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The Improvement of Shewhart-Stable Time Series Processes by Applying Jensen-Shannon Complexity Measures to Characterize Emergent Structure

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THE IMPROVEMENT OF SHEWHART-STABLE TIME SERIES PROCESSES BY APPLYING JENSEN-SHANNON COMPLEXITY MEASURES TO CHARACTERIZE EMERGENT STRUCTURE

by

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ABSTRACT

Despite the nearly universal imperative to continuously improve processes, Shewhart-stable time series are often ignored for improvement because of various generalized assumptions and "rules", many of which are actually very context dependent. The process control literature presents widely divergent views, including the extreme position that stable processes are completely random and therefore any further compensation ("tampering") can only increase process variability. The traditional reductionist approach to process improvement characterizes underlying factors using statistical variance measures, which has been very effective for unstable processes. However, this approach, especially when it involves Shewhart control charts, is generally much less effective for directing the improvement of stable time series, often resulting in a transition to passive monitoring to await a special cause of variation. A model-free strategy founded upon information theoretic quantifiers was researched to instead develop an emergence-based perspective for stable process improvement. Jensen-Shannon complexity was mapped temporally with permutation entropy to reveal structural patterns of order that could direct further improvement, challenging the notion of tampering. Stable processes disclosed informative nonrandom structure corresponding with relative degrees of randomness, also challenging the notion of a constant system of "chance causes." In the future, similar emergence-based methods could provide a useful supplement to reductionist methods during the improvement of virtually all types of processes.

Keywords: process improvement, Jensen-Shannon complexity, quality control, stable process, permutation entropy, emergence

I dedicate this research to my two miracles of complexity,

Avery and Brienna.

May you grow to appreciate the immense value (and fun) of scientific inquiry.

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LIST OF ABBREVIATIONS

aaPE	Amplitude Aware Permutation Entropy
AIE	Applied Information Economics
ANOVA	Analysis of Variance
ApEn	Approximate Entropy
AR	Auto Regressive
c-chart	Count Chart for Poisson Model
CBM	Condition-Based Maintenance
CECP	Complexity-Entropy Causality Plane
CGM	Continuous Glucose Monitoring
C_{JS}	Complexity, Jensen-Shannon
CL	Center Line of Control Chart
C_{max}	Complexity, maximum
C_{min}	Complexity, minimum
CPEI	Composite Permutation Entropy Index
CQI	Continuous Quality Improvement
CS	Chaotic Systems
CUSUM	Cumulative Sum Chart
df	Degrees of Freedom
D	Embedding Dimension
DE	Dispersion Entropy
Decr#	Decreasing Variance Pseudorandom Time Series, # of Distinct States
DispEn	Dispersion Entropy
DOE	Design of Experiments
ECD	Entropy Complexity Diagram
EEG	Electroencephalogram
EKG	Electrocardiogram
ETE	Estimate Talk Estimate Approach
EWMA	Exponentially Weighted Moving Averages
EvOp	Evolutionary Operations
fBm	Fractional Brownian Motion
fGm	Fractional Gaussian Motion
fgPE	Fine Grained Permutation Entropy
gBm	Geometric Brownian Motion
GIGO	Garbage In, Garbage Out
GP	Goal Programming
H_S	Information Uncertainty (or Entropy), Shannon
IAUQ	Integrated Approach Based on Uniform Quantization
iPE	improved Permutation Entropy
i.i.d.	Independent and Identically Distributed
IMPE	Improved Multiscale Permutation Entropy
ImR	Individual and Moving Range Control Chart
Incr#	Increasing Variance Pseudorandom Time Series, # of Distinct States
JS	Jensen-Shannon Divergence
K-L	Kullback–Leibler Divergence
KS	Kolmogorov–Sinai Entropy

LCL	Lower Control Limit of Control Chart
LE	Local Effect
L-Z	Lempel-Ziv
mpb	musica popular brasileira
miPE	Multiscale Improved Permutation Entropy
mPE	Modified Permutation Entropy
MPE	Multiscale Permutation Entropy
MPR	Martin-Plastino-Rosso Complexity Method
mR	Moving Range
np-chart	Count Chart for Binomial Model
NCDF	Normal Cumulative Distribution Function
p-chart	Rate Chart for Binomial Model
PDF	Probability Density Function
PE	Permutation Entropy
PeEn	Permutation Entropy
PE-LE	Permutation Entropy- Local Effect
PID	Proportional Integral Differential Controller
PM	Preventive Maintenance
Pml	Probability of Marble Location
PRNG	Pseudorandom Number Generator
Q-Q	Quartile-Quartile Plot
Q_J	Disequilibrium, Jensen
R#	Random Process, # = Distinct Values
RR	Interval distance between two consecutive heartbeats
SaEn	Sample Entropy
SD	System Dynamics
SFI	Santa Fe Institute
SP	Stochastic Processes
SPC	Statistical Process Control
sPE	Spatial Permutation Entropy
SR	Stability Ratio
stPE	Spatio-Temporal Permutation Entropy
Temp	Temperature
TIS	Theil's Inequality Statistic
TQM	Total Quality Management
TR	Tokai Rika
TSA	Time Series Analysis
u-chart	Rate Chart for Poisson Model
UCL	Upper Control Limit of Control Chart
V#	Vertical Funnel Experiment #
WGN	White Gaussian Noise
WIP	Work in Process
wPE	Weighted Permutation Entropy
XbarR	Average and Range Control Chart
XbarS	Average and Standard Deviation Control Chart
XmR	Individual and Moving Range Control Chart
ZCR	Zero Crossing Rate

CHAPTER ONE: INTRODUCTION

This research was designed to address a common problem associated with Shewhart-stable time series processes. That is, despite the universal imperative that process improvement should be conducted continuously, this guidance is often ignored when a process is Shewhart-stable. This happens because of three common interconnected perceptions. First, the level of randomness associated with stable processes is generalized to be completely due to a *constant* system of "chance causes", which implies that the process can reveal little to no useful information. Second, since a stable process is considered random, any attempt to improve it can only make things worse. Third, since attempted improvements can only make things worse, other methods beyond control charts that might proactively guide the continued improvement of stable processes are often abandoned. Standard control charts provide little useful insight into the low-dimensional dynamics at play within a process that remains Shewhart-stable (Wheeler, 2010, May 5&6). Therefore, a stable process is often simply monitored to await the next unusual variance event.

The statistical process control literature does not help to resolve this problem. Widely divergent views are expressed regarding the improvement of Shewhart-stable processes. It is common to find the extreme position that *any* additional compensation is "tampering", which can only increase process variability. Slightly less extreme perspectives offer that some minor tweaking may be okay, but nonetheless, any effort short of a "fundamental" change to the process will inevitably be uneconomical, a waste of time, and lead to frustration. A "fundamental" change is often characterized as significant re-engineering of the process, to include advancements like incorporating new equipment, new procedures, new materials, etc.

This decision is not taken lightly because the incorporation of these changes is considered likely to set processes back to Shewhart-instability, which can have expensive effects on process outcomes.

One of the perceived shortcomings of Shewhart control charts is the faulty assumption of process stationarity. It is well known that the process mean will continuously wander to some degree, even for a stable process (Bisgaard, 2008; Box & Paniagua-Quiñones, 2007). The effect can be mitigated somewhat by various methods, but the central problem involves awareness and compensation for dynamics that are constantly changing for a time series. This problem has become especially acute with the ubiquitous incorporation of high-rate in-process sensors, causing some practitioners to advocate the abandonment of control charts altogether (Gunter, 1998). New insights into process dynamics could advance the state of the process control practice.

Therefore, this research incorporated a review of diverse extant methods that have been employed to reveal causal dynamics for stable processes- without focusing on control charts. This review revealed that the traditional approach to process improvement usually seeks some form of partition for underlying factor dynamics as either signal or noise using measures of statistical variance. This has proven to be very effective for Shewhart-unstable processes. However, this approach has generally been much less effective to direct the improvement of Shewhart-stable processes, in part because signals are more difficult to discern when processes are Shewhart-stable. Since stable process performance tends to be relatively consistent, further improvement can seem challenging and the return on investment for such efforts can sometimes seem trivial. The result is therefore often a transition from active improvement to passive process monitoring with control charts.

A paper by Georgantzas & Orsini (2003) explored new territory in Shewhart-stable process improvement by employing system dynamics fundamentals to evaluate the traditional funnel experiment. Ultimately, the unique insights from this paper helped to define a strategic path for the research herein. Most of the other reviewed methods employed reductionist approaches to improve Shewhart-stable processes. The Georgantzas & Orsini paper instead relied on *emergence-based* insights from system dynamics. In consequence, a strategy was similarly developed for this dissertation to further explore emergence–based process improvement for Shewhart-stable processes by applying Shannon information and structural complexity measures. This research used empirical data collected from improved stable processes to answer the question: *Can a methodology based on emerging structural complexity provide information useful to direct the continued improvement of a Shewhart-stable process?*

To begin to answer this question, model-free permutation entropy methods were applied to various types of time series processes. However, Shewhart-stable processes were especially affected by shortcomings associated with identical values within permutation entropy tuples. Therefore, methods developed to mitigate this issue were reviewed in the literature. None of these methods were ideal, and so a different approach that incorporates the *local effect* of each tuple was investigated and validated for use in this research. Named the permutation entropy-local effect (PE-LE) method, its application provided evidence that all processes, including Shewhart-stable processes, are comprised of *inconstant* systems of chance causes. Instead of continuing to accommodate the blanket assumption of stable process randomness, this research instead determined the varying levels of nonrandom dynamics at play. Improvements that were intended to reduce process variation could thus be evaluated in terms of changing process homogeneity and uncertainty. The assessment of two highly stochastic processes also supported the conclusion of the Ramsey theory that no process is absolutely random.

The application of permutation entropy to evaluate process randomness was an important intermediate procedure, but it did not directly characterize emerging structural dynamics to answer the research question. So, the Martin-Plastino-Rosso (MPR)-method was evaluated for its provision of insights associated with structural complexity. The MPR-method applied probabilistic assessments of time series using permutation entropy and disequilibrium to arrive at measures of Jensen-Shannon complexity. When coupled with normalized Shannon entropy measures, processes could be compared equitably via relative positions on the Complexity-Entropy Causality Plane developed by Rosso et al. (2007). Results confirmed that Jensen-Shannon complexity measures disclose informative structural patterns that can be causally correlated with improved Shewhart-stable processes, thus answering the research question.

Although these results were sufficient to answer the research question, a potentially useful extension was also investigated for its capability to provide additional insights into continually evolving process dynamics. Named the Entropy-Complexity Change Diagram, it was designed to reveal how the structural complexity of a process changed in time relative to the maximum randomness condition for an equivalent time series. The visualization of decreasing relative structural complexity was correlated with the continued improvement of empirical Shewhart-stable processes.

Future research could further extend these emergence-based methodologies to direct continued process improvement. The PE-LE method could be improved, or may at least stimulate similar ideas to mitigate identical values within permutation entropy tuples. With the appropriate automation, algorithms could be developed to track structural complexity in near real time for virtually any kind of time series process. Similar methods could also be applied as a supplement to existing reductionist statistical methods to ultimately improve the practice of process control.

Research Question

Can a methodology based on emerging structural complexity provide information useful to direct the continued improvement of a Shewhart-stable process?

Figure 1. Complexity quote illustration. From Dan Hamilton Art, http://www.danhamiltonART.com.

Research Objectives

This research was *exploratory*, quantitative, and applied, and was guided by four objectives related to answering the research question. The first objective was to evaluate existing methods that have already been employed to improve Shewhart-stable processes to determine how emergence-based improvement might be developed to potentially complement these methods. Results are provided in the first two parts of the Literature Review section.

The second objective was to gather data from a highly-controlled, Shewhart-stable, incrementally-improved process to ensure that methods were tested and developed using empirical data applicable to real world process improvement scenarios. To support this objective, four experiments were conducted using vertical translation of a funnel, as described in the Methodology section.

The third research objective was to determine whether an information-theoretic method could measure degrees of randomness for Shewhart-stable processes, thereby quantifying ongoing nonrandom dynamics. This research would test the blanket assumption that stable processes are comprised a constant system of “chance causes”. Results are presented in the Process Randomness Study section of the Findings.

The final objective was to determine whether a chosen quantifier of structural complexity could direct the improvement of Shewhart-stable time series processes. While investigating permutation entropy as an underlying measure, it became apparent that the method to mitigate identical values within tuples was going to be an important consideration for this research. As a consequence, a method based on *local effect* within tuples was developed, which then created a sub-objective to characterize and validate this method. The MPR-method was selected to develop the structural complexity quantifier, but while investigating the evolution of Jensen-Shannon complexity, it also became evident that processes change in different ways relative to equivalent random time series. Therefore, a method based on log change calculations was explored, yielding another sub-objective to characterize the Entropy-Complexity Change Diagram.

Research Problems

The problem that motivated this research was the abandonment of Shewhart-stable time series for continuous process improvement. To better characterize the problem, this section explores the reasons that process monitoring often takes the place of process improvement. This section will also delve into some of the most questionable and conflicting perceptions that perpetuate this problem. The first assumption to be explored is the characterization of stable processes as constant systems of chance causes. The second is associated with the predictability of stable processes. The third section will explore the ubiquitous rules that are frequently offered using absolute terms “always” and “never”, regarding stable processes. The final section will explore differing perceptions about how best to evaluate the dynamics of stable processes.

In the course of this research, an unanticipated issue motivated the investigation of an additional research problem. That is, permutation entropy was found to suffer from limitations associated with identical values in tuples, requiring the determination of a countermeasure. The second part of this section will briefly introduce this additional research problem.

Abandonment of Stable Process Improvement

In the practice of statistical process control, Shewhart-stable time series processes are often ignored for improvement, despite the nearly universal mandate that improvement should be a continuous effort. This happens because stable processes are often generalized to be caused by a constant system of chance causes, which implies that any additional compensation ("tampering") can only increase process variability. Since many process owners accept this generalization at face value, they simply don't consider other methods that might proactively guide the continued improvement of stable processes. The common paradigms associated with randomness, control limits and predictability can be very context-dependent, which contributes to widely divergent perceptions in the process control literature. The majority of these perceptions are useful when actually attributed to specific contexts, but the ubiquitous promotion of absolute, generalized rules continues to remain problematic. Some practitioners have thus decided to abandon control charts altogether and instead focus on diagnosis and continuing performance improvement by other means. Examples of these methods and various factors contributing to this research problem are presented in more detail in the next four sections.

The Chance Causes Assumption

The idea that stable processes are characterized by a constant system of chance causes probably started when Dr. Walter Shewhart (1931) published *Economic Control of Quality of Manufactured Product*. His definition of process control required:

1. "That a controlled quality must be *variable* quality", and
2. That it must demonstrate, "constant variability *within limits*" (p.6).

He also stated that, "…any unknown cause of a phenomenon will be termed a chance cause" (p.7), and postulated that, "Constant systems of chance causes do exist in nature" (p.12). Shewhart offered a distinction between nature and production systems when he stated, "...in the majority of cases there are unknown causes of variability in the quality of a product which do not belong to a constant system" (p.14), which he called assignable causes. Many practitioners instead call these special causes today, and the remaining "chance causes" are instead called common causes. Following Shewhart, practitioners often consider stable processes, with only common causes of variability, to be *constant* systems of variability or randomness (chance causes).

Practitioners have often embraced these original ideas to further expound the absolute randomness of Shewhart-stable processes. One of the more influential proponents and a close associate of Dr. Shewhart was Dr. W. Edwards Deming. In *The Team Handbook*, written by Scholtes, et al. (2003) Deming stated:

> A stable process, one with no indication of a special cause of variation, is said to be, following Shewhart, in statistical control or stable with respect to the quality-characteristic measured. It is a random process. Its behavior in the near future is predictable. (p.2-15)

Then, a few paragraphs later:

> A system may be stable, yet turn out faulty items and mistakes. To take action on the system in response to production of a faulty item or a mistake is to tamper with the system. The result of tampering is only to increase in the future the production of faulty items and mistakes, and to increase costs- exactly the opposite of what we wish to accomplish (p.2-15).

In many of Deming's books and lectures, he repeatedly demonstrated the concepts of tampering and randomness in stable processes through two experiments. For Deming's (1994) red bead experiment, 800 red beads were mixed with 3200 white beads, and blindfolded participants withdrew samples of 50 beads with the stated goal to withdraw only white beads. Naturally, they failed. A key objective was to demonstrate the futility and invariably negative consequences of trying to manipulate a process that is based completely on randomness (Chapter 7).

In Deming's (1986) traditional funnel experiment, which he attributed to Dr. Lloyd S. Nelson (p.327), 50 marbles were dropped through a funnel maintained at a fixed height above a point target. The resting place of each marble was marked. Then, three different rules for lateral translation of the funnel were tried to compensate for the variation around the target. For all three rules, the variation increased compared to leaving the funnel alone. A key objective was again to demonstrate the futility and invariably negative consequences of trying to improve a process that is based completely on randomness (Chapter 11). Unfortunately, the foreseeable results of these two experiments have been widely generalized to represent any and all stable processes, regardless of the context. It is often simply assumed that the factors contributing to variance must be essentially random or else the process would not remain stable.

Dorian Shainin provided essentially the opposite perspective. In an October, 1994 interview (ReVelle Solutions LLC, 2015, September 8), Shainin stated, “Nothing happens without a reason in this world, and it’s not 10 things at once, or 15 things. It’s not common causes. It’s not things at random. It’s a single interaction or main effect.” He then presented a slide with the following phrase: “Axiom #1: Nothing in manufacturing is RANDOM (equally likely)”. He followed this by commenting, “In fact, the only thing that’s random in this world is a table of random numbers, artificially made that way by humans, in order to have the probabilities come out equal” (26:10-27:40). In agreement with his father’s earlier sentiment, Richard Shainin (2012) wrote, “Even when a system meets the Shewhart standard of equilibrium (all common cause), there is always one cause-effect relationship whose contribution to variation is stronger than the others” (p.174).

Interestingly, despite Deming’s positions on randomness and tampering, he was also an enthusiastic proponent, particularly in *Out of the Crisis*, for, “…the necessity to reduce constantly the variation from common causes” (p.136). Yet this position was essentially at odds with some of Shewhart’s (1931) writings such as, “When a phenomenon has been shown to exhibit control, we have likely gone about as far as we can in detecting the existence of assignable or discoverable causes by standard tests” (p.159). This conflict will be addressed in greater detail in the Literature Review section. The Findings section will also reveal evidence from this research that is contrary to the common generalizations of randomness for Shewhart-stable processes.

The Predictability Paradigm

Many, and perhaps most, practitioners also consider Shewhart-stable processes to be predictable. But again, there are context-dependent exceptions. Shewhart's (1931) initial teachings provide support for this idea: "A phenomenon will be said to be controlled when, through the use of past experience, we can predict, at least within limits, how the phenomenon will vary in the future" (p.6). And:

> …to glean what we can about the workings of unknown chance causes which are generally acknowledged to be controlled in the sense that they permit of prediction within limits. Perhaps no better examples could be considered than length of human life and molecular motion" (p.8)

Shewhart's reference to prediction based on perceived *natural* limits is certainly not a new concept. Ancient astronomers predicted the relative motions of the moon and planets based on observations of the limits of travel of these bodies over time (Wright, 2002). In developing a theory of color in the early 1800's, Johann Wolfgang von Goethe wrote that, "With light poise and counterpoise, Nature oscillates within her prescribed limits, yet thus arise all the varieties and conditions of the phenomena which are presented to us in space and time" (Matthaei, 1971). Problems arise when comparing such natural limits, based upon empirical observation, with Shewhart's (1931) method for determining control limits. This concern generally arises because his criteria for establishing 3-Sigma limits were also based upon a subjective personal view of economic correlations. Since Shewhart's basis for establishing control limits has repeatedly been questioned, the assessment of process predictability based upon these limits has also been questioned. This will be discussed in greater detail in a different context shortly.

When authors attempt to associate predictability with absolute randomness, another context-dependent problem arises because absolute randomness is physically unachievable (Kosko, 2006, p.66), in compliance with the Ramsey theory (Motzkin, 1967; Prömel, 2005; Stillwell, 2010). It is an unrealistic idealization to insist that none of the process components (factors) interact, or if they do, that all of their interactions remain perfectly balanced. Instead, accepting that no process is truly random means that nonrandom dynamics will necessarily be hidden in the noise, causing some factors and interactions to have more influence than others. The assessment of predictability can thus become a problem of understanding the extent and influence of nonrandom dynamics.

Conclusions about nonrandom dynamics and predictability can also depend upon the level of process observation. Contrary to Shewhart's focus on underlying chance causes, stable process predictability can also be characterized from a macro-level perspective in terms of emergent process behaviors that are not discernable from lower-level details in isolation. According to Waldrop (1992), it is possible to understand emergence-based processes yet not be able to make reliable predictions about them (p.306). Additionally, some chaotic processes are stable with macro-level behavior that is deterministic, yet these chaotic processes are unpredictable and very sensitive to initial conditions (Boeing, 2016). Processes can be Shewhart-stable, yet represent systems that have been known to reveal chaotic behaviors, including machinery (Litak et al., 2009; Priesmeyer, 1992; Redelico et al., 2017), production systems (Deshmukh, 2003; Sajid et al., 2015), and chemical reactions (Eiswirth, 1993; Elnashaie, 2006).

Depending upon the research, it is also possible to evaluate process behavior at the micro-level view. In this context, a random process can be considered the *least* predictable process. For instance, consider the physical analogy wherein the position of one particle is being tracked within the volume of an ideal gas: If the position of all of the particles was perfectly random, then the position of the particle of interest would be perfectly *un*predictable. A pdf representing all of the particles in this perfectly random system would be a uniform distribution. At the opposite extreme, the *most* predictable process would present only one possible answer, such as the position of a particle within an ideal crystal. Because its position is perfectly fixed (zero randomness), its position is perfectly predictable. A pdf of this system would show one state at unity and the remaining states at zero.

The contrary perspectives and paradigms elucidated in this section provide support for the idea that context can make a significant difference when characterizing processes and planning for process improvement. Similar characterizations of varying frames of reference will be discussed in the next section, but with a focus on generalized rules.

Rules, Instead of Tools

When Shewhart-stable processes are discussed in the process control literature, it is often in terms of control chart monitoring. Given this control chart-centric frame of reference, the focus often switches from proactive process improvement to specifying limitations. In general, the presentation often reverts to rules, instead of tools. If continuous process improvement remained the goal, the emphasis could simply shift to alternative techniques, new contexts, and new paradigms. But instead, guidance laden with absolute adverbs usually pervades, sometimes beginning to resemble irrefutable laws of science. Comments similar to those in Figure 2 are ubiquitous in the literature.

- A stable process is always predictable
- Improvement of a stable process always requires a fundamental change
- Looking for improvements for a stable process is always a waste of time/ low-payback strategy/ uneconomical/ will lead to frustration
- A stable process is always performing up to its highest potential
- Comparing only two results when they both come from a stable process will never yield insights of any importance.
- Adjustment of a stable process always increases variation/ waste/ costs
- Adjustment of a stable process always represents unnecessary adjustment
- A stable process is always operating in economic equilibrium
- A stable process is always a constantly random process
- A stable process is always operating with minimum variation for the conditions
- Adjusting a stable process for deviation (funnel rule 2), relative to the goal (funnel rule 3), or based on the previous result (funnel rule 4) always makes things worse
- A stable process is always required before designed experiments will be useful/ hypothesis testing will be accurate/ a statistical model can be specified/ measurements will have meaning
- A stable process should always be considered a mix of interacting factor relationships where no factor relationship is dominant

Figure 2. “Rules” associated with Shewhart-stable processes.

To be sure, each of these pronouncements about stable processes is true in a great many contexts, especially when the perspective is artificially restricted to one variety of control chart. Yet, none of them is always true, all of the time. Given this perspective, these rules could be considered a deterrent to progress. Perhaps the only absolute statement that can actually withstand the test of time is this: Absolute statements are *always* wrong in some context. Greater acknowledgement of this reasoning would encourage guidance incorporating specific context, instead of absolute rules based on assumptions and sweeping generalizations. This especially includes the pernicious position that improvement of a stable process *always* represents tampering. It would be more productive to consider the diverse methods that have proven effective for improving Shewhart-stable processes, as will be presented in the Literature Review section.

Numerous other problems arise from painting Shewhart-stable processes with such broad brushes. For example, Shewhart's (1931) basis for establishing the control limit *rule* of 3-Sigma to differentiate unstable from stable process behavior was somewhat subjective. Although he referred extensively to statistical principles and Chebyshev's theorem to substantiate his logic, he essentially determined through experience that, "…3 seems to be an acceptable economic value" (p.277). Decades earlier, Karl Pearson preceded Shewhart (without reference to economics), stating, "take 3σ as definitely significant" (Gosset, 1906), perhaps lending some credence to the basis for this rule. Although this 3-Sigma definition for control limits has unquestionably facilitated unstable process improvement over the years, its value has also been questioned in numerous process control situations (Woodall & Faltin, 2019). Despite Deming's (1986) creative index entry: "Modified limits, never" (p.501), a number of authors have advocated context-dependent alternatives, which will be presented in greater detail in the Literature Review section.

Evaluating Stable Process Dynamics

The constant flows of energy, matter, and information in physical systems means the relationships between all of the factors operating on the system are constantly changing. It is commonly stated in the process control literature, such as Box & Luceño (1997) and Hindle & Wheeler (2017), that processes will not stay stable long, thanks to entropy. It would be difficult to deny such sentiment, since this tendency toward disorder actually *is* a law of science (the 2nd Law of Thermodynamics). Mechanical parts will wear down over time, human service-providers will become tired or complacent, environmental factors will deviate from optimal conditions. Wheatley (1999) said, "If we want progress, then we must provide the energy to reverse decay" (p.19). The conundrum in the practice of process control is how to apply energy to most effectively leverage this awareness of constant change.

Process improvement efforts have traditionally taken a reductionist approach, attempting to isolate factors from each other statistically so their individual contributions (and interactions) to the whole may be better understood. However, problems can arise when these methods are used to evaluate stable processes. For example:

- Assumptions about process stationarity are generally wrong, even for stable processes (Box & Paniagua-Quiñones, 2007).

- Contributions to statistical variance can stem from initial conditions, intrinsic aspects of the process, environmental influences, and combinations of all three, which can be challenging to isolate and identify.
- Empirical insight into the underlying process dynamics can be inaccessible for a variety of reasons, such as insufficient measurement resolution.
- The dynamics of an ongoing process can change considerably by the time designed experiments are completed, limiting their relevance.

Nevertheless, this research assumed that *all process behavior is caused*, that it should not be generalized away to "randomness", and that new paradigms can be leveraged to continuously pursue process improvement. In a relevant lecture entitled, *There's No Such Thing as a Common Cause,* Pyzdek (1990) proclaimed: "Unexplained variation in a process is really just a measure of our level of ignorance about the process" (p.104). Also, Henri Poincare (1908) dedicated an entire book chapter entitled "Chance", presenting similar sentiment (Chapter 4).

The solution proposed in this research is that all physical processes possess a varying *mixture* of randomness and nonrandomness at all levels. Instead of *assuming* i.i.d. or perfect randomness for stable processes, this research evaluated methods to quantify the changing *level* of randomness for each process, to better direct improvement efforts. Instead of attempting to isolate the effects of individual factors for a stable process, the approach embraced in this research studied the overall effects of all interactions simultaneously.

Contrary to the reductionist approaches that attempt to partition signals and noise, this emergence-based approach leveraged the enduring constancy of change in processes by mapping the temporal and causal evolution of processes, which concurrently facilitated useful comparisons among diverse processes. It was evident that even stable processes reveal constantly changing patterns of order and disorder over time at the system level, which can be correlated with process improvements. Measures of Jensen-Shannon complexity provided tracking of nonrandom structural dynamics as process states evolved.

Additionally, the accumulated flows of process information were measured probabilistically over time with permutation entropy. However, the traditional permutation entropy methodology suffered from limitations associated with identical values in tuples. Thus, numerous methods to mitigate this problem were considered, yielding the development and validation of a solution. An overview of this unanticipated research problem is provided next.

Mitigation of Identical Values in Tuples

In their seminal paper establishing the permutation entropy method, Bandt & Pompe (2002) first identified the problem of identical values in tuples. They believed their method would work well as long as the same value did not appear within a tuple (an introductory explanation is provided below). Fortunately, such equal values are relatively rare in continuous series. However, equal values are typical in discrete series data, and discrete series are common with the ubiquitous digitization of data streams. Because Shewhart-stable processes demonstrate relatively low variability, they are naturally more affected by this problem than Shewhart-unstable processes.

A simple introduction to the problem is provided here for the unfamiliar reader. The Methodology section provides a more detailed description. Start by encoding the first group of *D* consecutive numbers in a time series based on their relative amplitudes. For this example, assume the number of values in a tuple (the embedding dimension) is D=3. Then jump τ numbers to the right and start over. For this example (and all of this research), each jump to the right (time lag) was $\tau = 1$. Continue this process iteratively in real time or to the end of the recorded time series.

Each tuple will represent one of $D!$=6 possible distinct symbols, as shown graphically in Figure 3. Possible distinct tuples are $\Omega_3 = \{(012)(021)(102)(120)(201)(210)\}$, presented in the same order as in the figure. Note that none of these symbols account for an equal value within a tuple, which would look like, for example, (001) or (101), and herein lies the problem. The ultimate question has been, "What is the best way to account for identical values within a tuple that will *minimize* the erroneous representation of emergent process dynamics?"

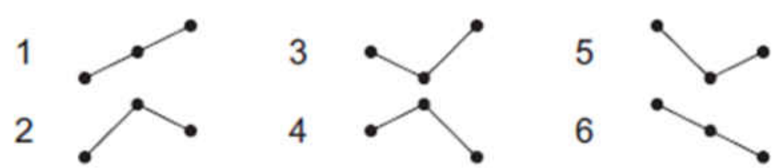


Figure 3. Distinct permutation tuples for embedding dimension D=3. From *Classifying Cardiac Biosignals using Ordinal Pattern Statistics and Symbolic Dynamics* by Parlitz et al. Copyright © 2012 by Elsevier. Reprinted with the permission of Elsevier. All rights reserved.

Bandt & Pompe recommended adding small amplitude noise to break the identical values. This dithering can actually improve the information throughput for some nonlinear time series via stochastic resonance (Kosko, 2006, p.149). However, in a paper published on the topic 15 years later, Zunino et al. (2017) stated that, in practice, this addition of noise (also known as random imputation), "has been rarely implemented" (p.1883). Additionally, Traversaro et al. (2018) pointed out that adding noise may fix one problem but add others: "Random imputation may overestimate the entropy, as it adds random noise to the series, and masks forbidden patterns by inducing those missing patterns to appear, concealing this dynamical property characteristic of chaotic dynamics" (p.075502-8).

Many methods have been devised to handle identical values within permutation entropy tuples, as presented in the Literature Review section. However, every method reviewed is perceived by other researchers to possess some sort of shortcoming, and there is little agreement on the ideal method to address the problem. Given the diversity of new methods proposed, this appears to be a fairly active area of research. Because the research herein was focused on applying permutation entropy to stable processes, the identical value problem was a very significant consideration. As a consequence, a method based on "local effect" within tuples was researched, validated, and applied to mitigate identical values for all processes evaluated in this research.

This Permutation Entropy- Local Effect (PE-LE) method is presented in detail in the Methodology section. Briefly, it is based on the local effect of relative amplitudes within tuples in concert with an existing method involving "statistically complete" parsing. This parsing removed from the analysis a varying percentage of tuples that were problematic (non-pattern following), based upon unique process characteristics such as measurement resolution and the corresponding quantity of distinct values in the series. Although the PE-LE method presented certain improvements, especially for low-dimensional stochastic processes, it was certainly not the ultimate fix. Indeed, two inherent shortcomings were immediately known before even starting the validation. However, PE-LE appeared to be comparable in some ways to existing methods, and may provide fertile ground along the path to eventually developing a more perfect solution.

Research Gaps

No literature was discovered that:

- Applied the vertical funnel experiment to accumulate stable process data.
- Applied structural complexity to continuously improve Shewhart-stable processes.
- Applied Jensen-Shannon complexity to continuously improve Shewhart-stable processes.
- Applied local effect to accommodate identical values in permutation entropy tuples.
- Simultaneously displayed changing structural complexity and entropy based on log change evaluations that were relative to the maximum randomness condition for an equivalent time series.

Methodology Overview

The Methodology is presented in six sections. The first part presents an overview including a flowchart and a simplified list of steps used to accomplish the analyses. The next section presents the data collection and validation effort associated with the four vertical translation funnel experiments. Then, the methods used to acquire or develop the data for 18 other process is presented. This is followed by the data visualization plan, which consisted of four primary display techniques. The next section provides the detailed theoretical background and associated mathematical computations applied in this research for information-theoretic quantifiers. The final section reveals the method developed for log change evaluations.

Findings Overview

Five interrelated sections of results were presented in the Findings. First, the results for the four vertical translation funnel experiments were presented. Specifically, the first three experiments were identified as Shewhart-stable, improved, and the last as Shewhart-unstable, improved. Next, a brief process randomness study provided evidence for differing levels of randomness associated to Shewhart-stable processes and supported the position that absolute randomness is an idealized impossibility.

The third section focused on validation of the PE-LE method. Three axioms for permutation entropy methods were developed and tested using different process types, showing promising results. Comparisons were then made with a published study that applied the time-ordered imputation method. PE-LE appeared to more accurately represent the probability space by visual inspection for two stochastic processes in specific situations. The measurement resolution effect was then characterized, with results supporting the assumption that measurement resolution could be applied to tailor and fine-tune an analysis based on research goals. In general, at lower measurement resolutions, beneficial trade-offs could be made if the quantity of distinct values can be optimized. Finally, numerous shortcomings were elucidated for the PE-LE method.

The next section provided PE-LE results for all of the processes evaluated in this research. These results consisted of pdfs for the PE-LE tuples at D=4 and counts/ densities based on local effect. This section provided reference support for later sections that made comparisons among these processes.

The fifth section focused on answering the research question by evaluating Jensen-Shannon complexity in various ways. Fundamental conclusions were supported by applying the Martin-Plastino-Rosso (MPR)-method to PE-LE results to display processes on the Complexity-Entropy Causality Plane (CECP). Decreased structural complexity and increased randomness corresponded with greater improvement to Shewhart-stable processes. A number of other process types were also displayed simultaneously on the CECP to provide more insight into the utility of this technique.

To explore relative change dynamics, time series results were then compared to maximally random equivalent time series. This normalization procedure was designed to allow equitable comparisons between very different processes while providing insight into how levels of randomness and complexity change over time. The log change method was applied to create “change charts” for normalized Shannon information and Jensen-Shannon complexity that displayed ongoing changes relative to a theoretical absolute randomness baseline. The three improved Shewhart-stable funnel experiments revealed similar patterns by turning toward decreased relative structural complexity corresponding with continued improvement. Various other processes were also evaluated to better characterize this methodology. Results emphasized the importance of applying the appropriate measurement resolution based on research goals when PE-LE is the underlying mechanism.

The results for both change charts were then combined to simultaneously display results on the Entropy-Complexity Change Diagram (ECCD). After plotting numerous process types, results suggested that improving processes migrate toward the origin, which represents maximum theoretical randomness and decreasing relative structural complexity. However, it was also discovered that many processes require the analysis of many thousands of tuples before their values stabilized. These results might find utility in future research, as discussed in the final section in the Findings.

Research Contributions

This research provided evidence to dispel the assumptions that Shewhart-stable processes should be represented as constant systems of chance causes and that improvement of Shewhart-stable processes always represents tampering. Evidence was also provided to support the conclusion that perfect randomness is an idealized impossibility.

However, probably the most significant contribution was associated with answering the research question regarding the application of structural complexity to direct the improvement of Shewhart-stable processes. Three techniques based on structural complexity displayed patterns corresponding with levels of process improvement: The Complexity-Entropy Causality Plane, the $\tilde{C}_{JS}$ change chart, and the Entropy-Complexity Change Diagram.

The Complexity-Entropy Causality Plane provided the most robust characterization of stable process improvement. However, with the appropriate software coding and sensing/measuring of process characteristic, the Entropy-Complexity Change Diagram (ECCD) could also provide near real time dynamic viewing of simultaneous changes in process randomness and structural complexity. Either diagrammatic technique could be used to monitor processes for unanticipated changes or for cause-and-effect analyses associated with intended process improvements, even for stable processes. Numerous processes could also be viewed simultaneously on the same plot to explore potential correlations.

Another research contribution involved the mitigation of identical values in permutation entropy tuples. Depending on the number of distinct values, stable processes can be especially prone to the appearance of identical values. Although imperfect, the PE-LE method employed in this research seemed to provide certain advantages when applied with an appropriate measurement resolution, in part by including more causal information in the analysis than some other methods. It is hoped that the PE-LE method developed in this research can be improved upon, or at least that it might stimulate similar ideas, as experts in this field continue to advance mitigations for identical values within tuples.

Finally, the vertically translated funnel experiments were applied to study Shewhart-stable processes. Perhaps this reliable empirical methodology will find utility beyond this research effort. Although Shewhart-stable processes were the focus of this research, the investigated information-theoretic, emergence-based techniques could direct continued process improvement in near real time for virtually any kind of ongoing time series process. Similar methods may find utility as a supplement to existing reductionist statistical methods, especially for near real time tracking of evolving time series processes.

Caveats and Unique Terminology

Some of the information theory terminology used in this dissertation is fairly unique, especially for a reader accustomed to topics in process control. Therefore, these terms are described up front, to hopefully alleviate confusion. At the same time, the target audience is a reader reasonably aware of SPC fundamentals and it would be inefficient to provide excessive background information. Hence, in addition to terminology clarifications, certain caveats are also included below.

Process Control- It is assumed throughout this manuscript that the reader has an intermediate level of SPC knowledge, especially regarding control charts and the lateral funnel experiment. Both of these topics are already well-developed and neither was the focus of this research. However, both are discussed and control charts were applied for a portion of the funnel experiment analysis. As such, the equations for XmR charts have been provided in the methodology section and a cursory overview has been provided at the end of this introduction section. Readers that need to dig deeper will find a great number of accessible resources. Popular books include Ott, et al.'s *Process Quality Control* (2000) and Wheeler's *Understanding Variation* (2000).

Stable Process- The notion of a stable process has numerous meanings in various contexts. In probability theory, a stable process is a certain type of stochastic process, defined by a family or distribution of random variables. Brownian motion and the Wiener process are examples of such stochastic processes. This assumption of near perfect randomness is too limiting for the research herein, which is intended to be applicable to the kind of high-rate time series processes that can be expected in say, chemical production or on a manufacturing line. A stable time series process in this context will have continuously varying degrees of randomness to include occasional outliers that momentarily exceed three standard errors of variation from the process mean (or median). Hence, the meaning of "stable" that is intended throughout this dissertation is specifically "Shewhart-stable" with respect to control chart applications.

To avoid overly pedantic and restrictive interpretations of process stability, this research assessed Shewhart-stability in terms of a stability ratio presented by Ramirez & Runger (2006). According to Ramirez (2017), the benefits of this method include the capability to, "Develop a more objective and consistent way to determine if a parameter is in a state of statistical control [and] evaluate the process stability in hundreds of a parameter in an efficient manner" (p.3). Also important: Whenever the word "stable" is used alone in this text, Shewhart-stable is still implied.

Process Improvement- For the purposes of this research, process improvement is defined herein as actions taken that reduce variation, which also appears as decreasing structural complexity and increasing process randomness over time. The proportional relevance of these three definitions depends upon the technique being applied for process analysis.

Within Limits- Means that process data is contained within the control limits on a Shewhart control chart. Also referred to as within bounds, in control, stable, predictable.

Common Causes- A category of variation associated with a Shewhart-stable process, which is sometimes referred to as noise, chance cause, non-assignable, natural, or routine variation.

Special Causes- A category of variation associated with a Shewhart-unstable process, which is sometimes referred to as signal, assignable, unnatural, or exceptional variation. It was not necessary to apply control chart zone test rules in this research to define special causes.

Graph Symbology- Some of the graphics were so information dense that the primary differentiation between datasets was provided by different coloring. Descriptive labels were included as required, but it is recommended to print copies in color to improve clarity.

Accent Symbology- Hats such as $\hat{S}$ represent estimators, bars such as $\overline{mR}$ represent means, twiddles such as $\widetilde{H}$ represent normalized data for a time series. Normalized H_s is sometimes presented without an accent, especially in imported graphics. In these instances, the word "normalized" is added in descriptions or the context reveals normalization because the value range extends from zero to one.

Shannon Entropy- Interchangeably referred to as information, homogeneity, randomness, or uncertainty throughout the text. Although thermodynamic entropy is similar in many respects, it is not the same. In the few instances where thermodynamic entropy was mentioned, an effort was made to reduce opportunities for confusion by applying unambiguous context.

Tuple- This phrase represents a symbolic permutation entropy grouping of adjacent data. Tuple length is based on a selectable "embedding dimension", D. Tuple is, unfortunately, also called motif, window, sliding window, slice, symbolic word, pattern, sequence of symbols, order, and vector by various authors.

Measurement Resolution- The permutation entropy- local effect method is affected significantly by measurement resolution. For processes that are otherwise identical, different quantities of distinct values available to permute was found to yield different results. The measurement resolution is specified in terms of, e.g., integers, tenths, hundredths, which would correspondingly results in an increasing quantity of distinct values as measurement resolution is increased. The quantity of distinct values can be determined via two methods. The first method, which was employed in this research, would determine the *actual* number of distinct values that resulted for each time series. An example is a series for which measurement resolution was to the thousandths place resulting in 3,021 actual distinct values on the interval [0.000, 345.860]. The second method would determine the *possible* or *accessibl*e number of distinct values. The previous example to the thousandths place would yield 345,861 possible distinct values on the interval [0.000, 345.860]. This distinction if not to be confused with the determination of the number of accessible distinct states, K, for permutation entropy calculations, which applied the maximum possible number of tuples (K=42 for the PE-LE method at D=4), regardless of how many distinct tuples actually appeared.

Identical values- The identical numbers within a tuple are also called equal values and ties in various places.

Logarithms- All appearances of "log" are base 2, with units in bits. As such, the subscript 2 was not added to log functions. Log base *e* was applied for log change computations only, and the "*ln*" nomenclature therefore appears in those two equations.

Reductionism vs. Emergence- Reductionist methods assume a process can be comprised of no more than a sum of its parts. Therefore, the underlying dynamics can be better understood by reducing a process into its constituent factors. The better the factors are understood, the better the process is understood. Emergence-based methods assume a process is comprised of complex interactions, where the resulting process behavior is greater than the sum of its constituent parts. The factors are assumed to not individually possess the properties of the process, but instead create those properties through interaction. Thus, the emerging dynamics can be better understood from an organizational or structural perspective. The better the overarching structure is understood, the better the process is understood. A reductionist often seeks to "provide a solution" whereas an emergentist seeks to "generalize from the solution space". Neither approach is necessarily better than the other, but this research has endeavored to show that both perspectives should be considered complimentary.

Entropic vs. Structural Complexity- Various papers presented in the Literature Review section applied the term *complexity* in reference to changing entropy. Examples include the foundational permutation entropy paper written by Bandt & Pompe (2002). Some authors also discuss complexity based on level of complication. However, this research focused on Jensen-Shannon complexity, which is one variety of *structural complexity*. Structural complexity transcends entropic considerations. In this regard, structural complexity is more like applications in non-equilibrium statistical thermodynamics, and fits in the category of emergence-based quantifiers. Therefore, when the word *complexity* is used in this dissertation without another descriptor, *structural complexity* is implied.

Process Control Overview

Statistical process control (SPC) includes various methods to understand, monitor, and improve processes. Quality practitioners often seek to improve the stability of manufacturing and service-related process outputs, to reduce costs and waste while improving profitability and customer satisfaction. SPC techniques can be applied to evaluate the amount of variation present in a process of interest and, of the available techniques, Shewhart control charts are probably the most popular. This is because they are comparatively simple and, as an observational method (vs. many experimental methods), can provide useful process information in near real time.

Dr. Walter A. Shewhart invented control charts on May 16, 1924, based on his quality improvement work at Bell Laboratories (Juran, 1997, p.13). His focus was to better understand variability in manufactured goods. He surmised that a source of variability could be most easily identified if the measured variation fell outside some limits defined by chance. If a process revealed variation that was not within these limits, it was characterized as "uncontrolled" and actions to bring the process into a better state of *statistical control* could logically be pursued.

Shewhart's (1931) control charts are somewhat unique compared to most other methods of statistical inference. Many statistical methods assume an appropriate probability model, establish a desired risk of false alarm (as an alpha value) prior to the study (hopefully), and then compare this value to the critical values calculated from the data to assess statistical significance. Shewhart control charts employ the opposite perspective in that critical values are fixed, the alpha value is allowed to wander, and no assumptions are made about representative probability models. Shewhart opined that probability models can only ever represent statistics of the data (not parameters) because there can never be enough data to establish a unique and completely accurate model. His method proposed a way around this problematic assumption of a probability model by instead basing the inferential process entirely on the original data themselves (Wheeler, 1995).

Pattern recognition enables the inferential analysis made accessible by Shewhart's control charts. To build a control chart, samples of a quality characteristic of interest are taken over time and displayed with respect to a centerline statistic (usually the mean) calculated from homogeneous groupings of these data. Control limits are drawn at three standard errors above and below the centerline statistic. Generally, a process is considered unstable if any data samples plot outside the control limits (special cause variation), and stable if they all lie within the limits (common cause variation).

In the traditional application of control charts, indications of high variance are correlating temporally with process events suspected of contributing to the excess variability. Sometimes the signal represents a favorable change that should be leveraged to establish a new normal, but most of the time, the sudden variation increase signaled by a special cause is detrimental to the desired process outcome. These signals provide time-based clues that hopefully point to root causes during subsequent investigation. As a consequence, analytic relevance is generally greater the closer the data are plotted to real-time. As causes are mitigated, control charts can be updated starting with new data, yielding a continuous cycle of process maintenance. If calculated control limits tighten over time, the process is actually being improved in terms of variation reduction.

However, it is challenging to determine opportunities for process improvement when all of the data remain within control limits. For many practitioners, no signals of high variance (special causes) equate to no process problems worth pursuing. However, this is contrary to the spirit of continuous process improvement, and provides some of the basis for continuing controversy. While there is comparatively little disagreement about the need to improve an unstable process, there is a great deal of disagreement regarding how or whether a stable process should be improved. Some of the relevant conflicts regarding methods, theoretical underpinnings, and the effectiveness of various statistical tools were provided in detailed studies by Woodall (2000) and Woodall & Montgomery (1999).

Numerous varieties of control charts have been developed. Examples include regression control charts, Cumulative Sum (CUSUM) charts, Exponentially Weighted Moving Average (EWMA) charts, real-time contrast charts, p-, u-, c-, and np- control charts, XbarR, and XbarS charts. Although only one type of control chart (XmR) was applied in this research, this cursory introduction into process control is nevertheless provided because control charts have historically been among the most popular tools for pattern-based assessment of process stability and improvement, and similar pattern-based assessment is conceptually fundamental to this research.

This research employed a limited analysis of the vertical funnel data with the individual value and moving range (XmR) control chart, an example of which is provided in Figure 4. The

XmR chart was chosen because it is one of the simplest and most flexible control charts, providing useful information for virtually any kind of process (Wheeler & Chambers, 2010, pp.48-50). The XmR chart displays every "individual" data point (vice subgrouped data) from a continuous stream of process information, facilitating the immediate viewing of micro-level process changes.

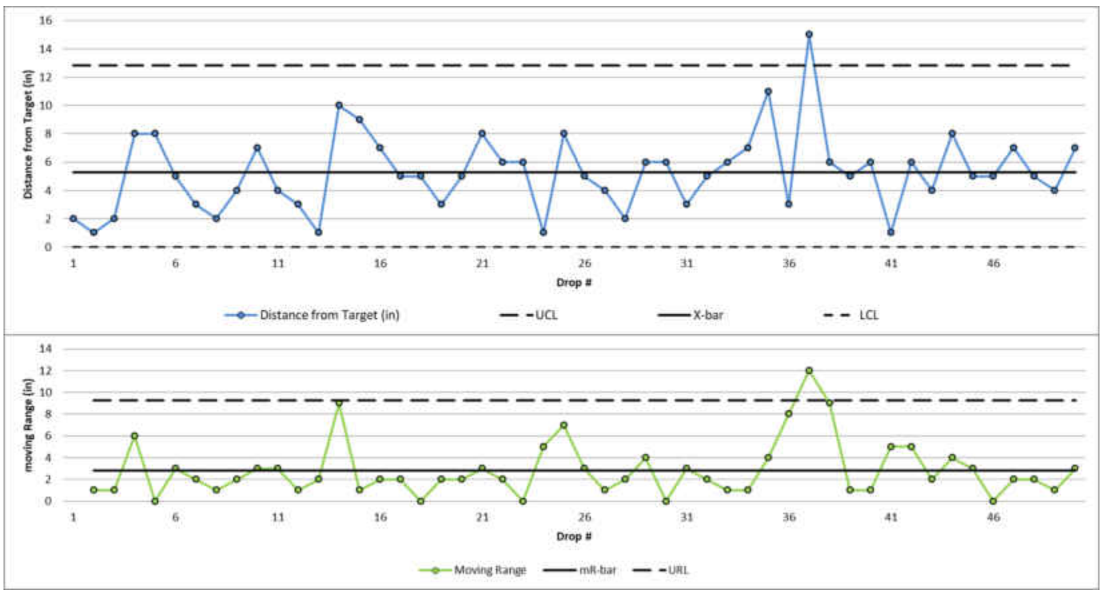


Figure 4. Example of XmR chart. Individuals (X) chart on top and moving range (mR) chart on the bottom. The point at marble drop # 37 represents a special cause of variation based on exceeding both UCL and URL control limits. Data represent the 23 inch drop height for vertical funnel experiment #2.

The three control limits (UCL, LCL, and URL) displayed in Figure 4 are typical of many control charts, and may be recalculated iteratively as each data point arrives. Alternatively, for stable processes, they may be recalculated only after significant process instability is indicated. The moving range (mR) control chart is usually presented directly below the control chart of individual run data and displays the calculated delta, or range, between each consecutive data point. The moving range control chart is considered useful because it provides better awareness of ongoing variation through sensitivity to sudden changes in time series data, especially when signals only show up on the range chart but not the average chart (Wheeler & Chambers, 2010, pp.49-50). However, some controversy about the usefulness of moving range charts has also been published, as detractors claim mR-charts are ineffective to show changes in process variability (Rigdon, et al., 1994; Sullivan & Woodall, 1996). Nevertheless, the challenging imperative most relevant to this research is to differentiate among improved-Shewhart-stable processes when all of the variation continues to remain within control limits.

Altogether, the control chart presents a graphic representation of process stability but effective implementation requires more background information than has been provided in this brief overview. The equations to calculate various kinds of control limits, tables for bias correction factors, and zone test criteria may be found in most textbooks written about statistical process control, but are beyond the scope presented here. Accessible information regarding SPC methodologies is available in Moen, et al. (1991), Ott, et al. (2000), and Wheeler (2000).

CHAPTER TWO: LITERATURE REVIEW

This Literature Review addresses a number of different topic areas that were specifically relevant to the research. First, the inconsistent perspectives regarding the improvement of Shewhart-stable processes are explored in some of the foundational continuous process improvement literature. Next, the notion that it is always tampering to try to improve an already Shewhart-stable process is challenged by reviewing some of the methods that have been shown to be effective for the improvement of stable processes. Following this section is a review of literature specifically relevant to the traditional funnel experiment, which again challenges the notion of tampering.

The next section reviews applications of complexity theory in process control, which is followed by a review of the most relevant permutation entropy (PE) literature. This section includes PE studies that were applied for process control and papers that discussed PE modifications to mitigate identical values in tuples. Next, two prevalent complexity-entropy diagrams are reviewed, followed by a literature review summary describing the several gaps that were most relevant to this research.

Continuous Improvement of Stable Processes

This section will present some of the foundational literature associated with the continuous improvement of Shewhart-stable processes and common causes of variation. Examples of contradictory positions will be provided regarding the randomness of stable processes and the notion that improving a stable process is always tampering. This section will also examine the differing opinions put forth about when to end "continuous" process improvement.

It is easy enough to recite the standard imperative that process improvement should continue forever. But, practically speaking, many practitioners decide how "much" continuous improvement they consider reasonable for the given set of circumstances. The corpus of process control literature does not provide consistent answers about how much improvement is prudent or when it should end. Widely divergent views are expressed, *especially* regarding the improvement of Shewhart-stable processes. Despite evidence to the contrary for a diversity of processes, it is still common to find opinions proclaiming that any attempted improvement of a stable process is "always" tampering, over-correction, or over-compensation. Stable processes are also frequently described as perfectly random, or constant systems of "chance causes", again despite evidence to the contrary.

The idea that stable processes are a constant system of chance causes probably started with Dr. Walter Shewhart (1931), who invented control charts in 1924. His definition of process control required that a controlled quality must be variable and it must demonstrate *constant variability* within limits. He stated that, "…any unknown cause of a phenomenon will be termed a chance cause" (p.7) and postulated that, "Constant systems of chance causes do exist in nature" (p.8). Shewhart (1939) frequently referred to "constant" systems of chance causes but also extensively discussed problems with definitions of randomness and the importance of applying experience and empirical results over models to make process improvement decisions (Chapter 1).

Many practitioners have interpreted Shewhart's original ideas to either proclaim or imply the *absolute* randomness of Shewhart-stable processes. One of the more influential proponents was an acolyte of Dr. Shewhart named Dr. W. Edwards Deming. *The Team Handbook*, written by Scholtes, et al. (2003) quoted one of Deming's letters, in which Deming stated:

> A stable process, one with no indication of a special cause of variation, is said to be, following Shewhart, in statistical control or stable with respect to the quality-characteristic measured. It is a random process. Its behavior in the near future is predictable. (p.2-15)

Then, a few paragraphs later:

> A system may be stable, yet turn out faulty items and mistakes. To take action on the system in response to production of a faulty item or a mistake is to tamper with the system. The result of tampering is only to increase in the future the production of faulty items and mistakes, and to increase costs- exactly the opposite of what we wish to accomplish (p.2-15).

In many of his books and lectures, Deming demonstrated these concepts of tampering and randomness in stable processes through two experiments. In his famous red bead experiment, 800 red beads were mixed with 3200 white beads, and blindfolded participants withdrew samples of 50 beads with the stated goal to withdraw only white beads. Naturally, they failed. A key objective was to demonstrate the futility and invariably negative consequences of trying to manipulate a process that is based completely on randomness.

In the Nelson funnel experiment, 50 marbles were dropped through a funnel maintained at a fixed height above a point target. The resting place of each marble was marked. Then, three different rules were tried that used lateral translation of the funnel to compensate for the variation around the target. For all three rules, the variation increased compared to leaving the funnel alone. A key objective was again to demonstrate the futility and invariably negative consequences of trying to manipulate a process that is based completely on randomness. Unfortunately, the predictable results of these two experiments have been widely generalized to represent any and all stable processes, regardless of the context. Some quality practitioners often simply assume that the factors contributing to common cause variation are perfectly random. But numerous interpretations and subtle differences in terminology abound.

According to Balestracci (2009), “Each source of common cause contributes a random, small amount of variation” and, “Common cause just means that the data points cannot be treated and reacted to differently” (p.142). A different view is provided by Hoerl & Snee (2002), who opine that it is called common cause variation, “because the causes of this variation tend to be common to all data points” (p.44). According to Juran & Gryna (1980), common causes are “random, i.e. due solely to chance” (p.289). Yet, according to Bart Kosko (2006) in *Noise*, true randomness exists only as a mathematical abstraction and is not physically achievable (p.66). Additionally, the mathematical validation of the Ramsey theory proposes that ideal randomness is impossible (Motzkin, 1967; Prömel, 2005; Stillwell, 2010).

Various opinions also appear in the literature about the appropriate time (if ever) to stop improving a process. Deming (1986) provided 14 points of guidance for the transformation of Western management. His 5th point reads, "Improve constantly and forever the system of production and service, to improve quality and productivity, and thus constantly decrease costs" (p.23). Deming expanded this principle, including the following phrases about the continuous and comprehensive nature of effort that he advocated: "Continual reduction of waste", "Continual improvement of quality in every activity of procurement, transportation, engineering, methods, maintenance, locations of activities, sales, methods of distribution, supervision, retraining, accounting, payroll, service to customers," "Continual improvement in methods to understand better each customer's needs", "Continual improvement of materials, of selection of new employees, of the skills of people at work on the job, and of repeated operations", "Continual improvement in planning and in operation", "Continual work with vendors" (pp.49-52).

Deming's (1986) opinion, which he credited to Joseph Juran, was that statistical control needs to be achieved and maintained *before* process improvement can begin (pp.51,321). Consequently, his position was that statistical control of a process was not an end in itself, but rather the starting point so that the, "…serious work to improve quality and economy of production can commence" (p.354). In other words, Deming did not consider removal of special causes to be improvement of the process.

Deming provided additional defining guidance for the causes of variation based on his extensive experience in quality control. He wrote, "We shall speak of faults of the system as common causes of trouble, and faults from fleeting events as special causes." Also, "The fact is that most troubles with service and production lie in the system" (p.314). These statements conform with Deming's position that mitigating special causes is not improving the process itself since special causes of variation are not an inherent part of a process.

Deming (1986) believed special causes represent only about 6% of all troubles. Conversely, he believed common causes constitute the remaining 94% of all troubles (p.315). If this estimate is embraced, it is interesting that leaving a stable process alone concurrently means that 94% of troubles should be left alone. But not everyone agrees with Deming' estimated 94/6 split. Davis Balestracci, Jr. (2009) advocated an 80/20 split based on the Pareto principle (p.123). According to Henry Neave (1990), Joseph Juran advocated an 85/15 split and Deming was rumored to have updated his estimate to 98/2 (p.69). Brian L. Joiner (1994) did not identify a specific percentage split but acknowledged that "most" of the problems arise from common causes (p.137). Shewhart's (1931) foundational book also indirectly answered this question when he used Chebyshev's theorem to define the probability that an observed value would lie within control limits (p.277). His choice of $\sigma = 3$ yields an 89/11 split when applied to the theorem.

Numerous authors concur with the importance of variation reduction but do not acknowledge the demarcation between common/special causes created by Shewhart. Thomas Pyzdek (1990) gave a presentation entitled, *There's no Such Thing as a Common Cause*, in which he proclaimed: "The division of variation into these categories is utterly artificial" (p.1). His position was that all variation is either visible or hidden, and that we should, "…refuse to accept any level of variation as acceptable" (p.1) Steiner & MacKay (2005) wrote extensively about variation reduction in *Statistical Engineering*, including the detailed elucidation of seven variation reduction approaches, but used the phrase "dominant causes" when discussing process improvement throughout the book.

In *Understanding Industrial Experimentation*, Dr. Donald Wheeler (1990) advocated control charts for analyzing production data but cautioned against using them in experimentation:

> …Control Charts have a potential shortcoming as a tool for analyzing experimental data. Industrial experiments will generally involve the exploratory analysis of a limited amount of data that is, *a priori*, thought to contain real differences. Control Charts are set up for the analysis of ongoing streams of data that, hopefully, contain no real differences. So, when a Control Chart is used to analyze experimental data, those differences identified as potential signals by the Control Chart are likely to represent real effects, but some real differences may be missed. (p.56)

Some experts go so far as to condemn the entire practice of control charting, regardless of context. In his article *Farewell Fusillade*, Bert Gunter (1998) wrote:

> Control charts have had a long and successful run, but it is time to move beyond these now archaic and simplistic tools (and this goes for the endless recent variations like EWMA, multi-variate, and robust versions, which are rarely used, of course). The reality of modern production and service processes has simply transcended the relevance and utility of this honored but ancient tool (p.3).

David Hartshorne (2019) provided a similar perspective among the responses to his online article, *Big Data, Big Disappointment*:

> There is an old joke that goes something like this: What is the difference between these two myths? There is someone called Santa who visits everyone on the "nice" list in a single night. The things that cause physical events are either "common" or "special". The answer is that one is a harmless story told to those that don't know the truth, perpetuated by those that do know, whilst the other is the work of the devil and has caused untold misery for as long as people can remember. But nobody is sure which is which. (8th comment)

Although Deming and Juran believed process improvement does not begin until a process reaches Shewhart-stability, the position of various authors is essentially the opposite. That is, they consider process improvement to be almost entirely relevant to Shewhart-unstable processes, achieved by removing special causes to establish statistical control. In an article dedicated to the application of control charts, Wheeler (2010, May 5&6) stated that special causes will typically be the dominant source of variation in the product stream and that common causes will tend to cancel each other out. To support his position, he provides a theoretical system in which four special causes of variation represents 79% of the total variation (p.3). This theoretical system seems to diverge considerably from Deming's experience that about 2-6% of causes are special.

Wheeler then wrote that it is, "unlikely to be economical" (p.3) to attempt to control any of the other causes of variation, all of which are common causes, and that, "looking for root causes of variation in a predictable process will lead to frustration" (p.14). He does, however, estimate that control of the next four most dominant causes- if they could be identified- would notionally yield another 9% reduction in variation. Wheeler further states that, "seeking to identify and control common causes is a low-payback strategy" (p.9), and that a process operating predictably will have "the minimum amount of variation that is consistent with economic production" and "will be operating at full potential" (p.10). He shows that as long as outcomes are satisfactory for a predictable process, there is "no need to improve the process" (p.13), which some practitioners might consider contrary to the idea of continuous improvement.

Wheeler does acknowledge the opportunity to make a fundamental change to a stable process by introducing, "new technology, new equipment, new procedures, or new materials" (p.10). He then cautions, however, that this would immediately change the cause-and-effect relationships, returning the process to the starting point for improvement- meaning the removal of new special causes. He also states that benign neglect is the result when, "…conventional wisdom also tells us to leave things alone as long as the outcomes are satisfactory" (p.12).

Elsewhere in his voluminous literature, Wheeler (2010) does advocate the improvement of stable ("predictable") processes, qualified by carefully defined "states" of the process in terms of specification compliance. In *Six Sigma Practitioners Guide to Data Analysis*, he opines that practitioners should, "Upgrade or adjust predictable processes in the threshold state [some non-conforming product produced]; and ignore or tweak the predictable processes in the ideal state [100% conforming product produced]" (p.242). He also defines a system he calls, "The Effective Cost of Production & Use" to apply practical economic factors to make such decisions (Chapters 14&15).

Many authors concur with Wheeler's position. In *The Trust Factor* (2003), Robert Whipple states, "If you try to fix common cause variation by going after why it occurred, you are stupid" (p.165). Although Joiner (1994) advocates extensively for the improvement of stable processes in *Fourth Generation Management*, he warns, as did Wheeler above, that trying to find out what is different among common causes using a control chart is a "low-yield strategy" (p.141). Instead, Joiner reveals various methods other than control charts to identify common causes, which will be reviewed in the next section of this Literature Review. Regarding process improvement however, Joiner's position about stable processes is unambiguous:

> Discovering that a process is stable does *not* mean we need be satisfied with its variation or its level. It does not mean we should settle for whatever the process is currently delivering. Leaving the process alone is not improvement. (p.140)

In *Statistical Thinking*, Hoerl & Snee (2002) discuss how the appearance of a special cause can actually by a sign of process improvement. The example they provide is a sudden increase in productivity in manufacturing. They state that, "Positive special causes must also be identified so they can be institutionalized" (p.45).

In *Unlocking Ford Secrets*, Sullivan & Manoogian (2009) revealed how practitioners at the Tokai Rika plant in Japan used control charts to continually monitor a very stable process producing car cigarette lighters (pp.164-175). The authors provide control chart data for 379 work days of production in 1980-1982 and narrate the changes made along the way as special causes arose. One of the more advantageous actions that the Tokai Rika personnel exploited involved the mitigation of a common cause of variation. They capitalized on an opportunity from information gathered about the failure dynamics of a part called a positioning collar (p.180). Instead of waiting for the same special cause of variation to arise again as it had multiple times before, they conducted preventive maintenance by replacing this part before it failed. Preventive maintenance (PM) is one of the more common methods used in industry to proactively maintain an already stable process, and, according to Schrader & Elshennawy (2000), "is a needed function that should be integrated into the manufacturing system" (p.733). Although the primary purpose of PM is proactive maintenance of process stability, some coincidental variation reduction is possible depending upon the nature of the event. Preventive maintenance will also be discussed in the next section of this Literature Review.

Shewhart (1931) was naturally among the first to address the issue of how "much" process control should be pursued. He stated, "When a phenomenon has been shown to exhibit control, we have likely gone about as far as we can in detecting the existence of assignable or discoverable causes by standard tests" (p.159). This comment suggests Shewhart believed it would be very difficult to discover the roots of common cause variation (at least in terms of "standard tests"- whatever this vague phrase means). It is interesting that Deming (1986) was such an effective advocate of most of Shewhart's ideas, but came to effectively the opposite conclusion about the improvement of common causes with his frequent advocacy for improving stable processes "constantly and forever" (p.23).

Despite Deming's (1994) "constantly and forever" position, he did eventually relax his viewpoint slightly in *The New Economics* by acknowledging that there can be a limit to improvement efforts based on economic considerations. He stated, "Once statistical control is achieved…the next step is improvement of the process, provided the economic advantage hoped for will be a good investment" (p. 177), and, "The cost of improvement may be … outlandish, not worth the foreseeable economic gains" (p. 177), and, "Consideration of economics may lead to a decision not to make any change at this time" (p. 183).

However, these types of "outlandish" economic decisions can also be subsumed to more important considerations, depending upon the process being managed. For instance, some processes must always be maintained in a state of maximum stability, almost completely without regard to cost. Such processes often involve aspects of human safety and the potential for huge financial loss, massive environmental damage, etc. Specific examples include mission assurance processes for space operations, cyber protection of financial markets, fire suppression systems, human surgery, or nuclear reactor operations. A specific example for which economic factors probably would not have mattered involved the Space Shuttle Columbia in January and February, 2003. Although all critical processes being monitored seemed stable for 16 days, a single insufficiently understood special cause event from the beginning of the mission ultimately caused complete mission failure and loss of the crew (www.century-of-flight.net, 2019). According to Hollingham (2014), NASA engineer David Baker stated that, had NASA been aware of the effects of this special cause, a rescue mission could have been launched using Space Shuttle Atlantis, without regard for the outlandish resource implications.

Another disputed topic related to the improvement of stable processes involves who has responsibility for the different phases of an improvement effort. Deming (1986) believed that management is primarily responsible for eliminating the common causes of variation (p.112). Arguably, Deming's central premise in *Out of the Crisis* is that management, not the workers, are responsible for process improvement, specifically because only management has the power to change the system creating the common causes. Indeed, one key point of his red bead experiment was to show that it is a waste of time to ask the workers to improve a stable system if only management had the power to change the system (such as removing some of the red beads, which represented defects, from the system).

However, based on implementing a quality program for process improvement, Deming's 14^{th} point for management states, "Put everybody in the company to work to accomplish the transformation" (p.24). Juran (1974) had a more refined answer, stating, "Responsibility is not clear unless it is stated in terms of decisions or actions. It is futile to ask, 'Who is responsible for quality?' since the question does not identify a decision or action" (p.11-4). In *The Team Handbook, 3rd Edition*, Scholtes, et al. (2003) suggested that, "…at least 85% of problems can only be corrected by changing systems (which are largely controlled by management) and fewer than 15% are under an employee's control" (p.XXV).

In summary, it is evident that the statistical process control literature provides very little consistency regarding guidance associated with Shewhart-stable processes. Naturally, the full context associated with every statement in this section is important to understand each author's complete position. However, at a minimum, this overview revealed that a diversity of contradictory opinions are abundant in the literature. Table 1 succinctly summarizes this diversity of opinion.

Table 1. Summary of diverse positions regarding stable processes.

Question/ Concept	Position 1	Position 2	Position 3	Position 4
Stable processes are:	Perfectly random	Near-random	Varying in degrees of randomness	Not random
Improving a Stable Process is:	Tampering	A Good Idea	It depends on the process	
Improvement of a process begins when:	Special causes are mitigated	Common causes are mitigated	Any negative causes are mitigated	A positive cause is amplified
Process improvement should continue:	Constantly and forever	Before stability is achieved	After stability is achieved	It depends on the process
Process improvement should continue:	Constantly and forever	Based on economic considerations	Based on all relevant considerations	Sometimes economic considerations matter little
Common causes represent what % of trouble	96-98%	85-89%	80	Most
Common causes are relevant to process improvement	Yes	No		
Control charts are relevant to process improvement	Yes	No		
Improvement of a stable process should depend upon whether or not non-conforming product is being produced	Yes	No		
Trying to find the root cause of stable process variation means:	You are stupid	You are smart	It depends on the process	
The responsibility for improving common causes lies with:	Management	Workers	Depends on responsibility for decisions or actions	Everyone

Methods for Improving Shewhart-Stable Processes

"No problem can be solved from the same consciousness that created it."

-Albert Einstein

This section presents a number of methods from the literature that can be relevant to the improvement of a Shewhart-stable process. The assumption that improvement of a stable system is *always* tampering can be challenged by the successful application of these methods. Indeed, many of these methods directly challenge one or more of the "rules" about Shewhart-stable processes presented in Figure 2. Generally, when many reviewed sources actually discussed stable processes, it was often couched in some variety of the "wait for trouble paradigm", instead of proactive, continuing pursuit of improvement. This was often because techniques other than control charts were not being considered.

Shewhart control charts are one of the simplest and most widely used tools for process improvement, but using them to improve Shewhart-stable processes is generally considered a low-yield strategy (Joiner, 1994; Wheeler, 2010). Even if improvement of a stable process is not pursued and control charts are only passively monitored, process owners are not necessarily being complacent. Rather, they may simply be unaware of the numerous viable techniques that are available for consideration and, they may be discouraged by the conventional platitude that "fundamental" process redesign is their only recourse. Also, designed experiments may be seen as the next step for dissecting stable processes so they can be improved. But, when a process is already operating in statistical control, the time commitment required to plan experiments, and then collect and analyze the data hypothesized to be most relevant can often seem prohibitive. Practically speaking, Shewhart-stable processes are often simply ignored for active improvement.

