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Talking Back: Mathematics Teachers Supporting Students' Engagement in a Common Core Standard for Mathematical Practice: A Case Study

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TALKING BACK: MATHEMATICS TEACHERS SUPPORTING STUDENTS' ENGAGEMENT IN A COMMON CORE STANDARD FOR MATHEMATICAL PRACTICE:

A CASE STUDY

by

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ABSTRACT

The researcher in this case study sought to determine the ways in which teachers support their students to create viable arguments and critique the reasoning of others (SMP3). In order to achieve this goal, the self-conceived classroom roles of two teachers, one experienced and one novice, were elicited and then compared to their actualized roles observed in the classroom. Both teachers were provided with professional development focused on supporting student engagement in SMP3. This professional development was informed by the guidelines that describe the behaviors students should exhibit as they are engaged in the standards for mathematical practice contained in the Common Core State Standards for Mathematics. The teachers were observed, video recorded, and interviewed during and immediately after the professional development. A final observation was performed four weeks after the PD. The marked differences in the teachers' characteristics depicted in each case added to the robustness of the results of the study. A cross-case analysis was performed in order to gauge how the novice and experienced teachers' roles compared and contrasted with each other. The comparison of the teachers' self-perception and their actual roles in the classroom indicated that they were not supporting their students as they thought they were. The analysis yielded specific ways in which novice and experienced teachers might support their students. Furthermore, the cross-case analysis established the support that teachers are able to provide to students depends on (a) teaching experience, (b) teacher content and pedagogical knowledge, (c) questioning, (d) awareness of communication, (e) teacher expectations, and (f) classroom management. Study results provide implications regarding the kinds of support teachers might need given their teaching experience and mathematics content knowledge as they attempt to motivate their students to engage in SMP3.

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CHAPTER 1: INTRODUCTION

Background

For many years, organizations such as the National Council of Teachers of Mathematics (NCTM), The National Governor's Association for Best Practices (NGA), the National Research Council (NRC) and the Council of Chief State School Officers (CCSSO) have voiced their concerns about the mathematics education that students receive. With publications such as *Principles and Standards for School Mathematics*, *Curriculum Focal Points*, *Adding it Up*, and many other timely reports, these organizations have attempted to disseminate information regarding mathematics teaching, students' expectations, and teacher preparation. American society has come to expect more from the education students receive, especially after comparing what we have termed "high-achieving" countries' performances in mathematics to that of American students (Achieve, 2010; NCTM, 2010b; NGA &CCSSO, 2010; Porter, 2002; Richardson, Ball, & Moses, 2009). Many studies, such as the ones described in *The Teaching Gap*, aimed to investigate how students learn, to develop more effective teaching methods, to provide more culturally relevant material, and, lately, to increase students' high stake testing scores by making teachers accountable for their students' performance (Lambdin & Walcott, 2007) have been performed. However, the reality is that besides all the elaborate methods and activities, "students' learning depends fundamentally on what happens inside the classroom as teachers and learners interact over the curriculum" (Ball & Forzani, 2011, p. 17).

This quest for higher achievement is not a new trend. On the contrary, the history of mathematics education is full of efforts pursued with the goal of increasing students' understanding of the mathematics they are learning. In the late 1950's and 1960's, the New Math movement sought to bring the performance of American students to a level high enough so that they would be prepared to enter careers focused on scientific discoveries and on engineering (NCTM, 1970). During this time, mathematicians recommended that

> "teachers should first promote the discovery of mathematical concepts through the manipulation of blocks, sticks, chips, or other objects; then present these concepts pictorially; and finally introduce the appropriate mathematical symbolism… educators emphasized the notion of a spiral curriculum – a curriculum in which ideas are returned to again and again in increasingly more complex and abstract forms" (Lambdin & Walcott, 2007, p. 11)

However, this movement was not as successful as the mathematicians that designed it thought it would be due to the fact that many parents were unable to help their children with the mathematics they were learning and teachers were unfamiliar with the recommended teaching methods (Lambdin & Walcott, 2007). Many criticized the new math. One of the most vocal critics of this educational movement was Morris Kline who expressed his concerns in his book, *Why Johnny Can't Add*. In there, Kline argues that the new mathematics put too much emphasis on formal definitions and axioms of mathematics, without letting students discover theorems and test them to see whether they were true or not. Additionally, he criticized the use of what he termed "pretentious terminology" in the mathematics classroom because it was not easily understood by students, the emphasis on mathematical structures that students were not able to understand, and he argued that the content was too abstract for students (Kline, 1973). This disapproval "led to recommendations that American mathematics education should go 'back to basics" (Lambdin & Walcott, 2007, p. 13) and initiated the 'back to basics' movement.

The back to basics movement was born from the need to give a real world function to the mathematics taught at schools, especially for career preparation. This educational movement was not backed up by any specific "psychological learning theories… though a return to connectionism (drill and practice) was common" (Lambdin &Walcott, 2007, p. 13). In addition, the start of the back to basics movement saw the first testing of students. Therefore, proficiency examinations became part of the learning practice (Kraus, 1978) as academic success was seen as the ultimate goal of education. Still, this goal was not achieved as the back to basics movement was characterized by “declining test scores, high levels of both functional and scientific illiteracy, the proliferation of remedial mathematics courses at the postsecondary level, and modest achievement on the part of the gifted” (Alexander & Pallas, 1984, p. 392). In 1983, The Commission on Excellence published *A Nation at Risk*, which equalized the educational crisis to a national threat and yet another educational cycle was over.

The end of the Back to Basics period saw the birth of the Standards Era and a renewed effort to bring consistency to what students learned in the classroom. Still, each state had its own standards and students’ mathematical education was dependent on where they lived (Byrd et al., 2010). In 2000, NCTM published *Principles and Standards for School Mathematics* and for the first time there was an effort to implement national standards. Even though the *Principles and Standards for School Mathematics* became the paradigm that mathematics educators and mathematics teacher educators tried to follow, many years went by before a concrete effort to have the same mathematics standards for all students became a reality. Currently, with the implementation of the Common Core State Standards for Mathematics, the curriculum will be alike for all students in most of the country (Achieve, 2010; Lambdin & Walcott, 2007).

Mathematics education research had always focused on the students’ achievement, in how they learn, and whether one method is more effective than others but that changed when Shulman (1986) put across the idea of teacher pedagogical knowledge, calling it the “missing paradigm” of educational research. He looked at several states’ teacher certification exams and realized that they indeed tried to gauge the content knowledge that teachers had, but he realized that “no one asked how subject matter was transformed from the knowledge of the teacher into the content of instruction" (p. 6). This notion changed the way researchers viewed teacher knowledge research. It was no longer enough to know what subject content knowledge teachers had, it was important to know how they transferred this knowledge to their students. Thus the research of pedagogical content knowledge of teachers and how it affects student learning was born.

The Student-Centered Mathematics Classroom

Within the school, the classroom becomes the place where learning takes place. Moreover, what happens there will determine how and what students will learn and in what ways they will use it in their future lives, at work or in everyday tasks (NCTM, 2000). Ideally, in this classroom, the teacher excels in engaging his or her students with meaningful mathematical tasks that promote the development of understanding while striving to teach the students how to express their mathematical ideas in an assortment of ways. The teacher steers students from conjectures and moves them to make generalizations in order to deepen their mathematical understanding. In a classroom such as this, there are many meaningful discussions lead by students. These conversations are centered in solving meaningful mathematics problems and explaining and evaluating solution methods and answers while the teacher is a catalyst for deepening understanding (Fennema et al., 1999; Hiebert et al, 1998). According to Hill et al. (2005), "knowledgeable teachers…provide better mathematical explanations, construct better representations, better 'hear' students' methods, and have a clearer understanding of the structures … of mathematics and how they connect" (p. 401). The teacher's role in a classroom such as this can be easily overlooked given its passive nature. However, this function is essential to what and at what level students learn (Hiebert et al., 2007; Shulman & Shulman, 2004).

A teacher's role in orchestrating this type of activity in the classroom requires a great amount of planning, thought, deliberateness, and, most importantly, depth of subject matter knowledge (Fennema & Romberg, 1999; Hiebert et al., 1998; NCTM, 2000; Stylianides & Ball, 2008). In addition, the teacher must give great thought to the selection of the problems the students will work on, as this is central to what will be learned in the classroom (NCTM 2000). The choice of the task the students are going to work on, argue many mathematics education researchers, is the most important decision that the teacher makes (Fennema et al., 1999; Hiebert et al., 1998; NCTM, 2000; Stigler & Hiebert, 1999). This decision must be made with the purpose of taking the students through the hypothetical learning trajectory that will take them to the next concept they are to learn or prepare them for the mathematics they are likely to encounter in later grades. "The trajectory is the teacher's vision of the mathematical path the class might take, and its hypothetical nature comes from the fact that it is based on the teacher's guess about how learning might proceed along the path" (Hiebert, et al., 1996, p. 34). Sometimes, this trajectory will take them to familiar concepts that they have already mastered but more times than none, students will face tasks that seem familiar but are really not and they will be forced to find ways to solve the problem in a way that fits the question itself and that is, at the same time, mathematically sound. In addition, teachers must possess mathematical knowledge that is beyond what their students might ever learn in their classrooms and knowledge of new technologies that might be available to use (NCTM, 2000). Currently, the preparation that teachers receive does not train them in these kinds of roles due to the increased amount of time it would require teacher candidates to graduate from such programs (Ball, 1990; Boyd et al., 2009; Hiebert et al., 2007). Nesbitt and Bright (1999) conducted a study with pre-service teachers that found the major factor influencing teacher behavior in the classroom was their own experiences when they were students themselves. This means that despite the completion of teacher preparation programs, teachers need the support of continuing education in order to become better educators so that they can support student learning (Hill et al., 2005) otherwise, they will revert to teaching the way they were taught. Consequently, the pedagogical knowledge a teacher needs to provide a learning environment such as the one previously described might best be obtained when the teacher is proactively looking to broaden and deepen his or her knowledge of teaching.

Researchers (Fennema et al, 1999; Hiebert, 1998; Stigler & Hiebert, 1999, 2009) have investigated the teacher-centered mathematics classroom and have attempted to describe it in contrast to the previously described image of the student-centered classroom. However, the most prominent of these research efforts is the Trends in International Mathematics and Science Study (TIMSS), which is conducted by the International Association for the Evaluation of Educational Achievement. TIMSS investigates the achievement of 4th and 8th grade students in mathematics and science. Currently, the study compares the achievement of students in 36 countries (Gonzales et al., 2009).

The Teacher-Centered Mathematics Classroom

TIMSS has been investigating classrooms and students' achievement since 1995 and it is designed to align broadly with mathematics and science curricula in the participating countries. The results, therefore, suggest the degree to which students have learned mathematics and science concepts and skills likely to have been taught in school. TIMSS also collects background information on students, teachers, and schools (Gonzales et al., 2009)

TIMSS (Mullis et al., 2008) found that mathematics classrooms in the United States were highly structured with a predictable pattern of behaviors. There was an emphasis on procedures, and unchallenging mathematics. The problems used in the US classrooms were found to be challenging but their challenging nature was lost when teachers perceive the problem too problematic for the students and offered help without giving students the chance to struggle and come up with an answer. This study found that 69% of the problems presented in the mathematics lesson used in the observed classroom were used to emphasize procedures. In addition, it also found that 13% of the problems used were employed to state or exemplify concepts, and 17% of the problems were used to make connections (Mullis et al., 2008). According to the 2007 TIMSS Report, the American classroom was characterized by a low level of mathematical challenges with a prevalence of routine exercises, problems were solved using "steps". Moreover, there seemed to be an emphasis placed on students' regular practice of familiar concepts often in preparation for standardized testing. The study also found that the elementary mathematics content was stripped to its barest elements in an effort to make the curriculum more accessible to all students. In addition, there was an emphasis to teach the steps involved in solving problems rather than promoting reasoning. This research study found that 69% of the problems presented in American classrooms were used to demonstrate procedures. Additionally, it found that 91% of the problems that were used during instruction in American classrooms were solved using procedures. According to the results of the study, teachers had no expectations that students would either reason or justify their answers. Students were solely judged by whether their answers were correct or not. Consequently, the reasonableness of the students' answers was of no importance. The study revealed that spiraling was still being widely used in schools. The authors argue that this might be in order to prepare students to take high-stakes standardized tests and because school administrators and the available resources support this practice. In addition, the report states that mathematics lessons were taught in a fragmented manner due to the misconception that single topics are learned more effectively. Thus, concepts were taught without making the connections that exist in mathematics. In addition to these, the report states that students' instruction was often interrupted by visitors to the classroom or school wide announcements (Mullis et al., 2008).

Traditionally, the secondary mathematics classroom in the United States has a high level of structure, with time blocks for different activities similar to the following. Students enter the classroom and there is some kind of work on the board for them to do while the teacher checks homework and attendance, this is referred to as either bell work or lesson openers. Then, the teacher goes over the answers of the bell work and introduces the lesson. At this point, the teacher shows examples of problems and demonstrates how to solve them. The students are assigned individual class work and they work on it while the teacher moves around the classroom observing students' work. After that, the teacher assigns the daily homework and the students leave. In the classroom just described, the teacher is at the center of the activities. The teacher demonstrates and explains processes and steps while the students are passive participants of this interaction. They listen and work independently (Stigler & Hiebert, 1999). The structure of how these mathematics lessons were planned and the routines followed were found to be very uniform from one grade to another (Stigler & Hiebert, 1999).

This type of classroom is characterized by the authoritative discourse that dominates the conversations within. The teacher dominates the conversation and the students passively accept it. There is no reflection in how the new information fits with what has been learned or with what will be learned in the future. Most importantly, there is no visible effort on the part of the students to make sense of what is being said. What the teacher says is accepted implicitly. In addition, students have to work within a set of conventions to which they have been accustomed.

For example, a common convention occurs when students are confused and they ask the teacher for help. Invariably, the teacher will tell them exactly how to solve the problem. In addition, students have the textbook examples to look at and follow. Still, these students work within very strict time limitations due to curricular goals and so not to be left behind. The urgency to finish the assigned problem supersedes any understanding that these students might or might not attain (Boaler, 1999).

In teacher-centered classrooms, students are limited to answer questions with few words and teachers tend to only look for the ‘right answer.’ No justification or reasoning is offered or expected. The purpose of this kind of questioning is to obtain the right answer only and to use it to quickly assess whether the students comprehend what the teacher had planned. Since the teacher is only seeking right answers, he or she continues to further assume that mutual understanding exists as long as he or she hears the desired answer. In this classroom, the students’ ideas and methods are not elicited and the learning is centered on the method demonstrated by the teacher. In this classroom, “the nature of mathematics is not opened to questioning by the students… the intended goal is for students to accept and understand mathematics as the teacher already knows it” (Wood et al., 1993, p. 58).

The role of communication is diminished by the nature of this kind of classroom. There is no expectation that students will communicate their thinking to each other or the teacher. In fact, the teacher dominates the communication and the extent of the students’ participation in this communication is to answer the questions posed by the teacher with few words. The established norms in the classroom environment prevent the flow of communication among students and there is a consensus that communication is a negative behavior in the classroom. The expectation of silence on the part of the students is evident in the posted classroom rules and in the consequences associated with breaking that silence (Pace & Hemmings, 2007).

Communication

Traditionally, the classroom has not been a place where a lot of active communication occurs. Usually,

> there is little or no mathematical discourse and communication typically consists of a lecture with recitation, where the teacher deviates very little from delivering a preplanned body of information and set of questions and students give very short answers… discourse is often choppy rather than coherent, and there is little follow-up of student responses. Such discourse can be viewed as the oral equivalent of fill-in-the-blank or short-answer questions. (Secada et al., 1995, p. 4)

NCTM argues that "when students are challenged to think and reason about mathematics and to communicate the results of their thinking to others orally, they learn to be convincing. Listening to others' explanations gives students opportunities to develop their own understandings" (NCTM, 2000, p. 60). The implication is that this is not just communication for the sake of talking, this communication is deliberate and planned. Its goal is for students to build their own mathematical reasoning as they construct and defend their rationale and as they listen to their peers' reasoning.

Verbal classroom interactions can be identified as either directive or supportive. According to Mehan (1987), directive interaction is centered on the assessment of the students' knowledge and teacher control. This spoken pattern is as follows: teacher initializes by asking a question (I), the student responds (R), and then the teacher evaluates the answer (E) as either being correct or not – this is referred to as IRE (Mehan, 1982, 1987). IRE is one of the most common directive interactions and it implies that the teacher evaluates while the students respond to questions with the answers they believe the teacher seeks. In addition, there is the unstated understanding that the students will correct their misunderstanding as they listen to these conversations (Van Bramer, 2003).

In contrast to directive interactions, supportive interactions are observed in classrooms that are more student-centered. In these types of conversations, students have control of the learning process and the exchange of ideas is viewed as a source of knowledge. The teacher's role shifts from the source of to the facilitator of the conversation. This interaction follows these steps: the teacher initializes by either asking a question or introducing a concept (I), one of the students responds to the question or comments on the given topic (R), and the answer given is used to elicit more responses (F) from the participants (Waring, 2008) – this is referred to as IRF. Conversations of this type can go on as long as the participants want (Van Bramer, 2003).

Communication in the classroom

Some education researchers (Schoenfeld, 1992; Steffe & Kieran, 1995; Von Glasserfeld, 1990, 1991) argue that the only way that students learn is to be active participants of the learning process, especially in mathematics. This means that students should work together in problem solving and should talk to each other in order to reach solutions. Additionally, when students explain their answers to each other it leads them to reflect and to look at their own logic critically. Moreover, to have their reasoning assessed by their peers is more readily acceptable to students than when it is corrected by the teacher (Schoenfeld, 1992; Steffe & Kieren, 1994; Von Glasserfeld, 1990, 1991). Unfortunately, this type of communication is not the norm in most United States classrooms (Mullis et al., 2008).

The concept of sharing ideas in the classroom is not a new notion. NCTM (2000) stresses the importance of classroom discourse, specifically through the Communication Standard. This standard states that communication is essential in the teaching and learning of mathematics and it should reflect the mathematical reasoning and thinking of students. Moreover, the teacher must bring forth this type of discussion from students without taking the lead. To be effective, this type of communication must be student centered, students must strive to explain their reasoning, and must listen carefully and assess others' reasoning. Students constructing arguments to justify their answers synthesize their ideas. When students listen to others' reasoning, they are given the opportunity to develop their own understanding (NCTM, 2000). Thus, the importance of allowing students to share their reasoning and assess the reasoning of their peers is crucial.

