STARS

University of Central Florida
STARS

Electronic Theses and Dissertations, 2004-2019

2015

Essays on Sales Force Career Incentives

Somnath Banerjee University of Central Florida

Part of the Marketing Commons Find similar works at: https://stars.library.ucf.edu/etd University of Central Florida Libraries http://library.ucf.edu

This Doctoral Dissertation (Open Access) is brought to you for free and open access by STARS. It has been accepted for inclusion in Electronic Theses and Dissertations, 2004-2019 by an authorized administrator of STARS. For more information, please contact STARS@ucf.edu.

STARS Citation

Banerjee, Somnath, "Essays on Sales Force Career Incentives" (2015). *Electronic Theses and Dissertations, 2004-2019.* 1202. https://stars.library.ucf.edu/etd/1202



ESSAYS ON SALES FORCE CAREER INCENTIVES

by

SOMNATH BANERJEE M.B.A., International Management Institute, New Delhi, India, 2005

B.Sc.(Physics Hons.), Patna University, Patna, India, 2002

A dissertation submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy in the Department of Marketing in the College of Business Administration at the University of Central Florida Orlando, Florida

Summer Term 2015

Major Professor: Axel Stock

© 2015 Somnath Banerjee

ABSTRACT

This dissertation uses game theoretic models in a principal-agent framework to study how firms optimally manage long term career related incentives for their sales people. When sales people put sales effort they face incentives not only from short term incentives like commissions and bonuses but also from long term rewards associated with progression in their career. In particular, sales people are often motivated to get promoted and avoid being laid off, to get selected to managerial positions and to form stronger relationships with customers so that they can bargain for higher wages in the future, respectively. Three different essays examine each of these three career related incentives and how firms can optimally manage them.

Essay 1 (Chapter 2) studies why and how firms use a type of promotion and layoff policy, called the Forced Ranking policy, to provide optimal long term career incentives to sales people. Findings from the essay suggests that when sales people are ambiguity averse and there is economic uncertainty regarding promotions and layoffs, firms are likely to commit to a promotion policy but may or may not commit to a layoff policy as part of Forced Ranking. Interestingly, it is shown that firms enjoying higher margins are more likely to commit to both promotion and layoffs, consistent with observations from industry practice. Results also suggest that in absence of costs from promoting and laying off employees, firms should use an up-or-out contract to motivate the sales force.

Essay 2 (Chapter 3) investigates how career incentives associated with promotion of sales employees to sales management roles may interfere with selection of the right sales managers. The essay was motivated by the common observation that organizations often promote their best sales people to sales managerial roles but after promotion find that the sales people are not as good as they were expected to be in their new roles, a phenomenon called Peter Principle. An alternative explanation for this phenomenon of adverse selection is provided and possible solutions are analyzed as part of the essay.

In essay 3 (Chapter 4) long term career incentives that sales reps face when they can form relationships with their customers are considered. Loyalty generated from customer-salesperson relationships is often "owned" by the sales person and it can be lost if the sales person moves to another firm. Therefore, firms compete for both customers as well as sales reps with the objective of poaching customers that are loyal to the sales reps. The essay analyzes how firms can deal with such a competition. Findings suggest that contrary to general beliefs, the presence of antiemployee poaching regulations like Non-Compete clauses, or tacit collusion to not poach each other's employees may hurt firm profits under some conditions.

Overall, the dissertation answers how firms can manage sales force career incentives to maximize profits.

I dedicate this dissertation to my mother Sunanda Banerjee, my father Shakti Kr. Banerjee, my wife Aditi Dutta, my mother-in-law Sumita Dutta, and my two daughters, Avni and Stuti Banerjee. Without the support, love and inspiration from my family, this dissertation would not have existed.

ACKNOWLEDGMENTS

I am deeply grateful to my advisor, Dr. Axel Stock, for guiding me through the dissertation research. This research originated from a term paper that I prepared for his seminar on Marketing Models and since then this research has benefitted immensely from his regular and detailed feedback. I would also like to thank my dissertation committee members, Dr. Anand Krishnamoorthy, Dr. Amit Joshi and Dr. Wilfred Amaldoss, for their support and for providing invaluable feedback during this research. Next, I would like to sincerely thank Dr. Gautham Vadakkepatt for inspiring and guiding me during the time I spent in the PhD program and for his contribution towards making me think more about managerial relevance of my dissertation research. I am also deeply indebted to Dr. Pradeep Bhardwaj, the PhD program coordinator of our department, for acting as a mentor at both professional and personal levels. I am thankful to Dr. Ronald Michaels and all the other faculty members in the department of Marketing at the University of Central Florida for creating a very supportive environment that was also very conducive for research. The financial support from Graduate Research Excellence Fellowship is gratefully acknowledged.

TABLE OF CONTENTS

LIST C	OF FIGU	JRES	xii
LIST C	OF TABI	LES	kiv
СНАР	TER 1:	INTRODUCTION	1
CHAP	PTER 2:	FORCED RANKING POLICIES - WHY AND HOW TO IMPLEMENT	
]	THEM TO MOTIVATE THE SALES FORCE	4
2.1	Introdu	ction	4
2.2	Literat	ure Review	9
2.3	Model		12
	2.3.1	Ambiguity Aversion and Ex Ante Uncertainty Regarding Ex Post Optimal	
		Rates of Promotion and Layoffs	13
	2.3.2	Firm's Forced Ranking Policy	15
	2.3.3	Sales Response Function	16
	2.3.4	Salesperson's Utility Function	17
	2.3.5	Game Sequence	18
2.4	Analys	is	19

	2.4.1	Optimal Committed Forced Ranking Policy	19
	2.4.2	Ambiguous vs. Committed Forced Ranking Policy	25
2.5	Extens	ions, Robustness and Further Discussion	28
	2.5.1	Commitment to Forced Ranking Policy	29
	2.5.2	Heterogeneity	31
	2.5.3	Interaction of Forced Ranking and Sales Compensation	32
2.6	Conclu	sion	38
	2.6.1	Managerial Implications	38
	2.6.2	Limitations and Directions for Future Research	40
CHAP	TER 3:	PETER PRINCIPLE IN SALES MANAGERIAL PROMOTIONS - AN AL-	
]	TERNATIVE EXPLANATION	45
3.1	Introdu	ction	45
3.2	Literat	ure Review	49
3.3	Model		51
	3.3.1	Sales Employee Characteristics	51
	3.3.2	Sales Response Functions	53
	3.3.3	Firm, its Decisions and Game Sequence	56

3.4	Analys	sis	57
	3.4.1	Adverse Selection in Sales Managerial Promotions	57
		3.4.1.1 Period 2	57
		3.4.1.2 Period 1	60
	3.4.2	Sales Training and Testing as a Screening Device	66
		3.4.2.1 Analysis	68
3.5	Extens	ions and Future Research	75
	3.5.1	Asymmetric Labor Market Conditions for Sales Managers and Sales Rep-	
		resentatives	75
	3.5.2	Uniform Salaries and Incentives for Sales Employees	77
3.6	Conclu	sions and Managerial Implications	77
CHAP	TER 4:	FIRM COMPETITION FOR SALES FORCE OWNED CUSTOMER LOY-	
	1	ALTY	82
4.1	Introdu	uction	82
4.2	Model		87
	4.2.1	Consumer Model	87
	4.2.2	Sales Representatives and their Decisions	91
	4.2.3	Firms, their Decisions and Game Sequence	93

4.3	Analys	sis and Results
	4.3.1	Firm Competition without Employee Poaching
		4.3.1.1 Period 2
		4.3.1.2 Period 1
	4.3.2	Firm Competition with Employee Poaching and Comparisons with the
		Case of No Employee Poaching
		4.3.2.1 Period 2
		4.3.2.2 Period 1
4.4	Extens	ions
	4.4.1	Empirical Evidence for Proposition 4.3
4.5	Conclu	ision
APPEN	NDIX A	PROOFS FOR THE RESULTS IN CHAPTER 2
A.1	Optima	al Forced Ranking Policy
A.2	Ambig	uous vs. Committed Forced Ranking Policy
A.3	Interac	tion between Forced Ranking and Sales Compensation
APPEN	NDIX B	PROOFS FOR THE RESULTS IN CHAPTER 3
APPEN	NDIX C	PROOFS FOR THE RESULTS IN CHAPTER 4

LIST OF REFERENCES		
--------------------	--	--

LIST OF FIGURES

Figure 2.1: Firms ex post staffing requirements at SE and SSE levels	42
Figure 2.2: Game Sequence	42
Figure 2.3: Ambiguous vs. Committed Forced Ranking Policy	43
Figure 2.4: Region where credible commitment to Forced Ranking Policy is not possible	43
Figure 2.5: Notation	44
Figure 3.1: Game Sequence Chapter 3	79
Figure 3.2: Adverse Selection Regions	80
Figure 3.3: Strategy Regions	80
Figure 3.4: Asymmetry in Labor Market and Adverse Selection Regions	81
Figure 4.1: State by State Survey of Non-Competes	113
Figure 4.2: Game Sequence Chapter 4	114
Figure 4.3: Employee Poaching and Non Employee Poaching Profits	114
Figure 4.4: Commercial Banking Industry	115
Figure 4.5: Software Services Industry	115
Figure 4.6: Insurance Industry	116

Figure 4.7: Energy Industry	•••				116
-----------------------------	-----	--	--	--	-----

Figure A.1:Exten	t of Approximation	in Taylor Approximation	
0	· · · · · · · · · · · · ·		

LIST OF TABLES

Table 3.1:	Period 2 Results
Table 4.1:	Regression Results
Table 4.2:	Net Profit Margin and Gross Profit Margin Accross Industries

CHAPTER 1: INTRODUCTION

This dissertation has been motivated by the observation that possibilities of career advancement and long term rewards associated with immediate selling effort acts as one of the strongest forms of incentive for most sales employees. For example, sales representatives routinely work hard to achieve promotion and avoid layoffs. Many sales representatives also put higher effort in order to get promoted to sales managerial positions to gain more responsibility and earn higher incentives through override commissions. Similarly, many of them put effort in acquiring new customers not just to earn direct commissions but in the hope that the acquired customer base will in future help them bargain for higher wages with the firm. Firms recognize these long term incentives and try to optimally manage them through levers like promotion and layoff policies or through policies regarding selection of sales managers or through use of legal clauses like Non-Compete laws, respectively. However, much of the current quantitative sales force literature is focused on studying short term incentive measures like commission pay etc. This dissertation consists of three essays on how firms can manage such long term career related incentives for sales employees and maximize profits.

In the first essay of this dissertation (Chapter 2) we examine why and how sales firms use a type of promotion and layoff policy called the Forced Ranking Policy to provide optimal long term career incentives to sales employees. Forced Ranking policies are a type of rank based performance appraisal policy in which employees are ranked and promoted or laid off on basis of their sales performance. A classic example is General Electric's 20-70-10 Forced Ranking policy which involved ranking of employees on basis of performance and then promotion and rewards for the top 20 percentile, inflation adjusted salaries and training for the middle 70 percentile and layoff or probation for the bottom 10 percentile of the workforce. Although these schemes are popular in industry practice, they raise some puzzling issues: Why do some firms commit to specific

proportions ex-ante, e.g. 20-70-10, and some do not? What percentage of sales people are to be promoted, laid off and left in the same position as part of such policies i.e. why use a 20-70-10 policy and not a 30-60-10 policy or something else? How do these policies interact with sales compensation? In this paper we use an agency theoretic framework to address these questions. We find that when sales people are ambiguity averse and there is economic uncertainty regarding promotions and layoffs, firms are likely to commit to a promotion policy but may or may not commit to a layoff policy. Interestingly, we show that firms enjoying higher margins are more likely to commit to both promotion and layoffs, consistent with observations from industry practice. Our results also suggest that in absence of costs from promoting and laying off employees, firms should use an up-or-out contract to motivate the sales force. Furthermore, we find that while promotions and performance pay can be complements, layoffs and performance pay are always substitutes. We identify conditions under which, counterintuitively, an increase in uncertainty may lead a firm to increase performance pay in the presence of Forced Ranking policies.

In the second essay (Chapter 3) we investigate career incentives associated with promotion of sales employees to sales management roles. The essay was motivated by the common observation that organizations often promote their best sales people to sales managerial roles but after promotion find that the sales people are not as good as they were expected to be in their new roles. This problem is attributed to Peter Principle and it is generally believed that lack of managerial skills in the promoted sales people is to be blamed for the problem. We show that the problem can arise even without consideration of managerial skills. We find that the problem can arise because promotion to managerial positions entails two different benefits associated with responsibilities of managing multiple revenue streams viz. higher sales potential due to multiple revenue streams and a possibility to diversify one's risk across multiple revenue streams. The two benefits generate asymmetric incentives in a sales force consisting of sales reps of heterogeneous risk and productivity characteristics such that the moderately risk averse reps put more effort than others to achieve promotion in spite of the fact that they are not the ones who would be the most suitable for managerial roles. Further, we find that there can even be conditions in which an employee with lower productivity and higher risk aversion gets promoted more often. The Peter Principle problem becomes more likely with an increase in productivity of the more risk averse employee. To deal with the problem that we identify we study and evaluate the different solutions being currently implemented by companies and find that under certain conditions sales training can be used by organizations to screen undesirable employees. Finally, we also study some sales force labor markets conditions that moderate the adverse selection problem.

In the third essay (Chapter 4) we consider long term career incentives that sales reps face when they can potentially form relationships with their customers. For example, firms often recruit sales representatives to build relationships with customers and to sell them products over time. In such a case of relationship marketing, customers build loyalty not only towards the firms but also towards their sales representatives. However, since the loyalty generated from customersalesperson relationships are often "owned" by the sales person they can be lost if the sales person moves to another firm. In this context, firms compete for both customers as well as sales reps with the objective of poaching customers that are loyal to the sales reps. We use a two period game theoretic model of duopolistic competition to study firms' salesforce compensation strategy and profits in this scenario. Our analysis interestingly reveals that under some conditions the possibility of poaching of sales reps can actually increase firm profits. We find that although the possibility of poaching of sales reps increases employee retention and wage costs, it also leads to a strategic benefit in form of softening of competition for acquisition of new customers. Our finding implies that contrary to general belief, the presence of anti-employee poaching regulations like Non-Compete clauses may hurt firms under some conditions. Furthermore, we find that if the intensity of competition in such a market is high then firm profits follow an inverted-u relationship with respect to profit margins. We report some empirical evidence for this result.

CHAPTER 2: FORCED RANKING POLICIES - WHY AND HOW TO IMPLEMENT THEM TO MOTIVATE THE SALES FORCE

2.1 Introduction

Policies related to promotions and layoffs are strategic levers that are often used by organizations to manage sales employees. The two primary functions of such policies are to motivate employees and to sort them according to their abilities. They are also often implemented through career tournaments in which employees are promoted, laid-off or retained in the same position as before on basis of their rank performance rather than absolute performance. Lately, many organizations have structured their rank based promotion and layoff policies together and integrated them with their yearly performance appraisal systems, collectively termed as the forced distribution ranking systems or Forced Ranking (henceforth abbreviated as FR) policies (Grote 2005). For example, the most well-known of such policies, the 20-70-10 Forced Ranking policy, popularized by Jack Welsh that he implemented at GE involved promotion and rewards for the top 20 percentile, inflation adjusted salaries and training for the middle 70 percentile and layoff or probation for the bottom 10 percentile of the workforce.

According to some estimates 60% of Fortune 500 companies, including Glaxo SmithKline, Hewlett-Packard, 3M, AIG, EDS and Ford, may be currently using similar policies (WSJ 2012). In the academic literature, Berger, Harbring and Siwaka (2012) show utilizing experiments that FR policies can lead to 12% higher productivity. Recent field experiments in sales organizations implementing policies like FR too provide similar results (Ahearne et al. 2012). Not surprisingly many of the organizations also use such policies to increase their sales force productivity (Zoltners, Sinha and Lorimer 2011a) considering that as opposed to most other functions, the sales function involves a

large extent of unobservable field effort that can be extremely costly, if not impossible, to monitor. For example, Sanofi-Aventis implemented a FR policy for approximately 1000 of its sales force professionals (Training 2008). Similarly, a presentation by Heinz HR made at the International Congress on Assessment Center Methods (Nye and Murphy 2002) suggests that the company's implementation of a FR policy helped increase its market share from 47% to 59% in the Ketchup market and achieve many more milestones. The use of FR tools in managing sales forces has become more prevalent than for other functions mainly because many of the criticisms about such policies do not hold true in general in case of sales forces. First, bias and discrimination are generally less possible as sales employees are ranked according to objective sales data rather than subjective assessment measures¹. Second, lack of collaboration or excessive competition among members of teams induced by such tools is unlikely to occur because many sales jobs do not require any form of team collaboration.

While it is clear that the use of FR in sales force management continues to be popular, the academic literature in Marketing has yet to analyze why and how firms should design FR policies to motivate sales employees. Therefore, we investigate the following research questions. First, while past research (DeVaro 2006, Baker, Jensen and Murphy 1988) suggests that most organizations make promotion and layoff decisions based on rank order performance data it is not clear why only a subset of these firms commit themselves to FR proportions ex ante, e.g. 20-70-10. Not committing to specific proportions in a FR policy creates ambiguity regarding the FR policy. Can such ambiguity be beneficial for firms? Past research in management and economics provide conflicting views on ambiguity in performance management systems with some arguing for a smoke-and-mirrors approach to performance management while others arguing for arbitrary but concrete rules (Gibbs 1991). Second, how do firms design FR policies, i.e. why does an organization like GE use a

¹Prior legal cases like Ropper vs. Exxon suggest that forced ranking is acceptable under law as long as assessment measures are objective (Grote 2005).

20-70-10 FR policy and not a 30-50-20 policy or any other policy for that matter? For example, AIG uses a 10-80-10 policy, Hewlett-Packard uses a 1-5 scale with 15% receiving the best grade of 5 and 15% receiving 1's. Which of these policies should be implemented? Moreover, how does such a policy change with salesforce characteristics and market conditions like total number of employees in the firm, uncertainty, etc.? Third, how is FR to be used with other instruments to motivate the sales force? More specifically, does FR complement or substitute performance pay? How do exogenous factors like individual specific uncertainty and costs associated with FR affect performance pay when performance pay is used in presence of FR?

To address the above questions we consider a tournament theory model similar to Kalra and Shi (2001) (henceforth, referred to as KS (2001)) in which a firm with a homogeneous sales force first decides whether to commit to a FR policy or to let it be ambiguous. We assume that the ex post optimal promotion and layoff policy is uncertain ex ante because of economic uncertainty. If the firm commits to a FR policy, it announces the proportions of promotions and probations (or layoffs) as part of the FR policy keeping in mind that it may have to incur additional costs if its ex ante commitment is different from the ex post optimal promotion and layoff policy. We assume that the wages associated with promotion and probation are exogenous to the firm's FR policy because wages for specific job designations are often determined through various factors that are not in control of the firm, e.g. competition in the labor market. Once the firm decides on its FR policy, risk and ambiguity averse sales employees compete with each other to get promoted and to avoid being put on probation. Promotion and probation decisions are made on basis of sales output generated which in turn are a function of effort made by the employees, individual specific uncertainty facing each employee and the common uncertainty that everyone faces.

We contribute to the existing sales force literature by extending it to include incentives from career tournaments. To our knowledge we are the first paper to introduce the concept of ambiguity aversion in the sales force literature. Our first contribution is, by incorporating ambiguity aversion of sales employees faced with economic uncertainty regarding the ex post optimal rate of promotions and layoffs, to show that firms are likely to ex ante commit to a promotion policy but may or may not commit to their layoff policy. We find this because ambiguity averse sales reps, when faced with uncertainty, put more weightage on a more pessimistic scenario. While a more pessimistic scenario in case of promotions leads to a less motivating contest structure, such a pessimistic scenario related to layoffs may lead to a more motivating contest structure. Furthermore, we show that firms keep their layoff policies ambiguous if the severity of layoffs is expected to be intermediate in case of an ex post adverse economic scenario. The rationale underlying this finding is that in this case keeping the layoff policy ambiguous is close to the incentive optimal contract and avoids having to pay any costs for deviation from the ex post optimal. In addition, we find that firms with higher profit margins are more likely to commit to a FR policy potentially explaining why we mostly hear about profitable firms like GE and Microsoft doing so. Finally, recognizing that a firm could renege on its FR policy in a one period model, we study an infinite horizon repeated game and identify conditions under which the firm is able to credibly commit to a FR policy.

Our second contribution is to identify the optimal design of a FR policy under commitment. We find that in contrast to the related theory from sales contests (KS 2001), which suggests that rank ordered tournaments are more efficient than multiple winner contests, in absence of costs of promoting and laying off employees the most effective form of a FR system is an up-or-out contest, a type of multiple winner contest. However, costs involved in promotion and layoff of employees lead to a situation where some of the middle ranked employees are left in the same position as before. The percentile range of these middle rank employees is determined by the impact of different exogenous factors like productivity of employees, uncertainty, profit margin, and number of employees. For example, we find that an increase in risk aversion of employees leads to an increase in proportion of promotion and layoffs as part of the optimal FR policy. We trace the divergence of

these results from those of KS (2001) to the difference in structures of FR contest and sales contest. More specifically, our results diverge because in case of FR the firm only determines the proportion of winners and losers given a flexible budget while in case of a sales contest it determines both the size of prizes as well as proportion of winners and losers, given a fixed budget.

Our third contribution is an analysis of the interaction between FR and sales compensation. We find that for small levels of risk aversion promotions and performance pay may be complements or substitutes depending on whether the cost of promotion is lower or higher than the change in reservation utility from promotions. On the contrary, we show that layoffs and performance pay are always substitutes. The rationale for this finding is as follows. While a marginal increase in rate of promotion increases the expected utility of participation for sales employees and thereby loosens the participation contract, a profit maximizing firm extracts this surplus by providing less risk premium i.e. by using higher performance pay. Fourthly, we contribute to the extant literature by finding conditions under which performance pay increases with uncertainty. We find that when risk aversion is small, an increase in employee specific uncertainty or unsystematic risk reduces the effectiveness of both performance pay as well as FR, a result consistent with the literature. However, the decrease in effectiveness of the two mechanisms need not necessarily be symmetric and when risk aversion is small, the FR mechanism becomes ineffective at a much faster rate than performance pay. If this happens, then the firm substitutes performance pay in place of FR along with an overall decrease in use of both FR and performance pay. When risk aversion is small the substitution effect is stronger than the effect of decrease in use of performance pay under some conditions and therefore, the use of performance pay counter intuitively increases with an increase in employee specific uncertainty in this case.

Our paper is organized as follows. In the next section we review the related literature streams in Marketing and Economics. In section 2.3 we introduce the modeling framework for the tournament that we employ in this paper. The analysis of the base model is presented in section 2.4. In section

2.5 we present a few extensions and robustness checks for the model, including our analysis of the interaction of the FR policy with performance pay. We conclude the paper by summarizing the results and by discussing limitations and avenues for future research in section 2.6.

2.2 Literature Review

Multiple streams of literature in Economics, viz. human-capital acquisition, job assignment, learning, efficiency wages and tournaments, provide a foundation for theories related to promotions and layoffs in organizations (for a broad review of these topics see Gibbons (1997)). Our research is primarily related to the literature on tournament theory.

Tournament theory studies why and how wage differences are determined on basis of an individual's relative performance compared to others rather than on basis of an individual's absolute performance. The seminal papers on tournaments (Lazear and Rosen 1981, Green and Stokey 1983, Nalebuff and Stiglitz 1983) identify conditions under which tournament based incentives are better than performance pay. In doing so, these papers distinguish between individual specific uncertainty and common uncertainty. Tournaments have an advantage over performance pay because they eliminate the common uncertainty, i.e. the uncertainty that is common to all the employees, from performance measurement. If the common uncertainty is high as compared to the individual specific uncertainty then tournaments are more appropriate than performance pay. Subsequent empirical literature confirms that promotion and layoff decisions in firms are made through tournaments (DeVaro 2006).

A number of papers in Marketing (KS 2001, Lim, Ahearne and Ham 2009, Lim 2010, Chen, Ham and Lim 2011, Ridlon and Shin 2013) and Economics (Moldovanu and Sela 2001) study how tournaments are designed optimally. KS (2001) study optimal design of a sales contest for a given

budget and find that rank ordered contests are superior to multiple winner contests if uncertainty is distributed logistically. Our tournament model is similar to that of KS (2001) but we assume the value of the prizes and costs associated with the promotions and layoffs to be exogenous to a firm's optimal tournament design decision. We do this because as opposed to sales contests, the prizes, i.e. the wages at different designations, are determined through a number of factors, like competition in the labor market and job design, which are not under the complete control of the firm. This is also consistent with empirical evidence (Lazear and Oyer 2004). For example, Lazear and Oyer (2004) conclude that "in the long run wages paid by the typical firm are determined by prevailing wages in the market, not by the conditions in the firm". Other Marketing papers study different aspects of contest design. Lim (2010) introduces the notion of social loss aversion in sales contests to show that under certain conditions the optimal contest may have a higher number of winners than losers. In a recent paper, Ridlon and Shin (2013) study how contests need to be handicapped when firms learn about asymmetric abilities of employees and when contests are carried out over time. Lim et al. (2009) and Chen et al. (2011) conduct laboratory experiments to test implications of sales contest theory. Our research models aspects of career tournaments that have not been analyzed and contributes to the literature on optimal contest design in three ways.

First, we show that the design of career tournaments like FR need not follow the predictions of the sales contest literature. For example, we find that the firm reduces the number of promotions as risk aversion increases in sales force while the sales contest literature predicts an increase in number of rewards (KS 2001).

Second, we study whether or not a firm would at all commit to a tournament design and show that under some conditions the firm may keep the design ambiguous. The driving forces for such a decision in our model are economic uncertainty regarding ex post optimal promotions and layoffs, and ambiguity aversion among employees. To our best knowledge, this is the first paper in the sales force literature to model ambiguity aversion (Ellsberg 1961, Gilboa and Schmeidler 1989). We conceptualize ambiguity as uncertainty about the probability distributions of promotion and layoff decisions by the firm if it does not commit to a FR policy. In such a circumstance it has been shown that the ambiguity averse decision maker (sales rep) violates the axiom of reduction of compound lotteries by putting more weight on less desirable outcomes (Ellsberg 1961). Weinschenk (2010) has studied moral hazard in presence of ambiguity aversion. However, he consider ambiguity in the performance measure while we consider ambiguity in design of contract.

Third, we also consider how career tournaments, i.e. FR, interact with performance pay. Our analysis of interaction between a FR policy and performance pay also extends the sales force compensation literature. The seminal paper in this literature, Basu, Lal, Srinivasan and Staelin (1985), finds that performance pays always decreases with uncertainty. While subsequent theoretical studies (Joseph and Thevarajan 1998, Lal and Srinivasan 1993) confirm this finding, empirical studies (John and Weitz 1989, Coughlan and Narsimhan 1992, Krishnamoorthy et al. 2005) report an ambiguous relationship between performance pay and uncertainty. Our research on interaction between performance pay and FR provides a potential explanation for this empirical observation by showing conditions under which performance pay can increase with uncertainty. The only other paper (Schöttner and Thiele 2010) that studies the interaction between performance pay and tournament considers risk neutrality of agents and selection aspects of tournaments and, therefore, provides insights about the usage of performance pay complementary to ours. More specifically, Schöttner and Thiele (2010) find that the optimal interplay of promotion tournaments and linear performance pay may involve low-powered individual incentives only when motivation and selection issues arise simultaneously because linear incentives may interfere with the selection of high ability managers by means of a promotion tournament.

Next, we outline our modeling framework. The notation used throughout the paper, presented in Figure 2.4, and all other Figures and Tables are given at the end of the Chapter.

2.3 Model

Consider a risk and ambiguity neutral sales organization that has three hierarchical levels, viz. senior sales executive (SSE), sales executive (SE) and sales executive on probation (SEp), for its sales employees. The higher designations are associated with a higher reservation wage, i.e. we assume the wage at SSE, SE and SEp levels to be U_P , U_S and U_L , respectively, where $U_P > U_S >$ $U_L \ge 0^2$. The exogeniety of the wages to the firm's FR decision can be due to multiple constraints on the firm's wage decisions. For example, the firm may set the wages for designations centrally, but changes its FR policy at a local level to account for regional differences in market conditions. Similarly, the firm may have longer term agreements on uniform wages for different designations and cannot change the wages according to FR policies conducted every period. Alternatively, it can be the case that a certain wage rate at a level is the market norm.

In the current model the firm is primarily concerned with motivating the employees at the SE level through a Forced Ranking (FR) policy in a single period. The principal (employer) employs N(i = 1, 2, ...N) identical and homogenous risk and ambiguity averse sales employees at the SE position each of whom are assigned to one of N identical territories. Moreover, there is no team activity and no collusion among SEs. As the territories are independent there is no possibility of SEs sabotaging each other's performances.

 $^{^{2}}$ We tie the wage levels to the job roles and not individuals. This is in line with empirical literature (Baker, Jensen and Murphy 1988).

2.3.1 Ambiguity Aversion and Ex Ante Uncertainty Regarding Ex Post Optimal Rates of Promotion and Layoffs

We assume that the firm's staffing requirements at different levels change from time to time. Let \widetilde{k}_P and \widetilde{k}_L be the proportions of SE employees to be promoted and laid off, respectively, at the end of the period according to the staffing requirement of the firm at the end of the period. These proportions are determined by market forces and situations within the firm at the end of the period. For example, if a number of employees from the SSE positions leave the firm creating vacancies at the higher position then \tilde{k}_P will be higher. Similarly, if market potential decreases then the firm may find it optimal to lay off at a higher rate at the end of the period, i.e. \tilde{k}_L will be higher. Further, we assume that the firm faces ex ante uncertainty about its staffing requirement at the end of the period, i.e. it does not know for sure at the start of the appraisal period how many employees it should promote or lay off at the end of the period, because that decision will depend on how economic and market forces play out over the period and on what the market potential is at the end of the period. Thus, we assume that there is uncertainty about ex post optimal rate of promotions, \widetilde{k}_P , and layoffs, \widetilde{k}_L , and, for simplicity, that both the firm and the salespeople have the same subjective beliefs about the probability distributions related to \tilde{k}_P and \tilde{k}_L . This subjective nature of uncertainty is crucial to our model as we discuss below and in subsection 2.3.3. We assume that $\tilde{k}_P \leq \frac{1}{2}$ and by construction $\tilde{k}_P + \tilde{k}_L \leq 1$. We assume $\tilde{k}_P \leq \frac{1}{2}$ because firms rarely have needs to promote more than 50% of employees (Average proportion of promotion in organizations tend to be less than 10% as per World at Work (2012, 2013) promotion surveys). We assume that $\tilde{k}_P = \{0, k_P\}$ and $\tilde{k}_L = \{0, k_L\}$. To explain our model further we refer to Figure 2.1.

Suppose there are two possible ex post scenarios related to staffing requirements for the SE and SSE levels denoted as "good" and "bad" scenario, respectively. For the staffing requirement at the SE level, the subjective beliefs about each of the scenarios are given by q and 1 - q, respectively.

Further, the favorable scenario is such that it involves less likelihood of ex post layoffs being optimal and the adverse scenario is such that it involves a higher likelihood of there being layoffs. The likelihoods for layoffs in "good" and "bad" scenarios are denoted as P_G and P_B , respectively, where $0 \leq P_G < P_B \leq 1$. For parsimony, we assume the distributions to be degenerate, i.e. $P_G = 0$ and $P_B = 1^3$. Similarly, we assume that there are two possible staffing requirements at the SSE level, leading to different likelihoods of ex post promotions being optimal. The likelihoods for promotions in "good" and "bad" scenarios are given as P_V and $P_N V$, respectively, where $0 \leq P_N V < P_V \leq 1$. For parsimony, we assume $P_V = 1$ and $P_N V = 0$. Parameter m is the subjective belief of there being a favorable scenario related to the staffing requirement for the SSE level. To keep the analysis simple, we assume that q and m are independent. Furthermore, we assume q and m to both be $\frac{1}{2}$ in order to reduce the number of parameters in the model, and because it implies that neither scenario is more likely to occur⁴. Facing this situation an ambiguity neutral decision maker, e.g. the firm, considers a weighted average of all the scenarios. However, an ambiguity averse individual, e.g. the sales representatives, puts more weight on the scenario that give them less expected value, i.e. they are pessimistic. This nature of decision making was first illustrated by the Ellsberg experiment (Ellsberg 1961). In modeling ambiguity aversion amongst sales representatives we assume, for the sake of simplicity, that they only consider the worst case scenario possible when faced with ambiguity, consistent with a MaxMin utility function (Gilboa and Schmeidler 1989). This is equivalent to assuming that the decision maker is infinitely ambiguity averse, a simpler formulation that helps us capture the phenomenon without the loss of much generality. The mathematical formulation of the employee's utility maximization problem is given in subsection 2.3.4.

³Qualitative results are not expected to change as long as $0 \leq P_G < P_B \leq 1$.

⁴Analysis for the general case where $q, m \in [0, 1]$ is available from the author.

2.3.2 Firm's Forced Ranking Policy

A firm can pursue two types of Forced Ranking (FR) policies viz. a), a committed FR policy, or b), an ambiguous FR policy. In a committed FR policy the firm declares at the beginning of the tournament that the first β_2 proportion of employees are to be promoted to the SSE position, the last β_1 proportion of employees are to be demoted to the SEp position and the middle $1 - \beta_2 - \beta_1$ proportion of employees are to be left in the SE position according to the sales performance ranks that the employees achieve. Contrary, in an ambiguous FR policy the firm only declares that it will make promotion and layoff decisions on basis of sales performance ranks that the employees achieve, but it does not commit to any proportion of promotion or layoff, i.e. it does not specify β_2 and β_1 . This allows the firm to make promotion and layoff decisions based on ex post optimal rates, i.e. $\beta_2 = \tilde{k}_P$ and $\beta_1 = \tilde{k}_L$, respectively. In our analysis, we also explore the case when the firm can commit to only one aspect of its FR policy, i.e. β_2 or β_1 and leave the other ambiguous.

Finally, we assume that the firm faces costs if it deviates from the ex post optimal rate of promotions and layoffs. We denote this cost by f and assume that it is linear in deviation of the firm's promotion and layoff policy from the ex post optimal. For example, if the ex post optimal rate of promotion is realized as $\tilde{k}_P = 0$ and the firm had committed to a Forced Ranking policy with $\beta_2 > 0$ then it will incur a cost of $(|\beta_2 - \tilde{k}_P|) f = \beta_2 f$. We also assume that there are absolute costs involved in promoting and laying off employees which increase linearly at the rate of c with the proportion of promotion and layoffs. For example, the firm incurs a cost of $(\beta_2 + \beta_1) c$ in case of a committed FR policy and a cost of $(\tilde{k}_P + \tilde{k}_L) c$ in case of an ambiguous FR policy. An example of cost c could be the expenditure on severance pay for laid off employees and training for promoted employees, while an example of cost f could be expenditure on recruiting additional employees if the firm committed to laying off more than the ex post optimal.

We first analyze the case when salespeople earn a fixed salary. Subsequently, in section 2.5.3, we

consider the case when a linear contract is used along with FR. The usage of linear contracts is common in industry practice and consistent with the academic literature in Marketing and Economics (Holmström and Milgrom 1991, Joseph and Thevaranjan 1998). Furthermore, we assume that the firm cannot renege on its FR policy else such a policy will lose its credibility. We relax this assumption in Section 2.5.1. Note that the model focuses on the motivation aspects and abstracts away from the internal selection aspects of promotions and layoffs to keep parsimony. However, our results are not expected to qualitatively change if there are selection aspects to the promotions as we discuss in section 2.5.2.

2.3.3 Sales Response Function

Effort on part of a salesperson is observable only to himself and the employer is only able to observe the sales output of each salesperson. The output of a salesperson is a function of his effort and a stochastic component capturing the uncertainty that the sales person faces in the environment. The sales response function is given as,

$$x(e_i) = e_i + \varepsilon_i + \gamma \tag{2.1}$$

where $x(e_i)$ represents sales given selling effort e_i . We denote ε_i as the risk⁵ unique to each territory such that $E[\varepsilon_i] = 0$, $Var[\varepsilon_i] = \sigma_{\varepsilon}^2$ with cumulative distribution function F(.) and density function f(.). Furthermore, we assume that ε_i follows a logistic distribution. This assumption has been primarily made for two reasons. Firstly, logistic distributions are symmetrical and very similar to normal distributions therefore capturing the distribution of sales territory uncertainty well, in

⁵The objective probability distribution of this uncertainty is common knowledge. Hence, there is no ambiguity about the sales task. This is reasonable because most sales reps spend enough time in a position to know the probability distribution associated with their task.

particular for firms with a large number of territories. Secondly, using the logistic distribution to model sales territory uncertainty follows past literature (KS 2001) thereby allowing us to both compare our results to theirs, and to simplify computations. In the model, γ is the risk common to all the territories such that $E[\gamma] = 0$ with cumulative distribution function G(.). We also assume that γ and ε_i are i.i.d. Note that there is a critical distinction between risk related to the sales response function and the uncertainty related to expost optimal promotion and layoff rates. In case of the individual specific risk related to sales response function everyone objectively knows the characteristics of the probability distribution while in case of the subjective uncertainty regarding ex post optimal promotion/layoff policy no one knows the probability distribution objectively and they only have subjective priors about what the distribution may look like. This distinction is critical because in case of risk associated with the sales response function the employees are risk averse while in case of the uncertainty associated with the expost promotion and layoff rates the employees are ambiguity averse. The rationale for assuming that the sales response function is objectively known is that employees and firms have more experience with selling and risks associated with it because they carry out the activity over again and again. In contrast, ex post optimal promotion and layoff rates are difficult to estimate and there is subjective uncertainty about them. The firm's profit margin from a unit of sales is δ .

2.3.4 Salesperson's Utility Function

When faced with objective risks as described above the i^{th} salesperson has a utility function given as $U(I_i, e_i) = u(I_i) - c(e_i)$ where u(.) is the von Neumann-Morgenstern utility function with a CARA coefficient of r and c(.) is the cost of effort or the disutility function such that $c(e) = \frac{e^2}{2}$. To model ambiguity aversion among employees we consider the MaxMin utility framework (Gilboa and Schmeidler 1989), which means that faced with subjective uncertainty employees consider the state of the world that gives them the minimum expected utility. According to empirical estimates most Americans are ambiguity averse (Dimmock et al. 2013). Thus, considering risk and ambiguity aversion, the incentive compatibility and individual rationality constraints are given as,

$$e_i^* = \arg\max_{e_i} \left(\min E[U(I_i, e_i)]\right) \quad (IC) \tag{2.2}$$

$$\min E[U(I_i, e_i^*)] \ge U_S \quad (\mathbf{IR})$$
(2.3)

Where,
$$E[U(I_i, e_i)] = \Pr(j \leq N\beta_2)u(U_P) + \Pr(N\beta_2 < j \leq N\beta_1)u(U_S)]$$

+ $\Pr(N(1 - \beta_1) < j)u(U_L) - c(e_i), \ j \text{ is the rank of the } i^{th} \text{ sales rep and},$
 $\beta_2 = \widetilde{k}_P \text{ and } \beta_1 = \widetilde{k}_L \text{ in case of an ambiguous FR policy}$

2.3.5 Game Sequence

Next, we describe the game sequence (See Figure 2.2). In stage 1, the firm decides on whether to commit to a FR policy or leave it ambiguous and in case of commitment it needs to determine the policy (β_1, β_2) . In stage 2 the employees accept the contract and put effort. In stage 3 nature resolves uncertainty (both objective risk and subjective uncertainty), sales performance is observed and the firm incurs costs associated with promotions and layoffs. Complete information regarding model parameters is assumed throughout the paper. We solve for the symmetric equilibrium using the notion of subgame perfect Nash Equilibrium. Next, we discuss our analysis of the model.

2.4 Analysis

2.4.1 Optimal Committed Forced Ranking Policy

Given the above set up, if the firm decides to commit to a FR policy then it solves the following constrained maximization problem:

$$\Pi_{CC} = \max_{\beta_1, \beta_2 \in [0,1]} \pi_{CC} = E \begin{bmatrix} \delta \sum (e_i^*(\beta_1, \beta_2) + \varepsilon_i + \gamma) - Nw_1 - N(\beta_1 + \beta_2)c \\ -fN(m(|\beta_2 - k_P|) + (1 - m)\beta_2) + q(|\beta_1 - k_L|) + (1 - q)\beta_1) \end{bmatrix}$$
(2.4)

subject to (2.2), (2.3) and $\beta_2 + \beta_1 \leq 1$

The firm benefits from the effort induced through FR, while it pays the current wage and the costs associated with FR. Since we consider different combination of committed and ambiguous Forced Ranking policy in our analysis we denote the firm's profit as Π_{ij} , where i(j) =

 $\left\{\begin{array}{c} C, \text{ if the firm commits to a promotion (layoff) policy} \\ A, \text{ if the firm leaves the promotion (layoff) policy ambiguous} \end{array}\right\}.$ Therefore, in case of a com-

mitted Forced Ranking policy profit is denoted as Π_{CC} . Next we present our first proposition.

Proposition 2.1 The optimal proportion of employees to be promoted, put on probation and kept in same position is given as β_2^* , β_1^* , $1 - \beta_2^* - \beta_1^*$, respectively. Where,

$$\beta_{2}^{*} = \left\{ \begin{array}{c} \underline{k}_{P} = \frac{1}{2} \left[1 - \frac{(N+1)d}{\delta N(U_{PS})} \left(c + f - \lambda_{1}^{*}U_{PS} \right) \right] \, if \, k_{P} < \underline{k}_{P} \\ k_{P} \, if \, \underline{k}_{P} < k_{P} < \overline{k}_{P} \\ \overline{k}_{P} = \frac{1}{2} \left[1 - \frac{(N+1)d}{\delta N(U_{PS})} \left(c + f \left(1 - 2m \right) - \lambda_{1}^{*}U_{PS} \right) \right] \, if \, \overline{k}_{P} < k_{P} \end{array} \right\}$$
(2.5)

$$\beta_{1}^{*} = \left\{ \begin{array}{c} \underline{k}_{L} = \frac{1}{2} \left[1 - \frac{(N+1)d}{\delta N(U_{SL})} \left(c + f + \lambda_{1}^{*} U_{SL} \right) \right] \text{ if } k_{L} < \underline{k}_{L} \\ k_{L} \text{ if } \underline{k}_{L} < k_{L} < \overline{k}_{L} \\ \overline{k}_{L} = \frac{1}{2} \left[1 - \frac{(N+1)d}{\delta N(U_{SL})} \left(c + f \left(1 - 2q \right) + \lambda_{1}^{*} U_{SL} \right) \right] \text{ if } \overline{k}_{L} < k_{L} \end{array} \right\}$$
(2.6)

Where
$$U_{SL} = u(U_S) - u(U_L), U_{PS} = u(U_P) - u(U_S), d = \frac{\sigma_{\epsilon}\sqrt{3}}{\pi}$$

In absence of any type of costs, i.e. c = 0, f = 0, and constraints, $\lambda_1^* = 0$, the optimal proportions of employees to be promoted and to be put on probation are $\beta_2^* = \frac{1}{2}$ and $\beta_1^* = \frac{1}{2}$, respectively, thereby leaving $1 - \beta_1^* - \beta_2^* = 0$ employees in the same position as before.

Proof. See Appendix A \blacksquare

In the optimal committed Forced Ranking (FR) system a firm promotes the top 50% of employees and puts the other 50% of employees on probation if it faces no costs i.e. c = 0, f = 0 and, constraints, $\lambda_1^* = 0$. However, as these costs increase the firm's optimal proportion of promotion and probation decreases from 50% leaving a void in the middle that is filled with employees who are allowed to stay in the same position as before. Further, in such a case when the ex post optimal rate of promotion (or probation) in case of the good (or bad) scenario is intermediate, $\underline{k}_P < k_P < \overline{k}_P$ (or $\underline{k}_L < k_L < \overline{k}_L$), then the firm just commits to that rate as the FR policy. The rationale for these findings are as follows.

A FR appraisal system has two effects on firm profitability. First, it increases profits by acting as an incentive for employees and thus, inducing higher effort from them. Second, it can increase costs due to promotions and probations as discussed above. If we only consider the incentive effects of the FR then we find that $\beta_2^* = \frac{1}{2}$ and $\beta_1^* = \frac{1}{2}$ i.e. an up-or-out contract is optimal. Such incentive structures are quite common in fields like consulting, law etc. where high powered incentives are used. While firms often work under fixed budgets and constraints, some firms face little recruitment

costs due to general economic conditions (and/or other factors like easy availability of skilled salesmen etc.). In such cases, it is optimal for the firms to implement an up-or-out FR policy in which the top half is promoted while the bottom half is put on probation, with no employee to be left in same position as before. For example, we find that firms rarely keep interns for an extended period of time and either promote them to permanent positions or let them go. Moreover, generally firms have easy availability of interns. Any other FR policy apart from the 50-50 up-or-out policy is a deviation from the incentive maximizing policy due to costs and constraints. The intuition behind the use of an up-or-out policy is to create an incentive structure that maximizes rewards and penalties at the same time so that the marginal benefit of putting up effort is maximized from the employee's perspective. Further, note that the result on proportion i.e. 50% promotion and 50% layoff is contingent on the distribution of the stochastic error term, i.e. logistic distribution of ε_i , which is symmetric. Our finding differs from KS (2001) who propose rank ordered tournaments to be more efficient than multiple-winner contests, while we advocate a multiple winner contest in which winners are promoted while non-winners are put on probation, with no employees left in the same position. The rationale for the difference in findings can be traced to the assumptions on structure of contests in both of the studies. While KS (2001) consider a fixed budget and a reward structure which is endogenously chosen, total costs in case of career tournaments are flexible and markets (or production technologies) rather than individual firms determine the value of the prizes, i.e. the value of specific job designations. In case of a sales contest as in KS (2001), the endogenous nature of the reward structure allows the organizer the flexibility to change the value of prizes as well the number of prizes. With such a flexibility, the organizer places higher rewards for top positions and progressively lower rewards for lower positions till $\frac{1}{2}$ the ranks are covered. This type of distribution of prizes is optimal because a marginal change in effort leads to greater changes in probability of achieving higher ranks than lower ranks amongst the top half of the ranks. Thus, a rank ordered tournament is optimal in this case. However, when the organizer lacks the flexibility to change the value of prizes, as in case of a career tournament, then she starts

rewarding from the top ranks and stops when her budget gets exhausted or $\frac{1}{2}$ the ranks are covered (whichever happens earlier). In absence of costs and constraints this leads to an up or out contract as opposed to a rank ordered tournament, but in presence of costs the career tournament structure converges to a rank ordered tournament. Our findings highlight the critical distinction between sales contest and career contests (and its implications for structure of the two contest) that often get overlooked i.e. while in case of sales contests the firm has control over the total structure of the contest, in case of a career contest the firm only controls who gets the prizes and not how much the value of the prizes should be since the value of the prizes are determined by market forces. In this line of reasoning KS (2001) assumptions better capture the realities of a sales contest while our assumptions reflect the realities of a career tournament.

