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Modeling Autocorrelation and Sample Weights in Panel Data: A Monte Carlo Simulation Study

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MODELING AUTOCORRELATION AND SAMPLE WEIGHTS IN PANEL DATA: A MONTE CARLO SIMULATION STUDY

by

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ABSTRACT

This dissertation investigates the interactive or joint influence of autocorrelative processes (autoregressive-AR, moving average-MA, and autoregressive moving average-ARMA) and sample weights present in a longitudinal panel data set. Specifically, to what extent are the sample estimates influenced when autocorrelation (which is usually present in a panel data having correlated observations and errors) and sample weights (complex sample design feature used in longitudinal data having multi-stage sampling design) are modeled versus when they are not modeled or either one of them is taken into account. The current study utilized a Monte Carlo simulation design to vary the type and magnitude of autocorrelative processes and sample weights as factors incorporated in growth or latent curve models to evaluate the effect on sample latent curve estimates (mean intercept, mean slope, intercept variance, slope variance, and intercept slope correlation). Various latent curve models with weights or without weights were specified with an autocorrelative process and then fitted to data sets having either the AR, MA or ARMA process. The relevance and practical importance of the simulation results were ascertained by testing the joint influence of autocorrelation and weights on the Early Childhood Longitudinal Study for Kindergartens (ECLS-K) data set which is a panel data set having complex sample design features.

The results indicate that autocorrelative processes and weights interact with each other as sources of error to a statistically significant degree. Accounting for just the autocorrelative process without weights or utilizing weights while ignoring the autocorrelative process may lead to bias in the sample estimates particularly in large-scale datasets in which these two sources of error are inherently embedded. The mean intercept and mean slope of latent curve models

without weights was consistently underestimated when fitted to data sets having AR, MA or ARMA process. On the other hand, the intercept variance, intercept slope, and intercept slope correlation were overestimated for latent curve models with weights. However, these three estimates were not accurate as the standard errors associated with them were high. In addition, fit indices, AR and MA estimates, parsimony of the model, behavior of sample latent curve estimates, and interaction effects between autocorrelative processes and sample weights should be assessed for all the models before a particular model is deemed as most appropriate. If the AR estimate is high and MA estimate is low for a LCAR model than the other models that are fitted to a data set having sample weights and the fit indices are in the acceptable cut-off range, then the data set has a higher likelihood of having an AR process between the observations. If the MA estimate is high and AR estimate is low for a LCMA model than the other models that are fitted to a data set having sample weights and the fit indices are in the acceptable cut-off range, then the data set has a higher likelihood of having an MA process between the observations. If both AR and MA estimates are high for a LCARMA model than the other models that are fitted to a data set having sample weights and the fit indices are in the acceptable cut-off range, then the data set has a higher likelihood of having an ARMA process between the observations. The results from the current study recommends that biases from both autocorrelation and sample weights needs to be simultaneously modeled to obtain accurate estimates. The type of autocorrelation (AR, MA or ARMA), magnitude of autocorrelation, and sample weights influences the behavior of estimates and all the three facets should be carefully considered to correctly interpret the estimates especially in the context of measuring growth or change in the variable(s) of interest over time in large-scale longitudinal panel data sets.

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LIST OF ABBREVIATIONS

AIC: Akaike Information Criterion

APIs: Asian Pacific Islanders

AR: Auto-regressive

AR(1): Auto-regressive lag 1

ARMA: Auto-regressive Moving Average

BIC: Bayesian Information Criterion

CFI: Comparative Fit Index

CV: Coefficient of Variation

CPI: Consumer Price Index

CPS: Current Population Survey

DEFF: Average Design Effects

DEFT: Average of the Square Root of the Design Effects

ECLS-K: Early Childhood Longitudinal Study for Kindergartens

ECI: Employment Cost Index

GC: Growth Curve

GFI: Goodness-of-Fit Index

GLS: Generalized Least Squares

HSFCES: Head Start Family and Child Experiences Survey

INT: Intercept

LC: Latent Curve

LCAR: Latent Curve Auto-regressive

LCARMA: Latent Curve Auto-regressive Moving Average

LCMA: Latent Curve Moving Average

MA: Moving Average

MA(1): Moving Average lag 1

MC: McDonald's Non-Centrality Index

NELS-88: National Educational Longitudinal Study of 1988

NHES: National Household Education Surveys Program

NPSAS: National Postsecondary Student Aid Study

PML: Pseudo Maximum Likelihood

PPS: Probability Proportional to Size

PSID: Panel Study on Income Dynamics

PSU: Primary Sampling Units

RMSE: Root Mean Square Error

SD: Standard Deviation

SE: Standard Error

SEM: Structural Equation Model

SRMR: Standardized Root Mean Square Residual

SSU: Secondary Sampling Units

RMSEA: Root Mean Square Error of Approximation

Wt: Weight

CHAPTER ONE: INTRODUCTION

This chapter will focus on defining the research problem, describing the background, purpose, and discussing the significance of the study under investigation.

Background of the Study

Large-scale survey data sets are pervasive in the field of social sciences. A number of organizations such as the U.S. Department of Education, National Council of Educational Statistics (NCES), and Bureau of Labor Statistics collect survey data and publish them in public domain for a variety of purposes, one of which includes utilization of the survey data by educational researchers. Some examples are The Early Childhood Longitudinal Study-Kindergarten Class of 1998-99 (ECLS-K), National Educational Longitudinal Study (NELS), National Household Education Surveys Program (NHES), Consumer Price Index (CPI), and Employment Cost Index (ECI). These data sets are usually longitudinal in nature and are collected at multiple points in time. Because these longitudinal surveys are collected on a national level, most of the data collected in these surveys have multi-stage sampling and usually employ complex sample designs (Hahs-Vaughn, 2006a, 2006b; Stapleton, 2002).

There has been extensive research on understanding the difficulties associated with analyzing data sets with complex sample designs during the past several decades (Hahs-Vaughn, 2003, 2005, 2006c; Hahs-Vaughn & Lomax, 2006; Graubard & Korn, 2002; Graubard & Korn, 1995a; Kish, 1968; Lumley, 2004; Matthew, 2008; Stapleton 2002). In large scale longitudinal surveys, some units in the population maybe oversampled (probability proportional to size sampling) in order to ensure that certain subgroups of the population are adequately represented in the population.

It is important to address two issues when a sample has to be analyzed having complex design features (Hahs-Vaughn, McWayne, Bulotsky-Shearer, Wen, Faria, 2011a):

- Non-random sample creates homogeneity between the units (non-independence)
- Disproportionate sampling to represent certain units in the population result in unequal selection probabilities (e.g., oversampling or adjustment for nonresponse) (Brick et al., 2000; Lee et al., 1989; Skinner et al., 1989).

This dissertation will focus on disproportionate sampling (which is one of the features of complex sample design) and non-independence of observations that is created due to repeated measures (feature of autocorrelation) and not due to clustering effects where the units within clusters are more similar to each other than units between clusters. Repeated measures is a form of data collection at multiple points in time that gives birth to another nuisance condition known as “autocorrelation”. Autocorrelation is a stochastic process that is usually present in time-series or panel data (also known as longitudinal or cross-sectional time-series data) when there is a relationship between successive observations in a sequence, that is, the score of the current observation in a sequence depends upon the scores of the previous observations in the same sequence (Box & Jenkins, 1976; Marsh, 1993). A stochastic process is a sequence of random variables that have some kind of autocorrelation/dependency between them. Time-series data is a sequence of observations that usually consist of repeated measures of the same units over specified time points (usually equidistant from each other) making the time series data a panel data set (Fredericksen & Rotondo, 1979). Hence, auto-correlation is present in time series or panel data that has been derived by some kind of stochastic process (Box & Jenkins, 1976; Glass et al., 1975; Rogosa, 1979; Sivo, 1997). Observations that are close together are more highly correlated than observations that are further apart in time because the observations are identical to each other. The correlation between the observations at different points in time also makes the errors associated with the observations correlated (Curran & Bollen, 2001; Harvey, 1981; Murphy et al., 2011; Rowley, 1989; Sivo, 1997; Sivo, 2001; Sivo & Willson, 2000). Autocorrelation also occurs "when the same measurement instruments are used over two or more occasions. Hence, there is a tendency for the measurement errors to correlate" (Schumacker & Lomax, 2004, p. 397). From this point onwards, the stochastic process will be referred to as autocorrelation or autocorrelative process

Sample weights are one of the components of complex multi-stage sampling design and autocorrelation is usually present in longitudinal survey data due to non-random assignment and they need to be properly modeled to obtain accurate estimates. The proposed dissertation will address the following two issues:

(a) Disproportionate sampling is utilized to select certain units that are under-represented in the population. This results in unequal selection probabilities where oversampling is usually utilized when certain units in the population are sampled at different rates. The proposed study is going to focus on oversampling certain units in the population which is a feature of disproportionate sampling (Hahs-Vaughn 2005, 2006a, 2006b; Hahs-Vaughn, McWayne, Bulotsky-Shearer, Wen, Faria, 2011b; Hahs-Vaughn & Lomax, 2006; Kish, 1974).

(b) Dependencies within the serial observations (autocorrelation) in the data that is created due to repeated measures data collected at multiple points in time (Marsh, 1993; Rogosa, 1979; Sivo et al., 2005; Sivo & Willson, 1997).

Failure to address and correct these issues results in standard errors and parameters estimates that are inaccurate. There is an increased probability of committing a Type I and Type II error. Type I error means that statistical significance is achieved when actually it is not achieved. Type II error means that statistical significance is not achieved when actually it is achieved. It then becomes important for researchers to understand the intricacies of complex survey sample designs. Failure to comprehend the details of these survey designs may lead to potential problems including biased and inaccurate estimates which then affect the inferences and conclusions made for research and practical purposes (Kalton, 1989; Kish, 1968; Kish & Frankel, 1973).

Sample weights have been established as the remedy towards correcting the bias created by disproportionate sampling within complex sample designs (Hahs-Vaughn 2005, 2006a; Hahs-Vaughn & Lomax, 2006; Stapleton, 2002; Stapleton 2006). A weight is simply defined as the reciprocal of the probability of selection of a particular unit in the population. The sample weight represents the number of units in the population that each unit represents (Kish, 1965). The utilization of sample weights helps to balance the problems associated with unequal selection probabilities and is often the most convenient method to deal with disproportionate sampling (Hahs-Vaughn, 2003, 2005, 2006a, 2011; Korn & Gruabard, 1995b; Kish, 1965, Muthen & Satorra, 1995; Stapleton, 2002).

On the other hand, researchers in the past have suggested the utilization of autocorrelative processes to account for the effects of autocorrelation on estimates (Huitema & McKean, 1991; Huitema & McKean, 2007; Huitema, et al., 1996; Sivo, 1997; Sivo & Willson, 2000; Sivo, et al, 2005). Autocorrelation is usually present in panel data which is collected at multiple points in time (on the same units and on same measures) because the values of the current observation and the error associated with that observation becomes serially dependent with the values of the observation and its error preceding that current observation. Since large-scale survey data (that usually have a complex sample design) are collected on a longitudinal basis, it is very likely that the condition of autocorrelation enters into the data. Hence, it becomes necessary to model the autocorrelation present in the longitudinal data to obtain accurate estimates. (Heitjan & Sharma, 1997; Murphy et al., 2011)

An example of an existing data set that has features of both complex sample design and autocorrelation is ECLS-K. The ECLS-K is a longitudinal study that follows the same cohort of a large, nationally representative and community-based cohort of kindergarten students through eighth grade. The fall of 1998 and spring of 1999 formed the baseline year of data collection. The final round of data collection was completed in spring 2004 when majority of the children went to the fifth grade. The ECLS-K base year data employs a complex sample design. First, the multi-purpose sampling frame of Primary sampling units (PSU's) was created which consisted of 1,404 geographic areas of counties or groups of counties derived from the 1990 county-level population data. Second, within each PSU, schools with less than a minimum number of kindergarten students were clustered together before sampling. Third, oversampling was used for children who were Asian and Pacific Islanders (API) to measure the size of PSU for selection purposes. Fourth, schools were selected with probability proportional to size (PPS). Fifth, stratification variables were created based on metropolitan statistical area (MSA), race-ethnicity, class size, and per-capita income. Sixth, three different kinds of weights (child, teacher, and school-level) were calculated to compensate for the unequal selection probabilities at each stage of the sampling design. The current dissertation is only concerned with oversampling certain units in the population (which is one of the issues associated with disproportionate sampling) and the weights associated with it that was utilized in the ECLS-K data (Tourangeau, Le, Sorongon & Najarian, 2009). The ECLS-K is a longitudinal panel data set because it has seven waves of data collected beyond kindergarten where the students have been followed as a cohort from 1998 (when they were in kindergarten) to 2007 (when they were in eight grade). Information was collected in fall and the spring of kindergarten (1998-99), the fall and spring of 1st grade (1999-2000), the spring of 3rd grade (2002), the spring of 5th grade (2004), and the spring of 8th grade (2007). Data in 2001, 2003, 2005, and 2006 was not collected. The ECLS-K can be assumed to have the phenomenon of autocorrelation because of two reasons. First, data has been collected on measures such as math, science and reading achievement scores, physical health and growth, social development, and emotional well-being of children from their teachers, schools, and families by utilizing the same instruments over multiple points in time. Second, data has been collected on variables such as mathematics and reading achievement scores by following the same cohort of kindergarteners over seven waves or seven time periods (four waves for general knowledge scores), thereby making the data a panel data set (Tourangeau et al., 2009). Furthermore, Brak and Brak (2011) had observed autocorrelation to a significant degree in the

ECLS-K data. They had to use a structural equation model that adjusted for the disturbances caused due to autoregressive process present in the observations of ECLS-K data.

Methodological Significance of the Study

Unequal selection probabilities influence the estimates and bias the results. Weights are used to correct for unequal selection probabilities (Hahs-Vaughn, 2011a, 2011b; Kish, 1965, 1968; Pfeffermann, 1993). It has already been established in the literature that autocorrelative processes influences the sample estimates, confidence intervals, and standard errors of the observations in a longitudinal data set (Bryk & Bryk, 2011; Huitema, 1985; Kutner, Nachtsheim & Neter, 2004). However, we do not know to what extent do the sample weights and the correct specification of the autocorrelative process present in the data can together correct for the bias arising due to oversampling and dependency that is created due to autocorrelation. Taking into consideration just the corrective effects of sample weights and neglecting the type of autocorrelative process or vice-versa present in the data may not always yield the best sample estimates.

If sample weights are utilized to correct for the bias created by unequal selection probabilities in a longitudinal data set, then it also becomes important to model the type of autocorrelative process present in the data in order to obtain the best sample estimates. When the autocorrelative process is present in the data set but is not correctly specified, then the merit of using sample weights can be undermined, resulting in less accurate estimates. Furthermore, it is also not known whether the autocorrelative process under-estimates or over-estimates the parameters when weights are applied versus when weights are not applied. Hence, the benefit of obtaining correct sample estimates by using sample weights can be improved if the correct type of autocorrelative process present in the data can be simultaneously addressed.

Research has not been located that has demonstrated the multiplicative or interactive effects of sample weights and the autocorrelative process on the estimates, standard errors, and confidence intervals. The studies that have investigated the effects of sample weights and autocorrelation on the estimates, standard errors and confidence intervals have been conducted in isolation. Vieira and Skinner (2008) indicated in their paper that "in the mainstream panel data modelling literature, there is little consideration of complex design sampling schemes other than through extensions of models to capture clustering effects" and they pointed that research on panel studies and complex sample designs has usually occurred in isolation. Furthermore, Sivo (1997) has recommended that it is important to test for all types of autocorrelative processes under investigation because it may happen that two or more processes may fit the same data equally well. It then becomes important and worthwhile to understand the extent to which sample weights influences the estimates by correcting for unequal selection probabilities in the presence of a particular autocorrelative process(s) that is existing in the longitudinal panel data set. The proposed dissertation study aims to fill these voids in the literature.

Research Questions

The present study is intended to answer the following research questions:

a) To what extent will the presence or absence of one or more than one autocorrelative process influence estimates in a panel data set when sample weights are applied versus ignored?

b) Is there any interactive or joint effect between the autocorrelative process and sample weights on the estimates in a longitudinal panel data set?

c) What kind of autocorrelative process(s) are present in the ECLS-K data and to what extent will the presence or absence of more than one autocorrelative process correct the estimates when sample weights are applied versus ignored?

Contributions of the Study

Over the years, there has been an increase in large-scale data sets that are collected at multiple points in time and involve a complex sample design (Rust & Rao, 1996; Saylor, Friedmann & Lee, 2012). There are an array of issues associated with longitudinal survey data analysis. Extensive work has been done on the application of sample weights to correct for the bias that occurs due to disproportionate sampling resulting in unequal selection probabilities (Binder, 1983; Hahs-Vaughn, 2003; Kish, 1965; Pfeffermann, Skinner, Holmes, Goldstein & Rashash, 1998; Stapleton, 2002). On the other hand, studies have also been conducted to model the type of autocorrelation present in a longitudinal data set and to assess the effect of the type of autocorrelative process on the estimates, confidence intervals, and standard errors (Duncan & Morgan, 1987; Marsh, 1993; Rogosa, 1979).

The focus of the proposed dissertation is on disproportionate sampling which arises from unequal selection and autocorrelation which arises from repeated measures of the same units over time (panel data), both of which are components of a longitudinal data having a complex sample design. The phenomenon of disproportionate sampling and autocorrelation can co-exist in a complex survey data that is longitudinal in nature. The proposed dissertation investigates the interaction between the two phenomena. Disproportionate sampling & autocorrelation both lead to bias in estimates and give erroneous results (Hahs-Vaughn et al., 2011a; Kish, 1968; Marsh, 1993; Rogosa, 1979; Sivo, 1997; Thomas & Heck, 2001). This dissertation is concerned about the joint influence of disproportionate sampling arising from complex sample design and autocorrelation arising from repeated measures.

Both disproportionate sampling and autocorrelation are nuisance conditions that should be taken into account to derive unbiased results. Hence, the simultaneous utilization of both over sampling and correct specification of the autocorrelative process(s) is pivotal to get the best estimates and accurate results. It is important to understand how the weights and the autocorrelative processes work in tandem to influence the estimates when it has already been established in the literature that both these factors have an important influence on the estimates.

This dissertation will be primarily a Monte-Carlo simulation based study. In the Monte Carlo method "properties of the distributions of random variables are investigated by use of simulated random numbers" (Gentle, 1985, p.612). "The Monte Carlo methods is an empirical method for evaluating statistics. Usually, the asymptotic properties of an estimator are known, but its finite sampling properties are not. Monte Carlo simulations allow researchers to assess finite sampling performance of estimators by creating controlled conditions from which sampling distributions of parameter estimates are produced. Knowledge of the sampling distribution is the key to evaluation of the behavior of a statistic" (Paxton, Curran, Bollen & Kirby, 2001, p. 289). In other words, the researcher creates a hypothetical population with parameter values already established by the researcher. The researcher then draws *N* number of samples each having *n* observations from the hypothetical population. The statistics of interest and its associated coefficients are estimated for each sample. A sampling distribution is created for each of the population parameters by assembling the sample statistics from each of the samples that have been derived from the hypothetical population. This sampling distribution has the attributes (e.g. mean, standard deviation, intercept, and slope) of the estimated sampling distribution. The coefficient estimates associated with the particular sample statistic is compared to the corresponding true population parameter value to estimate the amount of bias in the sample estimate when there is systematic violation of statistical rules. In the context of present study, population level parameter estimates (mean intercept, mean slope, intercept variance, slope variance, intercept slope correlation) will already be known for the simulated data sets specifying either the AR, MA and ARMA processes which would be then compared to the sample estimates to assess the change in the estimates when either the autocorrelative process or weights were accounted versus when both were accounted.

One of the advantages of a Monte Carlo study is to estimate the amount (or cut-off point) of autocorrelation that can be tolerated in the data before the estimates starts to deteriorate and make the results inaccurate and/or violate statistical assumptions (Gentle, 1985). For example: In regression analysis, how much multicollinearity can be tolerated before the regression coefficients become biased and inaccurate? A Monte Carlo study can be utilized to answer this type of question. In the context of the proposed dissertation, the Monte Carlo study can answer the following question: "In complex longitudinal, survey data, what is the magnitude of autocorrelation that can be tolerated when sample weights are considered before the estimates of interest become biased and inaccurate?"

In this simulation study, the estimates calculated after the weights were compared when the autocorrelative process had been taken into account with those estimates where the weights and/or autocorrelative process have not been taken into account. The comparison between the estimates will be done when both factors are modeled versus when either one of the factors is absent. The differences in the estimates will help to understand the extent to which weights and the autocorrelative process work hand-in-hand to derive the best possible sample estimates that are closer to the true population level estimates through the estimation of means, standard errors, and confidence intervals at the sample-level than in those situations when one or both the factors is ignored.

CHAPTER TWO: REVIEW OF LITERATURE

This chapter reviews the literature on complex survey design, sample weights, probability proportional to sampling, panel data, and then the types of autocorrelative processes to be utilized for the current study.

Complex Survey Design

Longitudinal survey data that is collected using complex survey design do not utilize simple random samples. Usually, complex sampling designs are longitudinal and include stratified multistage cluster sampling (Feder, Nathan & Pfefferman, 2000; Stapleton, 2002, 2006; Steele, 2007) and unequal selection probabilities (Hahs-Vaughn, 2006a, 2007; Kish, 1992; Pfefferman, 1993; Pfefferman et al., 1996; Pike, 2008). In addition, longitudinal survey data frequently utilize repeated measures where data is collected at multiple points in time on the same units. Disproportionate sampling leads to unequal selection of units and repeated measures create dependencies among the data points or observations that need to be modeled in order to yield accurate estimates (Bollen & Curran, 2004; Macurdy, 1982; Molenaar & Campbell, 2008; Sivo 1997).

Non-independence or dependencies in the data results when the data from the population is sampled using non-random sampling design which is usually employed in collecting complex survey data (Hahs-Vaughn & Onwuegbuzie, 2006; Hahs-Vaughn et al., 2011b). The assumption for statistical procedures is that the residual values are independent across the observations. Due to the utilization of non-random sampling design, the data collected becomes clustered. Clustering is separating the population into groups and sampling from a random subset of these groups (e.g. schools, districts). Hence, the level of analysis is the clusters. Clustering improves accuracy for a given cost but decreases precision for a given sample size (Lumley, 2004). Clustering of data disrupts the assumption of independence between the observations. This results in inaccurate estimates because the similarity within the units of a cluster is greater than between the clusters (Lee, Forthofer, & Lorimor 1989; Skinner, Holt, & Smith 1989). Ignoring the assumption of independence results in inaccurate estimates and leads to inflated Type I errors (Hox & Kreft 1994).

Stratification is another important feature of complex surveys. Stratification divides the population under observation into relatively homogenous groups (strata) based on a distinct set of characteristic(s). A predetermined number of units from each stratum (e.g. men and women, whites and Hispanics) is then sampled. The level of analysis are the units within the strata. The sub-populations are formed a priori before analysis. Stratification improves accuracy for a given sample size (Lumley, 2004). Stratification improves precision in the estimates by decreasing the mean square errors associated with the estimates in linear regression (Kott, 1991). Stratification is different from clustering because the former is utilized to segregate on variables at the macro level (such as districts, schools) based on the characteristics associated with those variables whereas the latter is applied to variables directly related to the features associated with the individual, unit or micro level (such as ethnicity, gender).

Unequal selection probabilities are generally an inherent component in complex surveys and usually occur when certain units in the population are sampled at different rates (e.g. females sampled more than males). When the sample is selected by simple random sampling, the sample is a true representation of population data as each unit in the population has an equal chance of selection. Complex sampling designs usually utilize unequal selection probabilities. Failure to account for the sample selection process might inflate the variances of estimated coefficients and bias the inference (Kalton, 1983; Pfeffermann, 1993). Disproportionate sampling generally occurs when subgroups of the population are oversampled to ensure that the group size of certain units in the population are sufficient for estimation, when the sampled units are non-responsive and do not participate in the survey and/or when probability proportional to size sampling is utilized (Hahs-Vaughn et al., 2011a, 2011b).

It is vital that researchers take the multi-stage sampling and/or disproportionate sampling into account because overlooking either one of them results in biased standard errors (that are underestimated), parameter estimates, confidence intervals and inferential test statistics (Diemer, 2008; Lumley, 2004; Hahs-Vaughn, 2003, 2005, 2006a; Stapleton, 2002). There are a number of national data sets that employ complex sample design and are longitudinal in nature. For example: The ECLS-K, NELS-88 (National Educational Longitudinal Study of 1988), NPSAS (National Postsecondary Student Aid Study), and Beginning Postsecondary Students Longitudinal Study (BPS).

Complex surveys have been widely studied in the past couple of decades. Some of the earliest works on complex surveys started with Kish (1965, 1968). In his paper, he calculated and applied standard errors, bias effects, ratio estimators, and the coefficient of variation for several different relative and index statistics that are important in social and economic surveys involving a complex multi-stage sampling design. He found out that the magnitude of sampling variability of the items in the surveys varied between the periodic surveys. He also concluded that standard errors calculated in accordance with the sample design was greater than the standard error calculated from random sampling because the observations were clustered in the survey. Kish and Frankel (1974) expanded the statistical methods and procedures to compute biases and design effects in three complex samples by utilizing five types of estimators (ratio means, simple correlations, regression coefficients, partial coefficients, and multiple correlations) as a function of number of strata in the sample. He also calculated sampling errors by using the Taylor expansion method, jackknife repeated replication method and balanced repeated replication method.

By the late 1970's and early 1980's, regression-based models and weighted estimators (that were design consistent) were developed, modified, and applied to analyze data from complex surveys (Binder, 1983; DuMouchel & Duncan, 1983; Hansen, Madow & Tepping, 1983; Holt, Smith, & Winter, 1980; Kalton, 1983). With the development of sophisticated computer program in 1980's such as SAS, LISREL, and MPlus, advanced statistical procedures such as bootstrapping (Kovar, Rao, & Wu, 1988), latent variable modeling (Mislevy & ETS, 1985), and cluster sampling (Graubard, Fears & Gail, 1989) began to be used. During the late 1980's and early 1990's, large-scale surveys conducted at multiple points in time that involved multi-stage stratification and/or clustering (such as ECLS-K, NELS: 88, CPI) began to proliferate which further provided impetus to research on issues related to complex sample design such as longitudinal weighting (Corder, Woodbury & Manton, 1990; Crocker, LaVange, Woodbury & Manton, 1990), and sampling weights (Pfeffermann, 1993). With the increase in complexity of large-scale survey data, different statistical models such as multi-level models (Pfeffermann et al. 1998; Stapleton, 2002), mixture-models (Wedel, Hofstede & Steenkamp, 1998), and structural equation models (Hahs-Vaughn, 2003, 2006b; Muthen & Satorra, 1995; Stapleton, 2006, 2008) began to be used to analyze complex sample data.

Sample Weights

Unequal weighting is usually employed when certain units in the population are sampled with unequal selection probability. It then becomes important to assign unequal weights to those units in the analysis. If 1,000 individuals are sampled from each county in Florida and the 1,000 people sampled from Orange County represent 1,000,000 people but the 1,000 sampled from Seminole County represents only 700,000 people. Therefore, each person from Seminole County should be weighted approximately 1.5 times as a person from Orange County in a valid analysis.

A sample weight is the inverse of the probability that the unit in question was sampled (Kish, 1965) and is used to obtain the best possible sample estimates that are close to population parameters when units have unequal probabilities of being included in a sample (Kaplan & Ferguson, 1999). When the unit of analysis (i.e., a person) is sampled at different rates, the sample weight signifies the number of persons in the population that each person represents (Korn & Graubard, 1995b). Unequal selection probability occurs when units in the population are sampled at different rates in order to select certain units that usually are under-represented in the population (Stapleton, 2002). Sample weights correct for the disproportionality of sample thereby making it more representative to the population characteristics under investigation. Ignoring disproportionate sampling may result in biased parameter estimates and inaccurate inferences (Pfeffermann, 1993). Incorporating weights in the sample is required to compensate for sampling at different rates (Kalton, 1983) and is usually the appropriate method to correct bias when certain units from the population are selected at unequal rates (Hahs-Vaughn, 2006a; Stapleton, 2002).

There are different kinds of weights utilized to adjust for unequal selection probabilities and other complex sample design features but the two most commonly used in the analysis of survey data are raw weights and relative (Hahs-Vaughn, 2005) or normalized weights (Kaplan & Ferguson, 1999). Raw weight is the inverse of an observations' likelihood of selection. Observations with a higher likelihood of getting selected (oversampling) will have a smaller raw weight value. Summing all the raw weights across all observations yield the population N ($\sum_{i=1}^{n} Yi = N$). Complex survey design usually are multi-stage. The raw weight is also calculated as the reciprocal of the product of the probabilities of selecting certain units at each stage of the sampling design. For example: If the probability of selection in stage 1 (state) is 1/5, and the probability of selection in stage 2 (district) is 1/15, then the final probability of selection is 1/75 and the corresponding raw weight is 75.

The estimates of interest (such as means, standard errors, confidence intervals, proportions) attained by utilizing raw weights is based on the population size and not on the total sample size (Kaplan & Ferguson, 1999) because adding all the raw weights across all the observations in the sample produces the total population size (West & Rathburn, 2004). Hence, any estimate that is directly influenced by sample size (such as test statistics) will be influenced when raw weight is used. The statistical test will have a higher likelihood of attaining significance because the population rather than sample size is being used for interpretation (Thomas & Heck, 2001).

In order to reduce the bias and ensure that standard error estimates reflect the actual sample size (Thomas & Heck, 2001), the raw weights are converted to relative weights. Relative weight is calculated in two ways. The raw weight is divided by its mean which reserves the sample size $\left(Y\text{i} = \frac{Yn}{\bar{Y}}\right)$ (Peng, 2000). Here, Y_i represents the normalized weight, Y_n is the raw weight for the nth observation and $\overline{Y}$ represents the mean weight. Relative weight can be calculated by first calculating the ratio of sample size to the population size and then multiplying the ratio with the raw weight $Y_i = Y_n\left(\frac{n}{i}\right)$ (West & Rathburn, 2004). The advantage of utilizing relative weight is that sample weights are incorporated and the concerns related to sample size sensitivity is also addressed simultaneously. The relative weights add up to the sample size *n* and is more accurate than the raw weight which is based on the total population size *N*.

Complex samples usually employ a multistage cluster sampling technique where units that are more similar to each other are grouped together. Clustering has a tendency to inflate the variance of estimates, because observations are not truly independent. This results in underestimation of the true variability in the population (Hox, 1998). If the units within the clusters are more similar than units between the clusters, then the estimates of variances and standard errors will be biased. In such situations, it becomes important to model the internal homogeneity of the cluster by modeling the intra-cluster correlation coefficient which estimates the degree of bias (that creeps into the data) created due to correlations present within the observations in the clusters. Hence, it is important to also consider the design effects (average design effects-DEFF and the average of the square root of the design effects-DEFT) in combination with relative weights (Hahs-Vaughn, 2005b; Hahs-Vaughn et al., 2011b). DEFF is the ratio between the square of standard error obtained from a complex sample to the square of standard error obtained from simple random sample (Kish, 1965). Both DEFF and DEFT are useful for making adjustments prior to hypothesis testing or after the hypothesis tests have been implemented. The proposed study is only focusing on oversampling certain units in the population and not taking into account the design effects which is generally calculated when cluster sampling is utilized (Hahs-Vaughn et al., 2011b).

Creel (2007) analyzed longitudinal survey data employing a complex, clustered sample design. He conducted a simulation study where repeated cross-section data and panel survey data was generated. Comparison was made between estimates for the mean and confidence intervals between the two data sets. He found estimates for contrast mean were the same but the standard error estimates for contrast mean was smaller for panel data than the repeated cross-section data. These differences were carried forward to the confidence intervals and t-tests where the panel data yielded statistically significant differences but the repeated cross-section data did not. He concluded that the type of analytical methodology used can result in making a different interpretation of the output.

The influence of sample weights has been studied in the context of single-level structural equation model (SEM) in which sample units have unequal chance of getting selected from the population. Kaplan and Ferguson (1999) conducted a simulation based study to investigate the effects of inclusion and non-inclusion of sample weights on the standard errors and fit statistics in SEM. The results revealed that ignoring sample weights can lead to large bias in the sample estimates and that the use of sample weights helps to diminish the bias. Sample weights have minimal influence on standard errors of the estimates when compared to the standard deviation of the sample when sampling variability is considered. The goodness-of-fit index and the likelihood chi-square statistic also improved with the inclusion of sample weights. The results are consistent with the findings of other studies in the literature on sample weights wherein ignoring weights can lead to considerable bias in the estimates (Hahs-Vaughn & Lomax, 2006; Holt, Smith & Winter, 1980).

Sample weights have also been studied in the context of multi-level SEM. Stapleton (2002) conducted a simulation based study and found out that non-inclusion of weights in the calculation of the covariance matrices can lead to biased parameter point estimates. The author also suggested that the utilization of an efficient sample size weight will sufficiently provide accurate estimates without adjusting the standard error. Sample weights when applied to survey data that has utilized multi-stage sampling design, produce meaningful estimates that are approximately equal to the population. In other words, application of sample weights to the longitudinal data using complex survey design ensures that the estimates obtained on the sample are representative of the population from which the sample was derived. If sample weights are not utilized, the results will reflect a bunch of observations that do not resemble the intended population level characteristics and will not be a true representation of the population (Kalton, 1989).

The next section reviews the literature on disproportionate sampling in relation to the current proposed study.

Disproportionate Sampling

Disproportionate sampling is a type of stratified sampling technique where the number of units sampled from each stratum is not proportional to their representation in the total population. This results in unequal selection probabilities because the units in the population are not given an equal chance of getting selected in the sample. The strata have different sampling segments due to which sample weights need to be applied to compensate for the disproportionality in the sample (Daniel, 2011).

Hansen and Hurwitz (1948) were the first to introduce the probability proportionate to the measure of size for 1^{st}-stage sampling unit, in their paper entitled "Theory of sampling from finite population." This paper explained a sampling scheme, in which the probability of sampling a certain number of observations is proportional to the sum of their sizes in the population without having any constraints on the number of sampling observations to be included from a single stratum. Midzuno (1952) described a sampling system with probability proportionate to the measure of size for 2^{nd}-stage sampling unit and also calculated a variance formula.

Disproportionate sampling is of three types depending on the objective of the allocation. The first type is disproportionate allocation for within strata where the investigator conducts detailed analyses on the observations within the strata of the sample. Usually, oversampling is employed to select observations that have a small number in the population. Although oversampling would create a disproportional distribution of the strata in the sample with respect to the population but there will be adequate number of units to conduct the within-strata analyses. The second type of disproportionate sampling is disproportionate allocation for between-strata analyses. In this type of allocation, comparison is made between the different strata. Hence, the investigator needs to increase the sample size of each stratum through equivalent allocation of observations in each strata. The third type of disproportionate sampling is optimum allocation. This type of allocation is used when both costs and precision or either one important. The size of the stratum will depend upon the cost and precision. This type of allocation is used when an overall accuracy needs to be achieved which is greater than the optimum allocation (Daniel, 2011).

Disproportionate sampling occurs when the certain groups in the population are oversampled in order to obtain sufficient sample size for estimation (Hahs-Vaughn et al., 2011b). Units that are oversampled (i.e., have a higher probability of being included in the sample) are given a smaller weight. Those oversampled units then have less effect on the sample estimates (Thomas, Heck & Bauer 2005). It also occurs when certain adjustments (such as non-response) needs to be done after clustering has taken place and probability proportional to size (PPS) sampling is utilized (such as sampling doctoral students for a study abroad program) (Pike, 2008, Hahs-Vaughn et al., 2011b).

Disproportionate sampling has been consistently shown to underestimate the standard errors and overestimate the test statistics, thereby increasing the chances of committing Type I errors (Hahs-Vaughn 2005, 2006a; Hahs-Vaughn & Lomax, 2006; Kish 1992; Korn & Graubard 1995; Stapleton 2002). The groups that are oversampled can inflate and bias the study results. Survey weights needs to be applied to correct for the bias created by disproportionate sampling in order to derive meaningful estimates that match to the population. Hence, weights ensures that the test statistics calculated from the sample is representative of the target population (Kalton, 1989). The proposed study is focusing only on oversampling which is one of the aspects of disproportionate sampling and not taking cluster sampling into consideration.

The next section focuses on panel data in the context of longitudinal, complex surveys.

Panel Data

Longitudinal survey data has four common designs: repeated surveys, panel survey, rotating panel survey, and split panel survey (Duncan & Kalton, 1987). This dissertation focuses on panel survey data. Panel data is a form of longitudinal data where the population, variable(s), people or units of interest are followed over a period of time (usually at specified and uniform time intervals). The most important feature of panel data that distinguishes it from longitudinal study is it involves repeated measures or observations made on the same units at different points in time which are almost equally spaced. A longitudinal study also involves repeated measures data observed at different time periods but the data is not necessarily collected on the same set of units. Longitudinal surveys are sometimes used interchangeably with “panel surveys” (Box & Jenkins, 1976; Forth, 2008; Hedeker & Gibbons, 2006).

Panel data has several advantages over cross-sectional data. First, the change in an individual’s scores can be assessed in an objective manner because repeated measurements are made on the same individual over multiple time periods. Second, panel data sets controls for individual heterogeneity which are the unmeasured person specific factors and/or effects that can bias the estimates. Third, panel data sets are capable of identifying and estimating effects that cannot be identified in cross-sectional data because of repeated measures at different time intervals. For example: Panel data sets can estimate what proportion of students who were dropouts in one time period remain dropouts in another time period. Cross-sectional data can only estimate the dropout rate at a specific point in time (Duncan, Juster & Morgan, 1987; Frees, 2004; Hsiao, 1986). Panel data sets also has limitations such as issues of coverage error (entire population of interest is unaccounted), non-response error (certain units and/or groups in the population do not participate in the study), various types of measurement errors that are caused due to inaccurate responses (possibly due to vague questions, unaware informants, mistakes in recording the responses), and recall issues where participants are not able to recollect historical information (Kasprzyk, Duncan, Kalton & Singh, 1989). Several studies in the literature have indicated that correlated measurement errors in longitudinal panel data are common (Joreskog, 1979; Marsh & Grayson, 1994; Rogosa, 1979; Sivo, 1997)

Panel data sets are pervasive especially in the field of econometrics and social sciences. The Panel Study on Income Dynamics (PSID) is a longitudinal survey that commenced in 1968 and surveyed approximately 4,800 families in U.S. The data set contains information on the socio-economic characteristics of each family. The PSID also oversampled poor whites, blacks, Hispanics, and youths in the military, thereby infusing a feature of complex sample design into the survey (PSID Main Interview User Manual, 2015). The U.S. Current Population Survey (CPS) is a monthly national household survey conducted by the Census Bureau which collects data on the unemployment rate and statistics on the job and labor market. Examples of panel data sets outside of U.S. include German Social Economic Panel, Panel Data for Organization for Economic Cooperation & Development (OECD) countries, and British Household Panel Survey.

Examples of panel data sets in the field of social sciences include the American Community Survey, American Time Use Survey, China Health and Nutrition Survey, and Cornell National Social Survey. There are a number of panel studies in the field of education (mentioned in the introduction chapter). The NELS-88 is a large-scale panel data set that surveyed eight graders in 1988 and followed the cohort in 1990, 1992, 1994, and 2000. The data contains detailed information on students, parents, teachers, and schools and include five waves of data. The primary purpose of the NELS: 88 was to study an array of interrelated educational policies that influence the attributes associated with schools, teachers, parents and students. The survey design was different for each year depending on the type and number of surveys, tests, and the respondents involved in each wave of data (Curtin, Ingels, Wu & Heuer, 2002). Similarly, the ECLS-K is another panel data set that followed a sample of students from kindergarten to 5th grade to collect information on early education programs, student's transitions, experiences and growth from kindergarten through 5th grade. It has four waves of data starting from 1998 and ending in 2004 (Tourangeau et al., 2009). For the present dissertation, ECLS-K would be analyzed because it has a complex sample design (Hahs-Vaughn, 2003; Stapleton, 2006) and a significant degree of autocorrelation has been found in the data set (Brak & Brak, 2011).

A number of studies have been conducted that have investigated estimations for panel data models under complex sample designs. Feder, Nathan, and Pfeffermann (2000) showed that the two level linear model (where both the first and second level random effects can develop autocorrelative over time) can be used to fit time series models to longitudinal series of short length and having missing data. They also found out that this kind of model can be used to account for longitudinal measurements obtained from multi-level populations. Sutradhar and Kovacevic (2000) crafted a simulation study to develop an estimating equation based on survey weights for estimating regression parameters involved in a multivariate polytomous longitudinal survey data.

Skinner and Holmes (2003) studied two methods for dealing with sampling effects by considering either repetitive observations as multivariate outcomes and adopting weighted estimators that explain the correlation structure, or considering a two-level longitudinal model and changing the weighting strategy proposed by Pfeffermann et al. (1998). Vieira and Skinner (2008) estimated model parameters and variances through a simulation study by analyzing a panel data having survey weights incorporated into the variance estimation. Linearization and point estimation procedures which included pseudo maximum likelihood (PML) and various forms of generalized least squares (GLS) were used. They found that linearization variance estimation performed better than the PML and GLS procedures.

There is a clear link established between time series designs and longitudinal panel designs in which the same units are observed at several points in time (Sivo, 2001). Rogosa (1979) indicated that "[longitudinal] panel designs are a combination of time-series and cross-sectional, with measurements obtained on a cross-section (wave) at each time point" (p. 275). The literature suggests that several models are available that specify correlated errors (Joreskog, 1979, 1981; Joreskog & Sorbom, 1977, 1989; Marsh, 1993, Rogosa, 1979). Time series models may prove to be appropriate for longitudinal data where the measurement errors are correlated across different points in time. Longitudinal data is considered equivalent to time series, when (a) the data on the same individuals is collected at multiple points in time thereby making a panel data set, (b) the occasions in time are separated at equal intervals, and (c) sufficient measurement occasions across several time periods are available (Fredericksen & Rotondo, 1979).

In the next section, the concepts and issues related to autocorrelation and autocorrelative processes is discussed.

Non-Independence of Observations (Autocorrelation)

Longitudinal panel data is often influenced with a condition known as "autocorrelation." Autocorrelation is a problematic condition because it usually occurs in panel time series data having serial observations. Autocorrelation in a sequence is generated by probabilistic models using random or stochastic processes. Several researchers have found that correlated measurement errors exist in longitudinal panel data (Joreskog, 1979, 1981; Joreskog & Sorbom, 1977, 1989; Marsh, 1993, Rogosa, 1979). It is possible to identify the type of stochastic process by examining the characteristics of autocorrelation (Box & Jenkins, 1976). Examples of panel data sets are The Panel Study on Income Dynamics, British Household Panel Survey, German Social Economic Panel, Panel Data for OECD countries, NELS-88, and ECLS-K. In traditional regression analyses, there are certain statistical assumptions. First, it is assumed that the observations and the errors associated with them are independent of each other, that is, there is serial independence between the observations. Second, it is assumed that there is a constant variance, that is, the errors associated with the observations have a common variance. Third, the errors associated with the observations are normally distributed.

In the presence of autocorrelation, the observations are not independent of each other and the amount of error in the data is underestimated. The estimated variances of the regression coefficients will also be biased and inconsistent. The value of R^2 and consequently the reliability coefficient would be usually overestimated leading to high value for t-statistics and greater chances of committing Type II error. The standard error will be seriously underestimated leading to positive autocorrelation and the prediction intervals will be excessively wide thereby biasing the other parameters estimates of theoretical interest. (Huitema & McKean, 1991; Johnston, 1984; Murphy, Beretvas & Pituch, 2011; Sivo, 2001; Sivo & Willson, 2000; Sivo, Fan & Witta, 2005; Rowley, 1989). In its simplest form, autocorrelation can be equated to a dependent t-test to correct for the sequential dependency between the pre and post-test scores. Ignoring the dependency between the pre and post-test scores would result in erroneous estimation of differences in means.

Box and Jenkins (1976) indicated that time series data may be modeled from two discrete autocorrelative processes: autoregressive (AR) and moving average (MA). Sivo (1997), Sivo and Willson (2000), Sivo et al. (2005), Sivo and Fan (2008) also studied the AR, MA and auto-regressive moving average (ARMA) models that were used to analyze longitudinal data. The need to account for the autoregressive and moving average autocorrelative processes increased with the finding of correlated errors in longitudinal panel data (Joreskog, 1979; Rogosa, 1979; Marsh & Grayson, 1994; Sivo & Willson, 2000).

Willson (1995b) explained the use of structural equation modeling (SEM) to test time series models against longitudinal panel data. Willson showed that a model specifying a MA process has not been tested yet. The MA process exists in the data when the errors associated with the observations are correlated with each other. He suggested that the errors may have an MA structure if a lag 1 autocorrelation among the errors were to be found, that is the error in a sequence at any time period is influenced by error immediately before it in the same sequence. Sivo and Willson (2000) investigated whether MA or ARMA models fit two longitudinal data sets (previously assumed to have quasi-simplex structures) better than quasi-simplex, one-factor, or AR models. They concluded that the fit, propriety, and parsimony of all five models should be considered simultaneously and compared before a particular model is deemed as appropriate for a particular panel data set.

In an autoregressive model, the previous value of an observation in a sequence influences the current value of the same variable in the same sequence. Autoregressive refers to models where a variable is regressed on itself at an earlier time period (Figure 1 from Sivo & Willson, 2000). The main feature of the autoregressive model is the regression of a variable on its earlier value. In autoregressive models, the variable is an additive function of its immediately preceding value plus a random disturbance. Autoregressive models answer the question, "How is the consistency of a variable over time influenced by the presence of correlation between the observations."

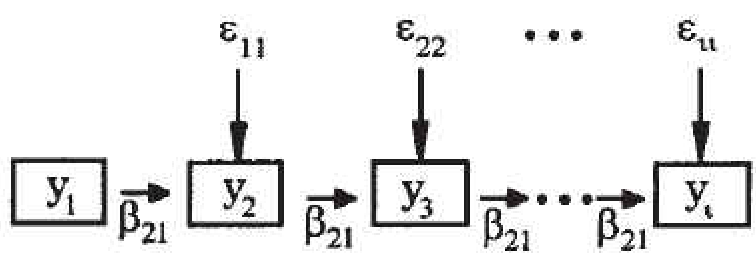

Figure 1: Autoregressive model (lag 1)

Anderson (1960), Bollen and Curran (2004), Box and Jenkins (1976), Humphreys (1960), Heise (1969), Wiley and Wiley (1970), Joreskog (1970, 1979), Rogosa (1979), and Werts, Joreskog, and Lin (1971) contributed towards the expansion, identification, and estimation of autoregressive models. In an autoregressive model, an observation in a particular sequence is an additive function of the observation(s) immediately before it in the same sequence plus a random error component associated with it (Bollen & Zimmer, 2010). The two important assumptions of autoregressive models are that the current observation in a particular series is correlated and influenced by observation(s) immediately preceding it within the same series (also known as lagged influence) and that coefficients of effects are same for all observations. The equation for the simplest autoregressive model is shown in equation 1.1.

$$Y_t = \beta_1 \gamma_{,t-1} + \beta_2 y_{t-2} + \cdots + \beta_i y_{t-i} + \ \xi_{it} \qquad (1.1)$$

Where, Y_t is observed score on a time point t diverged from initial value of Y_0 in the sequence, ξ denotes error component associated with a given time point t and β depicts the correlation among sequentially ordered scores at a particular time lag (e.g., t-1 = a lag of 1 where error associated with current observation is correlated to error associated with observation immediately before the current observation; t-2 = a lag of 2 where error associated with current observation is correlated to errors associated with two observations immediately preceding the current observation) (Bollen & Curran, 2004). Eideh and Nathan (2006) fitted the autoregressive model to longitudinal survey data (when the sampling design was informative) by using maximum-likelihood, pseudo-maximum-likelihood and sample-likelihood- based methods. The simulation study showed that the sample-likelihood-based method produces better estimators than the pseudo-maximum-likelihood method.

The Moving Average (MA) model is also a type of latent growth curve model. It answers the question, "How is the consistency of a variable over time influenced by the presence of correlation between the errors associated with the observations?" (Sivo et al. 2005) (p. 209). The MA models are often constructed to allow the present value in a time series sequence to be defined as a function of autocorrelated errors belonging to the same sequence. MA process is shown in equation 1.2.

$$Y_t = \varsigma_t - \beta_1\xi_{t-1} - \beta_2\xi_{t-2} - \cdots - \beta_q\xi_{t-q} \quad (1.2)$$

Where y_t represents an observed score taken on a particular time point (t) deviated from the initial value of Y_0 in the series, ξ represents error component associated with a time point (t), and ξ_{t-q} represents a correlation among the errors at a particular time lag. MA models suggest that errors correlate across occasions at some lag. For example, a lag 1 MA (having minimum of 4 waves) model would have the error for time period 1 correlate with the error from time period 2, and the error from time period 2 correlate with error in time period 3 (as shown in Figure 2 from Sivo & Willson, 2000). Usually, the error from time period 1 will not be correlated to error from time period 3.