In teacher-centered, text-dominated classrooms, the students listen and watch their teacher demonstrate procedures to solve problems. The students must usually raise their hands to ask questions in these classrooms. Students rely on the teacher's unquestioned power for most of what happens in the classroom, from instruction to the routines they follow every day. In these classrooms, students are limited to answer questions just with one word and teachers only look for the 'right answer.' No justification or reasoning is explained or expected. The purpose of this kind of questioning is to obtain the right answer only and to quickly assess whether the students comprehend the task or lesson. Since the teacher is only seeking right answers, he or she continues to further assume that mutual understanding exists as long as he or she hears the desired answer (Romberg & Caput, 1999). In this classroom, the students' ideas and methods are not elicited and the learning is centered on the method demonstrated by the teacher. In this classroom, "the nature of mathematics is not opened to questioning by the students… the intended goal is for students to accept and understand mathematics as the teacher already knows it" (Wood et al., 1993, p. 58).

In contrast, when discourse is used in the classroom, the teacher is compelled to direct students to create their own mathematical meanings and to ensure that those meanings support the students' productive disposition towards mathematics – they perceive mathematics as useful (NRC, 2001). It is important for students to engage in activities that create opportunities for them to reflect on their own mathematical reasoning as they listen and make sense of others' solution methods and logic. These class discussions created "opportunities to learn as the teacher and students negotiated mathematical meanings that would enable students to make connections from their individual mathematical constructions with the taken-as-shared meanings of the classroom" (Wood et al., 1993, p. 56). In order to be successful in this kind of classroom, teachers must understand the ways their students think and must have deep conceptual knowledge of mathematics (NCTM, 2000).

Transforming the Classroom

The process of changing from a teacher-centered classroom to one that promotes communication and reasoning, where all voices are heard and where students and teacher share the power is very complex. In the beginning of this transformation, the teacher must have an active role in transmitting the new expectations and behaviors that students must meet. This process is one that is not done rapidly. It is an ongoing process that will have the teacher "sustain positive norms and recognize when new norms arise or current norms become unstable" (Dixon et al., 2009b, p. 64). This change will create a "potential for transforming many relationships of power, including the relationships among members of the classroom and the relations between students and the subject matter being studied" (Cornelius & Herrenkohl, 2004, p. 468). Teachers can achieve this change by eliciting questions, arguments, and other discussion contributions from students. The purpose behind this is that students must receive some of the authority over what they learn by making sense of it and by justifying their reasoning (Carpenter & Leher, 1999). This way, students are held accountable for their learning and will hold their peers accountable for theirs. Furthermore, students' desire to argue their point of view moves from a social activity to a learning activity in which students struggle to learn. This kind of learning activity is critical as "the process of explaining one's thinking to one's partner can involve organizing or reorganizing and reformulating one's solution process in order to verbalize it, reconceptualizing one's solution in order to provide alternative explanations, and distancing oneself from one's thinking to try to take the perspective of another. The process of engaging in collaborative dialogue involves developing mathematical communication on the basis of commonly shared mathematical activity" (Yackel, Cobb, & Wood, 1993a, p. 46-47). When the conversations in a classroom are orchestrated in such a manner, the essence of group discussions will change and will deviate from the conversations contained in the aforementioned teacher-centered classroom and will transform students' roles from passive recipients of knowledge to that of architects of their own knowledge.

As students start to create and share their mathematical reasoning behind their solution methods and answers, they may build inaccurate or incomplete mathematical constructions. These incomplete constructs will serve as discussion points that will become the building blocks upon which solid and accurate mathematical conceptual knowledge will be built. As students discuss their reasoning, "they distance themselves from the activity in order to extend their own interpretation and thus make sense of the other's activity. The subject of the conversation at times revolves about differences of interpretation" (Yackel, Cobb, & Wood, 1993b, p. 41). It is important that the teacher monitors these closely and addresses misconceptions. This is because "analyzing student solutions – both correct and incorrect is just one example of the many mathematical tasks of teaching" (Suzuka et al, 2009, p. 9). In contrast, students in teacher-centered classrooms also build constructs which are inaccurate or incomplete. The difference is that in these classrooms, the teacher is not aware of these gaps because students do not share and talk about the logic behind their answers. So, because of their silence, students are allowed to take their misconceptions from one grade to another (Wood et al., 1993).

It is important to note here that as the classroom is transformed, new norms are conjured up in a collaborative effort between the teacher and the students. “Although the teacher is an authority figure in the classroom, the teacher can only initiate and guide the process of establishing the social norms” (Dixon et al., 2009a, p. 45). As time passes and students get used to the new social and sociomathematical norms in the classroom, the role of the teacher changes once again and goes from orchestrator of discourse to one that monitors concepts explained by the students by respecting the students’ ideas, by giving them time to make sense of concepts and express their ideas, and by causing “confusion” when a new concept is introduced (Goos, 2004; Cornelius & Herrenkohl, 2004).

This “confusion” is not to be mistaken with an attempt to frustrate the students. It has to do with posing problems that contain what Hiebert (1998) termed “problematic mathematics.” This “problematic mathematics” is at the center of the concept of cognitive dissonance which can be explained by Festinger’s theory of cognitive dissonance, “according to this theory, the discomfort caused by logical inconsistency or contradiction motivates the individual to modify his or her beliefs in order to bring them into a closer correspondence with reality…in mathematics education… beliefs are so overwhelming that reality is modified to fit beliefs” (Zaslavsky, 2005, p. 299). There are three types of tasks that can encourage student uncertainty that in turn will support the active learning of mathematics and will promote interesting and intriguing conversations in the classroom. All the tasks promote uncertainty by offering conflicting statements, by posing open-ended questions, and by posing problems which have answers that cannot be readily proven. These tasks have one thing in common: the fact that the students will question either process or answer because it is not readily apparent or it is confusing because it presents a situation that is apparently conflicting or implausible (Zaslavsky, 2005). The development of these tasks is important to the mathematics teacher and the mathematics teacher educator. “A critical factor in successfully implementing a task with the potential of generating unforeseen elements of uncertainty and doubt relies on appropriate classroom setting and climate” (Zaslavsky, 2005, p. 318). The chosen task cannot be one that is easily solved with a procedure, but it should be one that goes beyond the procedure and challenges the students’ conceptual knowledge. When this type of task is paired with the expectation that students share their mathematical thinking and reasoning as they attempt to solve these problems, then students will have powerful opportunities to develop their mathematical conceptual knowledge and will learn to communicate mathematical ideas effectively. This is because cognitive dissonance is closely related to the process of making sense. As students deal with the dissonance caused by the unfamiliar task, they engage in reasoning (Aronson, 1997). However, students cannot be expected to do this by themselves. They will need the support of their teacher. The only way a teacher can provide the type of support the students will require is that he or she is provided with learning opportunities as well. These opportunities must include attention to content knowledge for teaching and teacher change to support discourse and must be provided in effective ways (Franke et al., 1998).

Rationale

The mathematics education that students receive is at a pivotal point where change is inevitable. In answer to the less than perfect performance of the United States in international mathematics testing, such as the Trends in International Mathematics and Science Study

(TIMSS), and in order to prepare future generations to meet the challenges of a global workplace, the National Governors Association (NGA) along with the Council of Chief State School Officers (CCSSO) developed the Common Core Standards Initiative and the Common Core State Standards for Mathematics (CCSSM). The goal proposed for the CCSSM is an ambitious one. President Obama proposed it during a 2009 address and it is that students "possess 21st Century skills like problem solving, critical thinking, and entrepreneurship and creativity" (President Obama's Remarks to the Hispanic Chamber of Commerce, para. 22). This goal mirrors student goals expressed by the NRC and NCTM in *Adding it Up* and in *Principles and Standards for School Mathematics*, respectively. In order to achieve this goal, it is not enough that teachers know the mathematics described in the content standards. Teachers must be mathematically proficient because of the "emphasis on Mathematical Practices require students to be able to think mathematically, and apply the techniques they have learned to rich problems in diverse contexts. Achieving this requires changes in the way mathematics is taught and assessed in most schools" (MARS, 2012, para. 1).

The behavior expectations that the Standards for Mathematical Practice (SMPs) will place on students will dramatically change the manner in which mathematics is taught and learned. This might cause teachers to move from a teacher-centered classroom to one that centers on a community of learners dedicated to solving meaningful problems as they increase their mathematics skill and knowledge, where the teacher role is one of facilitator of this knowledge. This will entail teachers changing classroom routines, expectations, and tasks. Additionally, it may change the ways that teachers need to support their students as they develop mathematics reasoning and understanding because "a lack of understanding effectively prevents a student from engaging in the mathematical practices" (NGA & CCSSO, 2010, p. 8). The implementation of the CCSSM and especially the SMP might be a challenge because teachers must learn to support students in ways that are unfamiliar.

Standards for Mathematical Practice

One feature that is unique to the CCSSM is the SMPs. These are eight standards that describe specific behaviors that must be observed in all students at all grade levels. They are described as "processes and proficiencies" in the CCSSM document (p. 6). Table 1 lists the eight SMPs and NCTM's Process Standards that they most closely reflect. It is important to note the emphasis that the SMPs make on having students solving problems and in justifying the logic and reasonableness of their answers by explaining their answers. These are consistent with the goals of the Problem Solving and Reasoning and Proof process standards (NCTM, 2000, p. 52-59). As can be seen in Table 1, the Standards for Mathematical Practice are closely related to the process standards and each can be matched to the corresponding process standards.

Table 1 - CCSSM's Standards for Mathematical Practice

Standard	NCTM's Process Standards
Make sense of problems and persevere in solving them	Problem Solving, Reasoning and Proof
Reason abstractly and quantitatively	Connections, Reasoning and Proof
Create viable arguments and critique the reasoning of others	Reasoning and Proof and Communication
Model with mathematics	Representations and Connections
Use appropriate tools strategically	Problem Solving
Attend to precision	Problem Solving
Look for and make use of structure	Reasoning and Proof, Problem Solving
Look for and express regularity in repeated reasoning	Reasoning and Proof, Problem Solving

NCTM, 2000; NGA & CCSSO, 2010

According to the CCSSM document, "the Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise" (NGA & CCSSO, 2010, p. 8). The Standards for Mathematical Practice are an "expectation of performance" and should be actively used by the teacher to constantly assess students' knowledge and make instructional decisions, as it is supported by the literature (Hiebert, et al., 1997; NCTM, 2000; Shafer & Romberg, 1999).

For the purpose of this study, considerable emphasis was placed on Standard of Mathematical Practice 3 (SMP3), "construct viable arguments and critique the reasoning of others" (NGA & CCSSO, 2010, p. 6). According to this practice, students are expected to build their reasoning in ways that are mathematically sound and logical and be able to assess their peers' logic. In order to create these arguments, students can use any model or representation to support their reasoning, giving validity to their logic (NCTM, 2000; NGA & CCSSO, 2010). This reasoning could be expressed in writing or verbally. Through their own experiences constructing their arguments, students are able to analyze the logic of others. "Students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments" (NGA & CCSSO, 2001, p. 7). The expectations associated with this standard should be incorporated in the classroom norms and the classroom environment should be one that encourages discourse among the students.

Teachers must be ready to support their students as they build their mathematical reasoning, express it, and assess the logic of others. This standard, by its own definition, contains two critical tasks that are absent from the teacher-centered classroom (Hiebert, 1998; Stigler & Hiebert, 1999, 2009): reasoning making and communication. In order for students to successfully fulfill SMP3, the fundamental fabric of the classroom must change to a more student-centered one. In addition, the teacher must change his or her views regarding power in the classroom

(Cornelius & Herrenkohl, 2004), about communication (Wood et al., 1993; Yackel et al., 1993a), and their perceived role in the classroom (Forman & Ansell, 2001). Teachers must provide students with the tools they need to be able to explicate their reasoning and they should choose problems that contain significant and problematic mathematics as this promotes the dissonance necessary for students to engage in meaningful reasoning activities (Blanton & Kaput, 2005; Hiebert et al., 1998). This change will not be easy and it will have its own set of challenges for preservice teachers because teacher preparation programs are not of long enough duration to provide this type of instruction (Hiebert et al., 2009). In the case of practicing teachers, student-centered Professional Development (PD) opportunities must be made available (Franke, 2001).

The challenges associated with a shift from a teacher-centered to a more democratic mathematics classroom are complicated due to the complex nature of the mathematics classroom, the intricate interactions that happen in it, and the perceived roles that both teachers and students play (Hiebert et al, 1998; NRC, 2001). This does not mean that these challenges are unsurpassable. On the contrary, careful PD planning and appropriate teacher support can overcome these. Thus, "improving educational outcomes for young people depends on developing and supplying skilled instructional practice. Such practice is complex and involves much that is not natural or intuitive" (Ball & Forzani, 2011, p. 18). To achieve this change, the first task teachers must do is to face their own perceptions and compare them to the reality of everyday instruction in their classroom (Ernest, 1988; Hill et al., 2004). Then, they should be exposed to experiences that have them "struggle with important mathematical ideas, justify their thinking to peers, investigate alternative solutions proposed by others, and reconsider their conceptions of what it means to do mathematics" (Mewborn, 2003, p. 49). Just like students, inquiry and teacher mathematical reasoning must be at the center of PD opportunities (Franke et al., 1998). In addition, teachers should be able to practice what they learn by implementing it into their classrooms, as they will improve their practice when they develop “an understanding of their practices in relation to their students’ learning” (Franke et al., 1998, p. 68).

Research Questions

The need for teachers’ deeper understanding of discourse and how to encourage it in the classroom is great, given the body of research that confirms its efficacy in improving students’ conceptual knowledge (Akkus & Hand, 2010; Leher & Franke, 1992; Nathan & Knuth, 2003; Powell & Kalina, 2009; Wood et al., 1993) and the way that it will facilitate meeting the goals of SMP3 that specifies that students should be able to construct sound arguments for their solutions while being able to assess each other’s logic (Blanton & Kaput, 2005; Boaler, 2002; Carpenter, et al., 2000; NCTM, 2000, 2010; Phillips, 2008). By utilizing the 6th grade mathematics class as the setting, this study focused on the following questions:

1. What are teachers’ self perceptions regarding how they encourage students to share mathematical reasoning?
2. What are teachers actualized roles regarding how they encourage students to share mathematical reasoning?
3. How do teachers’ perceived and actualized roles related to how they encourage students to share mathematical reasoning differ?
4. How do teachers influence students’ discourse in ways that enable them to express their mathematical reasoning in the classroom?
5. How do teachers influence students’ abilities to critically assess the mathematical reasoning of others in the classroom?

The intent of this study was to inform mathematics teacher educators, school administrators, district leaders, and those who provide professional development so they could prepare and support inservice teachers for the implementation of the Common Core State Standards for Mathematics. More specifically, this research was centered on the notion of investigating the challenges mathematics teachers face as they make changes in their teaching in order to provide them with the tools and support necessary to enable them to create classroom environments that support their students to provide mathematically sound reasoning and to assess their peers' justifications.

Conclusion

This chapter has introduced the notion that for the successful implementation of the CCSSM and more specifically SMP3, essential changes must occur in the kinds of interactions that happen in the classroom, in classroom norms, and in teacher expectations when it comes to students explaining and justifying their answers. These changes can be a challenge for practicing teachers. However, with appropriate and on-going support, in-service teachers can be fully prepared to enable their students as they demonstrate their mathematics conceptual knowledge through the SMPs (Ball & Forzani, 2010). The research questions formulated guided the scope of this study as it was conducted.

In the following chapter, the main constructs associated with this research are defined and further explained. The teacher content and pedagogical knowledge necessary to teach 6th grade mathematics informed the development of the content portion of the PD that was used in this study. Moreover, discourse, socio-mathematical norms, and communication were explained as they are central to SMP3. The CCSSM were described as they related to integer operations and data analysis as these topics were used during the study. The SMPs, with emphasis on SMP3, were further analyzed and teachers' responsibilities involved were explained. The PD that was used with the teacher-participants was described as well as any other additional material that was used in order to complete the PD the teachers received as they started to prepare to teach in a new way.

CHAPTER 2: LITERATURE REVIEW

This chapter describes the main constructs associated with the research. It starts with a brief introduction followed by a description of the CCSSM and the SMPs. Then, the content knowledge necessary to teach integer and data analysis concepts is discussed along with the social norms that exist in a classroom that promotes discourse. Subsequently, SMP3 is described in detail with emphasis on its two main constructs, reasoning and communication. After that, effective PD is discussed as a means to support inservice teachers to meet the requirements of SMP3. Then, the PD that was used with the teachers is explained and its efficacy is documented. Finally, the pilot study that informed this study is discussed.

Introduction

First, a brief background illuminates the importance that teachers have on the interactions they create in the classroom and how they affect what and how students learn (Ball & Forzani, 2010). Next, an overview of the CCSSM relating to integer operations and data analysis is provided. This is followed by an analysis of the content knowledge necessary to teach these topics in a manner aligned to the CCSSM and the NCTM's *Principles and Standards for School Mathematics*. After that, an in-depth analysis of the pedagogy needed to create the elements necessary for classroom discourse follows. Then, SMP3 is described in detail, with emphasis on building mathematical reasoning and communication. In addition, PD is discussed as a way to support inservice teachers to facilitate student engagement in SMP3 and as a tool to enable teachers to support and to meet their students' needs. Finally, the pilot study that preceded this research is described and the way it informed this research is explained.

The focal point of this study was SMP3, "create viable arguments and critique the reasoning of others" (NGA & CCSSO, 2010, p. 6) and how teachers might help their students engage in mathematical reasoning to create and evaluate explanations and justifications. When students solve problems and explain and justify their answers, other SMPs are involved. For example, students' intrinsic desire to solve a problem (SMP1) motivates them to persist as they attain the solution. As they work, they may use different tools (SMP5), they might use what they know about the mathematical concepts related to the problem (SMP7), or they might use shortcuts they have identified as they worked similar problems (SMP8). As the students work through the calculations they ensure there are no errors by carefully recording and checking their work (SMP6). When students construct their arguments and justifications they might use real-world situations or use a graph to explain their logic (SMP4). As students solve problems and discuss their reasoning with each other, they are engaged in several of the SMPs at the same time as it is the case in most good teaching. However, this study solely focused on SMP3 and the ways teachers can support their students in demonstrating it.

Background

Traditionally, those who decided to become teachers took the college path and graduated after a few years of studies and usually one year of limited teaching experience (Hiebert & Morris, 2009). Today, however, there are many pathways available for those who wish to pursue this career. The popularity of alternative teacher certification programs combined with the need for teachers in specific areas, such as science and mathematics, come together to create a mixture of teachers who bring wide-ranging experiences and backgrounds to the classroom (Hiebert et al., 2007). Regardless of the program chosen, it is folly to believe that the conclusion of the certification program chosen is the end of the education a teacher should receive especially when the expectation is that each and every teacher becomes an expert (Nemser, 1983). Teachers' previous experiences, formal education, and years of practice shape the way they value instruction and the methods they employ (Hiebert, et al., 2007). Thus, the education of teachers goes beyond them obtaining a teaching certification and it should be an ongoing, life-long, process. This learning process also needs to address pedagogical and content changes as educational trends change over time (Franke et al., 1998).