Proposition 2.1 also shows that when the ex post optimal rate of promotion (probation) in case of the good (bad) scenario is intermediate then the firm just commits to that rate as the FR policy. The rationale for this finding is, for example in case of promotions, that when the expected level of promotions in the favorable scenario is intermediate then this level of promotions anyway motivates employees well and the firm does not have to incur cost related to deviation from the ex post optimal policy whenever such a scenario occurs. Next, we discuss how the FR policy changes with market conditions and how these comparative statics differ from the predictions of the sales contest literature.

Proposition 2.2 In the Optimal Committed Forced Ranking Policy,

a) Proportion of promotion, β_2 , and layoffs, β_1 , increase with the number of employees, N. b) Proportion of promotion, β_2 , and layoffs, β_1 , decrease with risk aversion, r and individual specific uncertainty, d. c) Proportion of promotion, β_2 , increases, and layoffs, β_1 , decreases, with an increase in marginal cost of participation, λ_1 .

Proof. See Appendix A

Promotion and layoff rates increase with the number of employees, thereby reducing the percentage of employees who remain at the SE level. The firm increases its use of FR with increase in Nbecause with an increase in the number of employees the marginal probability of getting promoted (and avoiding probation) due to effort increases. This is so because for fixed FR proportions (β_2 and β_1) the number of promoted and laid off employees also increase with an increase in N. This effect dominates the effect of decrease in employees' marginal probability of getting promoted or laid off resulting from a higher number of participants. Thus, employee effort increases with Nand the firm finds it profitable to increase its use of FR in response, i.e. it increases β_2 and β_1 . This result may in part explain why we see up-or-out contracts to be more routinely employed at larger companies than at smaller firms. However, this effect of an increase in N on employee effort starts diminishing as N increases because at a higher N the aforementioned positive effect starts decreasing while the negative effect becomes more pronounced. In KS (2001) too the number of rewards increase with an increase in number of employees. However, the reason why it does is different from that in our model. For example, in a sales contest with risk averse salespeople, the inter rank spread will be smaller with an increase in N, and hence, larger number of ranks will be awarded.

Promotion and layoff rates decrease with an increase in risk aversion, thereby increasing the percentage of employees who remain at the SE level. The rationale is that an increase in risk aversion decreases the expected utility associated with a promotion or with staying in same position. The decrease in utility decreases the incentive for the employees to put effort which in turn decreases the effectiveness of the FR policy. As the FR policy becomes less effective the marginal benefit of using it decreases and therefore, as discussed above, the FR policy now involves letting more employees stay in the same position. This result is in contrast to KS (2001) who find that the optimal number of winners in a contest goes up with an increase in risk aversion. The divergence in prediction is because in case of sales contests the firm can adjust the size of the prizes and reduce the inter rank spreads with an increase in risk aversion, and it will do so because risk aversion leads to more weight on lower rewards. Such a reduction in inter rank spread leads to an increase in number of prizes.

The use of FR decreases when individual specific uncertainty increases because an increase in uncertainty increases the noise in the contest and decreases the marginal benefit of putting higher effort for the employees. Due to reduced incentives employees put less effort and the FR policy becomes less effective which in turn makes the firm leave a greater percentage of employees at the SE level. This result suggests that firms are not well advised to apply Forced Ranking policies to a less experienced segment of a sales force because it may demotivate salespeople who experience much uncertainty about the selling task. This result stands in contrast to KS (2001) who find that uncertainty does not affect the contest structure. The reason for latter result is that in case of KS (2001) the contest budget is predetermined and therefore, a symmetric decrease in employee effort resulting from greater uncertainty does not impact the contest structure.

A tightening of the participation constraint i.e. an increase in λ_1 increases the firm's use of promotions and decreases layoffs. Note that when the constraint on promotion (or layoff) binds it also has an effect on layoff (or promotion). For example, if the firm is constrained to limit its promotions to a smaller proportion then the firm cannot layoff more because else the employees' participation constraint will not be met. Surveys on promotions (World at Work 2012) suggest that on an average firms only promote around 10 percent of their work force in a given year. This potentially may explain why we find layoff proportion as part of FR policies to be in similar ranges. We limit our analysis for rest of the paper to the case when the participation constraint is not binding.

2.4.2 Ambiguous vs. Committed Forced Ranking Policy

The first two propositions discussed above dealt with the question of how a committed FR policy is designed optimally. In this subsection we analyze why and when a firm commits to a FR policy vs. leaving it ambiguous. We do so by comparing the profits Π_{CC} , Π_{AA} , Π_{AC} , and Π_{CA} . Of course, it appears that a firm could choose to use other instruments like quotas and commissions to motivate its salesforce thereby staying away from any kind of FR. However, there are two reasons why a FR may still be implemented even when the strategy space includes the aforementioned alternatives. First, if common uncertainty i.e. σ_{γ} is high, then it has been shown in the past (Green and Stockey 1983) that a tournament is superior to individual based compensation measures like commissions and quotas. Second, even if the firm does not explicitly motivate employees with a FR policy, the firm by default will be resorting to an ambiguous FR policy as long as employees believe that they will be evaluated relative to each other, which as we show below may be motivating by itself. In the following proposition, we identify the firm's optimal FR policy as a function of the cost of deviation from the ex post optimal strategy.

Proposition 2.3 If $f > \overline{F}$ the firm uses an ambiguous promotion and layoff policy. If $\underline{F} < f < \overline{F}$ the firm commits to a promotion policy but not to a layoff policy. If $f < \underline{F}$ the firm commits to both a promotion and layoff policy, where \underline{F} and \overline{F} are given in appendix.

$$\max \{\Pi_{CC}, \Pi_{AA}, \Pi_{AC}, \Pi_{CA}\} = \begin{cases} \Pi_{AA} \text{ if } f > \overline{F} \\ \Pi_{CA} \text{ if } \underline{F} < f < \overline{F} \\ \Pi_{CC} \text{ if } f < \underline{F} \end{cases} \end{cases}$$

Proof. See Appendix A

If costs of deviation from the ex post optimal are very high then we find that the firm lets its promotion and layoff policies be ambiguous. This is intuitive because every time a firm commits to a FR policy amid uncertainty there is a chance that its committed policy may end up being different from what is expost optimal leading to large costs. However, when f is intermediate firms commit to a promotion policy but not to a layoff policy. The intuition is that when firms leave their FR policy ambiguous the employees put more weight on the pessimistic case scenario. The pessimistic case scenario entails lower rates of promotion which is further away from an up-or-out contract and thus, less motivating for employees. Therefore, the firm benefits from committing to a promotion policy. In contrast, the pessimistic case scenario entails greater number of layoffs and thus, the employees work according to an incentive system that is closer to the up-or-out contract when the firm leaves its layoff policy ambiguous. Thus, when f is intermediate the firms can achieve an incentive system closer to the up-or-out contract without having to incur costs associated with deviation from the ex post optimal by leaving its layoff policy ambiguous. Therefore, we find that the firm pursues an ambiguous layoff policy but a committed promotion policy in this case. This result implies that we would expect to see firms being more likely to communicate their promotion policies but less likely to communicate their layoff policy, consistent with industry practice. For example, a survey of 614 firms found that 35% of firms communicate their promotion policies (WorldatWork 2013) while we generally find very few firms committing to layoff policies. As firms are more likely to pursue a committed promotion policy but they may or may not pursue a committed layoff policy, henceforth, we call a FR policy as committed if it involves commitment to both promotion and layoff policy, and ambiguous if it involves an ambiguous policy either with respect to both layoffs and promotions, or with respect to layoffs only.

Proposition 2.4 If $f < \overline{F}$ and $k_L^L < k_L < k_L^H$ then the firm commits to a promotion policy but

leaves the layoff policy ambiguous, where k_L^L and k_L^H are given in the appendix.

$$If f < \overline{F} then \max \{\Pi_{CC}, \Pi_{AA}, \Pi_{AC}, \Pi_{CA}\} = \begin{cases} \Pi_{CC} if k_L < k_L^L \\ \Pi_{CA} if k_L^L < k_L < k_L^H \\ \Pi_{CC} if k_L^H < k_L \end{cases}$$

Proof. See Appendix A

We call k_L the severity of layoff in the adverse scenario. When the severity of layoff in the adverse scenario is in an intermediate range it is more profitable for firms to follow an ambiguous FR policy as compared to a committed policy. The rationale for this finding is that in case of an ambiguous layoff policy the employees consider the worst case scenario while putting effort. If the worst case scenario is such that it is closer to the ideal contest structure, which is the case when the severity of layoff is intermediate, then the firm is better off leaving the layoff policy ambiguous because in such a case it does not have to incur costs associated with committing to a layoff policy. However, when severity of layoff in the adverse case is too high or too low then the contest structure implied by the adverse case is less motivating for the employees and in such a case the firm is better off committing to a layoff policy. The intuition for why a contest structure with intermediate level of layoff is more motivating flows from our results on optimal contest structure in such a context. These results can be seen in Figure 2.3. The result on severity of layoffs leads us to speculate whether Microsoft's decision to abandon the committed FR model was a strategic one (WSJ 2013). Before 2013 a high proportion of layoffs were rare in Microsoft and this situation corresponds to a case in our model where k_L is low. In such a case committing to a FR was more profitable for the company as is also predicted by our model. However, around 2013 there was a management change at Microsoft which also bought Nokia. Both these events increased the likelihood of higher layoffs (higher k_L). In such a scenario, our model predicts a move from a committed FR to an

ambiguous FR policy which is what Microsoft adopted.

Proposition 2.5 If $f < \overline{F}$ and the profit margin is sufficiently great, $\delta > \delta^*$, then the firm pursues a committed FR policy. Contrarily, if $\delta < \delta^*$ then the firm pursues an ambiguous FR policy in which it commits to a promotion policy but leaves the layoff policy ambiguous, where δ^* is given in the appendix.

If
$$f < \overline{F}$$
 then $\max \{\Pi_{CC}, \Pi_{AA}, \Pi_{AC}, \Pi_{CA}\} = \begin{cases} \Pi_{CA} \text{ if } \delta < \delta^* \\ \Pi_{CC} \text{ if } \delta > \delta^* \end{cases}$

Proof. See Appendix A

Interestingly, we find that firms are more likely to commit to a FR policy if their profit margins are high. The rationale is that while higher profit margins increase profits for both ambiguous and committed FR model, the rate of increase in profit with profit margin in case of the committed FR model is higher, because in this case the firm can also modify its layoff proportions taking into consideration the higher marginal benefit. In other words, an increase in profit margin increases the effectiveness of the FR policy and a firm that commits to such a policy can take more advantage of it. Our result potentially explains why we find a high percentage of Fortune 500 companies pursuing committed FR policies (WSJ 2012) but do not hear much about other smaller or less profitable firms following such policies.

2.5 Extensions, Robustness and Further Discussion

In this section we relax some of the conceptual assumptions that we made in the main model and discuss how doing so may affect our results. We extend our analysis in three directions. First,

we model an infinite horizon game to identify the conditions under which the firm can credibly commit to FR when they have reputational concerns. Second, we discuss how our results may change if salespeople are heterogeneous with respect to ability. Third, we consider the possibility that the firm uses other incentive mechanisms in addition to the Forced Ranking System and study the interaction between FR and linear sales compensation.

2.5.1 Commitment to Forced Ranking Policy

In the previous sections we have assumed that the firm can commit to a FR policy if it wants. In a static game such a commitment may not be credible because the firm can renege on its promise at the end of the period by implementing the ex post optimal policy as long as the costs from doing so are sufficiently low. Knowing this, the employees would not be affected by the commitment the firm makes and they will only work according to the ambiguous policy. Our assumption about the firm's ability to commit to a FR policy is an abstraction from the repeated nature of interaction between the firm and its employees. However, if we assume that the firm and the employees interact over an infinite number of periods then we can still show that there exists an equilibrium in which the firm can commit to a FR policy and that in this equilibrium it will honor its commitment.

Assume that the firm has a discount factor of μ . In addition, assume that the employees attach no credibility to firm's commitment to FR once the firm reneges on its commitment. Furthermore, note we only consider the case when $f < \underline{F}$ (Proposition 2.3) because otherwise commitment to FR proportions is anyway not optimal for the firm. In such a circumstance the firm will be committing to a FR policy and honoring it if,

$$\Pi_{CC} \left(1 + \mu + \mu^2 \dots \right) \ge \Pi_{CC} + X \left(f, k_L \right) + \Pi_{CA} \left(\mu + \mu^2 \dots \right)$$
(2.7)

where
$$X(f, k_L) = \begin{cases} \frac{f}{2} \left(1 - \frac{d(c+f)(N+1)}{NU_{SL}\delta} - k_L \right) > 0 \text{ if } k_L < \underline{k}_L \\ \frac{fk_L}{2} > 0 \text{ if } k_L \ge \underline{k}_L \end{cases}$$
 and (2.8)
$$\underline{k}_L = \frac{1}{2} \left(1 - \frac{d(c+f)(N+1)}{NU_{SL}\delta} \right)$$

Where, Π_{CC} and Π_{CA} are the per period firm profits under the committed and ambiguous FR models and $X(f, k_L) \ge 0$ is the benefit that the firm gets from reneging on its commitment to FR.

Proposition 2.6 Firms are able to commit to a FR policy if,

$$\Pi_{CC} - \Pi_{CA} \ge \left(\frac{1-\mu}{\mu}\right) X\left(f, k_L\right)$$

Proof. Follows from inequality 2.7 above.

If $X(f, k_L)$ is finite and μ is close to 1, i.e. the firm cannot gain too much from reneging on its commitment to FR and it is sufficiently forward looking, then the firm will still choose the committed FR model if its profit from the committed FR model is sufficiently higher than that of the ambiguous FR model. Note that our current model set up is analogous to assuming $\mu \rightarrow 1$. However, if $\mu < 1$ then there will be a region at the margin between strategy regions for committed FR and ambiguous FR where now the firm will choose the ambiguous FR in spite of the fact that its profit from the committed model is higher than that of the ambiguous model. Note that the benefit that the firm gets from reneging on its commitment to a FR policy is increasing in cost of deviation from the expost optimal policy, f, i.e., $\frac{\partial X}{\partial f} > 0$ if $\underline{k}_L \leq k_L$ or if $f < \frac{NU_{SL}\delta}{d(N+1)} (\underline{k}_L - k_L)$. Therefore, in such cases the benefit in first committing to FR and then reneging on it will increase as f increases. We can see this region in the graphical plot in Figure 2.4. Remember that in a static model we found that the firm would pursue a committed FR model when severity of layoff is high or low and the cost of deviating from the ex post optimal is low. We find the same result in the graph given in Figure 2.4. However, now we also find a small region between the ambiguous and committed FR strategy regions where the firms has an incentive to first commit to a FR policy and then renege on its commitment. The size of this region increases with an increase in cost of deviation from the ex post optimal layoff policy when k_L is high or when f is low. In equilibrium the firm will not be able to commit to a FR policy in this region because the employees will not consider it credible. When f is higher and k_L is low then while the firm's benefit from reneging on committed FR still increases with cost of deviation from ex post optimal, f, the firm also heavily reduces its use of committed FR with an increase in f because of which the overall effect of increase in f is negative on benefit from reneging i.e. X.

2.5.2 Heterogeneity

In the current paper we study why and how FR policies are designed to optimally motivate sales people. While firms often have the dual objective of both motivation and selection when they use such policies we abstract away from the selection aspect by assuming sales employees to be homogeneous. We focus on the motivational effects due to the following reasons. First, as has been acknowledged in the most recent work in this area in Economics (Ryvkin 2013, Balafoutas et al. 2013) it is unfortunately not tractable to consider the case of full heterogeneity in *N* player contests. However, such a framework would be necessary to analyze the optimal proportions in a FR system when considering both motivation and selection aspects of FR, Second, the aforementioned literature (Ryvkin 2013, Balafoutas et al. 2013) which is at the frontier of *N* player contests has considered a weaker form of heterogeneity i.e. it has assumed that heterogeneity is such that the strongest and weakest player's abilities are not too different from that of the average player's ability. The assumption is made because, as discussed above, the case of full blown heterogeneity is not tractable and there are reasons to expect heterogeneity at organizational levels to be of this

weaker kind on account of attrition of employees with very different ability characteristics. For example, it is expected that over time highly able employees will get promoted or leave the firm and the very weak ones will also leave. With the assumption of weaker heterogeneity and using Taylor approximation up to the first order Balafoutas et al. (2013) find that the total effort from a contest stays the same even though more able employees now have higher chances of getting the prize while lower ability employees have higher likelihood of getting laid off. Based on these finding we do not expect our results to be sensitive to consideration of heterogeneity in ability as long as it is of the weaker kind. Third, in general, the past literature on contests also shows that heterogeneity weakens the incentives in a contest and that it is not advisable to use contests as a motivational device if heterogeneity is high (Lazear and Rosen 1981, Ridlon and Shin 2013). Due to this reason we expect that sales organizations are more likely to segment their work force when they use policies like FR and such policies are more likely implemented for sub segments of the salesforce for which heterogeneity in ability is very low. Fourth, empirical research (e.g., DeVaro 2006) has validated the predictions from tournament theory literature, much of which, including KS (2001), is based on motivation for homogeneous players.

2.5.3 Interaction of Forced Ranking and Sales Compensation

In this section we relax the assumption of the firm offering a fixed salary in the first period and study the case when the firm offers linear performance pay along with FR to consider the interaction between FR and sales compensation. In deriving Propositions 2.7, 2.8 and 2.9 we consider the case of weak risk aversion, i.e. when risk aversion is small, and use perturbation analysis. We do so because the case of full risk aversion is not tractable and under such conditions of intractability past research in Economics (e.g., see Judd 1996, Fibich et al. 2006) has employed perturbation techniques. Furthermore, in the Appendix we use simulation to illustrate the accuracy of our solutions. We also note that according to theoretical reasoning provided by Rabin (2000) the

coefficient of absolute risk aversion in the population is indeed very small, of the order of 10^{-4} , and econometric (Cohen and Einav 2007) and experimental (Gertner 1993) studies have confirmed this through their estimates. Empirical research in sales force too provide similar findings. For example, Misra and Nair (2011) find the average monthly risk premium to be only 341.22 as compared to an average annual salary of 67,632 and the risk aversion coefficient to be between 0.0018 and 0.33. Furthermore, we assume f = 0, i.e. we only consider a committed Forced Ranking policy in the following analysis because the solutions are implicit and they are not tractable if economic uncertainty regarding ex post optimal promotion and layoff rate is introduced in the model. Given these assumptions, we obtain the first result of this analysis.

Proposition 2.7 In the presence of weak risk aversion, promotion complements (substitutes) performance pay and probation if and only if the cost of promotion is lower (higher) than the utility that the employees derive from promotions, i.e.

$$\frac{\partial^2 \pi}{\partial b \partial \beta_2} = \frac{N(1 - 2\beta_2)}{(N+1)d} \left(U_P - U_S \right) > 0 \Leftrightarrow \left(U_P - U_S \right) - c > 0 \tag{2.9}$$

$$\frac{\partial^2 \pi}{\partial \beta_1 \partial \beta_2} = -\frac{N^2 (1 - 2\beta_2) (1 - 2\beta_1)}{(N+1)d} \left(U_P - U_S \right) \left(U_S - U_L \right) > 0 \Leftrightarrow \left(U_P - U_S \right) - c > 0 \quad (2.10)$$

Proof. See Appendix A \blacksquare

Proposition 2.8 In the presence of weak risk aversion, performance pay and layoffs are always substitutes.

$$\frac{\partial^2 \pi}{\partial b \partial \beta_1} = \frac{N(1 - 2\beta_1)}{(N+1)d} \left(U_S - U_L \right) > 0 \Leftrightarrow \left(U_S - U_L \right) + c < 0$$

Proof. See Appendix A \blacksquare

The rationale for the above findings, given small values of risk aversion, is as follows. While a marginal increase in the rate of promotion increases the expected utility for the sales employees and thereby loosens the participation contract, a profit maximizing firm extracts this surplus by providing less risk premium and fixed salary i.e. by increasing the amount of performance pay or the number of layoffs. The higher marginal promotion rate makes sense only when the benefit from higher performance pay or layoff rate outweighs the cost involved in the increase of the promotion rate. Thus, in this case the firm finds it profitable to marginally increase the promotion rate and performance pay or layoffs, thereby making them complements. In contrast, a marginal increase in the layoff rate leads to tightening of the participation constraint and the firm has to provide a higher risk premium and fixed salary, i.e. lower performance pay, making layoff and performance pay substitutes. The complementarity or the substitutability between promotion and performance pay (or probation) is amplified with an increase in number of employees, a decrease in individual specific uncertainty or an increase in utility from promotion (or disutility from being put on probation). These changes lead the firm to use more of FR, and with higher use of FR the firm also complements or substitutes performance pay to a greater extent depending on whether FR and performance pay are complements or substitutes. An implication of Proposition 2.7 is that we would expect to see higher performance pay in organizations that provide employees with greater opportunities for career growth and face low costs in doing so, particularly when employee risk aversion is low. These may be characteristics of the financial services industry where employees' salaries are largely commission based. In contrast, in organizations with limited upward mobility and substantial costs of hiring/firing employees we would expect lower performance pay. A recent empirical study based on data from a large Engineering multinational corporation (Ederhof 2011) that shows that promotion and performance pay are substitutes supports this finding.

Now, we discuss the comparative statics results for performance pay with respect to exogenous factors.

Proposition 2.9 *Performance pay may increase or decrease with individual specific uncertainty. Performance pay increases with individual specific uncertainty if,*

$$(e-b) > \frac{1}{2}rb\left(\sigma_{\varepsilon}^{2} + \sigma_{\gamma}^{2}\right) + \frac{1}{8}\left(\frac{N\left(U_{P} - U_{S}\right)}{(1+N)d} + \frac{N\left(U_{S} - U_{L}\right)}{(1+N)d}\right)$$

Proof. See Appendix A \blacksquare

We find that performance pay may increase (or decrease) with individual specific uncertainty, depending on whether the effectiveness of forced ranking in inducing effort, given by the term $\left(e - b = \frac{N(1-\beta_2)\beta_2}{(N+1)d} \bigtriangleup U_{PS} + \frac{N(1-\beta_2)\beta_2}{(N+1)d} \bigtriangleup U_{SL}\right)$ is high (or low) and the performance pay's risk premium, represented by the term $\frac{1}{2}rb\left(\sigma_{\varepsilon}^2 + \sigma_{\gamma}^2\right)$, is low (or high). This result enhances findings from past theoretical literature which suggests that performance pay always decreases with individual specific uncertainty. The rationale for our finding is given below.

Employee specific uncertainty reduces the effectiveness of both performance pay and forced ranking in inducing effort, suggesting a negative direct effect of uncertainty on the usage of these two mechanisms, consistent with past literature (Basu et al. 1985, KS 2001). However, under certain conditions the forced ranking mechanism becomes ineffective at a faster rate than the use of performance pay and therefore, the firm substitutes forced ranking with performance pay along with the overall decrease in use of incentives. This substitution effect outweighs the aforementioned negative direct effect if effectiveness of forced ranking is high and risk premium associated with performance pay is low. To understand why, note that for case of weak risk aversion an increase in individual specific uncertainty decreases the effectiveness of forced ranking and performance pay by two different routes. Increase in individual specific uncertainty decreases the effectiveness of forced ranking because it increases the spread in the distribution of sales performance ranking and thus, decreases the marginal probability of getting promoted in a forced ranking policy leading to lower effort spent. This impact is stronger when the effectiveness of forced ranking is already high and the firm is making more use of forced ranking. However, the increase in individual specific uncertainty also reduces the effectiveness of performance pay because now the firm needs to pay higher risk premium and can afford to pay less sales commissions. This negative impact on performance pay is softer when the risk premium is lower. Therefore, if effectiveness of forced ranking is high and risk premium associated with performance pay is low then we find the use of performance pay to counterintuitively increase with individual specific uncertainty. It also needs to be noted that such an effect is only possible if the effort from forced ranking is above a threshold given by the third term in the inequality in Proposition 2.9.

Further, we find that the above conditions for increase in use of performance pay with individual specific uncertainty are more likely if costs of forced ranking are lower and if performance pay and promotion are complements i.e. if costs of forced ranking are lower than the expected utility that employees derive from promotion. This is so because a lower cost of forced ranking means that the firm uses more of forced ranking and therefore, forced ranking is more effective. In addition, a lower cost of forced ranking also leads to decrease in use of performance pay because of substitution between forced ranking and performance pay, and this also decreases the risk premium associated with performance pay. Note that a change in cost of forced ranking has no effect on the constant term in the inequality given in Proposition 2.9.

$$\frac{\partial}{\partial c} \left(\frac{1}{8} \left(\frac{N \triangle U_{PS}}{(1+N)d} + \frac{N \triangle U_{SL}}{(1+N)d} \right) \right) = 0$$
(2.11)

$$\frac{\partial}{\partial c} \left(rb \left(\sigma_{\varepsilon}^2 + \sigma_{\gamma}^2 \right) \right) = r \left(\sigma_{\varepsilon}^2 + \sigma_{\gamma}^2 \right) \frac{\partial b}{\partial c} > 0$$
(2.12)

$$\frac{\partial}{\partial c} \left(e - b \right) < 0 \text{ if } \left(U_P - U_S \right) - c > 0 \tag{2.13}$$

The above result may provide a possible explanation for the ambiguous effect of uncertainty on performance pay found in empirical studies on sales force productivity (John and Weitz 1989, Coughlan and Narsimhan 1992, Misra et al. 2005, Krishnamoorthy et al. 2005). In particular, if it is known that a firm uses both forced ranking and performance pay then a study of the empirical relationship between uncertainty and performance pay needs to also consider the effect of uncertainty on forced ranking and its indirect effect on the use of performance pay.

Proposition 2.10 If employees are weakly risk averse then performance pay decreases with common uncertainty, i.e. $\frac{\partial b}{\partial \sigma_{\gamma}} < 0$, and increases with cost of forced ranking, i.e. $\frac{\partial b}{\partial c} > 0$. If performance pay and promotion are complements then an increase in cost of forced ranking leads to decrease in use of probation and increase in use of promotion in the forced ranking policy, i.e. $(U_P - U_S) - c > 0 \Rightarrow \frac{\partial \beta_1}{\partial c} < 0, \frac{\partial \beta_2}{\partial c} > 0.$

Proof. See Appendix A

We find that performance pay decreases with common uncertainty. The reason is that an increase in common uncertainty leads the firm to substitute forced ranking in place of performance pay. The rationale is that the effectiveness of the forced ranking policy, which can remove the uncertainty common to all employees while performance pay is unable to do so, increases with common uncertainty. However, an increase in common uncertainty decreases the effectiveness of performance pay because the firm needs to provide higher risk premium and therefore, offers fewer incentives. As the effectiveness of forced ranking increases while that of performance pay decreases with common uncertainty the firm finds it optimal to substitute performance pay with forced ranking. The substitution effect also arises when costs associated with forced ranking increase. We find that when the firm faces increasing costs from employee replacement, it substitutes forced ranking with performance pay. In other words, an increase in the possibility of adverse selection in promotions will make performance pay more attractive. Interestingly, because of this substitution the firm increases its use of promotions and decreases its use of layoffs with an increase in cost of forced ranking if promotion and performance pay are complements. The intuition is the following. An increase in cost of forced ranking reduces the effectiveness of forced ranking as a motivational tool because of which the firm shifts its use of promotion and probation away from the proportions of 50% and 50%. However, while probation has no other role in the incentive system other than as a motivational mechanism because of which its use is reduced as costs increases, promotion also plays the role of increasing expected utility of staying in the organization for the employees and thereby can support the firm in pursuing a more aggressive performance pay policy. Due to the later role of promotion as a way to increase the expected utility of staying in organization, the surplus of which is extracted by the firm through higher performance pay, the firm promotes more and more employees above 50%, which may be less motivating with respect to promotion but allows the firm to provide higher incentive pay and motivation through sales commissions. This later role of promotion is only possible when the cost of promotion is less than the expected utility derived by the employees from the promotion, i.e., under the conditions when performance pay and promotion are complements.

2.6 Conclusion

2.6.1 Managerial Implications

The increasing use of forced ranked distribution systems is inevitable and growing in sales organizations. However, despite the popularity of these policies the academic literature in Marketing provides little guidance how to construct them to motivate the salesforce and how to use them in combination with other sales incentives such as performance pay. In this paper we attempt to close this gap in the literature. We use an agency theoretic model of a company employing a salesforce and show that a forced ranked distribution system can be used to motivate the salespeople to exert more effort. In particular, we show that when firms face economic uncertainty and the employees are ambiguity averse, firms may benefit by keeping their layoff policies ambiguous. However, in such circumstances it is still beneficial to commit to a promotion policy. Further, we find that firms with higher profit margins are more likely to commit to both promotion and layoff policies as part of FR. When we consider design of a committed FR policy, interestingly, we find that, in the absence of costs of FR or other constraints, the optimal contract offered would be an up-or-out contract promoting and demoting portions of the salesforce, but leaving nobody employed on the same level as before. The reason for the structure of the contract is that it maximizes the incentive to put effort by eliminating an intermediate achievement level. The up-or-out contract we propose constitutes a special case of a multiple winner contest and the result is surprising because the closely related literature on sales contests (KS 2001) predicts the superiority of a rank ordered tournament over multiple winner contests. The difference between our result and theirs can be traced to the fact that in sales contests companies can control both number of rewards and amount of reward given a fixed budget, whereas in career tournaments the amount of a reward, i.e. the value of a promotion to a specific rank, is largely determined by market forces and the total cost of the policy is flexible. While our paper suggests the optimality of up-or-out contracts for sales organizations under some conditions, we also find the use of up-or-out contracts in other industries. For example such incentive structures are quite common in fields like consulting, law etc. where high powered incentives are used.

When thinking about introducing a forced ranked distribution system companies have to be concerned about properly adjusting other sales incentives. We investigate whether they should increase or decrease performance pay with FR and find that these two incentives mechanisms can be substitutes or complements depending on the value of the promotion relative to its cost. Furthermore, we show that counter-intuitively an increase in individual specific uncertainty can lead to an increase in the use of performance pay when risk averseness of the salesforce is sufficiently small. This happens because the effectiveness of FR decreases more than the one of performance pay under this condition and the company optimally substitutes one for another.

2.6.2 Limitations and Directions for Future Research

While this paper makes several contributions to the existing literature, it is important to also point out limitations of the analysis. We study how a firm can motivate a homogeneous salesforce using a FR. Although a homogeneous salesforce is a simplification of a real world situation we argue in section 2.5.2 that the results of our analysis are robust to alternative assumptions. However, it would be worthwhile to explore the case of heterogeneity in a N player contest.

While we study the interaction of the tournament design with the linear wage contract offered to the salesforce, there are other sales incentives that are routinely given to salespeople, for example bonuses for reaching a specific sales output (Steenburgh 2008). Moreover, we considered weakly risk averse sales people when analyzing the interaction between FR and performance pay. Relaxing this assumption can increase our understanding of how these incentive mechanisms interact. It would be worthwhile and of practical relevance to understand how these incentive schemes need to be adjusted when implementing a career tournament. Moreover, we argue in the paper that the salaries for specific positions are determined by market forces, but do not explicitly model this process. It may be interesting to study a model where firms compete in the product as well as in the labor market to determine the impact of these processes on the tournament design.

Furthermore, in reality career tournaments have an inherently dynamic structure where promotions and demotions do not necessarily occur after each contest period. Considering disclosure or non-disclosure of interim outcomes or rankings in such a dynamic model with repeated interactions of the same players may be interesting to explore. It may be also important to consider that tournaments have psychological effects on its participants that need to be taken into consideration when designing them. For example, a winner may obtain psychological benefits above and beyond the economic benefits of a promotion whereas a loser may suffer from the consequences and be demotivated after being put on probation (Yang, Syam and Hess 2013). Analyzing the impact of these dynamic consequences on the optimally and structure of a career tournament is a fruitful area for future research. Finally, testing the implications of the current study with experimental or empirical work would lend additional credibility to our findings.

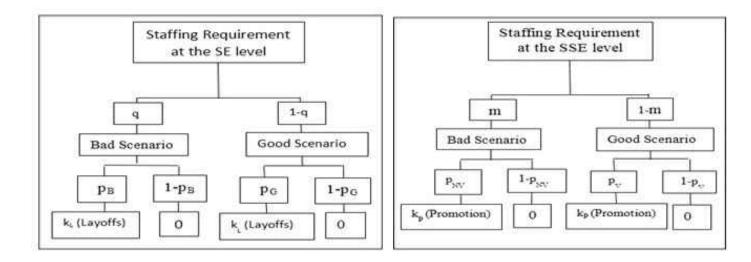


Figure 2.1: Firms ex post staffing requirements at SE and SSE levels

Stage 1	Stage 2	Stage 3
The Firm decides on whether to Commit to a Forced Ranking Policy or leave it as Ambiguous	The employees decide whether or not to accept the contract and put effort	Nature determines the random shocks and sales performance ranks are observed by all. Ex post Optimal proportion of Promotion and Layoffs are observed and Firm incurs costs depending on the Forced Ranking Policy it chose at stage 1.

Figure 2.2: Game Sequence

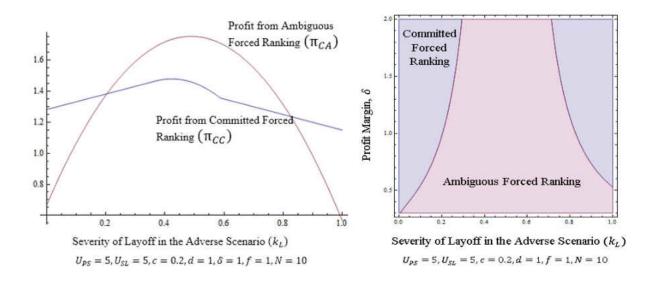


Figure 2.3: Ambiguous vs. Committed Forced Ranking Policy

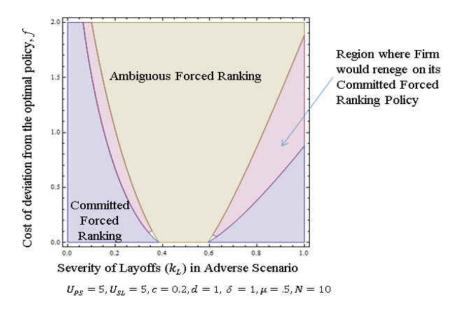


Figure 2.4: Region where credible commitment to Forced Ranking Policy is not possible

Notation	Description	
π_{ij}	Firm's Profit when the firm follows the <i>i</i> Promotion Policy and <i>j</i> Probation policy, where $i(\text{or } j) = \begin{cases} C, \text{ if the firm commits to a promotion (or probation)policy} \\ A, \text{ if the firm leaves the promotion (or probation)policy as ambiguous} \end{cases}$	
$egin{array}{c} eta_1 \ eta_2 \end{array}$	Proportion of employees to be put on probation (or laid off) as part of the forced ranking policy Proportion of employees to be promoted as part of the forced ranking policy	
$\sigma_\varepsilon(d)$	Variance (s.d.) of individual specific error term in sales response function, $d = \frac{\sigma_{\varepsilon}\sqrt{3}}{\pi}$	
σ_{γ}	Variance of common error term in sales response function	
U_k	Reservation Wage at position k, where $k = P$ (Promotion), $S = (\text{Same Position}), L(\text{Probation})$	
$U_k \\ c$	Cost of putting an employee on probation or promoting an employee	
δ	Profit Margin	
δ N	Number of employees	
f	Cost of deviation from ex post optimal probation and promotion policy	
	Ex post optimal probation (and promotion) policy	
w, b	Fixed salary and slope of linear incentive, respectively.	
λ_1	Marginal cost of reservation utility	
r	Risk aversion coefficient for employee <i>i</i>	
e_i	Effort by the i^{th} employee	
x_i	Sales generated by the i^{th} employee	
μ	Future discount factor for employees	

Figure 2.5: Notation

CHAPTER 3: PETER PRINCIPLE IN SALES MANAGERIAL PROMOTIONS - AN ALTERNATIVE EXPLANATION

3.1 Introduction

Sales managers play a very crucial role in sales organizations and their proper selection is very important. For example, Helmut Wilkie, VP Sales, Microsoft believes that "if your first-line management is broken, the entire sales force will be ineffective" (Zoltners, Sinha and Lorimer 2012a). Promotion to sales managerial positons, often the way by which new sales managers are selected, are also important career jumps for salespeople and thereby act as a significant source of motivation to exceed sales quotas and often the best salespeople in an organization are promoted to such sales management positions (Zoltners et al. 2012b). However, employees promoted on the basis of their superior sales performance are frequently found to be not as good as they were expected to be in their new role as managers and this phenomenon of adverse selection is often attributed to Peter Principle¹(Peter and Hull 1969) and lack of managerial skills (Anderson et al. 1999, Zoltners et al. 2012b, Hubspot blogs 2014). The general belief is that a high performing sales rep need not be a great sales manager because the two roles require different skills. To avoid this problem companies come up with different solutions. Some of them try to train and test their potential managers on managerial skills. For example, Boston Scientific has a formal corporate program for selecting and developing internal candidates for sales manager positions and it tests and trains candidates for competencies required at managerial levels (Zoltner et al. 2012c). Some suggest letting sales people self-select into managerial roles with the expectation that unsuitable candidates would decline opportunities to get into such roles once they know what the roles entail (Sales and

¹Peter Principle is a widely observed phenomenon that states that in any hierarchical organization, employees tend to rise to their level of incompetence. The term Peter Principle owes its name to Peter and Hull (1969) who coined it for the first time in their humorous book titled Peter Principle.

Marketing Management 2008). Some like Cardinal Health use dual career paths as a way to address the problem. They have separate vertical for management and individual sales roles (Zoltner et al. 2012c). However, in spite of the above issues being part of management wisdom for a long time we still see the problem arise.

A seemingly unrelated issue is that of risk aversion in middle management. For example, a study done by McKinsey Quarterly (2011) finds that 1500 executives from 90 countries demonstrated high levels of risk aversion with regard to investment decisions when asked to choose between two alternate scenarios with different risks and investment sizes. This is worrisome. Suppose a sales manager has the option of either investing \$2 million on a more conservative and less risky promotional campaign with a net expected return of \$1 million or investing the same amount on a more risky viral campaign with a net expected return of \$5 million. If the sales manager is fairly risk averse then he or she might choose the former option in spite of the fact that the latter option would be more beneficial for his or her company and sales executives. Some experts have suggested making companies and their managers more aware of behavioral biases like risk aversion to avoid such problems (McKinsey Quarterly 2012, WSJ 2012 etc.). While the advice is good, it can still fall short if more chronically risk averse individuals end up in management positions.

In this research, we use a two person tournament theory model similar to Lazear and Rosen (1981) to show that both the above problems can be related and can arise even without the consideration of lack of management skills in promoted sales employees and firms may be systematically promoting more risk averse sales employees to managerial positions. Further, we find that some of the proposed solutions like letting sales people self-select into roles may fall short of solving the problems that we identify.

We find that the Peter Principle problems can arise because promotion to managerial positions entails two different benefits associated with responsibilities of managing multiple revenue streams viz., one, higher sales potential due to multiple revenue streams and, two, a possibility to diversify one's risk across multiple revenue streams. For example, as sales managers as opposed to as sales reps, employees are often responsible for more number of revenue channels (e.g. key accounts, sales territories, other sales reps etc.) and therefore they can earn more incentives through override commissions if they perform well. At the same time their risks from business also get diversified to some extent because if one revenue channel performs badly then some other may possibly make up for it. These two benefits from sales management promotions generate asymmetric incentives in a sales force consisting of sales reps with unobservable and heterogeneous risk and productivity characteristics. This is so because the possibility of earning higher incentives through override commissions motivate the less risk averse sales reps more while the possibility to diversify ones risk across revenue channels motivate the more risk averse more. In our analysis we find that when such incentives are considered the moderately risk averse employees are most motivated to get promoted to sales managerial positions, put the highest effort and end up performing best under a wide range of conditions. However, for the companies selection of less risk averse employees to managerial positions would have been better and therefore, promotion of the best performer leads to the Peter Principle phenomenon. Moreover, this intuition suggests that the Peter Principle problem can arise even when all the employees are homogenous in terms of their managerial skills or when employees are heterogeneous with respect to their managerial skills and the firm promotes the most managerially skilled employee to the sales manager's position.

On further analysis we also find that there can even be conditions in which an employee with lower productivity and higher risk aversion gets promoted more often if the diversification benefit outweighs the benefits of managing higher sales at the managerial position. We also find that the adverse selection problem becomes more likely with an increase in ability of the more risk averse employee even though the firm's prefers higher productivity in its sales managers.

To deal with the Peter Principle problem that we identify we study and evaluate the different so-

lutions being currently implemented by companies. In doing so, we find that letting sales people self-select into roles may not be sufficient to deal with the problem because in our model employees prefer the role of sales manager. However, we find that under some conditions making promotion decisions on basis of an employee's sales educational qualifications or on basis of their interest in pursuing risky sales management training may allow less risk averse sales employees to signal their characteristics credibly to the companies. Interestingly, this result does not rely on the fact that employees undergoing sales education gain more sales skills but on the fact that their initiative in investing in a potentially risky skill acquisition itself suggests that they have superior risk characteristics than others. Our analysis also reveals that if the Peter Principle problem is not very severe a company may continue to make promotion decisions on basis of superior sales performance and allow some adverse selection because it benefits more from using the promotion as a carrot for motivating sales reps.

In an extension, we consider asymmetric labor market scenarios for sales managers and sales representatives and study how such asymmetry in the labor market may moderate the adverse selection problem that we have identified.

Our research contributes to the academic literature and business practice in Marketing by modeling sales managerial job assignment and by providing an alternate explanation for Peter Principle in sales managerial promotions. We also show that motivation and selection aspects of promotions in a hierarchy may not be aligned and thereby address an important research question identified in the literature (Prendergast 1999). For example, Prendergast (1999) asks whether "organizations can kill two birds with one stone" when tournaments are used, i.e. whether tournaments can help firms attain the dual objectives of higher motivation and better selection of employees. Furthermore, our results augment the argument that in sales management, employee selection should focus more on personality characteristics than performance or skill (Zoltners et al. 2012d).

3.2 Literature Review

Our research on adverse selection in sales managerial promotions is related to the quantitative marketing and economics literature on tournament and employee selection.

Empirical evidence suggests that firms primarily make internal selection and promotion decisions through tournament between employees (DeVaro 2006). A number of papers in the area of marketing study tournaments between sales employees as a source of motivation (Kalra and Shi 2001, Lim, Ahearne and Ham 2009, Lim 2010, Chen, Ham and Lim 2011, Ridlon and Shin 2013). Since, much of the focus of this literature has been on motivation of employees we believe that by studying tournaments as a selection device and finding conditions when it works well as a selection device and when it does not, we are making a contribution to this literature. We also believe that to the best of our knowledge we are the first paper in marketing to model tournament between employees with asymmetric risk characteristics. The papers in economics that study tournament between risk averse employees find that under limited liability the risk averse may put higher effort in a contest and under some conditions the outcome of the contest may be independent of risk aversion (Skaperdas and Gan 2004). While we also find that under some conditions the risk averse may have a higher probability of winning a contest, the mechanism behind our results are different from the ones in this literature.

Extant literature in marketing on the selection of sales employees finds that sales employees with heterogeneous ability and risk characteristics are likely to self-select into incentive plans that suit their characteristics (Lo et al. 2011, Lal and Staelin 1986). However, the focus of this literature has been on selection of outside employees for a sales rep role. We contribute to this stream of literature by studying selection of sales managers from within a sales force composed of heterogeneous sales representatives. Our research is also closely related to the Economics literature on alternative

explanations for a particular type of adverse selection in selection of employees, called the Peter Principle. The Peter Principle is defined as a phenomenon that states that in any hierarchical organization, employees tend to rise to their level of incompetence (Peter and Hull 1969). Fairburn and Malcomson (2001) show why firms may use promotions to motivate employees even when such promotions may conflict with efficient assignment of employees to jobs. Their explanation is based on the situation when performance is unverifiable and under such conditions they find that the use of promotion to provide incentives reduces the incentive for managers to be affected by influence activities even though such promotions may create Peter Principle. Lazear (2004) explains Peter Principle with regression to the mean of temporary productivity shocks. Koch and Nafziger (2007) find that it may be profitable for the principal to promote a less able agent if the agent puts extra effort after the promotion to overcome his incompetence. We contribute to this stream of literature by proposing an alternative explanation for Peter Principle in sales managerial promotions based on the effect that the different benefits at a sales manager's position has on the motivation of employees with asymmetric risk and productivity characteristics.

Finally, our paper is also related to use of screening to solve adverse selection problems associated with selection of employees. In a seminal work, Spence (1972) showed that investment in education can be used as a signal by which to credibly communicate one's higher ability in the job market. We show that investment in education can also credibly communicate one's higher willingness to take risks and therefore, under some conditions can solve another adverse selection problem that arises when employees with heterogeneous risk characteristics compete for a position.

3.3 Model

We consider a two period model. There is a firm that needs to internally fill a vacant sales manager's position that will come up in period 2. To keep the model tractable while being able to investigate the core issue outlined above we assume that the firm is considering sales management promotion (or selection) of one of the two salespeople that it employs as sales representatives. Sales employee characteristics and their decisions, the sales response functions, the characterisitics of the sales employee labor market and the firm's decisions are given below.

3.3.1 Sales Employee Characteristics

The two salespeople are characterized by different levels of risk aversion and productivity². Risk aversion is captured through a non-negative risk aversion coefficient and productivity is captured through a positive cost of effort coefficient. The employee 1 has risk aversion r_1 and productivity a_1 , while employee 2 has risk aversion r_2 and productivity a_2 . Both employees compete to get promoted to a single sales manager position. Without loss of generality, we normalize $r_2 = 0$ and $a_2 = 1$. The two employees are homogeneous in all other aspects. Their utility function is given as $U(I_i, e_i) = u(I_i) - c(e_i)$, where u(.) is the von Neumann-Morgenstern utility function, c(.) is the cost of effort or the disutility function, I_i is the income earned by the i^{th} by putting e_i effort. Furthermore, we assume the functional form for the cost function to be $c(e_i) = a_i \frac{e_i^2}{2}$, where a_i is the cost coefficient or a measure of the productivity of the employee, with $a_i > 0$. The productivity characteristics of the employee captures how the employee trades off between leisure and work. A high *a* would suggest that the employee values leisure more and therefore does not like working and is less productive. The productivity characteristics considered here is independent of the position

²We use the terms productivity and ability interchangeably though in more precise terms productivity is only a type of ability.

at which the employee is working. In other words, if an employee is less productive then he will be less productive both as a sales rep and as a sales manager. In computing the expected utility we consider the certainty equivalent form of the utility function (Milgrom and Roberts (1992), pgs. 246-247), i.e. $E[u(I)] \stackrel{def}{=} E[I] - \frac{r}{2} Var[I]$, where I is the income and r is the coefficient of constant absolute risk aversion. We denote the wage earned by the i^{th} employee at K^{th} position as $U_{K,i}$, where $i = \{1, 2\}$ and $K = \{P, S\}$. Here, P denotes the manager's position and S denotes the sales rep position.