The autoregressive and moving average processes can be simultaneously modeled being in the same data leading to formation of ARMA models. ARMA models are used to answer the following question, "How is the stability of a construct over time affected by autocorrelated observed scores and residuals?" (Sivo et al. 2005) (p. 209). ARMA process is shown in equation 1.3.

$$Y_t = \theta_1 Y_{t-1} + \theta_2 Y_{t-2} + \cdots + \theta_p Y_{t-p} - \beta_1 \xi_{t-1} - \xi_{t-2} - \cdots - \beta_q \xi_{t-q} + \xi_t \quad (1.3)$$

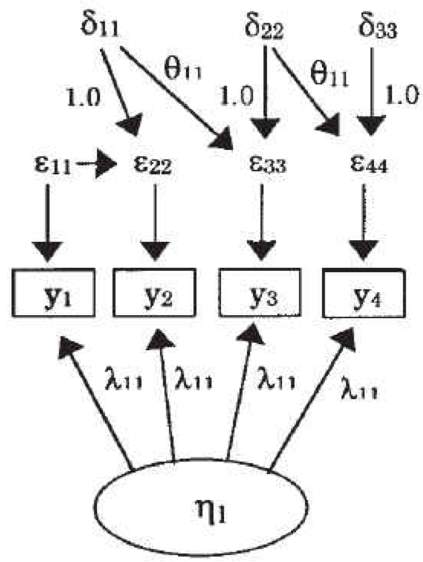

Figure 2: Moving Average (lag 1)

Latent or Growth Curve Model

Growth Curve Modeling (GCM) has become a popular alternative to the simplex and quasi-simplex, especially within the context of structural equation modeling (SEM). GCM through SEM offers more flexibility in testing various models about patterns in growth or decline and is a better alternative than either repeated measures ANOVA or hierarchical linear modeling because SEM helps to model measurement error (Duncan, Duncan, Strycker, Li, & Alpert, 1999; Fan, 2003; Yuan, Marshall & Bentler, 2003). The latent or growth trajectory model allows separate trajectories over time for variables which have been investigated at multiple points in time. The observed repeated measure is used to estimate a distinct growth trajectory for each observation across multiple time points. Each observation in the sample can have different time trends as marked by a different intercept or slope when tracked over time. The latent trajectory focuses on individual differences in trajectories over time and does not assume that all observations would have the same pattern in growth. The latent trajectory model has no impact on the lagged values of a variable on itself. The intercept and slope parameters that explain the trajectories differ over observations used in the model. The intercept and slope parameters are tested in the framework of two components which are vital to a latent curve model: the ***within-person sub-model*** and the ***between-person sub-model***. The ***within-person sub-model*** is represented in equation 1.4 (Bollen & Curran, 2004).

$$Y_{nt} = \propto_t + \Lambda_{12} + \beta_n + \epsilon_{nt} \tag{1.4}$$

Where, α_t is the random intercept for case 1 and β_n is the random slope for case n. The Λ_{12} is a constant within time t where $\Lambda_{12} = 0$, $\Lambda_{22} = 1$. The other values of Λ_{12} allow for the specification of linear or non-linear growth trajectories which usually varies for each unit in the model. The ε_{it} is the error term at time t. The latent trajectory model models each observation (n) to have a separate intercept and slope to explain the trajectory of a variable of interest over time (t). Linear latent curve models are usually specified for a pattern of growth across four occasions with the assumption of base parameters (0, 1, 2, 3). These base parameters indicate that the change over time begins at the first time point followed by steady linear growth at each successive time point. Nonlinear latent curve models specify in addition to the linear growth when this makes theoretical sense. A quadratic process may include the following base parameters [0 1 4 9 16 25 36 49]. The ***between-person sub-model*** depicts the basic latent curve model by specifying the starting point (intercepts- α) and the rate of change (slopes- β) as random variables represented by the equations 1.5 and 1.6:

$$\alpha_i = \mu_\alpha + \zeta_{\propto i} \tag{1.5}$$

$$\beta_i = \mu_\beta + \zeta_{\beta i} \tag{1.6}$$

The individual model parameters are used to represent the average of all the starting points (mean intercept-μ_α) and average rate of change (mean slope-μ_β) plus individual variation which is the error component ($\zeta_{\alpha i}$, $\zeta_{\beta i}$) for all observations in a particular group (Bollen & Zimmer, 2010).

Sivo (2001) added to the previous research on selecting the best model for time series and panel data by focusing on the appropriateness and practical benefits of specifying multivariate time series models in the context of SEM. This paper suggested that testing multiple indicator, latent curve models specifying AR, MA, and ARMA autocorrelative processes that are equivalents of each other to panel data is important. Such multiple indicator models can be used to utilize one of the major statistical advantages of SEM which is directly modeling and estimating the measurement error that exits between the observations. The paper also demonstrated the possibility for autocorrelated latent factor errors especially when autocorrelation among factor scores has been found to exist in panel data.

Sivo et al. (2005) combined the latent curve specification with the auto- correlated process known as autoregressive moving average (ARMA) resulting in the LCARMA model. They demonstrated that when growth curve (GC) models are fitted to longitudinal data, multiple competing models should also be considered which includes growth models that also specify

AR, MA, and ARMA processes. This simulation study integrated the GC, MA and ARMA models. It also demonstrated that the magnitude of autocorrelation in its various forms (AR, MA, and ARMA) can bias the growth curve parameter estimates of a GC model. The results of this study focus on the issue that GC model estimates become biased when autocorrelations are present in the manifest variables and the errors associated with them. The paper suggests that researchers using GC models should simultaneously consider different alternative rival models that are specified a priori, with AR, MA, or ARMA processes also because not doing so may result in biased growth curve estimates and difficulty to detect change over time.

Sivo and Fan (2008) extended the work done by Sivo et al. (2005). They explained how researchers can analyze longitudinal data in which change or growth is modeled to occur over time. The researchers showed that it is meaningful for researchers to model competing latent curve processes with some consideration of autocorrelative processes as well. This LCARMA model was equipped to simultaneously correlate not only the observations but also the associated errors over time, which is a common phenomenon with longitudinal data, so that parameter estimated were not biased. The latent curve ARMA (p, q) model is specified by combining a latent curve and AR and a latent curve MA panel model (Sivo, 2001; Sivo et al., 2005). The latent curve ARMA (p, q) model is specified in equation 1.7:

$$Y_t = \propto_i + \beta_i \lambda_t + Y_{t-1} + \gamma_1 + Y_{t-2} + \cdots + \gamma_p Y_{t-m} - \delta 1_{\epsilon t-1} - \delta 2_{\varepsilon t-2} - \cdots - \delta q_{et-n} + \epsilon_{ti} \tag{1.7}$$

where Y_t is an observed observation (Y) at time point (t) following after the initial value of Y_0 at time 1 of the series, $ß_i$ signifies a correlation among the scores at lag 1 where the current observation is influenced by the score immediately before it, and ε denotes residual error component associated with a given observation at a time point (t). In the ARMA model, not only the observations are correlated in the sequence but the errors associated with the observations also correlate (Figure 3 from Sivo & Willson, 2000).

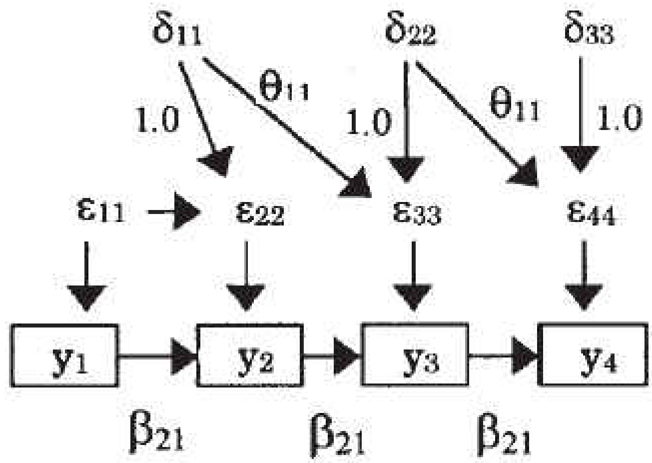

Figure 3: Autoregressive Moving Average (lag 1)

In the next chapter, the methodology for the current dissertation is explained.

CHAPTER THREE: METHODOLOGY

This chapter will focus on the methodology and research design utilized in the proposed study. The chapter will provide details of the Monte Carlo study design, the data generation process, the features of complex sample design (disproportionate sampling) that would be incorporated into the generated data, the sample selection procedure, and the calculation of sample weights. The first section of this chapter provides details of the Monte Carlo simulation which is a statistical experiment that was designed to investigate the interactive or joint influence of sample weights present in large-scale datasets and autocorrelative processes (Autoregressive-AR, Moving Average-MA; Autoregressive Moving Average-ARMA) by generating fictitious population-level data having a particular autocorrelative process. The data generation process is described which explains the design of the simulation. It also describes the steps to incorporate the varying values of autocorrelative processes and sample weights. The second section of the chapter describes the complex sample design aspects of the ECLS-K data set including creation of strata and the other components that were modeled in the simulation. The third section of the chapter describes the sample size selection from the generated population data by utilizing the SURVEY SELECT procedure in SAS. The fourth section focuses on calculation of sample weights based on the number of units sampled into each strata from the total population. The last section summarizes all the steps involved in the data generation process, complex sample design, sample size selection, and calculation of sample weights.

Data Generation Process

The design of Monte Carlo simulation in the present study was similar to the Monte Carlo study conducted by Sivo and Willson (2000) for generating the data sets having a particular autocorrelative process-AR, MA and ARMA. Sivo and Willson (1998) suggested that at least four waves of data is reasonable for fitting autoregressive (lag 1) and moving average (lag1) to a large panel data set. At least five or six waves of data is required at a minimum to fit a autoregressive moving average (ARMA) model (having lag 1 parameters each for the AR and MA portions). The values for latent curve estimates (mean intercept-10.0, mean slope-1.0, intercept variance-2.0, slope variance-0.10, and intercept slope correlation-0.05) were selected according to the values set by the simulation study conducted by Sivo, Fan, and Witta (2005) paper where they fitted latent curve model having AR, MA, and ARMA process to panel data. The random seed generated 252,000 data points that yielded similar means and standard deviations each time the data was generated in order to have consistency in the population-level estimates and to avoid change in these estimates with each iteration.

The Monte Carlo simulation used in the current dissertation had four conditions which were systematically varied to encompass all the possible range of conditions to the best possible extent. Three types of autocorrelative processes were examined as the first three conditions of the simulation. First, the autoregressive (AR) component varied according to three AR values (0.33, 0.67, or 0.85). Second, the moving average (MA) component varied according to three MA values (0.33, 0.67, or 0.85). Third, the ARMA values changed according to four values (with AR parameter values of 0.00, 0.33, 0.67, 0.85, and MA parameter values of 0.00, 0.30, 0.60, and 0.80). Finally, the fourth condition encompassed the application of sample weights versus no application of sample weights. The population size and the sample size calculations were computed to oversample Asian Pacific Islander students that matched the number of students sampled in the ECLS-K data set (which was 24 from each school).

Three eight-wave latent curve data sets having AR process (one for each AR lag1 values set to 0.33, 0.67, and 0.85), three eight-wave latent curve data sets having MA process (one for each MA lag1 values set to 0.33, 0.67, and 0.85), and four eight-wave latent curve ARMA data sets having ARMA process at lag1 (one for each AR values set to 0.00, 0.33, 0.67 & 0.85 and MA values set to 0.00, 0.30, 0.60 & 0.80 respectively) were generated each having 252,000 data points (Fan, Felsovalyi, Sivo, & Keenan, 2002) to obtain the population data. The value of 0.00 represents no autocorrelative process being modeled to observe how the models intended to fit a particular data set and the behavior of sample estimates when the respective process was not accounted. The latent curve estimates evaluated for the simulated population and sample data were mean intercept, mean slope, intercept variance, slope variance, intercept slope correlation, mean intercept standard error (SE), mean slope SE, intercept variance SE, slope variance SE, and intercept slope correlation SE. The model fit indices were used to evaluate the fit of a particular model(s) to the data. The fit indices consulted were chi-square, AIC, BIC- lower values indicate better fit; CFI > 0.95; McDonald Centrality Index > 0.90; RMSEA < 0.05; and RSMR < 0.08 (Hu & Bentler, 1999; Schumacker & Lomax, 2010; Sivo, Fan, Witta & Willse, 2006). The appropriateness of the model was assessed according to several benchmarks which includes the behavior of the estimates, model fit indices, and the successful convergence of the iterative estimation procedure (which is usually maximum likelihood).

Eight models were selected for each of the AR, MA and ARMA values for the sample-level data. These models are latent curve having sample weights but no ARMA, latent curve having no sample weights and no ARMA, latent curve AR with sample weights, latent curve AR without sample weights, latent curve MA with sample weights, latent curve MA without sample weights, LCARMA with sample weights and LCARMA without sample weights. All the eight models were fitted to each of the ten population data sets. A total of 200 samples were randomly selected for each model (1,600 samples from each data set and 16,000 total samples from 10 data sets) from each of the population data having an autocorrelative process and corresponding value. Each iteration calculated the latent curve estimates and the fit indices. For example: 200 samples were randomly selected for latent curve with sample weights model from the population data having the ARMA process of 0.33 and 0.30. This process was repeated for each population data set with a total of 10 runs to randomly select a total of 1,600 samples from each data set. The sample-level latent curve statistics were compared to the population-level parameters as a function of the eight models and the corresponding autocorrelative values.

Data having a the AR(1) autocorrelative process was generated to have data points for eight waves (or eight sets of scores) in which the observed data point was a function of a preceding data point plus a degree of random error with AR process values set at 0.33, 0.67, or 0.85. Data having the AR(1) autocorrelative process was generated to have data points for the eight waves in which the first data point had a direct influence on the second point. The AR(1) process means that the data points are at lag 1, that is the current data point is dependent on the consecutive data point preceding it. Data possessing the MA(1) autocorrelative process consist of data points for eight waves in which lag 1 measurement errors were correlated according to static MA process values set at 0.33, 0.67, or 0.85.

The MA(1) process means that the errors are at lag 1, that is the current error that is associated with the current data point is dependent on the consecutive error associated with the data point preceding it. The model suggests that errors were correlated at lag 1. In order to achieve a lag 1 error, the adjacent pair of errors should share a distinctive relationship that does not occur between the other pairs. Specifically, only errors belonging to adjoining waves were correlated in order to achieve a MA(1) autocorrelative process. Hence, an exclusive source of error impacts each error at a given wave, and that error is correlated with the next adjacent error at a given wave. Finally, the ARMA autocorrelative process was generated so that the data point was a function of a previous data point (AR process values set at 0.00, 0.33, 0.67, and 0.85) plus a random error portion and a correlated portion (MA process values set at 0.00, 0.30, 0.60, and 0.80). The autocorrelative values for the AR and MA processes were consistent with the values utilized in the simulation study by Sivo and Willson (2000).

The LINEQS option in the PROC CALIS (proc standing for procedure and CALIS representing Covariance Analysis of Linear Structural Equations) procedures in SAS was used to analyze the growth curve model estimates of the simulated data under study. The AR program was coded in a way so that the first observed data point had direct effect on the second observed data point plus a random error component. The phi coefficients for all the eight waves were forced to equal the first phi coefficient in the autocorrelative model. The MA program was coded so that each of the eight manifest variables of the growth curve model would be equivalent to the true score assessed for the first phi coefficient in the model plus some amount of random error.

“Hence, the manifest variables was constrained. The errors were correlated and all the error correlations were forced to equal to the correlation between the first and second error terms in the sequence. The ARMA process was simulated in a manner similar to the AR process, except the errors were permitted to correlate at lag 1, with all the error correlations constrained to equal the first error correlation in the sequence” (Sivo & Willson, 2000) (p. 180).

To demonstrate empirical evidence of the simulation results, ECLS-K was selected because it has been established from previously published research that ECLS-K data possesses autocorrelation (Bryk & Bryk, 2011) but the type of autocorrelative process existing in the data set has not yet been investigated. The ECLS-K has data collected over seven waves [(1st wave: fall of kindergarten (1998-99); 2nd wave: spring of kindergarten (1998-99); 3rd wave: fall of 1st grade (1999-2000); 4th wave: spring of 1st grade (1999-2000); 5th wave: spring of 3rd grade (2002); 6th wave: spring of 5th grade (2004); 7th wave: spring of 8th grade (2007)] which includes the math and reading achievement scores at seven time points. Hence, all the three types of autocorrelative processes (AR, MA, and ARMA) were fitted to the ECLS-K data for two reasons. First, it was important to test the competing autocorrelative models under study simultaneously in order to identify the best model that fits the data (Sivo & Willson, 1998). Second, it was essential to correct for the biasing influence of any autocorrelative process on the estimates (Sivo, 1997).

In order to assess the fit of the model(s) to the ECLS-K data set, fit indices of eight latent curve models, latent curve estimates with sample weights and without sample weights and their SE, means for math and reading scores, and number of iterations taken for model convergence were evaluated. If two models fit the data the ECLS-K data well, then model parsimony were used as a criteria to assess the best fit between the models. Results obtained by fitting each of the autocorrelative model to the ECLS-K data set under study were interpreted in light of the simulation findings.

Complex Sample Design

Complex sample design usually involve a multi-stage sampling design. The units that were sampled in the first stage are the primary sampling units (PSU's). In ECLS-K, the PSU's were counties or groups of counties with PPS sampling. The PSU size was determined by the number of 5-year olds in the population. There were total of 100 PSU's selected. Out of the 100 PSU's, 24 were self-representing (SR) and the remaining 76 PSU's were designated as non-SR. Asians and Pacific Islanders (APIs) were oversampled. The units that were sampled in the second sampling stage are the secondary sampling units (SSU). In ECLS-K, the SSU's were public and private schools. The schools were systematically selected with PPS sampling to a weighted measure of size based on the number of kindergartens enrolled (Tourangeau et al., 2009). "There were total of 1,413 schools selected. Out of the total 1,413 schools, there were 953 were public schools and 460 were private schools. The units that were sampled in the third-stage were children of kindergarten age selected within each sampled school. The objective of the child sample design was to achieve a minimum required sample size of APIs who were the only group to be oversampled. Equal probability systematic sampling was used to sample the children with a higher rate to sample the API children. In general, the number of children sampled at any one school was 72. Weight C7CW0 was utilized for calculation purposes because it is a longitudinal weight used to estimate the child-level characteristics or assessment scores across seven waves. In addition, the full sample weight, the sample design, and PSU variables are required. The full sample weight (C7CW0), the stratum variable (C7TCWSTR), and the PSU variable (C7TCWPSU) were utilized to calculate the standard errors and design effects across the seven waves of data collection" (Tourangeau et al., 2009) (p.100-160).

The ECLS-K not only employs a stratified clustered sampling design where the children, parents, teachers, and schools have their own design effects (that can be estimated from the survey data) but at the same time it also involves PPS sampling to account for oversampling. The number of 5-year olds and APIs (who were oversampled) in the first stage were used to select the PSU. In the second stage, the number of kindergartens were used to select the schools. Stratification and clustering occurred simultaneously not only at the PSU and school level but also at the school level. Clustering is inevitable in such kind on multistage design where grouping is occurring at each level based on the student's ethnicity and age.

The proposed study is only focusing on the complex design aspect with respect to oversampling and the dependence created due to repeated measures (not due to the clustering effect occurring because of complex multi-stage design). Design effects are not being considered in the present study because the districts and schools are being oversampled based on the number of API students and not because APIs students in one school share certain characteristics (such as gender, age) that make them different from the characteristic of API students in other schools, thereby creating a clustering effect. In other words, the cluster effects are not being modeled in the simulated data sets because the stratification is taking place at the student level only as a function of unequal selection probabilities arising from disproportionate sampling (oversampling) of API students.

As discussed earlier, a total of 100 districts and 1,413 schools were sampled in the ECLS-K data set. In order to incorporate the framework of a real-world data set such as ECLS-K into the simulation, fictitious districts and schools were created. However, for the current simulation study approximately 5.0% of the total number of sampled districts and 25.0% of schools were used as parameters to generate the population-level data resulting in 5 districts and approximately 350 schools. Each district had 70 schools with each school consisting of 720 students for a total district population size of 50,400 students and total population size of 252,000 students. There were ten data sets generated three each for AR, MA, and four for ARMA. Each data set had 252,000 data points. In order to incorporate the oversampling of APIs from each school, the number of APIs within each school was varied. In order to vary the number of APIs within each school and to create disproportionate number of students based on ethnicity, a race variable using a random uniform distribution was created. The variable was then divided into three strata-White (64%), Hispanic (20%), and Asian Pacific Islanders (17%) (APIs). These proportions were held constant so that each data set had consistency in terms of the distribution of units based on race.

The above parameters were selected to generate the simulated population-level data for three reasons. First, the calculations for selecting the number of districts, schools and students were made with the ultimate goal of sampling 72 students (as in ECLS-K data) from each school. The current study was focused on unequal selection probabilities resulting from disproportionate allocation occurring at the student-level variable (which is race) and no PSU, SSU and design effects were being modeled either at the district or school level. The weights were calculated based on stratification of race variable only. Hence, simulating a humongous data set like the

ECLS-K having 100 school districts and approximately 1,400 schools is not a necessary condition to demonstrate the interactive effects between autocorrelative processes and sample weights. Second, it is important to keep the size of the data set manageable in a simulation study because the increase in observations increased the complexity of the data generation process as each data point had eight waves of a particular autocorrelative process also being simultaneously generated along with the computations of variances and covariances of exogenous variables in the latent curve model. Third, the parameter estimate calculations for the latent curve model and the convergence process (sometimes leading to non-convergence issues) also become complex as the number of data points generated increases. The size of the data set was set to 252,000 data points to keep the simulation study under control, to emulate the ECLS-K stratum size of 24 and also to simultaneously model the joint influence of autocorrelation and sample weights on the estimates.

Sample size selection

Random probability sampling was implemented to oversample the APIs students from each school. The race variable was created for two reasons. First, this variable is directly related to student-level data. Second, the variable would help to incorporate oversampling into the generated autocorrelative data sets. The population from each of the ten data sets have fabricated districts, schools, students to resemble the data structure of ECLS-K data set. However, multi-stage sampling design in which PSU's or districts were used as first-stage sampling design and then schools were selected as second stage to sample APIs according to PPS sampling were not utilized in the current study. For the current study, the sample size in each school was 72 students (10% of total 720 students in each school) which equals to 24 students in each of the three race stratum.

The APIs which had disproportionate numbers in each school were selected using simple random sampling technique. "The PROC SURVEYSELECT procedure in SAS was used to randomly select APIs directly from each school. "This procedure provides a range of methods for selecting probability-based random samples. The procedure can select a simple random sample or a sample according to a complex multi-stage sample design that includes stratification, clustering, and unequal probabilities of selection. With probability sampling, each unit in the survey population has a known, positive probability of selection. This property of probability sampling avoids selection bias in order to make valid inferences from the sample estimates to the population estimates. The SURVEYSELECT procedure utilizes both equal probability sampling and probability proportional to size (PPS) sampling. In equal probability sampling, each unit in the stratum, has the same probability of being selected for the sample. In PPS sampling, a unit's selection probability is proportional to its size measure. PPS selection is often used in cluster sampling, where certain number of clusters of different size are first selected (based on primary sampling unit) in the first sampling stage" (An & Watts, 2011, p.3). This dissertation will only utilize the equal probability procedure available in SURVEYSELECT procedure because the districts (PSUs) and schools (SSUs) are not being used for the first and second stage sampling design respectively to create cluster effects. The number of observations to be randomly sampled in each strata was also specified in the procedure which was set to 24. Sample estimates, confidence intervals, standard errors and fit indices of the latent curve model were evaluated for each of the eight models. These estimates were the sample-level values of the observations in the sample data sets which were varied according to the AR, MA, and ARMA values.

Calculation of Sample Weights

The sample weights were calculated after randomly selecting the sample from each of the population data sets because now the underrepresented units in the population would have been oversampled thereby producing the bias in the sample estimates. The application of weights would revert back the sample estimates to approximately equal the population-level estimates. To reiterate, a weight is simply defined as the reciprocal of the probability of selection of a particular unit in the population (Kish, 1965). Three different weights were calculated, one for each of the race (White, Hispanic, and APIs) categories for each of the population-level data that varied according to the AR, MA and ARMA values. The total number of students in each of the three race categories at the population-level were computed. These numbers were constant across each iteration and for each AR, MA and ARMA lag values. The total number of students in each race was then divided by the total number of units randomly selected using the PROC SURVEYSELECT procedure for each stratum (which is 24). For example: If there were 200 APIs in the population and 72 students were randomly selected, then the sample weight would be $\frac{200}{24} = 8.33$. The adjusted weight was then calculated by dividing the weights by 100. The sample estimates were again computed after the application of weights and then compared to the population-level estimates as well as the sample estimates before the weights were applied.

For the current study, longitudinal weights in the ECLS-K data set were utilized to account for the differences that occured in student-level characteristics across the seven waves of data in the reading and math achievement scores. Longitudinal weights were used because the current study focuses on autocorrelative processes that exists in the scores of students observed over multiple periods of time. In order to apply and empirically demonstrate the relevance of simulation results in a real-world data set, the longitudinal weights present in the ECLS-K data set was modeled by accounting and then not accounting for the autocorrelative process present in the math and reading achievement scores collected over seven waves. The ECLS-K data was collected over seven waves (fall-kindergarten, spring-kindergarten, fall-first grade, spring-first grade, spring-third grade, spring-fifth grade, and spring-eight grade). The C7CW0 longitudinal weight accounted for the child assessment data from all the seven rounds of data collection and was used to model the weights along with the autocorrelative process.

Summary

To summarize, ten population-level data latent curve data sets were generated three each for autoregressive, moving-average, and four for the autoregressive moving-average autocorrelative values (with each data set having different values for AR, MA and ARMA processes). Each latent curve data set that specified a particular AR, MA and ARMA autocorrelative process was used to calculate the population-level means. These estimates were the true population-level means for the 252,000 observations generated in the data set having a particular autocorrelative process. A total of 16,000 samples were randomly drawn from the population-level data sets using the PROC SURVEYSELECT procedure in SAS which had the option of randomly allocating observations in each stratum based on the total sample size (24 in each stratum for a total sample size of 72 students). Each model was fitted to data specifying varying levels of AR, MA and ARMA processes. Fit indices, means, standard errors, and confidence intervals were calculated for each model fitted to each data set. The weights were computed based on the proportion of students sampled for each of the three race stratum in each sample from each of the population data sets.

The next chapter focuses on the results obtained from simulation, general linear model and analysis of ECLS-K data.

CHAPTER FOUR: RESULTS

This chapter focuses on the results of the proposed study. The present study intended to answer the following questions:

a) To what extent will the presence or absence of one or more than one autocorrelative process and corresponding value influence estimates in a panel data set when sample weights are applied versus ignored?

b) Is there any interactive or joint effect between the autocorrelative process and sample weights on the estimates in a longitudinal panel data set?

c) What kind of autocorrelative process(s) are present in the ECLS-K data and to what extent will the presence or absence of more than one autocorrelative process correct the estimates when sample weights are applied versus ignored?

The chapter is organized as follows: First, results are presented to assess the sample-level fit indices (χ^2, SRMR, RMSEA, AIC, Mc, and CFI) by the autocorrelative process (AR, MA, and ARMA) and their corresponding values (AR-0.33, 0.67, 0.85; MA-0.33, 0.67, 0.85, and ARMA with AR of 0.00, 0.33, 0.67 & 0.85, and corresponding MA values of 0.00, 0.30, 0.60 & 0.80) by eight model types (Latent Curve with sample weight-LC wt; Latent Curve with no ARMA and no sample weight-LC no ARMA & wt; Latent Curve AR with sample weight-LCAR wt; Latent Curve AR with no sample weight-LCAR no wt; LCARMA with sample weight-LCARMA wt; LCARMA with no sample weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no sample weight-LCMA no wt). These fit indices were used to assess the best model that fitted the simulated data. These eight models were fitted to data sets specifying an AR, MA and ARMA process.

The second section focuses on the population-level and sample-level estimates (mean slope- starting point; mean intercept- rate of change, intercept variance- average of all the starting points, slope variance- average rate of change, intercept slope correlation, mean slope standard error (SE), mean intercept SE, intercept variance SE, slope variance SE, and intercept slope correlation SE of the latent curve models fitted to each autocorrelative process and corresponding values. The sample-level latent curve estimates are presented for each autocorrelative process and values with the application of weights and without weights. The sample-level estimates were compared to the population-level estimates to evaluate the individual as well as interactive effects of autocorrelative processes and weights on the estimates. The latent curve estimates for the four models (latent curve-LC, latent curve autoregressive-LCAR, latent curve moving average-LCMA, and autoregressive moving average-ARMA) with weights were also compared to select the model that had the most stable estimates.

The third section focuses on the results from general linear model (GLM) procedure in SAS. The GLM procedure was utilized to evaluate the distributions of the sample-level estimates by LCARMA, LCAR, LCMA, and LC models with and without weights, to calculate the effect size (R^2 and η^2), coefficient of variation, mean square error (MSE), and Tukey comparisons by model type, sample weight, and interaction between model type and weight. Box plots and interaction plots were also utilized to assess the separate and joint influence of autocorrelative process and weights on the sample latent curve estimates. The GLM results are presented by each autocorrelative process-overall (combined for all the models), ARMA process (for each set of values-AR of 0.33 & MA of 0.30; AR of 0.67 & MA of 0.60; AR of 0.85 & MA of 0.80), AR process only, MA process only and finally the latent curve model only having no autocorrelative process. The ARMA process was analyzed by its different values through the GLM because it was the best fitting model and it was important to evaluate the influence of different ARMA values and sample weights on the sample LC estimates. The fourth section emphasizes on the results derived by analyzing the ECLS-K data in light of the simulation findings. The fifth section summarizes the results from the simulation study and analysis of ECLS-K data.

Simulation Results

This section presents results from the simulation. The fit indices for sample data results are presented by the AR, MA and ARMA processes and corresponding values for population estimates and then sample estimates.

Fit Indices for Sample Data

Table 1 provides the fit indices of eight latent curve models fitted to data with no autocorrelative process (AR of 0.00 & MA of 0.00). The chi-square values for the LCARMA no weight model was the lowest (34.6848) and the highest value was for the latent curve autoregressive (LCAR) with weight (175.9837). The SRMR, RMSEA and AIC values were lowest for the LCARMA no weight model and highest for LCAR with weight. The Mc Donald Centrality Index (Mc) and CFI value was best for the LCARMA no weight model and worst for the LCAR with weight. Overall, the best fitting model was the LCARMA no weight (χ^2-34.6848, SRMR-0.0182, RMSEA-0.0256, AIC-64.6748, Mc-0.9871, CFI-0.9982) followed by LCARMA with weight (χ^2-56.7465, SRMR-0.1540, RMSEA-0.0654, AIC-86.7465, Mc-0.9380, CFI-0.9926) model when fitted to data having no autocorrelative process. The worst fitting model was LCAR with weights. The LCARMA without weights model was the best fitting model out of all the eight models even after the AR and MA process set to 0.00. This suggests a strong ARMA process in the simulated data.

Table 1: Sample LC Model Fit Indices by No ARMA Process (AR & MA-0.00)

Model	χ^2	**SRMR**	**RMSEA**	**AIC**	**MC**	**CFI**
LC-wt	44.2032	0.01910	0.03990	70.2032	0.9702	0.9963
LC- no ARMA & wt	40.5607	0.01987	0.03269	66.5707	0.9784	0.9972
LCAR-wt	175.9837	0.1904	0.1495	203.9837	0.7148	0.9615
LCAR- no wt	38.7148	0.0195	0.0322	66.7284	0.9802	0.9975
LCARMA-wt	56.7465	0.1540	0.0654	86.7465	0.9380	0.9926
LCARMA- no wt*	34.6848	0.0182	0.0256	64.6748	0.9871	0.9982
LCMA-wt	39.6333	0.0191	0.0339	67.6333	0.9782	0.9971
LCMA-no wt	35.9023	0.0189	0.0256	63.9023	0.9866	0.9981

* Best Fitting Model; Latent Curve with weight-LC wt; Latent Curve with no ARMA and no weight-LC no ARMA & wt; Latent Curve AR with weights-LCAR wt; Latent Curve AR with no weight-LCAR no wt; LCARMA with weight-LCARMA wt; LCARMA with no weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no weight-LCMA no wt; χ^2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI-Comparative Fit Index; χ2- chi-square; SRMR-Standardized Root Mean Square Residual; RMSEA- Root Mean Square Error of Approximation; AIC- Akaike Information Criterion; MC- McDonald's Non-Centrality Index; CFI-Comparative Fit Index

Table 2 provides the fit indices of eight latent curve models fitted to data having ARMA process with AR of 0.33 and MA of 0.30. The best fitting model was LCARMA no weight (χ^2-37.3829, SRMR-0.0149, RMSEA-0.02387, AIC-67.3729, Mc-0.9860, CFI-0.9983) followed by LCARMA with weight (χ^2-52.3777, SRMR-0.0265, RMSEA-0.0406, AIC-82.3777, Mc-0.9556, CFI-0.9960). The worst fitting model was latent curve with no ARMA and no weight followed by LCMA no sample weights model. Table 3 provides the fit indices of eight latent curve models fitted to data having ARMA process with AR of 0.67 and MA of 0.60 with the LCARMA no weight (χ^2-40.1927, SRMR-0.0188, RMSEA-0.0286, AIC-70.1827, Mc-0.9805, CFI-0.9983) being the best fitting model followed by LCARMA with weight (χ^2-42.2662, SRMR-0.0206, RMSEA-0.0358, AIC-72.2662, Mc-0.9714, CFI-0.9980). The latent curve with no ARMA and no weight was again deemed the worst fitting model of all the eight models based on the fit indices followed by LCMA no sample weights model.

Table 2: Sample LC Model Fit Indices by ARMA Process (AR-0.33 MA-0.30)

Model	χ^2	SRMR	RMSEA	AIC	MC	CFI
LC-wt	1599.949	0.3313	0.4849	1625.549	0.0269	0.7464
LC- no ARMA & wt	1866.054	0.5502	0.5241	1892.064	0.0166	0.7013
LCAR-wt	415.9144	0.0678	0.2433	443.9144	0.4159	0.9376
LCAR- no wt	387.4536	0.0154	0.2347	415.4636	0.4394	0.9418
LCARMA-wt	52.3777	0.0265	0.0406	82.3777	0.9556	0.9960
LCARMA- no wt*	37.3829	0.0149	0.02387	67.3729	0.9860	0.9983
LCMA-wt	835.1201	0.2492	0.3530	863.1201	0.1564	0.8698
LCMA-no wt	1113.672	0.3011	0.4097	1141.672	0.0819	0.82365

* Best Fitting Model; Latent Curve with weight-LC wt; Latent Curve with no ARMA and no weight-LC no ARMA & wt; Latent Curve AR with weights-LCAR wt; Latent Curve AR with no weight-LCAR no wt; LCARMA with weight-LCARMA wt; LCARMA with no weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no weight-LCMA no wt; χ^2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI-Comparative Fit Index

Table 3: Sample LC Model Fit Indices by ARMA Process (AR-0.67 MA-0.60)

Model	χ^2	SRMR	RMSEA	AIC	MC	CFI
LC-wt	2148.821	0.1909	0.5634	2174.821	0.0076	0.7148
LC- no ARMA & wt	2140.085	0.2003	0.5620	2166.095	0.0089	0.7144
LCAR-wt	618.7014	0.0159	0.3018	646.7014	0.2573	0.9207
LCAR- no wt	613.6259	0.0160	0.3001	641.6359	0.2618	0.9209
LCARMA-wt	42.2662	0.0206	0.0358	72.2662	0.9714	0.9980
LCARMA- no wt*	40.1927	0.0188	0.0286	70.1827	0.9805	0.9983
LCMA-wt	1024.569	0.1488	0.3923	1052.569	0.1016	0.8662
LCMA-no wt	1033.975	0.1493	0.3942	1061.975	0.9925	0.8641

* Best Fitting Model; Latent Curve with weight-LC wt; Latent Curve with no ARMA and no weight-LC no ARMA & wt; Latent Curve AR with weights-LCAR wt; Latent Curve AR with no weight-LCAR no wt; LCARMA with weight-LCARMA wt; LCARMA with no weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no weight-LCMA no wt; χ^2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI-Comparative Fit Index

Table 4 provides the fit indices values of eight latent curve models fitted to data having ARMA process with AR of 0.85 and MA of 0.80. The LCARMA no weight (χ^2-33.0722, SRMR-0.0232, RMSEA-0.0216, AIC-63.0722, Mc-0.9908, CFI-0.9992) was the best fitting model followed by the LCARMA with weight (χ^2-44.4738, SRMR-0.0227, RMSEA-0.0393, AIC-74.4738, Mc-0.9662, CFI-0.9979).

Table 4: Sample LC Model Fit Indices by ARMA Process (AR-0.85 MA-0.80)

Model	**χ^2**	**SRMR**	**RMSEA**	**AIC**	**MC**	**CFI**
LC-wt	2013.702	0.0694	0.5452	2039.702	0.0103	0.7492
LC- no ARMA & wt	2422.499	0.1307	0.5988	2448.499	0.0040	0.6957
LCAR-wt	659.0156	0.0125	0.3120	687.0156	0.2345	0.9204
LCAR- no wt	660.4773	0.0103	0.3125	688.4733	0.2332	0.9197
LCARMA-wt	44.4738	0.0227	0.0393	74.4738	0.96625	0.9979
LCARMA- no wt*	33.0722	0.0232	0.0216	63.0722	0.9908	0.99928
LCMA-wt	788.8032	0.0793	0.3425	816.8032	0.1752	0.9040
LCMA-no wt	1256.523	0.0860	0.4358	1284.523	0.05951	0.8440

* Best Fitting Model; Latent Curve with weight-LC wt; Latent Curve with no ARMA and no weight-LC no ARMA & wt; Latent Curve AR with weights-LCAR wt; Latent Curve AR with no weight-LCAR no wt; LCARMA with weight-LCARMA wt; LCARMA with no weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no weight-LCMA no wt; χ^2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI-Comparative Fit Index

The latent curve with weight and without weight had the worst fit. An important trend to note in the fit indices for the LCARMA no weight and LCARMA with weight is the difference between the fit indices for both the models which is more in Table 2 (when the AR and MA process was 0.33 and 0.30) than in Table 3 (when the AR and MA process was 0.67 and 0.60). Furthermore, the fit indices for the LCARMA no weight and LCARMA with weight in Table 3 were better than the fit indices of both models in Table 1 and 2. Overall, the fit indices results from Tables 1 through 4 indicate that ARMA models fitted the ARMA data better than the other models. Although the LCARMA no weight models had the best fit but the fit was slightly better than LCARMA with weight model which could be due to some random noise associated with including weights in the latter model.

Tables 5 through 7 presents the fit indices values of eight latent curve models fitted to data having an AR process and no MA process modeled. Table 5 present the fit indices results for AR process of 0.33. The best fitting models was LCARMA no weight (χ^2-35.7735, SRMR-0.0096, RMSEA-0.0286, AIC-65.7735, Mc-0.9846, CFI-0.9987) and LCARMA with weight (χ^2-38.0987, SRMR-0.0090, RMSEA-0.0336, AIC-68.0987, Mc-0.9794, CFI-0.9983) with the latter fitting slightly better than the former. The worst fitting models were again latent curve with weight and latent curve without ARMA no weight models followed by latent curve MA models. Table 6 provides the fit indices for AR process of 0.67 with LCARMA with weights (χ^2-35.2155, SRMR-0.0077, RMSEA-0.0266, AIC-65.2155, Mc-0.9859, CFI-0.9989) and LCARMA without weights (χ^2-38.2735, SRMR-0.0079, RMSEA-0.0338, AIC-68.2735, Mc-0.9790, CFI-0.9985) having the best fit indices.

Table 5: Sample LC Model Fit Indices by AR Process (AR-0.33, MA-0.00)

Model	**χ^2**	**SRMR**	**RMSEA**	**AIC**	**MC**	**CFI**
LC-wt	999.6424	0.3449	0.3810	1025.642	0.1073	0.8360
LC- no ARMA & wt	1314.952	0.5257	0.4388	1340.952	0.0515	0.7809
LCAR-wt	68.2863	0.0449	0.0555	96.2863	0.9272	0.9934
LCAR- no wt	39.4568	0.0099	0.0337	67.4568	0.9786	0.9982
LCARMA-wt	38.0987	0.0090	0.0336	68.0987	0.9794	0.9983
LCARMA- no wt*	35.7735	0.0096	0.0286	65.7735	0.9846	0.9987
LCMA-wt	713.3745	0.2646	0.3252	741.274	0.2070	0.8843
LCMA-no wt	1016.337	0.3018	0.3909	1044.337	0.1025	0.8317

* Best Fitting Model; Latent Curve with weight-LC wt; Latent Curve with no ARMA and no weight-LC no ARMA & wt; Latent Curve AR with weights-LCAR wt; Latent Curve AR with no weight-LCAR no wt; LCARMA with weight-LCARMA wt; LCARMA with no weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no weight-LCMA no wt; χ^2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI-Comparative Fit Index

The latent curve MA were the worst fit models followed by LC models. Table 7 had the same trend as table 5 and 6 with the LCARMA no weight (χ^2-34.5953, SRMR-0.0074, RMSEA-0.0265, AIC-64.5953, Mc-0.9873, CFI-0.9991) and LCARMA with weight (χ^2-36.7912, SRMR-0.0076, RMSEA-0.0314, AIC-66.7912, Mc-0.9823, CFI-0.9988) having the best fit indices with the latter fitting slightly better than the former. The latent curve and LCMA models had the worst fit indices. The fit indices for the LCARMA with weight and without weights models had similar fit indices and fit the data well irrespective of the AR value in Tables 5 through 7 because the LCARMA is a more accommodating model as it specifies both the AR and MA processes. Hence, the fit of LCAR and LCARMA models are comparable but the LCAR with weights was the best fitting model to the data having AR process because the AR lag1 estimates (ranging from 0.5787 to 0.9474) of LCAR with weight models were higher than the LCARMA with weight (0.5763 to 0.9414) models. Furthermore, the MA lag1 estimate for LCAR with weights model fitted to data having no MA process was also close to zero (Tables 105 and 106).

Table 6: Sample Level Fit Indices by AR Process (AR-0.67, MA-0.00)

Model	**χ^2**	**SRMR**	**RMSEA**	**AIC**	**MC**	**CFI**
LC-wt	1301.718	0.2031	0.4364	1237.718	0.0535	0.8228
LC- no ARMA & wt	1308.207	0.2059	0.4375	1334.207	0.0526	0.8208
LCAR-wt	42.5303	0.0084	0.0395	70.5303	0.9717	0.9981
LCAR- no wt	39.1287	0.0082	0.0323	67.1287	0.9793	0.9986
LCARMA-wt	38.2735	0.0079	0.0338	68.2735	0.9790	0.9985
LCARMA- no wt*	35.2155	0.0077	0.0266	65.2155	0.9859	0.9989
LCMA-wt	910.1646	0.2216	0.3691	938.1646	0.1315	0.8772
LCMA-no wt	914.8995	0.2235	0.3702	942.8995	0.1299	0.8758

* Best Fitting Model; Latent Curve with weight-LC wt; Latent Curve with no ARMA and no weight-LC no ARMA & wt; Latent Curve AR with weights-LCAR wt; Latent Curve AR with no weight-LCAR no wt; LCARMA with weight-LCARMA wt; LCARMA with no weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no weight-LCMA no wt; χ^2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI-Comparative Fit Index

Tables 8 through 10 presents the fit indices values of eight latent curve models fitted to data having an MA process and no AR process modeled. Table 8 depicts the fit indices with MA process of 0.33. The best fitting model was the latent curve moving average (LCMA) no weight (χ^2-34.5165, SRMR-0.0208, RMSEA-0.0256, AIC-64.5165, Mc-0.9875, CFI-0.9982) and LCMA with weight (χ^2-122.6257, SRMR-0.1544, RMSEA-0.1216, AIC-152.6257, Mc-0.8062, CFI-0.97970) with the latter having better fit indices than the former. The LC and LCAR were the worst fitting models because they were being fitted to data having an MA process. The LCARMA no weight had similar fit indices as the latent curve MA because it is a more flexible model as it also specifies the MA process. The worst fitting models were latent curve with weight and latent curve without weights.

Table 7: Sample LC Model Fit Indices by AR Process (AR-0.85, MA-0.00)

Model	χ^2	SRMR	RMSEA	AIC	MC	CFI
LC-wt	1057.168	0.1062	0.3921	1083.168	0.0944	0.8679
LC- no ARMA & wt	1630.987	0.1248	0.4898	1656.987	0.0249	0.7929
LCAR-wt	41.3696	0.0079	0.0380	69.3696	0.9743	0.9984
LCAR- no wt	38.9792	0.0074	0.0338	66.9792	0.9796	0.9987
LCARMA-wt	36.7912	0.0076	0.0314	66.7912	0.9823	0.9988
LCARMA- no wt*	34.5953	0.0074	0.0265	64.5953	0.9873	0.9991
LCMA-wt	703.6092	0.0916	0.3228	731.609	0.2124	0.9133
LCA-no wt	1199.701	0.1452	0.4257	1227.701	0.0672	0.8486

* Best Fitting Model; Latent Curve with weight-LC wt; Latent Curve with no ARMA and no weight-LC no ARMA & wt; Latent Curve AR with weights-LCAR wt; Latent Curve AR with no weight-LCAR no wt; LCARMA with weight-LCARMA wt; LCARMA with no weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no weight-LCMA no wt; χ^2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI-Comparative Fit Index

Table 9 provides the fit indices for MA process of 0.67 with the LCARMA no weight (χ^2-34.8733, SRMR-0.0263, RMSEA-0.0262, AIC-64.8733, Mc-0.9866, CFI-0.9982) being the best fitting model followed by LCARMA with weight (χ^2-168.2259, SRMR-0.1762, RMSEA-0.1487, AIC-198.2259, Mc-0.7258, CFI-0.9655). Although the fit of LCARMA models were better than LCMA no weight (χ^2-36.4158, SRMR-0.0288, RMSEA-0.0267, AIC-64.4158, Mc-0.9854, CFI-0.9981) and LCMA with weight (χ^2-40.2370, SRMR-0.0292, RMSEA-0.0343, AIC-68.2370, Mc-0.9769, CFI-0.9972) but the fit was better because the LCARMA model had both AR and MA processes specified into it.

Table 8: Sample LC Model Fit Indices by MA Process (AR-0.00, MA-0.33)

Model	χ^2	SRMR	RMSEA	AIC	MC	CFI
LC-wt	428.3163	0.02265	0.2438	454.3163	0.4004	0.9027
LC- no ARMA & wt	424.2022	0.02241	0.2426	450.2022	0.4036	0.9028
LCAR-wt	162.9182	0.1519	0.1428	190.9182	0.7364	0.9674
LCAR- no wt	397.5036	0.0317	0.2384	425.5036	0.4283	0.9091
LCARMA-wt	122.6257	0.1544	0.1216	152.6257	0.8062	0.9770
LCARMA- no wt*	34.5165	0.0208	0.0256	64.5165	0.9875	0.9982
LCMA-wt	39.1043	0.02243	0.0325	67.1043	0.9795	0.9974
LCMA-no wt	35.7210	0.0219	0.0259	63.7210	0.9870	0.9982

* Best Fitting Model; Latent Curve with weight-LC wt; Latent Curve with no ARMA and no weight-LC no ARMA & wt; Latent Curve AR with weights-LCAR wt; Latent Curve AR with no weight-LCAR no wt; LCARMA with weight-LCARMA wt; LCARMA with no weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no weight-LCMA no wt; χ^2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI-Comparative Fit Index

Table 10 presents the fit indices for the MA process of 0.85 with similar trends as table 9 with the LCARMA no weight, LCMA no weight, and LCMA with weight having good fit indices. An important point to note in all the tables is the large differences in the fit indices of LCARMA with weight and LCARMA no weight when only the MA process was modeled which was not present when the data had only the AR process or the ARMA process modeled. The differences in the fit indices between both the models were marginal for the AR and ARMA models. In addition, the fit indices for the latent curve with weight and latent curve without ARMA and no weights were better when only the MA process was modeled than the AR and

ARMA processes with the worst fit in the ARMA models. The fit of LCMA and LCARMA models are comparable but the LCMA was the best fitting the data having MA process because the fit indices and MA lag1 estimates (ranging from 18.2172 to 28.5331) of LCAR with weight models were better and higher than the LCARMA with sample weight (14.3328 to 34.7541) models. Furthermore, the MA lag1 estimate for LCAR with sample weight model fitted to data having no MA process was also close to zero (Tables 105 and 106).

Overall, the trend in all the tables from 1 through 10 suggest a joint or interactive influence of both weights and autocorrelative processes on the model fit indices. The joint influence justifies the needs to further investigate this interactive effect of sample weights and the AR, MA, and ARMA processes on the sample latent curve estimates. The LCAR model with sample weight had the best fit indices when fitted to AR data. The LCMA model with sample weight had the best fit indices when fitted to MA data. The LCARMA model with sample weight had the best fit indices when fitted to AR data.