The Common Core State Standards (CCSS) is one of those changes that teachers must adapt to and prepare for. The CCSS contain new content standards for language arts and mathematics (NGA & CCSSO, 2010). The Common Core State Standards for Mathematics (CCSSM) include content standards across the grades, K to 12. However, they have a unique feature: the Standards for Mathematical Practice (SMP). The SMPs describe the behaviors that a proficient student, regardless of grade, must exhibit in the classroom. Teacher perception of what each mathematical practice intends might not truly reflect the intention behind the standard. Therefore, it is important to provide teachers with learning opportunities that enable them to unpack the meaning of each of the mathematical practices and how they relate to the content standards.

Common Core State Standards for Mathematics

In 2010, the National Governor's Association (NGA) jointly with the Council of Chief State School Officers (CCSSO) released the Common Core State Standards Initiative. Along with this document, the Common Core State Standards for Mathematics (CCSSM) were made public as well. The CCSSM are an answer to the increasing body of research that analyzed the mathematics standards of high-performing countries and found that, when compared with those countries' standards, American standards, for the most part were "a mile wide and an inch deep" (Porter, 2002, p. 3). Byrd et al. (2010) prepared a report that analyzed the standards for all 50 states, and found that for the most part these standards "vary dramatically—something we've known for more than a decade and have demonstrated on multiple occasions. A small handful of them are strong, but most lack the content and clarity needed to provide a solid foundation for effective curriculum, assessment, and instruction" (p. 21).

In comparison, the CCSSM have been found to be focused, coherent, rigorous, and very similar in content and learning trajectories to NCTM's *Curriculum Focal Points for Prekindergarten through Grade 8: A Quest for Coherence* and the SMP share goals (Achieve, 2010) with the NCTM's *Principles and Standards for School Mathematics'* Process Standards and the recommendations contained in the National Research Council's *Adding It Up* (Achieve, 2010; NGA & CCSSO, 2010).

Standards for Mathematical Practice and Mathematics Content

Standards for Mathematical Practice

One attribute to the CCSSM is the SMPs. These are eight standards that describe specific behaviors that must be observed in all students at all grade levels. These are described as "processes and proficiencies" in the CCSS document (p. 6). Table 1 lists the eight SMP and NCTM's (2000) corresponding Process Standards. It is important to note the emphasis that the SMP make on having students solving problems and in justifying the logic and reasonableness of their answer by explaining their answer. These are consistent with goals of the Problem Solving and Reasoning and Proof process standards. The SMPs are closely related to the process standards and each can be matched to the corresponding process standard (Achieve, 2010; NCTM, 2000).

According to the CCSSM document, "the Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise"

(NGA & CCSSO, 2010, p. 8). The Standards for Mathematical Practice are a complement to the CCSSM because they are an "expectation of performance" and should be actively used by the teacher to constantly assess students' knowledge and make instructional decisions, as it is supported by the literature (Hiebert, et al., 1997; NCTM, 2000; Shafer & Romberg, 1999).

The limited literature (Achieve, 2010) available on the CCSSM focus on the content standards and how they compare with individual state standards and to the standards set forth by reputable organizations, like NCTM. However, there is increased interest in the SMP and this will be more pronounced as they are implemented. The focus of this research will be limited to SMP3, "create viable arguments and critique the reasoning of others" (NGA & CCSSO, 2010, p. 6) and how teachers can support their students in demonstrating this SMP. Students engaged in this practice will be creating reasonable explanations for solutions and assessing their peers' logic. For the successful implementation of SMP3, teachers will have to identify and meet their students' needs as they relate to reasoning, constructing arguments, and assessing abilities in order to enable them to exhibit proficiency in this particular SMP given the complexity of the tasks associated with what SMP3 entails. One way to facilitate this process for inservice teachers is by providing them with PD opportunities that are generative – that is supported and ongoing as teachers engage in practices that serve as ongoing learning (Franke et al., 1998) - and that address the students' needs – as it relates to how to develop students' mathematical thinking.

This study was conducted in two different 6th grade classrooms. At that time, the teachers were at different place due to the level of the students. One of the classrooms had on-level and advanced students who were working with integer addition and subtraction. The other classroom had the lowest performing students at the school who were working with data analysis. In both classrooms, the expectation of evidence of the SMP was the same.

Integer Addition and Subtraction

According to NCTM, students starting middle school have had experiences with zero and have rudimentary knowledge of negative numbers. At this time, students should be able expand their knowledge of positive and negative numbers and should be able to translate real world contexts into mathematical situations that include negative numbers (NCTM, 2000). Furthermore, the CCSSM state that students should extend their knowledge of the basic operations to integers by "maintaining the properties of operations and the relationships between addition and subtraction, and multiplication and division" (NGA & CCSSO, 2010, p. 46). The understanding of integer operations, in particular addition and subtraction with negative numbers is essential for understanding how to solve one-step linear equations that students will later learn (NCTM, 2000; NGA & CCSSO, 2010). Table 2 depicts the standards that are associated with integer operations in middle school.

Table 2 - CCSSM The Number System, Grade 7

Critical Area	Cluster	Standard
Apply and extend previous understandings of operations with fractions to add, subtract, multiply and divide rational numbers	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.	*Describe situations in which opposite quantities combine to make 0.*
		Understand $p + q$ as the number located a distance $\|q\|$ from p, in the positive or negative direction depending on whether p is positive or negative
		Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers is the absolute value of the difference, and apply this principle in real world contexts
		Apply properties of operations as strategies to add and subtract rational numbers

NGA & CCSSO, 2010 (p. 48)

As Table 2 demonstrates, mastery of rational numbers, including absolute value, understanding of additive inverses and multiplicative inverses, and perceiving rational numbers as an extension of whole numbers is essential. Additionally, the use of the number line as a tool is emphasized in the cluster. The application of the properties of operations is embedded in the standards themselves. This means that students must extend their knowledge of these properties to solve new kinds of problems. According to the NGA and CCSSO, “students explain and interpret the rules for adding, subtracting, multiplying, and dividing with negative numbers… they use the arithmetic of rational numbers as they formulate expressions and equations in one variable and use these equations to solve problems.” (2010, p. 45). As stated before, the second group of students, despite being in the same grade, was working with statistics.

Statistics and Probability

The introduction of formal data analysis marks a milestone for the students as they “begin to develop their ability to think strategically” (NGA and CCSSO, 2010, p. 39). As students start to make sense of the different distributions that exists and the different meanings behind different measures of central tendency, they will use this information to answer a variety of questions. As they address these questions, they will start to rationalize their answers.

Table 3 depicts the standards associated with Statistics and Probability in the 6^{th} grade. The emphasis is on understanding statistical variability and on the ability to summarize distributions. Thus, students must analyze the data and make statements about its distribution and shape. Additionally, students should be able to comprehend how measures of central tendency and measures of variability describe different characteristics in a group of data.

Table 3 - CCSSM, Statistics and Probability, 6th grade

Critical Area	Cluster	Standard
	Develop understanding of statistical variability	*Recognize a statistical questions as one that anticipates variability in the data related to the question and accounts for it in the answers*
		Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape
		Recognize that a measure of center for numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number
	Summarize and describe distributions	*Display numerical data in plots in a number line, including dot plots, histograms, and box plots*
		Summarize numerical data sets in relation to their context, as

NGA & CCSSO, 2010 (p. 45)

The learning trajectory that is to be followed is of paramount importance because when the teacher focuses on the "identification of significant and recognizable clusters of concepts and connections in students' thinking that represent key steps forward, trajectories offer a stronger basis describing the interim goals that students should meet" (Daro et al., 2011, p. 12). The understanding of these concepts will enable students to be able to work with the higher mathematics that is coming in subsequent grades, so this concepts' comprehension is paramount to the students' success (NCTM, 2000). The way students can achieve this comprehension is through the instruction they receive from their teachers. "Skillful teaching can make the difference between students being at the top of the class or the bottom, completing high school or dropping out" (Ball & Forzani, 2011, p. 18).

It is very important to note that the standards in both areas use the term "understand." The CCSSM document is very specific on what "student understanding" means. It states that student understanding is demonstrated by the student's ability to explain and justify the mathematics he or she uses, being able to explain the rule, and say from where it comes. Procedures are important but the determining factor for understanding is the ability to rationalize actions (NGA & CCSSO, 2010).

The SMPs are part of the CCSSM and it is the teacher's responsibility to ensure that students develop and demonstrate these practices in order to meet the standards. As such they will be discussed in detail later in this chapter. It is up to the teacher to relate these practices to the content. However, those standards that have the "expectation of understanding are potential 'points of intersection' between the Standards for Mathematical Content and the Standards for Mathematical Practice" (NGA & CCSSO, 2010, p. 8). An understanding of the SMP together with a deep understanding of the standards along with their expectations, when combined with content and pedagogical knowledge will allow the teacher to create a classroom environment that will transform students into mathematics practitioners (Ball & Forzani, 2011). This pedagogical knowledge is mostly acquired through practice and it is usually beyond what most teachers experience in teacher preparation programs. This knowledge entails knowing about the challenges students are likely to encounter and how to adjust content instruction to meet the students' needs (NCTM, 2000).

Teachers

Teacher Content Knowledge

In order to teach effectively, teachers must know the subject they are teaching in depth. This is especially true in mathematics (NCTM, 2000). Teachers who themselves are not proficient in the mathematical concepts they teach, will be ineffective when teaching students (Ball et al, 2008; Hilbert et al., 1997; Shulman, 1986; Stigler & Hiebert, 1999). According to the National Council of Teachers of Mathematics (2010),

> Rich mathematical understanding guides teachers' decisions in much of their work, such as choosing tasks for a lesson, posing questions, selecting materials, ordering topics and ideas over time, assessing the quality of students' work, and devising ways to challenge and support their thinking (p. 2).

Teacher content knowledge, along with pedagogical knowledge and knowledge of students' thinking, comprises the cornerstone of what teachers bring into the classroom (Hiebert et al., 1998; Ball & Forzani, 2011). The tasks that require teacher knowledge include careful planning of instruction, deliberate selection of meaningful and interesting problems for students to solve, setting specific goals, specifying clear learning trajectories for students, implementing meaningful ways of assessing students during and after instruction, analyzing student work, ensuring content requirements remain high, and accommodating students' needs and challenges (Fennema & Romberg, 1999; Hiebert, et al., 1997; NCTM, 2000). This knowledge is required to teach all mathematics concepts (NCTM, 2000).

The subject of teacher content knowledge has been absent from the field of mathematics education for most of its history. This was probably because most research efforts focused on student performance. However, its importance is demonstrated in the statements made by NCTM and by the results of the aforementioned studies. In 1976, Skemp coined the term 'instrumental knowledge' to describe the knowledge that is based in memorization of procedures without understanding. He also stated that 'relational understanding' was that knowledge that was characterized with deep understanding of the concepts behind the procedures followed to solve problems. At the time, his description was to be applied to the students. However, he stated that "nothing else but relational understanding can ever be adequate for a teacher" (p. 24) even though he thought the reality was that probably a majority of teachers did not have it.

Denmark and Kepner (1980) conducted one of the first studies geared towards measuring the knowledge that teachers needed to teach and their opinions regarding learning mathematics.

This study demonstrated that a majority of teachers expressed interest in receiving additional professional development regarding the mathematics concepts they taught. Yet, 42% of the teachers (grades four through eight) agreed that the most important reason to learn mathematics was for it to be used as a basic skill. Later on, Shulman (1986) stated that teacher content knowledge was the "missing paradigm" in mathematics education research. That is, up to that point, there were no studies that focused on what teachers did, how they taught, how they explained concepts and where those explanations came from, and what they taught. Furthermore, Shulman suggested that there were three kinds of teacher knowledge: propositional, case, and strategic. Propositional knowledge refers to understanding that is based in "disciplined empirical or philosophical inquiry, practical experience, and moral or ethical reasoning" (p. 11). The focus of case knowledge is what is learned through recognized, proven, and complete descriptions of others' experiences. Strategic knowledge is centered on the professional behavior, rules, and expectations that are characteristic of the teaching profession. Mathematics teachers must understand the concepts they teach deeply and with flexibility. Furthermore, teachers should provide students with instances that have them give "in-depth explanation[s], the sophisticated use of argument and evidence, and the strategic employment of technology; and encouraging growth in interpersonal skills through whole-and small- group work, oral argument, and other opportunities for social interaction" (Ball & Forzani, 2011, p. 20). In order to meet SMP3, teachers must be mindful of the rationale behind the questions they ask, how to elicit students' reasoning, and to understand that reasoning: this is a "high level practice" (p. 21) according to Ball and Forzani (2011).

In the beginning of the 1990s, there was an increased interest in determining what kind of knowledge teachers brought to the classroom. However, there was not a consensus on what "content knowledge to teach" entailed. At that time, Ball (1990) conducted a study that determined that deep content knowledge was necessary to teach students with understanding, so they can make conjectures, explain and justify their reasoning, and understand the concepts behind the procedures. In her argument, Ball stated that

> Teachers should understand the subject in sufficient depth to be able to represent it appropriately and in multiple ways – with story problems, pictures, situations, and concrete materials. They need to understand their subject flexibly enough so they can interpret and appraise students' ideas, helping them to extend and formalize intuitive understandings and challenging incorrect notions (p. 458).

Still, she found that the content knowledge of the teachers she researched was inadequate and would not support students' mathematical proficiency. Additionally, she stated in her conclusions that research in the subject of teacher content knowledge, was greatly needed. The shift of the attention in mathematics education research to the content knowledge of teachers is demonstrated by other studies (Mullens, Murnane, & Willet, 1996; Rowan, Chiang, & Miller, 1997). These studies confirmed that teacher content knowledge had a dramatic effect on student achievement and suggested that more research on the subject was needed.

Ma (1999) conducted a study that compared and contrasted the mathematical content knowledge between teachers from China and the United States. The results of her study indicated that the American teachers were lacking what she termed a "profound understanding of fundamental mathematics" (p. 120). Ma considered the mathematics taught in elementary schools as fundamental since it is the foundation upon which other concepts will be built.

> Profound understanding of fundamental mathematics (PUFM) is more than a sound conceptual understanding of elementary mathematics – it is the awareness of the conceptual structure and basic attitudes of mathematics inherent in elementary mathematics and the ability to provide a foundation for that conceptual structure and instill those basic attitudes in students. A profound understanding is the capacity to connect a topic with topics of similar or less conceptual power. Depth of understanding is the capacity to connect to a topic with those of greater conceptual power. Thoroughness is the capacity to connect all topics. (Ma, 1999, p. 124)

This groundbreaking study brought the deficiencies in the mathematics content knowledge of teachers in the United States to center stage. Ma stated that, for the most part, American teachers were focused on teaching procedures and that their content knowledge was fragmented. Ma concluded that the low level of mathematical proficiency that American students have is due to the mediocre state of teachers' mathematical knowledge.

Also in 1999, Stigler and Hiebert published the results of the TIMSS video study comparing the teaching practices among Japan, Germany, and the United States. They challenged the notion that all teachers knew everything that they need to teach. The center of attention of their study was what happened in the classroom after the lesson started and they found that the teaching methods among the countries were very different. They attributed this to the idea that teaching is a cultural activity. American teachers, whether they are aware of it or not, tend to teach the same way they were taught. Additionally, they stated "curriculum developers often have tried to create 'teacher-proof curricula' – content that is to be presented to students in such a straightforward way that it could not be distorted by incompetent teachers" (p. 170). The repercussion of this is that teachers do not think of their students' abilities, learning styles, learning trajectories, and other important factors when they instruct in this manner. The solution, they thought, was to turn teachers into teacher-researchers. Their idea was to close the gap that exists between the changes that researchers propose and what teachers in the classroom enact. Based on their observations, they determined that the level of mathematics content taught in the American classroom was lower than in the other two countries, that U. S. lessons did not require a great deal of mathematical reasoning, and that the lessons were centered on procedures. However, being mathematically proficient requires more than just recalling steps to solve a problem. To be able to achieve "mathematical proficiency", a concise definition was needed.

The National Research Council (2001) defines ‘mathematical proficiency’ as being comprised of five strands that are necessary to learn mathematics concepts in depth and with understanding. These strands are conceptual understanding, procedural competence, strategic competence, adaptive reasoning, and productive disposition. Conceptual understanding has to do with the comprehension of the mathematical concept. Procedural fluency involves the carrying out of procedures to solve a problem. Strategic competence has to do with the formulation, depiction, and answering of a problem. Adaptive reasoning involves student reflection, explanation, and justification of a particular mathematical concept. Productive disposition is the way students regard mathematics as useful and important to their lives. The definitions of these strands might be different from each other but the strands exist together and cannot be separated. They build on and support each other and they are developed through time. It is important to note that if the teacher does not possess deep content knowledge, then students will not learn the content deeply because teachers’ mathematical proficiency has an impact on students’ learning. If the teacher possesses mathematics proficiency, the student will be more likely to make gains in mathematics achievement (Hill, Rowan, & Ball, 2005).

The RAND Mathematics Study Panel (2003) determined that a great number of students, especially those from disadvantaged conditions, are taught by teachers that lack the proper mathematics content knowledge to teach. This report also stated that even though professional development could help this situation, many of these programs lacked consistency and high level goals. Additionally, many of the teachers that needed this continuing education did not pursue it. This study, along with others (Ball, Hill, & Bass, 2005; Hill & Ball, 2004; and Hill, Rowan, & Ball, 2005) confirmed that there is a direct correlation between students’ achievement scores and teacher’s mathematical content knowledge. All of them agreed that further research on the subject was needed.

One project, (Hill, Schilling, & Ball, 2004) had as its main goal to develop an instrument that could measure teacher content knowledge, they recommended that further investigation was needed to determine the exact measure of the mathematics topics that made up the mathematics content knowledge that is necessary to teach. These and other efforts (Hiebert et al., 2007; Hiebert & Morris, 2009; Newton, 2005) to measure teacher content knowledge only confirm the shift of attention towards teachers’ mathematical content knowledge that is currently prevalent in mathematics education research. Thus, it is necessary for teachers to have deep understanding, not only of the mathematical concepts they teach, but also of the mathematical concepts their students are likely to encounter in the future. This will enable teachers to construct teaching trajectories that will facilitate their students to be ready for higher mathematics (Hiebert et al., 1998). According to NCTM (2010) the learning trajectory associated with reasoning starts with conjectures, then moves on to testing those conjectures. If the conjecture is true, it moves to the generalization stage, if the conjecture is false, then the process of conjecture starts all over again. This learning trajectory has students testing their conjectures in order to determine if they are true or not. If they are, they will be accepted as generalizations that can be applied to similar problems.