It is important to note that since the employees are assumed to be homogenous with respect to all other characteristics except for risk aversion and productivity they are equally qualified or not qualified in terms of managerial skills. This assumption helps us to abstract away from the issue of selection with respect to managerial skills. With the assumption of homogeneity of managerial skills if we still found that firm's promotion policy leads to selection of an undesirable employee then that would suggest that there is a deeper problem of adverse selection involved that does not get addressed even when the firm ensures that the promoted employees have the required managerial skills. Moreover, in our model set up if we find adverse selection with the assumption of homogeneity then it would also mean that if there is heterogeneity with respect to managerial skills and the firm promotes the most managerially skilled employee to the managerial position, even then there can be a problem of adverse selection because inherent managerial skills and, risk aversion or productivity are expected to be independent of each other. In other words, there is no reason to believe that a more inherently managerially skilled employee will be less or more risk averse or will have high or low productivity.

3.3.2 Sales Response Functions

The sales response function at the sales rep's position is given as,

$$x_{S,i} \stackrel{def}{=} e_i + e_P + \varepsilon_i + \gamma, \text{ where } \varepsilon_i \sim NIID\left(0, \sigma_{\epsilon}^2\right) \text{ and } \gamma \sim N\left(0, \sigma_{\gamma}^2\right).$$
(3.1)

When employee i working as a sales rep makes an effort of e_i he generates a sales of $x_{S,i}$. The sales generated also depends on the effort, e_P , put by the sales rep's manager, random factors in the sales rep's territory, ε_i , and random factors, γ , that are common to all territories in the firm. We explain the sales response function with an example. Suppose that the context is a bank branch that sells financial products in a neighborhood. The bank branch has three channels viz. an internet channel that caters to internet leads, a walk-in channel that caters to customers who directly walk in to the branch and a direct selling channel in which a sales rep has to directly approach businesses in surrounding areas to sell them products. Each of the channels has a dedicated sales representative while the sales manager is responsible for overall sales from the bank branch. In this context, the effort by the sales rep is in form of selling through his channel while the effort by the manager is in form of demand inducement activity like sales promotion for the overall branch. The random factor specific to a territory is the uncertainty that the sales rep faces related to his channel. For example, sometimes there is bad weather and branch walk-ins are very few, an uncertainty that need not affect the other channels. The common random factor captures the uncertainty that everyone faces. For example, if the corporate office of the bank runs a very successful campaign then everyone benefits or when there is a public relations debacle by the bank then everyone suffers.

We assume that the sales of x_i translates into a profit of $\delta_S x_{S,i}$ that is attributed to the sales rep's position, where δ_S is the profit per unit sales that the firm attributes to the sales rep. Note that the profit from sales rep's effort can be written as $\delta_S x_{S,i} = \delta_S (e_i + \varepsilon_i + \gamma) + \delta_S e_P$. It will be evident

in our analysis that for the sales rep $\delta_S e_P$ is essentially a constant and his motivation to put effort is only based on $\delta_S (e_i + \varepsilon_i + \gamma)$.

The sale manager manages n territories. The sales response function for the sales manager's position is given as,

$$x_{P,i} \stackrel{def}{=} \sum_{n} e_{S,l} + e_i + \varepsilon_l + \gamma = n \left(e_i + \gamma \right) + \sum_{n} \left(\varepsilon_l + e_{S,l} \right)$$
(3.2)

where ε_l is the random factor in l^{th} territory and $e_{S,l}$ is the effort made by the sales rep in that territory.

When the employee *i* working as a sales manager puts an effort of e_i he generates a sales of $x_{P,i}$ from *n* territories under his control. The sales generated also depends on the effort, $e_{S,l}$, put by the sales rep *l* under the sales manager, random factors in each of the *n* territories, ε_l , and random factors, γ , that are common to all territories in the firm. We assume that the sales of $x_{P,i}$ translates into a profit of $\delta'_P x_{P,i}$ that is attributed to the sales manager, where δ'_P is the profit per unit sales that the firm attributes to the sales manager's position. Note that profit from the sales manager's effort can be written as $\delta'_P x_{P,i} = \delta'_P n (e_i + \gamma) + \delta'_P \sum_n (\varepsilon_l + e_{S,l})$. It will be evident in our analysis that for the sales manager $\delta'_P \sum_n \varepsilon_l$. Further, for the sake of parsimony and to cut down on the number of parameters we assume that $Var\left(\sum_{n} \varepsilon_l\right) = \frac{\sigma_i^2}{n} \to 0$ or $n \to \infty$. Our results do not change qualitatively for other values of *n*, with n > 1. As $\delta'_P n$ is a constant we denote the constant by δ_P , i.e. $\delta_P = \delta'_P n$.

The sales manager's position is more desirable compared to the sales rep's position on account of two reasons. First, it provides the employee with an opportunity to earn a higher reservation

wage because he generates higher profits for the firm by managing multiple revenue channels or by managing multiple other sales reps. Second, it provides the employee with an opportunity to diversify some of the channel or individual specific risks associated with selling at the sales rep's position because the individual specific risks across territories or reps are independent. For example, in the context of the bank branch, if the sales manager runs a successful sales promotion campaign then he benefits from sales from all the channel. At the same time the sales manager also benefits from his role because he can diversify the risks that are specific to each of the channels. In other words, there is likelihood that if the walk-in channel underperforms then the internet channel may make up for it. In terms of model assumptions, the two differences between the job profiles of sales managers and sales reps have been captured in terms of differences in the profit generated per unit effort and the uncertainty associated with the jobs. More specifically, we have denoted the profit per unit of effort to be δ_P and δ_S , for the manager's and the rep's positions, respectively, such that $\delta_P \geq \delta_S > 0$. We assumed the profit margin δ_P to be higher than δ_S because the effort put by a sales manager contributes to profit generated in multiple territories as opposed to only one territory. Further, we assumed that the sales manager's position only involves the normally distributed common uncertainty (or systematic risk) γ while a sales rep's position involves both normally distributed common and individual specific uncertainty $\varepsilon + \gamma$. Therefore, if an employee puts an effort of e at the sales manager's position then the employee generates $\delta_P (e + \gamma)$ in profits but if the same employee puts the same effort as a sales rep, the employee generates $\delta_S (e + \varepsilon + \gamma)$ in profits³. Here, it needs to be added that our model set up is general enough to allow for any type of activity as part of the effort investment at the two positions. For example, the effort of e at the sales rep position may be in the form of prospecting and persuasion of customers while the same level of effort e at the managerial position may be in the form of running a better sales promotion campaign for all the sales employees under the manager's territory. We do not distinguish between

³Note that the variance in sales revenues generated by the sales manager is higher compared to the one generated by the sales rep if δ_P is sufficiently high.

what activity the position requires but only in terms of how much effort is invested in the activity, by whom and at what position.

3.3.3 Firm, its Decisions and Game Sequence

We endogenize the determination of wages for employees by considering competition in the labor market for sales employees. The wages are determined in the labor market where firms compete to recruit employees. The elasticity of the labor market for sales reps and sales managers is denoted as θ where $0 \le \theta < \infty$. Consistent with the literature on oligopsony in labor markets (Boal and Ransom 1997) we use θ to express the division of profit between the firm and its employees. More specifically, an employee's wage is $\frac{\theta}{1+\theta}$ and the firm's profit is $\frac{1}{1+\theta}$ of the profit that the employee generates at a position. Note that our treatment of the labor market is not restrictive because we consider all cases between monopsony ($\theta = 0$) and perfect labor market competition ($\theta \to \infty$). Further, we assume that external promotions are not possible i.e. a different firm cannot recruit a sales rep as its sales manager. This is largely true of most labor markets because of information asymmetries that exist between the employing organization and the external organization. In the current framework this is possible if we assume that the current employers know that both the employees have enough managerial skills to meet the minimum threshold required at the sales manager position but the other firms do not have this information and they only recruit employees at different positions laterally, i.e. only a sales manager but not a sales rep in a different firm is recruited as a sales manager. Given the firm's access to labor markets and in order to focus on the problem of selection of sales manager we consider the firm's composition of sales reps to be the same over both the periods. This implies that the firm needs to select a good type sales manager from its sales force and the proportion of good and bad type sales reps stays the same over time. This is reasonable because otherwise the firm can always recruit employees from the pool of candidates available in the market and the employees can also seek opportunities at other organizations. The ratio of different types of candidates in the market is expected to be stable over time.

The game sequence is as follows. In the first period, the firm conducts a tournament that promotes the employee with the better performance to the sales manager position and leaves the other employee at the sales rep position. As part of the tournament the firm only makes the decision on whom to promote but not the wage increase associated with the promotion. In the second period, reservation wages at both the manager and rep positions are determined by the labor market competition and optimal linear contracts are offered to all employees. The perfectly forward looking firm and the employees maximize their expected profits and expected utilities, respectively. The game is solved for the subgame perfect Nash Equilibrium using backward induction. The firm's and employees' objective functions for each of the periods are given as part of the analysis.

3.4 Analysis

3.4.1 Adverse Selection in Sales Managerial Promotions

3.4.1.1 Period 2

We first present the solutions for the second period. Firm's period 2 profit is given as Π_2 . For the *i* type employee at the K^{th} position, where $i = \{1, 2\}$ and $K = \{P, S\}$, the firm solves the following problem to offer its profit maximizing contract $(m_{K,i}, w_{K,i})$, where $w_{K,i}$ is the fixed salary and $m_{K,i}$ is the per unit commission. Since the analysis is only for period 2 we suppress the subscripts t = 2 in the effort and other variables. $\Pi_2 = \pi_{P,i} + \pi_{S,1} + \pi_{S,2}$, where *i* is the type of sales rep that was promoted by the firm in period 1

$$\pi_{K,i} = \max_{m_{K,i}, w_{K,i}} E[\delta_K x(e_{K,i}) - (w_{K,i} + m\delta_K x(e_{K,i}))]$$
(2.3)

$$e_{K,i}^* \epsilon \arg \max E[(w_{K,i} + m_{K,i}\delta_K x(e_{K,i}))] - c(e_{K,i}) - \frac{r_i}{2} Var[\delta_K m_{K,i} x(e_{K,i})]$$
(IC) (3.4)

$$E[(w_{K,i} + m_{K,i}\delta_K x(e_{K,i}))] - c(e_{K,i}) - \frac{r_i}{2}Var[\delta_K m_{K,i} x(e_{K,i})] \ge U_{K,i} \qquad (IR)$$
(3.5)

For an employee with characteristics (a_i, r_i) , the firm earns an expected profit of $\pi_{S,i}$ when the employee works as a sales rep and $\pi_{P,i}$ when the employee works as a sales manager. The employee with characteristics (a_i, r_i) earns a wage of $U_{S,i}$ as a sales rep and $U_{P,i}$ as a sales manager, and puts in an effort of $e_{S,i}$ and $e_{P,i}$, respectively. Derivations of optimal effort levels, wages and profits are shown in the Appendix and the terms are summarized in Table 3.1.

Lemma 3.1 Employees put more effort and earn higher wages in the managerial position as compared to the sales rep position i.e. $e_{S,i} < e_{P,i}$ and $U_{S,i} < U_{P,i} \forall i$. The more risk averse and less productive employees still earn less wages at each of the positions i.e. $U_{S,i} < U_{S,j}$ and $U_{P,i} < U_{P,j}$ $\forall i, j \in \{1, 2\}$ for $r_i > r_j$ and $a_i < a_j$.

Note that even without considering the effect of additional managerial skills and in spite of the fact that the employees' productivity stays the same, employees put more effort and earn higher wages in managerial positions as compared to sales rep position i.e. $e_{S,i} < e_{P,i}$ and $U_{S,i} < U_{P,i} \forall i$. This is so because a sales management position allows an employee's effort to have an effect on profit generated in multiple territories and at the same time allows the individual specific risks to be diversified. The multiplier and diversification effects make it more attractive for employees to put effort and also reduce their risk premiums. Because of the higher effort put by employees and their

need for lesser risk premium the firm provides greater incentives which in turn lead to higher effort and profits, part of which gets reflected in the employee's wage. This effect of promotion on wage and effort is irrespective of the level of productivity or risk aversion of the employee. However, $U_{S,i} < U_{S,j}$ and $U_{P,i} < U_{P,j} \forall i, j \in \{1, 2\}$ for $r_i > r_j$ and $a_i < a_j$, i.e. more risk averse and less productive employees still earn lower wages at each of the positions because they demand more risk premium and put less effort per unit of incentive. Note while the firm earns a higher profit from an employee when he works as a manager as compared to a rep, it cannot promote all the reps to a managerial position because it is constrained by the limited number of such positions available. Therefore, the firm would want to promote the employee who helps the firm earn the highest profit at the managerial position, i.e. the firm is interested in the best combination of employee characteristics (a_i, r_i) among the employees who compete where the set of characteristics (a_i, r_i) are better than (a_j, r_j) if and only if $\pi_{P,i} > \pi_{P,j}$ (Lemma 3.2):

Lemma 3.2 For the firm, selection of employee 2 is optimal when $(a_2 = 1, r_2 = 0) \succcurlyeq (a_1, r_1) \Leftrightarrow \pi_{P,2} \ge \pi_{P,1} \Leftrightarrow 1 > a_1 \ge a_1^T \ge 0 \Leftrightarrow \frac{1-a_1}{a_1^2 \sigma_\gamma^2} > r_1 > 0$, where $a_1^T = \frac{1}{2} \sqrt{\frac{1+4r_1\sigma_\gamma^2}{r_1^2\sigma_\gamma^4}} - \frac{1}{2r_1\sigma_\gamma^2}$

Proof. Proof follows from comparison of $\pi_{P,2}$ and $\pi_{P,1}$, which are given in Table 3.1.

The above condition implies that the firm will always prefer an employee with higher productivity and lower risk aversion over an employee with lower productivity and higher risk aversion at the managerial position. Further, higher productivity can substitute for the lack of lower risk aversion. More specifically, if the firm faces a choice between a risk averse employee with higher productivity (employee 1) and a risk neutral employee with lower productivity (employee 2) the firm will prefer the risk neutral employee as long as the risk averse employee's productivity is not too high i.e. $a_1 \ge a_1^T$. The threshold for productivity is higher for employees with greater risk aversion and when common uncertainty in the manager's position is higher.

3.4.1.2 Period 1

Next, we derive period 1 results. However, before we do so we define what is meant by adverse selection in the current model.

Definition 1 Adverse Selection is defined as the condition when the undesirable employee has a higher probability of being promoted to the management position. Among two employees the undesirable employee is the one who produces lower profits for the firm in period 2 at the managerial position.

Note that we do not specify promotion of a particular employee as adverse selection. Rather, we define adverse selection as a condition that occurs when an employee who is less suited for the management position has higher probability of getting promoted. In addition, note that while promotion of the winner in a tournament means that the less suited employee will always have some chance of being promoted we only consider the selection as problematic when the less suited has a higher chance of being promoted. To derive the conditions when adverse selection arises in the tournament between the employees we derive the period 1 efforts and winning probabilities for each of the employees. Period 1 efforts are given as,

$$e_1^* = \arg\max p_1 U_{P1} + (1 - p_1) U_{S1} - \frac{1}{2} r_1 (U_{P1} - U_{S1})^2 (1 - p_1) p_1 - \frac{a_1 e_1^2}{2}$$
(3.6)

$$e_2^* = \arg\max p_2 U_{P2} + (1 - p_2) U_{S2} - \frac{1}{2} r_2 (U_{P2} - U_{S2})^2 (1 - p_2) p_2 - \frac{a_2 e_2^2}{2}$$
(3.7)

Where, $p_1 = G(x_1 > x_2) = G(e_1 + \varepsilon_1 + \gamma \ge e_2 + \varepsilon_2 + \gamma) = G(e_1 - e_2 \ge \varepsilon_2 - \varepsilon_1) = G(e_1 - e_2 \ge \zeta)$, where $\zeta \sim N(0, 2\sigma_{\epsilon}^2)$ since $\varepsilon_i \sim NIID(0, \sigma_{\epsilon}^2)$, $p_2 = 1 - p_1$, $\frac{\partial p_2}{\partial e_2} = -\frac{\partial p_1}{\partial e_2} = -(-g(\Delta e)) = g(\Delta e)$,

$$U_{P1}, U_{P2}U_{S1}, U_{S2}$$
 are given in Lemma 3.1, $a_2 = 1$ and $r_2 = 0$

Using the results from Table 3.1 and Lemma 3.2 we derive the conditions under which the employee with undesirable productivity and risk characteristics has a higher probability of promotion to the sales manager's position in equilibrium. These conditions are summarized in Proposition 3.1. The complete derivations are given in the appendix.

Proposition 3.1 The less suitable employee has a higher probability of getting promoted to the sales manager's position, if,

(1)
$$0 < a_1 < 1, 1 < \frac{\delta_P}{\delta_S}, 0 < \frac{1 - a_1}{a_1^2 \sigma_\gamma^2} < r_1 < A + \frac{1}{2}\sqrt{B}$$
 (3.8)

or (2)
$$a_1 = 1, 1 \le \frac{\delta_P}{\delta_S} < \sqrt{\frac{\sigma_\gamma^2 + \sigma_\epsilon^2}{\sigma_\gamma^2}}, 0 < r_1 < A + \frac{1}{2}\sqrt{B}$$
 (3.9)

or (3)
$$a_1^U > a_1 > 1, 1 \le \frac{\delta_P}{\delta_S} < \sqrt{\frac{\sigma_\gamma^2 + \sigma_\epsilon^2}{\sigma_\gamma^2}}, 0 < A - \frac{1}{2}\sqrt{B} < r_1 < A + \frac{1}{2}\sqrt{B}$$
 (3.10)

where a_1^U , A and B are given in the Appendix.

Proof. See Appendix B

Condition (1) states that for any δ_P and δ_S with $\delta_P > \delta_S$ there exists a range of parameters in which the less suitable employee has a higher probability of getting promoted. However, this requires the risk aversion of the less suitable employee to be intermediate i.e. $\frac{1-a_1}{a_1^2 \sigma_\gamma^2} < r_1 < A + \frac{1}{2}\sqrt{B}$, and his productivity to be higher than that of the more suitable employee, i.e. $a_1 < 1$. On the other hand, when employees only differ in risk aversion, i.e. $a_1 = 1$, adverse selection only arises if the sales manager's benefits from diversification are greater than his benefits from earning a greater revenue due to more responsibilities at his position i.e. $1 \le \frac{\delta_P}{\delta_S} < \sqrt{\frac{\sigma_\gamma^2 + \sigma_c^2}{\sigma_\gamma^2}}$, and if risk aversion is below a threshold, i.e. $r_1 < A + \frac{1}{2}\sqrt{B}$. Finally, condition (3) characterizes the range of parameters for the interesting case in which the employee with lower productivity, i.e. $a_1 > 1$ and higher risk aversion, $r_1 > 0$, ends up with a higher probability of getting promoted. However, this requires the diversification effect to be stronger than the effect of earning higher revenues, $1 \le \frac{\delta_P}{\delta_S} < \sqrt{\frac{\sigma_{\gamma}^2 + \sigma_{\epsilon}^2}{\sigma_{\gamma}^2}}$, the productivity disadvantage to be sufficiently low, i.e. $1 < a_1 < a_1^U$, and the risk aversion coefficient to be intermediate.

To interpret these conditions we study the plots in Figure 3.2 which depict the regions where adverse selection occurs. The plots capture the parameter space of possible risk and productivity characteristics for employee 1 (a_1, r_1) under two different conditions related to the extent of individual specific uncertainty. In the first condition, $1 \leq \frac{\delta_P}{\delta_S} < \sqrt{\frac{\sigma_1^2 + \sigma_2^2}{\sigma_\gamma^2}}$, the benefit from possible diversification of individual specific risk associated with sales reps or individual channels is higher compared to the benefit from possibilities of higher revenue generation associated with the managerial position while in the second condition the relationship is reversed. The vertical line $a_1 = 1$ represents the case when both the employees have same productivity but different risk aversions, while the x-axis, i.e. $r_1 = 0$ represents the case when employees have different productivities but both are risk neutral.

We gather a number of insights from Proposition 3.1 and Figure 3.2. First, we find that adverse selection never happens when employees only differ in their productivity. In the plots, this is clear since the adverse selection regions do not touch the x-axis except for the case when productivity coefficients are equal i.e. $a_1 = 1$. This result validates the intuition suggested by the previous literature that though with increasing asymmetry in productivity the overall effort in the tournament decreases, the more productive player still outperforms the weaker one (Lazear and Rosen 1983, Kalra and Shi 2001 etc.). This suggests that by only considering heterogeneity in ability or productivity of employees, we may end up ignoring possibilities of adverse selection in employee promotions.

Second, when employees differ in risk characteristics, the possibility of adverse selection arises and this also depends on differences in the productivity of employees. In the case when employees have the same productivity but different risk aversions i.e. on the vertical line $a_1 = 1$, adverse selection arises only when the risk averse employee has a moderate level of risk aversion and the benefits from diversification of risk at higher managerial position outweigh the incentives from higher revenue possibilities. This is so because promotion to a managerial position has two asymmetric incentives for the two employees. The less risk averse employee is more motivated to get promoted because of prospects of earning a higher wage, while the more risk averse employee seeks the possibility of lesser risks experienced at the sales manager's position due to possibilities of diversification. When the incentive from the later effect is higher, then the more risk averse employee puts in more effort. However, beyond a certain level of risk aversion even the possibility of diversification of risk is not enough to motivate the risk averse employee and the employee underperforms compared to the less risk averse employee. Thus, we see this effect only for a limited range of risk aversion levels (in the above plot, $r_1 \in [0, 0.000025]$).

Third, it is interesting to note that there is a small parabolic region on the right of the vertical line $a_1 = 1$ where the employee with lower productivity and higher risk aversion (employee 1 in this case) puts in more effort than the more able and less risk averse employee. This region represents a case of severe adverse selection for the firm where it finds none of the good qualities in the employee who has a higher likelihood of promotion. This case occurs because in the region of moderate risk aversion, the asymmetry in motivation for promotion on account of higher possibilities of diversification of risks is so high that even lower productivity (or higher disutility of effort) is not enough to stop the risk averse employee from putting more effort and getting promoted. The parabolic projection on the right of the vertical line suggests that in this region employees with moderate risk aversion are the ones who are most motivated. However, the region ends when productivity of the risk averse employee falls below a threshold. These results suggest that the

notion that employees with higher productivity always have a higher chance of promotion may not hold true when employees differ with respect to risk aversion and the managerial position provides opportunities for diversification of risks.

Fourth, we find that there always exists a region in the parameter space in which adverse selection happens. This is clear from the adverse selection region on the left side of the vertical line on the each of the plots. We explain our findings related to the size of the adverse selection region in the following proposition.

Proposition 3.2 The region of adverse selection increases with an increase in productivity of the risk averse employee, ceteris paribus, i.e.,

$$\frac{\partial \Delta r_i}{\partial a_1} < 0, \text{ if condition } i \text{ (proposition 3.1) holds, where } i = \{1, 2, 3\} \text{ and}$$
$$\Delta r_1 = A + \frac{1}{2}\sqrt{B} - \frac{1 - a_1}{a_1^2 \sigma_\gamma^2}, \Delta r_2 = A + \frac{1}{2}\sqrt{B}, \Delta r_3 = A + \frac{1}{2}\sqrt{B} - \left(A - \frac{1}{2}\sqrt{B}\right) = \sqrt{B}$$

Proof. See Appendix B

With higher productivity of the risk averse employee the possibility of adverse selection becomes greater because the increase in productivity makes the possibility of diversification of risk even more lucrative for the risk averse employees. This is so because a possibility of diversification of risk decreases the risk premium that employees need as sales managers and the need for less risk premium induces the firm to offer more incentives, which have a stronger effect as productivity of the employees increase. Thus, the diversification effect that induces risk averse employees to put more effort in the first period becomes more pronounced with an increase in productivity of the employees. It needs to be noted that an increase in productivity of the employees also makes them more desirable for the firm and hence, may decrease adverse selection. However, the first effect of

higher effort by more risk averse employees with an increase in productivity dominates the latter effect of increase in desirability of the employee.

Taken together, the above results suggest the possibility of adverse selection in promotion tournament for sales manager positions for a wide range of risk aversion and productivity parameters when sales managerial positions provide employees with possibilities of diversification of risks that the employees face at sales rep levels. These results provide an explanation for the phenomenon of Peter Principle found in sales managerial positions and counterintuitively predict adverse selection in sales managerial promotions even when employees do not differ in their managerial skills. Further, these findings trace the source of adverse selection to personality traits like risk aversion and productivity rather than skills which can be acquired over time. It suggests that sales organizations should also consider personality traits in addition to skills when selecting sales managers (Zoltners et al. 2012c). Furthermore, our results also explain why middle level managers are often more risk averse (McKinsey Quarterly 2012).

Given the possibilities of adverse selection when implementing tournaments to select sales managers one might wonder whether the use of tournament is optimal for the firm. Note that the firm's optimization problem involves inducement of effort in the first period and selection of the employee with risk and productivity characteristics that maximize firm's profits for the managerial position in the second period. From the firm's perspective, the dual goals of effort inducement and selection of better sales manager are met when the employee with a better combination of risk and productivity characteristics has a higher probability of being promoted. However, if the tournament leads to a higher probability of promotion for the unsuitable employee then the firm faces adverse selection⁴. Since, at the equilibrium the firm will care about both selection and effort, it will trade off the benefits from effort in the current period and potential costs from adverse

⁴Note that even when this is not the case sometimes the unsuitable employee will get promoted due to randomness in the environment.

selection in the subsequent period, and for a sufficiently high need for current period effort, it will still use a tournament to motivate effort even though there may be adverse selection. This tradeoff has been shown in the following subsection in Proposition 3.4. However, before analyzing this tradeoff we study what other methods the firm can use to select sales managers from its sales reps and whether it can be optimal for the firm to choose one of such methods instead of tournaments. This has been analyzed in the following subsection.

3.4.2 Sales Training and Testing as a Screening Device

In the preceding section we identified an adverse selection problem associated with promotion of the best performing sales rep to a managerial role. In this section we explore possible solutions to the adverse selection problem, and in doing so we consider the solutions that firms are already implementing to prevent the occurrence of Peter Principle in sales management promotions.

The solution of letting sales rep self-select between different positions, i.e. sales rep and sales management, is inefficient in solving the adverse selection problem because, as we saw in Lemma 3.1, in our model all employees earn higher wages in management positions, and therefore, all of them want to get promoted. Similarly, the solution of training the best performing sales rep to be a better manager also need not solve the problem that we identified because while such a solution will ensure that the sales rep is not lacking in managerial skills, the promoted sales rep may still be the one with higher risk aversion. A possible solution can be to use sales training and testing as a screening device to screen employees. Since the root of the adverse selection problem is the higher incentive for moderately risk averse employees as compared lower risk averse employees,

the risk associated with sales training and testing can be potentially used to dissuade and separate the moderately risk averse from the low risk averse sales reps. We explore this avenue in the following analysis.

Suppose that the firm treats sales management training (or just management training, e.g. an EMBA degree) and associated quality of training (internal test scores or EMBA GPA or quality of business school from where EMBA is acquired) as a signal of preparedness of the sales rep to join a management position. If the firm decides to use sales training and test scores to select its sales managers then in period 1 the firm declares that it will make promotion decisions based on training and test scores, and subsequently, the two employees make the decision on whether to sign up for sales training, and if they do choose to pursue training then they undergo sales training and testing. Further, assume that the human capital in form of management ability that sales employees get from such management education is stochastic such that,

$$m \stackrel{def}{=} e + \mu \tag{3.11}$$

Where, *m* is the expected management ability that an employee may get from pursuing sales education at an effort investment of e, $\frac{a_i e^2}{2}$ is the disutility associated with making the effort, $\mu \sim N\left(0, \sigma_{\mu}^2\right)$ is a random factor that may affect acquisition of sales management skills. The random factor may reflect the fact that investment in education carries a risk for the employee. This risk can arise from the fact that the employee may not get the test scores required for promotion or it may represent the possibility that the employee misjudged his or her ability to pursue such an education.

Our assumptions reflect the empirical evidence that while investment in education leads to higher expected ability and associated wage increases, the returns are risky in nature. For example, Chen (2001), Hartog et al. (1994) and WSJ (2015) report that the riskiness of college attendance is an important factor in the choice of whether to pursue education. We have assumed the test score to be an exact measure of the management ability that the sales employee acquires. This assumption can be relaxed without changing the results by assuming another error term in the equation 3.11.

3.4.2.1 Analysis

We first derive the conditions that separate the employees according to their risk aversion. If employee i decides to invest in sales education then the returns from such an education are given as,

$$M_{i} = \max_{e} E\left[m = e + \mu - \frac{a_{i}e^{2}}{2} - \frac{r_{i}}{2}Var\left(\mu\right)\right] = \frac{1}{2}\left(\frac{1}{a_{i}} - r_{i}\sigma_{\mu}^{2}\right)$$

Hence, the total expected value of promotion for the employee 1 is given as $U_{P1} + \frac{1}{2} \left(\frac{1}{a_1} - r_1 \sigma_{\mu}^2 \right)$ and for employee 2 is given as $U_{P2} + \frac{1}{2}$. The individual rationality and incentive compatibility constraints for both the employees for the separation condition to hold are given as,

$$U_{P1} + \frac{1}{2} \left(\frac{\theta}{1+\theta} \right) \left(\frac{1}{a_1} - r_1 \sigma_{\mu}^2 \right) - U_{S1} \le 0 \text{ (IC1), } U_{P1} + \frac{1}{2} \left(\frac{\theta}{1+\theta} \right) \left(\frac{1}{a_1} - r_1 \sigma_{\mu}^2 \right) \ge 0 \text{ (IR1)}$$
$$U_{P2} + \frac{1}{2} \left(\frac{\theta}{1+\theta} \right) \ge U_{S2} \text{ (IC2), } U_{P2} + \frac{1}{2} \left(\frac{\theta}{1+\theta} \right) \ge 0 \text{ (IR2),}$$

where U_{P1}, U_{P2}, U_{S1} and U_{S2} are given in Lemma 3.1

The first incentive compatibility constraint (IC1) ensures that the employee 1 is better off by not pursuing education and the second incentive compatibility constraint (IC2) ensures that employee 2 is better off by pursuing education. These two constraints together ensure that there is separation in the actions of the two sales reps and that makes it possible for the firm to screen the undesirable employee. The other two constraints i.e. the individual rationality constraints (IR1) and (IR2) ensure that the employees are at least compensated for participating as much as their reservation wages.

Solving the above inequalities we derive the conditions for screening in Proposition 3.3.

Proposition 3.3 Sales training and testing can ensure screening of the better employee under the conditions given below i.e. employee 2 would choose to enroll in sales training and testing while employee 1 would not, if,

- 1. Employee 1's risk aversion is beyond a threshold level, i.e. if $r_1 > \overline{r}_1$. The threshold level for risk aversion decreases with a decrease in productivity, i.e. $\frac{\partial \overline{r}_1}{\partial a_1} < 0$.
- 2. The uncertainty associated with sales training is moderate and individual specific uncertainty is low or if uncertainty associated with sales training is high.

$$\frac{1+\delta_P^2-\delta_S^2+a_1r_1\sigma_{\gamma}^2}{a_1r_1+a_1^2r_1^2\sigma_{\gamma}^2} < \sigma_{\mu}^2 < \frac{1+\delta_P^2+a_1r_1\sigma_{\gamma}^2}{a_1r_1+a_1^2r_1^2\sigma_{\gamma}^2} \text{ and } 0 < \sigma_{\epsilon}^2 < \overline{\sigma}_{\epsilon}$$
(3.12)

or
$$\frac{1+\delta_P^2+a_1r_1\sigma_\gamma^2}{a_1r_1+a_1^2r_1^2\sigma_\gamma^2} < \sigma_\mu^2$$
 and $\overline{\sigma}_\epsilon < \sigma_\epsilon^2$ (3.13)

3. The uncertainty associated with sales training is moderate and common uncertainty is high or if uncertainty associated with sales training is high.

$$\frac{1}{a_1r_1} < \sigma_{\mu}^2 < \frac{1 + \delta_P^2 - \delta_S^2 + a_1r_1\sigma_{\varepsilon}^2 \left(1 + \delta_P^2\right)}{a_1r_1 + a_1^2r_1^2\sigma_{\varepsilon}^2} \text{ and } 0 < \overline{\sigma}_{\gamma} < \sigma_{\gamma}^2$$

$$(3.14)$$

or
$$\frac{1+\delta_P^2-\delta_S^2+a_1r_1\sigma_{\varepsilon}^2(1+\delta_P^2)}{a_1r_1+a_1^2r_1^2\sigma_{\varepsilon}^2} < \sigma_{\mu}^2 \text{ and } 0 < \sigma_{\gamma}^2 < \overline{\sigma}_{\gamma}$$
 (3.15)

4. The uncertainty associated with sales training is moderate and profit per unit of effort in management position is low,

$$\sigma_{\mu}^{2} > \frac{1 + 2a_{1}r_{1}\sigma_{\gamma}^{2} + a_{1}r_{1}\sigma_{\gamma}^{4} + a_{1}r_{1}\sigma_{\varepsilon}^{2} + a_{1}r_{1}\delta_{S}^{2}\sigma_{\varepsilon}^{2} + a_{1}r_{1}\sigma_{\gamma}^{2}\sigma_{\varepsilon}^{2}}{a_{1}r_{1} + 2a_{1}r_{1}\sigma\gamma^{2} + a_{1}^{3}r_{1}^{3}\sigma\gamma^{4} + a_{1}^{2}r_{1}^{2}\sigma_{\varepsilon}^{2} + a_{1}^{3}r_{1}^{3}\sigma_{\gamma}^{2}\sigma_{\varepsilon}^{2}} \text{ and } \delta_{S} < \delta_{P} < \overline{\delta}_{P} \quad (3.16)$$

Proof. See Appendix

Sales training and testing can screen the risk averse sales rep from becoming sales manager if the risk associated with investment in sales training is high and the risk aversion in the sales rep is also sufficiently high. The reason is that in such a case the risk averse employee would find it unattractive to undergo such training while the less risk averse employee would still find it attractive to acquire sales training because there are net expected benefits from acquisition of management skills. The threshold risk aversion beyond which the risk averse sales rep will be dissuaded from pursuing sales training decreases with decrease in productivity of the employee. This intuition being that while a lower productivity decreases the expected benefits from a undergoing sales training it does not decrease the risks associated with such a training. Thus, a decrease in benefit from sales training but no decrease in risk associated makes the risk averse employee more unlikely at the margin to undergo such training as productivity decreases. However, we also find that sales training is ineffective in addressing adverse selection when the risk averse employee is only moderately risk averse and productivity difference between the two employees is not too high. A result that suggests that screening employees is more difficult if they are not very different from each other. Interestingly, this result means that sales training may be ineffective in resolving the case of severe adverse selection when the risk averse employee is only moderately risk averse selection when the risk averse employee is not too high. A result that suggests that screening employees is more difficult if they are not very different from each other. Interestingly, this result means that sales training may be ineffective in resolving the case of severe adverse selection when the promoted employee is both more risk averse and less productive because that case arises when the risk averse employee is only moderately risk averse. These results can be seen in the following graphical plot in Figure 3.3. In the figure the region of adverse selection is same as the one that we discussed in Proposition 3.1. However, the region in the top half reflects the parameter range where training can help firms select the right sales manager.

As part of our analysis we also find that sales training is more effective in screening employees when the risk associated with effort investment is high because the risk is the main screening mechanism by which risk averse employees are screened. However, sales training is also effective when the risk associated with it is moderate and individual specific uncertainty is sufficiently low. The rationale is that if individual specific uncertainty is low then the benefits from diversification at the managerial position are limited and therefore, the incentives for the risk averse employees are low which reduces their willingness to pursue a risky sales training to get promoted to the managerial position. Thus, in some sense the reduction in individual specific uncertainty is a substitute for increase in uncertainty of the sales training, the main mechanism for screening. An increase in common uncertainty also decreases the incentive for the risk averse employee to get promoted because in spite of being promoted the employee would still face uncertainty at the management position. Therefore, the level of risk in sales training required to screen the employee is lower if common uncertainty is high. Finally, we also find that lower risk is required in the sales training to be effective in screening if the profit per unit of effort in the management position, δ_P , is below a threshold. The reason is that an increase in δ_P increases the benefits from promotion and a higher risk in sales training is required to screen the risk averse employees in such a case. Interestingly, when δ_P is quite high there is also a lower likelihood of adverse selection because in such cases the increase in benefit from promotion is even higher for the low risk averse employees. However, when δ_P is moderate and risk aversion of the risk averse employee is moderate too then we find presence of adverse selection that cannot be resolved by sales training because sales training is effective in screening when risk aversion is high and δ_P is low. These results can be seen in Figure 3.3.

In this subsection we analyzed when a firm would choose sales training over tournament in selection of sales managers. If selection of the right employee was the only motive of the firm then, as we saw in Figure 3.2, the firm would choose sales training when risk aversion of the risk averse employee is high and productivity is low. If the risk aversion of the risk averse employee is low or moderate and productivity is higher then a tournament would be better at preventing adverse selection. This can be seen from the white region in the Figure 3.3. However, while selection is one of the motives for the firm in promotion of sales reps to sales management positions the firm also cares about the increase in profits from higher effort induced because of the tournament, and the possibility of managerial skill acquisition through sales training. In the overall profit maximization the firm faces a tradeoff between the use of tournament and the use of sales training. This tradeoff can be observed by comparing the firm profit from the use of tournament and sales training in selection of sales managers. The firm's profit from tournament and sales training are given as,

$$\Pi_{Tournament} = \underbrace{\delta_{S} (e_{1} + e_{2})}_{\text{Period 1 Profit}} + \underbrace{\left(\Pr\left(e_{1} - e_{2} \ge \zeta\right) \pi_{P,1} + \left[1 - \Pr\left(e_{1} - e_{2} \ge \zeta\right)\right] \pi_{P,2}\right) + (\pi_{S,1} + \pi_{S,2})}_{\text{Period 2 Profit}}$$

where, e_1, e_2 are given in 3.6 and 3.7, respectively, $\zeta \sim N\left(0, 2\sigma_{\epsilon}^2\right)$

 $\pi_{P,2}, \pi_{P,1}$ are given in 3.1 and $(\pi_{S,1} + \pi_{S,2})$ is the profit from the sales reps

$$\Pi_{SalesTraining} = \underbrace{\left(\pi_{P,2} + \frac{1}{2\left(\theta + 1\right)}\right) + \left(\pi_{S,1} + \pi_{S,2}\right)}_{\text{Period 2 Profit}}$$

Proposition 3.4 The firm uses tournament over sales training to select sales managers if the effort generated from the tournament is higher than the loss from adverse selection of sales manager and lack of managerial training of the sales manager,

$$\Pi_{Tournament} \ge \Pi_{SalesTraining} \Leftrightarrow \delta_S \left(e_1 + e_2 \right) \ge \frac{1}{2 \left(\theta + 1 \right)} + \Pr \left(e_1 - e_2 \ge \zeta \right) \left(\pi_{P,2} - \pi_{P,1} \right)$$
(3.17)

The above proposition highlights the fundamental tradeoff between a tournament and a screening

procedure based on training and testing to select sales managers. A tournament for the sales managerial position acts as the costless source of motivation for the firm's employees, i.e. the firm would have anyway chosen an employee for the sales management position but by running a tournament it can extract effort from the two employees in the process of selecting a sales manager. However, if the motivation is asymmetric, as it is in case of a heterogeneous sales force, then the firm may be worse off because if the less desirable sales rep becomes more motivated in getting promoted then the firm may suffer from potentially lower profits in subsequent periods. In Proposition 3.1 we discussed the conditions when this happens. The tradeoff between these benefits and costs determine whether the firm chooses tournament or sales training to select its managers. In the above inequality 3.17, $\delta_S (e_1 + e_2)$ represents the profit from effort induced by the tournament, $\Pr (e_1 - e_2 \ge \zeta)$ represents the probability that the undesirable employee gets promoted, $(\pi_{P,2} - \pi_{P,1})$ represents the net loss for the firm if the undesirable employee is promoted and $\frac{1}{2(\theta+1)}$ represents the profits from higher management skills that the firm would have got had the sales reps gone through sales training.

$$\underbrace{\delta_{S}(e_{1}+e_{2})}_{\text{profit from effort induced by tournament}} \geq \underbrace{\frac{1}{2(\theta+1)}}_{\text{loss from lack of managerial training}} + \underbrace{\Pr\left(e_{1}-e_{2}\geq\zeta\right)\left(\pi_{P,2}-\pi_{P,1}\right)}_{\text{loss from adverse selection}}$$

3.5 Extensions and Future Research

We extend our analysis of the problem to identify natural conditions that moderate the adverse selection problem. In doing so, we consider the case of asymmetric labor market conditions in the market for sales reps and sales managers. Then we discuss a future research idea related to the use of uniform wages for all sales reps, a practice many firms are constrained to follow because of fairness concerns in sales force.

3.5.1 Asymmetric Labor Market Conditions for Sales Managers and Sales Representatives

It is plausible to consider a labor market scenario where the labor market elasticity for sales managers and sales reps are different. Such differences may arise because of asymmetry in supply of labor at the two levels or because of difference in the extent of competition for sales employees at the different levels. For example, if there are multiple firms that need sales reps and there is an undersupply of sales reps in the area but there is an oversupply of sales managers then the elasticity in the sales rep labor market would be high compared to that of the sales manager labor market. Similarly, variance in elasticities may also arise because of asymmetry in bargaining power at the two levels. For example, if sales reps unionize but sales managers do not, then sales reps may be able to get a larger share of the surplus that they create as compared to the sales managers, a likely scenario because we do see more unionization at lower levels. We can model such cases by allowing the labor market elasticity at the two levels to be different. Suppose the labor market elasticity is θ_m at the managerial level and θ_r at the sales rep level. Further, let $\gamma_m = \frac{\theta_m}{1+\theta_m}$, $\gamma_r =$

$$\frac{\theta_r}{1+\theta_r}$$
 and $\gamma = \frac{\gamma_r}{\gamma_m}$

Our analysis suggest that the labor market asymmetries moderate Peter Principle as summarized in the following proposition.

Proposition 3.5 When the elasticity in the sales rep labor market is high compared to the elasticity of labor in the market for sales managers, i.e. when $\gamma > \gamma^*$, then firms face heightened risks of adverse selection. A less elastic labor market for sales rep as compared to the market for sales managers, $\gamma < \gamma^*$, leads to decreased risk of adverse selection.

Proof. See Appendix.

The rationale for the result is as follows. As labor market elasticity increases the firm loses share of the profit generated by the employees because in such a labor market firms compete more aggressively to recruit employees. For the employee this is good because the employee gets a larger share of the profits in such a case. When elasticity is higher in the sales rep's labor market as compared to the sales manager's labor market then the benefits of working at the sales rep level increases for both the employees. However, this benefit increases more for the less risk averse employee because he generates more profits at the riskier sales rep position owing to his lower risk aversion. Since, there are asymmetrically higher benefits for the less risk averse employee from staying in the same position as the elasticity at the sales rep level increases, the employee's incentive to get promoted decreases more than that of the more risk averse employee. This causes an increase in the probability of the more risk averse employee to get promoted and hence, adverse selection is

more likely.

3.5.2 Uniform Salaries and Incentives for Sales Employees

Our primary analysis was based on the assumption that firms can customize sales contracts according to the characteristics of the sales employees once promotion decisions are made because after promotion decisions are made the incentives of the sales employees at both management and representative levels, and the firms are aligned as far as design of sales contract is concerned. However, some firms are constrained to offer uniform sales contracts to their heterogeneous sales force (Misra, Nair and Daljord 2013). There can be many reasons for this. Possible reasons include fairness concerns in sales force, preference for simplicity in design of contracts, or costs involved in design of customized contracts for each type of sales employee. It is not clear whether and how the Peter Principle problem would arise in such context. Future research can investigate such systems of wages and their effect on the Peter Principle problem discussed in this paper.

3.6 Conclusions and Managerial Implications

Firms often promote their best sales reps to sales management roles only to find that the promoted sales employees are not as good as they were expected to be based on their superior performance at sales representative level. This widely observed phenomenon, termed as the Peter Principle in sales managerial promotions, is believed to be caused by the lack of managerial expertise in the promoted sales representatives because their promotion was based on their excellence as sales reps.

In this research, we show that even if the promoted sales reps had the required managerial expertise the problem may still arise and firms may be overlooking a deeper problem associated with such promotions. In the process, we also show that firms may be promoting more risk averse sales employees to management position. We then analyze the current solutions being implemented by firms to solve the Peter Principle problem and find that while many of the current solutions, like letting sales employees self-select between roles, may not be effective in solving the problem that we identify with our model, the use of sales training and testing can be used as a screening mechanism to avoid the problem under some conditions. We also identify some natural circumstances that moderate the Peter Principle problem.

An important managerial implication of our research is that firms may not be able to solve the Peter Principle problem in sales managerial promotions by trying to only train the promoted employees or by letting employees self select between managerial and representative job roles. Further, the practice of promotions based on performance may increase risk aversion at the management level, something that may hurt firm profits because managers who are more risk averse take less risky decisions and in the process, overlook profitable but risky opportunities. Finally, we also find support for the argument made by some experts, "that in sales management, employee selection should focus more on personality characteristics than performance or skill" (Zoltners et al. 2012d).

Period 1		Period 2		
Stage 1	Stage 2	Stage 1	Stage 2	Stage 3
Employees compete with each other to get promoted	Nature determines the error and promotion decisions are made based on performance	Firm learns the characteristics of the employees. It provides contracts to the employees based on their designation.	Sales employees accept contracts and then put effort.	Nature determines the error and wages are paid.

