad - T_i\bar{g}(\hat{x}(t), \hat{x}(t-\tau)) - N_iT_iE\bar{x}(t) \\ &= N_ie + (T_i\bar{A}_i - L_i\bar{C}_i - N_iT_iE)\bar{x}(t) \\ &\quad + T_i\bar{A}_{di}\tilde{x}(t-\tau) + T_i\bar{D}_i\omega(t) \\ &\quad + T_i\bar{g}(\tilde{x}(t), \tilde{x}(t-\tau)) \end{aligned} \tag{11}$$

where

$$\tilde{x}(t-\tau) = x(t-\tau) - \hat{x}(t-\tau) \tag{12}$$

$$\begin{aligned} \bar{g}(\tilde{x}(t), \tilde{x}(t-\tau)) &= \bar{g}(x(t), x(t-\tau)) \\ &\quad - \bar{g}(\hat{x}(t), \hat{x}(t-\tau)) \end{aligned} \tag{13}$$

According to assumption (2),

$$\begin{aligned} &T_i\bar{A}_i - L_i\bar{C}_i - N_iT_iE \\ &\quad = T_i\bar{A}_i - L_i\bar{C}_i - N_i(I_{n+q+w} - Q_i\bar{C}_i) \\ &\quad = T_i\bar{A}_i - (L_i - N_iQ_i)\bar{C}_i - N_i = 0 \end{aligned} \tag{14}$$

Then the error dynamic system (11) can be simplified to the following formula:

$$\begin{aligned} \dot{e}(t) &= N_ie + T_i\bar{A}_{di}\tilde{x}(t-\tau) + T_i\bar{D}_i\omega(t) \\ &\quad + T_i\bar{g}(\tilde{x}(t), \tilde{x}(t-\tau)) \end{aligned} \tag{15}$$

Let $A = 1$, then A set of special solutions to equation (14) are as follows:

$$N_i = T_i\bar{A}_i - K_i\bar{C}_i, L_i = K_i + N_iQ_i \tag{16}$$

The following theorem is one of the main results of this paper, which not only gives sufficient conditions for the existence of the generalized observer, but also guarantees the robust stability of the error dynamic system (15).

In this paper, $*$ stands for symmetric transposition.

Theorem 1: *For the error dynamic system* (15), *given the scalars* $\gamma > 0$ *and* $\varepsilon_1 > 0$, *if there exist positive definite symmetry matrices* $P_i > 0$, $R_i > 0$, *a matrix* Y_i *and matrices* ρ_1 *and* ρ_2 *with appropriate dimensions, the following inequality holds:*

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ * & \Omega_{22} & \Omega_{23} \\ * & * & \Omega_{33} \end{bmatrix} < 0 \tag{17}$$

where

$$\begin{aligned} \Omega_{11} &= \bar{A}_i^TT_i^TP_i - \bar{C}_i^TY_i^T + P_iT_i\bar{A}_i - Y_i\bar{C}_i + 2\varepsilon_1\rho_1^T\rho_1I \\ &\quad + \varepsilon_1^{-1}P_iT_iT_i^TP_i + R_i + \sum_{j=1}^{N}\pi_{ij}P_j + I, \end{aligned}$$

$\Omega_{12} = P_i T_i \bar{A}_{di}, \Omega_{13} = P_i T_i \bar{D}_i,$

$\Omega_{22} = -R_i + 2\varepsilon_1 \rho_2^T \rho_2 I, \Omega_{23} = 0,$

$\Omega_{33} = -\gamma^2 I, \quad Y_i = P_i K_i.$

then there is an observer (9), *which can make the error dynamic system* (15) *asymptotically stable and can meet* H_∞ *performance* γ.

Proof: Defined the following Lyapunov-Krasovskii function

$$V(e(t), i) = e^T(t)P(r_t)e(t) + \int_{t-\tau}^{t} e^T(\theta)R(r_t)e(\theta)d\theta \tag{18}$$

Then along with the error dynamic system (15), the weak infinitesimal operator of $V(e(t), i)$ is as follows:

$$\begin{aligned}\ell V(e(t), i) = {} & e^T(t)(N_i{}^T P_i + P_i N_i)e(t) \\ & + 2e^T(t)P_i T_i \bar{A}_{di} e(t-\tau) \\ & + 2e^T(t)P_i T_i \bar{D}_i \omega(t) + e^T(t)\sum_{j=1}^{N} \pi_{ij} P_j e(t) \\ & + 2e^T(t)P_i T_i \bar{g}(\tilde{x}(t), \tilde{x}(t-\tau)) \\ & + e^T(t)R_i e(t) - e^T(t-\tau)R_i e(t-\tau)\end{aligned} \tag{19}$$

From lemma 1 and lemma 2, we can obtain that

$$\begin{aligned}& 2e^T(t)P_i T_i \bar{g}(\tilde{x}(t), \tilde{x}(t-\tau)) \\ & \le 2\varepsilon_1[e^T(t)\rho_1^T \rho_1 e(t) + e^T(t-\tau)\rho_2^T \rho_2 e(t-\tau)] \\ & + \varepsilon_1^{-1} e^T(t)P_i T_i T_i^T P_i e(t)\end{aligned} \tag{20}$$

Substituting the formula (16) and (20) into the formula (19), we can obtain:

$$\begin{aligned}& \ell V(e(t), i) \\ & \le e^T(t)(\bar{A}_i^T T_i^T P_i - \bar{C}_i^T K_i^T P_i + P_i T_i \bar{A}_i \\ & - P_i K_i \bar{C}_i)e(t) \\ & + 2e^T(t)P_i T_i \bar{A}_{di} e(t-\tau) + 2e^T(t)P_i T_i \bar{D}_i \omega(t) \\ & + e^T(t)\sum_{j=1}^{N} \pi_{ij} P_j e(t) + \varepsilon_1^{-1} e^T(t)P_i T_i T_i^T P_i e(t) \\ & + 2\varepsilon_1[e^T(t)\rho_1^T \rho_1 e(t) + e^T(t-\tau)\rho_2^T \rho_2 e(t-\tau)] \\ & + e^T(t)R_i e(t) - e^T(t-\tau)R_i e(t-\tau)\end{aligned} \tag{21}$$