Table 9: Sample LC Model Fit Indices by MA Process (AR-0.00, MA-0.67)

Model	χ^2	**SRMR**	**RMSEA**	**AIC**	**MC**	**CFI**
LC-wt	646.8244	0.0276	0.3037	672.8244	0.2417	0.8477
LC- no ARMA & wt	641.3281	0.0275	0.3024	667.3281	0.24436	0.8475
LCAR-wt	300.4958	0.1601	0.2043	328.4958	0.5362	0.9330
LCAR- no wt	609.3995	0.0364	0.2995	637.3995	0.2624	0.8552
LCARMA-wt	168.2259	0.1762	0.1487	198.2259	0.7258	0.9655
LCARMA- no wt*	34.8733	0.0263	0.0262	64.8733	0.9866	0.9982
LCMA-wt	40.2370	0.0292	0.0343	68.2370	0.9769	0.9972
LCMA-no wt	36.4158	0.0288	0.0267	64.4158	0.9854	0.9981

* Best Fitting Model; Latent Curve with weight-LC wt; Latent Curve with no ARMA and no weight-LC no ARMA & wt; Latent Curve AR with weights-LCAR wt; Latent Curve AR with no weight-LCAR no wt; LCARMA with weight-LCARMA wt; LCARMA with no weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no weight-LCMA no wt; χ^2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI-Comparative Fit Index

Table 10: Sample LC Model Fit Indices by MA Process (AR-0.00, MA-0.85)

Model	χ^2	SRMR	RMSEA	AIC	MC	CFI
LC-wt	660.593	0.0286	0.3071	686.593	0.2341	0.8401
LC- no ARMA & wt	658.195	0.0288	0.3066	684.1995	0.2349	0.8389
LCAR-wt	313.5649	0.1668	0.2093	341.5649	0.5202	0.9279
LCAR- no wt	624.8132	0.0370	0.3035	652.8132	0.2532	0.8473
LCARMA-wt	175.4033	0.1893	0.1525	205.4033	0.7139	0.9628
LCARMA- no wt*	32.5624	0.0279	0.0210	62.5624	0.9919	0.9986
LCMA-wt	37.3533	0.0302	0.0298	65.353	0.9834	0.9978
LCMA-no wt	33.7951	0.0300	0.0211	61.7951	0.9914	0.9986

* Best Fitting Model; Latent Curve with weight-LC wt; Latent Curve with no ARMA and no weight-LC no ARMA & wt; Latent Curve AR with weights-LCAR wt; Latent Curve AR with no weight-LCAR no wt; LCARMA with weight-LCARMA wt; LCARMA with no weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no weight-LCMA no wt; χ^2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI-Comparative Fit Index

Latent Curve Estimates for Population Data

Tables 11 through 13 summarizes the population-level latent curve estimates for the AR process only (0.33, 0.67, and 0.85 values), MA process only (0.33, 0.67, and 0.85 values), and ARMA process (AR-0.00, MA-0.00; AR-0.33, MA-0.30; AR-0.67, MA-0.60; AR-0.85, MA-0.80 values). The latent curve model was used in the simulation study because it can simultaneously model change across time and also specify AR, MA and ARMA processes to account for autocorrelation. The latent curve estimates were calculated from the 252,000 data points generated through the SAS macro having the PROC CALIS equations. The mean intercept, mean slopes, and intercept variance had consistent values across all the processes and values. The slope variance was slightly higher for the AR process than the MA and ARMA process while the intercept slope correlations were similar in MA and ARMA process but higher than the AR process.

Table 11: Population LC Estimates for AR Process

Parameter	AR-0.33	AR-0.67	AR-0.85
Mean Intercept	8.7267	8.7226	8.7393
Mean Slope	0.9984	0.9693	0.8804
Intercept Variance	13.3985	13.3549	13.3197
Slope Variance	0.0527	0.0518	0.0504
Intercept Slope Correlation	0.0182	-0.0182	-0.1393

AR-Autoregressive

Table 12: Population LC Estimates for MA Process

Parameter	MA-0.33	MA-0.67	MA-0.85
Mean Intercept	8.7298	8.7224	8.7195
Mean Slope	1.0033	1.0062	1.0070
Intercept Variance	13.5607	13.7468	13.8003
Slope Variance	0.0479	0.0442	0.0432
Intercept Slope Correlation	0.0215	0.0119	0.0087

MA-Moving Average

Table 13: Population LC Estimates for ARMA Process

Parameter	AR-0.00 MA-0.00	AR-0.33 MA-0.30	AR-0.67 MA-0.60	AR-0.85 MA-0.80
Mean Intercept	8.7337	8.7295	8.7286	8.7281
Mean Slope	1.0023	1.0019	1.0217	1.0494
Intercept Variance	13.4241	13.5423	13.7297	13.8082
Slope Variance	0.0529	0.0483	0.0453	0.0446
Intercept Slope Correlation	0.0218	0.0209	0.0382	0.0767

ARMA: Auto regressive moving average

Latent Curve Estimates for Sample Data with AR Process

The sample-level statistics were calculated by running 200 iterations through a SAS macro to randomly generate 200 samples (each having a sample size of 72). The results are presented for each autocorrelative process and corresponding values with and without weights.

Tables 14 and 15 provide results on the sample-level estimates for the LC, LCAR, LCMA, and LCARMA models having 0.33 AR value with weight and without weights respectively. All the estimates for the AR process having no weights were lower than the AR process having weights. The standard errors (SE) were also lower for each of the corresponding statistic. The intercept slope correlation for the LCAR had negative values of -3.2609 and -0.0174 for weights and without weights respectively. Large differences were observed between the statistic and associated standard error for intercept variance, slope variance, and intercept slope correlation in Tables 14 and 15. The pattern was similar across all the models with and without weights. The LCAR and LCARMA models had lower values than the LC and LCAR models. This trend was observed in both the tables. The values for mean intercept with weights for LCAR (8.2911) and LCARMA (8.7085) in Table 14 were closer to the population-level estimate of mean intercept (8.7267) in Table 11 but the values were lower without weights in Table 15 except for LCMA (8.3214).

Table 14: Sample LC Estimates for AR Process with Weights (AR-0.33, MA-0.00)

Statistic	LC	LCAR	LCMA	LCARMA
Mean Intercept	12.2776	8.2911	11.6903	8.7085
Mean Intercept SE	2.3569	1.4040	2.2139	1.4978
Mean Slope	3.2083	0.7567	3.2997	0.9938
Mean Slope SE	0.3006	0.1323	0.3075	0.1297
Intercept Variance	1181.601	422.57	1024.722	469.9269
Intercept Variance SE	115.6183	43.0062	101.7673	47.4934
Slope Variance	18.5712	2.1870	18.6521	1.8843
Slope Variance SE	1.8827	0.3127	1.9692	0.2837
Intercept Slope Correlation	88.1064	-3.2669	98.3448	0.2141
Intercept Slope Correlation SE	11.8909	3.0906	11.8169	3.2957

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

Tables 16 and 17 summarizes the statistics and SE of AR process of 0.67 with weights and without weights respectively. The latent curve, LCAR and LCARMA with AR of 0.67 with weights had lower values for mean intercept, mean slope, intercept variance, slope variance (except for the latent curve where the slope variance was higher for AR of 0.67), and intercept slope correlation than the AR process of 0.33 with weights.

Table 15: Sample LC Estimates for AR Process No Weights (AR-0.33, MA-0.00)

Statistic	LC	LCAR	LCMA	LCARMA
Mean Intercept	9.1511	6.3894	8.3214	6.3981
Mean Intercept SE	0.4285	0.2523	0.3748	0.2528
Mean Slope	2.7926	0.9856	2.9468	0.9973
Mean Slope SE	0.0484	0.0436	0.0515	0.0455
Intercept Variance	38.9976	13.2118	29.1666	13.2402
Intercept Variance SE	3.8126	1.3193	2.9177	1.3249
Slope Variance	0.4763	0.0549	0.5106	0.0532
Slope Variance SE	0.0487	0.0075	0.0555	0.0076
Intercept Slope Correlation	1.9535	-0.0174	2.7272	0.0017
Intercept Slope Correlation SE	0.3296	0.0798	0.3321	0.0815

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

Table 16: Sample LC Estimates for AR Process with Weights (AR-0.67, MA-0.00)

Statistic	LC	LCAR	LCMA	LCARMA
Mean Intercept	9.8236	8.7141	10.0177	8.7210
Mean Intercept SE	2.0543	1.4927	2.1361	1.4964
Mean Slope	7.1301	0.8970	7.0326	0.9340
Mean Slope SE	0.8028	0.1923	0.7627	0.1964
Intercept Variance	901.1067	468.2985	947.7205	469.5451
Intercept Variance SE	87.6414	47.1466	94.7631	47.3677
Slope Variance	138.2936	1.9464	123.2999	1.9008
Slope Variance SE	13.3834	0.3252	12.0803	0.3459
Intercept Slope Correlation	302.3346	-3.4864	302.0606	-1.9033
Intercept Slope Correlation SE	31.7163	6.2263	31.2495	6.4395

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

Similar trends were seen in the estimates between the models having weights and no weights as in Tables 14 and 15 having AR of 0.33. The values for mean intercept with weights for LCAR (8.7141) and LCARMA (8.7210) in Table 16 were closer to the population-level estimate of mean intercept (8.7226) in Table 11 but the values were lower without weights in Table 17. The intercept slope correlation for LCAR and LCARMA had negative values for both weights and without weights respectively in Table 16. The LCAR and LCARMA estimates were lower than the LC and LCMA estimates irrespective of weights but were similar between each other in both tables 16 and 17.

Table 17: Sample LC Estimates for AR Process No Weights (AR-0.67, MA-0.00)

Statistic	LC	LCAR	LCMA	LCARMA
Mean Intercept	6.7153	6.4066	6.7715	6.4086
Mean Intercept SE	0.3456	0.2507	0.3592	0.2513
Mean Slope	5.9623	0.9213	5.9345	0.9517
Mean Slope SE	0.1344	0.1290	0.1277	0.1329
Intercept Variance	25.5048	13.1907	26.7666	13.2238
Intercept Variance SE	2.4814	1.3285	2.6793	1.3348
Slope Variance	3.8788	0.0551	3.4550	0.0538
Slope Variance SE	0.3754	0.0009	0.3387	0.0096
Intercept Slope Correlation	8.4747	-0.0987	8.4711	-0.0536
Intercept Slope Correlation SE	0.8916	0.1737	0.8781	0.1798

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

Tables 18 and 19 provides the estimates and SE for AR process of 0.85 with and without weights respectively. The values for mean intercept with weights for LCAR (8.7704), and LCARMA (8.7645) in Table 18 were closer to the population-level estimate of mean intercept (8.7393) in Table 11 but the values were lower without weights in Table 17. The trend in the estimates was approximately the same for the other tables. The LC models has the highest estimates followed by the LCMA. The estimates of LCAR and LCARMA models were comparable with and without weights and lower than the LC and LCAR models.

Table 18: Sample LC Estimates for AR Process with Weights (AR-0.85, MA-0.00)

Statistic	LC	LCAR	LCMA	LCARMA
Mean Intercept	7.9956	8.7704	8.0868	8.7645
Mean Intercept SE	1.8356	1.4950	1.7906	1.4975
Mean Slope	9.7633	0.7582	9.7978	0.8154
Mean Slope SE	1.1718	0.3555	1.1579	0.3669
Intercept Variance	719.0429	469.6726	668.6531	470.0673
Intercept Variance SE	69.9586	47.1133	66.8645	47.2906
Slope Variance	295.0634	2.3364	287.1063	2.2665
Slope Variance SE	28.5016	0.7671	27.8345	0.8091
Intercept Slope Correlation	404.8461	-8.6952	402.4392	-6.2129
Intercept Slope Correlation SE	41.8648	13.1983	40.7309	13.7035

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

Comparisons were made within the statistics for AR process having 0.33, 0.67, and 0.85 values modeled with weights in tables 14, 16, and 18 respectively. The LC models estimates were unstable with increase in the mean slope & SE, mean slope variance & SE, and intercept correlation & SE but decrease in the mean intercept & SE, intercept variance & SE as the AR value increased from 0.33 to 0.85. The LCAR models estimates increased for mean slope SE, intercept variance, intercept slope correlation & SE. The other estimates increased, decreased or remained constant with change in AR values from 0.33 to 0.85. The LCMA model estimates increased for means slope & SE, slope variance & SE, intercept slope correlation & SE and decreased for mean intercept & SE, intercept variance & SE as the AR value increased from 0.33 to 0.85. The LCARMA values remained fairly constant for all the statistics except for mean slope & SE, slope variance & SE, intercept correlation & SE making it an acceptable and stable model amongst all the other models with regards to the latent curve estimates.

Table 19: Sample LC Estimates for AR Process No Weights (AR-0.85, MA-0.00)

Statistic	LC	LCAR	LCMA	LCARMA
Mean Intercept	5.3323	6.4260	4.9580	6.4230
Mean Intercept SE	0.3068	0.02535	0.3200	0.2536
Mean Slope	7.9840	0.9458	8.1492	0.9617
Mean Slope SE	0.1975	0.1480	0.1920	0.1527
Intercept Variance	20.2338	13.3493	20.7814	13.3293
Intercept Variance SE	1.9564	1.3245	2.1274	1.3264
Slope Variance	8.4006	0.0566	7.8689	0.0545
Slope Variance SE	0.8096	0.0099	0.7653	0.0104
Intercept Slope Correlation	11.2017	-0.0655	11.5495	-0.0357
Intercept Slope Correlation SE	1.1723	0.2101	1.1862	0.2168

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

Latent Curve Estimates for Sample Data with MA Process

Tables 20 and 21 provide results on the sample-level estimates for the latent curve, LCAR, LCMA, and LCARMA models having 0.33 MA value with weight and without weights respectively. The LC and LCMA had similar values with weights and without weights. However, the values were lower for all the models without weights than those with weights. The intercept variance and SE showed the largest differences between the LC models with higher values attributed to the LC having weights. The LCAR models had the smallest estimates for all the statistics with weights. The LCAR models have negative values for mean slope, intercept variance, and slope variance for weights. Overall, the estimates for LCMA remained fairly stable in Tables 20 and 21 except for the intercept variance and intercept variance SE. The LCARMA model with weights had negative values for mean slope and intercept variance. The values for mean intercept with weights for LC (8.7264) and LCMA (8.7254) in Table 20 were closer to the population-level estimate of mean intercept (8.7298) in Table 12 but the values were lower without weights in Table 21.

Table 20: Sample LC Estimates for MA Process with Weights (AR-0.00, MA-0.33)

Statistic	LC	LCAR	LCMA	LCARMA
Mean Intercept	8.7264	1.3959	8.7254	1.4607
Mean Intercept SE	1.5176	0.2630	1.5133	0.2618
Mean Slope	0.9976	-0.0902	0.9982	-0.1144
Mean Slope SE	0.1238	0.0672	0.1118	0.0677
Intercept Variance	486.2958	-16.8322	476.1705	-35.256
Intercept Variance SE	47.8128	2.3674	47.6007	3.8887
Slope Variance	2.7324	-0.7098	1.6204	-1.6169
Slope Variance SE	0.3213	0.1267	0.2781	0.2010
Intercept Slope Correlation	-2.5922	3.0093	0.4116	6.5294
Intercept Slope Correlation SE	2.7853	0.49938	2.5275	0.7810

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

Table 21: Sample LC Estimates for MA Process No Weights (AR-0.00, MA-0.33)

Statistic	LC	LCAR	LCMA	LCARMA
Mean Intercept	6.4104	6.2082	6.4090	6.4108
Mean Intercept SE	0.2564	0.2482	0.2538	0.2540
Mean Slope	0.9982	0.9121	0.9987	1.005
Mean Slope SE	0.0209	0.0261	0.0188	0.0270
Intercept Variance	13.6820	12.6392	13.3829	13.3997
Intercept Variance SE	1.3459	1.2640	1.3393	1.3412
Slope Variance	0.0779	0.0704	0.0463	0.0468
Slope Variance SE	0.0091	0.0087	0.0079	0.0079
Intercept Slope Correlation	-0.0825	-0.1308	0.0039	0.0039
Intercept Slope Correlation SE	0.0791	0.0746	0.0547	0.0719

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

Tables 22 and 23 summarizes the estimates of the four models having MA of 0.67 with weight and without weights respectively. Huge differences were seen in the mean slope for all the models in Tables 22 and 23 across all the models which was not the pattern seen in mean slopes for MA of 0.33 in Tables 20 and 21. The LCAR model had the lowest values for all the estimates with weights. The values for mean intercept with weights for LC (8.7143) and LCMA (8.7181) in Table 22 were closer to the population-level estimate of mean intercept (8.7224) in Table 12 but the values were lower without weights in Table 23. Tables 24 and 25 shows the statistics for MA process of 0.85 with weights and without weights respectively. Similar trends were seen as in the previous tables with the LCAR and LCARMA having the lowest values in Tables 24.

Table 22: Sample LC Estimates for MA Process with Weights (AR-0.00, MA-0.67)

Statistic	LC	LCAR	LCMA	LCARMA
Mean Intercept	8.7143	1.3893	8.7181	1.4543
Mean Intercept SE	1.5236	0.2630	1.5087	0.2576
Mean Slope	1.0016	-0.1036	1.0008	-0.1400
Mean Slope SE	0.1280	0.0715	0.1034	0.0725
Intercept Variance	490.6305	-19.9782	482.2630	-57.4748
Intercept Variance SE	48.1956	2.5496	47.45353	4.7428
Slope Variance	2.9300	-0.7979	1.5254	-2.6438
Slope Variance SE	0.3447	0.1435	0.2786	0.2507
Intercept Slope Correlation	-2.9875	3.4226	0.2186	10.5157
Intercept Slope Correlation SE	2.8946	0.5462	2.4440	0.9502

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

An important point to note in tables 20 through 25 is that the LCAR and LCARMA models had lower estimates with weights but similar estimates without weights among all the four models tested without weights. The values for mean intercept with weights for LC and LCMA were close to the population-level mean intercept value for all the MA values of 0.33, 0.67, and 0.85. There were some large variations in the intercept variance & SE, slope variance & SE, and intercept slope correlation & SE when compared within weights and without weights across all the models and values. Overall, the LCAR and LCARMA models with weights had lower values than the corresponding LC and LCMA models across all the MA values. Comparisons were made within the statistics for MA process having 0.33, 0.67, and 0.85 values modeled with weights in tables 20, 22, and 24 respectively. The mean slope and intercept slope correlation SE estimates increased whereas the intercept variance & SE decreased for the LC model. The mean slope SE, intercept variance, and slope variance SE increased whereas mean intercept & SE and mean slope decreased for LCAR model. The mean slope increased whereas intercept variance and slope variance decreased for LCMA model.

Table 23: Sample LC Estimates for MA Process No Weights (AR-0.00, MA-0.67)

Statistic	**LC**	**LCAR**	**LCMA**	**LCARMA**
Mean Intercept	6.3988	6.2059	6.4037	6.3947
Mean Intercept SE	0.2534	0.2492	0.2532	0.2534
Mean Slope	1.0103	0.8994	0.9992	1.0103
Mean Slope SE	0.0282	0.0274	0.0175	0.0282
Intercept Variance	13.5436	12.8201	13.573	13.5436
Intercept Variance SE	1.3343	1.2793	1.3366	1.3343
Slope Variance	0.04463	0.0761	0.0437	0.0446
Slope Variance SE	0.0082	0.0095	0.0080	0.0082
Intercept Slope Correlation	0.0134	-0.1650	-0.0004	0.0134
Intercept Slope Correlation SE	0.0754	0.0791	0.0697	0.0754

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

Table 24: Sample LC Estimates for MA Process with Weights (AR-0.00, MA-0.85)

Statistic	LC	LCAR	LCMA	LCARMA
Mean Intercept	8.7283	1.4069	8.7189	1.4594
Mean Intercept SE	1.5267	0.2626	1.5082	0.2562
Mean Slope	0.9967	-0.1181	0.9988	-0.1528
Mean Slope SE	0.1279	0.0733	0.1002	0.0744
Intercept Variance	493.2690	-23.2777	485.7283	-64.4611
Intercept Variance SE	48.3964	2.6920	47.4906	5.1707
Slope Variance	2.9158	-0.9618	1.4790	-3.0032
Slope Variance SE	0.3459	0.1536	0.2804	0.2753
Intercept Slope Correlation	-3.0511	4.03327	0.1155	11.8350
Intercept Slope Correlation SE	2.9026	0.5773	2.4169	1.0354

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

Table 25: Sample LC Estimates for MA Process No Weights (AR-0.00, MA-0.85)

Statistic	LC	LCAR	LCMA	LCARMA
Mean Intercept	6.4116	6.2409	6.4042	6.3959
Mean Intercept SE	1.5267	0.2502	0.2527	0.2536
Mean Slope	0.9967	0.8901	0.9977	1.0055
Mean Slope SE	0.1279	0.0279	0.0170	0.0289
Intercept Variance	493.2690	12.9832	13.6456	13.6156
Intercept Variance SE	48.3964	1.2921	1.3341	1.3339
Slope Variance	2.9158	0.0772	0.0433	0.0439
Slope Variance SE	0.3459	0.0097	0.0081	0.0083
Intercept Slope Correlation	-3.0511	-0.1837	-0.0056	0.0046
Intercept Slope Correlation SE	2.9026	0.0809	0.0691	0.0760

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

The intercept variance increased, the mean slope & SE decreased whereas the mean intercept & SE, and slope variance remained constant for the LCARMA model. All other estimates increased, decreased or remained constant with increase in the MA values from 0.00 to 0.85. The estimates for all the four models having the MA process did not have stability in the estimates with change in the MA values. Hence, it was difficult to select one model that could be deemed better than the other models by evaluating the latent curve estimates.

Latent Curve Estimates for Sample Data with ARMA Process

Tables 26 through 34 summarizes the latent curve estimates for the ARMA process. Tables 26 and 27 provides the estimates for AR process of 0.00 and MA process of 0.00 with weights and without weights respectively. The LCAR and LCARMA had lower estimates than the LC and LCAR model estimates with weights in Table 26 but similar estimates without weight in Table 27. The mean slope values for the LC, LCAR and LCMA were higher for the weights than without weights. The values for mean intercept with weights for LC (8.7242) and LCMA (8.7240) in Table 26 were closer to the population-level estimate of mean intercept (8.7337) in Table 13 but the values were lower without weights in Table 27.

Table 26: Sample LC Estimates with Weights (AR 0.00, MA 0.00)

Statistic	LC	LCAR	LCMA	LCARMA
Mean Intercept	8.7242	1.5402	8.7240	1.3464
Mean Intercept SE	1.4958	0.2375	1.4960	0.2348
Mean Slope	0.9977	-0.1259	0.9978	-0.0614
Mean Slope SE	0.1088	0.0581	0.1083	0.0561
Intercept Variance	472.2738	-36.3606	471.1967	-0.2788
Intercept Variance SE	46.5938	2.6991	46.6148	3.1301
Slope Variance	1.9385	-1.8064	1.8509	0.0043
Slope Variance SE	0.2507	0.1345	0.2526	0.1554
Intercept Slope Correlation	0.1239	7.2828	0.3829	0.1908
Intercept Slope Correlation SE	2.4113	0.5422	2.4065	0.6236

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

Tables 28 and 29 presents the ARMA values with AR of 0.33 and MA of 0.30 with weights and without weights respectively. The LC and LCMA had higher values than LCAR and LCARMA in Table 28 and Table 29. The values for mean intercept with weights for LCARMA (8.5034) in Table 28 was closer to the population-level estimate of mean intercept (8.7295) in Table 13 but the values were lower without weights in Table 29. The estimates for intercept variance and SE for ARMA in Table 28 and Table 29 were lower than the intercept variance when AR and MA were modeled individually. Tables 30 and 31 provides the ARMA values with AR of 0.67 and MA of 0.60 with weights and without weights respectively.

Table 27: Sample LC Estimates with No Weights (AR 0.00, MA0.00)

Statistic	LC	LCAR	LCMA	LCARMA
Mean Intercept	6.386656	6.370248	6.411404	6.428654
Mean Intercept SE	0.252065	0.253671	0.252218	0.261509
Mean Slope	0.992812	0.985544	0.998434	1.001001
Mean Slope SE	0.018666	0.02495	0.018403	0.026155
Intercept Variance	13.0901	13.19525	13.33623	15.9516
Intercept Variance SE	1.327385	1.334915	1.320412	1.58995
Slope Variance	0.04463	0.05287	0.052749	0.060785
Slope Variance SE	0.007857	0.00812	0.007251	0.008525
Intercept Slope Correlation	0.03568	-0.00481	0.001126	0.01136
Intercept Slope Correlation SE	0.071302	0.072419	0.068694	0.082318

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

The mean intercepts in Table 30 were higher for the LC and LCAR models than the mean intercepts in Table 28. The mean slope SE and intercept variance for all models with ARMA of 0.67 and 0.60 respectively were also higher than the estimates in Table 28 where the AR and MA was 0.33 and 0.30 respectively. The values for mean intercept with weights for LCAR (8.6559) and LCARMA (8.6766) in Table 30 were to the population-level estimate of mean intercept (8.7295) in Table 13 but the values were lower without weights in Table 31. Tables 32 and 33 provides the estimates for AR process of 0.85 and MA process of 0.80 with weights and without weights respectively.

Table 28: Sample LC Estimates for ARMA Process with Weights (AR-0.33, MA-0.30)

Statistic	LC	LCAR	LCMA	LCARMA
Mean Intercept	11.9870	7.9078	11.4231	8.5034
Mean Intercept SE	2.3325	1.3332	2.1808	1.4619
Mean Slope	3.2420	0.4750	3.3454	0.8718
Mean Slope SE	0.3240	0.1443	0.3244	0.1419
Intercept Variance	1161.1668	384.6776	987.0616	448.2207
Intercept Variance SE	113.7372	39.2959	99.3196	45.5090
Slope Variance	21.7955	2.8546	20.4229	2.0072
Slope Variance SE	2.1949	0.3635	2.2012	0.3614
Intercept Slope Correlation	81.2300	-12.2813	98.4192	-1.1760
Intercept Slope Correlation SE	12.3411	3.2529	12.0951	3.7155

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

Table 29: Sample LC Estimates for ARMA Process No Weights (AR-0.33, MA-0.30)

Statistic	LC	LCAR	LCMA	LCARMA
Mean Intercept	9.1211	6.3317	8.2741	6.4172
Mean Intercept SE	0.4433	0.2571	0.3793	0.2635
Mean Slope	2.7778	0.8924	2.9544	1.0162
Mean Slope SE	0.0536	0.0462	0.0544	0.0513
Intercept Variance	42.6548	15.2504	29.6982	17.9324
Intercept Variance SE	4.1814	1.52271	2.9908	1.8045
Slope Variance	0.5919	0.0798	0.5611	0.1548
Slope Variance SE	0.0601	0.0103	0.0617	0.0200
Intercept Slope Correlation	1.4596	-0.2016	2.6017	0.4853
Intercept Slope Correlation SE	0.3709	0.1050	0.3434	0.1501

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

The values for mean intercept with weights for LCAR (8.7696) and LCARMA (8.7685) in Table 32 was closer to the population-level estimate of mean intercept (8.7281) in Table 13 but the values were lower without weights in Table 29. The mean slope estimate for all the models with weights in Table 32 was highest among all the ARMA values when compared to Tables 26 and 28 whereas the means slope SE, and intercept variance were highest for the LC model only in Table 32 when compared to other ARMA values with weights.

Table 30: Sample LC Estimates for ARMA Process with Weights (AR-0.67, MA-0.60)

Statistic	LC	LCAR	LCMA	LCARMA
Mean Intercept	9.3955	8.6559	8.4414	8.6766
Mean Intercept SE	1.9037	1.4942	1.3948	1.4951
Mean Slope	7.2029	0.5656	7.3022	0.8903
Mean Slope SE	0.8468	0.2184	0.87088	0.21397
Intercept Variance	797.9542	472.6170	448.1349	472.6359
Intercept Variance SE	77.5975	47.1064	41.2766	46.9164
Slope Variance	154.8199	3.1582	162.5529	2.3931
Slope Variance SE	14.9741	0.51709	15.7990	0.5497
Intercept Slope Correlation	283.8732	-19.3273	251.0082	-2.7017
Intercept Slope Correlation SE	30.4655	7.2247	24.7994	7.7134

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

Comparisons were made within the statistics for ARMA process modeled with weights in tables 26, 28, 30 and 32 respectively. The LC, LCAR, and LCMA model estimates were unstable and had large fluctuations in the estimates with change in the ARMA values. The LCARMA model had the most stable estimates of all the four models with all the estimates remaining fairly constant with change in the ARMA values. Hence, the LCARMA model was deemed appropriate and acceptable for the AR and ARMA processes with weights when the latent curve estimates were evaluated.

Table 31: Sample LC Estimates for ARMA Process No Weights (AR-0.67, MA-0.60)

Statistic	LC	LCAR	LCMA	LCARMA
Mean Intercept	6.6256	6.3983	6.3277	6.4112
Mean Intercept SE	0.3358	0.2584	0.2348	0.2594
Mean Slope	5.9498	0.6489	6.0088	1.0219

Statistic	LC	LCAR	LCMA	LCARMA
Mean Slope SE	0.1420	0.1416	0.1465	0.1463
Intercept Variance	25.9702	15.8852	12.6910	15.8773
Intercept Variance SE	2.5336	1.5820	1.1675	1.557
Slope Variance	4.3008	0.0978	4.5828	0.9346
Slope Variance SE	0.4170	0.0172	0.4455	0.0981
Intercept Slope Correlation	7.9028	-0.5038	7.0753	1.3608
Intercept Slope Correlation SE	0.9078	0.2439	0.6996	0.3490

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

Table 32: Sample LC Estimates for ARMA Process with Weights (AR-0.85, MA-0.80)

Statistic	LC	LCAR	LCMA	LCARMA
Mean Intercept	8.8336	8.76961	8.9848	8.7685
Mean Intercept SE	1.4849	1.5133	1.4275	1.4999
Mean Slope	9.5730	0.2707	9.6149	0.6652
Mean Slope SE	1.2322	0.4113	1.2135	0.3904
Intercept Variance	480.9226	483.2602	463.5307	474.7282
Intercept Variance SE	45.8854	47.9928	42.7227	47.0314
Slope Variance	326.9094	4.8599	316.0710	5.9125
Slope Variance SE	31.5132	1.7591	30.5656	1.6280
Intercept Slope Correlation	372.8015	-32.7827	362.9114	-11.6879
Intercept Slope Correlation SE	37.0391	15.8700	35.5847	15.7581

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

Table 34 provides information on the number of iteration it took for the maximum likelihood procedure for model convergence and to reach at a proper solution. Higher number of iterations taken by the procedure to attain model convergence implies a greater probability of poor fit. The maximum likelihood procedure took 27 iterations each to converge the latent curve model having ARMA processes (AR-0.67, MA-0.60 & AR-0.85, MA-0.80) with 15 iterations taken by the procedure to reach a proper solution for the ARMA process of AR-0.33, MA-0.30.

Table 33: Sample LC Estimates for ARMA Process No Weights (AR-0.85, MA-0.80)

Statistic	LC	LCAR	LCMA	LCARMA
Mean Intercept	5.6951	6.4327	6.7231	6.4115
Mean Intercept SE	0.2997	0.2560	0.2429	0.2535
Mean Slope	7.8642	0.7557	7.8300	1.0272
Mean Slope SE	0.2037	0.1664	0.2021	0.1710
Intercept Variance	19.4915	13.7725	13.6858	13.7059
Intercept Variance SE	1.8940	1.3577	1.2499	1.3411
Slope Variance	8.9338	0.0933	8.7554	0.0502
Slope Variance SE	0.8621	0.0155	0.8484	0.0142
Intercept Slope Correlation	10.8993	-0.4520	10.3038	0.0312
Intercept Slope Correlation SE	1.1627	0.2521	1.0076	0.2720

SE-Standard Error; LC-Latent Curve; LCAR-Latent Curve Autoregressive; LCMA-Latent Curve Moving Average; LCARMA-Latent Curve Autoregressive Moving Average

Table 34: Iterations Taken for Model Convergence in Simulation for Population Data

Lag Process	Number of Iterations
AR-0.33	14
AR-0.67	19
AR-0.85	17
MA-0.33	11
MA-0.67	11
MA-0.85	12
AR-0.00; MA-0.00	12
AR-0.33; MA-0.30	15
AR-0.67; MA-0.60	27
AR-0.85; MA-0.80	27

AR-Autoregressive; MA-Moving Average

The minimum number of iterations taken for model convergence was with the MA processes and then by the AR processes. The number of iterations taken by the maximum likelihood procedure to converge latent curve models with MA process and ARMA process of 0.33 and 0.30 were comparable. A common pattern observed is the number of iterations increased as the values for the AR and ARMA processes increased which was not the case for the MA process (as it took 11 iterations each for MA of 0.33 & 0.67 and 12 for MA of 0.85). The results in Table 34 were later compared to the number of iterations maximum likelihood took to converge the latent curve model having an autocorrelative process to the ECLS-K data.

Overall General Linear Model Results

This section focuses on the results from the General Linear Model (GLM). The ANOVA models were run to measure the effect size, (R^2 and η^2), coefficient of variation, root mean square error (RMSE), and mean. Tukey comparisons were conducted to find out which means were statistically different from each other by model type (LC, LCAR, LCMA, and LCARMA) and weight. The joint influence of model type and weight on each of the latent curve estimates (Mean Intercept, Mean Intercept SE, Mean Slope, Mean Slope SE, Intercept Variance, Intercept Variance SE, Slope Variance, Slope Variance SE, Intercept Slope Correlation, and Intercept Slope Correlation SE) were also evaluated. The box plots were utilized to assess the distribution of latent curve estimates by model type. The interaction plots were consulted to evaluate the joint influence of model type and weight on the latent curve estimates.

A total of 10 ANOVA models were run in SAS PROC GLM procedure, one model for each of the latent curve estimate (Mean Intercept, Mean Intercept SE, Mean Slope, Mean Slope SE, Intercept Variance, Intercept Variance SE, Slope Variance, Slope Variance SE, Intercept Slope Correlation, and Intercept Slope Correlation SE) as the dependent variable with model type and weight as the independent variables. The ANOVA models were built by regressing each of the latent curve estimates on the model type and use of weight (0-as no weight and 1-as weight). GLM was used to test for the joint influence of model type and weight on each of the latent curve estimate and also to estimate the effect size. All the ANOVA models were significant including the interaction term between model type and weight.

Table 35 summarizes the overall fit indices by model type. The LCARMA model was the best fit (χ^2-56.9968, SRMR-0.0471, RMSEA-0.0477, AIC-85.9967, Mc-0.9447, CFI-0.9937). The worst fitting model was the latent curve (χ^2-1168.04, SRMR-0.1591, RMSEA-0.3818, AIC-1194.04, Mc-0.2082, CFI-0.8207) which did not have any autocorrelative process modeled into it. Table 36 provides the effect size (R^2), coefficient of variation, root mean square error (RMSE), and mean for each of the latent curve estimate. Interpreting the effect size in conjunction with the CV and RMSE is important to truly evaluate the variation in the latent curve estimates by the independent variables (model type and weight) because the residual values are smaller relative to the predicted values of the latent curve estimates. The coefficient of variation (CV) is obtained by dividing RMSE by the mean. The CV explains the spread or dispersion in the dependent variable (which are the latent curve estimates). It explains the model fit as a function of the relative size of the squared residuals (difference between the actual and predicted values). Lower values for CV indicate a good model fit. The RMSE is a measure to assess the goodness of fit of a model to the data because it estimates the closeness of a fitted line to the observed data points. Smaller RMSE values implies closer fit of the model to the data. Although the CV for mean intercept (27.8739), mean slope (94.1801), mean intercept SE (42.2943), and mean slope SE (107.3006) are high but the value of RMSE for these estimates are low thereby suggesting a good model fit and better predictability of the estimate by accounting for model type and weight as predictors. Hence, 33.9%, 37.6%, 75.5%, and 37.7% of the variation in mean intercept, means slope, mean intercept SE, and mean slope SE respectively was explained by model type and weight. Model type and weight together explained approximately a significant portion of the variation in each of the latent curve estimate ranging from 31.6% to 75.5%.

Table 35: Overall Fit Indices by Model Type

Model	χ^2	SRMR	RMSEA	AIC	Mc	CFI
LC	1168.04	0.1591	0.3818	1194.04	0.2082	0.8207
LCAR	312.6501	0.0509	0.1760	340.6501	0.6039	0.9473
LCMA	590.3385	0.1231	0.2380	618.3385	0.4697	0.9193
LCARMA	55.9968	0.0471	0.0477	85.9967	0.9447	0.9937

χ^2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI-Comparative Fit Index; LC-Latent Curve; AR-Auto regressive; MA-Moving Average

Table 36: Overall Effect Size for LC Estimates

Statistic	R^2	Coefficient of Variation	Root MSE	Mean
Mean Intercept	0.3394	27.8739	1.9729	7.0782
Mean Slope	0.3756	94.1801	2.2423	2.3808
Intercept Variance	0.6928	74.1536	181.6728	244.9951
Slope Variance	0.3171	241.257	59.6694	24.7327
Intercept Slope Correlation	0.4073	206.8394	79.3926	38.3837
Mean Intercept SE	0.7549	42.2943	0.3496	0.8267
Mean Slope SE	0.3768	107.3006	0.2275	0.2121
Intercept Variance SE	0.7108	68.8873	17.1165	24.8472
Slope Variance SE	0.3161	229.8854	5.7545	2.5032
Intercept Slope Correlation SE	0.4335	136.7207	7.8695	5.7559

SE-Standard Error; MSE-Mean Square Error

Overall ANOVA Results

Effect size is commonly utilized to tell us how strongly two or more variables are related, or how large the difference between groups is. According to SPSS for Windows, 9.0 (1998), eta squared is interpreted as the proportion of the total variability in the dependent variable that is accounted for by variation in the independent variable(s) (Levine & Hullett, 2002). It is the ratio of the between groups sum of squares to the total sum of squares. Semi-partial η^2 is used as a measure of effect size and it contains variance associated with treatment and the interaction between treatment and error. Partial η^2 is also a measure of effect size but only contains variance associated with interaction between treatment and error. The semi-partial η^2 was interpreted for the ANOVA models because it is a conservative measure of effect size than partial η^2. The partial η^2 is also reported in the ANOVA tables. Tables 37 and 38 provides the ANOVA results for each of the latent curve estimates and corresponding SE respectively. All the models were significant at $p<0.001$. The semi-partial η^2 was also reported in the ANOVA tables. The semi-partial η^2 values were higher for all the SE estimates whereas the η^2 values ranged from 1% to 1.7% for the latent curve estimates.

Table 37: Overall ANOVA Results for LC Estimates

Dependent Variable⁺	**Mean Intercept**	**Mean Slope**	**Intercept Variance**	**Slope Variance**	**Intercept Slope Correlation**
Model_Type	16837.743*	46910.916*	18554874*	9335068*	2542629*
Wt	4158.800*	15.8329*	83236387*	8747813*	21103403*
Model_type X Wt	10996.677*	1459.365*	17289611*	8364121*	22743534*
Semi-Partial η^2	0.1167	0.0113	0.1006	0.1003	0.1337
Partial η^2	0.1501	0.0178	0.2467	0.1281	0.1841

$^+$-Sum of Squares reported; *Significant at $p < .001$; η^2-Eta Square reported only for interaction effect between Model_type & weight; Wt-Weight

Table 38: Overall ANOVA Results for SE Estimates

Dependent Variable+	Mean Intercept SE	Mean Slope SE	Intercept Variance SE	Slope Variance SE	Intercept Slope Correlation SE
Model_Type	690.849*	119.957*	1515257*	81856.51	150454*
Wt	4806.733*	269.462*	8595122*	89829.89	474135*
Model_type X Wt	525.3649*	111.547*	1406008*	73127.05	133353*
Semi-Partial η^2	0.0659	0.0839	0.0935	0.0944	0.0763
Partial η^2	0.2118	0.1187	0.2444	0.1213	0.1187

+-Sum of Squares reported; *Significant at $p < .001$; η^2-Eta Square reported only for interaction effect between Model_type & weight; Wt-Weight

Overall Tukey Comparisons

Table 39 provides results of Tukey comparisons between the means of latent curve estimates by model type. The letters in the Tukey grouping column are different only when the means between two models are different from each other to a statistically significant degree. The means are significantly different between the LC, LCMA & LCARMA models for mean intercept, LC & LCARMA models for mean slope, and LC, LCMA, and LCARMA for intercept variance. With regards to the SE, the means are significantly different between the LC, LCMA & LCARMA models for intercept variance SE, LCMA & LCARMA for slope variance SE, and LC, LCMA & LCARMA models for intercept slope correlation SE. Overall, the LCAR and LCARMA had lower means than LC and LCMA for all the latent curve estimates.

Table 39: Overall Tukey Comparisons by Model Type

Statistic	Model	Mean	Tukey Grouping
Mean Intercept	LC	8.1759	A
	LCMA	8.0287	B
	LCARMA	6.0957	C
	LCAR	6.0125	C
Mean Slope	LCMA	4.1108	A
	LCMA	4.0742	A
	LCARMA	0.7325	B
	LCAR	0.6059	B
Intercept Variance	LC	371.068	A
	LCMA	332.565	B
	LCARMA	139.326	C
	LCAR	137.021	C
Slope Variance	LC	49.687	A
	LCMA	48.074	A
	LCAR	0.689	B
	LCARMA	0.481	B
Intercept Slope Correlation	LC	78.415	A
	LCMA	78.042	A
	LCARMA	0.281	B
	LCAR	-3.203	B
Mean Intercept SE	LC	1.0613	A
	LCMA	1.0057	B
	LCARMA	0.6253	C
	LCAR	0.6144	C
Mean Slope SE	LC	0.3017	A
	LCMA	0.2956	A
	LCARMA	0.1257	A
	LCAR	0.1253	A
Intercept Variance SE	LC	36.2302	A
	LCMA	32.7632	B
	LCARMA	15.6185	C
	LCAR	14.7769	C
Slope Variance SE	LC	4.8204	A
	LCMA	4.7091	A
	LCARMA	0.2480	B
	LCAR	0.2354	B
	LC	9.0823	A
	LCMA	8.5500	B

Statistic	Model	Mean	Tukey Grouping
Intercept Slope Correlation SE	LCARMA	2.7716	C
	LCAR	2.6198	C

SE-Standard Error; LC-Latent Curve; AR-Auto regressive; MA-Moving Average

Table 40 provides results of Tukey comparisons by weight with 1 and 0 signifying the presence and absence of weights respectively. The means are significantly different between all the latent curve estimates except mean slope. The means were higher for the estimates with weights than without weights with wide differences between the means for intercept variance, slope variance, intercept slope correlation, intercept variance, and intercept slope correlation SE.

Table 40: Overall Tukey Comparisons by Weight

Statistic	Model	Mean	Tukey Grouping
Mean Intercept	1	7.5880	A
	0	6.5684	B
Mean Slope	1	2.4123	A
	0	2.3494	A
Intercept Variance	1	473.080	A
	0	16.910	B
Slope Variance	1	48.1151	A
	0	1.3502	B
Intercept Slope Correlation	1	74.701	A
	0	2.066	B
Mean Intercept SE	1	1.3748	A
	0	0.2786	B
Mean Slope SE	1	0.3418	A
	0	0.0823	B
Intercept Variance SE	1	48.0247	A
	0	1.6697	B
Slope Variance SE	1	4.8727	A
	0	0.1338	B
Intercept Slope Correlation SE	1	11.1996	A
	0	0.3123	B

SE-Standard Error; 0-With Weight; 1-Without Weight

This trend was also seen in the simulation results. Tables 41 and 42 summarizes the ANOVA results for the interactive effect between model type and weight on the means of LC estimates. In general, the means were lower for the LC, LCAR, and LCMA, and LCARMA models without weights than for models with weights. In general, the means for the LC estimates are closer to the population-level estimates when the weights are not applied even in the presence of the autocorrelative process as compared to the estimates with weights and autocorrelative process as they are further away from the population-level estimates.

A wide fluctuation is observed in the intercept variance and slope variance between the models that have weights versus those that do not have weights especially in mean intercept variance and slope variance for the models with weights than without weights which is similar to the simulation results. The standard errors were compared to quantify the degree of precision in estimating the true mean of the population for the various latent curve estimates. The highest values for SE was for the LC with weight model followed by LCMA with weight models. The lowest SE was for the LCARMA and LCAR no weight model. The low SE for the LCAR and LCARMA models without weights as compared to the LCAR and LCARMA models with weights may be due to random noise because of the ARMA process that is present in the data that biases the latent curve estimates. This pattern was clearly evident in the simulation results also.

Table 41: Overall LC Estimates by Model_Type*Weights

Model	Weight	Mean Intercept	Mean Slope	Intercept Variance	Slope Variance	Intercept Slope Correlation
LCARMA	0	6.4053	0.9954	13.4357	0.0500	-0.0022
	1	5.78016	0.4696	265.2172	0.9118	0.5644
LCAR	0	6.3411	0.8839	13.1773	0.0700	-0.1870
	1	5.6838	0.3278	260.8642	1.3080	-6.2196
LCMA	0	6.7004	3.7819	18.6728	2.5922	4.2796
	1	9.3569	4.4397	646.4573	93.5550	151.8115
LC	0	6.8265	3.7363	22.3546	2.6885	4.1809
	1	9.5252	4.4121	719.7812	96.6855	152.6485

0-With Weight; 1-Without Weight; LC-Latent Curve; AR-Auto regressive; MA-Moving Average

Table 42: Overall SE Estimates by Model_Type*Weights

Model	Weight	Mean Intercept SE	Mean Slope SE	Intercept Variance SE	Slope Variance SE	Intercept Slope Correlation SE
LCARMA	0	0.2535	0.0804	1.3356	0.0095	0.1353
	1	0.9972	0.1711	29.9014	0.4865	5.4078
LCAR	0	0.2517	0.0786	1.3131	0.0100	0.1310
	1	0.9770	0.1725	28.2406	0.4607	5.1085
LCMA	0	0.2923	0.0846	1.8463	0.2547	0.4727
	1	1.7191	0.5065	63.6799	9.1635	16.6273
LC	0	0.3168	0.0861	2.1837	0.2608	0.5099
	1	1.8058	0.5172	70.2766	9.3799	17.6547

0-With Weight; 1-Without Weight; LC-Latent Curve; AR-Auto regressive; MA-Moving Average

Box Plots for Latent Curve Estimates by Model Type

This section utilizes the box plots for each of the latent curve estimates to assess their distribution as a function of model type and weight. Figures 4 through 13 shows the box plots for each latent curve estimate by model type. Figures 14 through 18 shows the distribution of fit indices by model type. The box plots were consulted to evaluate the variability in the data as a function of model type. Figures 18 through 28 shows the box plots by both model type and weight. Figure 4 displays the distribution of mean-intercept by model and it can be seen that the upper and lower quartile spread in mean intercept values were approximately equal as the width of each box plot was similar for each model. However, the range for minimum and maximum observations were higher within the LCMA and LC models than LCARMA and LCAR models. Figure 5 displays the mean slope distribution by model. The width of the plots for LCARMA and LCAR models was narrower than the LCMA and LC models suggesting lesser variation in the mean slopes estimates.

Figure 6 depicts the intercept variance distribution by model. The ranges for the maximum observation was highest for the LC model followed by LCMA model. However, the ranges for minimum observation were higher within the LCARMA and LCAR models than the LCMA and LC models. Figure 7 illustrates the distribution in slope variance by model with a vast range for the maximum observations in the LCMA and LC model. There was a minimal difference in the upper and lower quartiles within the LCARMA and LCAR models. The means for the LCMA and LC models were above the maximum observation suggesting it an outlier. Figure 8 displays the distribution of intercept slope correlation by model with a high number of data points lying outside the minimum and maximum observations. The highest number of data points were lying outside the maximum observations for the LCMA and LC models. The LCARMA and LCAR models were cluttered together with more outliers lying below the minimum observation. Figure 9 provides the distribution of mean intercept SE with the box plots of approximately the same size indication equal spread of data points within each model. The means for LCMA and LC models were higher than the median. The maximum observation was higher for the LCMA and LC models and the minimum observation were similar for each model. Figure 10 displays the box plot for mean slope SE with high number of outliers outside the maximum observation for the LCMA and LC models. The width of box plots for the LCMA and LC model was wider (indicating more spread) than the LCARMA and LCAR models but the width were similar. Figure 11 illustrates the distribution in the intercept variance SE with equal spread in the data across all the four models. However, the maximum observation was higher for the LCMA and LC models with high number of outlier lying outside the maximum value for the LC model. There were no outliers for the LCARMA and LCAR models.

The mean was higher than median for LCMA and LC. Figure 12 shows the slope variance SE with large number of outliers for the LCMA and LC models. The mean for both the models were higher than the maximum observations. The outliers in the LCARMA and LCAR models were lower than the other two models with minimal differences in the upper and lower bounds. Figure 13 shows the intercept slope correlation SE with more variability in the data for the LCMA and LC models than the LCARMA and LCAR models. The means were also higher for the former two models with high number of outliers. Figure 14 through 18 shows the box plots for the fit indices in order to evaluate the variability within the data as a function of model.