Perhaps it not surprising then that the improvement of Shewhart-stable processes is not a widespread topic in the process control literature. Despite this relatively slight treatment, its relevance to this research effort is significant, as can be confirmed by the second half of the research question: Can a methodology based on emergent structural complexity provide information useful to direct the continued improvement of a Shewhart-stable process? Since continued improvement of a Shewhart-stable process is the focus of this research, this review will provide insights to better comprehend the gap in the state of the practice.

Ultimately, no books were discovered that were dedicated specifically to improving Shewhart-stable processes, although Joiner (1994) dedicated seven pages to the topic (pp.140-146). Balestracci provided six papers and two videos focused on identifying common causes, and declared that his inspiration came from Juran's (1981) 16-video series *Juran on Quality Improvement*. Torbeck (2012) provided a single page summary of relevant ideas while Wu (2014) provided numerous relevant ideas in terms of *teaching* process variation.

A variety of papers discussed concepts that could be applied to improve stable processes that were subsumed under the general category of experimentation techniques. For instance, many papers were found that discussed the identification of common causes in terms of statistical disaggregation of variance components (Satterthwaite, 1946; Yashchin, 1994). Experimentation is a very relevant, diverse, and encompassing topic, which is already highly developed in the literature. As such, experimentation will only be described briefly and in very general terms so that the focus can remain on reviewing the other lesser used and lesser understood techniques that can be considered for stable process improvement. Additionally, a few papers will be reviewed in the next section that specifically analyzed the traditional funnel experiment. They were deemed especially relevant for dissecting the tampering assumption, and so were discussed separately.

More can almost always be done to improve a process than to simply monitor a control chart to wait for the next special cause. And, contrary to the platitude, many of the available techniques do not represent a "fundamental" process change. For instance, some proactive problem-avoidance strategies are simpler to implement than control charts, can achieve variation reduction, and represent minor, non-fundamental process changes. Moreover, many of the methods presented are based on direct observation (vice time-lagged data transformation techniques) which is generally advantageous, especially when it is possible to avoid "assuming a model". All models are wrong by varying degrees, but many practitioners consider them necessary for process characterization strategies. Shewhart (1939) acknowledged this limitation but also claimed models could be useful with his sentiment that, "any model is always an incomplete though useful picture of the conceived physical thing…" (p.19). Shewhart's comment pre-dates similar sentiment about models often attributed to Dr. George Box.

This section will first present some of the more obvious and simple methods and move progressively on to techniques of increasing complication or esotericism. Starting with the very simplest method, practitioners can leverage their knowledge of the system to look for obvious improvements. This could include certain applications of Deming's (1994) idea of "Profound Knowledge of the System" (pp.92-115). For instance, new insights could be gained by knowing which data to focus on, or common sense changes with a very high chance of success could be implemented. Torbeck (2012) called this approach, "Control what can be controlled" (p.32). Balestracci (2012, September 19) called these, "obvious 'no-brainer' solutions", but also cautioned that the usual problem is, "these ideas needed management support to be implemented" (p.1). Steiner & MacKay (2005) provided eight real-world examples of successful obvious solutions in manufacturing but cautioned that it is important to first consider the possible side effects and, that the key consideration is usually cost (p.214). For instance, obvious improvements can often be implemented by replacing troublesome machinery, but this expensive solution may not be available in the budget.

Rudimentary examples of this approach are easily explained with training systems that represent stable processes, such as the funnel experiment, quincunx, and red bead experiment. For the funnel experiment, Deming (1994) offered up lowering the funnel as a free solution and using a fuzzier tablecloth as a cheap solution (p.196). For the quincunx, Joiner (1994) stated that row of central pegs could be removed (p.141), which would allow fewer of the beads to become outliers, thereby decreasing kurtosis. In the red bead experiment, the beads could be optically sorted by a machine before sampling, or the sampling paddles could be modified, or the beads made magnetic (Hunter, 2014).

These kinds of solutions often extend to the next method, which Juran (1988) called "fool-proofing". He included examples of fail-safe designs and automation including robotics (p.228). The equivalent Japanese term for fail-safe designs is poka-yoke, which translates to "mistake-proofing" in English. Examples often include hardware that allows production processes to proceed in only one possible way. By design, these solutions prevent certain special causes from appearing in the first place, and they are often very simple applications. Classical examples include a mechanical stop that prevents a drillbit from making a hole that is too deep or different colored pages for different paperwork processes (Torbeck, 2012). It is common to witness variation reduction following application of these techniques.

Another problem avoidance concept is preventive maintenance (PM), which applies proactive measures based on historical knowledge of process performance to prevent special causes before they are most likely to happen. PM, according to Schrader & Elshennawy (2000), "is a needed function that should be integrated into the manufacturing system" (p.733). As will be seen in the Tokai Rika analysis, PM addressed a common cause of variation to provide minor variation reduction for this Shewhart-stable process on work day 355. Also, a closely related concept is condition-based maintenance (CBM), which tracks the condition of critical components to drive preventive maintenance priorities.

Juran (1964) provided many ideas useful to address common causes, including techniques for what he called "Common-Use Tools" including making use of trends, comparisons, and summaries that underscore the vital few causes with the "Pareto Matrix" (p.317). Similarly, while presenting the concept of process optimization, Juran (1988) stated, "The starting point is to analyze the data on human errors and to apply the Pareto principle" (p.226), where he again demonstrated his Pareto Matrix concept. An example of Juran's Pareto Matrix is provided in Figure 5.

	Policy writer						
Error type	A	B	C	D	E	F	Total
1	0	0	1	0	2	1	4
2	1	0	0	0	1	0	2
3	0	16	1	0	2	0	19
4	0	0	0	0	1	0	1
5	2	1	3	1	4	2	13
6	0	0	0	0	3	0	3
27							
28							
29							
Totals	6	20	8	3	36	7	80

Figure 5. Pareto Matrix. From *Juran on Planning for Quality* by Joseph M. Juran (p.177). Copyright © 1988 by Juran Institute, Inc. Reprinted with the permission of Free Press, a Division of Simon & Schuster, Inc. All rights reserved.

In this example, Juran made a distinction between process performance (actually does) and capability (could do). For instance, Worker B's performance was 20 errors, but her capability was 4 error types, whereas Worker E's 36 errors across many error types represented both performance and capability (data for error types 7 through 29 were not presented to keep the figure simple). Three focus areas to improve the process immediately become apparent. First, Worker E may not be suited to this type of work, and would probably benefit from reassignment to work better aligned with individual skills and abilities. Second, Worker B may need retraining based on error type 3 (it was found that she misunderstood one part of the procedure). Lastly, error type 5 had a relatively high rate across all of the workers, suggesting additional causal investigation.

Juran's Pareto Matrix concept can be extended beyond human error tracking to any kind of common cause improvement, as advocated in an online video by Davis Balestracci (2017, February 23a). Balestracci (2012) also augmented many of Juran's techniques for identifying common causes in his insightful five-paper series in *Quality Digest*. These techniques, following Juran, included exhausting in-house data, studying the current process in more depth, and "cutting new windows" (process dissection). Balestracci employed these concepts to look at data differently in terms of stratification and disaggregation, which Brian Joiner also advocated in *Fourth Generation Management*.

Joiner (1994) defined stratification as, "sort data into groups or categories based on different factors; look for patterns in the way the data points cluster or do not cluster". In doing so, "We end up localizing common cause variation, pinpointing it at its source." Further, stratification helps us by, “Revealing patterns in the data that point to the source of trouble” and this focusing, “Allows us to identify the leverage points where a little effort brings major improvement” (p.143). According to Hoerl & Snee (2002), “You can never do too much stratification. If the variable you used to stratify turns out not to be important, cross it off the list and move on to other variables” (p185).

In a real-world example revealing the utility of exhausting in-house data via stratification, Balestracci (2014, July 28) provides a control chart, Figure 6, representing only common cause variation, for which, “the wide limits were discouraging to the point of being ridiculous” (p.1).

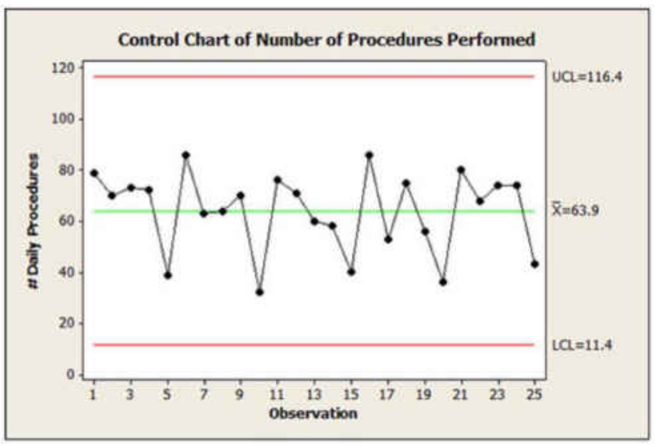


Figure 6. Control chart of Shewhart-stable process with wide control limits. From *More common cause subtlety. You've got a chart, and its common cause. Now what?* by Davis Balestracci. Copyright © 2014 by author. Reprinted with the permission of author. All rights reserved.

The process owner was seeking insights from the control chart in Figure 6 to update staffing but couldn’t ascertain any benefit, since all of the variation was well within the wide control limits. Balestracci offered that such wide limits often suggest that questions should be asked about the sampling method. In doing so, he discovered that these data represented consecutive daily counts, excluding weekends. It was now possible to label the data based on day of the week, as presented in Figure 7.

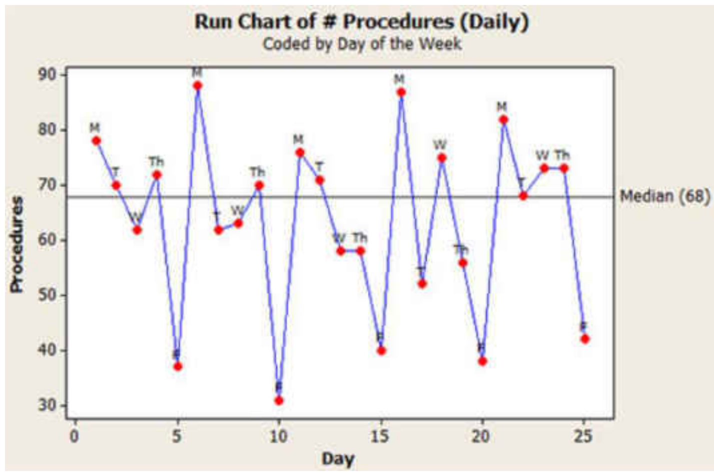


Figure 7. Common cause control chart stratified by day of the week. From *More common cause subtlety. You've got a chart, and its common cause. Now what?* by Davis Balestracci. Copyright © 2014 by author. Reprinted with the permission of author. All rights reserved.

Based on this stratification, a pattern appeared among the common causes for all of the high values and all of the low values, which could be applied to optimize staffing based on day of the work week. However, Balestracci gained the most detailed insight by then applying of simultaneous, multi-dimensional viewing of non-variation data, similar to Juran's Pareto matrix concept, in Figure 8.

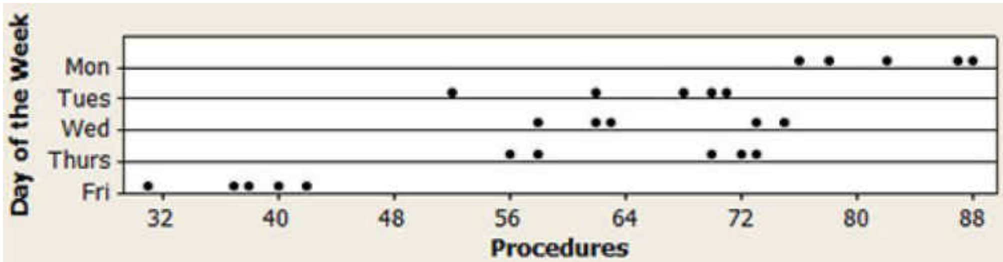


Figure 8. Stratified histogram based on common cause control chart stratified by day of the week. From *More common cause subtlety. You've got a chart, and its common cause. Now what?* by Davis Balestracci.

Note in Figure 8 that Balestracci stratified the y-axis by categories, which is different from the typical histogram presentation. Based on the initial Shewhart-stable control chart, the process owner could only comprehend that staffing needed to accommodate 11 to 166 procedures per day. By applying Balestracci's *Stratification Histogram*, she was empowered with data to more precisely conclude that staffing needed to accommodate 50 to 103 procedures on Mondays, 40 to 80 procedures on Tuesdays to Thursdays and 23 to 63 procedures on Fridays. For more detail about this technique, Balestracci described a similar study in his book, *Data Sanity* (pp.131-135).

This multi-dimensional concept of stratification can add another dimension by also incorporating temporal information associated with the time series. Numerous methods will be reviewed in this section that apply reductionist approaches with simultaneous viewing of temporal clues. For the research herein, important conceptual parallels are apparent, in that new insights became discoverable by adding new dimensions of visibility. This advantage will be conspicuous when the Complexity-Entropy Causality Diagram and the Entropy-Complexity Change Diagram are introduced.

Regarding the specific concept of disaggregation, Joiner (1994) defined the activity as, "Divide the process into component pieces and manage the pieces" (p.141). As an example, Joiner presented an order-entry-to-shipping process that had been Shewhart-stable for weeks but nonetheless suffered many late shipment complaints. By beginning to track the performance of three sub-processes (order entry, assembly, and packaging/labeling), a time lag due to workspace layout was made visible. Rearranging the workspace provided immediate process improvement and provided a concurrent benefit by easing the effort of the workers. Joiner claimed that disaggregation is, "More complex than stratification or experimentation, but is also more powerful" (p.146). However, he also cautioned that, "Disaggregation only works when each piece of the process has an aim tied to serving the next step and is consistent with the overall aim of the process" (p.146).

Another broad category of methods for improving Shewhart stable processes focuses on factors of practical, instead of statistical significance. This concept can take a risk/opportunity optimization approach, as provided in the following allegory. Suppose a house is being designed by a fictitious architect named Momoko Zoristern. The design includes a fireplace that opens into two rooms and Ms. Zoristern tells the owner that based on her experience, pecuniary considerations, and, especially, extensive statistical analysis, brick thresholds extending three-feet to either side of the fireplace will be optimal in case any sparks fly out. The house is then built to her specifications. The question is whether occupants should always sleep soundly at night, being warmed by the fire, because they have three-foot brick thresholds? Some would argue that the answer depends most on practical risk considerations such as the size of the fire, whether the wood has a high resin content, whether flammable debris is in the vicinity of the fireplace, whether open windows or air conditioning creates a draft, whether leaves are being burned, etc. The point of this story is that practical concerns are often more relevant to process questions than the results of statistical analyses.

One source for this emphasis on practical considerations involves the criticisms that have been increasingly leveled at statistical hypothesis testing (McShane et al., 2019; Ziliak & McCloskey, 2008), including those by the American Statistical Association (Wasserstein & Lazar, 2016). The essential concern is that consistent, statistically significant results do not necessarily imply practical significance. According to Ziliak & McCloskey (2008), "Statistical significance is, we argue, a diversion from the proper objects of scientific study. Significance, reduced to its narrow statistical meaning only, has little to do with a defensible notion of scientific inference, error analysis, or rational decision making" (p.2). Ostensibly in jest, they also proposed an alternative F-test, where the "F" stands for floccinaucinihilipilification (p.321).

Advancing this concern back to process control, Juran (1997) characterized Shewhart's control charts as, "A perpetual test of significance" (p.12), implying that control charts inherently provide an ongoing hypothesis test of process stability. But Deming (1986) believed control charts had utility specifically because they did *not* involve tests of significance. Deming stated, "Avoid passages in books that treat confidence intervals and tests of significance, as such calculations have no application in analytic problems" (p.369). In other works, Deming (1938) stated "Statistical 'significance' is by itself not a rational plan for action" (p.30) and, (1961):

> The standard error of a result does not measure the usefulness thereof. The standard error, however helpful in the use of data from samples, only gives us a measure of the variation between repeated samples…It does not mean that the persistent components of

> the non-sampling errors are small. It is important, for such reasons, I believe, not to focus attention on the standard error alone. (p.55)

Finally, Deming (1961) provided overall support for a *practical* synthesis of information, stating, "Statistical theory shows how mathematics, judgment, and substantive knowledge work together to best advantage" (p.v).

Many authors have provided additional support for a practical synthesis while also eschewing what they consider an artificial distinction between cause types based on 3-Sigma of variation. They offer that maintaining the standard distinction between special and common causes has become increasingly difficult with the astronomically high volume of data created by in-process sensors throughout manufacturing lines (Woodall & Faltin, 2019). In addition to these physical difficulties, numerous conceptual difficulties have also been presented.

In a presentation entitled, *There's No Such Thing as a Common Cause*, Pyzdek (1990) described a real-world process that was in dire need of improvement despite being represented by only common causes on a control chart. He stated:

> Some people even mistakenly believe that when variation is from a system of common causes that it has no cause. The belief in common causes has become so established that it has become taboo to suggest that they don't actually exist. This paper discusses a bold new idea: there is really no such thing as a common cause of variation. The division of variation into these categories is utterly artificial. By maintaining this artificial distinction we often cripple problem solving and process improvement efforts. (p.102)

In defining the effects of this 3-Sigma classification scheme, he listed desirable effects, which focused on tampering, economic thresholds, simple guidelines, and assigning responsibility to management. He also listed the undesirable effects, which were:

- People are taught to believe that finding and fixing causes of variation within the control limits is difficult and expensive and usually involves making major changes to the process. Because of this belief, simple, elegant, and inexpensive solutions are often overlooked.
- Intelligent thought is replaced by mindless conformance to arbitrary rules.
- Creative analysis of variation within control limits, which includes most process variation, is curtailed.
- Continuous improvement is inhibited. (p.103)

Pyzdek then discussed his perspective on continuous improvement: "The result is that many companies focus all of their attention on bringing processes into a state of statistical control and fail to see that statistical control is just the first step on the path to continuous improvement" and, "A new approach is necessary that makes it clear that one *never* leaves variation to chance. Reduction of variation is continuous" (p.104). He then described in detail the numerous improvements that were made to a Shewhart-stable, real-world process, which reduced the defect rate by a factor of 10 (from 50 defects to 5 per 1,000), without making any "fundamental" changes to the process and after spending only a few dollars on improvements. To better support his imperative for continuous improvement of stable processes, one of

Pyzdek's key recommendations was to change the variation terminology to "visible and hidden" to remove the artificial distinction created by the 3-Sigma rule.

Steiner & MacKay (2005) published an algorithm for variation reduction which focuses on finding and verifying "dominant" causes of variation so they can be mitigated. The methods they advocate utilize a combination of practical and statistical techniques, including: Fix the obvious, desensitize the process, apply feedforward control, apply feedback control, make processes robust, and 100% inspection. The authors do not avoid using control charts altogether but advocate more diverse applications such as run charts, acceptance control charts, and regular calculations of process capability (p.310), stating, "Data summaries such as the average, standard deviation, histogram, and box plots do not show how the process output varies over time. To show this behavior, we use a run chart, a simple plot of the output values against the order or time of collection" (p.21). Also, "We do not require the process to be stable, as defined by the control chart, to estimate the problem baseline" (p81).

Other methods have also focused on variation reduction based on practical considerations, in this case by repositioning the control limits. The determination of statistical thresholds like control limits involves a standard trade-off between sensitivity and false alarm rates. A false alarm is an inappropriate signal of a special cause when the process is actually in control. Applying a conservative threshold means fewer false alarms, but also that many signals of interest can be missed. Conversely, using an aggressive threshold increases the false alarm rate but results in fewer missed signals.

According to Woodall & Faltin (2019), selecting the appropriate threshold involves numerous considerations that balance practical importance with statistical significance. For instance, when multiple control charts are being monitored simultaneously, the multiple testing effect can dramatically increase the overall false alarm rate. Additionally, numerous small process shifts may not be of practical importance, suggesting a rationale for desensitizing the threshold. Moreover, traditional performance metrics are sometimes complicated by covariance structures that become increasingly complex. Ultimately, any performance metric will increasingly lose relevance as the false alarm rate approaches unity.

The authors go on to state that a control chart for a Shewhart-stable process (assuming that the in-control parameters are known and that the data are independent over time) can share some of the same shortcomings as hypothesis tests. They emphasize that "acceptable" rates of false alarm depends on practical concerns by discussing many real-world process problems. For example, they compare the difference between an errant smoke alarm and an errant heat-sensitive sprinkler system, wherein the former is a nuisance but the latter can be highly destructive.

Woodall & Faltin then propose adding a zone between the in-control and out-of-control zones to provide better flexibility for false alarm rates. They called this the *indifference zone*, a term they credited to Freund (1960). They carefully acknowledged that many other authors have advocated similar three-region approaches. They also emphasized that the regions do not have to be symmetric about a target value, and that a *deadband* can be created by combining the in-control zone and indifference zone. They provide numerous applications including modified control charts and acceptance control charts. Ultimately, they advocate this approach by discussing how Shewhart's rule of 3-Sigma was founded on dated economic considerations from the 1920's that they believed to have lost relevance in today's high data rate environments.

However, Wheeler (2013, May 1) provides another perspective in a paper entitled, *Contra Two Sigma*. The focus of this paper was to show why using 2-Sigma control limits is not the appropriate way to increase sensitivity and he instead advocates adding four Western Electric zone test rules to signal special causes. He compares sensitivity/false-alarm results using power function charts, ARL charts, and a chart depicting probability of false alarm vs. number of subgroups. However, he also states that the problem with a 3-Sigma chart is not usually too little sensitivity, but rather too much. He wrote that once people new to control charts learn how to use them, comments about too little sensitivity often turn into, "We have too many signals to get around them all" (p.1) Later in the paper, he wrote,

> In practice, most people who use process behavior charts effectively find that they have plenty of signals using Detection Rule One [data outlying control limits]. In fact, the problem is usually one of needing a procedure that is *less* sensitive, rather than more sensitive. (p.8)

The two comments suggest that 3-Sigma control limits often provide more sensitivity to process changes than desired, even if additional zone test rules are abandoned. Yet, Wheeler only offers corrections for the condition of too little sensitivity. Wheeler instead maintains the position that 3-Sigma limits always provide the optimal tradeoff between false alarms and valid signals as evidenced by his closing comments: "…while theory suggests that three-sigma limits should work, over 80 years of practice has proven beyond any doubt that they do work as expected. Make no changes and accept no substitutes" (p.9).

Some authors believe other factors are more important for the improvement of process performance than focusing on gradual variation reduction. In *Diagnosis Performance and Reliability* (2019), David Hartshorne describes looking for non-random patterns with respect to a spatio-temporal framework. He advocates trying to determine what's wrong through a *symptomatic diagnosis* and, as needed, to determine what's happening through a *topographic diagnosis*. Diagnosis is often accomplished through a process of elimination via a progressive search and *Matryoshka strategy* (nested-doll), dividing the analysis into a stratified hierarchy of contrasts to converge upon one of four variation types (elemental, cyclical, structural, or temporal) that contributes most to performance (p.110).

Hartshorne does not find relevance for the traditional common/ special cause designation, nor even the normal view of continuous improvement, stating, "Achieving … excellence through lots of small incremental improvements is a fallacy, only a step change from controlling the Steep X [strongest cause-effect relationship] will get there." (p.41) To arrive at a causal explanation, his diagnostic methods focus more on the analysis of geometry and physics than statistics, while applying thermodynamic conjugate pairs to better understanding the flows of entropy, power, and energy through machinery of interest (pp.84-85). He summarizes this energy conversion using a *Z-diagram* to evaluate inertance, resistance, and compliance. Ziliak & McCloskey (2008) support a similar focus on practical considerations, stating, "…in economics and biology and physics, the big ideas come chiefly from nonstatistical sources" (p.147).

Nevertheless, designed experimentation has been in continuous successful use since the original statistical methods were created by Sir Ronald Fisher in the 1920's. Experimentation has been highly developed in the literature and widely utilized to identify causes of variation.

Contrary to the belief that common causes cannot be isolated because they are random, DoE routinely identifies the main effects and interactions of process factors associated with stable processes (Moen et al., 1991). Indeed, an advantage of analyzing stable processes is that the impact of nuisance variables is generally decreased since processes are already in a state of statistical control before starting. But, according to some authors, this prior state of statistical control should not be taken as a requirement, which is contrary to some prevailing guidance listed in Figure 2. Bisgaard (2008) cautioned that, “The need for statistical control as a prerequisite for conducting industrial experiments is misconceived” (p.143). He believed, following Box & Luceño (1997), that statistical control is a fiction anyway, so this notional prerequisite would imply that experiments could never actually be conducted (p.149).

Despite experimentation’s wide success in many applications, DoE can also present disadvantages in certain situations that can discourage use. For example, experiments can be time-consuming, complicated, and require destructive testing. Application usually requires significant statistical expertise or else the experiments may be inconclusive, or prone to mismeasurement or contamination of the data. Additionally, experiments can be prohibitively expensive to conduct, especially if production lines need to be shut down during testing and modification. Results may also lack statistical significance, yield false positives/ negatives, or lack replicability (Shuttleworth & Wilson, 2008).

Nonetheless, for stable processes, designed experiments are generally more effective than control charts for separating signals from noise, which explains why the literature generally favors designed experimentation as the primary method to identify common causes. However, designed experiments do not usually provide results in as timely a fashion as observation-based methods like control charts. In consequence, experimental results are sometimes already overcome by process changes before they become available for review. This timeliness characteristic often serves as a deterrent to the continued improvement of a stable process.

In the era when designed experiments were invented, slow and deliberate data collection and analysis provided adequate monitoring of the vast majority of processes. However, most production processes have now become fast-moving, generating massive quantities of data thanks to the proliferation of computation, automation, and inexpensive in-process sensors. The volume of information available often becomes problematic for effective process monitoring. Methods that can accommodate rapid time series are gaining importance, which was a consideration when evaluating measures of structural complexity for this research.

One of these rapid monitoring methods was presented by Dr. George Box in 1957 and is called evolutionary operations (EvOp). With EvOp, the effects of minor experimental modifications can be monitored during normal production to determine if they are making things better or worse. These minor changes are still large enough to determine optimum process ranges for variables of interest but not large enough to create non-conforming product. In practice, this has proven to be a very powerful technique to continuously optimize stable processes, without ever having to make a "fundamental" change.

Another useful approach incorporates a sequential test of hypothesis, and is called the cumulative sum (CUSUM) technique (Page, 1954). CUSUM acknowledges the inherent non-stationarity of time series data but also assumes that the underlying distribution is well-defined, which is generally advantageous for the improvement of stable processes more than for unstable processes. Similar to techniques employed in this research, CUSUM accumulates information over time to progressively arrive at more precise representations of the time series. Also similar, displays can be tailored to the research, including V-mask charts and Manhattan plots. CUSUM is also more sensitive to process changes than control charts for shifts less than about 2.3 standard errors of the mean, based on average run length calculations by Wheeler (1995, p.310).

Another category of methods that can facilitate stable process improvement acknowledges the inherent non-stationarity of time series data, but focuses on modeling the serial dependence between successive values. Following Holt (1957) and Muth (1960), Box & Jenkins (1963) described an approach called *Autoregressive Integrated Moving Average* (ARIMA), which can be applied flexibly to forecast future points in the time series and extends easily to the *Exponentially Weighted Moving Average* (EWMA) method. EWMA uses a recursive calculation to apply weighting factors that decrease exponentially and achieves a form of controller that is insensitive to the choice of initial value. But according to ARL calculations by Wheeler (1997), is no more sensitive to changes than control charts that employ zone test rules (p.322). Similar to methods developed in this research, EWMA accumulates information about stable processes over time to refine results.

Box & Paniagua-Quiñones (2007) extended the serial dependence concept by creating a *bounded adjustment chart*. They acknowledged Pyzdek's (1990) observation that the difference between common and special causes can be dependent upon the level of residual noise. They then proposed that non-stationarity is a consequence of the accumulation of undetected permanent components of noise. To challenge the assumption that noise is comprised only of transitory components, they provided physical examples of permanent components of noise such as a tire that is slightly damaged by a sharp stone or a driveline that has developed corrosion.

Their bounded adjustment chart incorporates EWMA principles with no assumptions required about the mean. Superficially, the bounded adjustment chart looks like a control chart. But, the limits are chosen so that after adjustments are made, the smallest standard deviation is applied for a given average adjustment interval. A variable λ can be modified by the user to allocate percentages of new vs. old data into the analysis, providing a flexible means for targeted investigations. The authors recommended monitoring the bounded EWMA with the adjustment chart while concurrent monitoring the residuals with a control chart. They propose that the control chart will be more useful for detecting special causes with the misleading non-stationarity data removed in this manner.

The next method encompasses a wide-variety of techniques and will therefore be reviewed here in fairly general terms. Simulation has been applied in the context of stable process improvement to test and compare preconceived improvement alternatives without having to commit physical resources to experimentation. Adams et al. (1999) provided two case studies where simulation was used to successfully improve stable processes by focusing on reducing work in process (WIP), improving operator utilization, reducing cycle time at certain stations, and improving part handling and routing. One of their conclusions revealed the specificity of improvement that can be targeted:

> To be most effective, simulation models should be developed that apply continuous improvement concepts. For example, rather than merely modeling the total cycle time for each machine, much more insight can be gained by separating run time, setup and changeover times, downtime, break times, defect rates, and material handling into and out of the machine. (p.772)

For process control, simulations are often be applied to spatio-temporal work and information flow scenarios, similar to the *traveling salesman problem* or the *piano mover's problem*, where the measured factors can facilitate continuous optimization and redesign. Some authors state that predictable processes benefit more from applications of simulation than unpredictable processes, in part because the models tend to more accurately represent the actual processes (Balakirsky, 2009).

Predictive analytics also encompass a wide diversity of techniques that can facilitate stable process improvement. Computational techniques include "knowledge discovery in databases" and "machine learning algorithms". Crowdsourcing techniques include prediction markets and applications of the Delphi method. Methods from decision theory also include risk analysis and *Applied Information Economics*. Some of the more promising methods will be presented here.

Machine "learning" algorithms apply families of classification techniques that can be tailored to develop novel insights about datasets. A few categories of classifiers will be summarized to reveal the extent of analytic flexibility available. *Linear Regression* is a relatively simple technique that is already familiar to many statistically-minded quality practitioners, which can quickly reveal output variables worthy of further investigation. *Logistic Regression* provides the probability that data belong to one of two classes, but when more than two classes are useful to the study, *Linear Discriminant Analysis* reveals the mean of each class and the variance across all classes. *Decision Trees* output class values for decision nodes of interest while the *Random Forest* technique introduces randomness into these decision trees and aggregates statistical methods to improve predictive outputs.

Naïve Bayes Analysis provides probabilities for each class along with conditional probabilities given each input (after assuming inputs are independent). *K Nearest Neighbor* determines the similarity between the K data instances considered relevant to predicting the desired output for a training dataset. *Learning Vector Quantization* is very similar, but summarizes the training dataset into vectors (which uses less memory). *Support Vector Machines* are popular for discriminating between data classes by specifying a hyperplane to separate them. Finally, *Boosting* is a technique designed to improve the overall predictive output by correcting errors in a number of weaker classifiers.

Although they can be effective to glean novel insights from large datasets, probably the biggest concern common to these classifiers is the concept of garbage in, garbage in (GIGO). For instance, if the dataset applied to train an algorithm isn't "clean," outputs may be inaccurate and misleading. According to CIKLUM.com (2019, January 11), problematic examples include when the dataset:

- Is incomplete, to include missing values
- Is inaccurate
- Contains samples and examples in addition to real data
- Mixes nominal, ordinal, interval, and continuous data
- Includes unnecessary data
- Includes data missing the necessary attributes
- Includes misaligned components

- Inappropriately mixes measured and unmeasured values
- Is excessively difficult to process
- Includes duplicated categories, attributes, or values
- Mixes standard and non-standard formatting

The next methods for consideration to improve stable processes are prediction markets, which are a type of crowdsourcing technique that exchanges information instead of physical goods. This technique can find applicability in virtually any industry to include service, software, and manufacturing. Prediction markets can be exploited by allowing the people directly involved with a stable process to anonymously exchange online value tokens based on their perceptions of the likely outcome of events. The purpose is to elicit aggregate beliefs about an unknown future outcome, motivated by a confidential financial (or other) reward for being right. Usually, a prediction market provides a binary option that will expire at the price of either 0 or 100%. Market equilibrium prices ultimately indicate what the crowd thinks about the probabilities of outcomes, which can leveraged to modify the appropriate influencers associated with those outcomes.

The accuracy of various prediction markets has been favorably described. According to Kamp & Koen (2009), the reported accuracy of prediction markets is 94-99%, whereas preference markers are 70-85% accurate and idea markets are 10-46% accurate. Berg et al. (2008) conducted a study comparing prediction market results to 964 polls over the five U.S. Presidential elections since 1988, determining that market results were closer to the eventual outcome 74% of the time. They also concluded that when prediction markets were started more than 100 days in advance of a Presidential election, they consistently outperformed the polls.

The Delphi method has certain potential advantages over the prediction market approach. The Delphi method uses anonymous forecasts from a more structured group of individuals to develop predictions. Also called the Estimate-Talk-Estimate (ETE) approach, the Delphi method asks experts to answer questionnaires in two or more rounds. After each round, a facilitation team provides an anonymized summary of the experts' forecasts from the previous round, and includes their reasoning. Participants are thus encouraged to incorporate the replies of other members to ultimately revise and improve their own earlier answers. A goal is often to ease the communication process among experts by shifting a significant portion of the overall effort to the facilitation team. The desired outcome is that the group will eventually converge towards the most "correct" answer, which could be applied to identify optimal stable process improvements.

Khorramshahgol (1999) merged the Delphi technique with Shewhart control charts to reduce the subjectivity associated with the Goal Programming (GP) multiple criteria decision making process. The author provided an illustrative example of a bank focusing on four objectives: To increase net profit, reduce operating cost, increase savings deposits, and increase community services. Experts provided numerical aspiration level rankings associated with these goals over three rounds. Then means and control limits were calculated to build control charts. Any outliers could then easily be identified, which the experts could focus on to help establish consensus and improve a stable process. Solutions to the corresponding GP functions determined funding allocation among the four objectives, with control charts helping to establish target levels and upper and lower limits.

Parente & Anderson-Parente (2011) conducted a long term study of Delphi accuracy. In 1981, 600 participants were polled twice about 18 mental health scenarios that could possibly occur in the next 30 years. In 2011, the accuracy of the group predictions was assessed. Participants correctly predicted the occurrence of 14 of 18 scenarios and, for the scenarios that did occur, the predictions were temporally accurate to within approximately 1-5 years. These results suggest that the Delphi technique tends to provide accuracy sufficient for useful predictions.

Another potentially useful decision analysis technique that can be applied to improve stable processes is *Applied Information Economics* (AIE), presented by Hubbard (2010) in *How to Measure Anything*. The overarching assumption is that if something is causing an effect, it must be observable and therefore measurable. AIE incorporates:

1. Calibrated assessment by estimators trained to generate probabilities that are neither overconfident nor underconfident. Some important factors may still be missed and some assessors may still introduce biases, but at least the factors included are calibrated so that further reduction of uncertainty is optimized by applied measurement efforts to the right factors.

2. Information value calculations (similar to this research) for uncertain variables to reveal where to focus efforts for further uncertainty reduction. When starting with little uncertainty (such as a Shewhart-stable process), a lot of new data is required to reduce uncertainty a little. Conversely, when starting with a lot of uncertainty (Shewhart-unstable), little new data is required to reduce uncertainty a lot. Also, no information is conveyed when the outcome is already known.

3. Monte Carlo modeling including additional measurements with empirical methods according to the information value of measurements. However, if unknown covariance is present between factors of interest, the Monte Carlo distribution may not fit reality.
4. Optimization methods to determine ideal risk/ return positions for a set of alternatives.

Few articles about AIE are available in the academic literature. Many other decision and risk methods have also received little research regarding long term benefits (Hubbard, 2020). However, the components of AIE have a well-developed theoretical basis including information value calculations from decision theory, empirical methods for scientific measurement, Monte Carlo simulation, and calibration training.

The next applicable method is system dynamics (SD), created in the 1950's by Dr. Jay Forrester (1971), which applies stocks, flows, feedback loops, table functions and time delays to better understand the behavior of complex systems. The basic premise is that the many interconnected and time-delayed relationships among structural components are often just as important in determining process behavior as the individual components themselves. This method is similar to the research herein, in that both are founded on emergence-based analysis. More specifically, in applying SD, important properties of the whole often emerge, which cannot be discovered by dissecting the properties of the components. One of the studies reviewed in the next section applied SD to the traditional funnel experiment, with results that were very relevant to this research (Georgantzas & Orsini, 2003).

In her landmark SD publication *Thinking in Systems*, Donella Meadows (2008) defined 12 different leverage points as "points of power" for SD, but cautioned, following Forrester, that, "Although people deeply involved in a system often know intuitively where to find leverage points, more often than not they push the change in the *wrong direction*" (p.145). She offered that, "Leverage points frequently are not intuitive", especially since the most significant problems are associated with complex and dynamic systems. Especially relevant to this research, she defined the sixth most important of these leverage points as *Information Flows*, stating,

> It's not a parameter adjustment, not a strengthening or weakening of an existing feedback loop. It's a new loop, delivering feedback to a place where it wasn't going before. Missing information flows is one of the most common causes of system malfunction (p.157).

Importantly, Meadows also considered information to be a highly variable stock, "From which to select possible patterns- and a means for experimentation, for selecting and testing new patterns" (p.160). Furthermore:

> Any system, biological, economic, or social, that gets so encrusted that it cannot self-evolve, a system that systematically scorns experimentation and wipes out the raw material of innovation, is doomed over the long term on this highly variable planet (p.160).

Finally, she focused on defining the purposes or functions of the entire system, stating that they are, "Larger, less obvious, and higher-leverage" (p.161) than lower-level factors. Her sentiments provide support for not only continuing to improve stable systems but also for considering emergence-based insights alongside reductionist methods. This idea is also relevant in light of Meadows' two most powerful leverage points: Paradigms and the Power to Transcend Paradigms.

Nobel laureate Ilya Prigogine (1997) discussed new paradigms in terms of dissipative structures in chapter two of *The End of Certainty*. Paradigm transcendence is sometimes required to embrace his ideas about how disorder is the source of new order and therefore some level of disequilibrium is necessary for new growth. These ideas, based in thermodynamics, are relevant to this research regarding stable process improvement. Finally, while contemplating new paradigms for predicting change, a paragraph by Wheatley (1999) offers an appropriate conclusion to this section:

> The challenge for us is to see past the innumerable fragments to the whole, stepping back far enough to appreciate how things move and change as a coherent entity. We live in a very fuzzy world, where boundaries have an elusive nature and seldom mean what we expect them to mean. The illusory quality of the boundaries will continue to drive us crazy as long as we focus on trying to specify them in more detail, or to decipher clear lines of cause and effect between concepts that we treat as separate but aren't. (p.43)

Table 2. Summary of methods presented to facilitate Shewhart-stable process improvement.

Method	Variations Presented
Obvious Improvements	System of Profound Knowledge
Fail-Safe Designs	Poka-Yoke, Automation
Preventive Maintenance	Condition-Based Maintenance
Stratification	Pareto Matrix, Stratified Control Chart, Stratified Histogram
Disaggregation	Sub-Process Analysis
Practical over Statistical Significance	Risk/Opportunity Optimization, Hidden/Visible Variation, Dominant Variation Reduction via "Statistical Engineering", Hartshorne Strategies, Modified Control Charts
Designed Experiments	Evolutionary Operations
Sequential Test of Hypothesis	CUSUM
Assumes Non-Stationarity	ARIMA, EWMA, Bounded Adjustment Chart
Simulation	Spatio-Temporal, Information Flow
Predictive Analytics	Machine Learning Algorithms, Prediction Markets, Delphi Method, Applied Information Economics
System Dynamics	Emergence-based
New Paradigms	Information and Disequilibrium-based

Traditional Funnel Experiment Analyses

The traditional lateral translation funnel experiment is often used to demonstrate that compensation for variation, when a process is already stable, will always increase variation compared to just leaving the funnel alone. However, as will be seen, this conclusion is controversial and based on assumptions that are not always valid. This section will present critical literature reviews of the funnel experiment for three purposes. One, to justify the use in this research of the vertical translation funnel experiment to represent an improving stable process. Two, to elucidate the assumptions underlying the traditional funnel experiment that prevent extension of the experiment's lessons to many real-world applications. And most importantly, to support the case for the continuous improvement of Shewhart-stable processes in actual practice.

The conduct of the funnel experiment, to include descriptions of the four rules, and application of the data to control charts is well described in the literature, such as Deming (1986, Chapter 11). Therefore, the standard experimental setup will not be described here. It is also assumed that the reader has an intermediate understanding of how SPC fundamentals relate to the funnel experiment. Readers requiring more background information are encouraged to review any of the numerous explanatory videos available on the internet, such as Crostic (2015), and consider critically the desired learning outcomes.

Sparks & Field (2000) discussed the assumptions that underlie Shewhart control charts, and how they are violated when charting the funnel experiment. For example, they stated that it is assumed for the $\bar{X}$ chart, that the observations are independent and that the means of the observations are approximately normally distributed with constant variance. Also, for the R (range) chart, that the observations are independent and, "…that the observations themselves (not just the means) are approximately normally distributed with constant variance" (p.295).

The authors then use Q-Q plots, developed by Wilk & Gnanadesikan (1968), to demonstrate that the data are approximately normal with homogeneous variance for rules 1 and 2, but either non-normal or with heterogeneous variance for rules 3 and 4. Additionally, they evaluate autocorrelation functions to determine that the observations are not independent for rules 2, 3, and 4. Their conclusion is that the funnel experiment does not meet the assumptions of use for Shewhart control charts under rules 2, 3, and 4. They then decry the blind use of Shewhart charts for this purpose and thus advocate the need to always check that the assumptions are not violated in the process data before using Shewhart charts in any application. The authors also recommend avoiding the use of generic examples when training because trainees often have trouble extending these examples to an application in their own setting (p.300).

Another perspective was provided by MacGregor (1990). He makes the case that adjustments to a stable process will increase the variance in certain situations, but that, "these situations do not often occur in the continuous process industries where active process control has been applied with excellent results for decades" (p.255). He states that his paper is only relevant to rules 1 (hands-off policy) and 2 (active control policy), because the other rules are not used in practice. His application of rule 2 is based on proportional compensation instead of full compensation.

He identifies that the problem with applying the funnel experiment to practical situations is the assumption that the true process mean stays fixed. He states, "A much more general situation in the process control industries is that the process mean is not a constant, but is varying in time due to stochastic disturbances" (p.256). He does some evaluations using performance ratios and compares rule 2 deviation with and without control actions, concluding that:

> By taking corrective action every time (rule 2), the performance is never worse than that of rule 1 (no action) and it gets progressively better in situations where the random measurement error is a reduced percentage of the total variance, and where the disturbances are more autocorrelated or drifting in nature… (p.258)

MacGregor characterizes the funnel experiment as "idealized" by the assumption of purely random measurement errors. In concluding, he states, "A policy which actively controls the process to target will often result in substantial reduction of the output variance" (p.259).

The final paper, written by Georgantzas & Orsini (2003), uses systems dynamics modeling to evaluate the funnel experiment causally. This is unique from the other approaches, which look at measures that are coincidental, relative to randomness. Also, instead of the typical two dimensional plots, they add the probability of marble location (Pml) to create three dimensional plots for each rule, based on simulated data.

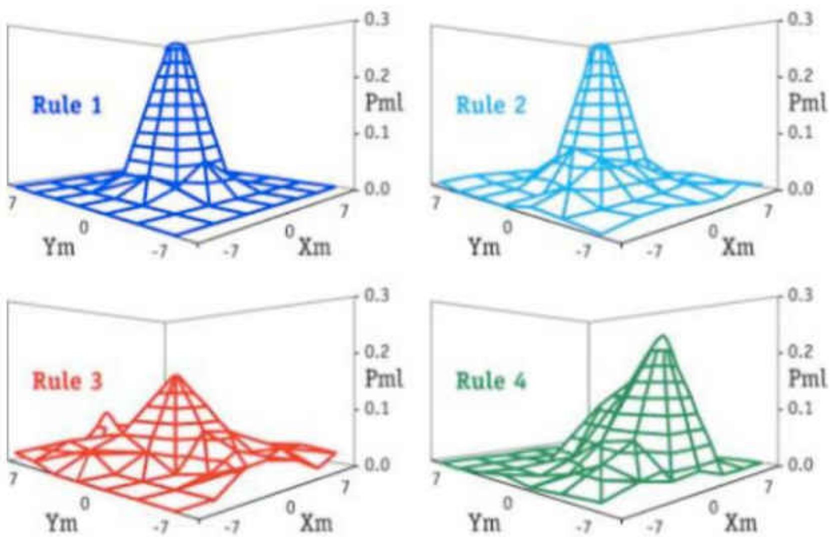


Figure 9. Three-dimensional plots of probable marble location based on simulated data. From *Tampering Dynamics* by Nicholas Georgantzas & Joyce Orsini. Copyright © 2003 by author. Reprinted with permission of author. All rights reserved.

Applying Theil's (1966) inequality statistic (TIS), they focus on analyzing the residual errors to decompose the mean squared error into its three components: bias, unequal variation, and unequal covariation. This helps them detect and describe the nonstationarity of the mean. They then compare the TIS results for rule 1 with the other 3 rules to specifically verify five conclusions made in the Sparks & Field (2000) paper described above.

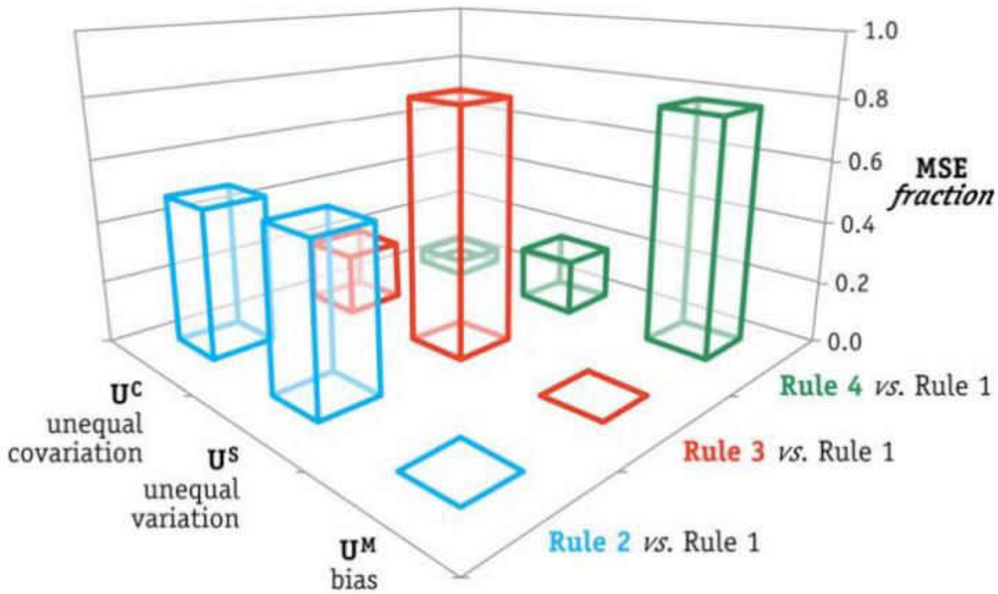


Figure 10. Theil's inequality statistic applied to the traditional funnel experiment. From *Tampering Dynamics* by Nicholas Georgantzas & Joyce Orsini. Copyright © 2003 by author. Reprinted with permission of author. All rights reserved.

Their next analysis compares each of the rules in terms of Shannon information, revealing significantly higher values for rules 2, 3, and 4 than for rule 1. This confirms what logic would suggest about the level of uncertainty or surprisal associated with the three variance-generating rules.

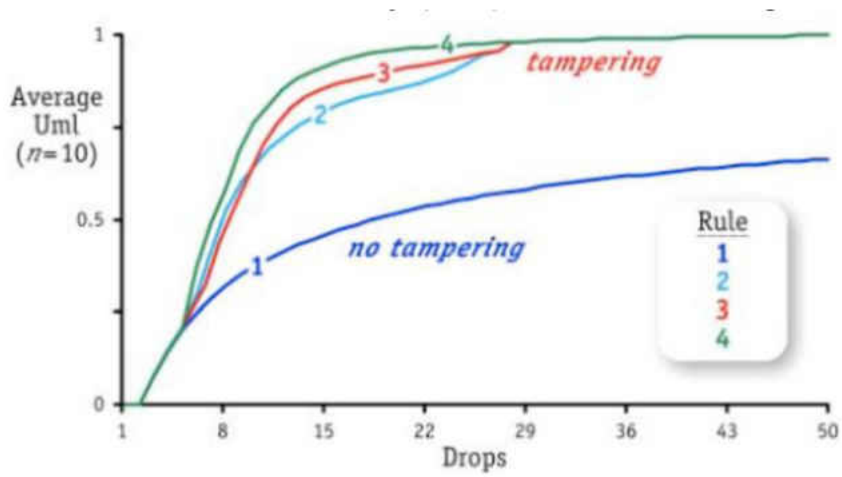


Figure 11. Shannon information plots for the four funnel rules. From *Tampering Dynamics* by Nicholas Georgantzas & Joyce Orsini. Copyright © 2003 by author. Reprinted with permission of author. All rights reserved.

Most importantly, they dedicate many pages to decomposing the structural dynamics that were revealed for each funnel rule in their simulation. They advocate the merging of complexity theory with simulation modeling to help examine the, "Multiple, nonlinear, generative mechanism embedded in the processes…" (p.24). The authors also advocate using system dynamics modeling to help, "Detect, explain, and prevent tampering" (p.24). Their final comment advocates the integration of emergence-based analysis with contemporary reductionist methods: "Complementing statistics with system dynamics can help managers sequester the entropy (uncertainty) of the processes they must manage" (p.24).

This paper was probably the most relevant source found in relation to the research question in that it discussed complexity theory and the kinds of emergent structural dynamics that can be leveraged to gradually improve a time series process. The key differences were their use of simulated data instead of empirical, and their use of systems dynamics modeling instead of permutation entropy and Jensen-Shannon complexity measures. Nevertheless, this paper provided the initial inspiration for the direction of this research.

Research Gap

Table 3. Research gap in funnel experiment literature.

Author	Uses Empirical Data	Counters Randomness, Tampering Assumptions	Applies Information Theory	Applies Emergence-Based Analysis	Tests Vertical Translations
Sparks & Field		X			
MacGregor	X	X			
Georgantzas & Orsini		X	X	X	
Lorimer	X	X	X	X	X

Complexity Theory Applications for Industrial Process Control

This section provides a review of the literature applying complexity theory to process control. However, the word "complexity" possesses many different meanings depending upon the context. An incomplete list of different meanings and concepts is provided in Appendix E. To improve the applicability of this review toward relevant research, these meanings will first be winnowed into three categories: *Complicated Complexity*, *Entropic Complexity*, and *Structural Complexity*. Complicated complexity measures processes in terms of how cumbersome or convoluted they are, as caused by many intricate, interconnected constituent parts. Entropic complexity measures processes in terms of the variability of entropic measures such as Shannon information, which generally corresponds with the heterogeneity or randomness of a process.

Structural complexity measures processes in terms of emergence-based dynamics, which may not be directly discerned from the constituent parts.

The general *concept* of complexity has been applied to many process control applications. However, of the three categories, only emergence-based structural complexity in the form of Jensen-Shannon complexity (C_{JS}) was directly applicable to this research. To better characterize structural complexity, Feldman et al. (2008) state,

> Measures of randomness do not capture the property of organization. This led to the recent efforts to develop measures that are, on the one hand, as generally applicable as the randomness measures but which, on the other, capture a system's complexity—its organization, structure, memory, regularity, symmetry, and pattern" (p.1).

This paper is reviewed more fully in the section of this Literature Review entitled, "Complexity-Entropy Diagrams". Another helpful characterization was provided by Crutchfield & Young (1989), when they applied statistical mechanics to infer statistical [structural] complexity. The resulting definition for structural complexity correlated physical structure with system randomness.

Complicated Complexity. Examples of papers applying complicated complexity to process control topics include *The Role of Variation, Mistakes, and Complexity in Producing Nonconformities* by Hinckley & Barkan (1995), *Finding a Complexity Measure for Business Process Models* by Latva-Koivisto (2001), and *Management and Control of Complexity in Manufacturing* by Wiendahl & Scholtissek (1994). However, complicated complexity was not relevant to this research so papers applying this type of complexity measure were not reviewed critically or for research gaps.

Entropic Complexity. Although Shannon entropy was certainly relevant to this research, it was derived in the application of permutation entropy methods, which in turn partially supported the assessment regimen for Jensen-Shannon (structural) complexity. Since measures of Shannon entropy provided a supporting role, entropic complexity was not a fundamental measure in this research. Therefore, papers discussing this type of complexity also were not reviewed. However, *Assessing the Structural Complexity of Manufacturing Systems Configurations* by Kuzgunkaya & ElMaraghy (2006) was interesting because the authors used the phrase "structural complexity" in the title, described manufacturing in terms of complicated complexity but applied entropic complexity as a reference measurement.

Structural Complexity. In the literature, structural complexity quantifiers were often applied to *characterize* time series processes. A number of papers were also discovered that applied complexity principles to various aspects of quality control. For instance, Dooley et al. (1995) advocated applying emergence-based complexity concepts to Total Quality Management (TQM) to improve the performance of complex organizations. Hutchinson (1994) advocated applying complexity theory and chaos theory principles to reduce errors in health care quality management when SPC is inadequate. Stroebel et al. (2005) advocated the improvement of primary health care service processes using a theoretical framework called *Multimethod Assessment Process/Reflective Adaptive Process*, which was informed by complexity science principles. The authors included a case study, which demonstrated improvements over Continuous Quality Improvement (CQI) for communication and training concepts such as, "Adaptive practices, increased capacity for learning, improved systems, richer connections and relationships, improved patient outcomes" (p.443). These studies were applying complexity *principles* to improve activities such as training, organizational performance, learning, etc., which can be contrasted with the focus of this research on applying ongoing measurements of Jensen-Shannon complexity to seek empirical evidence of improvement for verified Shewhart-stable time series.

Some literature applied the phrase "structural complexity" but with a much different definition of structural complexity in mind. In *Linking Service Structural Complexity to Customer Satisfaction*, Martínez-Tur (2001) defined structural complexity as, "The diversity of services offered by an organization" (p.1). Lindemann, et al. (2008) published a book entitled, *Structural Complexity Management: An Approach for the Field of Product Design* but used the word "structural" in reference to product structures, which the authors analyzed in terms of complicated complexity. In *The Structural Complexity of Software an Experimental Test*, Darcy et al (2005) contrasted algorithmic complexity with structural complexity, providing a definition for the structural complexity of a program as, "The organization of program elements within a program". The paper specifically examines, "The potential interaction of the two dominant dimensions of structural complexity, coupling and cohesion" (p.1). Finally, in *Structural Complexity Evolution in Free Software Projects: A Case Study*, Terceiro (2009) similarly defined structural complexity as, "The product of average module coupling and average module lack of cohesion" (p.4). These alternative conceptions of structural complexity were not relevant to this research.

Many sources were also discovered that applied the same definition of structural complexity that was used in this research. In general, the most relevant of this literature was associated with the *MPR-Method* or *Computational Mechanics*. After reviewing many papers based on these methods, none were found that measured changing structural complexity of an empirical time series to continuously inform the improvement of that time series as might be seen in an industrial engineering process control application. Continuous improvement is defined herein as actions taken that reduce variation, which also appears as decreasing structural complexity and increasing process randomness over time. Instead, the majority of these papers were focused on process *characterization*.

Numerous process characterization studies applied permutation entropy for fault detection and diagnosis, especially regarding rotary components like bearings, but very few of these also calculated changing structural complexity to facilitate new insights for process monitoring. The research presented by Radhakrishnan & Kamarthi (2016, December) and Radhakrishnan et al. (2019) did actually take the next step by applying the Complexity-Entropy Causality Plane for industrial component health monitoring to inform preventive maintenance. Of the literature reviewed, these studies were the closest in theme to this dissertation's research, but a minor distinction remains. Although both research efforts addressed common causes of variation, the distinction lies in the purpose of the research. That is, this dissertation's research was focused on informing the continued improvement (such as continuous complexity reduction) of a stable process, whereas the reviewed studies were instead focused on *characterizing* faults to inform actions that can better *maintain* the status quo by preventing special causes. Similar to the Tokai Rika process evaluated elsewhere in this research, preventive maintenance can achieve generally minor variation reduction, but its ultimate purpose is prevention, not improvement. An analogue that makes a similar distinction is the difference between risk monitoring/ prevention versus opportunity analysis.

Research Gap

No literature was discovered that measured changing structural complexity (such as Jensen-Shannon complexity) for an empirical time series to specifically inform the continuous improvement of that time series.

Permutation Entropy

This section will briefly introduce the foundational paper for permutation entropy, provide some of the resource literature, and list some of the diverse applications. The focus will then shift to three primary topics of interest. The first topic will cover permutation entropy's use in process control. The second will cover modifications that have been made to traditional permutation entropy methods, with emphasis on the mitigation of identical values within tuples. The final section will briefly introduce the foundational literature that established the Complexity-Entropy Causality Plane and provide a few of the resources that were referenced to seek more detailed explanations.

Before reviewing the literature, a brief overview of PE will first be provided to establish some basic familiarity. PE uses an ordinal scheme to extract a probability distribution associated with an input, which yields an information entropy quantifier. The ordinal symbolic method arises naturally from the time series based on the relative amplitude of adjacent values and therefore requires no model assumptions. Permutation entropy inherits the causal information that stems from the temporal structure of the system, revealing complex, nonrandom dynamic content. A more detailed discussion of the advantages and disadvantages of PE is provided in the Methodology section.

In 2002, Bandt and Pompe published the seminal paper entitled, *Permutation Entropy: A Natural Complexity Measure for Time Series*, which initiated a new field of information theory research. An online *Google Scholar* review on April 12, 2020, revealed 2,537 citations for this paper. Permutation entropy has been applied to many fields that traditionally monitor time series data. However, as will be revealed shortly, there have been relatively few instances of researchers applying this method to traditional industrial engineering problems for process control.

Bandt and Pompe (2002) introduced permutation entropy as, "Complexity parameters for time series based on neighboring values", which they claimed is, "Particularly useful in the presence of dynamical or observational noise." They stated advantages of, "Simplicity, extremely fast calculation, robustness, and invariance with respect to nonlinear monotonous transformations" (p.174102-1). They then introduce the procedures and basis for their method, which will be presented in the Methodology section. Finally, they provided some examples that validated permutation entropy's advantages.

The first example was a speech signal, which they compared with the traditional zero crossing rate (ZCR) parameter. PE was more robust to noise perturbations, choice of tuple length, and sampling frequency. PE also sometimes indicated voice transitions better. The chaotic time series of the logistics map was the second example, which they compared with the Lyapunov exponent. In the presence of significant dynamical and observation noise, PE demonstrated its invariance to nonlinear monotonic transformations. They claim this advantage is, “In contrast to all known complexity parameters”. They close by stating that this method, “Seems preferable when there are huge data sets and no time for preprocessing and fine-tuning of parameters” (p.174102-4).

Numerous overviews and advancements have been published since Bandt and Pompe’s seminal paper. Detailed theoretical background information can be found in Amigo (2010) and Martin, Plastino & Rosso (2006). A practical tutorial providing an overview of applications is provided by Riedl, Müller & Wessel (2013) based on, “Exhaustive literature research summarizing and classifying the experiences of experts” (p.250). Foundational concepts underlying PE are provided in Lopez-Ruiz, Sañudo, Romera, & Calbet (2012) and in Kowalski, Martin, Plastino, Rosso, and Casas (2011).

As a tool for characterizing complex time series, PE has been applied in a wide range of scientific areas and for diverse purposes. The following list briefly demonstrates the heterogeneity of application without being exhaustive:

- Rotary Machines. Redelico et al. (2017), Yan et al. (2012)
- Vehicle Behavior. Aquino et al. (2015)
- Electric Load. Aquino et al. (2017)
- EEG Neuronal Activity. Cao et al. (2004), Li et al (2007), Liang et al. (2015), Montani et al. (2015), Morabito et al. (2012), Azami & Escudero (2016a)

- EKG Heart Activity. Bian et al. (2012)
- Optical Semiconductor Laser Chaos. Liu et al. (2016), Toomey et al. (2014), Zunino et al. (2011), Soriano et al. (2011)
- Biometrics. Rosso et al. (2016), Zunino et al. (2017)
- Geophysics. Sippel et al. (2016), Saco et al. (2010), Consolini et al. (2014)
- Econophysics. Zunino et al. (2009), Bariviera et al. (2015)[22–25]
- Hydrology. Serinaldi et al. (2014), Lange et al. (2013), Stosic et al. (2016)

Permutation Entropy for Industrial Process Control

Many permutation entropy papers were discovered representing fields that conduct process monitoring, such as medicine (e.g. EKG, EEG processes) or finance (e.g., stock market inefficiency, gold pricing). In addition, numerous process characterization studies applied permutation entropy for fault detection and diagnosis, especially regarding rotary components like bearings. In general, the focus of most of the papers surveyed was process characterization and monitoring. A smaller subset applied the results to make adjustments relevant to process performance, such as mitigating special causes. Ultimately, no papers were discovered that applied permutation entropy to improve stable process performance through continuous improvement, resulting in ongoing variation reduction. Since some applications attempted to at least maintain a stable process, three papers will be reviewed that applied permutation entropy results to control theory to assess and modify complex industrial process performance. Control charts were only discussed in one of these three papers.

The first relevant paper, written by Martinez & de Prada (2007) discusses control loops in process plants, to include real-time optimizers, model predictive controllers, proportional integral derivative controllers, and inferential loops. The authors lament the normal degradation of process operation that occurs over time and advocate permutation entropy as a method to reveal, "Predictability patterns in the error time series" (p.2) to supplement other process improvement efforts. Their focus was on the ordinal relationships between error values instead of between the values themselves. They refer to two other papers that will be mentioned here because they are closely related to the topic, but do not meet review criteria. The first, by Bandt (2005), discussed permutation entropy techniques for error time series but did not address application to process control. The other, by Ghraizi, et al. (2007) also discussed error time series analysis, but did not address the application of permutation entropy.

Martinez & de Prada go on to discuss the method and merits of permutation entropy. They consider the state when all the permutations have equal probability (random error disclosed via a uniform probability distribution) as the ideal for any controller that is providing process stability over the control horizon. With this as the reference, they then propose a performance index, which is the normalization of the current Shannon information measure. They conclude with promotion of ordinal methods for other process supervision tasks, to include batch processes and hybrid control systems. The authors did not extend their presentation beyond permutation entropy into structural complexity research.

The second paper was written by Wu, et al. (2019), which also discusses control loop performance supervision. However, these authors advocate a modified version of permutation entropy that introduces data amplitude weighting (wPE) to establish a performance index. This method was designed to facilitate closed loop performance monitoring using Shewhart and EWMA control charts. The wPE technique was devised by Fadlallah, et al. (2013), in part to better detect spiky processes, and will be discussed in greater detail in the next section of this Literature Review.

This paper by Wu, et al. creates numerous opportunities for confusion, which should be explained up front. First, they refer numerous times to a paper by “Rachid” which is actually the paper by Ghraizi, et al. (2007) mentioned above. This occurred because they confused first and last names in their reference #15. However, their point was to show that “Rachid” contributed to the analysis of time series predictability, to include proposing a performance index. Second, they referred to Ghraizi correctly elsewhere, but this time referencing a symposium presentation two years after the “Rachid” paper which *did* apply permutation entropy (Ghraizi, et al., 2009). This is their reference #18, and it will be reviewed shortly in this section. Finally, they refer to a paper by Zhang, et al. (2014) which proposed a performance assessment benchmark based on minimum information entropy. Although this paper applied entropy techniques, it did not specifically apply the permutation entropy methodology.

Wu, et al. established control chart control limits for their processes and used simulated data to evaluate whether the Shewhart control chart would capture two abrupt changes in the process data using both PE and wPE-derived performance indices. Both spikes were captured with wPE and neither for PE. They then used EWMA to assess changing pressure for natural gas pipeline control loop data. Only wPE was evaluated, but it demonstrated the ability to correctly track pressure performance changes. Their research verified that a version of permutation entropy could accurately track process performance with a control chart. Although these authors referred to complexity many times, their context was either to describe permutation entropy as a measure of entropic complexity or to describe complicated complexity. As such, Wu et al. did not expand their presentation from PE into the emergence-based structural complexity relevant to this research.

The third and final source, Ghraizi et al. (2009), looked at the predictability of error patterns for control loop performance. They established a performance index by dividing the variance of the residuals by the variance of the controller errors. They acknowledged that a large confidence interval is generated when the loop performance degrades based on *t*-distribution assumptions, limiting its usefulness. Therefore, their tests used ordinal patterns for the residuals to relax the statistical assumptions. This allowed them to compute a new performance index based on permutation entropy. They compared the performance of the new index with the old one using three industrial case studies. The PE-based performance index provided better and more consistent results, even when the control loop was in saturation. They closed by advocating their method to automatically supervise all process plant control loops in real time. They mentioned the term dynamic complexity, which was a reference to entropic complexity, and not to the structural complexity relevant to this research.

Research Gap

Table 4. Research gap in permutation entropy literature for process control.

Author	SPC Related	Process Improvement Beyond Feedback Control	Use Permutation Entropy Generally	Use Modified Permutation Entropy	Apply Structural Complexity Theory
Martinez & de Prada	X		X		
Wu, et al.	X		X	X	
Ghraizi, et al.	X		X		
Lorimer	X	X	X	X	X

Mitigation of Identical Values within Tuples

In their seminal paper, Bandt & Pompe (2002) identified a potential problem with their permutation entropy method. They believed their method would work well as long as identical values did not appear within a tuple. Fortunately, such equal values are relatively rare in continuous series. However, equal values are much more common for discrete series data. With the ubiquitous digitization of data streams, this problem has become more significant. When a discrete time series represents a Shewhart-stable process, equal values are even more likely.

Bandt & Pompe recommended for discrete series to, "Numerically break equalities by adding small random perturbations" (p.174102-1). Their assumption was that the varying noise would map with more or less equal probability to each suitable symbol. However, in a paper published on the topic 15 years later, Zunino et al. (2017) stated that, in practice, this addition of a small amount of noise, "Has been rarely implemented" (p.1883). Additionally, Traversaro et al. (2018) pointed out that adding noise fixes one problem but creates another: "Random imputation may overestimate the entropy, as it adds random noise to the series, and masks forbidden patterns by inducing those missing patterns to appear, concealing this dynamical property characteristic of chaotic dynamics" (p.075502-8). It should be noted that having equal values within a tuple is not the same problem as "forbidden" patterns, which are non-deterministic patterns within chaotic processes (Rosso et al., 2012). Nor are they "missing" patterns, which are unobserved patterns in stochastic time series resulting from finite sample size (Borges et al., 2019).

Identical values are especially a problem for discrete processes with low amplitude or low dimensionality data. Not only do Shewhart-stable processes typically fall in this category, but also highly repetitive processes such as EKG, and periodic processes such as data from oscillatory machines. This could be problematic in process control when evaluating the effect of intended process improvements. The concept of equal values within a tuple may seem nebulous at this point so a very brief introduction to the problem will be provided here. The Methodology section will provide more detail.

Encoding each group of D consecutive numbers based on their relative amplitudes. In this case, assume a tuple length (embedding dimension) of D=3. Each tuple will represent one of $D!$=6 possible distinct symbols, as shown graphically in Figure 12. Possible distinct tuples are Ω_3 = {(012)(021)(102)(120)(201)(210)}, presented in the same order as in the figure. Note that none of these symbols account for an equal value within the tuple, which would look like, for example, (001) or (101). Also note that some authors prefer to use the numbers (123) instead of, equivalently, (012). So the most relevant question is: "What is the best way to account for identical values within a tuple that would minimize the erroneous representation of emergent process dynamics?”

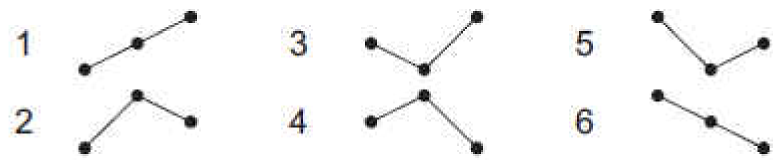


Figure 12. Distinct permutations for embedding dimension, *D*=3. From *Classifying Cardiac Biosignals using Ordinal Pattern Statistics and Symbolic Dynamics* by U. Parlitz et al. Copyright © 2012 by Elsevier. Reprinted with the permission of Elsevier. All rights reserved.

Zunino et al. (2017) stated that, “Equal values in the time series are very usually ranked according to their temporal order” (p.1883). This method is also called time-ordered imputation. Returning to the example above, if the pre-coded data were {6.0, 6.1, 6.0} the equivalent coded symbol using this temporal ordering scheme would become (021), creating a viable symbol. Although this simple and popular method of time-ordering mitigates all identical values, Zunino et al. conducted a comprehensive empirical analysis, which revealed that, “This way of dealing with ties introduces non-negligible spurious temporal correlations that can potentially lead to erroneous conclusions about the true underlying dynamic nature” (p.1890). The results of this paper are very relevant to the research herein, and are studied in more detail in the section of the Methodology discussing permutation entropy local effect validation.

The next method to mitigate identical values is to simply remove tuples that contain identical values, since they are not “statistically complete.” This method is analogous to a “complete case analysis” in statistics. According to Traversaro (2018, Table II), the statistically complete method preserves the missing or forbidden patterns but creates problems because throwing out some of the data will likely bias the presentation of the emerging dynamics. The statistically complete method is embraced for the research herein, to handle non-pattern following tuple types (Red & Wizard), and will be discussed further in the Methodology section.