Figure 3.1: Game Sequence Chapter 3

Table 3.1: Period 2 Results

	Sales Rep Position	Sales Manager's Position	
Firm's Profit	$\pi_{S,i} = \frac{1}{1+\theta} \frac{1}{2a_i} \left(\frac{\delta_S^2}{1+a_i r_i \left(\sigma_{\varepsilon}^2 + \sigma_{\gamma}^2\right)} \right)$	$\pi_{P,i} = \frac{1}{1+\theta} \frac{1}{2a_i} \left(\frac{\delta_P^2}{1+a_i r_i(\sigma_\gamma^2)} \right)$	
Employee's Wage	$U_{S,i} = \frac{\theta}{1+\theta} \frac{1}{2a_i} \left(\frac{\delta_S^2}{1+a_i r_i \left(\sigma_{\varepsilon}^2 + \sigma_{\gamma}^2\right)} \right)$	$U_{P,i} = \frac{\theta}{1+\theta} \frac{1}{2a_i} \left(\frac{\delta_P^2}{1+a_i r_i(\sigma_\gamma^2)} \right)$	
Employee's Effort	$e_{S,i} = \frac{1}{a_i} \left(\frac{\delta_S}{1 + a_i r_i \left(\sigma_{\varepsilon}^2 + \sigma_{\gamma}^2\right)} \right)$	$e_{P,i} = \frac{1}{a_i} \left(\frac{\delta_P}{1 + a_i r_i(\sigma_\gamma^2)} \right)$	

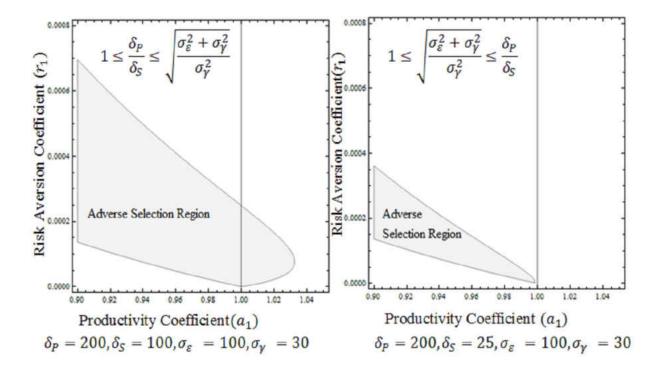


Figure 3.2: Adverse Selection Regions

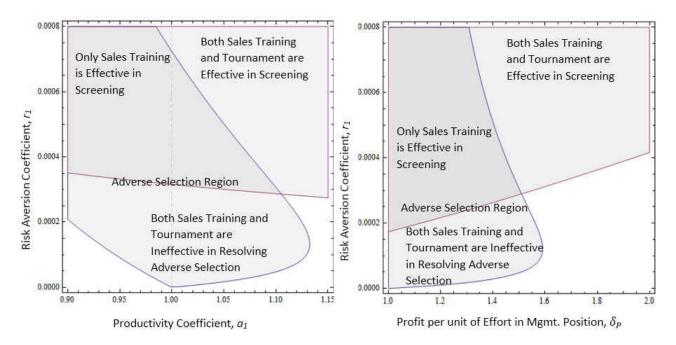


Figure 3.3: Strategy Regions

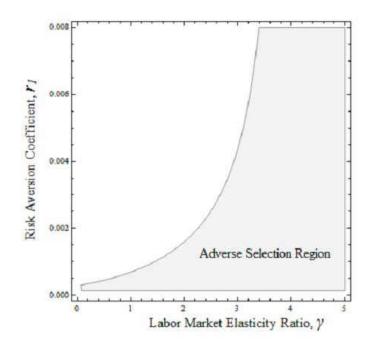


Figure 3.4: Asymmetry in Labor Market and Adverse Selection Regions

CHAPTER 4: FIRM COMPETITION FOR SALES FORCE OWNED CUSTOMER LOYALTY

4.1 Introduction

Customer relationship management (CRM) involves maximizing the net present value of customer relationships instead of maximizing profit from discrete transactions with customers. Over the last two decades firms have increasingly focused on building long term customer relationships as opposed to profiting from transactional relationships, and this has been particularly the case in industries like Banking, Insurance and Software Services where personal selling plays an important role. At the heart of CRM is the focus on acquisition and retention of customers (Bowman and Narayandas 2004, Gupta et al. 2004, Musalem and Joshi 2009, Verhoef 2003), and in industries in which personal selling plays a key role the task of customer acquisition and retention rests with the sales representatives of the firms. In the process of retaining customers one of the primary goals of these sales reps is to build loyalty in the customer towards their firm. However, in such long term relationships customers often end up building loyalty not only to the firm but also towards the sales rep who deals with the customer (Zoltner, Sinha and Lorimer 2011b, Palmatier, Sheer and Steenkamp 2007, Palmatier 2008, Sirdeshmukh, Singh, and Sabol 2002, Macintosh and Lockshin 1997, Beatty et al. 1996, Crosby and Stephens 1987). This often creates the possibility of customer defection to competition when a competitor poaches a sales rep. For example, Palmatier et al. (2006) state that customer-sales rep relationships are often stronger than firm-customer relationships and the loyalty generated from customer-salesperson relationships are often "owned" by the sales person and can be lost if the sales person moves to another firm" (Palmatier 2008). According to Tax and Brown (1998), American Express reports that 30% of customers would follow their financial advisor to a new firm.

To deal with the above problem, the existing literature (Palmatier, Sheer and Steenkamp 2007, Palmatier 2008) and conventional belief suggest that firms can benefit from the existence of noncompete laws. In this research we study whether this is indeed the case. More specifically, we study the strategic implications of poaching of sales reps and ask, whether non-compete laws are always beneficial for firms in the above contexts. This is an important subject on two accounts. First, it is not clear what the strategic implications of stopping the possibility of sales rep poaching in a market are. On one hand, it is expected that firms will benefit because they will not have to spend more to retain their sales reps from getting poached and they can extract the surplus created by customer's loyalty to sales rep through higher sales, but on the other hand, the effects of absence of sales rep poaching on incentives for sales representatives to put effort and competition between firms are not clear. Would a sales rep put higher effort to acquire and retain customers when non-compete agreements are allowed by the law or when they are not allowed? How would the firms shift their expenditure between acquisition and retention of customers when they can enforce non-compete agreements? Second, it is important to find answers to these questions because there exists significant debate about the efficacy of Non-compete laws and there is also a notion among many firms that tacit collusion to stop employee poaching is beneficial to them. Figure 4.1 (based on data from Beck Reed Riden 2013) provides the distribution of states where non-compete

agreements are enforceable versus not enforceable. For example, non-compete agreements can be enforced in Florida but not in California. The reason for the divergence among states with respect to this policy may be the belief that non-compete agreements benefit firms, but hurt the employees (NY Times 2014, CNBC 2014). However, if contrary to this belief these policies would also hurt firms then abolishing them would be a win-win for everyone. In particular, firms spend a significant amount of resources to lobby for non-compete laws and they can save money if we find that non-compete laws actually hurt them. Finally, some firms also tacitly collude to stop employee poaching. For example, recently Google and Apple were found to be colluding to not poach each other's employees and they had to pay damages to the tune of \$ 2.3 billion (WSJ 2014). If it can be shown that such collusion is sub-optimal for the firms even when they do not get penalized by anti-trust agencies then such insight would benefit firms.

To answer the above questions, we consider a two period model in which two firms with an given profit margin offer linear contracts to forward looking sales reps in each of the periods and the sales rep put effort to sell to new and existing customers. In our model we capture the strength of relationship between a sales rep and a customer as a switching cost that the customer faces with respect to the sales rep in the second period apart from the switching cost that the customer faces with respect to the firm. The competing firm sells to the customers and it can also poach or provide wage contract offers to the focal firm's sales rep if non-compete clauses are not enforced.

Counterintuitively, we show that the possibility of enforcement of non-compete clauses may actually hurt firms under a range of conditions. The rationale for this finding is that while a direct effect of employee poaching is that firms earn less because their employees can bargain for a higher wage on account of their relationship with customers and the possibility of better offers from the competing firms, an indirect effect from employee poaching is that firms are less motivated to capture market share in the acquisition market. The reason is that the value of capturing a higher market share in the period 1 is diminished by through bargaining by their sales employees on basis of customer relationship and competition. An implication of the indirect effect is that the competition in the market for acquisition of new customers, i.e. in period 1, gets softened due to possibility of employee poaching. If the indirect effect of softening of competition is sufficiently strong as compared to the direct effect of loss in profits due to bargaining by the employee then we find that the possibility of employee poaching can actually benefit firms. Such a case occurs when firm switching costs are high, sales rep customer relationships are strong or the profit margin is sufficiently high. Under these conditions the competition in the acquisition market is sufficiently high and the softening of such competition is particularly valuable for firms. Our finding has implications for firms that spend effort on lobbying for anti-employee poaching regulations or tacitly collude to enforce non-compete clauses. Our model implications are also important for policy makers and regulators.

Furthermore, our model also predicts that firm profits would increase in a convex manner with an increase in profit margin if competition in the market is less intense and the market is expanding. On the contrary, firm profits are expected to have a inverted-u shaped relationship with profit margin if markets are competitive and saturated, and if firm switching costs or sales rep switching costs are high. We find evidence of such a relationship in COMPUSTAT data for last 10 years. For

example, in case of software services industry, an industry where monopolistic power is high and market expansion has taken place during the last 10 years, we find that net profit margins increase in a convex manner with gross profit margins, while in case of commercial banking industry, an industry with saturated market, a more commoditized product, and significant firm and sales rep switching costs net profit margins increase follow a concave relationship with gross profit margins. This increases the confidence in predictive power of our model and other results.

Our paper is related to the literature on game theoretic models of customer relationship management and the empirical literature on sales person-customer relationships.

The literature on game theoretic models of customer relationship management studies the effect of factors like firm switching costs, future discount factors and market time horizon on dynamic competitive strategy for firms (Klemperer, 1987a and Klemperer, 1987b, Farrell and Shapiro, 1988 and Farrell and Shapiro, 1989, Villas-Boas 2004, Villas-Boas 2006, Shi 2013, etc.). For a review of this literature refer to Villas Boas (2015) and, Farrel and Klemperer (2007). Some important findings in this literature are that firms want to charge higher prices to their existing consumers due to their switching costs and firms compete aggressively by lowering price to acquire consumers. We contribute to this stream of literature by modeling the role of the sales person in a firm's relationship marketing strategy. In doing so we introduce the notion of a new type of switching cost that customer faces if she has to switch to a different sales person. Moreover, we model firm competition for both sales persons and customers in this context. Our results contribute to the literature by identifying conditions in which introduction of a new additional switching cost, based on the relationship between sales reps and customers, decreases market competition and thereby increases firm profitability.

The empirical literature on sales person-customer relationships finds that such relationships may be strong and they may arise independent of the relationship between the firm and customers which in turn can become a risk for firms (Palmatier, Sheer and Steenkamp 2007, Palmatier 2008). For a review of this literature refer to Palmatier (2006). We contribute to this literature by showing that, under certain conditions, the presence of sale force owned customer loyalty and the possibility of poaching of sales employees can be good for firms.

Next, we discuss the modeling framework of the study.

4.2 Model

Consider a two period model in which two firms compete to sell their products to customers. The product category is such that firms require sales reps to carry out the activity of selling to customers in each of the periods. The consumers and their characteristics, the sales reps and their characteristics, the firm and its decisions, and the game sequence for the model are outlined below.

4.2.1 Consumer Model

In modeling consumer demand we borrow the approach used in Subramanian, Raju and Zhang (2014). There are N customers in the market. The k^{th} customer derives a utility of U_{iT}^k from

purchase of the product from firm i in period T. The utility U_{iT}^k is given as,

$$U_{iT}^{k} = V + e_{iT} + \gamma_{i} (s + f) + \xi_{iT}^{k}, \text{ where, } i, j, T = \{1, 2\}, i \neq j,$$

$$\left\{ \begin{array}{cc} 0 & if \ T = 1 \\ 1 & if \ T = 2 \text{ and customer bought product from } i \text{ in Period } 1 \\ 0 & if \ T = 2 \text{ and customer bought product from } j \text{ in Period } 1 \end{array} \right\},$$

$$\left\{ \xi_{1T}^{k} - \xi_{2T}^{k} \sim U \left[-v, v \right] \text{ and } e_{iT} \text{ is the sales effort made by firm } i \text{'s sales rep in period } T. \right\}$$

We assume that in both periods the customers have the same base utility, V, for the product that both firms sell. However, in each period customers experience a random preference shock that makes them more favorable towards one of the firms. This preference shock may be due to situational or contextual factors. For example, the customer may have had a very urgent need for the product at the exact time when a sales rep from one of the firm visits the customer or the customer may make her decision to choose a firm because she saw an ad from the firm just before the purchase. The net effect of the random preference shock is uniformly distributed, i.e. $\xi_{1T}^k - \xi_{2T}^k \sim U[-v, v]$. Thus, v captures the extent of random factors on a consumer's decision making process. This approach of modeling consumer demand is also consistent with random utility discrete choice models and Hoteling line models. It is also assumed that the net effect of the random preference, $\xi_{1T}^k - \xi_{2T}^k$, is independent across the two periods. The assumption is supported by the fact that the random preference is due to situational or contextual factors and such factors are expected to occur independently across time. The modeling assumption is also consistent with past literature (Subarmanian, Raju and Zhang 2014, Shi 2013).

At the end of the first period customers form loyalty not only towards firms' products but also towards the sales employee who served them in the first period. We capture customer loyalty towards the firm by a fixed increase in preference for the firm given by f, and towards the sales rep by a fixed increase in preference for purchase from the sales rep given by s. The parameters f and s can be interpreted as the switching costs the customer needs to incur if she shifts to a different firm or a different sales rep, respectively. The reasoning for exogeneity of firm switching cost is in line with past literature (Klemperer 1986 etc.). We also model sales rep specific switching costs because empirical literature in this area makes a distinction between customer's loyalty towards the firm and the sales rep. For example, Palmatier et al. (2007) make such a distinction and find a significant amount of customer loyalty is specific to the sales rep serving the customer, and such a loyalty towards sales rep can exist independently of any loyalty towards the firm. Similar finding appear in other empirical studies in this area. According to Tax and Brown (1998), American Express reports that 30% of customers would follow their financial advisor to a new firm". We assume the sales rep switching costs to be exogenous to the model because a rational customer will attach no commitment power to additional effort made by the sales rep to generate switching costs. To elaborate on this rationale, note that a sales rep switching cost will only be generated if there are affective costs of leaving a brand or a personal relationship with a sales rep. A sales rep cannot commit to providing better benefit in terms of firm's product offerings just because the customer repeat purchases the product from that specific sales rep instead of any other sales rep from the same firm. This rationale is supported by the typology of sources of switching costs presented in Burnham et al. (2004). In the absence of any real product related benefits of purchasing from a specific sales rep, the customer's incentive to buy from the same sales rep will arise from the personal relationship loss if she does not purchase from the same rep. The costs related to such loss in personal relationships will be affective and exogenous to the sales rep's effort in first period. More specifically, in our model the customer only benefits from higher effort by the salesperson. If the extent of personal relationship was endogenous to the sales rep's effort then the rep will invest effort in building a stronger personal relationship in period 1, but he will have no incentive to provide the sales rep makes to build the relationship in period 1, because that is no guarantee for same level of effort in period 2. Therefore, the only switching cost based on the personal relationship between the customer and the sales rep will be affective and exogenous to the sales rep's effort.

We assume that customers are myopic and make their purchase decisions to maximize their current period utility in each period. This enables us to study the strategic considerations that arise solely due to the competitive interaction between firms and introduce our main insights. Given the above utility function, the demand for each of the firms in each of the periods is computed as below.

Customer k choose firm i iff $U_{iT}^{k} = V + e_{iT} + \gamma_{i} (s+f) + \xi_{iT}^{k} > U_{jT}^{k} = V + e_{jT} + \gamma_{j} (s+f) + \xi_{jT}^{k}$ (4.2)

or,
$$\Pr\left(U_{iT}^{k} > U_{jT}^{k}\right) = \Pr\left(e_{iT} - e_{jT} + (\gamma_{i} - \gamma_{j})(s+f) + \xi_{iT}^{k} - \xi_{jT}^{k} \ge 0\right)$$
 (4.3)

or,
$$\Pr\left(e_{iT} - e_{jT} + (\gamma_i - \gamma_j)(s+f) + \xi_{iT}^k - \xi_{jT}^k \ge 0\right) = \frac{1}{2} + \frac{e_{iT} - e_{jT} + (\gamma_i - \gamma_j)(s+f)}{2v}$$

(4.4)

Suppose, X_{iT} = Demand for firm *i* in period *T*,

 P_i = Probability that a customer purchased from firm i in period 1,

 $P_{i|j}$ = Probability that a customer purchased from firm i in period 2 conditional on having purchased from firm j in Period 1 and $i, j = \{1, 2\}$.

Therefore,
$$P_i = \frac{1}{2} + \frac{e_{i1} - e_{j1}}{2v}$$
, $P_{i|j} = \frac{1}{2} + \frac{e_{i2} - e_{j2} - (s+f)}{2v}$, $P_{i|i} = \frac{1}{2} + \frac{e_{i2} - e_{j2} + (s+f)}{2v}$, $X_{11} = P_1, X_{21} = P_2 = 1 - P_1, X_{12} = P_1P_{1|1} + P_2P_{1|2} = P_1P_{1|1} + (1 - P_1)P_{1|2}$
and $X_{22} = P_2P_{2|2} + P_1P_{2|1} = (1 - P_1)P_{2|2} + P_1P_{2|1}$

Next, we discuss the characteristics of the sales reps and their decisions.

4.2.2 Sales Representatives and their Decisions

We assume that each of the firms have access to m number of homogenous perfectly forward looking risk neutral sales reps in both the periods. Their utility functions are as given below.

$$EU_{iT} \stackrel{Def}{=} E[y_{iT}] - a\left(\frac{\left(e_{iT}^{N}\right)^{2}}{2} + \frac{\left(e_{iT}^{E}\right)^{2}}{2}\right)$$
(4.5)

Where, EU_{iT} is the expected utility, y_T is the income and, e_{iT}^N and e_{iT}^N are the effort of the sales rep in selling to new and existing customers, respectively, for firm *i* in period *T*. Subscripts $i, T = \{1, 2\}$. Parameter *a* is the cost coefficient of effort.

Sales reps have a reservation wage of $\overline{U} = 0$ and a per period limited liability of l = 0. They maximize their expected utility and participate only if both the participation constraint and the limited liability constraints are met. The limited liability assumption is common in the sales literature. (e.g., Bester and Krähmer 2008; Bergmann and Friedl 2008; Shin 2008; Simester and Zhang 2010, and Zhang and Simester 2014). In the current scenario the limited liability assumption ensures that the firm cannot extract the benefits of increased wage in the subsequent periods by paying negative wages in the first period. This is consistent with business practice because we rarely find firms paying negative wages to employees. A rationale for the assumption can be the fact that employees have limited access to credit and wealth and that they have recurring expenses per period because of which they cannot accept a negative wage in any given period. The limited liability assumption also rules out the possibility that the firm sells its business to the sales rep, which is plausible because employees generally retain the right to leave the firm ex post at any time.

Sales reps are forward looking and make decisions on whether to accept contract and how much

effort to put in each period. Therefore, the individual rationality and incentive compatibility constraints for the sales employees working for firm i in period 1 and period 2 are given as,

$$e_{i1}^{N} \in \underset{e_{i1}^{N}}{\operatorname{arg\,max}} EU_{i1} = E\left[y_{i1}\right] - a\left(\frac{\left(e_{i1}^{N}\right)^{2}}{2}\right) + EU_{2}$$

$$(4.6)$$

$$EU_{i1} = \max_{e_{i1}^N} EU_{i1} = E\left[y_{i1}\right] - a\left(\frac{\left(e_{i1}^N\right)^2}{2}\right) + EU_2 \ge 0$$
(4.7)

$$EU_{i1} = \max_{e_{i1}^{N}} EU_{i1} = E\left[y_{i1}\right] - a\left(\frac{\left(e_{i1}^{N}\right)^{2}}{2}\right) \ge 0$$
(4.8)

$$e_{i2}^{N}, e_{i2}^{E} \in \underset{e_{i2}^{N}, e_{i2}^{E}}{\operatorname{arg\,max}EU_{i2}} = E\left[y_{i2}\left(e_{i1}^{N}, e_{i2}^{N}, e_{i2}^{E}\right)\right] - a\left(\frac{\left(e_{i2}^{N}\right)^{2}}{2} + \frac{\left(e_{i2}^{E}\right)^{2}}{2}\right)$$
(4.9)

$$EU_{2} = \max_{\substack{e_{i2}^{N}, e_{i2}^{E}}} EU_{i2} = E\left[y_{i2}\left(e_{i1}^{N}, e_{i2}^{N}, e_{i2}^{E}\right)\right] - a\left(\frac{\left(e_{i2}^{N}\right)^{2}}{2} + \frac{\left(e_{i2}^{E}\right)^{2}}{2}\right) \ge 0$$
(4.10)

Where, EU_1 and EU_2 are the expected utility from period 1 and 2, respectively.

In the following section we discuss how the firms decide on wage contracts and why the second period wage for the employee depends on his first period effort.

4.2.3 Firms, their Decisions and Game Sequence

The two risk neutral and perfectly forward looking firms earn a profit margin of δ over the sale of every product and they provide linear sales contracts to the sales reps in each of the periods to sell their products. Price or price margin is assumed to be fixed to abstract away from the issue of two period pricing and to keep parsimony. We do not expect the results to be substantially impacted by making pricing endogenous because sales contract and price are substitutes, and it is expected that the firm's pricing decisions will be aligned with its sales incentive decisions. Our model is more well suited to capture realities of industries in which non-price competition plays a much bigger role than price competition, customers are relatively less sensitive to price changes and frequent price changes by firms are not observed. For example, in case of commercial banking average balances do not change frequently and customers rarely place much importance on account interest rates when shopping for a bank account. Therefore, most of the changes in price margins are determined on basis of relatively exogenous factors like Federal Reserve policy etc. In case of Airlines, it has been argued that there is high incidence of tacit price collusion and Airlines seem to be primarily competing on non-price factors (Cilberto and Williams 2013).

We limit our analysis to linear contracts because past literature suggests that linear contracts can mimic optimal nonlinear contracts when aggregated over a period of time (Holmstrom and Milgrom 1997). Further, we assume that the firm offers different linear contracts for selling to new and existing customers. This is optimal from firm's perspective and is in line with anecdotal evidence. While the firm writes incentive contracts on profit from new customer acquisition, it also provides incentives to sell to existing customers on basis of retention rate and associated profit. This structure of the incentive contract is most closely aligned with the effort made by sales reps towards acquisition and retention, and hence optimal for the firm. Consequently, the firm i offers the following linear sales contracts in period 1 and period 2, respectively.

$$y_{i1} = w_{i1} + b_{i1}^N \delta X_{i1} = w_{i1} + b_{i1}^N \delta P_i$$
(4.11)

$$y_{i2} = w_{i2} + b_{i2}^N \delta P_{i|j} + b_{i2}^E \delta P_{i|i}$$
(4.12)

Where, w_{i1} and w_{i2} are the fixed salaries, and b_{i1}^N and b_{i2}^N are

new customer acquisition incentives for period 1 and period 2, respectively,

and b_{i2}^E is the repeat customer sales incentive for period 2.

The firms maximize their profits across the periods subject to the incentive compatibility and individual rationality constraints.

$$\Pi_{i1} = \max_{w_{i1}, b_{i1}^N} \pi_{i1} = E \left[\delta X_{i1} - \left(w_{i1} + b_{i1}^N \delta P_i \right) \right] + \Pi_{i2}$$
(4.13)
Subject to (4.6) and (4.7)

$$\Pi_{i2} = \max_{w_{i2}, b_{i2}^N} \pi_{i2} = E \left[\delta X_{i2} - \left(w_{i2} + b_{i2}^N \delta P_{i|j} + b_{i2}^E \delta P_{i|i} \right) \right]$$
(4.14)
Subject to (4.9) and (4.10)

It is assumed that the firms have free access to the labor market from where they can recruit sales reps at the beginning of either of the periods. The game sequence for strategic interaction in the market is given in Figure 4.2. In stage 1 of period 1 both firms offer sales contracts to employees. In stage 2 of period 1 the employees accept the sales contracts and put effort towards converting customers. Subsequently customers make purchase decision based on the realization of the random preference shock in stage 3. In stage 1 of period 2 firms first make competing sales contract offers

to the sales reps who sold products to customers. In stage 2 of period 2 the sales rep decide on which offer to accept and then put sales effort for the firm from which they accept the contract offer. In stage 3 of period 2 the customers make their choices based on the realization of random preference shock for period 2, and firms honor the contracts that they agreed on.

4.3 Analysis and Results

To begin our analysis we study the case when firms compete for customers but employee poaching is not possible. Subsequently, we analyze the case when firms compete for both customers and sales reps, and they can poach each other's sales reps. As part of the second analysis we also compare the case of firm competition without employee poaching with the case of firm competition with employee poaching.

4.3.1 Firm Competition without Employee Poaching

4.3.1.1 Period 2

At the beginning of period 2 the firm faces the choice between employing the existing sales rep and a new sales rep. The existing sales rep too faces a choice between continuing to work in the same firm and leaving it. Since the presence of the sales rep in the firm increases the probability of conversion of an existing customer by a factor of $\frac{s}{2v}$ the firm prefers to hire the same sales executive. The sales executive too realizes that the additional probability of sales conversion is due to him and if he leaves the firm the additional probability of conversion will vanish. As employee poaching is not possible in the current scenario the sales rep cannot threaten to go to another firm and increase the other firm's probability of conversion of the focal firm's existing customer. However, in the current case the sales rep can still leave the firm and the additional surplus due to sales rep specific switching cost would be destroyed. This leads to bargaining between the sales rep and the firm over the surplus created from sales specific switching cost and both, the sales rep and the firm, are on equal footing. The reason is that the surplus is destroyed if anyone of them disagrees to work with each other. Therefore, the surplus is equally divided among the firm and the sales rep. This ensures that the sales rep earns a wage higher than his outside option and the firm still prefers to employ the sales rep because otherwise it loses a part of the profit.

Lemma 4.1 An existing sales rep earns an expected utility which is higher than the market wage. For an existing sales rep working for firm *i* the expected utility is given as $\Delta_{i2} = \frac{\delta s X_i}{4v}$. The firm prefers to retain an existing sales rep as opposed to hiring a new one in period 2.

Proof. See Appendix.

The higher the sales rep switching cost, profit margin and period 1 customer base the higher is the expected utility that the sales rep can earn by working for the same firm. This is so because an increase in sales rep switching cost increases the likelihood of sales conversion of the existing customers of the sales rep and the firm. Further, an increase in profit margin increases the value of the higher likelihood of sales conversion due to sales rep specific switching cost and an increase in period 1 customer base increases the number of existing customers. The above dynamics increases the incentive for the sales rep to put effort in period 1 and also, induces the firm to provide higher powered incentives to capture market share in period 1. Next we report our period 1 results.

4.3.1.2 Period 1

In period 1 the firm maximizes long term profits by investing in sales incentives so that the sales executives put higher effort to garner market share and the firm can reap the benefits from its own switching costs as well as sales rep switching costs. The sales reps too put higher effort to increase their period 1 customer base so that they benefit from higher wage in period 2. We see these intuitions in Lemma 4.2.

Lemma 4.2 Sales rep effort and sales incentives increase with sales rep switching cost and firm switching costs.

$$e_{21}^{N} = e_{11}^{N} = \frac{\left(s + 4b_{11}^{N}v\right)\delta}{8av^{2}} = \frac{\left(4f + 3s + 4v\right)\delta}{8av^{2}}$$
(4.15)

$$b_{11}^N = b_{21}^N = \frac{2f + s + 2v}{2v} \tag{4.16}$$

Proof. See Appendix. ■

The firm increases its period 1 sales incentives as its own firm switching costs increase and as sales rep switching costs increase. The intuition is that it can derive long term benefit from its current investment in form of sales incentive and higher market coverage. Such long term benefits increase with increase in switching costs. Furthermore, the firm provides stronger incentives as uncertainty in the sales process v decreases because with lesser uncertainty the effectiveness of the incentives increase. Moreover, as profit margin and productivity of the employees increase the firm uses more of sales incentives because these increase the value in terms of per unit of incentives provided.

Proposition 4.1 *Firm profits increase with increase in uncertainty in the sales process, decrease in firm switching costs and sales rep switching costs, and decrease in productivity of the employees.*

$$\Pi_{11} = \Pi_{12} = -\frac{\delta \left(16a(s-8v)v^3 + (16f^2 + 24fs + 9s^2 + 32fv + 24sv + 24v^2)\delta\right)}{128av^4} \qquad (4.17)$$
$$\frac{\partial \Pi_{11}}{\partial v} > 0, \frac{\partial \Pi_{11}}{\partial f} < 0, \frac{\partial \Pi_{11}}{\partial s} < 0, \frac{\partial \Pi_{11}}{\partial a} > 0$$

Proof. See Appendix.

The firm profits increase as the uncertainty in the sales process increase. The rationale is that an increase in v decreases the effectiveness of the incentives that the firms provide to the sales executives to sell products and a decrease in the sales incentives softens the competition in the market for acquisition and retention of customers. Because the market is covered a decrease in incentives and a softening of competition leads to higher profits for both firms. A decrease in firm switching costs and sales rep switching costs increase firm profits because they lead to a decrease in level of competition for acquisition of new customers. A decrease in productivity of sales reps too decreases the effectiveness of incentives and therefore, an associated decrease in use of incentives softens the competition and leads to higher profits. **Proposition 4.2** *Firm switching costs and sales rep switching costs interact to increase competition and decrease firm profits, i.e.* $\frac{\partial^2 \Pi_{11}}{\partial f \partial s} < 0.$

Proof. See Appendix.

In the consumer's demand function sales rep switching costs and firm switching costs enter as independent factors that increase customer's probability of purchase of the product and therefore, we would expect there to be no interaction between the two switching costs. The extra demand generated due to the switching costs are also independent of each other. Interestingly, the firm investment in period 1 market share to reap long term benefits from firm and sales rep switching costs are complements. The rationale is that if the firm's own switching costs increase then the firm would invest more in acquiring new customers in period 1 to reap the benefits from higher firm switching costs. This increase in investment in acquisition of customers has a positive spill over effect on sales rep switching costs too because the effect of sales rep switching costs is on a greater customer base. Similar to these intuitions, the sales rep switching cost has a positive spill over effect on the effect of firm switching costs on period 2 firm profits. Since, in a covered market an increase in switching cost leads to higher competition for acquisition of customers and lower profits, the interactive effect of the two switching costs leads to lower profits. This result implies that the customer's loyalty towards the firm and the sales rep interact and work together.

Proposition 4.3 Firm profits follow an inverted-u relationship with profit margins. Firm profits decrease with profit margin if productivity of sales reps is sufficiently high, uncertainty in the

market is low and profit margin is high, i.e.,

$$\begin{aligned} 0 < a < \frac{72f^2 + 128fs + 57s^2}{64f^4 + 248f^3s + 360f^2s^2 + 232fs^3 + 56s^4}, f + s < v < v^* \\ \frac{-8asv^3 + 64av^4}{16f^2 + 24fs + 9s^2 + 32fv + 24sv + 24v^2} < \delta < 1 \end{aligned}$$

Where, v^* *is given in appendix.*

Proof. See Appendix.

Interestingly, profits follow an inverted-u relationship with profit margin. The reason is that an increase in profit margin impacts the firm profits in two opposite ways. While on one hand an increase in profit margin leads to more profit per customer acquired or retained, on the other hand it leads to higher effectiveness of sales incentives for the firms because now the same unit sales is more valuable to the firm. The second effect induces the firms to invest more in sales incentives and this leads to heightened competition in the acquisition market. The effect of increase in competition in the acquisition market is a decrease in firm profits. The first effect, i.e. the effect of more profit per customer acquired or retained, is more linear in nature because the overall market size stays the same and a higher margin increases the value of the market linearly. However, the second effect, i.e. the negative effect of heightened competition in the acquisition market linearly. However, the second effect, i.e. the negative effect of heightened competition in the acquisition market linearly. However, the second effect, i.e. the negative effect of heightened competition in the acquisition market linearly. However, the second effect, i.e. the negative effect of heightened competition in the acquisition market. The effect of market, increases at an increasing rate with profit margin. Therefore, when profit margin is smaller the first effect. This causes the inverted-u shaped relationship between profits and profit margin. There can also be a

case in which firm profits do not fall with an increase in profit margin. This scenario arises when competition in the market is sufficiently low and hence, the second effect of increasing competition is much lower compared to the first effect of increase in profits. The conditions for competition to be sufficiently low are given in Proposition 4.1.

4.3.2 Firm Competition with Employee Poaching and Comparisons with the Case of No Employee Poaching

Next, we consider the case when employee poaching is possible in the market and firms compete for both, customers and sales reps. In the process, we focus much of our discussion on contrasting the scenario in this subsection with that of subsection 4.3.1.

4.3.2.1 Period 2

In period 2 the sales reps for firm 1 can be poached by firm 2. If a sales rep is poached by firm 2 then that firm gains from the sales rep switching cost that the customer developed towards the sales rep who is poached. However, firm 1 suffers a decrease in probability of purchase for customers if the same sales rep is not employed to convert the existing customers. As the sales rep and the customer's loyalty towards him is just as valuable for both the firms and because the employee would make the decision to join either of the firms solely on the basis of who is offering him a better wage the firms engage in a Bertrand competition on wage. Therefore, the sales rep captures the whole surplus generated from the sales rep specific switching cost. The situation for an existing

sales rep from firm 2 is analogous. This brings us to our next proposition.

Proposition 4.4 An existing sales rep earns an expected utility of $\Delta_{i2} = \frac{\delta s X_i}{2v}$, which is higher than the expected utility without employee poaching. The period 2 profits for both firms are lower than in the case when employee poaching is possible.

Proof. See Appendix.

When employees can be poached firms compete more in order to retain them. The higher competition translates into better wage offers to the sales reps and lower profits for the firm. These results underscore the main opposition to laxer employee poaching regulations. This is also the reason why firms often make their sales employees sign non-compete agreements and in some instances firms have tried to tacitly collude to refrain from hiring each other's sales employees. It is important to note that the sales rep not only earns expected utility as compared to the case of no employee poaching in period 2 but also accross both the periods. The reason is that in both the cases the sales rep earn the reservation wage of 0 in period 1. We next consider firm's maximization problem in period 1.

4.3.2.2 Period 1

We first report results on period 1 effort levels of the sales reps and incentives offered by the firms.

Lemma 4.3 Sales rep effort and sales incentives increase with sales rep switching cost and firm

switching costs.

$$e_{21}^{N} = e_{11}^{N} = \frac{\left(s + 2b_{11}^{N}v\right)\delta}{4av^{2}} = \frac{\left(f + v\right)\delta}{2av^{2}}$$
(4.18)

$$b_{11}^N = b_{21}^N = 1 + \frac{f}{v} - \frac{s}{2v}$$
(4.19)

Proof. See Appendix.

In period 1 sales reps put more effort in acquiring customers not just to earn greater incentives in the current period but to also garner a bigger customer base so that they become more valuable in the eye of the competing firm and the current firm in period 2. This results also highlights the fact that sales reps often put effort not just to earn more sales incentives but to also acquire customer relationships because those relationships can help them earn more in the future. Interestingly, when employee poaching is possible, the firms offer lower incentives than what they would have offered otherwise. The rationale is that in the current case the main benefit from sales rep switching cost goes to the sales reps and it is less valuable for the firms to invest more in acquiring customers in period 1. Therefore, they invest less in sales incentives to acquire market share in period 1. Lemma 4.3 suggests that there can be a silver lining to the situation when employee poaching is possible in the market. Next, we study whether the decrease in sales incentive can have a strategic effect on the firms.

Proposition 4.5 *Firms make more profits when employee poaching is possible in the market if the sales rep switching cost is high or firm switching cost is high and, if productivity of employees is*

high or profit margin is high, i.e. if,

$$\begin{aligned} (i) \ 0 < s < -\frac{8v}{3} + 4v\sqrt{\frac{2}{3}}, \ \frac{1}{8}(-3s - 8v) + \sqrt{\frac{3}{2}}v < f < -s + v \ , \\ 0 < a \leq \frac{1}{2v^2}\sqrt{\frac{3}{2}}, \ \frac{16av^3}{8f + 3s + 8v} < \delta < 2\sqrt{\frac{2}{3}}av^2 \\ (ii) \ 0 < s < -\frac{8t}{3} + 4v\sqrt{\frac{2}{3}}, \ \frac{1}{8}(-3s - 8v) + \sqrt{\frac{3}{2}}v < f < -s + v \ , \\ \frac{1}{2v^2}\sqrt{\frac{3}{2}} < a < \frac{8f + 3s + 8v}{16v^3}, \ \frac{16av^3}{8f + 3s + 8v} < \delta < 1 \\ (iii) \ -\frac{8v}{3} + 4v\sqrt{\frac{2}{3}} \leq s < v, \ 0 < f < -s + v, \\ 0 < a < \frac{1}{2v^2}\sqrt{\frac{3}{2}}, \ \frac{16av^3}{8f + 3s + 8v} < \delta < 2\sqrt{\frac{2}{3}}\sqrt{a^2v^4} \\ (iv) \ -\frac{8v}{3} + 4v\sqrt{\frac{2}{3}} \leq s < v, \ 0 < f < -s + v, \\ \frac{1}{2v^2}\sqrt{\frac{3}{2}} \leq a < \frac{8f + 3s + 8v}{16v^3}, \ \frac{16av^3}{8f + 3s + 8v} < \delta < 2\sqrt{\frac{2}{3}}\sqrt{a^2v^4} \\ (iv) \ -\frac{8v}{3} + 4v\sqrt{\frac{2}{3}} \leq s < v, \ 0 < f < -s + v, \\ \frac{1}{2v^2}\sqrt{\frac{3}{2}} \leq a < \frac{8f + 3s + 8v}{16v^3}, \\ \frac{16av^3}{16v^3}, \\ \frac{16av^3}{8f + 3s + 8v} < \delta < 1 \end{aligned}$$

Proof. See Appendix.

Interestingly, firm profits may increase or decrease when employee poaching is possible in the market as compared to the case when it is not. Firms make more profits when employee poaching is possible if firm specific switching costs like brand loyalty or sales rep switching costs like customer's loyalty to the sales rep and, productivity of the employees or profit margin are higher. The rationale for our finding is given below.

While a direct effect of possibility of employee poaching is that firms earn less because they have to compete more in order to retain their employees and this causes an increase in costs through higher wage for sales employees on account of their relationship with customers, an indirect strategic effect of employee poaching is that firms are less motivated to capture market in the acquisition

market because the value of capturing a higher market in the period 1 is diminished through bargaining by their sales employees on basis of customer relationship and competition. An implication of the indirect effect is that the competition in the market for acquisition of new customers, i.e. in period 1, gets softened due to possibility of employee poaching. If the indirect effect of softening of competition is sufficiently strong as compared to the direct effect of loss in profits due to bargaining by the employee then the possibility of employee poaching can actually benefit firms. Such a case occurs when firm switching costs are high, sales rep customer relationships are strong or profit margin is sufficiently high because under these conditions competition in the acquisition market is sufficiently high and the softening of such competition is particularly valuable for the firms. We illustrate these intuitions in the Figure 4.3.

Our finding has implications for firms that spend effort on lobbying for anti-employee poaching regulations or tacitly collude to enforce non-compete clauses. Our model implications are also important for policy makers and regulators. The result suggests that removal of anti-employee poaching regulations can be win-win for both sales employees and firms if the above conditions hold for the market. Moreover, under the above conditions if firms tacitly collude to non poach each other's employees then their strategy is sub optimal. Furthermore, our results are also interesting because they suggest that counter to general beliefs it is better to compete in two markets than one, i.e., in the current case firms actually make more profits when they compete for both employees as well as customers as opposed to the case when they compete for customers only.

4.4 Extensions

4.4.1 Empirical Evidence for Proposition 4.3

We conducted preliminary robustness checks on predictive power of our results in Proposition 4.3 to gauge whether our model captures reality. This was particularly important because the proposition results are counter intuitive and they are a function of the fact that we assumed competition to be based on sales incentives as opposed to prices. For the empirical analysis we collected COM-PUSTAT data from different industries for a period of 10 years. Since our current analysis is only based on the case when there is competition in the market and the markets are covered but in our data and empirical context we may encounter situations where firms are monopolistic and in growing industries, we extend our model to derive the relationship between firm profits and profit margin in the case when there is only one firm and the market is not completely covered¹. Our analytical model predicts that firm profits would increase in a convex manner with an increase in profit margin in case of monopolistic firms operating in industries where market expansion is possible, while firm profits are expected to have a inverted-u shaped relationship with profit margin if markets are competitive and saturated, and if firm switching costs or sales rep switching costs are high. Therefore, we hypothesize the following.

Hypothesis 4.1 If market competition is low and market is expanding then firm profits follow an increasing and convex relationship with profit margin. If market competition is high and market is

¹The analysis for the monopoly model is presented in Appendix C along with the rest of the proofs.

saturated then firm profits follow an inverted-u shaped relationship with profit margin.

To carry out the empirical test we focused on four different industries that follow closely the context that we have modeled. We chose Commercial Banking and Insurance industry for the data because they are markets where competition is moderate and market growth is low, and sales rep and firm specific switching costs play an important role. Moreover, in both these markets much of the competition between the firms is based on sales incentives as opposed to prices. To contrast the results from the above two industries with those where monopolistic power is higher and market expansion has taken place during the last 10 years we considered Software Services and Energy industries.

For the analysis we considered the following fixed effects panel data model.

Netmargin_{*it*} =
$$\alpha_i + \alpha_t + \beta_1 + \beta_2 \text{Grossmargin}_{it} + \beta_3 (\text{Grossmargin}_{it})^2 + \beta_4 \text{SG} \& A_{it} + \varepsilon_{it}$$

Where, Netmargin = $\frac{\text{Net Income}}{\text{Revenue}}$, Grossmargin = $\frac{\text{Revenue} - \text{COGS}}{\text{Revenue}}$ SG&A = $\frac{\text{SG}\&\text{A Expenses}}{\text{Revenue}}$ and subscripts *i* and *t* denore firm and time, respectively.

(4.20)

The results for the analysis are summarized in Table 4.1.

As part of the empirical analysis we find that reduced form evidence confirms our hypothesis on effect of profit margin on firm's profit. More specifically, in industries in which sales effort competition plays an important role and markets are saturated firm profit follows an inverted-u relationship with profit margin (our competition results). This is consistent with our hypothesis because the second order effect of gross margin on net profit is negative in Commerical Banking and Insurance industries ($\beta_3 = -0.9233, p < .001$ in Commercial Banking and $\beta_3 = -0.6350, p < .001$.05 in Insurance). The presence of customer switching costs with respect to firm and/or its sales force accentuates this relationship. In industries in which sales effort competition is limited or/and market growth is present firm profit follows a convex relationship with profit margin (our monopoly results). This is consistent with our hypothesis because the second order effect of gross margin on net profit is positive or not significant in Software Services and Energy industries ($\beta_3 = 0.2152$, not significant, in Software Services and $\beta_3 = 0.2868, p < .001$ in Energy). These relationships between profit and profit margins are partly mediated by the extent selling expenditures made by the firms in form of sales incentives. For example, in case of a highly competitive and saturated industry an increase in profit margin induces firms to compete even more for individual customers by providing higher sales incentives. This increase in sales expenditure decreases the profit beyond a certain level of profit margin. The results are interesting because industry executives always want the profit margins for their industries to increase but we show that this objective may be suboptimal from a profit maximization standpoint.

The empirical results also give us confidence about our model and provide an alternative approach of viewing competition. The model suggests that the primary driving force can be non-price competition in the market. This is so because an alternative simple one period Hoteling line model with endogenous price does not provide results consistent with the empirical evidence that we get. In such a hoteling line model price margins can only change if competition is softened but if that is the case then profits increase monotonically with increase in profit margins.

One of the empirical issues with our approach is that it may be argued that profit margins are endogenous and there can be a third driving force that affects both profit margins and net profits. While this is plausible, it does not seem to be the case for our analysis. Firstly, as argued in the earlier paragraph, it seems that an alternative model with endogenous prices may not be able to explain our results and it can be reasonable to assume that cost changes are exogenous. In addition, an alternative model will have to explain the second order effect of price margin and the mediation of selling expenditures. Secondly, in most of the industries that we study prices can indeed be considered to be relatively stable. For example, in case of commercial banking and insurance companies heavily compete on sales effort.

Though, we consider the above factors, our results may still need to be interpreted with care and further research using more structural approaches is required to affirm the mechanism that we suggest behind our reduced form results.

4.5 Conclusion

In this research we study the effect of the possibility of poaching of sales reps on firm profits in a competitive market when sales reps can form relationship with customers. Findings suggest that

such a possibility can have strategic benefits for the firms in the form of softening of competition and these strategic benefits may counter the negative effects of increase in costs of retaining sales reps. This result has important implications for firms and policy makers because it implies that contary to general beliefs the existence of non-compete laws may hurt firm profits under some conditions and it may then be suboptimal for firms to tacitly collude to not poach each other's employees.

While the above results are important the limitations of the research need to be noted. First, future research can consider price competition along with competition on sales incentives and thereby relax the assumption of exogenous profit margins. Second, we have not considered the possibility of presence of only a limited number of sales reps in the sales force labor market. Future research can relax this assumption. Third, the research assumes risk neutrality of sales reps. Further research is required to examine the effect of risk aversion of sales rep in the current research context.

Table 4.1: Regression Results

	Commercial Banking Net Margin	Software Services Net Margin	Insurance Net Margin	Energy Net Margin
Gross Margin	1.897^{***} (.1098)	.4792** (.2397)	1.600^{***} (.2544)	$.5318^{**}$ (.0741)
$(Gross Margin)^2$	9233^{***}	.2152 $(.1894)$	6350^{**}	$.2868^{***}$
SG & A	8925^{***} (.1237)	6904^{***}	9781^{***}	5884^{**}
R-Square	0.6398	0.3916	0.4594	0.3192

Table 4.2: Net Profit Margin and Gross Profit Margin Accross Industries

Industries	Extent of Competition	Switching Costs	Relationship between	
	& Market Growth		Profit Margin and Profit	
Commercial Banking	Moderate & Low	High	Inverted-u	
Insurance	High & Low	High	Inverted-u	
Energy	Low & Moderate	High	Convex	
Software & Services	Moderate & High	Moderate	Linear	

Number of States and Enforceability of Non Compete Agreements for Customer Relationships

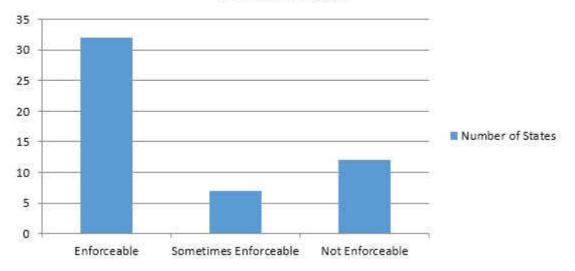


Figure 4.1: State by State Survey of Non-Competes

Period 1		Period 2			
Stage 1	Stage 2	Stage 3	Stage 1	Stage 2	Stage 3
Firms offer sales contracts to sales reps.	Sales reps accept contract and put effort.	Nature determines the random preference shocks for customers and final sales are realized. Sales incentives are paid.	Firms offer sales contracts to existing sales reps. If employee poaching is possible then they also offer contracts to competitor's sales reps. Firms can also offer contracts to new sales reps.	Sales reps decide on whether to accept the contracts offered by the firms and then put effort.	Nature determines the random preference shocks for customers and final sales are realized. Sales incentives are paid.