If assuming that $\omega(t) = 0$, then we can obtain that

$$\begin{aligned}& \ell V(e(t), i) \\ & \le e^T(t)(\bar{A}_i^T T_i^T P_i - \bar{C}_i^T K_i^T P_i \\ & + P_i T_i \bar{A}_i - P_i K_i \bar{C}_i)e(t) \\ & + 2e^T(t)P_i T_i \bar{A}_{di} e(t-\tau) + e^T(t)\sum_{j=1}^{N} \pi_{ij} P_j e(t) \\ & + 2\varepsilon_1[e^T(t)\rho_1^T \rho_1 e(t) + e^T(t-\tau)\rho_2^T \rho_2 e(t-\tau)] \\ & + \varepsilon_1^{-1} e^T(t)P_i T_i T_i^T P_i e(t) + e^T(t)R_i e(t) \\ & - e^T(t-\tau)R_i e(t-\tau) \\ & = \xi^T(t)\Omega_1 \xi(t)\end{aligned} \tag{22}$$

where

$$\Omega_1 = \begin{bmatrix} \Omega'_{11} & \Omega_{12} \\ * & \Omega_{22} \end{bmatrix}, \quad \xi(t) = \begin{bmatrix} e(t) \\ e(t-\tau) \end{bmatrix},$$

$$\begin{aligned}\Omega'_{11} = {} & \bar{A}_i^T T_i^T P_i - \bar{C}_i^T Y_i^T + P_i T_i \bar{A}_i - Y_i \bar{C}_i \\ & + 2\varepsilon_1 \rho_1^T \rho_1 I + \varepsilon_1^{-1} P_i T_i T_i^T P_i + R_i + \sum_{j=1}^{N} \pi_{ij} P_j.\end{aligned}$$

It can be seen from the formula (17) that $\Omega_1 < 0$, that is $\ell V(e(t), i) \le 0$. As can be seen from definition 1, the error dynamic system (15) is asymptotically stable.

It is discussed below that under the zero initial condition, the error dynamic system (15) meets H_∞ performance γ.

Under the zero initial conditions, for any non-zero external disturbance $\omega(t) \in L^2[0, \infty]$, the weak infinitesimal operator of $V(e(t), x(t), i)$ along the error dynamic system (15) is as follows:

$$\begin{aligned}& \ell V_\omega(e(t), i) \\ & \le e^T(t)(N_i{}^T P_i + P_i N_i)e(t) \\ & + 2e^T(t)P_i T_i \bar{A}_{di} e(t-\tau) \\ & + 2e^T(t)P_i T_i \bar{D}_i \omega(t) + e^T(t)\sum_{j=1}^{N} \pi_{ij} P_j e(t) \\ & + 2\varepsilon_1[e^T(t)\rho_1^T \rho_1 e(t) + e^T(t-\tau)\rho_2^T \rho_2 e(t-\tau)] \\ & + \varepsilon_1^{-1} e^T(t)P_i T_i T_i^T P_i e(t) + e^T(t)R_i e(t) \\ & - e^T(t-\tau)R_i e(t-\tau)\end{aligned} \tag{23}$$

Introduce the following performance function:

$$J = e^T(t)e(t) - \gamma^2 \omega^T(t)\omega(t) + \ell V_\omega(e(t), i) \tag{24}$$

then

$$\begin{aligned}& e^T(t)e(t) - \gamma^2 \omega^T(t)\omega(t) + \ell V_\omega(e(t), i) \\ & = \varsigma^T(t)\Omega_2 \varsigma(t)\end{aligned} \tag{25}$$

where

$$\Omega_2 = \begin{bmatrix} \Omega_{11} & \Omega_{22} & \Omega_{13} \\ * & \Omega_{12} & \Omega_{23} \\ * & * & \Omega_{33} \end{bmatrix},$$

$$\varsigma(t) = \begin{bmatrix} e(t) \\ e(t-\tau) \\ \omega(t) \end{bmatrix}$$

It can be seen from the formula (17), $\Omega_2 < 0$, that is, $J = e^T(t)e(t) - \gamma^2 \omega^T(t)\omega(t) + \ell V_\omega(e(t), i) < 0$.

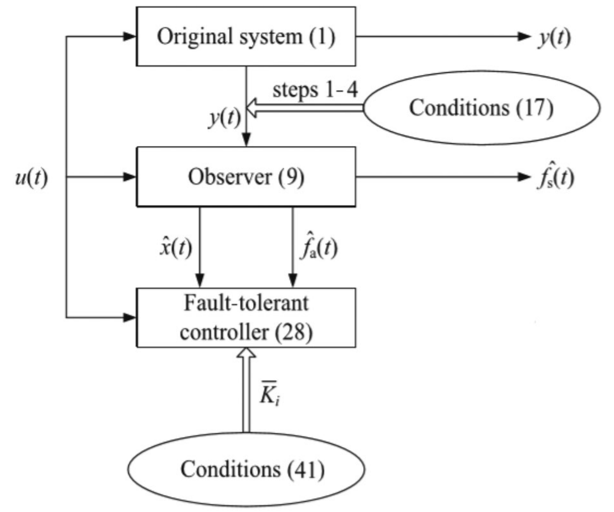

Figure 1. Design mechanism block diagram.

According to Dynkin's formula,

$$E\left\{\int_0^\infty [e^T(t)e(t) - \gamma^2\omega^T(t)\omega(t)]dt\right\} + E\{V_\omega(e(t), i)\} - E\{V_\omega(e_0, r_0)\} < 0 \tag{26}$$

where e_0, r_0 are the initial values of corresponding variables, respectively.