Bian, et al. (2012) presented a method called modified permutation entropy (mPE) to better characterize heart rate variability signals in empirical studies. They employed three groups of heart rate data to evaluate their method. Equal values within tuples were mapped to the same symbol, considerably increasing the number of possible tuples. According to Zunino et al. (2017), one problem with mPE is that it, “Does not reach its maximum value for … totally random signals (white noise) as this actually happens for the standard PE” (p.1884). In other words, mPE is not generalizable to existing permutation entropy analysis. Another perspective regarding mPE was provided by Azami & Escudero (2016a), who recognized that mPE has an advantage for long signals. However, they offered that mPE would be unreliable in the case of short signals because of the number of potential tuples created.

Olofsen et al. (2008) developed a Composite PE Index (CPEI) to measure anesthetic drug effect for EEGs. They recognized that PE’s sensitivity to high frequencies makes it a poor measure for drug effects. Since their focus was on low frequency process behaviors, they created a tie tuple to use when the difference between any two of the data values within a tuple was less than 0.5 mV. Since D=3 has 6 nominal tuples, they created a seventh tuple for ties. They also recognized that tie tuples evaluated at $\tau = 1$ could distinguish periods of delta waves and those at $\tau = 2$ could distinguish mid-frequency from slow waves. Both of these features were considered useful to measure drug effect, so they took advantage of the natural summation of entropies to create their composite index. To normalize their index, they used the typical logarithm of the maximum possible quantity of distinct symbols, which in their case was 7x7= 49. This feature made CPEI ungeneralizable to existing permutation entropy analysis with WGN not equal to unity. After evaluating their method using real EEG signals, they determined that their index was better than spectral entropy and approximate entropy methods. They closed by claiming that CPEI, “Allows the graded differentiation of different anaesthesia-induced EEG patterns, and the handling of low-amplitude measurement noise” (p.820).

Chen et al. (2019) proposed improved PE (iPE) and multiscale improved PE (miPE) to incorporate amplitude information, mitigate equal values within a tuple, and perform better under noisy conditions. The method they identified for multiscale iPE (miPE) was to organize the time series under any multiscale course graining method, then feed the result into their iPE algorithm. Their iPE algorithm was composed of two parts, definition of pattern and entropy estimation. They considered their method for definition of pattern an improvement upon the integrated approach based on uniform quantization (IAUQ) of Porta et al. (2007), previous used to evaluate heart period variability. Their iPE algorithm accommodates potentially useful amplitude information, which the normal PE coding scheme misses and many other methods ignore. Their figure, presented below, reveals the problem graphically.

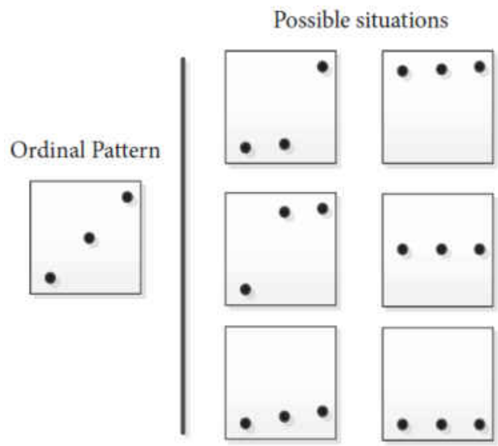


Figure 13. Possible Permutation Amplitude Situations. From *Improved Permutation Entropy for Measuring Complexity of Time Series under Noisy Conditions* by Zhe Chen et al. Copyright © 2019. Reprinted under Creative Commons Attribution License v4.0. https://creativecommons.org/licenses/by/4.0/

Chen et al (2019) identified four improvements that their method makes over the normal PE method. First, the same symbol is assigned to equal values. They stated that the way iPE processes repeated values, "Will not cause overestimating of permutation patterns thus a more precise complexity measure can be obtained" (p.3). Second, iPE provides better accommodation of amplitude and fluctuation information. Third, iPE provides more robustness to noise interference. And fourth, better information is provided by L^D possible tuples instead of just $D!$, where L denotes the discretization level. They claim the selection of L involves a trade-off between accurate entropy estimation and high noise immunity. After analysis, they established L=4 as optimal for subsequent study.

The authors acknowledged some weaknesses of iPE. For example, iPE provides unreliable entropy estimation for short time series with $N < 4L^D$, where N = total number of data. Also, the extra distinct tuples means iPE does not reach the maximum entropy value for WGN.

They dismiss this problem by stating that iPE, "Obtains large enough entropy measurements (>0.93) when $L \geq 4$, which is generally identical to the fact" (p.4). The fact they reference is $\widetilde{H}_{\max} = 1$ for WGN.

To validate their algorithm, they analyzed spiky data, heart rate variability data, an autoregressive model, and ship noise. For all four time series, they compared the results of iPE with the results from five other methods, numerically demonstrating the advantages of their algorithm. Using abbreviated terminology, the other five methods were PE, wPE, aaPE, mPE, and IAUQ. It should be noted that each of these methods is described elsewhere in this literature review, and any of these methods that do not directly address identical values within tuples are presented in the next section. Chen et al. (2019) close by recommending the application of iPE in future work such as, fault diagnosis, acoustic signal processing, and stock market analysis.

The paper by Porta et al (2007) was mentioned in the previous review of Chen et al. (2019) with respect to the IAUQ method. Although IAUQ did not focus on permutation entropy per se, the ordinal analytic methods described in Porta for handling identical values within an entropy sample "family" merit comment. Porta et al (2001) preceded Bandt and Pompe (2002) in a line of thought about entropy analysis that is similar with respect to the symbolic discretization of time series data. The IAUQ method that Porta et al (2001) developed created a large number of possible distinct entropy samples, which they recognized could make sample classification unmanageable. They therefore developed a scheme to group such patterns into a smaller number of "families", to ease the monitoring effort.

For D=3, the pattern "families" that Porta et al (2001) created were: (1) Patterns with no variation, all the symbols are equal; (2) Patterns with one variation, two consecutive symbols are equal and the remaining one is different; (3) Patterns with two like variations, the three symbols form an ascending or descending ramp; (4) Patterns with two unlike variations, the three symbols form a peak or a valley (p.1285). Their methodology to reduce tuple redundancy was exploited in future studies, including Guzetti et al. (2005) and Porta et al (2007). However, the only paper discovered that applied their IAUQ signal quantization methodology to advance permutation entropy was Chen et al. (2019), reviewed above.

Another method, which is based on data-driven imputation, was proposed by Traversaro et al. (2018). The authors started by assuming that the visible discrete data are a corrupted version of an underlying, unobserved continuous time series with, by definition, no ties. Their method determines a probability distribution using the information available from the observed, corrupted data to estimate the "true" patterns. An elaborate scheme of symbol weighting is introduced and then the data-driven method is compared to three other methods. These methods were the statistically complete, time-ordered, and random imputation methods. Analysis was accomplished using decimal expansion of three irrational numbers, the logistic map, and a delayed logistic map. Their conclusion was that random imputation and statistically complete methods should not be used at all, and that the data driven method outperforms the time-ordered method in some cases. They also recommended that none of the imputation versions should be applied when 35% or more of the vectors have ties, due to the misleading estimates they generate (p.075502-13). For the research herein, some of the processes had greater than 35% ties in vectors (such as V4), which limited the applicability of this otherwise promising method.

Xiao-Feng & Yue (2009) introduced fine-grained PE (fgPE). This method takes into account the magnitude of the difference between neighboring values when symbolizing the time series. Their method represents certain advancements from two other entropy modification techniques developed before PE was introduced in 2002. The first was approximate entropy (ApEn) (Pincus, 1991) and the second was sample entropy (SaEn) (Richman & Moorman, 2000). They proposed that their fgPE algorithm, based on the Lyapunov exponent (λ), could overcome the equal value problem. They also suggested that their technique, "Improves the performance for detecting the dynamical change of time series and approximates more closely to the Lyapunov exponent for the chaotic time series" (p.2691). Azami & Escudero (2016a) acknowledged that fgPE can improve the performance of PE, but found the precision regulation factor (α) that Xiao-Feng & Yue created to be too sensitive (p.4).

The method that Azami & Escudero (2016a) instead proposed is called amplitude aware permutation entropy (aaPE). Their focus was signal segmentation and spike detection for EEG data, but the mitigation of equal values was also addressed by their method. They acknowledged that wPE (Fadlallah et al., 2013) is a powerful tool but suffers from two weaknesses. First, wPE cannot distinguish when a constant value is added to the original signal because variance would not change. Second, wPE does not allow customization of the importance given to the amplitude information, based on the purpose of the analysis.

Largely to address these shortcomings, Azami & Escudero built an algorithm that considers the mean value in each tuple, the difference between consecutive samples, and adds a customizable variable for the amplitude. Customization can be made depending upon whether the mean is more relevant to a study or instead the differences between consecutive samples. However, they also warned against using a time lag greater than unity ($\tau > 1$), without considering the characteristics of the signal. They thought this could lead to aliasing, which is when different signals become indistinguishable. They also recommended using methods other than aaPE when evaluating series with multiple temporal scales (multiscale series).

The authors provide an example for using aaPE to mitigate identical values, based on half contributions by adjoining values and a combination of two algorithms. They stated that this method can discriminate between types of ascending and descending sequences. They closed by claiming that aaPE is, “A powerful tool to segment signals and detect spikes. It can be applied in different applications where the mean values of neighboring samples and changes in amplitude values are important” (p.30). For mitigating identical values within a tuple, both of these considerations can be relevant.

The final paper reviewed was significant not only because the authors evaluated numerous processes that would be considered Shewhart-stable, but also because their viewpoint contradicted a substantial body of analytical work regarding identical values within tuples. Specifically, Cuesta-Frau et al. (2018) researched whether identical values within a tuple are really even an issue. They evaluated four different biomedical datasets: EEG, Body Temperature (Temp), interval distance between heartbeats (RR), and continuous glucose monitoring (CGM) data. These four were selected because they, “Often include ties due to the intrinsic nature of the records (RR records), lack of resolution of the measuring devices (CGM), slow variations in the underlying signal (Temp), or just by chance (EEG)” (p.13). They counted up the percent of tuples containing equal values for embedding dimensions ranging from D=3 to 9 in these datasets. Their results are presented in Table 5.

Table 5. Percent of tuples containing equal values for four stable datasets. From *Patterns with Equal Values in Permutation Entropy: Do They Really Matter for Biosignal Classification?* by Cuesta-Frau et al. Copyright © 2018 by author. CC0 1.0 license, https://creativecommons.org/publicdomain/zero/1.0/

	3 (%)	4 (%)	5 (%)	6 (%)	7 (%)	8 (%)	9 (%)
EEG	2.54	3.29	3.91	4.44	4.91	5.35	5.75
Temp	9.26	11.71	13.81	15.55	16.88	18.02	19.48
RR	16.48	22.28	27.08	31.01	34.27	37.10	39.90
CGM	20.58	23.77	26.40	27.90	30.19	31.00	31.87

The authors condemned random imputation, mPE, and aaPE methods because, "All the methods try to account for ties from a mathematical perspective, but not from a conceptual perspective: What ties really represent in a time series, since equal values may be an intrinsic part of the dynamics of the records" (p.1) They also took issue with the use of, "Synthetic or very specific types of records, which do not provide a complete picture of the real influence of ties in PE" (p.2).

They thought the questions that should instead be considered are (p.2):

- Is there a correlation with the percentage of ties and PE performance?
- Is it preferable to account for ties in the PE algorithm or try to remove them in advance?
- Is the time series location of ties making any difference on their influence?

Cuesta-Frau et al. (2018) pointed out that only one study has attempted to answer these questions: Zunino et al. (2017). They acknowledged that Zunino et al. concluded that equal values introduce a PE bias. However:

> …this study [Zunino et al.] was focused on assessing the changes in the results of PE compared to those obtained with no ties, in absolute terms. In signal classification tasks, the main driver of PE performance is not the specific PE value, but its intra- and interclass distribution (p.2).

Similarly, Cuesta-Frau et al. claimed the main objective of their study was, “To assess the influence of ties in relative terms, not to propose a signal classification scheme” (p.2). They thought the continuing problem was:

> PE bias may be distributed more or less uniformly among all the classes, and therefore, the differences may still remain apparent. Based on this hypothesis, the present study aims to gain a more practical insight into the real influence of equal values in PE. (p.2)

Cuesta-Frau et al. then intentionally corrupted each experimental dataset by inserting synthetic equal and independent values. They specifically increased the percentage of ties in the EEG and CGM datasets up to 50% to, “Obtain PE as a function of the level of ties and plot the results to visually determine the degree of variation that such changes entail” (p.6). Their results showed that adding random perturbations to break ties improved results by only about 1%. Also, that mPE improved the baseline case by about 2%, and aaPE was worse than no mitigation at all. They hypothesized that stochastic resonance may be involved, which they defined as, “An effect by which an external disturbance and the internal dynamics of a signal have a positive collaborative interaction that results in enhanced signal detection” (p.13).

They concluded that ties are not all equal. More specifically, “Those due to consecutive values seem to exert a lower influence than those due to unconnected equal values in the same pattern” (p.14). And a final suggestion:

> When ties are involved, if PE had to be applied to a classification task and the results were poor, we would suggest maximising [the embedding dimension, *D*] in accordance with the computational resources available (memory and time cost) and the classification performance achieved, using the standard PE algorithm. (p.14)

Although Cuesta-Frau et al. focused on stochastic effects, chaotic influences were not provided in their analysis, which could be a significant concern depending upon the nature of the process. For all four types of data series that Cuesta-Frau tested, other authors have provided insights regarding chaotic process behaviors. Some relevant studies that could be contemplated to address this research gap are listed in Appendix D.

Research Gap

Table 6. Research gap for mitigation of equal values within tuples.

Author	Proposal reaches H_{max} for WGN	Assumes Equal Values are Problematic	Applies Statistically Complete Method	Applies Local Effect
Bandt & Pompe (2002)	X	X		
Zunino et al (2017)	X	X		
Traversaro et al. (2018)	X	X	X	
Bian et al. (2012)		X		
Olosen et al. (2008)		X		
Chen et al. (2019)		X		
Porta et al. (2001)		X		
Xiao-Feng & Yue (2009)	X	X		
Azami & Escudero (2016a)	X	X		
Cuesta-Frau et al. (2018)				
Lorimer (2020)		X	X	X

Modifications to Permutation Entropy

The literature for traditional permutation entropy, including the seminal paper by Bandt & Pompe (2002), describes weaknesses in certain applications. However, one of PE's strengths- the ubiquity of application to diverse time series- can be an additional weakness when processes possess certain characteristics such as spikiness. The previous section reviewed methods devised to mitigate various problems associated with equal values in tuples. This section will look more broadly at other relevant aspects of permutation entropy that suggest the need for modifications. Topics include mitigating tuple redundancy, incorporating spatio-temporal information, dealing with spiky processes, incorporating amplitude information, and handling multiscale applications.

Many variations to the original method were discovered in the permutation entropy literature. An attempt was made in this section to review the most salient of PE modifications, since it was impractical to review every possible variation. An illuminating paper, written by Zanin et al. (2012), provides a similar review of alternative PE methods, but is focused specifically on biomedical and econophysics applications. For readers that would like more detail, this paper is recommended as a supplement to this section of the dissertation.

Fadlallah et al. (2013) proposed the weighted PE (wPE) method. They thought it was important to, “Differentiate between distinct patterns of a certain motif and the sensitivity of patterns close to the noise floor” because patterns may be too disparate in amplitudes and variances (p.022911-1). They conducted simulations which showed, “Better robustness and stability in the presence of higher levels of noise”, and the capability to, “Extract complexity information from data with spiky features or having abrupt changes in magnitude” (p.022911-1). However, according to Azami & Escudero (2016a), wPE cannot distinguish the case where a constant value is added to the original signal, because its variance would not change. They also opined that wPE could be better if the amplitude information could be customizable, depending upon the application. The previous section of this Literature Review presented Azami & Escudero’s amplitude aware Permutation Entropy (aaPE) methodology.

Berger et al. (2017) introduced the centroid of a weighted power spectrum to represent an epoch of EEG for redundancy reduction. This centroid was based on the ordinal patterns of three consecutive samples, which they thought could, “Aid the interpretation of PeEn [permutation entropy] in EEG, and may increase its comparability with other techniques of EEG analysis” (p.1). Their proposal was specific to EEG, but their weighting scheme could ostensibly be applied to many other applications.

Schlemmer et al. (2018) introduced spatial PE (sPE) and Spatio-Temporal PE (stPE) to extract three-dimensional order patterns from images of EKG video. The purpose was to capture the entropic complexity of spatial and temporal structures at the same time. They claim that, “SPE and STPE are robust against noise”, and usefulness for, “Extracting complexity features at different spatial scales” (p.1).

Porta et al. (2007) proposed the integrated approach based on uniform quantization (IAUQ) method, to improve analysis under noisy conditions and reduce redundancy when there are many distinct types of symbols. The work of Porta et al. in 2001 was described in the previous section, as it pertained to identical values within a tuple. This later work in 2007 continued to use methods that differ from traditional permutation entropy but nonetheless applied entropy estimation via analysis of symbolic dynamics. Application of the IAUQ method to human heart data in their 2007 study allowed differentiation between normal subjects and heart failure patients.

Rostaghi & Azami (2016) introduced dispersion entropy (DispEn) as an improvement over sample entropy (SaEn) and traditional permutation entropy. They opined that SaEn is not fast enough, and PE has numerous shortcomings that can be addressed by their method. Instead of permutation patterns, DispEn uses patterns based on the dispersion of the data. They analyzed synthetic signals, EEG, blood pressure, and roller bearing data. They then compared their results to analyses of the data accomplished with SaEn and PE. The authors claimed that DispEn, unlike PE, can detect the noise bandwidth and simultaneous frequency and amplitude change.

Also, that it can quantify the "regularity" of time series, and outperform PE in both signal discriminatory power and computation time.

Azami & Escudero (2018) further advanced the dispersion entropy method. They investigated the effect of linear and nonlinear mapping approaches, evaluated DispEn's sensitivity to noise, and developed a new measure that deals only with the fluctuations of time series (FDispEn). FDispEn was promoted as a method to discriminate deterministic from stochastic time series. Using two physiological datasets, they compared the performance of DispEn, FDispEn, permutation entropy, sample entropy, and Lempel–Ziv (LZ) complexity. Based on their results, they opined that DispEn is, "The most consistent technique to distinguish various dynamics of the biomedical signals" (p.1).

Numerous methods have been developed to analyze multiscale processes, likely because many physical and biological systems reveal structure on multiple spatio-temporal scales. Of the relevant papers found, the first to address the topic was Costa, Goldberger & Peng (2002a), but without using permutation entropy methods. They created a multiscale entropy (MSE) method that used sample entropy (SaEn) to quantify the complexity of time series. Three other papers by this same group of authors (2002b; 2003; 2005), applied MSE but did not incorporate permutation entropy.

The first paper found that examined multiscale permutation entropy (MPE) was Aziz & Arif (2005). They proposed advancing multiscale research with the permutation entropy method because of its robustness in the presence of dynamical and observational noise. They assessed a number of coarse-grained sequences representing different temporal scales in terms of permutation entropy. Their first step was to make multiple successive coarse-grained versions by averaging the time data points within non-overlapping windows of increasing length. This created a scale factor. The second step was to calculate the PE for each coarse-grained time series and plot them as a function of the scale factor.

Aziz & Arif compared their results to the MSE method for heart RR interval data. They determined that MPE was better able to handle nonstationarity, outliers, and artifacts than MSE. They also noted that MPE's analysis of heart data allowed the authors to distinguish the between healthy and pathological groups. Morabito et al. (2012) used MPE for a new biomedical application. Specifically, they evaluated EEG recordings for Alzheimer's disease research. But they did not make any significant modifications to the MPE method.

Azami & Escudero (2016b), on the other hand, noticed some improvements that could be made. The authors offered that the "stability" may be compromised for short time series when applying conventional MPE techniques. More specifically, when the scale factor is high, the number of samples in the coarse-graining sequence decreases, which may result in an unstable entropy measurement. They also noted that MPE is not symmetric, in that tuples artificially separate adjacent values, forming an inconsistent analysis. As a result of these deficiencies, they proposed the improved MPE (IMPE) method to, "Reduce the variability of entropy measures over long temporal scales, leading to more reliable and stable results" (p.28).

Azami & Escudero used synthetic signals and EEG data to compare results between MPE and IMPE. Using the synthetic signals, both methods were investigated to better understand their behavior in terms of signal processing concepts. These concepts included frequency, amplitude, noise power, and signal bandwidth. The results showed that, "A signal with noise, larger bandwidth and/or higher frequency would generally lead to higher values of entropy.

[However,] MPE and IMPE results were not sensitive to slow changes in amplitude" (p.40). They closed by claiming that the results from both synthetic and EEG processes, "Support the idea that IMPE has a better performance than MPE, although both are useful and informative tools to calculate the complexity of a time series" (p.40).

Zunino et al. (2012) also focused on the problem of multiscaled time series. However, their approach extended beyond characterizing the permutation entropy. They also estimated Jensen-Shannon complexity as a function of the embedding time lag (τ), and explicitly determined the scale by changing this embedding delay. In so doing, they were able to infer the emerging dynamics of the complex time series and, importantly, "Identify the range of scales where deterministic or noisy behaviors dominate the system's dynamics" (p.046210-9). Their method advanced the applicability of a representational diagram developed in 2007, called the Complexity-Entropy Causality Plane, which is the next topic in this Literature Review.

Table 7. Summary of modification methods reviewed for permutation entropy.

Author	Method	Abbreviation
Fadlallah et al. (2013)	Weighted PE	wPE
Berger et al. (2017)	Centroid of Weighted Power Spectrum	
Schlemmer et al. (2018)	Spatial and Spatio-Temporal PE	sPE / stPE
Porta et al. (2007)	Integrated Approach on Uniform Quantization	IAUQ
Rostaghi & Azami (2016)	Dispersion Entropy	DispEn
Azami & Escudero (2018)	Fluctuation Dispersion Entropy	FDispEn
Aziz & Arif (2005)	Multiscale PE	MPE
Morabito et al. (2012)	Applied MPE to EEG	
Azami & Escudero (2016b)	Improved MPE	IMPE
Zunino et al. (2012)	Applied C_{JS} to Complexity-Entropy Causality Plane	
Lorimer	PE Local Effect	PE-LE

Complexity-Entropy Diagrams

Numerous varieties of complexity-entropy diagram concepts were discovered in the literature. No sources were discovered that applied log change calculations similar to the Entropy-Complexity Change Diagram (ECCD) developed in this research. The ECCD will be described in the Methodology section. The Complexity-Entropy Causality Plane (CECP) was used extensively to evaluate research data. Therefore, this section will review the CECP along with another relevant idea to facilitate research gap comparisons. This review will also present some of the key literature associated with the CECP, since it was integral to this research. Both of the concepts in this review have unique strengths and it seems evident that either could have been applied productively to the research herein. However, in the interest of assuming a reasonable scope of research, only the CECP was chosen to evaluate research data. It should be noted that different measures of both complexity and entropy were applied in the two diagrammatic concepts reviewed.

The h_μ vs. E Approach

The first complexity-entropy diagram to be reviewed was proposed by Feldman, McTague, & Crutchfield (2018) in a paper entitled, *The Organization of Intrinsic Computation: Complexity-Entropy Diagrams and the Diversity of Natural Information Processing*. One of this paper's authors (Crutchfield) co-wrote a seminal paper entitled, *Inferring Statistical Complexity* (Crutchfield & Young, 1989), which introduced the first similar complexity-entropy concept.

The authors acknowledged the diversity of these diagrams, writing, "there is not a universal complexity-entropy curve, there is not a general complexity-entropy transition, nor is it [the] case that complexity-entropy diagrams for different systems are even qualitatively similar" (p.3). They acknowledge that a single complexity-entropy diagram concept can portray different processes in substantially different ways, and that some researchers may see this as a detriment given the perceived "inconsistency" in presentation. However, the authors instead argue this is an asset because it better demonstrates the, "Genuine diversity of distinct kinds of intrinsic computation" (p. 1), and the, "Richness in nature's organization" (p.3).

The authors present one of the key advantages of these diagrams by stating, "Since complexity-entropy diagrams are a function only of observed configurations, they can be used to compare systems without reference to system coordinates or parameters" (p.1). They then justify structural complexity as a unique measure by mentioning that process behaviors cannot be meaningfully summarized by the entropy rate or by the fractal dimension, alone. This is likely mentioned because entropy measures the changing randomness of a process and fractal dimension provides the level of complication associated with certain self-similar structures. Instead, to truly understand a process, they advocate adding structural complexity, a measure that reveals the, "Organization, structure, memory, regularity, symmetry, and pattern" (p.1).

They then apply two bins to distinguish between all the various conceptions of complexity. The first bin includes "deterministic complexities", which capture only randomness. These measures include the Shannon entropy rate, Lyapunov characteristic exponents, and Kolmogorov-Chaitin complexity. See Appendix E for more detail about these and other information-theoretic complexity concepts. The other bin includes, "statistical or structural complexities", which capture a property distinct from, and complementary to randomness. Their paper focuses exclusively on complexity of the structural and statistical sort, as does the research in this dissertation.

For the entropy term to be represented on the x-axis, the authors chose the entropy rate (h_μ), in bits per symbol (not per second), which exists for all stationary sequences. They then connect this term with other similar variables, stating, "The entropy rate is also known as the *metric entropy* in dynamical systems theory and is equivalent to the *thermodynamic entropy density* familiar from equilibrium statistical mechanics" (p.4).

Regarding the complexity term to be represented on the y-axis, the authors state that they do not prescribe how complexity depends upon entropy, and instead define complexity by function. They state, "A useful complexity measure should have an unambiguous interpretation that accounts in some direct way for how correlations are *organized* in a system" (p. 2). They then chose excess entropy (E) as their measure of complexity which, by its name, may sound like it contradicts the authors' statement about the distinction between structural complexity and entropy. However, they write:

> The excess entropy tells us how much information must be gained before it is possible to infer the actual per-symbol randomness (h_μ). It is large if the system possesses many regularities or correlations that manifest themselves only at large scales. As such, the excess entropy can serve as a measure of global structure or correlation present in the system. (p.5)

The authors also relate this term to other known variables, stating excess entropy, "Goes by a number of different names, including 'stored information'; 'effective measure complexity'; 'complexity'; 'predictive information'; and 'reduced Renyi entropy of order 1' (p.5). They also present a definition for the word *process*, which is, "The distribution over all possible sequences generated by a system" (p4).

They then examined excess entropy's relationship to entropy for a wide assortment of process types, including, "A broad array of deterministic nonlinear and linear stochastic processes, including maps of the interval, cellular automata and Ising spin systems in one and two dimensions, Markov chains, and probabilistic minimal finite-state machines" (p.1). For some of these processes, they estimated the various information-theoretic quantities by simulation. In several, measures were estimated from sequences generated by the temporal or spatial process. They claim:

> …the entropy rate and excess entropy can be reliably estimated via simulation, given access to a reasonably large amount of data. Moreover, this estimation is purely inductive—one does not need to use knowledge of the underlying equations of motion or the hidden states that produced the sequence. (p.6)

Ten revealing figures were presented based on their analyses of each process. It is beyond the scope of this review to present them all, and interested readers are encouraged to examine the paper. However, one figure will be discussed here because it represents, very simply, some of the advantages that are inherent to complexity-entropy diagrams. Figure 14 displays the tent map, which is defined by:

$$f_\mu := \mu \, min\{x, 1 - x\} \quad (1)$$

where := means that the left side is defined by the right side and μ is a positive real constant.

When changed, μ yields a wide range of dynamical behavior for the tent map. Note that Feldman et al. (2018) used the symbol a in their paper to represent μ in this equation.

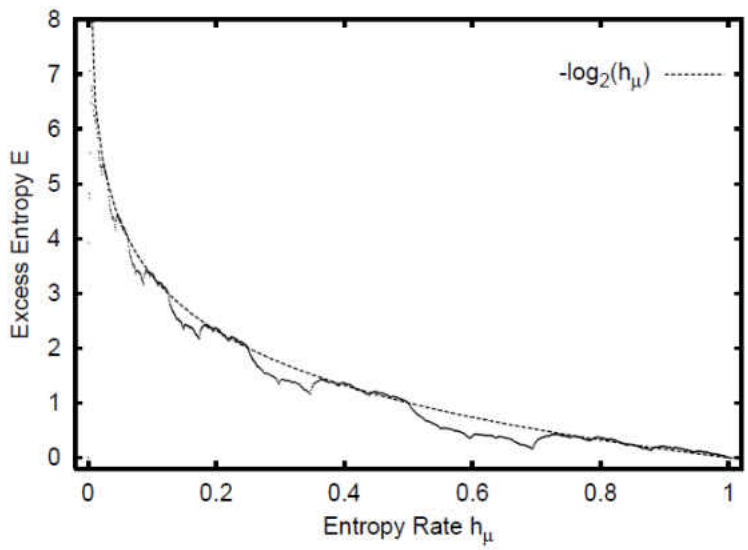


Figure 14. Complexity entropy diagram for tent map. From *The Organization of Intrinsic Computation: Complexity-Entropy Diagrams and the Diversity of Natural Information Processing* by David Feldman et al. Copyright © 2018 by American Institute of Physics. Reprinted with permission of AIP Publishing. All rights reserved.

It is useful to note that although the tent map can represent an astonishing diversity of dynamical behaviors, the complexity-entropy relationship for band mergings based on this plotting method revealed a simple representation via the expression:

$$E = -log_2\, h_\mu \quad (2)$$

The other point of interest is that a great richness of organizational structure is revealed in the vicinity of this simple approximation, perhaps presenting fertile grounds for additional research to better understand the natural processes associated with the tent map. Given the adaptable complexity-entropy landscape, it seems the authors have revealed new insights by establishing a new viewpoint on existing knowledge. It is worth mentioning that the basis for this methodology is analytic and inductive, which is also the basis for Wheeler's (1995) interpretation of a distinguishing attribute of Shewhart control charts. That is, in contrast to the various deductive statistical methods used for enumeration of process characteristics.

The authors close by discussing some perceptions of their research. First, the exploration of each class of process caused the adjustment of very different parameters and yielded very different effects, dramatically illustrating that the complexity-entropy diagram, "Allows for a common comparison across rather varied systems" (p.15). Next, the diagram revealed some of the fundamental differences between processes in terms of structure/ intrinsic computation/ organization, which could be exploited to reveal even more by simply changing input assumptions. They provided the example of a topological *Є-machine*, which they intentionally biased to reveal certain structural characteristics. The authors also determined that mapping of process characteristics could be used to provide clues about the appropriate representative model class, and to classify process behaviors within a model class. Their concluding comment was that the large diversity of representation provided by complexity-entropy diagrams, "Optimistically points to the richness of information processing available in the mathematical and natural worlds" (p.15).

The H_s vs C_{JS} Approach

This review will discuss the foundational paper that synthesized numerous information-theoretic concepts to create the Complexity-Entropy Causality Plane (CECP). The CECP was applied extensively to this dissertation's research, as evident in the Findings section. The CECP was selected because it provided graphically illuminating depictions of the dynamics associated with different Shewhart-stable time series processes. This review will provide some conceptual depth from the literature, but leave much of the theoretical and mathematical exposition for the Methodology section. A multitude of diverse process characterizations have benefited from this analytic tool since its inception in 2007, and a number of process characterizations via the CECP are displayed in the Methodology section to reveal the utility of this display format. These sources will not be reviewed here because no studies were discovered that applied the CECP for continuous stable process improvement as would be useful in industrial process control.

Rosso, Larrondo, Martin, Plastino, and Fuentes (2007) published the foundational paper for the CECP, entitled *Distinguishing Noise from Chaos*. As the title suggests, their motivation for developing this new graphical representation space was to create a tool to clearly distinguish between chaotic and stochastic processes. They declared that chaotic and stochastic processes share several properties that make them almost indistinguishable, and that most of the existing methods provided little relief. The authors started by reviewing some of the fundamental characteristics of time series data. They stated that real data, "always possess a stochastic component due to omnipresent dynamical noise" (p.154102-1). They then referenced key lessons from Wold's landmark 1938 paper entitled, *A Study in the Analysis of Stationary Time Series*:

> Indeed, Wold proved that any (stationary) time series can be decomposed into two different parts. The first (deterministic) part can be exactly described by a linear combination of its own past; the second part is a moving average component of a finite order. Hence it may seem superfluous to ask whether a time series generated by ''natural processes'' is either deterministic, chaotic, or stochastic. However, having in mind Wold's theorem [Kurths & Herzel (1987), Cambanis et al. (1987)] it makes sense to ask, with respect to the deterministic part (predictable from the past), whether (i) it is dominant vis-a-vis the unpredictable stochastic part or (ii) it is of a regular or chaotic nature. (p.154102-1)

The authors then claim that chaotic systems always produce time series with a physical structure, and that several statistical complexity measures based on disequilibrium have been introduced to define process structure. They point to Lamberti et al. (2004) for a superior definition of complexity measure. It is, "(i) able to grasp essential details of the dynamics, (ii) an intensive quantity, and (iii) capable of discerning among different degrees of periodicity and chaos" (p.154102-1).

Jensen-Shannon complexity (C_{JS}) was put forth as the superior measure, and as a "functional" of the probability distribution associated with the time series. The functional components they discussed include Jensen disequilibrium (Q_J), which specifically uses Jensen-Shannon divergence to calculate the true metric distance between distributions. The technical reasons precipitating their abandonment of the popular Kullback-Leibler divergence method will be presented in the Methodology section.

Rosso et al. made six more critical points regarding their proposed CECP methodology. For improved clarity, a simplified reasoning has also been added to each of their six points:

1. The probability distribution is calculated in terms of permutation entropy. This is because PE is perhaps the only entropy measure that incorporates the temporal structure of the emerging physical process.
2. For any chaotic system, permutation entropy yields a positive, finite Kolmogorov-Sinai (KS) entropy. This is important because KS entropy reveals how the information in a process evolves in time or, for a chaotic map, under iteration.
3. The progression of the iteratively calculated normalized Shannon entropy can be regarded as an arrow of time. This is because the second law of thermodynamics generally specifies that entropy grows monotonically.
4. Jensen-Shannon complexity (C_{JS}) is not a trivial function of the Shannon entropy (H_S). This is because C_{JS} is based on the interplay of normalized entropy and normalized disequilibrium, yielding variability between a range of possible values defined by C_{min} and C_{max} (which are also variable).
5. Jensen disequilibrium (Q_J) is a quantity different from zero only if there exist more likely states among those that are accessible. This is because Q_J provides a measure of Jensen

divergence as the distance between the current pdf and the uniform reference distribution (perfect randomness).

6. As an information complexity quantifier, C_{JS} provides greater insight than Q_J alone. This is because it provides maximum values for hidden structural dynamics that would otherwise be lost. Also, C_{JS} provides a zero reference value at completely random and completely deterministic systems.

To demonstrate the functionality of the CECP, Rosso et al. selected five kinds of chaotic maps, and two kinds of stochastic processes to analyze. The results of those plots are displayed in Figure 15.

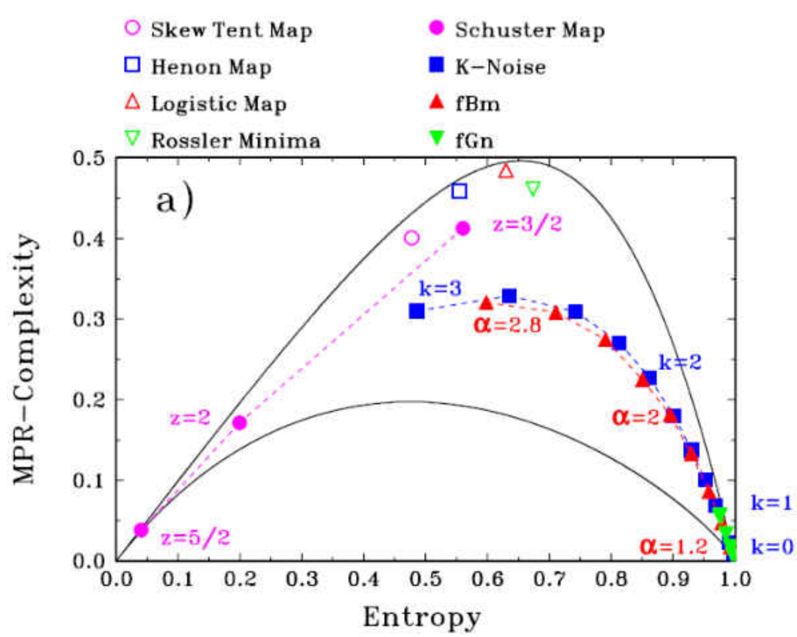


Figure 15. Complexity-Entropy Causality Plane. The lower curved line is C_{min} and the upper curved line is C_{max}. From *Distinguishing Noise from Chaos* by Rosso et al. Copyright © 2007 by The American Physical Society. Reprinted with the permission of The American Physical Society. All rights reserved.

The authors then summarized their perceptions. They noted that the CECP accommodates noise and chaos at different planar locations. Stochastic processes tended toward C_{min} and chaotic processes toward C_{max}. Also, because real data always contain a stochastic component due to omnipresent dynamical noise, the CECP helped classify different degrees of what they called "stochasticness". They also noticed that the CECP distinguishes Gaussian from non-Gaussian processes and different degrees of correlations (colored noise). Consequently, "This representation plane is an effective tool for revealing the sometimes subtle difference between noise and chaos" (p.154102-4).

Research Gap

Table 8. Research gap for complexity-entropy diagrams.

Author	Applied Structural Complexity	Applied Shannon Information	Applied Log Change
Feldman et al (2018)	X		
Rosso et al. (2007)	X	X	
Lorimer	X	X	X

Literature Review Summary

This review began by establishing the contested basis for the continuous improvement of Shewhart-stable processes. Positions were provided from some of the foundational literature regarding the absolute randomness of stable processes and whether improving a stable process is always tampering. The next section challenged the notion of tampering by providing numerous methods from the literature that have indeed been used to improve Shewhart-stable processes. Next, papers were reviewed that statistically analyzed the traditional funnel experiment. The purpose was two-fold: To provide additional insight into the tampering assumption, but also to review concepts potentially relevant to the vertical funnel experiment used to gather empirical data for this research.

In the next section, studies that have applied complexity concepts for process control were reviewed. None were discovered that applied Jensen-Shannon complexity. The next part focused on literature relevant to permutation entropy (PE). A few studies applied PE methods for process control applications but only one used modifications to the traditional PE method and control charts and none applied structural complexity measures. Next, methods were reviewed that modified PE to mitigate identical values within tuples. Various shortcomings were evident for all reviewed methods and none applied the concept of *local effect*.

The next section reviewed two prevalent diagrams that simultaneously display structural complexity vs. entropy for various processes. The h_μ vs. E diagram introduced some fundamental concepts and advantages. The Complexity-Entropy Causality Plane provided a different perspective, based on similar underlying concepts. Neither paper applied the relative change methodology developed for the Entropy-Complexity Change Diagram. To recap the most relevant research gaps, no literature was discovered that:

- Applied the vertical funnel experiment
- Applied structural complexity to continuously improve Shewhart-stable processes

- Applied Jensen-Shannon complexity to continuously improve Shewhart-stable processes
- Modified permutation entropy based on local effect
- Simultaneously displayed changing structural complexity and entropy based on log change evaluations relative to the maximum randomness condition for an equivalent time series.

CHAPTER THREE: METHODOLOGY

The Methodology section is presented in five parts. The first presents the data collection and validation effort associated with the four vertical translation funnel experiments. The second presents methodological information for the other 18 processes that were evaluated in this research. The third presents the four basic data visualizations applied in this research. The fourth progressively builds upon underlying information-theoretic methodologies whose synthesis yielded the Complexity-Entropy Causality Plane. The final section provides the log change methodologies used to evaluate processes in terms of changing entropy and complexity relative to the maximum randomness condition for equivalent time series.

Overview

Numerous methods were synthesized to ultimately answer the research question: Can a methodology based on emerging structural complexity provide information useful to direct the continued improvement of a Shewhart-stable process? The flowchart in Figure 16 provides a high-level overview of the analytic methodology applied in this research.

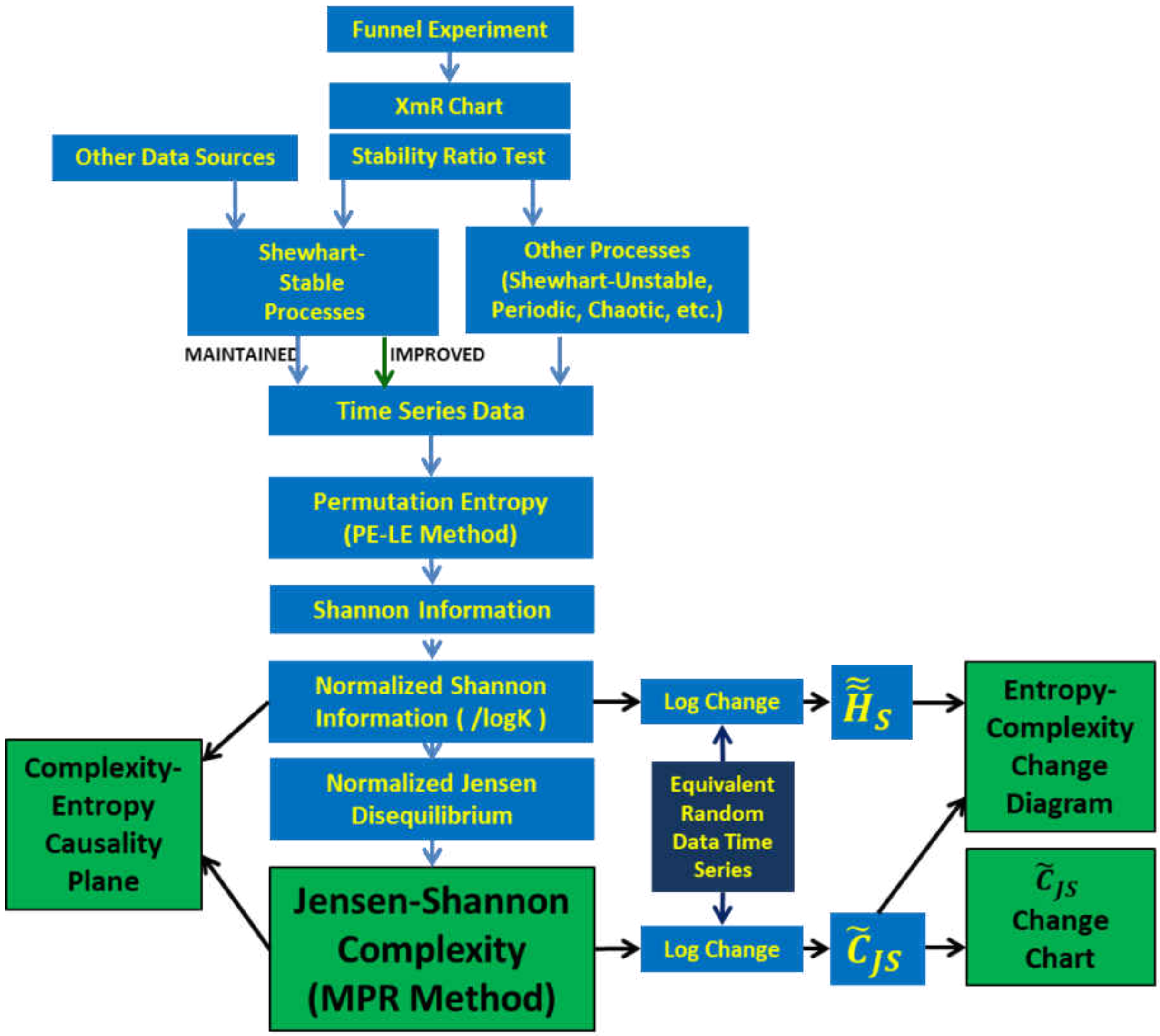


Figure 16. Methodology Flowchart. Green boxes represent primary outputs.

In addition, the essential methodology used to answer the research question can also be distilled into the following six steps. As the time series data progresses:

1. Use permutation entropy- local effect method to establish tuple allocations. (Red & Wizard tuple sets were parsed for this research. If the parsing policy had been

established at the outset, the analytic method could have ignored them, obviating the need for proactive parsing after the fact.)

2. Calculate updated probability densities for PE-LE tuple set (using 42 tuples for D=4 without Red or Wizard tuple sets).

3. Follow traditional permutation entropy method to calculate Shannon entropies.

4. Follow MPR-method to calculate $\widetilde{H}_S$ and C_{JS} as time series progresses (except that H_{max} and P_e are different due to PE-LE's extra tuples). View structural complexity results on Complexity-Entropy Causality Plane.

5. Use log change to calculate $\widetilde{\widetilde{H}}_S$ and $\tilde{C}_{JS}$ relative to equivalent random series.

6. Use $\tilde{C}_{JS}$ change chart and Entropy-Complexity Change Diagram to observe emerging complexity dynamics as time series progresses.

Vertical Translation Funnel Experiment

The four vertical translation funnel experiments were considered central to this research because they were empirically-derived time series instead of idealized synthetic models, and because they represented Shewhart-stable processes that were progressively improved. Moreover, process improvement over the course of each experiment could be deemed reliable. Also, the corresponding decreasing trend for variation could be verified for each time series.

Participants

The experimental setup was custom built and modified by the author. The execution of the experimental process and associated data recording was fairly simple and repetitive, with >7,500 drops recorded in seven months, and was also accomplished by the author.

Experimental Design

The design of this experiment was based on two requirements. First, the experiment needed to faithfully represent a process that was generally Shewhart-stable. To accomplish this, the design attempted to control conditions that could introduce biases in the data. The second requirement was that the stable process could be improved in a consistent and verifiable manner so that the analytic method could be evaluated for its capability to answer the research question.

The design of the experimental apparatus is shown in the figure below.

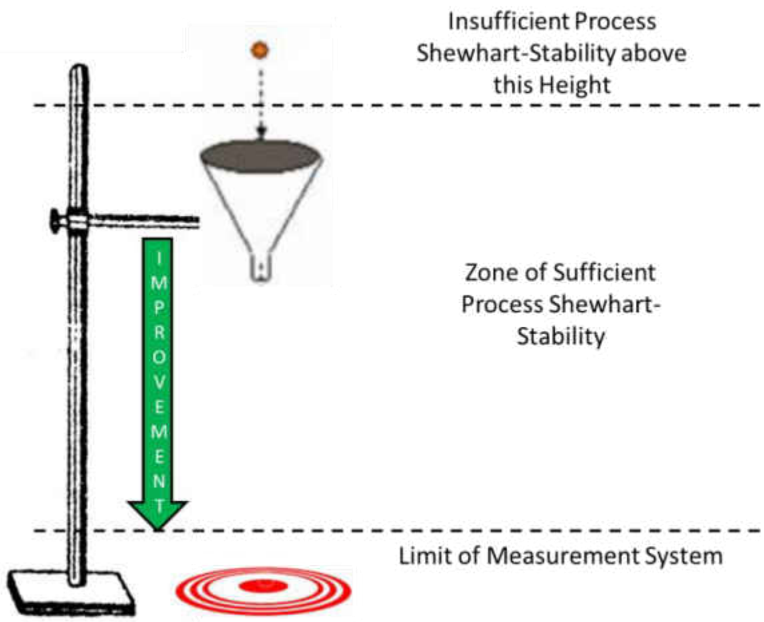


Figure 17. Funnel experiment apparatus concept.

Equipment

The basic equipment consisted of a marble, a funnel, a vertically adjustable stand for the funnel, a 4 ft x 6 ft panelboard landing surface (mounted atop an adjustable-level table made from lumber), a target consisting of concentric one-inch wide circles, and sheets of packing paper for marking results- one sheet for each drop height. The packing paper was marked to create a permanent record to allow review and facilitate further research. A six-foot carpenter's level was used to determine level for the table. A funnel was used for the first experiment but this was replaced with a vacuum pump and hose for the remaining experiments (ostensibly to improve consistency by reducing potential marble spin and vibration effects). The short length of the vacuum hose required mounting a tray table for the pump to rest upon.

Experimental Laboratory Setup

The experiment was conducted inside an air-conditioned house. The effects of temperature and airflow changes were considered negligible.

Figure 18. Experimental laboratory setup. Left picture shows the configuration for experiment 1 (55 to 22 inch heights). Right picture shows the configuration used for 38 inch heights and lower.

Variables

Dependent Variable

The dependent variable was the resting spot of each marble, measured in integer inches from the center of the target.

Independent Variable

The independent variable was the drop height above target, in integer inches.

Experimental Procedure

The traditional funnel experiment was modified to approximate a Shewhart-stable process. Rule one was maintained (no horizontal translations) and only vertical translations that incrementally changed the drop height directly above the target were permitted. Four experiments were conducted using a varying range of heights above the target and with different conditions in place that were intended to minimize bias and variability in the data. Data chunkiness standards were used to determine the minimum acceptable drop height. XmR charts and Stability Ratio calculations were used to assess process stability.

The target consisted of concentric one-inch rings from 1 to 11 inches. For marbles coming to rest beyond 11 inches, a tape measure centered on the target was used to record the distance (in integer inches). Similar to shooting sports, any marble coming to rest on a dividing line was considered to be at the interior distance, not the exterior distance.

The process that was repeated for each experiment was:

1. Leveled the drop table using the four adjustable jackscrews mounted to the base of the table. Level was determined in both horizontal dimensions and diagonally using a six-foot carpentry level.
2. Taped one sheet of thin packing paper over the centered, taped down target for each drop height. The resting position of each marble was recorded in pencil on these sheets.
3. Conducted 50 marble drops using the funnel/ vacuum pump from each drop height.
4. For each drop, recorded the resting place of the marble in the following ways:
    - On packing paper as a pencil dot. One sheet of paper per drop height, per experiment. (This yielded 143 sheets total).
    - Hand-written in research journal, along with any observations deemed potentially relevant.
    - As data in Microsoft Excel. This software was used for computational analysis and to create related charts.

The experiments were modified based on progressive learning. Many changes were intended to reduce the variability of the process studied. The relevant procedures for each experiment are described as follows.

Funnel Experiment #1

- Started at 55 inches drop height and progressed down to 22 inches.
- Reconfigured drop tower and reestablished table level and target center.
- Collected drop data from 21 inches down to 5 inches. Configuration of drop tower would not physically allow drop heights lower than 5 inches.
- Conducted data chunkiness calculations, thus determining the lowest practical height as 7 inches above target. This calculation nullified any concerns for mitigating the physical limitation at 5 inches. The lower limit of 7 inches was used for the remainder of experimentation.
- XmR charts for heights greater than 38 inches revealed Shewhart-instability, meaning that the heights from 38-55 inches did not add information useful to addressing the research question. This also meant that the drop tower did not need to be reconfigured to achieve these greater drop heights for the next three experiments.

Funnel Experiment #2

- To mitigate variation before collecting data:
    - Replaced funnel with vacuum pump
    - Secured vacuum pump orifice with tape to keep it pointed straight down for each height
    - Affixed the panelboard on the tabletop surface better with contact cement and more screws.

    - Straightened the drop tower to vertical with base shims and by reconfiguring both side ropes.
    - Changed the recording papers to thinner sheets of packing paper.
- Started at 7 inches and progressed upward to 24 inches. After finishing drops at 24 inches, relocated the vacuum pump tray table to the edge of the landing platform table after noting that the marble occasionally touched the vacuum pump platform legs. This prevented future interference with marble roll.
- Continued from 25 inches upward to 38 inches drop height.

<u>Funnel Experiment #3</u>

- To mitigate variation before data collection:
    - Added 7 sheets of packing paper under the target to reduce marble bounce. The tradeoff was that some outlier marbles would experience a small gravity assist if they rolled off the edge of the paper stack.
    - Conducted 30 test drops to assess the likelihood that outliers would reach the edge of the paper stack. Three marbles from 38 inches rolled off the edge, so another 4 sheets of packing paper were added under the target for a total of 12 including the target sheet. Conducted another 30 test drops from 38 inch height. No marbles rolled off the edge.
- Started at 38 inches and progressed down to 27 inches.
- Noted that the drops were slowly developing a consistent bias off-center as drop height was reduced. Therefore, after finishing drops for 27 inch height, moved the target stack

by 0.5 inches to re-center. This compensation appeared to arrest the perceived bias in the drops at lower heights. Later, presumed that much of the directional bias may have been caused by dimpling of the paper as the marble repeatedly hit the same spot when dropped from upper heights.

- Continued marble drops from 26 inches down to 7 inches.

Funnel Experiment #4

- To mitigate variation before data collection:
  - Based upon the paper dimpling presumption, removed the top 3 sheets of packing paper and replaced them with 10 unused sheets. Including the target sheet, there were now 18 sheets of packing paper in the stack. Also, to mitigate future dimpling, conducted remaining experimentation from low to high heights.
  - Used the results of test drops from low heights to level the table instead of relying on carpentry level.
- Started at 7 inches and progressed up to 9 inches. These data were recorded in the research journal, but not deemed useful on the packing paper because almost all marble drops came to rest within the 1 inch circle.
- Continued recording drops on packing paper from 10 inches up to 37 inches. Noted that the marble occasionally deviated from its roll path slightly for a ridge or valley in the paper stack. Although the 18-sheet stack reduced marble bounce considerably, the tradeoff was that the paper stack was also less flat than before.
- Concluded experimentation by disassembling the drop table and experimental apparatus.

Analytic Methodology

` This section will present three analytic methodologies for the funnel experiment. The first is the use of data chunkiness standards to determine measurement limits. The second is analysis of process stability using XmR charts. The third is analysis of process stability using the Stability Ratio (SR) test.

Measurement Limits

The measurement limits were assessed in terms of data chunkiness standards. Data chunkiness becomes a problem when the measurement resolution increment (which was one inch for all four funnel experiments at all heights) gets too large for the data. When this chunkiness is relevant, many of the data outlying the Upper Range Limit on a moving range control chart will be false alarms, which can make a Shewhart-stable process appear Shewhart-unstable.

When control charts are based on individual values, as with this research, the detection standard for data chunkiness requires that a minimum of four possible values appear within the URL of the moving range chart. In this research, the measurement limit was assessed based on the results of the first experiment, which also determined the lowest usable height for the three remaining funnel experiments. All four experiments were considered equivalent with respect to data chunkiness results.

Figure 19 helps clarify the data chunkiness problem. The chart happens to represent results from the 6-inch drop height for experiment #1, which will be discussed further in the Findings section. Notice that only three of the five distinct values remain within the URL, which violates the rule requiring four or more values within the URL. In addition to not meeting the chunkiness standards, these results bring the values associated with the 32nd and 33rd marble drops into question as probable false alarms of process instability.

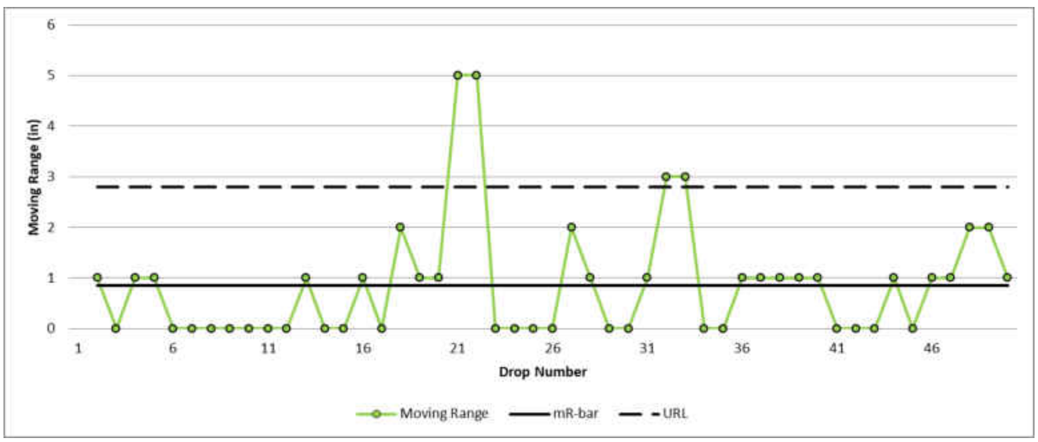


Figure 19. Sample moving range (mR) chart to demonstrate data chunkiness. Three of five distinct mR values remained within the URL. From vertical funnel experiment #1, 6 inch drop height.

Shewhart-Stability of Process

XmR charts were used to conduct ongoing, high level assessments of process stability for marble drops at each height. These assessments, during the conduct of the experiment, provided feedback for process performance in terms of variability. This feedback was used to modify the experimental setup between experiments with the intention to reduce process variability.

Standard equations were used to create the XmR charts, based on the average (not median), which were:

$$Control\ Limits = \bar{X} \pm (2.66\overline{mR}) \quad (3)$$

$$Upper\ Range\ Limit = 3.27\overline{mR} \quad (4)$$

where $\bar{X}$ equals the average of process data and $\overline{mR}$ equals the average moving range of process data. The lower range limit for the mR chart rested at zero.

Stability Ratio Test

The stability ratio test was the primary method to assess process Shewhart-stability for the vertically-translated funnel experiments. This was accomplished by comparing the global standard deviation statistic to a within-subgroup measure of dispersion, σ(x) for the set of 50 data for each height. The equation used for the SR test was:

$$Stability\ Ratio = \frac{s^2}{[\sigma(x)]^2}\text{ , where } s = \frac{\sum_i (x_i - \bar{x})^2}{(x_k - 1)}\ and\ \sigma(x) = \frac{\overline{mR}}{d_2} \quad \textbf{(5)}$$

For this application i = 1 to 50 and k=50 because 50 measurements were conducted at each height, and d_2 is a bias correction factor for measures of dispersion, required due to nonlinear manipulations of the data (e.g., exponential squaring). Bias correction factors are found in tables, such as Wheeler (1995, p.416), where d_2 = 1.128 for XmR charts. Further derivation of d_2 is provided later in this section.

The stability ratio test is based on a pseudo F-statistic described by Ramirez & Runger (2006). It is called a "pseudo" F-statistic because it violates the typical assumption for an F-test that random samples are selected in an independent manner from two populations. It further assumes that the relative frequency distribution of the sample will be approximately normal. The basic assumption for the SR test is that any two estimators of process variance should be similar when a process is stable (SR ≈ 1). For instance, the authors provided an example (p.55) for which SR can be defined as any of the following:

$$SR = \frac{\boldsymbol{s^2}}{\boldsymbol{s^2_{R,S,mR}}} \text{ or } \frac{\boldsymbol{s^2_{global}}}{\boldsymbol{s^2_{local}}} \text{ or } \frac{\boldsymbol{s^2_{long\ term}}}{\boldsymbol{s^2_{short\ term}}} \tag{6}$$

Ramirez and Runger (2006) recommend using enough data such that df_1 and df_2 are both greater than 50. The degrees of freedom for the funnel data were $df_1 = 49$ and $df_2 = 30$. The consequence of not fully meeting this recommendation was that the SR-test was not as sensitive to departures from stability as it could be. The equations for calculating the degrees of freedom were:

$$df_1 = (k-1) \; and \; df_2 = 0.62(k-1) \qquad , where \; k = \# \; of \; subgroups \tag{7}$$

For the funnel experiments, the critical value assuming a 1% level of significance was a table lookup in Wheeler (2019). For N=50, the SR critical value was SR ≤ 1.59. Although df_1 and df_2 remained the same throughout the experiment, a new SR value was calculated for each height using equation (6). Applying each SR value and both *df* values to the F-tables yielded an F-statistic for each height that could be compared to the critical value above. Any p-values above the 1% level of significance failed to provide strong evidence of process instability. The critical value, depicted in the typical statistical nomenclature was H_0 : SR ≤ 1.59 (Stable) and H_1 : SR > 1.59 (Unstable).

The bias correction factor d_2 may also be calculated using the following equation:

$$d_2 = \int_0^{\infty} w\, f(w) dw\,, \qquad where\ f(w) = \frac{n(n-1)}{[\sqrt{2\pi}]^n} \int_{-\infty}^{\infty} \left[\int_x^{x+w} e^{-\left(\frac{u^2}{2}\right)} du \right]^{n-2} e^{-\left(\frac{(x+w)^2}{2}\right)} e^{-(x^2/2)} dx \tag{8}$$

For n=2, applicable to XmR charts and SR tests in this research, this equation can be simplified to:

$$d_2 = \frac{2}{\sqrt{\pi}} \tag{9}$$

Comparative Time Series

The vertical translation funnel experiments were considered central to this research because they were empirically-derived time series instead of idealized models, and because they represented Shewhart-stable processes that were progressively improved. More specifically, process improvement could be considered reliable because variation repetitively decreased throughout the time series. However, data from other process types were also desired to validate the PE-LE methodology and to evaluate complexity-entropy diagram results. Most importantly, these other processes were needed for pattern comparisons, to help define how Jensen-Shannon complexity may or may not provide indications of process improvement for Shewhart-stable processes. The other process categories used in this research were Shewhart-stable (maintained with minor improvement), Shewhart-unstable, chaotic, periodic, quasi-random (Pi), and pseudorandom (PRNG). This section will focus on the methodology associated with the generation and identification of the different process datasets.

Tokai Rika Time Series

In March 1982, a group of executives from Ford Motor Company visited the Tokai Rika manufacturing plant in Japan. Tokai Rika had been focused for years on controlling the variability of manufactured cigarette lighters. This was because a critical safety problem had been identified years earlier when a lighter popped out of its socket and burned a car driver's leg (Sullivan & Manoogian, 2009). During their visit, the Ford team received translated copies of control charts representing 18 months of data, which represented production of nearly 6.5 million lighters. The leader of the Ford team, Lawrence P. Sullivan (2009), analyzed these charts and wrote about them in his book, *Unlocking Ford Secrets*. These charts represented a process that was Shewhart-stable, such that it was only considered necessary to collect one subgroup of four measurements daily, even though about 17,000 lighters were produced each day.

The dataset represents one subgroup mean for each of 379 consecutive work-days of production. This time series process was evaluated because it was Shewhart-stable, and because seven significant process events were documented that could potentially be correlated with structural complexity results. A run chart for this process is presented in Figure 20. The larger colored data points represent the seven significant events, which are identified in Table 9.

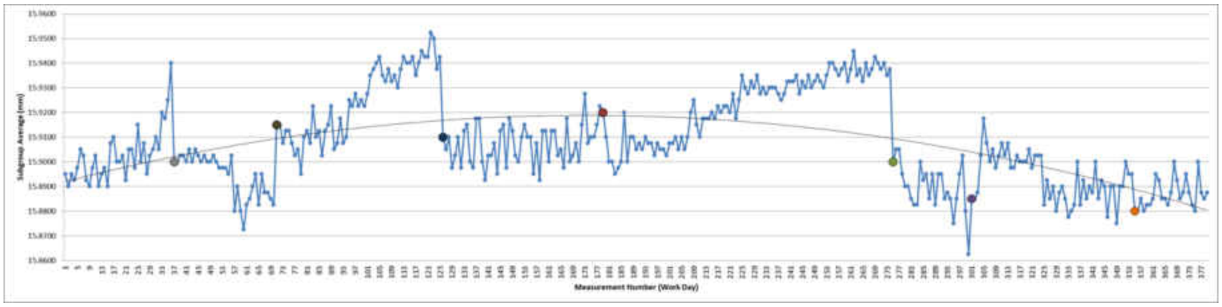


Figure 20. Tokai Rika run chart for production of cigarette lighters from August 1980 to March 1982. Trendline is 2nd order polynomial fit. Colored dots represent significant process events. Data reused with the permission of the Tokai Rika intellectual property division.

Table 9. Significant events for Tokai Rika lighter manufacturing process. Data reused with the permission of the Tokai Rika intellectual property division.

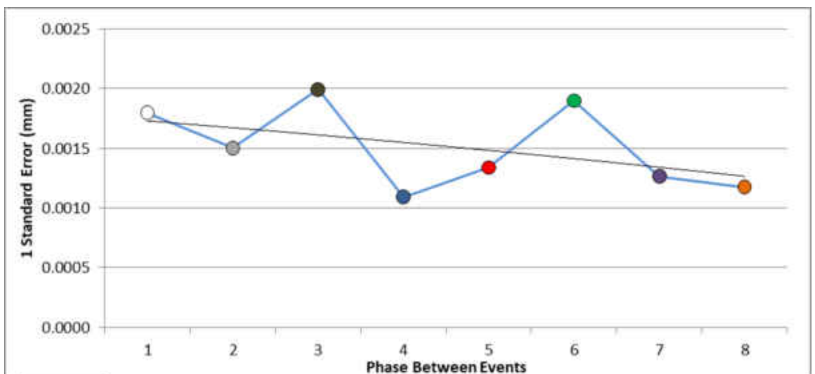
Event #	Color	Event Description	Improvement or Mitigation?
37	Gray	Replaced a worn collar	Improvement
71	Brown	Repaired a die causing wrinkle on flange	Mitigation
126	Blue	Installed a new type of positioning collar	Improvement
179	Red	Adjusted a blanking punch causing wrinkle on flange	Mitigation
275	Green	Replaced a worn positioning collar	Mitigation
301	Purple	Tightened a loose bolt	Improvement
355	Orange	Preventively replaced a positioning collar	Improvement

The variation was calculated for each phase between significant process events using the equation for the standard error of the mean:

$$SE = \frac{s}{\sqrt{n}} \;, \tag{10}$$

where s is the standard deviation and n is the sample size.

One standard error was calculated for each phase to determine whether the event caused variation reduction (process improvement) or simply mitigated a special cause. The overall variation trend was portrayed with a 2^{nd} order polynomial fit trendline in Figure 21.



	1-36	37-70	71-125	126-178	179-274	275-300	301-354	355-379
1SE	0.0018	0.0015	0.0020	0.0011	0.0013	0.0019	0.0013	0.0012

Figure 21. Variation for phases between significant process events. Trendline is 2^{nd} order polynomial fit, suggesting minor overall variation reduction (process improvement). Data reused with the permission of the Tokai Rika intellectual property division. All rights reserved.

In addition to this variation study, a moving range (mR) chart is provided in Figure 22. The mR chart displays the range variability from measurement to measurement in the time series. The displayed average moving range (mR-bar) and the upper range limit (URL) is computed for the entire dataset and is provided to view the overall trend for process dispersion.

Many of the high variability values directly correspond with significant process events, when an improvement or mitigation immediately caused considerable change to variability.

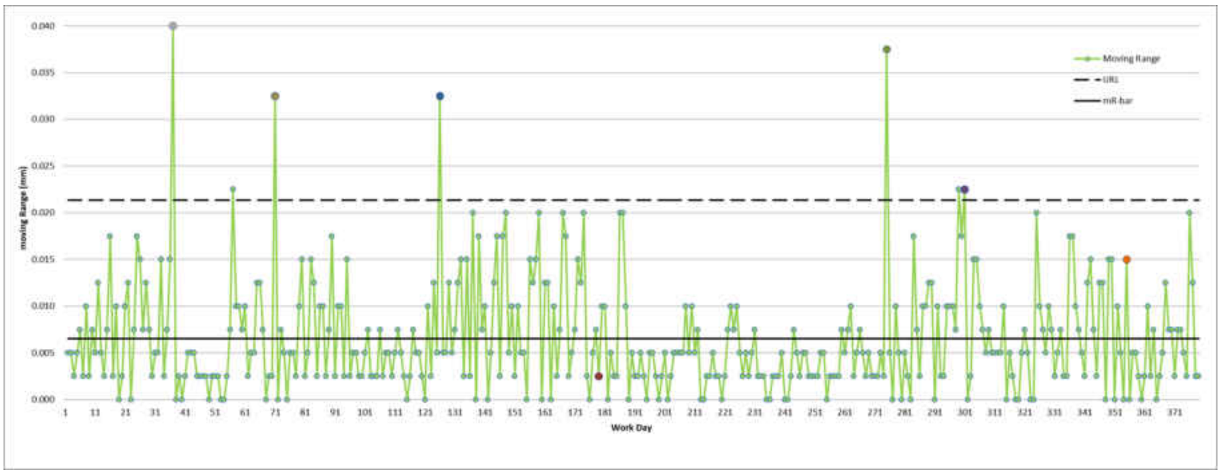


Figure 22. Tokai Rika moving range chart. Colored dots represent significant process events. Data reused with the permission of the Tokai Rika intellectual property division.

This research was designed to seek complexity correlations associated with physical process changes. Therefore, for the purposes of this research, an *improvement* is specifically defined herein as actions taken that reduce variation (which can also appear as decreasing structural complexity and increasing process randomness over time). In contrast, a response to a special cause that results in little change to downstream variation is considered a mitigation, not an improvement. This differs from the state of the practice, in which process improvement has many valid meanings, even including events external to the process such as re-centering the process according to specifications. Also, unlike many quality engineering studies, the decision about when to redraw control limits was not relevant to this research.

These points are emphasized here because all but one (355) of the seven significant events in the Tokai Rika dataset were responses to special causes, focused on bringing the variation back into control. However, some of the events (37, 126, & 301) did yield some variation reduction. Overall, Figure 20 displays a process that is Shewhart-stable the vast majority of the time and, like any real-world process, experiences occasional variation events that require mitigation.

For a manufacturing line producing 17,000 lighters per day, maintaining this level of variation control for such an extended period represents a significant accomplishment. Beyond the simple monitoring and process maintenance that typically accompanies most Shewhart-stable processes, it was also laudable that Tokai Rika personnel proactively conducted a preventive maintenance event, even though the process was performing well within specifications. However, Tokai Rika's Shewhart-stable process differs from the vertical funnel experiments in that the process was not continuously and progressively improved through iterative variation reduction events. Nonetheless, the Tokai Rika dataset provided utility as a real-world comparison for the results developed in this research.

Distillate Flowrate Time Series

The distillate flowrate time series represents a Shewhart-unstable process. These data were downloaded from an open-source dataset provided by Kevin Dunn (2020) at www.openmv.net with no restrictions on use. The distillate dataset contained over 40,000 data. Since the column limit for Microsoft Excel is slightly above 16,000, the final 16,000 data of the time series were selected for this research. However, as other analyses were eventually conducted, many of the processes were evaluated for only 10,000 data. Thus, for consistency, the first 10,000 of the final 16,000 distillate data were therefore chosen for evaluation, as presented in Figure 23. Many of the data in the second half of the time series were at 0.0, which ostensibly represents shutdown of the distillation process. A few outliers were also present including the 5,320th point (value ≈ 346), which is above the frame of the figure. This outlier was not included in the figure to allow better visibility of process characteristics.