The Teaching Principle described by NCTM (2000) states that “effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well” (p. 16). When students can relate what they know to the new concepts they are learning, connections are built. These connections become the basis of new knowledge. Furthermore, if the previous knowledge is deep and cohesive, it will be easier for students to understand the new material (NCTM, 2010). Thus, it is important that students get the opportunity to relate and extend what they know of whole numbers (Hiebert et al., 1997; NCTM, 2010; Saxe, Gearhart, & Seltzer, 1999; Stigler & Hiebert, 1999). The teacher needs to create context by either writing word problems or by modifying textbook problems in order to focus the instruction towards the needs and abilities of the students (Ball, 1990) and to follow the learning trajectories set for the students. Additionally, the teacher must be able to construct multiple representations of the concepts they teach with or without context and provide meaning to the units used in the problems (NCTM, 2000).

Pedagogical Knowledge of Teachers

The combination of what teachers know about teaching and their mathematical content knowledge determines the interactions that will occur in the classroom between teacher and students (Ball & Forzani, 2011). Shulman (1986) described this knowledge as having three different components: propositional knowledge, case knowledge, and strategic knowledge. The propositional knowledge refers to research-based principles, for example, the five steps that should be present in every lesson plan or reading strategies. Case knowledge refers to what the teacher knows about teaching by observing other teachers or by reading descriptions of events that are detailed as to show its characteristics. Strategic knowledge refers to the teacher's ability to put into practice either propositional or case knowledge when faced with a specific situation. In other words, the teacher will need the ability to move from strategic to case knowledge in real time. In addition, they will need to proactively anticipate instances when either case or strategic knowledge will be needed. At the time when Shulman put forth his ideas, he stated that there was not enough on the subject of teacher knowledge as a whole, including both content and pedagogy.

Eventually, some of the focus of mathematics education research moved away from student and school performance and centered on the teacher. Hiebert et al. (1997) described many of the decisions that are based on teacher pedagogical knowledge. The most important undertaking that a teacher has is to be able to select mathematical tasks that are challenging to the students. This is critical because the teacher needs to determine whether the residual knowledge that the students will retain after the task is completed fits the learning trajectory set for the students. Choosing the task appropriately is a process. First, the teacher needs to formulate goals based on the students' needs. In other words, the teacher needs to be able to determine what the students' thinking is so he or she can determine tools and any other needs to achieve learning goals while keeping aligned with the trajectories. Second, while the students work on the task, the teacher has to be able to give some information about the problem to the students without "watering down" the level of difficulty. The duty for the teacher is not to solve the problem; it is to give students enough time and support to solve it for themselves. "Regardless of the context, worthwhile tasks should be intriguing, with a level of challenge that invites speculation and hard work" (NCTM, 2000, p. 19).

Research (Fennema, Sowder, & Carpenter, 1999; Hiebert et al., 1997; Saxe, Gearhart, & Seltzer, 1999) supports the notion that classroom environments that support understanding as the main goal, have in place set structures that encourage students' discourse and that have an emphasis on fairness. This type of classroom environment encourages the students to want to understand. This is the first step for students' ownership of their own learning. In these classrooms, understanding is important to both teacher and students. Conceptual knowledge is important. However, teachers would not be able to promote deep understanding from their students without having teaching knowledge.

Discourse

The use of discourse in the classroom entails the teacher to elicit students to express their ideas. As students create different meanings, the differences within those meanings will give way to discussions that will enable students to argue their reasoning (Wood, 1999). This argument is defined as "discursive exchange among participants for the purpose of convincing others through the use of certain models of thought" (Wood, 1999, p. 172). It is important for students to engage in activities that create opportunities for them to reflect on their own mathematical reasoning as they listen and make sense of others' solution methods and logic. These class discussions created "opportunities to learn as the teacher and students negotiated mathematical meanings that would enable students to make connections from their individual mathematical constructions with the taken-as-shared meanings of the classroom" (Wood et al., 1993, p. 56). In order to be successful in this kind of classroom, teachers must understand the ways their students think and must have deep conceptual knowledge of mathematics (Ball, Hill, & Hyman, 2005; Carpenter & Leher, 1999; Hill et al., 2009; NCTM, 2000).

In 2010, Akkus and Hand developed the mathematics reasoning approach (MRA). The MRA is a method developed to increase the dialogue among students in order to promote the sharing of reasoning and to help students construct their own justifications using templates that resemble Polya's problem solving heuristic or Schoenfeld's phases of problem solving. Polya's problem solving heuristic refers to the activities of guessing, insight, and discovery (Schoenfeld, 1992) and Schoenfeld's phases of problem solving refer to the "use of five heuristic strategies:

- Draw a diagram if at all possible;
- If there is an integer parameter, look for an inductive argument;
- Consider a logical alternative: arguing by contradiction or contrapositive;

- Consider a similar problem with fewer variables;
- Try to establish subgoals" (Schoenfeld, 1980, p. 796).

The characteristic that sets this approach aside from the aforementioned methods is that "it specifically asks students to compare their solutions with their peers and to reflect on their problem solution after a classroom discussion" (Akkus & Hand, 2011, p. 976). The MRA has students both writing and talking about their reasoning. This research focused on investigating the struggles that teachers suffer as they attempt to implement more student-centered methods. It found that there are two main categories that encompass the struggles which teachers go through when relinquishing power to their students. Those factors are internal and external. The internal refers to the teachers' beliefs, and pedagogical and content knowledge. The external refers to curriculum, teaching resources, school support, and parent support. "The role of the teacher is crucial in providing sufficient opportunities for all students to be able to take part in negotiations, in individual, small-group, and whole-class settings" (Akkus & Hand, 2011, p. 994). According to the authors, the main role of the teacher in these instances is to provide problems that provoke discussion, thought, and that create conflict. This created conflict will be the catalyst that will enable conversations centered in making sense of the problems and reasonable solution methods (Akkus & Hand, 2011). Furthermore, these conversations will facilitate the consensus that is necessary when discussing problems and their solution methods.

In 2009, Powell and Kalina discussed in greater depth the idea of incorporating social constructivism into the classroom as a means of increasing student understanding. Social constructivism was developed by Vygotsky, with Piaget's ideas (Powell & Kalina, 2009), and it centers on the notion that social interaction enables students to learn. Within the many ideas put forth by social constructivism, the concept of cooperative learning is very prominent as it is, according to Vygotsky, an essential component of developing deeper understanding. Besides knowing the theory, teachers must be aware of the diversity that is prevalent in the classroom and how it affects the communication that exists in the classroom. Some argue that 'language' refers to the mode of talking. However, constructivists see 'language' as a consensus of meaning. The members of the classroom develop the meanings in the communication. The norms established in the classroom, combined with its members, and their thinking make communication possible in the classroom. Moreover, the way teachers establish norms that dictate the ways students are going to be listening and responding to each other is paramount as the focal point of discourse is in the interaction between the person that talks and those who listen to him or her (NCTM, 2000; Wood, 1999).

Leher and Franke (1992) investigated how the components of teaching, like teacher personality, content knowledge, and pedagogical knowledge, interact together in order to create a philosophy of teaching that is unique to each teacher. It is interesting to note that this notion makes teaching as personal as learning. Personal views, in addition to experiences, content knowledge, expectations, and beliefs come together to create the individual teacher. This group of factors come together and affect how teachers perceive student thinking. This is noteworthy because teachers need to be able to assess their students' reasoning and logic (Leher & Franke, 1992). For instance, a teacher who is a member of a minority group brings into the classroom the experiences associated with being a minority and would be more able to better comprehend and empathize with the frustrations of those students who share his or her background (Ehrenberg et al., 1995)

The classroom is the community inside which many meaningful interactions should occur in order to enable students to achieve higher levels of understanding. In this community, the role of the teacher is not passive, on the contrary, it is active. However, it is characterized by its discrete subtleness.

> Rather than primarily explaining and demonstrating, the teacher is asked to craft instruction in a nontraditional way, at times leading from behind, at times stepping in as a mathematical authority, and at other times carefully guiding the discussion and activities and seeding ideas (Nathan & Knuth, 2003, p. 176).

This way of teaching creates many challenges to teachers because their professional training likely did not prepare them to teach in this manner (Ball & Forzani, 2011; Hiebert et al, 2007); additionally, their experiences as students were likely very different to what they are trying to do in the classroom (Ball & Forzani, 2011). In addition to teachers' perceptions and beliefs, what they have experienced as students themselves will influence what happens in the mathematics classroom (Hiebert & Morris, 2007). In addition, classroom environments depend on the interaction between the participants (teacher/student and student/student), and the levels of scaffolding, both analytical and social, that exist in the classroom. Analytical refers to the mathematical concepts that are being learned and the social scaffolding to the acceptable behavior within that community. Both types of scaffolding, along with the teacher's beliefs and perceptions, have a major component in the classroom environment. In more teacher-centered classrooms, the teacher dominates both analytical and social scaffolding. In order to have students lead discussions and share their ideas with each other, teachers must let go of the control they have over these scaffolds (Akkus & Hand, 2011; Amit & Fried, 2005).

There are challenges associated with trying to implement this kind of reform in the classroom. The first one is associated with the complexity of the classroom itself since it has intricate structure, it will change very slowly. When the teacher changes his or her practice of instruction, then changes also must occur in how the teacher and students work together, students' disposition towards mathematics, and teacher expectations. "These changes also must occur in the larger context of accountability measures, standardized testing, and parent and community influences, to name but a few" (Nathan & Knuth, 2003, p. 202).

Cobb, Wood, Yackel, and McNeal (1992) conducted an in-depth study of the classroom environment as it related to the creation of justifications and explanations and how teachers elicited those explanations. They realized that the classroom environment and the teachers' beliefs were critical to the implementation of such a social activity. Additionally, they observed how the classrooms were set up and how norms placed in the classroom affected what kind of interactions happened there. They described five different kinds of norms in the classroom: regulations, conventions, morals, truths, and instructions. Regulations are those behaviors that have been historically a part of the classroom and are those, when broken, will cause some kind of punishment to the person who breaks them. One example of a regulation could be set seats for the students. Conventions, like regulations, have been present in the classroom for a very long time and they continue to be. However, when conventions are broken, the result would be the disapproval from the other members of the classroom. An example of a convention could be that only one student from each group will get up to get the materials needed for the lesson. Another type of convention that can be present in the classroom is the expectation that students explain their reasoning and justify it when questioned by their peers. The rest of the norms, morals, truths and instructions are interrelated. For example, if a student cheats, the teacher might make him feel guilty for his dishonest behavior. The punishment for breaking any of these is failure. In some instances, the "conviction that it is impermissible to use any methods other than the standard procedures taught in school to solve school-like tasks and that the use of these procedures is the rational and objective way to solve mathematical tasks in any situation whatsoever" (Cobb et al., 1992, p. 589). Moreover, the interpretations that both groups, teachers and students, gave to the classroom norms and the self perceptions of their roles in this community, along with the meaningfulness of the mathematical task, contribute to the conversations and justifications and explanations that are constructed in the classroom.

Dixon, Egendoerfer, and Clements (2009) examined the dialogue that happened in a classroom and how the roles of both teachers and students and the classroom norms affected this discourse. This study found that there was a struggle between teachers' preconceived ideas regarding change in the classroom and their support of innovative methods. However, this study found that the quality of students' discourse increased when they were allowed to share their ideas freely with each other. In the classroom where this study was conducted, "the teacher was no longer seen as the only one with the ability to impart knowledge and student ownership of ideas was evident in social interactions as well as written responses" (p. 1074). In order to be effective, the teacher must create "an intellectual environment where serious mathematical thinking is the norm (NCTM, 2000). The choice of mathematics problems, which is one of the tasks a teacher must perform, is essential for encouraging students to think critically, to struggle with mathematical ideas, and to vocalize that struggle in an attempt to build conceptual understanding. Another important factor controlled by the teacher is the hypothetical learning trajectory that the teacher has in mind when teaching a particular concept to students. The students, on the other hand, have to put aside their beliefs that the teacher will direct their learning and take an active role in that learning. For this to happen, the socio-mathematical norms that are instituted in the classroom need to support the teacher and the students in their new roles. The change in norms may cause challenges. The teacher's main struggle in this changing methodology is giving up the control of the classroom and the behavioral problems that can arise from taking out the set of rules that every classroom has posted on the walls. This struggle causes a disconnection between what the teacher knows she needs to do versus what she does. The results of this research indicate that the quality of the students' discourse as it related to mathematical reasoning increased dramatically especially when the classroom environment supported a system that encouraged students to talk freely

Hufferd-Ackles, Fuson, and Sherin (2004) conducted a year-long case study that took an in-depth look at the changes teachers must go through and the challenges they must overcome in order to implement reform-based mathematics instruction that supports students' discourse. A reform-based classroom is characterized by students listening to their teacher and each other. This is because the teacher is not the sole source of knowledge. This is a classroom where students are allowed to discuss their own mathematical reasoning in order to strengthen their own conceptual knowledge (Baxter et al., 2001). At the center of Hufferd-Ackles et al.'s rationale is the notion that all those that compose the classroom would "be constructing their own knowledge and reflecting on and discussing this knowledge" (Hufferd-Ackles et al., 2004, p. 83). One of the most important teacher-led activities in this kind of classroom is questioning (NCTM, 2000). According to Hufferd-Ackles and colleagues, questioning is divided in three different levels. The first level of questioning has the teacher asking questions because the mathematical conceptual understanding of students must be gauged. Therefore, students must be questioned to assess the reasonableness of their answers. During this level, only the teacher is the one posing questions. The second level of questioning starts when students begin to build their questions and start posing those questions to each other. This phase is complicated because the teacher must stop his or her impulse to correct students or to give them cues and instead give them time to formulate their questions and explain what they are trying to find out. In this level, both teacher and students are posing questions. The third level of questioning happens when students pose their questions confidently to each other. This phase usually starts with the teacher asking the students if they have any questions for their peers and it ends when students pose questions of their own, without prompting. Moreover, students in this stage strive to ensure that their peers understand the concepts being explored in the classroom. This stage is characterized by the teacher just monitoring the students while they actively learn. Additionally, it means that the power in the classroom has shifted from the teacher to the students. At this level, all students

> asked questions, with the lower-achieving students often only mimicking what they had heard their teacher ask in a previous class… the fact that these students were asking questions gave evidence of their comfort with being a participant in the math-talk learning community and confirmed their engagement with the discussion (Hufferd-Ackels et al., 2004, p. 96)

It is important to mention that, even though it is not apparent, the teacher's role at this level is essential because this role entails time management, resolving differing points of view, and clarification of unclear concepts. One key element of all levels is the patience the teacher needs to have to cope with the long periods of silent pauses that will result as students attempt to make sense of the problems they are solving. In addition, the students might feel uncomfortable with the shift in their role, from passive listener to active participant in the classroom. In addition, the students had to learn how to listen to each other, as each explained their reasoning. As the members of the classroom get used to their new roles in instruction, they will become accustomed and their functions will become second nature. However, this will require time and perseverance. Moreover, teachers must understand that this process is an evolving effort that requires monitoring. Teachers must constantly evaluate those norms that appear to be established and modify those that are not getting the desired results (Dixon, Andreasen, & Stephan, 2009).

Socio-Mathematical Norms for Discourse

The classroom is a social structure which is made up of the teacher and the students. Whenever there is a group of people, all their differences, commonalities, feelings, friendships, and rivalries will affect the environment (Hadjioannou, 2007). Conversely, the environment that teachers create in the classroom will affect the interactions that happen there (Turner & Meyer, 2004). One of the most influential factors in the norms that are established in classrooms are the teachers' beliefs – "the way they conceptualize their instruction, and learn from experience" (Brody, 1998, p. 25). In addition, the opinions that teachers have of their students and the rapport they create with them will affect the conversations that happen there (Hadjioannou, 2007).

Traditionally, it has been thought that a classroom with a teacher at the front of it, instructing students, while they sit listening quietly, was a classroom in which students learned (Stigler & Hiebert, 1999).

> Society traditionally entrusts teacher with the formal right and responsibility to take charge in the classroom and expects students to obey. The character of teacher-student authority relations has great bearing on the quality of students' education experience and teachers' work (Pace & Hemmings, 2007, p. 4)

The perception of what the classroom has to look like is a result of the many educational philosophies that had been developed through the years. These philosophies might be based on different theories that may disagree on what was pedagogically sound. However, all, in their own way, had the ultimate goal of making instruction more effective and meaningful for students. Within that body of work, there are those who believe that the most effective learning environment is one in which both, teacher and students, share the power (Hufferd-Ackles, Fuson, & Sherin, 2004). Teachers have several types of power within a classroom environment. They are the authority figure, therefore, they can rule over the students, punish them, or reward them. This type of power is a traditional power that has been bestowed to teachers by society's expectations. Nonetheless, this is not the only kind of power present in the classroom. Some teachers influence students in ways that have nothing to do with traditional power. For example, there are teachers that have personalities so appealing to students that they will do anything to gain the teacher's approval. This kind of power depends largely on the teacher's personality, on the students' needs, and on the ability of the teacher to fulfill those needs. Another example of power in the classroom would be the autocratic teacher. This is a teacher who will use inherited traditional power to install themselves as the "boss" of the classroom. In these classrooms students are compelled to do the teacher's will with a system of rewards or punishments and their tone when talking to students is usually authoritative (Cornelius & Herrenkohl, 2004; Pace & Hemmings, 2007). In these teacher-controlled classrooms, student discourse is not an expectation because the teacher controls the communication (Boostrom, 1991).

According to Boostrom (1991) there are four kinds of rules in the teacher-centered classroom. There are rules about non-academic procedures, like "put your finished work in the basket." There are other rules about how to work in the classroom, like "work on your assignment quietly." There are rules about behavior, like "keep your hands and things to yourself." The last kind of rules has to do with the subject being learned in the classroom. In mathematics, a rule could be "your answer must be in simplest form" when dealing with certain problems involving fraction operations. These rules provide students with the expected behavior they are to have while in the classroom. Likewise, these rules allow students to create the reality of what the classroom is and the way one should behave while in it. Regardless of the type of rule, the teacher's role in orchestrating these norms and regulations is central to the way students behave and learn.

Amit and Fried (2005) discuss the fact that many reform movements in mathematics education shift the authority in the classroom from the teacher to the students and the implications that this conveys. One of the interesting facts that they found was that students tended to ask the person who is physically nearer to them when they run into difficulties and have a question. This person is usually one of their peers, as a matter of fact, when the person closer to them is busy, they will almost always turn to the second closest person to them. Still, curiously, even though the shift in the reform mathematics classroom shifts the attention from the teacher to the student, the teacher still has immense amounts of authority in the class, not only as the enforcer of rules and order, but as the mathematics expert. Students tend to see their teachers as “a strong figure with powers they lack” (Amit & Fried, 2005, p. 157). This fact is not really that surprising when the typical power associated with the traditional role of the teacher is taken into account (Cornelius & Herrenkohl, 2004; Pace & Hemmings, 2007).