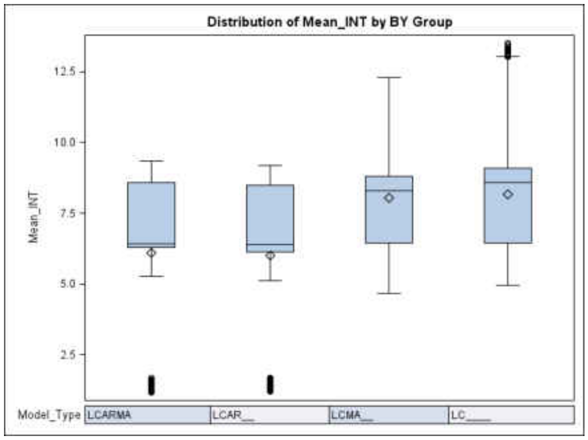

Figure 4 : Mean Intercept by Model

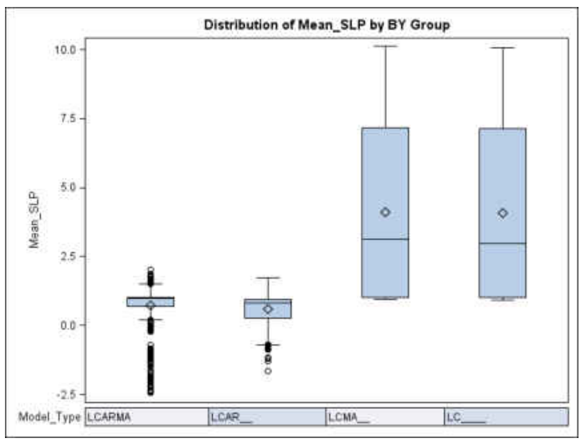

Figure 5: Mean Slope by Model

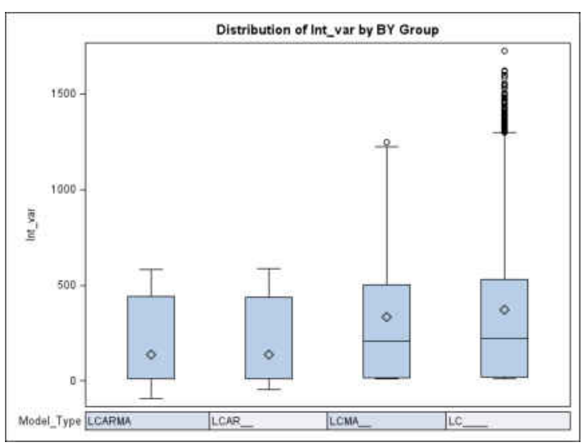

Figure 6: Intercept Variance by Model

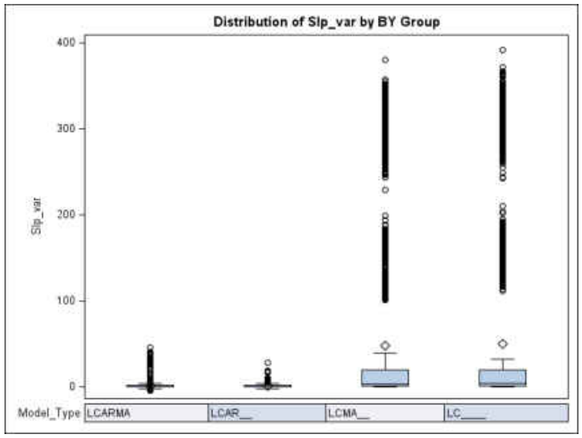

Figure 7: Slope Variance by Model

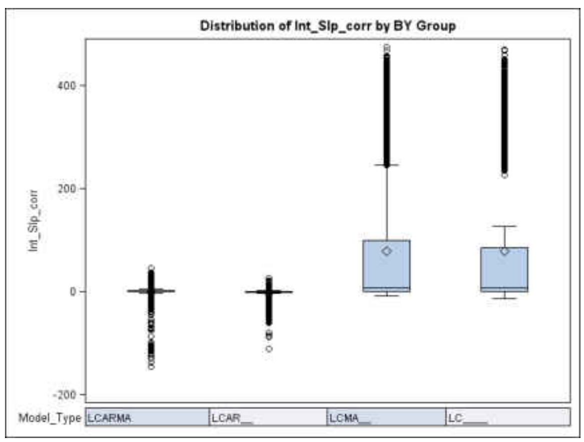

Figure 8: Intercept Slope Correlation by Model

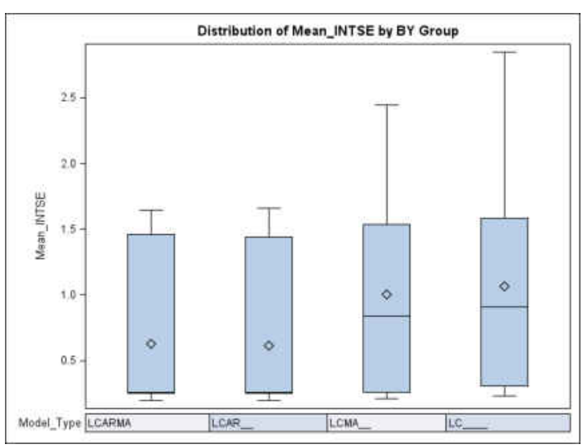

Figure 9: Mean Intercept SE by Model

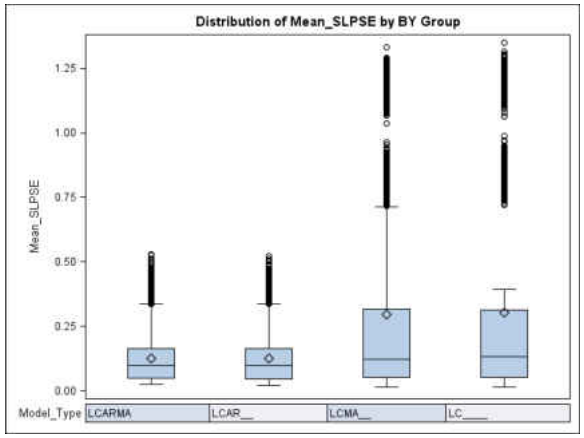

Figure 10: Mean Slope SE by Model

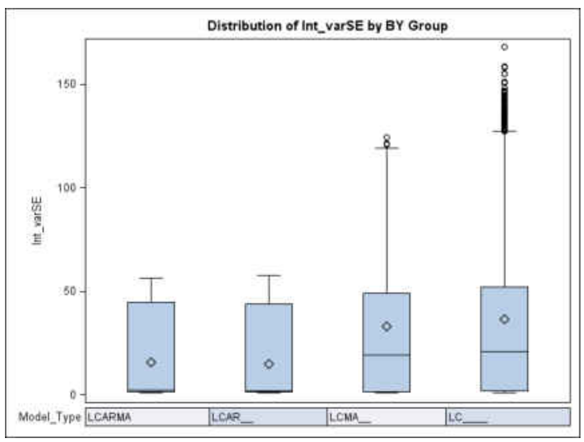

Figure 11: Intercept Variance SE by Model

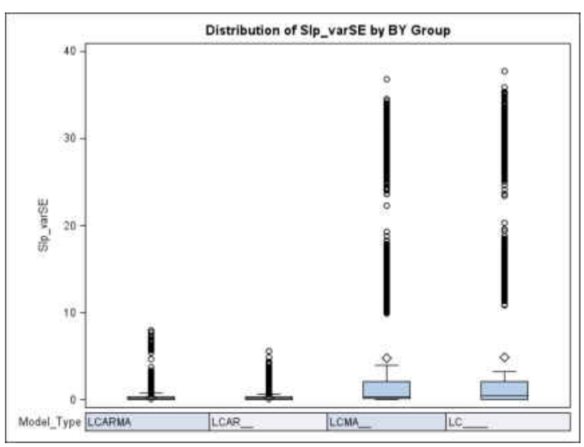

Figure 12: Slope Variance SE by Model

Higher variability in the fit indices indicate instability and a higher likelihood of the fit indices to fall above or below the established cut-off values. It can be seen from all the figures that the width of box plots is smallest for the LCARMA model followed by the LCAR models. The outliers were more for the SRMR and Mc in LCARMA model. The LCMA model has the largest width for all the fit indices followed by the LC model. Overall, the results from the box plots indicate that the LCARMA and LCAR models fitted the data better than the LCMA and LC models. Tables 1 through 10 on the fit indices by model further add support to the better fit of LCARMA and LCAR models. Figures 19 through 28 presents the overall distributions of latent curve estimates by model type*weight. The distributions of all the LC estimates were lower for the LCARMA model and LCAR model (both with weights and without weights) than the LCMA and LC models suggesting lesser variability in the data for the first two models. The box plots also indicate that the LCARMA and LCAR models were lower and closer to the population-level estimates.

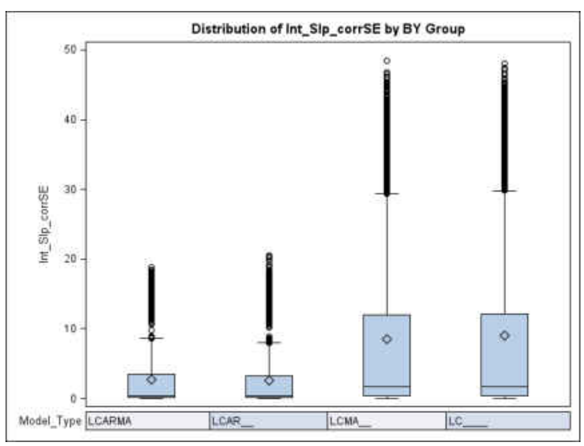

Figure 13: Intercept Slope Correlation SE by Model

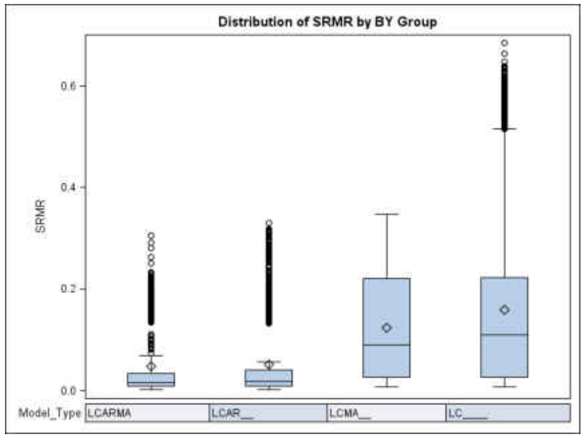

Figure 14: SRMR by Model

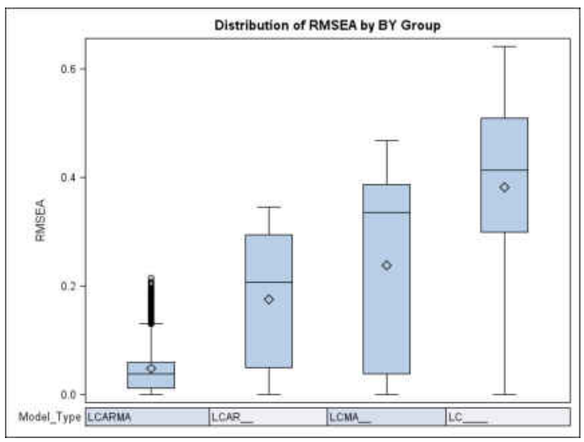

Figure 15: RMSEA by Model

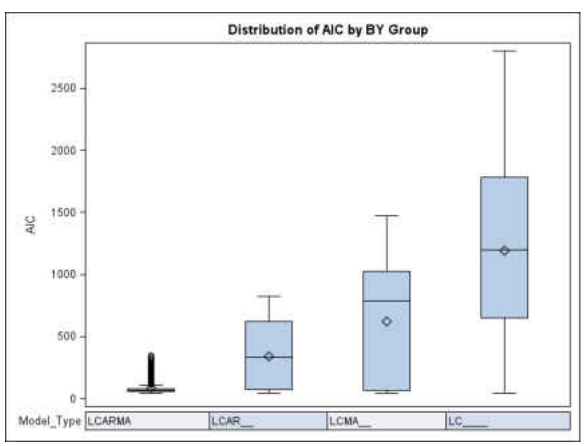

Figure 16: AIC by Model

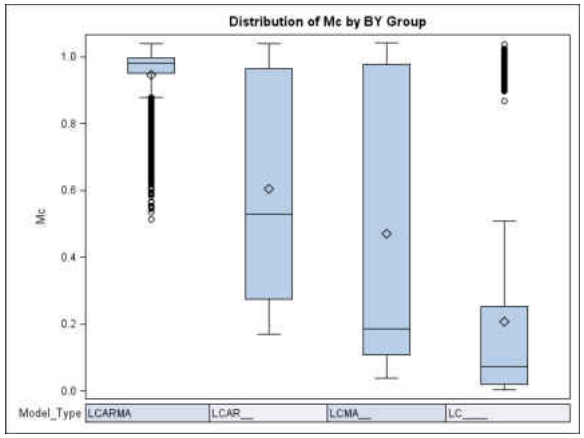

Figure 17: Mc by Model

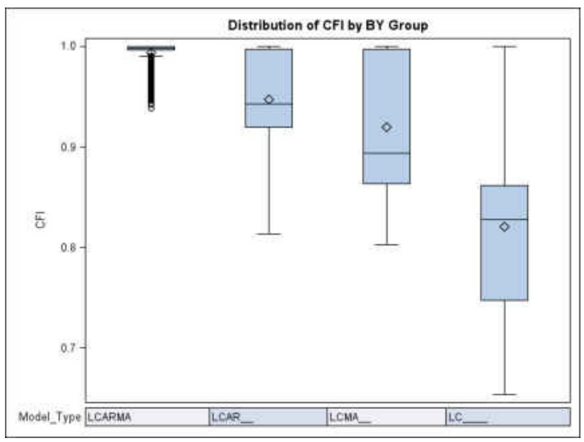

Figure 18: CFI by Model

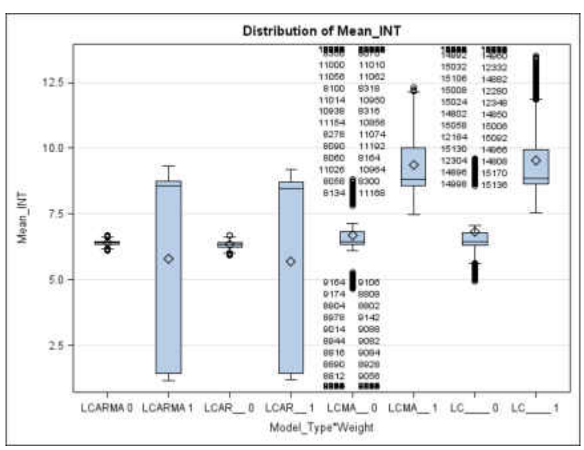

Figure 19: Mean Intercept by Model*Weight

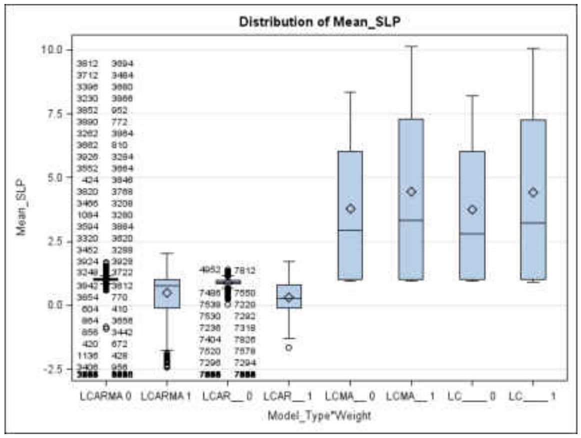

Figure 20: Mean Slope by Model*Weight

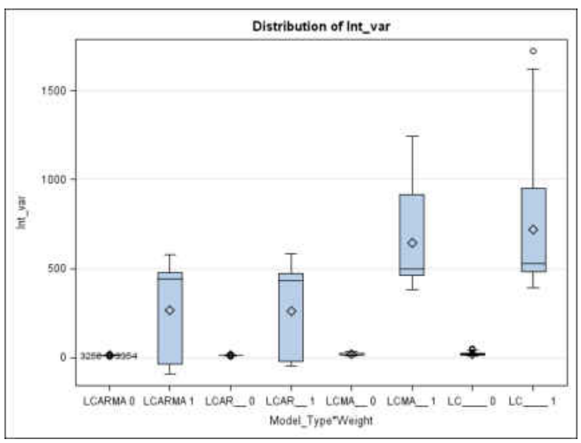

Figure 21: Intercept Variance by Model*Weight

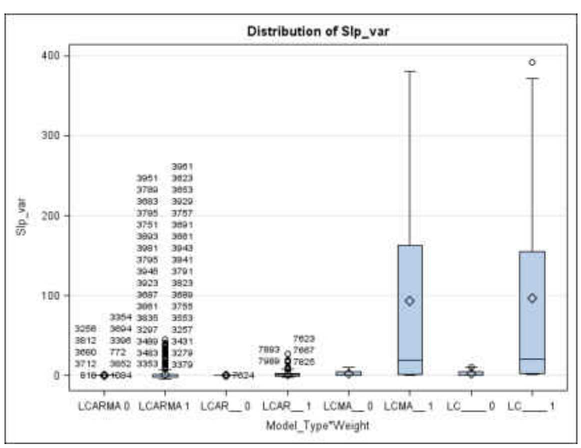

Figure 22: Slope Variance by Model*Weight

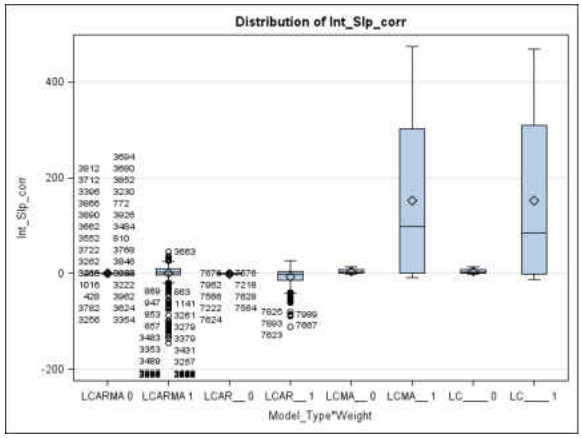

Figure 23: Intercept Slope Correlation by Model*Weight

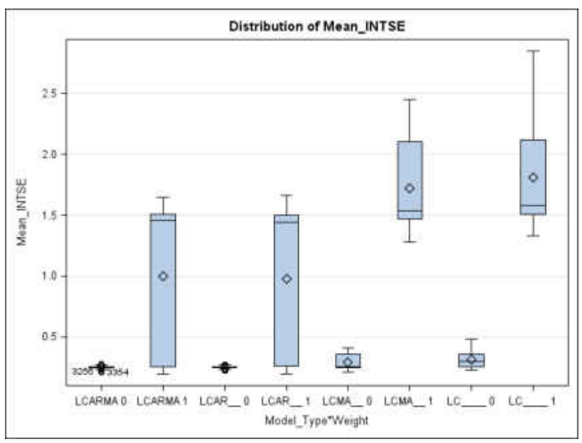

Figure 24: Mean Intercept SE by Model*Weight

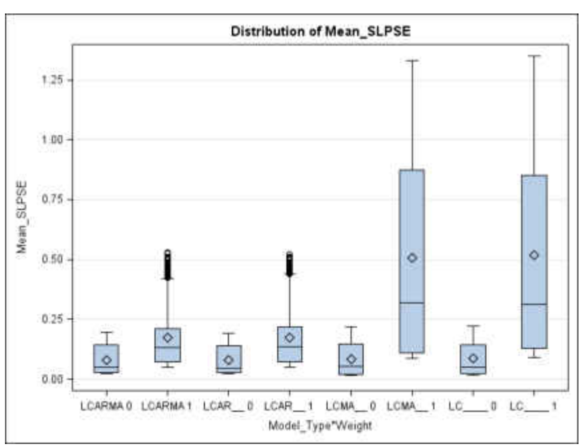

Figure 25: Mean Slope SE by Model*Weight

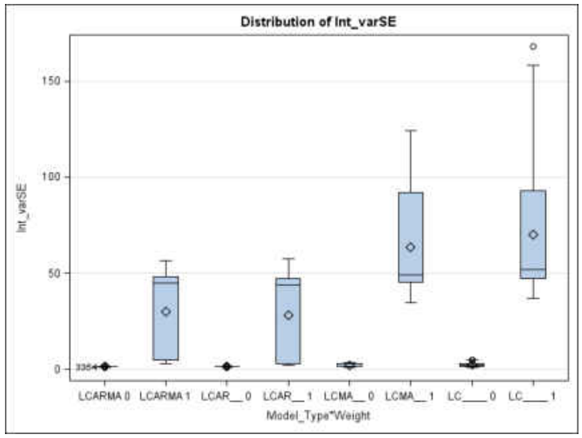

Figure 26: Intercept Variance SE by Model*Weight

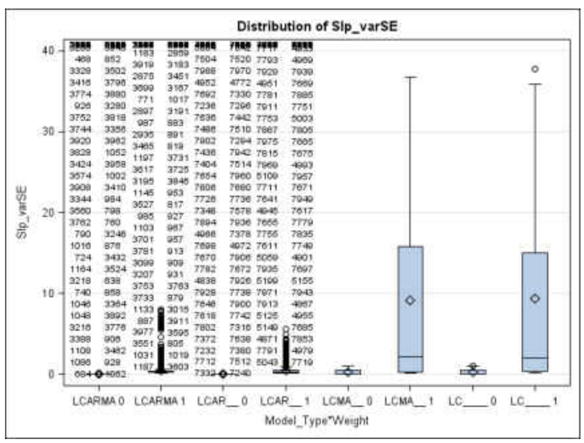

Figure 27: Slope Variance SE by Model*Weight

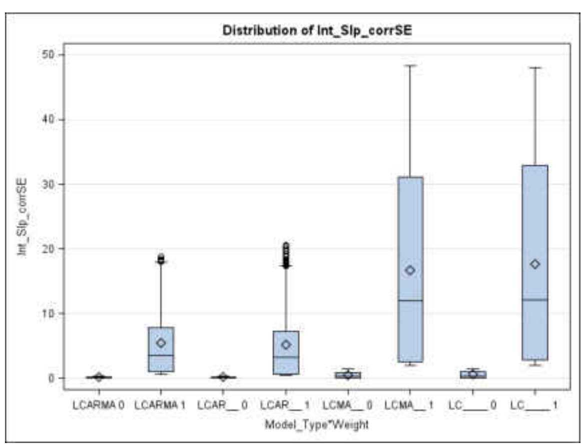

Figure 28: Intercept Slope Correlation SE by Model*Weight

The results in next section focuses on General Linear Model (GLM) with the data having an ARMA process only where both the AR and MA processes were modeled simultaneously. The fit indices, effect sizes, ANOVA results, means, tukey results, interaction plots for LC estimates of LCARMA model were compared with the other models because it was the best fitting model. Furthermore, the GLM was run for each of the AR and MA process values (AR = 0.33, MA = 0.30; AR = 0.67, MA = 0.60; AR = 0.85, MA = 0.80) because it was important to assess the behavior of fit indices, effect sizes, ANOVA results, means, tukey results and plots for LC estimates in relation to the change in AR and MA values and weights.

General Linear Model Results with ARMA Process (AR 0.33 & MA 0.30)

Table 43 provides the fit indices for the ARMA process with the LCARMA model having a much better fit (χ^2-42.8673, SRMR-0.0201, RMSEA-0.0314, AIC-72.8673, Mc-0.9729, CFI-0.9975) than the other models. Table 44 provides a summary of the effect size with ARMA process (AR 0.33 & MA 0.30). As indicated earlier, lower values for CV indicate a good model fit.

Table 43: Fit Indices with ARMA Process (AR 0.33 & MA 0.30)

Model	χ^2	SRMR	RMSEA	AIC	Mc	CFI
LC	1735.98	0.4420	0.5052	1761.98	0.0205	0.7233
LCAR	402.3876	0.0416	0.2394	430.3876	0.4265	0.9396
LCMA	973.7137	0.2751	0.3813	1001.71	0.1193	0.8468
LCARMA	42.8673	0.0201	0.0314	72.8673	0.9729	0.9975

χ^2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI-Comparative Fit Index; LC-Latent Curve; AR-Auto regressive; MA-Moving Average

The RMSE for all the estimates were low except for intercept variance, intercept slope correlation and intercept variance SE suggesting a good model fit and better predictability of the estimate by accounting for model type and weight as predictors. Hence, 92.4%, 94.3%, 97.3%, 97.4%, 98.7%, 97.3%, and 99.3% of the variation in mean intercept, means slope, slope variance, mean intercept SE, mean slope SE, slope variance SE, and intercept slope correlation SE respectively was explained by model type and weight.

Table 44: Effect Size for LC Estimates with ARMA Process (AR 0.33 & MA 0.30)

Statistic	**R^2**	**Coefficient of Variation**	**Root MSE**	**Mean**
Mean Intercept	0.9241	6.4002	0.5599	8.7483
Mean Slope	0.9431	14.6105	0.2844	1.9470
Intercept Variance	0.9585	23.3802	90.4017	386.660
Slope Variance	0.9734	23.9702	1.4552	6.0712
Intercept Slope Correlation	0.9705	32.8951	7.0232	21.3503
Mean Intercept SE	0.9743	12.2487	0.1326	1.0828
Mean Slope SE	0.9868	9.1080	0.0130	0.1427
Intercept Variance SE	0.9620	22.1724	8.5643	38.6262
Slope Variance SE	0.9728	22.9172	0.1513	0.6604
Intercept Slope Correlation SE	0.9931	10.1665	0.4121	4.0543

SE-Standard Error; MSE-Mean Square Error

ANOVA Results with ARMA Process (AR 0.33 & MA 0.30)

Tables 45 and 46 summarizes the ANOVA results with ARMA process (AR 0.33 & MA 0.30). All the models were significant at $p<.001$ including the interaction between model type and weight. The semi-partial η^2 values for the slope variance (0.2632), intercept slope correlation (0.3444), and slope variance SE (0.2392) were high as compared to the η^2 values for other estimates and thereby greater variation in these LC estimates was explained by model type and weight.

Table 45: ANOVA Results for LC Estimates with ARMA Process (AR 0.33 & MA 0.30)

Dependent Variable⁺	**Mean Intercept**	**Mean Slope**	**Intercept Variance**	**Slope Variance**	**Intercept Slope Correlation**
Model_Type	3542.77*	2083.32*	486286*	37315*	1005888*
Wt	2383.71*	2.17*	2101361*	52995*	663344*
Model_type X Wt	153.20*	53.22*	4241465*	33465*	918137*
Semi-Partial η^2	0.0233	0.0235	0.1350	0.2632	0.3444
Partial η^2	0.2349	0.2923	0.7653	0.9085	0.9212

⁺-Sum of Squares reported; *Significant at p < .001; η^2-Eta Square reported only for interaction effect between Model_type & weight; Wt-Weight

Table 46: ANOVA Results for SE Estimates with ARMA Process (AR 0.33 & MA 0.30)

Dependent Variable⁺	**Mean Intercept SE**	**Mean Slope SE**	**Intercept Variance SE**	**Slope Variance SE**	**Intercept Slope Correlation SE**
Model_Type	106.98*	3.53*	458159*	359.64*	8180.36*
Wt	904.97*	13.50*	209971*	626.87*	23525.90*
Model_type X Wt	51.15*	3.10*	399302*	321.55*	7264.47*
Semi-Partial η^2	0.0469	0.1520	0.1299	0.2392	0.1851
Partial η^2	0.6462	0.9202	0.7737	0.8981	0.9641

⁺-Sum of Squares reported; *Significant at p < .001; η^2-Eta Square reported only for interaction effect between Model_type & weight; Wt-Weight

Tukey Comparisons with ARMA Process (AR 0.33 & MA 0.30)

Tables 47 and 48 provides the Tukey comparisons between the means of LC estimates with ARMA process (AR 0.33 & MA 0.30) by model type and weight respectively. A different letter implies that means between any two models is different from each other to a statistically significant degree. The means of each model were significantly different from each other for all the LC estimates except mean slope SE and slope variance SE. The means of LC estimates for the LCARMA and LCAR model were lower than the LC and LCMA models. The means were significantly different between the models with weight and without weights in Table 48 with the former having higher means.

Table 47: Tukey Comparisons by Model Type with ARMA Process (AR 0.33 & MA 0.30)

Statistic	**Model**	**Mean**	**Tukey Grouping**
Mean Intercept	LC	10.5647	A
	LCMA	9.8559	B
	LCARMA	7.4537	C
	LCAR	7.1189	D
Mean Slope	LCMA	3.1509	A
	LC	3.0161	B
	LCARMA	0.9378	C
	LCAR	0.6831	D
Intercept Variance	LC	604.280	A
	LCMA	510.616	B
	LCARMA	231.851	C
	LCAR	199.893	D
Slope Variance	LC	11.2396	A
	LCMA	10.5440	B
	LCAR	1.4697	C
	LCARMA	1.0317	D
Intercept Slope Correlation	LCMA	50.7419	A
	LC	41.5174	B
	LCARMA	-0.5908	C
	LCAR	-6.2674	D
Mean Intercept SE	LC	1.3907	A
	LCMA	1.2844	B
	LCARMA	0.8610	C
	LCAR	0.7950	D
Mean Slope SE	LCMA	0.1901	A
	LC	0.1893	A
	LCARMA	0.0961	B
	LCAR	0.0953	B

Statistic	Model	Mean	Tukey Grouping
Intercept Variance SE	LC	59.1878	A
	LCMA	51.3817	B
	LCARMA	23.5332	C
	LCAR	20.4021	D
Slope Variance SE	LCMA	1.1371	A
	LC	1.1320	A
	LCAR	0.1869	B
	LCARMA	0.1857	B
Intercept Slope Correlation SE	LC	6.3786	A
	LCMA	6.2483	B
	LCARMA	1.9121	C
	LCAR	1.6782	D

SE-Standard Error; LC-Latent Curve; AR-Auto regressive; MA-Moving Average

Table 48: Tukey Comparisons by Weight with ARMA Process (AR 0.33 & MA 0.30)

Statistic	Model	Mean	Tukey Grouping
Mean Intercept	1	9.9688	A
	0	7.5277	B
Mean Slope	1	1.9838	A
	0	1.9102	B
Intercept Variance	1	749.062	A
	0	24.258	B
Slope Variance	1	11.8264	A
	0	0.3161	B
Intercept Slope Correlation	1	41.7118	A
	0	0.9888	B
Mean Intercept SE	1	1.8349	A
	0	0.3307	B
Mean Slope SE	1	0.2346	A
	0	0.0508	B
Intercept Variance SE	1	74.8435	A
	0	2.4089	B
Slope Variance SE	1	1.2863	A
	0	0.0345	B
Intercept Slope Correlation SE	1	7.8891	A
	0	0.2195	B

0-With Weight; 1-Without Weight; SE-Standard Error

Tables 49 and 50 provides the means for LC estimates with ARMA process (AR 0.33 & MA 0.30) when both model and weights were together accounted along with the confidence intervals (CI). The CI for the mean intercept and mean slope were narrower and tighter indicating more precision in both the estimate when the interaction between weights and autocorrelation is accounted than the other three estimates (intercept variance, slope variance, and intercept slope correlation). It can be seen that the means of the latent curve estimates and associated SE for the models are higher for the estimates having weights than those not having weights. This suggests an upward bias in the LC estimates when the autocorrelative ARMA process (AR 0.33 & MA 0.30) is modeled with weights. The bias is more pronounced for the LCMA and LC models. The only latent curve sample estimate for the models with weights that were closer to population estimate was mean intercept (8.5141 versus 8.7295 respectively). The differences between the SE of the LCMA and LC models with weighs and without weights was also more than the differences between the LCARMA and LCAR models. Larger differences were observed in the intercept variance estimate and SE between models with weights and without weights.

Table 49: Means for LC Estimates with ARMA Process
by Model_Type*Weights (AR 0.33 & MA 0.30)

Model	Weight	Mean Intercept	Mean Slope	Intercept Variance	Slope Variance	Intercept Slope Correlation
LCARMA	0	6.3932	1.0045	13.3732	0.0484	0.0097
	1	8.5141	0.8711	450.3282	2.0149	-1.1912
LCAR	0	6.3202	0.8926	13.1421	0.0722	-0.1875
	1	7.9176	0.4736	386.6436	2.8671	-12.3471
LCMA	0	8.2733	2.9546	29.6872	0.5614	2.6006
	1	11.4384	3.3473	991.5446	20.5265	98.8831
LC	0	9.1241	2.7889	40.8298	0.5821	1.5324

Model	Weight	Mean Intercept	Mean Slope	Intercept Variance	Slope Variance	Intercept Slope Correlation
	1	12.0052	3.2433	1167.730	21.8971	81.5023

0-With Weight; 1-Without Weight; LC-Latent Curve; AR-Auto regressive; MA-Moving Average.

Table 50: Means for SE Estimates with ARMA Process
by Model_Type*Weights (AR 0.33 & MA 0.30)

Model	Weight	Mean Intercept SE	Mean Slope SE	Intercept Variance SE	Slope Variance SE	Intercept Slope Correlation SE
LCARMA	0	0.2541	0.0499	1.3428	0.0085	0.0909
	1	1.4680	0.1423	45.7236	0.3629	3.7332
	0 (C.I)	6.8912 5.8952	1.1023 0.9067	16.0051 10.7413	0.0651 0.0317	0.1879 -0.1685
	1 (C.I)	11.3914 5.6368	1.1500 0.5922	539.9465 360.7099	2.7262 1.3036	6.1259 -8.5083
LCAR	0	0.2511	0.0458	1.3080	0.0088	0.0874
	1	1.3390	0.1448	39.4960	0.3650	3.2690
	0 (C.I)	6.8124 5.8280	0.9824 0.8028	15.7058 10.5784	0.0894 0.0550	-0.0162 -0.3588
	1 (C.I)	10.5420 5.2932	0.7574 0.1898	464.0558 309.2314	3.5825 2.1517	-5.9399 -18.7543
LCMA	0	0.3792	0.0544	2.9897	0.0617	0.3434
	1	2.1896	0.3258	99.7736	2.2124	12.1531
	0 (C.I)	9.0165 7.5301	3.0612 2.8480	35.5470 23.8274	0.6823 0.4405	3.2737 1.9275
	1 (C.I)	15.7300 7.1468	3.9859 2.7087	1187.1009 795.9883	24.8628 16.1902	122.7032 75.0630
LC	0	0.4385	0.0531	3.9949	0.0588	0.3562
	1	2.3429	0.3253	114.3806	2.2052	12.4010
	0 (C.I)	9.9836 8.2646	2.8930 2.6848	48.6598 32.9998	0.6973 0.4669	2.2306 0.8342
	1 (C.I)	16.5973 7.4131	3.8809 2.6057	1391.9160 943.5440	26.2193 17.5749	105.8083 57.1963

0-With Weight; 1-Without Weight; SE-Standard Error; C.I.-Confidence Interval; LC-Latent Curve; AR-Autoregressive; MA-Moving Average; SE-Standard Error

Interaction Plots with ARMA Process*Weight (AR 0.33 & MA 0.30)

Figures 29 through 38 presents the interaction plots between model type and weights for each of the LC estimates and associated SE for models that were fitted to the LCARMA data. An examination of how the four models (LCARMA, LCAR, LCMA, and Latent Curve models fitted the LCARMA (0.33, 0.30) data revealed that the mean intercept (Figure 29) were consistently underestimated for all four models when sample weights were not incorporated (mean intercept for population was 8.7295). While the LCAR model's mean intercept was approximately equal to that of the LCARMA model, the plot clearly depicts the Mean Intercept as underestimated for both the LCMA and Latent Curve models. This finding suggest that ignoring the AR component of the ARMA process downwardly biases the mean intercept growth parameter.

Figure 30 for mean slope had mixed results when weights were present versus not present in the models. The means were slightly overestimated for all the four models with weights than without weights because the population estimate for intercept variance was 1.0019. Figure 31 for intercept variance (population value of 13.5423) had similar trends as mean intercept with the means being overestimated for the models with weights with variability in the estimates within the models. The means for slope variances (Figure 32) (population value of 0.0483) were almost constant for all the models without weights. However, the overestimation was less in the LCARMA and LCAR models with weights than the LCMA and MC models. Figure 33 shows the intercept slope correlation being overestimated for the LCMA and LC models with weights and slight underestimation for LCAR model with weights. There were minimal differences in the intercept slope correlation means between the LCARMA models with weights and without weights.

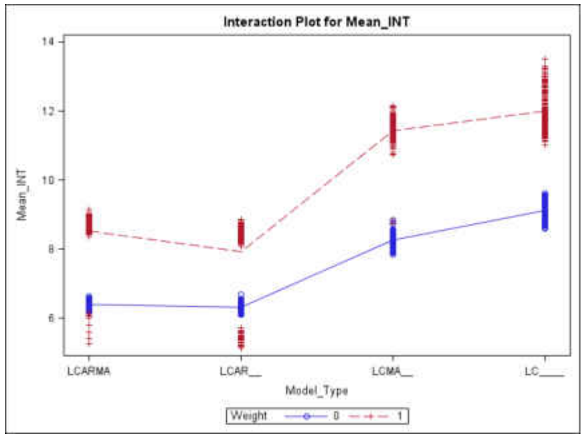

Figure 29: Interaction Plot of Mean Intercept with ARMA (AR 0.33 & MA 0.30)

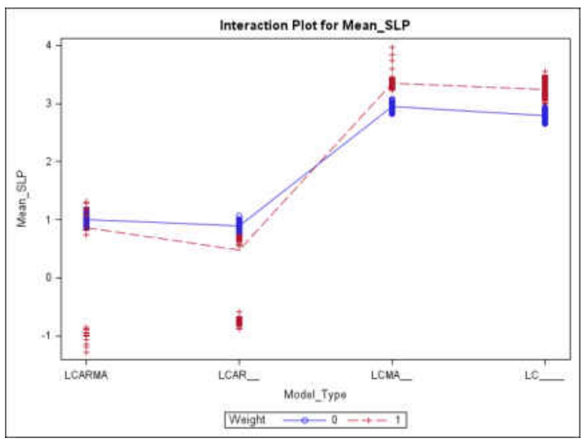

Figure 30: Interaction Plot of Mean Slope with ARMA (AR 0.33 & MA 0.30)

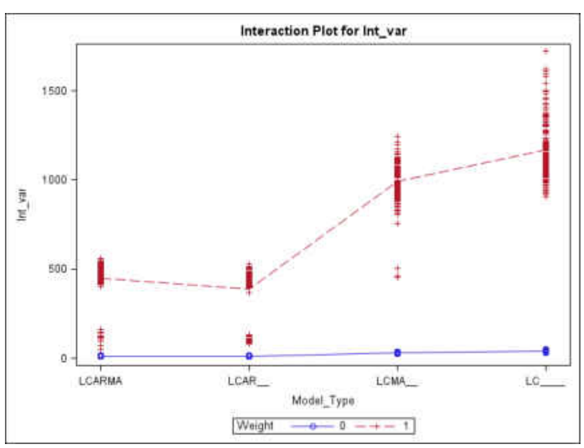

Figure 31: Interaction Plot of Intercept Variance with ARMA (AR 0.33 & MA 0.30)

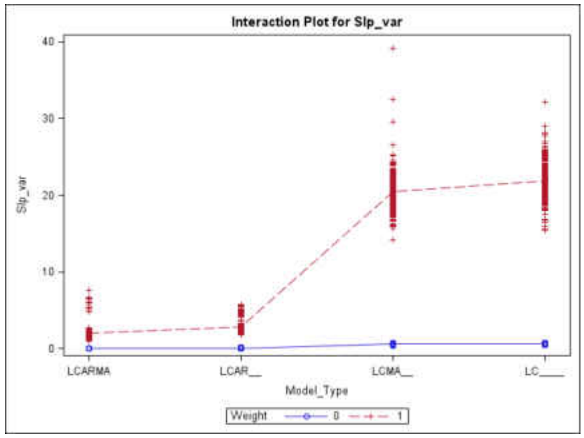

Figure 32: Interaction Plot of Slope Variance with ARMA (AR 0.33 & MA 0.30)

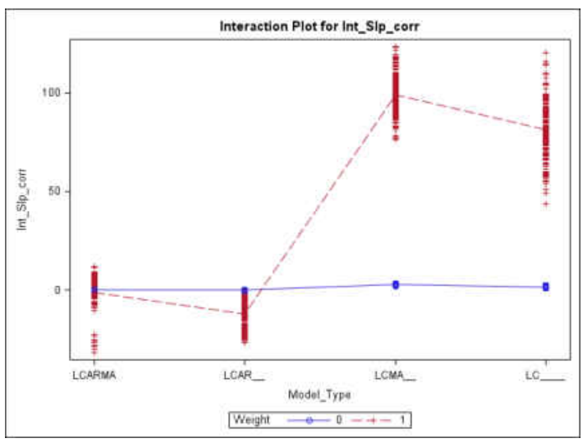

Figure 33: Interaction Plot of Intercept Slope Correlation with ARMA (AR 0.33 & MA 0.30)

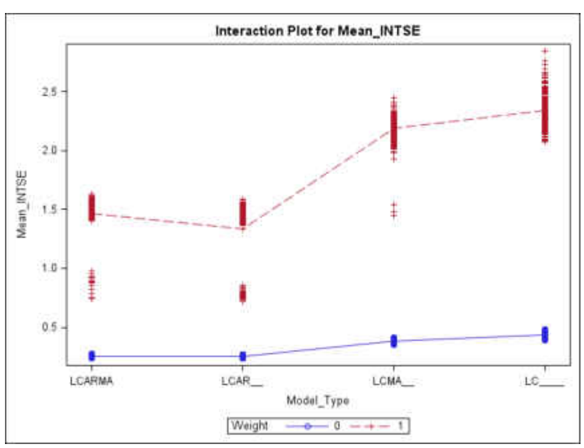

Figure 34: Interaction Plot of Mean Intercept SE with ARMA (AR 0.33 & MA 0.30)

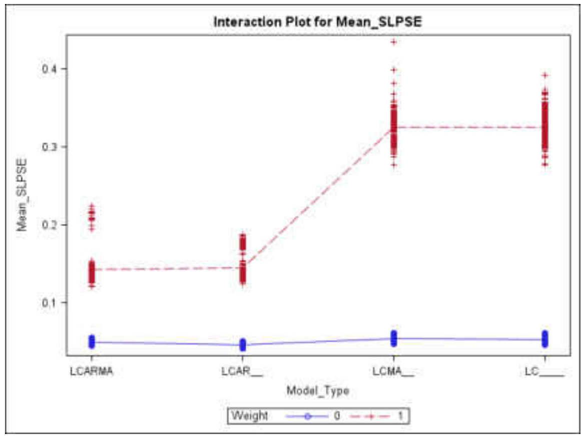

Figure 35: Interaction Plot of Mean Slope SE with ARMA (AR 0.33 & MA 0.30)

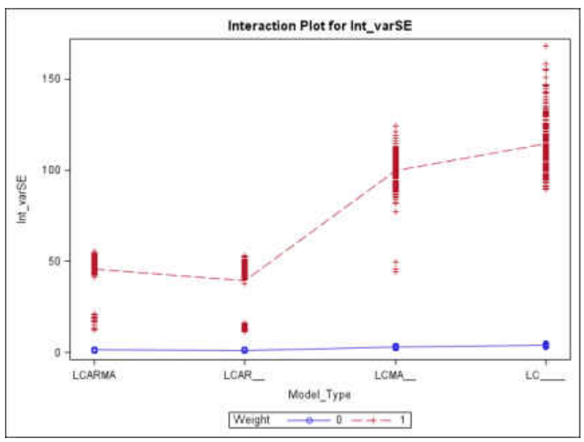

Figure 36: Interaction Plot of Intercept Variance SE with ARMA (AR 0.33 & MA 0.30)

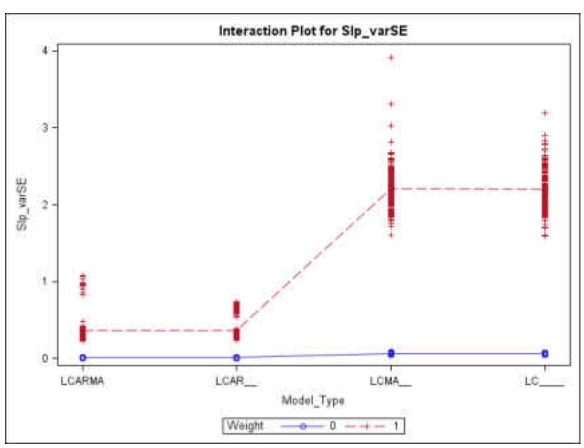

Figure 37: Interaction Plot of Slope Variance SE with ARMA (AR 0.33 & MA 0.30)

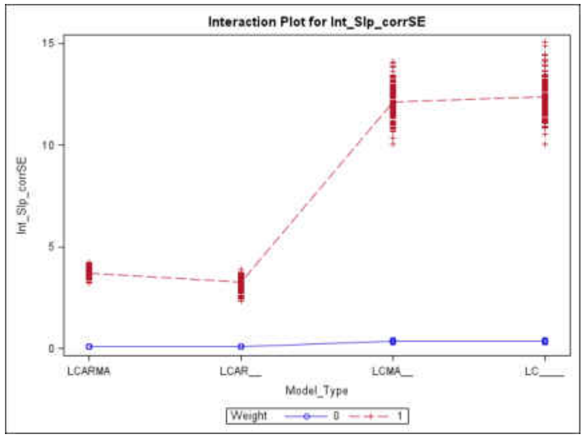

Figure 38: Interaction Plot of Intercept Slope Correlation SE with ARMA (AR 0.33 & MA 0.30)

Similar trends were seen as in Figure 29 for mean intercept SE (population value of 0.0075) (Figure 34), mean slope SE (population value of 0.0013) (Figure 35), intercept variance SE (population value of 13.5423) (Figure 36), slope variance SE (population value of 0.0483) (Figure 37), and intercept slope correlation SE (population value of 0.0030) (Figure 38). Overall, the highest latent curve estimates within the models with weights were for the LCMA and LC models. The overestimation in latent curve estimates for the LCARMA and LCAR models with weights was less as compared to the LCMA and LC models with weights implying that the sample latent curve estimates for the first two models were closer to the population latent curve estimates. Further support is added to this trend by the lower SE values of LCARMA and LCAR models than the other two in Table 49. The CI for the mean intercept and mean slope were narrower and tighter indicating more precision in both the estimate when the interaction between weights and autocorrelation is accounted than the other three estimates (intercept variance, slope variance, and intercept slope correlation). The CI for mean intercept and mean slope were also different from each other to a statistically significant degree when model with weights and without weights were compared.

General Linear Model Results with ARMA Process (AR 0.67 & MA 0.60)

The results in this section focuses on the data having an ARMA process only where both the AR and MA processes were modeled (AR = 0.67, MA = 0.60). The fit indices, effect sizes, ANOVA results, means, tukey results and plots for LC estimates of LCARMA model were compared with the other models because it was the best fitting model. Table 51 provides the fit indices for the ARMA process with the LCARMA model having a much better fit (χ^2-39.0791, SRMR-0.0195, RMSEA-0.0314, AIC-69.0791, Mc-0.0.9780, CFI-0.9984) than the other models. Table 52 provides a summary of the effect size of LC estimates with ARMA process (AR 0.33 & MA 0.30). The RMSE for all the estimates were low except for intercept variance, slope variance, intercept slope correlation, and intercept variance SE suggesting a good model fit and better predictability of the estimate by accounting for model type and weight as predictors. Hence, 95.8%, 99.2%, 96.9%, 99.4%, 98.7%, and 98.6% of the variation in mean intercept, mean slope, slope variance, mean intercept SE, mean slope SE, slope variance SE, and intercept slope correlation SE respectively was explained by model type and weight.

Table 51: Fit Indices with ARMA Process (AR 0.67 & MA 0.60)

Model	χ^2	SRMR	RMSEA	AIC	Mc	CFI
LC	2148.14	0.1961	0.5634	2174.14	0.0076	0.7141
LCAR	617.5599	0.0159	0.3016	645.5599	0.2579	0.9206
LCMA	1028.96	0.1490	0.3932	1056.96	0.1005	0.8651
LCARMA	39.0791	0.0195	0.0314	69.0791	0.9780	0.9984

χ^2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI-Comparative Fit Index

Tables 53 and 54 summarizes the ANOVA results with ARMA process (AR 0.67 & MA 0.60). All the models were significant at $p<.001$ including the interaction between model type and weight. The semi-partial η^2 values for the slope variance (0.3096), intercept slope correlation (0.3444), mean slope SE (0.2816), and slope variance SE (0.3023) were high as compared to the η^2 values for other estimates and thereby greater variation in these LC estimates was explained by model type and weight.

Table 52: Effect Size for LC Estimates with ARMA Process (AR 0.67 & MA 0.60)

Statistic	**R^2**	**Coefficient of Variation**	**Root MSE**	**Mean**
Mean Intercept	0.9584	3.3330	0.2539	7.6187
Mean Slope	0.9929	6.7776	0.2507	3.6996
Intercept Variance	0.9121	31.4825	89.183	283.2791
Slope Variance	0.9875	18.3668	7.6593	41.7019
Intercept Slope Correlation	0.9897	18.1174	12.0078	66.2772
Mean Intercept SE	0.9690	13.0076	0.1200	0.9232
Mean Slope SE	0.9940	6.9036	0.0235	0.3412
Intercept Variance SE	0.9104	31.8703	8.7713	27.5219
Slope Variance SE	0.9872	18.1677	0.7470	4.1116
Intercept Slope Correlation SE	0.9868	14.3636	1.3029	9.0710

SE-Standard Error; MSE-Mean Square Error

ANOVA Results with ARMA Process (AR 0.67 & MA 0.60)

Table 53: ANOVA Results for LC Estimates with ARMA Process (AR 0.67 & MA 0.60)

Dependent Variable+	**Mean Intercept**	**Mean Slope**	**Intercept Variance**	**Slope Variance**	**Intercept Slope Correlation**
Model_Type	88.61*	13728.17*	9023100*	2598479*	8351870*
Wt	2251.46*	138.22*	1144419*	2489109*	6278338*
Model_type X Wt	25.62*	183.88*	7959125*	2323635*	7466304*
Semi-Partial η^2	0.0104	0.0130	0.0552	0.3096	0.3344
Partial η^2	0.1997	0.6475	0.3860	0.9614	0.9702

+-Sum of Squares reported; *Significant at $p < .001$; η^2-Eta Square reported only for interaction effect between Model_type & weight; Wt-Weight

Table 54: ANOVA Results for SE Estimates with ARMA Process (AR 0.67 & MA 0.60)

Dependent Variable+	Mean Intercept SE	Mean Slope SE	Intercept Variance SE	Slope Variance SE	Intercept Slope Correlation SE
Model_Type	21.833*	42.0540	87653*	23593*	45202.64*
Wt	687.777*	62.8948	1080255*	24201*	117598.90*
Model_type X Wt	10.2463*	41.4787	77297*	21092*	40322.33*
Semi-Partial η^2	0.0138	0.2816	0.0565	0.3023	0.1959
Partial η^2	0.3086	0.9791	0.3869	0.9596	0.9372

+-Sum of Squares reported; *Significant at $p < .001$; η^2-Eta Square reported only for interaction effect between Model_type & weight; Wt-Weight; SE-Standard Error

Tukey Comparisons with ARMA Process (AR 0.67 & MA 0.60)

Tables 55 and 56 provides the Tukey comparisons between the means of LC estimates with ARMA process (AR 0.67 & MA 0.60) by model type and weight respectively. A different letter implies that means between any two models is different from each other to a statistically significant degree. The means of each model were significantly different from each other for mean slope, intercept slope correlation, and intercept slope correlation SE latent curve estimates but not for mean intercept, intercept variance, slope variance, intercept slope correlation, mean intercept SE, mean slope SE, intercept variance SE, and slope variance SE latent curve estimates. Overall, the LC and LCMA models had higher means than the LCARMA and LCAR models. The means were significantly different between the models with weights and without weights in Table 56 with the former having higher means. There were striking differences between the means of intercept variance, slope variance, intercept slope correlation, intercept variance SE, slope variance SE, and intercept slope correlation SE for models with weights and without weights.