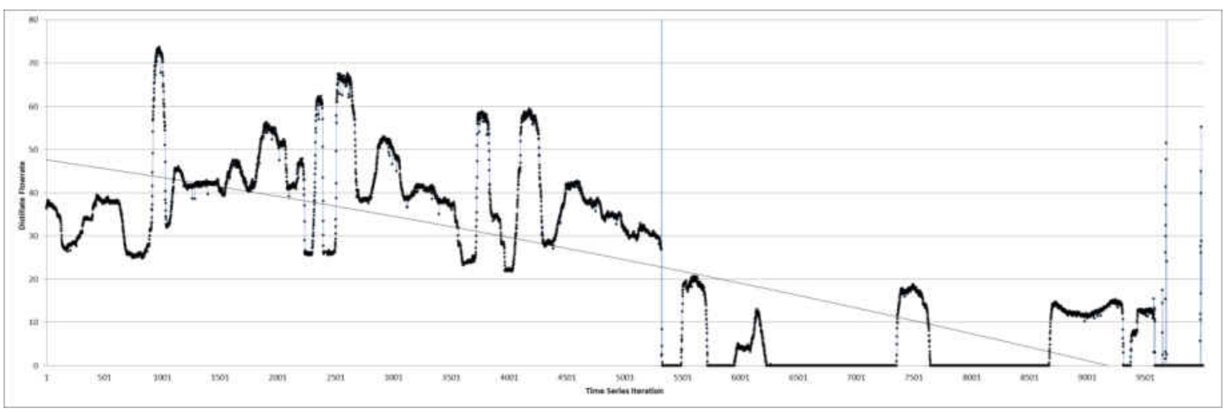


Figure 23. Distillate flowrate data. Measurement resolution was to the thousandths place resulting in 3,021 distinct values within the interval [0.000, 345.860]. Trendline is 2nd order polynomial fit. From www.openmv.net by Kevin Dunn (2020). Dataset provided open source with no restrictions on use.

Electrical Usage Time Series

The electrical usage time series represents a Shewhart-unstable process. These data were downloaded from an open-source dataset provided by Kevin Dunn (2020) at www.openmv.net with no restrictions on use. The dataset contained 2,712 measurements, as presented in Figure 24.

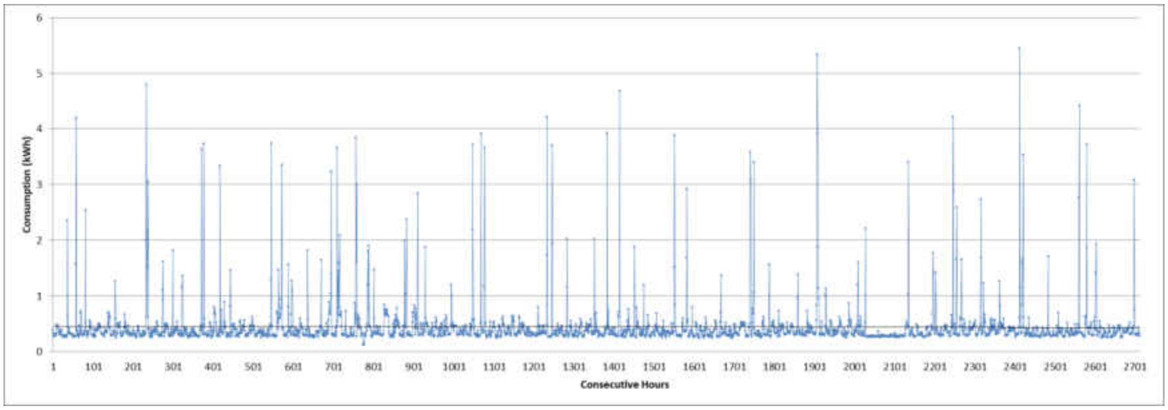


Figure 24. Electrical usage data. Consecutive hourly measurements. Measurement resolution was to the hundredths place resulting in 164 distinct values within the interval [0.12, 5.45]. Trendline is 2nd order polynomial fit. From www.openmv.net by Kevin Dunn (2020). Dataset provided open source with no restrictions on use.

Logistic Map Time Series

A chaotic process was desired for comparing results with Shewhart-stable processes and to help validate the utility of the PE-LE method. A chaotic region of the logistic map was selected with rate of growth r=3.888 and initial population x_n =0.02. The first 10,000 data values were tested but only the first 1,500 data values are shown in Figure 25 for clarity. The equation for the logistic map is:

$$x_{n+1} = rx_n\,(1 - x_n)\ , \tag{11}$$

where x_n is a number between zero and one that represents the ratio of the current population to the maximum possible population and r is the growth rate.

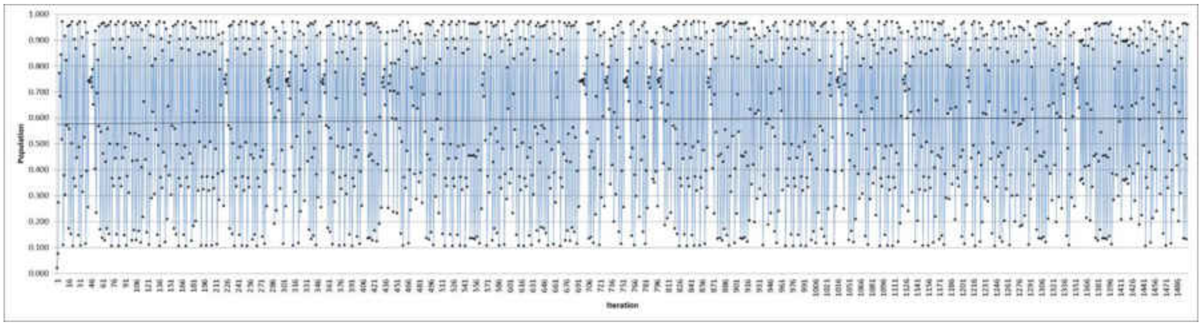


Figure 25. Logistic map for r=3.888 and initial population x_n=0.02. Measurement resolution was to the thousandths place resulting in 866 distinct values on the interval [0.020, 0.972].

Sine Wave Time Series

A periodic process was desired for comparing results with Shewhart-stable processes and to help validate the utility of the PE-LE method. The wave was discretized every 7.5 degrees and then degrees were converted into radians. The first 1,500 values are presented to maintain clarity in Figure 26, although 10,000 values were used for research.

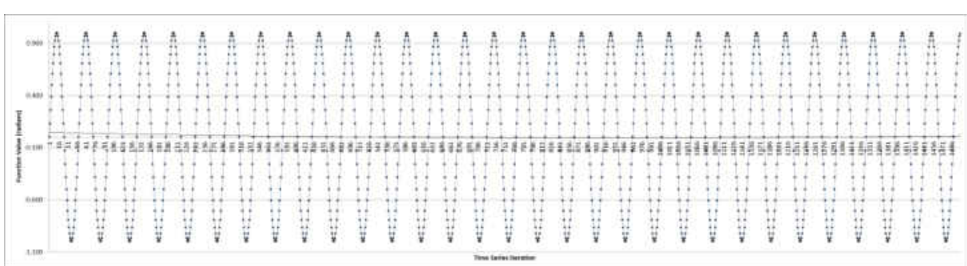


Figure 26. Sine wave discretized every 7.5 degrees with values in radians. Measurement resolution was to the thousandths place resulting in 25 distinct values on the interval [-1.000, +1.000].

Pi as Quasi-Random Time Series

A quasi-random process was desired to compare to other series generated by Microsoft Excel's pseudorandom number generator and to help validate the utility of the PE-LE method. The first 1,500 values of Pi are presented to maintain clarity in Figure 27, although 10,000 values were used for research.

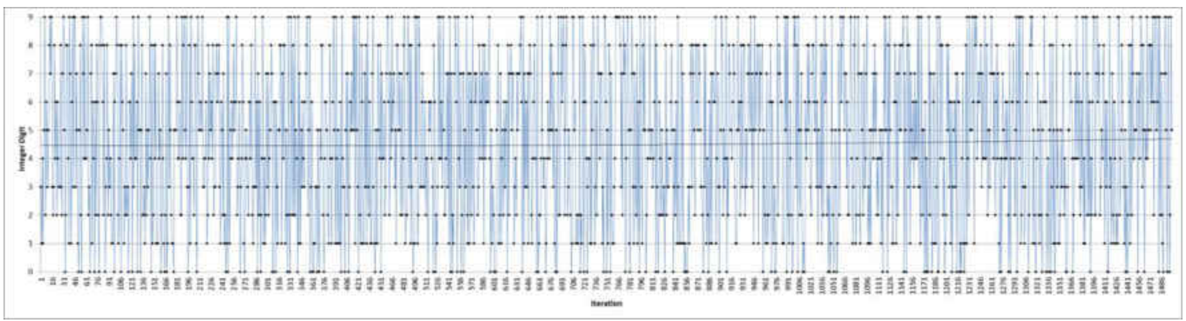


Figure 27. Pi. The first 1,500 values presented for clarity. Measurement resolution was to the integer place resulting in 10 distinct values on the interval [0,9].

Pseudorandom Time Series

Numerous pseudorandom series were desired to evaluate measurement resolution effects, to help validate the utility of the PE-LE method, and to develop a complexity-entropy diagram based on log change computations. All pseudorandom series were generated by Microsoft Excel's Mersenne Twister pseudorandom number generator (Matsumoto and Nishimura, 1998). Unimproved random processes were represented by R#, where R meant random and the # indicated the quantity of distinct values. Example included R10, R25, R50, R100, R500, and

R1000. Graphs for these processes, being random, would not add value and are therefore not presented here. The decreasing variation pseudorandom series (Decr#) are presented in Figure 28 and the single increasing variation series (Incr#) in Figure 29.

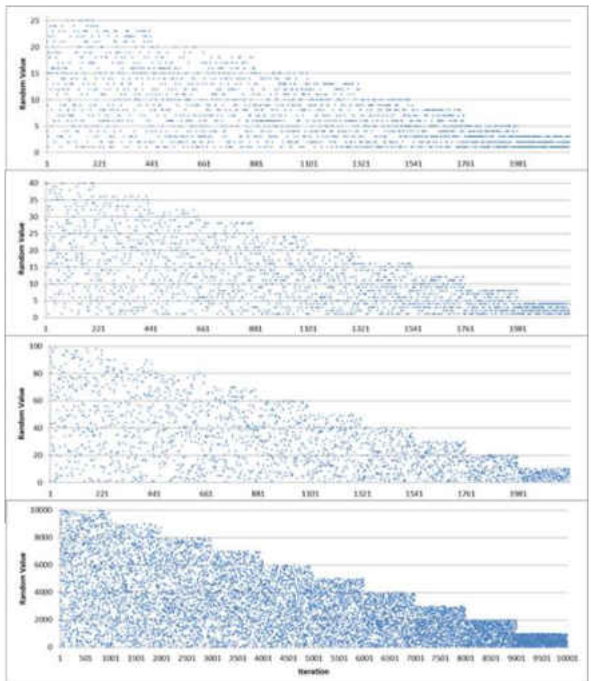


Figure 28. Pseudorandom generated improved processes, Decr #, where # represents the quantity of distinct values in the first iteration. From top to bottom: Decr25, Decr40, Decr100, Decr10K.

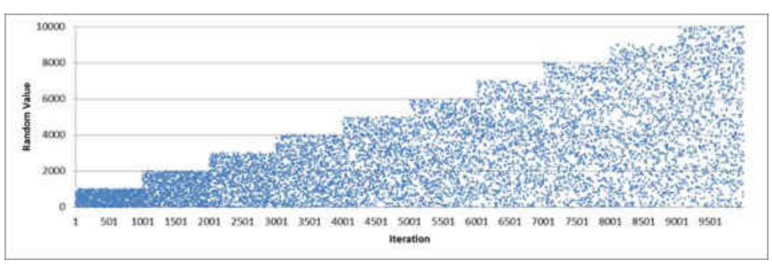


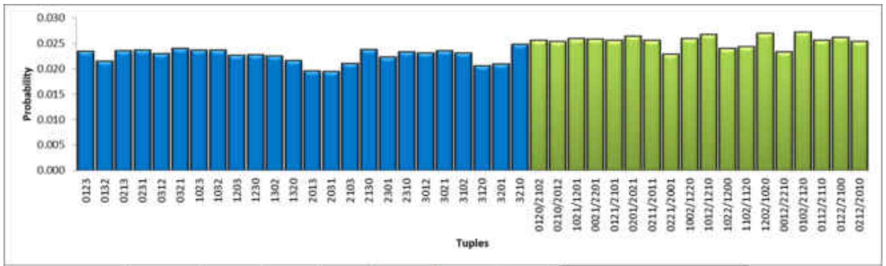
Figure 29. Incr10K pseudorandom generated series.

Data Visualization Plan

Four data visualizations were central to this research, and each is described along with its purpose. The descriptions of each data visualization include a sample figure.

PE-LE Probability Density Function

The Permutation Entropy- Local Effect pdf visualization includes the pdf for the 42 distinct tuples, a numerical summary of the counts for tuple categories and local effects, and a pdf for the local effect results. This visualization served numerous purposes concurrently. First, it provided a visual indication of process homogeneity/randomness. Second, it revealed the prevalence of tuples containing identical values (green tuples). Third, it revealed local effect results. Fourth, it provided the degree of local effect balance associated with a time series. Fifth, it facilitated comparisons among processes with different characteristics or at different points in a time series to reveal distinctive information content. Finally, it facilitated the characterization and validation of the PE-LE method.



Category	Count	Density
Parlitz	5089	0.509
Forest	4315	0.432
Wizard	263	0.026
Red	330	0.033
Sum	9997	1.000

LE	Count	Density
-3	426	0.045
-2	1475	0.157
-1	2433	0.259
0	723	0.077
1	2467	0.262
2	1439	0.153
3	441	0.047
Sum	9404	1

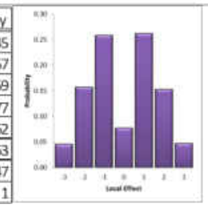


Figure 30. Sample PE-LE pdf data visualization.

Complexity-Entropy Causality Plane

The primary purpose of the CECP was to define and compare processes in terms of $\widetilde{H}_S$ and C_{JS} simultaneously. The CECP was devised using the MPR-method, which includes application of the traditional permutation entropy method. However, the PE-LE method was applied to this research to mitigate identical values within tuples, and necessarily yields different results from the traditional method based on a differing number of distinct tuples. The $\widetilde{H}_S$ plot was an extraction from results establishing the CECP visualization. The C_{min} and C_{max} lines define the possible region for the data plot. For this research, C_{max} was displayed based on the traditional 24 tuples but would be slightly higher for PE-LE's 42 tuples. The next standard basis for C_{max} would be based on 5! = 120 tuples, so the 24 tuple line was retained instead. Also, close-ups are provided for the lower-right region of interest in the CECP diagram.

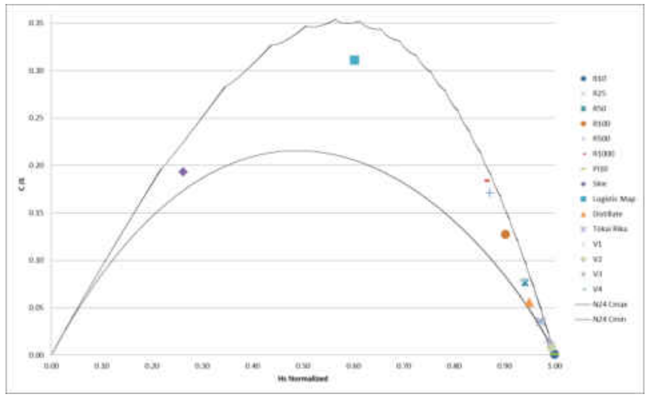


Figure 31. Sample CECP data visualization.

$\tilde{C}_{JS}$ Change Chart

The $\tilde{C}_{JS}$ change chart had two purposes. First, to reveal how the structural complexity of a process changed in time relative to the maximum randomness condition for an equivalent time series. Second, to allow equitable comparisons of changing structural complexity for numerous processes when plotted on the same chart.

Figure 32. Sample C_{JS} change chart for data visualization.

Entropy-Complexity Change Diagram

The primary purpose of the ECCD was to compare different processes in terms of simultaneously changing randomness and structural complexity to evaluate the effects of process improvements. Both $\tilde{\tilde{H}}_S$ and $\tilde{C}_{JS}$, were evaluated relative to the maximum randomness condition for an equivalent time series, and both were naturally linked to temporal causal dynamics.

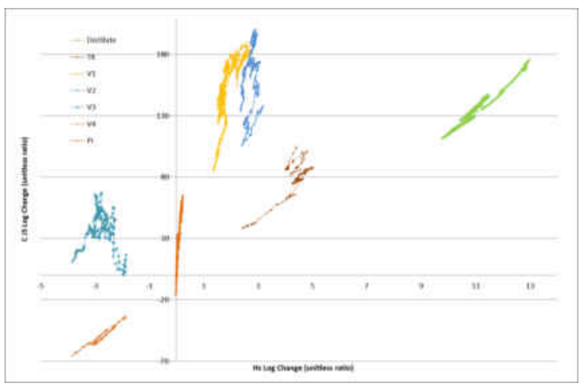


Figure 33. Sample Entropy Complexity Change Diagram (ECCD) data visualization.

Information Theoretic Quantifiers

"We inhabit a world that co-evolves as we interact with it."

-Margaret J. Wheatley

A vast array of quantifiers are associated with the concept of information, with only certain ones selected for this research effort. This section will progressively discuss the mathematical theory that defines the quantifier elements, to ultimately build up to the Complexity-Entropy Causality Plane. The first section will briefly describe Shannon information- the methodological foundation upon which everything else was built. The second will define the permutation entropy methodology in detail. Part of this section will also discuss the PE-LE method, developed to mitigate identical values within permutation entropy tuples. The third section will present the MPR-method for Jensen-Shannon complexity. Finally, all of the methodologies will be tied together to reveal the Complexity-Entropy Causality Plane.

Information-theoretic quantifiers are often based on observational studies, which attempt to infer the properties of a process by analyzing a measured time record of process behavior. Even the simplest processes evolve in time, demonstrating dynamic behaviors that can be quantified. The underlying dynamics for a scalar time series $X(t)$ can often be described as a function of variables $V= \{v_1, v_2, \ldots, v_k\}$, where $dV/dt = f(V)$. The problem is to decide which analytic method for $X(t)$ is most likely to reveal the dynamics of the process. To help make this decision, a relevant first question is: How much information is this observable time series encoding about the dynamics of the process?

These perceptions about the observable time series and the objectives of the research allow a researcher to better choose from a diverse family of information-theoretic quantifiers. For example, the Fisher information measure (FIM) is based on the asymptotic theory of maximum likelihood estimation, which determines an expected value for the observed information. FIM is considered asymptotic because it is not based on prior information. This measure represents the curvature of the log-likelihood support curve, and as such can be a useful element for "relative entropy" quantification.

However, if a precisely defined prior is more relevant to the study, the principle of maximum entropy developed by Jaynes (1957) can be used to determine the probability distribution that best encodes process behavior, based on the relative information content of every possible distribution. Algorithmic entropy is yet another measure, which is often used to assess the information attributes of computer programs. The list could go one, with the interested reader directed to Appendix E for more details.

Unfortunately, most information-theoretic quantifiers suffer from the same fundamental problem: The loss of causal information associated with both the time scaling and the order relations of the data. Usually, a "noncausal coarse-grained" description of the time series is generated by creating a symbolic sequence. Specifically, a symbol from a finite alphabet $\boldsymbol{A}$ is assigned to each time point of the series $X(t)$. However, the inextricable linkage between the probability density function (P) and the sample space Ω complicates the determination of the pdf that most appropriately encodes process behavior.

For instance, consider the histogram, and the loss of time and ordering information that corresponds with its generation. Indeed, many other commonly used analytic methods are of this *noncausal* coarse-grained type, and each establishes an appropriate probability space (Ω, P) in a different way. Common examples include Fourier analysis (Powell & Percival, 1979), binary symbolic dynamics (Mischaikow et al., 1999), frequency counting (Rosso et al., 2009), amplitude statistics procedures (De Micco et al., 2008), and wavelet transforms (Rosso, et al., 2001). All these different approaches capture the global process dynamics, but they are not equivalent in their ability to discern relevant physical details or the emerging dynamics (Zanin et al., 2012). Because each method establishes an appropriate probability space (Ω, P) in a different way, the relevant characteristics of the data can suggest the choice of method that will best optimize the probability space. Such characteristics include variation, noise contamination, stationarity, and length of the series.

As such, the data from the vertically translated funnel experiment was assessed to have low variation, low dynamical error, medium observational error, medium stationarity, and low quantity of time series data. This assessment was accomplished in subjective terms, based on previous experience with other time series processes. However, the researcher's experience with the diversity of information-theoretic quantifiers was insufficient to suggest an obvious choice that would optimize the probability space. Therefore, the initial analysis of time series data consisted of applying the most basic and foundational of the information-theoretic quantifiers- Shannon information.

Shannon Information

In 1948, Claude Shannon published *A Mathematical Theory of Communication*, which initiated the age of information theory. Shannon initially created information entropy to measure the limits associated with data compression and signal processing. However, researchers quickly found other uses in a multitude of fields beyond communications and computation. For virtually any process, Shannon information can be used at its most basic level to quantify the uncertainty, homogeneity, or randomness of that process. However, more than the information content of the process can be quantified. Additionally, the flows of information can be quantified- spatially, temporally, and symbolically. Comparisons can thus be made among and within processes. Areas of research for Shannon information also include the transmission, processing, extraction, and utilization of information.

As an analytic concept, entropy got its start in thermodynamic applications. For instance, *Gibbs Entropy* is often used to characterize the macroscopic state of a system using the probability distribution of a set of distinct "microstates". According to the 2nd Law of Thermodynamics, entropy is often considered an unrelenting trend toward "disorder". In the field of process quality control, undesired increases in statistical variance are often attributed to "entropy", such as Box & Luceño (1997) and Hindle & Wheeler (2017). Although they are not strictly identical, many close parallels exist between the thermodynamic and informational conceptions of entropy. In statistical thermodynamics, Gibbs entropy (S) measures heat capacity. In statistical complexity analysis, Shannon entropy (H_s) measures information flows and content. Nonetheless, the fundamental form is identical for both measures of entropy:

$$H = -k \sum_i P(x_i) \, log_b \, P(x_i) \, , \qquad (12)$$

where k is a constant ($k = 1$ in this research), $P(x)$ are measured probabilities for each state, and b is the chosen logarithmic base ($b = 2$ for this research with all units in bits)

Shannon information can measure the average rate at which information is "produced" by a process. Low probability events are more surprising and therefore carry more information than high probability events. The changing probabilities associated with the "states" of a process can be measured over time. By measuring the Shannon information immediately before and after a process event, an increase in process randomness can provide a measure corresponding with improved process stability.

Ongoing probability measurements will eventually cause H_s to approximately stabilize in the vicinity of an information quantity that is representative of a stable process. For example, in this research with vertical funnel data, it took the first 20 or so measurements before H_s started to stabilize around a value, with the representative value assessed after the full 50 measurements. A typical progression for each measurement height is provided in Figure 34, whose form follows the curve of *information accumulation*.

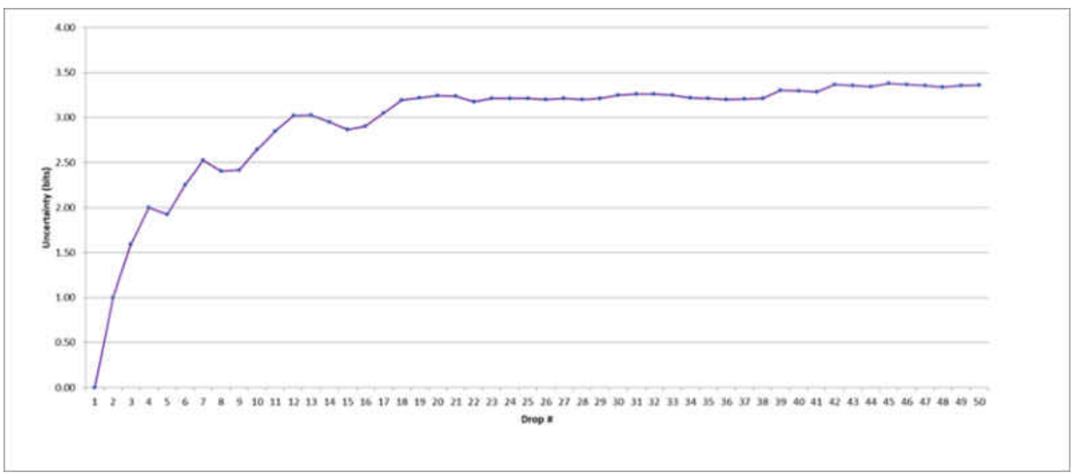


Figure 34. Shannon information accumulation. Experiment 2, 31 inch drop height.

As drop heights decreased, less uncertainty or surprise was expected for each data measurement. It was therefore reasonable to expect generally lower Shannon information values for lower heights. This was indeed the case, as displayed later in Figure 60. Interestingly, if the funnel was lowered until it rested on the target, and the distance recorded for each marble drop was 0.0 inches, an information calculation of $H_s = 0$ bits would *not* be accurate. Zero Shannon entropy ($H_{min} = 0$) is theoretically unachievable because this would imply that a measured process has no information content. Instead, some uncertainty about a process is introduced with the act of measurement, even if a process is in (theoretically) perfect equilibrium. This statement is considered true for two reasons. First, all measurements possess some level of error. Second, the act of measurement always changes the system being measured (the observer effect).

Interesting parallels exists with reference to statistical process control literature. Deming (1986) repeatedly opined that there can never be a "true" measurement of anything (pp.279-285). Similarly, Shewhart (1931) discussed how there can never be perfect certainty about any measured system or process, such as the speed of light (p.62). Both of these influential researchers deferred repeatedly to the writings of C.I. Lewis (1929), especially Lewis' chapters 5 & 6, for the foundational philosophical support of these conclusions.

Similar to the discussion above about H_{min}, Shannon information can approach, but never quite reach its theoretical maximum H_{max}. This is because the data would have to be perfectly random (an impossibility), creating a perfectly uniform distribution to represent all states of the process. This impossibility is discussed elsewhere in this dissertation in terms of the *Ramsey Theory*, with supporting evidence provided in Figure 61. However, H_{max} does provide a useful reference value to allow a common measurement basis. H_{max} is calculated with the following equation:

$$H_{max} = log\,(\#\ of\ Possible\ Distinct\ States) \tag{13}$$

Shannon information measurements are often normalized to allow for equitable comparisons, in which case $\widetilde{H}_{max}$ is defined to equal unity, although theoretically unachievable. The normalized Shannon entropy $\widetilde{H}_S$ was used as a benchmark throughout this research to quantify the changing randomness associated with processes and to better understand the effects of improvements. This measure is calculated by dividing the measured Shannon information H_s by H_{max}, as follows:

$$\widetilde{\boldsymbol{H}}_{\boldsymbol{S}} = \frac{\boldsymbol{H_S}}{\boldsymbol{H_{max}}} = \frac{-\sum_i \boldsymbol{P}(x_i)\, \boldsymbol{log}\, \boldsymbol{P}(x_i)}{\boldsymbol{log}\,(K)} \quad , \tag{14}$$

where K = # of distinct states.

As seen in the numerator, the information encoded in a process is defined by the relative probabilities of the states that make up that process, mathematically revealing how the pdf P and the sample space Ω are inextricably linked. This feature has made H_s a powerful tool for the analysis of time series data since its inception in 1948. However, H_s is not without certain shortcomings. Methods based on Shannon entropy often poorly describe highly nonlinear and chaotic processes (Zanin et al., 2012). Although they are better at describing linear processes, it is often difficult to truly know how linear or chaotic a process is until the structural complexity of the system is understood. Also, as was mentioned in the introduction to this section of the Methodology, almost all of the methods for developing the probability density function cause the abandonment of the structural causal information that is encoded in the temporal, ordered dynamics during construction. Fortunately, a method was developed that can overcome these shortcomings, as follows in the next section.

Permutation Entropy

In 2002, Bandt and Pompe published a new method that accounts for temporal dynamics, more faithfully describes nonlinear processes, and alleviates many concerns about how to describe the probability space for time series. They called this method permutation entropy. Their idea was to analyze permutation patterns that were based on an ordering relationship among adjacent values.

Methodological Advantages

Of the numerous methods suitable for this research, the advantages of permutation entropy determined its selection. Permutation entropy can harness any of a variety of information measures during an ongoing observational study while preserving the scalar dynamics associated with data order. The method is simple and robust to both dynamic and observational noise, yielding slight computational overhead compared with many similar methods. Causal information is retained by including the past dynamics of the system as ordinal symbolic sequences, which arise naturally from the time series. This makes it possible to empirically reconstruct the underlying phase-space, even though noise may be present. No model-based assumptions are required and, for the traditional method, data amplitudes need not be considered. Also, the ready evaluation of a wide variety of time series process types is easily extended to various measures of disequilibrium and complexity.

Permutation entropy is considered a “causal coarse-grained method”. Causal information is incorporated into the analysis by including the past dynamics of the system in the symbolic sequencing. Probabilities can be worked with directly without consideration for any interference terms. Further, nonlinear drifts or scaling induced by measurement will not modify the estimation of quantifiers, which is a helpful feature when analyzing experimental data. This robustness is achieved because the ordinal patterns are invariant with respect to nonlinear monotonous transformations. Instead of being limited to low dimensional dynamical processes, permutation entropy has utility for any type of time series (stochastic, chaotic, regular, periodic, etc.). It is also not necessary to assume the existence of an attractor in chaotic phase space, as is required of other methods.

General Methodology

To introduce the method, the simple methodological example provided by Bandt & Pompe (2002) is a suitable starting point here as well. Assume a time series of seven values, x = (4, 7, 9, 10, 6, 11, 3). Then organize the six pairs of neighboring values in accordance with their relative amplitudes:

Table 10. Permutation pattern example, D=2.

Time Series Pair	Permutation Pattern
(4,7)	01
(7,9)	01
(9,10)	01
(10,6)	10
(6,11)	01
(11,3)	10

The permutation entropy based on this embedding dimension D = 2 is a measure of the probabilities of the permutations 01 and 10.

$$H(2) = -\frac{4}{6} log \frac{4}{6} - \frac{2}{6} log \frac{2}{6} = 0.918\, bits$$

For D = 3, the permutations for this same time series would be:

Table 11. Permutation pattern example, D=3.

Time Series Pair	Permutation Pattern
(4,7,9)	012
(7,9,10)	012
(9,10,6)	120
(10,6,11)	102
(6,11,3)	120

And the probability computation:

$$H(3) = -2\left(\frac{2}{5} log \frac{2}{5}\right) - \frac{1}{5} log \frac{1}{5} = 1.522\ bits$$

For the previous example with D=3, the following figure reveals all of the possible $D!$=6 permutation patterns. Following this figure, similar visualizations are presented for D=4 and D=5. These figures are presented to clarify the pattern apportionment, but also because these specific ordering schemes, from Parlitz et al. (2012), were treated as the convention in this dissertation for the presentation of results.

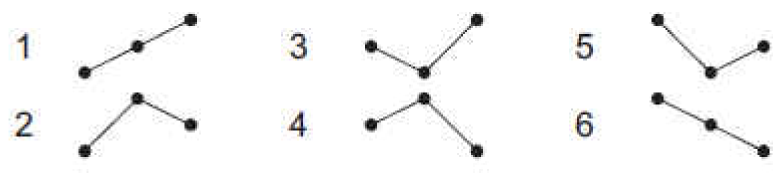


Figure 35. Distinct permutations for embedding dimension, D=3. From *Classifying Cardiac Biosignals using Ordinal Pattern Statistics and Symbolic Dynamics* by U. Parlitz et al. Copyright © 2012 by Elsevier. Reprinted with the permission of Elsevier. All rights reserved.

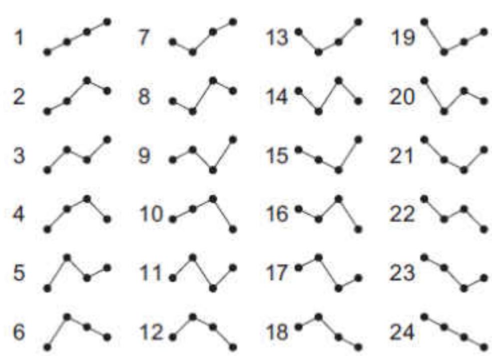


Figure 36. Distinct permutations for embedding dimension, D=4. Total number of possible distinct patterns is $D!$ = 24. From *Classifying Cardiac Biosignals using Ordinal Pattern Statistics and Symbolic Dynamics* by U. Parlitz et al. Copyright © 2012 by Elsevier. Reprinted with the permission of Elsevier. All rights reserved.

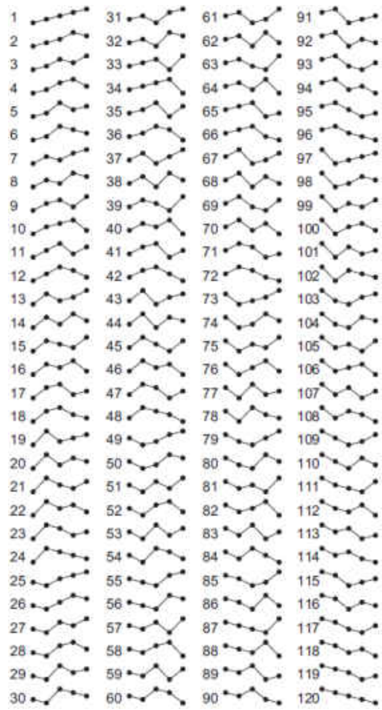


Figure 37. Distinct permutations for embedding dimension, *D*=5. Total number of possible distinct patterns is *D!* = 120. From *Classifying Cardiac Biosignals using Ordinal Pattern Statistics and Symbolic Dynamics* by U. Parlitz et al. Copyright © 2012 by Elsevier. Reprinted with the permission of Elsevier. All rights reserved.

DEFINITION: Consider a time series $\{x_t\}_{t=1,\ldots,N}$. Study all possible *D!* permutations π of order *D*. For each π, determine the relative frequency:

$$\boldsymbol{p(\pi)} = \frac{\#\{t\mid t \le N-D, (x_{t+1},\ldots,x_{t+D})\ has\ type\ \pi\}}{N-D+1} \quad , \qquad (15)$$

where N is the cardinal number (# of data).

M is the number of tuples in a string, defined in the denominator above, *M=N-D+1*. This equation estimates the frequency of π for a finite series of values. To determine *p*(π) exactly, assume an infinite time series and take the limit as $N \to \infty$ in the above equation. This limit exists with $p(\pi) = 1$ when the underlying stochastic process fulfills the stationarity condition: For $k \leq D$ the probability for $x_t < x_{t+k}$ should not depend on t.

The permutation entropy of order $D \geq 2$ is defined as,

$$\boldsymbol{H(D) = -\sum p(\pi) \log p(\pi)}\,, \qquad (16)$$

where the sum runs over all $D!$ permutations π of order D.

H(D) is the information contained in comparing *D* consecutive values of the time series. For $0 \leq H(D) \leq \log D!$ the lower bound is attained for a strictly increasing or decreasing sequence of values, and the upper bound for a completely random sequence where all *D!* possible distinct permutations appear with the same probability. H_{max} is used to normalize the permutation entropy calculation of Shannon entropy and is based on the logarithm of the maximum number of possible distinct states (which are distinct tuple types).

Graphical insight regarding maximum Shannon entropy calculations is provided in Figure 38. Some of the values provided are associated with different PE embedding dimensions (*D!*).

# Distinct States	Hmax (bits)
2	1.00
4	2.00
6	2.58
8	3.00
10	3.32
24	4.58
42	5.39
48	5.58
96	6.58
120	6.91
192	7.58
384	8.58
720	9.49
5040	12.30

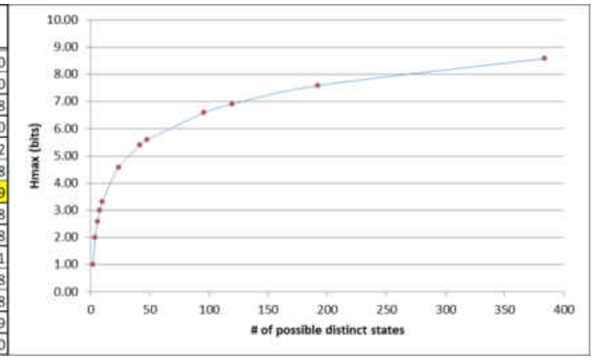


Figure 38. Some common H_{max} values for PE. The number of distinct states (K) corresponds with embedding dimensions (D). The yellow highlighted cell is the value for 42 distinct tuples, used for the PE-LE method at D=4.

Bandt and Pompe (2002) recommended using only embedding dimensions ranging from D={3,…,7} (p.174102-1). $D \geq 8$ rapidly gets unwieldy with the number of possible distinct permutation types, $D!$ They also recommended $N>>D!$ to facilitate differentiation between stochastic and chaotic processes. Finally, they recommended using τ =1, which is the “time lag” or “embedding delay”. This represents how many values to skip before encoding the next tuple. The research in this dissertation only used τ =1, but some authors have discovered that applying different time lags can provide additional information about processes, as related to intrinsic time scaling (Soriano et al., 2011; Zunino et al., 2010). In addition, larger time lags can be used to mitigate certain digitization-induced errors such oversampling, where states are measured in multiple successive repetitions for time series (Daw et al., 2003).

Another variable δ corresponds to how much time is encompassed by each tuple. This variable can be modified to search for corresponding process dynamics or to mitigate digitization-induced errors. However, for the research in this dissertation, and for most of the research literature that was reviewed, this variable was ignored. The following figure from

Zunino et al. (2011) provides a graphical example of the interplay between *D*, τ, and δ, as applied to permutation assignments.

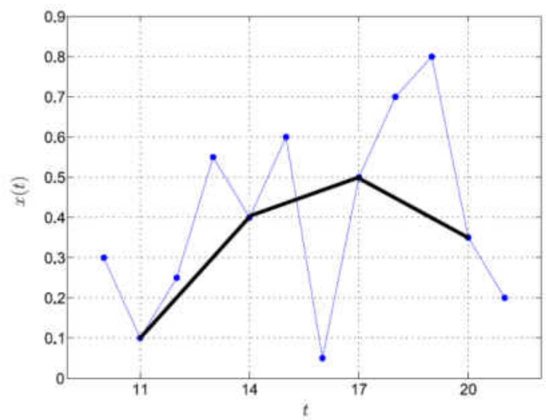


Figure 39. Procedure to identify ordinal patterns from a time series. Embedding dimension $D = 4$, embedding delay $\tau = 3$ and time $\delta = 20$. From *Commodity Predictability Analysis with a Permutation Information Theory Approach* by Zunino et al. Copyright © 2011 by Elsevier B.V. Reprinted with the permission of Elsevier. All rights reserved.

Evaluation of Process Randomness

The stability of a process generally increases as its randomness increases. In statistics, this randomness is sometimes based on the assumption that factors are independent and identically distributed (i.i.d.). An assumption of perfect randomness requires two impossibilities: That none of the factors interact and that the factors contribute equally to process behaviors. This impossibility could be represented by a perfect uniform probability distribution. In reality, all processes possess varying degrees of randomness, based on their relative magnitudes of departure from the assumption of perfect randomness.

Normalization of Shannon entropy allows an "equitable" assessment of degrees of randomness or relative stability for different processes. What's more, the change in process randomness after an intended process improvement can provide a measure of the impact the improvement made to process stability. In this research, the assessment of the degree of randomness was calculated as follows. First permutation entropy was applied to characterize the probability space (Ω, P). The resulting pdf was used to calculate Shannon information H_s. Then Shannon information was normalized to $\widetilde{H}_S$ based on the number of possible distinct states that made up the process.

$$\widetilde{\boldsymbol{H}}_{\boldsymbol{S}} = \frac{\boldsymbol{H}_{\boldsymbol{S}}}{\boldsymbol{H}_{\boldsymbol{max}}} = \frac{-\sum \boldsymbol{P}(\boldsymbol{\pi}) \, \boldsymbol{log} \, \boldsymbol{P}(\boldsymbol{\pi})}{\boldsymbol{log} \, (\mathrm{K})} \quad , \tag{17}$$

where K = # of distinct states.

The equation repeated here is identical in form to the normalization equation (14) presented in the Methodology section for Shannon information, with one key difference. The ongoing calculation of probabilities is now based specifically on permutation entropy tuples $P(\pi)$ instead of some other definition of process states. For traditional permutation entropy, the number of possible distinct states is usually just $D!$ based on the embedding dimension chosen. However, in this research, the PE-LE method was used to mitigate identical values within tuples. As a consequence, the number of possible distinct tuples for D=4 was actually 42, not 4! = 24, as will be discussed in the next section of the Methodology.

Methodological Disadvantages

The traditional permutation entropy methodology loses the actual amplitude information from the original series. Although relative magnitudes within tuples are maintained, the amplitude information could help detect certain process characteristics including abrupt changes in magnitude and spiky features (Fadlallah et al., 2013). The incorporation of amplitude information could also provide better robustness in the presence of noise. Numerous methods have been devised to incorporate amplitude information, as presented in the Literature Review section. Concurrent observation of the process amplitude data with a control chart could also help to mitigate some of these time series monitoring issues.

Another disadvantage is the impact of identical values within tuples, which are not accommodated in the traditional permutation symbology scheme. Numerous methods for handling identical values by modifying the traditional permutation entropy methodology were presented in the Literature Review. It was revealed there the sort of undesirable effects that were caused by each method. Because identical values within tuples are naturally more likely for stable processes, which was the focus of this research, an alternative method was investigated that added distinct permutations based on the "local effect" within tuples. That methodology is described in the next section.

Permutation Entropy- Local Effect

The standard permutation entropy symbology does not account for equal values within tuples. For example, there are no equal value configurations of tuples in Figure 35, Figure 36, or

Figure 37. Unfortunately, equal values are common for discrete series data, which are increasingly prevalent with the ubiquitous digitization of data streams. Parsing tuples from the analysis that do not meet the symbology scheme equates to throwing away information that may help to reveal process dynamics (Traversaro et al., 2018, Table II). Worse, various mitigation strategies may skew the probability distributions of process states, thereby promoting the wrong conclusions about process behavior (Zunino et al., 2017). Although at least one paper disagrees (Cuesta-Frau et al., 2018), this problem is widely seen to be a significant limitation of the permutation entropy methodology.

This problem was assessed to have a significant impact on this dissertation's research effort. Because Shewhart-stable processes demonstrate less variability, they are naturally more likely to be affected by this problem than Shewhart-unstable processes. As such, it was important to select a method to mitigate identical values. In the Literature Review section, numerous methods were examined but each one presented a shortcoming. While considering the pros and cons of the various methods, it became apparent that a slightly different approach, based on measuring the "local effect" in each tuple, might have merit. When this Permutation Entropy-Local Effect (PE-LE) method was tested against axiomatic validation standards, it appeared to perform at least as well as some of the other methods, and perhaps better for low dimensional stochastic processes. It was therefore decided to embrace this method for the analyses conducted herein. This section will identify the PE-LE methodology, and present the validation approach.

Assume D=3 and $\tau = 1$. According to traditional permutation entropy convention, there are $D!$=6 possible distinct symbols, $\Omega_3 = \{(012)(021)(102)(120)(201)(210)\}$. Note that none of these symbols account for an equal value within a tuple, which would be represented symbolically as, for example, (001) or (101), and herein lies the problem. The ultimate question has been, “What is the best way to account for identical values within a tuple that will *minimize* the erroneous representation of emerging process dynamics?”

Other methods have been developed to mitigate identical values, each possessing various pros and cons. After review, some desirable attributes were contemplated to guide the development of a new method:

1. Retain the causal influences associated with temporal ordering
2. Retain the causal influences associated with magnitude ordering
3. Minimize the parsing of problematic tuples via *statistically complete rules/ complete case analysis*
4. Define rules for distinct tuple types so that exceptionally long data series would not be required to:
    a. See all possible distinct tuples types
    b. Achieve balance among probability densities for among all possible distinct tuple types for stochastic series
    c. Achieve imbalance among probability densities for all possible distinct tuple types for complex nonlinear series
5. Approximate H_{max} such that $\widetilde{H}_S$ is nearly unity for WGN.
6. Allow equitable comparisons between processes when they are analyzed similarly.

A simple example will introduce the local effect concept. Consider the D=4 permutation tuple (0212). Maintaining the temporal ordering from left to right, the difference between the first two values (02) is +2, between the second two values (21) is -1, and between the final two values (12) is +1. The local effect for the tuple is the sum of these three differences:

LE=+2-1+1= 2

The following table provides a few more examples for tuples with identical values.

Table 12. Local effect calculation examples.

Tuple	1st Difference	2nd Difference	3rd Difference	Local Effect
0212	2	-1	1	2
0121	1	1	-1	1
0110	1	0	-1	0
2001	-2	0	1	-1
2100	-1	-1	0	-2

The next step was to determine all of the possible tuples, assuming that identical values were permitted. Note that PE-LE was developed in this research for D=4 only, but the basic methodology could be extended to any embedding dimension. The easiest starting point was to list all of the possible permutations for the values (0,1,2,3) with repetition allowed, and order matters. The total number of such permutations is given by:

$$\boldsymbol{n^r} = \textbf{\textit{permutations with repetition, order matters}}\ , \qquad (18)$$

where n= values to choose from, and r= # values chosen.

For n=4 and r=4, there are 256 permutations, listed in Table 13.

Table 13. All 256 permutations for the values (0,1,2,3). Generated at www.mathisfun.com. Website maintained by Pierce (2020).

{0,0,0,0}	{0,0,0,1}	{0,0,0,2}	{0,0,0,3}	{0,0,1,0}	{0,0,1,1}	{0,0,1,2}	{0,0,1,3}	{0,0,2,0}	{0,0,2,1}	{0,0,2,2}	{0,0,2,3}	{0,0,3,0}	{0,0,3,1}	{0,0,3,2}	{0,0,3,3}
{0,1,0,0}	{0,1,0,1}	{0,1,0,2}	{0,1,0,3}	{0,1,1,0}	{0,1,1,1}	{0,1,1,2}	{0,1,1,3}	{0,1,2,0}	{0,1,2,1}	{0,1,2,2}	{0,1,2,3}	{0,1,3,0}	{0,1,3,1}	{0,1,3,2}	{0,1,3,3}
{0,2,0,0}	{0,2,0,1}	{0,2,0,2}	{0,2,0,3}	{0,2,1,0}	{0,2,1,1}	{0,2,1,2}	{0,2,1,3}	{0,2,2,0}	{0,2,2,1}	{0,2,2,2}	{0,2,2,3}	{0,2,3,0}	{0,2,3,1}	{0,2,3,2}	{0,2,3,3}
{0,3,0,0}	{0,3,0,1}	{0,3,0,2}	{0,3,0,3}	{0,3,1,0}	{0,3,1,1}	{0,3,1,2}	{0,3,1,3}	{0,3,2,0}	{0,3,2,1}	{0,3,2,2}	{0,3,2,3}	{0,3,3,0}	{0,3,3,1}	{0,3,3,2}	{0,3,3,3}
{1,0,0,0}	{1,0,0,1}	{1,0,0,2}	{1,0,0,3}	{1,0,1,0}	{1,0,1,1}	{1,0,1,2}	{1,0,1,3}	{1,0,2,0}	{1,0,2,1}	{1,0,2,2}	{1,0,2,3}	{1,0,3,0}	{1,0,3,1}	{1,0,3,2}	{1,0,3,3}
{1,1,0,0}	{1,1,0,1}	{1,1,0,2}	{1,1,0,3}	{1,1,1,0}	{1,1,1,1}	{1,1,1,2}	{1,1,1,3}	{1,1,2,0}	{1,1,2,1}	{1,1,2,2}	{1,1,2,3}	{1,1,3,0}	{1,1,3,1}	{1,1,3,2}	{1,1,3,3}
{1,2,0,0}	{1,2,0,1}	{1,2,0,2}	{1,2,0,3}	{1,2,1,0}	{1,2,1,1}	{1,2,1,2}	{1,2,1,3}	{1,2,2,0}	{1,2,2,1}	{1,2,2,2}	{1,2,2,3}	{1,2,3,0}	{1,2,3,1}	{1,2,3,2}	{1,2,3,3}
{1,3,0,0}	{1,3,0,1}	{1,3,0,2}	{1,3,0,3}	{1,3,1,0}	{1,3,1,1}	{1,3,1,2}	{1,3,1,3}	{1,3,2,0}	{1,3,2,1}	{1,3,2,2}	{1,3,2,3}	{1,3,3,0}	{1,3,3,1}	{1,3,3,2}	{1,3,3,3}
{2,0,0,0}	{2,0,0,1}	{2,0,0,2}	{2,0,0,3}	{2,0,1,0}	{2,0,1,1}	{2,0,1,2}	{2,0,1,3}	{2,0,2,0}	{2,0,2,1}	{2,0,2,2}	{2,0,2,3}	{2,0,3,0}	{2,0,3,1}	{2,0,3,2}	{2,0,3,3}
{2,1,0,0}	{2,1,0,1}	{2,1,0,2}	{2,1,0,3}	{2,1,1,0}	{2,1,1,1}	{2,1,1,2}	{2,1,1,3}	{2,1,2,0}	{2,1,2,1}	{2,1,2,2}	{2,1,2,3}	{2,1,3,0}	{2,1,3,1}	{2,1,3,2}	{2,1,3,3}
{2,2,0,0}	{2,2,0,1}	{2,2,0,2}	{2,2,0,3}	{2,2,1,0}	{2,2,1,1}	{2,2,1,2}	{2,2,1,3}	{2,2,2,0}	{2,2,2,1}	{2,2,2,2}	{2,2,2,3}	{2,2,3,0}	{2,2,3,1}	{2,2,3,2}	{2,2,3,3}
{2,3,0,0}	{2,3,0,1}	{2,3,0,2}	{2,3,0,3}	{2,3,1,0}	{2,3,1,1}	{2,3,1,2}	{2,3,1,3}	{2,3,2,0}	{2,3,2,1}	{2,3,2,2}	{2,3,2,3}	{2,3,3,0}	{2,3,3,1}	{2,3,3,2}	{2,3,3,3}
{3,0,0,0}	{3,0,0,1}	{3,0,0,2}	{3,0,0,3}	{3,0,1,0}	{3,0,1,1}	{3,0,1,2}	{3,0,1,3}	{3,0,2,0}	{3,0,2,1}	{3,0,2,2}	{3,0,2,3}	{3,0,3,0}	{3,0,3,1}	{3,0,3,2}	{3,0,3,3}
{3,1,0,0}	{3,1,0,1}	{3,1,0,2}	{3,1,0,3}	{3,1,1,0}	{3,1,1,1}	{3,1,1,2}	{3,1,1,3}	{3,1,2,0}	{3,1,2,1}	{3,1,2,2}	{3,1,2,3}	{3,1,3,0}	{3,1,3,1}	{3,1,3,2}	{3,1,3,3}
{3,2,0,0}	{3,2,0,1}	{3,2,0,2}	{3,2,0,3}	{3,2,1,0}	{3,2,1,1}	{3,2,1,2}	{3,2,1,3}	{3,2,2,0}	{3,2,2,1}	{3,2,2,2}	{3,2,2,3}	{3,2,3,0}	{3,2,3,1}	{3,2,3,2}	{3,2,3,3}
{3,3,0,0}	{3,3,0,1}	{3,3,0,2}	{3,3,0,3}	{3,3,1,0}	{3,3,1,1}	{3,3,1,2}	{3,3,1,3}	{3,3,2,0}	{3,3,2,1}	{3,3,2,2}	{3,3,2,3}	{3,3,3,0}	{3,3,3,1}	{3,3,3,2}	{3,3,3,3}

Many of these permutations can be simplified. For instance, the permutation patterns (0000), (1111), (2222), and (3333) all represent the same magnitude situation for the underlying data. Therefore, all four tuples simplify to the lowest common tuple (0000). Similarly, (0101), (0202), (0303), (1212), (1313), and (2323), all simplify to the lowest common tuple (0101). Carrying on this simplification scheme results in 75 distinct lowest common tuples.

Table 14. The 75 distinct lowest common tuples for *D*=4, allowing identical values.

0000	0001	0010	0011	0012	0021	0100	0101	0102	0110	0111	0112	0120	0121	0122
0123	0132	0201	0210	0211	0212	0213	0221	0231	0312	0321	1000	1001	1002	1010
1011	1012	1020	1021	1022	1023	1032	1100	1101	1102	1110	1120	1200	1201	1202
1203	1210	1220	1230	1302	1320	2001	2010	2011	2012	2013	2021	2031	2100	2101
2102	2103	2110	2120	2130	2201	2210	2301	2310	3012	3021	3102	3120	3201	3210

The local effect was then calculated for each of these 75 distinct tuples. Three observations were made about the results, as will be presented in the tables and figures to follow.

1. The range of local effect was from -3 to +3 for all of the *D*=4 tuples.
2. Every tuple other than 0000 had an "opposite twin". In other words, a mirror effect was observed for the sign of the deltas between adjacent values, which also yielded opposite signs for the resulting calculated local effect.
3. Probability density functions for "mirror sets" were perfectly balanced.

The 24 tuples used in traditional *D*=4 permutation entropy do not have any identical values within the tuples. They are displayed in local effect mirror set configuration in Table 15.

Table 15. Local effect mirror sets for the 24 tuples used in traditional *D*=4 permutation entropy.

Tuple	Δ1	Δ2	Δ3	LE	Tuple	Δ1	Δ2	Δ3	LE
0231	2	1	-2	1	3102	-2	-1	2	-1
0321	3	-1	-1	1	3012	-3	1	1	-1
1032	-1	3	-1	1	2301	1	-3	1	-1
1302	2	-3	2	1	2031	-2	3	-2	-1
2103	-1	-1	3	1	1230	1	1	-3	-1
2013	-2	1	2	1	1320	2	-1	-2	-1
0132	1	2	-1	2	3201	-1	-2	1	-2
0312	3	-2	1	2	3021	-3	2	-1	-2
1023	-1	2	1	2	2310	1	-2	-1	-2
1203	1	-2	3	2	2130	-1	2	-3	-2
0123	1	1	1	3	3210	-1	-1	-1	-3
0213	2	-1	2	3	3120	-2	1	-2	-3

Extracting these traditional 24 tuples from the original set of 75 distinct lowest common tuples leaves the 51 tuples with identical values. Table 16 summarizes the local effect for these remaining tuples in mirror set configuration.

Table 16. Local effect mirror sets for the 51 tuples that contain identical values.

Tuple	Δ1	Δ2	Δ3	LE	Tuple	Δ1	Δ2	Δ3	LE
0000	0	0	0	0					
0010	0	1	-1	0	1101	0	-1	1	0
0100	1	-1	0	0	1011	-1	1	0	0
0110	1	0	-1	0	1001	-1	0	1	0
0120	1	1	-2	0	2102	-1	-1	2	0
0210	2	-1	-1	0	2012	-2	1	1	0
1021	-1	2	-1	0	1201	1	-2	1	0
0001	0	0	1	1	1110	0	0	-1	-1
0011	0	1	0	1	1100	0	-1	0	-1
0101	1	-1	1	1	1010	-1	1	-1	-1
0111	1	0	0	1	1000	-1	0	0	-1
0021	0	2	-1	1	2201	0	-2	1	-1
0121	1	1	-1	1	2101	-1	-1	1	-1
0201	2	-2	1	1	2021	-2	2	-1	-1
0211	2	-1	0	1	2011	-2	1	0	-1
0221	2	0	-1	1	2001	-2	0	1	-1
1002	-1	0	2	1	1220	1	0	-2	-1
1012	-1	1	1	1	1210	1	-1	-1	-1
1022	-1	2	0	1	1200	1	-2	0	-1
1102	0	-1	2	1	1120	0	1	-2	-1
1202	1	-2	2	1	1020	-1	2	-2	-1
0012	0	1	1	2	2210	0	-1	-1	-2
0102	1	-1	2	2	2120	-1	1	-2	-2
0112	1	0	1	2	2110	-1	0	-1	-2
0122	1	1	0	2	2100	-1	-1	0	-2
0212	2	-1	1	2	2010	-2	1	-1	-2

The probability densities of the local effects for the various sets of tuples were then computed. Results in Table 17 show that all sets are balanced around zero local effect.

Table 17. Probability densities for tuple sets based on local effect.

	24 tuples		51 tuples		75 tuples	
LE	count	density	count	density	count	density
-3	2	0.08	0	0.00	2	0.03
-2	4	0.17	5	0.10	9	0.12
-1	6	0.25	14	0.27	20	0.27
0	0	0.00	13	0.25	13	0.17
1	6	0.25	14	0.27	20	0.27
2	4	0.17	5	0.10	9	0.12
3	2	0.08	0	0.00	2	0.03

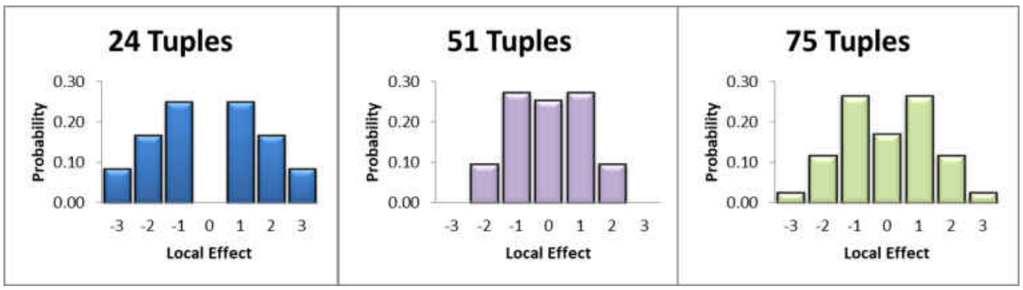


Figure 40. Probability densities based on local effect for various tuple sets.

Since balance around zero local effect was demonstrated, the next question was: Could the tuples with zero local effect be abandoned with little change to the analytic results? To answer this question, it was assumed that the needed insight would be revealed by the distribution of probability densities among the 75 tuples. Since the focus of this research was the improvement of Shewhart-stable processes, a highly stochastic process was analyzed using the PE-LE methodology. The first 10,000 digits of Pi were considered a suitable quasi-random process, which would serve as the stochastic baseline for evaluating PE-LE. The resulting distribution of the 9,997 tuples was divided between the traditional 24 tuples in Table 18 and the remaining 51 identical value tuples in Table 19.

Table 18. Distribution of tuple counts for 24 traditional tuples based on 10,000 digits of Pi and PE-LE method. Total tuple count out of 9997 = 5095.

Tuple	LE	Count	Tuple	LE	Count
0231	1	202	3102	-1	221
0321	1	247	3012	-1	205
1032	1	205	2301	-1	213
1302	1	217	2031	-1	220
2103	1	194	1230	-1	224
2013	1	215	1320	-1	198
0132	2	216	3201	-2	208
0312	2	216	3021	-2	221
1023	2	202	2310	-2	210
1203	2	212	2130	-2	206
0123	3	210	3210	-3	206
0213	3	210	3120	-3	217

Table 19. Distribution of tuple counts for 51 identical value tuples based on 10,000 digits of Pi and PE-LE method. Total tuple count out of 9997 = 4902. The far right column represents the sum for each pair of "opposite twins".

Tuple	LE	Count	Tuple	LE	Count	Σpair
0000	0	10				10
0010	0	39	1101	0	46	85
0100	0	46	1011	0	48	94
0110	0	47	1001	0	47	94
0120	0	126	2102	0	135	261
0210	0	122	2012	0	138	260
1021	0	126	1201	0	120	246
0001	1	42	1110	-1	37	79
0011	1	40	1100	-1	38	78
0101	1	49	1010	-1	42	91
0111	1	33	1000	-1	46	79
0021	1	108	2201	-1	125	233
0121	1	123	2101	-1	119	242
0201	1	136	2021	-1	126	262
0211	1	115	2011	-1	114	229
0221	1	110	2001	-1	102	212
1002	1	117	1220	-1	116	233
1012	1	115	1210	-1	103	218
1022	1	119	1200	-1	119	238
1102	1	110	1120	-1	119	229
1202	1	133	1020	-1	123	256
0012	2	127	2210	-2	104	231
0102	2	118	2120	-2	130	248
0112	2	125	2110	-2	113	238
0122	2	110	2100	-2	107	217
0212	2	114	2010	-2	125	239

In this evaluation of 10,000 quasi-random values, some interesting characteristics became apparent for the 51 tuples containing identical values. First, the distribution of tuple counts for non-highlighted cells was generally about half what it was for the 24 traditional tuples. More specifically, the mean count for the 24 tuples was 212.3 and the mean count for the 36 identical value tuples of interest (non-highlighted) was 119.2. Therefore, when the counts for each pair of opposite twins were summed, the results were approximately consistent with the counts for individual tuple types in the traditional set of 24.

ASSUMPTION: "Approximately consistent" is assumed to mean that the tuple count for opposite twins is within about ± 15% of the traditional tuples for a stochastic process with low measurement resolution.

This inconsistency was probably caused by the abandonment of the highlighted cells from the calculations. To further evaluate the overall balance of local effect, which could be extended to any process, the "centerpoint" equation was devised, as follows:

$$LE\ Centerpoint_{D=4} = -3(Count) - 2(Count) - 1(Count) + 1(Count) + 2(Count) + 3(Count) \qquad (19)$$

The LE centerpoint result including all 75 tuples for this process was:

-3(423)-2(1424)-2610+2630+2(1440)+3(420) = 43

This answer represents less than 0.5% of the 10,000 values in the time series. Based on these results, the decision was made to continue the investigation with each pair of opposite twins combined to become a new, distinct tuple type comprised of the pair.

The second observation was that the highlighted tuples do not follow the observed pattern. Summing certain sets of highlighted tuples with the same LE does achieve values within about 15% of the mean for the traditional tuples for this specific stochastic process. For instance,

Zero LE Tuples: (0000)+(0100)+(0110)+(1011)+(1001) = 198

±1 LE Tuples: (0001)+(0011)+(0111)+(1110)+(1100)+(1000) = 236

However, these relationships are currently unclear. Therefore, the results for all of the *non-pattern-following* zero LE and ±1 LE tuple groupings were summed to allow this pattern-based investigation to continue. The results were:

Zero LE Tuples: (0000)+(0010)+(0100)+(0110)+(1101)+(1011)+(1001) = 283

±1 LE Tuples: (0001)+(0011)+(0101)+(0111)+(1110)+(1100)+(1010)+(1000) = 327

The difficulty in labeling these two groupings of 7 and 8 tuples quickly became apparent. It was too unwieldy to type all 7 and 8 tuples each time they were referenced, especially when displayed in figures. Same for calling them something like, "Zero LE non-pattern-following tuple group" and, "±1 LE non-pattern-following tuple group". Short names were therefore chosen, more or less at random, as labels for these two groups. The ±1 LE non-pattern following tuple group was labeled, “Red” and the zero LE non-pattern following group was labeled “Wizard”.

While working with all of the original set of 75 tuples in various analyses, it became apparent that it was also somewhat unwieldy to refer to the remaining 51-15= 36, 36/2= 18 pairs of, “Pattern-following opposite twin LE tuple pairs”, so the label “Forest” was chosen for these 18 pairs. Finally, the, “24 traditional *D*=4 tuples” were assigned the short label “Parlitz” in deference to the paper written by Parlitz et al., (2012) in which the authors had the foresight to publish a logical, graphical ordering standard that could ease comparisons among different PE studies. As such, for the remainder of this research, these 24 traditional tuple types are presented in the “Parlitz-order”. This short label naming convention for PE-LE is summarized below.

Table 20. Short label assignments for PE-LE tuple categories.

Label	Long Title	# Tuples
Red	±1 LE non pattern-following tuple group	1
Wizard	0 LE non pattern-following tuple group	1
Forest	Pattern-following opposite twin LE tuple pairs	18
Parlitz	Traditional D=4 tuples	24
	Total	44

The evaluation of 10,000 digits of Pi is displayed in the following pdf, assuming 44 tuples for the PE-LE method.

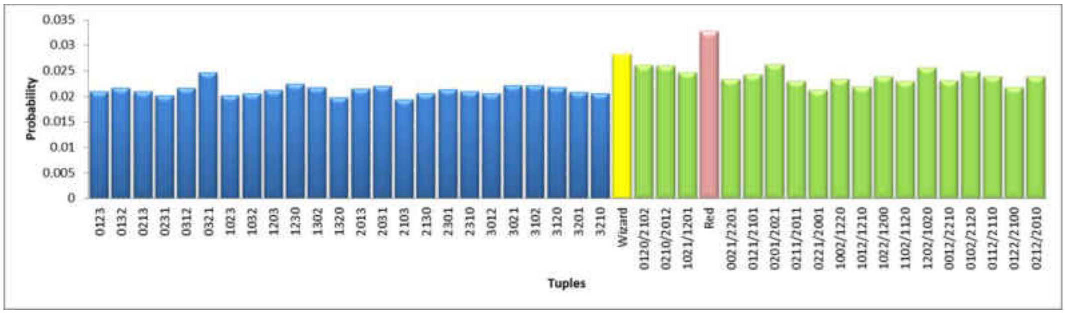


Figure 41. PDF for first 10,000 digits of Pi assuming 44 tuples for the PE-LE method.

Given the stochastic nature of the digits of Pi, the distribution is expected to appear approximately uniform. However the Red and Wizard types are clearly not following the pattern expected for a highly stochastic process. Next, to continue to assess the Red and Wizard tuples, a Shewhart-unstable process type was selected. The goal was to assess whether the PE-LE method could be generalized to any process type while considering the six desirable features delineated above. In Figure 42, the PE-LE method was applied to the first 10,000 data of the final 16,000 values of the Distillate Flowrate dataset available open-source without restriction at www.openmv.net (Dunn, 2020).

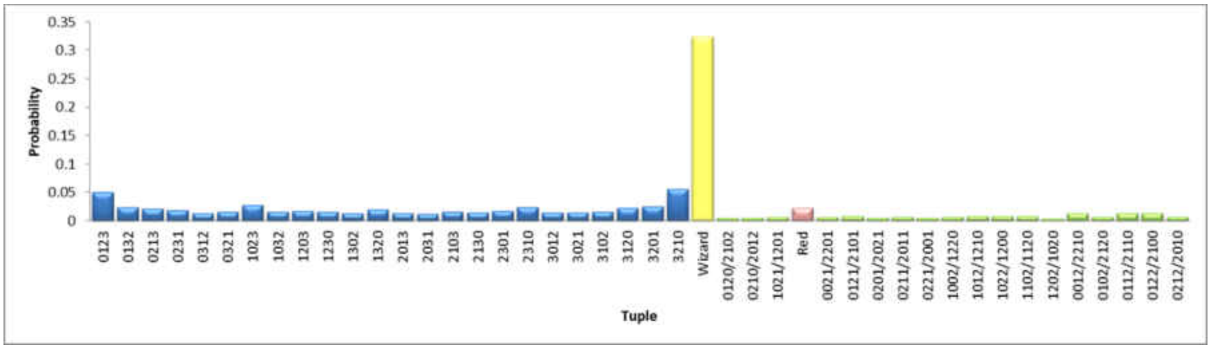


Figure 42. PDF for 10,000 Distillate Flowrate measurements assuming 44 tuples for the PE-LE method. From www.openmv.net by Kevin Dunn (2020). Dataset provided open source with no restrictions on use.

Inspection of the dataset in Figure 23 reveals instances where the distillate process appeared to be shutting down, settling out at zero flowrate for a time, and then starting again. This would likely explain the high relative density of the Wizard tuples. To continue the assessment of the Red and Wizard tuples, a Shewhart-stable process type was selected for evaluation. The pdf in Figure 43 represents the 1,600 time series measurements for the second vertical-translation funnel experiment. During this experiment, the funnel was incrementally raised above the target by one inch following every 50 measurements. At the lower funnel heights, the measurements tended to vary much less, which likely explains the prevalence of the Red and Wizard tuples.

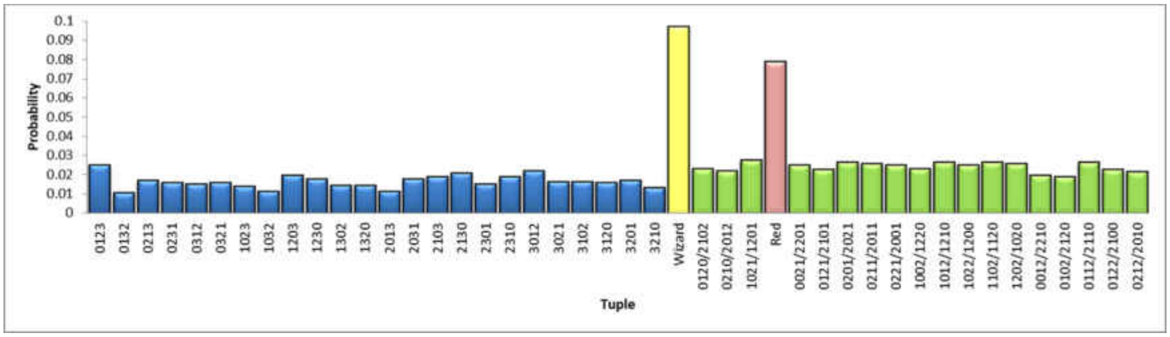


Figure 43. PDF for 1,600 measurements from the second vertical translation funnel experiment, assuming 44 tuples for the PE-LE method.

These insights from evaluating these three processes supported a decision for how to handle the Red and Wizard tuple types, as provided in the next section.

Red & Wizard Tuple Decision

To advance the PE-LE methodology, a decision was required for whether to retain or to eliminate the Red and Wizard tuples. Decision criteria were available from the six desirable attributes defined above:

1. Retain the causal influences associated with temporal ordering

   Yes. True whether or not they are retained.