One unique result Amit and Fried found was the interaction of one group of students as they worked collaboratively to solve problems, these students were observed as they “consulted with one another, raised possibilities on their own, revised opinions, and seemed to arrive at common conclusions” (Amit & Fried, 2005, p. 159). It is important to note here that during this interaction, the teacher did not participate in the reasoning process as the students worked together. However, this observation was the exception. For the most part of this study, the groups formed by the students worked to solve the problems and answers were shared with the exclusive purpose of finishing the task as soon as possible. These observations only confirm that the mere act of grouping students will not achieve the conversations and interactions that will demonstrate their mathematical reasoning as they solve problems. The expectation that students will share their reasoning as they work in groups should be embedded in the classroom norms and environment as this not only let students to act like “mathematicians but also allows the teacher to gain insight into students’ misconceptions and ways of thinking through a problem” (Gavin & Moylan, 2012, p. 187).

Teachers initially set the classroom environment when they let their expectations and beliefs be known, and by the rules they have put in place in their classrooms. As students enter the classroom environment, more variables come into play. The students’ experiences, beliefs, expectations, and the way they feel about this particular environment all come into play in the daily interactions and any incidents that happen in the classroom. One main factor in this dynamic atmosphere is the way students react towards the classroom rules and norms that the teacher has decided will be part of the classroom setting. In the mathematics classroom, besides the rules that apply to behavior and procedures, students also have to abide by the rules of mathematics. Sometimes students are agreeable and follow and accept these rules, the problem arises when students do not adhere to the rules. Moreover, this scenario becomes more complicated when we take into account the teacher’s reactions to students’ behaviors. For the students, these rules and expectations create a perspective of what a classroom should be like (Broostrom, 1991; Cornelius & Herrenkohl, 2004; Pace & Hemmings, 2007).

A new idea of what a classroom should be is emerging, one in which students share their thinking and pose questions to one another and their teacher. The teacher is not at the front of the class, but moves around, asking questions that ignite class wide discussions (Herbel-Einsenmann & Ottel, 2011). Many want to move away from the teacher-centered classroom into this new collaborative learning environment but when teachers introduce new ways of learning and sharing information, they change the underlying structure of the classroom in a fundamental way. Moving from a teacher-centered classroom setting to a more student-centered one will forever change the norms and regulations that were previously instituted in the classroom (Cornelius & Herrenkohl, 2004). When a classroom is geared towards discourse and mathematical tasks center on students sharing the logic of their answers with each other, students' reasoning skills increase and as they do, students can construct solid arguments based on mathematical concepts.

Reasoning

Mathematical reasoning is one of the cornerstones of mathematics education as it is fundamental for students to learn and it is difficult for teachers to teach and assess (NCTM, 2010). The importance of possessing mathematical reasoning has been recognized throughout the years and this is evident in the efforts that mathematics educators have put forth by developing new learning theories in order to increase their students' understanding (NCTM, 2010). According to NCTM (2000), activities that promote the reasoning of students should be an integral part of the everyday mathematics classroom. This is because students "who reason and think analytically tend to note patterns, structure, or regularities in both real-world situations and symbolic objects; they ask if those patterns are accidental or if they occur for a reason; and they conjecture and prove" (p. 56). The ability to reason is intricately related to mathematics proficiency.

The NRC (2001) defined mathematical proficiency as a group of characteristics necessary to be able to learn and perform mathematics successfully. 'Mathematical proficiency' is not just one umbrella concept, it is made up of 5 different strands that are entangled together and are mutually supportive of each other. These strands are conceptual understanding, productive disposition, procedural fluency, strategic competence, and adaptive reasoning. Of the five strands, adaptive reasoning is the adhesive that holds the others together as it "refers to the capacity to think logically about the relationships among concepts and situations" (NRC, 2001, p. 129). Reasoning is, therefore, central to comprehension and how to justify solutions. The clearest indicator of adaptive reasoning is when a student can justify his or her solution with sound mathematical ideas and concepts by using either informal or formal proofs. It was thought that the ability to describe mathematical reasoning was a characteristic of advanced students engaged in higher level mathematics. However, research (Blanton & Kaput, 2005; Boaler, 2002; Carpenter, et al., 2000; Fennema et al., 1999; Hiebert, 1982; Hiebert et al., 1997; NCTM, 2000, 2010; Phillips, 2008; Stigler & Hiebert, 1999, 2009) categorically demonstrates that students at all grade levels are able to justify their work if they possess enough knowledge of the concept, if the task they are working with is termed "mathematics worth doing," and if the classroom environment is geared towards promoting understanding. In order to support students' reasoning, teachers must have more than common mathematics content knowledge. They need specialized mathematics content knowledge, which is used to teach indirectly (Hill, et al., 2005). This indirect teaching involves practices that are both "complex and involves much that is not natural or intuitive" (Ball & Forzani, 2011, p. 18); for example, not telling the students the answer to a particular problem but allowing them to struggle as they attempt to solve it. In addition, teachers must "reflect upon their own learning experiences as they relate to students' thinking" (Phillips, 2008, p. 17). Most importantly, they must have a classroom environment that supports mathematical discourse (Carpenter et al., 2000; Fennema et al., 1999; Hiebert et al., 1997; Stigler & Hiebert, 2009). Moreover, teachers must be able to assess the reasoning of their students and understand how students think about the mathematics they are doing (Phillips, 2008).

The process of developing mathematical reasoning is characterized by a hypothesis making and testing process. Mathematical reasoning involves conversations that include justifications and refutations of those justifications. There is more to justifications than just reciting the process of solving a problem, it involves “more than the observation of a pattern. Although a pattern may explain some property of a sequence of values, the pattern itself requires an explanation that exposes the structure underlying it” (Reid, 2002, p. 25). The process of reasoning starts with conjectures. These can be constructed by students of all grade levels, from kindergarten to calculus (NCTM, 2010). Conjectures are “statements that are tentatively thought to be true but are not known to be true” (p. 13). Conjectures need to be proven or contested by further testing. Therefore, conjectures become the access point by which students start developing mathematical reasoning while doing mathematical tasks. Conjectures can be made from almost any concept that is taught in the mathematics classroom so it is important that the teacher is cognizant of this fact so he or she can integrate conjecturing into his or her daily practice. Figure 1 depicts the beginning of the reasoning process, as students make conjectures, test them, then either accept them or refute them and finally generalize them.

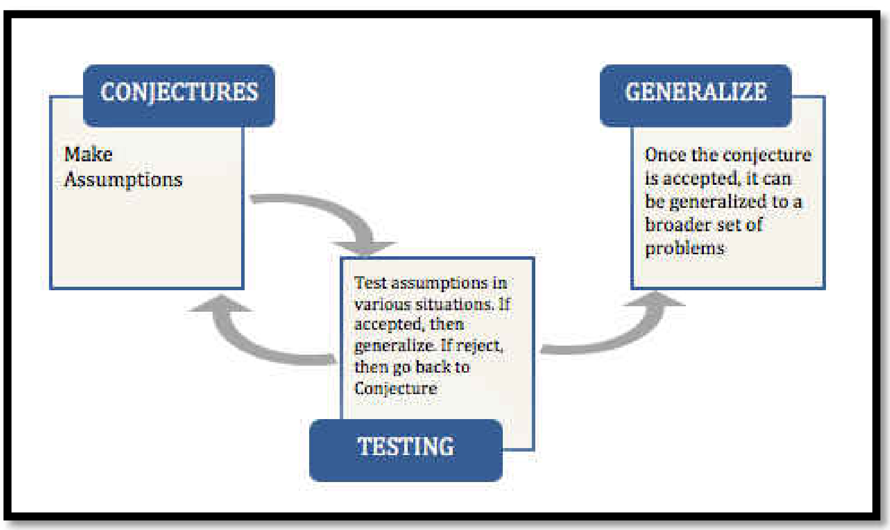


Figure 1 - The Process of Reason Making
NCTM, 2010

As students test their conjectures, they will either accept them or refute them. When they refute them, then they go back to making another conjecture that might or might not be true. This is a process that requires students to have interesting problems to solve (Hiebert et al., 1998), opportunities to communicate with each other, and enough time. “Conjecturing serves as an introduction to further mathematical reasoning since the development of conjectures requires verifying or refuting statements” (NCTM, 2010, p. 16). As students test their conjectures, they should make arguments about why the conjecture is valid or not. The goal of argumentation is to convince others of the reasonableness of the conjectures (Wood, 1999). The process of reasoning cannot happen without argumentation, that is, students justifying and assessing each other’s logic (Lithner, 2000). When the classroom environment supports argumentation, then these discussions will clarify the meanings of terms, symbols, and illustrations and they will facilitate the “investigations of important ideas – ideas that are perhaps not apparent to learners but draw on and build their reasoning” (NCTM, 2010, p. 29).

The mathematics content needed to teach is one topic that is affected by how the teachers perceive what they teach. There is a tendency to see the teaching of elementary mathematics concepts as easy. However, this is not the case (Ma, 1999; NCTM, 2000). Teachers not only have to have mathematical knowledge that is well beyond the topics they teach, they must possess and understand mathematical reasoning, be able to recognize new ways of solving problems, be able to follow their students’ reasoning, be able to understand mathematics deeply, and be able to understand how mathematical concepts connect to one another. Moreover, teachers’ mathematical reasoning must reach far beyond the mathematics they teach (NCTM, 2010) and it is essential for teaching.

> Rich mathematical understanding guides teachers’ decisions in much of their work, such as choosing tasks for a lesson, posing questions, selecting materials, ordering topics and ideas over time, assessing the quality of students’ work, and devising ways to challenge and support their thinking. (NCTM, 2010, p. 2)

In 2005, Blanton and Kaput conducted a yearlong study that looked at how a teacher implemented a classroom environment that promoted the development of mathematical reasoning. They found that teachers must be very thoughtful about the tasks they give their students to do and they must provide their students tools for reasoning as these tools facilitated the students in the study to be able to generalize about algebraic concepts they were learning (Blanton & Kaput, 2005). This was achieved when the teacher created "a conversation that called on students to engage in some form of generalizing or formalizing or to reason with generalizations" (p. 432). Students must be given ways to express their reasoning either with objects or processes. Therefore, students used charts, diagrams, number lines, and representations, among others, to express their reasoning. They found that "these objects became referents around which students reasoned mathematically" (p. 432). Consequently, teachers must be knowledgeable in multiple ways of representing and expressing reasoning, not only formally, but also by using tools. The researchers identified three instructional practices that support the development of reasoning in the classroom: conversations, spiraling of key topics and the design of the task or activity for the students. Spiraling refers to the practice of revisiting of the same topics in an increasingly deep manner over time. The teacher must provide students with the opportunities to communicate their reasoning with each other. Moreover, teachers must include these conversations in a seamless and natural way during daily instruction. Teachers must choose key concepts and include them in activities and conversations over periods of time during the school year. This allows the students to reason about these topics in an increasingly complex way. The nature of the task is of paramount importance not only for the development of reasoning in the students, but because if the teacher can develop their own meaningful task it demonstrates "growth because it shows a capacity to generate resources beyond the finite resource base provided by professional development" (p. 434). The researchers found that the teacher got better at developing meaningful tasks as time went by. As the teacher saw the increased understanding the students were developing. Reasoning in the classroom must be "woven into instruction over sustained periods of time in ways that allow the complexity of ideas to be deeply developed. Moreover, robustness is captured by the teacher's ability to either identify, modify, or adapt resources for planned instructional purposes" (p. 440). The successful implementation of the norms that will convert a classroom into one that promotes the development of mathematical reasoning through discourse requires significant changes in the teacher and his or her beliefs. This is beyond any teacher preparation program and will only be achieved through meaningful professional development (Blanton & Kaput, 2005).

In 1999, Bowers, Cobb, and McClain performed a teaching experiment that was geared to identify the instructional practices that were needed in a classroom that promoted reasoning. One of the aspects that they discussed was the creation of contexts that supported students as they were making sense of the mathematical concepts they were learning. By providing the students with the framework of a candy factory, the teacher was able to provide students with the tools they needed to make conjectures and test them. Furthermore, this context allowed elementary students to explain and justify their reasoning to others, to construct their logic in a mathematically sound manner, and, most importantly, the same context enabled them to support the ability to analyze other's reasoning. One interesting component of this study is that the students helped create the infrastructure of the candy factory, specifically, how the candies were to be packed (in rolls of ten and boxes of 100). This gave the students ownership of the context while they used it to explain and justify their reasoning. The context created for the students also had a secondary benefit: it made the task of assessing the students' reasoning easier for the teacher.

The topic of assessing student understanding is an important one as teachers must be able to perform this task effectively. This critical skill is seldom taught in education programs even though it is essential to teaching (Hiebert, 2007; NCTM, 2000). Morris (2010) conducted a study to gauge the factors that influence how pre-service teachers evaluate mathematical reasoning. The conclusion of the study was that the preservice teachers failed to evaluate the reasonableness of students' arguments in a consistent manner. The results of this study suggested that pre-service teachers employed a large variety of assessments when evaluating their students' mathematical reasoning. "Many pre-service teachers did not appear to understand the relationships among mathematical proof, explaining why, and inductive arguments" (Morris, 2010, p. 510). The findings supported the notion that preservice teachers must be exposed to instances where they need to evaluate a variety of explanations and justifications by using different types of activities that mirror situations encountered in the classroom. In the case of practicing teachers, the only way to acquire this kind of experience is through professional development opportunities (Ball & Forzani, 2011; Birman et al., 2000).

There are specific things that teachers can do to promote mathematical thinking, especially when students are grouped and working simultaneously. Teachers must monitor the conversations that are happening within these groups and learn to identify those occasions where students are showing difficulties sharing with each other. The purpose of teacher intervention in this instance is to facilitate student interaction, not to help them solve the problem. This is termed process help (Dekker & Elshout-Mohr, 2004). Another intervention aimed to improve the reasoning process while students are working within collaborative groups is called product help

(Dekker & Elshout-Mohr, 2004). Like process help, the goal of product help is to increase reasoning without giving direct help. While a teacher gives product help, he or she

> play[s] the role of flexible part-time assistant for the collaborating students. They may perform regulating activities, such as asking the students to explain and justify their work. They may provide hints and scaffolding when key activities become too difficult for the students (Dekker & Elshout-Mohr, 2004, p.44)

The uses of these two interventions carry their own set of challenges. First, it is essential that the teacher communicate with the group at large instead of with just the student that is being helped. Also, the teacher must possess strong content knowledge in order to be able to comprehend the mathematics the students are doing within the group swiftly, as not to spend too much time looking at the work. In other words, teachers must be able to assess the reasoning of their students in real time, without having to spend too much time analyzing it. Moreover, the teacher must be aware of the dangers of the ease with which one can transition from the role of facilitator to one of full participant (Dekker & Elshout-Mohr, 2004).

In addition to the aforementioned challenges, there are some associated with setting up a classroom in a way that supports the development of reasoning. One is related to the task that the students need to complete. When the teacher removes the most challenging aspects of the problems, then the students tend to diminish the time they spend making sense of the problem. Teachers must get used to allowing students to struggle with mathematical ideas in a way that motivates them to solve the problem using innovative ways and methods. When students struggle, they will be more apt to create conjectures, test them, and make generalizations (Henningsen & Stein, 1997). Another challenge that teachers face is the fact that they need to shift their attention from students “getting the right answer” to “developing reasoning.” This might be a major hurdle for some teachers since tradition dictates that we just look at the students’ answers (Henningsen & Stein, 1997). When teachers solve the difficult part of problems, "the cognitive demands of the task are weakened and students' cognitive processing, in turn, becomes channeled into more predictable and (often) mechanical forms of thinking" (Henningsen & Stein, 1997, p. 535). Another important factor in the successful implementation of developing mathematical reasoning as a classroom goal is time. It seems that, when teachers move quickly from one concept to the next, there is efficiency. However, this fast pace robs students of the time necessary to make sense of the mathematics in ways that promote reasoning (Henningsen & Stein, 1997). One way that inservice teachers can meet the challenges related to changing their classroom to one that is student-centered and where reasoning is the focus of instruction is to attend professional development opportunities geared to address those specific concerns and challenges.

Professional Development

Many states, due to the shortage of qualified teachers, especially in mathematics and science, have hired teachers that possess temporary teaching certificates in the hopes that they will complete the requirements necessary to be awarded a professional teaching certificate. This ease of entry, allows teachers with no classroom experience into the classroom (Darling-Hammond, 1999). In addition, the short length of some college of education degrees produce teachers with very limited or no classroom experience (Hiebert & Morris, 2009). The retention rate of new teachers is low, when compared to the number of new teachers entering the classroom (RAND Mathematics Study Panel, 2003). The pressure put on teachers regarding students' outcomes measured by standardized testing by policy makers, parents, and society at large make many of them feel disillusioned and frustrated. This causes them to leave the teaching profession for good. For those who stay, professional development (PD) becomes the only way, other than graduate coursework, to acquire the continuous education required to be informed and to be able to add to their professional practice (Birman et al., 2000; Crockett, 2007; Mewborn, 2003; Porter et al., 2000). Teachers must continue their education throughout their careers. Specifically, effective mathematics instruction "requires reflection and continual efforts to seek improvement" (NCTM, 2000, p. 17). The importance of professional development geared to support teachers as they meet students' needs has never been more vital. Research (Birman et al., 2000; Franke et al., 1998, 2001; Porter el al., 2001; Wei et al., 2010) indicates that PD that is framed around higher-order teaching activities, that is focused on teachers working in the same school, and that is centered on reform activities, yield higher results in teacher learning and in practice improvement. Higher-order teaching activities are those activities that are centered not only on mathematics knowledge, but on how mathematics is learned (Davis & Simmt, 2006). Reform activities are those activities that promote conceptual knowledge, that decrease the emphasis placed on procedural knowledge, and that encourage students to solve problems by using what they know rather than by giving them a solution path (Smith et al, 2005). In addition, teachers must be able to apply what they learn during PD in the context of their own classroom as they receive that PD (Mewborn, 2003).

Unfortunately, teachers' experiences with PD vary greatly and the numbers of teachers exposed to high-quality PD is indeed very small (Birman et al, 2000; Porter el al., 2001). The kind of "intense, collaborative, content-rich, and practice-focused professional learning, which leads to better student outcomes is not typical in the U. S." (Wei et al., 2010, p. 1). The reality of teaching is that "most U. S. teachers work in isolation, take a heavy dose of workshops, and do not receive effective learning opportunities in many areas" (Wei et al., 2010, p. 1). It is critical that districts and school systems provide their teachers with first-rate PD that help them meet students' needs, especially when facing the transition to the Common Core State Standards that were adopted by a majority of the states during 2010.

Franke et al. (1998) argue that the teachers' attitudes and previous knowledge affect the way in which teachers will apply the knowledge they receive during PD. Researchers (Birman et al., 2000; Darling-Hammond et al., 2010; Franke et al., 1998, 2001; Porter el al., 2001) concluded that PD is only effective when certain aspects are met. Most importantly, it should be curriculum centered. The discombobulated curriculum that has been typical of the United States, where each state has its own, is one of the factors that influence the effectiveness of PD (Ball & Forzani, 2011). The adoption of the CCSS will "offer the possibility of a common foundation on which a stronger educational infrastructure could be built" (Ball & Forzani, 2011, p. 18). The challenge now becomes making sure that PD provides teachers with the appropriate skilled instructional practice methods. The disjointed curriculum that existed in the United States has been replaced by the Common Core State Standards, which, for the first time, will give a large part of the country the same standards. In a way, this is the first step towards imitating the conditions that exist in high achieving countries. The CCSSM brings cohesiveness to the curriculum students should master. However, these standards also carry their own challenge posed in the Standards for Mathematical Practice. With respect to SMP3, PD must support teachers as they generate ways to support their students' reasoning development while they encourage students to share explanations and justifications with each other.