Table 55: Tukey Comparisons by Model Type with ARMA Process (AR 0.67 & MA 0.60)

Statistic	Model	Mean	Tukey Grouping
Mean Intercept	LC	8.0131	A
	LCARMA	7.5445	B
	LCAR	7.5267	B
	LCMA	7.3905	C
Mean Slope	LCMA	6.6593	A
	LC	6.5934	B
	LCARMA	0.9403	C
	LCAR	0.6056	D
Intercept Variance	LC	413.041	A
	LCARMA	244.351	B
	LCAR	244.145	B
	LCMA	231.579	B
Slope Variance	LCMA	83.9979	A
	LC	79.9537	B
	LCAR	1.6301	C
	LCARMA	1.2261	C
Intercept Slope Correlation	LC	146.7125	A
	LCMA	129.7089	B
	LCARMA	-1.3295	C
	LCAR	-9.9829	D
Mean Intercept SE	LC	1.1212	A
	LCARMA	0.8776	B
	LCAR	0.8762	B
	LCMA	0.8179	C
Mean Slope SE	LCMA	0.5106	A
	LC	0.4960	B
	LCAR	0.1800	C
	LCARMA	0.1784	C
Intercept Variance SE	LC	40.1693	A
	LCAR	24.3346	B
	LCARMA	24.2531	B
	LCMA	21.3307	C
	LCMA	8.1640	A
	LC	7.7333	B

Statistic	Model	Mean	Tukey Grouping
Slope Variance SE	LCARMA	0.2825	C
	LCAR	0.2667	C
Intercept Slope Correlation SE	LC	15.7512	A
	LCMA	12.8152	B
	LCARMA	3.9848	C
	LCAR	3.7330	D

SE-Standard Error; LC-Latent Curve; AR-Auto regressive; MA-Moving Average

Table 56: Tukey Comparisons by Weight with ARMA Process (AR 0.67 & MA 0.60)

Statistic	Model	Mean	Tukey Grouping
Mean Intercept	1	8.8049	A
	0	6.4325	B
Mean Slope	1	3.9936	A
	0	3.4058	B
Intercept Variance	1	550.723	A
	0	15.835	B
Slope Variance	1	81.1443	A
	0	2.2597	B
Intercept Slope Correlation	1	128.9188	A
	0	3.6357	B
Mean Intercept SE	1	1.5788	A
	0	0.2675	B
Mean Slope SE	1	0.5395	A
	0	0.1430	B
Intercept Variance SE	1	53.5057	A
	0	1.5381	B
Slope Variance SE	1	8.0085	A
	0	0.2225	B
Intercept Slope Correlation SE	1	17.6442	A
	0	0.4979	B

0-With Weight; 1-Without Weight; SE-Standard Error

Tables 57 and 58 provides the means for LC estimates with ARMA process (AR 0.67 & MA 0.60) when both model and weights were together accounted along with the confidence intervals. It can be seen that the means of the latent curve estimates and associated SE for the models are higher for the estimates having weights than those not having weights.

Table 57: Means for LC Estimates with ARMA Process
by Model_Type*Weights (AR 0.67 & MA 0.60)

Model	Weight	Mean Intercept	Mean Slope	Intercept Variance	Slope Variance	Intercept Slope Correlation
LCARMA	0	6.4003	0.9897	13.5532	0.0515	-0.0078
	1	8.6887	0.8909	475.1495	2.4007	-2.6511
LCAR	0	6.3859	0.6459	13.3667	0.0904	-0.5578
	1	8.6674	0.5653	474.9237	3.1697	-19.4079
LCMA	0	6.3284	6.0096	12.6953	4.5850	7.0786
	1	8.4526	7.3090	450.4631	163.4108	252.3391
LC	0	6.6151	5.9777	23.7259	4.3118	8.0298
	1	9.4109	7.2091	802.3553	155.5956	285.3950

0-With Weight; 1-Without Weight; LC-Latent Curve; AR-Auto regressive; MA-Moving Average

Table 58: Means for SE Estimates with ARMA Process
by Model_Type*Weights (AR 0.67 & MA 0.60)

Model	Weight	Mean Intercept SE	Mean Slope SE	Intercept Variance SE	Slope Variance SE	Intercept Slope Correlation SE
LCARMA	0	0.2535	0.1425	1.3411	0.0125	0.2154
	1	1.5016	0.2143	47.1649	0.5525	7.7543
	0 (C.I)	6.8972 5.9034	1.2690 0.7104	16.1818 10.9246	0.0760 0.0270	0.4144 -0.4300
	1 (C.I)	11.6318 5.7456	1.3109 0.4709	567.5927 382.7063	3.4836 1.3178	12.5473 -17.8495
LCAR	0	0.2518	0.1413	1.3330	0.0144	0.2030
	1	1.5005	0.2187	47.3361	0.5190	7.2630
	0 (C.I)	6.8794 5.8924	0.9228 0.3690	15.9794 10.7540	0.1186 0.0622	-0.1599 -0.9557
	1 (C.I)	11.6084 5.7264	0.9940 0.1366	567.7025 382.1449	4.1869 2.1525	-5.1724 -33.6434
LCMA	0	0.2349	0.1465	1.1680	0.4457	0.6999
	1	1.4009	0.8746	41.4932	15.8823	24.9304
	0 (C.I)	6.7888 5.8680	6.2967 5.7225	14.9846 10.4060	5.4586 3.7114	8.4504 5.7068

Model	Weight	Mean Intercept SE	Mean Slope SE	Intercept Variance SE	Slope Variance SE	Intercept Slope Correlation SE
	1 (C.I)	11.1984 5.7068	9.0232 5.5948	531.7898 369.1364	194.5401 132.2815	301.2027 203.4755
LC	0	0.3300	0.1416	2.3100	0.4173	0.8732
	1	1.9123	0.8504	78.0286	15.0495	30.6292
	0 (C.I)	7.2619 5.9683	6.2552 5.7002	28.2535 19.1983	5.1297 3.4939	9.7413 6.3183
	1 (C.I)	13.1590 5.6628	8.8759 5.5423	955.2914 649.4192	185.0926 126.0986	345.4282 225.3618

0-With Weight; 1-Without Weight; SE-Standard Error; C.I.-Confidence Interval; LC-Latent Curve; AR-Autoregressive; MA-Moving Average

This suggests an upward bias in the LC estimates when the autocorrelative ARMA process (AR 0.67 & MA 0.60) is modeled with weights. The bias is more pronounced for the LCMA and LC models. The only latent curve sample estimate closer to population estimate was mean intercept (8.6887 versus 8.7286 respectively). The differences between the SE of the LCMA and LC models with weighs and without weights was also more than the differences between the LCARMA and LCAR models. Larger differences were observed in the intercept variance estimate and SE and intercept slope correlation SE between models with weights and without weights.

Interaction Plots with ARMA Process*Weight (AR 0.67 & MA 0.60)

Figures 39 through 48 presents the interaction plots between model type and weights for each of the LC estimates and associated SE for models that were fitted to the LCARMA data. An examination of how the four models (LCARMA, LCAR, LCMA, and Latent Curve models fitted the LCARMA (0.67, 0.60) data revealed that the mean intercept (Figure 39) were consistently

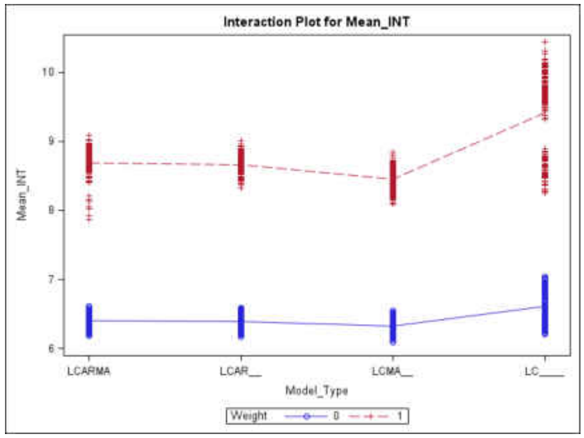

Figure 39: Interaction Plot of Mean Intercept with ARMA (AR 0.67 & MA 0.60)

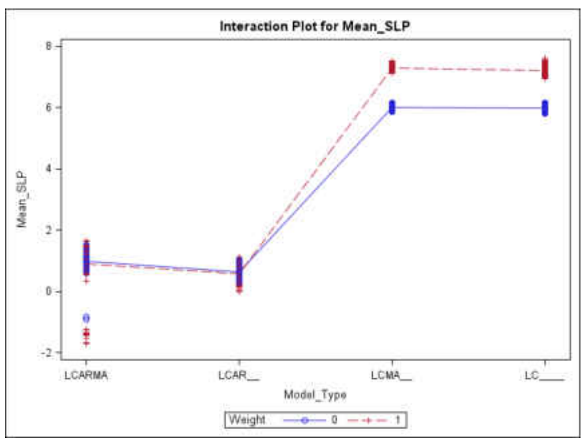

Figure 40: Interaction Plot of Mean Slope with ARMA (AR 0.67 & MA 0.60)

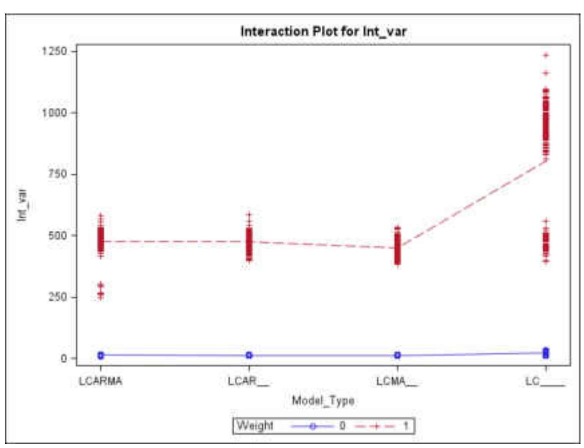

Figure 41: Interaction Plot of Intercept Variance with ARMA (AR 0.67 & MA 0.60)

underestimated for all four models when sample weights were not incorporated (mean intercept for population was 8.7286). The mean intercept were approximately equal within all the models with weights and without weights except for the LC model with weights where the means were higher than the other three models having weights. This finding suggest that ignoring the AR component of the ARMA process downwardly biases the mean intercept growth parameter. Figure 40 shows that the mean slope (population value of 1.0217) were overestimated for LCMA and LC models with weights but the difference between the mean slopes was minimal for the LCARMA and LCAR models with weights and no weights.

Figure 41 for intercept variance (population value of 13.7297) shows that the mean was overestimated when weights were present versus when they were not present in the four models. The means were constant for intercept variance of all models without weights. Figure 41 for intercept variance (population value of 13.5423) had similar trends as mean intercept with the means being overestimated for the models with weights. Similar estimates were observed within the models except for LC model with weights which had higher estimates than the other three models with weights. The means for intercept variances were almost constant for all the models without weights. Figure 42 shows the slope variance (population value of 0.0453) being overestimated for the LCMA and LC models with weights and slight underestimation for LCARMA and LCAR model with weights. Figure 43 shows the intercept slope correlation (population value of 0.0382) interaction plot with LCMA and LCAR means with weights being overestimated and the LCAR mean slightly underestimated. There were minimal differences in the intercept slope correlation means between the LCARMA models with weights and without weights.

The sample latent curve values for mean intercept SE (population value of 0.0075) (Figure 44), mean slope SE (population value of 0.0042) (Figure 45), intercept variance SE (population value of 0.0397) (Figure 46), slope variance SE (population value of 0.0002) (Figure 47), and intercept slope correlation SE (population value of 0.0065) (Figure 48) having ARMA process (0.67, 0.60) were overestimated for models with weights. However, the overestimation in models with weights was less for mean slope SE and intercept slope SE. Minimal differences

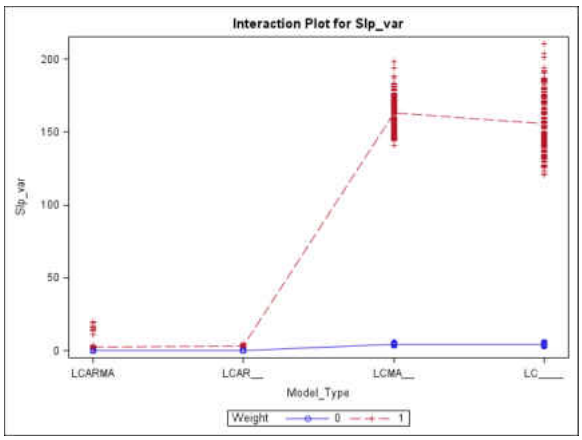

Figure 42: Interaction Plot of Slope Variance with ARMA (AR 0.67 & MA 0.60)

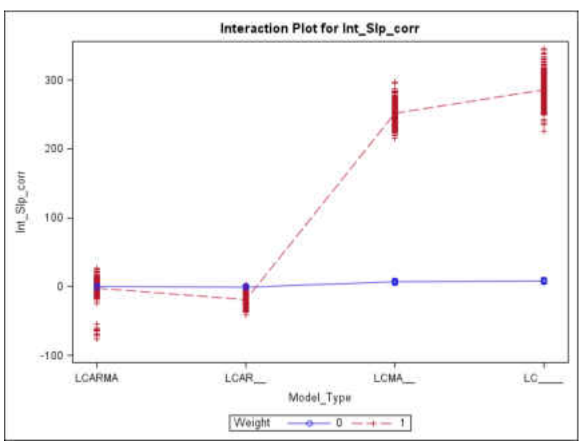

Figure 43: Interaction Plot of Intercept Slope Correlation with ARMA (AR 0.67 & MA 0.60)

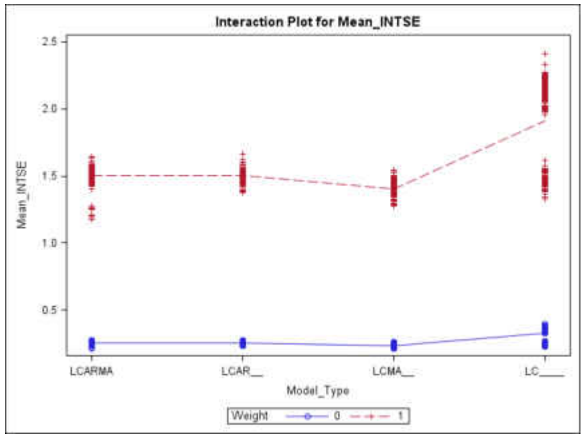

Figure 44: Interaction Plot of Mean Intercept SE with ARMA (AR 0.67 & MA 0.60)

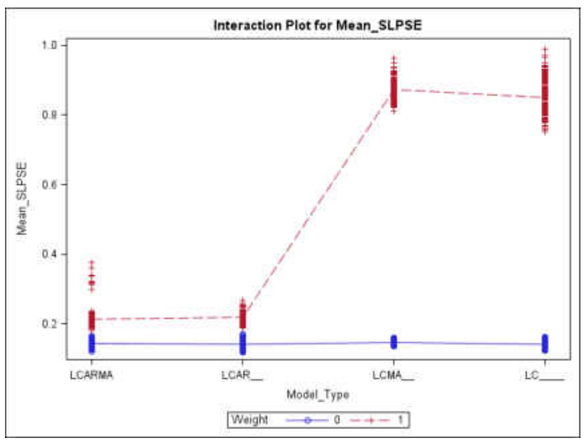

Figure 45: Interaction Plot of Mean Slope SE with ARMA (AR 0.67 & MA 0.60)

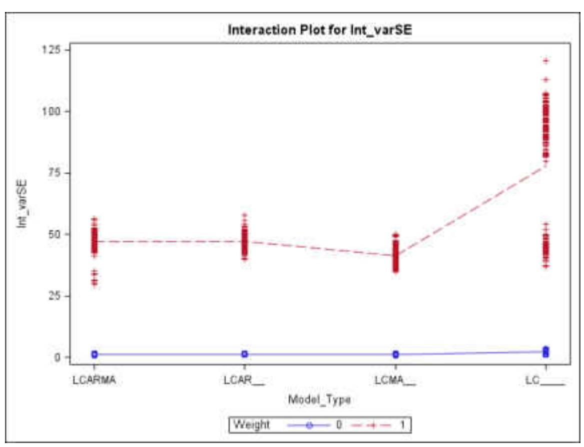

Figure 46: Interaction Plot of Intercept Variance SE with ARMA (AR 0.67 & MA 0.60)

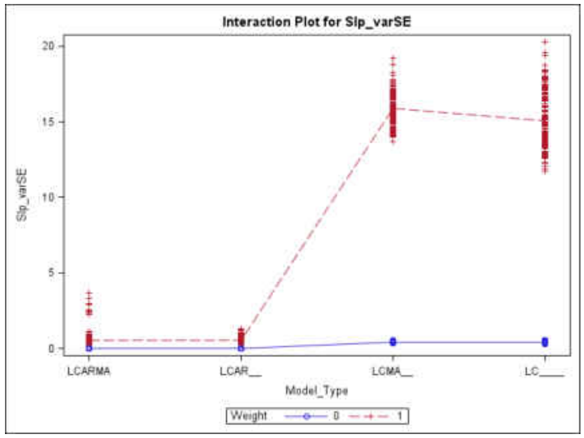

Figure 47: Interaction Plot of Slope Variance SE with ARMA (AR 0.67 & MA 0.60)

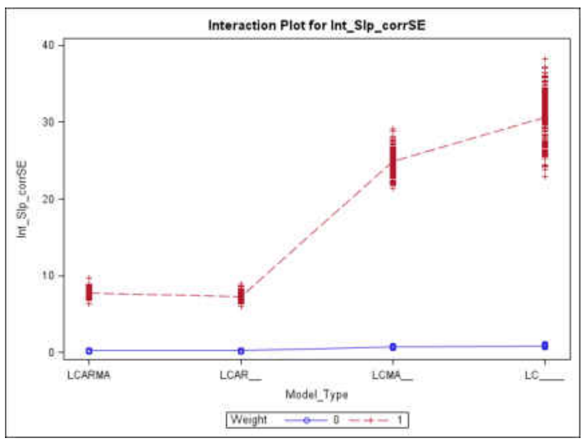

Figure 48: Interaction Plot of Intercept Slope Correlation SE with ARMA (AR 0.67 & MA 0.60)

were present in the slope variance SE estimates between the LCARMA and LCAR models with weighs and without weights. Overall, the highest latent curve estimates within the models with weights were for the LCMA model. The overestimation in latent curve estimates for the LCARMA and LCAR models with weights was less as compared to the LCMA and LC models implying that the sample latent curve estimates for these models were closer to the population latent curve estimates. The differences in the estimates between the LCARMA and LCAR models with and without weights were less for the slope variance, intercept slope correlation, mean slope SE, slope variance SE, and intercept slope correlation SE. The CI for the mean intercept and mean slope were narrower and tighter indicating more precision in both the estimate when the interaction between weights and autocorrelation is accounted than the other three estimates (intercept variance, slope variance, and intercept slope correlation). The CI for mean intercept and mean slope were also different from each other to a statistically significant degree when model with weights and without weights were compared.

<u>General Linear Model Results with ARMA Process (AR 0.85 & MA 0.80)</u>

The results in this section focuses on the data having an ARMA process only where both the AR and MA processes were modeled (AR = 0.85, MA = 0.80). The fit indices, effect sizes, ANOVA results, means, tukey results and plots for LC estimates of LCARMA model were compared with the other models because it was the best fitting model. Table 59 provides the fit indices for the ARMA process with the LCARMA model having a much better fit (χ^2-38.7730, SRMR-0.0230, RMSEA-0.0305, AIC-68.77, Mc-0.0.9785, CFI-0.9985) than the other models. Table 60 provides a summary of the effect size of LC estimates with ARMA process (AR 0.33 & MA 0.30). The RMSE for all the estimates were low except for intercept variance, slope variance, and intercept slope correlation suggesting a good model fit and better predictability of the estimate by accounting for model type and weight as predictors. Hence, 96.6%, 98.7%, 99.6%, 99.6%, 99.1%, 99.1%, and 99.3% of the variation in mean intercept, mean slope, mean intercept SE, mean slope SE, intercept variance SE, slope variance SE, and intercept slope correlation SE respectively is explained by model type and weight.

Tables 59 and 60 summarizes the ANOVA results with ARMA process (AR 0.85 & MA 0.80). All the models were significant at p<.001 including the interaction between model type and weight. The semi-partial η2 values for the slope variance (0.2632), intercept slope correlation (0.3444), and slope variance SE (0.2392) were high as compared to the η2 values for other estimates and thereby greater variation in these LC estimates was explained by model type and weight. Tables 61 and 62 summarizes the ANOVA results with ARMA process (AR 0.85 & MA 0.80). All the models were significant at p<.001 including the interaction between model type and weight. The semi-partial η^2 values for the slope variance (0.3119), intercept slope correlation (0.3405), and slope variance SE (0.2941) were high as compared to the η2 values for other estimates and thereby greater variation in these LC estimates was explained by model type and weight.

Table 59: Fit Indices with ARMA Process (AR 0.85 & MA 0.80)

Model	χ^2	**SRMR**	**RMSEA**	**AIC**	**Mc**	**CFI**
LC	2218.10	0.1001	0.5720	2244.10	0.0072	0.7224
LCAR	659.7465	0.0114	0.3122	687.7465	0.2339	0.9201
LCMA	1022.66	0.0827	0.3892	1050.66	0.1174	0.8740
LCARMA	38.7730	0.0230	0.0305	68.7730	0.9785	0.9985

χ^2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI-Comparative Fit Index; LC-Latent Curve; AR-Auto regressive; MA-Moving Average

Table 60: Effect Size for LC Estimates with ARMA Process (AR 0.85 & MA 0.80)

Statistic	R^2	**Coefficient of Variation**	**Root MSE**	**Mean**
Mean Intercept	0.9660	3.2050	0.2428	7.5774
Mean Slope	0.9886	9.3068	0.4374	4.7001
Intercept Variance	0.9896	9.6069	23.5742	245.3872
Slope Variance	0.9941	12.5809	10.5614	83.9482
Intercept Slope Correlation	0.9864	21.3533	19.0051	89.0031

Statistic	R^2	Coefficient of Variation	Root MSE	Mean
Mean Intercept SE	0.9964	4.1624	0.0363	0.8722
Mean Slope SE	0.9957	5.5856	0.0278	0.4988
Intercept Variance SE	0.9914	8.7292	2.0674	23.6844
Slope Variance SE	0.9913	14.6282	1.2288	8.4008
Intercept Slope Correlation SE	0.9926	9.4167	1.2588	13.3683

SE-Standard Error; MSE-Mean Square Error

ANOVA Results with ARMA Process (AR 0.85 & MA 0.80)

Table 61: ANOVA Results for LC Estimates with ARMA Process (AR 0.85 & MA 0.80)

Dependent Variable⁺	Mean Intercept	Mean Slope	Intercept Variance	Slope Variance	Intercept Slope Correlation
Model_Type	70.08*	25883.86*	32138.25*	1056062*	16101166*
Wt	2547.26*	175.12*	8480441*	1010986*	11237911*
Model_type X Wt	50.94*	472.07*	19544.31*	9450988*	14410885*
Semi-Partial η^2	0.0184	0.176	0.003	0.3119	0.3405
Partial η^2	0.3517	0.6078	0.0216	0.9816	0.9616

⁺-Sum of Squares reported; *Significant at $p < .001$; η^2-Eta Square reported only for interaction effect between Model_type & weight; Wt-Weight

Table 62: ANOVA Results for SE Estimates with ARMA Process (AR 0.85 & MA 0.80)

Dependent Variable⁺	Mean Intercept SE	Mean Slope SE	Intercept Variance SE	Slope Variance SE	Intercept Slope Correlation SE
Model_Type	0.7783	73.32*	1663.70*	91167.8*	45588.27*
Wt	593.7997	156.75*	790229*	101523*	257848.61*
Model_type X Wt	0.4548	62.10*	1538.72*	81300.8*	38794.89*
Semi-Partial η^2	0.0008	0.2116	0.0019	0.2941	0.1125
Partial η^2	0.1781	0.9805	0.1844	0.9713	0.9389

⁺-Sum of Squares reported; *Significant at $p < .001$; η^2-Eta Square reported only for interaction effect between Model_type & weight; Wt-Weight; SE-Standard Error

Tukey Comparisons with ARMA Process (AR 0.85 & MA 0.80)

Tables 63 and 64 provides the Tukey comparisons between the means of LC estimates with ARMA process (AR 0.85 & MA 0.80) by model type and weight respectively. A different letter implies that means between any two models is different from each other to a statistically significant degree. The means of each model were not significantly different from each other for all the LC estimates except intercept slope correlation, mean intercept SE, and mean slope SE. Overall, the means of LC estimates for the LCARMA and LCAR model were lower than the LC and LCMA models. The means were significantly different between the models with weights and without weights in Table 64 with the former having higher means. There were striking differences between the means of intercept variance, slope variance, intercept slope correlation, intercept variance SE, slope variance SE, and intercept slope correlation SE for models with weights and without weights.

Table 63: Tukey Comparisons by Model Type with ARMA Process (AR 0.85 & MA 0.80)

Statistic	**Model**	**Mean**	**Tukey Grouping**
Mean Intercept	LCMA	7.8540	A
	LCAR	7.6012	B
	LCARMA	7.5900	B
	LC	7.2644	C
Mean Slope	LCMA	8.7225	A
	LC	8.7186	A
	LCARMA	0.8462	B
	LCAR	0.5134	C
Intercept Variance	LC	250.207	A
	LCAR	248.516	A
	LCARMA	244.217	B
	LCMA	238.608	C
	LC	167.9217	A

Statistic	Model	Mean	Tukey Grouping
Slope Variance	LCMA	162.4133	B
	LCARMA	2.9814	C
	LCAR	2.4766	C
Intercept Slope Correlation	LC	191.85	A
	LCMA	186.608	B
	LCARMA	-5.828	C
	LCAR	-16.617	D
Mean Intercept SE	LC	0.8923	A
	LCAR	0.8846	B
	LCARMA	0.8767	C
	LCMA	0.8352	D
Mean Slope SE	LC	0.7179	A
	LCMA	0.7078	B
	LCAR	0.2889	C
	LCARMA	0.2807	D
Intercept Variance SE	LCAR	24.6753	A
	LCARMA	24.1863	B
	LC	23.8898	B
	LCMA	21.9864	C
Slope Variance SE	LC	16.1877	A
	LCMA	15.7070	B
	LCAR	0.8873	C
	LCARMA	0.8211	C
Intercept Slope Correlation SE	LC	19.1009	A
	LCMA	18.2962	B
	LCAR	8.0611	C
	LCARMA	8.0151	C

SE-Standard Error; LC-Latent Curve, AR-Autoregressive, MA-Moving Average

Table 64: Tukey Comparisons by Weight for ARMA Process (AR 0.85 & MA 0.80)

Statistic	Model	Mean	Tukey Grouping
Mean Intercept	1	8.8392	A
	0	6.3156	B
Mean Slope	1	5.0310	A
	0	4.3693	B
Intercept Variance	1	475.610	A
	0	15.164	B
Slope Variance	1	163.4382	A

Statistic	Model	Mean	Tukey Grouping
	0	4.4582	B
Intercept Slope Correlation	1	172.8106	A
	0	5.1956	B
Mean Intercept SE	1	1.4814	A
	0	0.2630	B
Mean Slope SE	1	0.8118	A
	0	0.1858	B
Intercept Variance SE	1	45.9081	A
	0	1.4607	B
Slope Variance SE	1	16.3665	A
	0	0.4351	B
Intercept Slope Correlation SE	1	26.0630	A
	1	0.6736	B

0-With Weight; 1-Without Weight; SE-Standard Error

Tables 65 and 67 provides the means for LC estimates with ARMA process (AR 0.85 & MA 0.80) when both model and weights were together accounted. It can be seen that the means of the latent curve estimates and associated SE for the models are higher for the estimates having weights than those not having weights. This suggests an upward bias in the LC estimates when the autocorrelative ARMA process (AR 0.85 & MA 0.80) is modeled with weights. The bias is more pronounced for the LCMA and LC models. The only latent curve sample estimate for the models with weights that were closer to population estimate was mean intercept (8.7685 versus 8.7281 respectively). The differences between the SE of the LCMA and LC models with weighs and without weights was also more than the differences between the LCARMA and LCAR models. Larger differences were observed in the intercept variance estimate & SE, slope variance, and intercept slope correlation & SE between models with weights and without weights. The CI for the mean intercept and mean slope are narrower and tighter indicating more precision in both the estimate when the interaction between weights and autocorrelation is accounted than the other three estimates (intercept variance, slope variance, and intercept slope correlation). The CI for mean intercept and mean slope were also different from each other to a statistically significant degree when model with weights and without weights were compared.

Table 65: Means for LC Estimates with ARMA Process
by Model_Type*Weights (AR 0.85 & MA 0.80)

Model	Weight	Mean Intercept	Mean Slope	Intercept Variance	Slope Variance	Intercept Slope Correlation
LCARMA	0	6.4115	1.0272	13.7057	0.0502	0.0312
	1	8.7685	0.6652	474.7282	5.9124	-11.6879
LCAR	0	6.4327	0.7557	13.7725	0.0933	-0.4520
	1	8.7696	0.2707	483.2601	4.8599	-32.7827
LCMA	0	6.7231	7.8300	13.6860	8.7554	10.3038
	1	8.9848	9.6149	463.5306	316.0710	362.9114
LC	0	5.6951	7.8642	19.4913	8.9338	10.8993
	1	8.8336	9.5730	480.9229	326.9094	372.8015

0-With Weight; 1-Without Weight; LC-Latent Curve, AR-Autoregressive, MA-Moving Average

Table 66: Means for SE Estimates with ARMA Process
by Model_Type*Weights (AR 0.85 & MA 0.80)

Model	Weight	Mean Intercept SE	Mean Slope SE	Intercept Variance SE	Slope Variance SE	Intercept Slope Correlation SE
LCARMA	0	0.2535	0.1710	1.3411	0.0143	0.2720
	1	1.4999	0.3904	47.0314	1.6280	15.7581
	0 (C.I)	6.9084 5.9146	1.3624 0.6920	16.3343 11.0771	0.0782 0.0222	0.5643 -0.5019
	1 (C.I)	11.7083 5.8287	1.4304 -0.1000	566.9097 382.5467	9.1033 2.7215	19.1980 -42.5738
LCAR	0	0.2560	0.1664	1.3577	0.0155	0.2521
	1	1.5133	0.4114	47.9928	1.7591	15.8700
	0 (C.I)	6.9345 5.9309	1.0818 0.4296	16.4336 11.1114	0.1237 0.0629	0.0421 -0.9461
	1 (C.I)	11.7357 5.8035	1.0770 -0.5356	577.3260 389.1942	8.3077 1.4121	-1.6775 -63.8879
	0	0.2429	0.2022	1.2499	0.8484	1.0076
	1	1.4276	1.2135	42.7227	30.5656	35.5847

Model	Weight	Mean Intercept SE	Mean Slope SE	Intercept Variance SE	Slope Variance SE	Intercept Slope Correlation SE
LCMA	0 (C.I)	7.1992 6.2470	8.2263 7.4337	16.1358 11.2362	10.4183 7.0925	12.2787 8.3289
	1 (C.I)	11.7829 6.1867	11.9934 7.2364	547.2671 379.7941	375.9796 256.1624	432.6574 293.1654
LC	0	0.2998	0.2037	1.8940	0.8621	1.1627
	1	1.4849	1.2321	45.8854	31.5132	37.0391
	0 (C.I)	6.2827 5.1075	8.2635 7.4649	23.2035 15.7791	10.6235 7.2441	13.1782 8.6204
	1 (C.I)	11.7440 5.9232	11.9879 7.1581	570.8583 390.9875	388.6753 265.1435	445.3981 300.2049

0-With Weight; 1-Without Weight; SE-Standard Error; C.I.-Confidence Interval; LC-Latent Curve, AR-Autoregressive; MA-Moving Average;

Interaction Plots with ARMA Process*Weight (AR 0.85 & MA 0.80)

Figures 49 through 58 presents the interaction plots between model type and weights for each of the LC estimates and associated SE for models that were fitted to the LCARMA data. An examination of how the four models (LCARMA, LCAR, LCMA, and Latent Curve models fitted the LCARMA (0.85, 0.80) data reveals that the Mean Intercept (population value of 8.7281) (Figure 49) were consistently underestimated for all four models when sample weights were not incorporated. This finding suggest that ignoring the AR component of the ARMA process downwardly biases the mean intercept growth parameter. Figure 50 shows that the mean slope (population value of 1.0494) were underestimated for all the models with weights but the difference between the mean slopes was less between the LCARMA and LCAR model with weights and no weights.

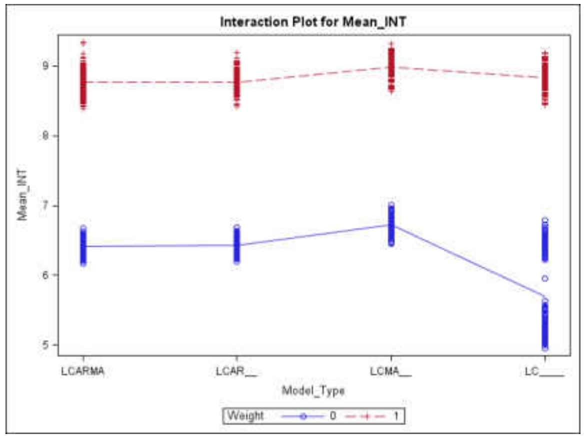

Figure 49: Interaction Plot of Mean Intercept for ARMA (AR 0.85 & MA 0.80)

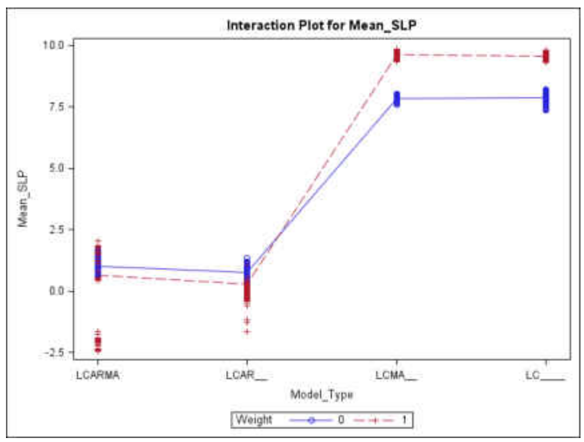

Figure 50: Interaction Plot of Mean Slope for ARMA (AR 0.85 & MA 0.80)

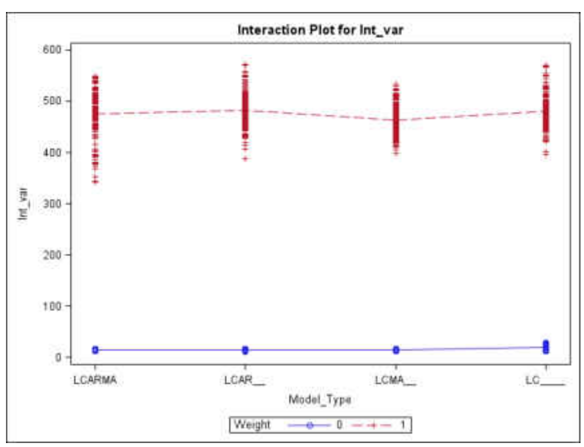

Figure 51: Interaction Plot of Intercept Variance for ARMA (AR 0.85 & MA 0.80)

Figure 51 for intercept variance (population value of 13.8082) for models with weights were overestimated for all the models. The means for intercept variance for models without weights were constant across all the models and were closer to the population latent curve estimate. The plot for slope variance (population value of 1.0446) in Figure 52 shows the means were overestimated between all the models having weights and no weights. However, the differences in the means between the LCARMA and LCAR models with and without weights were minimal. Figure 53 again shows the intercept slope correlation being overestimated for the LCMA and LC models and underestimated for LCAR models with weights. There was not much difference in the estimate within the LCARMA models with weights and no weights. Similar trends were seen as in Figure 49 for mean intercept SE (population value of 0.0074) (Figure 54), mean slope SE (population value of 0.0081) (Figure 55), intercept variance SE (population value of 0.03949) (Figure 56), slope variance SE (population value of 0.004) (Figure 57), and intercept slope correlation SE (population value of 0.0127) (Figure 58). The means for all the estimates were almost constant for all the models without weights. Overall, the highest latent curve estimates within the models with weights were for the LCMA and LC models. The overestimation in latent curve estimates for the LCARMA and LCAR models with weights was less as compared to the LCMA and LC models implying that the sample latent curve estimates for these models were closer to the population latent curve estimates. The differences in the estimates between the LCARMA and LCAR models with and without weights were less for the

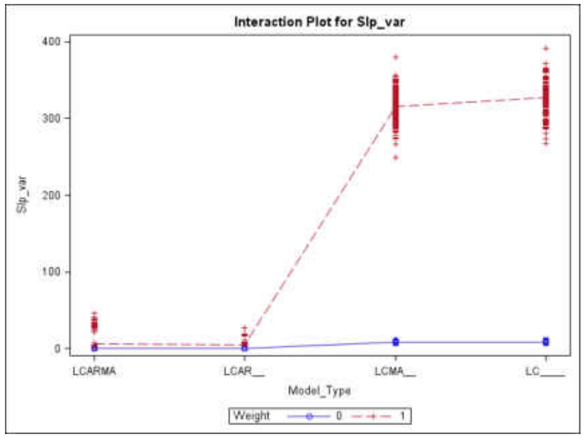

Figure 52: Interaction Plot of Slope Variance for ARMA (AR 0.85 & MA 0.80)

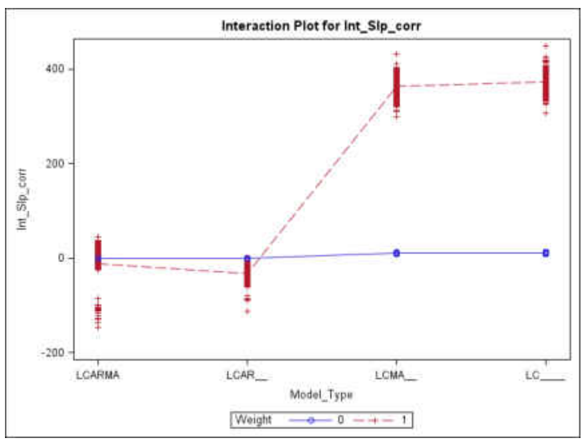

Figure 53: Interaction Plot of Intercept Slope Correlation for ARMA (AR 0.85 & MA 0.80)

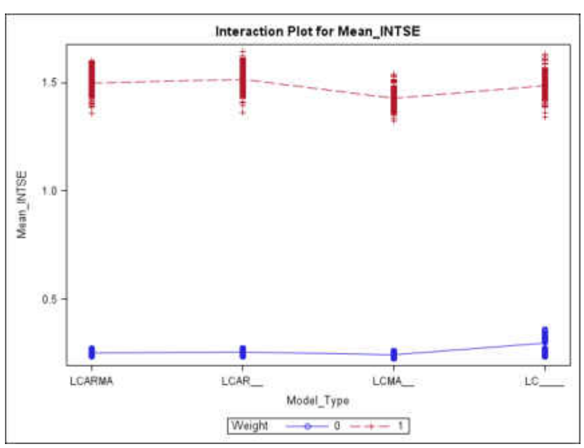

Figure 54: Interaction Plot of Mean Intercept SE for ARMA (AR 0.85 & MA 0.80)

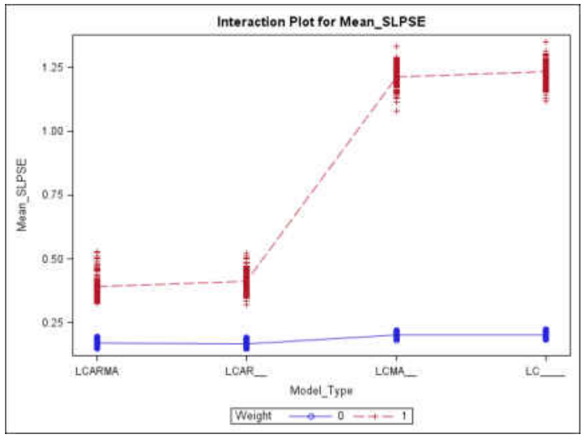

Figure 55: Interaction Plot of Mean Slope SE for ARMA (AR 0.85 & MA 0.80)

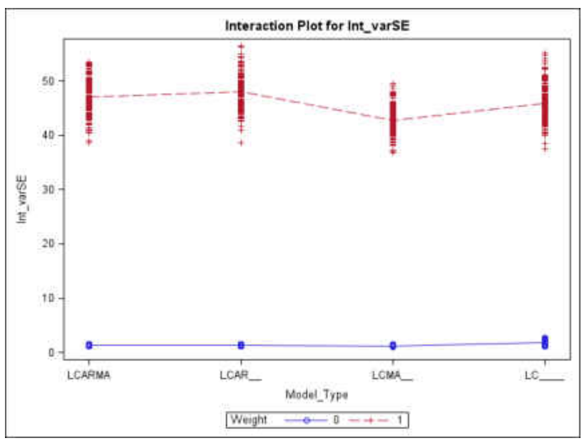

Figure 56: Interaction Plot of Intercept Variance SE for ARMA (AR 0.85 & MA 0.80)

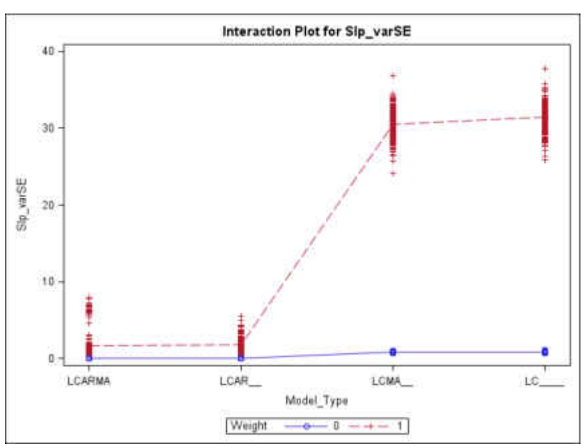

Figure 57: Interaction Plot of Slope Variance SE for ARMA (AR 0.85 & MA 0.80)

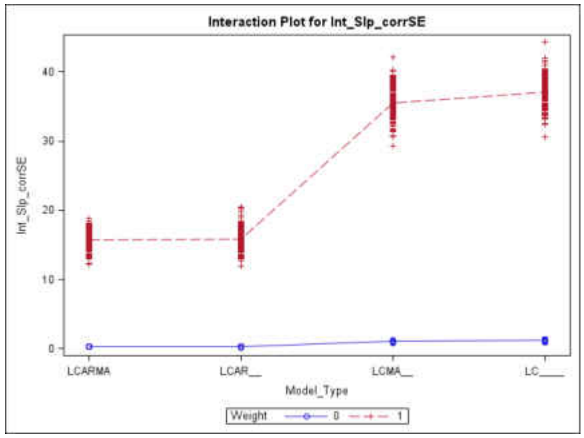

Figure 58: Interaction Plot of Intercept Slope Correlation SE for ARMA (AR 0.85 & MA 0.80)

mean slope, intercept slope correlation, mean slope SE, and slope variance SE with minimal differences in slope variance.

General Linear Model Results with AR Process (MA 0.00)

The results in this section focuses on the data having AR process only where the MA process was set to 0.00. The fit indices, effect sizes, ANOVA results, means, tukey results and plots for LC estimates of LCARMA model were compared with the other models because it was the best fitting model. Table 67 provides the fit indices for the ARMA process with the LCARMA model having a much better fit (χ^2-36.4579, SRMR-0.0082, RMSEA-0.0301, AIC-66.4579, Mc-0.9831, CFI-0.9985) than the other models. Table 68 provides a summary of the effect size of LC estimates with AR process (MA 0.00). The RMSE for all the estimates were low except for intercept variance, slope variance, and intercept slope correlation suggesting a good model fit and better predictability of the estimate by accounting for model type and weight as predictors. Hence, 61.1%, 70.3%, 97.3%, and 66.2%, of the variation in mean intercept, mean slope, mean intercept SE, and mean slope SE respectively is explained by model type and weight.

Table 67: Overall Fit Indices with AR Process (MA 0.00)

Model	χ^2	SRMR	RMSEA	AIC	Mc	CFI
LC	1268.78	0.2518	0.4293	1294.78	0.0640	0.8202
LCAR	44.9584	0.0145	0.0388	72.9584	0.9684	0.9976
LCMA	909.6643	0.2080	0.3673	937.6643	0.1418	0.8718
LCARMA	36.4579	0.0082	0.0301	66.4579	0.9831	0.9987

χ^2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI-Comparative Fit Index; LC-Latent Curve; AR-Auto regressive; MA-Moving Average

Table 68: Effect Size for LC Estimates with AR Process (MA 0.00)

Statistic	R^2	Coefficient of Variation	Root MSE	Mean
Mean Intercept	0.6144	14.2267	1.1335	7.9818
Mean Slope	0.7038	48.7329	1.7243	3.5383
Intercept Variance	0.9306	28.5381	100.5248	352.2472
Slope Variance	0.5540	147.1632	56.3161	38.2678
Intercept Slope Correlation	0.7548	97.1534	65.5336	67.4537
Mean Intercept SE	0.9738	12.0531	0.1251	1.0383
Mean Slope SE	0.6621	61.8339	0.1875	0.3033
Intercept Variance SE	0.9315	28.1717	9.8597	34.9985
Slope Variance SE	0.5552	143.2268	5.4273	3.7893
Intercept Slope Correlation SE	0.7500	71.3048	6.5646	9.2064

SE-Standard Error; MSE-Mean Square Error

ANOVA Results with AR Process (MA 0.00)

Tables 69 and 70 summarizes the ANOVA results with AR process (MA 0.00). All the models were significant at $p<.001$ including the interaction between model type and weight. The semi-partial η2 values for the slope variance (0.1745), intercept slope correlation (0.2468), mean slope SE (0.1534), slope variance SE (0.1702), and intercept slope correlation SE (0.1441) were high as compared to the η^2 values for other estimates and thereby greater variation in these LC estimates was explained by model type and weight.

Table 69: ANOVA Results for LC Estimates with AR Process (MA 0.00)

Dependent Variable⁺	Mean Intercept	Mean Slope	Intercept Variance	Slope Variance	Intercept Slope Correlation
Model_Type	995.693*	33166.153*	63748894*	6650776*	23132464*
Wt	8618.668*	285.580*	52959857*	6287058*	19524648*
Model_type X Wt	233.526*	419.036*	56365801*	5947020*	20721704*
Semi-Partial η^2	0.0146	0.0087	0.0807	0.1745	0.2468
Partial η^2	0.0364	0.0286	0.5379	0.2813	0.5017

⁺-Sum of Squares reported; *Significant at $p < .001$; η^2-Eta Square reported only for interaction effect between Model_type & weight; Wt-Weight

Table 70: ANOVA Results for SE Estimates with AR Process (MA 0.00)

Dependent Variable⁺	Mean Intercept SE	Mean Slope SE	Intercept Variance SE	Slope Variance SE	Intercept Slope Correlation SE
Model_Type	142.2280	87.0278	588328*	60480.9*	134815.7*
Wt	2587.7559	166.7738	5228488*	61702.6*	365723.6*
Model_type X Wt	69.5390	76.5378	519255*	54015*	118997.8*
Semi-Partial η^2	0.0242	0.1534	0.0763	0.1702	0.1441
Partial η^2	0.4809	0.3122	0.5271	0.2768	0.3656

⁺-Sum of Squares reported; *Significant at $p < .001$; η^2-Eta Square reported only for interaction effect between Model_type & weight; Wt-Weight; SE-Standard Error

Tukey Comparisons with AR Process (MA 0.00)

Tables 71 and 72 provides the Tukey comparisons between the means of LC estimates with AR process (MA 0.00) by model type and weight respectively. A different letter implies that means between any two models is different from each other to a statistically significant degree. The means of each model were not significantly different from each other for all the LC estimates except mean intercept SE. The means of LC estimates for the LCARMA and LCAR model were lower than the LC and LCMA models. The means were significantly different between the models with weight and without weights in Table 72 with the former having higher means.