2. Retain the causal influences associated with magnitude ordering

   Yes. The Red and Wizard tuples added information associated with magnitude ordering in terms of categories of local effect within tuples.

3. Minimize the parsing of problematic tuples via "statistically complete" rules/ "complete case analysis"

   Retaining Red and Wizard tuples would require no parsing. Eliminating them would require some parsing, the amount depending upon the likelihood of identical values within the time series.

4. Define rules for distinct tuple types so that exceptionally long data series would not be required to:

   a. See all possible distinct tuples types

      Longer data series would be required to see the Red & Wizard tuple types.

   b. Achieve balance among probability densities for all possible distinct tuple types for stochastic series

      The densities of the Red & Wizard tuples were contrary to achieving balance for stochastic processes.

   c. Achieve imbalance among probability densities for all possible distinct tuple types for complex nonlinear series.

      The densities of the Red & Wizard tuples added to the tuple imbalance present for nonlinear series as witnessed with the Distillate Flowrate data.

5. Approximate H_{max} such that $\widetilde{H}_S$ is nearly unity for WGN.

   Eliminating two of 44 tuples via statistically complete parsing would move $\widetilde{H}_S$ further away from unity for WGN.

6. Allow equitable comparisons between processes when they are analyzed similarly.

   The Red & Wizard tuples may provide undue influence over the results to allow equitable comparisons between diverse process types.

Based on these criteria, the decision was made to eliminate the Red & Wizard tuples from the PE-LE methodology for this research. The consideration of stochastic processes in criterion 4b was a primary determinant since this research was focused on Shewhart-stable processes. This decision meant that the 15 tuples represented by Red & Wizard would be parsed from any analysis before starting entropy calculations.

Justification: For a highly stochastic process (10,000 digits of Pi), the Red & Wizard tuple groups did not follow the pattern of density distribution evidenced by the rest of the tuples. These pattern differences were ultimately not understood, and could potentially provide inequitable influence on the results. Furthermore, grouping seven tuples together because they were all zero LE and didn't follow the pattern of the other zero LE tuples would be arbitrary without a better understanding of the dynamics. Similarly so for grouping eight tuples together because they were all ±1 LE and didn't follow the pattern of the other ±1 LE tuples.

This decision was significant both in terms of the negative effects associated with statistically complete parsing and because the decision would impact how the probability space (Ω, P) would be defined for the entire research effort. Before the decision to eliminate the Red and Wizard tuple sets was considered final, the new PE methodology required validation to better characterize its applicability for answering the research question. The results of this effort are provided in the Findings section.

Structural Complexity

Permutation entropy measures the uncertainty of a process. Although this can provide a feel for process randomness, the underlying process mechanisms often remain opaque, especially for Shewhart-stable processes. If the complex interactions among the components that make up the process are understood, process knowledge increases. The quest for process knowledge lies at the intersection of uncertainty, opacity, and complexity.

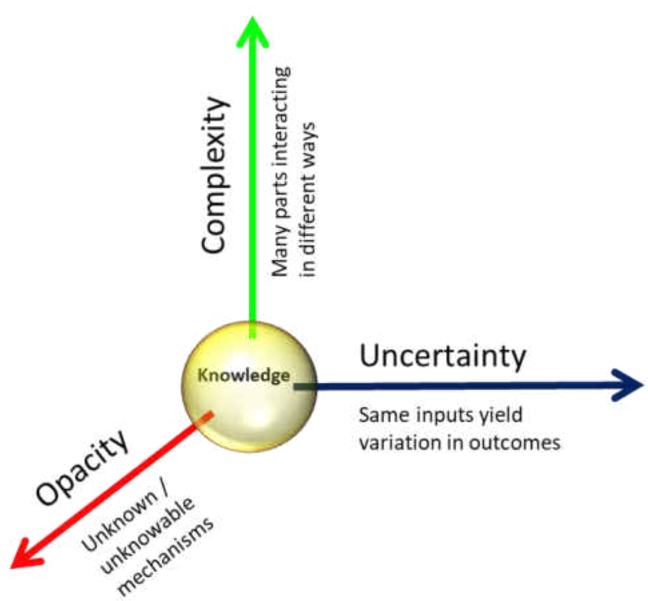


Figure 44. The bubble of process knowledge. Adapted from a figure attributed to Dr. Myron Tribus in *Understanding Industrial Experimentation* (p.x) by Donald Wheeler.

A typical approach to building process knowledge is to first address the "Complexity" arrow, above, which is defined by many parts interacting in different ways. It is thought that breaking a process apart statistically will allow the component parts to reveal their contribution to the greater whole. By comparing these varying contributions, the researcher can then bin some of them as "signals", and the rest as "noise". This is typical of the reductionist approach, and it has worked effectively since the 19^{th} century. However, there is another approach, which is, in many ways, the opposite of the reductionist approach. The emergence-based approach examines problems from the perspective that some overall process behaviors cannot be understood through analysis at the component level. A simple example is the pressure of a system, which cannot be understood by attempting to measure attributes of each component particle. This example would suggest the question: What is the pressure of a particle? Conceptually, the emergence-based approach is much more recent than the reductionist approach, beginning in the 1940's, due in part to Bertalanffy's (1945) *General Systems Theory* and Wiener's (1948) *Cybernetics*.

It may seem somewhat contradictory for this research effort to focus concurrently on stable processes and complexity. Shouldn't a stable process be among the least complex of all processes? This question demonstrates the important distinction between the words complicated and complex. It is true that a stable process is usually less complicated than other processes. In fact, a stable process may seem relatively simple. But in terms of *structural complexity*, the analysis of an apparently simple process may reveal interesting and unexpected intrinsic dynamics. The logistic map is an example of a seemingly simple process with amazing emergent structural complexity, with ties to not only population dynamics but also to chaos theory and fractal dynamics.

Nonlinearity Paradigm

The next logical comment might be, “But the logistic map is largely nonlinear, whereas a Shewhart-stable process is most likely a linear phenomenon since the variation remains within such tight limits for so long”. However, as supported below, it turns out that underlying nonlinear dynamics are actually overwhelming prevalent, even for stable systems. For a process to be approximately linear implies that the sum of all of the inputs produces a consistent and proportional response in the output. This is usually a rare and passing phenomenon in our dynamic world. According to De Canete et al. (2011):

> Nearly all systems are inherently nonlinear in nature, since most of the relationships in physics are nonlinear. Moreover, most of the linear systems are a special case of nonlinear systems in limited ranges of operation. Besides this, the nonlinear nature of the physical elements constituting the system may be an essential feature of the system, causing the overall system behavior to be nonlinear… (p. 46).

It also turns out that even distinguishing a linear phenomenon is problematic. According to Bickel & Bühlmann (1996), although linear processes can be approximated, “Given any infinitely long data sequence, it is impossible (with any test statistic) to distinguish perfectly between linear and nonlinear processes (including slightly noisy chaotic processes)” (p.12128).

Another pertinent perspective regarding nonlinear chaotic processes is provided by Rosso et al. (2012):

> The concept of low-dimensional deterministic chaos, derived from the modern theory of nonlinear dynamical systems, has changed our way of understanding and analyzing observational data *S(t)* (time series), leading to a paradigm-change from linear to nonlinear approaches. Linear methods interpret observational signals from an underlying dynamical system that is regarded as being governed by a linear regime under which small perturbations lead to small effects. Consequently, all irregular behavior must be attributed to random external inputs [Kantz & Schreiber, 2002]. However, chaos theory has shown that random inputs are not the only possible source of irregularities in a system's outputs. (p.42)

Dooley et al. (1995) provided additional insights into chaotic nonlinearity for quality control, focusing on the potential impacts to statistical hypothesis testing:

> Statistical tests of hypotheses look at signal to noise ratios; they question if the magnitude of an observed signal is significantly larger than the "background noise", or experimental error. If it is not, then the effect is not statistically significant, because it may have arisen from other random and unpredictable causes. If, however, the system being measured is actually deterministically chaotic (or at least a majority of the observed variance is from deterministic chaos), then the statistical test is not comparing the effect (usually linear) to background noise, but rather to other nonlinear effects. Thus the presence of statistical significance *could* in some situations only mean that the observed linear effect is bigger

than the unpredictable nonlinear effect. If this indeed were the case, it may change the way decisions are made on the basis of statistical hypothesis testing. (p.20)

Simple-appearing processes, including stable processes based on machinery or chemical reactions can be extremely information-rich. Also, signals with the same degree of statistical variability can possess very different complexity properties (Goldberger et al., 2012). Some simple processes have also been studied that generate complex dynamics in particular parameter regimes. Amaral et al. (2004) found that, "under general conditions, complex dynamics can be generated by [simple] systems fulfilling the following two requirements, (i) a ''small-world'' topology and (ii) the presence of noise" (p.15551). Given these perspectives about the varying nature of simple processes, methods that provide insight into nonlinear process dynamics can still be useful to direct the improvement of stable processes. Such methods can assist with the recognition of changing patterns to estimate future process performance, even when reductionist statistical methods like control charts suggest predictable performance. The dilemma is which emergence-based method to choose for this insight. To assist with selection, it is useful to first review more detailed characterizations of the two broad categories: Reductionism vs. Emergentism.

Reductionism vs. Emergentism

Reductionist methods assume a process can be comprised of no more than the sum of its parts. Therefore, the underlying dynamics can be better understood by reducing a process into its constituent factors. The better the factors are understood, the better the process is understood. For a nonlinear process example, a complex waveform can be decomposed into individual signals through Fourier analysis.

Emergence-based methods assume a process is comprised of complex interactions, where the resulting process behavior is greater than the sum of its constituent parts. The factors are not assumed to necessarily possess the properties of the process, but instead create those properties through interaction. Therefore, underlying dynamics are understood from a higher-level perspective where information storage can be reviewed to determine structural and organizational characteristics of processes. The better the overarching structure is understood, the better the process behavior is understood. For a nonlinear process example, by expanding the 2-D Mandelbrot set in one more dimension, the logistic map comes into view from the side.

More relevant to this research, assume that the goal is to iteratively assess process predictability, which will change naturally and as process improvements are made. A reductionist might seek to "provide a solution" through data manipulations that separate active factors (signals) from inert factors (noise). An emergentist might seek to "generalize from the solution space" in terms of how the dynamic structure of the process changes coincident with the changing uncertainty. Generalizing from the solution space is not necessarily better or worse than providing a solution, with such optimization decisions being context-dependent. However, this research has endeavored to show that both perspectives should be considered complimentary.

Various emergence-based quantifiers provide insights into the structural dynamics of processes. In this research, Jensen-Shannon complexity (C_{JS}) has successfully revealed emergent organizational content that can be exploited for process awareness and process improvement. C_{JS} was chosen because of specific strengths, including a natural correspondence with permutation entropy and the Complexity-Entropy Causality Plane. These correspondences will be elucidated momentarily. However, other quantifiers similar to C_{JS} have been discussed in the literature. Quantifiers from computational mechanics (Crutchfield, 2017; Crutchfield & Young, 1989) and other structural complexity research could have undoubtedly provided useful and relevant insights.

Methods associated with the emergence approach often begin the analysis by quantifying flows of information entropy (H_S) via the changing probability distributions for applicable "states". In this way, H_s measures the randomness of the process. Unfortunately, many researchers equate measures of varying entropy with the "complexity" of the process. This confuses things because structural complexity transcends entropy conceptually. This dissertation does not uses the phrase "complexity" in the first sense, and the expression *structural complexity* is applied frequently to help alleviate this confusion.

Moreover, the research herein applied a specific complexity calculation method called the *MPR-method* to measure C_{JS}. The MPR-method is distinctive in that it provides important additional insights about the probability distribution not already detected by the entropy. This method was defined by Martin, Plastino, and Rosso (2006), in a paper entitled, *Generalized Statistical Complexity Measures: Geometrical and Analytical Properties*. The authors used the expression statistical complexity in the same sense that structural complexity is used in this research. This foundational paper provides detailed computational derivations surpassing the summary provided in this Methodology section.

Jensen Disequilibrium

To understand Jensen-Shannon complexity (C_{JS}) first requires an understanding of the concept of Jensen disequilibrium (Q_J). A uniform probability distribution represents a theoretical system with no disequilibrium (Q_{Jmin}). All distinct factors that comprise the system are equally likely and represent perfectly randomness. The Shannon entropy is maximized but the disequilibrium is minimized. A theoretical physical analogy is an ideal gas comprised of a finite number of point particles moving randomly via perfectly elastic collisions.

The opposite theoretical extreme is maximum disequilibrium (Q_{Jmax}), which is represented by a probability distribution with one factor maximized and all of the other factors at zero. Because only one distinct factor is likely, there is no randomness. The Shannon entropy in this situation would be zero. A theoretical physical analogy is the position of a particle within an ideal crystal. Its position is perfectly fixed with zero randomness. There is no chance that the particle is anywhere else in the crystal other than its known position.

This application of "dis"-equilibrium may seem odd instead of just measuring "equilibrium". This is so because all processes have some level of disequilibrium, but no processes are in true equilibrium. Equilibrium (minimum disequilibrium) is a theoretically abstraction and reality is therefore represented by the deviation from this abstraction. Given this logic, Q_{Jmin} serves as the baseline reference from which to measure disequilibrium.

To measure the Jensen disequilibrium (Q_J) associated with each process, it is necessary to compute the distance between the reference uniform distribution at Q_{Jmin} and the current probability distribution. Quality professionals are familiar with statistical variance, especially as applied to process control. Statistical variance is a true metric distance measure, satisfying the triangle inequality. However, many emergence-based methods instead use Kullback- Leibler (K-L) divergence to calculate the distance between probability distributions, which is asymmetric and does not satisfy the triangle inequality. Yet, K-L divergence remains popular because it satisfies some properties that are canonical extensions to characterizations of Shannon information such as redundancy, additivity, maximality, and continuity (Hobson, 1971).

Jensen disequilibrium uses neither statistical variance nor K-L divergence, but rather Jensen divergence, the square root of which is a *true* metric distance measure. Jensen divergence can be applied to different forms of entropy calculation, including Shannon, Renyi, and Tsallis. For this reason, the selected form of entropy measure is often applied after the word Jensen, such as Jensen-Shannon divergence. Only Shannon entropy was used as the reference entropy in this research.

Computation of Q_J is a two part process. First, the Jensen-Shannon divergence $J[P, P_e]$ is calculated between the probability distribution P and the reference uniform distribution P_e:

$$J[P, P_e] = H\left[\frac{P+P_e}{2}\right] - \frac{H[P]}{2} - \frac{H[P_e]}{2} \qquad (20)$$

P_e can represent an infinite number of uniform distributions, depending upon the number of distinct states (K) that comprise the probability distribution. The form for P_e is:

$$P_e = \frac{1}{K} \; from\ the\ uniform\ distribution \left\{\frac{1}{K}, \ldots \frac{1}{K}\right\} \quad , \tag{21}$$

where K = # of distinct states.

Next, Q_{Jmin} is calculated, which is inverse of the maximum possible value of $J[P, P_e]$. This value is the normalization constant, obtained when one of the states of P is equal to one and the remaining states are equal to zero.

$$Q_{Jmin} = -2 \; \left\{ \left(\frac{K+1}{K}\right) \log(K+1) - 2\log(2K) + logK \right\}^{-1} \quad , \tag{22}$$

where K = # of distinct states.

Finally, $\tilde{Q}_J$ is calculated by multiplying the normalizing constant by the Jensen-Shannon divergence, as follows:

$$\tilde{Q}_J[P, P_e] = Q_{Jmin} \cdot J[P, P_e] \tag{23}$$

Jensen-Shannon Complexity

So far, $\tilde{H}_S$ and $\tilde{Q}_J$ have been defined in different parts of this Methodology section. The next step is to develop an intuitive understanding of how these variables relate to C_{JS}. In 1995, Lopez-Ruiz, Mancini, and Calbet published a seminal paper entitled, *A Statistical Measure of Complexity*, in which they related the three concepts of entropy, disequilibrium, and complexity.

The authors began by presenting these three concepts in physical terms. They state, “The notion of ‘complexity’ in physics [Anderson, 1991; Parisi, 1993] starts by considering the perfect crystal and the isolated ideal gas as examples of simple models and therefore as systems with zero ‘complexity’” (p.321). They then make the case that complexity cannot be defined just in terms of order and information. A crystal represents maximum order, which can be perfectly described by minimum information. An ideal gas, on the other hand, represents maximum disorder but, “The system can be found in any of its accessible states with the same probability. All of them contribute in equal measure to the ‘information’ stored in the ideal gas. It has therefore a maximum ‘information’” (p.321). Zero complexity would therefore be defined by extrema, as either maximum or minimum information and as either minimum or maximum disorder, which they found unsatisfactory.

The authors then discussed the idea of defining complexity in direct correspondence with disequilibrium. They established that, “Disequilibrium would be different from zero if there are privileged, or more probable, states among those accessible” (p.321). But they said a direct correspondence would not work because an ideal gas is at minimum disequilibrium and a crystal is concurrently at maximum disequilibrium, yet both represent zero complexity. Having thus eliminated this option, they then propose that complexity should be defined as the product of disequilibrium and information. They provided a sketch, reproduced in Figure 45, to represent an intuitive notion of the relative magnitudes of these three quantities. Lopez-Ruiz et al. (1995) then developed equations for their intuitive arguments and provided analytic support for this definition of complexity by plotting two complex processes out of equilibrium- the Lorenz map and the logistic map.

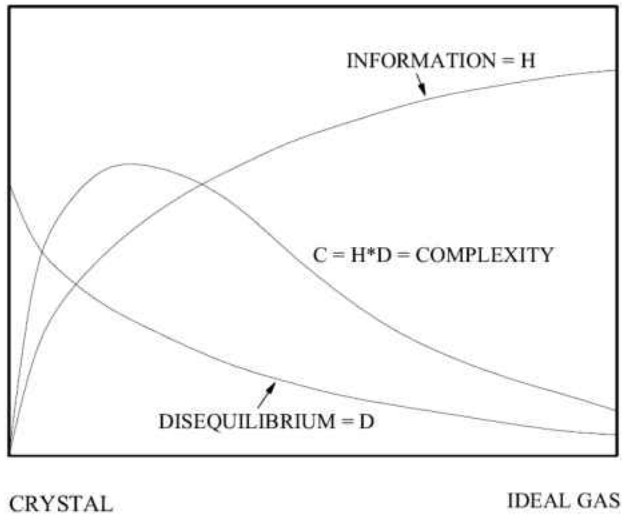


Figure 45. Information disequilibrium complexity diagram. From *Statistical Complexity and Fisher-Shannon Information. Applications* by Lopez-Ruiz et al. Copyright © 2011 by Springer Nature. Reprinted with the permission of Springer Nature. All rights reserved.

Based on the seminal insights provided by Lopez-Ruiz et al. (1995), Martin, Plastino, and Rosso (2006) developed the MPR-method to compute Jensen-Shannon complexity C_{JS}, as follows:

$$C_{JS}\,[P] = \tilde{Q}_J\,[P, P_e]\,\cdot\,\tilde{H}_S\,[P] \tag{24}$$

As an information-theoretic complexity quantifier, C_{JS} reveals the hidden structural dynamics of processes via stored information. These are insights that would otherwise be lost if only viewing Q_J and H_s alone. This equation also reveals why C_{JS} is not a trivial function of the entropy. C_{JS} is clearly based concurrently upon normalized entropy and normalized disequilibrium, and the interplay of these interactions results in a higher order of emergence.

C_{JS} quantifies emerging dynamics between a range of possible complexity values. This range of complexity is dependent upon the number of accessible states K that make up the probability distribution. In Figure 46, a complexity diagram depicts the minimum complexity (C_{min}) and maximum complexity (C_{max}) in terms of normalized H_s and the number of accessible states (using N in place of K).

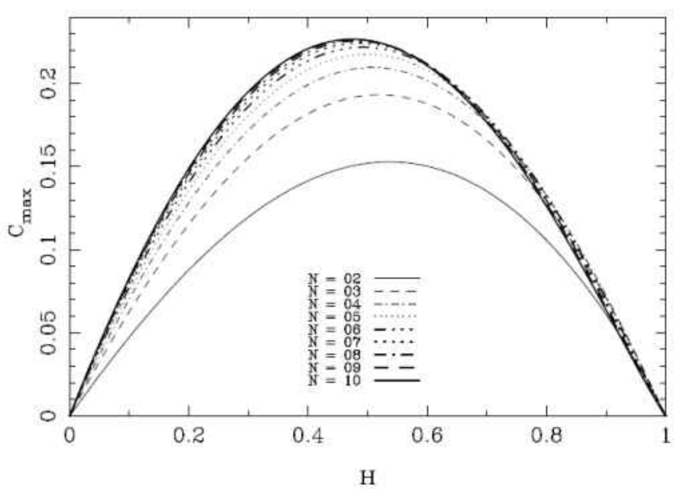


Figure 46. Complexity-entropy diagram. C_{min} for N=2, C_{max} for N= (3,…,10), where N is the number of accessible states. From *Statistical Complexity and Fisher-Shannon Information. Applications* by Lopez-Ruiz et al. Copyright © 2011 by Springer Nature. Reprinted with the permission of Springer Nature. All rights reserved.

The methodology for calculating C_{min} and C_{max} will be summarized succinctly here. More detailed mathematical foundations can be found in Martin et al. (2006). C_{min} is obtained when one state has a probability P_a and all other states have equal probabilities with P_b = (1 -

P_a)(N - 1), where N is the number of tuples. C_{max} is calculated from a set of K - 1 probability distributions. In this probability set, one state has a probability P_a, where $0 \leq P_a \leq 1 / (K - l + 1)$; l takes values $l = 1, \ldots, K - 1$. Other $K - l$ states have equal probabilities $P_b = (1 - P_a)(K - l)$.

Given the number of states, an entropy-complexity diagram can be drawn with C_{min} and C_{max} appropriately plotted. Then, the iteratively calculated ($\widetilde{H}_S$, C_{JS}) points may be plotted within the complexity bounds. As the plot is developed, a pattern will emerge with temporal dependence. This is because the 2nd Law of Thermodynamics specifies that entropy grows monotonically. As such, the progression of the iteratively calculated normalized Shannon entropy can be regarded as an arrow of time. As a process evolves in time toward equilibrium (as $\widetilde{H}_S$ *tends* to stabilize), the system *tends* to approach its maximum complexity path. Figure 47 depicts this evolutionary progression of complexity.

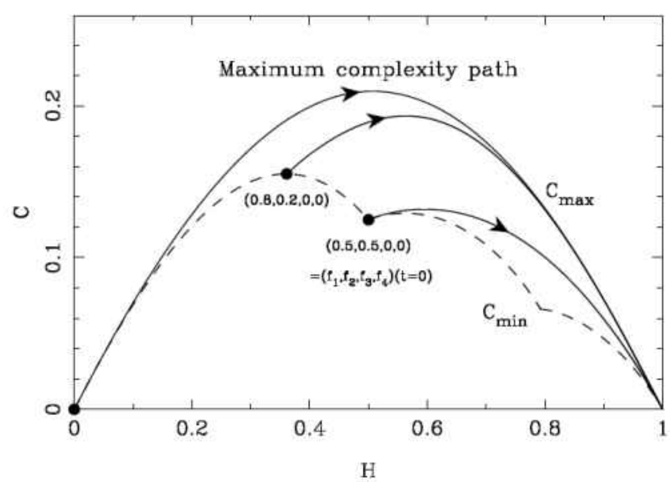


Figure 47. Complexity-entropy diagram. Time evolution of the system for three different initial conditions starting at t = 0. The system tends to approach the maximum complexity path as it evolves in time toward equilibrium. C_{min} for N=2, C_{max} for N= (3,…,10). From *Statistical Complexity and Fisher-Shannon Information. Applications* by Lopez-Ruiz et al. Copyright © 2011 by Springer Nature. Reprinted with the permission of Springer Nature. All rights reserved.

For the MPR-method, the probability distributions [P] and [P_e] are calculated in terms of permutation entropy. This is the chosen technique because PE has the advantage of being perhaps the only entropy measure that incorporates the temporal structure of the underlying physical process. The probability distributions are constructed based upon the number of distinct states. For traditional PE, this is the number of accessible tuple types defined by the factorial of the embedding dimension (D!).

<u>Complexity Entropy Causality Plane</u>

The same three authors that established the MPR-method in 2006 were also contributing authors for another foundational paper a year later. In 2007, Rosso, Larrondo, Martin, Plastino, and Fuentes published *Distinguishing Noise from Chaos*. Their motivation for this paper was to develop the graphic representational space, since called the Complexity-Entropy Causality Plane (CECP), to clearly distinguish between chaotic and stochastic processes. They perceived that chaotic and stochastic processes share several properties that make them almost indistinguishable, and that most of the existing methods provided little relief.

More specifically, they referred to Wold (1938), who proved that any (stationary) time series can be decomposed into two different parts: The first (deterministic) part can be exactly described by a linear combination of its own past; the second part is a moving average component of a finite order. The authors stated that real data, "always possess a stochastic component due to omnipresent dynamical noise" and that, "it makes sense to ask, with respect to the deterministic part (predictable from the past), whether (i) it is dominant vis-a-vis the unpredictable stochastic part or (ii) it is of a regular or chaotic nature" (p.154102-1).

One of the distinct advantages of the MPR-method, and the associated application of permutation entropy, involves chaotic processes. PE yields a positive Kolmogorov-Sinai (KS) entropy value for chaotic processes, but zero for regular dynamics, and infinity for stochastic processes. KS entropy reveals how the information in a process evolves in time. In consequence, dynamical processes can be reliably categorized as deterministic-chaotic if they exhibit at least one positive Lyapunov exponent and a finite positive KS entropy (Rosso et al, 2007).

To further develop their CECP plots, the authors analyzed five kinds of chaotic maps, and two kinds of stochastic processes. They noted that the CECP accommodates noise and chaos at different planar locations. Stochastic processes tended toward C_{min} and chaotic processes toward C_{max}. Also, because real data always contain a stochastic component due to omnipresent dynamical noise, the CECP helped classify different degrees of what they called "stochasticness". They also noted that the CECP distinguishes different degrees of correlations (colored noise) and distinguishes Gaussian from non-Gaussian processes. Consequently, "this representation plane is an effective tool for revealing the sometimes subtle difference between noise and chaos" (p.154102-4). The results of those plots are displayed in Figure 48 and Figure 49.

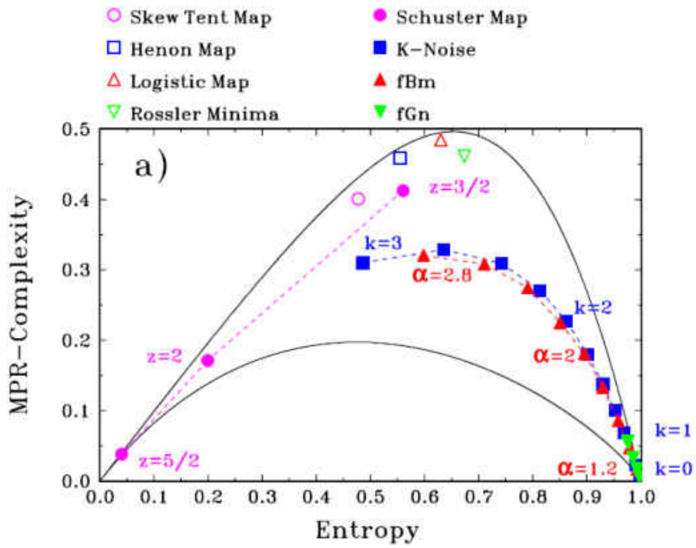


Figure 48. Complexity-Entropy Causality Plane. From *Distinguishing Noise from Chaos* by Rosso et al.

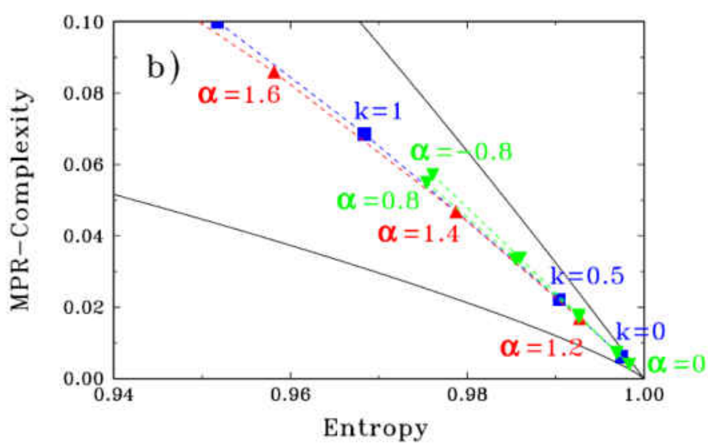


Figure 49. CECP close up. From *Distinguishing Noise from Chaos* by Rosso et al.

Since then, permutation entropy and the MPR-method have been applied to a wide variety of process types with results plotted on CECP diagrams. Definite trends have emerged, as certain process types tend to plot in certain regions of the CECP diagram. Generally, C_{JS} and $\widetilde{H}_S$ often stabilize approximately over time, helping to define process behaviors and characteristics.

The capability of the CECP to distinguish stochastic from chaotic processes can be beneficial to process control applications. It is possible for processes to be Shewhart-stable, yet associated with systems that have been known to reveal chaotic behaviors, such as machinery (Litak et al., 2009; Priesmeyer (1992); Redelico et al., 2017), production systems (Deshmukh, 2003; Sajid et al., 2015), and chemical reactions (Elnashaie, 2006; Eiswirth, 1993). In these instances, the CECP could be applied to provide insight into the emerging nature of process behaviors. Over time, the CECP regions associated with stochastic and chaotic processes have been refined, as Figure 50 reveals.

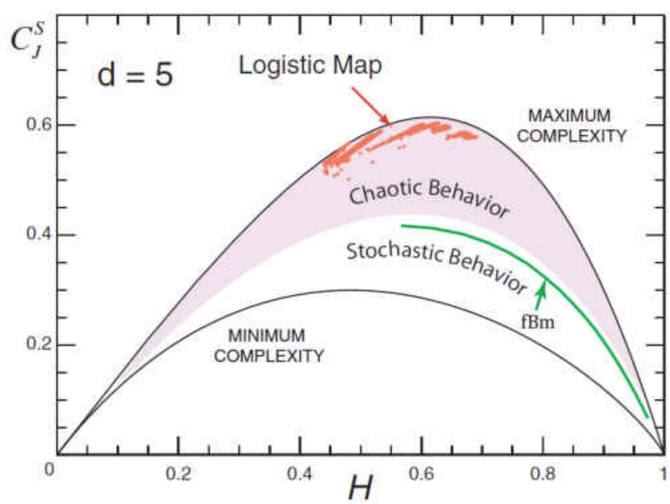


Figure 50. Complexity-entropy diagram. Stochastic and chaotic regions. fBm is fractional Brownian motion. The permutation entropy embedding dimension used to plot the logistic map is D=5. From *Permutation Entropy Analysis of Temperature Fluctuations from a Basic Electron Heat Transport Experiment* by Maggs & Morales. Copyright © 2013 by IOP Publishing Ltd. Reprinted with permission of IOP Publishing Ltd. All rights reserved.

The MPR-method and the associated CECP provide illuminating projections of the emerging dynamics associated with different types of time series processes. Because the time-evolution of processes is depicted, the CECP can be harnessed to track changing Jensen-Shannon complexity in near real-time. As processes are improved or changed, the resulting effects can be monitored with the CECP. Figure 51 reveals the time evolution of a van der Pol's oscillator at four different levels of noise.

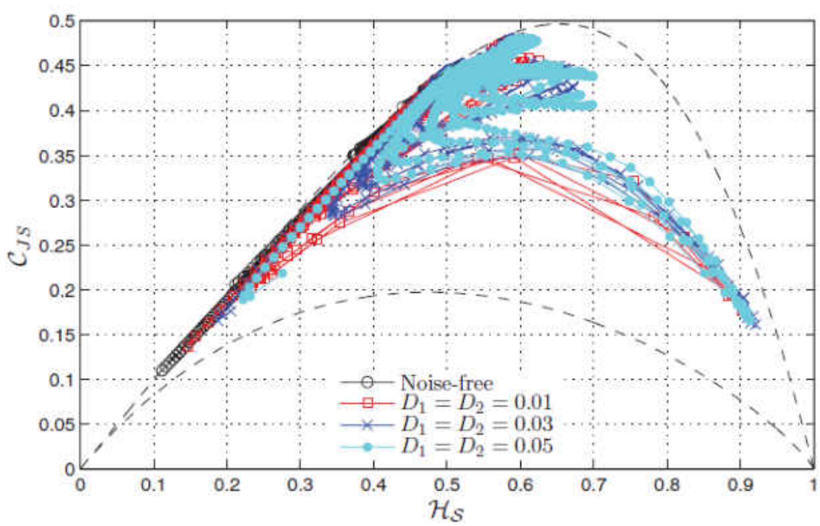


Figure 51. CECP. Time evolution of van der Pol's Oscillator at different noise levels. Embedding dimension *D*=6. From *Distinguishing Chaotic and Stochastic Dynamics from Time Series by Using a Multiscale Symbolic Approach* by Zunino et al. Copyright © 2012 by American Physical Society. Reprinted with permission of American Physical Society. All rights reserved.

The MPR-method can also be applied to understand the complexity dynamics associated with different varieties of the same process. In this research, this idea was applied to pseudorandom number-generated processes at different measurement resolutions, to better characterize the measurement resolution effect. For SPC applications, processes could be evaluated in terms of different materials, or methods, personnel, machines, measurements, environmental conditions, etc. An illuminating example of an analysis comparing process complexity characteristics is provided in the next two figures, as different varieties of music were evaluated in terms of amplitude and intensity. The first figure, Figure 52, reveals probability density functions and the second, Figure 53, provides CECP results.

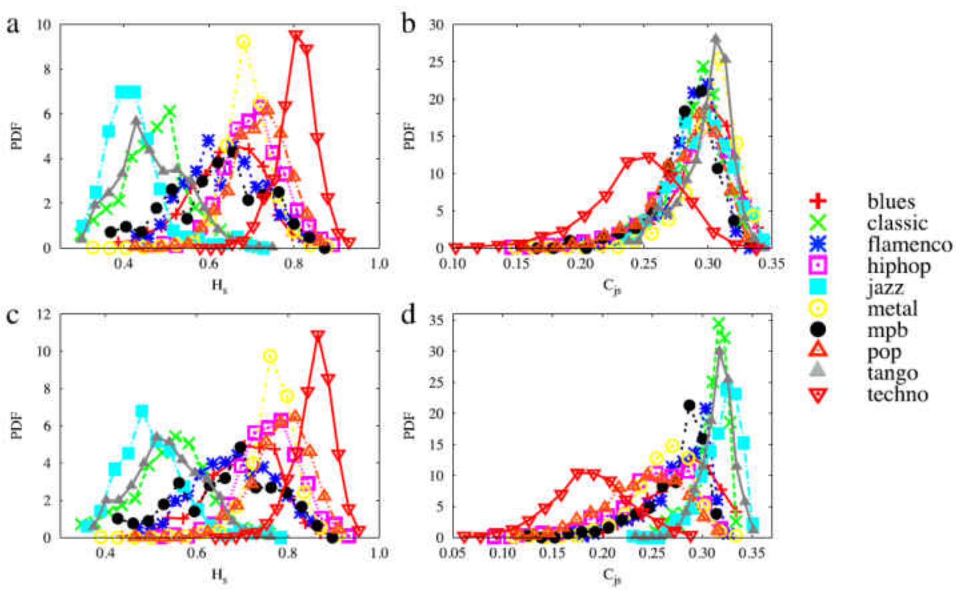


Figure 52. Permutation entropy probability density functions for a variety of music types. Top- amplitude series. Bottom- intensity series. *D*=5. From *Complexity-Entropy Causality Plane: A Useful Approach for Distinguishing Songs* by Ribeiro et al. Copyright © 2011 by Elsevier B.V. Reprinted with the permission of Elsevier. All rights reserved.

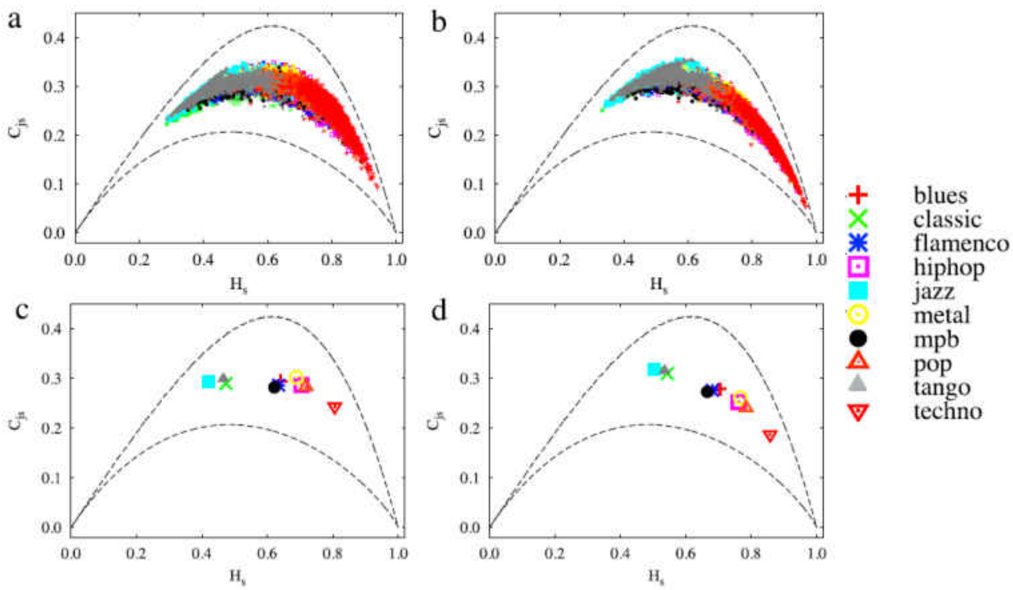


Figure 53. CECPs for music. (a) amplitude series (b) intensity series (c) mean values amplitude (d) mean values intensity. *D*=5. From *Complexity-Entropy Causality Plane: A Useful Approach for Distinguishing Songs* by Ribeiro et al.

Many other studies based on Jensen-Shannon complexity and the MPR-method were discovered that similarly *characterized* processes, some of which could even be considered Shewhart-stable, as discussed in the Literature Review section. Of the process characterizations reviewed, none were discovered that applied the MPR-method for the purpose of continuously improving an industrial engineering type of process. Various information-theoretic methods are presented in Appendix E, which could be applied similarly to extend structural complexity research.

For readers desiring additional insight, some extensions to the MPR-method are described in the Future Research part of the Conclusion section. Additionally, practical application summaries can be found in Riedl et al. (2013). Other studies of potential interest to industrial engineering applications include research correlating noise interference to different embedding dimensions *D*, in Borges, et al. (2019). Also, studies applying the Hurst exponent *H* to measure the long term memory of a time series are found in Bariviera et al. (2019), and studies of subtle measurement shifts at high sampling rates in Zunino et al. (2012).

Log Change Evaluations

During research, the various process types were evaluated extensively using Jensen-Shannon complexity measures. In particular, the results for the improved processes (vertical funnel experiments) were compared with unimproved processes to seek exploitable pattern differences. However, as will be seen in the Findings section, measurement resolution definitely affected results associated with the PE-LE method. To explore compensation for some of these effects, C_{JS} and $\widetilde{H}_S$ for each measured process were compared to the C_{JS} and $\widetilde{H}_S$ results for a pseudorandom series generated with the same number of data and at the same measurement resolution.

Various methods were initially evaluated to quantify the relative change, but the log change method provided three distinct advantages. First, the results for a series of changes can be summed to yield a precise (not approximate like percentages) total change. Second, normalization with the factor 100 aligns with the definition for percentage change for very small changes, which is often the situation for Shewhart-stable series compared with random series. Third, the *magnitude* of change is consistent regardless of which comparator is chosen as the reference. The equations used for this research were:

$$\tilde{C}_{JS} = -100 * ln \frac{C_{JS\ for\ process\ series}}{C_{JS\ for\ equivalent\ random\ series}} \tag{25}$$

$$\tilde{\tilde{H}}_S = 100 * ln \frac{\tilde{H}_{S\ for\ process\ series}}{\tilde{H}_{S\ for\ equivalent\ random\ series}} \tag{26}$$

In both of these equations, the word "equivalent" specifically means for the same number of data and at the same measurement resolution.

$\tilde{\tilde{H}}_S$ Change Chart

This chart was not used directly to answer the research question. However, $\tilde{\tilde{H}}_S$ is used for one axis of the Entropy-Complexity Change Diagram (ECCD) so this chart reveals how this variable is operating independently. In effect, $\tilde{\tilde{H}}_S$ has been normalized in two independent and complementary ways. The first normalization method was associated with permutation entropy (PE-LE) as the Shannon entropies were divided by H_{max} = log K, where K is the available number of distinct tuple types. This was considered a "static" normalization in that the normalizing factor does not change throughout the entire analysis. Specifically, for all of this research, this normalization factor was log(42) = 5.3923 bits. The second normalization method made a relative comparison of these $\tilde{H}_S$ values in time series to the equivalent randomly generated time series by applying the log change calculation. This can be considered a "dynamic" normalization in that the normalizing factor continues to change equivalently throughout the entire time series, based on how many measurements have accumulated thus far in the series.

$\tilde{C}_{JS}$ Change Chart

Jensen-Shannon complexity leverages the interplay between normalized Jensen disequilibrium and normalized Shannon entropy to reveal emerging structural dynamics for processes. For the MPR-method, which is based on permutation entropy, the Shannon information was normalized based on the maximum number of distinct tuples comprising the process. This was one component of the C_{JS} calculation, in tandem with the normalized Jensen disequilibrium. However, the overall result for C_{JS} can itself be "dynamically" normalized by quantifying the relative change between process results and another equivalent series representing maximum randomness. This is accomplished with the log change equation. This effect allows processes to be compared equivalently, although the maximum randomness possible for each process differs due to unique characteristics such as time series run lengths. In the case of the PE-LE method, this normalization procedure also allowed equitable comparisons among processes with differing measurement resolutions.

A process plotted on a $\tilde{C}_{JS}$ run chart reveals how structural complexity changes in time relative to the maximum randomness condition for an equivalent time series. Numerous processes can also be plotted on the same chart to allow equitable comparisons of changing normalized structural complexity. A sample of this chart is provided in Figure 32.

Entropy-Complexity Change Diagram

Since the calculations of both $\tilde{\tilde{H}}_S$ and $\tilde{C}_{JS}$ just described were linked to temporal causal dynamics, different processes could be compared in terms of simultaneous changes in both randomness and structural complexity on a Entropy-Complexity Change Diagram. A sample of this diagram is provided in Figure 33.

CHAPTER FOUR: FINDINGS

Six interrelated sections of results are presented. First, the four vertical translation funnel experiments are characterized, and their intended applicability to this research was confirmed. Specifically, the first three experiments were identified as Shewhart-stable, improved and the last as Shewhart-unstable, improved. Next, a brief process randomness study provided evidence for differing levels of randomness associated with different Shewhart-stable processes. Results for the first 10,000 digits of Pi and 10,000 pseudorandomly generated digits also supported the position that absolute randomness is an idealized impossibility.

The third section focused on validation of the PE-LE method. Three axioms for permutation entropy methods were developed and tested using different process types. Two specific comparisons were made with a study that evaluated a commonly-applied method called time-ordered imputation, revealing advantages in this specific case. The measurement resolution effect was then characterized for the PE-LE method, with results supporting the assumptions that measurement resolution could be misused, but also applied to tailor and fine-tune analyses based on research goals. Finally, numerous shortcomings of the PE-LE method were elucidated.

The next section provided PE-LE results for all of the 22 processes evaluated in this research. These results consisted of pdfs for the PE-LE tuples at D=4 and counts/ densities based on local effect. This section was intended as a reference for later sections that made comparisons among these processes. The fifth section focused on answering the research question by evaluating Jensen-Shannon complexity in various ways. Fundamental conclusions were supported by applying the Martin-Plastino-Rosso (MPR)-method with PE-LE results, to display processes on the Complexity-Entropy Causality Plane (CECP). Decreased structural complexity and increased randomness corresponded with the gradual improvement of Shewhart-stable processes, answering the research question. A number of other process types were also displayed simultaneously on the CECP to provide more insight into the utility of this technique.

Time series results were then compared to maximally random equivalent time series to explore relative change dynamics for entropy and structural complexity. This normalization procedure provided different insights into how levels of randomness and complexity change over time and facilitated comparisons between different processes. Results emphasized the importance of determining the appropriate measurement resolution based on research goals when PE-LE is the underlying mechanism. Both change charts were then combined to evaluate the simultaneous display of changing results in two dimensions on the Entropy-Complexity Change Diagram (ECCD). After plotting numerous process types, results suggested that improving processes migrate toward the origin, which represents maximum theoretical randomness and increasingly random structural complexity.

Vertical Translation Funnel Experiments

The four vertical translation funnel experiments provided the primary data for this research because they were empirically-derived time series instead of idealized models, and because they represented Shewhart-stable processes that were progressively improved. Moreover, process improvement over the course of each experiment could be deemed reliable as the corresponding decreasing trend for location and variation was verified for each time series, as provided in Figure 54-Figure 59. Each funnel experiment is presented as a complete time series in Figure 54. Drop heights changed after every 50^{th} measurement. Measurements were the distance at which the marble came to rest from the center of the target in integer inches. All trendlines represent 2^{nd} order polynomial fits and all y-axes are represented at the same scale.

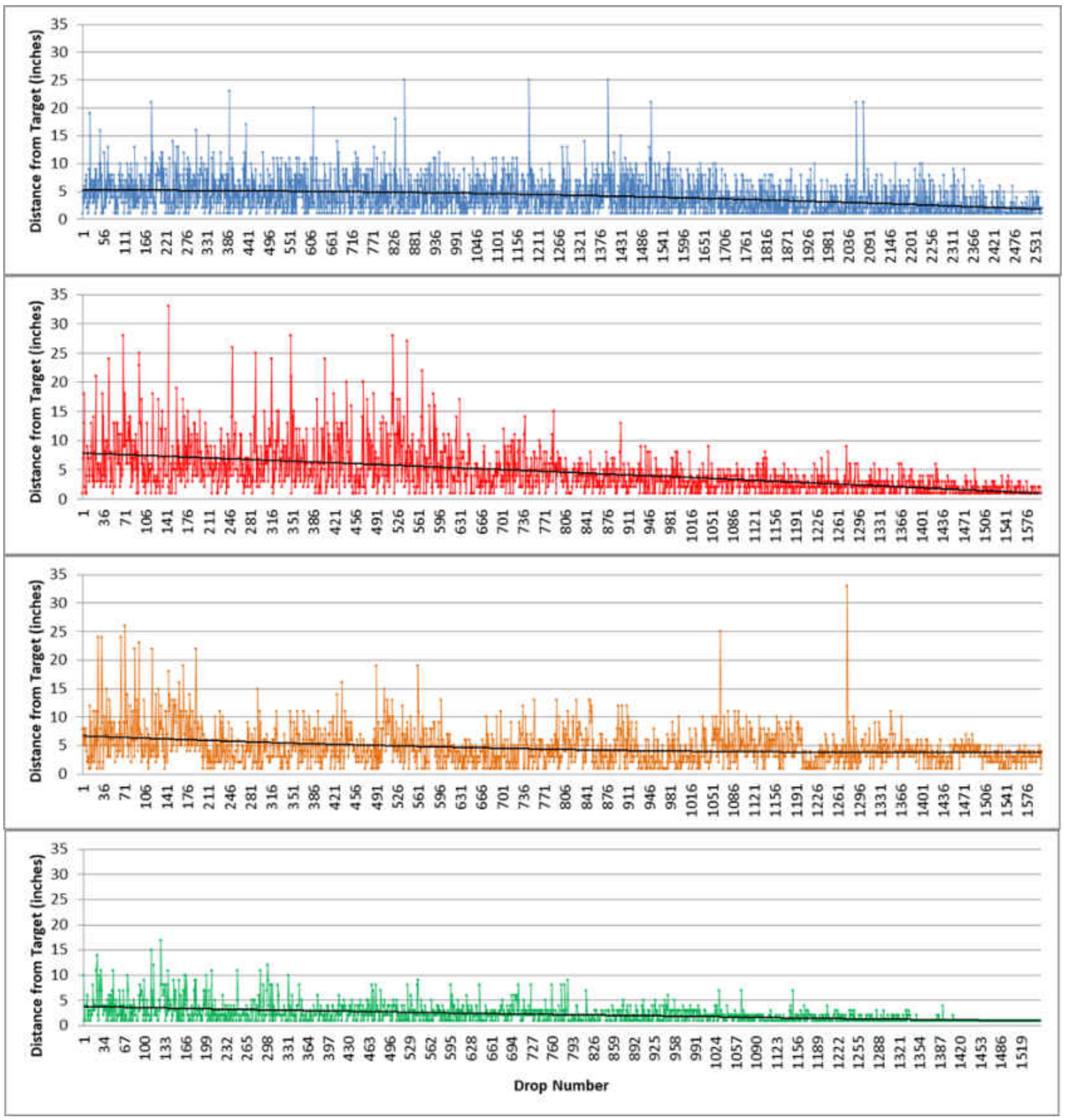


Figure 54. Complete time series for each of the four funnel experiments. Presented in order with V1 at the top and V4 at the bottom. Trendlines are 2^{nd} order polynomial fit.

In Figure 55, by removing the trendlines and the lines between data points, the discrete integer nature of the time series is made more visible.

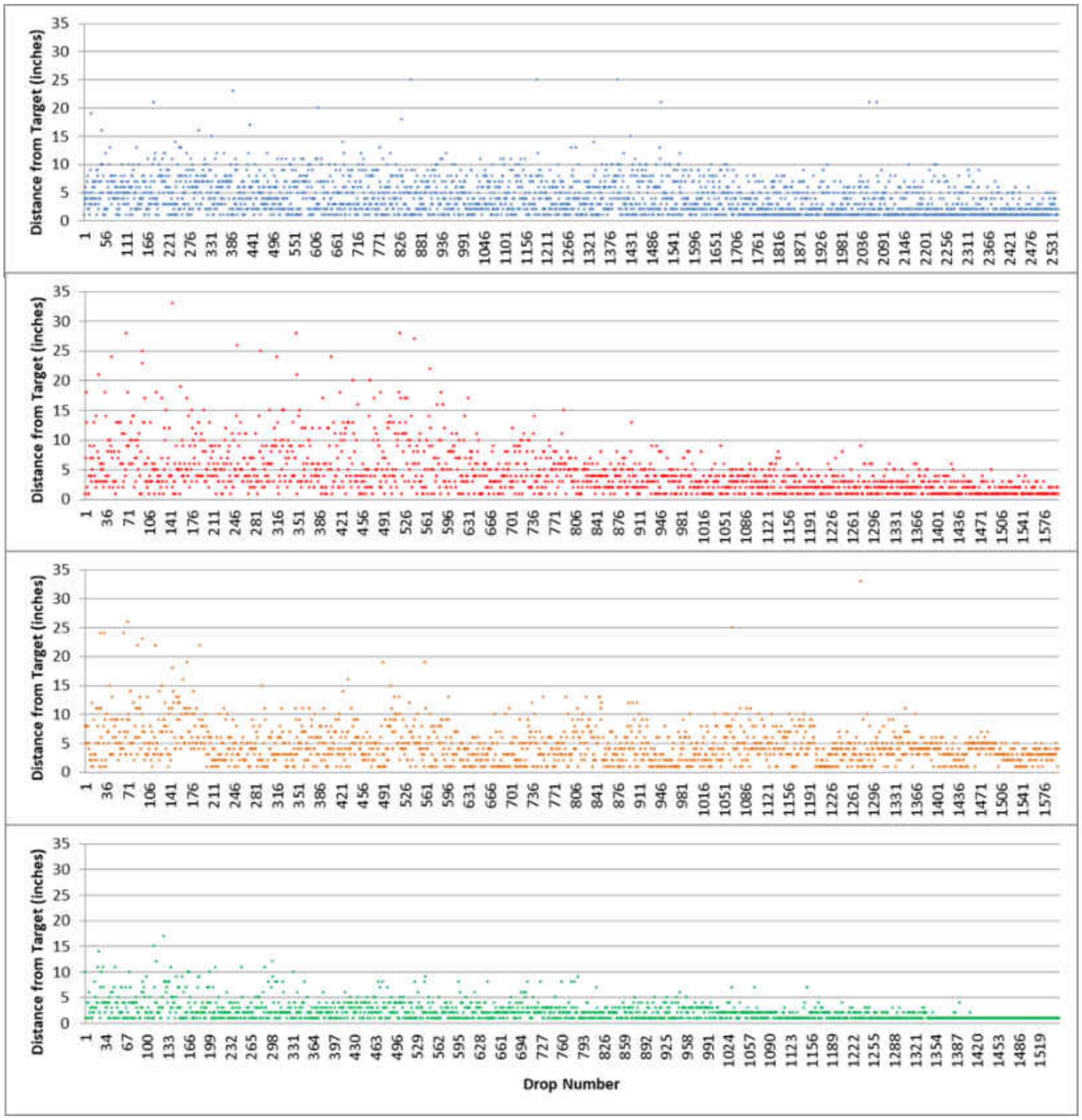


Figure 55. Complete time series for each of the four funnel experiments. Presented in order with V1 at the top and V4 at the bottom.

At the conclusion of each experiment, the data were analyzed using XmR charts to assess process variability. The mean (X-bar) was calculated for drops at each height, Figure 56, as was the mean moving range (mR-bar), Figure 57. Data values are presented in Appendix A in Table 34.

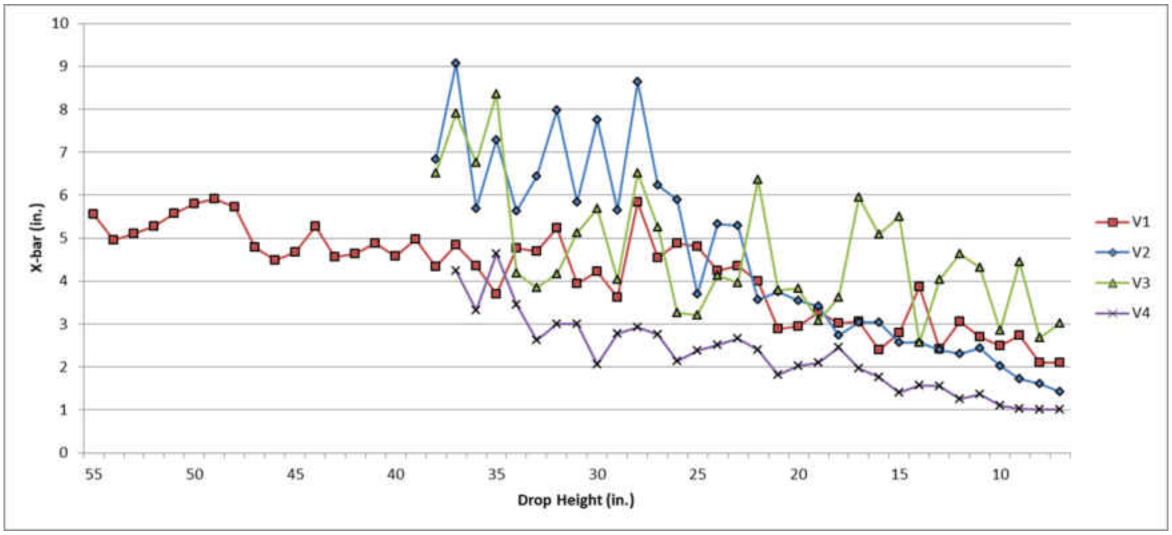


Figure 56. X-bar for each drop height for four vertical funnel experiments.

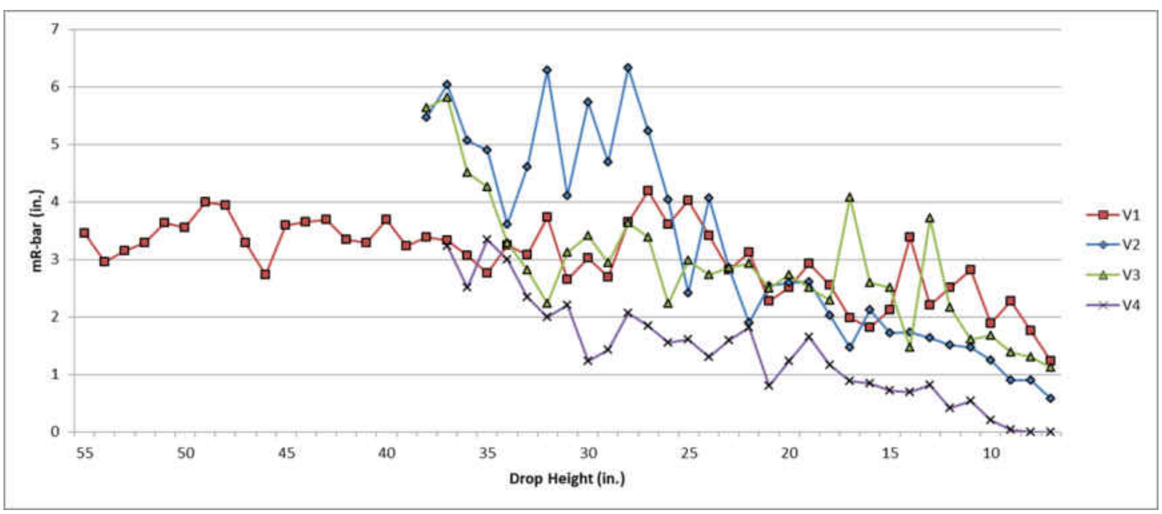


Figure 57. mR-bar for each drop height for four vertical funnel experiments.

Between each iteration of funnel experiments 1-4, changes to experimental apparatus were made that were deemed likely to reduce variability before the experiment was repeated. In an ideal situation, there would be no special causes, and the variation would get progressively smaller in magnitude each time the funnel was lowered. In the actual conduct of these experiments, there were outliers and the variation sometimes increased temporarily at a lower drop height. Table 21 summarizes the results of the XmR charts based on the number of special causes that were counted at each drop height for heights from 37 to 7 inches above the target. No additional control chart zone test rules were applied in the determination of special causes.

Table 21. XmR chart outlier counts for the four funnel experiments. Green highlighted cells represent drop heights in which no special causes were indicated for both the Individual values (X) and the moving Range (mR) chart.

	Experiment 1		Experiment 2		Experiment 3		Experiment 4	
Height (in)	# X Outliers	# mR Outliers	# X Outliers	# mR Outliers	# X Outliers	# mR Outliers	# X Outliers	# mR Outliers
37	0	0	1	2	2	0	1	1
36	0	0	1	2	1	2	0	0
35	0	0	0	0	1	1	2	3
34	0	0	0	0	0	0	0	0
33	0	0	2	3	1	3	2	3
32	1	2	1	3	2	1	3	3
31	0	1	1	1	0	0	1	1
30	2	2	1	0	1	0	1	1
29	1	1	1	3	1	2	0	0
28	1	2	2	1	0	0	0	0
27	0	0	1	0	1	1	3	3
26	0	0	1	1	0	0	1	2
25	1	1	0	0	0	2	1	2
24	0	0	0	0	1	0	0	0
23	0	0	1	1	2	2	2	2
22	0	0	0	0	0	1	4	4
21	0	0	1	1	2	0	1	2
20	0	0	0	0	1	1	0	0
19	0	0	0	0	1	0	0	0
18	0	0	1	3	1	0	0	0
17	2	2	0	0	1	2	1	2
16	0	0	0	0	0	0	1	2
15	0	0	0	0	0	0	2	4
14	2	4	1	2	1	1	1	2
13	0	0	1	2	1	2	1	1
12	1	2	0	0	1	0	2	4
11	0	0	0	0	1	2	2	1
10	2	2	1	0	0	0	3	6
9	2	1	1	2	0	0	1	2
8	1	1	0	1	0	0	0	0
7	1	2	4	5	0	0	0	0
# with no Outliers		18		12		10		10
Pass Rate		**58%**		**39%**		**32%**		**32%**

In terms of XmR chart performance, experiment 1 provided the best pass rate at 58% and the remaining three experiments had pass rates below 40%. Next, in Figure 58, the mean values for each drop height are presented.

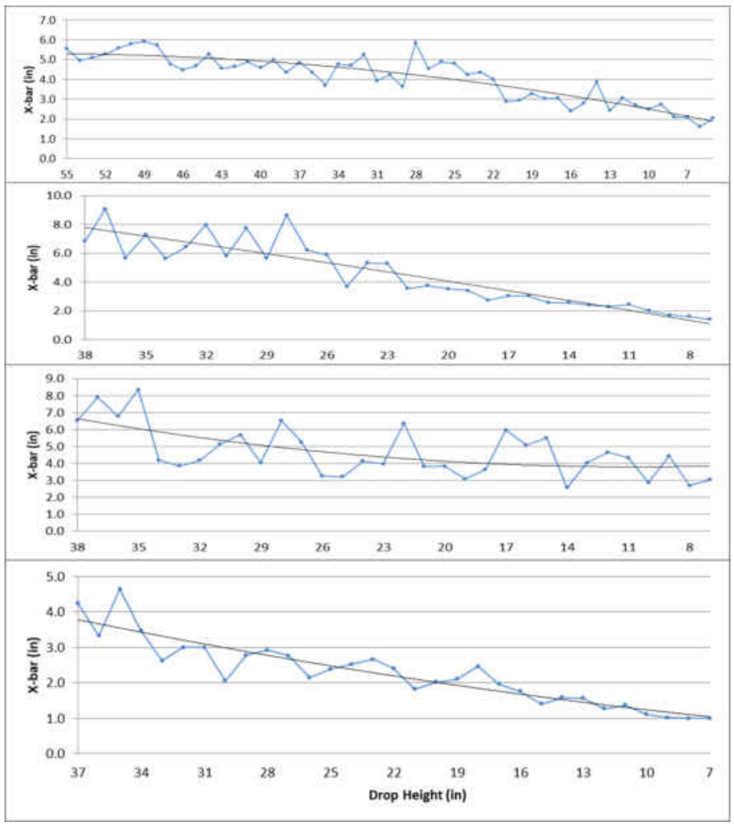


Figure 58. Mean value for each drop height. Statistic presented for funnel experiments 1 through 4 from top to bottom, respectively. Trendlines are 2nd order polynomial fit.

All four experiments revealed an overall decreasing trend for the mean distance from target as the drop heights decreased. The changing standard deviation was also tracked with $\hat{\sigma}$ values presented in Figure 59. The equation used for sigma-hat was:

$$\hat{\sigma} = \frac{\bar{R}}{d_2} \, , \qquad (27)$$

where $\bar{R}$ is the average range and d_2 is a bias correction factor for measures of dispersion.

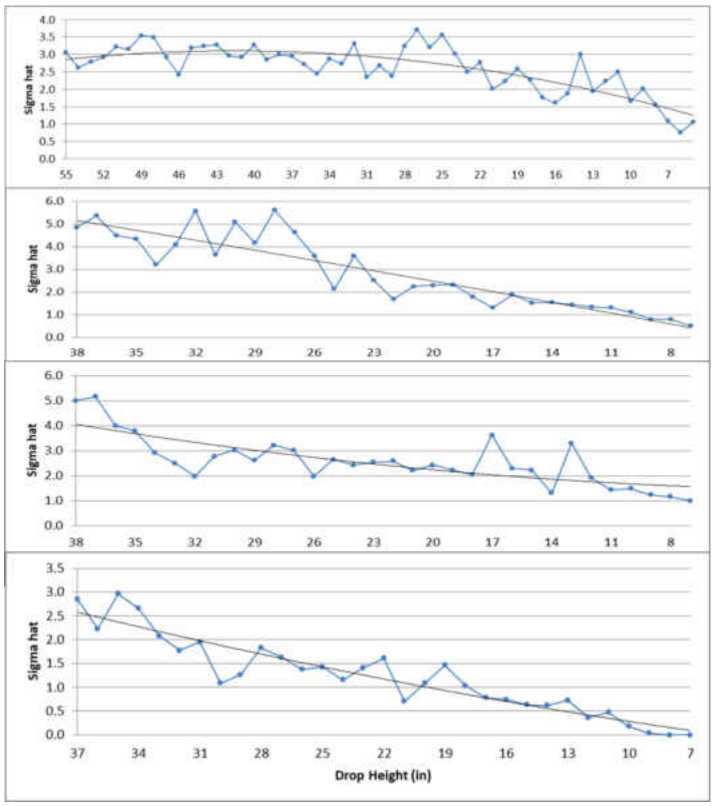


Figure 59. Sigma hat for each drop height presented for funnel experiments 1 through 4 from top to bottom, respectively. Trendlines are 2nd order polynomial fit.

In Figure 59, the trendlines for experiments 2-4 show a consistent decreasing trend. However, the trendline for experiment 1 displayed a slight increase for approximately the first third of the experiment before a decreasing trend began in the vicinity of drop height 38 inches. The data were also analyzed using the stability ratio (SR) test method, with results presented in Table 22.

Table 22. Stability ratio test results for all four funnel experiments. Red highlighted cells failed the test.

	Experiment 1		Experiment 2		Experiment 3		Experiment 4	
Height (in)	SR ≤ 1.59?	SR Stable? p > 0.05	SR ≤ 1.59?	SR Stable? p > 0.05	SR ≤ 1.59?	SR Stable? p > 0.05	SR ≤ 1.59?	SR Stable? p > 0.05
37	1.14	0.359	1.24	0.268	1.17	0.330	1.26	0.255
36	0.86	0.682	1.59	0.090	1.31	0.218	1.32	0.214
35	1.10	0.401	0.92	0.605	1.09	0.411	1.53	0.107
34	1.03	0.471	1.01	0.503	0.83	0.723	1.24	0.270
33	1.08	0.414	1.50	0.118	1.34	0.199	1.76	0.051
32	1.55	0.101	1.01	0.503	1.55	0.102	2.62	0.003
31	1.04	0.468	1.23	0.274	1.01	0.498	1.69	0.063
30	1.20	0.303	1.08	0.422	1.08	0.414	5.34	0.000
29	1.35	0.191	1.29	0.230	1.27	0.242	2.48	0.005
28	1.45	0.140	1.19	0.310	1.00	0.505	1.53	0.107
27	0.88	0.663	1.14	0.359	1.42	0.154	2.33	0.008
26	0.86	0.684	1.00	0.515	0.87	0.675	3.55	0.000
25	1.18	0.316	1.00	0.515	0.91	0.620	2.86	0.001
24	0.99	0.522	0.90	0.637	1.27	0.246	3.51	0.000
23	1.10	0.399	1.20	0.305	1.13	0.364	2.56	0.004
22	1.02	0.483	1.13	0.369	1.54	0.104	2.81	0.002
21	0.91	0.627	1.03	0.470	1.44	0.144	14.11	0.000
20	0.89	0.644	0.82	0.735	1.33	0.203	5.39	0.000
19	0.83	0.725	0.69	0.875	1.22	0.285	3.11	0.001
18	1.01	0.499	0.91	0.618	1.12	0.374	4.01	0.000
17	1.85	0.038	1.18	0.321	1.18	0.317	10.72	0.000
16	1.21	0.291	0.99	0.521	0.95	0.573	13.00	0.000
15	1.26	0.255	0.83	0.720	1.30	0.226	23.17	0.000
14	1.86	0.037	1.00	0.506	1.42	0.153	20.38	0.000
13	1.17	0.331	1.44	0.144	2.02	0.021	15.10	0.000
12	0.99	0.520	0.92	0.615	1.20	0.304	71.33	0.000
11	1.04	0.460	1.11	0.386	1.07	0.432	39.68	0.000
10	1.55	0.101	1.29	0.231	1.02	0.486	313.90	0.000
9	1.32	0.208	1.48	0.126	0.92	0.609	8095.73	0.000
8	1.12	0.379	1.09	0.410	1.08	0.420	#DIV/0!	#DIV/0!
7	1.78	0.048	1.60	0.085	1.01	0.499	#DIV/0!	#DIV/0!
# Stable		28		30		30		4
Pass Rate		**90%**		**97%**		**97%**		**13%**

The first three experiments each exhibited pass rates at or above 90%, and were considered Shewhart-stable for the purposes of this research. Experiment 4 was not considered stable, although its decreasing variation suggests that the process was improving over time. Nonetheless, experiment 4 still provided utility for various comparisons with the other three experiments during exploratory research. Also, some of the data collected for experiment 1 were above and below the drop heights evaluated for the SR test. Nonetheless, all of experiment 1's data was used for permutation entropy analyses, again to support the exploratory nature of this research.

Shannon information was continuously calculated for all drop heights in all funnel experiments. With the decision to embrace the permutation entropy method, these calculations were not actually used to develop an answer to the research question. However, since these results were available, Figure 60 is included to reveal the (non-normalized) Shannon entropy measurements for the funnel experiments. Corresponding data are provided in Appendix A.

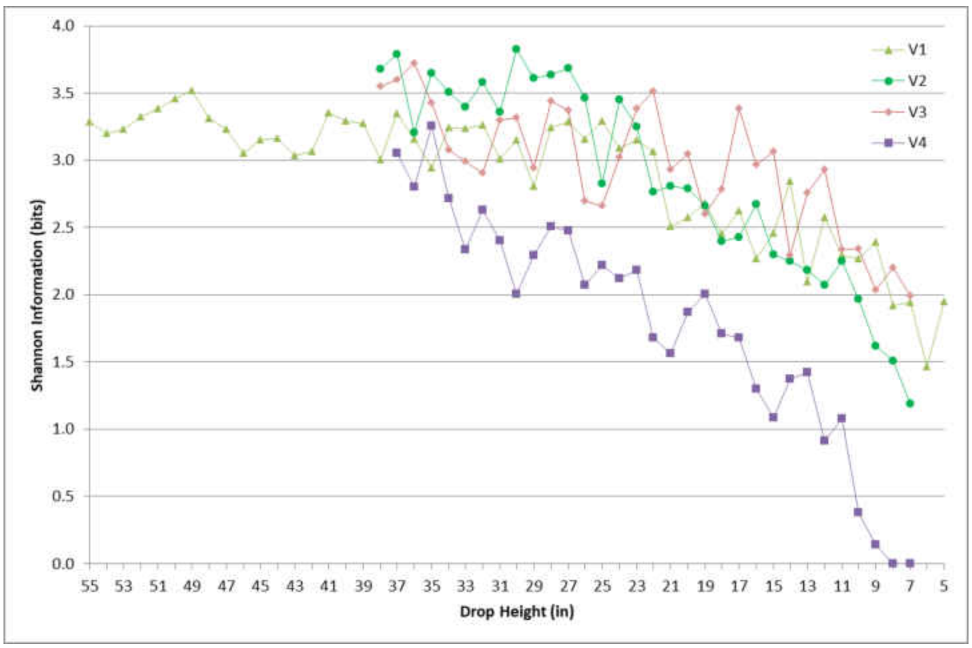


Figure 60. Shannon information at the final (50th) drop for each height. Funnel experiments 1-4.