Figure 4.2: Game Sequence Chapter 4

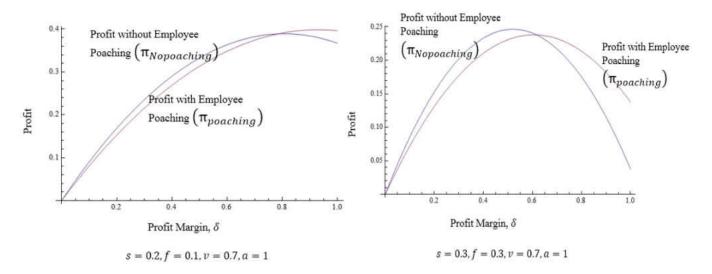


Figure 4.3: Employee Poaching and Non Employee Poaching Profits

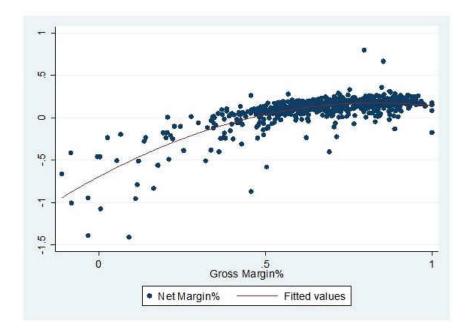


Figure 4.4: Commercial Banking Industry

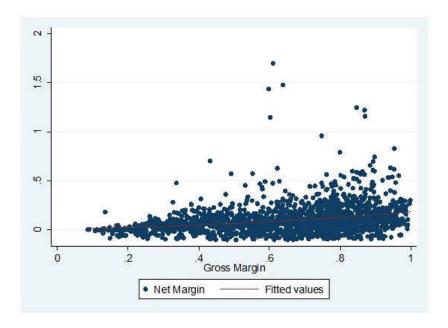


Figure 4.5: Software Services Industry

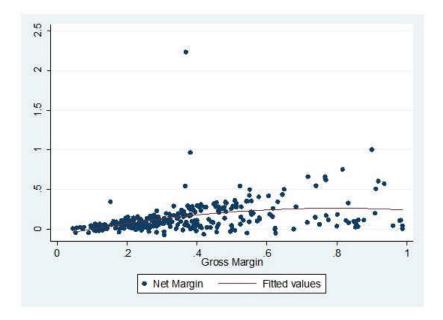


Figure 4.6: Insurance Industry

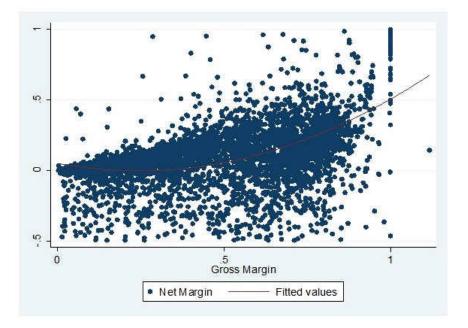


Figure 4.7: Energy Industry

APPENDIX A: PROOFS FOR THE RESULTS IN CHAPTER 2

A.1 Optimal Forced Ranking Policy

To prove Proposition 2.1 we use the result from Lemma A.1. Lemma A.1 and its proof are given below.

Lemma A.1 The marginal increase in probability of avoiding a layoff and acheving a promotion due to an increase in effort is given as,

$$\frac{\partial \Pr(j < N(1-\beta_1))}{\partial e_i} = \frac{N(1-\beta_1)N\beta_1}{N(N+1)d}s'(e_i), \\ \frac{\partial \Pr(j \leq N\beta_2)}{\partial e_i} = \frac{N(1-\beta_2)N\beta_2}{N(N+1)d}s'(e_i)$$
(A.1)

Proof. These derivations have largely been based on the derivations done by Kalra and Shi (2001; See Technical Appendix, page 191). ■

In the given tournament model,

$$\Pr(Rank \ of \ i^{th} \ employee = j) = \int_{\epsilon_i} \left(\begin{array}{c} N-1 \\ j-1 \end{array} \right) [1 - (F(y)]^{j-1} F^{N-j}(y) f(\epsilon_i) d\epsilon_i \qquad (A.2)$$

Hence,
$$\Pr(j < N(1 - \beta_1)) = \sum_{j=1}^{N(1 - \beta_1)} \int_{\epsilon_i} \begin{pmatrix} N - 1 \\ j - 1 \end{pmatrix} [1 - (F(y))^{j-1} F^{N-j}(y) f(\epsilon_i) d\epsilon_i \quad (A.3)$$

Where $y = s(e_i) - s(e_i^*) + \epsilon_i + \gamma - \gamma$ (In the current research, $s(e_i) = e_i$). The above function is a summation of the probabilities of getting each of the ranks from 1 to $N(1 - \beta_1)$ for the *i*th employee. The above probabilities have been derived using rules of order statistics. (The structure of the game discussed here is analogous to the multiple-winner sales contest that was discussed in Kalra and Shi 2001). The derivative of the probabilities with respect to effort is given below,

$$\frac{\partial \Pr(j < N(1 - \beta_1))}{\partial e_i} = \frac{\partial}{\partial e_i} \left(\sum_{j=1}^{N(1 - \beta_1)} \int_{\epsilon_i} \left(\begin{array}{c} N - 1\\ j - 1 \end{array} \right) [1 - (F(y)]^{j-1} F^{N-j}(y) f(\epsilon_i) d\epsilon_i \right)$$
(A.4)

Now the expression in the above eq. can be simplified as below using the Leibniz rule,

$$\frac{\partial}{\partial e_i} \left(\sum_{j=1}^{N(1-\beta_1)} \int_{\epsilon_i} \left(\begin{array}{c} N-1\\ j-1 \end{array} \right) [1-(F(y)]^{j-1} F^{N-j}(y) f(\epsilon_i) d\epsilon_i \right)$$
(A.5)

$$= \int_{\epsilon_i} \begin{pmatrix} N-1\\ j-1 \end{pmatrix} \{(j-1)[1-(F(y)]^{j-2}(-f(\epsilon_i)s'(e_i))F^{N-j}(y)$$
(A.6)

$$+[1 - (F(y)]^{j-1}(N-j)F^{N-j-1}(y)f(\epsilon_i)s'(e_i)\}f(\epsilon_i)d\epsilon_i$$
(A.7)

Since $y = s(e_i) - s(e_i^*) + \epsilon_i + \gamma - \gamma$, $\frac{\partial y}{\partial e_i} = s'(e_i)$ and $\frac{\partial F(y)}{\partial y} = f(y) = f(\epsilon_i)$. Rearranging the right hand side of the equation (A.7), we get, $s'(e_i) \int_{\epsilon_i} {\binom{N-1}{j-1}} [(N-j) - (N-1)F(y)][1 - (j-1)F(y)][1 - (j-$

 $F(y)]^{j-2}F^{N-j-1}(y)f^2(\epsilon_i)d\epsilon_i$. Recognizing that $y = s(e_i) - s(e_i^*) + \epsilon_i + \gamma - \gamma$ and at equilibrium $e_i = e_i^*$, since all the agents or employees are symmetrical in their characteristics, $y = \epsilon_i$. Hence,

$$s'(e_{i}) \int_{\epsilon_{i}} \binom{N-1}{j-1} [(N-j) - (N-1)F(\epsilon_{i})][1-F(\epsilon_{i})]^{j-2}F^{N-j-1}(\epsilon_{i})f^{2}(\epsilon_{i})d\epsilon_{i} \qquad (A.8)$$

$$= \binom{N-1}{j-1} (N-j) \int_{\epsilon_{i}} [1-F(\epsilon_{i})]^{j-2}F^{N-j-1}(\epsilon_{i})f^{2}(\epsilon_{i})d\epsilon_{i}$$

$$-\binom{N-1}{j-1} (N-1) \int_{\epsilon_{i}} [1-F(\epsilon_{i})]^{j-2}F^{N-j}(\epsilon_{i})f^{2}(\epsilon_{i})d\epsilon_{i} \qquad (A.9)$$

In the above results the pdf and cdf of a logistically distributed random variable ϵ_i with mean 0 and variance $\frac{\pi^2 d^2}{3}$ are taken as, $f(x) = \frac{1}{d} \frac{\exp\left(\frac{-x}{d}\right)}{[1+\exp\left(\frac{-x}{d}\right)]^2}$ and $F(x) = \frac{1}{1+\exp\left(\frac{-x}{d}\right)}$ respectively. For any integer $m \ge 2, n \ge 2$ and m > n, the distribution satisfies, $\int_x [1 + \exp\left(\frac{-x}{d}\right)]^{-m} [\exp\left(\frac{-x}{d}\right)]^n dx = \frac{n-1}{m-1} \int_x [1 + \exp\left(\frac{-x}{d}\right)]^{-(m-1)} [\exp\left(\frac{-x}{d}\right)]^{(n-1)} dx$. Then for any k = 2, 3..., N, repeating the transformation, we get, $\int_x [1 + \exp\left(\frac{-x}{d}\right)]^{-(N+2)} [\exp\left(\frac{-x}{d}\right)]^k dx = \frac{(k-1)!(N-k+1)!}{(N+1)!} d$. We use this transformation on eq. (A.9).

We can rewrite (A.9) as,
$$= \binom{N-1}{j-1} \frac{(N-j)}{d^2} \int_{\epsilon_i} [1 + \exp\left(\frac{-\epsilon_i}{d}\right)]^{N+1} [\exp\left(\frac{-\epsilon_i}{d}\right)]^j d\epsilon_i$$
$$-\binom{N-1}{j-1} \frac{(N-1)}{d^2} \int_{\epsilon_i} [1 + \exp\left(\frac{-\epsilon_i}{d}\right)]^{N+2} [\exp\left(\frac{-\epsilon_i}{d}\right)]^j d\epsilon_i$$
(A.10)

$$= \binom{N-1}{j-1} \left[\frac{(N-j)(j-1)!(N-j)!}{d.N!} - \frac{(N-1)(j-1)!(N-j+1)!}{d.N!} \right] = \frac{N-2j+1}{N(N+1)d}$$

Summing up, we get,
$$\frac{\partial \Pr(j < N(1 - \beta_1))}{\partial e_i} = \sum_{j=1}^{N(1 - \beta_1)} \frac{N - 2j + 1}{N(N+1)d} s'(e_i) = \frac{N(1 - \beta_1)N\beta_1}{N(N+1)d} s'(e_i)$$

(A.12)

Similarly, we get,
$$\frac{\partial \Pr(j \le N\beta_2)}{\partial e_i} = \sum_{j=1}^{N\beta_2} \frac{N-2j+1}{N(N+1)d} s'(e_i) = \frac{N(1-\beta_2)N\beta_2}{N(N+1)d} s'(e_i)$$
 (A.13)

Proof. Poposition 2.1.

The firm's objective function in case of a committed FR policy is given as,

$$\Pi_{CC} = \max_{\beta_1, \beta_2 \in [0,1]} \pi_{CC} = E \begin{bmatrix} \delta \sum (e_i^*(\beta_1, \beta_2) + \varepsilon_i + \gamma) - Nw_1 - N(\beta_1 + \beta_2)c \\ -fN(m(|\beta_2 - k_P|) + (1 - m)\beta_2 + q(|\beta_1 - k_L|) + (1 - q)\beta_1) \end{bmatrix}$$

(A.14)

$$e_i^* = \arg\max_{e_i} E[U(I_i, e_i)] \quad (IC)$$
(A.15)

$$E[U(I_i, e_i^*)] \ge U_S \quad (IR) \tag{A.16}$$

The effort for the i^{th} employee is given as,

$$e_i(\beta_1, \beta_2) = \arg\max E[U(I_i, e_i)]$$
(A.17)

 $= \arg \max\{\Pr(j \le N\beta_2)u(U_P) + \Pr(N\beta_2 < j \le N\beta_1)u(U_S)] + \Pr(N(1-\beta_1) < j)u(U_L) - c(e_i)\}$ (A.18)

$$= \arg \max\{\Pr(j \le N\beta_2)u(U_P) + (1 - \Pr(j \le N\beta_2) - \Pr(N(1 - \beta_1) < j)u(U_S) + \Pr(N(1 - \beta_1) < j)u(U_L) - c(e_i)\}$$
(A.19)

where
$$j$$
 is the rank of the i^{th} employee

$$= \arg \max\{\Pr(j \leqslant N\beta_2)[u(U_P) - u(U_S)] + u(U_S) + \Pr(N(1 - \beta_1) < j))[u(U_L) - u(U_S)] - c(e_i)\}$$
(A.20)

But,
$$\Pr(N(1 - \beta_1) < j)) = 1 - \Pr(N(1 - \beta_1) \ge j))$$
 and
hence, $\frac{\partial \Pr(N(1 - \beta_1) < j))}{\partial e_i} = -\frac{\partial \Pr(N(1 - \beta_1) \ge j))}{\partial e_i}$. Also, $\frac{\partial [u(U_S)]}{\partial e_i} = 0$
 $FOC : \frac{\partial \Pr(j \le N\beta_2)}{\partial e_i} (u(U_P) - u(U_S)) + \frac{\partial \Pr(N(1 - \beta_1) \ge j))}{\partial e_i} (u(U_S) - u(U_L)) - c'(e_i) = 0$
(A.21)

In the above FOC,
$$\frac{\partial \Pr(j < N(1 - \beta_1))}{\partial e_i} = \sum_{j=1}^{N(1 - \beta_1)} \frac{N - 2j + 1}{N(N+1)d} s'(e_i) = \frac{N(1 - \beta_1)N\beta_1}{N(N+1)d} s'(e_i)$$

$$\frac{\partial \Pr(j \leqslant N\beta_2)}{\partial e_i} = \sum_{j=1}^{N\beta_2} \frac{N-2j+1}{N(N+1)d} s'(e_i) = \frac{N(1-\beta_2)N\beta_2}{N(N+1)d} s'(e_i) \text{ according to Lemma A.1.}$$

(A.23)

(A.22)

Subsituting the derivatives from Lemma A.1 and, $s(e_i) = e_i$ and $c(e_i) = \frac{(e_i)^2}{2}$ in the FOC., we get

$$e_i(\beta_1,\beta_2) = \frac{(1-\beta_2)N\beta_2}{(N+1)d} \left[u(U_P) - u(U_S) \right] + \frac{(1-\beta_1)N\beta_1}{(N+1)d} \left[u(U_S) - u(U_L) \right]$$
(A.24)

Further, substituting the above equation in the firm's objective function, we get,

$$\max_{\beta_{1},\beta_{2}\in[0,1]}\pi_{CC} = E \begin{bmatrix} \delta \sum (e_{i}^{*}(\beta_{1},\beta_{2}) + \varepsilon_{i} + \gamma) - Nw_{1} - N(\beta_{1} + \beta_{2})c \\ -fN\left((m\left(|\beta_{2} - k_{P}|\right) + (1 - m)\beta_{2}\right) + q\left(|\beta_{1} - k_{L}|\right) + (1 - q)\beta_{1}) \end{bmatrix}$$
(A.25)

Since the above objective function is not differentiable at $k_P = \beta_2$ and $k_L = \beta_1$, we first assume $k_P \le \beta_2$

and
$$k_L \geq \beta_1$$
 and then solve the problem,

$$\max_{\beta_{1},\beta_{2}} E \begin{bmatrix} \pi = \delta Ne(\beta_{1},\beta_{2}) - Nw_{1} - N(\beta_{1} + \beta_{2})c \\ -fN\left((m\left(|\beta_{2} - k_{P}|\right) + (1 - m)\beta_{2}\right) + q\left(|\beta_{1} - k_{L}|\right) + (1 - q)\beta_{1}\right) \end{bmatrix}$$
(A.26)
s.t $\beta_{1},\beta_{2}\epsilon\left[0,1\right], \ \beta_{1} + \beta_{2} \leq 1,$
 $\beta_{1}u(U_{L}) + \beta_{2}u(U_{P}) + (1 - \beta_{1} - \beta_{2})u(U_{S}) \geq u(U_{S})$ (A.27)

To solve the constrained optimization problem we define the Lagrangian as,

$$\max_{\beta_{1},\beta_{2}} E \begin{bmatrix} \delta e(\beta_{1},\beta_{2}) - w_{1} - (\beta_{1} + \beta_{2})c - f \begin{pmatrix} (m(\beta_{2} - k_{P}) + (1 - m)\beta_{2}) \\ +q(\beta_{1} - k_{L}) + (1 - q)\beta_{1} \end{pmatrix} \\ +\lambda_{2}(1 - \beta_{1} - \beta_{2}) + \lambda_{1} \begin{pmatrix} \beta_{1}u(U_{L}) + \beta_{2}u(U_{P}) \\ +(1 - \beta_{1} - \beta_{2})u(U_{S}) - u(U_{S}) \end{pmatrix} \end{bmatrix}$$
(A.28)

We ignore the constraints $\beta_1, \beta_2 \leq 1$ because $0 \leq 1 - \beta_1 - \beta_2$. The FOCs for the langragian are,

$$\beta_1 \ge 0, L_{\beta_1} \le 0, \beta_1 L_{\beta_1} = 0, \ \beta_2 \ge 0, L_{\beta_2} \le 0, \beta_2 L_{\beta_2} = 0 \text{ and } \lambda_j \ge 0, L_{\lambda_j} \ge 0, \lambda_j L_{\lambda_j} = 0 \text{ where } j = 1, 2$$

We assume $\beta_1>0$ and $\beta_2>0$ (the assumption is satisfied if δ is large enough enough),

and $\lambda_2 = 0$ i.e. $\beta_1 + \beta_2 < 1$, (this assumption always holds, as we later show)

$$L_{\beta_1} = \frac{\delta(1 - 2\beta_1)N}{(N+1)d} U_{SL} - (c + f(1 - 2q) + \lambda_1 U_{SL}) = 0$$
(A.29)

$$L_{\beta_2} = \frac{\delta(1 - 2\beta_2)N}{(N+1)d} U_{PS} - (c + f - \lambda_1 U_{PS}) = 0$$
(A.30)

Solving the above equations we get the following solutions.

$$\beta_2^* = \frac{1}{2} \left[1 - \frac{(N+1)ad}{\delta N (U_{PS})} \left(c + f - \lambda_1^* U_{PS} \right) \right] \text{ if } \beta_2^* > k_P \tag{A.31}$$

$$\beta_1^* = \frac{1}{2} \left[1 - \frac{(N+1)ad}{\delta N(U_{SL})} \left(c + f \left(1 - 2q \right) + \lambda_1^* U_{SL} \right) \right] \text{ if } k_L > \beta_1^* \tag{A.32}$$

Where $U_{SL} = u(U_S) - u(U_L), U_{PS} = u(U_P) - u(U_S)$ and λ_1^* is the marginal cost of reservation utility.

Following the above process, the complete solution for the problem is given below,

$$\beta_{2}^{*} = \left\{ \begin{array}{c} \underline{k}_{P} = \frac{1}{2} \left[1 - \frac{(N+1)ad}{\delta N(U_{PS})} \left(c + f - \lambda_{1}^{*} U_{PS} \right) \right] \text{ if } k_{P} < \underline{k}_{P} \\ k_{P} \text{ if } \underline{k}_{P} < k_{P} < \overline{k}_{P} \\ \overline{k}_{P} = \frac{1}{2} \left[1 - \frac{(N+1)ad}{\delta N(U_{PS})} \left(c + f \left(1 - 2m \right) - \lambda_{1}^{*} U_{PS} \right) \right] \text{ if } \overline{k}_{P} < k_{P} \end{array} \right\}$$
(A.33)

$$\beta_{1}^{*} = \left\{ \begin{array}{c} \underline{k}_{L} = \frac{1}{2} \left[1 - \frac{(N+1)ad}{\delta N(U_{SL})} \left(c + f + \lambda_{1}^{*} U_{SL} \right) \right] \text{ if } k_{L} < \underline{k}_{L} \\ k_{L} \text{ if } \underline{k}_{L} < k_{L} < \overline{k}_{L} \\ \overline{k}_{L} = \frac{1}{2} \left[1 - \frac{(N+1)ad}{\delta N(U_{SL})} \left(c + f \left(1 - 2q \right) + \lambda_{1}^{*} U_{SL} \right) \right] \text{ if } \overline{k}_{L} < k_{L} \end{array} \right\}$$
(A.34)

Proof. Proposition 2.2. ■

Based on results from Proposition 2.1 it can be verified that $\frac{\partial \beta_i^*}{\partial N} > 0, \frac{\partial \beta_i^*}{\partial d} < 0, \frac{\partial \beta_i^*}{\partial r} < 0, \frac{\partial \beta_1^*}{\partial U_S} > 0, \frac{\partial \beta_2^*}{\partial U_S} < 0, \frac{\partial \beta_1^*}{\partial \lambda_1} < 0, \frac{\partial \beta_2^*}{\partial \lambda_1} > 0$ where $i = \{1, 2\}$.

A.2 Ambiguous vs. Committed Forced Ranking Policy

Let profit (effort) for the firm (employees) under a FR policy be denoted by Π_{ij} (e_{ij}),

where $i = \begin{cases} C, \text{ if the firm commits to a promotion policy} \\ A, \text{ if the firm leaves the promotion policy ambiguous} \end{cases}$ and $j = \begin{cases} C, \text{ if the firm commits to a Layoff policy} \\ A, \text{ if the firm leaves the Layoff policy ambiguous} \end{cases}$

To find the firm's optimal FR policy, we first find Π_{CC} , Π_{CA} , Π_{AC} and Π_{AA} . To calculate the profits we first compute the effort put by employees under each of the four policies. The effort e_{CC} directly follows from eq. A.24. The effort e_{CA} is derived below.

$$e_{CA} = \arg \max \left(\min \left\{ \begin{array}{l} \Pr(j \leqslant N\beta_2)u(U_P) + \Pr(N\beta_2 < j)u(U_S)] - c(e), \\ \Pr(j \leqslant N\beta_2)u(U_P) + \Pr(N\beta_2 < j \le Nk_L)u(U_S)] \\ + \Pr(N(1 - k_L) < j)u(U_L) - c(e) \end{array} \right\} \right)$$
(A.35)
$$e_{CA} = \frac{(1 - \beta_2)N\beta_2}{(N+1)d} \left[u(U_P) - u(U_S) \right] + \frac{(1 - k_L)Nk_L}{(N+1)d} \left[u(U_S) - u(U_L) \right]$$
(A.36)

Following the above process, we get,

$$e_{CC} = \frac{(1-\beta_2)N\beta_2}{(N+1)d} \left[u(U_P) - u(U_S) \right] + \frac{(1-\beta_1)N\beta_1}{(N+1)d} \left[u(U_S) - u(U_L) \right]$$
(A.37)

$$e_{CA} = \frac{(1-\beta_2)N\beta_2}{(N+1)d} \left[u(U_P) - u(U_S) \right] + \frac{(1-k_L)Nk_L}{(N+1)d} \left[u(U_S) - u(U_L) \right]$$
(A.38)

$$e_{AC} = \frac{(1 - \beta_1)N\beta_1}{(N+1)d} \left[u(U_S) - u(U_L) \right]$$
(A.39)

$$e_{AA} = \frac{(1-k_L)Nk_L}{(N+1)d} \left[u(U_S) - u(U_L) \right]$$
(A.40)

The firm's problems under the different FR systems are given as,

$$\Pi_{CC} = \max_{\beta_1, \beta_2} E \begin{bmatrix} \pi_{CC} = \delta N e_{CC} - N w_1 - N(\beta_1 + \beta_2)c \\ -fN\left((m\left(|\beta_2 - k_P|\right) + (1 - m)\beta_2\right) + q\left(|\beta_1 - k_L|\right) + (1 - q)\beta_1\right) \end{bmatrix}$$
(A.41)

$$\Pi_{CA} = \max_{\beta_2} E\left[\pi_{CC} = \delta N e_{CA} - N(k_L + \beta_2)c - fN\left((m\left(|\beta_2 - k_P|\right) + (1 - m)\beta_2\right))\right] \quad (A.42)$$

$$\Pi_{AC} = \max_{\beta_1} E\left[\pi_{AC} = \delta N e_{AC} - N(\beta_1 + k_P)c - fN\left(+q\left(|\beta_1 - k_L|\right) + (1-q)\beta_1\right)\right]$$
(A.43)

$$\Pi_{AA} = E \left[\pi_{AA} = \delta N e_{AA} - N (k_L + k_P) c \right]$$
(A.44)

Since $\Pi_{\beta_1\beta_2} = 0$

$$\Pi_{CC} = \Pi_{CP} + \Pi_{CL}, \Pi_{CA} = \Pi_{CP} + \Pi_{AL}, \Pi_{AC} = \Pi_{AP} + \Pi_{CL}, \Pi_{AA} = \Pi_{AP} + \Pi_{AL}$$
(A.45)

where,
$$\Pi_{CP} = \frac{\delta(1-\beta_2^*)N\beta_2^*}{(N+1)d} \left[u(U_P) - u(U_S) \right] - (\beta_2^*)c - f\left(m\left(|\beta_2^* - k_P|\right) + (1-m)\beta_2^*\right) \right]$$

(A.46)

$$\Pi_{AL} = \frac{\delta(1-k_L)Nk_L}{(N+1)d} \left[u(U_S) - u(U_L) \right] - (k_L)c$$
(A.47)

$$\Pi_{CL} = \frac{\delta(1-\beta_1^*)N\beta_1^*}{(N+1)d} \left[u(U_S) - u(U_L) \right] - (\beta_1^*)c - f\left(q\left(|\beta_1^* - k_L|\right) + (1-q)\beta_1^*\right)$$
(A.48)

$$\Pi_{AP} = 0$$
, and, β_2^* and β_1^* are given in Proposition 2.1. (A.49)

Proof. Proposition 2.3. ■

As part of the proposition we prove that,

$$\Pi_{CC} > \Pi_{CA} > \max \{\Pi_{AC}, \Pi_{AA}\} \text{ if } 0 < f < \underline{F}$$
$$\Pi_{CA} > \Pi_{CC} > \max \{\Pi_{AC}, \Pi_{AA}\} \text{ if } \overline{F} > f > \underline{F}$$
$$\Pi_{AA} > \Pi_{CA} > \max \{\Pi_{AC}, \Pi_{CC}\} \text{ if } f > \overline{F}$$

To prove the above first consider the case $\overline{k}_L < k_L$.

We find that $\Pi_{CA} > \Pi_{CC}$ if $f > \underline{F}$ and $\Pi_{CC} > \Pi_{CA}$ if $f < \underline{F}$

or,
$$\Pi_{AL} > \Pi_{CL}$$
 if $f > \underline{F}$ and $\Pi_{CL} > \Pi_{AL}$ if $f < \underline{F}$.

where,
$$\underline{F} = \frac{(cd(1+N) - N(1-2k_L)U_{SL}\delta)^2}{2dk_L N(1+N)U_{SL}\delta}$$
 (A.50)

Now, we show that $\min \{\Pi_{CA}, \Pi_{CC}\} > \max \{\Pi_{AC}, \Pi_{AA}\}$ if $0 < f < \overline{F}$

If
$$f < \underline{F}$$
, $\Pi_{CA} = \min \{\Pi_{CA}, \Pi_{CC}\}$ and $\Pi_{CA} > \max \{\Pi_{AC}, \Pi_{AA}\}$

First note that if $f < \underline{F}$, $\Pi_{AC} > \Pi_{AA}$ because $\Pi_{CL} > \Pi_{AL}$

Now, $\Pi_{CA} > \Pi_{AC}$ because $\Pi_{CL} > \Pi_{AL}$ and $\Pi_{CP} > \Pi_{AP}$

If
$$\overline{F} > f > \underline{F}$$
, $\Pi_{CC} = \min \{\Pi_{CA}, \Pi_{CC}\}$ and $\Pi_{CC} > \max \{\Pi_{AC}, \Pi_{AA}\}$

First note that if $f > \underline{F}$, $\Pi_{AA} > \Pi_{AC}$ because $\Pi_{AL} > \Pi_{CL}$

Now,
$$\Pi_{CC} > \Pi_{AA}$$
 if $f < F$
where, $\overline{F} = \frac{\left(cd\left(1+N\right)-NU_{PS}\delta\right)^2}{2dk_PN\left(1+N\right)U_{PS}\delta}$ (A.51)

Finally, note that $\Pi_{CA} < \Pi_{AA}$ if $\overline{F} < f$

The proofs for the cases for $k_L < \underline{k}_L$ and $\underline{k}_L < k_L < \overline{k}_L$ are similar.

Proof. Proposition 2.4.

If $f < \overline{F}$, then $\Pi_{CC} > \Pi_{CA} > \max \{\Pi_{AC}, \Pi_{AA}\}$ if $k_L < k_L^L, \Pi_{CA} > \Pi_{CC} > \max \{\Pi_{AC}, \Pi_{AA}\}$ if $k_L^L < k_L < k_L^H$ and $\Pi_{CC} > \Pi_{CA} > \max \{\Pi_{AC}, \Pi_{AA}\}$ if $k_L^H < k_L$. Since in Proposition 2.3 we show that if $f < \overline{F}$ then $\min \{\Pi_{CA}, \Pi_{CC}\} > \max \{\Pi_{AC}, \Pi_{AA}\}$ we only compare Π_{CC} and Π_{CA} . As part of the proof we show that if $k_L^L < k_L < k_L^H$ then $\Pi_{CA} > \Pi_{CC}$.

It can be shown that $\frac{\partial^2 (\Pi_{CA} - \Pi_{CC})}{\partial k_L^2} < 0 \ \forall k_L$, $\Pi_{CA} - \Pi_{CC} < 0$ at $k_L = 0$ and $k_L = 1$ and $\Pi_{CA} - \Pi_{CC} = 0$ at $k_L = k_L^L$, k_L^H where k_L^L , k_L^H are the roots of the equation $\Pi_{CA} - \Pi_{CC} = 0$. Therefore, if $k_L^L < k_L < k_L^H$ then $\Pi_{CA} > \Pi_{CC}$. Also, $k_L^L < \underline{k}_L$ and $\overline{k}_L < k_L^H$

Proof. Proposition 2.5. ■

If $f < \overline{F}$, then $\Pi_{CA} > \Pi_{CC} > \max \{\Pi_{AC}, \Pi_{AA}\}$ if $\delta < \delta^*$ and $\Pi_{CC} > \Pi_{CA} > \max \{\Pi_{AC}, \Pi_{AA}\}$ if $\delta^* < \delta$. As part of Proposition 2.4 we show that if $\delta^* < \delta$ then $\pi_{CA} < \pi_{CC}$. Since in Proposition 2.3 we show that if $f < \overline{F}$ then $\min \{\Pi_{CA}, \Pi_{CC}\} > \max \{\Pi_{AC}, \Pi_{AA}\}$ we only compare Π_{CC} and Π_{CA} . As part of the proof we show that $\Pi_{CC} > \Pi_{CA}$ if $\delta^* < \delta$.

It can be shown that $\frac{\partial^2 (\pi_{CA} - \pi_{CC})}{\partial \delta^2} < 0 \ \forall \delta$, at $\delta = \underline{\delta}, \pi_{CA} - \pi_{CC} > 0$ and at $\delta = \overline{\delta}, \pi_{CA} - \pi_{CC} < 0$ where $\underline{\delta} = \frac{cd + df + cdN + dNf - 2dfq - 2dfNq}{NU_{SL}}$ is the lowest value of δ for all conditions to be met,

therefore, there exists a $\delta = \delta^*$ such that $\pi_{CA} - \pi_{CC} < 0 \forall \delta > \delta^*$

A.3 Interaction between Forced Ranking and Sales Compensation

Derivation of the equilibrium contract for the tournament with performance pay

In the first period an employee is motivated by two potential sources of utilities viz. one, from possibilities of promotion and avoidance of probation, and two, from linear incentives associated with sales. Since both these incomes are random variables we represent them with the notations \tilde{z} and \tilde{y} respectively such that,

$$\widetilde{z} = \left\{ \begin{array}{c} U_P \text{ if } j \leq N\beta_2 \\ \\ U_S \text{ if } N\beta_2 \leq j \leq N(1-\beta_1) \\ \\ U_L \text{ if } j \geq N(1-\beta_1) \end{array} \right\} \text{ and } \widetilde{y} = w_1 + b\widetilde{x}(e_i)$$

where $j(\tilde{x}(e_i))$ is the rank of the i^{th} employee and

$$\widetilde{x}(e_i) = e_i + \varepsilon_{\epsilon} + \varepsilon_{\gamma} \ \exists E[\widetilde{x}(e_i)] = e_i, Var[\widetilde{x}(e_i)] = \sigma_{\epsilon}^2 + \sigma_{\gamma}^2 = \sigma^2$$

In the current problem $(\beta_1, \beta_2, b, w_1)$ are the firm's decision variables and the problem for the firm in the case when it uses performance pay along with FR is be given below. In this part of analysis we normalize $\delta = 1$ to keep parsimony. Moreover, since we solve for a symmetric equilibrium we supress the subscript *i* whenever the subscript is not essential.

Analysis

$$\max_{(\beta_2,\beta_1,b,w)} Ne^* - N.E[w + be^*] - N(\beta_2 + \beta_1)c \quad \text{(Firm's Problem)}$$
(A.52)

$$e = \arg\max EU (IC) \tag{A.53}$$

$$EU(e^*) \ge \overline{U}$$
 (IR) (A.54)

$$EU = \underbrace{\int_{\gamma} \int_{\epsilon_{i}} \Pr\left(j \le N\beta_{2}\right) u\left(U_{P} + y\right) f(\epsilon_{i} + \gamma) d\epsilon_{i} d\gamma}_{EU1}}_{EU1}$$

$$+ \underbrace{\int_{\gamma} \int_{\epsilon_{i}} \Pr\left(N(1 - \beta_{1}) \ge j > N\beta_{2}\right) u\left(U_{S} + y\right) f(\epsilon_{i} + \gamma) d\epsilon_{i} d\gamma}_{EU2}}_{EU2}$$

$$+ \underbrace{\int_{\gamma} \int_{\epsilon_{i}} \Pr\left(N(1 - \beta_{1}) < j\right) u\left(U_{L} + y\right) f(\epsilon_{i} + \gamma) d\epsilon_{i} d\gamma}_{EU1}$$

$$+ \underbrace{\int_{\gamma} \int_{\epsilon_{i}} (1 - \Pr\left(j \le N\beta_{2}\right) - \Pr\left(N(1 - \beta_{1}) < j\right)\right) u\left(U_{S} + y\right) f(\epsilon_{i} + \gamma) d\epsilon_{i} d\gamma}_{EU2}}_{EU3}$$
or $EU = \underbrace{\int_{\gamma} \int_{\epsilon_{i}} \Pr\left(N(1 - \beta_{1}) < j\right) u\left(U_{L} + y\right) f(\epsilon_{i} + \gamma) d\epsilon_{i} d\gamma}_{EU3}$

$$eV2$$

$$+ \underbrace{\int_{\gamma} \int_{\epsilon_{i}} \Pr\left(N(1 - \beta_{1}) < j\right) u\left(U_{L} + y\right) f(\epsilon_{i} + \gamma) d\epsilon_{i} d\gamma}_{EU3}$$
or $EU = \int_{\gamma} \int_{\epsilon_{i}} \Pr\left(j \le N\beta_{2}\right) \left[u\left(U_{P} + y\right) - u\left(U_{S} + y\right)\right] f(\epsilon_{i} + \gamma) d\epsilon_{i} d\gamma$

$$+ \underbrace{\int_{\gamma} \int_{\epsilon_{i}} \Pr\left(N(1 - \beta_{1}) < j\right) \left[u\left(U_{S} + y\right) - u\left(U_{L} + y\right)\right] f(\epsilon_{i} + \gamma) d\epsilon_{i} d\gamma}_{FU}$$

$$EU = EU1 + EU2 + EU3, such that,$$

$$EU1 = \sum_{j=1}^{N\beta_2} \left(\begin{array}{c} \int_{\gamma} \int_{\epsilon_i} \left[u \left(U_P + y \right) - u \left(U_S + y \right) \right] f(\epsilon_i + \gamma) \\ \left(\begin{array}{c} N - 1 \\ j - 1 \end{array} \right) \left[1 - F(Y) \right]^{j-1} F^{N-j}(Y) f(\epsilon_i) d\epsilon_i d\gamma \end{array} \right)$$
(A) (A.58)

$$EU2 = \left(\int_{\gamma} \int_{\epsilon_{i}} u\left(U_{S} + y\right) f(\epsilon_{i} + \gamma) d\epsilon_{i} d\gamma\right)$$
(B) (A.59)
$$EU3 = \sum_{j=N(1-\beta_{1})}^{N} \left(\int_{\gamma} \int_{\epsilon_{i}} \left[u\left(U_{S} + y\right) - u\left(U_{L} + y\right)\right] f(\epsilon_{i} + \gamma) \\ \left(\int_{\gamma} \int_{\epsilon_{i}} \left[u\left(U_{S} + y\right) - u\left(U_{L} + y\right)\right] f(\epsilon_{i} + \gamma) \\ \left(\int_{\gamma} \int_{\gamma} \int_{\epsilon_{i}} \left[u\left(U_{S} + y\right) - u\left(U_{L} + y\right)\right] f(\epsilon_{i} + \gamma) \\ \left(\int_{\gamma} \int_{\gamma} \int_{\epsilon_{i}} \left[u\left(U_{S} + y\right) - u\left(U_{L} + y\right)\right] f(\epsilon_{i} + \gamma) \\ \left(\int_{\gamma} \int_{\gamma} \int_{\epsilon_{i}} \left[u\left(U_{S} + y\right) - u\left(U_{L} + y\right)\right] f(\epsilon_{i} + \gamma) \\ \left(\int_{\gamma} \int_{\gamma} \int_{\epsilon_{i}} \left[u\left(U_{S} + y\right) - u\left(U_{L} + y\right)\right] f(\epsilon_{i} + \gamma) \\ \left(\int_{\gamma} \int_{\gamma} \int_{\epsilon_{i}} \left[u\left(U_{S} + y\right) - u\left(U_{L} + y\right)\right] f(\epsilon_{i} + \gamma) \\ \left(\int_{\gamma} \int_{\gamma} \int_{\epsilon_{i}} \left[u\left(U_{S} + y\right) - u\left(U_{L} + y\right)\right] f(\epsilon_{i} + \gamma) \\ \left(\int_{\gamma} \int_{\gamma} \int_{\gamma} \int_{\epsilon_{i}} \left[u\left(U_{S} + y\right) - u\left(U_{L} + y\right)\right] f(\epsilon_{i} + \gamma) \\ \left(\int_{\gamma} \int_{\gamma} \int_{\gamma} \int_{\gamma} \left[u\left(U_{S} + y\right) - u\left(U_{L} + y\right)\right] f(\epsilon_{i} + \gamma) \\ \left(\int_{\gamma} \int_{\gamma} \int_{\gamma} \int_{\gamma} \int_{\gamma} \left[u\left(U_{S} + y\right) - u\left(U_{L} + y\right)\right] f(\epsilon_{i} + \gamma) \\ \left(\int_{\gamma} \int_{\gamma} \int_{\gamma} \int_{\gamma} \int_{\gamma} \int_{\gamma} \left[u\left(U_{S} + y\right) - u\left(U_{L} + y\right)\right] f(\epsilon_{i} + \gamma) \\ \left(\int_{\gamma} \int_{\gamma} \int_{\gamma}$$

$$Where, u (U_{K} + y) = -\exp\left[-r\left(U_{K} + w + be + b\varepsilon_{i} + b\gamma - \frac{e^{2}}{2}\right)\right], \{K = P, S, L\}$$

$$y = w + be + b\varepsilon_{i} + b\gamma - \frac{e^{2}}{2}, Y = e_{i} - e_{i}^{*} + \varepsilon_{i} + \gamma - \gamma$$

$$f(\varepsilon) = \frac{1}{d} \frac{\exp\left(\frac{-\varepsilon}{d}\right)}{[1 + \exp\left(\frac{-\varepsilon}{d}\right)]^{2}} \text{ and } F(\varepsilon) = \frac{1}{1 + \exp\left(\frac{-\varepsilon}{d}\right)}$$

$$\frac{\partial EU}{\partial e} = 0, EU (e^{*}) = \overline{U} \text{ for IC and IR.}$$

$$EU1 = \sum_{j=1}^{N\beta_{2}} \int_{\gamma} \left(\underbrace{\begin{pmatrix} N - 1\\ 0 \\ \frac{1}{j-1} \end{pmatrix} [1 - (F(Y)]^{j-1}F^{N-j}(Y)f(\varepsilon_{i})d\varepsilon_{i}}{\frac{1}{j-1}}\right)}_{v} d\gamma \qquad (A.61)$$

$$\frac{\partial EU1}{\partial e} = \underbrace{v}_{dG1} \frac{\partial u}{\partial e} + \underbrace{u}_{F2} \frac{\partial v}{dG2} \qquad (A.62)$$

$$\begin{split} \frac{\partial EU1}{\partial e} &= \sum_{j=1}^{N\beta_2} \int_{\gamma} \int_{\epsilon_i} \underbrace{\left(\frac{\partial \left[u\left(U_P+y\right)-u\left(U_S+y\right)\right]}{\partial e}\right) f(\epsilon_i+\gamma)}{\left(\frac{\partial \left[u\left(U_P+y\right)-u\left(U_S+y\right)\right] f(\epsilon_i+\gamma)\right]}{iG^{1}}} \right)}{I(\epsilon_i+\gamma)} \\ &= \underbrace{\left(\begin{pmatrix} N-1\\ j-1 \end{pmatrix} \left[1-(F(Y)]^{j-1}F^{N-j}(Y)f(\epsilon_i)d\epsilon_i\right)\right]}_{dG^1} \right)}_{dG^1} \\ &= \int_{\gamma} \int_{\epsilon_i} \underbrace{\left(\frac{\partial \left[u\left(U_P+y\right)-u\left(U_S+y\right)\right]}{\partial e}\right) f(\epsilon_i+\gamma)}}{\left[1-(F(Y)]^{j-1}F^{N-j}(Y)f(\epsilon_i)d\epsilon_i\right]} \\ &= \int_{\gamma} \int_{\epsilon_i} \underbrace{\left(\frac{\partial \left[u\left(U_P+y\right)-u\left(U_S+y\right)\right]}{\partial e}\right) f(\epsilon_i+\gamma)}_{dG^1} \right)}_{dG^1} \\ &= \int_{\gamma} \int_{\beta_2} \left[\left(\underbrace{\left(\frac{\partial \left[u\left(U_P+y\right)-u\left(U_S+y\right)\right]}{\partial e}\right) f(\epsilon_i+\gamma)} \right)}_{dG^2} f(\epsilon_i+\gamma) d\epsilon_i d\gamma \\ &= \int_{\gamma} \int_{\beta_2} \left(\underbrace{\left(\frac{\partial \left[u\left(U_P+y\right)-u\left(U_S+y\right)\right]}{\partial e}\right) f(\epsilon_i+\gamma)} \right)}_{dE^2} f(\epsilon_i+\gamma) d\epsilon_i d\gamma \\ &= -\int_{\gamma} \int_{\beta_2} \left(\underbrace{\left(\frac{\partial \left[u\left(U_P+y\right)-u\left(U_S+y\right)\right]}{\partial e}\right)}_{dE^2} f(\epsilon_i+\gamma) d\epsilon_i d\gamma \\ &= -\int_{\gamma} \int_{\beta_2} \left(\underbrace{\left(\frac{\partial \left[u\left(U_P+y\right)-u\left(U_S+y\right)\right]}{\partial e}\right)}_{dE^2} f(\epsilon_i+\gamma) d\epsilon_i d\gamma \\ &= -\int_{\gamma} \int_{\beta_2} \left(\underbrace{\left(\frac{\partial \left[u\left(U_P+y\right)-u\left(U_S+y\right)\right]}{\partial e}\right)}_{dE^2} f(\epsilon_i+\gamma) d\epsilon_i d\gamma \\ &= -\int_{\gamma} \int_{\beta_2} \left(\underbrace{\left(\frac{\partial \left[u\left(U_P+y\right)-u\left(U_S+y\right)\right]}{\partial e}\right)}_{dE^2} f(\epsilon_i+\gamma) d\epsilon_i d\gamma \\ &= -\int_{\gamma} \int_{\beta_2} \left(\underbrace{\left(\frac{\partial \left[u\left(U_P+y\right)-u\left(U_S+y\right)\right]}{\partial e}\right)}_{dE^2} f(\epsilon_i+\gamma) d\epsilon_i d\gamma \\ &= -\int_{\gamma} \int_{\beta_2} \left(\underbrace{\left(\frac{\partial \left[u\left(U_P+y\right)-u\left(U_S+y\right)\right]}{\partial e}\right)}_{dE^2} f(\epsilon_i+\gamma) d\epsilon_i d\gamma \\ &= -\int_{\gamma} \int_{\beta_2} \left(\underbrace{\left(\frac{\partial \left[u\left(U_P+y\right)-u\left(U_S+y\right)\right]}{\partial e}\right)}_{dE^2} f(\epsilon_i+\gamma) d\epsilon_i d\gamma \\ &= -\int_{\gamma} \int_{\beta_2} \int_{\beta_2} \left(\underbrace{\left(\frac{\partial \left[u\left(U_P+y\right)-u\left(U_S+y\right)\right]}{\partial e}\right)}_{dE^2} f(\epsilon_i+\gamma) d\epsilon_i d\gamma \\ &= -\int_{\gamma} \int_{\beta_2} \int_{\beta_2} \left(\underbrace{\left(\frac{\partial \left[u\left(U_P+y\right)-u\left(U_S+y\right)\right]}{\partial e}\right)}_{dE^2} f(\epsilon_i+\gamma) d\epsilon_i d\gamma \\ &= -\int_{\gamma} \int_{\beta_2} \int_{\beta$$