Table 71: Tukey Comparisons by Model Type with AR Process (MA 0.00)

Statistic	**Model**	**Mean**	**Tukey Grouping**
Mean Intercept	LC	8.5493	A
	LCMA	8.3073	B
	LCARMA	7.5707	C
	LCAR	7.4996	C
Mean Slope	LCMA	6.1935	A
	LC	6.1401	A
	LCARMA	0.9424	B
	LCAR	0.8775	B
Intercept Variance	LC	481.081	A
	LCMA	452.968	B
	LCARMA	241.556	C
	LCAR	233.384	C
Slope Variance	LC	77.447	A
	LCMA	73.482	A
	LCAR	1.106	B
	LCARMA	1.036	B
Intercept Slope Correlation	LCMA	137.599	A
	LC	136.153	A
	LCARMA	-1.332	B
	LCAR	-2.605	B
Mean Intercept SE	LC	1.2213	A
	LCMA	1.1991	B
	LCARMA	0.8749	C
	LCAR	0.8580	D
Mean Slope SE	LC	0.4426	A
	LCMA	0.4332	A
	LCARMA	0.1706	B
	LCAR	0.1668	B
	LC	46.9115	A

Statistic	Model	Mean	Tukey Grouping
Intercept Variance SE	LCMA	45.1866	B
	LCARMA	24.3563	C
	LCAR	23.5398	C
Slope Variance SE	LC	7.5003	A
	LCMA	7.1740	A
	LCARMA	0.2444	B
	LCAR	0.2386	B
Intercept Slope Correlation SE	LC	14.6443	A
	LCMA	14.3657	A
	LCARMA	3.9862	B
	LCAR	3.8299	B

SE-Standard Error; LC-Latent Curve, AR-Autoregressive, MA-Moving Average

Table 72: Tukey Comparisons by Weight with AR Process (MA 0.00)

Statistic	Model	Mean	Tukey Grouping
Mean Intercept	1	9.3218	A
	0	6.6418	B
Mean Slope	1	3.7823	A
	0	3.2944	B
Intercept Variance	1	684.411	A
	0	20.083	B
Slope Variance	1	74.459	A
	0	2.077	B
Intercept Slope Correlation	1	131.232	A
	0	3.676	B
Mean Intercept SE	1	1.7726	A
	0	0.3041	B
Mean Slope SE	1	0.4897	A
	0	01169	B
Intercept Variance SE	1	68.0026	A
	0	1.9945	B
Slope Variance SE	1	7.3747	A
	0	0.2040	B
Intercept Slope Correlation SE	1	13.9353	A
	0	0.4777	B

0-With Weight; 1-Without Weight; SE-Standard Error

Tables 73 and 74 provides the means for LC estimates with AR process (MA 0.00) when both model and weights were together accounted. It can be seen that the means for the models are lower for the estimates without weights than those with weights. This suggests an under estimation in the LC estimates when the autocorrelative AR process (MA 0.00) is present along with the incorporation of weights. The only latent curve sample estimates for the models with weights that were closer to population estimate was mean intercept (8.7267, 8.7226, and 8.7393 for 0.33, 0.67, and 0.85 AR values respectively) and mean slope (0.9984, 0.9693, and 0.8804 for 0.33, 0.67, and 0.85 AR values respectively). The differences between the SE of the LCMA and LC models with weighs and without weights was also more than the differences between the LCARMA and LCAR models. Larger differences were observed in the intercept variance estimate & SE between models with weights and without weights. The CI for the mean intercept and mean slope are narrower and tighter indicating more precision in both the estimate when the interaction between weights and autocorrelation is accounted than the other three estimates (intercept variance, slope variance, and intercept slope correlation). The CI for mean intercept and mean slope were also different from each other to a statistically significant degree when model with weights and without weights were compared. The CI for the mean intercept and mean slope are narrower and tighter indicating more precision in both the estimate when the interaction between weights and autocorrelation is accounted than the other three estimates (intercept variance, slope variance, and intercept slope correlation). The CI for mean intercept and mean slope were also different from each other to a statistically significant degree when model with weights and without weights were compared.

Table 73: Means for LC Estimates with AR Process by Model_Type*Weights (MA 0.00)

Model	Weight	Mean Intercept	Mean Slope	Intercept Variance	Slope Variance	Intercept Slope Correlation
LCARMA	0	6.4099	0.9702	13.2647	0.0539	-0.0291
	1	8.7314	0.9144	469.8465	2.0172	-2.6340
LCAR	0	6.4073	0.9509	13.2505	0.0556	-0.0605
	1	8.5918	0.8040	453.5169	2.1566	-5.1495
LCMA	0	6.6836	5.6768	25.5715	3.9448	7.5826
	1	9.9316	6.7100	880.3651	143.0194	267.6148
LC	0	7.0662	5.5796	28.2453	4.2519	7.2100
	1	10.0322	6.7006	933.9169	150.6426	265.0957

0-With Weight; 1-Without Weight; LC-Latent Curve, AR-Autoregressive, MA-Moving Average

Table 74: Means for SE Estimates with AR Process by Model_Type*Weights (MA 0.00)

Model	Weight	Mean Intercept SE	Mean Slope SE	Intercept Variance SE	Slope Variance SE	Intercept Slope Correlation SE
LCARMA	0	0.2526	0.1102	1.3287	0.0092	0.1593
	1	1.4972	0.2310	47.3839	0.4795	7.8129
	0 (C.I)	6.9050 5.9148	1.1862 0.7542	15.8690 10.6604	0.0719 0.0359	0.2831 -0.3413
	1 (C.I)	11.6659 5.7969	1.3672 0.4616	562.7189 376.9741	2.9570 1.0774	12.6793 -17.9473
LCAR	0	0.2521	0.1069	1.3241	0.0088	0.1545
	1	1.4639	0.2267	45.7554	0.4684	7.5051
	0 (C.I)	6.9014 5.9132	1.1604 0.7414	15.8457 10.6553	0.0728 0.0384	0.2423 -0.3633
	1 (C.I)	11.4610 5.7226	1.2483 0.3597	543.1975 363.8363	3.0747 1.2385	9.5605 -19.8595
LCMA	0	0.3513	0.1237	2.5748	0.3865	0.7988
	1	2.0468	0.7427	87.7983	13.9613	27.9324
	0 (C.I)	7.3721 5.9951	5.9193 5.4343	30.6181 20.5249	4.7023 3.1873	9.1482 6.0170
	1 (C.I)	13.9433 5.9199	8.1657 5.2543	1052.449 708.280	170.3835 115.6553	322.3623 212.8673
LC	0	0.3603	0.1267	2.7501	0.4112	0.7978
	1	2.0823	0.7584	91.0728	14.5892	28.4907
	0 (C.I)	7.7724 6.3600	5.8279 5.3313	33.6355 22.8551	5.0579 3.4459	8.7737 5.6463

Model	Weight	Mean Intercept SE	Mean Slope SE	Intercept Variance SE	Slope Variance SE	Intercept Slope Correlation SE
	1 (C.I)	14.1135 5.9509	8.1871 5.2141	1112.419 755.4142	179.2374 122.0478	320.9375 209.2539

0-With Weight; 1-Without Weight; LC-Latent Curve, AR-Autoregressive, MA-Moving Average; C.I.-Confidence Interval; SE-Standard Error

Interaction Plots with AR Process*Weight (MA 0.00)

Figures 59 through 68 presents the interaction plots between model type and weights for each of the LC estimates and associated SE for models that were fitted to the LCAR data (MA 0.00). An examination of how the four models (LCARMA, LCAR, LCMA, and Latent Curve models fitted the LCAR data reveals that the Mean Intercept (population values of 8.7267, 8.7226, and 8.7393 for AR processes of 0.33, 0.67, and 0.85 respectively) were consistently underestimated for all four models when sample weights were not incorporated (Figure 59). This finding suggest that ignoring the AR component of the ARMA process downwardly biases the mean intercept growth parameter. Figure 60 shows that the mean slope (population values of 0.9984, 0.9693, and 0.8804 for AR processes of 0.33, 0.67, and 0.85 respectively) was slightly overestimated for LCMA and LC models with weights but the difference between the mean slopes was minimal for the LCARMA and LCMA models with weights and no weights. Intercept variance (population value of 13.3985, 13.3549, and 13.3197 for 0.33, 0.67, and 0.85 AR values respectively) for models with weights were overestimated for all the models (Figure 61). However, the overestimation was less for the LCARMA and LCAR models with weights. The means for intercept variance for models without weights were constant across all the models and were closer to the population latent curve estimate respectively). The means being overestimated between all the models having weights and no weights. Figure 62 shows the slope variance (population value of 0.0527, 0.0518, and 0.0504 for 0.33, 0.67, and 0.85 AR values respectively) being overestimated between the LCMA and LC models having weights. However, the differences in the means between the LCARMA and LCAR models with and without weights were minimal.

The means for slope variance for models without weights were constant across all the models. Figure 63 again shows the intercept slope correlation being overestimated for the LCMA and LC models with almost no difference between the LCARMA and LCAR models with and without weights. The means were constant within all the models with no weights for all the latent curve estimates except mean slope. Similar trends were seen as in Figure 59 for mean intercept SE (population value of 0.0074) (Figure 64), mean slope SE (population value of 0.0012, 0.0037, and 0.0073 for 0.33, 0.67, and 0.85 AR values respectively) (Figure 65), intercept variance SE (population value of 0.0389, 0.0391, and 0.0389 for 0.33, 0.67, and 0.85 AR values respectively) (Figure 66), slope variance SE (population value of 0.0002, 0.0024, and 0.0002 for 0.33, 0.67, and 0.85 AR values respectively) (Figure 67), and intercept slope correlation SE (population value of 0.0026, 0.0052, and 0.0101 for 0.33, 0.67, and 0.85 AR values respectively) (Figure 68).

Overall, the highest latent curve estimates within the models with weights were for the LCMA and LC models. The overestimation in latent curve estimates for the LCARMA and LCAR models with weights was less as compared to the LCMA and LC models with weights implying that the sample latent curve estimates for the models with weights were closer to the population latent curve estimates. The differences in the estimates between the LCARMA and

LCAR models with and without weights were approximately similar for all the estimates except mean intercept, intercept variance, mean intercept SE, intercept variance SE, and intercept slope correlation SE.

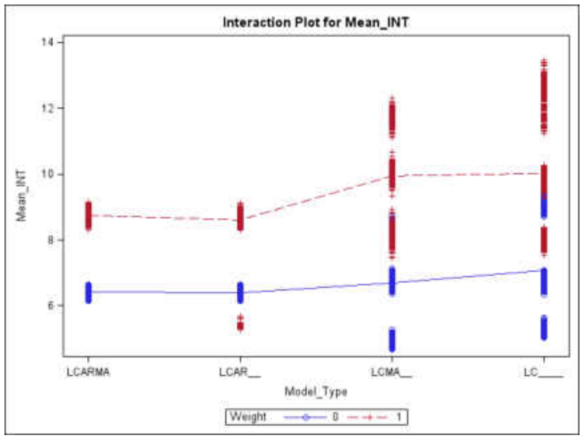

Figure 59: Interaction Plot of Mean Intercept for AR (MA 0.00)

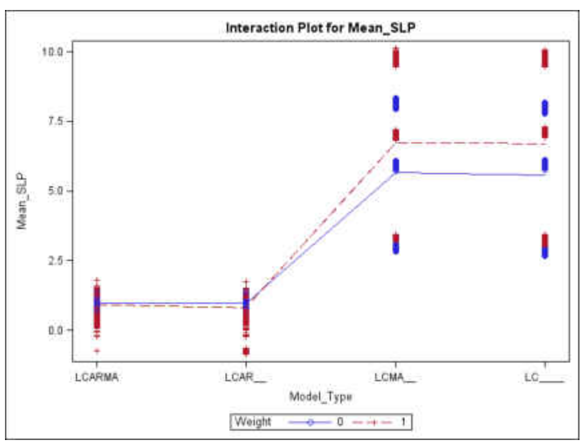

Figure 60: Interaction Plot of Mean Slope for AR (MA 0.00)

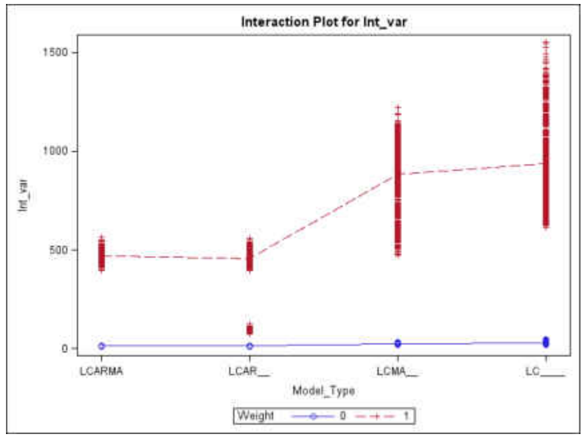

Figure 61: Interaction Plot of Intercept Variance for AR (MA 0.00)

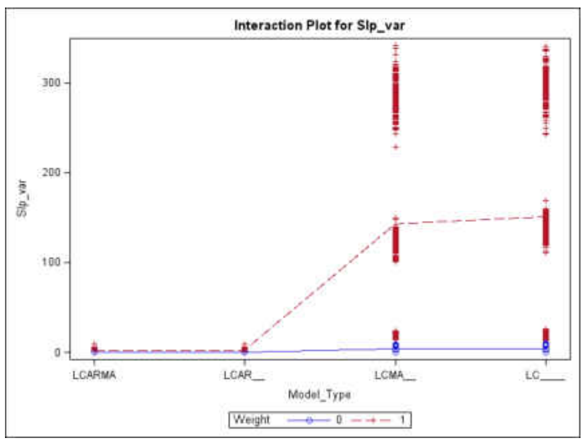

Figure 62: Interaction Plot of Slope Variance for AR (MA 0.00)

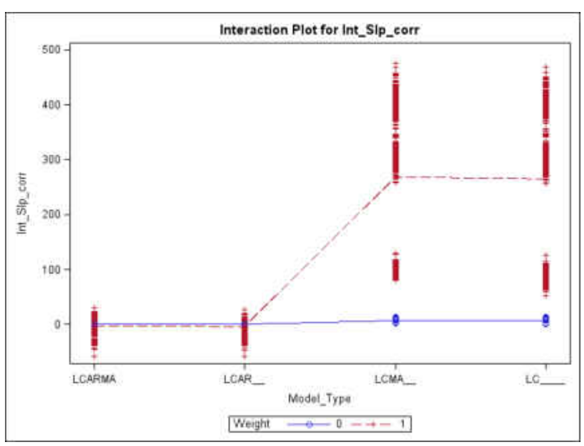

Figure 63: Interaction Plot for Intercept Slope Correlation for AR (MA 0.00)

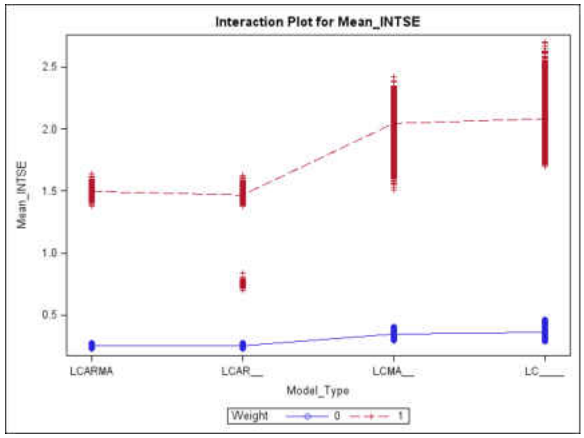

Figure 64: Interaction Plot for Mean Intercept SE for AR (MA 0.00)

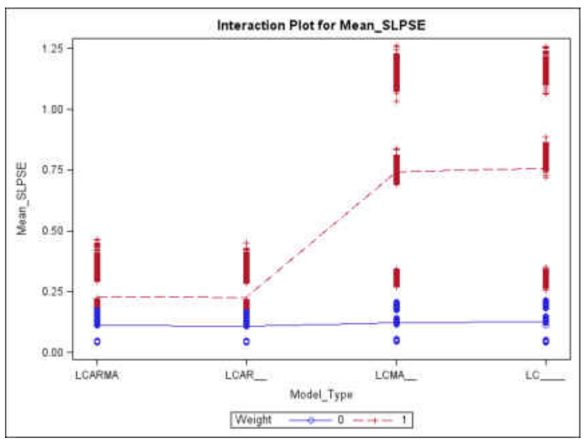

Figure 65: Interaction Plot for Mean Slope SE for AR (MA 0.00)

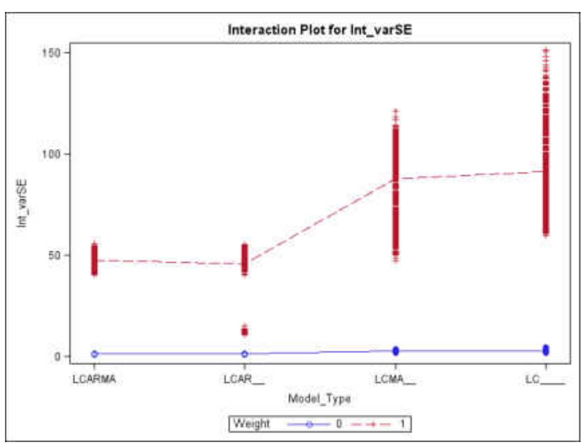

Figure 66: Interaction Plot for Intercept Variance SE for AR (MA 0.00)

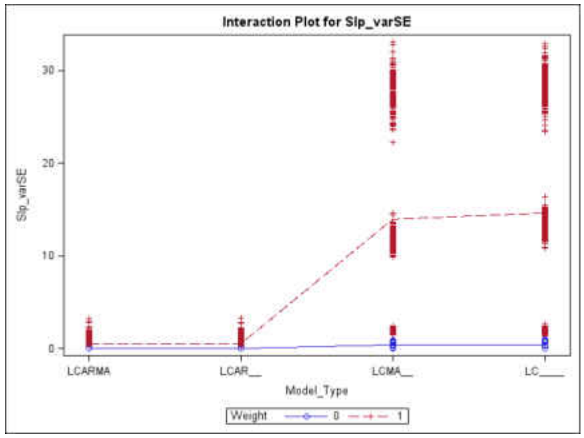

Figure 67: Interaction Plot for Slope Variance SE for AR (MA 0.00)

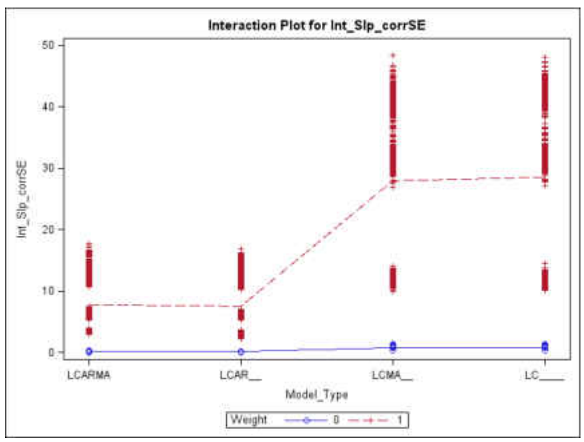

Figure 68: Interaction Plot for Intercept Slope Correlation SE for AR (MA 0.00)

General Linear Model Results with MA Process (AR 0.00)

The results in this section focuses on the data having MA process only where the AR process was set to 0.00. The fit indices, effect sizes, ANOVA results, means, tukey results and plots for LC estimates of LCMA model were compared with the other models because it was the best fitting model. Table 75 provides the fit indices for the MA process with the LCMA model having a much better fit (χ^2-37.1044, SRMR-0.0271, RMSEA-0.0284, AIC-65.1044, Mc-0.9839, CFI-0.9979) than the other models. A closer examination of the LCARMA model fit indices (χ2-

94.7012, SRMR-0.0992, RMSEA-0.0826, AIC-124.7012, Mc-0.8687, CFI-0.9834) revealed that it fitted the data fairly well with the RMSEA and Mc values slightly away from the cut-off values. Table 76 provides a summary of the effect size of LC estimates with MA process (AR 0.00). The RMSE for all the estimates were low except for intercept variance suggesting a good model fit and better predictability of the estimate by accounting for model type and weight as predictors. Hence, 99.8%, 99.7%, 95.8%, 80.9%, 99.8%, 98.6%, 99.5%, 97.4%, and 99.0%, of the variation in mean intercept, mean slope, slope variance, intercept slope correlation, mean intercept SE, mean slope SE, intercept variance SE, slope variance SE, and intercept slope correlation SE respectively is explained by model type and weight.

Table 75: Overall Fit Indices with MA Process (AR 0.00)

Model	χ^2	SRMR	RMSEA	AIC	Mc	CFI
LC	576.5773	0.0263	0.2844	602.5772	0.2931	0.8633
LCAR	401.4493	0.0973	0.2330	429.4493	0.4561	0.9067
LCMA	37.1044	0.0271	0.0284	65.1044	0.9839	0.9979
LCARMA	94.7012	0.09918	0.0826	124.7012	0.8687	0.9834

χ2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald’s Non-Centrality Index; CFI-Comparative Fit Index; LC-Latent Curve; AR-Auto regressive; MA-Moving Average

Table 76: Effect Size for LC Estimates with MA Process (AR 0.00)

Statistic	R^2	Coefficient of Variation	Root MSE	Mean
Mean Intercept	0.9984	1.8289	0.1045	5.7163
Mean Slope	0.9973	3.4771	0.0246	0.7078
Intercept Variance	0.9946	13.1739	15.6883	119.0861
Slope Variance	0.9579	173.6869	0.3045	0.1753
Intercept Slope Correlation	0.8092	134.2498	1.7190	1.2804
Mean Intercept SE	0.9980	4.2420	0.0242	0.5707
Mean Slope SE	0.9862	7.8343	0.0045	0.0585
Intercept Variance SE	0.9948	10.5343	1.4233	13.5118

Statistic	R^2	Coefficient of Variation	Root MSE	Mean
Slope Variance SE	0.9743	16.3997	0.0212	0.1294
Intercept Slope Correlation SE	0.9905	11.8048	0.1046	0.8865

SE-Standard Error; MSE-Mean Square Error

ANOVA Results with MA Process

Tables 77 and 78 summarizes the ANOVA results with MA process (AR 0.00). All the models were significant at $p<.001$ including the interaction between model type and weight. The semi-partial η2 values for the mean intercept (0.4576), mean slope (0.3160), intercept variance (0.3740), slope variance (0.4725), intercept slope correlation (0.3429), mean intercept SE (0.3284), intercept variance SE (0.3092), and intercept slope correlation SE (0.2104) were high as compared to the η^2 values for mean slope SE and slope variance SE and thereby greater variation in these LC estimates was explained by model type and weight.

Table 77: ANOVA Results for LC Estimates with MA Process (AR 0.00)

Dependent Variable+	Mean Intercept	Mean Slope	Intercept Variance	Slope Variance	Intercept Slope Correlation
Model_Type	16397.04*	407.04*	82044593*	5065.39*	25972.94*
Wt	1975.47*	345.47*	53597921*	62.12*	8634.55*
Model_type X Wt	15542.30*	349.05*	81745108*	4990.80*	25451.83*
Semi-Partial η^2	0.4576	0.3160	0.3740	0.4725	0.3429
Partial η^2	0.9966	0.9918	0.9858	0.9182	0.6425

+-Sum of Squares reported; *Significant at $p < .001$; η^2-Eta Square reported only for interaction effect between Model_type & weight; Wt-Weight

Table 78: ANOVA Results for SE Estimates with MA Process (AR 0.00)

Dependent Variable⁺	Mean Intercept SE	Mean Slope SE	Intercept Variance SE	Slope Variance SE	Intercept Slope Correlation SE
Model_Type	475.243*	0.5005*	589353*	6.108*	1163.274*
Wt	484.840*	5.851*	712671*	69.809*	3150.533*
Model_type X Wt	470.822*	0.878*	587217*	6.101*	1163.429*
Semi-Partial η^2	0.3284	0.1198	0.3092	0.0725	0.2104
Partial η^2	0.9941	0.8970	0.9837	0.7387	0.9568

⁺-Sum of Squares reported; *Significant at $p < .001$; η^2-Eta Square reported only for interaction effect between Model_type & weight; Wt-Weight; SE-Standard Error

Tukey Comparisons with MA Process

Tables 79 and 80 provides the Tukey comparisons between the means of LC estimates with MA process (AR 0.00) by model type and weight respectively. A different letter implies that means between any two models is different from each other to a statistically significant degree. The means of each model were significantly different from each other for all the LC estimates except mean intercept, mean slope and mean intercept SE. The means of LC estimates for the LCARMA and LCAR model were lower than the LC and LCMA models. The means were significantly different between the models with weight and without weights in Table 80 with the former having higher means.

Table 79: Tukey Comparisons by Model Type with MA Process (AR 0.00)

Statistic	Model	Mean	Tukey Grouping
Mean Intercept	LC	7.5649	A
	LCMA	7.5632	A
	LCARMA	3.9293	B
	LCAR	3.8078	C
Mean Slope	LCMA	0.9989	A
	LC	0.9985	A
	LCARMA	0.4355	B

Statistic	Model	Mean	Tukey Grouping
	LCAR	0.3982	C
Intercept Variance	LCMA	251.9303	A
	LCMA	247.4606	B
	LCARMA	-3.6077	C
	LCAR	-19.4387	D
Slope Variance	LC	1.4708	A
	LCMA	0.7930	B
	LCAR	-0.3744	C
	LCARMA	-1.1881	D
Intercept Slope Correlation	LCARMA	4.8181	A
	LCAR	1.6643	B
	LCMA	0.1239	C
	LC	-1.4844	D
Mean Intercept SE	LC	0.8891	A
	LCMA	0.8817	B
	LCARMA	0.2561	C
	LCAR	0.2560	C
Mean Slope SE	LC	0.0740	A
	LCMA	0.0614	B
	LCARMA	0.0498	C
	LCAR	0.0489	D
Intercept Variance SE	LC	24.7453	A
	LCMA	24.4258	B
	LCARMA	2.9686	C
	LCAR	1.9074	D
Slope Variance SE	LC	0.1735	A
	LCMA	0.1435	B
	LCARMA	0.1253	C
	LCAR	0.0753	D
Intercept Slope Correlation SE	LC	1.4712	A
	LCMA	1.2665	B
	LCARMA	0.4988	C
	LCAR	0.3096	D

SE-Standard Error; LC-Latent Curve, AR-Autoregressive, MA-Moving Average

Table 80: Tukey Comparisons by Weight with MA Process (AR 0.00)

Statistic	Model	Mean	Tukey Grouping
Mean Intercept	1	6.3578	A

Statistic	Model	Mean	Tukey Grouping
	0	5.0748	B
Mean Slope	1	0.9761	A
	0	0.4395	B
Intercept Variance	1	224.7565	A
	0	13.4157	B
Slope Variance	1	0.2891	A
	0	0.0616	B
Intercept Slope Correlation	1	2.6217	A
	0	-0.0607	B
Mean Intercept SE	1	0.8885	A
	0	0.2529	B
Mean Slope SE	1	0.0934	A
	0	0.0236	B
Intercept Variance SE	1	25.6968	A
	0	1.3268	B
Slope Variance SE	1	0.2500	A
	0	0.0083	B
Intercept Slope Correlation SE	1	1.6967	A
	0	0.0764	B

0-With Weight; 1-Without Weight; SE-Standard Error

Tables 81 and 82 provides the means for LC estimates with MA process (AR 0.00) when both model and weights were together accounted. It can be seen that the means for the LCARMA and LCAR models in Table 81 were lower for the estimates with weights (except for intercept slope correlation) than those with weights. This suggests an overestimation in the LC estimates when the autocorrelative MA process (AR 0.00) is present but weights are absent. This trend was not observed in Table 82. The values for latent curve estimates for the models being fitted to the data having MA process are in contrast to the latent curve estimates for the models being fitted to the data having AR and ARMA processes because the estimates were lower in the latter two models with no weights. The LCMA and LC model estimates with weight for mean intercept (population values of 8.7298, 8.7224, and 8.7195 for MA values of 0.33, 0.67, and 0.85 respectively) and mean slope (population values of 1.033, 1.0062, and 1.0070 for MA values of 0.33, 0.67, and 0.85 respectively) was close to the population estimate. There were minimal differences in the SE of latent curve estimates in Table 82 across all the models with weights and without weights (except for intercept variance SE with larger differences observed between the LCMA and LC models). The CI for the mean intercept and mean slope are narrower and tighter indicating more precision in both the estimate when the interaction between weights and autocorrelation is accounted than the other three estimates (intercept variance, slope variance, and intercept slope correlation). The CI for mean intercept and mean slope were also different from each other to a statistically significant degree when model with weights and without weights were compared.

Table 81: Means for LC Estimates with MA Process by Model_Type*Weights

Model	Weight	Mean Intercept	Mean Slope	Intercept Variance	Slope Variance	Intercept Slope Correlation
LCARMA	0	6.4005	1.0069	13.5195	0.0451	0.0094
	1	8.7249	-0.1357	-52.3969	-2.4213	9.6267
LCAR	0	6.2184	0.9005	12.8140	0.0744	-0.1598
	1	8.7164	-0.1040	-20.0294	-0.8232	3.4884
LCMA	0	6.4057	0.9985	13.5337	0.0444	-0.0007
	1	8.7208	0.9993	481.3875	1.5416	0.2486
LC	0	6.4069	0.9983	13.7956	0.0823	-0.0918
	1	8.7230	0.9987	490.0659	2.8594	-2.8769

0-With Weight; 1-Without Weight; LC-Latent Curve, AR-Autoregressive, MA-Moving Average

Table 82: Means for SE Estimates with MA Process by Model_Type*Weights (AR 0.00)

Model	**Weight**	**Mean Intercept SE**	**Mean Slope SE**	**Intercept Variance SE**	**Slope Variance SE**	**Intercept Slope Correlation SE**
LCARMA	0	0.2537	0.0281	1.3365	0.0082	0.0755
	1	0.2585	0.0716	4.6007	0.2424	0.9222
	0 (C.I)	6.8978 5.9032	1.0620 0.9518	16.1390 10.9000	0.0612 0.0290	0.1574 -0.1386
	1 (C.I)	1.9648 0.9514	0.0046 -0.2760	-43.3795 -61.4143	-1.9462 -2.8964	11.4342 7.8192
LCAR	0	0.2492	0.0271	1.2784	0.0093	0.0782
	1	0.2629	0.0707	2.5364	0.1412	0.5409
	0 (C.I)	6.7068 5.7300	0.9536 0.8474	15.3197 10.3083	0.0926 0.0562	-0.0065 -0.3131
	1 (C.I)	1.9127 0.8821	0.0346 -0.2426	-15.0581 -25.0007	-0.5464 -1.1000	4.5486 2.4282
LCMA	0	0.2532	0.0178	1.3367	0.0080	0.0703
	1	1.5101	0.1049	47.5149	0.2791	2.4628
	0 (C.I)	6.9020 5.9094	1.0334 0.9636	16.1536 10.9138	0.0601 0.0287	0.1371 -0.1385
	1 (C.I)	11.6806 5.7610	1.2049 0.7937	574.5167 388.2583	2.0886 0.9946	5.0757 -4.5785
LC	0	0.2555	0.0215	1.3557	0.0097	0.0816
	1	1.5227	0.1266	48.1349	0.3373	2.8608
	0 (C.I)	6.9077 5.9061	1.0404 0.9562	16.4528 11.1384	0.1013 0.0633	0.0681 -0.2517
	1 (C.I)	11.7075 5.7385	1.2468 0.7506	584.4103 395.7215	3.5205 2.1983	2.7303 -8.4841

0-With Weight; 1-Without Weight; C.I.-Confidence Interval; SE-Standard Error; LC-Latent Curve, AR-Autoregressive, MA-Moving Average;

Interaction Plots for MA Process*Weight (AR 0.00)

Figures 69 through 78 presents the interaction plots between model type and weights for each of the LC estimates. An examination of how the four models (LCARMA, LCAR, LCMA, and

Latent Curve models fitted the LCMA (AR 0.00) data reveals that the mean intercept (population values of 8.7298, 8.7224, and 8.7195 for MA processes of 0.33, 0.67, and 0.85 respectively) was overestimated for the LCARMA and LCAR models and underestimated for the LCMA and LC models when sample weights were not incorporated (Figure 69). This finding suggest that ignoring the MA process downwardly biases the mean intercept growth parameter in the ARMA and AR process. Figure 70 shows that the mean slope (population values of 1.0033, 1.0062, and 1.0070 for MA processes of 0.33, 0.67, and 0.85 respectively) were underestimated for LCARMA and LCAR models with weights but the difference between the mean slopes was minimal for the LCMA and LC models with weights and no weights. Figure 71 for intercept variance (population value of 13.5607, 13.7468, and 13.8003 for MA values of 0.33, 0.67, and 0.85 MA values respectively) for models with weights were underestimated for the LCARMA and LCAR models but overestimated for the LCMA and LC models. The means for intercept variance for models without weights were constant across all the models and were closer to the population latent curve estimate. Figure 72 displays the interaction plot of slope variance (population value of 0.0479, 0.0442, and 0.0432 for MA values of 0.33, 0.67, and 0.85 MA values respectively) with underestimated values for the LCARMA and LCAR models with weights and overestimated values for the LCMA and LC models with weights. Figure 73 shows the intercept slope correlation being overestimated for the LCARMA and LCAR models with weights and underestimated for the LC models. The LCMA model estimate was similar between the models with and without weights. Mean intercept SE (population value of 0.0075) was similar for LCARMA and LCMAR models with weights and without weights (Figure 73).

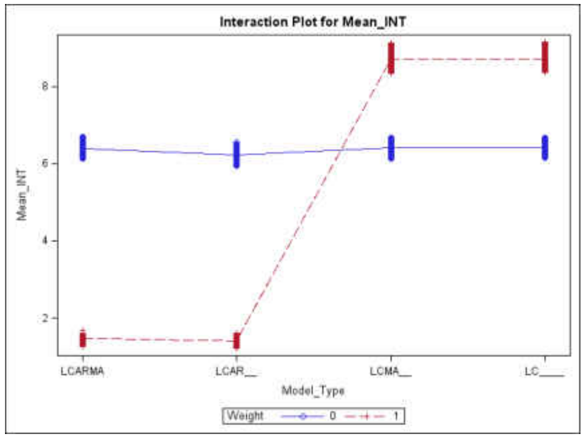

Figure 69: Interaction Plot for Mean Intercept for MA (AR 0.00)

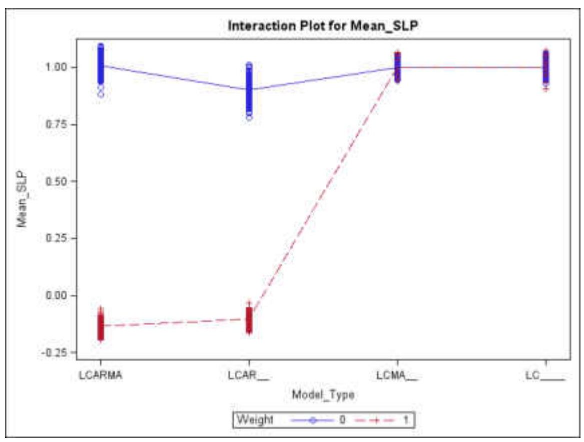

Figure 70: Interaction Plot for Mean Slope for MA (AR 0.00)

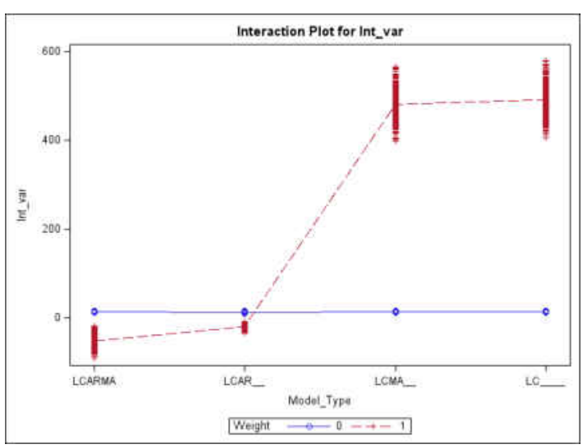

Figure 71: Interaction Plot for Intercept Variance for MA (AR 0.00)

Mean intercept SE was overestimated for LCMA and LC models with weights. Mean slope SE (population value of 0.0007for MA values of 0.33 MA values) (Figure 74), intercept variance SE (population value of 0.0396, 0.0396, and 0.0432 for MA values of 0.33, 0.67, and 0.85 MA values respectively) (Figure 75) (means similar for LCARMA and LCAR models with weights and without weights), slope variance SE (population value of 0.0002) (Figure 76), and intercept slope correlation SE (population value of 0.0027, 0.0028, and 0.0028 for MA values of 0.33, 0.67, and 0.85 MA values respectively) (Figure 77) estimates were overestimated for all the models with weights.

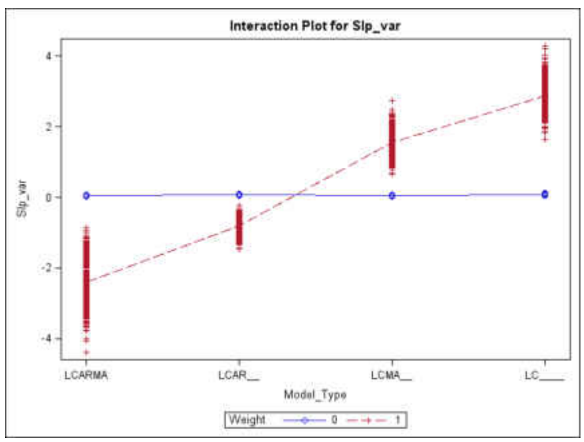

Figure 72: Interaction Plot for Slope Variance for MA (AR 0.00)

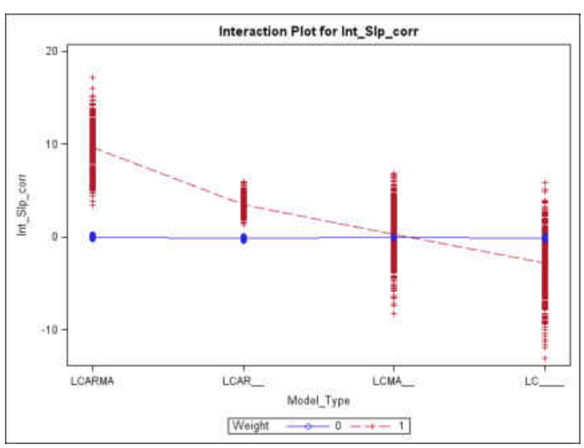

Figure 73: Interaction Plot for Intercept Slope Correlation for MA (AR 0.00)

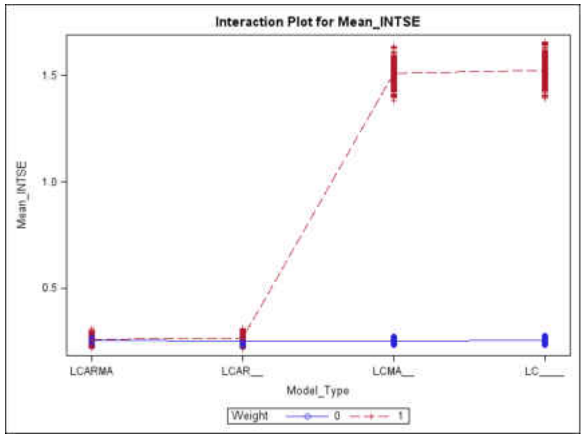

Figure 74: Interaction Plot for Mean Intercept SE for MA (AR 0.00)

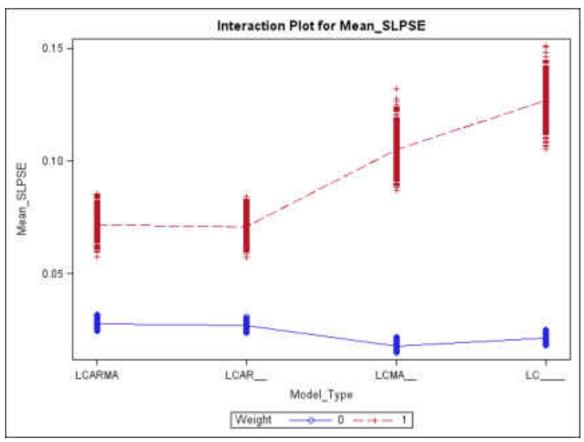

Figure 75: Interaction Plot for Mean Slope SE for MA (AR 0.00)

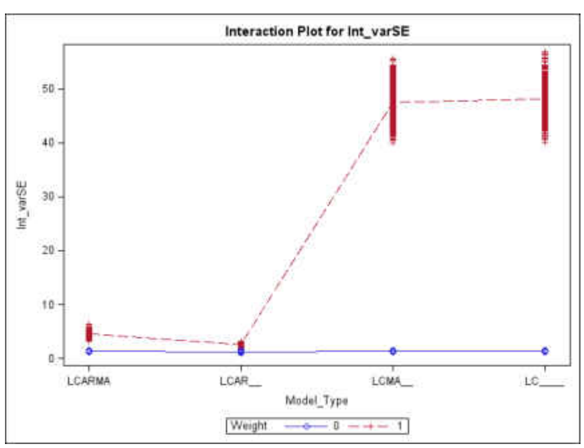

Figure 76: Interaction Plot for Intercept Variance SE for MA (AR 0.00)

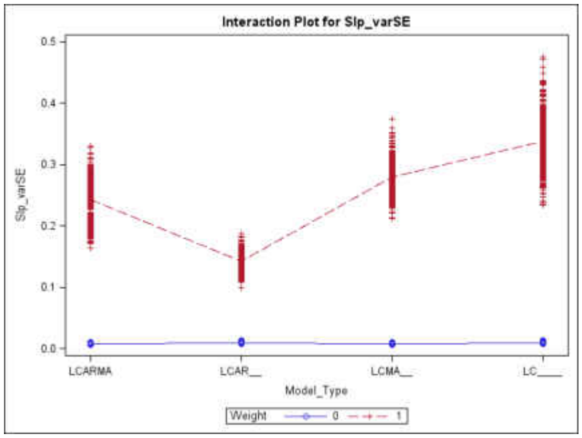

Figure 77: Interaction Plot for Slope Variance SE for MA (AR 0.00)

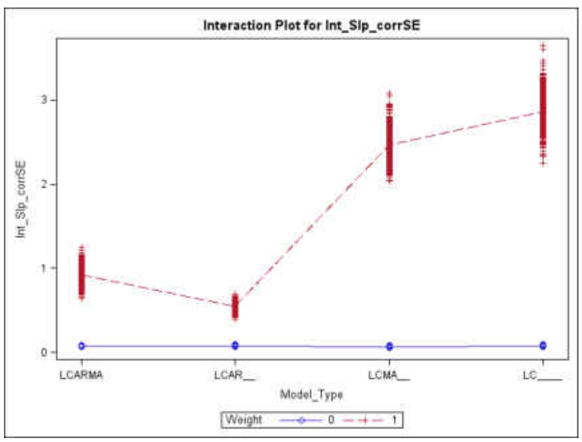

Figure 78: Interaction Plot for Intercept Slope Correlation SE for MA (AR 0.00)

The mean estimate values in Figure 75, 76 and 77 were similar between models without weights. Large differences were observed in mean SE between models with weights and without weights except for intercept variance SE (Figure 75).

<u>General Linear Model Results for LC Model (AR 0.00 & MA 0.00)</u>

The results in this section focuses on the data having no AR and MA process where the both AR process and MA process was set to 0.00. Table 83 provides the fit indices with the

LCMA model having a much better fit (χ^2-37.7466, SRMR-0.0190, RMSEA-0.0297, AIC-65.7466, Mc-0.9825, CFI-0.9976) than the other models. A closer examination of the LC model fit indices (χ^2-42.1125, SRMR-0.0189, RMSEA-0.0360, AIC-68.1125, Mc-0.9749, CFI-0.9968) and LCARMA model (χ^2-45.7685, SRMR-0.0864, RMSEA-0.0457, AIC-75.7685, Mc-0.9624, CFI-0.9954) revealed that it fitted the data well and were within the cut-of values. Table 84 provides a summary of the effect size of LC estimates without AR and MA process (AR 0.00 & MA 0.00).

Table 83: Overall Fit Indices of LC Model (AR 0.00 & MA 0.00)

Model	χ^2	SRMR	RMSEA	AIC	Mc	CFI
LC	42.1125	0.0189	0.0360	68.1125	0.9749	0.9968
LCAR	107.5840	0.1051	0.0910	135.5840	0.8471	0.9794
LCMA	37.7466	0.0190	0.0297	65.7466	0.9825	0.9976
LCARMA	45.7685	0.0864	0.0457	75.7685	0.9624	0.9954

χ2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI-Comparative Fit Index; LC-Latent Curve; AR-Auto regressive; MA-Moving Average

Table 84: Effect Size for LC Estimates of LC Model (AR 0.00 & MA 0.00)

Statistic	R^2	Coefficient of Variation	Root MSE	Mean
Mean Intercept	0.9987	1.6342	0.0938	5.7430
Mean Slope	0.9978	3.0388	0.0219	0.7232
Intercept Variance	0.9960	10.7460	12.9623	120.6245
Slope Variance	0.9739	66.2312	0.1827	0.2758
Intercept Slope Correlation	0.7505	137.7635	1.3828	1.0037
Mean Intercept SE	0.9984	3.8310	0.0215	0.5613
Mean Slope SE	0.9889	7.2339	0.0037	0.0525
Intercept Variance SE	0.9960	9.4205	1.2348	13.1082
Slope Variance SE	0.9779	15.0604	0.0155	0.1032
Intercept Slope Correlation SE	0.9930	10.3529	0.0814	0.7863

SE-Standard Error; MSE-Mean Square Error

The RMSE for all the estimates were low except for intercept variance suggesting a good model fit and better predictability of the estimate by accounting for model type and weight as predictors. Hence, 99.9%, 99.8%, 97.4%, 75.0%, 99.8%, 98.9%, 99.6%, 97.8%, and 99.3%, of the variation in mean intercept, mean slope, slope variance, intercept slope correlation, mean intercept SE, mean slope SE, intercept variance SE, slope variance SE, and intercept slope correlation SE respectively was explained by model type and weight.

ANOVA Results of LC Model (AR 0.00 & MA 0.00)

Tables 85 and 86 summarizes the ANOVA results with no AR and MA process (AR 0.00 & MA 0.00). All the models were significant at $p<.001$ including the interaction between model type and weight. The semi-partial η^2 values for the mean intercept (0.4578), mean slope (0.3314), intercept variance (0.3614), slope variance (0.4678), intercept slope correlation (0.3097), mean intercept SE (0.3401), intercept variance SE (0.3163), and intercept slope correlation SE (0.2239) were high as compared to the η^2 values for mean slope SE and slope variance SE and thereby greater variation in these LC estimates was explained by model type and weight.

Table 85: ANOVA Results for LC Estimates of LC Model (AR 0.00 & MA 0.00)

Dependent Variable⁺	**Mean Intercept**	**Mean Slope**	**Intercept Variance**	**Slope Variance**	**Intercept Slope Correlation**
Model_Type	5364.90*	121.16*	2435694*	955.48*	3757.72*
Wt	708.98*	120.07*	1841331*	78.83*	1622.67*
Model_type X Wt	5350.17*	119.97*	243514*	955.83*	3780.58*
Semi-Partial η^2	0.4678	0.3314	0.3614	0.4678	0.3097
Partial η^2	0.9974	0.9936	0.9891	0.9473	0.5539

⁺-Sum of Squares reported; *Significant at $p < .001$; η^2-Eta Square reported only for interaction effect between Model_type & weight; Wt-Weight

Table 86: ANOVA Results for SE Estimates of LC Model (AR 0.00 & MA 0.00)

Dependent Variable⁺	**Mean Intercept SE**	**Mean Slope SE**	**Intercept Variance SE**	**Slope Variance SE**	**Intercept Slope Correlation SE**
Model_Type	159.8835	0.2016*	192969*	1.1668	336.7458*
Wt	151.7502	1.5035*	222134*	14.7479	822.5981*
Model_type X Wt	160.9613	0.3437*	193129*	1.1712	337.5377*
Semi-Partial η^2	0.3401	0.1659	0.3163	0.0670	0.2239
Partial η^2	0.9954	0.9374	0.9876	0.7525	0.9697

⁺-Sum of Squares reported; *Significant at $p < .001$; η^2-Eta Square reported only for interaction effect between Model_type & weight; Wt-Weight; SE-Standard Error

Tukey Comparisons of LC Model (AR 0.00 & MA 0.00)

Tables 87 and 88 provides the Tukey comparisons between the means of LC estimates with no AR and MA process (AR 0.00 & MA 0.00) by model type and weight respectively. A different letter implies that means between any two models is different from each other to a statistically significant degree. The means of each model were not significantly different from each other for all the LC estimates except for slope variance. The means of LC estimates for the LCARMA and LCAR model were lower than the LC and LCMA models. The means were significantly different between the models with weight and without weights in Table 88 with the former having higher means.

Tables 89 and 90 provides the means for LC estimates with no AR and MA process (AR 0.00 & MA 0.00) when both model and weights were together accounted. It can be seen that the means for LCARMA and LCAR models in Table 89 were higher for the estimates without weights (except for intercept slope correlation) than those with weights. This suggests an overestimation in the LC estimates when both the autocorrelative process (AR 0.00 & MA 0.00) and weights are absent. This trend was not observed in Table 90. The SE for latent curve models with weights were mostly higher than the models without weights (except for mean intercept SE for LCARMA and LCAR with weights). The values for latent curve estimates for the models being fitted to the data having no autocorrelative process are in contrast to the latent curve estimates for the models being fitted to the data having AR, MA and ARMA processes because the estimates were lower in the AR and ARMA models with no weights. The CI for the mean intercept and mean slope are narrower and tighter indicating more precision in both the estimate when the interaction between weights and autocorrelation is accounted than the other three estimates (intercept variance, slope variance, and intercept slope correlation). The CI for mean intercept and mean slope were also different from each other to a statistically significant degree when model with weights and without weights were compared.