Although each of these results was based on a relatively low quantity of data, they revealed a generally decreasing trend for randomness/ uncertainty as the funnel was lowered toward the target. In conclusion, the funnel experiments were deemed to represent improving processes in terms of decreasing location, dispersion, and randomness over time, with all but V4 representing a Shewhart-stable process.

Process Randomness Study

Normalized Shannon Entropy $\widetilde{H}_S$ was calculated for various processes using the PE-LE method, with results presented in Figure 61. Due to normalization, this plot reveals the *relative* degree of randomness associated with each process. Three of the funnel experiments (V1, V2, and V3) were Shewhart-stable, but each demonstrated varying degrees of randomness relative to idealized absolute randomness, $\widetilde{H}_S = 1$. Consequently, none of these three stable time series could be considered completely due to "chance causes". Moreover, the highly stochastic process based on the first 10,000 digits of π also did not achieve perfect randomness (the blue diamond labeled Pi10), nor did 10,000 pseudorandom digits on the interval [0,9] generated by Excel's Mersenne Twister (the orange dash labeled R10). These results suggest that there are always emergent nonrandom dynamics at play in processes that could potentially be exploited if only these dynamics were visible and understood.

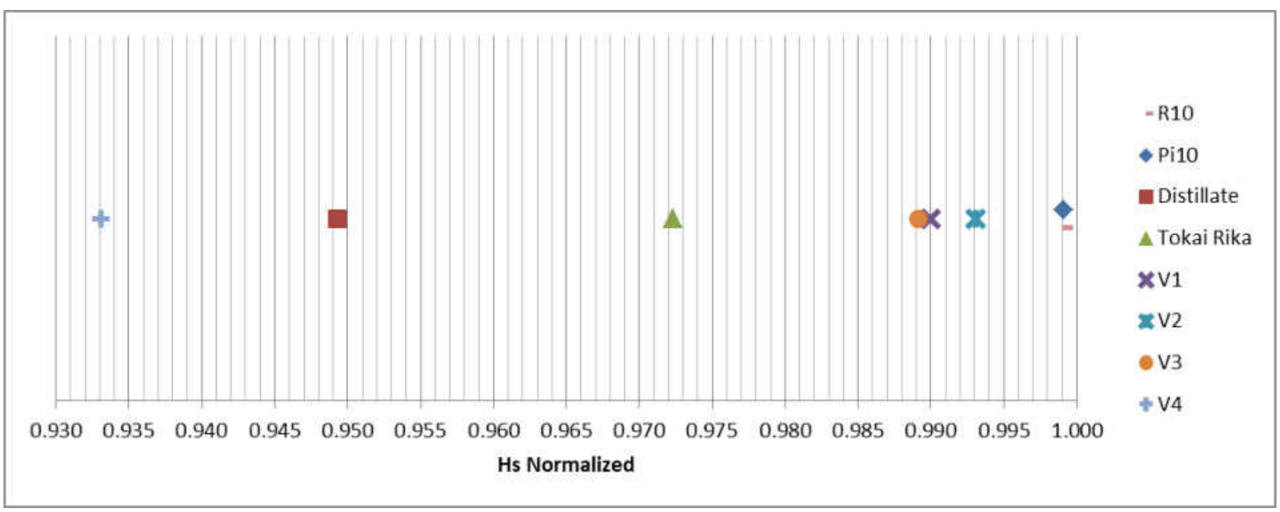


Figure 61. Normalized Shannon information plot based on PE-LE method.

Figure 61 is based on the mean value of normalized Shannon entropy for the final ~10% of $\widetilde{H}_S$values in each of the time series, as provided in Table 23.

Table 23. Mean and one standard error for the final ~10% of Hs normalized values in process time series.

Process	Distinct #s	Hs norm Mean	Hs norm 1SE
R10	10	0.999094574	0.000002192
Pi10	10	0.999069041	0.000000317
Distillate	3021	0.949292797	0.000017577
TR	33	0.972279282	0.000212773
V1	23	0.989960822	0.000049601
V2	29	0.993052053	0.000060106
V3	24	0.989116527	0.000080474
V4	15	0.933049074	0.000288577

Additionally, $\widetilde{H}_S$ calculations from the V2 and V3 time series are presented in Figure 62. Temporally, they tend to build from lesser to greater values (left to right) based on two contributions: The accumulation of information, and any improvement to the process as increasing process randomness corresponds with ongoing variation reduction. This plot suggests that Shewhart-stable processes are perhaps better represented as, "Inconstant systems of mostly chance causes".

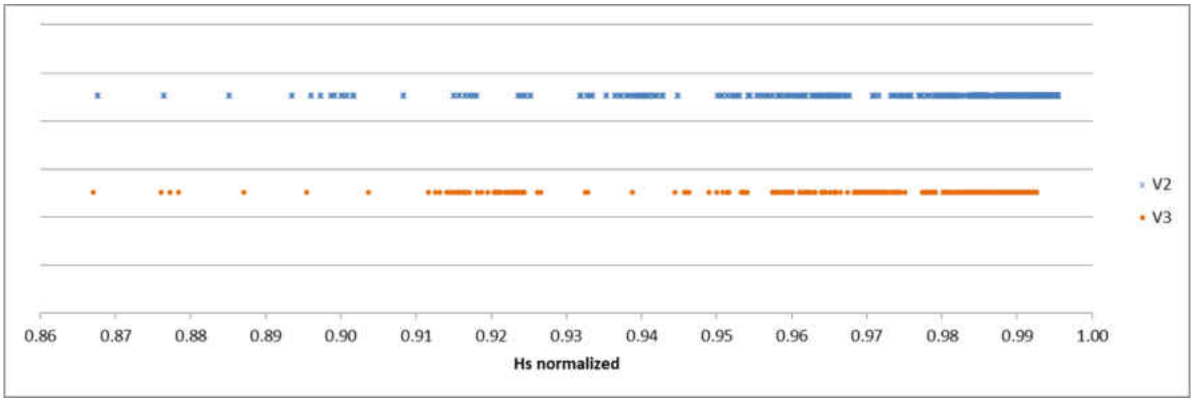


Figure 62. Progression of normalized Hs from left to right for two Shewhart-stable processes (V2 and V3) as improvements were made to the processes. The normalized *Hs* values are presented for each time series starting with the 45th tuple.

PE-Local Effect Validation

Numerous modifications have been devised by various authors to address shortcomings associated with the traditional permutation entropy (PE) method. Many of these were assessed in the Literature Review section. In the Methodology section, the traditional PE method was introduced in detail, as was the PE-LE methodology, devised to mitigate identical values within tuples by focusing on local effects. This section will now reveal validation results provided by the PE-LE method. The purpose of this validation was to assess PE-LE's strengths and weaknesses to ultimately determine appropriateness for this research.

Validation of a methodology generally requires formal requirements, instead of merely stating desires. As such, three axioms were developed to provide validation standards for the PE-LE method. An axiom is a proposition considered self-evidently true. Mathematicians might instead say an axiom is a proposition upon which an abstractly defined structure is based. It is assumed that these axioms could be applied to validate any other technique that has modified the traditional permutation entropy methodology to mitigate identical values.

AXIOM ONE: Although non-traditional ordinal patterns may vary, every traditional permutation entropy ordinal pattern will appear with the same probability for sufficiently large time series representing unconstrained, uncorrelated stochastic processes.

AXIOM TWO: The sum of local effects for all tuple types will balance near zero for sufficiently large time series representing unconstrained, uncorrelated stochastic processes.

AXIOM THREE: Any permutation entropy method meeting the requirements of the first two axioms will faithfully represent the probability space for any process type, given sufficiently large time series.

Two tests were conducted to *objectively* validate the PE-LE method with respect to Axioms One and Two:

1. The first 10,000 digits of Pi. Measurement resolution was integers resulting in 10 distinct values on the interval [0,9].
2. 10,000 random digits provided by Microsoft Excel's pseudorandom generator. Measurement resolution was integers resulting in 10 distinct values on the interval [0,9].

Axiom Three was *subjectively* assessed for extension to "any" process type by evaluating a representative sample from three other process categories.

ASSUMPTION: The strategy for Axiom Three was to assume concurrence until proven otherwise, as assessment of additional process types grows over time.

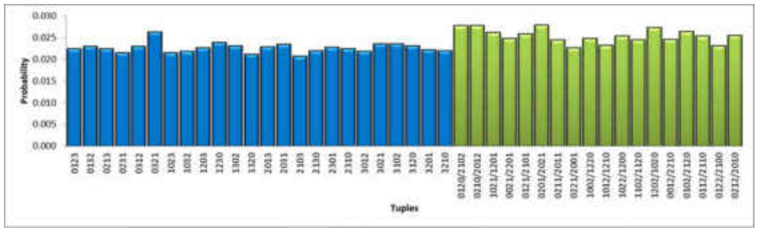
Three different categories of processes were selected to initially assess Axiom Three. All were tested with 10,000 measurements in the time series.

1. Periodic. A sine wave was discretized every 7.5 degrees and converted to radians. Measurement resolution was to the thousandths place resulting in 25 distinct values on the interval [-1.000, 1.000].
2. Shewhart-Unstable. Distillate flowrate data was tested. Measurement resolution was to the thousandths place resulting in 3,021 distinct values within the interval [0.000, 345.860].
3. Chaotic. The logistic map was tested with rate of growth r=3.888 and initial population x_n=0.02. Measurement resolution was to the thousandths place resulting in 866 distinct values on the interval [0.020, 0.972]. The equation for the logistic map is:

$$x_{n+1} = rx_n(1 - x_n) \quad , \tag{28}$$

where x_n is a number between zero and one that represents the ratio of the current population to the maximum possible population and r is the growth rate.

The results for the first two stochastic processes are presented in Figure 63 and Figure 64, and both facilitated the evaluation of Axioms One and Two.



Category	Count	Density
Parlitz	5095	0.510
Forest	4292	0.429
Wizard	283	0.028
Red	327	0.033
Sum	9997	1.000

LE	Count	Density
-3	423	0.045
-2	1424	0.152
-1	2447	0.261
0	767	0.082
1	2466	0.263
2	1440	0.153
3	420	0.045
Sum	9387	1

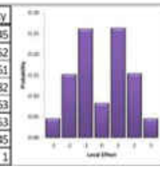
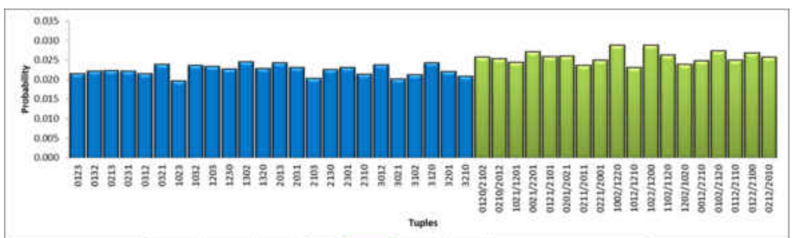


Figure 63. PDF for first 10,000 values of Pi using PE-LE. Measurement resolution was integers resulting in 10 distinct values on the interval [0,9].



Category	Count	Density
Parlitz	5013	0.501
Forest	4326	0.433
Wizard	294	0.029
Red	364	0.036
Sum	9997	1.000

LE	Count	Density
-3	421	0.045
-2	1397	0.150
-1	2507	0.268
0	704	0.075
1	2476	0.265
2	1425	0.153
3	409	0.044
Sum	9339	1

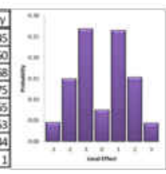


Figure 64. PDF for 10,000 random values generated with Microsoft Excel's pseudorandom generator using PE-LE. Measurement resolution was integers resulting in 10 distinct values on the interval [0,9].

Axiom One requires that every traditional permutation entropy ordinal pattern will appear with the same probability for sufficiently large time series representing unconstrained, uncorrelated stochastic processes. In both studies, the Parlitz tuples appeared to have an approximately uniform distribution, meeting this requirement.

Coincidentally, in both studies, the Parlitz tuples had slightly lower densities than the Forest tuples. Comparing the density means, the Parlitz sets both came out to $\bar{p}$= 0.021 and the Forest sets both came out to $\bar{p}$ = 0.024. The Red & Wizard sets also represented about 6% of the tuple count data in both studies. In combination, these results suggest that there were more identical value tuples than traditional tuples for both stochastic processes, which a complete case analysis would have discarded.

Axiom Two required the sum of local effects for all tuple types to balance near zero for sufficiently large time series representing unconstrained, uncorrelated stochastic processes.

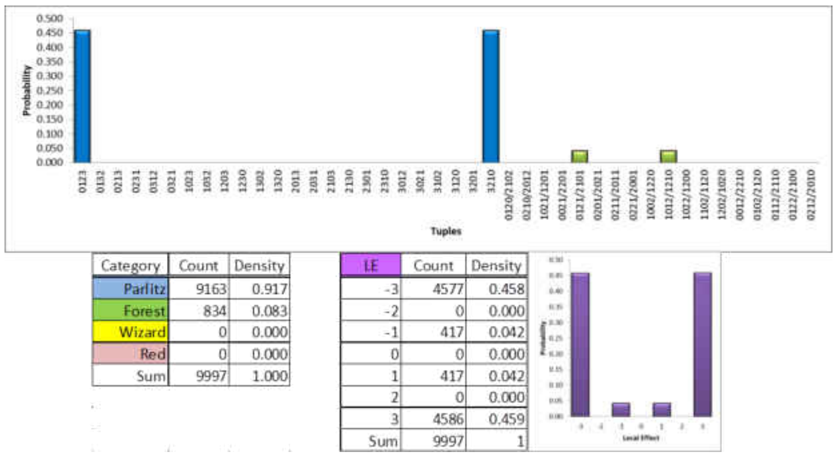
ASSUMPTION: "Near zero" cannot be precisely defined until numerous evaluations of different stochastic processes are conducted. Therefore, for the purposes of this exploratory research, a notional limit of ±1% of the time series length will be assumed based on the LE centerpoint calculation method.

For 10,000 data, this notional limit equates to a count of ±100 tuples. An approximate assessment of local effect balance is made by visually examining both purple Local Effect pdfs. To get a more precise answer, the LE centerpoint was calculated for both studies by multiplying the counts and local effect values.

Pi: -3(423)-2(1424)-2447+2466+2(1440)+3(420) = 42

Pseudorandom Values: -3(421)-2(1397)-2507+2476+2(1425)+3(409) = -11

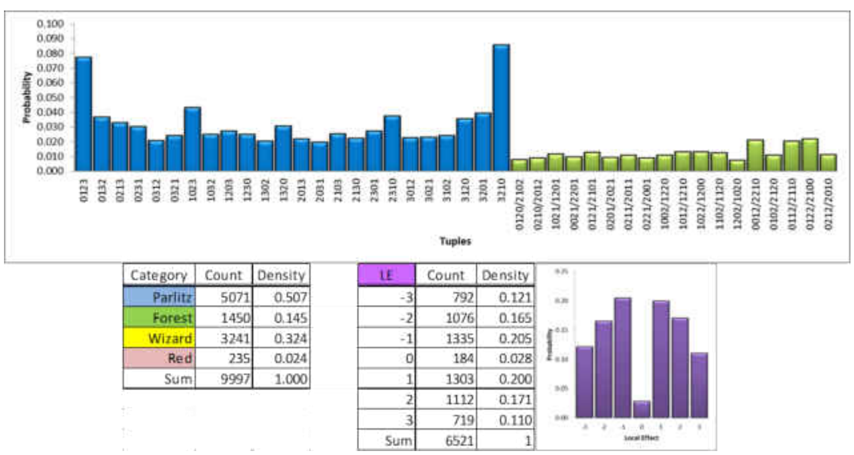
Based on these results, the requirement of Axiom Two was met for both processes. The next three studies will address Axiom Three.



Category	Count	Density
Parlitz	9163	0.917
Forest	834	0.083
Wizard	0	0.000
Red	0	0.000
Sum	9997	1.000

LE	Count	Density
-3	4577	0.458
-2	0	0.000
-1	417	0.042
0	0	0.000
1	417	0.042
2	0	0.000
3	4586	0.459
Sum	9997	1

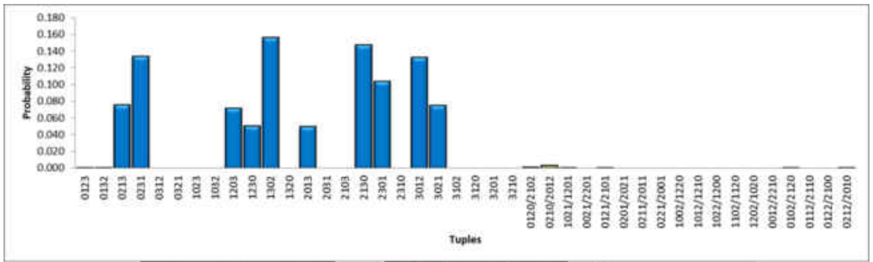
Figure 65. Periodic process type. A sine wave was discretized every 7.5 degrees and converted to radians. Measurement resolution was to the thousandths place resulting in 25 distinct values on the interval [-1.000, 1.000].



Category	Count	Density
Parlitz	5071	0.507
Forest	1450	0.145
Wizard	3241	0.324
Red	235	0.024
Sum	9997	1.000

LE	Count	Density
-3	792	0.121
-2	1076	0.165
-1	1335	0.205
0	184	0.028
1	1303	0.200
2	1112	0.171
3	719	0.110
Sum	6521	1

Figure 66. Shewhart-unstable process type: Distillate flowrate data 10,000 values. Measurement resolution was to the thousandths place resulting in 3021 distinct values on the interval [0.000, 345.860]. From www.openmv.net by Kevin Dunn (2020). Dataset provided open source with no restrictions.



Category	Count	Density
Parlitz	9954	0.996
Forest	43	0.004
Wizard	0	0.000
Red	0	0.000
Sum	9997	1.000

LE	Count	Density
-3	0	0.000
-2	2224	0.222
-1	2865	0.287
0	39	0.004
1	3396	0.340
2	719	0.072
3	754	0.075
Sum	9997	1

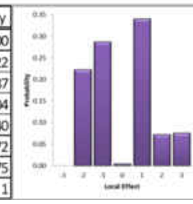


Figure 67. Chaotic process type. The logistic map was tested with r=3.888 from x_n=0.02. Measurement resolution was to the thousandths place resulting in 866 distinct values on the interval [0.020, 0.972].

Axiom Three requires that any permutation entropy method meeting the requirements of the first two axioms will faithfully represent the probability space for any process type, given sufficiently large time series. The pdf for the sine wave was appropriate and added beneficial peak and trough information not typically seen in methods that discard or corrupt identical values. The pdf for the distillate data faithfully represented a Shewhart-unstable process. The Wizard set alone would have represented ~32% of the data if included, probably because the distillation process was at zero value for long periods. Information apparently associated with distillation startup (0123) and shut down (3210) was also evident. The pdf for the logistic map was appropriate for a chaotic process and this was the first (purple) local effect pdf seen that was not approximately balanced. Since this was not a stochastic process, this result was not deemed problematic.

In summary, the PE-LE method met the requirements of Axioms One and Two objectively. The PE-LE method also subjectively met the requirements of Axiom Three for five processes representing four process types. Consequently, the PE-LE method was selected to represent the probability space (Ω, P) for all of this research effort.

PE-LE Performance Comparison

One paper discussed in the Literature Review (Zunino et al., 2017) was especially relevant to the assessment of the PE-LE methodology for this research effort. Some of the results from this study were compared to the results herein. The authors summarized their purpose: "In this work, we carefully study the effect that the presence of equalities has on permutation entropy estimated values when these ties are symbolized, as it is commonly done, according to their order of appearance" (p.1883). This is also called the time-imputation method. Two of their figures were used to compare results between the time-imputation method and the PE-LE method.

Figure 68 displays pdf results for the first 10,000 values of Pi. The top figure is based on the time-imputation method from Zunino et al. and the bottom is based on the PE-LE method. The relevant comparison is between the pdfs for the 24 traditional tuples, which are blue in both figures. For both studies, measurement resolution was integers resulting in 10 distinct values on the interval [0,9].

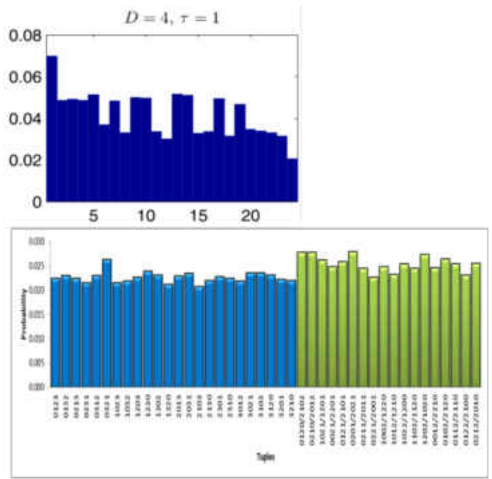


Figure 68. Top) PDF for first 10,000 digits of π evaluated traditionally with Parlitz-ordered $D! = 24$ tuples and time-ordered imputation to mitigate identical values. Measurement resolution was integers resulting in 10 distinct values on the interval [0,9]. From *Permutation Entropy Based Time Series Analysis: Equalities in the Input Signal Can Lead to False Conclusions* by Zunino et al. Bottom) PDF for first 10,000 digits of π evaluated with PE-LE to mitigate identical values. D=4, τ=1. The first 24 tuples (in blue) are Parlitz-ordered. Measurement resolution was integers resulting in 10 distinct values on the interval [0,9].

Figure 69 provides pdf results for 10,000 pseudorandomly-generated values. The top figure is based on the time-imputation method from Zunino et al. and the bottom is based on the PE-LE method. The relevant comparison is again between the pdfs for the 24 traditional tuples, which are blue in both figures. For all four pdfs, measurement resolution was integers resulting in 10 distinct values on the interval [0,9].

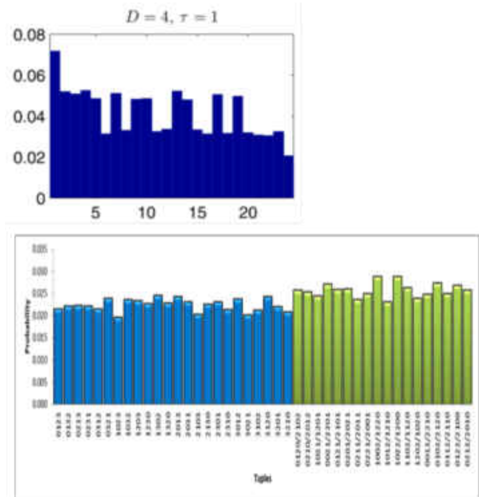


Figure 69. Top) PDF for 10,000 pseudorandom digits evaluated traditionally with Parlitz-ordered *D!* = 24 tuples and time-ordered imputation to mitigate identical values. Measurement resolution was integers resulting in 10 distinct values on the interval [0,9]. From *Permutation Entropy Based Time Series Analysis: Equalities in the Input Signal Can Lead to False Conclusions* by Zunino et al. Copyright © 2017 by Elsevier B.V. Reprinted with the permission of Elsevier. All rights reserved. Bottom) PDF for 10,000 pseudorandom digits evaluated with PE-LE to mitigate identical values. D=4, τ=1. The first 24 tuples (in blue) are Parlitz-ordered. Measurement resolution was integers resulting in 10 distinct values on the interval [0,9].

For both of these highly stochastic processes, the pdf should approximate a uniform distribution. By visual inspection, and ONLY for these two specific comparisons with $D = 4$, $\tau = 1$, PE-LE appears to more accurately represent the probability space than time-ordered imputation. Zunino et al. (2017) presented these pdfs for the purpose of showing that the time-imputation method can lead to false conclusions.

PE-LE's inclusion of the pattern-following opposite twin local effect tuple pairs ("Forest" tuples) appears to add useful causal information in many cases, but also brings disadvantages. Perhaps the most significant issue noted is associated with measurement resolution effects, which will be discussed in the next section.

Measurement Resolution Effects

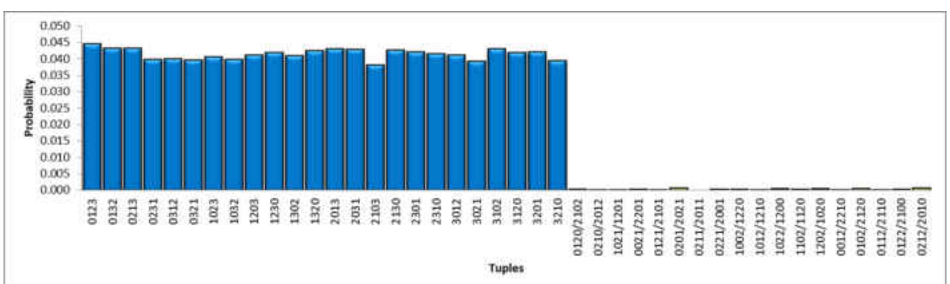
Some initial research attention was focused on strategies for preventing identical values in the first place. A brief example will clarify the thought process. To make this example seem less artificial, assume these data were actually collected at increased resolution, but rounded to a lower resolution for the first permutation entropy pdf analysis. Then, recognizing the identical value problem, the analysis (not the data collection) was repeated, but using the fully available resolution.

For the first round, at D=3, assume pre-coded data were {6.0, 6.1, 6.0} with equivalent coded symbol (010) for identical values. By removing the artificial truncation to realize the available measurement resolution, these pre-coded data were now actually {6.01,6.11,6.04} with equivalent coded symbol (021). This is now a viable symbol under the traditional PE methodology so the problem would be resolved for this tuple, and ostensibly for many others in the time series.

This example explained the concept, but a realistic scenario would be the repetitive evaluation of an ongoing process: When numerous identical values are evident, the measurement resolution could be progressively increased until identical values mostly fall outside, not inside tuples. Additionally, as measurement resolution increases, the number of distinct numbers available to permute increases, and therefore the likelihood of equal values decreases. The assumption was that selection of a fine enough measurement resolution before starting an analysis could largely obviate the need to mitigate equal values in tuples.

This assumption is related to a comment in the seminal PE paper by Bandt & Pompe (2002). They specified that their definitions were justified if values, "have a continuous distribution so that equal values are very rare. Otherwise, we can numerically break equalities by adding small random perturbations" (p.174102-1). The limit associated with continuously increasing a discretized measurement resolution would be the continuous distribution that Bandt & Pompe recommend. The value of increased measurement resolution could also naturally depend upon other process characteristics, such as the statistical variance of the time series, any periodic or chaotic tendencies, etc.

To test the idea using the PE-LE methodology, 10,000 pseudorandom digits were generated on the interval [0, 999] instead of [0,9]. An increase in the quantity of distinct values is a natural consequence of increasing the measurement resolution, which yields fewer identical values in tuples. Results are provided in Figure 70, which can be compared with Figure 64.



Category	Count	Density
Parlitz	9939	0.994
Forest	58	0.006
Wizard	0	0.000
Red	0	0.000
Sum	9997	1.000

LE	Count	Density
-3	812	0.081
-2	1670	0.167
-1	2546	0.255
0	6	0.001
1	2430	0.243
2	1655	0.166
3	878	0.088
Sum	9997	1

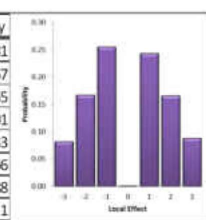


Figure 70. PDF for 10,000 pseudorandom digits evaluated with PE-LE to mitigate identical values. *D*=4, τ=1. The first 24 tuples (in blue) are Parlitz-ordered. Measurement resolution was integers resulting in 1000 distinct values on the interval [0,999].

As expected, most of the identical value problem went away. The probability densities for the Forest tuples disappeared and the Parlitz tuples saw a corresponding increase in probability densities. Also, the requirements of all three Axioms still appear to be met. For Axiom Two, a slight imbalance in the LE pdf is apparent but the centerpoint calculation comes out to 52, which is within the notional ±1% assumption.

It's apparent that measurement resolution influences PE-LE's probability space characterization more significantly than it does for traditional PE because of the extra identical value (Forest) tuples. Perhaps application of the measurement resolution effect could be used to tailor and fine-tune a PE-LE analysis, benefitting the goals of the research. To determine whether this effect could be used to advantage, another test was conducted. For this test the previous logistic map evaluation was presented at two different measurement resolutions. One resolution produced 866 distinct values and the other produced 11, as presented in Figure 71 and Figure 72.

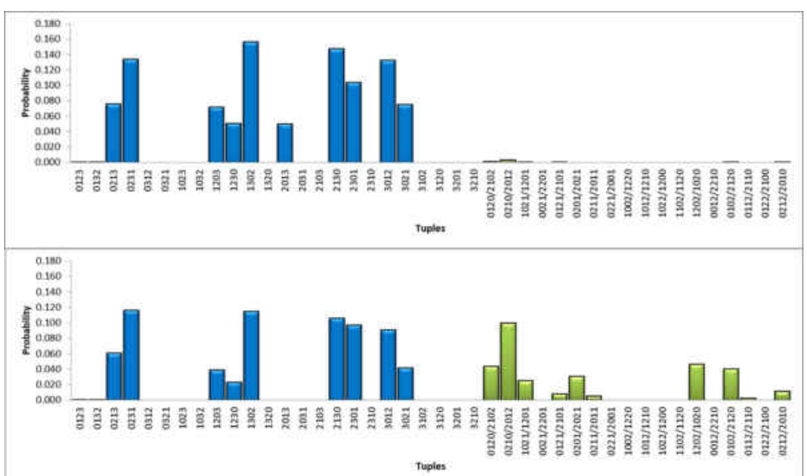
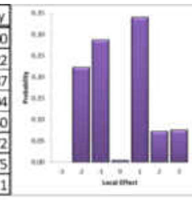


Figure 71. PDFs for the logistic map with r=3.888, x_o=0.02. Top) Measurement resolution to the thousandths place resulted in 866 distinct values on the interval [0.020, 0.972]. Bottom) Measurement resolution to the tenths place, resulting in 11 distinct values on the interval [0.0, 1.0].

Category	Count	Density
Parlitz	9954	0.996
Forest	43	0.004
Wizard	0	0.000
Red	0	0.000
Sum	9997	1.000

LE	Count	Density
-3	0	0.000
-2	2224	0.222
-1	2865	0.287
0	39	0.004
1	3396	0.340
2	719	0.072
3	754	0.075
Sum	9997	1



Category	Count	Density
Parlitz	6626	0.663
Forest	3007	0.301
Wizard	65	0.007
Red	299	0.030
Sum	9997	1.000

LE	Count	Density
-3	0	0.000
-2	1749	0.182
-1	2367	0.246
0	1622	0.168
1	2742	0.285
2	568	0.059
3	585	0.061
Sum	9633	1

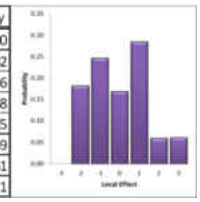


Figure 72. Local effect statistics corresponding with the Logistic Map pdfs in Figure 71.

Obvious differences are evident when comparing the results from both measurement resolutions. Comparing the top pdf to the lower pdf, the Parlitz set saw decreased tuple densities, but (2301) stayed about the same and (2013) disappeared altogether. On the contrary, in the Forest set, new tuples appeared and old tuples grew. The case could be made, at least for this instance of the logistic map, that fine-tuning the measurement resolution can provide access to a richer understanding of causal dynamics for the same time series.

PE-LE Shortcomings

At least three significant shortcomings were realized for the PE-LE methodology during research. First, the addition of extra tuples (at total of 42 instead of 24 for D=4) skews the measurement of Shannon entropy. This means that a theoretically absolute random process (such as white Gaussian noise), would not be represented correctly at $\widetilde{H}_S = 1$, but rather at a somewhat lesser value.

The second shortcoming includes numerous potential problems associated with the measurement resolution effect. Although fine-tuning the resolution might provide certain advantages, the appropriate measurement resolution may not be physically achievable with the available measurement devices. And some studies simply cannot be repeated to establish a basis for adjustment. Also, if a finer measurement resolution is required, many studies in the field probably wouldn't have "wasted" any available measurement resolution in the first place.

Worse, it is foreseeable that results could be dramatically altered, intentionally or unintentionally, by the choice of measurement resolution. Without enough investigation, use of the improper measurement resolution could inadvertently skew results, potentially leading to the wrong conclusions. Similar to flawed hypothesis testing and data dredging (McShane et al., 2019; Wasserstein & Lazar, 2016; Ziliak & McCloskey, 2008), the measurement resolution could potentially be abused by modifying an analysis iteratively until the desired, but knowingly incorrect, results are achieved.

The final disadvantage is related to the first. By choosing to parse the Red & Wizard tuples from the analysis before computing $\widetilde{H}_S$ the overall results are necessarily changed. The Red & Wizard tuples are more likely to appear at lower measurement resolutions and less likely at higher resolutions. In some tests with 1,000 distinct values, they did not appear at all.

A corresponding study evaluated the trend between the number of discrete random numbers and $\widetilde{H}_S$ for otherwise comparable time series with 10,000 values generated by Excel's PRNG. After the Red & Wizard tuples were parsed, as necessary, the mean was calculated for the final 1,000 $\widetilde{H}_S$ values. Since C_{JS} is partially dependent upon $\widetilde{H}_S$, the mean was also calculated for the final 1,000 values of C_{JS}. Results are presented in Figure 73.

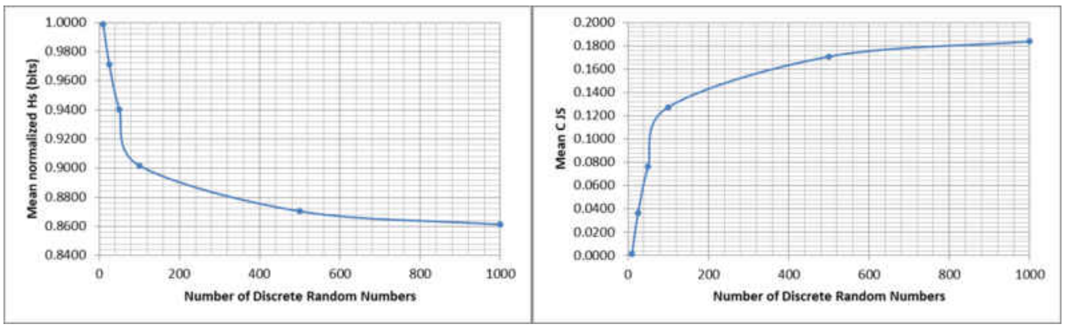


Figure 73. Mean normalized *Hs* (left) and C_{JS} (right) versus the number of discrete numbers available in a pseudorandom series. Each series started with 10,000 values and mean calculations were made for the final 1,000 values after the Red & Wizard tuples were parsed using the PE-LE methodology.

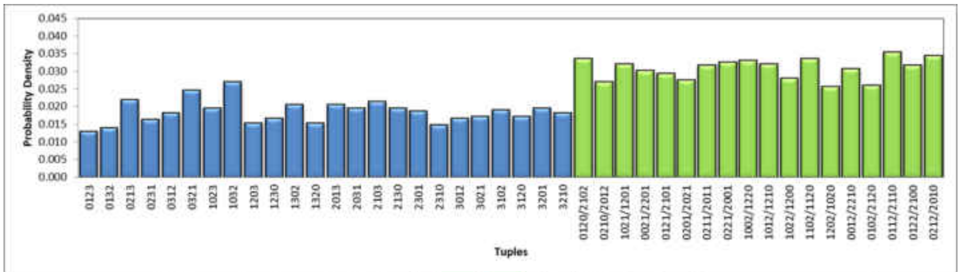
Both trends appear logarithmic. It was not deemed necessary to build in algorithmic compensation for this effect to accomplish the objectives of the research. As such, this finding is presented here for informational purposes, in case another researcher is interested in pursuing this further.

Summary of PE-LE Results for Processes

The primary time series of interest for this research were the funnel experiments (V1-V4), and the Tokai Rika (TR) series. V1, V2, and V3 represented Shewhart-stable processes that were being improved, V4 represented a Shewhart-unstable process that was being improved (V4), and TR represented a Shewhart-stable process that experienced minor improvements but overall variation was essentially in stasis. The distillate flowrate and electrical usage series were included to represent Shewhart-unstable processes. The supplementary time series were the logistic map at two measurement resolutions, sine wave, and Pi, which were useful for various comparisons with the processes of interest. Additionally, different pseudorandom series generated by Excel's PRNG were used as comparative baselines to evaluate the changing dynamics of entropy and complexity in processes.

Table 24. Data summary for time series processes.

Process	Starting Amount of Data	Tuples after Parse	Forest Tuples	Qty of Distinct #s
V1	2,550	2,144	1,191	23
V2	1,700	1,309	690	29
V3	1,600	1,353	744	24
V4	1,550	761	589	15
TR	379	337	175	33
Pi	10,000	9,387	4,295	10
Sine	10,000	9,885	834	25
Logistic11	10,000	9,633	3,007	11
Logistic866	10,000	9,997	43	866
Distillate	10,000	6,521	1,450	3,021
Electrical	2,712	2,566	820	164



Category	Count	Density
Parlitz	956	0.375
Forest	1191	0.468
Wizard	185	0.073
Red	215	0.084

LE	Count	Density
-3	76	0.035
-2	320	0.149
-1	546	0.254
0	199	0.093
1	614	0.286
2	317	0.148
3	75	0.035
	2147	1

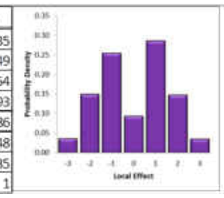
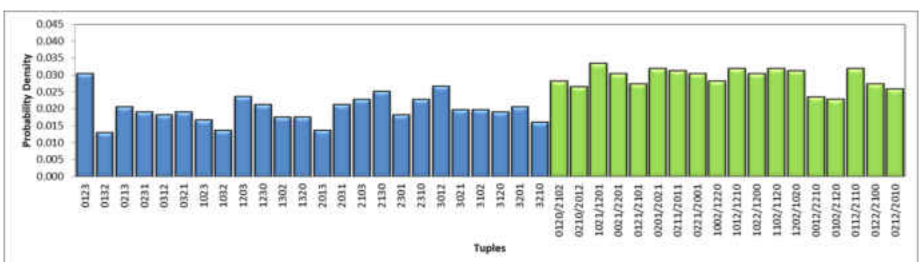


Figure 74. Funnel Experiment 1 (V1) pdfs and local effect data.



Category	County	Density
Parlitz	626	0.392
Forest	690	0.432
Wizard	155	0.097
Red	126	0.079

LE	Count	Density
-3	46	0.035
-2	208	0.158
-1	358	0.272
0	116	0.088
1	346	0.263
2	175	0.133
3	67	0.051
Sum	1316	1

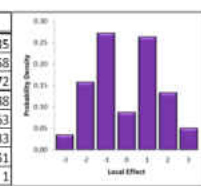
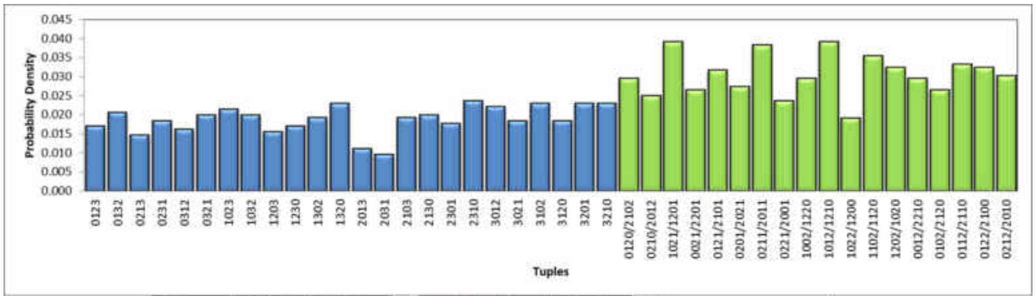


Figure 75. Funnel Experiment 2 (V2) pdfs and local effect data.



Category	Count	Density
Parlitz	612	0.383
Forest	744	0.466
Wizard	110	0.069
Red	131	0.082

LE	Count	Density
-3	56	0.041
-2	211	0.156
-1	353	0.260
0	127	0.094
1	356	0.263
2	210	0.155
3	43	0.032
Sum	1356	1

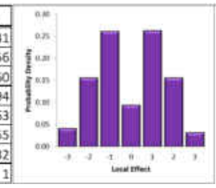
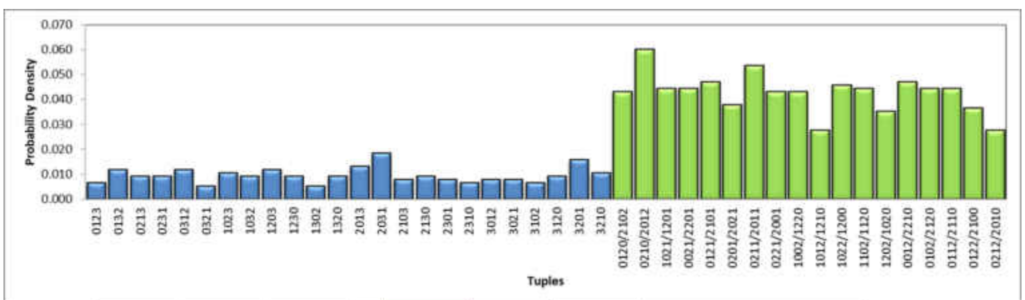


Figure 76. Funnel Experiment 3 (V3) pdfs and local effect data.



Category	Count	Density
Parlitz	175	0.113
Forest	589	0.381
Wizard	511	0.330
Red	272	0.176

LE	Count	Density
-3	15	0.020
-2	108	0.141
-1	203	0.266
0	113	0.148
1	203	0.266
2	110	0.144
3	12	0.016
Sum	764	1

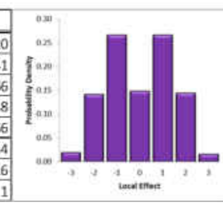
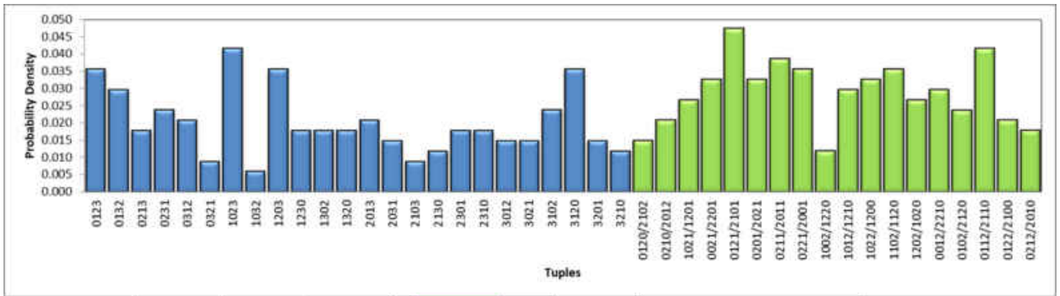


Figure 77. Funnel Experiment 4 (V4) pdfs and local effect data.



Category	Count	Density
Parlitz	162	0.431
Forest	175	0.465
Wizard	19	0.051
Red	20	0.053

LE	Count	Density
-3	16	0.047
-2	49	0.145
-1	85	0.252
0	21	0.062
1	89	0.264
2	59	0.175
3	18	0.053
Sum	337	1

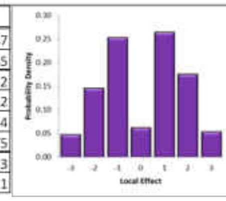
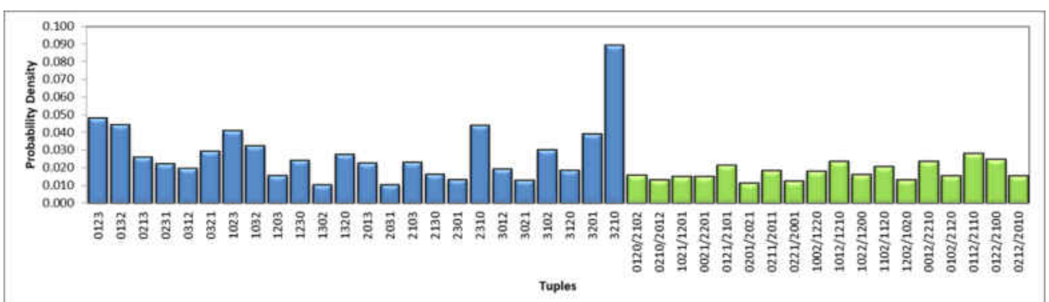


Figure 78. Tokai Rika pdfs and local effect data. Data reused with the permission of the Tokai Rika intellectual property division. All rights reserved.

Distillate pdfs and local effect data are provided in Figure 66.

Electrical usage pdfs and local effect data are provided in Figure 79.



Category	Count	Density
Parlitz	1746	0.645
Forest	820	0.303
Wizard	69	0.025
Red	74	0.027

LE	Count	Density
-3	277	0.108
-2	432	0.168
-1	539	0.210
0	112	0.044
1	577	0.225
2	439	0.171
3	190	0.074
Sum	2566	1

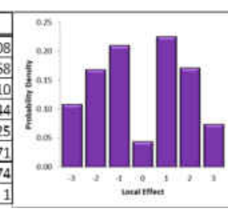


Figure 79. Electrical usage pdfs and local effect data. From www.openmv.net by Kevin Dunn (2020). Dataset provided open source with no restrictions on use.

Supplementary Time Series

Logistic map (866 and 11 distinct values) pdfs are provided in Figure 71 and associated local effect data is provided in Figure 72.

Sine wave pdfs and local effect data is provided in Figure 65.

Pi pdfs and local effect data is provided in Figure 63.

Pseudorandom Time Series

Microsoft Excel's Mersenne Twister Pseudorandom Number Generator was used to build the random series used in this research for comparison with other time series. These random series facilitated the evaluation of the PE-LE method, were especially useful for the log change evaluations when an equivalent random series with the same number of distinct values was required. They were also used to approximate Shewhart-stable processes. Two types of pseudorandom series were generated- unchanging (R#) and changing (Decr#/Incr#), where the number sign represents the number of distinct values in each series. The data summary for unchanged pseudorandom series is provided in Table 25 and associated pdfs follow in Figure 80.

Table 25. Data summary for unimproved pseudorandom processes.

Pseudo-Random Process	Starting Amount of Data	Tuples after Parse	Forest Tuples	Qty of Distinct #s
R10	10,000	9,404	4,315	10
R25	10,000	9,883	2,075	25
R50	10,000	9,965	1,239	50
R100	10,000	9,991	139	100
R500	10,000	9,996	141	500
R1000	10,000	9,997	76	1000

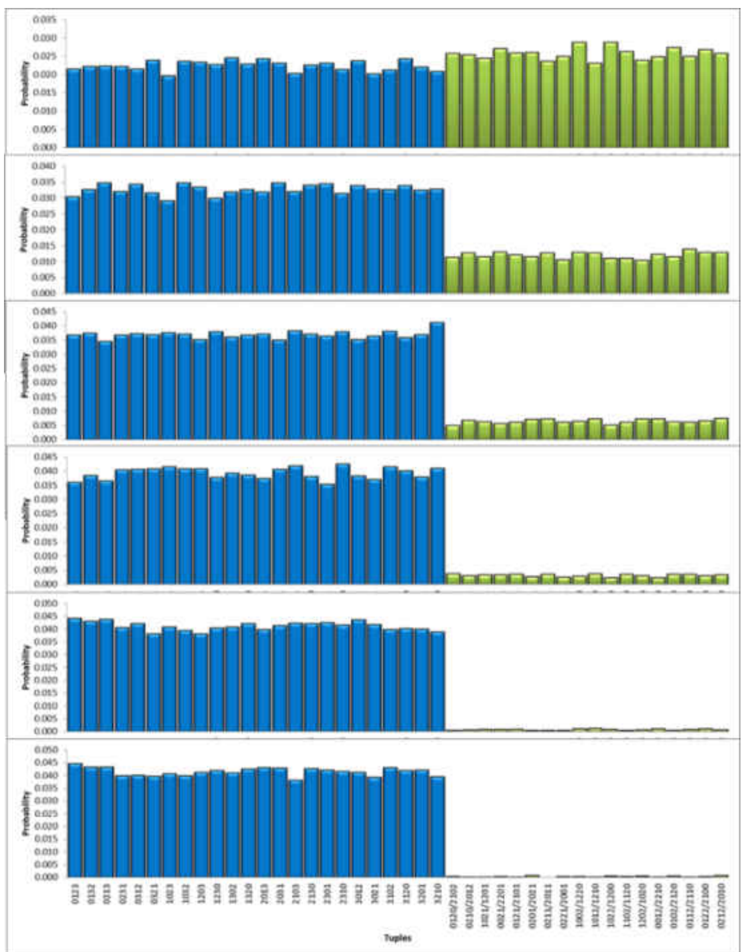


Figure 80. PDFs for baseline pseudorandom series based on numbers of distinct values using PE-LE method. From top to bottom: R10, R25, R50, R100, R500, R1000. Each series started with 10,000 values. Probability densities represent allocation after Red & Wizard tuple parsing. See Table 25 for details.

Data summary for changing pseudorandom series is provided in Table 25. Variation calculations and associated pdfs follow.

Table 26. Data summary for improved pseudorandom processes (Decr#) plus one increasing pseudorandom process (Incr10K) for comparison. Decr is short for decreasing variability and Incr is short for increasing variability.

Pseudo-Random Process	Starting Amount of Data	Tuples after Parse	Forest Tuples	Qty of Distinct #s
Decr25	2,200	1,940	778	25
Decr40	2,200	2,074	715	40
Decr100	2,200	2,192	115	99
Decr10K	10,000	9,997	23	5,252
Incr10K	10,000	9,997	8	5,221

Table 27. Variation calculations for five pseudorandom processes.

	Decr25		Decr40		Decr100	
Iteration Range	Random Value Range	1SE	Random Value Range	1SE	Random Value Range	1SE
1-220	1-25	0.48	1-40	0.80	1-100	2.03
221-440	1-23	0.47	1-36	0.69	1-90	1.71
441-660	1-20	0.39	1-32	0.61	1-80	1.56
661-880	1-18	0.35	1-28	0.54	1-70	1.29
881-1100	1-15	0.29	1-24	0.45	1-60	1.11
1101-1320	1-13	0.26	1-20	0.40	1-50	0.96
1321-1540	1-10	0.20	1-16	0.31	1-40	0.78
1541-1760	1-8	0.15	1-12	0.23	1-30	0.57
1761-1980	1-5	0.09	1-8	0.16	1-20	0.39
1981-2200	1-3	0.06	1-4	0.07	1-10	0.20

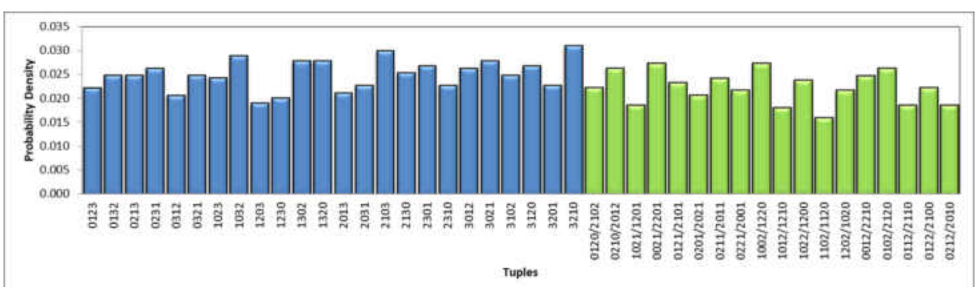
	Decr10K		Incr10K	
Iteration Range	Random Value Range	1SE	Random Value Range	1SE
1-1K	1-10K	93.2	1-1K	9.2
1K-2K	1-9K	84.2	1-2K	18.4
2K-3K	1-8K	72.1	1-3K	27.4
3K-4K	1-7K	64.4	1-4K	36.7
4K-5K	1-6K	54.1	1-5K	45.5
5K-6K	1-5K	45.6	1-6K	54.6
6K-7K	1-4K	36.4	1-7K	63.9
7K-8K	1-3K	27.2	1-8K	74.7
8K-9K	1-2K	18.2	1-9K	81.9
9K-10K	1-1K	9.5	1-10K	94.0

Since Decr25 most resembled the characteristics of experiments V1-V3, its data were subjected to a stability ratio (SR) test to characterize the level of stability of this pseudorandom process. Results for each integer value range passed the test, as presented in Table 28. Since all ranges were pseudorandomly generated, it was assumed that all other Mersenne Twister processes would also pass the SR test.

Table 28. SR test results for Decr25 pseudorandom process.

Integer Values	Xbar	mR-bar	SR ≤ 1.59?	SR Stable? p >0.05
1-25	13.40	7.75	1.08	0.305
1-23	12.52	7.28	1.15	0.184
1-20	9.46	6.57	0.98	0.565
1-18	9.69	6.22	0.89	0.771
1-15	8.05	4.82	0.99	0.540
1-13	6.83	4.47	0.96	0.599
1-10	5.65	3.25	1.06	0.353
1-8	4.46	2.62	0.97	0.571
1-5	2.92	1.57	1.00	0.506
1-3	1.96	0.88	1.17	0.160

Each of the processes were evaluated using the PE-LE method, as presented in the following figures.



Category	Count	Density
Parlitz	1162	0.529
Forest	778	0.354
Wizard	109	0.050
Red	148	0.067

LE	Count	Density
-3	112	0.058
-2	287	0.148
-1	508	0.262
0	130	0.067
1	522	0.269
2	290	0.149
3	91	0.047
Sum	1940	1

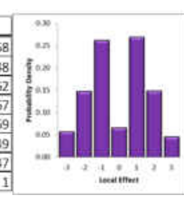
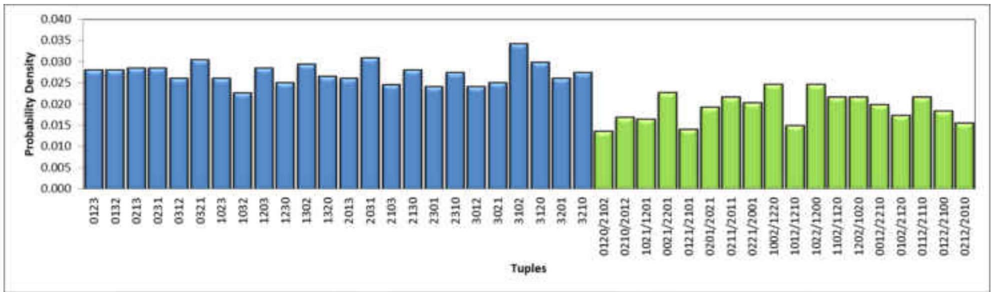


Figure 81. Decr25 pdfs and local effect data.



Category	Count	Density
Parlitz	1359	0.619
Forest	715	0.325
Wizard	49	0.022
Red	74	0.034

LE	Count	Density
-3	119	0.057
-2	318	0.153
-1	555	0.268
0	97	0.047
1	548	0.264
2	320	0.154
3	117	0.056
Sum	2074	1

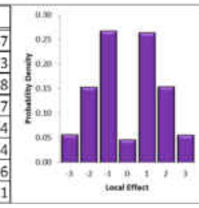
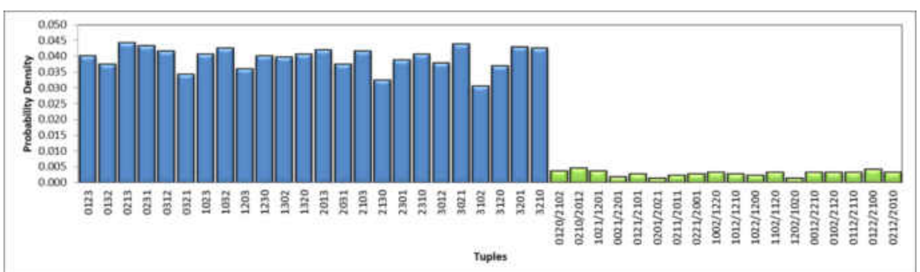


Figure 82. Decr40 pdfs and local effect data.



Category	Count	Density
Parlitz	2077	0.945
Forest	115	0.052
Wizard	1	0.000
Red	4	0.002

LE	Count	Density
-3	174	0.079
-2	365	0.167
-1	521	0.238
0	26	0.012
1	558	0.255
2	363	0.166
3	185	0.084
Sum	2192	1

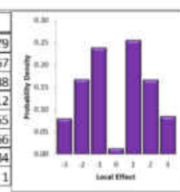
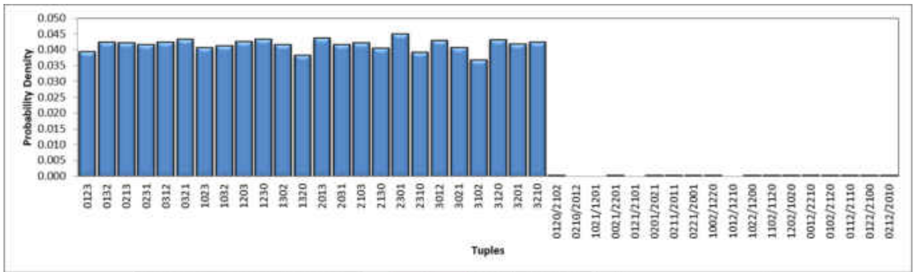


Figure 83. Decr100 pdfs and local effect data.



Category	Count	Density
Parlitz	9974	0.998
Forest	23	0.002
Wizard	0	0.000
Red	0	0.000

LE	Count	Density
-3	854	0.085
-2	1627	0.163
-1	2482	0.248
0	2	0.000
1	2535	0.254
2	1683	0.168
3	814	0.081
Sum	9997	1

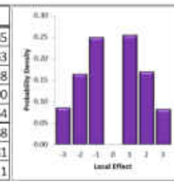
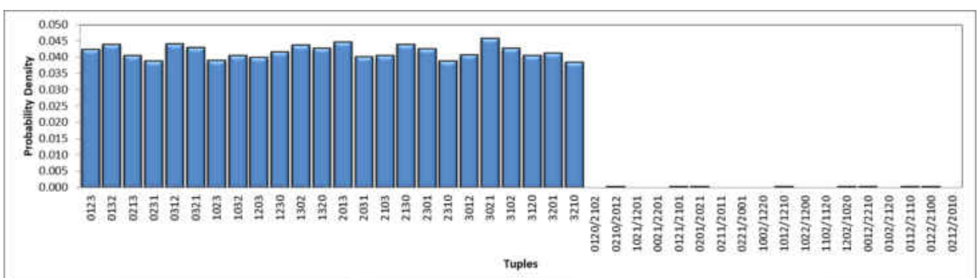


Figure 84. Decr10K pdfs and local effect data.

To allow further comparison of the four pseudorandom processes with decreasing variation, one pseudorandom series was created with increasing variation, as presented in Figure 85.



Category	Count	Density
Parlitz	9989	0.999
Forest	8	0.001
Wizard	0	0.000
Red	0	0.000

LE	Count	Density
-3	789	0.079
-2	1696	0.170
-1	2505	0.251
0	1	0.000
1	2511	0.251
2	1667	0.167
3	828	0.083
Sum	9997	1

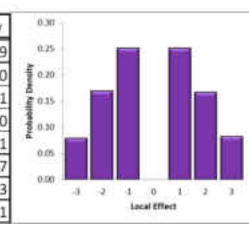


Figure 85. Incr10K pdfs and local effect data.

Characterizations of Improved Processes

Given that the previous sections of the Findings characterized the process data for the 22 time series evaluated in this research, to include PE-LE results, this section is now focused on answering the research question: Can a methodology based on emerging structural complexity provide information useful to direct the continued improvement of a Shewhart-stable process? Three techniques based on the temporal plotting of Jensen-Shannon complexity were researched for their capability to display patterns corresponding with varying levels of process improvement: The Complexity-Entropy Causality Plane, the $\tilde{C}_{JS}$ change chart, and the Entropy-Complexity Change Diagram.

Complexity-Entropy Causality Plane

Results to support the generation of CECP plots for all 22 processes are displayed in Table 29. These time series had varying run lengths due to starting conditions and statistically complete parsing. Therefore, the two columns of mean values were based on the final ~10% of calculations for each time series. Values were presented to nine decimal places because (1) Some of the standard errors were quite small, and (2) For some comparisons, differences in mean values did not appear until the 5th decimal place.

Table 29. Results that supported generation of CECP plots for all 22 processes.

Process	Distinct #s	Hs norm Mean	Hs norm 1SE		C JS Mean	C JS 1SE
R10	10	0.999094574	0.000002192		0.001124741	0.000002655
R25	25	0.971166234	0.000003950		0.036170330	0.000005341
R50	50	0.940201034	0.000015093		0.076554748	0.000019617
R100	100	0.901738503	0.000009450		0.127369218	0.000012329
R500	500	0.870330805	0.000006313		0.170803076	0.000008844
R1000	1000	0.861262311	0.000009364		0.183778882	0.000013654
Pi10	10	0.999069041	0.000000317		0.001145954	0.000000398
Sine	25	0.262196267	0.000000366		0.192950642	0.000022968
Logistic11	11	0.712000000	0.000004860		0.328000000	0.000002200
Logistic866	866	0.601904227	0.000004158		0.310779664	0.000001482
Distillate	3021	0.949292797	0.000017577		0.055895541	0.000017113
Electrical	164	0.963565682	0.000013400		0.039565441	0.000012976
TR	33	0.972279282	0.000212773		0.035253640	0.000297859
V1	23	0.989960822	0.000049601		0.011535342	0.000061596
V2	29	0.993052053	0.000060106		0.008678302	0.000075358
V3	24	0.989116527	0.000080474		0.013511132	0.000094456
V4	15	0.933049074	0.000288577		0.079432269	0.000335664
Decr25	25	0.997120000	0.000004420		0.003595000	0.000004342
Decr40	40	0.993645455	0.000011267		0.008031818	0.000013931
Decr100	99	0.941578006	0.000373702		0.074650607	0.000483302
Decr10K	5246	0.853363866	0.000009126		0.195622652	0.000013783
Incr10K	5221	0.853000000	0.000009130		0.196000000	0.000013780

All 22 processes are plotted concurrently on the CECP presented in Figure 86. As expected, the sine wave displayed the least randomness, followed by both versions of the (deterministic) logistic map. The logistic map also displayed the greatest structural complexity, with 11 distinct values providing greater complexity than the identical logistic map at 866 distinct values. C_{max} is plotted based on the standard 24 tuples for D=4 instead of the non-standard 42, causing it to appear slightly lower than it should. This artifact caused Logistic11 to incorrectly appear out of bounds.

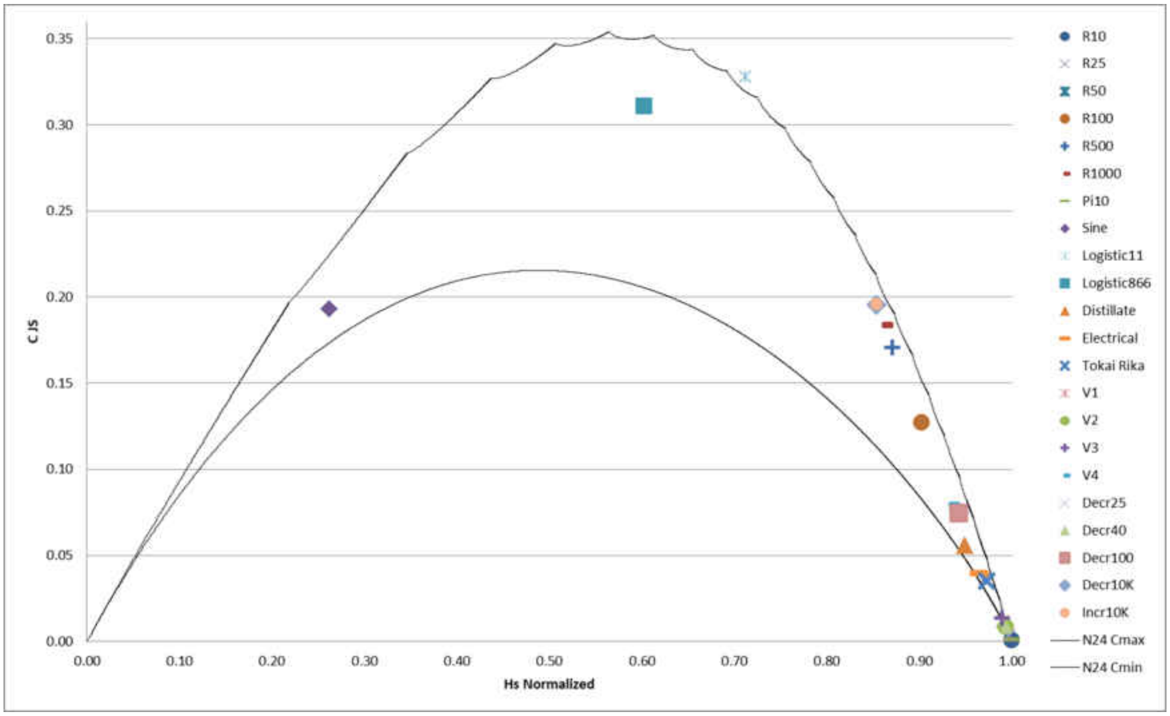


Figure 86. CECP for all 22 evaluated processes. C_{max} is based on 24 distinct tuples instead of 42 causing it to appear slightly lower than it should.

Abandoning the sine wave and both logistic map plots allowed better visibility of the relative positions of the remaining 19 time series, as depicted in the CECP close-up in Figure 87.

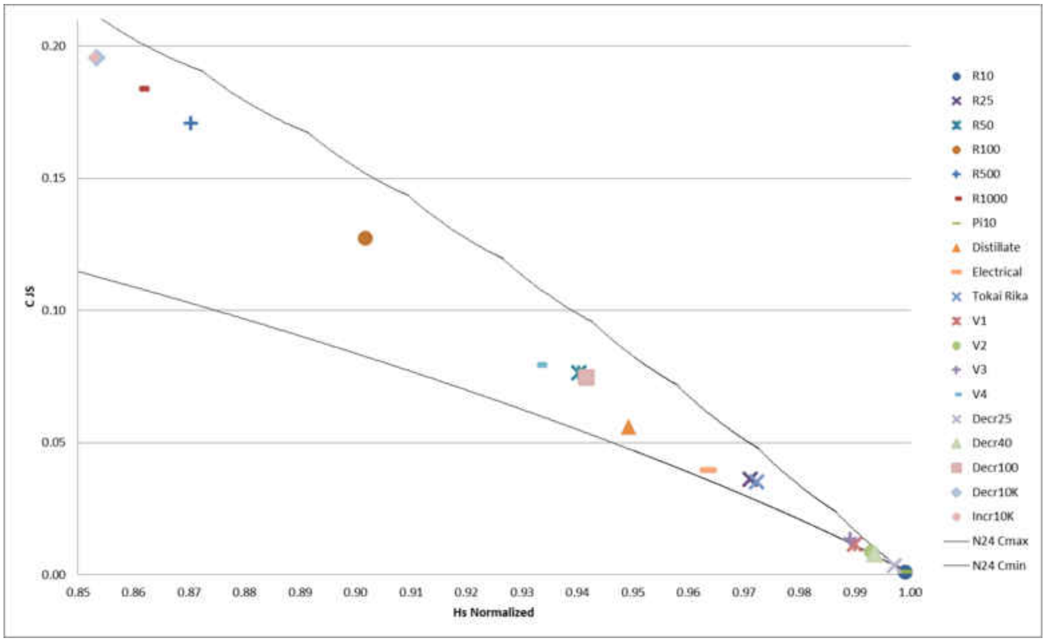


Figure 87. CECP close-up for all processes evaluated except sine and both logistic map plots.

Additional focusing and dissection in the CECP close up facilitated comparative evaluations. The plot in Figure 88 includes only those processes that were pseudorandomly generated. The observation that Decr10K and Incr10K appear in almost the identical position highlights an important characteristic of C_{JS} when PE-LE is the underlying method. That is, instead of directly considering the changing variance, the more important consideration appears to be the effect that process characteristics are having on process homogeneity, which will be reflected in effect of the 42-tuple pdf on entropy measurements. This is relevant because the C_{JS} calculation applies the changing pdf to measure process homogeneity via entropy and to measure the metric probability distance from perfect theoretical equilibrium.

Momentarily reviewing definitions may be useful. *Variance* defines the magnitude of spread among process outcomes by calculating the expectation of the squared deviation from the mean for a random variable of interest. *Entropy* instead defines the homogeneity/ uncertainty/ randomness of the process based on the *probabilities* of the distinct states Ω required to characterize the process. For a Gaussian distribution, variance and entropy can have similar meanings. However, as the probability distribution progressively deviates further from the Gaussian, variance begins to obfuscate the structural information, whereas entropy more accurately reflects the changing structural dynamics.

So, Decr10K and Incr10K probably plot in the same $\widetilde{H}_S$, C_{JS} space because their nearly identical probability distributions under the PE-LE methodology establish their equivalent homogeneity and their equivalent distances from ideal equilibrium as calculated by Jensen-Shannon divergence. More specifically, given that both processes have more than 5,200 distinct values, inspection of Figure 84 and Figure 85 confirms they both have almost no identical value tuples and the remaining Parlitz tuples are in a nearly uniform distribution. This effect suggests that the PE-LE methodology may provide less utility for informing process improvement when there are a high number of distinct values and that this utility progressively increases as the number of distinct values decreases.