Similarly A2 can be written as,

$$= \int_{\gamma} \frac{N(1-\beta_{2})\beta_{2}}{(N+1)d} \left[\left(u \left(U_{P} + y \right) - u \left(U_{S} + y \right) \right) f(\epsilon_{i} + \gamma) \right]_{-\infty}^{\infty} d\gamma - \int_{\gamma} \int \frac{N(1-\beta_{2})\beta_{2}}{(N+1)d} \left[u \left(U_{P} + y \right) - u \left(U_{S} + y \right) \right] f(\epsilon_{i} + \gamma) d\epsilon_{i} d\gamma$$
(A.68)

$$= -\int_{\gamma} \int \frac{N(1-\beta_2)\beta_2}{(N+1)d} \left[u\left(U_P + y\right) - u\left(U_S + y\right) \right] f(\epsilon_i + \gamma) d\epsilon_i d\gamma$$
(A.69)

$$\frac{\partial EU1}{\partial e} = -\frac{\partial}{\partial e} \left(\int_{\gamma} \int \beta_2 \left(\left[u \left(U_P + y \right) - u \left(U_S + y \right) \right] \right) f(\epsilon_i + \gamma) d\epsilon_i d\gamma \right) - \int_{\gamma} \int \frac{N(1 - \beta_2)\beta_2}{(N+1)d} \left[u \left(U_P + y \right) - u \left(U_S + y \right) \right] f(\epsilon_i + \gamma) d\epsilon_i d\gamma,$$
(A.70)

$$EU2 = \int_{\gamma} \int_{\epsilon_i} u \left(U_S + y \right) f(\epsilon_i + \gamma) d\epsilon_i d\gamma$$
(A.71)

Following the above process

$$\frac{\partial EU3}{\partial e} = -\int_{\gamma} \int \beta_1 \left(\frac{\partial \left[u \left(U_S + y \right) - u \left(U_L + y \right) \right]}{\partial e} \right) f(\epsilon_i + \gamma) d\epsilon_i d\gamma$$

$$+ \int_{\gamma} \int \frac{N(1 - \beta_2)\beta_2}{(N+1)d} \left[u \left(U_S + y \right) - u \left(U_L + y \right) \right] f(\epsilon_i + \gamma) d\epsilon_i d\gamma$$
In addition, $\int_{\gamma} \int_{\epsilon_i} u \left(U_K + y \right) f(\epsilon_i + \gamma) d\epsilon_i d\gamma =$

$$- \exp \left[-r \left(U_K + w + be - \frac{r}{2}b^2\sigma^2 - \frac{e^2}{2} \right) \right], \quad \{K = P, S, L\}$$
and $\frac{\partial}{\partial e} \left(\int_{\gamma} \int_{\epsilon_i} u \left(U_K + y \right) f(\epsilon_i + \gamma) d\epsilon_i d\gamma \right)$

$$= \frac{\partial}{\partial e} \left(\exp \left[-r \left(U_K + w + be - \frac{r}{2}b^2\sigma^2 - \frac{e^2}{2} \right) \right] \right)$$
(A.74)

Doing First Order Taylor Approximation at r = 0 i.e. $\exp\left[-r\left(U_K + w + be - \frac{r}{2}b^2\sigma^2 - \frac{e^2}{2}\right)\right]$

$$= 1 - r\left(U_K + w + be - rb^2\sigma^2 - \frac{e^2}{2}\right) + R_2, \text{ we get,}$$
(A.75)

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 \dots$$
(A.76)

where, $x = r, f(x) = \exp[-x(A - xB)], a = 0, f(a) = \exp[0] = 1$

$$f'(a) = -(A - 2rB), \frac{f'(a)}{1!}(x - a) = -r(A - 2rB)$$
(A.77)

Hence,
$$\exp\left[-r\left(U_{K}+w+be-\frac{r}{2}b^{2}\sigma^{2}-\frac{e^{2}}{2}\right)\right] = 1-r\left(U_{K}+w+be-rb^{2}\sigma^{2}-\frac{e^{2}}{2}\right)+R_{2}$$
(A.78)

$$\frac{\partial EU1}{\partial e} = -\frac{\partial}{\partial e} \left(\beta_2 b \left[-r \left(U_P - U_S\right)\right]\right) - \frac{N(1 - \beta_2)\beta_2}{(N+1)d} \left[-r \left(U_P - U_S\right)\right]$$
(A.79)

$$\begin{split} EU2 &= -r\left(U_S + w + be - rb^2\sigma^2 - \frac{e^2}{2}\right) \text{ because } EU2 = U_S + E\left[y\right],\\ &\frac{\partial EU2}{\partial e} = -r\left(b - e\right)\\ &\frac{\partial EU3}{\partial e} = -\frac{\partial}{\partial e}\left(\beta_1 b\left[-r\left(U_S - U_L\right)\right]\right) - \frac{N(1 - \beta_1)\beta_1}{(N+1)d}\left[-r\left(U_S - U_L\right)\right]\\ &e^* = \frac{N(1 - \beta_2)\beta_2}{(N+1)d}\left(U_P - U_S\right) + \frac{N(1 - \beta_2)\beta_2}{(N+1)d}\left(U_S - U_L\right) + b \end{split}$$

Individual Rationality

$$EU1 = \sum_{j=1}^{N\beta_2} \left(\begin{array}{c} \int_{\gamma} \int_{\epsilon_i} \left[u \left(U_P + y \right) - u \left(U_S + y \right) \right] f(\epsilon_i + \gamma) \\ \left(\begin{array}{c} N - 1 \\ j - 1 \end{array} \right) \left[1 - (F(Y))^{j-1} F^{N-j}(Y) f(\epsilon_i) d\epsilon_i d\gamma \end{array} \right)$$
(A) (A.80)

$$EU2 = \left(\int_{\gamma} \int_{\epsilon_i} u \left(U_S + y\right) f(\epsilon_i + \gamma) d\epsilon_i d\gamma\right)$$
(B) (A.81)

$$EU3 = \sum_{j=N(1-\beta_1)}^{N} \begin{pmatrix} \int_{\gamma} \int_{\epsilon_i} \left[u \left(U_S + y \right) - u \left(U_L + y \right) \right] f(\epsilon_i + \gamma) \\ \begin{pmatrix} N-1 \\ j-1 \end{pmatrix} \left[1 - (F(Y))^{j-1} F^{N-j}(Y) f(\epsilon_i) d\epsilon_i d\gamma \right] \end{pmatrix}$$
(C) (A.82)

Assuming symmetric Nash Equilibrium,

$$EU1 = \sum_{j=1}^{N\beta_2} \int_{\gamma} \int_{\epsilon_i} \underbrace{\left[u\left(U_P + y\right) - u\left(U_S + y\right)\right] f(\epsilon_i + \gamma)}_{F}$$

$$\underbrace{\left(\begin{array}{c}N-1\\j-1\end{array}\right) \left[1 - (F(Y)]^{j-1} F^{N-j}(Y) f(\epsilon_i) d\epsilon_i d\gamma\right.}_{dG}$$
(A.83)

$$= \int_{\gamma} \beta_2 \left[\left(\left(\frac{\partial \left[u \left(U_P + y \right) - u \left(U_S + y \right) \right]}{\partial e} \right) f(\epsilon_i + \gamma) \right) \right]_{-\infty}^{\infty} d\gamma - \int_{\gamma} \int \beta_2 \left(\left(\frac{\partial \left[u \left(U_P + y \right) - u \left(U_S + y \right) \right]}{\partial e} \right) \right) f(\epsilon_i + \gamma) d\epsilon_i d\gamma$$
(A.84)

$$EU1 = -\int_{\gamma} \int \beta_2 \left(\left(\frac{\partial \left[u \left(U_P + y \right) - u \left(U_S + y \right) \right]}{\partial e} \right) \right) f(\epsilon_i + \gamma) d\epsilon_i d\gamma$$
(A.85)

$$EU2 = \left(\int_{\gamma} \int_{\epsilon_i} u \left(U_S + y\right) f(\epsilon_i + \gamma) d\epsilon_i d\gamma\right)$$
(A.86)

$$EU3 = -\int_{\gamma} \int \beta_1 \left(\frac{\partial \left[u \left(U_S + y \right) - u \left(U_L + y \right) \right]}{\partial e} \right) f(\epsilon_i + \gamma) d\epsilon_i d\gamma$$
(A.87)

$$EU = \beta_2 \left[u \left(U_P + y \right) - u \left(U_S + y \right) \right] + u \left(U_S + y \right) - \beta_1 \left[u \left(U_S + y \right) - u \left(U_L + y \right) \right]$$
(IR)

(A.88)

After First Order Taylor Approximation

$$EU = \beta_2 \left[(U_P - U_S) \right] + U_S + w + be - rb^2 \sigma^2 - \frac{e^2}{2} - \beta_1 \left[(U_S - U_L) \right] \ge \overline{U}$$
(A.89)

$$w + be = \overline{U} + rb^2\sigma^2 + \frac{e^2}{2} + \beta_1 \left[(U_S - U_L) \right] - \beta_2 \left[(U_P - U_S) \right] - U_S$$
(A.90)

$$E[\Pi^*] = \max_{(\beta_1, \beta_2, b, w_1)} E[\pi = N\widetilde{x}(e) - N\widetilde{y} - N(\beta_2 + \beta_1)c]$$
(A.91)

or
$$\max_{(\beta_1,\beta_2,b,w_1)} E[\pi = N\left(\widetilde{x}(e) - \widetilde{y} - (\beta_2 + \beta_1)c\right)]$$
(A.92)

$$\max_{(\beta_1,\beta_2,b,w_1)} E[\pi] = N \left(E[\widetilde{x}(e)] - E[\widetilde{y}] - (\beta_2 + \beta_1)c \right)$$
(A.93)

$$\max_{(\beta_1,\beta_2,b,w_1)} E[\pi] = e^* - E[\tilde{y}] - (\beta_2 + \beta_1)c$$
(A.94)

$$\max_{(\beta_1,\beta_2,b)} E[\pi] = e^* - \left(\overline{U} + rb^2\sigma^2 + \frac{(e^*)^2}{2} + \beta_1\left[(U_S - U_L)\right] - \beta_2\left[(U_P - U_S)\right] - U_S\right) - (\beta_2 + \beta_1)c$$

where,
$$e^* = \frac{N(1-\beta_2)\beta_2}{(N+1)d} (U_P - U_S) + \frac{N(1-\beta_2)\beta_2}{(N+1)d} (U_S - U_L) + b$$
 (A.96)

$$\frac{\partial^2 \pi}{\partial b \partial \beta_2} = \frac{N(1 - 2\beta_2)}{(N+1)d} \left(U_P - U_S \right) > 0 \Leftrightarrow \left(U_P - U_S \right) - c + E > 0 \tag{A.97}$$

$$\frac{\partial^2 \pi}{\partial b \partial \beta_1} = \frac{N(1 - 2\beta_1)}{(N+1)d} \left(U_S - U_L \right) > 0 \Leftrightarrow \left(U_S - U_L \right) + c < 0 \tag{A.98}$$

$$\frac{\partial^2 \pi}{\partial \beta_1 \partial \beta_2} = -\frac{N^2 (1 - 2\beta_2)(1 - 2\beta_1)}{(N+1)d} \left(U_P - U_S \right) \left(U_S - U_L \right) > 0 > 0 \Leftrightarrow \left(U_P - U_S \right) - c > 0$$

(A.99)

An explicit solution for the firm's objective function is non tractable because of which we solve it implicitly. However, note that there exists a unique maximum since the objective function is a sum of strictly concave and linear functions of all the decision variables, for sufficiently small values of r. We assume that the constraints $(\beta_1, \beta_2, b) \epsilon R_{[0,1]}$ and $\beta_1 + \beta_2 \leq 1$ hold at the equilibrium. Let $b = b^*(\alpha), \beta_2 = \beta_2^*(\alpha), \beta_1 = \beta_1^*(\alpha)$ be the unique solution for the above objective function, where $\alpha = (c, r, \sigma_{\varepsilon}, \sigma_{\gamma}, \Delta U_{PS} \stackrel{Def}{=} U_P - U_S, \Delta U_{SL} \stackrel{Def}{=} U_S - U_L, N).$

Since, there exists a unique maximum for the problem we know that $\begin{bmatrix} \pi_{bb} & \pi_{b\beta_2} & \pi_{b\beta_1} \\ \pi_{\beta_2b} & \pi_{\beta_2\beta_2} & \pi_{\beta_2\beta_1} \\ \pi_{\beta_1b} & \pi_{\beta_1\beta_2} & \pi_{\beta_1\beta_1} \end{bmatrix}$ is

Negative Definite

Proof. A Check for accuracy of Taylor Approximation.

Taylor Approximation

$$R_2(r) = \frac{1}{2} \int_0^r (r-t)^2 f^3(t) dt$$
 (A.100)

where
$$f(t) = -\exp\left(-t\left(U_K + w + be - \frac{t}{2}b^2\sigma^2 - \frac{e^2}{2}\right)\right)$$
 (A.101)

$$R_2(r) = \frac{1}{2} \left(2 - 2 \exp\left(-r \left(A - rB\right)\right) - 2Ar + \left(A^2 + 2B\right)r^2\right)$$
(A.102)

Where,
$$A = U_K + w + be - \frac{e^2}{2}, B = b^2 \sigma^2$$
 (A.103)

In our analysis $R_2(r)$ is the extent of approximation that we are making with respect to the utility

function. In the following graph we plot the proportion of approximation as compared to the value of the utility function (in the y-axis) as a function of the risk aversion (on x axis). As the graph suggest, the extent of approximation increases as risk aversion increases. However, for risk aversion less than.2 the approximation is only of around 5% of the value of the utility function. These similation results were dervied assuming A = 1 and B = 1.

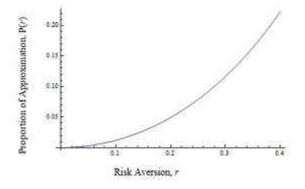


Figure A.1: Extent of Approximation in Taylor Approximation

Where,
$$P(r) = \frac{R_2(r)}{|-r(A - Br)|}$$
, $A = 1$ and $B = 1$

Proof. Poposition 2.7 and 2.8 - Whether performance pay and FR policies are complements or substitutes

To ascertain whether b, β_2 and β_1 are complements or substitutes, we evaluate signs for the following cross derivatives.

Relationship between b and β_2

$$\frac{\partial^2 \pi}{\partial b \partial \beta_2} = -\frac{N(1-2\beta_2)}{(N+1)d} \Delta U_{PS}$$
(A.104)

$$\frac{\partial^2 \pi}{\partial b \partial \beta_2} = -\frac{N(1-2\beta_2)}{(N+1)d} \Delta U_{PS}$$
(A.105)

$$\frac{\partial^2 \pi}{\partial b \partial \beta_2} > 0 \Leftrightarrow \beta_2 \epsilon \left(\frac{1}{2}, 1\right] \text{ and } \frac{\partial^2 \pi}{\partial b \beta_2} \le 0 \Leftrightarrow \beta_2 \epsilon \left[0, \frac{1}{2}\right]$$

Relationship between b and β_1

$$\frac{\partial^2 \pi}{\partial b \partial \beta_1} = -\frac{N(1-2\beta_1)}{(N+1)d} \Delta U_{SL}$$

$$\frac{\partial^2 \pi}{\partial b \partial \beta_1} > 0 \Leftrightarrow \beta_1 \epsilon \left(\frac{1}{2}, 1\right] \text{ and } \frac{\partial^2 \pi}{\partial b \partial \beta_1} \le 0 \Leftrightarrow \beta_1 \epsilon \left[0, \frac{1}{2}\right]$$
(A.106)

Relationship between β_2 and β_1

$$\frac{\partial^2 \pi}{\partial \beta_1 \partial \beta_2} = -\left(\frac{N^2 (1 - 2\beta_2)(1 - 2\beta_1)}{(N+1)^2 d^2} \Delta U_{PS} \Delta U_{SL}\right)$$

$$\frac{\partial^2 \pi}{\partial b \partial \beta_1} > 0 \Leftrightarrow \beta_1 \epsilon \left(\frac{1}{2}, 1\right] \text{ and } \frac{\partial^2 \pi}{\partial b \partial \beta_1} \le 0 \Leftrightarrow \beta_1 \epsilon \left[0, \frac{1}{2}\right]$$
(A.107)

If the above cross derivatives A.104,A.105 and A.107 take positive sign then the two decision variables are complements, else they are substitutes. To sign the above dervatives the only information that is required is whether $\beta_2 > \frac{1}{2}$ or $\beta_1 > \frac{1}{2}$. In order to ascertain whether these inequalities are satisfied, we put constraints on the objective function and derive the underlying exogenous conditions satisfying the inequalities. To do so we define the Lagrangian as,

$$L(\beta_1, \beta_2, b, \mu_1, \mu_2, \alpha) = E[Ne - N\left(rb^2\sigma^2 + \frac{e^2}{2} - \beta_2\Delta U_{PS} + \beta_1\Delta U_{SL}\right) - N(\beta_2 + \beta_1)c] + \mu_1\left(\frac{1}{2} - \beta_2\right) + \mu_2\left(\frac{1}{2} - \beta_1\right)$$
(A.108)

The Kuhn Tucker Conditions for the above Lagrangian are given as,

$$b \ge 0, \frac{\partial L}{\partial b} \le 0, b\frac{\partial L}{\partial b} = 0, \beta_2 \ge 0, \frac{\partial L}{\partial \beta_2} \le 0, \beta_2 \frac{\partial L}{\partial \beta_2} = 0, \beta_1 \ge 0, \frac{\partial L}{\partial \beta_1} \le 0, \beta_1 \frac{\partial L}{\partial \beta_1} = 0$$
$$\mu_1 \ge 0, \frac{\partial L}{\partial \mu_1} \ge 0, \mu_1 \frac{\partial L}{\partial \mu_1} = 0, \mu_2 \ge 0, \frac{\partial L}{\partial \mu_2} \ge 0, \mu_2 \frac{\partial L}{\partial \mu_2} = 0$$

Since we study the model only in the region $(\beta_1, \beta_2, b) \epsilon R_{++}$, the first three sets of equations can be reduced to $\frac{\partial L}{\partial b} = 0$, $\frac{\partial L}{\partial \beta_2} = 0$, $\frac{\partial L}{\partial \beta_1} = 0$.Now to study the conditions when only $\beta_2 < \frac{1}{2}$ we assume $\mu_2 = 0$ i.e. β_1 to be unconstrained and the Lagrangian to be constrained i.e. $\beta_2 = \frac{1}{2}$ and $\mu_1 > 0$. With these we solve the following first order conditions,

$$\frac{\partial L}{\partial b} = 0, \beta_2 = \frac{1}{2}, \frac{\partial L}{\partial \beta_1} = 0, \mu_1 > 0, \frac{\partial L}{\partial \mu_1} = 0$$

Solving the above FOCs we get $\mu_1 = \Delta U_{PS} - c$

As we have assumed that $\beta_2 = \frac{1}{2}$ therefore the constraint is binding and $\mu_1 > 0$, else there would be a contradiction. $\mu_1 > 0 \Leftrightarrow U_{PS} - c > 0$.

Hence,
$$\beta_2 > \frac{1}{2} \Leftrightarrow \Delta U_{PS} - c > 0$$
 and $\beta_2 \le \frac{1}{2} \Leftrightarrow \Delta U_{PS} - c \le 0$

By the same logic, to study the conditions when only $\beta_1 < \frac{1}{2}$ we assume $\mu_1 = 0$ i.e. β_2 to be unconstrained, and the Lagrangian to be constrained i.e. $\beta_1 = \frac{1}{2}$ and $\mu_2 > 0$. With these assumptions we solve the following first order conditions,

$$\frac{\partial L}{\partial b} = 0, \beta_1 = \frac{1}{2}, \frac{\partial L}{\partial \beta_2} = 0, \mu_2 > 0, \frac{\partial L}{\partial \mu_2} = 0$$

Solving the above FOCs we get $\mu_2 = -\Delta U_{SL} - c$

As we have assumed that $\beta_1 = \frac{1}{2}$ therefore the constraint is binding and $\mu_2 > 0$, else there would be a contradiction. $\mu_2 > 0 \Leftrightarrow -\Delta U_{SL} - c > 0$. However, $-\Delta U_{SL} - c < 0$ since $\Delta U_{SL} > 0$ and $c > 0, \mu_2 \not\geq 0$. Therefore, the constraint $\beta_1 < \frac{1}{2}$ is never binding. On basis of the above conditions we can sign the cross derivatives as,

$$\frac{\partial^2 \pi}{\partial \beta_2 \partial b} = -\frac{N^2 (1 - 2\beta_2)}{(N+1)d} \Delta U_{PS} > 0 \Leftrightarrow \beta_2 \ge \frac{1}{2} \Leftrightarrow U_{PS} - c > 0 \tag{A.109}$$

$$\frac{\partial^2 \pi}{\partial \beta_2 \partial b} = -\frac{N^2 (1 - 2\beta_2)}{(N+1)d} \Delta U_{PS} \le 0 \Leftrightarrow \beta_2 \le \frac{1}{2} \Leftrightarrow U_{PS} - c \le 0$$
(A.110)

$$\frac{\partial^2 \pi}{\partial \beta_1 \partial b} = -\frac{N^2 (1 - 2\beta_1)}{(N+1)d} \Delta U_{SL} < 0 \Leftrightarrow \beta_1 \le \frac{1}{2} \forall \alpha \epsilon \mathbf{R}_+$$
(A.111)

(A.112)

$$\frac{\partial^2 \pi}{\partial \beta_2 \partial \beta_1} = -\frac{N^2 (1 - 2\beta_2) (1 - 2\beta_1)}{(N+1)d} \Delta U_{PS} \Delta U_{SL} > 0 \Leftrightarrow \beta_2 \ge \frac{1}{2} \& \beta_1 \le \frac{1}{2} \Leftrightarrow \Delta U_{PS} - c > 0.$$

$$\frac{\partial^2 \pi}{\partial \beta_2 \partial \beta_1} = -\frac{N^2 (1 - 2\beta_2) (1 - 2\beta_1)}{(N+1)d} \Delta U_{PS} \Delta U_{SL} \le 0 \Leftrightarrow \beta_2 \le \frac{1}{2} \& \beta_1 \le \frac{1}{2} \Leftrightarrow \Delta U_{PS} - c \le 0.$$
(A.113)

Proof. Proposition 2.9 and 2.10. The comparative statics for the problem are derived below,

The comparative statics for the problem can be derived as below,

$$\left| \begin{array}{cccc}
\pi_{bb} & \pi_{b\beta_{2}} & \pi_{b\beta_{1}} \\
\pi_{\beta_{2}b} & \pi_{\beta_{2}\beta_{2}} & \pi_{\beta_{2}\beta_{1}} \\
\pi_{\beta_{1}b} & \pi_{\beta_{1}\beta_{2}} & \pi_{\beta_{1}\beta_{1}}
\end{array} \right| \left| \begin{array}{c}
\frac{\partial b^{*}}{\partial \alpha} \\
\frac{\partial \beta_{2}^{*}}{\partial \alpha} \\
\frac{\partial \beta_{1}^{*}}{\partial \alpha}
\end{array} \right| = \left[\begin{array}{c}
-\pi_{b\alpha} \\
-\pi_{\beta_{2}\alpha} \\
-\pi_{\beta_{1}\alpha}
\end{array} \right]$$
(A.114)
$$\frac{\partial b}{\partial \alpha} = \frac{1}{H} \left| \begin{array}{c}
-\pi_{b\alpha} & \pi_{b\beta_{2}} & \pi_{b\beta_{1}} \\
-\pi_{\beta_{2}\alpha} & \pi_{\beta_{2}\beta_{2}} & \pi_{\beta_{2}\beta_{1}} \\
-\pi_{\beta_{1}\alpha} & \pi_{\beta_{1}\beta_{2}} & \pi_{\beta_{1}\beta_{1}}
\end{array} \right| = -\frac{\pi_{b\alpha}D1 - \pi_{\beta_{2}\alpha}D2 + \pi_{\beta_{1}\alpha}D3}{H} \qquad (A.115)$$

where, $D1 = \pi_{\beta_2\beta_2}\pi_{\beta_1\beta_1} - (\pi_{\beta_2\beta_1})^2$, $D2 = \pi_{b\beta_2}\pi_{\beta_1\beta_1} - \pi_{\beta_1\beta_2}\pi_{b\beta_1}$, $D3 = \pi_{b\beta_2}\pi_{\beta_2\beta_1} - \pi_{\beta_2\beta_2}\pi_{b\beta_1}$

$$\frac{\partial \beta_2}{\partial \alpha} = \frac{1}{H} \begin{vmatrix} \pi_{bb} & -\pi_{b\alpha} & \pi_{b\beta_1} \\ \pi_{\beta_2 b} & -\pi_{\beta_2 \alpha} & \pi_{\beta_2 \beta_1} \\ \pi_{\beta_1 b} & -\pi_{\beta_1 \alpha} & \pi_{\beta_1 \beta_1} \end{vmatrix} = \frac{\pi_{b\alpha} D' 1 - \pi_{\beta_2 \alpha} D' 2 + \pi_{\beta_1 \alpha} D' 3}{H}$$
(A.116)

where, $D'1 = \pi_{\beta_2 b} \pi_{\beta_1 \beta_1} - \pi_{\beta_2 \beta_1} \pi_{\beta_1 b}, D'2 = \pi_{\beta_1 \beta_1} \pi_{bb} - (\pi_{\beta_1 b})^2, D'3 = \pi_{bb} \pi_{\beta_2 \beta_1} - \pi_{\beta_2 b} \pi_{b\beta_1}$

$$\frac{\partial \beta_1}{\partial \alpha} = \frac{1}{H} \begin{vmatrix} \pi_{bb} & \pi_{b\beta_2} & -\pi_{b\alpha} \\ \pi_{\beta_2 b} & \pi_{\beta_2 \beta_2} & -\pi_{\beta_2 \alpha} \\ \pi_{\beta_1 b} & \pi_{\beta_1 \beta_2} & -\pi_{\beta_1 \alpha} \end{vmatrix} = -\frac{\pi_{b\alpha} D'' 1 - \pi_{\beta_2 \alpha} D'' 2 + \pi_{\beta_1 \alpha} D'' 3}{H}$$
(A.117)

where,
$$D''1 = \pi_{\beta_{2}b}\pi_{\beta_{1}\beta_{2}} - \pi_{\beta_{1}b}\pi_{\beta_{2}\beta_{2}}, D''2 = \pi_{bb}\pi_{\beta_{1}\beta_{2}} - \pi_{b\beta_{2}}\pi_{\beta_{1}b}, D''3 = \pi_{\beta_{2}\beta_{2}}\pi_{bb} - (\pi_{\beta_{2}b})^{2}$$

Since $\begin{bmatrix} \pi_{bb} & \pi_{b\beta_{2}} & \pi_{b\beta_{1}} \\ \pi_{\beta_{2}b} & \pi_{\beta_{2}\beta_{2}} & \pi_{\beta_{2}\beta_{1}} \\ \pi_{\beta_{1}b} & \pi_{\beta_{1}\beta_{2}} & \pi_{\beta_{1}\beta_{1}} \end{bmatrix}$ is Negative Definite, $|H| = \begin{vmatrix} \pi_{\beta_{1}\beta_{1}} & \pi_{\beta_{1}\beta_{2}} & \pi_{\beta_{1}b} \\ \pi_{\beta_{2}\beta_{1}} & \pi_{\beta_{2}\beta_{2}} & \pi_{\beta_{2}b} \\ \pi_{b\beta_{1}} & \pi_{b\beta_{2}} & \pi_{bb} \end{vmatrix} < 0$

and D1, D'2, D''3 > 0

$$\frac{\partial b}{\partial \sigma_{\varepsilon}} = \frac{-1}{H} \left(\pi_{b\sigma_{\varepsilon}} D1 - \pi_{\beta_{1}\sigma_{\varepsilon}} D2 + \pi_{\beta_{1}\sigma_{\varepsilon}} D3 \right)$$
(A.118)

and hence,
$$sign\left(\frac{\partial b}{\partial \sigma_{\varepsilon}}\right) = sign\left(\pi_{b\sigma_{\varepsilon}}D1 - \pi_{\beta_{1}\sigma_{\varepsilon}}D2 + \pi_{\beta_{1}\sigma_{\varepsilon}}D3\right)$$
 (A.119)

The term, $\pi_{b\sigma_{\varepsilon}}D1 - \pi_{\beta_1\sigma_{\varepsilon}}D2 + \pi_{\beta_1\sigma_{\varepsilon}}D3$, can be simplified and written as,

$$=\frac{1}{k^{2}(1+N)^{2}\sigma_{\epsilon}^{3}}2N^{5}U_{PS}U_{SL} (1-e))rb\sigma_{\epsilon}^{2}\left(8(e-b)-4(1-e)-\left(\frac{N\Delta U_{PS}}{(1+N)d}+\frac{N\Delta U_{SL}}{(1+N)d}\right)\right)$$
(A.120)

As
$$\frac{1}{k^2(1+N)^2\sigma_{\epsilon}^3}2N^5\Delta U_{PS}\Delta U_{SL} \ (1-e)) > 0, \ \frac{\partial b}{\partial\sigma_{\epsilon}} > 0$$

iff $2(e-b) - (1-e) > \frac{1}{4}\left(\frac{N\Delta U_{PS}}{(1+N)d} + \frac{N\Delta U_{SL}}{(1+N)d}\right)$ (A.121)

Therefore,
$$\frac{\partial b}{\partial \sigma_{\varepsilon}} > \inf 2\left(e - b\right) > \frac{1}{2}rb\left(\sigma_{\varepsilon}^{2} + \sigma_{\gamma}^{2}\right) + \frac{1}{4}\left(\frac{N\Delta U_{PS}}{(1+N)d} + \frac{N\Delta U_{SL}}{(1+N)d}\right)$$
(A.122)
or
$$2\left(\frac{N(1-\beta_{2})\beta_{2}\Delta U_{PS}}{(1+N)d} + \frac{N(1-\beta_{1})\beta_{1}\Delta U_{SL}}{(1+N)d}\right)$$
$$> \frac{1}{2}rb\left(\sigma_{\varepsilon}^{2} + \sigma_{\gamma}^{2}\right) + \frac{1}{4}\left(\frac{N\Delta U_{PS}}{(1+N)d} + \frac{N\Delta U_{SL}}{(1+N)d}\right)$$
(A.123)

The above conditions for increase in use of performance pay with individual specific uncertainty are more likely fulfilled if costs of FR are lower and if performance pay and promotion are complements i.e. if costs of FR are lower than the expected utility that employees derive from promotion. A lower cost of FR means that the firm uses more of FR and therefore, FR is more effective, i.e. $\frac{\partial}{\partial c}(e-b) < 0$ if $(U_P - U_S) - c > 0$. In addition, a lower cost of FR also leads to decrease in use of performance pay because of substitution between FR and performance pay, and this also decreases the risk premium associated with performance pay, i.e. $\frac{\partial}{\partial c} \left(rb \left(\sigma_{\varepsilon}^2 + \sigma_{\gamma}^2 \right) \right) = r \left(\sigma_{\varepsilon}^2 + \sigma_{\gamma}^2 \right) \frac{\partial b}{\partial c} > 0$. For a proof of these statements we differentiate the above inequality with respect to c on both the sides and show that the inequality is more likely to hold if costs, c, are lower. First we start from the right side.

Note that
$$\frac{\partial}{\partial c} \left(\frac{1}{8} \frac{N \Delta U_{PS}}{(1+N)d} + \frac{N \Delta U_{SL}}{(1+N)d} \right) = 0$$
 (A.124)

and hence,
$$\frac{\partial}{\partial c} \left(rb \left(\sigma_{\varepsilon}^2 + \sigma_{\gamma}^2 \right) + \frac{1}{8} \left(\frac{N \Delta U_{PS}}{(1+N)d} + \frac{N \Delta U_{SL}}{(1+N)d} \right) \right) = r \left(\sigma_{\varepsilon}^2 + \sigma_{\gamma}^2 \right) \frac{\partial b}{\partial c}.$$
 (A.125)

$$\frac{\partial b}{\partial c} = -\frac{\pi_{bc}D1 - \pi_{\beta_2 c}D2 + \pi_{\beta_1 c}D3}{H}$$
(A.126)

$$= \frac{-1}{H} \left(\frac{4N^5 \left(1 - \beta_1 - \beta_2\right) \Delta U_{PS} \triangle U_{SL} \left(1 - e^*\right)}{d^2 (1 + N)^2} \right) > 0 \tag{A.127}$$

because $H < 0, (1 - \beta_1 - \beta_2) > 0$ and $(1 - e^*) > 0$.

Therefore,
$$\frac{\partial}{\partial c} \left(rb \left(\sigma_{\varepsilon}^2 + \sigma_{\gamma}^2 \right) + \frac{1}{8} \left(\frac{N \Delta U_{PS}}{(1+N)d} + \frac{N \Delta U_{SL}}{(1+N)d} \right) \right) > 0$$
 (A.128)

Now, consider the left side of the inequality.

$$\frac{\partial}{\partial c} (e - b) = \frac{N \Delta U_{PS}}{(1 + N)d} (1 - 2\beta_2) \frac{\partial \beta_2}{\partial c} + \frac{N \Delta U_{SL}}{(1 + N)d} (1 - 2\beta_1) \frac{\partial \beta_1}{\partial c} - r \left(\sigma_{\varepsilon}^2 + \sigma_{\gamma}^2\right) \frac{\partial b}{\partial c} < 0$$
(A.129)
because $\frac{\partial b}{\partial c} > 0$ as shown above,

$$\frac{\partial \beta_1}{\partial c} = \frac{-1}{H} \left[-\pi_{\beta_2 c} D_2 + \pi_{\beta_1 c} D_3 \right]$$
(A.130)

$$= \frac{-1}{H} [D_2 - D_3] < 0, D_2 < 0, D_3 > 0 \text{ if } \beta_2 > \frac{1}{2},$$

$$\frac{\partial \beta_2}{\partial c} = \frac{1}{H} \left[-\pi_{\beta_2 c} D'_2 + \pi_{\beta_1 c} D'_3 \right]$$

$$= \frac{1}{H} \left[D'_2 - D'_3 \right] > 0, D'_2 > 0, D'_3 < 0 \text{ if } \beta_2 > \frac{1}{2}.$$

(A.131)

Since by Proposition 2.7, $\beta_2 > \frac{1}{2} \Leftrightarrow (U_P - U_S) - c > 0, \frac{\partial}{\partial c} (e - b) < 0$ if $(U_P - U_S) - c > 0$

$$\frac{\partial b}{\partial \sigma_{\gamma}} = \frac{-1}{H} \left(\pi_{b\sigma_{\gamma}} D1 - \pi_{\beta_{2}\sigma_{\gamma}} D2 + \pi_{\beta_{1}\sigma_{\gamma}} D3 \right) = \frac{-1}{H} \left(\pi_{b\sigma_{\gamma}} D1 \right)$$
(A.132)
as $\pi_{\beta_{2}\sigma_{\gamma}} = 0$ and $\pi_{\beta_{1}\sigma_{\gamma}} = 0$.
$$\frac{\partial b}{\partial \sigma_{\gamma}} < 0 \text{ because } \pi_{b\sigma_{\gamma}} < 0, H < 0 \text{ and } D1 > 0.$$

APPENDIX B: PROOFS FOR THE RESULTS IN CHAPTER 3

Proof. Lemma 3.1.

Consider the optimal contract for a sales rep i at position K. Since the contracts for each of the two employees is derived in the same manner and the proof is only for period 2 and we suppress the subscripts i, K and t = 2 in the effort and sales variables. The firm's second period problem can be stated as

$$\max_{m,w} E[\delta x(e) - (w + m\delta x(e))]$$
(B.1)

$$e^* \epsilon \arg \max E[(w + m\delta x(e))] - c(e) - \frac{r}{2} Var[\delta m x(e)]$$
 (IC) (B.2)

$$E[(w + m\delta x(e))] - c(e) - \frac{r}{2}Var[\delta mx(e)] \ge U_K \qquad (IR)$$
(B.3)

Solving the IC constraint in the second period problem we get,

$$e^* \epsilon \arg \max E[(w + m\delta x(e))] - c(e) - \frac{r}{2} Var[\delta m x(e)]$$
 (B.4)

$$= \arg\max w + m\delta s(e) - c(e) - \frac{r}{2} \left(\delta m\right)^2 \left(\sigma_{\gamma}^2 + \sigma_{\epsilon}^2\right)$$
(B.5)

$$FOC: m\delta s'(e) = c'(e)$$
, and hence, $e^* = \frac{\delta m}{a}$. (B.6)

Also, from the IR constraint we have,

$$E[(w + m\delta x(e))] - c(e) - \frac{r}{2} Var[\delta m x(e)] \ge U_K$$
(B.7)

Since IR constraint binds,

$$w + m\delta s(e) = c(e) + \frac{r}{2} \left(\delta m\right)^2 \left(\sigma_{\gamma}^2 + \sigma_{\epsilon}^2\right) + U_k \tag{B.8}$$

Substituting the value of $w + m\delta s(e)$ in firm's problem, we get,

$$\max_{m} E[\delta x(e)] - c(e) - \frac{r}{2} \left(\delta m\right)^2 \left(\sigma_{\gamma}^2 + \sigma_{\epsilon}^2\right) - U_k \tag{B.9}$$

but we know that,
$$c(e) = a \frac{(e^*)^2}{2} = a \frac{(\delta m)^2}{2a^2}$$
 (B.10)

$$\max_{m} \delta\left(\frac{\delta m}{a}\right) - a \frac{\left(\delta m\right)^{2}}{2a^{2}} - \frac{r}{2} \left(\delta m\right)^{2} \left(\sigma_{\gamma}^{2} + \sigma_{\epsilon}^{2}\right) - U_{k}$$
(B.11)

$$FOC: 1 - m - mar\left(\sigma_{\gamma}^{2} + \sigma_{\epsilon}^{2}\right) = 0 \text{ or } m = \frac{1}{1 + ar\left(\sigma_{\gamma}^{2} + \sigma_{\epsilon}^{2}\right)}$$
(B.12)

Note that the marginal profitability of effort does not affect the optimal slope of the contract. Substituting the value of m on the RHS of eq. $w_2 + m\delta s(e) = c(e) + \frac{r}{2} (\delta m)^2 (\sigma_{\gamma}^2 + \sigma_{\epsilon}^2) + U_k$, we get,

$$E[(w + m\delta x(e))] = a\frac{(\delta m)^2}{2a^2} + \frac{r}{2}(\delta m)^2(\sigma_{\gamma}^2 + \sigma_{\epsilon}^2) + U_k$$
(B.13)

$$= \frac{\left(\delta m\right)^2}{2a} \left(1 + ar\left(\sigma_{\gamma}^2 + \sigma_{\epsilon}^2\right)\right) + U_k = \frac{\delta^2 m}{2a} + U_k \tag{B.14}$$

$$E[\delta x(e)] - E[(w + m\delta x(e))] = \frac{\delta^2 m}{a} - \frac{\delta^2 m}{2a} - U_k$$
(B.15)

$$= \frac{\delta^2 m}{2a} - U_k, Where \ m = \frac{1}{1 + ar\left(\sigma_\gamma^2 + \sigma_\epsilon^2\right)}.$$
(B.16)

Proof. Table 3.1 Expressions.

As per Lemma 3.1 the firm's profit generated by the i^{th} employee at k^{th} position is given as $\frac{1}{2a_i} \left(\frac{\delta_k^2}{1+a_i r_i (\sigma_{\varepsilon}^2 + \sigma_{\gamma}^2)} \right)_i - U_k$ when only linear contract is used, where $k = \{S, P\}$ and U_k is the net utility that the employee earns. Therefore, the total profit to be shared between the employee and the firm at position k is given as $\frac{1}{2a_i} \left(\frac{\delta_k^2}{1+a_i r_i (\sigma_{\varepsilon}^2 + \sigma_{\gamma}^2)} \right) - U_k + U_k = \frac{1}{2a_i} \left(\frac{\delta_i^2}{1+a_i r_i (\sigma_{\varepsilon}^2 + \sigma_{\gamma}^2)} \right)$. In the sales manager's position the total profit will be $\frac{1}{2a_i}\left(\frac{\delta_P^2}{1+a_ir_i(\sigma_\gamma^2)}\right)$ since $\delta = \delta_P$ and $\sigma_{\varepsilon}^2 = 0$ in such a case. In the sales rep's position the profit will be $\frac{1}{2a_i}\left(\frac{\delta_S^2}{1+a_ir_i(\sigma_{\varepsilon}^2+\sigma_\gamma^2)}\right)$ since $\delta = \delta_S$ and $\sigma_{\varepsilon}^2 > 0$ in such a case. In the current case an employee's wage at a position will be $\left(\frac{\theta}{1+\theta}\right)$ times the total profit generated by the employee at that position and the firm's profit from the employee at that position will be $\left(\frac{1}{1+\theta}\right)$ times the total profit generated by the total profit generated by the employee at that position and the firm's profit from the employee at that position will be $\left(\frac{1}{1+\theta}\right)$ times the total profit generated by the employee. Where θ represents the elasticity of labor supply. Accordingly profits and wages for sales rep and sales manager's positions are determined in Table 3.1. Effort at the rep and manager's position are determined on basis of eq. (B.6) and (B.12) in Appendix proof of Lemma 3.1.

Proof. Proposition 3.1. ■

In solving the first period game for the employees we denote the first period effort level for employee *i* by e_i , where $i = \{1, 2\}$. Performance is measured based on the sales done by the employees in the first period which is a linear function of employee's effort and other random factors that are normally distributed. Hence, the sales done by the employee *i* is given as $x_i = e_i + \varepsilon_i + \gamma$ where $\varepsilon_i \sim NIID(0, \sigma_{\epsilon}^2)$ and $\gamma \sim N(0, \sigma_{\gamma}^2)$. The firm promotes the employee with higher sales performance in the first period. The period 1 model is similar to Lazear and Rosen (1981).

In the following proof we first find the first period effort for both employees and then compare the effort levels. The conditions under which the employee with higher level of first period effort, i.e. the employee with higher probability of getting promoted, is less desirable for the firm in period

2, where desirability conditions are given in Lemma 3.2, gives us conditions for adverse selection stated in Proposition 3.1.

In the given case, employee effort for employee 1 is a solution to the following utility maximization problem.

$$e_1^* = \arg\max p_1 U_{P1} + (1 - p_1) U_{S1} - \frac{1}{2} r_1 (U_{P1} - U_{S1})^2 (1 - p_1) p_1 - \frac{a_1 e_1^2}{2}$$
(B.17)

Where,
$$p_1 = G(x_1 > x_2) = G(e_1 + \varepsilon_1 + \gamma \ge e_2 + \varepsilon_2 + \gamma)$$
 (B.18)

$$= G(e_1 - e_2 \ge \varepsilon_2 - \varepsilon_1) = G(e_1 - e_2 \ge \zeta),$$
(B.19)

where
$$\zeta \sim N\left(0, 2\sigma_{\epsilon}^{2}\right)$$
 since $\varepsilon_{i} \sim NIID\left(0, \sigma_{\epsilon}^{2}\right), U_{P1} = \frac{\theta}{1+\theta} \frac{1}{2a_{1}} \left(\frac{\delta_{P}^{2}}{1+a_{1}r_{1}\sigma_{\gamma}^{2}}\right),$ (B.20)

$$U_{S1} = \frac{\theta}{1+\theta} \frac{1}{2a_1} \left(\frac{\delta_S^2}{1+a_1 r_1 \left(\sigma_\gamma^2 + \sigma_\varepsilon^2\right)} \right), \tag{B.21}$$

$$\frac{1}{2}r_1 \left(U_{P1} - U_{S1}\right)^2 (1 - p_1)p_1 \text{ is the risk premium for employee 1 and}$$
$$\frac{a_1 e_1^2}{2} \text{ is the disutility or cost of effort for employee 1.}$$

Hereafter $G(e_1 - e_2 > \zeta)$ is denoted by $G(e_1 - e_2)$ and the pdf is denoted by $g(e_1 - e_2)$.

$$FOC_{1}: \frac{\partial p_{1}}{\partial e_{1}} \left\{ U_{P1} - U_{S1} - \frac{1}{2} r_{1} \left(U_{P1} - U_{S1} \right)^{2} (1 - p_{1}) p_{1} \right\} - a_{1} e_{1} = 0$$
(B.22)

$$\frac{1}{a_1}\frac{\partial p_1}{\partial e_1}\left\{U_{P1} - U_{S1} - \frac{1}{2}r_1\left(U_{P1} - U_{S1}\right)^2(1-p_1)p_1\right\} = e_1$$
(B.23)

$$\frac{\partial p_1}{\partial e_1} = \frac{\partial G(e_1 - e_2)}{\partial e_1} = \frac{\partial G(e_1 - e_2)}{\partial (e_1 - e_2)} \frac{\partial (e_1 - e_2)}{\partial e_1} = g(\Delta e), \text{ Where } \Delta e = e_1 - e_2 \qquad (B.24)$$

$$\frac{1}{a_1}g(\Delta e)\left\{U_{P1} - U_{S1} - \frac{1}{2}r_1\left(U_{P1} - U_{S1}\right)^2\left(1 - p_1\right)p_1\right\} = e_1$$
(B.25)

The effort for employee 2 is similarly derived by solving the optimization problem

$$e_2^* = \arg\max p_2 U_{P2} + (1 - p_2) U_{S2} - \frac{1}{2} r_2 (U_{P2} - U_{S2})^2 (1 - p_2) p_2 - \frac{a_2 e_2^2}{2}$$
(B.26)

Where
$$p_2 = 1 - p_1$$
, $\frac{\partial p_2}{\partial e_2} = -\frac{\partial p_1}{\partial e_2} = -(-g(\Delta e)) = g(\Delta e)$ and $r_2 = 0$ (B.27)

Hence,
$$FOC_2 : \frac{g(\Delta e)}{a_2} \{ U_{P2} - U_{S2} \} = e_2$$
 (B.28)

We assume that a pure strategy equilibrium exists for the above game. However, as noted by Lazear and Rosen (1981, p845), such an equilibrium only exists if σ^2 is sufficiently large. Assuming a pure strategy Nash equilibrium at (e_1^*, e_2^*) , we have the following second order conditions,

$$SOC_{1} : \frac{1}{a_{1}}g'(\Delta e) \left\{ U_{P1} - U_{S1} - \frac{1}{2}r_{1}(U_{P1} - U_{S1})^{2}(1 - 2p_{1}) \right\} + \frac{r_{1}}{a_{1}}(U_{P1} - U_{S1})^{2}(g(\Delta e))^{2} < 1$$
(B.29)

$$\frac{1}{a_1}g'(\Delta e)\left\{U_{P1} - U_{S1} - \frac{1}{2}r_1(U_{P1} - U_{S1})^2(1 - 2p_1)\right\} < 1 - g(\Delta e)^2 \frac{r_1}{a_1}(U_{P1} - U_{S1})^2 \quad (B.30)$$

$$SOC_2: \frac{g'(\Delta e)}{a_2} \{ U_{P2} - U_{S2} \} < 1$$
 (B.31)

To compare the effort levels of the two employees, we subtract the FOCs from

one another and derive an expression for difference between the effort levels.