Table 87: Tukey Comparisons by Model (AR 0.00 & MA 0.00)

Statistic	Model	Mean	Tukey Grouping
Mean Intercept	LC	7.5739	A
	LCMA	7.5739	A
	LCAR	3.9556	B
	LCARMA	3.8686	C
Mean Slope	LC	0.9980	A
	LCMA	0.9980	A
	LCARMA	0.4670	B
	LCAR	0.4296	C
Intercept Variance	LC	244.1173	A
	LCMA	243.5605	A
	LCARMA	6.495	B
	LCAR	-11.6747	C
Slope Variance	LC	1.0017	A
	LCMA	0.9557	B
	LCARMA	0.0277	C
	LCAR	-0.8812	D

Statistic	Model	Mean	Tukey Grouping
Intercept Slope Correlation	LCAR	3.6568	A
	LCMA	0.1961	B
	LCARMA	0.1004	B
	LCAR	0.0618	B
Mean Intercept SE	LCMA	0.8774	A
	LC	0.8773	A
	LCAR	0.2456	B
	LCARMA	0.2448	B
Mean Slope SE	LC	0.0638	A
	LCMA	0.0636	A
	LCAR	0.0415	B
	LCARMA	0.0410	B
Intercept Variance SE	LCMA	24.0955	A
	LC	24.0844	A
	LCARMA	2.2374	B
	LCAR	2.0154	B
Slope Variance SE	LCMA	0.1305	A
	LC	0.1295	A
	LCARMA	0.0818	B
	LCAR	0.0712	C
Intercept Slope Correlation SE	LC	1.2460	A
	LCMA	1.2436	A
	LCARMA	0.3485	B
	LCAR	0.3073	C

SE-Standard Error; LC-Latent Curve; AR-Auto regressive; MA-Moving Average

Table 88: Tukey Comparisons by Weight (AR 0.00 & MA 0.00)

Statistic	Model	Mean	Tukey Grouping
Mean Intercept	1	6.4086	A
	0	5.0773	B
Mean Slope	1	0.9971	A
	0	0.4492	B
Intercept Variance	1	227.9014	A
	0	13.3477	B
Slope Variance	1	0.4978	A
	0	0.0538	B
Intercept Slope Correlation	1	2.0108	A
	0	-0.0033	B

Statistic	Model	Mean	Tukey Grouping
Mean Intercept SE	1	0.8692	A
	0	0.2533	B
Mean Slope SE	1	0.0831	A
	0	0.0218	B
Intercept Variance SE	1	24.8910	A
	0	1.3254	B
Slope Variance SE	1	0.1992	A
	0	0.0073	B
Intercept Slope Correlation SE	1	1.5033	A
	0	0.0693	B

0-With Weight; 1-Without Weight; SE-Standard Error

Table 89: Means for LC Estimates of LC Model
by Model_Type*Weights (AR 0.00 & MA 0.00)

Model	Weight	Mean Intercept	Mean Slope	Intercept Variance	Slope Variance	Intercept Slope Correlation
LCARMA	0	6.4164	1.001	13.3722	0.0529	0.0035
	1	8.7158	-0.0669	-0.3823	0.0025	0.1973
LCAR	0	6.3956	0.9908	13.2978	0.0546	-0.0114
	1	8.7072	-0.1316	-36.6471	-1.817	7.3251
LCMA	0	6.4114	0.9983	13.3440	0.0527	0.0012
	1	8.7364	0.9978	473.7770	1.8588	0.3909
LC	0	6.4112	0.9984	13.3767	0.0552	-0.0064
	1	8.7365	0.9977	474.8578	1.9470	0.1300

0-With Weight; 1-Without Weight; LC-Latent Curve; AR-Auto regressive; MA-Moving Average

Table 90: Means for SE Estimates of LC Model
by Model_Type*Weights (AR 0.00 & MA 0.00)

Model	Weight	Mean Intercept SE	Mean Slope SE	Intercept Variance SE	Slope Variance SE	Intercept Slope Correlation SE
LCARMA	0	0.2549	0.0257	1.3349	0.0074	0.0703
	1	0.2346	0.0563	3.1399	0.1563	0.6267
	0 (C.I)	6.9160	1.0514	15.9886	0.0674	0.1413

Model	Weight	Mean Intercept SE	Mean Slope SE	Intercept Variance SE	Slope Variance SE	Intercept Slope Correlation SE
		5.9168	0.9506	10.7558	0.0384	-0.1343
	1 (C.I)	1.7806 0.8610	0.0434 -0.1772	5.7719 -6.5365	0.3088 -0.3038	1.4256 -1.0310
LCAR	0	0.2538	0.0248	1.3250	0.0073	0.0693
	1	0.2373	0.0583	2.7057	0.1352	0.5447
	0 (C.I)	6.8930 5.8982	1.0394 0.9422	15.8948 10.7008	0.0689 0.0403	0.1244 -0.1472
	1 (C.I)	1.9807 1.0505	-0.0173 -0.2459	-31.3439 -41.9503	-1.5520 -2.0820	8.3927 6.2575
LCMA	0	0.2522	0.0184	1.3211	0.0072	0.0686
	1	1.5026	0.1088	46.8700	0.2537	2.4185
	0 (C.I)	6.9057 5.9171	1.0344 0.9622	15.9334 10.7546	0.0668 0.0386	0.1357 -0.1333
	1 (C.I)	11.6815 5.7913	1.2110 0.7846	565.6422 381.9118	2.3561 1.3615	5.1312 -4.3494
LC	0	0.2522	0.0185	1.3205	0.0072	0.0688
	1	1.5024	0.109	46.8482	0.2518	2.4233
	0 (C.I)	6.9055 5.9169	1.0347 0.9621	15.9649 10.7885	0.0693 0.0411	0.1284 -0.1412
	1 (C.I)	11.6812 5.7918	1.2113 0.7841	566.6803 383.0353	2.4405 1.4535	4.8797 -4.6197

0-With Weight; 1-Without Weight; SE-Standard Error; C.I.-Confidence Interval; LC-Latent Curve; AR-Autoregressive; MA-Moving Average

Interaction Plots of LC Model (AR 0.00 & MA 0.00)

Figures 79 through 88 presents the interaction plots between model type and weights for each of the LC estimates. An examination of how the four models (LCARMA, LCAR, LCMA, and Latent Curve models fitted the LCMA (AR 0.00 & MA 0.00) data reveals that the mean intercept (population value of 8.7337) was overestimated for the LCARMA and LCAR models when sample weights were not incorporated (Figure 79). The mean intercept was underestimated for LCMA and LC models with no weights. This finding suggest that ignoring the AR and MA process upwardly biases the mean intercept growth parameter for LCARMA and LCAR models. Figure 80 shows that the mean slope (population value of 1.0023) was underestimated for LCARMA and LCAR models with weights but the difference between the mean slopes was minimal for the LCMA and LC models with weights and no weights.

Figure 81 and Figure 82 for intercept variance (population value of 13.4241) and slope variance (population value of 0.0529) also had the same trends as mean intercept with the means having an downward bias when weights were present for the LCARMA and LCAR models and upward bias for the LCMA and LC models. The difference between the LCARMA and LCAR model intercept variance with weights and without weights was less as compared to the difference between the LCMA and LC models. The means for intercept variances and slope (population value of 0.0529) variances were almost constant for all the models without weights. Figure 83 again shows the intercept slope correlation (population value of 0.0218) being slightly biased for the LCARMA model with weights and having upward bias for the LCAR model with weights. Similar means for the LCMA and LC models were observed with weights and without weights. Mean intercept SE (population value of 0.0075) was similar between the LCARMA and LCAR models with and without weights and upwardly biased for the LCMA and LCAR models with weights (Figure 84). Mean slope SE (population value of 0.0071) (Figure 85), intercept variance SE (population value of 0.0390) (Figure 86), slope variance SE (population value of 0.0002) (constant for all models with no weights) (Figure 87), and intercept slope correlation SE (population value of 0.0024) (Figure 88) mean estimates were overestimated for all the models with weights.

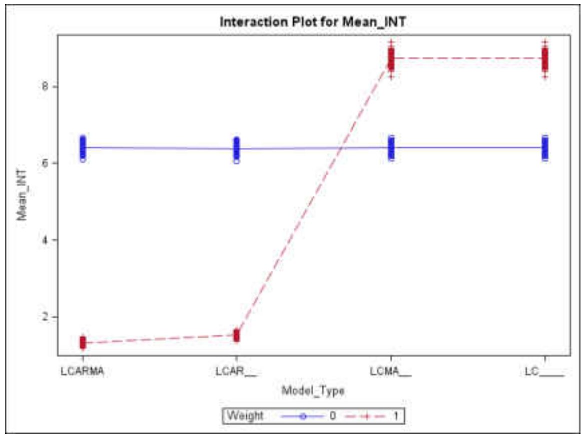

Figure 79: Interaction Plot for Mean Intercept (AR 0.00 & MA 0.00)

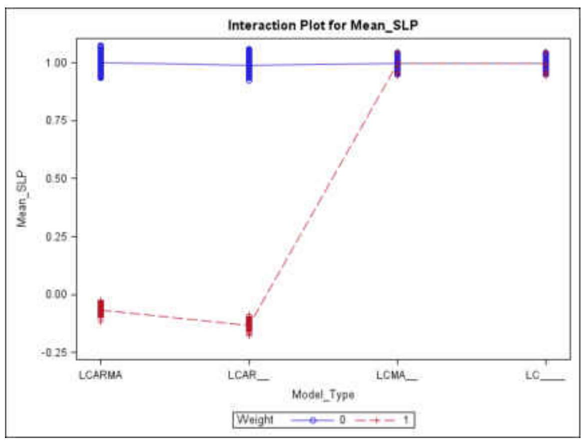

Figure 80: Interaction Plot for Mean Slope (AR 0.00 & MA 0.00)

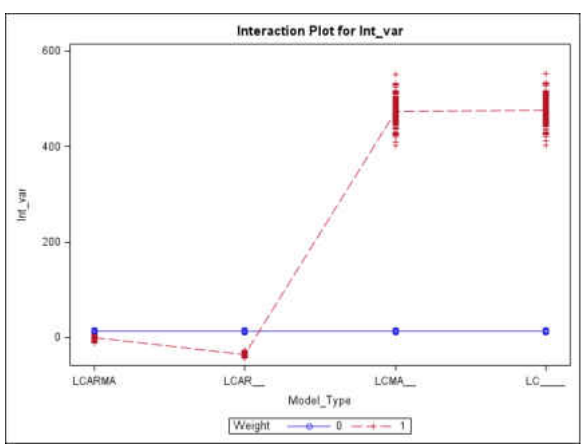

Figure 81: Interaction Plot for Intercept Variance (AR 0.00 & MA 0.00)

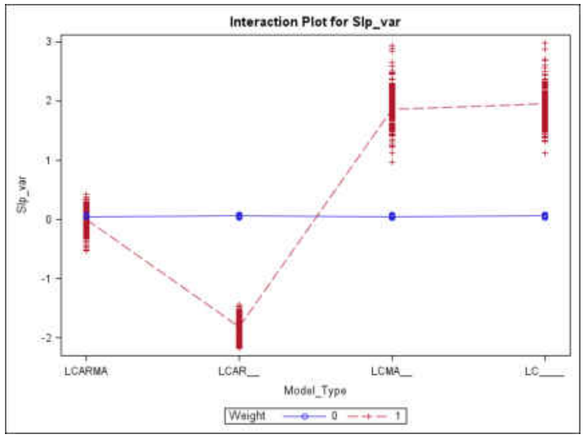

Figure 82: Interaction Plot for Slope Variance (AR 0.00 & MA 0.00)

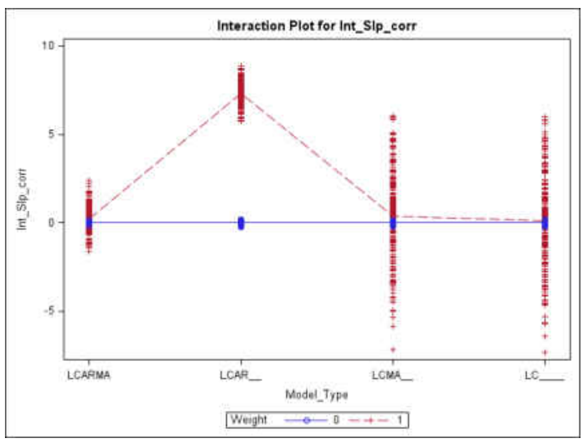

Figure 83: Interaction for Intercept Slope Correlation (AR 0.00 & MA 0.00)

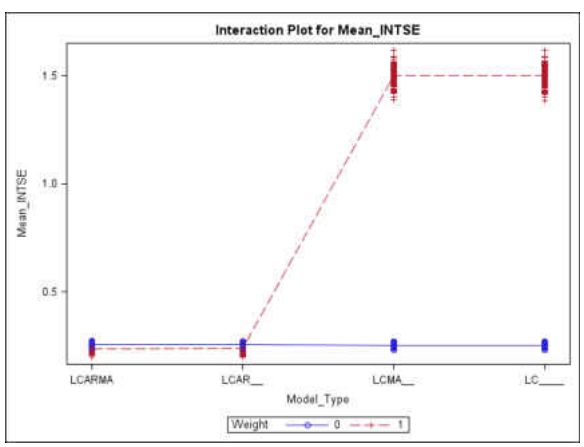

Figure 84: Interaction Plot for Mean Intercept SE (AR 0.00 & MA 0.00)

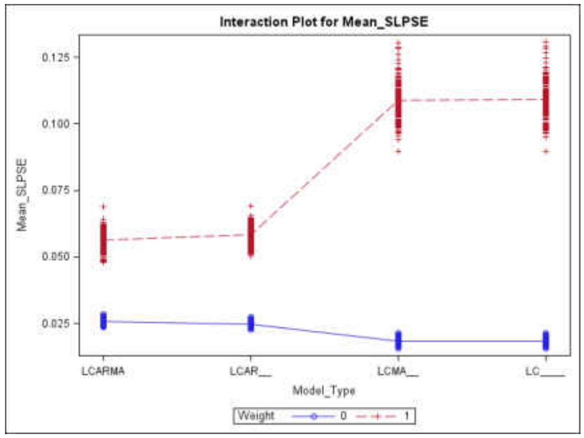

Figure 85: Interaction Plot for Mean Slope SE (AR 0.00 & MA 0.00)

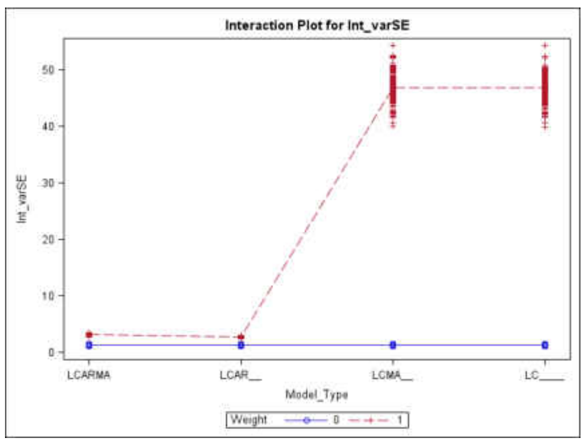

Figure 86: Interaction Plot for Intercept Variance SE (AR 0.00 & MA 0.00)

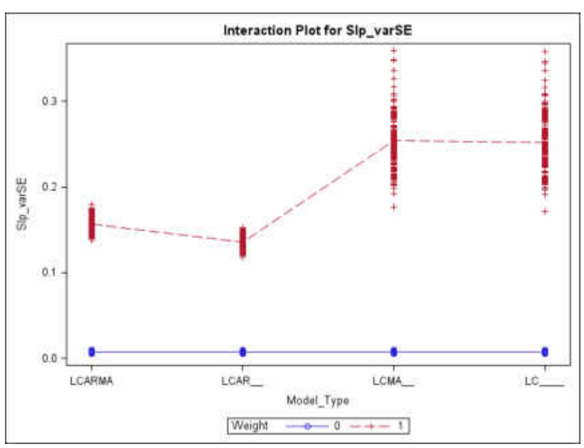

Figure 87: Interaction Plot for Slope Variance SE (AR 0.00 & MA 0.00)

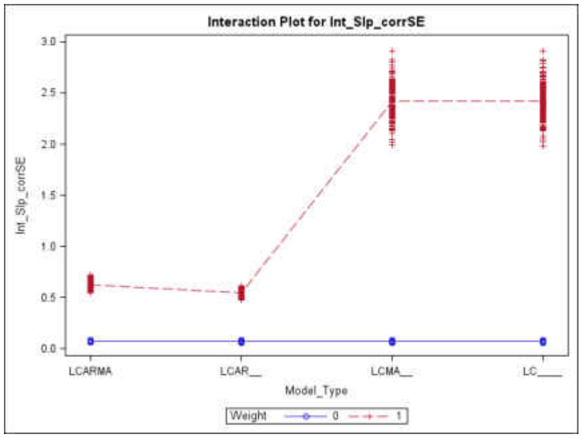

Figure 88: Interaction Plot for Intercept Slope Correlation SE (AR 0.00 & MA 0.00)

ECLS-K Data Results

This section focuses on the analysis of math and reading achievement scores from ECLS-K data which were collected over seven time periods with each time period representing one wave of data. Table 90 provides the fit indices for Math achievement scores. The best fitting model was LCARMA with weights (χ^2-424.8563, SRMR-13964.98 RMSEA-0.0902, AIC-452.856, Mc-0.9182, CFI-0.9762). All the fit indices except SRMR and RMSEA (lowest for all the models) were within the cut-offs. The worst fitting model was LC with no ARMA and no weights (χ^2-11705.91, SRMR-0.1119 RMSEA-0.4635 AIC-11729.91, Mc-0.0846, CFI-0.2919). None of the models were even close to the cut-offs for the fit indices except for the LCARMA with weight. The models with weights had a better fit to the data than the models without weights. This is in contrast to the fit indices for the simulation data where the values were lower for the models without weights.

Table 91: Fit Indices for Math Achievement Scores

Model	χ^2	SRMR	RMSEA	AIC	MC	CFI
LC-wt	1632.899	0.1119	0.1721	1656.899	0.7115	0.9052
LC- no ARMA & wt	11705.91	0.4258	0.4635	11729.91	0.0846	0.2919
LCAR-wt	717.9572	0.0961	0.1157	743.9572	0.8632	0.9590
LCAR- no wt	7685.973	0.6199	0.3839	7711.973	0.1978	0.5374
LCMA-wt	1274.380	0.1081	0.1552	1300.380	0.7674	0.9263
LCMA- no wt	8845.795	0.3448	0.4119	8871.795	0.1548	0.4674
LCARMA-wt*	424.8563	13964.98	0.0902	452.856	0.9182	0.9762
LCARMA- no wt	7659.724	0.5070	0.3923	7687.724	0.1989	0.5390

* Best Fitting Model; Latent Curve with weight-LC wt; Latent Curve with no ARMA and no weight-LC no ARMA & wt; Latent Curve AR with weights-LCAR wt; Latent Curve AR with no weight-LCAR no wt; LCARMA with weight-LCARMA wt; LCARMA with no weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no weight-LCMA no wt; χ^2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI- Comparative Fit Index

Table 92 provides the means and standard deviations for the Math achievement scores. C1R4MSCL, C2R4MSCL, C3R4MSCL, C4R4MSCL, C5R4MSCL, C6R4MSCL, and, C7R4MSCL represents the Math scores collected in time period 1, 2, 3, 4, 5, 6, and 7 respectively. The means and S.D. for the models having weights were similar to each other and so were the means and S.D for models without weights. The means and S.D. increased with each wave of Math achievement scores. Furthermore, the means and SE for the models without autocorrelative process modeled and with weights were higher than the models without autocorrelative process modeled and no weights. This trend was observed in all the seven waves of math scores. This might be because the C7CW0 longitudinal weight in the ECLS-K data set that accounted for the child assessment data from all the seven rounds of data collection probably did not account for the ARMA process that exists in the data due to correlation between the math observations.

Table 92: Means for Math Achievement Scores for LC Model

Variable	LC: Weights without Autocorrelative Process			LC: No Weights & Without Autocorrelative Process		
	Mean	S.D.	S.E.	Mean	S.D.	S.E
C1R4MSCL	25.7563	181.1658	3.7252	26.5068	10.3428	0.2126
C2R4MSCL	35.8954	234.5063	4.8221	37.8271	12.9477	0.2662
C3R4MSCL	43.0983	273.8815	5.6317	45.0142	14.9377	0.3071
C4R4MSCL	60.9937	336.2459	6.9141	63.5411	18.2721	0.3757
C5R4MSCL	98.3602	471.1405	9.6880	101.4520	24.5601	0.5050
C6R4MSCL	122.5117	487.1483	10.0171	126.0263	24.4390	0.5025
C7R4MSCL	138.8994	454.1151	9.3379	142.5513	23.0986	0.4749

C1R4MSCL-Math Score for 1st time period, C2R4MSCL-Math Score for 2nd time period, C3R4MSCL-Math Score for 3rd time period, C4R4MSCL-Math Score for 4th time period, C5R4MSCL-Math Score for 5th time period, C6R4MSCL--6th Score for 6th time period, and, C7R4MSCL-Math Score for 7th time period; SD-Standard Deviation; SE-Standard Error; LC-Latent Curve

Table 93 provides the results of fit indices for reading achievement scores. The best fitting model was again LCARMA with weights (χ^2-479.474, SRMR-0.0590, RMSEA-0.0961, AIC-507.474, Mc-0.9076, CFI-0.9705). The chi-square and AIC values were higher whereas the SRMR value was lower than the Math scores. The worst fitting model was LC with no ARMA and no weights (χ^2-11822.439, SRMR-0.4508, RMSEA-0.4658, AIC-11846.458, Mc-0.0.0825, CFI-0.2609). Similar trends were seen in the fit indices for reading where the models with weights fitted better than models without weights which is in contrast to the simulation fit indices. The fit indices were worse for the LC no ARMA and no weight model for the reading scores than the math scores. The chi-square, SRMR, RMSEA and AIC fit indices for the reading scores were higher than the math scores indicating a poor fit. In addition, the MC and CFI values for the reading scores were lower than the math scores again implying a poor fit of the models to the data. Table 94 provides the means and standard deviations for the reading achievement scores. C1R4RSCL, C2R4RSCL, C3R4RSCL, C4R4RSCL, C5R4RSCL, C6R4RSCL, and, C7R4RSCL represents the reading scores collected in time period 1, 2, 3, 4, 5, 6, and 7 respectively.

Table 93: Fit Indices for Reading Achievement Scores

Model	χ^2	SRMR	RMSEA	AIC	MC	CFI
LC-wt	2575.776	0.114	0.2167	2599.776	0.5829	0.8356
LC-no ARMA & wt	11822.439	0.4508	0.4658	11846.458	0.0825	0.2609
LCAR-wt	526.649	0.0622	0.0985	552.6491	0.8988	0.9675
LCAR- no wt	7935.763	0.5926	0.3901	7961.763	0.1877	0.5043
LCARMA-wt*	479.474	0.0590	0.0961	507.474	0.9076	0.9705
LCARMA- no wt	7527.338	0.3592	0.3888	7555.338	0.2045	0.5298
LCMA-wt	1456.857	0.1015	0.1661	1482.857	0.7383	0.9076
LCMA-no wt	8402.532	0.3020	0.4014	8428.532	0.1700	0.4571

* Best Fitting Model; Latent Curve with weight-LC wt; Latent Curve with no ARMA and no weight-LC no ARMA & wt; Latent Curve AR with weights-LCAR wt; Latent Curve AR with no weight-LCAR no wt; LCARMA with weight-LCARMA wt; LCARMA with no weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no weight-LCMA no wt; χ^2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI- Comparative Fit Index

Table 94: Means for Reading Achievement Scores for LC Model

Variable	LC: Weight without Autocorrelative Process			LC: No Weights & Without Autocorrelative Process		
	Mean	S.D.	S.E	Mean	S.D.	S.E.
C1R4RSCL	32.3504	252.9221	5.200813	33.5307	14.2613	0.2932
C2R4RSCL	43.9753	314.0529	6.457839	46.0254	17.6871	0.3636
C3R4RSCL	50.2749	357.7382	7.356136	52.7937	20.4414	0.4203
C4R4RSCL	74.8831	488.4879	10.04473	78.7133	26.7198	0.5494
C5R4RSCL	124.3871	590.0493	12.13313	239.1312	29.9782	0.6164
C6R4RSCL	148.0300	538.5375	11.07389	152.4926	27.8692	0.5730
C7R4RSCL	165.4558	634.8422	13.0542	170.9667	30.9805	0.6370

C1R4MSCL-Reading Score for 1st time period, C2R4MSCL- Reading Score for 2nd time period, C3R4MSCL- Reading Score for 3rd time period, C4R4MSCL- Reading Score for 4th time period, C5R4MSCL- Reading Score for 5th time period, C6R4MSCL-Reading Score for 6th time period, C7R4MSCL-Math Score for 7th time period, SD-Standard Deviation; SE-Standard Error; LC-Latent Curve; SD-Standard Deviation; SE-Standard Error; LC-Latent Curve

The means and S.D. for the models having weights were similar to each other and so were the means and S.D for models without weights. The means and S.E. increased with each wave of reading achievement scores (except between C5R4RSCL and C6R4RSCL). Furthermore, the means and SE for the models without autocorrelative process modeled and weights were higher than the models without autocorrelative process modeled and no weights. This trend was observed in all the seven waves of reading scores. This might be because the C7CW0 longitudinal weight in the ECLS-K data set that accounted for the child assessment data from all the seven rounds of data collection probably did not account for the ARMA process that exists in the data due to correlation between the reading observations.

Table 95 presents the results for the number of iterations it toll for the maximum likelihood procedure to converge a model. For the ECLS-K data the highest number of iterations taken for model convergence were for the LCARMA no weight, both for the math and reading scores followed by the LCAR no weight model (with less iteration required for the reading scores). The iteration results from the simulation study also had less number of iterations for the ARMA model of 0.33 and 0.30. The moving average models also took less iterations for model convergence which was similar to the MA models in Table 34.

Table 95: Iterations Taken for Model Convergence in ECLS-K Data

Model	**ECLS-K (Math)**	**ECLS-K (Reading)**
LC-wt	9	8
LC-no ARMA & wt	48	45
LCAR-wt	17	13
LCAR- no wt	114	98
LCARMA-wt	12	10
LCARMA- no wt	188	73
LCMA-wt	11	6
LCMA-no wt	34	30

LC-Latent Curve; AR-Auto regressive; MA-Moving Average; Wt-Weight

Table 96 provides the latent curve estimates for the four models (LC, LCAR, and LCMA and LCARMA) with weights and without weights respectively that were fitted to the ECLS-K math scores. A general comparison between the latent curve estimates reveals that estimates and their standard errors were lower for models without weights than models with weights. It is not possible to ascertain which model had accurate latent curve estimate because population level latent curve parameter values were not available. The SE were not high for the estimates except for intercept variance and slope variance for all models with weights and without weights. Table 97 provides summary information on the standard errors associated with the latent curve estimates. The confidence intervals for the latent curve models fitted to math scores were narrower for mean intercept and mean slope estimates indicating more precision in the estimates than the intercept variance, slope variance, and intercept slope correlation. The means itself were not included in the confidence intervals indicating that they were different from each other between the models with weights and without weights to a statistically significant degree. The confidence intervals were wider for mean intercept and mean slope in the LCMA and LC models with weights and no weights.

Table 98 provides the latent curve estimates for the four models (LC, LCAR, and LCMA and LCARMA) with weights and without weights respectively that were fitted to the ECLS-K reading scores. A general comparison between the latent curve estimates reveals that estimates and their standard errors were lower for models without weights than models with weights especially for intercept variance, slope variance, and intercept slope correlation. It is not possible to ascertain which model had accurate latent curve estimate because population level latent curve parameter values were not available.

Table 96: LC Estimates for Math Scores

Model	Weight	Mean Intercept	Mean Slope	Intercept Variance	Slope Variance	Intercept Slope Correlation
LCARMA	1	12.0317	2.9150	13689	189.5616	-1.27709
	0	12.9255	3.4243	-48.2914	-6.0282	14.0625
LCAR	1	18.0598	10.3240	20356	713.1644	0.2311
	0	11.8710	3.1818	-30.4346	-4.6181	9.6648
LCMA	1	21.1953	17.9744	27627	3559	0.4427
	0	18.9666	20.0324	-1.5164	1.7827	31.1479
LC	1	21.2483	17.5422	30123	3764	0.3847
	0	17.7016	70.7202	119.3329	10.2705	0.1309

0-With Weight; 1-Without Weight; LC-Latent Curve; AR-Auto regressive; MA- Moving Average

Table 97: Standard Errors & Confidence Intervals-LC Estimates of Math Scores

Model	Weight	Mean Intercept SE	Mean Slope SE	Intercept Variance SE	Slope Variance SE	Intercept Slope Correlation SE
LCARMA	1	2.5030	0.6893	13.3134	27.5858	0.0578
	0	0.1523	0.1509	3.9582	0.2883	0.8676
	1 (C.I)	16.9376, 7.1258	4.2660, 1.5639	13715.09, 13662.91	243.6298, 135.4934	-1.1638, -1.3904
	0 (C.I)	13.2240, 12.6270	3.7200, 3.1285	-40.5333, -56.0495	-5.4631, -6.5933	15.763, 12.362
LCAR	1	3.3543	0.8721	1.0505	61.9201	0.0405
	0	0.1490	0.1229	2.5212	0.1919	0.5696
	1 (C.I)	24.6342, 11.4854	12.0332, 8.6146	20358.94, 20353.94	834.5278, 591.801	0.3105, 0.1517
	0 (C.I)	12.1630, 11.5789	3.4227, 2.9409	-25.493, -35.3762	-4.2419, -4.9942	10.7812, 8.5483
LCMA	1	3.8178	1.3827	22.7413	132.0405	0.0216
	0	0.2360	0.0708	4.7583	0.4056	1.0226
	1 (C.I)	28.6782, 13.7124	20.6845, 15.2643	27671.57, 27582.43	3817.799, 3300.201	0.4850, 0.4003
	0 (C.I)	19.4292, 18.5040	20.1712, 19.8936	7.8098, -10.8427	2.5776, 0.9877	33.1522, 29.1436
	1	3.7816	1.3833	24.3293	131.8392	0.0194
	0	0.2503	0.0756	4.5321	0.4024	0.0284

Model	Weight	Mean Intercept SE	Mean Slope SE	Intercept Variance SE	Slope Variance SE	Intercept Slope Correlation SE
LC	1 (C.I)	28.6602, 13.8364	20.2535, 14.8309	30170.69, 30075.31	4022.405. 3505.595	0.4223, 0.3466
	0 (C.I)	18.1922, 17.2110	70.8684, 70.5720	128.2158, 110.45	11.0592, 9.4818	0.1865, 0.0752

0-With Weight; 1-Without Weight; SE-Standard Error; C.I.-Confidence Interval; LC-Latent Curve; AR-Autoregressive; MA-Moving Average

The SE were not high for the estimates except for intercept variance, slope variance, and intercept slope correlation for all models with weights and without weights. Table 99 provides summary information on the standard errors associated with the latent curve estimates of models fitted to reading scores. The confidence intervals for the latent curve models fitted to reading scores were narrower for mean intercept and mean slope estimates indicating more precision in the estimates than the intercept variance, slope variance, and intercept slope correlation. The means itself were not included in the confidence intervals indicating that they were different from each other between the models with weights and without weights to a statistically significant degree.

Table 98: LC Estimates for Reading Scores

Model	Weight	Mean Intercept	Mean Slope	Intercept Variance	Slope Variance	Intercept Slope Correlation
LCARMA	1	15.2422	5.8291	21634	-65.0592	-1689
	0	19.6076	8.7083	-72.2639	-11.2485	30.8025
LCAR	1	16.8361	7.0974	21386	-172.868	-744.3113
	0	14.6486	5.6880	1.6518	-4.3015	6.2119
LCMA	1	23.8549	22.3442	52354	5825	3582
	0	24.0149	24.0655	43.8504	12.1992	1.7172
LC	1	23.4579	22.0070	65579	6377	1905
	0	21.0010	25.2179	316.8679	21.4435	-23.3488

0-With Weight; 1-Without Weight; LC-Latent Curve; AR-Auto regressive; MA- Moving Average

The results in Tables 96 and 98 are quite similar to the simulation results where the latent curve estimates for the models with no weights were lower than the estimates for models having weights. Larger differences were observed within the estimates for models with weights and without weights. The LCARMA model with weights had the lowest latent curve estimates and associated SE when compared to the other three models with weights. The fit indices and the number of iterations taken by maximum likelihood procedure for model convergence lend support to the stability of the model and also to the latent curve estimates. One possible reason for high standard errors in models with weights for math and reading scores can be due to increase in sample size which might be inflating the standard errors.

Table 99: Standard Errors & Confidence Intervals-LC Estimates of Reading Scores

Model	**Weight**	**Mean Intercept SE**	**Mean Slope SE**	**Intercept Variance SE**	**Slope Variance SE**	**Intercept Slope Correlation SE**
LCARMA	1	3.8326	1.0361	24.4763	62.5425	170.5982
	0	0.2477	0.2604	7.4541	0.5147	1.4754
	1 (C.I)	22.7541 7.7303	7.8599 3.7983	21681.973 21586.026	57.524 -187.642	-1354.6275 -2023.3725
	0 (C.I)	20.0931 19.1221	9.2187 8.1979	-57.6539 -86.8739	-10.2397 -12.2573	33.6943 27.9107
LCAR	1	4.0313	1.0598	23.8830	68.9416	195.5641
	0	0.2257	0.1728	5.1382	0.3655	1.0912
	1 (C.I)	24.7374 8.9348	9.1746 5.0202	21432.810 21339.189	-37.742 -307.993	-361.0057 -1127.6169
	0 (C.I)	15.0910 14.2062	6.0267 5.3493	11.7227 -8.4191	-3.5851 -5.0179	8.3507 4.0731
LCMA	1	5.4776	1.8596	0.4731	236.5962	8.7730
	0	0.3258	0.0947	1.9271	9.0758	0.7257
	1 (C.I)	34.5910 13.1188	25.9890 18.6994	52354.927 52353.072	6288.728 5361.271	3599.1951 3564.8049

Model	Weight	Mean Intercept SE	Mean Slope SE	Intercept Variance SE	Slope Variance SE	Intercept Slope Correlation SE
	0 (C.I)	24.6535 23.3763	24.2511 23.8799	47.6275 40.0733	29.9878 -5.5894	3.1396 0.2948
LC	1	5.6283	1.8522	0.0795	235.1663	7.0918
	0	0.3792	0.1060	10.6132	0.7929	0.4452
	1 (C.I)	34.4894 12.4264	25.6373 18.3767	65579.155 65578.844	6837.925 5916.074	1905.0380 1904.9620
	0 (C.I)	21.7442 20.2578	25.4257 25.0101	337.6698 296.0660	22.9976 19.8894	-22.4762 -24.2214

0-With Weight; 1-Without Weight; SE-Standard Error; C.I.-Confidence Interval; LC-Latent Curve; AR-Autoregressive; MA-Moving Average

Summary

Tables 100, 101 and 102 provides information on the fit indices, growth curve estimates and their associated standard errors of the eight models for ARMA process of 0.33, 0.30; 0.85, 0.80; 0.00, 0.00 respectively. Tables 103 and 104 provides the summary information on models fitted to data sets having MA process of 0.33 and 0.85 respectively. Tables 105 and 106 provides the summary information on models fitted to data sets having AR process of 0.33 and 0.85 respectively. The tables would help to compare the change in the model fit and behavior of the estimates when a particular model with weights versus no weights is fitted to a data set having different ARMA values. The AR and MA estimates are also reported for each model. Overall, the LCARMA no weight was the best fitting model but the fit was slightly better when the model was fitted to data having ARMA process of 0.67, 0.60 than 0.33, 0.30. The LCARMA model with weights was the second best fitting model but the fit was marginally less than LCARMA with weights. The chi-square statistic was lowest for the LCARMA no weight models when fitted to data having MA of 0.33 and highest for LC model (AR 0.00, MA 0.00) fitted to ARMA data (AR 0.85, MA 0.80). The SRMR had lowest values for LCARMA models fitted to data having AR process of 0.85 and ARMA of 0.85, 0.80. RMSEA values were lowest for the LCARMA models and highest for the LC models. RMSEA values were high for LC models fitted to MA and AR data sets. RMSEA values was highest when the LCARMA models were fitted to MA data than when they were fitted to AR and MA data. Similar pattern was observed with the AIC values where LCARMA models had high values when fitted to MA data. The Mc Donald non-centrality index fit statistic was highest when the AR, MA and ARMA models were fitted to AR, MA and ARMA data sets respectively. The fit statistic was also good for LCARMA model fitted to AR data. The CFI fit index was highest for LCAR and LCARMA models fitted to AR and ARMA data sets respectively. Latent curve models performed poorly when fitted to AR, MA and ARMA data sets. Overall, the MA estimates for the LCMA model with weights were high when fitted to MA data sets whereas the AR estimates for LCAR models with weights were high when fitted to AR data sets. Similarly, both the AR and MA estimates were high for ARMA models with weights fitted to ARMA data sets.

The LCARMA model with weight having the ARMA process of 0.85 and 0.80 (8.7685) had the mean intercept estimate closest to the population estimate of 8.7281 and also had a SE of 0.6653 associated with it. All LCARMA and LCAR models with weights having different values for AR and MA processes had mean intercept values closest to the population value amongst all the models. The LC model with weight having just the AR process of 0.85 also had the mean intercept latent curve estimate (8.7242) closest to the population value (8.7393) with a SE of 0.9978. The LCMA and LC models with weights had higher mean intercept estimates than the population values irrespective of the autocorrelative values and weights when fitted to AR and

ARMA data sets. These models with weights also had a higher SE. The mean slope values were consistently underestimated for the LCARMA models with weights and the underestimation was higher for the LC model with no AR and MA processes. Wider differences were observed in the mean slope values within the latent curve models with weights and no weights fitted to AR, MA and ARMA data sets (except in the latent curve models fitted to MA process).

The intercept variance, slope variance, and intercept slope correlation estimates were consistently being overestimated with models having weights but the associated SE with these estimates were lower for the LCARMA and LCAR models than the LCMA and LC models. An important trend noted in these three estimates was an increase in SE with increase in the AR and MA processes from 0.33 & 0.30 to 0.85 & 0.80. The SE was lowest for 0.33 & 0.30 processes and highest for 0.85 and 0.80 processes suggesting an increase in the level of multicollinearity between the observations which might be creating the masking effects and preventing the true interactive, linear effects between autocorrelative processes and weights to blossom in the latent curve models with weights. Furthermore, the box plots for mean intercept and mean slope had less number of outliers than the intercept variance, slope variance, and intercept slope correlation estimates again suggesting greater variability in the observations and less precision in the estimates. The slope variance was underestimated in LCARMA and LCAR models with weights and overestimated in all the LCMA and LC models fitted to data sets having MA values of 0.33, 0.67, and 0.85. Although the values of intercept variance were overestimated for the LCMA and LC models with weights, the values remained fairly constant with change in the MA values. The intercept slope correlation was overestimated in the LCARMA models with weights and the magnitude of overestimation increased as the MA values increased from 0.33 to 0.85. All the latent curve models without weights had downward bias in mean intercept and upward bias in the slope variance, and intercept slope correlation estimates for latent curve models having weights.

The preliminary results obtained by analyzing the ECLS-K data showed that just correctly applying the weights is not sufficient to obtain the most accurate estimates because the fit indices for the latent curve model having an ARMA process without weights had poor fit indices. All the eight models (latent curve having weights but no ARMA, latent curve having no weights and no ARMA, latent curve AR with weights, latent curve AR without weights, latent curve MA with weights, latent curve MA without weights, LCARMA with weights and LCARMA without weights) were fitted to the math and reading scores in the ECLS-K data set. The fit indices for the LCARMA model with weights fitted to the math and reading achievement scores were the best out of all the eight models. The standard deviations and associated standard errors were high for the latent curve models with weights than without weights because the C7CW0 weights that was constructed in the ECLS-K data set to account for all the child-level characteristics across seven waves of data most likely did not take the ARMA process into consideration. The latent curve estimates of the models without weights were lower than the models with weights which was similar to the simulation results. Wider differences were observed in the mean slope values within the latent curve models with weights and no weights fitted to AR, MA and ARMA data sets (except in the latent curve models fitted to MA process). The number of iterations taken for model convergence should be used as a secondary measure for assessing model fit.

Table 100: Summary for ARMA Process (AR-0.33, MA-0.30)

Model (AR-0.33; MA-0.30)	**ARlag1**	**Mean Intercept**	**Mean Slope**	**Intercept Variance**	**Slope Variance**	**Int. Slope Corr.**	**MA1**	**χ^2**	**SRMR**	**RMSEA**	**AIC**	**Mc**	**CFI**
LC-wt		11.9870 (2.3325)	3.2420 (0.3240)	1161.167 (113.7372)	21.7955 (2.1949)	81.2300 (12.3211)		1599.5485	0.3314	0.4850	1625.5485	0.0269	0.7464
LC-nowt	0.66896	9.1211 (0.4434)	2.7778 (0.0536)	42.6548 (4.1814)	0.5919 (0.0601)	1.4596 (0.3709)		1866.0536	0.5503	0.5242	1892.0636	0.0167	0.7013
LCAR-wt	0.7107723	7.9078 (1.3332)	0.4750 (0.1443)	384.6776 (39.2959)	2.8546 (0.3635)	-12.2813 (3.2529)		415.9144	0.0678	0.2434	443.9144	0.4159	0.9376
LCAR-nowt	0.6050821	6.3317 (0.2571)	0.8924 (0.0462)	15.2540 (1.5227)	**0.0549** (0.0103)	**-0.0174** (0.1050)	17.3366	387.4536	0.0155	0.2347	415.4636	0.4395	0.9418
LCARMA_wt	0.6083922	**8.7085** (1.4619)	**0.9938** (0.1419)	448.2208 (45.5090)	2.0072 (0.3614)	-1.1760 (3.7155)	18.3760	52.3778	0.0266	0.0406	82.3778	0.9557	0.9960
LCARMA_nowt*	0.5737523	6.4172 (0.2635)	**0.9973** (0.0513)	17.9324 (1.8045)	**0.0532** (0.0200)	0.4853 (0.1501)	0.6411	37.3830	0.0149	0.0239	67.3730	0.9860	0.9984
LCMA_wt		11.4231 (2.1808)	3.3454 (0.3244)	987.0616 (99.3196)	20.4229 (2.2012)	98.4192 (12.0951)	26.0313	835.1201	0.2493	0.3531	863.1201	0.1564	0.8699
LCMA_nowt		8.2741 (0.3793)	2.9544 (0.0544)	29.6982 (2.9908)	0.5611 (0.0617)	2.6017 (0.3434)	0.8827	1113.6719	0.3012	0.4098	1141.6719	0.0819	0.8236

* Best Fitting Model; Latent Curve with weight-LC wt; Latent Curve with no ARMA and no weight-LC no ARMA & wt; Latent Curve AR with weights-LCAR wt; Latent Curve AR with no weight-LCAR no wt; LCARMA with weight-LCARMA wt; LCARMA with no weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no weight-LCMA no wt; χ2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI- Comparative Fit Index. Int. Slope Corr.-Intercept Slope Correlation. Numbers in brackets are standard errors associated with that LC estimate. **Bold numbers are values close to population estimates**. Population Value of Mean Intercept: 8.7295; Population Value of Mean Slope: 1.0019; Population Value of Intercept Variance: 13.5423; Population Value of Slope Variance: 0.0483; Population Value of Intercept Slope Correlation: 0.0209; ARlag1-Autoregressive estimate at lag 1; MA-Moving average estimate at lag 1.

Table 101: Summary for ARMA Process (AR-0.85, MA-0.80)

Model (AR-0.85; MA-0.80)	ARlag1	Mean Intercept	Mean Slope	Intercept Variance	Slope Variance	Int. Slope Corr.	MA1	χ^2	SRMR	RMSEA	AIC	Mc	CFI
LC-wt		8.8337 (9.5730)	9.5730 (1.2321)	480.9227 (45.8854)	326.9095 (37.0391)	372.8016 (37.0392)		2013.7021	0.0695	0.5453	2039.7021	0.0104	0.7492
LC-nowt		5.6951 (7.8642)	7.8642 (0.2037)	19.4915 (1.8940)	8.9338 (0.8621)	110.8993 (1.1627)		2422.4987	0.1307	0.5988	2448.4987	0.0041	0.6958
LCAR-wt	0.9989764	**8.7696** (0.2708)	0.2707 (0.4113)	483.2603 (47.9928)	4.8599 (1.7591)	-32.7828 (15.8700)		659.0156	0.0125	0.3121	687.0156	0.2346	0.9204
LCAR-nowt	0.954429	6.4327 (0.7558)	0.7557 (0.2560)	13.7725 (1.3577)	0.0933 (0.0155)	**0.0155** (0.2521)		660.4773	0.0104	0.3125	688.4773	0.2333	0.9198
LCARMA_wt	0.9568346	**8.7685** (0.6653)	0.6652 (0.3904)	474.7283 (47.0315)	5.9125 (1.6280)	-11.688 (15.7581)	28.824903	44.4739	0.0228	0.0394	74.4739	0.9663	0.9979
LCARMA_nowt*	0.9185099	6.4116 (1.0273)	1.0272 (0.1710)	13.7059 (1.3411)	0.0502 (0.0142)	**0.0143** (0.2720)	0.800269	33.0722	0.0233	0.0216	63.0722	0.9908	0.9993
LCMA_wt		8.9849 (9.6149)	9.6149 (1.4275)	463.5308 (42.7227)	316.071 (30.5656)	362.9114 (35.5847)	41.557471	788.8032	0.0794	0.3426	816.8032	0.1753	0.9040
LCMA_nowt		6.7232 (7.8301)	7.8300 (0.2429)	13.6858 (1.2499)	8.7555 (0.8484)	10.6038 (1.0076)	1.74895	1256.5226	0.0860	0.4358	1284.5226	0.0595	0.8440

* Best Fitting Model; Latent Curve with weight-LC wt; Latent Curve with no ARMA and no weight-LC no ARMA & wt; Latent Curve AR with weights-LCAR wt; Latent Curve AR with no weight-LCAR no wt; LCARMA with weight-LCARMA wt; LCARMA with no weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no weight-LCMA no wt; χ2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI- Comparative Fit Index. Int. Slope Corr.-Intercept Slope Correlation. Numbers in brackets are standard errors associated with that LC estimate. **Bold numbers are values close to population estimates**. Population Value of Mean Intercept: 8.7281; Population Value of Mean Slope: 1.0494; Population Value of Intercept Variance- AR 0.85, MA 0.80: 13.8082; Population Value of Slope Variance: 0.0446; Population Value of Intercept Slope Correlation: 0.0767; ARlag1-Autoregressive estimate at lag 1; MA-Moving average estimate at lag 1.

Table 102: Summary for ARMA Process (AR-0.00, MA-0.00)

Model (AR-0.85; MA-0.80)	**ARlag1**	**Mean Intercept**	**Mean Slope**	**Intercept Variance**	**Slope Variance**	**Int. Slope Corr.**	**MA1**	**χ^2**	**SRMR**	**RMSEA**	**AIC**	**Mc**	**CFI**
LC-wt		**8.7242** (1.4958)	0.9977 (0.1088)	472.2739 (46.5938)	1.9385 (0.2507)	0.1239 (2.4113)		44.20325	0.019106	0.039905	70.20325	0.970291	0.99635
LC-nowt	1.0118	6.3866 (0.2520)	0.9928 (0.0186)	13.0901 (1.3273)	0.0446 (0.0078)	0.0356 (0.0713)		40.56075	0.019874	0.032698	66.57075	0.978471	0.99729
LCAR-wt	1.0047	1.5402 (0.2375)	-0.1259 (0.0581)	-36.3607 (2.6991)	-1.8064 (0.1345)	7.2828 (0.5422)		175.9837	0.190479	0.149529	203.9837	0.71488	0.961546
LCAR-nowt	0.0101	6.3702 (0.2536)	0.9855 (0.0249)	13.19525 (1.3349)	**0.0528** (0.0081)	-0.0048 (0.0724)	-24.285	38.71485	0.019522	0.032201	66.72485	0.980249	0.997501
LCARMA_wt	0.9945	1.3464 (0.2348)	-0.0615 (0.0561)	-0.2788 (3.1301)	0.0043 (0.1554)	0.1908 (0.6236)	-24.0263	56.7465	0.154076	0.065426	86.7465	0.938071	0.992687
LCARMA_nowt*	-0.0017	6.4286 (0.2615)	**1.0010** (0.0261)	15.9516 (1.5899)	0.0607 (0.0085)	0.0113 (0.0823)	0.068129	34.6848	0.018253	0.025655	64.6748	0.987142	0.998202
LCMA_wt		**8.7240** (1.4960)	0.9978 (0.1083)	471.1968 (46.6148)	1.8509 (0.2526)	0.3829 (2.4065)	1.751712	39.6333	0.019174	0.033902	67.6333	0.978286	0.997199
LCMA_nowt		6.4114 (0.2522)	0.9984 (0.0184)	13.3362 (1.3204)	**0.0527** (0.0072)	0.0011 (0.0686)	0.051551	35.90236	0.018935	0.025635	63.90236	0.986651	0.998139

* Best Fitting Model; Latent Curve with weight-LC wt; Latent Curve with no ARMA and no weight-LC no ARMA & wt; Latent Curve AR with weights-LCAR wt; Latent Curve AR with no weight-LCAR no wt; LCARMA with weight-LCARMA wt; LCARMA with no weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no weight-LCMA no wt; χ2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI- Comparative Fit Index. Int. Slope Corr.-Intercept Slope Correlation. Numbers in brackets are standard errors associated with that LC estimate. **Bold numbers are values close to population estimates**. Population Values of Mean Intercept: 8.7337; Population Values of Mean Slope: 1.0023; Population Values of Intercept Variance: 13.4241; Population Values of Slope Variance: 0.0529; Population Values of Intercept Slope Correlation: 0.0218; ARlag1-Autoregressive estimate at lag 1; MA-Moving average estimate at lag 1.