It can be obtained from the formula (26) that

$$E\left\{\int_0^\infty [e^T(t)e(t) - \gamma^2\omega^T(t)\omega(t)]dt\right\} < E\{V_\omega(e_0, r_0)\} \tag{27}$$

It can be seen from definition 2 that the error dynamic system (15) is asymptotically stable and meets H_∞ performance γ. This completes the proof.

According to the conclusion of theorem 1, the algorithm for solving the control law is given as follows, and the specific design mechanism in Figure 1.

Step 1: From assumption 2, we can obtain the matrices T_i and Q_i.

Step 2: Apply the LMI toolbox to solve the inequality (17). If there is a solution, we can obtain the matrix K_i through $K_i = P_i^{-1}Y_i$.

Step 3: Substituting K_i into the formula (16), we can obtain matrices N_i and L_i.

Step 4: The coefficient matrices of the observer have been solved so that the error of the observer converges to zero. In addition, the estimation of the state, actuator faults and sensor fault can be obtained from $\hat{x}(t) = \begin{bmatrix} I_n & 0 & 0 \end{bmatrix}\hat{\bar{x}}(t), \hat{f}_a(t) = \begin{bmatrix} 0 & I_q & 0 \end{bmatrix}\hat{\bar{x}}(t)$ and $\hat{f}_s(t) = \begin{bmatrix} 0 & 0 & I_w \end{bmatrix}\hat{\bar{x}}(t)$, respectively. ■

3.2. Fault-tolerant controller design

Based on the observer design, we propose a method of state feedback fault-tolerant control design, which makes the system relatively stable.

The design of state feedback fault-tolerant controller is as follows:

$$u(t) = \bar{K}_i\hat{x}(t) - B_i^T E_i\hat{f}_a(t) \tag{28}$$

where $\hat{x}(t)$ and $\hat{f}_a(t)$ are the estimated values obtained by the observer formula (9).

According to the assumption 3, substituting formula (28) into formula (1), the closed-loop system is as follows:

$$\begin{aligned} E\dot{x}(t) &= A_ix(t) + A_{di}x(t-\tau) + B_i\bar{K}_i\hat{x}(t) - E_i\hat{f}_a(t) \\ &\quad + D_i\omega(t) + E_if_a(t) + g(x(t), x(t-\tau)) \\ &= (A_i + B_i\bar{K}_i)x(t) - B_i\bar{K}_i\tilde{x}(t) + A_{di}x(t-\tau) \\ &\quad + D_i\omega(t) + E_i\tilde{f}_a(t) + g(x(t), x(t-\tau)) \\ &= (A_i + B_i\bar{K}_i)x(t) - \Psi_ie(t) + A_{di}x(t-\tau) \\ &\quad + D_i\omega(t) + g(x(t), x(t-\tau)) \end{aligned} \tag{29}$$

where

$$\tilde{x}(t) = x(t) - \hat{x}(t), \quad \tilde{f}_a(t) = f_a(t) - \hat{f}_a(t),$$
$$\Psi_i = \begin{bmatrix} B_i\bar{K}_i & -E_i & 0 \end{bmatrix}.$$

Theorem 2: *For the closed-loop system* (29), *given the scalars* $\mu > 0, \varepsilon_1 > 0$ *and* $\varepsilon_2 > 0$, *if there exist positive definite symmetry matrices* $U_i > 0$, $V_i > 0$, *a matrix* $\bar{K}_i$ *and matrices* ρ_1 *and* ρ_2 *with appropriate dimensions, the following inequality holds:*

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & \Xi_{23} \\ * & * & \Xi_{33} \end{bmatrix} < 0 \tag{30}$$

where

$$\Xi_{11} = (A_i + B_i\bar{K}_i)^TU_i + U_i(A_i + B_i\bar{K}_i) + 2\varepsilon_2\rho_1^T\rho_1I + \varepsilon_2^{-1}U_iU_i^T + V_i + I + \sum_{j=1}^N \pi_{ij}U_j,$$

$$\Xi_{12} = U_iA_{di}, \Xi_{13} = U_iD_i, \Xi_{22} = -V_i + 2\varepsilon_2\rho_2^T\rho_2I,$$

$$\Xi_{23} = 0, \quad \Xi_{33} = -\mu^2I.$$

then there is a robust fault-tolerant controller (28), *which can make the closed-loop system* (29) *asymptotically stable and can meet* H_∞ *performance* $\delta = \sqrt{\mu^2 + \beta\gamma^2}$, β *is a positive scalar given in the proof below.*

Proof: Defined the following Lyapunov-Krasovskii function:

$$V(x(t), i) = x^T(t)U(r_t)x(t) + \int_{t-\tau}^t x^T(\theta)V(r_t)x(\theta)d\theta \tag{31}$$

Then along the closed-loop system (29), the weak infinitesimal operator of $V(x(t), i)$ is as follows:

$$\ell V(x(t), i) = x^T(t)[(A_i + B_i\bar{K}_i)^TU_i + U_i(A_i + B_i\bar{K}_i)]x(t)$$

$$- 2x^T(t)U_i\Psi_i e(t) + 2x^T(t)U_iA_{di}x(t-\tau)$$
$$+ 2x^T(t)U_iD_i\omega(t) + x^T(t)\sum_{j=1}^{N}\pi_{ij}U_jx(t)$$
$$+ 2x^T(t)U_ig(x(t), x(t-\tau))$$
$$+ x^T(t)V_ix(t) - x^T(t-\tau)V_ix(t-\tau) \quad (32)$$