Subtracting
$$FOC_2$$
 from FOC_1 , we get,

$$g(\Delta e)\left(\frac{1}{a_1}\left(U_{P1} - U_{S1}\right) - \frac{1}{2}r_1(U_{P1} - U_{S1})^2(1 - 2p_1) - \frac{1}{a_2}\left(U_{P2} - U_{S2}\right)\right) = e_1 - e_2 \quad (B.32)$$

$$g(\Delta e)\left(\left(\frac{U_{P1} - U_{S1}}{a_1} - \frac{U_{P2} - U_{S2}}{a_2}\right) - \frac{1}{2}(1 - 2p_1)\left(r_1\frac{(U_{P1} - U_{S1})^2}{a_1}\right)\right) = e_1 - e_2 \quad (B.33)$$

$$g(\Delta e)\left(\left(\frac{U_{P1} - U_{S1}}{a_1} - \frac{U_{P2} - U_{S2}}{a_2}\right) + \left(G(\Delta e) - \frac{1}{2}\right)\left(r_1\frac{(U_{P1} - U_{S1})^2}{a_1}\right)\right) = \Delta e \quad (B.34)$$

$$\frac{U_{P1} - U_{S1}}{a_1} - \frac{U_{P2} - U_{S2}}{a_2} = \frac{\Delta e}{g(\Delta e)} \left(1 - \frac{g(\Delta e) \left(G(\Delta e) - \frac{1}{2}\right)}{\Delta e} \left(r_1 \frac{(U_{P1} - U_{S1})^2}{a_1}\right) \right)$$
(B.35)

If
$$1 - \frac{g(\Delta e)\left(G(\Delta e) - \frac{1}{2}\right)}{\Delta e}\left(r_1\frac{(U_{P1} - U_{S1})^2}{a_1}\right) > 0$$
 then (B.36)

$$\operatorname{Sign}\left(\frac{U_{P1} - U_{S1}}{a_1} - \frac{U_{P2} - U_{S2}}{a_2}\right) = \operatorname{Sign}\left(\Delta e\right) \text{ since } g(\Delta e) > 0 \tag{B.37}$$

We first show that
$$1 - \frac{\left(G(\Delta e) - \frac{1}{2}\right)}{\Delta e} \left(g(\Delta e) \cdot r_1 \frac{\left(U_{P1} - U_{S1}\right)^2}{a_1}\right) > 0$$
 (B.38)

Consider SOC_1 ,

$$\frac{1}{a_1}g'(\Delta e)\left\{U_{P1} - U_{S1} - \frac{1}{2}r_1(U_{P1} - U_{S1})^2(1 - 2p_1)\right\} < 1 - g(\Delta e)^2 \frac{r_1}{a_1}(U_{P1} - U_{S1})^2 \quad (B.39)$$

The above inequality also holds if the employees had symmetric characteristics and in a symmetric

equilibrium $e_1^* = e_2^*$ and $\Delta e = 0$. Evaluating the SOC_1 in such a hypothetical equilibrium, we see that,

$$0 < 1 - g(0) \left(g(0) \frac{r_1}{a_1} (U_{P1} - U_{S1})^2 \right) \text{ since } g'(0) = 0$$
 (B.40)

Hence,
$$0 < 1 - g(0) \cdot g(0) \frac{r_1}{a_1} (U_{P1} - U_{S1})^2$$
 is implied by SOC_1 (B.41)

Comparing inequality (B.38) with the above inequality, we find that

if
$$\left(g(\Delta e)r_1\frac{(U_{P1}-U_{S1})^2}{a_1}\right) \le g(0)\frac{r_1}{a_1}(U_{P1}-U_{S1})^2$$
 and $\frac{\left(G(\Delta e)-\frac{1}{2}\right)}{\Delta e} < g(0)$ then (B.38) holds
Since, $g(0) = \max_{\Delta e} \left\{g\left(\Delta e\right)\right\}, \left(g(\Delta e)r_1\frac{(U_{P1}-U_{S1})^2}{a_1}\right) \le g(0)\frac{r_1}{a_1}(U_{P1}-U_{S1})^2$

$$\begin{aligned} & \text{To show } \frac{\left(G(\Delta e) - \frac{1}{2}\right)}{\Delta e} \leq g(0), \text{ consider the case when } \Delta e \geq 0 \\ & \text{Note that } G(\Delta e) = \int_{-\infty}^{\Delta e} g(x) dx = \int_{0}^{\Delta e} g(x) dx + \frac{1}{2} \text{ or } \frac{\left(G(\Delta e) - \frac{1}{2}\right)}{\Delta e} = \frac{\left(\int_{0}^{\Delta e} g(x) dx\right)}{\Delta e} \quad (B.42) \\ & \text{However, } \left(\int_{0}^{\Delta e} g(x) dx\right) \leq g(0) \Delta e \text{ as } g(0) = \max_{\Delta e} \left\{g\left(\Delta e\right)\right\} \\ & \text{Since } \left(\int_{0}^{\Delta e} g(x) dx\right) \leq g(0) \Delta e \forall \Delta e > 0, \\ & \left(\frac{G(\Delta e) - \frac{1}{2}}{\Delta e}\right) \leq g(0) \forall \Delta e > 0, \\ & \text{If } \Delta e < 0 \text{ then } G(\Delta e) = \frac{1}{2} - \int_{\Delta e}^{0} g(x) dx, \\ & G(\Delta e) - \frac{1}{2} = -\int_{\Delta e}^{0} g(x) dx \\ & \left(\frac{G(\Delta e) - \frac{1}{2}}{\Delta e}\right) = \frac{-|\Delta e|}{-|\Delta e|} = \frac{0}{|\Delta e|} \leq g(0) \forall \Delta e. \\ & \text{Hence, } (B.38) \text{ holds if } SOC_1 \text{ holds.} \\ & \text{Thus, } \text{Sign} \left(\frac{U_{P1} - U_{S1}}{a_1} - \frac{U_{P2} - U_{S2}}{a_2} > 0 \iff \Delta e = e_1 - e_2 > 0 \end{aligned}$$

The firm faces adverse selection in the domain (a_1, r_1) when $0 \le a_1, r_1 > 0$,

$$e_1(a_1, r_1) - e_2(a_2 = 1, r_2 = 0) > 0 \text{ and } \pi_{P,2}(a_2 = 1, r_2 = 0) \ge \pi_{P,1}(a_1, r_1)$$

or $\frac{U_{P1} - U_{S1}}{a_1} - \frac{U_{P2} - U_{S2}}{a_2} > 0 \text{ and } \frac{1 - a_1}{a_1^2 \sigma_{\gamma}^2} < r_1$

from (B.45) and Lemma 3.2, respectively.

The domain of (a_1, r_1) for which the above two inequalities (B.45 and Lemma 3.2) hold represent the domain in which the firm the firm faces adverse selection. To identify these regions in the (a_1, r_1) space we consider the inequalities under three conditions viz. $0 \le a_1 < a_2, a_1 = a_2$ and $a_1 > a_2$.

Condition 1, $0 \le a_1 < 1$.

Under the this condition
$$\frac{1-a_1}{a_1^2 \sigma_{\gamma}^2} > 0$$
 and hence, $0 < \frac{1-a_1}{a_1^2 \sigma_{\gamma}^2} < r_1$ and $\frac{U_{P1} - U_{S1}}{a_1} - \frac{U_{P2} - U_{S2}}{a_2} > 0$
or $\frac{1}{a_1^2} \left(\frac{\delta_P^2}{1+a_1 r_1 \sigma_{\gamma}^2} - \frac{\delta_S^2}{1+a_1 r_1 (\sigma_{\gamma}^2 + \sigma_{\varepsilon}^2)} \right) - \frac{1}{2} \left(\delta_P^2 - \delta_S^2 \right) > 0$ (B.46)
or $2 \left(\delta_P^2 - \delta_S^2 \right) \left(1 + a_1 r_1 \sigma_{\gamma}^2 \right) + a_1 r_1 \delta_P^2 \sigma_{\varepsilon}^2 - a_1^2 \left(\delta_P^2 - \delta_S^2 \right) \left(1 + a_1 r_1 \sigma_{\gamma}^2 + \sigma_{\varepsilon}^2 \right) \right) > 0$

since
$$2a_1^2 \left(1 + a_1 r_1 \sigma_{\gamma}^2 \right) \left(1 + a_1 r_1 \left(\sigma_{\gamma}^2 + \sigma_{\varepsilon}^2 \right) \right) > 0$$
 (B.48)

solving the above inequality we get bounds on r_1 given by

$$A - \frac{1}{2}\sqrt{B} < r_{1} < A + \frac{1}{2}\sqrt{B},$$
where, $A = \frac{\delta_{P}^{2}\sigma_{\gamma}^{2} - 2a_{1}^{2}\delta_{P}^{2}\sigma_{\gamma}^{2} - \delta_{S}^{2}\sigma_{\gamma}^{2} + 2a_{1}^{2}\delta_{S}^{2}\sigma_{\gamma}^{2} + \delta_{P}^{2}\sigma_{\epsilon}^{2} - a_{1}^{2}\sigma_{\epsilon}^{2} + a_{1}^{2}\delta_{S}^{2}\sigma_{\epsilon}^{2}}{2a_{1}^{3}\left(\delta_{P}^{2} - \delta_{S}^{2}\right)\sigma_{\gamma}^{2}\left(\sigma_{\gamma}^{2} + \sigma_{\epsilon}^{2}\right)}$

$$B = \frac{\delta_{P}^{2}\sigma_{\gamma}^{4} - 2\delta_{P}^{2}\delta_{S}^{2}\sigma_{\gamma}^{4} + \delta_{S}^{4}\sigma_{\gamma}^{4} + 2\delta_{P}^{4}\sigma_{\gamma}^{2}\sigma_{\epsilon}^{2} - 2a_{1}^{2}\delta_{P}^{4}\sigma_{\gamma}^{2}\sigma_{\epsilon}^{2} - 2\delta_{P}^{2}\delta_{S}^{2}\sigma_{\gamma}^{2}\sigma_{\epsilon}^{2} + 2a_{1}^{2}\delta_{S}^{4}\sigma_{\gamma}^{2}\sigma_{\epsilon}^{2}}{\left(a_{1}^{6}\left(\delta_{P}^{2} - \delta_{S}^{2}\right)^{2}\sigma_{\gamma}^{4}\left(\sigma_{\gamma}^{2} + \sigma_{\epsilon}^{2}\right)^{2}\right)} + \frac{\delta_{P}^{4}\sigma_{\epsilon}^{4} - 2a_{1}^{2}\delta_{P}^{4}\sigma_{\epsilon}^{4} + a_{1}^{4}\delta_{P}^{4}\sigma_{\epsilon}^{4} + 2a_{1}^{2}\delta_{P}^{2}\delta_{S}^{2}\sigma_{\epsilon}^{4} - 2a_{1}^{4}\delta_{P}^{2}\delta_{S}^{2}\sigma_{\epsilon}^{4} - 4a_{1}^{4}\delta_{S}^{4}\sigma_{\epsilon}^{4}}{\left(a_{1}^{6}\left(\delta_{P}^{2} - \delta_{S}^{2}\right)^{2}\sigma_{\gamma}^{4}\left(\sigma_{\gamma}^{2} + \sigma_{\epsilon}^{2}\right)^{2}\right)}$$
(B.50)

$$\frac{1-a_1}{a_1^2\sigma_\gamma^2} < A + \frac{1}{2}\sqrt{B} \text{ and } A - \frac{1}{2}\sqrt{B} < 0 \; \forall \; 1 \leq \frac{\delta_P}{\delta_S} \text{ when } a_1 < 1$$

Thus, parameter space that satisfies the conditions $0 \le a_1 < 1$, $1 \le \frac{\delta_P}{\delta_S}$, $0 < \frac{1-a_1}{a_1^2 \sigma_{\gamma}^2} < r_1 < A + \frac{1}{2}\sqrt{B}$

represents the region of adverse selection.

The region of adverse selection for the other conditions are given below.

Condition 2,
$$a_1 = 1$$
. Under this condition $\frac{1 - a_1}{a_1^2 \sigma_{\gamma}^2} = 0$ and hence, $0 < r_1$ and $\frac{U_{P1} - U_{S1}}{a_1} - \frac{U_{P2} - U_{S2}}{a_2} > 0$
or $\frac{1}{a_1^2} \left(\frac{\delta_P^2}{1 + a_1 r_1 \sigma_{\gamma}^2} - \frac{\delta_S^2}{1 + a_1 r_1 (\sigma_{\gamma}^2 + \sigma_{\varepsilon}^2)} \right) - \frac{1}{2} \left(\delta_P^2 - \delta_S^2 \right) > 0$

solving the above inequality we get an upper bound on r_1 given by

 $A - \frac{1}{2}\sqrt{B} < r_1 < A + \frac{1}{2}\sqrt{B}$, where A and B are the same as calculated before.

However, when
$$a_1 = 1$$
, $A + \frac{1}{2}\sqrt{B} > 0 \Leftrightarrow 1 \le \frac{\delta_P}{\delta_S} < \sqrt{\frac{\sigma_\gamma^2 + \sigma_\epsilon^2}{\sigma_\gamma^2}}$ and $A - \frac{1}{2}\sqrt{B} < 0$

Thus, the region for adverse selection in the current case is given as,

$$a_1 = 1, 1 \le \frac{\delta_P}{\delta_S} < \sqrt{\frac{\sigma_\gamma^2 + \sigma_\epsilon^2}{\sigma_\gamma^2}}, 0 < r_1 < A + \frac{1}{2}\sqrt{B}$$

 $\begin{array}{l} \text{Condition 3, } a_1 > 1. \text{Under this condition } \frac{1 - a_1}{a_1^2 \sigma_{\gamma}^2} < 0 \text{ and hence, } 0 \le r_1 \text{ and } \frac{U_{P1} - U_{S1}}{a_1} - \frac{U_{P2} - U_{S2}}{a_2} > 0 \\ \\ \text{or } \frac{1}{a_1^2} \left(\frac{\delta_P^2}{1 + a_1 r_1 \sigma_{\gamma}^2} - \frac{\delta_S^2}{1 + a_1 r_1 \left(\sigma_{\gamma}^2 + \sigma_{\varepsilon}^2\right)} \right) - \frac{1}{2} \left(\delta_P^2 - \delta_S^2\right) > 0 \end{array}$

+solving the above inequality we get an upper bound on r_1 given by

$$A - \frac{1}{2}\sqrt{B} < r_1 < A + \frac{1}{2}\sqrt{B}$$
, where A and B are the same as calculated before.

However, when $a_1 > 1$, $A + \frac{1}{2}\sqrt{B} > 0$ or $A - \frac{1}{2}\sqrt{B} > 0 \Leftrightarrow 1 \le \frac{\delta_P}{\delta_S} < \sqrt{\frac{\sigma_{\gamma}^2 + \sigma_{\epsilon}^2}{\sigma_{\gamma}^2}}$

and $A + \frac{1}{2}\sqrt{B} > A - \frac{1}{2}\sqrt{B} \forall a_1^U > a_1 > 1$, where, a_1^U is a root of the inequality B > 0.

Thus, the region of adverse selection is given as,

$$a_1^U > a_1 > 1, A - \frac{1}{2}\sqrt{B} < r_1 < A + \frac{1}{2}\sqrt{B}, 1 \le \frac{\delta_P}{\delta_S} < \sqrt{\frac{\sigma_{\gamma}^2 + \sigma_{\epsilon}^2}{\sigma_{\gamma}^2}}$$

Proof. Proposition 3.2.

First, we show that $\frac{\partial \Delta r}{\partial a_1} < 0$ for condition 1 and 2 (Proposition 3.1), where $\Delta r = \Delta r_1 = A + \frac{1}{2}\sqrt{B} - \frac{1-a_1}{a_1^2\sigma_\gamma^2}$. Note that $\frac{\partial \Delta r}{\partial a_1} < 0$ for condition 1 and 2 (Proposition 3.1) implies $\frac{\partial \Delta r_2}{\partial a_1} = \frac{\partial \left(A + \frac{1}{2}\sqrt{B}\right)}{\partial a_1} < 0$ for condition 2 (Proposition 3.1).

Let
$$X = Z\left(\frac{1-a_1}{a_1^2\sigma_\gamma^2}\right), Z = 2a_1^3\left(\delta_P^2 - \delta_S^2\right)\sigma_\gamma^2\left(\sigma_\gamma^2 + \sigma_\epsilon^2\right),$$
 (B.51)

$$\overline{A} = \delta_P^2 \sigma_\gamma^2 - 2a_1^2 \delta_P^2 \sigma_\gamma^2 - \delta_S^2 \sigma_\gamma^2 + 2a_1^2 \delta_S^2 \sigma_\gamma^2 + \delta_P^2 \sigma_\epsilon^2 - a_1^2 \sigma_\epsilon^2 + a_1^2 \delta_S^2 \sigma_\epsilon^2$$
(B.52)

$$\overline{B} = \delta_P^2 \sigma_\gamma^4 - 2\delta_P^2 \delta_S^2 \sigma_\gamma^4 + \delta_S^4 \sigma_\gamma^4 + 2\delta_P^4 \sigma_\gamma^2 \sigma_\epsilon^2 - 2a_1^2 \delta_P^4 \sigma_\gamma^2 \sigma_\epsilon^2 - 2\delta_P^2 \delta_S^2 \sigma_\gamma^2 \sigma_\epsilon^2$$

$$+2a_{1}^{2}\delta_{S}^{4}\sigma_{\gamma}^{2}\sigma_{\epsilon}^{2}+\delta_{P}^{4}\sigma_{\epsilon}^{4}-2a_{1}^{2}\delta_{P}^{4}\sigma_{\epsilon}^{4}+a_{1}^{4}\delta_{P}^{4}\sigma_{\epsilon}^{4}+2a_{1}^{2}\delta_{P}^{2}\delta_{S}^{2}\sigma_{\epsilon}^{4}-2a_{1}^{4}\delta_{P}^{2}\delta_{S}^{2}\sigma_{\epsilon}^{4}+a_{1}^{4}\delta_{S}^{4}\sigma_{\epsilon}^{4}$$
(B.53)

Note that
$$A = \frac{A}{Z}, B = \frac{B}{Z^2}$$
 and $\Delta r = \frac{1}{Z} \left(\overline{A} + \sqrt{\overline{B}} - X\right)$. (B.54)

$$\cdot \frac{\partial \Delta r}{\partial a_1} = -\frac{1}{Z^2} \left(\overline{A} + \sqrt{\overline{B}} - X \right) \frac{\partial Z}{\partial a_1} + \frac{1}{Z} \frac{\partial \left(\overline{A} + \sqrt{\overline{B}} - X \right)}{\partial a_1}$$
(B.55)

If
$$Z > 0$$
, $\frac{\partial Z}{\partial a_1} > 0$, $\left(\overline{A} + \sqrt{\overline{B}} - X\right) \ge 0$, $\frac{\partial \left(\overline{A} + \sqrt{\overline{B}} - X\right)}{\partial a_1} < 0$ then $\frac{\partial \Delta r}{\partial a_1} < 0$

$$Z = 2a_1^3 \left(\delta_P^2 - \delta_S^2\right) \sigma_\gamma^2 \left(\sigma_\gamma^2 + \sigma_\epsilon^2\right) > 0 \text{ and } \frac{\partial Z}{\partial a_1} = 6a_1^2 \left(\delta_P^2 - \delta_S^2\right) \sigma_\gamma^2 \left(\sigma_\gamma^2 + \sigma_\epsilon^2\right) > 0 \text{ since } \delta_P \ge \delta_S$$
(B.56)

Note that $\sqrt{\overline{B}} > 0$ in $\overline{A} + \sqrt{\overline{B}} - X$. Therefore, the only negative component for the term $\overline{A} + \sqrt{\overline{B}} - X$. will come from $\overline{A} - X$. If we show that $\overline{A} + \sqrt{\overline{B}} - X \ge 0$ at min $(\overline{A} - X)$ then $\overline{A} + \sqrt{\overline{B}} - X \ge 0$. Since $\frac{\partial \left(\overline{A} - X\right)}{\partial a_1} = -\left(\delta_P^2 - \delta_S^2\right) \left(\left(1 - a_1\right)\sigma_\gamma^2 + \sigma_\epsilon^2\right) \le 0 \ \forall a_1 \in (0, 1],$ $\min\left(\overline{A} - X\right) \text{ is attained at } a_1 = 1. \ \overline{A} + \sqrt{\overline{B}} - X \ |_{a_1=1} = 0 \text{ hence, } \overline{A} + \sqrt{\overline{B}} - X \ge 0 \forall a_1 \in (0, 1].$ $\frac{\partial \left(\overline{A} + \sqrt{\overline{B}} - X\right)}{\partial a_1} = \frac{\partial \left(\overline{A} - X\right)}{\partial a_1} + \frac{1}{2\sqrt{\overline{D}}} \frac{\partial \overline{B}}{\partial a_1}, \text{ where } \frac{\partial \left(\overline{A} - X\right)}{\partial a_1} \le 0 \forall a_1 \in (0, 1].$ $\frac{\partial \overline{B}}{\partial a_1} = -4a_1 \left(\delta_P^2 - \delta_S^2\right) \sigma_\epsilon^2 \left(\delta_S^2 \left(\sigma_\gamma^2 + a_1^2 \sigma_\epsilon^2\right) + \delta_P^2 \left(\sigma_\gamma^2 + \left(1 - a_1^2\right) \sigma_\epsilon^2\right)\right) < 0 \forall a_1 \in (0, 1].$ Hence, $\frac{\partial \left(\overline{A} + \sqrt{\overline{B}} - X\right)}{\partial a} < 0$. Therefore, $\frac{\partial \Delta r}{\partial a} < 0 \forall a_1 \in (0, 1]$. This proves $\frac{\partial \Delta r}{\partial a_1} < 0$ for condition 1 and 2 of Proposition 3.1. To prove $\frac{\partial \Delta r_3}{\partial a_1} < 0$ where $\Delta r_3 = \sqrt{B} = \frac{1}{Z} \left(\sqrt{\overline{B}} \right)$, note that $\frac{\partial \Delta r_3}{\partial a_1} = -\frac{1}{Z^2} \left(\sqrt{\overline{B}} \right) \frac{\partial Z}{\partial a_2} + \frac{1}{Z} \frac{\partial \left(\sqrt{\overline{B}} \right)}{\partial a_2}$ Since Z > 0, $\frac{\partial Z}{\partial a_1} > 0$, $\sqrt{\overline{B}} \ge 0$, $\frac{\partial\sqrt{B}}{\partial a_1} = \frac{-4a_1\left(\delta_P^2 - \delta_S^2\right)\sigma_\epsilon^2\left(\delta_S^2\left(\sigma_\gamma^2 + a_1^2\sigma_\epsilon^2\right) + \delta_P^2\left(\sigma_\gamma^2 + (1 - a_1^2)\sigma_\epsilon^2\right)\right)}{2\sqrt{B}} < 0$ (B.57) $\frac{\partial \Delta r_3}{\partial a_1} < 0 \forall a_1 \in (1,\infty)$. Hence, $\frac{\partial \Delta r_3}{\partial a_1} < 0$ for condition 3 of Proposition 3.1.

Proof. Proposition 3.3.

The individual rationality and incentive compatibility constraints for both the employees for the separation condition to hold are given as,

$$U_{P1} + \frac{1}{2} \left(\frac{\theta}{1+\theta} \right) \left(\frac{1}{a_1} - r_1 \sigma_{\mu}^2 \right) - U_{S1} \le 0 \text{ (IC1)}$$
(B.58)

$$U_{P1} + \frac{1}{2} \left(\frac{\theta}{1+\theta} \right) \left(\frac{1}{a_1} - r_1 \sigma_\mu^2 \right) \ge 0 \text{ (IR1)}$$
(B.59)

$$U_{P2} + \frac{1}{2} \left(\frac{\theta}{1+\theta} \right) \ge U_{S2} \text{ (IC2)} \tag{B.60}$$

$$U_{P2} + \frac{1}{2} \left(\frac{\theta}{1+\theta} \right) \ge 0 \text{ (IR2)} \tag{B.61}$$

where U_{P1}, U_{P2}, U_{S1} and U_{S2} are given in Lemma 3.1

From Lemma 3.1 we know that $U_{P2} \ge U_{P1}, U_{P2} \ge U_{S2}, U_{P1} \ge U_{S1}$.

Hence, eq. (B.60) and (B.61) always hold. The inequality (B.59) always holds if (B.58) holds.

The condition for separation is given by the inequality (B.58). Solving (B.58), we get,

$$\frac{U_{P1} - U_{S1}}{\frac{1}{2} \left(\frac{\theta}{1+\theta}\right) r_1} + \frac{1}{r_1 a_1} \le \sigma_{\mu}^2 \tag{B.62}$$

or,
$$\frac{1}{r_1 a_1} \left(\frac{\delta_P^2}{1 + a_1 r_1 \sigma_\gamma^2} - \frac{\delta_S^2}{1 + a_1 r_1 \left(\sigma_\gamma^2 + \sigma_\varepsilon^2\right)} + 1 \right) \le \sigma_\mu^2$$
(B.63)

In eq. (B.63) $\frac{\delta_P^2}{1+a_1r_1\sigma_\gamma^2} - \frac{\delta_S^2}{1+a_1r_1(\sigma_\gamma^2+\sigma_\varepsilon^2)} > 0$ and therefore, there exists a threshold \overline{r}_1 beyond which the inequality holds. Further, as a_1 increases the threshold \overline{r}_1 decreases. Solving the inequality (B.63) we get the following conditions that ensure that the inequality hold.

$$(1.) \ \frac{1+\delta_P^2-\delta_S^2+a_1r_1\sigma_{\gamma}^2}{a_1r_1+a_1^2r_1^2\sigma_{\gamma}^2} < \sigma_{\mu}^2 < \frac{1+\delta_P^2+a_1r_1\sigma_{\gamma}^2}{a_1r_1+a_1^2r_1^2\sigma_{\gamma}^2} \text{ and } 0 < \sigma_{\epsilon}^2 < \overline{\sigma}_{\epsilon}$$
(B.64)

or
$$\frac{1+\delta_P^2+a_1r_1\sigma_\gamma^2}{a_1r_1+a_1^2r_1^2\sigma_\gamma^2} < \sigma_\mu^2$$
 and $\overline{\sigma}_\epsilon < \sigma_\epsilon^2$ (B.65)

$$(2.) \ \frac{1}{a_1 r_1} < \sigma_{\mu}^2 < \frac{1 + \delta_P^2 - \delta_S^2 + a_1 r_1 \sigma_{\varepsilon}^2 \left(1 + \delta_P^2\right)}{a_1 r_1 + a_1^2 r_1^2 \sigma_{\varepsilon}^2} \text{ and } 0 < \overline{\sigma}_{\gamma} < \sigma_{\gamma}^2 \tag{B.66}$$

or
$$\frac{1+\delta_P^2-\delta_S^2+a_1r_1\sigma_{\varepsilon}^2\left(1+\delta_P^2\right)}{a_1r_1+a_1^2r_1^2\sigma_{\varepsilon}^2} < \sigma_{\mu}^2 \text{ and } 0 < \sigma_{\gamma}^2 < \overline{\sigma}_{\gamma}$$
(B.67)

$$(3.) \ \ \sigma_{\mu}^{2} > \frac{1 + 2a_{1}r_{1}\sigma_{\gamma}^{2} + a_{1}r_{1}\sigma_{\gamma}^{4} + a_{1}r_{1}\sigma_{\varepsilon}^{2} + a_{1}r_{1}\delta_{S}^{2}\sigma_{\varepsilon}^{2} + a_{1}r_{1}\sigma_{\gamma}^{2}\sigma_{\varepsilon}^{2}}{a_{1}r_{1} + 2a_{1}r_{1}\sigma\gamma^{2} + a_{1}^{3}r_{1}^{3}\sigma\gamma^{4} + a_{1}^{2}r_{1}^{2}\sigma_{\varepsilon}^{2} + a_{1}^{3}r_{1}^{3}\sigma_{\gamma}^{2}\sigma_{\varepsilon}^{2}} \text{ and } \delta_{S} < \delta_{P} < \overline{\delta}_{P} \quad (B.68)$$

Proof. Proposition 3.5.

The condition for adverse selection is given by eq. (B.45), i.e., $\frac{U_{P1}-U_{S1}}{a_1} - \frac{U_{P2}-U_{S2}}{a_2} > 0 \iff \Delta e = e_1 - e_2 > 0$ and Lemma 3.2.

Eq. (B.45) can be written as
$$\frac{1}{a_1^2} \left(\frac{\theta_m}{1+\theta_m} \frac{\delta_P^2}{1+a_1r_1\sigma_\gamma^2} - \frac{\theta_r}{1+\theta_r} \frac{\delta_S^2}{1+a_1r_1(\sigma_\gamma^2+\sigma_\varepsilon^2)} \right) - \frac{1}{2} \left(\frac{\theta_m}{1+\theta_m} \delta_P^2 - \frac{\theta_r}{1+\theta_r} \delta_S^2 \right) = \frac{1}{a_1^2} \left(\frac{\delta_P^2}{1+a_1r_1\sigma_\gamma^2} - \frac{\frac{\theta_r}{1+\theta_r}}{\frac{\theta_m}{1+\theta_m}} \frac{\delta_S^2}{1+a_1r_1(\sigma_\gamma^2+\sigma_\varepsilon^2)} \right) - \frac{1}{2} \left(\delta_P^2 - \frac{\frac{\theta_r}{1+\theta_r}}{1+\theta_m} \delta_S^2 \right) = \frac{1}{a_1^2} \left(\frac{\delta_P^2}{1+a_1r_1\sigma_\gamma^2} - \gamma \frac{\delta_S^2}{1+a_1r_1(\sigma_\gamma^2+\sigma_\varepsilon^2)} \right) - \frac{1}{2} \left(\delta_P^2 - \frac{\frac{\theta_r}{1+\theta_r}}{1+\theta_m} \delta_S^2 \right) = \frac{1}{a_1^2} \left(\frac{\delta_P^2}{1+a_1r_1\sigma_\gamma^2} - \gamma \frac{\delta_S^2}{1+a_1r_1(\sigma_\gamma^2+\sigma_\varepsilon^2)} \right) - \frac{1}{2} \left(\delta_P^2 - \gamma \delta_S^2 \right) \iff \Delta e = e_1 - e_2 > 0.$$
 If γ is above a threshold γ^* then $\Delta e = e_1 - e_2 > 0.$

APPENDIX C: PROOFS FOR THE RESULTS IN CHAPTER 4

Firm Profits without Employee Poaching

Period 2

The period 2 market shares for firm 1 and firm 2 are denoted by X_{12} and X_{22} , respectively, and they are computed below.

$$X_{12} = P_1 P_{1|1} + P_2 P_{1|2} = P_1 P_{1|1} + (1 - P_1) P_{1|2}$$
(C.1)

and
$$X_{22} = P_2 P_{2|2} + P_1 P_{2|1} = (1 - P_1) P_{2|2} + P_1 P_{2|1}$$
 (C.2)

Where,
$$P_1 = \frac{1}{2} + \frac{e_{11}^N - e_{21}^N}{2v}$$
, (C.3)

$$P_{1|2} = \frac{1}{2} + \frac{e_{12}^N - e_{22}^E - (s+f)}{2v},$$
(C.4)

$$P_{2|1} = \frac{1}{2} + \frac{e_{22}^N - e_{12}^E - (s+f)}{2v},$$
(C.5)

$$P_{1|1} = \frac{1}{2} + \frac{e_{12}^E - e_{22}^N + (s+f)}{2v},$$
(C.6)

and
$$P_{2|2} = \frac{1}{2} + \frac{e_{22}^E - e_{12}^N + (s+f)}{2v}$$
 (C.7)

Given the above market shares the firms simultaneously solve the following optimization problems,

$$\Pi_{12} = \max_{w_{12}, b_{12}^N, b_{12}^E} \pi_{12} = E \left[\delta X_{12} - \left(w_{12} + b_{12}^N \delta P_{1|2} + b_{12}^E \delta P_{1|1} \right) \right]$$
(C.8)

Subject to

$$EU_{12} = \max_{e_{12}^N, e_{12}^E} EU_{12} = E\left[y_{12}\left(e_{12}^N, e_{12}^E\right)\right] - a\left(\frac{\left(e_{12}^N\right)^2}{2} + \frac{\left(e_{12}^E\right)^2}{2}\right) \ge 0$$
(C.9)

$$e_{12}^{N}, e_{12}^{E} \in \underset{e_{12}^{N}, e_{12}^{E}}{\operatorname{arg\,max}} EU_{12} = E\left[y_{12}\left(e_{12}^{N}, e_{12}^{E}\right)\right] - a\left(\frac{\left(e_{12}^{N}\right)^{2}}{2} + \frac{\left(e_{12}^{E}\right)^{2}}{2}\right)$$
(C.10)

Where,
$$y_{12} \left(e_{12}^N, e_{12}^E \right) = w_{12} + b_{12}^N \delta P_{1|2} + b_{12}^E \delta P_{1|1}$$
 (C.11)

$$\Pi_{22} = \max_{w_{22}, b_{22}^N, b_{22}^E} \pi_{22} = E \left[\delta X_{22} - \left(w_{22} + b_{22}^N \delta P_{2|1} + b_{22}^E \delta P_{2|2} \right) \right]$$
(C.12)

Subject to

$$EU_{22} = \max_{e_{22}^N, e_{22}^E} EU_{22} = E\left[y_{22}\left(e_{22}^N, e_{22}^E\right)\right] - a\left(\frac{\left(e_{22}^N\right)^2}{2} + \frac{\left(e_{22}^E\right)^2}{2}\right) \ge 0$$
(C.13)

$$e_{22}^{N}, e_{22}^{E} \in \underset{e_{22}^{N}, e_{22}^{E}}{\operatorname{arg\,max}} EU_{22} = E\left[y_{22}\left(e_{22}^{N}, e_{22}^{E}\right)\right] - a\left(\frac{\left(e_{22}^{N}\right)^{2}}{2} + \frac{\left(e_{22}^{E}\right)^{2}}{2}\right)$$
(C.14)

Where,
$$y_{22} \left(e_{22}^N, e_{22}^E \right) = w_{22} + b_{22}^N \delta P_{2|1} + b_{22}^E \delta P_{2|2}$$
 (C.15)

Solving the sales reps' incentive compatibility constraint we get the effort levels for the sales reps as $e_{12}^N = \frac{b_{12}^N}{2av}, e_{12}^E = \frac{b_{12}^E}{2av}, e_{22}^N = \frac{b_{22}^N}{2av}, e_{22}^E = \frac{b_{22}^E}{2av}$. These effort levels are irrespective of whether the sales rep worked for the firm in period 1or not. Substituting the effort levels in firm's objective function and maximizing the firm's period 2 commissions we find the profit for the firm with an existing sales rep and a new sales rep to be as given below. Period 2 profit for firm i with an existing sales rep

$$\Pi_{12}(\text{existing sales rep}) = \overline{\Pi}_{12} + \frac{\delta s X_1}{2v} - \overline{w}_{12} - \Delta_{12}$$
(C.16)

Period 2 profit for firm i with a new sales rep

$$\Pi_{12}(\text{a new sales rep}) = \overline{\Pi}_{12} - \overline{w}_{12}$$
(C.17)

Where,
$$\overline{w}_{12} = a \left(\frac{\left(e_{12}^N\right)^2}{2} + \frac{\left(e_{12}^E\right)^2}{2} \right) - \left(b_{12}^N \delta P_{1|2} + b_{12}^E \delta P_{1|1}\right)$$
 (C.18)

$$\overline{\Pi}_{i2} = \max_{b_{12}^N, b_{12}^E} E\left[\delta X_{12} - \left(b_{12}^N \delta P_{1|2} + b_{12}^E \delta P_{1|1}\right)\right]$$
(C.19)

$$(b_{12}^N, b_{12}^E) \in \underset{b_{12}^N, b_{12}^E}{\operatorname{arg\,max}} E\left[\delta X_{12} - (b_{12}^N \delta P_{1|2} + b_{12}^E \delta P_{1|1})\right],$$

$$X_{12}$$
 is given in eq. (C.1)

and, $P_{1|2} \ {\rm and} \ P_{1|1}$ are given in eq. $(C.4) \ {\rm and} \ (C.6) \ , {\rm respectively.}$

In eq. (C.4) and (C.6),
$$e_{12}^N = \frac{b_{12}^N}{2av}, e_{12}^E = \frac{b_{12}^E}{2av}, e_{22}^N = \frac{b_{22}^N}{2av}, e_{22}^E = \frac{b_{22}^E}{2av}$$

Since under the current analysis we are not considering the possibility of competition for sales reps it is not possible for the competing firm j to offer a better wage to firm i's sales rep and lure the sales rep. However, the sales rep can still choose to leave the firm. If the sales rep leaves the firm then the sales rep's outside option gives him an expected utility of $\overline{U} = 0$.

$$EU_{12}(\text{same firm}) = EU_{12} = \overline{U} + \Delta_{12} \tag{C.20}$$

$$EU_{12}(\text{outside option}) = \overline{U} = 0$$
 (C.21)

Under the above conditions the firm and the sales rep would bargain over the surplus created because of the incremental sales that is possible due to the switching cost that customers form specific to the existing sales rep. Assuming equal bargaining power between the firm and the sales rep the bargaining solution can be attained by solving the following problem.

 $\max_{\Delta_1} G = (\Pi_{12}(\text{existing sales rep}) - \Pi_{12}(\text{new sales rep}))^{\frac{1}{2}} (EU_{12}(\text{same firm}) - EU_{12}(\text{outside option}))^{\frac{1}{2}}$

$$\operatorname{or,max}_{\Delta_1} G = \left(\overline{\Pi}_{i2} + \frac{\delta s X_1}{2t} - \overline{w}_{12} - \Delta_1 - \left(\overline{\Pi}_{i2} - \overline{w}_{12}\right)\right)^{\frac{1}{2}} \left(\overline{U} + \Delta_1 - \overline{U}\right)^{\frac{1}{2}}$$
(C.23)

$$\max_{\Delta_1} G = \sqrt{\left(\frac{\delta s X_1}{2v} - \Delta_1\right) \Delta_1}$$
(C.24)

Hence,
$$\Delta_1 = \frac{\delta s X_1}{4v}$$
 (C.25)

Therefore, in the equilibrium the firm will retain its existing sales rep and the sales rep would accept the offer. The period 2 profits and wages for firm i and its sales reps will be given as,

$$\Pi_{12}(\text{existing sales rep}) = \overline{\Pi}_{12} + \frac{\delta s X_1}{2v} - \Delta_1 = \overline{\Pi}_{12} + \frac{\delta s X_1}{4v}$$
(C.26)

$$EU_{12}(\text{same firm}) = EU_{12} = \Delta_1 = \frac{\delta s X_1}{4v}$$
 (C.27)

Since the situation in firm 2 would be symmetric, the firm 2's profit and its existing sales rep's expected utility would be given as,

$$\Pi_{22}(\text{existing sales rep}) = \overline{\Pi}_{22} + \frac{\delta s X_2}{2v} - \Delta_2 = \overline{\Pi}_{22} + \frac{\delta s X_2}{4v}$$
(C.28)

$$EU_{22}(\text{same firm}) = EU_{22} = \Delta_2 = \frac{\delta s X_2}{4v}$$
 (C.29)

Where,
$$\overline{\Pi}_{22} = \max_{b_{22}^N, b_{22}^E} E\left[\delta X_{12} - \left(b_{22}^N \delta P_{2|1} + b_{22}^E \delta P_{2|2}\right)\right]$$
 (C.30)

$$\left(b_{22}^{N}, b_{22}^{E}\right) \in \underset{b_{22}^{N}, b_{22}^{E}}{\arg\max E} \left[\delta X_{12} - \left(b_{22}^{N}\delta P_{2|1} + b_{22}^{E}\delta P_{2|2}\right)\right]$$
(C.31)

 X_{22} is given in eq. (C.2)

and, $P_{2|1}$ and $P_{2|2}$ are given in eq. $\left(C.5\right)\,$ and $\,\left(C.7\right),$ respectively.

In eq. (C.5) and (C.7),
$$e_{12}^N = \frac{b_{12}^N}{2av}, e_{12}^E = \frac{b_{12}^E}{2av}, e_{22}^N = \frac{b_{22}^N}{2av}, e_{22}^E = \frac{b_{22}^E}{2av}$$

Solving the optimization problems for $\overline{\Pi}_{12}$ and $\overline{\Pi}_{22}, we get,$

$$\overline{\Pi}_{12} = \frac{\delta \left(2av(-2s+2v+3sP_1+f(-2+4P_1))+(1-6P_1+6P_1^2)\delta\right)}{8av^2} - \frac{\delta sP_1}{4v} \qquad (C.32)$$

$$\overline{\Pi}_{22} = \frac{\delta \left(2av(s+2v+f(2-4P_1)-3sP_1)+(-3+10P_1-10P_1^2)\delta\right)}{8av^2} - \frac{\delta sP_2}{4v}$$
(C.33)

Period 1 In the period 1 the firms simultaneously solve the following optimization problems,

$$\Pi_{11} = \max_{w_{11}, b_{11}^N} \pi_{11} = E\left[\delta X_{11} - \left(w_{11} + b_{11}^N \delta P_1\right)\right] + \overline{\Pi}_{12} + \frac{\delta s X_1}{4v}$$
(C.34)

Subject to

$$EU_{11} = \max_{e_{11}^N} EU_{11} = E\left[y_{11}\left(e_{11}^N\right)\right] - a\frac{\left(e_{11}^N\right)^2}{2} + \frac{\delta s X_1}{4v} \ge 0$$
(C.35)

$$EU_{1} = \max_{\substack{e_{i_{1}}^{N}}} EU_{11} = E\left[y_{11}\right] - a\frac{\left(e_{11}^{N}\right)^{2}}{2} \ge 0$$
(C.36)

~

$$e_{11}^N \in \underset{e_{11}^N}{\operatorname{arg\,max}} EU_{11} = E\left[y_{11}\left(e_{11}^N\right)\right] - a\frac{\left(e_{12}^N\right)^2}{2} + \frac{\delta s X_1}{4v}$$
 (C.37)

Where,
$$E\left[y_{11}\left(e_{11}^{N}\right)\right] = w_{11} + b_{11}^{N}\delta P_{1}, X_{11} = P_{1} = \frac{1}{2} + \frac{e_{11}^{N} - e_{21}^{N}}{2v}$$
 (C.38)

and $\overline{\Pi}_{12}$ is given in $\,(C.32)$

$$\Pi_{21} = \max_{w_{21}, b_{21}^N} \pi_{21} = E\left[\delta X_{21} - \left(w_{21} + b_{21}^N \delta P_2\right)\right] + \overline{\Pi}_{22} + \frac{\delta s \left(1 - X_1\right)}{4v}$$
(C.39)

Subject to

$$EU_{21} = \max_{\substack{e_{21}^{N}}} EU_{21} = E\left[y_{21}\left(e_{21}^{N}\right)\right] - a\frac{\left(e_{21}^{N}\right)^{2}}{2} + \frac{\delta s\left(1-X_{1}\right)}{4v} \ge 0$$
(C.40)

$$EU_{21} = \max_{\substack{e_{21}^N}} EU_{21} = E\left[y_{21}\right] - a\left(\frac{\left(e_{21}^N\right)^2}{2}\right) \ge 0 \tag{C.41}$$

$$e_{21}^{N} \in \underset{e_{21}^{N}}{\operatorname{arg\,max}} EU_{21} = E\left[y_{21}\left(e_{21}^{N}\right)\right] - a\frac{\left(e_{21}^{N}\right)^{2}}{2} + \frac{\delta s\left(1-X_{1}\right)}{4v}$$
 (C.42)

Where,
$$E\left[y_{21}\left(e_{21}^{N}\right)\right] = w_{21} + b_{21}^{N}\delta P_{2}, X_{21} = P_{2} = \frac{1}{2} + \frac{e_{21}^{N} - e_{11}^{N}}{2v}$$
 (C.43)

and $\overline{\Pi}_{22}$ is given in (C.33)

Solving the sales reps' incentive compatibility constraint we get the effort levels for the sales reps as,

$$e_{11}^{N} = \frac{\left(s + 4b_{11}^{N}v\right)\delta}{8av^{2}} \tag{C.44}$$

$$e_{21}^{N} = \frac{\left(s + 4b_{21}^{N}v\right)\delta}{8av^{2}} \tag{C.45}$$

Since $\frac{\delta s X_1}{4v} \ge 0$ and $\frac{\delta s (1-X_1)}{4v} \ge 0$ the limited liability constraint will bind in period 1. Therefore, $w_{11} + b_{11}^N \delta P_1 = a \frac{(e_{11}^N)^2}{2}$ and $w_{21} + b_{21}^N \delta P_2 = a \frac{(e_{21}^N)^2}{2}$.

Or,
$$w_{11} = a \frac{\left(e_{11}^N\right)^2}{2} - b_{11}^N \delta P_1$$
 (C.46)

$$w_{21} = a \frac{\left(e_{21}^{N}\right)^{2}}{2} - b_{21}^{N} \delta P_{2}$$
(C.47)

Substituting the above eq. (C.46), (C.44), (C.3) in eq. (C.34) and optimizing, we get,

$$b_{11}^N = b_{21}^N = \frac{2f + s + 2v}{2v} \tag{C.48}$$

$$\Pi_{11} = \Pi_{12} = -\frac{\delta \left(16a(s-8v)v^3 + \left(16f^2 + 24fs + 9s^2 + 32fv + 24sv + 24v^2\right)\delta\right)}{128av^4}$$
(C.49)

Proof. Lemma 4.1.

Eqs. C.26 and C.27 provide proof for Lemma 4.1.

Proof. Lemma 4.2.

Eqs. C.44, C.45 and C.48 provide proof for Lemma 4.2.

Proof. Proposition 4.1.