Table 103: Summary for MA Process (AR-0.00, MA-0.33)

Model (AR-0.00; MA-0.33)	ARlag1	Mean Intercept	Mean Slope	Intercept Variance	Slope Variance	Int. Slope Corr.	MA1	χ^2	SRMR	RMSEA	AIC	Mc	CFI
LC-wt		**8.7264** (1.5176)	**0.9976** 0.1238	486.2958 (47.8128)	2.7324 (0.3213)	-2.5922 (2.7853)		428.3163	0.0227	0.2438	454.3163	0.4004	0.9028
LC-nowt		6.4104 (0.2564)	**0.9982** 0.0209	13.6820 (1.3459)	0.0779 (0.0091)	-0.0825 (0.0791)		424.2022	0.0224	0.2427	450.2022	0.4036	0.9028
LCAR-wt	1.0037079	1.3959 (0.2630)	-0.0902 0.0672	-16.8322 (2.3674)	-0.7098 (0.1267)	3.0093 (0.4993)		162.9182	0.1519	0.1429	190.9182	0.7365	0.9675
LCAR-nowt	0.0608429	6.2082 (0.2482)	0.9121 0.0261	12.6392 (1.264)	0.0704 (0.0087)	-0.1308 (0.0746)		397.5036	0.0317	0.2385	425.5036	0.4283	0.9092
LCARMA_wt	1.0082169	1.4607 (0.2618)	-0.1144 (0.0677)	-35.256 (0.0678)	-1.6169 (0.2010)	6.5294 (0.7810)	14.332835	122.6257	0.1545	0.1216	152.6257	0.8062	0.9771
LCARMA_nowt*	-0.003139	6.4108 (0.2540)	1.005 (0.0270)	**13.3997** (1.3412)	**0.0468** (0.0081)	0.0103 (0.0751)	0.5245855	34.5165	0.0208	0.0257	64.5165	0.9875	0.9983
LCMA_wt		**8.7254** (1.5133)	**0.9982** (0.1111)	476.1705 (47.6007)	1.6204 (0.2781)	0.4116 (2.5275)	18.217294	39.1043	0.0224	0.0326	67.1043	0.9795	0.9975
LCMA_nowt		6.409 (0.2538)	**0.9987** (0.0188)	**13.3829** (1.3393)	**0.0463** (0.0079)	0.0039 (0.0719)	0.523461	35.7211	0.0220	0.0259	63.7211	0.9871	0.9982

* Best Fitting Model; Latent Curve with weight-LC wt; Latent Curve with no ARMA and no weight-LC no ARMA & wt; Latent Curve AR with weights-LCAR wt; Latent Curve AR with no weight-LCAR no wt; LCARMA with weight-LCARMA wt; LCARMA with no weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no weight-LCMA no wt; χ2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI- Comparative Fit Index. Int. Slope Corr.-Intercept Slope Correlation**.** Numbers in brackets are standard errors associated with that LC estimate. **Bold numbers are values close to population estimates**. Population Values of Mean Intercept: 8.7298; Population Values of Mean Slope: 1.0033; Population Values of Intercept Variance: 13.5607; Population Values of Slope Variance: 0.0479; Population Values of Intercept Slope Correlation: 0.0215; ARlag1-Autoregressive estimate at lag 1; MA-Moving average estimate at lag 1.

Table 104: Summary for MA Process (AR-0.00, MA-0.85)

Model (AR-0.00; MA-0.85)	**ARlag1**	**Mean Intercept**	**Mean Slope**	**Intercept Variance**	**Slope Variance**	**Int. Slope Corr.**	**MA1**	χ^2	**SRMR**	**RMSEA**	**AIC**	**Mc**	**CFI**
LC-wt		**8.7283** (1.5267)	0.9967 (0.1279)	493.2691 (48.3964)	2.9158 (0.3459)	-3.0512 (2.9026)		660.5930	0.0287	0.3072	686.5930	0.2341	0.8401
LC-nowt		6.4116 (0.2561)	0.9964 (0.0218)	13.8762 (1.3622)	0.0849 (0.0100)	**0.0101** (0.0832)		658.1995	0.0289	0.3066	684.1995	0.2349	0.8390
LCAR-wt	1.0131261	1.4069 (0.2626)	-0.1181 (0.0733)	-23.2777 (2.6920)	-0.9618 (0.1536)	4.0332 (0.5773)		313.5649	0.1669	0.2093	341.5649	0.5202	0.9280
LCAR-nowt	0.0676297	6.2409 (0.2502)	0.8901 (0.0279)	12.9832 (1.2921)	0.0772 (0.0097)	-0.1837 (0.0809)		624.8132	0.0370	0.3035	652.8132	0.2532	0.8473
LCARMA_wt	1.0223313	1.4594 (0.2562)	-0.1528 (0.0744)	-64.4612 (5.1707)	-3.0032 (0.2753)	11.8350 (1.0354)	34.754179	175.4033	0.1893	0.1526	205.4033	0.7139	0.9628
LCARMA_nowt*	-0.002994	6.3959 (0.2536)	1.0055 (0.0289)	13.6156 (0.2528)	0.0439 (0.0083)	0.0046 (0.0692)	0.82356	32.5625	0.0279	0.0210	62.5625	0.9920	0.9986
LCMA_wt		**8.7188** (1.5082)	0.9988 (0.1002)	485.7284 (47.4906)	1.479 (0.2804)	0.1155 (2.417)	28.533112	37.3530	0.0303	0.0299	65.3530	0.9834	0.9978
LCMA_nowt		6.4042 (0.2527)	0.9977 (0.0170)	13.6456 (1.3341)	0.0433 (0.0081)	-0.0056 (0.0761)	0.82316	33.7951	0.0300	0.0211	61.7951	0.9915	0.9986

* Best Fitting Model; Latent Curve with weight-LC wt; Latent Curve with no ARMA and no weight-LC no ARMA & wt; Latent Curve AR with weights-LCAR wt; Latent Curve AR with no weight-LCAR no wt; LCARMA with weight-LCARMA wt; LCARMA with no weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no weight-LCMA no wt; χ2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI- Comparative Fit Index. Int. Slope Corr.-Intercept Slope Correlation. Numbers in brackets are standard errors associated with that LC estimate. **Bold numbers are values close to population estimates**. Population Values of Mean Intercept: 8.7195; Population Values of Mean Slope: 1.0070; Population Values of Intercept Variance: 13.8003; Population Values of Slope Variance: 0.0432; Population Values of Intercept Slope Correlation: 0.00087; ARlag1-Autoregressive estimate at lag 1; MA-Moving average estimate at lag 1.

Table 105: Summary for AR Process (AR-0.33, MA-0.00)

Model (AR-0.33; MA-0.00)	**ARlag1**	**Mean Intercept**	**Mean Slope**	**Intercept Variance**	**Slope Variance**	**Int. Slope Corr.**	**MA1**	χ^2	**SRMR**	**RMSEA**	**AIC**	**Mc**	**CFI**
LC-wt		12.2776 (2.3569)	3.2083 (0.3006)	1181.601 (115.6183)	18.5712 (1.8827)	88.1064 (11.8909)		999.6424	0.3450	0.3810	1025.6424	0.1074	0.8360
LC-nowt		9.1511 (0.4285)	2.7926 (0.0484)	38.9976 (3.8126)	0.4763 (0.0487)	1.9535 (0.3296)		1314.9521	0.5257	0.4388	1340.9521	0.0516	0.7809
LCAR-wt	0.6401133	8.2911 (1.404)	0.7567 (0.1323)	422.57 (43.0062)	2.187 (0.3127)	-3.2669 (3.0906)		68.2863	0.0450	0.0556	96.2863	0.9272	0.9934
LCAR-nowt	0.5787549	6.3894 (0.2523)	0.9856 (0.0436)	13.2118 (1.3193)	**0.0549** (0.0075)	**-0.0174** (0.0798)		39.4569	0.0099	0.0337	67.4569	0.9786	0.9983
LCARMA_wt	0.5763265	**8.7085** (1.4978)	**0.9938** (0.1297)	469.9269 (47.4934)	1.8843 (0.2837)	0.2141 (3.2957)	1.700285	38.0987	0.0090	0.0336	68.0987	0.9794	0.9983
LCARMA_nowt*	0.5754653	6.3981 (0.2528)	**0.9973** (0.0455)	13.2402 (1.3249)	**0.0532** (0.0076)	0.0017 (0.0815)	0.048684	35.7735	0.0097	0.0286	65.7735	0.9846	0.9987
LCMA_wt		11.6903 (2.2139)	3.2997 (0.3075)	1024.722 (101.7673)	18.6521 (1.9692)	98.3448 (11.8169)	16.755748	713.2745	0.2646	0.3252	741.2745	0.2071	0.8843
LCMA_nowt		8.3214 (0.3748)	2.9468 (0.0515)	29.1666 (2.9177)	0.5106 (0.0555)	2.7272 (0.3321)	0.65675	1016.3370	0.3018	0.3909	1044.3370	0.1025	0.8317

* Best Fitting Model; Latent Curve with weight-LC wt; Latent Curve with no ARMA and no weight-LC no ARMA & wt; Latent Curve AR with weights-LCAR wt; Latent Curve AR with no weight-LCAR no wt; LCARMA with weight-LCARMA wt; LCARMA with no weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no weight-LCMA no wt; χ2- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI- Comparative Fit Index. Int. Slope Corr.-Intercept Slope Correlation. Numbers in brackets are standard errors associated with that LC estimate. **Bold numbers are values close to population estimates**. Population Values of Mean Intercept: 8.7267; Population Values of Mean Slope: 0.9984; Population Values of Intercept Variance: 13.3985; Population Values of Slope Variance: 0.0527; Population Values of Intercept Slope Correlation: 0.0182; ARlag1-Autoregressive estimate at lag 1; MA-Moving average estimate at lag 1.

Table 106: Summary for AR Process (AR-0.85, MA-0.00)

Model (AR-0.85; MA-0.00)	**ARlag1**	**Mean Intercept**	**Mean Slope**	**Intercept Variance**	**Slope Variance**	**Int. Slope Corr.**	**MA1**	**χ^2**	**SRMR**	**RMSEA**	**AIC**	**Mc**	**CFI**
LC-wt		7.9956 (1.8356)	9.7633 (1.1718)	719.0429 (69.9586)	295.0634 (28.5016)	404.8461 (41.8648)		1057.1684	0.1062	0.3921	1083.1684	0.0945	0.8679
LC-nowt		5.3323 (0.3068)	7.984 (0.1975)	20.2338 (1.9564)	8.4006 (0.8096)	11.2017 (1.1723)		1630.9869	0.1248	0.4899	1656.9869	0.0249	0.7930
LCAR-wt	0.9474406	**8.7704** (1.4950)	0.7582 (0.3555)	469.6726 (47.1133)	2.3364 (0.7671)	-8.6952 (13.1983)		41.3696	0.0079	0.0380	69.3696	0.9743	0.9985
LCAR-nowt	0.929092	6.426 (0.0253)	0.9458 (0.148)	**13.3493** (1.3245)	**0.0566** (0.0099)	-0.0655 (0.2101)		38.9792	0.0075	0.0339	66.9792	0.9796	0.9987
LCARMA_wt	0.9414791	**8.7645** (1.4975)	0.8154 (0.3669)	470.0673 (47.2906)	2.2665 (0.8091)	-6.2129 (13.7035)	1.854138	36.7912	0.0077	0.0314	66.7912	0.9824	0.9989
LCARMA_nowt*	0.9270263	6.423 (0.2536)	0.9617 (0.1527)	**13.3293** (1.3264)	**0.0545** (0.0104)	-0.0357 (0.2168)	0.0538595	34.5953	0.0075	0.0265	64.5953	0.9873	0.9991
LCMA_wt		8.0868 (1.7906)	9.7978 (1.1579)	668.6531 (66.8645)	287.1063 (27.8345)	402.4392 (40.7309)	18.733275	703.6092	0.0916	0.3228	731.6092	0.2125	0.9133
LCMA_nowt		4.958 (0.3200)	8.1492 (0.1920)	20.7814 (2.1274)	7.8689 (0.7653)	11.5495 (1.1862)	0.783224	1199.7009	0.1452	0.4257	1227.7009	0.0673	0.8486

* Best Fitting Model; Latent Curve with weight-LC wt; Latent Curve with no ARMA and no weight-LC no ARMA & wt; Latent Curve AR with weights-LCAR wt; Latent Curve AR with no weight-LCAR no wt; LCARMA with weight-LCARMA wt; LCARMA with no weight-LCARMA no wt; Latent Curve MA-LCMA wt; Latent Curve MA with no weight-LCMA no wt; $\chi 2$- chi square; SRMR-Standardized root mean square residual; RMSEA-Root Mean Square Error of Approximation; AIC-Akaike Information Criterion; MC-McDonald's Non-Centrality Index; CFI- Comparative Fit Index. Int. Slope Corr.-Intercept Slope Correlation. Numbers in brackets are standard errors associated with that LC estimate. **Bold numbers are values close to population estimates**. Population Values of Mean Intercept: 8.7393; Population Values of Mean Slope: 0.8804; Population Values of Intercept Variance: 13.3197; Population Values of Slope Variance: 0.0504; Population Values of Intercept Slope Correlation: -0.1393; ARlag1-Autoregressive estimate at lag 1; MA-Moving average estimate at lag 1.

Tables 107 and 108 provides information on the semi-partial eta square for latent curve estimates and associated standard errors respectively. The SE for the MA and LC models were higher than the SE of ARMA and AR models with the ARMA models having lowest SE. The eta-square was more reliable for the mean intercept and mean slope latent curve estimates because the SE were lowest for these two estimates when compared to SE of intercept variance, slope variance and intercept slope correlation.

Table 107: Summary of Semi-Partial Eta square (η^2) for Latent Curve Estimates

Model	Mean Intercept	Mean Slope	Intercept Variance	Slope Variance	Intercept Slope Correlation
Overall ANOVA	0.1167	0.0113	0.1006	0.1003	0.1337
ARMA (0.33, 0.30)	0.0233	0.0235	0.1350	0.2632	0.3444
ARMA (0.67, 0.60)	0.0104	0.0130	0.0552	0.3096	0.3344
ARMA (0.85, 0.80)	0.0184	0.176	0.003	0.3119	0.3405
AR (MA 0.00)	0.0146	0.0087	0.0807	0.1745	0.2468
MA (AR 0.00)	0.4576	0.3160	0.3740	0.4725	0.3429
LC (AR 0.00, MA 0.00)	0.4678	0.3314	0.3614	0.4678	0.3097

Note: η^2 reported for only interaction effects between model*weight

Table 108: Summary of Semi-Partial Eta square (η^2) for SE Estimates

Model	Mean Intercept SE	Mean Slope SE	Intercept Variance SE	Slope Variance SE	Intercept Slope Correlation SE
Overall ANOVA	0.0659	0.0839	0.0935	0.0944	0.0763
ARMA (0.33, 0.30)	0.0469	0.1520	0.1299	0.2392	0.1851
ARMA (0.67, 0.60)	0.0138	0.2816	0.0565	0.3023	0.1959
ARMA (0.85, 0.80)	0.0008	0.2116	0.0019	0.2941	0.1125
AR (MA 0.00)	0.0242	0.1534	0.0763	0.1702	0.1441
MA (AR 0.00)	0.3284	0.1198	0.3092	0.0725	0.2104
LC (AR 0.00, MA 0.00)	0.3401	0.1659	0.3163	0.0670	0.2239

Note: η^2 reported for only interaction effects between model*weight

CHAPTER FIVE: CONCLUSIONS, DISCUSSION & FUTURE WORK

Over the past several decades, there has been an increase in the availability of large-scale, longitudinal data sets that have been collected across multiple time periods (Duncan & Kalton, 1987) in the field of social sciences (such as NHES, PSID, NPSAS, HSFCES, NELS-88, and ECLS-K). There has also been growth in research related to the issues of multi-stage sample design (clustering, stratification and sample weights) which is an inherent part of these data sets (Hahs-Vaughn, 2005, 2011a). An important aspect of these data sets that has not been widely researched is examining the sample weights in context of time series processes and autocorrelation which is a common characteristic of panel data sets (Eideh & Nathan, 2006). Researchers analyzing the ECLS-K data set have come across issues related to autocorrelation (correlated errors) (Kaplan, 2002; Bryk & Bryk, 2011). Swankoski (2011) noted that the error terms for each child across time were correlated since ECLS-K is a panel data set and a standard ordinary least square regression model cannot be used for data having covariances between the data points and the errors attached to it. Rogosa (1979) indicated that "longitudinal panel designs are a combination of time-series and cross-sectional data, with measurements obtained on a wave at each time point" (p.275). It becomes important to simultaneously examine the influence of sample weights and multiple competing time series models (having a particular autocorrelative process) because assessing the behavior of estimates as a function of sample weights alone and ignoring time series models specifying AR, MA and ARMA processes or vice-versa does not provide accurate estimates. There has been extensive research to examine the autocorrelative processes underlying in panel data sets in the field of econometrics. However, there has been limited research that explore the autocorrelative processes existing between the observations in large-scale survey panel data sets in the field of education. More specifically, no research has been cited in the field of education which investigates the fit of multiple competing latent curve models having varying autocorrelative processes and weights (that have been specified a priori) to a survey panel data set and how the latent curve estimates are influenced by the interaction between autocorrelative processes and sample weights. The present dissertation aims to fill this void in the literature.

The current study was planned to achieve two objectives. The first objective was to ascertain the autocorrelative processes (auto-regressive-AR, moving average-MA, and autoregressive moving average-ARMA) underlying in ECLS-K data. Four facts supported the need for conducting research on the first objective. First, autocorrelation is present in longitudinal panel data that have correlated errors (Rogosa, 1979; Jöreskog, 1979; Marsh, 1993; Marsh & Grayson, 1994; Huitema & McKean, 2007). Second, Brak and Brak (2011) found AR process in the reading achievement scores of ECLS-K data set. They found the observations to be correlated and they had to make certain adjustments to the AR disturbances in order to obtain accurate growth estimates. This article adds support to the need for correctly modeling the type of autocorrelative process existing in the ECLS-K data set because it is a longitudinal panel data set (collected at seven time periods) and can be considered similar to time series because "the same group of individuals over seven occasions or time points are measured (i.e. panel study), the occasions for repeated measurements are equidistant in time, and enough measurement occasions over time are included" (p.603) (Sivo, 2001). Third, since ECLS-K is a panel data set where data has been collected on the same individuals at seven points in time (such as for math and reading achievement scores), it is ideal to model growth curve models (Kaplan, 2002) that specify AR, MA and ARMA processes because failure to account for autocorrelative processes results in biased growth or latent curve estimates. Sivo, Fan and Witta (2005) stated that "In practice, growth curve models are fitted to longitudinal data, alternative rival hypotheses to consider would include growth models that also specify autoregressive (AR), moving average (MA), and autoregressive moving average (ARMA) processes. AR (i.e., simplex) processes are commonly found in longitudinal data and may diminish the ability of a researcher to detect growth if not explicitly modeled leading to bias in the estimates. MA and ARMA processes do not affect the fit of growth models, but do notably bias some of the parameters" (p. 215). Fourth, Sivo and Willson (1998) indicated that at least five or six occasions are recommended when testing ARMA (1,1) models. It was found in the current study that the full ARMA process is expressing itself in the ECLS-K data set when the latent curve models were fitted to the math and reading scores.

The second objective was to investigate the joint or interactive effects of sample weights and autocorrelation. Sample weights are commonly applied in large-scale panel survey data that have been usually collected at multiple points in time on the same individuals and have complex multi-stage sample design features. Two types of bias are usually introduced in such a data set. First, the complex sample design features leads to unequal selection probabilities where each unit in the population does not have an equal chance of selection and are sampled at different rates (DuMochel & Ducan, 1983; Stapleton, 2002; Hahs-Vaughn, 2006). The sample then is not a true representation of the population. The second source of bias occurs due to the dependencies that is created in the data. To be clear, the scope of this dissertation is to model stratification as a feature of complex sample design and not the dependency that is created due to clustering (as the units within the clusters are similar to each other when compared to between clusters). In order to model the dependencies between the observations, autocorrelation was considered in the current study that occurs when data is collected on the same individuals over multiple occasions in time (Macurdy, 1982; Rowley, 1989; Sivo, 1997). Both disproportionate sampling and autocorrelation leads to bias in the estimates (Pfeffermann, 1993; Hahs-Vaughn, 20011b; Rogosa, 1979; Sivo, 2005). However, there are very few studies that have examined the influence of both these aspects simultaneously on the estimates. Specifically, no study has been located that evaluates how the presence or absence of one or both of these aspects biases the estimates.

Based on the objectives of the present study, the following three research questions were formulated:

a) To what extent will the presence or absence of one or more than one autocorrelative process and corresponding value influence the estimates in a longitudinal panel data set when sample weights are applied versus ignored?

b) Is there any interactive or joint effect between the autocorrelative process and sample weights on the estimates in a longitudinal panel data set?

c) What kind of autocorrelative process(s) is present in the ECLS-K data and to what extent will the presence or absence of more than one autocorrelative process correct the estimates when sample weights are applied versus ignored?

In order to answer the above research questions, a Monte Carlo simulation study was conducted in which the autocorrelative processes (AR-0.33, 0.67 & 0.85; MA-0.33, 0.67 & 0.85 and ARMA-0.33 & 0.30; 0.67 & 0.60; 0.85 & 0.80) were varied to create ten population data sets in which each observation had eight waves. Three disproportionate stratums were created so that unequal selection could be applied. PROC SURVEYSELECT procedure in SAS was utilized to randomly select 16,000 samples (each sample size of 72) after the disproportionate allocation. Sample weights were then applied to the sample data to account for the unequal selection probabilities that occurred due to disproportionate sampling. Eight competing models were specified a priori and fitted to a data having AR, MA and ARMA process with varying values. General linear models were constructed for each of the latent curve estimates (mean slope, mean intercept, intercept variance, slope variance, intercept slope correlation) to account for the variance explained by the individual and joint influence of autocorrelative process and weights on the estimates. The propriety of a latent curve model for each sample was assessed by consulting the model fit indices (AIC, BIC, CFI, RMSEA and Mc Donald Non-Centrality Index), the closeness (overestimation or underestimation) of the sample level latent curve estimates to population level population parameters, and the AR, MA and ARMA estimate values. The ECLS-K data set was interpreted in light of the simulation results.

The model that had the best fit indices was the LCARMA model without weights for all the three AR (0.33, 0.67, and 0.85), MA (0.33, 0.67, and 0.85) and ARMA (0.33 & 0.30; 0.67 & 0.60; 0.85 & 0.80) values. The LCARMA model without sample weights was the best fitting model than the one with sample weights but the fit was slightly better. The mean intercept was consistently underestimated for all the LC models having sample weights but no autocorrelative process fitted to data having ARMA process. The mean slope was closer to the population value for all the latent curve models having MA process for both with sample weight and without sample weights. However, the values were generally biased for the LCMA and LC models for the AR, MA and ARMA processes. The intercept variance, slope variance and intercept slope correlation latent curve estimate was overestimated biased for LCARMA models with sample weights for all the three processes. All the ANOVA models were significant at the 0.001 level including the interaction term of model type and weight. The root MSE (RMSE) associated with R^2 was smallest for mean intercept and mean slope. There was a high degree of variability observed in intercept variance, slope variance, and intercept slope correlation estimates and their associated standard errors (Table 36). Consequently, the coefficient of variation (CV) for these three latent curve estimates were also high. One possible reason for intercept variance, slope variance, and intercept slope correlation estimates not being close to population level latent curve values maybe due to the high RMSE and CV values as a significant portion of the variation in the estimates (which were regressed against model type and weights) was consumed by the error component in the general linear model and little variation was left to explain the unique interactive influence of ARMA process and weights on these three estimates. All the eight models were fitted to the math and reading scores in the ECLS-K data set. The fit indices for the LCARMA model with sample weights fitted to the math achievement scores were the best out of all the eight models. The standard deviations and associated standard errors were high for the latent curve models with sample weights than without sample weights.

Discussion

The discussion is arranged around the insights derived from the simulation study, general linear model results and ECLS-K data set. The fit indices and latent curve estimates for the eight models has been summarized for the lowest and highest AR (0.33 & 0.85), MA (0.33 & 0.85) and ARMA (0.33, 0.30 & 0.85, 0.80) values only. Sivo and Willson (2000) stated that “whenever evaluating longitudinal panel data are evaluated, the fit, propriety, and parsimony of all models should be considered jointly and compared before a particular model is endorsed as most suitable" (p.174). **The insight gained from the fit indices results is that it is important to assess the interplay between type and magnitude of autocorrelative processes and weights in longitudinal panel data sets having complex sample design by utilizing model fit indices in conjunction to AR and MA estimates of the models having weights versus no weights. Researchers who analyze such data sets should always investigate the autocorrelative process underlying the data and fit multiple competing models (modeling AR, MA and ARMA components) that have been specified a priori to the data set. In addition, the fit indices alone should not be the only criterion to judge the fit of the model. The fit indices, AR and MA estimates, propriety and parsimony of the model should be assessed together along with the presence or absence of weights for all the models before a particular model is deemed as most appropriate.**

The LCARMA model without sample weights was the best fitting model than the one with sample weights when fitted to AR data but the fit was slightly better. Although the ARMA model (with sample weights and without sample weights) was a slightly better fitting model when it was fitted to data having AR process than the LCAR model (with sample weights and without weights) but it is important to consider the MA estimate for both the models in conjunction with the fit indices. The LCARMA is a more accommodating model because it has both the AR and MA components specified into it. The MA estimate was close to zero (MA estimate of 0.048684-no weight & 1.70028-with weight when fitted to data having AR process of 0.33 and MA estimate of 0.05386-no weight & 1.854138-with weight when fitted to data having

AR process of 0.85) for the LCARMA model indicating that the LCAR model is a better fit to the data having AR process. In addition, the AR estimates for the LCAR model with sample weights were higher than the LCAR model without sample weights and LCARMA models when fitted to data sets having AR process of 0.33 and 0.85 making the LCAR model with sample weights to be the best fitting model for the data having AR process.

Similarly, the ARMA model without sample weights was the best fitting model to the MA data when compared to the LCMA model. The AR estimates were close to zero (AR estimate of 1.00821685-no weight & -0.0031385-with sample weight when fitted to data having MA process of 0.33 and AR estimate of 1.02233125-no sample weight & -0.0029936-with sample weight when fitted to data having MA process of 0.85) for the LCARMA model indicating that the LCMA model with sample weights is a better fit to the data having MA process as the MA estimates for this model is highest among all the models after the AR estimates were considered. A model having the ARMA component in it and fitted to data having AR or MA processes will have better fit indices than the LCAR and LCMA models because the LCARMA is an all-encompassing model. The MA estimates for LCMA model with sample weights were higher than LCMA model without sample weights and LCARMA models when fitted to data set having MA process of 0.33 but not for data having 0.85 MA **suggesting that fit indices of LCMA model with sample weights can be more deceptive when used alone to evaluate the fit to a data when the MA process is high (MA 0.85)**. Although the AR estimates for the LCAR model with sample weights were high as compared to the LCARMA model with sample weights but there was not much difference between the estimates especially when the AR process was high (AR 0.85). **For practical purposes, weights, fit indices, AR and MA**

estimates of a particular model should be consulted more intuitively when the AR and MA process is high in a panel data set in order to declare a model to be the best fit. The LCARMA model had the best fit indices when fitted to data having ARMA process. Although the LCARMA model without sample weights had a slightly better fit than LCARMA with sample weights but the AR and MA processes were high for the latter making it the best fitting model. Sivo and Willson (2000) evaluated the quasi-simplex, one-factor, AR, MA, and ARMA models in terms of adequate fit, propriety, and parsimony. They noted that "In most cases, the models matching the generated data types fit better than any of the four remaining models" (p. 180). The simulation results of the current study is consistent with Sivo and Willson (2000) results because AR, MA and LCARMA models with sample weights fitted well to the data having AR, AR and ARMA processes respectively.

The LCARMA models with sample weights had the best fit indices out of all the eight models for the math ($\chi2$-424.8563, SRMR-13964.98 RMSEA-0.0902, AIC-452.856, Mc-0.9182, CFI-0.9762) and reading ($\chi2$-479.474, SRMR-0.0590, RMSEA-0.0961, AIC-507.474, Mc-0.9076, CFI-0.9705) achievement scores in the ECLS-K data. However, SAS PROC CALIS procedure used the Moore-Penrose matrix pseudoinverse for model convergence which is a type of inverse matrix used in linear algebra to calculate the best fit solution for a set of linear equations that has difficulty attaining a proper solution indicating a lack of sufficient fit to some degree even though the fit indices were the best for LCARMA model (Barata & Hussein, 2012). In addition, the MA estimates were enormously negative, large and statistically significant and that alternative competing models might be a better fit to the ECLS-K data than the LCARMA model.

The second insight from this study focuses on the type of autocorrelative processes which should be considered as an important criterion along with the standard errors and confidence intervals when interpreting growth curve estimates in the context of complex sample weights. Furthermore, autocorrelative processes and sample weights interact with each other as sources of error to a statistically significant degree. Sivo and Willson (2000) stated that "The propriety of a solution generated for a model should be evaluated according to several criteria, including the behavior of the parameter estimates and associated standard errors, model identification, and the iterative estimation procedure's attainment of successful convergence" (p.180). In the current simulation study, the mean intercept for the LCARMA model and LCAR model with sample weights was close to the population mean values (AR .33-8.7267; AR .67-8.7226; AR .85-8.7393; MA .33-8.7298; MA .67-8.7224; MA .85-8.7195) when fitted to the AR data. The mean intercept was overestimated for the LC model with sample weights when fitted to the AR data. The magnitude of overestimation decreased when the LC model with sample weight was fitted to data having AR process of 0.85. However, the mean intercept was underestimated when the LC model with no sample weights was fitted to the AR data (except when it was overestimated when LC model fitted to data having AR of 0.33). The mean intercept was overestimated for LCMA model with sample weights and underestimated for LCMA model without sample weights when fitted to AR data (except when it was overestimated when LCMA model without sample weights was fitted to data having AR process of 0.33 and 0.67). The estimate for mean intercept was closest to population values for LCARMA model with sample weights when it was fitted to ARMA data having AR 0.85, MA 0.80. The mean intercept was overestimated for LCMA and LC models with sample weights fitted to ARMA data having AR 0.33, MA 0.30 and AR 0.85, MA 0.80. However, it was underestimated for LCMA and LC models with no sample weights when fitted to ARMA data (except for LC model with no sample weights where it was overestimated). All the models had biased mean intercept values when fitted to data where no autocorrelative process was specified (except for LCMA and LC models with sample weights). The magnitude of bias was more for LCARMA and LCAR models with sample weights than without sample weights when fitted to data having no autocorrelative process. The mean intercept values were close to population values when the LCMA and LC models with sample weights were fitted to data having MA process but the values were underestimated for both models without sample weights. Overall, the mean intercept was underestimated when the LCARMA and LCAR models were fitted to MA data. It was upwardly biased when LCMA and LC models with weights were fitted to AR data and downwardly biased for LCMA and LC models without sample weights. **Hence, it can be inferred that sample level latent curve estimate for mean intercept is more influenced when LCAR and LCARMA models with sample weights are fitted to data having AR process. The mean intercept is closer to population estimate only when LC and LCMA models with sample weights is fitted to data having MA process. High levels of ARMA process in the data has some influence on mean intercept of LCAR and LCARMA models with sample weights.**

The mean slope was overestimated for the LCMA and LC models irrespective of sample weights when fitted to data having AR process. This trend was visible in all the LC models fitted to data having AR processes of 0.67 and 0.85. Although the mean slope was underestimated for LCARMA and LCAR models but the values were close to the population value (1.0033, 1.0062, and 1.0070 for MA processes of 0.33, 0.67, and 0.85 respectively) only for the LCARMA model with sample weights when fitted to data having AR process of 0.33. The magnitude of underestimation was more in the LCMA and LC models fitted to AR of 0.33 than with AR of 0.85. The mean slope was overestimated for all the models when fitted to ARMA data. This suggests that mean slope is overestimated when the data has an ARMA process probably because the mean slope has almost equal values for the estimate as well as RMSE resulting in high coefficient of variation (CV). The root mean square error (RMSE) and CV for mean intercept (Mean, 7.0782, RMSE-1.9729, CV-27.8739) was low when compared to the intercept variance, slope variance, and intercept slope correlation latent curve estimates suggesting precision and more reliability in the estimates. Furthermore, examination of confidence intervals for means of mean intercept show that they were tighter than the intervals for other latent curve estimates and that means for these two estimates were not present within the confidence intervals itself. This suggests statistical significance and accuracy in the estimates between the means of both these estimates with sample weights and without sample weights. Although the RMSE values for mean slope were also low, the high CV values for the estimate might be a possible reason for the upward bias in all the latent curve models and the data they were fitted to irrespective of sample weights. **Hence, it can be inferred that sample level latent curve estimate for mean slope is more influenced by MA process than the AR or ARMA processes. The mean slope in LCMA and LC models having sample weights would have a higher likelihood of approximating to population level value when they are fitted to data having a low MA process. The mean slope can be influenced to some degree when the LCARMA model is fitted to data having a low AR process.**

Sivo, Fan and Witta (2005) conducted a Monte Carlo simulation study to investigate the effects of unmodeled ARMA processes on growth curve (GC) parameter estimates. They found that the fit indices of GC model was poor as compared to the lag 1 GC autoregressive (GCAR) model. GC estimates were biased when the GC model was fitted to data having ARMA process. The intercept variance was upwardly biased that was due to the downward bias in the intercept slope correlation, a trend that was similar in the current simulation study. Sivo and Fan (2008) applied the simulation findings from Sivo, Fan and Witta (2005) paper to provide guidelines to researchers analyzing longitudinal data in which the change is occurring over time. They found that not utilizing the correct model underlying the data can underestimate or overestimate the true rate of change in reading scores of students based on a reading intervention. The intercept slope correlation was underestimated for the latent curve AR model and overestimated for the latent curve MA model. They also compared the fit of GC model and the GC moving average (GCMA) model to data having an MA process. The mean intercept, mean slope, and slope variance were all unbiased and the estimates were approximately same for the GC model and the GCMA model. The intercept variance was biased upward. The intercept slope correlation was modestly underestimated in the presence of an MA process. The intercept variance increased slightly, the slope variance was not affected, and consequently the intercept slope correlation decreased in the latent curve models fitted to data having an MA process.

In the current simulation study, the mean intercept variance was upwardly biased for all the latent curve models with sample weights when fitted to AR data. The magnitude of overestimation was higher for the LC, LCAR, and LCARMA models than the LCAR model thereby adding support to the better fit of LCAR model to AR data. The mean intercept variance of latent curve models without sample weights were also overestimated for the LC and LCMA models fitted to AR data. The mean intercept variance was closest to population estimate having AR process (13.3985-AR 0.33, 13.3549-AR 0.67, 13.3197-AR 0.85) for the LCAR (13.3493) and LCARMA (13.3293) models without sample weights fitted to AR data of 0.85. The best fitting model to AR data was LCAR with sample weights after evaluating the fit indices, AR and MA estimates followed by the LCARMA model. On the other hand, similar trends were observed where the mean intercept variance was closest to population estimate having MA process (13.5607- MA 0.33, 13.7468-MA 0.67, 13.8003-MA 0.85) when the LCMA (13.3829) and LCARMA (13.3997) models fitted to MA data of 0.33. The mean intercept variance was generally overestimated for LC and LCMA models with sample weights and underestimated for all LC models (AR 0.00, MA 0.00) without sample weights when fitted to ARMA data. None of the sample estimates for any model were close to the population estimate. Intercept variance is close to population estimate in the presence of AR process and almost equal when AR process is high in the data but it is near population estimate only when the MA process is low in the data. Although the intercept variance is underestimated for LCAR and LCARMA no sample weights models fitted to AR data but it is closer to the population estimate. **Hence, it can be inferred that sample level latent curve estimate for mean intercept variance is more influenced when LCAR and LCARMA models with no sample weights are fitted to data having high AR process whereas when LCMA and LCARMA models with no sample weights are fitted to data having low MA process.**

The mean slope variance was overestimated for the LC and LCAR models with sample weights fitted to data having AR, MA and ARMA processes (except for LCAR and LCARMA models with sample weights fitted to data having no autocorrelative process). The mean slope variance was closest to population estimate having AR process (0.0527-AR 0.33, 0.0518-AR 0.67, and 0.0504-AR 0.85) for the LCAR and LCARMA models without sample weights fitted to AR data. The mean slope variance was close to population level data (0.0479- MA 0.33, 0.0442-MA 0.67, 0.0432-MA 0.85) having MA process for LCMA and LCARMA models fitted to data having MA process of 0.33. The mean slope variance was biased upwardly for all the models with sample weights fitted to ARMA data with the magnitude of bias increasing from ARMA (0.33, 0.30) to ARMA (0.85, 0.80). The estimate was close to population level estimates (0.0483- ARMA 0.33, 0.30; 0.0453-AR 0.67, MA 0.60; 0.0446-AR 0.85, MA 0.80) all the latent curve no sample weight models fitted to ARMA (0.33, 0.30). None of the latent curve models had mean slope variance close to population values when fitted to ARMA of 0.67, 0.60 and 0.85, 0.80. **Hence, it can be inferred that sample level latent curve estimate for slope variance is more influenced by the AR process than the MA or ARMA processes. The intercept variance in LCAR and LCARMA models without sample weights would have a higher likelihood of approximating to population level value when they are fitted to data having an AR process than the data having an MA or ARMA processes. Slope variance can be influenced to some degree when the ARMA process is low in the data and have a higher chance of being closer to population-level estimate.**

The mean intercept slope correlation had mixed results when the latent curve models with sample weights and without sample weights were fitted to data having varying levels of AR, MA and ARMA processes. In general, the population level estimates of mean intercept slope correlation for AR (0.0182 -AR 0.33, -0.0182 -AR 0.67, and -0.1393 -AR 0.85), MA (0.0215 -

MA 0.33, 0.0119 -MA 0.67, 0.0087 -MA 0.85) and ARMA (0.0209- ARMA 0.33, 0.30; 0.0382 - AR 0.67, MA 0.60; 0.0767-AR 0.85, MA 0.80) data sets were not close to the sample level estimate of mean intercept slope correlation and were mostly overestimated when the latent curve models were fitted to the data specifying a autocorrelative process. An important point to note in the intercept variance, slope variance and intercept slope correlation estimates is that the bias is higher in models with weights than no sample weights irrespective of the type of data specifying AR, MA or ARMA process. The RMSE and CV for these estimates were high which indicates a large error component which might be a possible cause for masking the true interactive effects between the autocorrelative and weights, as noted in Chapter 4 summary results section. This could also be a reason for the population-level estimate to be close to sample level estimates of latent curve models without sample weights.

The results of the present study are consistent with the results reported by Sivo, Fan and Witta (2005) where they noted that “certain GC parameters are closely related to AR parameters, whereas others are more closely related to MA parameters as the degree of bias for given GC parameters depends on which process is at hand. Interpretation of the GC parameters without recognizing and modeling AR or MA processes will lead to a misunderstanding of the GC results. (p. 229). Examination of mean intercept estimates in the ECLS-K data set indicated that models with sample weights had higher values than models without sample weights (except for LCARMA model without weights where the estimate was higher than with weights). The mean slope estimate was higher for all models without sample weights (except for LCAR model where estimate with weights was higher). The intercept variance had negative values for all models without sample weights except LC model. The slope variance was higher in all models with sample weights with negative values for LCARMA and LCAR models without sample weights. The intercept slope correlation was higher for all models without sample weights except for LC model. The standard errors associated with the estimates were statistically significant between the models with sample weights and no sample weights as reflected in the confidence intervals. However, the standard errors were narrower for the latent curve estimates of models with no weights. Generally, the variance associated with the error terms increased with each wave of Math scores. The MA estimates for the LCARMA model with sample weights was extremely high and negative. On the other hand, LCARMA model with no sample weights had positive MA estimates having low values for standard errors when compared to LCARMA model with weights. The MA estimates and associated standard errors for the LCMA models with sample weights were also high when compared to the LCMA model with no sample weights a trend similar with the LCARMA models. The standard error of the means for math (Table 92) and reading (Table 94) variables having sample weights and no autocorrelative processes modeled were high than the latent curve model having no sample weights and no autocorrelation indicating lesser precision in the mean estimates when sample weights are accounted but the appropriate autocorrelative process is not modeled. One possible reason for high standard errors in the models with sample weights maybe be due to modeling the C7CW0 longitudinal sample weight which might not be accounting for the ARMA process between the math scores and also because the weighting increased the sample size and consequently the standard error. Similar trends were seen in the reading scores also where the MA estimates for LCARMA model with sample weights were very large and negative as compared to LCARMA no sample weights model. The confidence intervals for mean intercept and mean slope were narrower than the confidence intervals for intercept variance, slope variance and intercept slope correlation in both math and reading scores, a trend similar to the simulation results.

In the ECLS-K data, the standard errors associated with the means of reading and math achievement scores at each time point was higher for the latent curve model with sample weights and not accounting for the autocorrelative process than the standard errors of latent curve having no sample weights and no autocorrelative process. The standard error of latent curve estimates with sample weights were higher without sample weights (except for intercept slope correlation SE of models with weights) probably because the C7CW0 weight in ECLS-K data set did not account for the ARMA process underlying in the data. Out of all the LC estimates, the LCARMA model had the lowest values for SE out of all the other models whereas the LC had the highest values for standard errors. This trend was true for latent curve models with weights and without sample weights which further adds support to the presence of ARMA process in the ECLS-K data.

The fourth insight from this study centers around the effect sizes associated with latent curve estimates due to the interactive effects between the autocorrelative processes and sample weights. The effect sizes should be interpreted in light of the SE associated with the estimates, type and magnitude of autocorrelative process. The general linear models were significant (including the interaction effects between latent curve model type and weights) for each latent curve estimate. It is important to assess the ANOVA results in tandem with the effect sizes, root mean square error (RMSE), and coefficient of variation (CV) values. Although the semi-partial eta square (η^2) for latent curve estimates for mean intercept and mean slope had low values but the estimates could be considered accurate based on the low values of SE and their proximity to the population-level estimates as noted in the simulation results. Examination of the Tukey results also indicate that the means of mean intercept and mean slope differed to a statistically significant degree when only models and sample weights were separately considered. Assessment of confidence intervals derived from the standard errors and means of estimates associated with data having AR, MA and ARMA processes also show that means of mean intercept and mean slope were different to a statistically significant degree when the interaction between the type of model and sample weights were considered. The interaction plots add support to the results obtained from simulation and general linear model. In addition, it is difficult to account for the influence of interaction between model type and sample weight when the RMSE and CV values are high which was true for intercept variance, slope variance and intercept slope correlation.

Recommendations

The following entails some guidelines for researchers analyzing large-scale panel data sets:

1. The literature on time series processes suggests that fitting multiple competing models that have been specified a priori is essential to understand the true autocorrelative process underlying in the data in order to correct for the bias originating due to non-independent observations and to obtain accurate estimates. Identifying and modeling the correct type of dependency in the data is as important as recognizing dependency. The results from the current study recommends that biases from both autocorrelation and weights needs to be simultaneously modeled to obtain the accurate estimates. Accounting for just the autocorrelative process without weights or utilizing sample weights while ignoring the

autocorrelative process may lead to bias in the sample estimates particularly in large-scale datasets in which these two sources of error are inherently embedded.

2. The type of autocorrelation, magnitude of autocorrelation, and weights influences the behavior of certain estimates and all the three facets should be carefully considered to correctly interpret these estimates especially in the context of measuring growth or change in the variable of interest over time. The simulation results show that all growth curve estimates especially mean intercept (starting point) and mean slope (rate of change) are influenced by the interactive effects of AR, MA and ARMA processes of different magnitudes and when sample weights were considered.
3. The AR and MA estimates for the models should be used in conjunction with the fit indices, the consideration of sample weights versus no sample weights and the data to which the model(s) are fitted before a model can be declared appropriate for a data set. If the AR estimate is high and MA estimate is low for a LCAR model than the other models that are fitted to a data set having sample weights and the fit indices are in the acceptable cut-off range, then the data set has a higher likelihood of having an AR process between the observations. If the MA estimate is high and AR estimate is low for a LCMA model than the other models that are fitted to a data set having sample weights and the fit indices are in the acceptable cut-off range, then the data set has a higher likelihood of having an MA process between the observations. If both AR and MA estimates are high for a LCARMA model than the other models that are fitted to a data set having sample weights and the fit indices are in the acceptable cut-off range, then the data set has a higher likelihood of having an ARMA process between the observations.

4. Longitudinal weights in ECLS-K needs refinement to account for ARMA processes or a better fitting model. The sample weights in the simulation results were built from the population data after correctly accounting for the ARMA process. The C7CW0 longitudinal sample weight in the ECLS-K data set that accounted for the child assessment data from all the seven rounds of data collection most likely did not account for the ARMA process. It is not clear whether the appropriate autocorrelative process was accounted during the construction of C7CW0 longitudinal sample weight in the ECLS-K data set. There is a need to refine the C7CW0 weight that takes into account time series processes in order to account for the autocorrelative effects existing between reading and math achievement scores of students collected at seven time points. This would help researchers to model the correct nature of dependency when this longitudinal sample weight is utilized.

Delimitations & Future Research

The current simulation study was designed to incorporate only the stratification aspect of complex sample design and not the clustering effects. In the literature, research has been conducted on complex sample design by investigating the clustering effects on sample estimates. Pfeffermann et al. (1998) studied the influence of various weighting procedures on the standard estimators obtained from level 1 and level 2 units of sample data derived from a survey having multi-stage and multi-level survey design. Stapleton (2002) analyzed the clustering effects and sample weights in the context of multi-level structural equation modeling. Although the presence of clusters is an important feature of real-world data sets such as ECLS-K, it was not investigated in the current study because it is not a necessary condition to test for the interactive effects between the AR, MA and ARMA processes and weights. The LINEQS in SAS PROC CALIS which were used to build latent curve models specifying a particular autocorrelative process did not take the clustering effects into account which might be a reason for the high standard errors in the latent curve estimates when sample weights were applied with autocorrelation than when no autocorrelation and no sample weights were considered. In ECLS-K data sets, the design effects were incorporated to account for the clusters that first occurred at the primary sampling unit (PSU) level (district) and then at secondary sampling unit (SSU) (school). The stratification usually occurred on the variable that is directly related to the individual. One possible area of future research is to consider the clustering effects in order to understand the interactive effects of autocorrelation and sample weights from a multi-level model context. Future research should also consider investigating the effects of increased levels of multicollinearity that creates masking effects and prevents the true variation between the interaction of time series processes and sample weights to be displayed on the sample estimates. A certain level of correlation is desired to obtain interactive effects. However, multicollinearity subdues the unique variation in the estimates explained by the predictor variables when the correlations between them are extremely high.

The current simulation study was modeled with an assumption of lag 1 process existing between the observations wherein the value of current observation and associated error term is dependent on the observation immediately preceding it. A third possible area of future research is to fit latent curve models specifying AR, MA and ARMA processes at lag2 to the ECLS-K data as the data set has seven waves as it might be possible that the current observation and associated error term is dependent on the last two observations and their error terms immediately preceding it. A third promising area of research is to fit additional and multiple competing models to the ECLS-K data because the LCARMA model had an inverse matrix and enormously high MA values even though it had best fit indices than the other models.

A fourth promising area of research is to construct a bias parameter in the simulation study. Sivo, Fan and Witta (2005) had calculated percent parameter bias for the mean intercept, mean slope, intercept variance, slope variance, and intercept slope correlation. They found that percent parameter bias on average increased by 77% for mean intercept, and the mean slope increased by more than 200% when the AR lag 1 process was not modeled. The intercept variance increased by more than 300% while intercept variance and intercept slope correlation was greater than 300% and 200% respectively. In future simulation research, the percentage of bias can be calculated by taking the population-level true parameter values and subtracting it from the estimated value obtained from the sample data and then dividing by the true population level values to estimate the amount of bias when both the stochastic process and sample weights were properly modeled versus when either the stochastic process or weight is modeled. In other words, the parameters of interest can be compared when the AR, MA and/or ARMA process is modeled and sample weights are applied versus when the processes are not modeled and weights are applied. Similar effects can be investigated when the processes are modeled and sample weights are not applied versus when processes are not modeled and sample weights are applied.

Lastly, research can also be conducted to estimate the effects of missing values and different imputation techniques in the ECLS-K data set in the context of growth curve and autocorrelation. The ECLS-K data had approximately 21,400 observations across seven waves for the math and reading achievement scores. However, only about 2,360 observations were taken into account for modeling purposes because the remaining observations did not have complete data across all the seven time points.

APPENDIX A: SAS ARMA PROGRAM

```
LINEQS
Var_X1 = 0 * Intercept + 1 F_INT + 0 F_SLP + E1,
Var_X2 = 0 * Intercept + 1 F_INT + 1 F_SLP + ARlag1 Var_X1 + E2,
Var_X3 = 0 * Intercept + 1 F_INT + 2 F_SLP + ARlag1 Var_X2 + E3,
Var_X4 = 0 * Intercept + 1 F_INT + 3 F_SLP + ARlag1 Var_X3 + E4,
Var_X5 = 0 * Intercept + 1 F_INT + 4 F_SLP + ARlag1 Var_X4 + E5,
Var_X6 = 0 * Intercept + 1 F_INT + 5 F_SLP + ARlag1 Var_X5 + E6,
Var_X7 = 0 * Intercept + 1 F_INT + 6 F_SLP + ARlag1 Var_X6 + E7,
Var_X8 = 0 * Intercept + 1 F_INT + 7 F_SLP + ARlag1 Var_X7 + E8,
 F_INT = Mean_INT INTERCEPT + D1,
 F_SLP = Mean_SLP INTERCEPT + D2;
```

APPENDIX B: SAS MA PROGRAM

```
LINEQS
Var_X1 = 0 * Intercept + 1 F_INT + 0 F_SLP + E1,
Var_X2 = 0 * Intercept + 1 F_INT + 1 F_SLP + E2,
Var_X3 = 0 * Intercept + 1 F_INT + 2 F_SLP + E3,
Var_X4 = 0 * Intercept + 1 F_INT + 3 F_SLP + E4,
Var_X5 = 0 * Intercept + 1 F_INT + 4 F_SLP + E5,
Var_X6 = 0 * Intercept + 1 F_INT + 5 F_SLP + E6,
Var_X7 = 0 * Intercept + 1 F_INT + 6 F_SLP + E7,
 F_INT = Mean_INT INTERCEPT + D1,
 F_SLP = Mean_SLP INTERCEPT + D2;
```

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