The analysis process of theorem 2 is similar to that of theorem 1, then

$$\begin{aligned}
J' &= x^T(t)x(t) - \mu^2\omega^T(t)\omega(t) + \ell V(x(t), i) \\
&\leq x^T(t)x(t) - \mu^2\omega^T(t)\omega(t) \\
&\quad + x^T(t)[(A_i + B_i\bar{K}_i)^T U_i \\
&\quad + U_i(A_i + B_i\bar{K}_i)]x(t) + 2x^T(t)U_iA_{di}x(t-\tau) \\
&\quad + 2x^T(t)U_iD_i\omega(t) + x^T(t)\sum_{j=1}^{N}\pi_{ij}U_jx(t) \\
&\quad + \varepsilon_2^{-1}x^T(t)U_iU_i^Tx(t) - 2x^T(t)U_i\Psi_ie(t) \\
&\quad + 2\varepsilon_2[x^T(t)\rho_1^T\rho_1x(t) + x^T(t-\tau)\rho_2^T\rho_2x(t-\tau)] \\
&\quad + x^T(t)V_ix(t) - x^T(t-\tau)V_ix(t-\tau) \\
&= \eta^T(t)\Omega_3\eta(t) - 2x^T(t)U_i\Psi_ie(t) \quad (33)
\end{aligned}$$

where

$$\Omega_3 = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & & \\ * & \Xi_{22} & \Xi_{23}* & * & \Xi_{33} \end{bmatrix},$$
$$\eta(t) = \begin{bmatrix} x(t) \\ x(t-\tau) \\ \omega(t) \end{bmatrix}$$

As can be seen from the formula (30),

$$\begin{aligned}
J' &= x^T(t)x(t) - \mu^2\omega^T(t)\omega(t) + \ell V(x(t), i) \\
&\leq -\min_{i\in N}\lambda_{\min}(-\Omega_3)(||x(t)||^2 + ||x(t-\tau||^2 \\
&\quad + ||\omega(t)||^2) - 2x^T(t)U_i\Psi_ie(t) \\
&\leq -\sigma_0(||x(t)||^2 + ||x(t-\tau||^2 + ||\omega(t)||^2) \\
&\quad + 2\sigma_1||x(t)||||e(t)|| \quad (34)
\end{aligned}$$

where

$$\sigma_0 = -\min_{i\in N}\lambda_{\min}(-\Omega_3), \sigma_1 = \max_{i\in N}||U_i\Psi_i|| \quad (35)$$

Defined a new Lyapunov-Krasovskii function as follows:

$$V(x(t), e(t), i) = V(x(t), i) + \beta V(e(t), i) \quad (36)$$

Given a positive scalar $\beta > (\sigma_1^2/\sigma_0)$, as can be seen from the formula (34),

$$\begin{aligned}
&lV(x(t), e(t), i) + x^T(t)x(t) - \mu^2\omega^T(t)\omega(t) \\
&= lV(x(t), i) + \beta lV(e(t), i) + x^T(t)x(t) \\
&\quad - \mu^2\omega^T(t)\omega(t) \\
&\leq -\min_{i\in N}\lambda_{\min}(-\Omega_3)(||x(t)||^2 \\
&\quad + ||x(t-\tau||^2 + ||\omega(t)||^2) \\
&\quad - 2x^T(t)U_i\Psi_ie(t) - \beta||e(t)||^2 - \beta||e(t-\tau)||^2 \\
&\quad + \beta\gamma^2||\omega(t)||^2 \\
&\leq -\sigma_0(||x(t)||^2 + ||x(t-\tau||^2 + ||\omega(t)||^2) \\
&\quad + 2\sigma_1||x(t)||||e(t)|| - \beta||e(t)||^2 \\
&\quad - \beta||e(t-\tau)||^2 + \beta\gamma^2||\omega(t)||^2 \\
&\leq -\sigma_0||x(t)||^2 + 2\sigma_1||x(t)||||e(t)|| \\
&\quad - \beta||e(t)||^2 + \beta\gamma^2||\omega(t)||^2 \\
&\leq -(\sqrt{\sigma_0}||x(t)|| - \sqrt{\beta}||e(t)||)^2 + \beta\gamma^2||\omega(t)||^2 \\
&\leq \beta\gamma^2||\omega(t)||^2 \quad (37)
\end{aligned}$$

then

$$\begin{aligned}
&lV(x(t), e(t), i) + x^T(t)x(t) \\
&\quad \leq \beta\gamma^2||\omega(t)||^2 + \mu^2||\omega(t)||^2 = \delta^2||\omega(t)||^2 \quad (38)
\end{aligned}$$

According to Dynkin's formula,

$$E\left\{\int_0^\infty [x^T(t)x(t) - \delta^2\omega^T(t)\omega(t)]dt\right\}$$
$$+ E\{V(e(t), x(t), i)\} - E\{V(e_0, x_0, r_0)\} < 0 \quad (39)$$

where e_0, x_0 and r_0 are the initial values of the corresponding variables.

According to the formula (39), we can obtain that

$$E\left\{\int_0^\infty [x^T(t)x(t) - \delta^2\omega^T(t)\omega(t)]dt\right\}$$
$$< E\{V(e_0, x_0, r_0)\} \quad (40)$$

It can be seen from definition 2 that the closed-loop system (29) is asymptotically stable and meets H_∞ performance δ. This completes the proof. ■

Theorem 3: *For the closed-loop system* (29), *given the scalars $\mu > 0, \varepsilon_1 > 0$ and $\varepsilon_2 > 0$, if there exist positive definite symmetry matrices $W_i > 0$, $U_i > 0$ and $V_i > 0$, a matrix $\bar{K}_i$ and matrices ρ_1 and ρ_2 with appropriate dimensions, the following inequality holds:*

$$\begin{bmatrix}
\hat{\Xi}_{11} & A_{di}W_i & D_iW_i & W_i\rho_1^T & I & W_i \\
* & \hat{\Xi}_{22} & 0 & 0 & 0 & 0 \\
* & * & -\mu^2 I & 0 & 0 & 0 \\
* & * & * & -\frac{1}{2}\varepsilon_2^{-1}I & 0 & 0 \\
* & * & * & * & -\varepsilon_2 I & 0 \\
* & * & * & * & * & -I
\end{bmatrix}$$
$$< 0 \quad (41)$$

where

$$\hat{\Xi}_{11} = W_iA_i{}^T + \bar{Y}_i^TB_i{}^T$$
$$+ A_iW_i + B_i\bar{Y}_i + \hat{V}_i + \sum_{j=1}^{N}\pi_{ij}\hat{U}_j,$$

$$\hat{\Xi}_{22} = -V_i + 2\varepsilon_2 \rho_2^T \rho_2 I.$$

Then $\bar{K}_i = \bar{Y}_i W_i^{-1}$ *is a robust fault-tolerant control law of the closed-loop system* (29).