Research (Ball et al, 2008; Hill & Ball, 2004; Hill et al., 2005; NCTM, 2000; Porter, 2000) demonstrates that there exists a direct correlation between teacher content knowledge and students' outcomes, so PD must be centered on improving content knowledge for teaching mathematics. In order to be effective, PD must contain the same inquiry elements to which teachers want to expose students. PD should be "inquiry-based, collaborative, subject-matter specific, and grounded in teaching and learning" (Crockett, 2007, p. 613). PD must allow teachers to analyze the mathematics they currently teach in depth, in order to identify the connections among different mathematical topics. In addition, teachers should be able to implement what they learn during PD in their classroom. Moreover, "teachers need professional development that affords them the opportunity to cultivate their listening skills and their ability to analyze children's ideas" (Mewborn, 2003, p. 49).

The body of research involving discourse, reasoning, classroom environment, communication, teacher change, and PD is extensive. In order to measure the state of a teacher-centered classroom at the time prior to this research study, an exploration of the middle school mathematics classroom was performed.

Pilot Study

An investigation was conducted in preparation for this study. Its goal was to gauge the amount of discourse present in the teacher-centered classroom. This was an ethnographic study of the mathematics classroom. Three mathematics classrooms, 6th, 7th, and 8th grades, were observed for a period of 4 weeks during the month of November 2011. Field notes and audio recordings were taken from each of the observations. In addition, a brief informal interview was conducted after each observation. What follows is a scene from a typical day in a middle school mathematics class. It depicts the interaction and communication between a teacher and her student.

> The teacher has shown the students how to write equivalent fractions by describing what steps to follow. The students are instructed to work individually on a worksheet that has been distributed after the lesson. One student gets up from his seat and approaches the teacher, he says, "*I do not know how to do this*" pointing to the worksheet. The teacher looks at him and says, "*It is what we just did.*" The student looks crestfallen but says, "*I don't get it.*" The teacher summarizes the lesson in one sentence "*You multiply the*

> *denominators or you find the least multiple and use either. This is like what we just did. Remember?*" The student remains silent and leaves. However, a few minutes later, he raises his hand and asks for assistance because he cannot understand. The teacher goes over to him and says, "*Just multiply these two*" pointing at the denominators of the problem in the worksheet. "*You get it*?" The student nods but does not do any of the problems on the worksheet. It is clear he does not understand but the teacher has moved on to another student and does not notice.

This vignette was recorded in a middle school mathematics classroom during a pilot study conducted in November, 2011.

The results are very similar to what was reported by the TIMSS study in 1999. According to observations of this limited number of teachers, problems are still used to demonstrate procedures and practice and drill is still one of the main activities done in the classroom. Communication is very limited and is teacher centered. The students receive instructions about the steps to follow to solve a certain problem and then they practice. There is a huge emphasis on the steps and not the logic behind those steps.

The pilot study yielded results that aligned with Stigler and Hiebert's (1999) results, even more than a decade after. The results revealed that the mathematics lesson starts with bell work that the students completed while the teacher was taking care of housekeeping issues like recording attendance or returning work. Then, the teacher solved bell work problem for the students. After that, the teacher announced what mathematical concept students were going to learn that day and demonstrated how to solve problems emphasizing procedures (steps) and usually without context. After the demonstration, students were paired with each other and concentrated on solving the worksheet that connected with the lesson. During this time, the teacher answered questions or just observed students. It is interesting to point out here the fact that when asked, all the teachers stated that the fact that they grouped students encouraged them to share their mathematical thinking. Upon close observation, it was noted that students just shared answers, in some cases they divided the problems with each other to finish faster. In not one instance was reasoning shared by them and once they were done with the work, which did not take long, they would converse with their friends (from other groups) about other subjects. At this point the teacher would stop doing whatever she was doing and redirect the students to their own work. A few minutes before the class period ended, the work was collected, the homework for the day was assigned, and the students were asked to write the goal and homework in their planners. After that, they waited for the dismissal bell. In the classrooms observed, the majority of the problems used by both teachers and students were solved using procedures. The emphasis was placed on the procedure as teachers made a point of writing the "steps for solving" problems on the board. Additionally, there was urgency, on the part of the teachers, to have students finish the daily work. This was apparent when they stated "you must finish this in class." When asked about this, the teachers stated that they would like to go over the material with greater depth but that they felt pressured by the school district to finish within the time constraint in order to provide students with the instruction needed to pass standardized tests.

The pilot study yielded a great amount of information regarding the interactions and routines that happened in the three classrooms that were observed. First, it was observed that all three classrooms' schedules were almost identical, regardless of the grade and teacher. This is something that is interesting and it suggests an effort to provide uniformity of structure across the grades. However, there was no evidence of observing the teacher addressing any individual student's need. Additionally, in all three classrooms the instruction was teacher-centered, with an emphasis on procedures rather than the concepts behind them. All three teachers observed taught using either the students' or the teacher edition textbook. The problems used came from that one resource and most of the problems had no context. There was no expectation of students having to explain and justify their answers in any of the three observed classrooms. Additionally, the social norms in the three classrooms did not include any kind of group (small or whole) discussion of the reasonableness of the answers given for the worksheets. However, the three teachers stated, when asked, that they believed the students just needed to be grouped to enable them to discuss the mathematics they were doing. Unfortunately, the three teachers in the pilot study were unable to hear the conversations that happened in each of the groups because they were busy trying to redirect students who were not on task.

If the students in these three classrooms were to be evaluated on their engagement with SMP3, there would be no evidence that they were engaged in the practice of building, exchanging, and assessing each other's reasoning. Inservice teachers similar to those in the pilot study need PD that will enable them to support the students and meet their needs as they engage students in SMP3.

Conclusion

This chapter described the constructs associated with the research that was conducted. The importance that teachers have in the kinds of interactions that happen in the classroom and how teachers affect the way students learn was explored. In addition, classroom norms that support students' engagement in SMP3 were described. The content standards associated with specific topics in 6^{th} grade mathematics were described along with the expectations related to SMP3. In addition, the mathematics reasoning process and communication were described in detail given their prominent role when students are engaged in SMP3.

The next chapter deals with the methodology that was used in the research study. The design, population, setting, and sampling methods utilized are discussed. In addition, the data collection process is described in detail. Additionally, an explanation of the content of the PD is provided. The plan used for data analysis is described and then, biases, limitations, and possible contributions of the study are discussed.

CHAPTER 3: METHODOLOGY

Research

In order to meet the specifications of SMP3, teachers must support and enable students to have the opportunities to engage in the type of discourse that will have students create, share, and evaluate each other's explanations and justifications. Teachers must address students' needs specifically, as they relate to the development of their mathematical reasoning and the ability to assess their peers' explanations. In addition, teachers must implement classroom norms that support a classroom where communication of mathematical ideas is an expectation. The research questions associated with this study were:

1. What are teachers' self perceptions regarding how they encourage students to share mathematical reasoning?
2. What are teachers actualized roles regarding how they encourage students to share mathematical reasoning?
3. How do teachers' perceived and actualized roles related to how they encourage students to share mathematical reasoning differ?
4. How do teachers influence students' discourse in ways that enable them to express their mathematical reasoning in the classroom?
5. How do teachers influence students' abilities to critically assess the mathematical reasoning of others in the classroom?

Research Design

The methodology used for this study was qualitative with a collective case study design. A collective case study is one of the three types of case studies specified by Stake (2003). It entails looking at several cases in order to "provide insight into an issue or to redraw a generalization” (Glesne, 2011, p. 22). This type of case “allows the investigation of a phenomenon, a population or a general condition” (Glesne, 2011, p. 22). The use of a collective case study design allowed the researcher to look at how two teachers who received similar PD implemented those strategies in their practice. According to Creswell (2007) “the inquirer selects multiple cases to illustrate the issue” (p. 74) in an effort to increase the ability to replicate and, ultimately, generalize to similar situations (Yin, 2003). A case study is a type of ethnography that allows the researcher to investigate complex settings. Given the fact that the mathematics classroom is a very complex environment, this method seemed appropriate to describe the interactions, conversations, and reasoning that happened there (Merriam, 1998).

According to Creswell (2007) and Yin (2003), the collective case study entails extensive data collection from multiple sources like observations, surveys, papers, archives, interviews, participant annotations, and artifacts. This research method provided an in-depth snap shot of the classroom as teachers attempted to create an atmosphere that enabled their students to engage in SMP3. This was achieved by collecting data from multiple sources.

Setting, Population and Sampling

Setting

The school district that was selected to perform this research was located in a large, metropolitan area located in the southeast United States. This district was among the top twenty largest districts in the nation. It had 182 schools, which offered classes ranging from Pre-Kindergarten to 12th grade. This district served over 180,000 students and employed around 12,750 classroom teachers. The middle school selected within which to conduct this research was located in the southeastern part of the school district. At the time of the study, it served approximately 900 students from diverse socio-economic levels ranging from middle-high to low. At that time, the ethnic breakdown of the school was as follows: 57% Hispanic, 23% Caucasian, 14% African American, 5% Asian/Pacific Islander, and 1% Multiracial. This school instructs students in the 6th, 7th, and 8th grades and offers special programs for English language learners, exceptional education students, and gifted students. According to the latest state standardized scores information, from the school year 2012-2013, 53% of the 6th grade students at this school scored on or above the minimum acceptable mathematics proficiency levels (FDOE, 2013).

This school was selected because I gained entrance by being recommended by one of the teachers who worked there. I met with the principal and explained the research to him. In turn, the principal allowed the researcher access to all mathematics teachers in the school. Additionally, the principal provided the researcher with the infrastructure necessary for conducting the PD sessions, such as room, time, and the technology needed to present the material.

Population and Sampling

The sampling strategy used in this study was purposeful sampling. According to Creswell (2007), "…the inquirer selects individuals and sites for study because they can purposefully inform an understanding of the research problem and central phenomenon of the study" (p. 125). I used the following criteria to choose the teachers: (1) the teacher must teach mathematics, (2) the teacher must demonstrate a willingness to participate in the study by accepting to participate in it, (3) no evidence of student engagement in constructing viable arguments and critiquing the reasons of others should be evident in the base line observation, and (4) length of teaching experience, ideally one teacher within his or her first two years of teaching and one experienced teacher (five or more years of experience). There was a chance that more than two teachers might fit these criteria. Therefore, a preference was given to those teachers who were teaching 6th grade content. The two teachers selected fit the criteria mentioned. One was in her first year teaching mathematics (Ms. Jane) and the other was into his 6th year teaching middle school mathematics (Mr. John). Both teach 6th grade mathematics and showed interest in participating in the study. An initial interview was conducted with each of the two teachers chosen for the study. The purpose of the interview was to gauge the teachers' self perceptions about how they encourage their students to talk about their mathematics reasoning and make sense of others' reasoning before the first observation. This observation served as the base line I used when I went into the classroom for successive observations. The initial observations were documented using ethnographic style field notes, as "the most important element of field work is being there… to write down what is seen and heard" (Fetterman, 2010, p. 9). The data collected in the initial interview combined with the initial classroom observation informed the researcher in the planning of the professional development that was used to support teachers as they enabled their students to communicate their mathematics reasoning as they justified their answers and as they motivated their students to assess their peers' logic.

Professional Development

The body of research about professional development (Birman et al., 2000; Darling-Hammond et al., 2010; Franke et al., 1998, 2001; Mewborn, 2003, NCTM, 2000; Porter el al., 2001) states that in order to be meaningful for teachers, professional development must include important mathematics, must promote change, must be grounded in sound theories, and must be ongoing. The professional development used engaged the teachers in activities that had them make sense of the methods related to eliciting students' reasoning and the justification of their answers with sound mathematical reasoning, provide students with opportunities to assess each others' justifications and explanations, plan questioning schemes that would enable students to share their mathematical thinking, be actively involved in the process of developing a body of knowledge regarding choosing appropriate problems, and identifying different manners of engaging students in sharing their answers with each other.

The PD that was used in this study was developed by the Mathematics Assessment Resource Service (MARS) in order to assist practicing teachers to prepare for the CCSSM and its effectiveness was discussed in Chapter 2. The Mathematics Assessment Project (MAP) was developed by the MARS Shell Center for Mathematics Education, the University of Nottingham, and the University of California at Berkeley and it consists of three main parts: summative tests or tasks, classroom challenges, and professional development modules. The rationale behind the MAP PD series is that in order to teach the CCSSM focusing on engaging students in the SMPs, teachers will need to teach in manners that are not traditional thus taking them out of their comfort zone. The method used in the PD was one that focuses on classroom activities, guided discussions, lesson preparation, and teacher reflection. The MAP PD is divided into five different modules. Module 1 is about formative assessment and how to use it to make instructional decisions. Moreover, it focuses on non-traditional types of assessment. Module 2 focuses on concept development with an emphasis on transforming typical textbook problems and tasks into non-structured tasks that promote inquiry and which are open ended. Module 3 concentrates on creating lessons that are centered on problem solving. Module 4 focuses on using questioning as a vehicle to promote discourse in the classroom. Module 5 centers on using student collaboration and how to implement it. The PD and its materials are free to use for teachers. Moreover, these materials were designed to be used by a group of teachers with a PD facilitator. The MAP project is directed by Hugh Burkhardt, Malcolm Swan, Daniel Pead, Phil Daro, and Alan Schoenfeld.

One of the most important PD sessions was the first one, as it set the tone for the rest of the PD sessions. The PD was offered to all the mathematics teachers at the school. However, only the two teachers who agreed to participate in the study attended. The data collection focused on these two teachers for the collective case study. It was important to create rapport between the teachers and I since ethnographic style observations were conducted, it was important that the teachers were not self-conscious of my presence in their classrooms during observations. Another purpose in creating rapport with these two teachers was to lay the groundwork for this small team of mathematics teachers to create a community of learners, where they can continue to support each other as they prepare to meet the demands of the CCSSM and the SMP. Tashana Howse, another researcher, and I conducted the PD sessions. Ms. Howse was a doctoral candidate and she was interested in investigating how minority student populations demonstrate SMP3 when their instruction is infused with Culturally Responsive Teaching (CRT) methods (Howse, 2013). CRT principles delineate a way for teachers to infuse their practice with the culture of his or her students and for the students' culture to be reflected in the classroom appearance, norms, and behaviors (Gay, 2000; Tutak et. al, 2011). As classroom environment is one of the main factors affecting what happens inside the classroom and the disposition of the students, Ms. Howse investigated the effects that a CRT infused classroom and task have on the ability of students in creating viable arguments and critiquing the reasoning of others. Table 4 depicts the synopsis of the content and activities that were planned for the seven one-hour sessions of professional development that spanned 4 weeks.

Table 4 - PD Schedule

Session	Content
Session One	• Participant and facilitator introductions • Overview of the study • Introduction to the CCSSM • Introduction to the SMP/Culturally Responsive Teaching (CRT) • In-depth look at SMP 3/CRT • Video relating to SMP 3/CRT
Session Two	• Formative Assessment (MAP module 1) • Activity – Video and whole group discussion • Identify practical classroom application of topics presented
Session Three	• In-depth discussion of mathematical reasoning • Concept Development (MAP module 2) • Activity – Video and whole group discussion • Identify practical classroom application of topics presented • Classroom Observation
Before Session Four	• Classroom Observation • Interview
Session Four	• In-depth discussion of choosing worthwhile mathematics tasks • Problem Solving (MAP module 3) • Activity – Video and whole group discussion • Identify practical classroom application of topics presented
Session Five	• Improving Learning Through Questioning (MAP module 4) • Activity – enabling student discourse/CRT/video • Identify practical classroom application of topics presented • Homework: Lesson Planning
Before Session Six	• Classroom Observation • Interview
Session Six	• Students Working Collaboratively (MAP module 5) • Activity – video and discussion • Group discussion and reflection of experience • Identify practical classroom application of topics presented
Session Seven	• Summary of topics discussed: mathematical reasoning, communication, discourse, and CRT • Review study plans for implementation within learning community • Thank you to the teachers for their participation

During the PD time period, teachers were provided with materials and instruction that supported the content of the professional development, including videos. These videos were added to the body of the PD. They were created by Dr. Juli Dixon who is an expert in the field of mathematics education and who is at the forefront of educating teachers about the CCSSM and the SMPs. The videos used demonstrated the techniques shown in the PD in the context of a US classroom as the videos that accompanied the MARS modules were done in British schools. The videos used depicted a typical US middle school classroom. This was important because it gave the teachers participating in the PD a context that was familiar to them and related the material presented in the PD into a habitual situation. The videos allowed the teachers to appreciate a practical example of what a classroom where discourse was encouraged looked like. Moreover, they were able to observe a master teacher demonstrate the techniques taught in the PD in a real live classroom. In addition to this, the teachers started building a body of knowledge that they can share, add to, and reflect about as a learning community. The intention was that they continue to use this body of knowledge to create a source of information that can be accessed by all the mathematics teachers in the school. The goal of the PD was that teachers acquired the basis with which to start to change their teaching practice by eliciting student-generated conjectures and encouraging students to respond to those of their classmates in order to better support the development of the SMPs, with emphasis on SMP3.

Data Collection Artifacts

During the research study, four different types of data collection artifacts were used to gather information.

Interviews

The teachers who participated in the study were interviewed using an Initial Interview Protocol (see Appendix A) as a manner of gauging (1) their opinions regarding their classroom environment, (2) their perceptions of students' interactions, (3) how they elicit active participation from their students, and (4) how they enable students to share their mathematical thinking. In addition, the teachers were asked to describe their students' level and mathematical ability. The initial interview protocol items were modified from items contained in the Horizon Interview Protocol (Appendix B) (Horizon Inc, 2000). The questions modified centered on classroom environment, student participation, and students' conversations. The teachers were first interviewed after the first observation.

In addition, the teachers participated in a phenomenological-type interview within forty-eight hours after each classroom observation that was performed during PD. During these interviews, video elicitation was used as a method to bring forth the participating teachers' reactions to their own teaching. Video eliciting is an integral part of what Schon (1983, 1987) termed "reflection-in-action." Furthermore, it involves "critically reflecting upon the experience after the fact" (Sewall, 2009, p. 12). Many studies (Beck, King, & Marshall, 2002; Moore, 1988; Sherin & Van Es, 2005; Wang & Hartley, 2003; Westerman & Smith, 1993) have featured Video Eliciting Research (VER) and find that this is a powerful means of allowing teachers to understand, evaluate, and discover those practices that will allow them to become better teachers. In addition, VER enables teachers to deepen their self-reflection practices and learn from each other (Sewall, 2009). The clips that were used during these interviews depicted the teachers at a moment where there were missing opportunities and the intent was that they reflected upon their practice. All interviews were audio recorded to ensure accuracy.