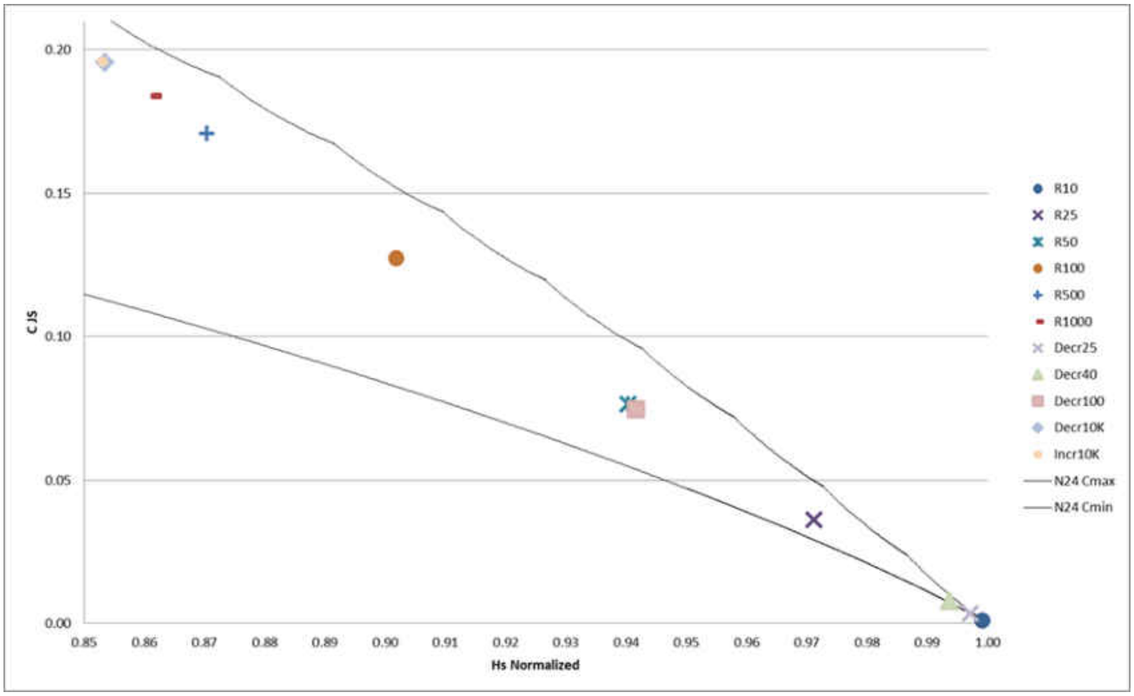
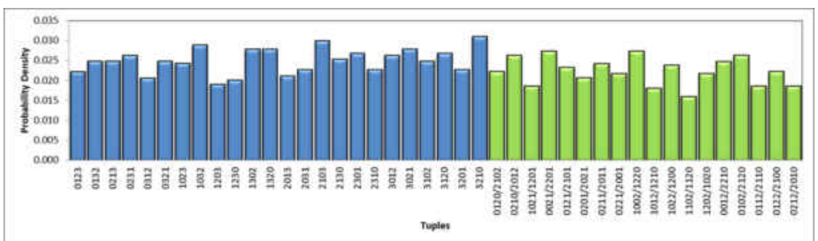


Figure 88. CECP close up for only pseudorandom processes.

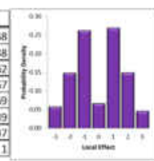
Given the dependence of PE-LE's calculation methodology upon measurement resolution and corresponding distinct values, a trend can be observed in Figure 88 showing that a decreasing number of distinct values corresponds with increasing $\widetilde{H}_S$ and decreasing C_{JS}. It follows logically that the most equitable comparisons can be made among processes possessing a similar number of distinct values. For instance, the results for Decr25 can be most reliably compared with R25 and, similarly, Decr100 can be most reliably compared with R100. In both cases, the Decr# processes plotted significantly closer to (1,0) than the equivalent R# processes.

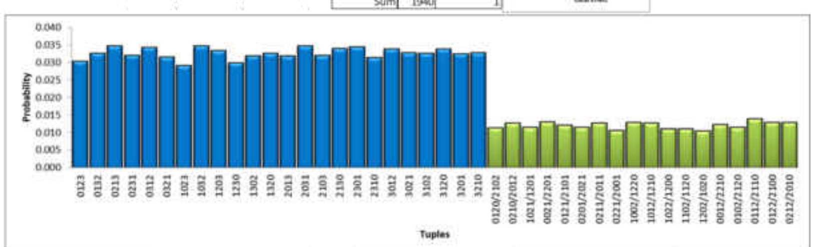
The pdfs for Decr25 and R25 are simultaneously displayed in Figure 59. Decr25 was based on 2,200 measurements to facilitate comparison with the funnel experiments, whereas R25 was based on 10,000 measurements to facilitate comparison with other R# processes. However, the probability densities still provide useful comparisons. Decr25 had higher densities of Forest Wizard and Red tuple groups, as might be expected, and therefore the 42-tuple pdf displays greater homogeneity than the 42-tuple pdf for R25. This would explain why Decr25 is closer to (1,0) in the CECP than R25. Contrary to the comparison results for Incr10K and Decr10K, at this lower quantity of distinct numbers (25), Jensen-Shannon complexity would better inform process improvement efforts, as also revealed on the CECP.



Category	Count	Density
Parlitz	1162	0.529
Forest	778	0.354
Wizard	109	0.050
Red	148	0.067

LE	Count	Density
-3	112	0.058
-2	287	0.148
-1	508	0.262
0	130	0.067
1	522	0.269
2	290	0.149
3	91	0.047
Sum	1940	1





Category	Count	Density
Parlitz	7725	0.773
Forest	2141	0.214
Wizard	59	0.006
Red	72	0.007
Sum	9997	1.000

LE	Count	Density
-3	656	0.066
-2	1568	0.159
-1	2544	0.258
0	352	0.036
1	2483	0.252
2	1621	0.164
3	642	0.065
Sum	9866	1

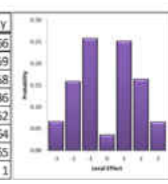


Figure 89. Comparison of two pseudorandom processes, Decr25 on top and R25 on bottom.

The next plot in Figure 90 examines the seven remaining empirical processes. The previous correlation with the number of distinct values now appears attenuated in that the

Distillate process with 3,021 distinct values is actually closer to (1,0) than V4 with only 15 distinct values. However, reviewing pdfs for these two Shewhart-unstable processes in Figure 66 and Figure 77 reveals that the Distillate process had more homogeneity than V4. The V4 pdf displays mostly identical value tuples and few Parlitz tuples. This comparison is useful because V4 demonstrated that the PE-LE method also loses utility to inform process improvement when there is little diversity in the data. A similar problem also limited control chart utility based on chunky data, as was displayed in Figure 19.

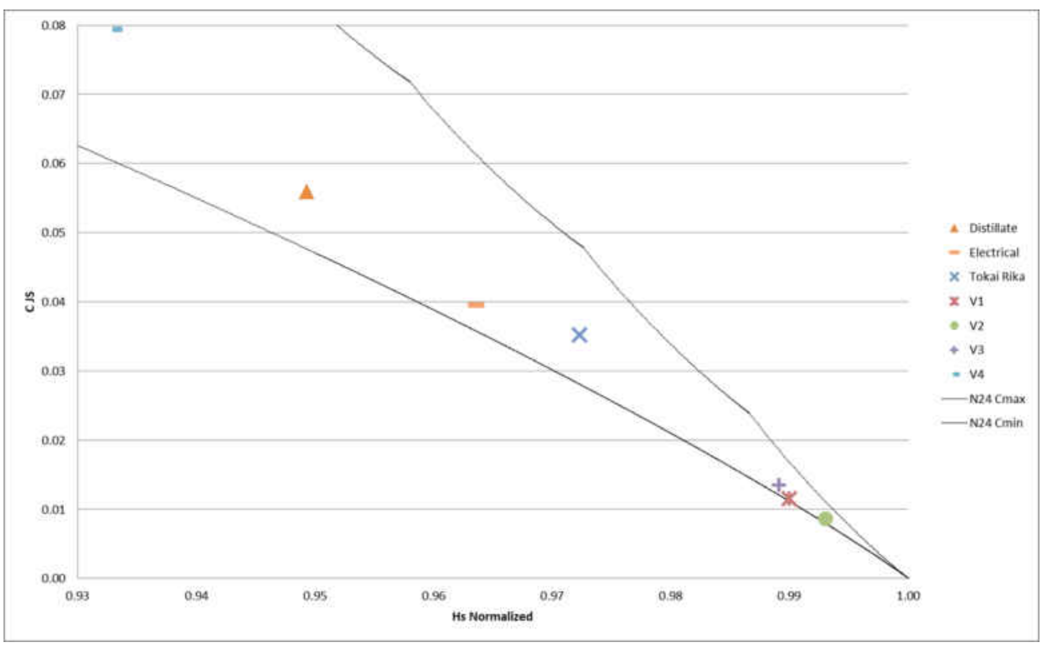


Figure 90. CECP close up for the empirical processes of interest.

Switching to V1, V2, and V3 displayed in Figure 90, these processes were fairly equivalent in terms of quantity of distinct tuples (23, 29, and 24 respectively). Comparing their pdfs in Figure 91 reveals that homogeneity was greatest for V2, then V1, then V3. As expected, this order corresponded with their increasing distance from (1,0) on the CECP.

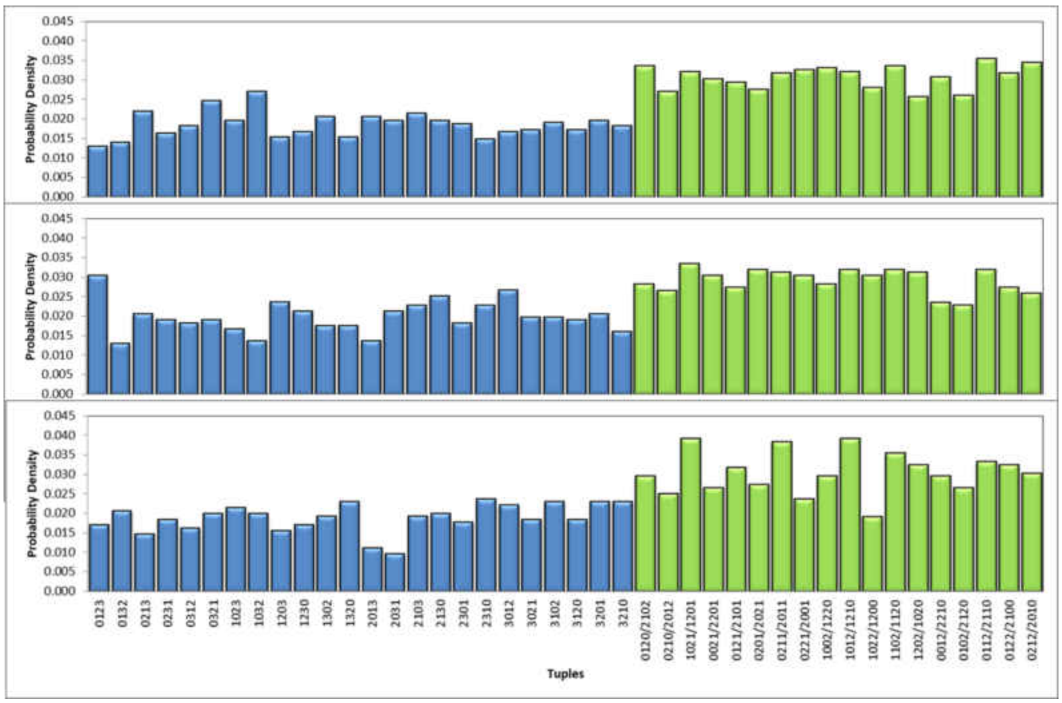


Figure 91. PDFs for V1, V2, and V3, in order top to bottom.

Observing V1, V2, and V3, the trend for quantity of distinct tuples was the opposite of the general trend seen overall. This observation, in coordination with the similar observation previously made about V4, suggests that PE-LE's utility to inform process improvement is based more on homogeneity than quantity of distinct tuples. Nonetheless, these results suggest that the quantity of distinct tuples could be tailored to the goals of the analysis when the measurement resolution can be chosen or modified.

The results described above provide evidence to support the conclusion that a methodology based on emerging structural complexity can provide information useful to direct the continued improvement of a Shewhart-stable process. The greater the process homogeneity, the greater the process improvement, and the closer the Jensen-Shannon complexity plot will appear to point (1,0) on the CECP. Processes plotted on the CECP can demonstrate time evolution corresponding with intended improvements and to facilitate comparisons among different processes. For example, the time evolution of various time series are presented in Figure 92 and Figure 93.

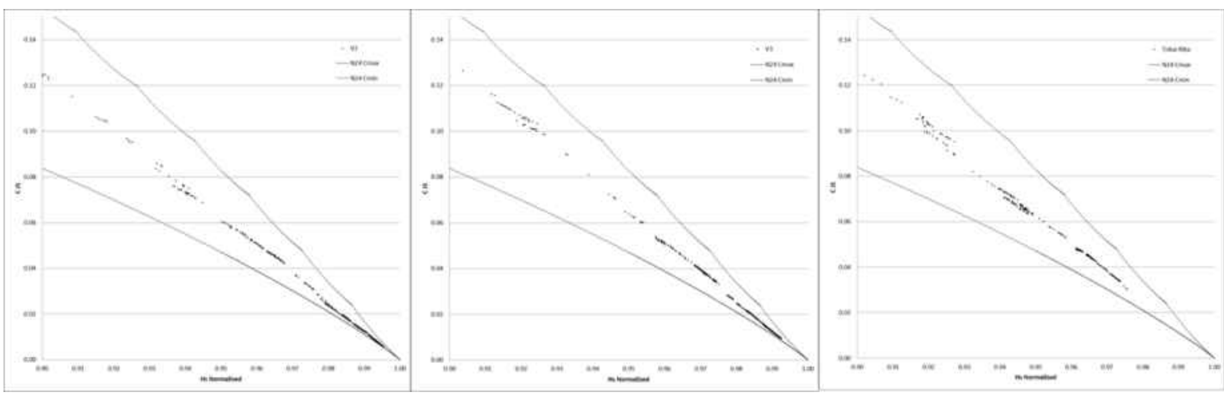


Figure 92. Example of C_{JS} time evolution plotted on CECP for V2, V3, and Tokai Rika time series from left to right, respectively.

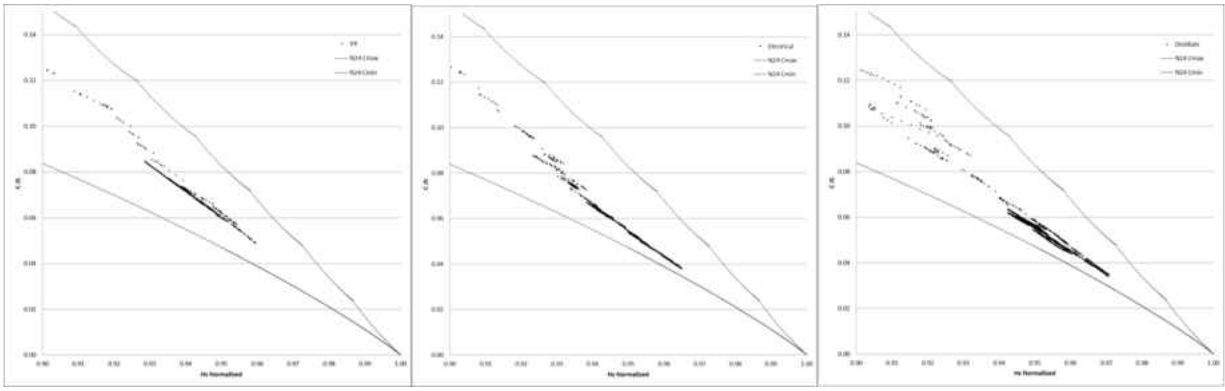


Figure 93. Example of C_{JS} time evolution plotted on CECP for V4, Electrical, and Distillate time series from left to right, respectively.

The Complexity-Entropy Causality Plane was considered the primary tool for generating results in this research and displayed process patterns providing the answer to the research question. However, an extension to the MPR-method was also explored to seek different insights for the same processes. The initial motivation was to seek compensation for some of the measurement resolution effect. This extended methodology was based on calculating relative changes for $\widetilde{H}_S$ and C_{JS} relative to maximally random equivalent time series.

Log Change Evaluations

This section provides results using three visualizations based on relative change evaluations. More specifically, log change equations were used to compare time series information relative to the maximum randomness condition for an equivalent time series. The word “equivalent” specifically means for the same number of measurements in the series and using the same quantity of distinct numbers.

Four points will assist with chart interpretation. First, since each time series is being compared to a random equivalent time series, a process that plots on the centerline would be as random as the comparator series. Thus, the further from the centerline, the less random the series. Second, the computation of Shannon entropy involves an accumulation of probability information, so the results naturally present no useful trends at the start of the accumulation but continue to gain relevance over time. Third, the foundation for the entropy calculations remains the permutation entropy- local effect (PE-LE) method, which means the changing probability distributions include the Forest tuples that contain identical values. Finally, because the underlying relative change calculation applies log change, the magnitude of change is consistent regardless of which side of the centerline the results appear.

The first visualization is the $\tilde{\tilde{H}}_S$ change chart, which does not possess direct utility for answering the research question. However, $\tilde{\tilde{H}}_S$ is used for one axis of the Entropy-Complexity Change Diagram (ECCD), so the $\tilde{\tilde{H}}_S$ change chart is presented to reveal how $\tilde{\tilde{H}}_S$ is operating independently. The second visualization is the $\tilde{C}_{JS}$ change chart, which was evaluated via pattern recognition for its potential to directly answer the research question. The final visualization is the ECCD, which was designed to compare the ongoing behavior of one or more time series in terms of entropy and structural complexity simultaneously.

$\tilde{\tilde{H}}_S$ Change Chart

This chart depicts the changing $\tilde{\tilde{H}}_S$, which has been normalized in two independent and different ways. The first normalization method was associated with permutation entropy (PE-

LE) when the Shannon entropy values were divided by H_{max} = log K, where K is the available number of distinct tuple types. This was considered a “static” normalization in that the normalizing factor did not change throughout the entire analysis. Specifically, for all of this research, the normalization factor was log(42) = 5.3923 bits. The second normalization method made a relative comparison of these $\widetilde{H}_S$ values in time series to the equivalent randomly generated time series using the log change calculation. This was considered a “dynamic” normalization in that the normalizing factor continues to change equivalently throughout the entire time series, based on a continuing accumulation of measurements in the series.

Figure 94 simultaneously depicts many of the processes evaluated during this research. The sine wave, being a periodic process with relatively little randomness, resides at a comparatively large distance from the centerline. The logistic map is a chaotic process, which is more random than sine and so resides closer to the zero line. The distillate series was unstable and, reviewing Figure 66, its pdf was not especially close to uniform, comprised of higher Parlitz tuples than Forest tuples along with two exceptionally high Parlitz tuples. The remaining series appear close together in this view, because depicting the sine wave yielded a wide y-axis. This is suboptimal for viewing, so Figure 95 focuses more closely on these remaining processes.

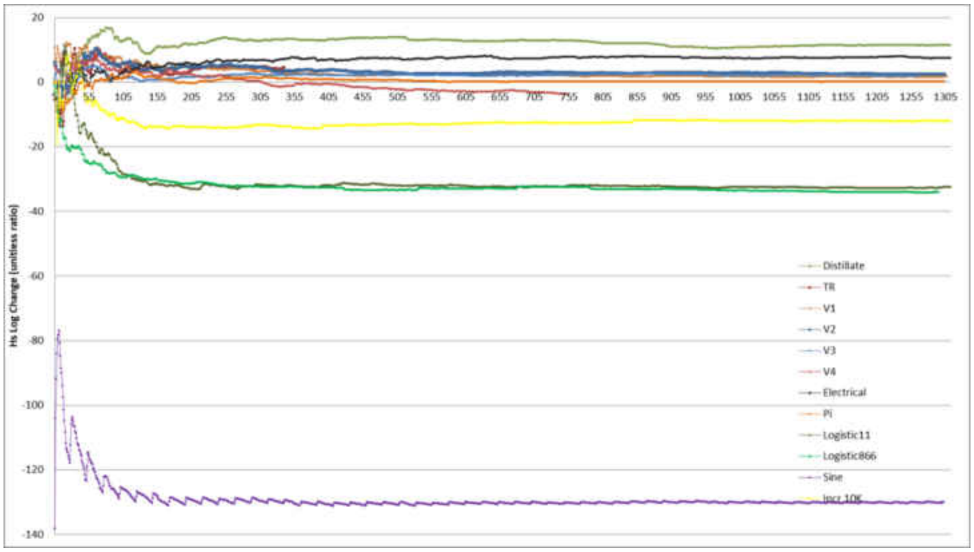


Figure 94. $\widetilde{\widetilde{H}}_S$ change chart for various time series processes.

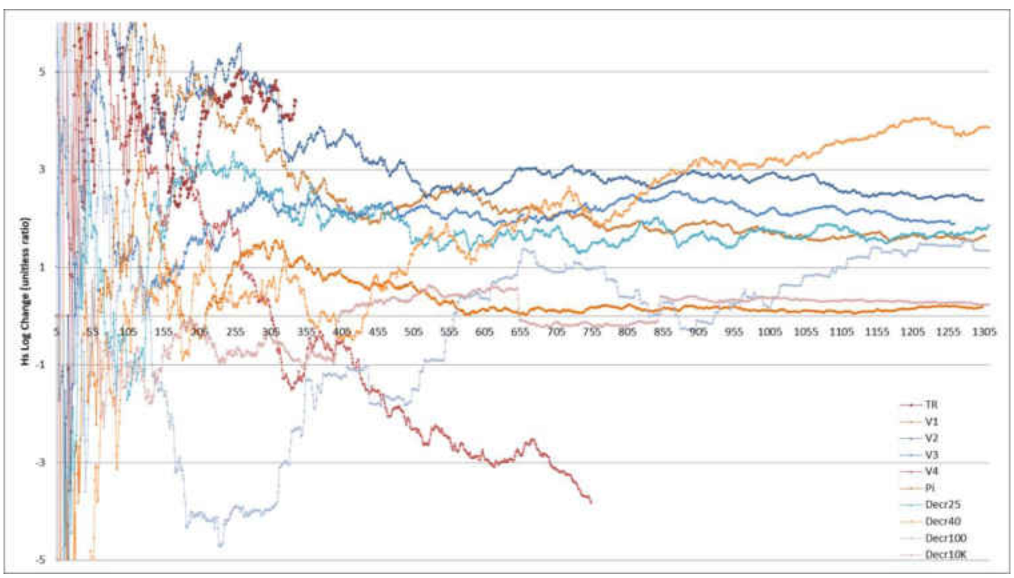


Figure 95. $\widetilde{\widetilde{H}}_S$ change chart for various time series processes.

Figure 95 is too busy to facilitate much pattern recognition and so further focusing will be accomplished in the next few figures. However, it is evident that at least 500 tuple calculations were necessary before many series accumulated enough information to begin to settle around a relative magnitude value. In Figure 96, the focus will shift to the four Decr# pseudorandom time series.

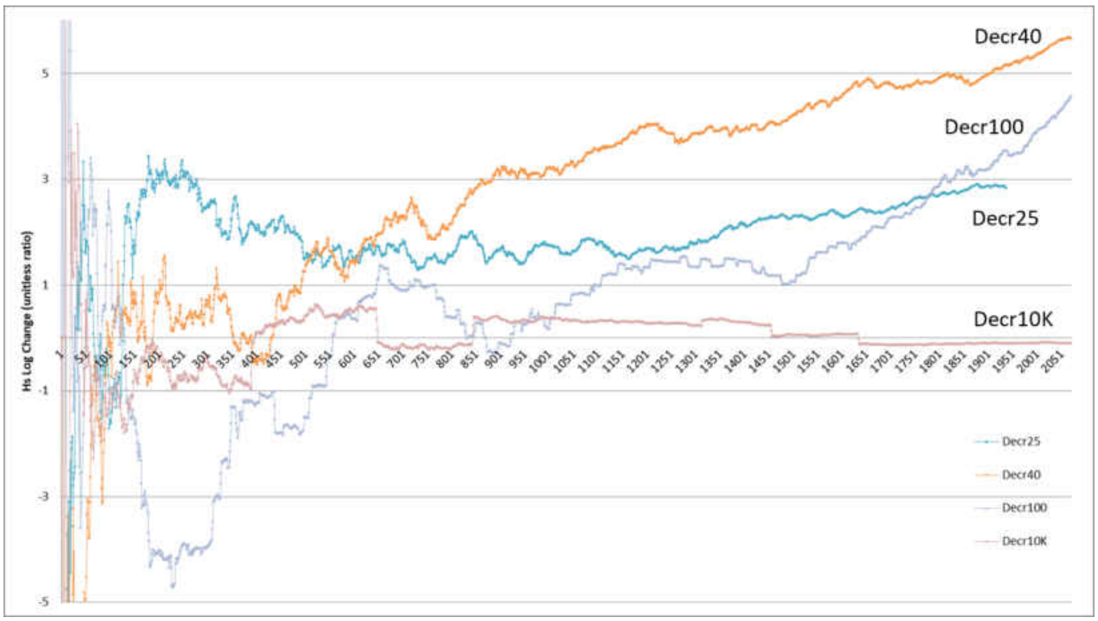


Figure 96. $\tilde{\tilde{H}}_S$ change chart for pseudorandom Decr# time series.

Figure 96 was built to focus on the behavior of Decr25, Decr40, and Decr100 because they were similar in having run lengths of 2,200 values and decreasing value ranges every 220 iterations, as presented in Table 27. These three series do not appear to have settled into a value range yet and are probably still "seeking" their representative relative magnitude value. This observation suggests that run lengths greater than 2,200 are desired when the quantity of distinct values is relatively low (~100 or less) and the process is changing considerably. Decr10K quickly settled into the vicinity of maximal randomness at the centerline, which is logical because the first 1000 values were based on a random value range of 1-10,000 and the next 1000 values were based on a random value range of 1-9,000. The small step changes are of unknown origin.

The four funnel experiments and the Tokai Rika (TR) series are presented in Figure 97. TR appears to have insufficient accumulation of probability information to achieve a stabilized value. Funnel experiments 1, 2 and 3 all appear to have stabilized after the first 500 or so calculations and in the same region near two magnitudes of relative change. The actual difference in magnitude between processes was minor enough that no distinguishing characteristics were evident.

Experiment 4 is clearly different from the others. Referring to Figure 77, V4's pdf was the only pdf evaluated during this research that had much higher probability densities for the identical value tuples (Forest, Red & Wizard) than the traditional Parlitz tuples. Additionally, reviewing Figure 54, although V4 was Shewhart-unstable, it's run chart reveals many identical values, especially in the second half of the experiment at lower drop heights. Although the progression in this change chart is steadily away from maximum randomness, it would not be prudent to offer any conclusions about V4. This is because the analysis is based on fewer than 800 tuples, and because V4 was such a unique series compared to the others.

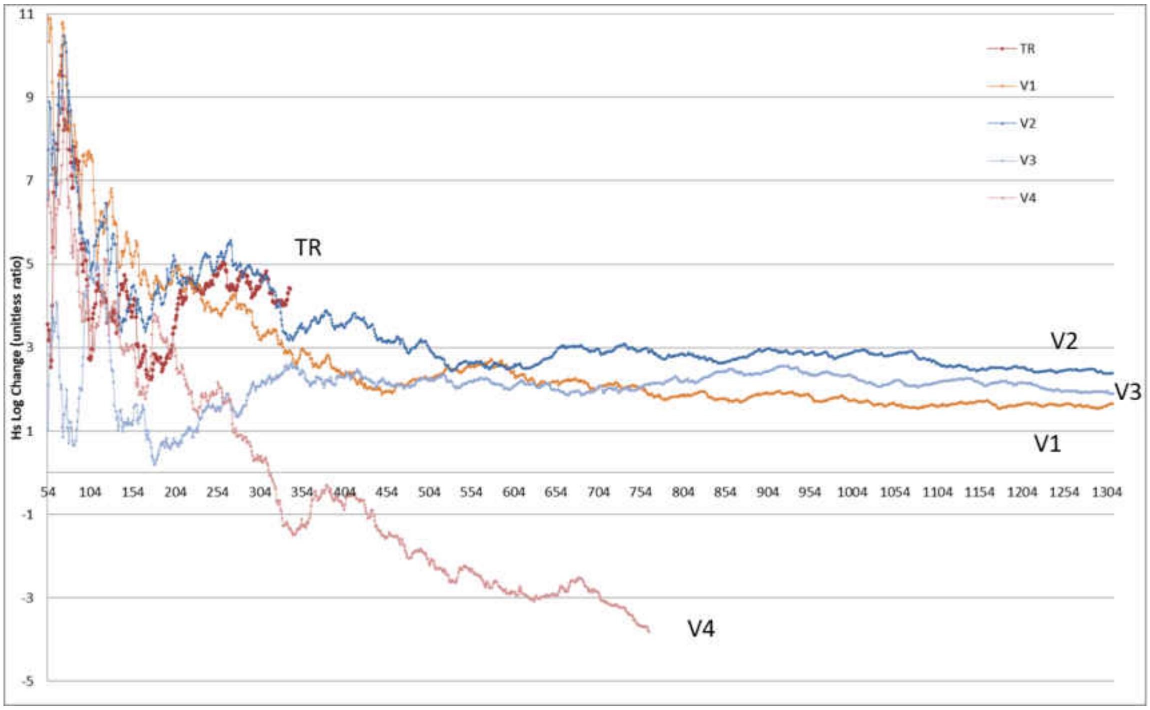


Figure 97. $\tilde{\tilde{H}}_S$ change chart for funnel and TR series.

<u>$\tilde{C}_{JS}$ Change Chart</u>

Unlike the $\tilde{\tilde{H}}_S$ change chart, the $\tilde{C}_{JS}$ change chart provides information that could be directly relevant for answering the research question. The normalized structural complexity that this chart plots also provides one dimension for the dynamic Entropy-Complexity Change Diagram. Three considerations will aid interpretation of this chart. First, the zero reference line represents maximally random Jensen-Shannon complexity, not entropic randomness as seen in the $\tilde{\tilde{H}}_S$ change charts. This may be difficult to conceptualize because C_{JS} presents emerging process dynamics based on the interplay between randomness *and* disequilibrium, both of which have been normalized using alternate methods. At the micro level, the bumpiness of the series plot can be considered an indication of changing local structural complexity. At the macro level, the overall progression of the series can be considered an indication of the evolving process complexity. The second point: Since C_{JS} is still partially dependent upon the computation of Shannon entropy, the results naturally present no useful trends at the start but continuously gain relevance over time. Third is a point that bears repeating: The underlying relative change calculation applies log change, which means the magnitude of change is consistent regardless of which side of the centerline the results appear.

Five charts are presented in this section. The first chart simultaneously depicts all of the processes evaluated. The next two charts home in on Tokai Rika's structural complexity, to seek patterns that can be correlated with known process events. The fourth chart depicts four comparator processes that were not improved, to look for common patterns of behavior. The last chart depicts the four funnel experiments, which were improved, to again look for common patterns of behavior.

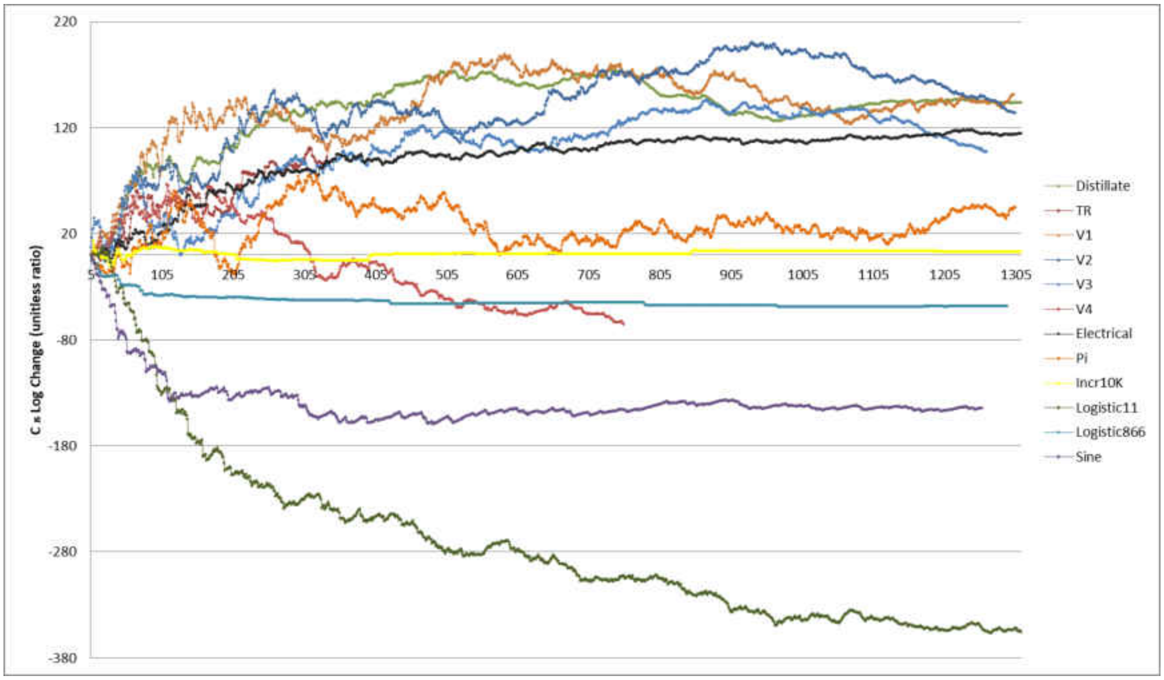


Figure 98. $\tilde{C}_{JS}$ change chart for various processes except Decr#.

The global view in Figure 98 reveals how differently these processes evolved in terms of normalized structural complexity. Perhaps some additional knowledge could be gleaned from this plot, but the processes are difficult to discern individually in this overlay. As such, other figures in this section will focus on certain process types to ease the analysis. Figure 99 and Figure 100 focus on analysis of the Tokai Rika process.

Figure 99. $\tilde{C}_{JS}$ change chart to facilitate the Tokai Rika investigation.

In Figure 99, a maximum run length of only 380 calculations is presented since, per Table 24, the Tokai Rika series presented only 337 tuples for analysis following parsing. The first 380 points for V2 and V4 were included to facilitate comparison. Seeking pattern correlations, there probably isn't much to be gleaned from this plot. Previously, in Figure 97, the TR processes did not start to stabilize until about 350 tuples. Nonetheless, the investigation will continue in Figure 100 since Tokai Rika included well-documented process events that could facilitate correlational analysis.

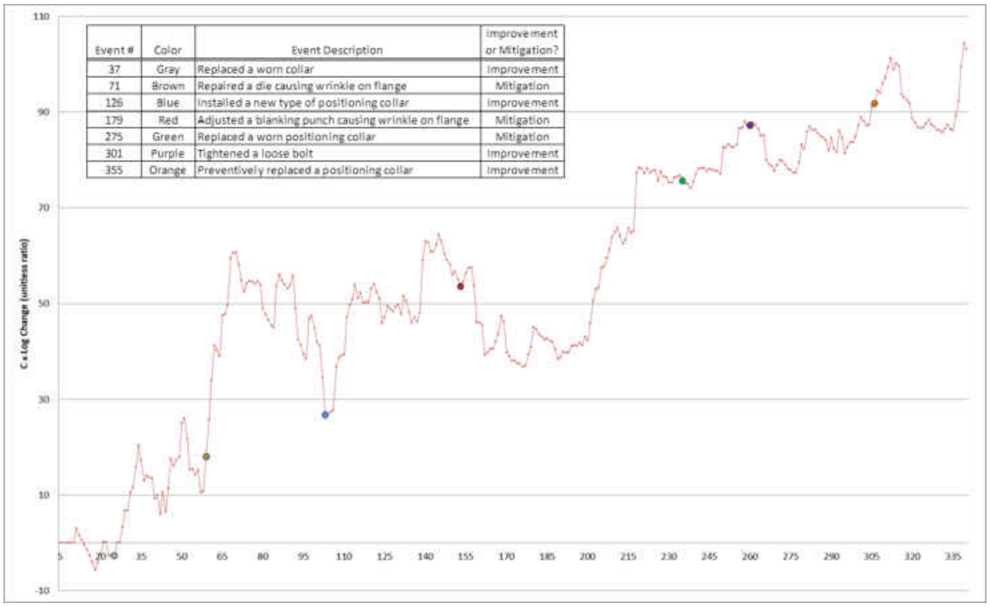


Figure 100. $\tilde{C}_{JS}$ change chart of the Tokai Rika process. Larger colored dots correspond to significant process events listed in the included table. Events are correct sequentially and based on the color coding, although event numbers no longer match up exactly due to tuple parsing. Data reused with the permission of the Tokai Rika intellectual property division.

The larger, colored data points in Figure 100 correspond with the process events described in the inset table. The focus is to correlate improvements, or even just significant events in general, with the behavior of the emerging structural complexity. No consistent patterns are noted. Globally, it appears that the complexity is diverging from the maximally random complexity baseline, but the run length is too short to put much credence into any trend observed. Overall, although Tokai Rika is a very stable and well-documented process, investigation of these data did not help to answer the research question. The next evaluation in Figure 101 will considered comparator processes that contributed considerably more tuples for analysis.

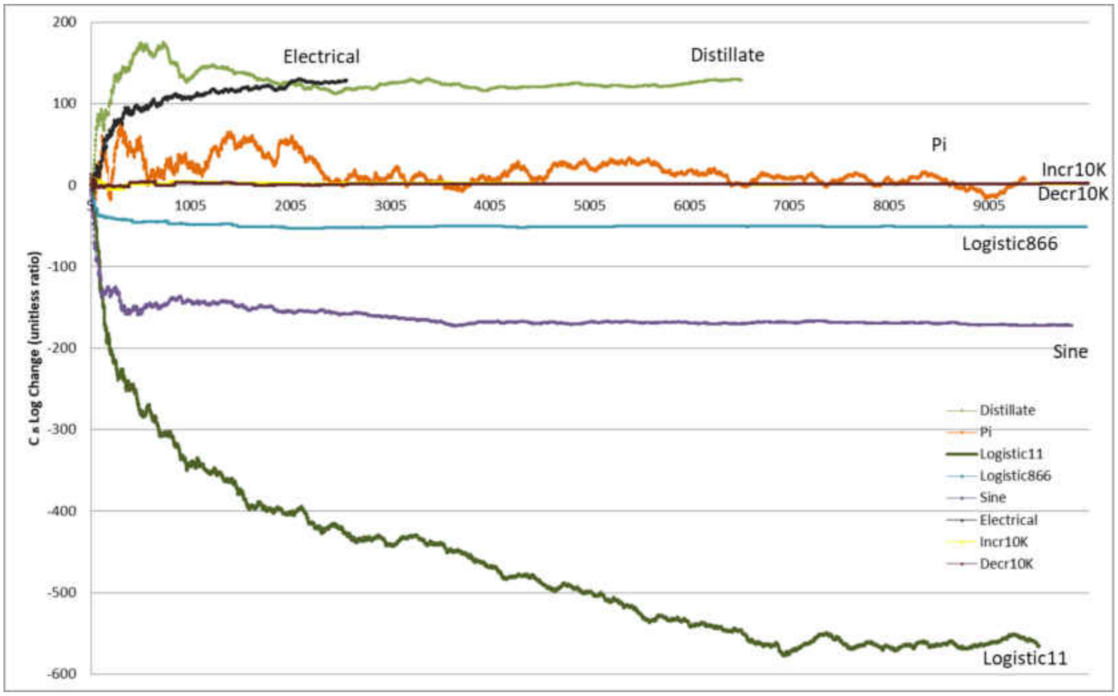


Figure 101. $\tilde{C}_{JS}$ change chart for comparator processes.

All of the processes depicted in Figure 101 were not being improved (in terms of variation), except Decr10K. Additionally, all but Electrical started with 10,000 measurements before parsing. Most of these series settled out at a gross distance from random equivalent complexity within about 1,000 tuples, except Pi and Logistic11. Logistic 11 continuously trended away from the baseline until potentially settling out at about 7,000 tuples. This behavior is consistent with the results from Figure 86, which revealed that Logistic 11 demonstrated the most Jensen-Shannon complexity of any of the 22 processes evaluated. Also, Incr10K plotted almost identically coincident with the Decr10K series, which is consistent with the results from Figure 88. The consistent pattern for all of these processes, save perhaps Electrical, is that they eventually settled out in a region of representative magnitude. More specifically, they settled out at a distance that represents the magnitude of relative change with respect to an equivalent random time series. In the case of Electrical, the plot seemed to still be rising and it's useful to also consider that a total of only 2,566 tuples were presented for analysis.

Regarding Pi, on the macro level it remained in the vicinity of the baseline as expected, since it is quasi-random. However, on the micro level, it exhibited significantly more relative complexity than might be expected. No explanation for this behavior is offered, but the CECP plot of Pi is presented in Figure 102 to compare both of these display methods in terms of the detail revealed by the time evolution of the process.

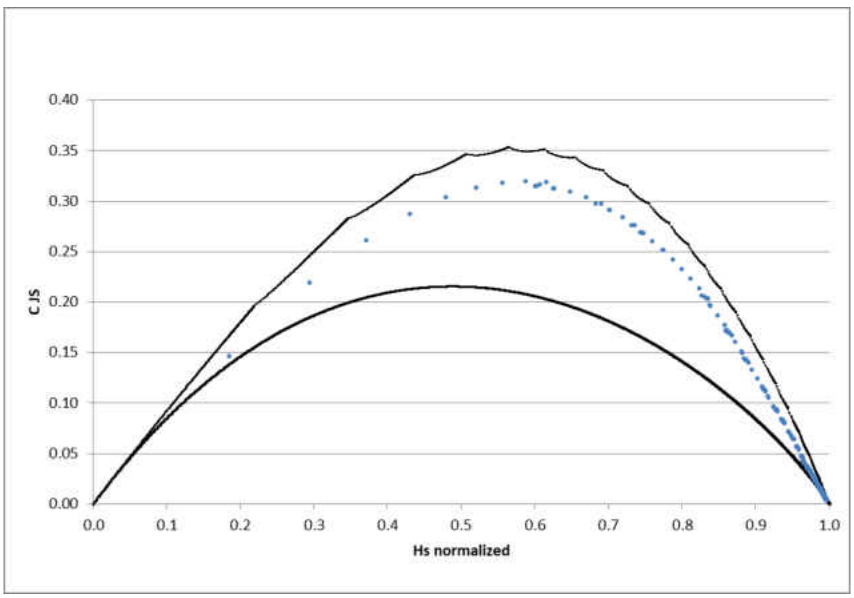


Figure 102. CECP plot for Pi.

Next, Figure 103 will present the results for the funnel experiments and some of the pseudorandom comparator processes.

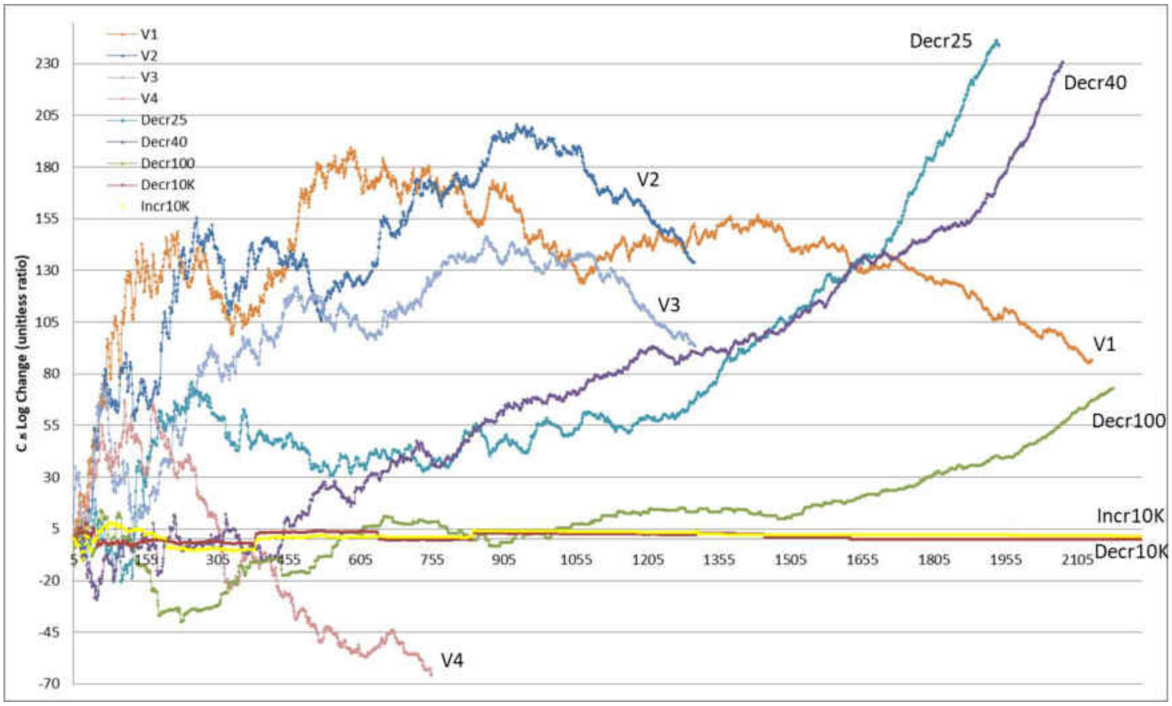


Figure 103. $\tilde{C}_{JS}$ change chart for vertical funnel experiments and pseudorandom comparator processes.

Figure 103 presents run lengths of 2,200 calculations or less and facilitates three observations. First, V4 is very different from the other processes, ostensibly due to the exceptional prevalence of identical value tuples. Second, V1, V2, and V3 all follow a trend in the later stages of process evolution toward the maximum random complexity baseline. Third, Decr25, Decr40, and Decr100 all follow a trend of process evolution away from the maximum random complexity baseline.

Although V1, V2, V3, Decr25, Decr40, and Decr100 all demonstrated decreasing *variation* in their original, unmanipulated time series, the opposite trends in Figure 103 between empirical and pseudorandom processes suggest further investigation. Clearly, the funnel processes V1-V3 are all decreasing in relative complexity whereas the pseudorandom Decr#

processes are all increasing in relative complexity. When all six of their 42-tuple pdfs are compared, it is apparent that the three funnel experiment pdfs present greater density among the Forest tuples than the Parlitz tuples, whereas the three Decr# processes present the opposite density prevalence. This result seems to confirm that variation is no longer the primary consideration when the PE-LE method is employed to evaluate C_{JS}. Instead, the interplay between randomness and disequilibrium is at least partially explained by the relative density prevalence between Forest and Parlitz tuple types. Ultimately, it seems that the quantity of distinct values is a significant consideration for the PE-LE method. Therefore, the most equivalent comparison between processes types appears to be when compared processes employ approximately the same quantity of distinct values. For the processes displayed in Figure 103, V1, V2, V3, and Decr25 all had comparable quantities of distinct values. Next, Figure 104 focuses on characterizing the funnel experiments.

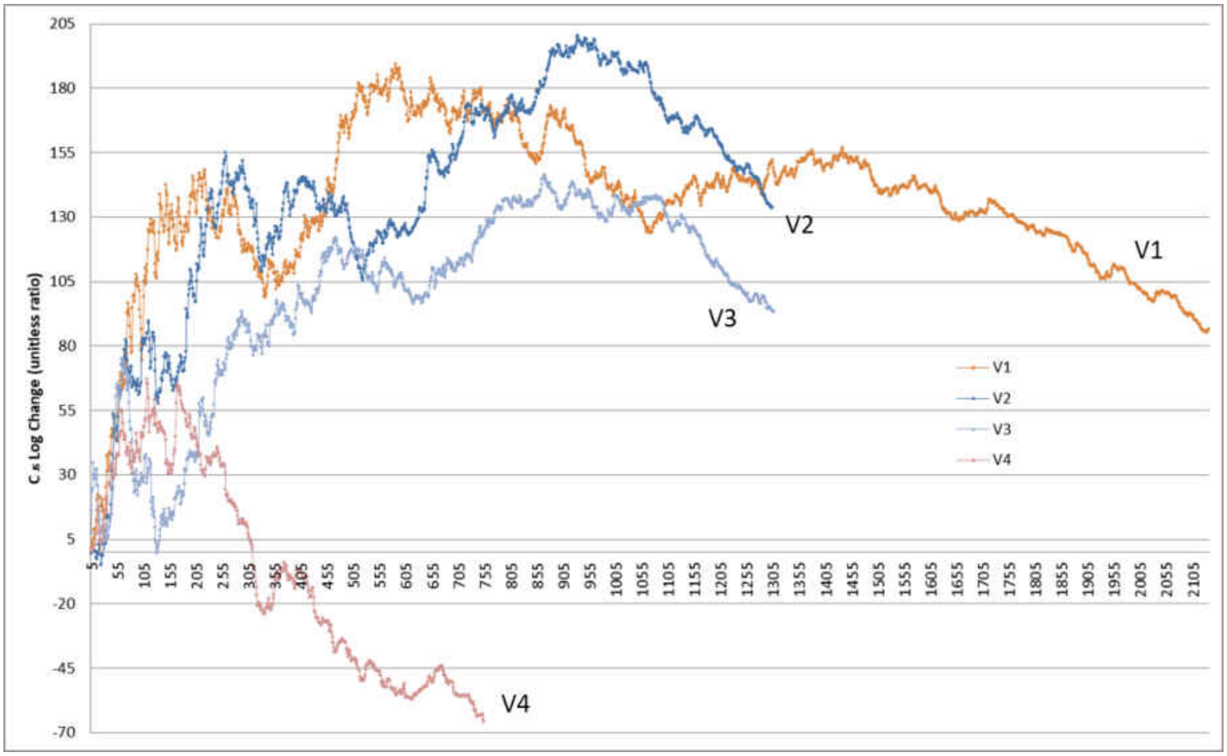


Figure 104. $\tilde{C}_{JS}$ Change Chart for vertical funnel experiments. V1, V2, and V3 represent improved Shewhart-stable processes. V4 is an improved Shewhart-unstable process.

In Figure 104, V1, V2, and V3 represent Shewhart-stable processes that were improved repeatedly over the course of the time series. V4 represents a Shewhart-unstable process that was also improved repeatedly over the course of the time series, but also suffered from significantly more parsing of Red & Wizard tuple types. “Repeatedly” means that every 50th data point, an event occurred that usually (but not always) resulted in progression of the mean value toward the “less is better” target value of 1 inch, and a decrease of statistical variation about the mean. These statistics were presented in Figure 57. These variation reduction results appeared to correspond with the eventually decreasing relative structural complexity observed in the vicinity of ~900 tuples for V1-V3. If the pdfs were observed ab initio and continuously for

V1, V2, and V3, they would initially have higher Parlitz tuple densities which would gradually be traded for higher Forest tuple densities as process improvement continued.

It is possible to be more specific about the observed decreasing progression toward the random complexity baseline. That is, V1 started around 600, V3 started around 900, and V2 started around 1,000. Naturally, the next question is why these three processes took differing amounts of time to progress toward the baseline when they were so similar. Reviewing their pdfs and local effect data, they each had similar density allocations among the four tuple categories of Parlitz, Forest, Red & Wizard. Also, their tuple and local effect pdfs all looked similar. However, a serial correlation was evident between the inflection point tuple and the measurement resolution. Regarding the quantity of distinct numbers, V1 had 23, V3 had 24, and V2 had 29. The correlation between trend start and distinct numbers is weak with R-sq(adj) = 27%, as depicted in the linear regression plot in Figure 105. It therefore remains undetermined why these three processes took varying times to turn toward the baseline.

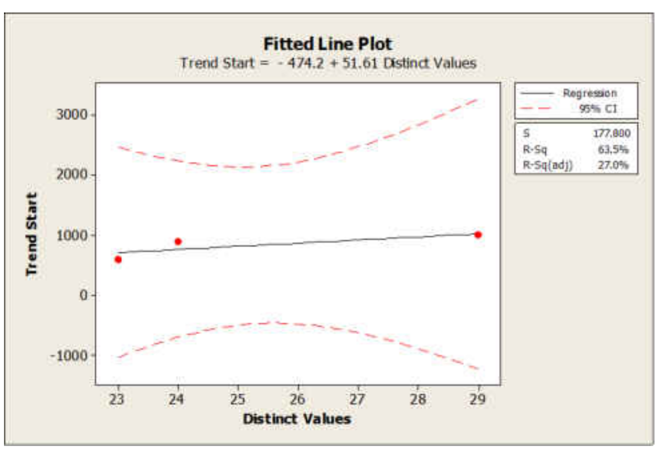


Figure 105. Linear regression analysis to correlate decreased relative complexity trend with quantity of distinct tuples for V1, V3, and V2, from left to right.

Overall, results provided mixed evidence that the $\tilde{C}_{JS}$ change chart methodology can present information useful to direct the continued improvement of a Shewhart-stable process. The best results seem to be correlated with 1) Smaller quantities of distinct values except that fewer than ~10 distinct values for D=4 may yield excessive statistically complete parsing (as demonstrated for the V4 series). 2) Longer runs to allow sufficient accumulation of probability density information for the 42-tuple pdf at D=4. V1, V2, and V3 demonstrated that the lower practical limit seems to be around 1000 tuples. Electrical, Decr25, Decr40, and Decr100 all demonstrated that even 2000 tuples can be insufficient depending upon process characteristics. Logistic11 demonstrated that about 7,000 tuples were required before the relative change potentially settled out near a gross magnitude value. These results in combination suggest a minimum of approximately 5,000 tuples would be desirable for non-chaotic process types. The next section combines the results of the previous two sections to demonstrate a simultaneous graphical presentation of both change chart results.

Entropy-Complexity Change Diagram

In the previous two sections, the relative change calculations for both $\tilde{\tilde{H}}_S$ and $\tilde{C}_{JS}$ were linked to temporal causal dynamics. These results can be displayed simultaneously on the Entropy-Complexity Change Diagram. This depiction can be used to graphically track the changing randomness and structural complexity for a process, and to facilitate comparisons among the different processes. Approximately the first 10-30% of calculations were abandoned for each process to facilitate depictions of the parts of the time series where values began to settle out at their gross relative magnitudes. The most random processes naturally plotted very close to the origin. Therefore, all of the R# processes appeared as a small dot at the origin and hence are not plotted. The remaining 16 of the original 22 processes are plotted together in Figure 106.

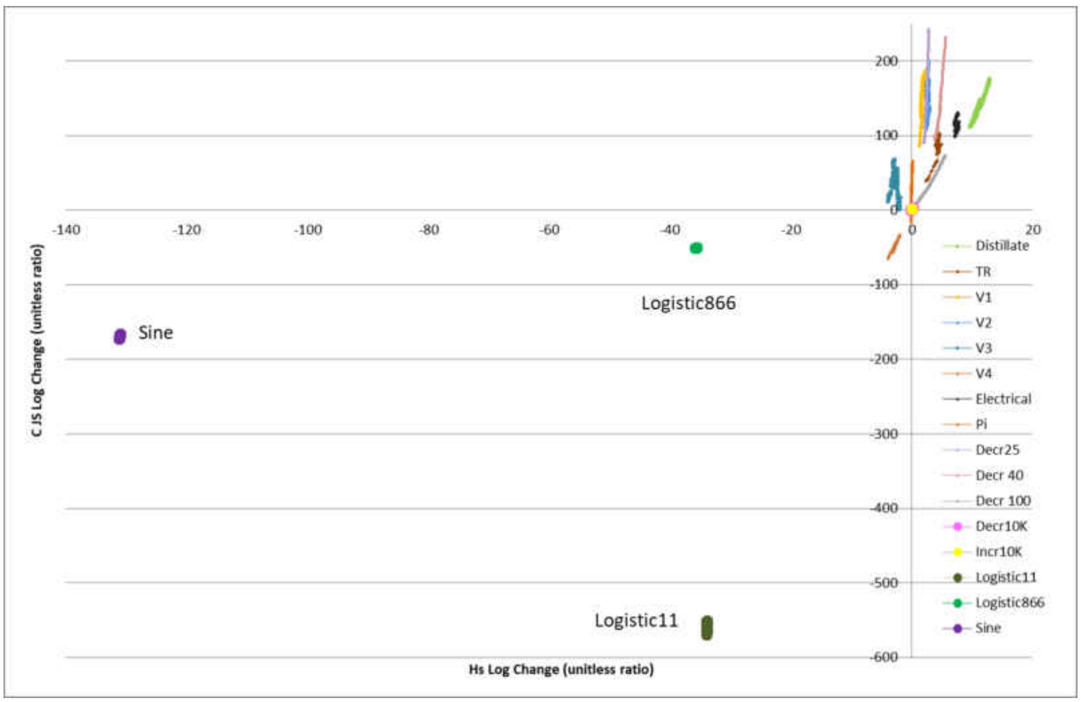


Figure 106. Entropy-Complexity Change Diagram for all processes except R#.

Figure 106 reveals the relative positions of these 16 processes. Of note, the identical logistic map with different quantities of distinct numbers (11 vs. 866) retained the same relative entropy value in either case but demonstrated dramatically different relative structural complexity values. The different quantities of distinct numbers naturally yielded many more

Forest tuple types for Logistic11 than Logistic866 (3007 vs. 43) and parsed Red/Wizard tuples (364 vs. 0).

Sine demonstrated highly non-random relative entropy, as might be expected given its periodic nature. Finally, Incr10K appears at the origin given its random nature. Removing these four processes from the depiction facilitates closer inspection of the remaining 12 processes in Figure 107.

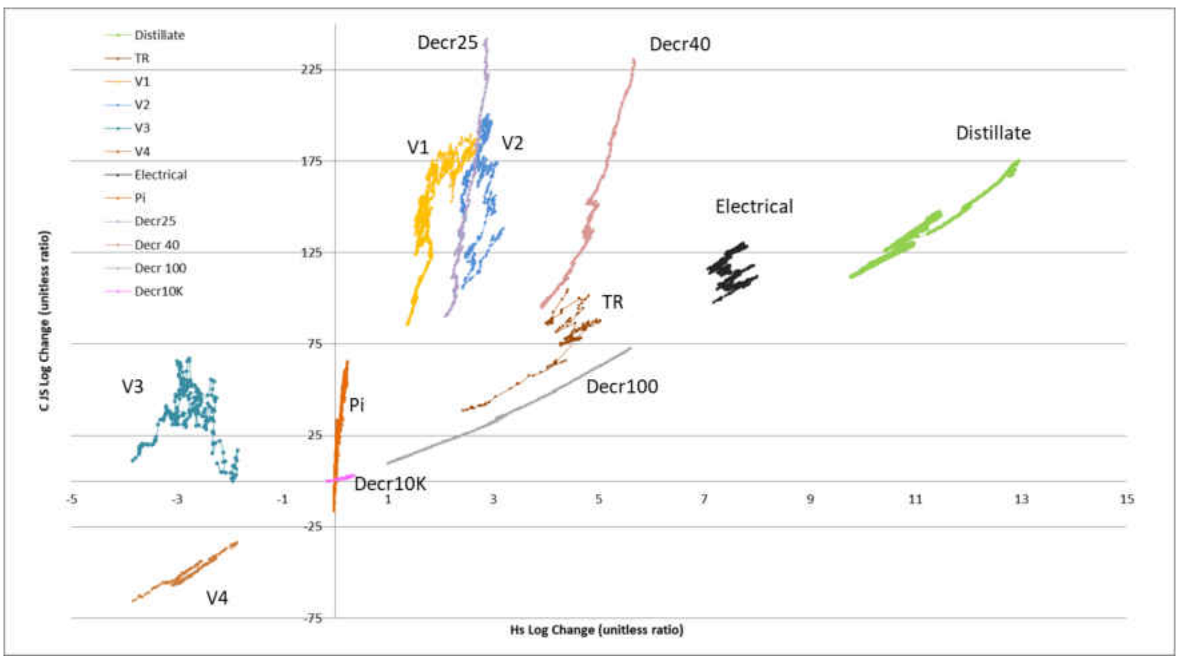


Figure 107. Entropy-Complexity Change Diagram for processes of interest.

In Figure 107, the following processes all demonstrated a progression away from the origin as the calculations were plotted: V4, TR, Decr25, Decr40, and Decr100. Electrical and Distillate settled approximately in place over time. The following processes demonstrated a progression toward the origin as the calculations were plotted: V1, V2, V3, Pi and Decr10K. For the random processes Pi and Decr10K, this progression was considered correlated to the accumulation of information to achieve maximum random complexity. For V1, V2, and V3, this progression was correlated to process improvement in terms of a pattern of decreasing relative structural complexity. Overall, the results for the ECCD provided evidence that the $\tilde{C}_{JS}$ change chart methodology can present information useful to direct the continued improvement of a Shewhart-stable process.

CHAPTER FIVE: CONCLUSION

Three conclusions are presented for this research. First, the application of structural complexity C_{JS} is discussed as it was applied to answer the research question. Next, challenges to prevailing assumptions about absolute randomness are reviewed. Finally, the utility of the PE-LE method is presented.

Conclusion One- The Research Question

The most significant contribution of this research was associated with answering the research question: Can a methodology based on emerging structural complexity provide information useful to direct the continued improvement of a Shewhart-stable process? Three pattern discovery techniques based on the temporal plotting of Jensen-Shannon complexity provided evidence that corresponded with varying levels of process improvement: The Complexity-Entropy Causality Plane, the $\tilde{C}_{JS}$ change chart, and the Entropy-Complexity Change Diagram.

The Complexity-Entropy Causality Plane (CECP) has been rigorously tested with a multitude of time series processes since its creation by Rosso, et al. in 2007. Therefore, Shewhart-stable processes such as V1, V2, and V3 that migrated concurrently toward increasing randomness and decreasing structural complexity were considered to be providing evidence of improvement in accordance with a reasonably well-established methodology. Furthermore, Shewhart-stable processes that settled out closer to the lower right corner of the CECP provided evidence corresponding with being more improved than processes that were further away. This provided a visual pattern correlation between decreasing variation, decreasing structural complexity and increasing randomness.

A number of processes were also plotted on the $\tilde{C}_{JS}$ change chart, but vertical funnel experiments V1, V2, and V3 were the only empirical processes that demonstrated a decreasing trend toward the maximally random structural complexity baseline. Although this was encouraging because it appeared to follow a logical pattern of behavior, it could not be considered reliable evidence corresponding with process improvement for at least three reasons. First, other comparable Shewhart-stable processes that were improving in terms of variation reduction (Decr25 and Decr40) did not display the same pattern over time. Yet, these two processes were also fundamentally different in that they were pseudorandomly generated. Second, V1, V2, and V3 were very similar in terms of measurement resolution and length of time series, whereas the same results for highly dissimilar time series would support a more confident conclusion. Finally, V1, V2, and V3 presented a relatively low number of permutation entropy tuples for analysis (around 2,000 each), whereas greater confidence could be lent to the results had these processes presented say, 10,000 or 15,000 tuples each. Overall, the evidence from the $\tilde{C}_{JS}$ change chart was not supported sufficiently to draw any reliable conclusions.

Although the $\tilde{C}_{JS}$ change chart comprises one of the two axes of the Entropy-Complexity Change Diagram, results from this pattern discovery technique were slightly more encouraging. This is because the concurrent incorporation of $\tilde{\tilde{H}}_S$ log change information caused improved processes to appear closer to the origin than unimproved processes.

However, the plot for the first 10,000 digits of Pi was curious. Being highly stochastic, the first 10,000 digits of Pi plotted close to the origin, as expected, but also demonstrated more overall $\tilde{C}_{JS}$ log change variability in the first ~2500 calculations than in the remaining ~7500 calculations. Each process demonstrated unique run lengths before stabilizing around a general magnitude, but it was not discerned why Pi would take so long. The reason behind this behavior might be worthy of additional investigation.

Moreover, processes identical in all respects other than measurement resolution plotted in significantly different regions, suggesting that measurement resolution could be optimized in conjunction with this technique to support a specific research goal, such as the continued improvement of a Shewhart-stable process.

Conclusion Two- Stable Process Randomness

The more Shewhart-stable a process was, the closer it plotted to $\widetilde{H}_S$=1.0. These normalized Shannon entropy calculations provided evidence for varying levels of randomness, which consequently implies a correspondence with nonrandom dynamics. That is, the higher the $\widetilde{H}_S$ value, the higher the level of randomness, and the lesser the influence of nonrandom dynamics. These results challenged the prevailing assumption that a Shewhart-stable process should be represented as a constant system of chance causes. Instead, Shewhart-stable processes could be more accurately characterized as inconstant systems of mostly chance causes.

Additionally, the values for $\widetilde{H}_S$ gradually increased as various Shewhart-stable processes were improved, demonstrating increasing homogeneity and challenging the assumption that improvement of Shewhart-stable processes always represents tampering. Finally, $\widetilde{H}_S$ measurements for two highly stochastic processes (10,000 digits of Pi and 10,000 digits of a pseudorandom generated process) provided evidence to support the conclusion of the Ramsey theory that perfect randomness ($\widetilde{H}_S$=1.0) is an idealized impossibility.

Conclusion Three- The PE-LE Method

The final research contribution is focused on the PE-LE method, which was devised to mitigate identical values in permutation entropy tuples by allocating tuple types in accordance with the local effect empirically observed in tuples. The PE-LE method was axiomatically validated based on the assumption that appropriate results for stochastic processes implies that results will be appropriate for all process types, by extension. However, relatively few diverse processes were actually tested since this research was focused on Shewhart-stable processes (which naturally tend to be stochastic). Complexity-Entropy Causality Plane (CECP) plots for processes evaluated with PE-LE plotted comparably to CECP plots in the literature for similar processes evaluated with traditional permutation entropy methods. Furthermore, resulting $\widetilde{H}_S$ and C_{JS} calculations did not demonstrate significant divergence from similar processes described in the literature.

Nonetheless, the PE-LE method was unquestionably imperfect. The addition of tuple types (e.g., 42 instead of 24 for D=4) changed the resulting $\widetilde{H}_S$ and C_{JS} calculations compared with the traditional permutation entropy methodology. These variations in calculation results corresponded with the prevalence of identical value tuples, the quantity of distinct values made available for analysis (measurement resolution), and the number of Red & Wizard tuple types parsed from the analysis. In general, a review of $\widetilde{H}_S$ and C_{JS} calculations suggested that the most relevant comparisons were among processes with similar measurement resolutions.

PE-LE's inclusion of identical value tuples appeared to include more causal information in the analysis than some other methods, especially for highly stochastic processes at low dimensionality. Additionally, it appeared that measurement resolution could be tailored when using PE-LE to better support research goals. For instance, lower measurement resolution appeared to increase the *sensitivity* of the analysis to changing structural complexity in part by including more identical value tuple information that would otherwise be hidden or abandoned. It is hoped that the PE-LE method developed in this research can be improved upon, or at least that it might stimulate similar ideas, as experts in this field continue to advance mitigations for identical values within tuples.

Limitations

Researcher Experience Concern- The author of this exploratory research had no prior experience applying permutation entropy, the MPR-method, or structural complexity calculations. Therefore, it is possible that an experienced scholar would find faults within some of the associated analyses and conclusions.

Previous Research Concern- No studies were discovered that measured changing structural complexity (such as Jensen-Shannon complexity) to inform the continuous improvement of an empirical time series, as is commonly pursued in industrial engineering process control applications. A sizeable body of research is available in the literature despite the relatively recent introduction of permutation entropy in 2002 and the MPR-method in 2006. Although a few studies were discussed in the Literature Review that used permutation entropy methods for process control applications, the next steps were not taken to also evaluate structural complexity. Because no directly comparable research was discovered, it is difficult to ascertain whether the results of this exploratory research will have much merit in the field of quality control.

Sampling Concern- The four vertical funnel experiment time series datasets were most relevant to this research, but were comprised of only 1,550 to 2,550 measurements. After applying the methodologies to analyze structural complexity, it became apparent that many of the processes (to include even Pi) displayed higher variability in the structural complexity values within the first ~2500 calculations before "settling out". Ostensibly, this effect is related to the accumulation of probability information needed to build a stable pdf comprised of 42 distinct states. Similarly, the initial hope was that the Tokai Rika dataset, being well-documented, would facilitate the discovery of correlations between complexity patterns and known improvement events. However, 379 measurements were ultimately considered an insufficient analytic basis for this research methodology.

Selection Bias/Diversity Concerns- Only 11 empirical and 11 pseudorandom processes were evaluated in this research. Of these, only three empirical processes (V1-V3) were improved-Shewhart-stable- the type most relevant to the research question- and all three were fairly similar to each other in terms of data variability and number of measurements. Also, there was little diversity in the comparator processes. For example, only one process was periodic and only one was chaotic. Thus, a more comprehensive study may yield contrary conclusions.

Data Collection Concerns- The vertical funnel experiments demonstrated some excess variability and were not perfectly Shewhart-stable. Based on the stability ratio (SR) test results, they were deemed sufficiently Shewhart-stable for the purposes of this research (except V4). Other processes that are more stable might yield different results.

Complication of Methodology Concern- The calculation methodology to achieve the Complexity-Entropy Causality Diagram plot was significantly more involved than developing a control chart. The $\tilde{C}_{JS}$ change chart and Entropy-Complexity Change Diagram were more elaborate still. This complication may limit their utility in some applications.

PE-LE Validation Concern- The PE-LE validation methodology was not sufficiently encompassing of other application scenarios to be universally applicable. For example, the PE-LE method was only researched for embedding dimension D=4 & time lag τ=1. Also, only six processes were examined relative to the validation. Thus, PE-LE method may yield different results in other scenarios or when applying other parameters.

Statistical Parsing Concern- The decision to parse Red & Wizard tuples was based on a pattern-following scheme for local effect results. Another researcher might find cause to retain some or all of these tuples in the PE-LE analytic method.