Proof: Let

$$W_i = U_i^{-1}, \quad \bar{K}_i = \bar{Y}_i W_i^{-1}, \quad \hat{V}_i = W_i V_i W_i,$$
$$\hat{U}_j = W_i U_j W_i$$

Multiplying $diag\{\, W_i \quad I \quad I\}$ on the left and right of the formula (30), according to Schur complement theorem, formula (30) is equivalent to formula (41). This completes the proof.

The fault-tolerant controller design procedure is listed following the Step 4 in the previous section.

Step5: Solving the formula (41), we can obtain matrix $\bar{K}_i$, then according to the estimated state $\hat{x}(t)$ and actuator faults $\hat{f}_a(t)$, the fault-tolerant controller (28) is constructed. ■

4. Simulation analysis

4.1. Numerical examples

The parameters of the nonlinear Markov jump time-delayed system are selected as follows [22,27]:

Mode 1

$$A_1 = \begin{bmatrix} -5 & 0 & 1 \\ 0 & -7.5 & 0 \\ 2 & 0 & -5 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} 0.2 & 0 & 0.1 \\ 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.2 \\ 0.2 \\ 0 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 0.01 \\ -2 \end{bmatrix}.$$

Mode 2

$$A_2 = \begin{bmatrix} -6 & 0 & 1.1 \\ 0 & -8 & 0 \\ 0 & 0 & -5 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} 0.1 & 0 & 0.05 \\ 0.05 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.2 \\ 0.2 \\ 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.3 \\ 0.05 \\ 0.1 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.01 \\ -2 \end{bmatrix}.$$

In mode 1 and mode 2, we assume that $\varepsilon_1 = \varepsilon_2 = 1$, the delay time is 3s, and the transition probability matrix is

$$\pi_{ij} = \begin{bmatrix} -0.4 & 0.4 \\ 0.3 & -0.3 \end{bmatrix}.$$

According to step 1, we can obtain

$$T = \begin{bmatrix} 0.5 & 0 & -0.0017 & 0 & -0.0017 \\ 0 & 1 & 0 & 0 & 0 \\ -0.0017 & 0 & 0.8333 & 0 & 0.3333 \\ 0 & 0 & 0 & 1 & 0 \\ -0.0017 & 0 & 0.3333 & 0 & 0.3333 \end{bmatrix},$$

$$Q = \begin{bmatrix} 0.5000 & 0.0017 \\ 0 & 0 \\ 0.0017 & 0.1667 \\ 0 & 0 \\ 0.0017 & -0.3333 \end{bmatrix}$$

Apply the LMI toolbox to solve the inequality (17). According to steps 2 ∼ 3, we can obtain

$$N_1 = \begin{bmatrix} -1.4149 & 0 & -0.4126 & 0.5000 & 1.8531 \\ -1.5287 & -7.5000 & -0.1558 & 0 & 0.2963 \\ 0.2460 & 0 & -3.2661 & -0.0017 & -1.8185 \\ -0.9653 & 0 & -0.2343 & 0 & 0.4589 \\ -1.6792 & 0 & -0.7953 & -0.0017 & -1.7693 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} -2.0500 & 0 & 0.4129 & 0.5000 & 0.3008 \\ -1.2437 & -8.0000 & -0.0098 & 0 & 0.0072 \\ -0.5914 & 0 & -3.3561 & -0.0017 & -1.6306 \\ -3.8215 & 0 & -0.0412 & 0 & 0.0441 \\ -0.3103 & 0 & -0.8245 & -0.0017 & -1.6910 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} -1.7935 & 0.2323 \\ 0.7646 & 0.0285 \\ 1.5434 & -0.8400 \\ 0.4831 & 0.0406 \\ 1.5103 & -0.4186 \end{bmatrix},$$

$$L_2 = \begin{bmatrix} -1.9738 & 0.1107 \\ 0.6218 & 0.0037 \\ 0.2974 & -0.8293 \\ 1.9107 & 0.0131 \\ 0.1611 & -0.4183 \end{bmatrix}.$$

Finally, according to step 5, we can obtain $\delta = 2.4062$, and the state feedback gain matrix is as follows:

$$K_1 = \begin{bmatrix} 0.5704 & -0.4129 & 2.6731 \end{bmatrix},$$
$$K_2 = \begin{bmatrix} 24.7483 & 2.4475 & 22.7443 \end{bmatrix}.$$

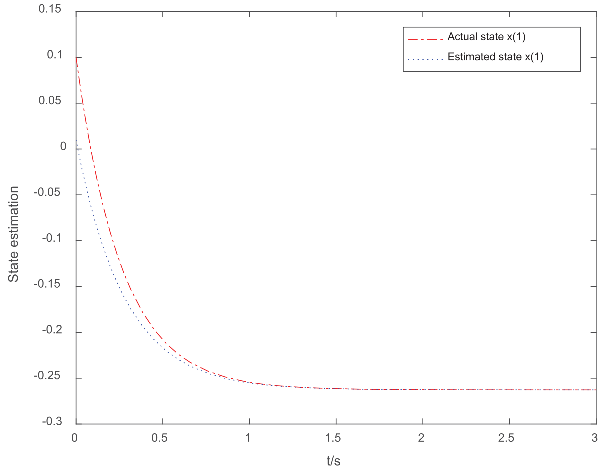

Figure 2. Actual and estimated value of state x_1.