Observation Field Notes

All observations were recorded in the form of field notes by the two researchers. Fetterman (1995) states that the researcher "writes down in regular, systematic ways what she observes and learns" (p. 1). The observation field notes, in addition to the audio and video recordings, enabled me to construct a more complete picture of the interactions that happened in the classroom during the observations. Observations were conducted during the instruction of the PD. Then, the teachers were observed for an entire week after the PD. The final observation was performed four weeks after the PD had been administered.

Video and Audio Recordings

All classroom observations were video and audio recorded. Two video cameras were used, one was fixed on the front of the classroom where the teacher usually stood and the other followed the teacher as he or she moved around the classroom. Audio recording devices were placed in each group of students to record the conversations they had as they worked through problems or tasks given by the teacher.

Reflections

As part of the PD, teachers were asked to write reflections of their experiences as they attempted to implement the methods that were described in the PD for the weeklong observations. The teachers wrote about their thoughts, challenges, frustrations, about what worked, and how they felt their students were doing with the new expectations of expressing their mathematical reasoning and assessing their peers' logic. The teachers were given explicit directions regarding what had to be included in the weeklong lessons. These requirements were the central focus of the PD. During that week, the teachers used lessons that included: (1) formative assessment, (2) a focus on concept development, (3) problem solving to assess student knowledge, (4) targeted questioning to develop and assess student comprehension, and (5) students working collaboratively. The last two requirements also had the goal of promoting student mathematical discourse with the teacher, when they had to justify their answer, and with each other, when they had to assess the justifications of their peers.

Data Collection

For the chosen methodology, case study, a variety of data collection tools are recommended (Yin, 2003). The data collection tools that were used in this study were interviews, classroom observations, and video and audio recordings of the teachers as they taught, and teacher reflections. Multiple modes of collecting data increased the fidelity of the study (Glesne, 2011). The data collected were used to create a coded matrix that yielded the themes that emerged as the data were analyzed. Data were collected in four different stages.

Stage 1 of Data Collection

Teachers were observed teaching a typical mathematics lesson and then they were formally interviewed before the first session of PD. The data collected from these sources enabled the researcher to determine the teachers' self perceptions versus the reality in the classrooms. Both, the initial interview and initial observation, informed the researcher of the teachers' individual needs as I assessed their deficits in engaging students in SMP3. The initial interview included modified items from the Horizon Interview Protocol (Appendix B). The protocol was constructed with modified questions and observable behaviors indicators that centered on classroom environment, teacher perception, student participation, and how the teacher facilitates student participation. The interview was recorded to ensure accuracy. The initial observation was noted using ethnographic style field notes as they allow the researcher to hone in on the different aspects of the classroom environment (Fetterman, 2010). Moreover, audio and video recording were used during the initial observation to ensure all aspects of the lesson were properly documented. To ensure impartiality in the field notes and to decrease judgmental orientation, two researchers observed and took field notes and these notes were compared for accuracy. Both observers were experienced mathematics teachers. In addition, both were doctoral candidates in mathematics education with experience in classroom observations and writing field notes. The recordings and both sets of notes served to triangulate the data collected in stage one of data collection.

Stage Two of Data Collection

The teachers were observed in two different instances during the period of the professional development as they attempted to put into practice the concepts they were learning. One observation was performed after session three and the other after session five of PD. The same data collection methods used during stage one of data collection were used during this phase as teachers integrated what they learned in the PD into their classroom as they started to encourage their students to share their reasoning with each other. The teachers were interviewed within 48 hours of each observation and VER was used to elicit responses from the teachers. The selection of the clips that were shown to the teachers focused on those that contained instances of missed opportunities for SMP3, and successful examples of the implementation of SMP3. The teachers observed clips that showed them teaching and then they commented on how they could improve, the missed opportunities they had, and talked about "what if" scenarios. The content captured in the videos drove the interviews and these were phenomenological in nature. The interviews were audio recorded to ensure accuracy. As before, observation data triangulation was performed with two observations, the audio recording, and the video recording. The teachers were asked to write a reflection regarding their experiences as they attempted to change the way they teach. These reflections provided powerful insights on the perceived and actual roles that teachers had to face as they changed the way their students learn.

Stages Three and Four of Data Collection

Both of these phases involved classroom observations and teacher reflections. Stage three observations were performed immediately after the PD was finished. Stage four observations were performed four weeks after the PD sessions were completed. During phase three, Mr. John was observed 5 times and Ms. Jane was observed 7 times. Originally, both teachers were going to be observed 5 times but when Ms. Jane was observed after the PD had concluded, the researchers did not observe instances that showed her students work with mathematics that allowed them to engage in SMP3. Therefore, the observation period was increased to 7 days in order to give Ms. Jane the opportunity to engage her students in SMP3. This matter was discussed in Chapter 5. Both of the teachers were observed once for phase 4. As before, all the observations were audio and video recorded and the two researchers used field notes to record their observations. The teachers were observed teaching the same class periods as in prior observations. The teachers selected the periods within which they wanted to be observed at the beginning of the study. After each observation, the researcher met with the teachers for a brief period of time, 5 or 10 minutes in order to elicit their opinion regarding the lesson they just taught except for the observation conducted four weeks after the PD. In addition, all the observations were audio and video recorded to ensure accuracy. At the end of the lessons, teacher reflections were collected. Table 5 shows the data collection methods that were used in each of the four phases of data collection.

Table 5 - Stages of Data Collection

	Purpose	Questions
Stage One of Data Collection		
Initial Observations	Establish baseline for PD	What are teachers' self perceptions regarding how they encourage students to share mathematical reasoning? What are teachers actualized roles regarding how they encourage students to share mathematical reasoning? How do teachers' perceived and actualized roles related to how they encourage students to share mathematical reasoning differ?
Initial Interviews	Establish baseline for PD	What are teachers' self perceptions regarding how they encourage students to share mathematical reasoning? What are teachers actualized roles regarding how they encourage students to share mathematical reasoning? How do teachers' perceived and actualized roles related to how they encourage students to share mathematical reasoning differ?
Stage Two of Data Collection		
Classroom Observation During PD	Document implementation of learned topics in real-life settings	How do teachers influence students' discourse in a way that enables them to express their mathematical reasoning in the classroom? How do teachers influence students' abilities to critically assess the reasoning of others in the classroom?
Reflections	Understand teacher struggles, doubts, successes, and roadblocks as they implement new methods of learning and expectations for their students.	What are teachers' self perceptions regarding how they encourage students to share mathematical reasoning? What are teachers actualized roles regarding how they encourage students to share mathematical reasoning? How do teachers' perceived and actualized roles related to how they encourage students to share mathematical reasoning differ? How do teachers influence students' discourse in ways that enable them to express their mathematical reasoning in the classroom? How do teachers influence students' abilities to critically assess the mathematical reasoning of others in the classroom?
Stage Three of Data Collection		
Classroom Observation After PD	Document implementation of learned topics in real-life settings	How do teachers influence students' discourse in a way that enables them to express their mathematical reasoning in the classroom? How do teachers influence students' abilities to critically assess the reasoning of others in the classroom?
Reflections	Understand teacher struggles, doubts, successes, and roadblocks as they implement new methods of learning for their students.	What are teachers' self perceptions regarding how they encourage students to share mathematical reasoning? What are teachers actualized roles regarding how they encourage students to share mathematical reasoning? How do teachers' perceived and actualized roles related to how they encourage students to share mathematical reasoning differ? How do teachers influence students' discourse in ways that enable them to express their mathematical reasoning in the classroom? How do teachers influence students' abilities to critically assess the mathematical reasoning of others in the classroom?
Stage Four of Data Collection		
Classroom Observation 4 Weeks After PD	Document implementation of learned topics in real-life settings	How do teachers influence students' discourse in a way that enables them to express their mathematical reasoning in the classroom? How do teachers influence students' abilities to critically assess the reasoning of others in the classroom

Table 5 depicts each of the data collection sources, their contribution to the research study, and the research question they helped address. The audio recordings, observations, and interviews, were transcribed and these, along with the field notes and reflections were coded into a matrix that was used to construct the recurring themes that emerged from the data gathered. The digital copies of the transcripts, video recordings, reflections, and any other documentation associated with the study were kept at a safe deposit box located at Bank of America.

The researcher recognized her position during the classroom observations that occurred. The researcher is a mathematics teacher educator with 8 years teaching experience in both the secondary and college levels. The researcher has a masters' degree in Mathematics Education, and, at the time of the study, was a doctoral candidate in Mathematics Education. The researcher has experience observing classrooms and evaluating teachers and the effectiveness of mathematics lessons. This background made the researcher a knowledgeable source for content and methods for teaching mathematics. During the duration of the 16 observations, the researcher did not advise, suggest, or offer any assistance to the teachers as they taught their students. Additionally, during the observations, the researcher kept her distance from the students and did not obtain any opinions from them or interact with them in any way. Due to the number of observations, the students and the teachers got used to the researcher's presence in the classroom, so her role as an observer went from one of outsider to one of "becoming an insider over time" (Creswell, 2007, p. 134). Nevertheless, the researcher acknowledged the fact that she was a visitor in the classroom, and by virtue of being an outsider, her presence impacted the behavior of both students and teachers.

Data Analysis

Qualitative research, such as this, can be a daunting task. Consequently, Glesne (2011) recommends that the researcher starts a codebook soon after the data collection starts. A codebook is a personal journal that the researcher keeps during the data collection phases of the study. It is highly personal so it will only work with the researcher that creates it. The researcher created a codebook for this study and in it she assigned a major code (theme), such as, questioning, constructing reasoning, and others to a page. Then, the researcher assigned sub-codes (sub-themes) to the major code, such as missed opportunities and conjectures. Both major codes and sub-codes had an explanation of what they were, this definition is what is called a "definitional drift" (Glesne, 2011) and the researcher used it to keep track of the topics as she wrote in the codebook. As the data were collected, the researcher wrote on each of the pages the dates and artifacts that were connected to each of the codes and sub-codes along with notes and reflections of my observations. This codebook facilitated the process of coding the main themes that surfaced as the data were analyzed. One of the sub-codes evolved with time and became a major code by virtue of time. The codebook was used in the process of analyzing the data. It helped me to group them under the codes and sub-codes to which they belong.

After the data were collected, the audio recordings and the video recordings were indexed using the codebook and NVivo. This is software that allowed the researcher to analyze and manage audio and video data. The interviews were transcribed and the reflections were coded using the codebook. The researcher used a thematic analysis and the codebook in order to look for themes and patterns. Once the data were arranged in codes (or themes) she used the codebook to help her arrange these codes into clumps that went together in a logical order. This process took some time because, as Glesne (2011) states, "it is a time when you think with your data, reflecting upon what you have learned, making new connections and gaining new insights, and imagining how the final write-up will appear" (p. 197-198). After this was done, each case emerged and was arranged in logical themes. Each case will be presented in depth, using narrative, tables, and figures (Creswell, 2007) in the following chapter.

Limitations

The limitations related to this study were the typical ones related to case studies. The first limitation was credibility. This refers to whether the results were trustworthy. The triangulation that was performed, because of the multiple sources of data collection, counteracted this limitation. The second limitation related to transferability. In other words, the likelihood that the results can be generalized. The constructs and themes that emerged from the study were based on teacher self perception and attitude, social norms, reasoning, and communication. Since the basis of the discussion of those themes was the same, the results could be generalized as far as it concerns a teacher with the same general characteristics as the ones observed in the study. The third limitation of the study dealt with the dependability of the findings. The reliability of the instruments (given by triangulation) paired with the consistencies that exist in the available literature concerning the description of classroom norms and practices that promote reasoning and discourse offset this limitation. The fourth limitation involved the conformability of the information gathered. The data gathering instruments that were used in the study were structured in a way that left room for interpretation. However, in order to neutralize this limitation, two researchers conducted the observations at the same time and the observations were compared to ensure uniformity. The fifth limitation concerned the subjects themselves: the participating teachers. I recognize the fact that the teachers that were observed might have tried to engage in a practice known as "potential deception" (Creswell, 2007). That is they might have attempted to alter their behavior due to my presence in their classroom. This could have caused an inaccurate depiction of what really happened in the classroom. Creswell (2007) advices that the observer should be "passive and friendly" (p. 134) during observations in order to counteract the possibility of this kind of deception. Therefore, this was the stance I had during all the observations. The last limitation of the study came to light after the study itself. This limitation was time. The complex nature of the classroom environment, daily routines, and expectations cannot be changed in a few weeks. The changes attempted should be implemented in the beginning of the school year and should be supported through the school year in order to be successful.

Summary

This chapter described the research study that was conducted and reiterated the research questions that guided this study. In addition, the research design, setting, population, and sampling were described. Moreover, a description of the PD that was used in this study was discussed. Then, the data collection artifacts that were used in this investigation were described along with the data collection process. The four stages of the data collection process were explained along with a plan for data analysis. Then, limitations of this study were discussed. The next chapter will analyze the findings of this study, by describing in detail each of the two cases and the themes that emerged in each.

CHAPTER 4: ANALYSIS

This research study mirrors others that investigate the interactions between students and teachers as they implement changes (Akkus & Hand, 2011; Birman et al., 2000; Bowers et al., 1999; Carpenter et al., 2000: Davis & Simmt, 2006; Dekker & Elshoot-Mohr, 2004; Dixon et al., 2009b; Hiebert et al., 2007; Hill & Ball, 2004a; Howse, 2013; Nathan & Knuth, 2003; Phillips, 2008; Porter et al., 2000; Wei et al., 2010). This study was divided into four phases; (1) initial interviews and classroom observations; (2) the instruction of PD and observations during PD; (3) week long classroom observations after PD was concluded, and; (4) classroom observation four weeks after the PD. The data from each phase will be presented and the following research questions will be addressed as each phase is analyzed:

1. What are teachers' self perceptions regarding how they encourage students to share mathematical reasoning?
2. What are teachers actualized roles regarding how they encourage students to share mathematical reasoning?
3. How do teachers' perceived and actualized roles related to how they encourage students to share mathematical reasoning differ?
4. How do teachers influence students' discourse in ways that enable them to express their mathematical reasoning in the classroom?
5. How do teachers influence students' abilities to critically assess the mathematical reasoning of others in the classroom?

Initial Observation and Interview

In this section, both cases will be described based on the observations that were conducted prior to PD and responses to the initial interview protocol, as this was the goal of phase 1. To this end, initial classroom observations were performed in order to obtain a clear picture of each of the teacher's teaching style, the environment of the classroom, and the interactions that happened there. In addition, an initial interview was performed to gauge their perceptions and opinions.

Case One: Ms. Jane

Background

Ms. Jane has a bachelor's degree in Elementary Education and a master's degree in Exceptional Education. Ms. Jane was in her third year teaching but this was her first year in middle school. Her previous two years of teaching experience were limited to Pre-Kindergarten. This was also Ms. Jane's first year teaching mathematics. Therefore, we considered her to be a first year mathematics teacher. The class that was observed in this study was made up of 21 students, 13 boys and 8 girls. The students in this class were considered to be the lowest performers in the 6^{th} grade. The school divided students by their performance. Because of Ms. Jane's master degree, she had these students along with all the ESE students in 6^{th} grade.

A typical Day in Ms. Jane's Class

Ms. Jane's classroom was cheery and colorful. The seats were arranged in rows facing the board. There were a couple of bookcases and four computers in the classroom and there was some student work on the walls. The bell opener and the daily goal were displayed on the board. The students entered the classroom and started taking their seats and Ms. Jane activated an electronic timer. Ms. Jane kept reminding them that they should start working on their bell opener. This work consisted of close-ended questions about converting percentages into decimals and fractions without context. This material was reviewing the previous day's class. Figure 2 is a depiction of the problems used this day

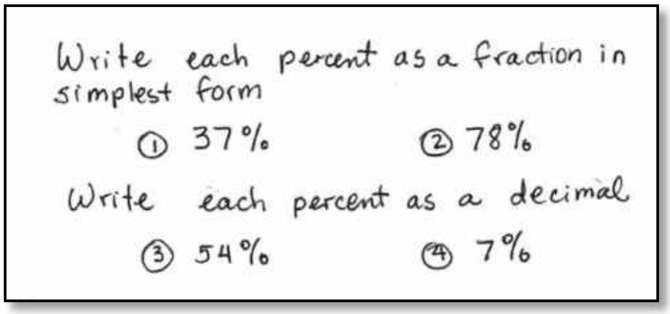


Figure 2 - Bell Opener in Initial Observation – Ms. Jane

The students had a hard time focusing on the task in front of them. They were restless and talkative. Ms. Jane moved around the room trying to keeping students on task, she reminded them that they needed to finish, and she answered individual questions. At the end of the five minutes allowed by Ms. Jane for the students to finish the board opener, she went over the problems with the whole class. What follows is an excerpt of one of the conversations that happened as the students were going over the bell opener (the students' names are pseudonyms).

Ms. Jane	So… look at number four… Alicia?
Alicia	hum… its 0.07
Ms. Jane	Yes, that is right. So, she knows that there is nothing there, so she adds some zeros to get 0.07, okay?

This was the last problem of the bell opener and Ms. Jane already mentioned the fact that students had to move the decimal point two spaces to the left in order to convert the percentage into a decimal number. Ms. Jane was content with the answer the student gave her and she briefly reviewed the procedure for converting the percentage to a decimal number. The expectation of explanation and justification for the answer did not exist.

After the bell opener, Ms. Jane instructed the students to take out their homework and stressed the fact that they could not lose any of the pages in their packs because they were going to be used those for subsequent lessons. Then, she proceeded to ask the students if anyone had a question regarding the homework assignment. While she repeated the phrase: "question about your homework” several times, the students talked amongst themselves. The following is one of the conversations that happened at this point in the observation.

Ms. Jane	Anyone has a question about the homework? Roberto?
Roberto:	Number 18 and 19.
Ms. Jane	Number 18 and 19? Ok, we will not do 18 because I told you to do the odd numbers. So, we will look at 19 only. OK, page 291 in your workbooks. Ok, it says “Order the percentages from least to greatest.” So, you have 92%, 8%, 52%, and 62%. How do you think you need to order those? Just like the percents in the test you got. How do you think you are going to order them from least to greatest? What does “least to greatest” mean?
Clara	You convert them into decimal and then order them from the smallest to the largest.
Ms. Jane	Do we have to convert them to decimals?
Roberto	Well, not really
Ms. Jane	Not really. You could but since they are all in percents you can look at them because they are all the same, OK? So, just like if you had decimals, they have to be all the same. So, if you have all decimals or if you have all percents they are all the same. If you have different, then you convert the fractions into decimals. OK? So, what does it mean “least to greatest”? Alicia?
Alicia	Small to big
Ms. Jane	That is right, smallest to biggest. So, if we have 92%, 8%, 52% and 62%. Which one is the least? Jim?
Jim	8%
Ms Jane	8%. Next one, what is it?
Clara	52%
Ms. Jane	52%. What is what the next one?
Carlos	62%.
Ms. Jane	62%. So what is the last one Alicia?
Alicia	Are those crossed out? (pointing at the percentages on the board)
Ms. Jane	No, but let’s cross them. Now, what is left?
Alicia	92%
Ms. Jane	Does this make sense? Carlos? Do you have any other questions?