Future Research

Follow-on research could be designed to overcome some of the limitations defined above while simultaneously attempting to replicate results. For instance, improved-Shewhart-stable processes that have a larger number of measurements in the time series (such as >5,000 values) could be evaluated. Also, a greater diversity of process types could be evaluated. Additionally, results achieved using different methods to mitigate identical values in tuples could be compared with PE-LE results and with results achieved using the traditional permutation entropy method with no mitigations in place. Finally, the PE-LE method could be validated more thoroughly with inclusion of some or all of the Red & Wizard tuple types.

Ultimately, future researchers might apply certain aspects of the PE-LE method to develop a more perfect solution to the problems associated with identical values in permutation entropy tuples. Additionally, researchers might explore the concurrent use of emergence-based complexity measures alongside traditional reductionist methods to gain greater awareness of process behaviors, including for Shewhart-stable processes.

Near Real-Time Analysis Research

With the appropriate computer coding and sensing/measuring equipment, the methodology applied in this research could incorporate near real time process analysis. As each new value was included in the analysis, the pdf for the tuples would update, causing the entropy calculations to update, causing the Jensen-Shannon complexity calculations to update, ultimately causing the plots on diagrams to update. Process data projected in near real time onto the Entropy-Complexity Change Diagram (ECCD) would provide dynamic viewing of simultaneous changes in randomness and structural complexity, which could be used to monitor processes for unanticipated changes or for cause-and-effect analyses associated with intended process improvements.

Also, numerous processes could be viewed simultaneously on the same plot to explore potential correlations between processes. Zunino et al. (2012) provided a similar correlational study on the price evolution of commodities using the Complexity-Entropy Causality Plane, Figure 108. In the center window of the figure, the co-evolution of both variables was tracked iteratively as each time series value was incorporated, helping to define potential temporal correlations in commodity pricing. The Entropy-Complexity Change Diagram could evaluate the same data on a different basis (relative to the maximum randomness condition for an equivalent time series), potentially providing complementary insights.

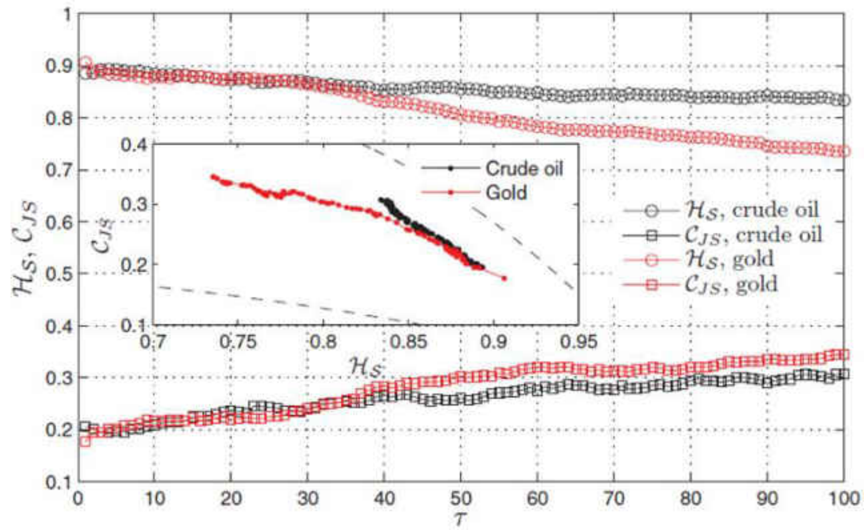


Figure 108. Correlation study between two variables based on commodity pricing. From *Distinguishing Chaotic and Stochastic Dynamics from Time Series by Using a Multiscale Symbolic Approach* by Zunino et al. Copyright © 2012 by American Physical Society. Reprinted with the permission of American Physical Society. All rights reserved.

Ordinal Network Research

Ordinal network diagrams could be applied to time series processes to gain insights associated with the *successions* of permutations, which could be correlated with process improvements. An example of this basic methodology is provided in Figure 109.

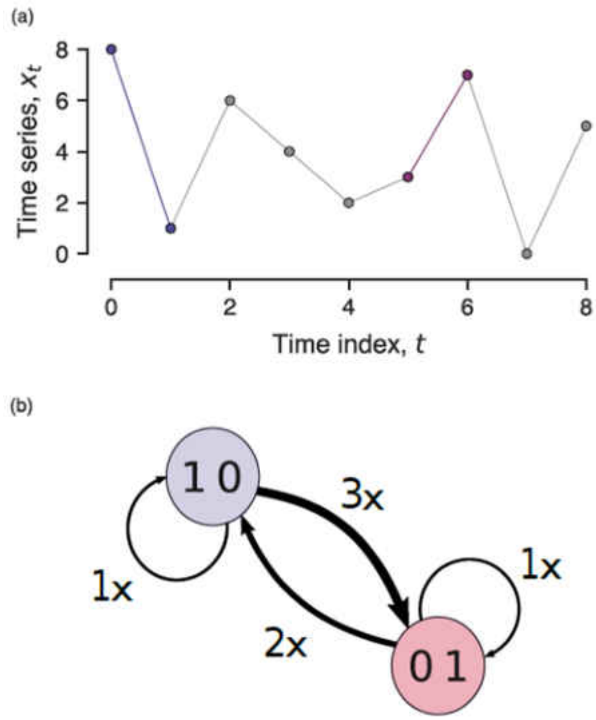


Figure 109. Ordinal network mapping of time series x_t = {8, 1, 6, 4, 2, 3, 7, 0, 5} (a) Illustration of the time series. For D = 2, the corresponding symbolic sequence is {(10)(01)(10)(10)(01)(01)(10)(01)}. (b) Ordinal network associated with the time series x_t. Arrows represent temporal succession of permutations in the symbolic sequence, and line weighting reflects the relative frequency of each possible succession. Self-loops appear when a permutation is followed by itself in the symbolic sequence. From *Characterizing Stochastic Time Series with Ordinal Networks* by Pesse & Ribeiro.

Improvements could be network mapped for virtually any kind of process to include stochastic processes like those that are Shewhart-stable or idealized White Gaussian Noise (WGN). Furthermore, results from different embedding dimensions could provide different insights into process behaviors. Figure 110 displays ordinal network maps for two processes at various embedding dimensions, D.

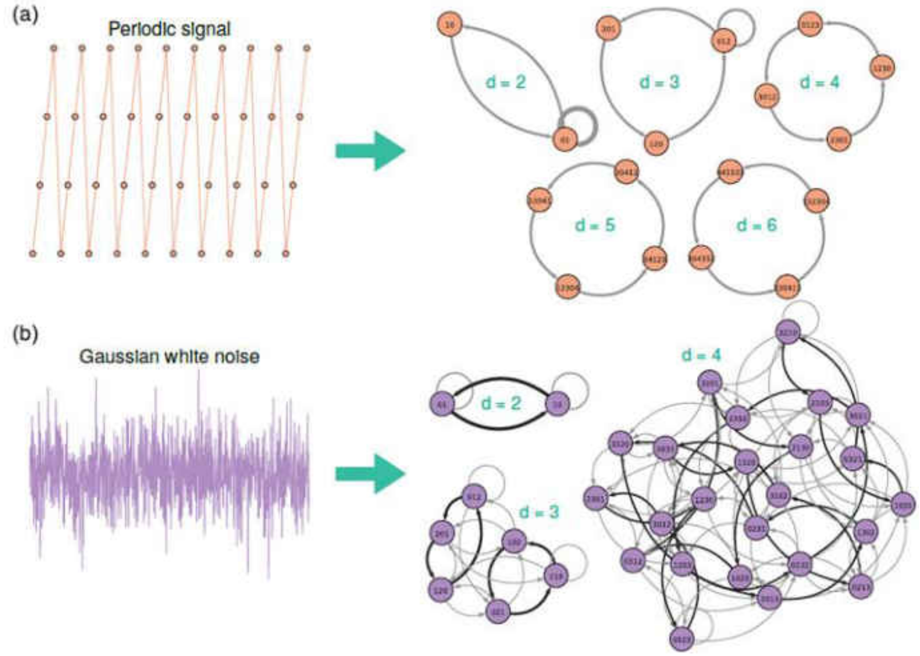


Figure 110. Ordinal networks of periodic and random time series. (a) Illustration of the mapping of a periodic signal into ordinal networks with different embedding dimensions *d*. (b) Mapping of white Gaussian noise into ordinal networks with different embedding dimensions *d*. Dark black arrows indicate transitions that occurred twice as often as those represented by light black arrows. From *Characterizing Stochastic Time Series with Ordinal Networks* by Pesse & Ribeiro. Copyright (c) 2019 by American Physical Society. Reprinted with the permission of American Physical Society. All rights reserved.

Embedding Delay Research

The research herein was accomplished using only time lag τ=1. However, some authors have discovered that applying different time lags can provide additional information about processes, as related to intrinsic time scaling (Soriano et al., 2011; Zunino et al., 2010). Figure 111 provides results for simulated geometric Brownian motion for ($1 \leq \tau \leq 500$). Similar research could be accomplished to reveal potential intrinsic time scaling for processes of interest to quality control applications.

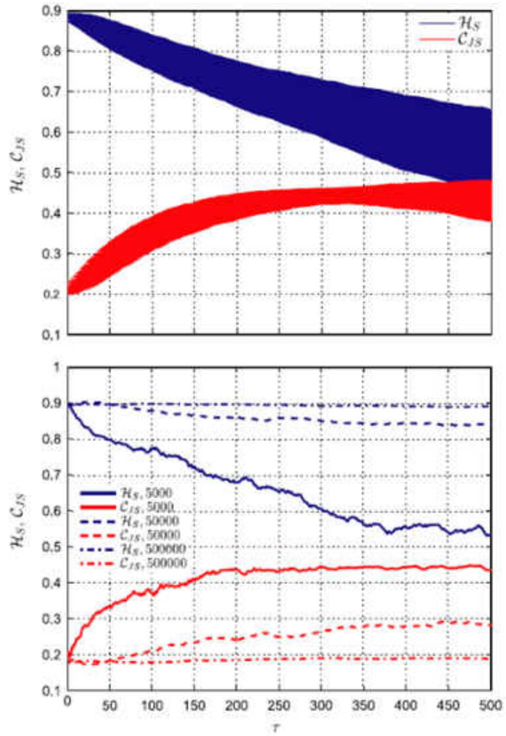


Figure 111. Permutation entropy H_S and structural complexity C_{JS} of geometric Brownian motion simulations as a function of the time lag τ ($1 \leq \tau \leq 500$) for embedding dimension D = 6. Top: Mean and standard deviation of both permutation quantifiers for 100 independent realizations of length N = 4673. Bottom: Estimated quantifiers for simulations with N = 5000, 50 000 and 500 000 data points. From *Commodity Predictability Analysis with a Permutation Information Theory Approach* by Zunino et al. Copyright © 2011 by Elsevier. Reprinted with the permission of Elsevier. All rights reserved.

High Sample-Rate Research

In-process sensors installed throughout manufacturing lines are producing astronomically large volumes of data at high sampling rates. Permutation entropy could be employed in methods similar to Figure 112 to gain insights into process performance and to evaluate the effects corresponding with process improvements.

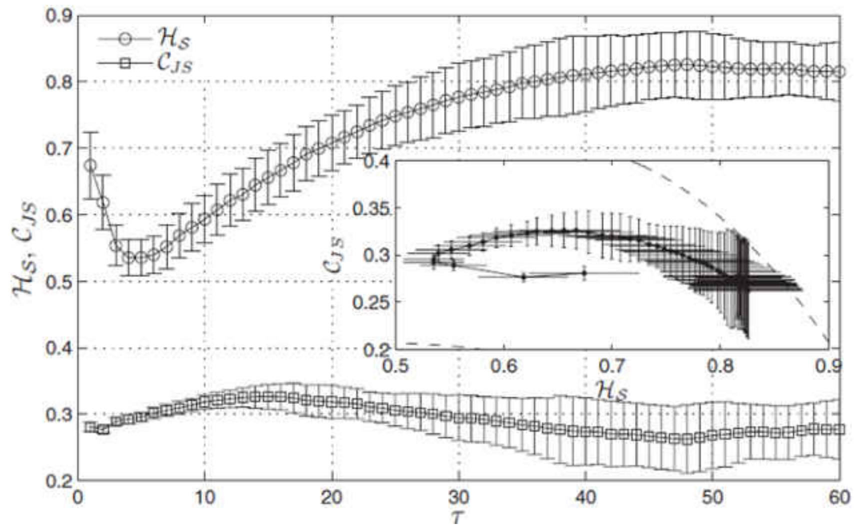


Figure 112. Postural sway measurements of high sample-rate data. Subtle shifts can be made visible for analysis of a complex multiscale time series. Permutation quantifiers (*HS* and *CJS*) presented as a function of time lag τ with embedding dimensions $D = 5$. Mean and standard deviation of the permutation quantifiers for ten independent trials associated to the same volunteer are depicted. Curve described by the symbolic quantifiers in the CECP is shown in the inset. The dashed lines represent the maximum and minimum complexity values for a fixed value of the entropy. From *Distinguishing Chaotic and Stochastic Dynamics from Time Series by Using a Multiscale Symbolic Approach* by Zunino et al. Copyright © 2012 by American Physical Society. Reprinted with the permission of American Physical Society. All rights reserved.

APPENDIX A: DATA

Table 30. Vertical Funnel Experiment 1 Data.

Height (in) \ Drop #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
55	5	1	4	4	6	3	4	5	4	7	8	2	2	2	3	5	1	4	9	3	19	4	5	6	7	4	7	6	3	7	3	4	4	7	9	1	8	9	2	3	4	7	5	4	10	7	16	5	10	10
54	5	1	8	2	1	1	2	2	12	2	2	7	2	6	5	10	5	4	13	5	7	8	8	3	7	5	8	4	4	4	4	7	6	8	4	5	1	2	6	5	1	4	3	2	1	6	5	7	8	10
53	4	6	7	5	3	4	8	5	5	6	3	1	7	5	5	4	7	3	6	5	10	6	1	6	2	6	8	8	1	10	9	4	5	2	7	5	2	2	13	6	7	2	2	3	10	3	4	6	2	4
52	3	4	4	8	7	5	4	7	8	1	1	5	7	6	2	2	2	11	5	3	1	4	3	2	6	5	7	8	5	3	10	4	4	21	11	7	3	12	1	4	4	4	3	2	3	1	8	5	10	8
51	8	8	9	7	8	8	11	7	3	5	12	3	7	12	6	8	9	3	6	3	5	1	8	9	5	3	6	1	3	4	1	11	5	3	5	1	2	1	3	3	14	1	7	6	3	1	4	10	8	2
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48	5	3	7	5	7	6	1	3	4	6	3	4	7	6	1	7	7	12	4	3	8	8	7	7	8	7	1	7	6	4	6	7	8	2	10	1	8	7	2	1	23	2	4	1	4	3	11	9	3	10
47	2	1	9	2	6	4	5	2	6	4	5	3	5	2	7	1	4	1	7	6	5	6	3	1	4	2	2	9	10	4	2	12	6	8	17	6	4	4	6	6	2	2	7	5	3	1	1	2	10	7
46	4	2	4	8	2	7	7	5	2	6	9	5	5	1	5	3	4	5	6	3	3	2	4	6	8	3	2	2	4	3	12	7	4	4	6	4	2	6	5	1	4	2	4	9	2	3	6	1	4	8
45	3	6	4	2	6	7	7	4	11	4	2	1	3	1	5	2	3	2	11	9	4	2	4	9	2	6	2	7	11	4	7	1	4	5	1	2	1	2	6	10	3	10	5	2	1	1	9	7	2	11
44	6	7	6	1	9	7	1	6	1	2	1	5	2	7	8	1	8	2	1	11	5	5	7	7	8	4	11	5	7	5	7	8	7	4	8	6	2	9	10	7	1	2	10	1	2	4	6	4	1	9
43	1	4	2	4	2	4	11	5	2	3	11	7	2	20	3	3	3	2	2	7	7	10	11	1	7	2	6	4	4	5	1	7	2	2	5	2	2	1	5	7	3	3	1	10	4	8	2	4	2	2
42	1	7	3	5	3	3	1	10	6	9	3	5	1	1	2	5	5	5	2	5	3	9	1	5	3	2	3	14	8	6	12	3	10	1	2	1	6	5	5	5	3	3	9	8	7	2	1	2	8	3
41	9	5	9	5	5	6	3	6	2	2	7	1	2	1	4	3	1	6	3	10	3	2	9	7	7	6	12	2	2	10	9	5	11	4	1	3	3	4	2	9	8	1	4	2	2	6	1	3	8	8
40	5	3	8	2	8	4	9	4	8	4	5	2	9	1	3	1	1	2	3	9	9	4	5	2	13	7	6	1	3	3	5	1	11	4	1	4	5	1	3	6	6	1	2	10	2	5	6	7	1	4
39	6	12	3	3	8	8	3	8	8	2	2	4	5	1	4	4	8	10	3	4	6	8	1	1	2	1	1	5	9	4	8	18	2	8	7	7	5	1	3	3	5	6	3	1	1	9	5	2	5	6
38	2	2	5	1	5	25	6	9	1	1	1	1	1	4	1	9	5	1	7	2	5	5	8	2	6	6	1	2	2	4	5	3	1	7	2	4	2	3	3	7	5	8	2	5	2	2	6	7	7	6
37	9	7	1	1	6	5	2	1	8	3	3	4	9	3	5	1	1	7	8	9	10	5	5	6	10	2	1	4	4	3	8	3	3	3	11	5	1	7	11	1	4	1	4	1	5	2	4	7	12	6
36	2	3	1	8	1	1	3	2	4	5	5	8	2	7	10	2	2	7	2	4	1	10	3	6	6	8	3	4	5	1	4	2	1	4	6	7	4	9	5	5	6	1	6	5	3	7	5	3	7	2
35	5	2	8	1	2	4	1	3	1	2	5	2	1	9	9	1	2	3	1	4	4	1	1	2	2	3	3	3	8	2	6	3	5	2	3	3	7	4	11	2	2	1	6	6	2	4	8	4	3	8
34	9	8	2	3	3	8	6	7	2	7	3	6	5	7	1	4	3	8	1	2	10	1	2	2	5	3	7	4	6	1	2	1	2	4	2	6	4	7	6	4	3	11	3	1	8	7	8	2	11	10
33	4	4	3	5	4	4	4	2	1	2	5	6	5	7	2	1	9	3	11	1	2	2	4	9	2	1	4	5	1	8	1	10	7	7	8	2	6	6	2	10	8	4	2	11	7	6	6	4	4	3
32	9	2	4	4	3	11	8	5	1	4	2	2	1	1	3	5	1	7	8	8	8	3	2	7	2	7	2	3	4	3	1	2	5	2	8	1	25	8	12	6	4	8	9	1	5	9	7	4	7	8
31	5	2	1	2	6	2	1	2	5	5	4	4	7	4	1	1	4	6	2	2	9	1	4	3	2	6	3	6	7	6	7	3	1	2	6	5	3	2	1	6	10	1	8	1	4	5	2	3	7	7
30	5	2	3	7	6	2	1	1	3	5	3	2	2	1	4	3	2	1	3	5	4	3	1	6	2	13	6	2	8	9	8	2	4	7	4	5	4	13	1	7	4	4	1	2	10	5	7	4	1	3
29	1	3	1	1	1	1	4	3	3	2	7	3	4	1	7	3	1	6	5	4	7	6	7	6	2	5	3	2	1	1	3	1	6	2	8	14	4	1	9	6	1	1	3	6	1	1	1	5	6	1
28	3	3	7	5	4	10	4	4	1	6	2	8	6	2	7	2	9	6	10	3	2	1	8	2	6	2	2	4	6	3	3	10	10	7	2	5	4	7	8	6	7	7	8	8	4	10	25	10	5	8
27	4	2	5	9	1	10	6	2	1	4	8	7	3	12	3	4	1	2	3	2	2	1	4	4	6	1	4	1	5	15	3	2	8	3	1	10	1	9	4	2	8	1	1	11	3	3	1	8	7	9
26	7	5	6	2	3	10	2	1	7	2	9	4	8	7	1	2	8	3	1	10	1	9	8	10	2	5	6	6	1	7	8	2	2	2	4	7	1	3	8	8	2	1	6	3	2	4	6	6	10	6
25	2	2	1	6	2	1	13	6	2	11	3	21	8	7	4	1	4	5	5	5	2	10	5	2	4	3	8	1	4	1	1	8	10	10	4	2	4	2	1	4	5	2	7	2	9	1	2	7	7	3
24	3	1	2	9	7	10	1	10	1	6	12	5	5	2	7	4	1	9	8	2	3	2	4	3	7	9	4	3	2	4	4	4	2	8	1	2	3	1	1	9	2	1	7	2	2	3	7	2	3	2
23	1	1	3	4	2	5	3	9	9	7	2	4	8	4	2	6	4	4	1	5	5	2	2	8	3	3	1	6	1	1	2	5	5	7	3	4	9	3	8	1	2	2	10	5	8	7	8	2	6	5
22	4	1	2	1	8	2	2	2	1	1	4	3	6	7	1	9	3	9	3	1	5	2	1	5	2	8	10	1	5	4	1	10	6	4	9	5	6	5	2	3	2	2	4	2	1	5	3	1	7	9
21	2	1	4	4	4	4	2	2	8	2	5	3	1	6	1	5	1	1	5	5	4	1	2	2	3	2	1	2	1	4	3	1	4	1	1	1	8	4	3	7	1	4	1	2	1	4	2	2	5	1
20	3	3	8	5	8	1	4	1	3	7	2	4	1	2	2	1	7	5	2	2	1	2	5	2	1	2	7	1	3	1	2	6	1	2	1	4	2	1	1	5	3	2	4	1	3	1	1	2	7	2
19	3	6	7	1	6	1	1	7	5	1	2	1	5	8	3	3	1	1	5	2	5	1	7	2	4	1	2	8	2	1	1	2	8	6	1	4	1	6	6	2	1	2	5	1	3	1	3	2	1	6
18	1	2	6	1	7	1	5	1	1	2	2	1	1	4	1	5	1	5	1	1	7	1	1	5	1	1	2	7	4	2	3	5	7	9	7	4	1	2	4	4	1	4	1	5	6	1	2	2	1	2
17	4	2	1	1	3	2	2	1	8	5	2	2	2	2	4	1	1	3	2	2	1	1	1	2	6	6	1	2	2	7	2	2	5	3	9	7	5	1	1	1	1	1	1	5	2	10	6	6	1	5
16	1	2	1	1	1	1	1	2	3	1	1	5	3	1	7	1	1	1	1	2	2	1	5	1	2	1	1	1	7	2	1	2	3	3	1	2	5	2	4	6	1	1	5	2	4	1	4	4	3	6
15	1	3	1	6	5	5	1	4	2	1	1	2	1	1	1	6	5	1	3	1	1	1	1	3	1	1	2	6	1	1	7	8	3	6	5	2	1	2	7	1	2	4	4	3	6	1	5	1	2	1
14	6	1	7	4	1	1	21	3	2	1	1	1	3	2	4	4	1	2	5	2	1	1	1	4	1	21	8	5	7	2	2	1	4	3	2	2	4	2	9	5	4	5	1	4	5	1	3	6	5	2
13	1	1	2	2	1	1	1	7	2	1	8	4	1	1	4	1	1	2	1	1	3	1	1	1	8	5	1	1	1	1	2	6	1	6	1	2	6	2	4	4	1	2	1	1	4	1	2	7	1	1
12	4	5	2	2	5	6	2	2	1	10	1	6	2	1	1	1	1	2	5	1	1	6	1	7	1	6	5	2	1	1	1	2	6	3	1	5	2	2	3	4	3	6	1	2	3	7	4	5	1	1
11	1	2	1	1	1	2	2	1	2	3	2	1	9	5	1	1	4	2	3	7	1	1	1	2	10	1	8	1	2	1	2	10	1	1	2	7	2	1	7	1	4	1	5	2	1	2	1	1	4	1
10	8	1	1	7	1	1	3	2	2	7	1	1	2	6	5	1	1	1	2	1	1	2	2	3	1	1	1	2	1	6	5	1	1	1	1	3	3	1	8	4	5	1	1	2	5	2	1	1	3	2
9	5	2	1	1	1	2	2	6	1	2	3	1	5	6	9	2	8	2	2	1	6	1	1	2	1	2	4	7	1	1	1	1	2	1	1	3	2	1	1	6	6	2	9	1	3	1	4	1	1	2
8	2	4	2	1	2	1	1	1	4	1	4	1	1	1	2	1	6	1	1	1	1	1	1	3	4	1	5	1	1	1	2	1	4	7	1	1	2	2	5	2	1	1	6	1	1	5	1	1	2	2
7	4	4	1	1	1	2	1	2	1	1	1	1	2	1	1	1	3	5	5	2	2	1	2	2	2	1	2	1	1	5	4	1	5	2	1	2	1	1	6	1	1	1	1	2	3	1	2	4	5	2
6	2	1	1	2	1	1	1	1	1	1	1	1	2	2	2	1	1	3	2	1	6	1	1	1	1	1	3	2	2	2	1	4	1	1	1	2	1	2	1	2	2	2	2	1	1	2	1	3	1	2
5	2	2	1	1	3	1	2	1	1	2	4	1	1	1	1	2	2	5	1	2	1	1	1	5	1	2	1	4	3	2	2	2	1	3	5	2	3	4	4	3	1	4	2	2	1	1	1	1	1	2

Table 31. Vertical Funnel Experiment 2 Data.

Height (in) \ Drop #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
38	1	7	13	18	2	1	1	4	9	7	4	3	5	7	9	13	4	3	14	2	3	8	21	7	3	6	5	6	3	5	6	2	3	18	4	14	3	8	10	4	2	10	9	8	24	7	3	2	8	3
37	3	1	13	13	2	7	8	13	13	6	6	11	2	6	3	11	5	4	28	9	9	18	9	6	6	10	13	10	14	6	14	2	7	10	3	7	9	10	5	11	1	4	8	1	25	23	13	13	6	17
36	3	6	3	2	7	1	4	3	13	3	6	3	5	5	1	6	5	18	5	3	4	3	1	3	4	2	17	7	3	1	5	12	10	15	2	3	6	7	12	4	4	4	4	33	2	1	3	3	6	1
35	9	5	5	5	5	1	6	19	5	10	4	9	4	6	3	3	5	17	5	4	14	2	6	9	5	15	3	12	10	6	7	9	11	3	4	4	13	13	6	4	10	11	4	4	8	10	15	3	8	5
34	3	5	4	2	6	13	9	4	7	9	8	4	6	5	9	1	5	4	6	3	3	9	5	7	5	1	1	6	3	5	10	12	2	6	7	8	3	9	4	6	1	3	11	2	2	10	5	5	4	14
33	26	4	6	9	11	13	5	5	5	3	3	7	7	11	4	11	4	6	3	1	4	2	7	3	7	5	2	7	4	5	2	11	3	1	4	3	14	8	8	25	3	9	2	7	6	2	5	4	9	6
32	3	4	6	3	15	6	8	9	6	9	3	12	9	10	9	24	2	13	1	4	8	3	10	5	15	9	10	15	3	4	7	6	2	8	10	7	9	3	3	9	4	8	13	4	7	4	7	28	1	21
31	3	14	15	9	3	2	12	3	9	6	4	5	12	3	3	6	7	10	10	2	9	3	4	6	12	6	2	2	4	5	5	4	6	3	4	6	12	5	1	2	2	17	6	4	1	2	6	2	1	12
30	5	12	9	6	24	5	1	2	3	13	3	11	4	4	9	7	5	2	2	18	10	11	8	3	2	13	10	11	1	12	4	13	13	5	13	7	3	4	2	6	13	20	11	8	3	4	10	3	4	16
29	2	1	1	1	4	3	1	6	4	3	2	5	1	4	2	5	5	14	2	20	4	2	1	1	10	17	4	12	5	11	5	6	10	9	2	3	18	2	5	8	7	4	1	7	4	3	5	12	13	6
28	4	13	14	2	1	4	5	13	4	3	11	5	2	9	10	11	18	13	28	17	1	11	4	8	11	17	3	13	11	6	17	10	3	5	9	10	14	2	2	6	8	3	27	6	4	1	9	8	6	10
27	3	11	3	4	2	1	1	2	8	5	2	10	10	1	5	14	9	22	12	1	5	9	5	1	7	7	3	4	4	16	7	10	6	2	1	10	18	5	16	5	3	12	3	6	2	2	5	8	1	3
26	8	5	2	11	8	8	9	9	3	7	2	11	5	7	1	10	7	1	4	3	4	3	1	10	14	3	7	4	4	17	3	1	6	8	7	7	8	3	5	4	1	6	4	4	11	8	6	10	1	4
25	1	1	1	1	5	4	6	6	2	6	4	2	2	5	1	3	5	3	2	8	3	9	1	3	3	3	5	5	1	3	6	1	2	3	2	4	1	4	5	4	8	2	6	4	5	8	5	6	2	3
24	4	3	2	12	3	9	8	1	4	6	9	3	6	1	9	8	10	4	1	10	3	11	3	6	6	4	5	8	10	2	10	2	2	2	4	2	8	11	14	1	3	7	3	4	2	4	1	5	4	7
23	2	1	2	8	8	5	3	2	4	7	4	3	1	10	9	7	5	5	3	5	8	6	6	1	8	5	4	2	6	6	3	5	6	7	11	3	15	6	5	6	1	6	4	8	5	5	7	5	4	7
22	3	3	2	5	5	3	5	7	1	5	3	2	1	1	1	1	1	2	5	3	5	6	5	3	4	2	5	3	6	4	7	2	3	4	6	4	4	2	1	4	2	3	6	2	4	4	2	4	8	4
21	4	1	5	2	4	4	5	2	4	2	2	3	2	6	1	2	2	3	2	5	1	5	4	4	8	2	4	6	2	3	4	5	2	5	7	6	3	3	7	2	1	5	1	2	4	5	3	8	13	2
20	5	4	2	2	6	4	2	3	1	2	1	1	6	2	5	2	3	3	5	2	5	5	4	3	2	4	3	5	3	6	1	9	3	3	5	3	4	1	4	1	9	3	1	2	8	2	1	4	8	4
19	4	1	4	4	1	4	1	7	4	3	3	5	5	1	4	3	5	3	4	1	1	3	4	1	5	2	2	3	3	1	4	4	2	7	2	7	4	1	5	1	8	4	4	8	2	4	1	3	6	2
18	2	2	5	3	2	3	1	2	2	3	2	3	3	8	1	2	1	4	1	3	3	3	1	3	2	1	4	1	4	1	5	1	2	3	3	6	3	4	3	3	2	3	4	1	9	1	1	2	4	1
17	6	4	2	2	1	2	2	1	2	3	5	4	2	3	4	5	3	1	2	3	5	3	3	4	1	4	5	2	1	3	4	5	3	1	4	1	2	3	4	4	4	4	6	2	5	3	1	2	2	4
16	5	3	3	5	2	1	3	2	1	1	3	2	5	1	2	3	3	4	2	4	3	2	2	1	5	4	6	2	6	4	1	3	5	2	1	6	7	4	1	8	1	1	7	2	1	1	3	3	4	1
15	1	3	2	1	5	4	2	5	2	1	4	2	2	1	4	3	2	3	4	2	1	6	1	2	3	3	1	5	5	2	2	4	2	1	4	1	2	1	2	3	5	1	2	2	4	3	1	1	2	3
14	2	2	2	4	2	2	4	2	1	1	1	3	2	1	2	2	2	4	2	1	2	6	2	2	1	2	5	2	4	2	3	2	4	1	7	3	1	2	4	2	1	3	1	1	8	3	4	3	2	3
13	1	1	1	1	3	2	2	3	1	5	1	3	1	2	2	1	2	2	2	1	1	3	4	2	2	9	1	1	1	3	6	3	6	2	2	1	2	2	3	6	1	2	1	1	6	4	3	2	2	1
12	1	2	3	1	1	2	2	3	2	2	1	3	2	3	4	2	5	1	2	3	1	1	6	3	2	1	3	2	3	1	5	5	1	3	1	2	1	3	1	3	2	2	5	3	2	3	1	1	2	1
11	3	2	4	1	2	2	1	2	1	1	2	3	2	6	1	2	2	6	2	2	5	4	4	2	1	3	1	1	3	3	4	2	2	5	5	1	2	3	3	2	1	1	2	1	2	3	1	4	1	3
10	1	1	3	4	1	2	1	1	1	2	1	4	1	3	4	1	1	1	1	4	2	2	1	3	6	2	1	5	3	3	2	1	1	1	3	1	1	1	1	1	3	1	2	2	4	3	2	2	1	2
9	3	3	2	1	3	2	3	1	1	2	2	2	1	2	1	1	1	1	1	2	1	1	1	3	4	1	3	3	1	1	1	1	3	1	1	2	2	2	1	5	1	1	3	1	1	1	2	1	1	1
8	1	1	2	1	1	2	2	1	3	1	2	1	1	1	2	1	2	1	3	3	1	2	3	1	1	2	1	1	2	1	2	1	1	2	4	2	2	2	1	1	1	1	3	1	1	4	2	1	1	1
7	3	1	1	1	1	2	1	2	1	2	2	2	3	1	1	3	1	1	1	1	2	1	1	1	1	1	3	1	1	1	1	1	1	2	1	2	2	1	1	1	2	1	1	1	1	1	2	2	2	1

Table 32. Vertical Funnel Experiment 3 Data.

Height (in) \ Drop #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
38	8	5	8	8	5	8	4	5	2	3	1	2	12	1	3	9	2	3	5	11	2	8	5	11	1	24	11	6	2	1	8	5	24	9	1	2	10	7	5	7	15	3	9	3	3	13	6	3	9	8
37	6	5	9	9	2	5	5	7	4	3	5	9	4	4	24	7	3	6	3	9	7	26	10	9	5	14	3	5	5	5	12	2	8	6	11	6	3	22	11	6	11	13	4	10	23	7	8	5	2	7
36	3	3	5	13	3	4	5	5	8	2	1	7	5	3	6	3	22	4	5	5	3	9	14	2	7	3	3	15	7	9	9	8	12	4	8	7	3	1	10	11	3	2	7	18	11	14	12	6	4	4
35	13	5	9	12	13	9	12	4	6	6	10	16	5	11	5	5	10	11	19	10	7	5	7	11	4	6	10	5	14	10	4	8	5	5	11	6	6	4	9	22	5	5	8	9	8	6	4	5	10	8
34	3	1	7	3	6	6	5	1	6	2	4	2	1	4	2	2	1	6	2	1	3	10	2	6	3	5	2	2	5	11	2	2	9	3	10	4	6	1	4	6	6	2	3	9	5	4	1	5	5	8
33	3	1	3	3	2	4	4	4	5	6	2	2	2	2	2	2	5	5	3	1	1	4	1	5	4	9	4	8	3	4	5	4	7	6	1	2	2	8	3	2	2	1	15	2	5	11	1	9	1	1
32	1	2	6	4	6	4	6	7	3	3	3	2	5	2	3	3	3	6	6	3	6	8	6	11	7	3	5	5	3	5	1	2	3	1	1	5	3	7	3	3	2	2	1	4	1	3	11	9	3	6
31	4	5	9	5	5	4	8	7	11	1	2	7	7	4	2	5	4	2	11	6	4	3	8	9	6	1	4	4	10	6	1	3	6	6	5	3	8	6	8	2	5	5	11	3	2	5	8	1	3	1
30	4	7	3	4	6	8	3	5	4	5	7	7	7	9	1	3	4	9	10	3	6	1	4	7	14	3	3	2	4	6	1	7	16	7	4	1	3	4	11	5	8	5	9	3	6	5	9	6	6	9
29	6	4	5	4	5	3	2	3	6	3	2	2	3	7	5	5	2	3	4	6	5	1	2	2	4	5	2	5	5	5	3	4	1	1	5	6	1	8	3	1	19	1	3	9	1	4	2	4	8	2
28	8	8	9	15	11	4	7	9	13	7	10	9	10	5	5	5	13	2	10	4	6	4	5	9	4	3	5	4	8	2	5	2	5	12	7	1	7	6	2	6	8	5	9	5	1	5	5	10	5	6
27	4	5	4	8	3	2	4	3	1	19	11	4	5	4	6	1	6	10	5	8	1	3	1	8	7	2	2	2	1	6	8	8	5	5	5	9	8	9	3	4	5	2	1	9	6	3	1	9	13	4
26	1	1	6	5	3	3	2	1	7	3	7	3	7	4	4	4	2	5	2	5	5	1	2	4	4	1	1	4	2	2	4	3	1	1	5	1	3	1	6	3	6	1	3	2	2	4	6	3	5	2
25	5	1	2	3	1	4	5	1	2	3	3	3	2	4	4	4	4	6	1	5	1	5	2	10	3	1	7	5	1	1	1	1	7	1	2	6	1	1	3	1	10	1	3	3	1	3	1	11	1	3
24	5	5	3	5	6	3	1	2	2	6	1	2	1	2	9	5	1	2	5	4	1	5	1	1	2	5	1	4	9	8	5	6	7	7	12	5	1	1	10	6	5	7	1	5	1	3	4	3	7	3
23	3	1	6	13	3	2	5	5	1	1	1	6	3	3	6	6	4	6	4	1	1	3	2	1	5	4	3	5	2	5	4	3	6	3	5	3	2	5	7	4	13	3	6	1	10	1	3	3	2	3
22	3	9	6	7	4	8	4	4	7	11	9	1	6	6	3	5	2	9	10	7	5	2	3	13	5	8	4	9	8	9	7	7	7	6	1	3	2	6	2	2	8	5	5	6	6	13	13	11	9	12
21	2	6	2	1	2	3	4	2	1	1	5	4	4	1	7	2	7	3	2	1	3	4	3	6	3	2	1	1	4	1	5	2	4	3	2	1	4	6	6	2	5	8	6	12	4	4	1	8	7	12
20	5	4	9	3	2	1	6	3	12	1	10	7	8	4	2	9	5	3	2	3	6	4	5	9	8	3	5	2	1	2	4	2	1	1	1	3	4	2	3	1	6	1	1	2	6	3	1	1	1	3
19	3	2	1	8	1	2	2	2	6	1	6	2	2	1	1	2	4	5	1	1	1	1	1	1	1	2	7	1	6	4	3	2	5	9	4	6	2	7	6	1	2	1	1	6	10	2	1	5	1	2
18	3	4	2	3	5	4	5	2	3	3	3	1	8	4	2	5	6	1	2	3	2	3	8	7	2	1	4	1	4	2	1	3	4	5	10	6	3	8	2	8	4	4	2	2	2	1	4	2	4	3
17	6	10	5	6	8	6	10	8	3	8	7	2	8	25	9	6	4	6	4	1	10	4	5	5	5	3	11	1	8	5	1	2	5	7	4	5	2	11	6	2	3	4	1	9	11	4	4	8	4	6
16	5	5	8	4	5	4	3	10	5	3	4	5	9	9	4	8	8	6	4	3	6	2	5	5	4	1	3	5	10	5	6	8	3	4	4	1	5	8	1	4	7	4	7	3	3	8	4	6	6	4
15	7	6	3	5	6	7	3	3	10	7	3	1	9	1	1	4	8	5	1	7	3	3	3	8	9	8	4	4	5	8	9	10	6	3	7	4	3	3	5	5	4	8	9	5	9	8	6	6	5	8
14	1	4	3	2	1	1	4	1	1	1	2	1	2	1	4	1	1	1	1	2	1	1	3	3	1	1	3	3	4	1	5	4	3	5	1	4	3	4	1	7	6	4	2	5	4	3	2	3	3	3
13	3	4	7	5	1	4	5	2	5	5	1	3	2	4	3	4	5	3	6	2	1	1	5	3	2	33	3	2	1	9	1	1	5	3	1	4	10	3	2	8	3	1	5	4	1	4	3	2	2	5
12	6	6	5	4	4	7	4	3	3	4	5	5	8	1	4	4	4	5	3	1	3	9	4	4	3	4	5	3	9	7	3	6	1	6	2	4	5	5	6	4	2	6	3	5	5	3	3	7	11	8
11	7	3	5	5	7	5	4	4	3	4	1	5	4	5	4	10	3	5	4	3	4	3	4	6	4	4	4	6	3	4	5	4	3	4	5	4	4	3	4	5	2	5	6	5	4	1	4	4	5	6
10	5	4	1	4	3	4	5	6	1	6	3	3	1	3	3	5	1	4	3	3	3	3	3	1	1	4	3	5	2	1	6	2	1	3	2	3	4	2	1	1	1	3	1	2	2	3	3	5	1	3
9	1	5	4	5	4	6	5	5	4	5	5	5	1	5	4	5	6	4	3	5	3	5	4	7	4	5	5	5	6	4	5	3	5	5	7	4	4	5	3	5	4	4	5	4	5	3	5	5	3	3
8	3	1	3	4	4	3	5	2	3	3	3	2	3	5	2	2	3	3	2	2	2	4	3	2	3	4	2	5	4	1	1	3	2	5	1	1	2	1	2	1	3	4	3	1	1	4	4	1	3	3
7	3	2	3	3	2	3	3	1	4	2	3	4	5	4	2	3	1	3	4	4	3	5	2	1	3	3	3	4	4	3	2	4	3	2	3	3	4	3	3	2	2	3	3	3	4	5	3	4	1	4

Table 33. Vertical Funnel Experiment 4 Data.

Height (m) \ Drop #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
37	10	1	1	1	4	4	6	1	2	3	1	1	3	2	2	8	3	5	3	4	4	11	14	2	1	7	10	4	11	7	4	4	2	4	3	3	4	1	5	4	1	6	3	2	6	1	7	1	11	4
36	1	3	4	5	1	2	2	2	7	2	1	4	1	4	1	7	3	1	2	1	4	10	7	4	1	1	4	1	4	4	1	5	3	4	2	2	1	1	1	3	6	3	8	6	7	5	2	1	9	2
35	3	1	1	5	2	2	2	1	1	15	1	7	4	4	12	2	2	3	2	3	1	1	3	5	3	17	8	8	8	6	7	8	4	1	8	2	3	11	5	3	4	5	4	1	2	5	9	3	7	7
34	3	1	3	1	9	1	1	6	1	1	2	7	1	2	10	4	10	5	3	2	1	1	2	1	1	4	2	2	1	7	9	2	9	1	2	2	3	4	2	3	1	2	1	3	7	4	4	7	2	10
33	4	1	1	7	2	1	2	11	1	4	1	3	2	5	1	1	3	2	1	2	3	2	1	2	2	6	2	1	2	4	1	2	1	4	1	3	3	1	4	1	2	3	1	4	2	2	3	1	1	11
32	5	2	1	2	1	1	2	2	3	2	3	1	1	2	4	1	2	2	1	1	1	3	5	1	3	3	5	5	3	1	1	4	3	1	6	2	11	3	3	1	2	8	1	1	2	2	2	7	12	9
31	4	1	2	8	2	1	8	1	4	2	1	2	3	2	2	8	6	2	2	2	2	1	2	2	3	4	2	3	1	1	1	10	4	1	4	3	1	2	6	2	2	1	3	2	1	4	3	4	4	8
30	1	1	1	5	2	2	1	1	2	1	2	3	1	1	1	1	3	1	3	2	1	1	2	4	3	2	2	2	3	6	1	1	3	2	4	3	2	2	1	2	3	1	1	2	4	1	4	3	1	1
29	1	2	1	4	4	3	1	1	3	3	2	3	1	4	6	2	2	4	1	4	3	5	3	4	4	3	1	1	2	3	2	4	2	4	5	3	3	1	2	4	3	1	1	1	4	2	4	4	5	3
28	1	2	3	4	1	2	3	3	2	5	1	2	5	4	2	3	8	2	1	7	1	2	1	8	3	4	3	3	1	3	1	7	4	1	2	1	4	5	2	2	2	3	2	4	4	4	2	1	1	4
27	1	4	5	4	4	3	5	1	4	2	1	1	2	1	1	4	2	1	1	4	3	1	2	1	1	8	1	1	3	3	6	4	1	3	1	3	3	1	2	2	8	9	2	2	1	1	4	5	2	3
26	1	1	2	4	1	2	1	1	2	2	1	1	1	5	1	1	1	3	2	2	2	2	4	3	2	4	3	1	3	2	1	4	1	2	3	1	2	2	1	1	3	1	2	1	8	1	1	6	4	1
25	4	4	2	1	3	1	5	1	3	3	1	3	3	1	4	4	2	1	2	1	6	2	3	4	3	1	3	1	4	3	1	2	2	2	2	1	4	4	2	1	2	8	1	1	2	1	1	1	1	1
24	2	4	1	3	2	2	2	2	2	1	3	2	1	2	3	2	3	1	2	2	2	1	2	3	3	2	4	3	5	2	2	3	1	2	2	1	4	1	3	2	2	4	1	2	6	4	3	2	6	6
23	5	6	2	3	8	3	2	2	1	1	3	1	1	5	2	3	2	3	2	2	3	4	2	2	8	2	2	3	1	3	1	2	3	3	3	3	1	3	1	2	3	2	3	3	1	1	2	3	5	1
22	1	3	3	1	2	2	3	1	8	2	3	2	2	1	1	3	2	2	1	4	2	2	1	3	8	1	1	3	1	8	2	2	2	3	9	1	1	1	1	1	2	2	1	1	2	1	2	1	4	4
21	2	1	2	1	1	2	1	1	1	1	2	2	2	1	7	3	2	1	2	2	1	1	2	2	1	2	1	1	3	3	3	1	1	2	2	1	1	2	2	3	2	2	2	2	3	2	1	1	1	3
20	3	3	1	1	1	1	4	3	3	2	1	2	1	1	3	3	2	3	3	1	2	3	1	4	5	1	1	2	1	3	1	4	2	1	1	4	1	1	1	2	1	1	2	1	4	3	1	3	1	1
19	1	1	1	2	4	1	5	1	3	1	2	2	1	1	3	1	3	1	1	1	1	4	1	2	4	5	2	3	1	3	2	1	1	4	1	1	1	1	1	4	1	5	2	1	4	3	6	1	1	2
18	3	1	3	3	3	3	2	5	1	2	3	3	1	1	3	3	3	3	2	1	1	4	3	1	1	3	3	1	3	3	3	1	4	3	3	2	3	3	1	3	3	3	1	3	2	4	3	1	3	2
17	1	1	1	1	2	3	3	3	2	1	2	1	1	2	1	2	2	2	2	2	1	1	4	2	2	4	1	2	1	7	2	2	2	3	2	2	1	1	2	1	1	1	4	2	2	2	2	2	2	2
16	1	2	2	1	2	3	2	2	2	2	1	1	2	2	1	7	1	2	1	3	2	1	2	2	2	1	2	1	1	2	1	1	1	2	1	2	1	1	2	2	2	2	2	2	1	2	2	1	2	2
15	2	1	1	1	1	1	1	1	1	1	2	2	1	3	1	1	1	1	1	1	1	2	2	1	1	1	1	2	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	3	1	4	1	1	7	1
14	1	1	2	1	2	2	1	1	2	2	1	1	3	2	2	2	1	4	1	1	3	2	1	3	2	2	2	2	2	1	1	2	2	1	1	2	1	1	1	1	1	1	2	1	1	1	2	1	2	1
13	1	2	1	2	4	1	1	2	1	1	1	1	1	1	2	2	2	3	1	1	1	3	2	1	2	1	1	2	1	2	1	1	2	2	1	1	1	1	1	1	3	1	2	2	3	1	2	1	3	1
12	1	1	1	1	1	2	2	1	1	2	1	1	1	1	1	1	1	2	2	1	1	1	2	1	1	2	2	1	1	1	1	1	1	1	3	1	1	1	1	1	1	1	2	1	1	1	3	1	1	1
11	1	1	2	1	1	1	1	1	2	1	2	1	2	1	1	1	2	1	2	1	2	1	1	1	1	1	1	2	1	1	1	2	3	1	2	2	3	2	2	1	1	1	1	1	1	1	1	1	2	1
10	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	1	1	2	1	1	1	1	1	4	1	1	1	1	1	1	1	1
9	1	1	1	1	1	1	1	1	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Table 34. X-bar and mR-bar for each drop height of vertical funnel experiments.

X-bar				
Height	V1	V2	V3	V4
55	5.56			
54	4.96			
53	5.10			
52	5.28			
51	5.58			
50	5.80			
49	5.92			
48	5.72			
47	4.78			
46	4.48			
45	4.68			
44	5.28			
43	4.56			
42	4.64			
41	4.88			
40	4.58			
39	4.98			
38	4.34	6.84	6.52	
37	4.84	9.08	7.90	4.24
36	4.36	5.68	6.76	3.32
35	3.70	7.28	8.36	4.64
34	4.76	5.64	4.18	3.46
33	4.70	6.44	3.84	2.62
32	5.24	7.98	4.16	3.00
31	3.94	5.84	5.12	3.00
30	4.22	7.76	5.68	2.06
29	3.62	5.66	4.04	2.78
28	5.84	8.64	6.52	2.92
27	4.54	6.24	5.26	2.76
26	4.88	5.90	3.26	2.14
25	4.80	3.70	3.20	2.38
24	4.24	5.34	4.12	2.52
23	4.36	5.30	3.96	2.66
22	4.00	3.56	6.36	2.40
21	2.88	3.76	3.80	1.82
20	2.94	3.54	3.82	2.02
19	3.28	3.42	3.08	2.10
18	3.02	2.74	3.62	2.46
17	3.06	3.04	5.96	1.96
16	2.40	3.04	5.08	1.76
15	2.80	2.56	5.50	1.40
14	3.86	2.56	2.56	1.58
13	2.42	2.40	4.04	1.56
12	3.06	2.30	4.64	1.26
11	2.70	2.44	4.32	1.36
10	2.50	2.02	2.86	1.10
9	2.74	1.72	4.44	1.02
8	2.10	1.62	2.68	1.00
7	2.10	1.42	3.02	1.00

mR-bar				
Height	V1	V2	V3	V4
55	3.45			
54	2.96			
53	3.14			
52	3.29			
51	3.63			
50	3.55			
49	4.00			
48	3.94			
47	3.29			
46	2.73			
45	3.59			
44	3.65			
43	3.69			
42	3.35			
41	3.29			
40	3.69			
39	3.22			
38	3.39	5.47	5.63	
37	3.33	6.04	5.82	3.22
36	3.06	5.06	4.51	2.51
35	2.76	4.90	4.27	3.35
34	3.24	3.61	3.29	3.00
33	3.08	4.61	2.82	2.35
32	3.73	6.29	2.22	2.00
31	2.65	4.10	3.12	2.20
30	3.02	5.73	3.41	1.22
29	2.69	4.69	2.94	1.43
28	3.65	6.33	3.63	2.06
27	4.18	5.22	3.39	1.84
26	3.61	4.04	2.22	1.55
25	4.02	2.41	2.98	1.61
24	3.41	4.06	2.73	1.31
23	2.82	2.84	2.86	1.59
22	3.12	1.90	2.92	1.82
21	2.27	2.53	2.49	0.80
20	2.51	2.59	2.73	1.22
19	2.92	2.61	2.51	1.65
18	2.55	2.02	2.29	1.16
17	1.98	1.47	4.08	0.88
16	1.82	2.12	2.59	0.84
15	2.12	1.71	2.51	0.71
14	3.39	1.73	1.47	0.69
13	2.20	1.63	3.71	0.82
12	2.51	1.51	2.16	0.41
11	2.82	1.47	1.61	0.53
10	1.88	1.24	1.67	0.20
9	2.27	0.90	1.39	0.04
8	1.76	0.90	1.31	0.00
7	1.22	0.57	1.12	0.00

Table 35. Shannon Information in bits (non-normalized, non-permutation entropy) for funnel experiments. Values recorded at final (50th) drop for each drop height. Height in inches.

Height	V1	V2	V3	V4
55	3.29			
54	3.20			
53	3.23			
52	3.32			
51	3.39			
50	3.46			
49	3.52			
48	3.31			
47	3.23			
46	3.06			
45	3.15			
44	3.16			
43	3.04			
42	3.06			
41	3.36			
40	3.29			
39	3.28			
38	3.00	3.68	3.55	
37	3.35	3.79	3.60	3.05
36	3.16	3.21	3.72	2.80
35	2.94	3.65	3.43	3.26
34	3.24	3.51	3.08	2.72
33	3.24	3.40	2.99	2.33
32	3.26	3.58	2.91	2.63
31	3.01	3.36	3.30	2.40
30	3.15	3.83	3.32	2.00
29	2.81	3.61	2.94	2.30
28	3.25	3.64	3.44	2.51
27	3.29	3.69	3.37	2.48
26	3.16	3.46	2.70	2.07
25	3.29	2.83	2.66	2.22
24	3.09	3.45	3.02	2.12
23	3.15	3.25	3.38	2.19
22	3.07	2.77	3.51	1.68
21	2.51	2.81	2.93	1.56
20	2.58	2.79	3.05	1.87
19	2.68	2.66	2.60	2.00
18	2.45	2.40	2.78	1.71
17	2.62	2.43	3.38	1.68
16	2.27	2.68	2.97	1.30
15	2.46	2.30	3.06	1.09
14	2.84	2.25	2.29	1.38
13	2.10	2.18	2.76	1.42
12	2.58	2.07	2.93	0.91
11	2.29	2.25	2.33	1.08
10	2.27	1.97	2.34	0.38
9	2.39	1.62	2.03	0.14
8	1.92	1.51	2.20	0.00
7	1.94	1.19	2.00	0.00
6	1.47			
5	1.95			

Table 36. Tokai Rika Data. Reused with the permission of the Tokai Rika intellectual property division.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1-20	15.8950	15.8900	15.8950	15.8925	15.8975	15.9050	15.9025	15.8925	15.8900	15.8975	15.9025	15.8900	15.8950	15.8975	15.8900	15.9075	15.9100	15.9000	15.9000	15.9025
21-40	15.8925	15.9050	15.9050	15.8975	15.9150	15.9000	15.9075	15.8950	15.9025	15.9050	15.9100	15.9050	15.9200	15.9175	15.9250	15.9400	15.9000	15.9000	15.9025	15.9025
41-60	15.9000	15.9050	15.9000	15.9050	15.9025	15.9000	15.9025	15.9000	15.9000	15.9025	15.9000	15.8975	15.8975	15.8975	15.8950	15.9025	15.8800	15.8900	15.8800	15.8725
61-80	15.8825	15.8850	15.8900	15.8950	15.8825	15.8950	15.8875	15.8875	15.8850	15.8825	15.9150	15.9150	15.9075	15.9125	15.9125	15.9075	15.9025	15.9050	15.8950	15.9100
81-100	15.9125	15.9075	15.9225	15.9100	15.9125	15.9025	15.9125	15.9150	15.9225	15.9050	15.9075	15.9175	15.9075	15.9100	15.9250	15.9225	15.9275	15.9225	15.9250	15.9225
101-120	15.9275	15.9350	15.9375	15.9400	15.9425	15.9350	15.9325	15.9375	15.9325	15.9350	15.9300	15.9375	15.9425	15.9400	15.9400	15.9425	15.9350	15.9400	15.9450	15.9425
121-140	15.9425	15.9525	15.9500	15.9375	15.9425	15.9100	15.9050	15.9100	15.8975	15.9025	15.9100	15.8975	15.9125	15.9150	15.9000	15.8975	15.9175	15.9175	15.9000	15.8925
141-160	15.9025	15.9025	15.9075	15.8950	15.9125	15.9150	15.8975	15.9175	15.9125	15.9025	15.9000	15.9100	15.9150	15.9100	15.9100	15.8950	15.9075	15.8925	15.9125	15.9125
161-180	15.9000	15.9125	15.9125	15.9025	15.9050	15.8975	15.9175	15.9000	15.9025	15.9075	15.9000	15.9150	15.9275	15.9075	15.9100	15.9100	15.9150	15.9225	15.9200	15.9100
181-200	15.9000	15.9000	15.8950	15.8975	15.9000	15.9200	15.9000	15.9100	15.9100	15.9050	15.9075	15.9050	15.9100	15.9075	15.9075	15.9025	15.9075	15.9050	15.9050	15.9025
201-220	15.9075	15.9075	15.9100	15.9050	15.9100	15.9050	15.9100	15.9200	15.9250	15.9150	15.9100	15.9175	15.9175	15.9175	15.9200	15.9175	15.9225	15.9200	15.9225	15.9225
221-240	15.9200	15.9275	15.9175	15.9250	15.9350	15.9300	15.9275	15.9325	15.9300	15.9350	15.9275	15.9300	15.9275	15.9300	15.9300	15.9300	15.9275	15.9250	15.9275	15.9325
241-260	15.9325	15.9325	15.9350	15.9275	15.9325	15.9300	15.9350	15.9300	15.9325	15.9350	15.9325	15.9300	15.9350	15.9400	15.9400	15.9375	15.9350	15.9375	15.9400	15.9325
261-280	15.9375	15.9450	15.9350	15.9375	15.9325	15.9400	15.9350	15.9375	15.9425	15.9400	15.9375	15.9400	15.9350	15.9375	15.9000	15.9050	15.9050	15.8950	15.8900	15.8900
281-300	15.8850	15.8825	15.8825	15.9000	15.8925	15.8950	15.8850	15.8950	15.8825	15.8950	15.8950	15.8850	15.8875	15.8850	15.8750	15.8850	15.8950	15.9025	15.8800	15.8625
301-320	15.8850	15.8850	15.8875	15.9025	15.9175	15.9075	15.9000	15.9050	15.8975	15.9025	15.9075	15.9025	15.9075	15.8975	15.8975	15.9025	15.9000	15.9000	15.9000	15.9050
321-340	15.8975	15.9025	15.9025	15.9025	15.8825	15.8925	15.8850	15.8900	15.8800	15.8875	15.8900	15.8850	15.8775	15.8800	15.8825	15.9000	15.8825	15.8925	15.8850	15.8900
341-360	15.8875	15.9000	15.8850	15.8925	15.8900	15.8775	15.8900	15.8900	15.8750	15.8900	15.8900	15.9000	15.8950	15.8950	15.8800	15.8800	15.8850	15.8800	15.8825	15.8825
361-379	15.8850	15.8950	15.8925	15.8850	15.8850	15.8825	15.8875	15.9000	15.8925	15.8850	15.8875	15.8950	15.8875	15.8825	15.8800	15.9000	15.8875	15.8850	15.8875	

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Nicholas C. Georgantzas
Professor, Information, Technology and Operations
Director, System Dynamics Advisory
Gabelli Business School, Fordham University
140 West 62nd Street, Suite 402
New York, NY 10023-7484, U.S.A.

On Thu, Apr 9, 2020 at 1:08 PM Blaine Lorimer <blaine.lorimer@knights.ucf.edu> wrote:
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I've remained intrigued by the 2003 Tampering Dynamics conference paper you wrote with Dr. Orsini. My own dissertation work (nearly complete) was inspired in part by this paper and has certain similarities. Thus, your paper is included in my literature review section. I'm pretty sure the article is open source online but I am requesting your permission to include figures 11, 12, and 13 in my dissertation to cover my bases. Do you approve?
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Process Data from Tokai Rika (1982)

kazuhito.yaragami@exc.tokai-rika.co.jp
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Blaine Lorimer
Dear Mr.Loriemer,
We confirmed the published Tokai Rika control charts from your information.
It is no problem to use it in your PhD dissertation.
Thank you for contacting our company.
Best regards,
Kazuhito Yaragami

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The story that is published is: In March 1982, a group of executives from Ford Motor Company were visiting the Tokai Rika plant. They noticed workers actively discussing a control chart for cigar lighter manufacturing, and asked about it. The Tokai Rika hosts translated these control charts to English and presented them to the executives to take home and study. The charts tracked about 400 work days of cigar lighter production. These data are from nearly 40 years

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Thank you for your consideration,
Blaine Lorimer
Student
University of Central Florida

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Blaine Lorimer
Dear Mr.Loriemer,
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I am KAZUHITO YARAGAMI, Manager of Intellectual Property Department.
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Please contact us by e-mail.
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TECHNICAL ADMINISTRATION DIV.
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Thank you, Blaine

APPENDIX C: SOFTWARE CODE

Software Code From
Classifying Cardiac Biosignals Using Ordinal Pattern Statistics and Symbolic Dynamics by Parlitz et al.

Available at https://doi.org/10.1016/j.compbiomed.2011.03.017

The following Python and MATLAB (TheMathWorks,Inc.) functions *perm_indices* compute the sequence *indcs* of permutation indices from a time series *ts* for a given word length *wl* and a given lag *lag*

Python function perm_indices for computing permutation indices.

```
import numpy asnp
def perm_indices(ts, wl=4, lag=1):
m = len(ts)-(wl-1)*lag
indcs = np.zeros(m, dtype=int)
for i in xrange(1,wl):
  st = ts[(i-1)*lag : m+((i-1)*lag0]
  for j in xrange (i,wl):
      indcs + =st>ts[j*lag : m+ j*lag]
  indcs*= wl-i
return indcs + 1
```

Listing 2. Matlab™ function perm_indices.m for computing permutation indices.

```
function indcs = perm_indices(ts, wl,lag) ;
m = length(ts)-(wl-1)*lag ;
 indcs = zeros(m,1) ;
for i = 1: wl-1 ;
   st = ts(1+(i-1)*lag : m+(i-1)*lag) ;
   for j = i:wl-1 ;
      indcs= indcs+(st>ts(1+j*lag : m+j*lag)) ;
   end
   indcs = indcs*(wl-i) ;
end
indcs=indcs + 1 ;
```

APPENDIX D: BIOLOGICAL CHAOTIC TIME SERIES STUDIES

Some literature is listed related to the application of chaos theory for biological processes that might frequently be considered Shewhart-stable. Some of these references could be useful in follow-on research for mitigating identical values in permutation entropy tuples but were not directly applicable to the research herein. They were therefore not specifically discussed in the Literature Review, nor included in the List of References.

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Denton, T. A., Diamond, G. A., Helfant, R. H., Khan, S., & Karagueuzian, H. (1990). Fascinating rhythm: a primer on chaos theory and its application to cardiology. *American Heart Journal*, 120(6), 1419-1440.

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Holt, T. A. (2002). A chaotic model for tight diabetes control. *Diabetic Medicine*, 19(4), 274-278.

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Varela, M., Jimenez, L., & Fariña, R. (2003). Complexity analysis of the temperature curve: new information from body temperature. *European Journal of Applied Physiology*, 89(3-4), 230-237.

Varela, M., Ruiz-Esteban, R., & De Juan, M. J. M. (2010). Chaos, fractals, and our concept of disease. *Perspectives in Biology and Medicine*, 53(4), 584-595.

Wang, X., Meng, J., Tan, G., & Zou, L. (2010). Research on the relation of EEG signal chaos characteristics with high-level intelligence activity of human brain. *Nonlinear Biomedical Physics*, 4(1), 2.

APPENDIX E: VARIOUS INFORMATION-THEORETIC QUANTIFIERS

Numerous information theoretic quantifiers have been developed since Claude Shannon published his seminal paper in 1948. Many of these apply entropy as a measure of uncertainty or randomness in a process. Also, many researchers view the dynamics associated with changing entropy to be a measure of complexity. However, so-called "structural" or "statistical" complexity extends beyond simple measures of entropy by incorporating physical structural correlations and can be used to quantify relationships between past information and future predictions (Crutchfield & Young, 1989). Many authors also discuss complexity in terms of complications associated with a process. Clearly, complexity has many meanings, depending upon the context. This appendix is not an attempt to exhaustively survey all possible information-theoretic measures. Some authors have already contributed to compilation efforts, and are listed below. Instead, this section is intended to summarize some of the salient literature associated with prevalent measures, to provide a more comprehensive list of resources for the interested reader, who might well be a quality engineer with little background in information and complexity theory.

- Statistical Complexity. Another name for the structural complexity determined by the MPR-method, which was discussed at length in this research.

Martin, M. T., Plastino, A., & Rosso, O. A. (2006). Generalized statistical complexity measures: Geometrical and analytical properties. *Physica A: Statistical Mechanics and its Applications, 369*(2), 439-462.

- Complexity Compilations. A few authors have compiled various definitions and measures of complexity.

Bennett, C. H. (1990). How to define complexity in physics, and why. In *From complexity to life: On the emergence of life and meaning*; Gregersen, N.H., Ed.; Oxford University Press: Oxford, UK, 2003; pp. 34–47.

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Feldman, D. P., & Crutchfield, J. P. (1998). Measures of statistical complexity: Why?. *Physics Letters-Section A, 238*(4), 244-252.

- Computational Mechanics.

Ay, N., & Crutchfield, J. P. (2005). Reductions of hidden information sources. *Journal of Statistical Physics, 120*(3-4), 659-684.

Clarke, R. W., Freeman, M. P., & Watkins, N. W. (2003). Application of computational mechanics to the analysis of natural data: an example in geomagnetism. *Physical Review E, 67*(1), 016203.

Crutchfield, J. P., & Young, K. (1989). Inferring statistical complexity. *Physical Review Letters, 63*(2), 105.

Shalizi, C. R., & Crutchfield, J. P. (2001). Computational mechanics: Pattern and prediction, structure and simplicity. *Journal of Statistical Physics, 104*(3-4), 817-879.

- Logical Depth. Based on algorithmic information and computational time complexity.

Bennett, C. H. (1986). On the nature and origin of complexity in discrete, homogeneous, locally-interacting systems. *Foundations of physics, 16*(6), 585-592.

- Basic Entropy Measures.

Rényi, A. (1961). On measures of entropy and information. In *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability, Volume 1: Contributions to the Theory of Statistics*. The Regents of the University of California.

Shannon, C. E. (1948). A mathematical theory of communication. *Bell System Technical Journal, 27*(3). 379–423. doi:10.1002/j.1538-7305.1948.tb01338.x

Tsallis, C. (1988). Possible generalization of Boltzmann-Gibbs statistics. *Journal of Statistical Physics. 52*(1–2). 479–487.

- Relative entropies- Well-developed in thermodynamics and quantum information theory, provides a reference for "available work". Also related to measuring differences for the Fisher Information Metric.

Kullback, S., & Leibler, R. A. (1951). On information and sufficiency. *The Annals of Mathematical Statistics, 22*(1), 79-86.

Scalassara, P. R., Dajer, M. E., Maciel, C. D., Guido, R. C., & Pereira, J. C. (2009). Relative entropy measures applied to healthy and pathological voice characterization. *Applied Mathematics and Computation, 207*(1), 95-108.

- Mutual Information.

Gu, F., Meng, X., Shen, E., & Cai, Z. (2003). Can we measure consciousness with EEG complexities? *International Journal of Bifurcation and Chaos, 13*(03), 733-742.

- Fractal dimension. Quantifies fractal patterns or sets by their complexity as a ratio of the change in detail to the change in scale.

Mandelbrot, B. (1967). How long is the coast of Britain? Statistical self-similarity and fractional dimension. *Science, 156*(3775), 636-638.

Mandelbrot, B. B. (1983). *The fractal geometry of nature*. New York: WH Freeman.

- Cross-Correlation Sum Analysis. A method to quantify the closeness of fractal measures.

Kantz, H. (1994). Quantifying the closeness of fractal measures. *Physical Review E, 49*(6), 5091.

- Lyapunov exponents. The rate of separation for two chaotic trajectories in phase space. The largest of these on the spectrum of exponents determines the predictability of the dynamical process.

Lyapunov, A. M. (1892). The general problem of the stability of motion. Probleme Géneral de la Stabilité de Mouvement. *Annales de la Faculté des Sciences de Toulouse, 9*, 203-474. English Translation (1992).

Pesin, Y. B. (1977). Characteristic Lyapunov exponents and smooth ergodic theory. *Uspekhi Matematicheskikh Nauk, 32*(4), 55-112.

- Algorithmic Complexity. The complexity of a particular string of data, in terms of all possible algorithms that can generate it.

Chaitin, G. J. (1975). A theory of program size formally identical to information theory. *Journal of the ACM, 22*(3), 329-340.

Kolmogorov, A. N. (1965). Three approaches to the quantitative definition of information. *Problems of Information Transmission, 1*(1), 1-7.

Solomonoff, R. (1978). Complexity-based induction systems: comparisons and convergence theorems. *IEEE Transactions on Information Theory, 24*(4), 422-432.

- Lempel–Ziv (L-Z) Complexity. Represents a lossless data compression algorithm usually associated with individual sequences, as opposed to probabilistic distributions. This is relevant in part because complexity is considered in many contexts to represent the "compressibility" of the data and because the purpose of much scientific research is to reduce the arbitrariness associated with an observed process. No *regularity* in process data would mean it has minimal redundancy and minimal predictability. Such a process could not be represented by any rules which, through algorithmic computation, allow its arbitrariness to be reduced. The Lempel-Ziv algorithm can measure the amount of data compression possible, thereby providing a measure of complexity. Similar to permutation entropy, L-Z often applies a sliding window concept to evaluate the data.

Amigó, J. M., Szczepański, J., Wajnryb, E., & Sanchez-Vives, M. V. (2004). Estimating the entropy rate of spike trains via Lempel-Ziv complexity. *Neural Computation, 16*(4), 717-736.

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- Nonlinear Cross-Prediction Analysis.

Schreiber, T. (1997). Detecting and analyzing nonstationarity in a time series using nonlinear cross predictions. *Physical Review Letters, 78*(5), 843.

- Recurrence Plots.

Eckmann, J. P., Kamphorst, S. O., & Ruelle, D. (1995). Recurrence plots of dynamical systems. *World Scientific Series on Nonlinear Science Series A, 16*, 441-446.

Marwan, N., Romano, M. C., Thiel, M., & Kurths, J. (2007). Recurrence plots for the analysis of complex systems. *Physics Reports, 438*(5-6), 237-329.

Vasconcelos, D. B., Lopes, S. R., Viana, R. L., & Kurths, J. (2006). Spatial recurrence plots. *Physical Review E, 73*(5), 056207.

- Recurrence Quantification Analysis.

Gao, J., & Cai, H. (2000). On the structures and quantification of recurrence plots. *Physics Letters A, 270*(1-2), 75-87.

- Statistical Testing in Reconstructed Phase Space.

Kennel, M. B. (1997). Statistical test for dynamical nonstationarity in observed time-series data. *Physical Review E, 56*(1), 316.

- Topological Methods.

Zhang, J., & Small, M. (2006). Complex network from pseudoperiodic time series: Topology versus dynamics. *Physical Review Letters, 96*(23), 238701.

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