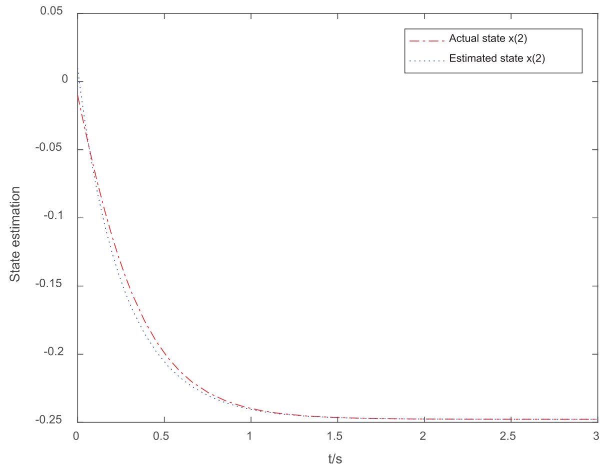

Figure 3. Actual and estimated value of state x_2.

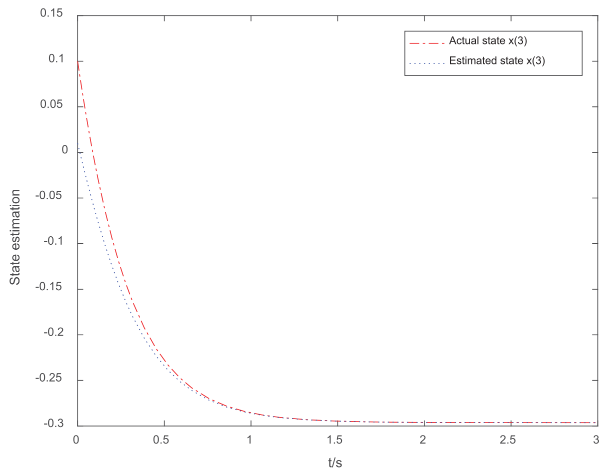

Figure 4. Actual and estimated value of state x_3.

For the convenience of simulation, we set the initial state as $x_0(\ 0.1 \quad -0.01 \quad 0.1\ )$, the actuator faults and the sensor faults as $f_a(t) = \sin(t)$ and $f_s(t) = \cos(t)$, respectively. The system state variable estimation is shown in Figures 2–4.

It can be seen from Figures 2–4 that the observer designed in this paper has a good estimation effect

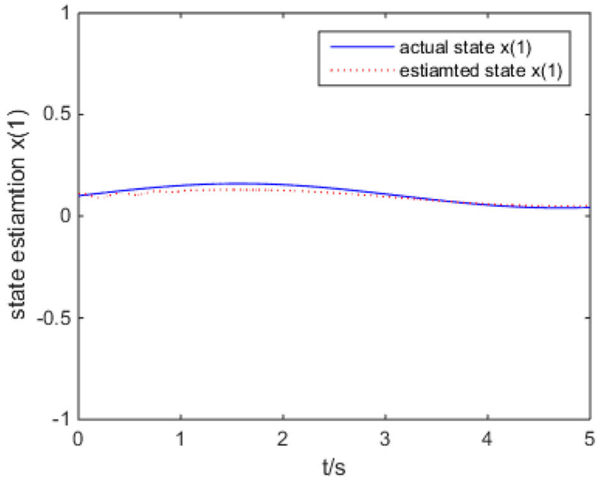

Figure 5. Actual and estimated value of state x_1.

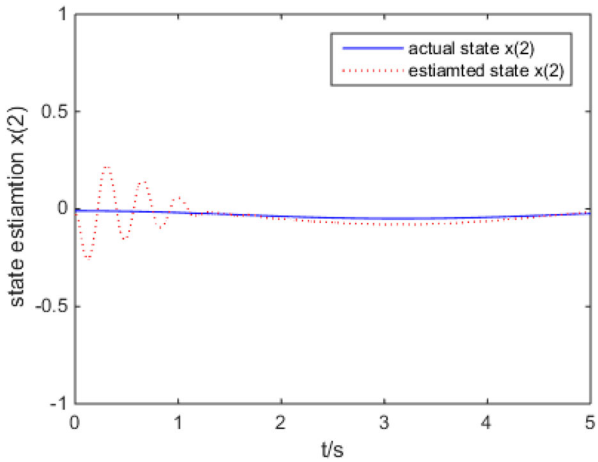

Figure 6. Actual and estimated value of state x_2.

on the state variables. The fault-tolerant controller is designed to make sure the system under the condition of actuator and sensor faults has a certain anti-interference ability and fault tolerance, the simulation results prove the effectiveness of the method.

In addition, it is compared with the method in [26]. But in this paper, the sensor failure and actuator failure are considered, which is more consistent with the actual control system. Considering that the states of the jump system cannot be directly obtained, in [26], under the action of the observer and the controller, the closed-loop state curves x_1, x_2 and x_3 are shown in Figures 5–7, respectively. After about 2s, the system reached stability. If the method in this paper is adopted, the state curvesx_1, x_2 and x_3 are shown in Figures 2–4, respectively. After about 1.5s, the system is stable, and the oscillation of the state trajectory curves is smaller. It can be seen that the controller designed in this paper is slightly better than [26] in terms of dynamic response speed. At the same time, in this paper, the sensor failure and actuator failure are also considered, more in line with the actual control system.