Eq. C.49 provides proof for profit in Proposition 4.1. Comparative statics for the profit function are given below.

$$\frac{\partial \Pi_{11}}{\partial v} = \frac{\delta \left(4asv^3 + \left((4f+3s)^2 + 6(4f+3s)v + 12v^2\right)\delta\right)}{32av^5} > 0, \\ \frac{\partial \Pi_{11}}{\partial s} = -\frac{\delta \left(8av^3 + 3(4f+3s+4v)\delta\right)}{64av^4} < 0, \\ \frac{\partial \Pi_{11}}{\partial f} = -\frac{(4f+3s+4v)\delta^2}{16av^4} < 0, \\ \frac{\partial \Pi_{11}}{\partial a} = \frac{\left((4f+3s)^2 + 8(4f+3s)v + 24v^2\right)\delta^2}{128a^2v^4} > 0$$

Proof. Proposition 4.2. ■

$$\frac{\partial^2 \Pi_{11}}{\partial s \partial f} = -\frac{3\delta^2}{16av^4} < 0.$$

Proof. Proposition 4.3. ■

$$\frac{\partial^2 \Pi_{11}}{\partial \delta^2} = -\frac{2f^2 + 4fv + 3v^2}{8av^4} < 0, \\ \frac{\partial \Pi_{11}}{\partial \delta} = -\frac{8a(s - 8v)v^3 + \left((4f + 3s)^2 + 8(4f + 3s)v + 24v^2\right)\delta}{64av^4} + \frac{1}{2}\left(\frac{1}{2}\right)^2 + \frac{1}{2}\left(\frac{$$

$$0 < a < \frac{72f^2 + 128fs + 57s^2}{64f^4 + 248f^3s + 360f^2s^2 + 232fs^3 + 56s^4}, f + s < v < v^* \text{ and } \frac{-8asv^3 + 64av^4}{16f^2 + 24fs + 9s^2 + 32fv + 24sv + 24v^2} < \delta < 1,$$
 where, v^* is a root of $16f^2 + 24fs + 9s^2 + 32fv + 24sv + 24v^2 = 0.$

Firm Profits with Employee Poaching

Period 2

The period 2 market shares for firm 1 and firm 2 are denoted by X_{12} and X_{22} , respectively, and they are given in eq. (C.1) and (C.2), respectively. The expressions for the conditional probabilities and probabilities in the demands are given by eqs. (C.6), (C.4), (C.5), (C.39) and (C.3). The firm 1's objective function is given below.

$$\Pi_{12} = \max_{w_{12}, b_{12}^N, b_{12}^E} \pi_{12} = E \left[\delta X_{12} - \left(w_{12} + b_{12}^N \delta P_{1|2} + b_{12}^E \delta P_{1|1} \right) \right]$$
(C.50)

Subject to

$$EU_{12} = \max_{e_{12}^N, e_{12}^E} EU_{12} = E\left[y_{12}\left(e_{12}^N, e_{12}^E\right)\right] - a\left(\frac{\left(e_{12}^N\right)^2}{2} + \frac{\left(e_{12}^E\right)^2}{2}\right) \ge 0$$
(C.51)

$$e_{12}^{N}, e_{12}^{E} \in \underset{e_{12}^{N}, e_{12}^{E}}{\operatorname{arg\,max}} EU_{12} = E\left[y_{12}\left(e_{12}^{N}, e_{12}^{E}\right)\right] - a\left(\frac{\left(e_{12}^{N}\right)^{2}}{2} + \frac{\left(e_{12}^{E}\right)^{2}}{2}\right)$$
(C.52)

Where,
$$y_{12}\left(e_{12}^{N}, e_{12}^{E}\right) = w_{12} + b_{12}^{N}\delta P_{1|2} + b_{12}^{E}\delta P_{1|1}$$
 (C.53)

$$\Pi_{22} = \max_{w_{22}, b_{22}^N, b_{22}^E} \pi_{22} = E \left[\delta X_{22} - \left(w_{22} + b_{22}^N \delta P_{2|1} + b_{22}^E \delta P_{2|2} \right) \right]$$
(C.54)

.

Subject to

$$EU_{22} = \max_{e_{22}^N, e_{22}^E} EU_{22} = E\left[y_{22}\left(e_{22}^N, e_{22}^E\right)\right] - a\left(\frac{\left(e_{22}^N\right)^2}{2} + \frac{\left(e_{22}^E\right)^2}{2}\right) \ge 0$$
(C.55)

$$e_{22}^{N}, e_{22}^{E} \in \underset{e_{22}^{N}, e_{22}^{E}}{\operatorname{arg\,max}} EU_{22} = E\left[y_{22}\left(e_{22}^{N}, e_{22}^{E}\right)\right] - a\left(\frac{\left(e_{22}^{N}\right)^{2}}{2} + \frac{\left(e_{22}^{E}\right)^{2}}{2}\right)$$
(C.56)

Where,
$$y_{22} \left(e_{22}^N, e_{22}^E \right) = w_{22} + b_{22}^N \delta P_{2|1} + b_{22}^E \delta P_{2|2}$$
 (C.57)

Solving the sales reps' incentive compatibility constraint we get the effort levels for the sales reps as $e_{12}^N = \frac{b_{12}^N}{2av}$, $e_{12}^E = \frac{b_{12}^E}{2av}$, $e_{22}^N = \frac{b_{22}^N}{2av}$. These effort levels are irrespective of whether the sales rep worked for the firm in period 1 or not. Substituting the effort levels in firm's objective function and maximizing the firm's period 2 commissions we find the profit for the firm with an existing sales rep and a new sales rep to be as given below.

Period 2 profit for firm 1 with an existing sales rep

$$\Pi_{12}(\text{existing sales rep}) = \overline{\Pi}_{12} + \frac{\delta s X_1}{2v} - \overline{w}_{12} - \Delta_{12}^{retain}$$
(C.58)

Period 2 profit for firm i with a new sales rep

$$\Pi_{i2}(\text{new sales rep}) = \overline{\Pi}_{12} - \overline{w}_{12}$$
(C.59)

Where,
$$\overline{w}_{12} = a \left(\frac{\left(e_{12}^N\right)^2}{2} + \frac{\left(e_{12}^E\right)^2}{2} \right) - \left(b_{12}^N \delta P_{1|2} + b_{12}^E \delta P_{1|1}\right)$$
 (C.60)

$$\overline{\Pi}_{12} = \max_{b_{12}^N, b_{12}^E} E\left[\delta X_{12} - \left(b_{12}^N \delta P_{1|2} + b_{12}^E \delta P_{1|1}\right)\right]$$
(C.61)

$$(b_{12}^N, b_{12}^E) \in \underset{b_{12}^N, b_{12}^E}{\operatorname{arg\,max}} E\left[\delta X_{12} - \left(b_{12}^N \delta P_{1|2} + b_{12}^E \delta P_{1|1}\right)\right],$$

 X_{12} is given in eq. (C.1)

and, $P_{1|2} \ {\rm and} \ P_{1|1}$ are given in eq. $(C.4) \ {\rm and} \ (C.6) \ , {\rm respectively.}$

In eq. (C.4) and (C.6),
$$e_{12}^N = \frac{b_{12}^N}{2av}, e_{12}^E = \frac{b_{12}^E}{2av}, e_{22}^N = \frac{b_{22}^N}{2av}, e_{22}^E = \frac{b_{22}^E}{2av}$$

Since under the current analysis we are considering the possibility of competition for sales reps it is possible for the competing firm j to offer a better wage to firm i's sales rep and lure the sales rep. However, the sales rep can still choose to leave the firm and not join firm j. If the sales rep leaves the firm then the sales rep's outside option gives him an expected utility of $\overline{U} = 0$.

$$EU_{12}(\text{same firm}) = EU_{12} = \overline{U} + \Delta_{12}^{retain}$$
(C.62)

$$EU_{22}(\text{at firm } 2) = EU_{12} = \overline{U} + \Delta_{22}^{poach}$$
(C.63)

$$EU_{12}(\text{outside option}) = \overline{U} = 0$$
 (C.64)

Period 2 profit for firm 2 with a poached sales rep would be,

$$\Pi_{22}(\text{poached sales rep}) = \overline{\Pi}_{22} + \frac{\delta s X_1}{2v} - \overline{w}_{12} - \Delta_{22}^{poach}$$
(C.65)

Period 2 profit for firm i with a new sales rep

$$\Pi_{22}(\text{new sales rep}) = \Pi_{22} - \overline{w}_{12} \tag{C.66}$$

The sales rep will work for firm 1 iff $EU_{12} \ge EU_{22} \Leftrightarrow \Delta_{12}^{retain} \ge \Delta_{22}^{poach}$. Firm 1's profit is given as Π_{12} (existing sales rep) = $\overline{\Pi}_{12} + \frac{\delta s X_1}{2v} - \overline{w}_{12} - \Delta_{12}^{retain}$ with the sales rep and Π_{12} (new sales rep) = $\overline{\Pi}_{12} - \overline{w}_{12}$ without the rep. Therefore, as long as $\frac{\delta s X_1}{2v} - \Delta_{12}^{poach} \ge 0$ the firm would prefer to keep the existing sales rep. Firm 2's profits from poaching and not poaching the firm 1's sales rep are given as Π_{22} (poached sales rep) = $\overline{\Pi}_{12} + \frac{\delta s X_1}{2v} - \overline{w}_{12} - \Delta_{22}^{poach}$ and Π_{i2} (new sales rep) = $\overline{\Pi}_{12} - \overline{w}_{12}$, respectively. Therefore, firm 2 would prefer to poach the same firm 1's sales rep iff $\frac{\delta s X_1}{2v} - \Delta_{22}^{poach} \ge 0$. Since the sales rep will work for firm 1 iff $\Delta_{12}^{Poaching} \ge \Delta_{22}^{Poaching}$ both the firm 1 and firm 2 will be engaged in a Bertrand competition on wages Δ_{12}^{retain} and Δ_{22}^{poach} and they will be willing to retain and poach the sales rep if $\frac{\delta s X_1}{2v} - \Delta_{12}^{retain} = \epsilon_1 \ge 0$ and $\frac{\delta s X_1}{2v} - \Delta_{22}^{poach} = \epsilon_2 \ge 0$, respectively. This would ensure that $\epsilon_1 \to 0$ and $\epsilon_2 \to 0$, or $\Delta_{12}^{retain} = \Delta_{22}^{poach} = \frac{\delta s X_1}{2v}$. Therefore, at equilibrium the sales rep will be indifferent between working for either of the firms and he will earn a wage of $\Delta_{12}^{retain} = \Delta_{22}^{poach} = \frac{\delta s X_1}{2v}$ while the firms will be indifferent between retaining or poaching each other's sales reps and their profits will be given as $\overline{\Pi}_{12} - \overline{w}_{12}$.

At the equilibrium the firm will retain its existing sales rep and the sales rep would accept the offer. The period 2 profits and wages for firm i and its sales reps will be given as,

$$\Pi_{12}(\text{existing sales rep}) = \overline{\Pi}_{12} + \frac{\delta s X_1}{2v} - \Delta_{12}^{retain} = \overline{\Pi}_{12}$$
(C.67)

$$EU_{12}(\text{same firm}) = EU_{12} = \Delta_{12}^{retain} = \frac{\delta s X_1}{2v}$$
(C.68)

Since the situation in firm 2 would be symmetric, the firm 2's profit and its existing sales rep's expected utility would be given as,

$$\Pi_{22}(\text{existing sales rep}) = \overline{\Pi}_{22} + \frac{\delta s X_2}{2v} - \Delta_{22}^{retain} = \overline{\Pi}_{22}$$
(C.69)

$$EU_{22}(\text{same firm}) = EU_{22} = \Delta_{22}^{retain} = \frac{\delta s X_2}{2v}$$
 (C.70)

Where,
$$\overline{\Pi}_{22} = \max_{b_{22}^N, b_{22}^E} E\left[\delta X_{12} - \left(b_{22}^N \delta P_{2|1} + b_{22}^E \delta P_{2|2}\right)\right]$$
 (C.71)

$$\left(b_{22}^{N}, b_{22}^{E}\right) \in \underset{b_{22}^{N}, b_{22}^{E}}{\operatorname{arg\,max}} E\left[\delta X_{12} - \left(b_{22}^{N}\delta P_{2|1} + b_{22}^{E}\delta P_{2|2}\right)\right]$$
(C.72)

 X_{22} is given in eq. (C.2)

and, $P_{2|1} \mbox{ and } P_{2|2} \mbox{ are given in eq. } (C.5) \mbox{ and } (C.7) \mbox{ , respectively.}$

In eq. (C.5) and (C.7),
$$e_{12}^N = \frac{b_{12}^N}{2av}, e_{12}^E = \frac{b_{12}^E}{2av}, e_{22}^N = \frac{b_{22}^N}{2av}, e_{22}^E = \frac{b_{22}^E}{2av}$$

Solving the optimization problems for $\overline{\Pi}_{12}$ and $\overline{\Pi}_{22}, we get,$

$$\overline{\Pi}_{12} = \frac{\delta \left(4av(v+s(-1+P_1)+f(-1+2P_1))+(1-6P_1+6P_1^2)\delta\right)}{8av^2}$$
(C.73)

$$\overline{\Pi}_{22} = \frac{\delta \left(4av(f+v-2fP_1-sP_1) + \left(-3+10P_1-10P_1^2\right)\delta\right)}{8av^2} \tag{C.74}$$

Period 1 In the period 1 the firms simultaneously solve the following optimization problems,

$$\Pi_{11} = \max_{w_{11}, b_{11}^N} \pi_{11} = E\left[\delta X_{11} - \left(w_{11} + b_{11}^N \delta P_1\right)\right] + \overline{\Pi}_{12}$$
(C.75)

Subject to

$$EU_{11} = \max_{\substack{e_{11}^N \\ e_{11}^N}} EU_{11} = E\left[y_{11}\left(e_{11}^N\right)\right] - a\frac{\left(e_{11}^N\right)^2}{2} + \frac{\delta s X_1}{2v} \ge 0$$
(C.76)

$$EU_{1} = \max_{\substack{e_{i_{1}}^{N}}} EU_{11} = E\left[y_{11}\right] - a\frac{\left(e_{11}^{N}\right)^{2}}{2} \ge 0$$
(C.77)

$$e_{11}^N \in \underset{e_{11}^N}{\operatorname{arg\,max}} EU_{11} = E\left[y_{11}\left(e_{11}^N\right)\right] - a\frac{\left(e_{12}^N\right)^2}{2} + \frac{\delta s X_1}{2v}$$
 (C.78)

Where,
$$E\left[y_{11}\left(e_{11}^{N}\right)\right] = w_{11} + b_{11}^{N}\delta P_{1}, X_{11} = P_{1} = \frac{1}{2} + \frac{e_{11}^{N} - e_{21}^{N}}{2v}$$
 (C.79)

and $\overline{\Pi}_{12}$ is given in (C.73)

$$\Pi_{21} = \max_{w_{21}, b_{21}^N} \pi_{21} = E\left[\delta X_{21} - \left(w_{21} + b_{21}^N \delta P_2\right)\right] + \overline{\Pi}_{22} + \frac{\delta s \left(1 - X_1\right)}{4v}$$
(C.80)

Subject to

$$EU_{21} = \max_{\substack{e_{21}^{N} \\ e_{21}^{N}}} EU_{21} = E\left[y_{21}\left(e_{21}^{N}\right)\right] - a\frac{\left(e_{21}^{N}\right)^{2}}{2} + \frac{\delta s\left(1 - X_{1}\right)}{4v} \ge 0$$
(C.81)

$$EU_{21} = \max_{\substack{e_{21}^N\\e_{21}}} EU_{21} = E\left[y_{21}\right] - a\left(\frac{\left(e_{21}^N\right)^2}{2}\right) \ge 0$$
(C.82)

$$e_{21}^{N} \in \underset{e_{21}}{\operatorname{arg\,max}} EU_{21} = E\left[y_{21}\left(e_{21}^{N}\right)\right] - a\frac{\left(e_{21}^{N}\right)^{2}}{2} + \frac{\delta s\left(1 - X_{1}\right)}{4v}$$
(C.83)

Where,
$$E\left[y_{21}\left(e_{21}^{N}\right)\right] = w_{21} + b_{21}^{N}\delta P_{2}, X_{21} = P_{2} = \frac{1}{2} + \frac{e_{21}^{N} - e_{11}^{N}}{2v}$$
 (C.84)

and
$$\overline{\Pi}_{22}$$
 is given in (C.74)

Solving the sales reps' incentive compatibility constraint we get the effort levels for the sales reps

as,

$$e_{11}^{N} = \frac{\left(s + 2b_{11}^{N}v\right)\delta}{4av^{2}} \tag{C.85}$$

$$e_{21}^{N} = \frac{\left(s + 2b_{21}^{N}v\right)\delta}{4av^{2}} \tag{C.86}$$

Since $\frac{\delta s X_1}{2v} \ge 0$ and $\frac{\delta s (1-X_1)}{2v} \ge 0$ the limited liability constraint will bind in period 1. Therefore, $w_{11} + b_{11}^N \delta P_1 = a \frac{(e_{11}^N)^2}{2}$ and $w_{21} + b_{21}^N \delta P_2 = a \frac{(e_{21}^N)^2}{2}$.

Or,
$$w_{11} = a \frac{\left(e_{11}^N\right)^2}{2} - b_{11}^N \delta P_1$$
 (C.87)

$$w_{21} = a \frac{\left(e_{21}^{N}\right)^{2}}{2} - b_{21}^{N} \delta P_{2}$$
(C.88)

Substituting the eq. (C.87), (C.85), (C.3) in eq. (C.75) and optimizing, we get,

$$b_{11}^N = b_{21}^N = 1 + \frac{f}{v} - \frac{s}{2v}$$
(C.89)

$$\Pi_{11} = \Pi_{12} = -\frac{\delta \left(8a(s-2v)v^3 + (2f^2 + 4fv + 3v^2)\delta\right)}{16av^4}$$
(C.90)

Proof. Proposition 4.4. ■

Eqs. C.67 and C.68 provide proof for Proposition 4.4.

Proof. Lemma 4.3.

Eqs. C.85,C.86 and C.89 provide proof for Lemma 4.3.

Proof. Proposition 4.5.

Let
$$\Pi_{Nopoaching} = -\frac{\delta \left(16a(s-8v)v^3 + \left(16f^2 + 24fs + 9s^2 + 32fv + 24sv + 24v^2\right)\delta\right)}{128av^4}$$
 based on eq. C.49 and $\Pi_{poaching} = -\frac{\delta \left(8a(s-2v)v^3 + \left(2f^2 + 4fv + 3v^2\right)\delta\right)}{16av^4}$ based on eq. C.90.

$$\begin{aligned} \Pi_{poaching} > \Pi_{Nopoaching}, \text{if,} \\ & -\frac{\delta \left(8a(s-2v)v^3 + \left(2f^2 + 4fv + 3v^2\right)\delta\right)}{16av^4} > \\ & -\frac{\delta \left(16a(s-8v)v^3 + \left(16f^2 + 24fs + 9s^2 + 32fv + 24sv + 24v^2\right)\delta\right)}{128av^4} \end{aligned}$$

or

$$\begin{aligned} \text{(i)} \ 0 < s < -\frac{8v}{3} + 4v\sqrt{\frac{2}{3}}, \ \frac{1}{8}(-3s - 8v) + \sqrt{\frac{3}{2}}v < f < -s + v \ , \\ 0 < a \leq \frac{1}{2v^2}\sqrt{\frac{3}{2}}, \ \frac{16av^3}{8f + 3s + 8v} < \delta < 2\sqrt{\frac{2}{3}}av^2 \\ \text{(ii)} \ 0 < s < -\frac{8t}{3} + 4v\sqrt{\frac{2}{3}}, \ \frac{1}{8}(-3s - 8v) + \sqrt{\frac{3}{2}}v < f < -s + v \ , \\ \frac{1}{2v^2}\sqrt{\frac{3}{2}} < a < \frac{8f + 3s + 8v}{16v^3}, \ \frac{16av^3}{8f + 3s + 8v} < \delta < 1 \\ \text{(iii)} \ -\frac{8v}{3} + 4v\sqrt{\frac{2}{3}} \leq s < v, \ 0 < f < -s + v, \\ 0 < a < \frac{1}{2v^2}\sqrt{\frac{3}{2}}, \ \frac{16av^3}{8f + 3s + 8v} < \delta < 2\sqrt{\frac{2}{3}}\sqrt{a^2v^4} \\ \text{(iv)} \ -\frac{8v}{3} + 4v\sqrt{\frac{2}{3}} \leq s < v, \ 0 < f < -s + v, \\ \frac{1}{2v^2}\sqrt{\frac{3}{2}} \leq a < \frac{8f + 3s + 8v}{16v^3}, \ \frac{16av^3}{2} \leq a < \frac{8f + 3s + 8v}{8f + 3s + 8v} < \delta < 2\sqrt{\frac{2}{3}}\sqrt{a^2v^4} \\ \text{(iv)} \ -\frac{8v}{3} + 4v\sqrt{\frac{2}{3}} \leq s < v, \ 0 < f < -s + v, \\ \frac{1}{2v^2}\sqrt{\frac{3}{2}} \leq a < \frac{8f + 3s + 8v}{16v^3}, \\ \frac{16av^3}{16v^3}, \\ \frac{16av^3}{8f + 3s + 8v} < \delta < 1 \end{aligned}$$

Monopoly Analysis for Hypothesis Tested In Section 4.4.1

Proof. Monopoly profits increase in a convex manner with increase in gross profit margins.

The period 2 market shares for firm 1 is denoted by X_{12} and it is computed below.

$$X_{12} = P_1 P_{1|1} + P_2 P_{1|2} = P_1 P_{1|1} + (1 - P_1) P_{1|2}$$
(C.91)

Where,
$$P_1 = \frac{e_{11}^N}{2v}$$
 (C.92)

$$P_{1|1} = \frac{e_{12}^E + (s+f)}{2v},\tag{C.93}$$

and
$$P_{1|2} = \frac{e_{22}^E}{2v}$$
 (C.94)

Given the above market share the firm's solves the following optimization problem,

$$\Pi_{12} = \max_{w_{12}, b_{12}^N, b_{12}^E} \pi_{12} = E \left[\delta X_{12} - \left(w_{12} + b_{12}^N \delta P_{1|2} + b_{12}^E \delta P_{1|1} \right) \right]$$
(C.95)

Subject to

$$EU_{12} = \max_{e_{12}^N, e_{12}^E} EU_{12} = E\left[y_{12}\left(e_{12}^N, e_{12}^E\right)\right] - a\left(\frac{\left(e_{12}^N\right)^2}{2} + \frac{\left(e_{12}^E\right)^2}{2}\right) \ge 0$$
(C.96)

$$e_{12}^{N}, e_{12}^{E} \in \underset{e_{12}^{N}, e_{12}^{E}}{\operatorname{arg\,max}} EU_{12} = E\left[y_{12}\left(e_{12}^{N}, e_{12}^{E}\right)\right] - a\left(\frac{\left(e_{12}^{N}\right)^{2}}{2} + \frac{\left(e_{12}^{E}\right)^{2}}{2}\right)$$
(C.97)

Where,
$$y_{12}\left(e_{12}^{N}, e_{12}^{E}\right) = w_{12} + b_{12}^{N}\delta P_{1|2} + b_{12}^{E}\delta P_{1|1}$$
 (C.98)

Solving the sales reps' incentive compatibility constraint we get the effort levels for the sales reps as $e_{12}^N = \frac{b_{12}^N}{2av}$, $e_{12}^E = \frac{b_{12}^E}{2av}$. These effort levels are irrespective of whether the sales rep worked for the firm in period 1 or not. Substituting the effort levels in firm's objective function and maximizing

the firm's period 2 commissions we find the profit for the firm with an existing sales rep and a new sales rep to be as given below.

Period 2 profit for firm 1 with an existing sales rep

$$\Pi_{12}(\text{existing sales rep}) = \overline{\Pi}_{12} + \frac{\delta s X_1}{2v} - \overline{w}_{12} - \Delta_{12}$$
(C.99)

Period 2 profit for firm 1 with a new sales rep

$$\Pi_{12}(\text{a new sales rep}) = \overline{\Pi}_{12} - \overline{w}_{12}$$
(C.100)

Where,
$$\overline{w}_{12} = a \left(\frac{\left(e_{12}^N\right)^2}{2} + \frac{\left(e_{12}^E\right)^2}{2} \right) - \left(b_{12}^N \delta P_{1|2} + b_{12}^E \delta P_{1|1}\right)$$
 (C.101)

$$\overline{\Pi}_{i2} = \max_{b_{12}^N, b_{12}^E} E\left[\delta X_{12} - \left(b_{12}^N \delta P_{1|2} + b_{12}^E \delta P_{1|1}\right)\right]$$
(C.102)

$$\left(b_{12}^{N}, b_{12}^{E}\right) \in \underset{b_{12}^{N}, b_{12}^{E}}{\arg\max} E\left[\delta X_{12} - \left(b_{12}^{N}\delta P_{1|2} + b_{12}^{E}\delta P_{1|1}\right)\right],$$
(C.103)

 X_{12} is given in eq. (C.91)

and, $P_{1|2}$ and $P_{1|1}$ are given in eq. (C.94) and (C.93), respectively.

In eq. (C.94) and (C.93),
$$e_{12}^N = \frac{b_{12}^N}{2av}, e_{12}^E = \frac{b_{12}^E}{2av}$$

The sales rep can choose to leave the firm. If the sales rep leaves the firm then the sales rep's outside option gives him an expected utility of $\overline{U} = 0$.

$$EU_{12}(\text{same firm}) = EU_{12} = \overline{U} + \Delta_{12}$$
(C.104)

$$EU_{12}(\text{outside option}) = \overline{U} = 0$$
 (C.105)

Under the above conditions the firm and the sales rep would bargain over the surplus created because of the incremental sales that is possible due to the switching cost that customers form specific to the existing sales rep. Assuming equal bargaining power between the firm and the sales rep the bargaining solution can be attained by solving the following problem.

$$\max_{\Delta_1} G = (\prod_{12} (\text{existing sales rep}) - \prod_{12} (\text{new sales rep}))^{\frac{1}{2}} (EU_{12}(\text{same firm}) - EU_{12}(\text{outside option}))^{\frac{1}{2}}$$

$$\operatorname{or,max}_{\Delta_1} G = \left(\overline{\Pi}_{12} + \frac{\delta s X_1}{2v} - \overline{w}_{12} - \Delta_1 - \left(\overline{\Pi}_{12} - \overline{w}_{12}\right)\right)^{\frac{1}{2}} \left(\overline{U} + \Delta_1 - \overline{U}\right)^{\frac{1}{2}}$$
(C.107)

$$\max_{\Delta_1} G = \sqrt{\left(\frac{\delta s X_1}{2v} - \Delta_1\right) \Delta_1}$$
(C.108)

Hence,
$$\Delta_1 = \frac{\delta s X_1}{4v}$$
 (C.109)

Therefore, in the equilibrium the firm will retain its existing sales rep and the sales rep would accept the offer. The period 2 profits and wages for firm i and its sales reps will be given as,

$$\Pi_{12}(\text{existing sales rep}) = \overline{\Pi}_{12} + \frac{\delta s X_1}{2v} - \Delta_1 = \overline{\Pi}_{12} + \frac{\delta s X_1}{4v}$$
(C.110)

$$EU_{12}(\text{same firm}) = EU_{12} = \Delta_1 = \frac{\delta s X_1}{4v}$$
 (C.111)

Solving the optimization problems for $\overline{\Pi}_{12}, \mbox{we get},$

$$\overline{\Pi}_{12} = \frac{1}{8av^2} \delta \left(2av(2v + 4fX_1 + 3sX_1 - 2vX_1) + \left(1 - 2X_1 + 4X_1^2\right) \delta \right)$$
(C.112)

Period 1 Analysis

In the period 1 the firm solves the following optimization problem,

$$\Pi_{11} = \max_{w_{11}, b_{11}^N} \pi_{11} = E\left[\delta X_{11} - \left(w_{11} + b_{11}^N \delta P_1\right)\right] + \overline{\Pi}_{12} + \frac{\delta s X_1}{4v}$$
(C.113)

Subject to

$$EU_{11} = \max_{e_{11}^N} EU_{11} = E\left[y_{11}\left(e_{11}^N\right)\right] - a\frac{\left(e_{11}^N\right)^2}{2} + \frac{\delta s X_1}{4v} \ge 0$$
(C.114)

$$EU_{1} = \max_{\substack{e_{i1}^{N} \\ e_{i1}}} EU_{11} = E\left[y_{11}\right] - a \frac{\left(e_{11}^{N}\right)^{2}}{2} \ge 0$$
(C.115)

$$e_{11}^N \in \underset{e_{11}^N}{\operatorname{arg\,max}} EU_{11} = E\left[y_{11}\left(e_{11}^N\right)\right] - a\frac{\left(e_{12}^N\right)^2}{2} + \frac{\delta s X_1}{4v}$$
 (C.116)

Where,
$$E\left[y_{11}\left(e_{11}^{N}\right)\right] = w_{11} + b_{11}^{N}\delta P_{1}, X_{11} = P_{1} = \frac{e_{11}^{N}}{2v}$$
 (C.117)

and $\overline{\Pi}_{12}$ is given in (C.112)

Solving the sales reps' incentive compatibility constraint we get the effort levels for the sales reps as,

$$e_{11}^{N} = \frac{\left(s + 4b_{11}^{N}v\right)\delta}{8av^{2}} \tag{C.118}$$

Since $\frac{\delta s X_1}{4v} \ge 0$ and $\frac{\delta s (1-X_1)}{4v} \ge 0$ the limited liability constraint will bind in period 1. Therefore, $w_{11} + b_{11}^N \delta P_1 = a \frac{(e_{11}^N)^2}{2}$ and $w_{21} + b_{21}^N \delta P_2 = a \frac{(e_{21}^N)^2}{2}$.

Or,
$$w_{11} = a \frac{\left(e_{11}^N\right)^2}{2} - b_{11}^N \delta P_1$$
 (C.119)

Substituting the eq. (C.119), (C.118), (C.92) in eq. (C.113) and optimizing, we get,

$$b_{11}^{N} = \frac{a^{2}t^{4}(8f + 5s + 4t) - 2at^{3}\delta + s\delta^{2}}{4a^{2}t^{5} - 4t\delta^{2}}$$
(C.120)
$$\prod_{11} = \frac{\left(\delta \left(16a^{3}t^{6} + a^{2}t^{2}\left(16f^{2} + 9s^{2} + 12st + 8t^{2} + 8f(3s + 2t)\right)\delta\right) - 2at(4f + 3s + 10t)\delta^{2} - 3\delta^{3}\right)}{32t^{2}\left(a^{3}t^{4} - a\delta^{2}\right)}$$
(C.121)

We normalize a = 1 to keep the analysis tractable.

$$\therefore \Pi_{11} = \frac{1}{32t^2 (t^4 - \delta^2)} \delta \begin{pmatrix} 16t^6 + t^2 (16f^2 + 9s^2 + 12st + 8t^2 + 8f(3s + 2t)) \delta \\ -2t(4f + 3s + 10t)\delta^2 - 3\delta^3 \end{pmatrix}$$
(C.122)

$$\frac{\partial \Pi_{11}}{\partial \delta} \mid_{\delta=0} = \frac{1}{2} > 0 \tag{C.123}$$

$$\frac{\partial^2 \Pi_{11}}{\partial \delta^2} = \frac{6 + \frac{t^4 (4f + 3s + t)^2}{(t^2 - \delta)^3} + \frac{t^4 (4f + 3(s + t))^2}{(t^2 + \delta)^3}}{32t^2} > 0$$
(C.124)

Hence,
$$\frac{\partial \Pi_{11}}{\partial \delta} > 0$$
 and $\frac{\partial^2 \Pi_{11}}{\partial \delta^2} > 0$ if the firm is a monopolist. (C.125)

LIST OF REFERENCES

- Ahearne, M., T. Steenburgh, J. Boichuk, F. Kraus, C. Liutec. 2012. Man on the Bench: The Power of Stick in the Sales Force. *Working Paper*.
- Anderson, R. E., A. J. Dubinsky, R. Mehta. 1999. Sales Managers:Marketing's Best Example of the Peter Principle? *Bus. Horizons*. 42 (1) 19–26.
- Baker, G., M. Jensen, K. Murphy. 1988. Competition and Incentives: Practice vs Theory. J. of Finance. 43(3) 593-616.
- Baker, G., M. Gibbs, B. Holmström. 1994. The Wage Policy of a Firm. *Quarterly J. of Econom.*. 4(November) 921-55.
- Balafoutas, L., G. Dutcher, F. Lindner, D. Ryvkin. 2013. The optimal allocation of prizes in tournaments of heterogeneous players. *Working Paper*.
- Basu, A., R. Lal, V. Srinivasan, R. Staelin. 1985. Salesforce compensation plans: an agency theoretic perspective. *Marketing Sci.*. 4(4) 267–291.
- Beatty, S. E., M. Mayer, J. E. Coleman, K. E. Reynolds, J. Lee. 1996. Customer-Sales Associate Retail Relationships. J. Retailing. 72 (3) 223–47.

Beck Reed Riden. 2013. Employee Non Competes - A State by State Survey.

- Berger, J., C. Harbring, D. Sliwka. 2012. Performance Appraisals and the Impact of Forced Distribution: An Experimental Investigation. *Management Sci.*, 59(1) 54-68.
- Bester H., D. Krähmer. 2008. Delegation and incentives. RAND J. Econom. 39(3):664-682.
- Boal, W. M., M. R. Ransom. 1997. Monopsony in the Labor Market. J. Econom. Literature. 35(1) 86-112.
- Bowman, D., D. Narayandas. 2004. Linking customer management effort to customer profitability in business markets. *J. Marketing Res.*. 41(4) 433-447.
- Burnham, T. A., J. K. Frels, V. Mahajan. 2003. Consumer Switching Costs: A Typology, Antecedents, and Consequences. J. Academy of Marketing Sci. 31 (2) 109–126.
- Chen, H., S. H. Ham, N. Lim. 2011. Designing Asymmetric Multi-Person Tournaments: An Experimental Study. *Management Sci.*, 57 (5) 864-883.
- Chen, S. 2001. Is Investing in College Education Risky? University of Rochester and SUNY Albany. *Mimeo*.
- Ciliberto, F., J. W. Williams, 2014. Does multimarket contact facilitate tacit collusion? Inference on conduct parameters in the airline industry. *RAND J. Econom.*, 45(4) 764-791, 12.
- CNBC. 2014. Are non-compete clauses out of control. June 25.

- Cohen, A., L. Einav. 2007. Estimating Risk Preferences From Deductible Choice. *Amer. Econom. Rev.*. 97(3) 745-788.
- Coughlan, A., C. Narasimhan. 1992. An Empirical Analysis of Sales-force Compensation Plans. J. Business. 65 (1) 93-121.
- Crosby, L. A., N. Stephens. 1987. Effects of relationship marketing on satisfaction, retention, and prices in the life insurance industry. *J. Marketing Res.*. 24, 404-411.
- Daljord, Ø., S. Misra, H. S. Nair. 2014. Homogenous Contracts for Heterogeneous Agents: Aligning Salesforce Composition and Compensation. *Working Paper*.
- DeVaro, J. 2006. Internal promotion competitions in firms. RAND J. Econom. 37(3) 521-542, 09.
- Dimmock, S. G., R. Kouwenberg, O. S. Mitchell, K. Peijnenburg. 2013. Ambiguity aversion and household portfolio choice: Empirical evidence. *NBER Working Paper*.
- Ederhof, M. 2011. Incentive compensation and promotion-based incentives of mid-level managers: Evidence from a multinational corporation. *The Accounting Rev.*. 86(1) 131-153.
- Einhorn, H., R. Hogarth. 1986. Decision making under ambiguity. J. Business. 59(4), 225-250.
- Ellsberg, D. 1961. Risk, Ambiguity, and the Savage Axioms. Quarterly J. of Econom. 75 (4) 643–669.

- Fairburn, J., J. Malcomson. 2001. Performance, Promotion, and the Peter Principle. *Rev. Econom. Studies*. 68(1) 45-66.
- Farrell, J., P. Klemperer. 2007. Coordination and lock-in: Competition with switching costs and network effects. In M. Armstrong, R. Porter (Eds.), *Handbook of Industrial Organization*. Amsterdam, The Netherlands: Elsevier.
- Farrell, J., C. Shapiro. 1988. Dynamic competition with switching costs. *RAND J. Econom.* 19, 123–137.
- Farrell, J., C. Shapiro. 1989. Optimal contracts with lock-in. Amer. Econom. Rev. 79, 51-68.
- Gertner, R. 1993. Game Shows and Economic Behavior: Risk-Taking On 'Card Sharks'. *Quarterly* of J. Econom. 108(2) 507–21.
- Gibbons, R. 1997. Incentives and Careers in Organizations. D. Kreps and K. Wallis (eds), Advances in Economic Theory and Econometrics. 2(Cambridge University Press).
- Gibbs, M. 1991. An Economic Approach to Process in Pay and Performance Appraisals. *Manuscript*, University of Chicago, Graduate School of Business.
- Gilboa, I., D. Schmeidler. 1989. Maxmin expected utility with a non-unique prior. J. Mathematical Economics. 18(2), 141–154.

- Green, J. R., N. L. Stokey. 1983. A Comparison of Tournaments and Contracts. J. of Political Economy. University of Chicago Press 91(3) 349-64.
- Grote, D. 2005. Forced Ranking: Making Performance Management Work. *Harvard Business School Press.*
- Gupta, S., D. R. Lehmann, J. A. Stuart. 2004. Valuing customers. J. Marketing Res., 41(1) 7–18.
- Hartog, J., H. V. Ophem, S. M. Bajdechi. 2004. How risky is investment in human capital? *CESifo Working Paper Series*. 1261, http://ideas.repec.org/p/ces/ceswps/1261.html.
- Ho, J., L. R. Keller, P. Keltyka. 2002. Effects of outcome and probabilistic ambiguity on managerial choices. J. Risk and Uncertainty. 24(1), 47–74.
- Holmström., B., P. Milgrom. 1987. Aggregation and Linearity in the Provision of Intertemporal Incentives. *Econometrica*. 55 (2) 303-328.

Hubspot Blogs. 2014. How to select Sales Managers who can actually manage?. October 04.

- John, G., B. A. Weitz. 1989. Sales Force Compensation: An Empirical Investigation of Factors Related to Use of Salary versus Incentives Compensation. *J. Marketing Res.* 26 (February) 1-14.
- Joseph, K., A. Thevaranjan. 1998. Monitoring and Incentives in Sales Organizations: An Agency Theoretic Perspective. *Marketing Sci.*. 17(2)107-123.

- Joseph, K., M. Kalwani. 1998. The Role of Bonus Pay in Sales force Compensation Plans. *Industrial Marketing Management*. 27, 147-159.
- Kalra, A., M. Shi. 2001. Designing Optimal Sales Contests: A Theoretical Perspective. *Marketing Sci.*, 20(3) 170-193.
- Kellner, M. 2011. Tournaments as response to ambiguity aversion in incentive contracts. *Working Paper*.
- Klemperer, P. 1987a. The competitiveness of markets with switching costs. *RAND J. Econom.*. 18, 138–150.

Klemperer, P. 1987b. Markets with consumer switching costs. Quarterly J. of Econom. 102, 375–394.

- Koch, A. K., J. Nafziger. 2012. Job Assignments under Moral Hazard: The Peter Principle Revisited.*J. Econom. Management Strategy*. 21(4) 1029–1059.
- Krishnamoorthy A, S. Misra, A. Prasad. 2005. Scheduling Sales Force Training: Theory and Evidence. *International J. of Research in Marketing* 22(4) 427-40.
- Lal, R., V. Srinivasan. 1993. Compensation plans for single and multi-product salesforces: An application of the Holmstrom–Milgrom model. *Management Sci.*. 39(7) 777–793.
- Lazear, E. P., S. Rosen. 1981. Rank-Order Tournaments as Optimum Labor Contracts. J. of Political Economy. University of Chicago Press 89(5) 841-64.

- Lazear, E. P., P. Oyer. 2004. Internal and External Labor Markets: A Personnel Economics Approach. *Labour Econom.*. 11 (October) 527-554.
- Lim, N., M. J. Ahearne, S. H. Ham. 2009. Designing Sales Contests: Does the Prize Structure Matter? J. Marketing Res., 46(3) 356-371.
- Lim, N. 2010. Social Loss Aversion and Optimal Contest Design. J. Marketing Res., 47 (4) 777-787.
- Macintosh, G., L. S. Lockshin. 1997. Retail relationships and store loyalty: A multi-level perspective. *International J. of Research in Marketing*. 14(5), 487-497.

McKinsey Quarterly. 2012. Overcoming a bias against risk. (August).

- Milgrom, P., J. Roberts. 1992. *Economics, Organization and Management*. Prentice Hall, Englewood Cliffs.
- Misra, S., A. Coughlan, C. Narasimhan. 2005. Sales-force Compensation: An Analytical and Empirical Examination of the Agency Theoretic Approach. *Quantitative Marketing Econom.* 3(1), 5-39.
- Misra, S., H. Nair. 2011. A Structural Model of Sales-Force Compensation Dynamics: Estimation and Field Implementation. *Quantitative Marketing Econom.*, 9(3), 211-225.
- Moldovanu, B., A. Sela. 2001. The Optimal Allocation of Prizes in Contests. *Amer. Econom. Rev.*. 91(3) 542-558.

- Musalem, A., Y. V. Joshi. 2009. How much should you invest in each customer relationship? A competitive strategic approach. *Marketing Sci.*. 28(3) 555–565.
- Nalebuff, B. J., J. E. Stiglitz. 1983. Prizes and Incentives: Towards a General Theory of Compensation and Competition. *Bell Journal of Economics* 14(1) 21-43.
- Nye, M., J. Murphy. 2002. Forced Ranking and Development Planning, *International Congress* on Assessment Center Methods (October) (Accessed on August 05, 2013) [Available at http://www.assessmentcenters.org/pdf/Murphy_Nye.pdf]
- NY Times. 2014. Noncompete Agreements Hurt Workers and Stifle Competition. June 10.
- Palmatier, R. W., R. P. Dant, D. Grewal, K. R. Evans. 2006. Factors Influencing the Effectiveness of Relationship Marketing: A Meta-Analysis. J. Marketing. 70 (October), 136-153.
- Palmatier, R. W., S. Gopalakrishna, M. B. Houston. 2006. Returns on Business-to-Business Relationship Marketing Investments: Strategies for Leveraging Profits. *Marketing Sci.* 25 (September-October), 477-493.
- Palmatier, R. W., L. K. Scheer, J. E. M. Steenkamp. 2007. Customer Loyalty to Whom? Managing the Benefits and Risks of Salesperson-Owned Loyalty. *J. Marketing Res.*. 44 (2) May, 185-199.

Palmatier, R. 2008. Relationship Marketing. Marketing Science Institute. Cambridge, MA, (1-140).

Peter, L., J. R. Hull. 1969. *The Peter Principle: Why Things Always Go Wrong*. New York: William Morrow and Company.

Prendergast, C. 1999. The provision of incentives in firms. J. Econom. Literature. 37(1) 7-63.

- Rabin, M. 2000. Risk Aversion and Expected-utility Theory: A Calibration Theorem. *Econometrica*.68(5) 1281-1292.
- Ridlon, R., J. Shin. 2013. Favoring the Winner or Loser in Repeated Contests. *Marketing Sci.*. 32(5) 768-785.
- Ryvkin, D. 2013. Heterogeneity of players and aggregate effort in contests. J. Econom. and Management Strategy. 22(4), 728-743.
- Schöttner, A., V Thiele. 2010. Promotion Tournaments and Individual Performance Pay. J. Econom. Management Strategy. 19(3) 699–731.
- Sirdeshmukh, D., J. Singh, B. Sabol. 2002. Consumer Trust, Value, and Loyalty in Relational Exchanges. *J. Marketing*. 66 (January), 15–37.
- Shin D. 2008. Information acquisition and optimal project management. *International J. Industrial Organization*. 26(4):1032–1043.
- Shi, M., 2013. A Theoretical Analysis of Endogenous and Exogenous Switching Costs. *Quantitative Marketing Econom.*, 11, 205-230.

- Simester D., J. Zhang. 2010. Why are bad products so hard to kill? *Management Sci.*. 56(7):1161–1179.
- Simester, D., J. Zhang. 2014. Why do sales people spend so much time lobbying for low prices? *Marketing Sci.*. 33(6) 796-808.
- Steenburgh, T. 2008. Effort or Timing: The Effect of Lump-sum Bonuses. Quantitative Marketing Econom.. 6(3) 235–256.
- Subramanian, U., J. S. Raju, Z. J. Zhang. 2014. The Strategic Value of High-Cost Customers. *Management Sci.*. 60 (2), 494-507.
- Tax, S. S., S. W. Brown. 1998. Recovering and learning from service failure. *Sloan Management Rev.*.40, (Fall), 75-88.

Training. 2008. Keeping Forced Ranking out of court. 45(June), 40-43.

- Villas-Boas, J. M. 2004. Consumer learning, brand loyalty, and competition. *Marketing Sci.* 23, 134–145.
- Villas-Boas, J. M. 2006. Dynamic competition with experience goods. *J. Econom. Management Strat*egy. 15, 37–66.
- Villas Boas, J. M. 2015. A short survey on switching costs and dynamic competition. *International J. of Research in Marketing*. 32(2) 219–222.

Wall Street Journal. 2012. "Rank and Yank" Retains vocal fans. (January 31).

Wall Street Journal. 2013. Microsoft Abandons Stack Ranking of Employees. (November 12).

Wall Street Journal. 2014. Silicon Valley Tech Giants Struck Deals on Hiring, Pay. (Apr 24).

Wall Street Journal. 2015. How College Has Become a Riskier Investment. (January 29).

- Weinschenk, P. 2010. Moral Hazard and Ambiguity. *Working Paper*. Max Planck Institute for Research on Collective Goods, Bonn 2010/39.
- *World at Work*. 2012. Promotion Guidelines. December. (Accessed on June 06, 2014.) [Available at http://www.worldatwork.org/waw/adimLink?id=68185]
- Yang, Y., N. Syam, J. Hess. 2013. Thrill of Victory and Agony of Defeat: Emotional Rewards and Sales Force Compensation. *Quantitative Marketing Econom.* 11(4) 379-402.
- Zoltners, A. A., P. K. Sinha, S. E. Lorimer. 2011a. How to Manage Forced Sales Rankings. *Harvard Business Review Blog* (July 27).
- Zoltners, A. A., P. K. Sinha, S. E. Lorimer. 2011b. Who Owns Your Customer Relationships: Your Salespeople or Your Company?. *Harvard Business Review Blog*. (December 21).
- Zoltners, A. A., P. K. Sinha, S. E. Lorimer. 2012a. *Building a Winning Sales Management Team: The Force Behind the Sales Force*. Z S Associates.

- Zoltners, A. A., P. K. Sinha, S. E. Lorimer. 2012b. Think Twice Before Promoting Your Best Salesperson. *Harvard Business Review Blog*. (July 06).
- Zoltners, A. A., P. K. Sinha, S. E. Lorimer. 2012c. To Build a Great Sales Team, You Need a Great Manager. *Harvard Business Review Blog.* (July 23).
- Zoltners, A. A., P. K. Sinha, S. E. Lorimer. 2012d. In Sales, Hire for Personality, then Train for Skill. *Harvard Business Review Blog*. (August 29).