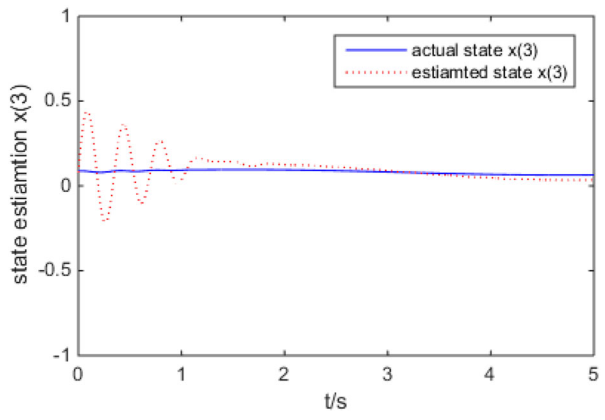

Figure 7. Actual and estimated value of state x_3.

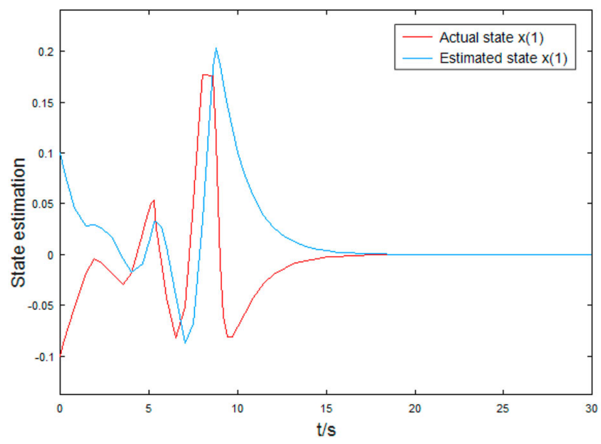

Figure 8. Actual and estimated value of state x_1.

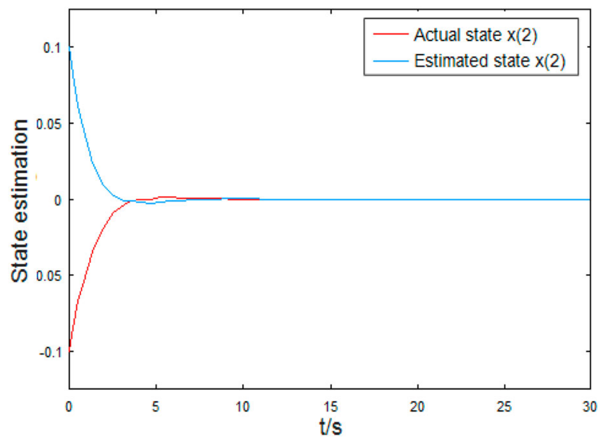

Figure 9. Actual and estimated value of state x_2.

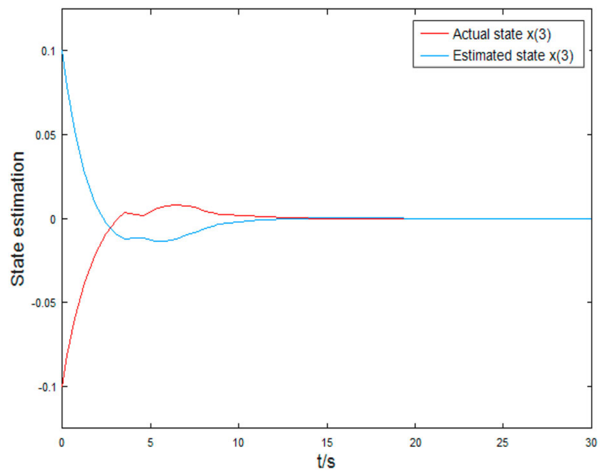

Figure 10. Actual and estimated value of state x_3.

4.2. Practical examples

In order to further verify the method designed in this paper, a simulation is carried out for a practical example to verify the effectiveness of the designed method. A linearized quadcopter model is selected [22, 27], the parameter matrix is as follows:

$$A(t) = \begin{bmatrix} -1.46 & 0 & 2.428 \\ 0.1643 + 0.5\beta(t) & -0.4 + \beta(t) & -0.3788 \\ 0.3107 & 0 & -2.23 \end{bmatrix}$$

$$A_{d1} = \begin{bmatrix} 0.1 & 0 & 0.1 \\ 0.1 & 0 & 0 \\ 0 & 0.1 & 0.2 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} 0.1 & 0 & 0.05 \\ 0.03 & 0 & 0 \\ 0 & 0.05 & 0.1 \end{bmatrix}$$

$$B_1 = B_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad D_1 = D_2 = \begin{bmatrix} 0.2 \\ 0.2 \\ 0 \end{bmatrix},$$

$$E_1 = \begin{bmatrix} 0 \\ -0.1 \\ 0 \end{bmatrix}, \quad E_2 = \begin{bmatrix} -0.1 \\ 0 \\ -0.3 \end{bmatrix}$$

$$C_1 = C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix},$$

$$F_1 = F_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

wherein $\beta(t)$ is the uncertain model parameter, which is subjected to a Markov process r(t) with $N = 2$:

$$\beta(t) = \begin{cases} -1, r(t) = 1 \\ -2, r(t) = 2 \end{cases}$$

The selection of other parameters is the same as that of the numerical example in Section 4.1.

For the convenience of simulation, we set the initial state as $x_0(\ -0.1 \quad -0.1 \quad -0.1\ )$, the actuator faults and the sensor faults as $f_a(t) = \sin(t)$ and $f_s(t) = \cos(t)$, respectively, and the delay time as 3s. The system state variable estimation is shown in Figures 8–10.

It can be clearly seen from Figures 8–10 that the system has certain fluctuations in the initial operating state. After a short adjustment process, the system reaches a stable state. The simulation results verify the effectiveness of the proposed method.

5. Conclusion

In this paper, robust fault estimation and fault-tolerant control for a class of nonlinear Markov jump time-delayed systems with both actuator and sensor faults were studied. Firstly, a generalized system is constructed by expanding the system state. Then a generalized observer design method is proposed for the system, and the simultaneous estimation of the original state, actuator fault and sensor fault of the nonlinear Markov jump time-delayed system is realized. In addition, based on the estimation of observer, a design method of state feedback fault-tolerant controller is proposed to make the system stable and meet certain performance indexes under the condition of simultaneous actuator and sensor failure and external interference. The sufficient conditions for the existence of generalized observers and fault-tolerant controllers are given by linear matrix inequalities. At last, a numerical example is given to demonstrate the effectiveness of the proposed method.

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References

[1] Cui KY, Zhu JF, Li CL. Exponential stabilization of Markov jump systems with mode-dependent mixed time-varying delays and unknown transition rates. Circ Syst Signal Process. 2019;38(10):4526–4547.
[2] Park BY, Kwon NK, Park PG. Stabilization of Markovian jump systems with incomplete knowledge of transition probabilities and input quantization. J Frankl Inst. 2015;352(10):4354–4365.
[3] Song Y, Dong H, Fei M. Mean square exponential stabilization of Markov networked control systems based on switching frequentness. Acta Autom Sin. 2012;38(05):876–881.
[4] Kao YG, Li W, Wang CH. Nonfragile observer-based H_∞ sliding mode control for *itô* stochastic systems with Markovian switching. Int J Robust Nonlinear Control. 2014;24(15):2035–2047.
[5] Kchaou M, El Hajjaji A, Toumi A. Non-fragile H∞ output feedback control design for continuous-time fuzzy systems. ISA Trans. 2015;54:3–14.
[6] Zhu LJ, Yin YY, Liu F, et al. H_∞ filtering for uncertain periodic Markov jump systems with periodic and partly unknown information. Circ Syst Signal Process. 2019;13(09):1309–1319.
[7] Wang HJ, Xue AK, Wang JH, et al. Event-based H-infinity filtering for discrete-time Markov jump systems with network-induced delay. J Frankl Inst Eng Appl Math. 2017;354(14):6170–6189.
[8] Shen H, Xu SY, Zhou JP, et al. Fuzzy H_∞ filtering for nonlinear Markovian jump neutral systems. Int J Syst Sci. 2011;42(5):767–780.
[9] Li FB, Wu LG, Shi P, et al. State estimation and sliding mode control for semi-Markovian jump systems with mismatched uncertainties. Automatica. 2015;51:385–393.
[10] Han CY, Wang W. Linear state estimation for Markov jump linear system with multi-channel observation delays and packet dropouts. Int J Syst Sci. 2019;50(01):163–177.
[11] Li XH, Choon KA, Lu DK, et al. Robust simultaneous fault estimation and nonfragile output feedback fault-tolerant control for Markovian jump systems. IEEE Trans Syst Man Cybern Syst. 2019;49(09):1769–1776.
[12] Li H, Shi P, Yao D. Adaptive sliding-mode control of Markov jump nonlinear systems with actuator faults. IEEE Trans Autom Control. 2017;62(4):1933–1939.
[13] Liu M, Cao XB, Shi P. Fuzzy-model-based fault-tolerant design for nonlinear stochastic systems against simultaneous sensor and actuator faults. IEEE Trans Fuzzy Syst. 2013;21(5):789–799.
[14] Chen LH, Huang XL, Fu SS. Fault-tolerant control for Markovian jump delay systems with an adaptive observer approach. Circuits Syst Signal Process. doi:10.1007/s00034-016-0277-8.
[15] Li HY, Gao HJ, Shi P, et al. Fault-tolerant control of Markovian jump stochastic systems via the augmented sliding mode observer approach. Automatica. 2014;50(7):1825–1834.
[16] Liu M, Shi P, Zhang LX, et al. Fault-tolerant control for nonlinear Markovian jump systems via proportional and derivative sliding mode observer technique. IEEE Trans Circ Syst Regul Pap. 2011;58(11):2755–2764.
[17] Ren W, Wang C, Lu Y. Fault estimation for time-varying Markovian jump systems with randomly occurring nonlinearities and time delays. Frankl Inst. 2017;354(3):1388–1402.
[18] Lu D, Li X, Liu J, et al. Fault estimation and fault-tolerant control of Markovian jump system with mixed mode-dependent time-varying delays via the adaptive observer approach. Dyn Syst Meas Contr. 2017;139:3.
[19] Lan J, Patton RJ. A new strategy for integration of fault estimation within fault-tolerant control. Automatica. 2016;69(1):48–59.
[20] Gao Y, Wu L, Shi P, et al. Sliding mode fault-tolerant control of uncertain system: A delta operator approach. Int J Robust Nonlinear Contr. 2017;27(18):4173–4187.
[21] Li X, Zhu F. Fault-tolerant control for Markovian jump systems with general uncertain transition rates against simultaneous actuator and sensor faults. Robust Nonlinear Contr. 2017;27:4245–4274.
[22] Li X, Zhu F. Simultaneous estimation of actuator and sensor faults for uncertain time-delayed Markovian jump systems. Acta Autom Sin. 2017;43(1):72–83.
[23] Shi P, Liu M, Zhang LX. Fault-tolerant sliding-mode-observer synthesis of Markovian jump systems using quantized measurements. IEEE Trans Ind Electron. 2015;62(9):5910–5918.
[24] Liu M, Shi P. Sensor fault estimation and tolerant control for *itô* Stochastic systems with a descriptor sliding mode approach. Automatica. 2013;49(5):1242–1250.
[25] Jiang B, Gao C, Xie J. Passivity based sliding mode control of uncertain singular Markovian jump systems with

time-varying delay and nonlinear perturbations. Appl Math Comput. 2015;271:187–200.

[26] He S, Liu F. Observer-based passive control for nonlinear uncertain time-delay jump systems. J Math Phys. 2009;29A(2):334–343.

[27] Li X, Karimi HR, Wang Y, et al. Robust fault estimation and fault-tolerant control for Markovian jump systems with general uncertain transition rates. J Frankl Inst. 2018;355:3508–3540.