As we can see from this dialogue, Ms. Jane was diligently trying to help her students make sense of the concept. However, she asked many questions and did not leave enough wait time for the students to answer. Additionally, many students were talking amongst themselves while she was attempting to review the homework. This made the environment very distracting. It was obvious that Ms. Jane was having some classroom management issues. After the homework was reviewed, Ms. Jane started the lesson. Ms. Jane began by quickly reviewing the previous day's lesson. While she was doing this, she asked a great number of questions very fast. Some of the students participated but some did not and continued their own conversations.

As she began, she stressed the fact that the students must follow the format for notes that was displayed on the board. She instructed the students to take out their table of contents (in their mathematics notebooks) and number the page with a 10. Figure 3 depicts the diagram that students followed for their notes. The students were strongly encouraged to follow this format.

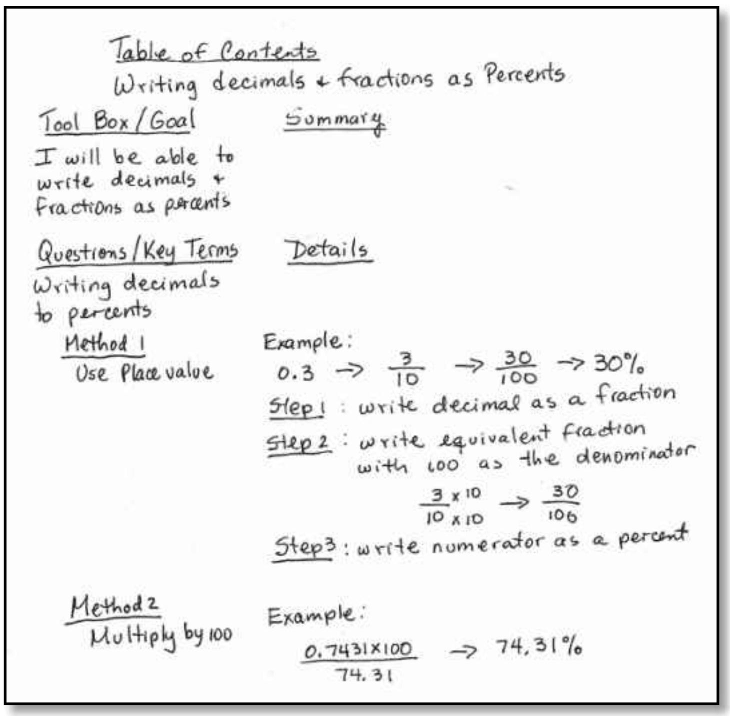


Figure 3 - Cornell Notes Format – Ms. Jane

While the students got ready to take notes, there was a lot of activity. Some students were not ready and Ms. Jane started to count down from ten to indicate that students should be ready to start as soon as she got to one. Still, after she was done with the countdown, students were not ready to start.

Ms. Jane announced that the projector was broken that that she would write on the board what they need to write in their notes. This became a “fill in the blanks” using the format on the board. Ms. Jane started to fill out the empty parts of the template. However, her writing was very small thus it was difficult to see from far away. The following is an example of the conversations that happened during this time.

Ms. Jane	Alright, I am going to need help with my goal (pointing at the template on the board). What is my goal? Carlos?
Carlos	Converting percentages into decimals and fractions (read from the board)
Ms. Jane	(writing on the “toolbox/goal” section of the template as she speaks) OK, then, I will write, “I will convert percentages into decimals and fractions” in my toolbox.

Throughout the lesson, Ms. Jane indicated what to write and where it went in the notes. There were a few pauses here because the students got loud with their conversations and Ms. Jane sat down and crossed her arms as a way to call attention to the fact that the students needed to focus on what she was saying. The following is an example of how Ms. Jane instructed her students. At this point Ms. Jane was discussing how to convert a decimal into a percent. She wrote in the empty spaces on the template as she spoke.

Ms. Jane — Method one: Use place value.

(Several students complain that they cannot see the board)

Ms. Jane — There is plenty of room up here. Move if you need to see. Henry, you can sit right up here, my dear.

Henry — But I like the back

Ms. Jane — You can sit right up here. Henry? Thank you.

(In the meantime, there was a lot of talk and noise. Students talked to one another across the classroom)

Ms. Jane — Ok, method one: place value! We have an example of 0.3. OK, so the first thing you are going to do is to write the decimal as a fraction. Write the decimal as a fraction. So, our decimal is point three, how do you write this as a fraction? Henry?

Henry — Three tenths

Ms. Jane — How did you come up with that?

Henry — Because the three is in the tenths place

Ms. Jane	Ok, so it is three over ten. OK? Your next step is to write an equivalent fraction with a hundred as the denominator. So write "write an equivalent fraction with a hundred as the denominator".

(There were more complaints from students that could not see the board, Ms. Jane repeated what the students needed to write several times and spelled out denominator for the students who could not see the board)

Ms. Jane	So, we have 3/10. We are going to write an equivalent fraction with a denominator of 100. How can I go from 10 to 100? Henry?
Henry	Add one zero?
Ms. Jane	Ok, adding one zero. Or, I can multiply by…?
Henry	Ten
Ms. Jane	Thank you. Or I multiply by 10. Whatever I do to the bottom I have to do to the top, so I multiply this (pointing to the 3 in 3/10) by 10. What is 3 times 10 David? (David does not answer) 3 times 10?
David	30
Ms. Jane	30! Ok, so now I have 30/100. My last step is to write the numerator as a percentage. (writing as she speaks) Write the numerator as a percent. OK, so my numerator is what?
Henry	30
Ms. Jane	That is right, 30! So, that is my percent.

The lesson continued in the same manner, some students answered questions and some talked about different things. Ms. Jane showed the students what they needed to write and repeated herself several times. There were several pauses during instruction because Ms. Jane stopped whatever she was doing and sat to wait for the students to start paying attention. Ms. Jane was making an effort to make the concepts more understandable for students by chunking down procedures into steps and having the students write the steps in their notes. Still, there was no reasoning associated with the concept and procedures were emphasized. Ms. Jane did not get a chance to finish the lesson. The bell rang before she concludes, so she announced it would continue the following day. The students left immediately. They were not dismissed.

Self Perceptions

Ms. Jane saw her role as a teacher as an important one. She also believed that she provided a positive and nurturing environment that promoted respect for students' ideas, questions and contributions as shown in the following excerpt from her initial interview

> I definitely want them to be comfortable, especially with math because I know that it can be frustrating and sometimes discouraging. It's not easy sometimes and often emotions are a struggle with math so I definitely want them to be able to try and not worry about getting the correct answer. I am not really worried about the correct answer but more of the process, um that they're getting it. Um, and I don't want them frustrated but I know it can be so I try to have yeah know a happy environment, comfortable.

Ms. Jane recognized her shortcomings in the classroom management of her students. She usually sat and waited for her students to calm down or quiet down before continuing her lesson. During the initial interview, Ms. Jane stated that she had been trying to use several reward systems, like tickets and "Classroom Dojo" to encourage good behavior and participation from the students. Still, she recognized the importance of having a positive learning environment and she worked very hard in order to obtain it.

When asked about the levels of her students and how they compared to the rest of the school, Ms. Jane stated that:

> Um, I have the lowest kids. I have the ones and twos on FCATS and all of the ESE kids. So um I have the lowest because we are divided by teams

Ms. Jane believed that her students were low performers and this was something that she repeated when she was asked about 'rigor'.

Supporting Students Sharing Mathematical thinking

When asked about how she perceived students' interactions with each other, with groups and as a whole classroom, she expressed that she varies how her students work either individual, paired, or in groups.

> I try a lot of different activities working either in partners, sometimes individually, and sometimes in groups, because I think it is important to you know interact.

Ms. Jane had the expectation that by pairing or grouping students was enough for them to share their mathematical thinking. Still, she expressed her concerns about how her students talked to each other especially when they disagreed with each other. This clearly demonstrated a lack of set rules and behavior expectations from the students. As demonstrated in the following excerpt, she was struggling with her students' behaviors.

> If they disagree with something, I respectfully disagree. Not I just, I don't say your answer is wrong because I know in math sometimes they get different answers so I try that. Um, but my kids are hard. You know it is tough because I do have middle schoolers and they are, they're making that change now and um they are disrespectful to each other and it is frustrating.

This response triggered a question about how she controlled students' conversations when they disagreed with each other. Ms. Jane stated that sometimes she would talk to the student individually but that she also had stopped the class to talk to students about behavior.

> I might bring a situation um if it is a problem, I will just sit them down and be like ok guys, you know I'm not blaming any fingers here and there but like if we are in a situation where this happens, what is a better way to do it? Or if something has happened like you know we have had you know notes being tossed around or you know stuff like that, that I'll sit them down and just kind of talk to them about it and maybe ways that they could do it differently um, because some of them come up to me and will ask me about you know discipline.

It was interesting to note here that some of the students asked about the discipline in the classroom.

When asked how she encouraged students to express their mathematical thinking and reasoning, Ms. Jane expressed that this was one area where she struggles. Still she asked her students to show the steps they used to solve a particular problem and sometimes she had students write reflections. However, this was not a consistent practice. As demonstrated in the following excerpt, Ms. Jane knew some ways of eliciting mathematical thinking from her student. However, she was not using these techniques effectively.

> Um because I do have them you know, I'll have them, I'll ask them you know, why did you do that or why did you do that? Or I'll have them show their steps so that way I do know. I also have reflections. Like um how you think you did, what steps on corrections. I have them explain what they did to me in writing, um to see what they did wrong or what they did, that they have to work on. But I do struggle with that.

By the way Ms. Jane described the explanations she asked from her students, she concentrated in the procedures associated with the problems her students solved. Then, Ms. Jane was asked about how she ensured that rigor stays high in the discussions and explanations she asked of her students. Here it was interesting to see that she did not truly understand what "rigor" meant as demonstrated by her answer. Rigor refers to the ways proficient students can demonstrate a deep mastery of the tasks presented to them. These tasks are used to develop students' high level skills such as analysis, creativity, and synthesis.

> Hum, for rigor? Um, I try to build off of what they know their fundamental skills and it is hard for me um for these classes because they are so low that I, I do find trouble sometimes reaching everybody because I do have a few high ones and it is hard um to try and reach them too because I feel like sometimes they know it and I don't really know how to scaffold as well because this is a new kind of curriculum so I'm kind of still getting my feet wet so I'm trying still, because I have had some of the higher team transfer to my team and so now they're at that point where they're not really here but they're not so I have kind of drastic I guess.

It was interesting to note here that Ms. Jane admitted that she had no idea how to scaffold the content for her students. In addition, she had the belief that some of her students were "too low" and others were "high" and she believed she was not being effective in reaching either group.

Initial Analysis of Case One: Ms. Jane

Ms. Jane's case was a typical representation of that of a novice teacher even though this was not her first year teaching. The fact that this was her first experience outside of Pre-K and in teaching mathematics made her experience very much like that of a first year teacher. Like many first year teachers, Ms. Jane struggled with her students' bad behavior during class, she had no set of daily routines that the students could follow, and there were no clear expectations from the students or the teacher. Her chosen method of classroom management disrupted the flow of the instruction as she stopped the lesson and remained silent until students finished or stopped whatever they were doing to disturb the class.

Ms. Jane was insecure about the curriculum she was following and struggled with maintaining rigor for her students. In addition, she was not sure whether she was reaching all of them and did not know how to scaffold to meet her students' needs. She was cognizant of the fact that her students were struggling but seemed unsure how to help them. When asking students to explain their answer, Ms Jane's expectations were centered about the students showing their steps.

Like the teacher-centered classroom explained in Chapter 1, Ms. Jane's classroom centered on textbook problems that lacked content and which had no connection to real world contexts. The instruction centered on the steps for solving the problems and the note taking required by the students reflect this. The conversations in the classroom were teacher centered and focused on the answers and in a lesser way on the procedures associated with solving a problem. The expectation of the students sharing and assessing their mathematical reasoning was not present.

Case Two: Mr. John

Background

Mr. John has a bachelor's degree in Secondary Mathematics Education and has six years teaching experience, all in the same school. He has only taught mathematics. During that time, he had taught all grades in the school and different levels of students. The class observed for this study was made up of 21 students 16 girls and 5 boys. The students in this class were in the Advancement Via Individual Determination (AVID) program. This means that these students had higher expectations of performance and that they were challenged with a higher-level curriculum.

A Typical day in Mr. John's Class

The students were lining outside Mr. John's door before the bell rang as they came back from lunch at this time. Mr. John greeted them at the door and the students entered quietly, took their seats, took out their materials, and started working on the bell opener work immediately. All the students were on task, doing what they were supposed to do, and there was no talk besides Mr. John welcoming them to class. The bell opener was a review of several lessons the students have already mastered. Figure 4 depicts the bell opener work that was waiting for the students as they walked into the class.

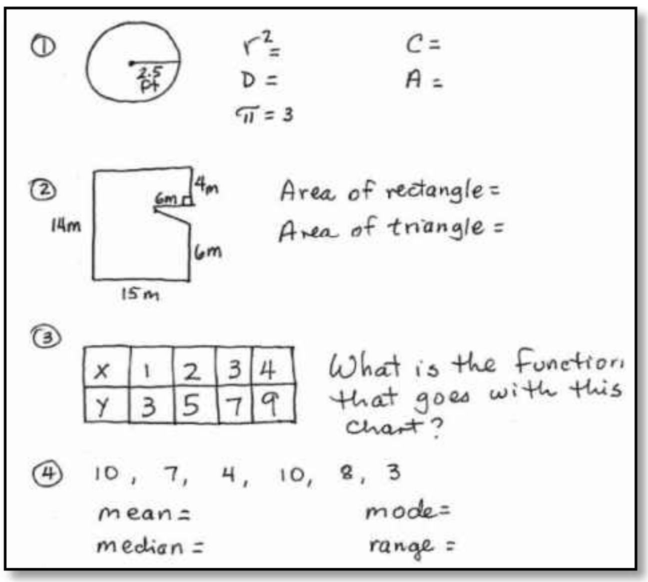


Figure 4 - Bell Opener in Initial Observation – Mr. John

As it can be seen in Figure 4, the bell opener had a variety of concepts like functions, area of complex polygons, measures of central tendency, and area and circumference of circles. While the students work, Mr. John took care of housekeeping issues like attendance, collecting homework, and giving back graded work. After the allotted ten minutes to finish the work, students, by pairs, were asked to go to the board and solve the problems. The students wrote their answers silently, some showed their work while others just wrote the answer next to the problem.

After all problems were answered, Mr. John went over each one by repeating the steps associated with solving each one. The following excerpt demonstrates his method:

Mr. John	Alright! Number three. The answer to that is $y = 2 \times x + 1$. This is right, that is correct but how do we write it correctly? Fabiola?
Fabiola	$y = 2x + 1$
Mr. John	That is a better way to write it. So, it is going to be $y = 2x + 1$. Right, you always put your number in front of your variable. When we are doing algebra remember we do not have to put the multiplication sign in between your variable and your number. The algebra way is just to squish them all together. Right, they become a little pair with the number in front of the letter. Can anybody tell me how I got that? How did I get that? (silence) Come on guys, time erased your memory banks? Christy, how did I get this?
Christy	You multiply 1 times 2 and then add 1 to get 3.
Mr. John	That is right but how do I know this works for the whole table?
Christy	Because you work through each number
Mr. John	That is right, I have to check everything, right? Now, how they got this 2 is a little more detailed. When you are first doing these remember, you need to look at your x number and find out what am I doing to my x number to get to my y number. If it does not work for the rest of my numbers then I need to go back; maybe I have to multiply something with that top number to get the bottom number. Try to find patterns like that; if none of those work then go back to the top and think what you can add or subtract after you multiply that top number to get to the bottom number. That is your thought process. Most of the time, it works out nice and neat; you multiply the top number by another number and that will give you the bottom number. OK? So try the easy ones first, then go back and if that doesn't work, go to the complicated ones.

The conversation in this excerpt was a typical exchange between Mr. John and his students. Even though he took a moment to ask his students how they figured the answer. Mr. John dominated most of the conversation as he tried to explain different methods for getting the answer. After they finished reviewing the bell work, the students passed their homework assignment to the front. Mr. John said, “quick groups” and the students moved their desks to the person next to them. In less than 15 seconds the students were paired for the next part of the lesson. Mr. John had excellent classroom management and this was apparent in the familiar way they performed any of the classroom routines during the observation. All the students were on task, working, and very few low conversations were heard.

The students were working on geometry topics and to this end Mr. John had them complete a geometry notebook. As he taught new topics, the students were to take specific notes that they placed in their geometry notebooks for future reference. Figure 5 depicts the notes that were given to the students that day.

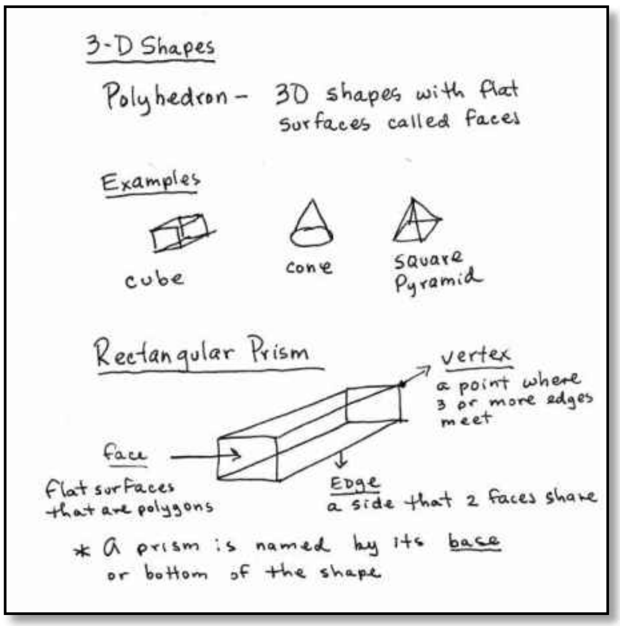


Figure 5 - Geometry notes – Initial Observation Mr. John.

These notes are known as guided notes. Mr. John wrote on the overhead what the students need to write in order to complete the notes. The students needed to write everything that Mr. John displayed in their notes. The following excerpt demonstrated Mr. John's manner of instruction at that time of the initial observation.

Mr. John So, 3D shapes have a special name. In math books they do not call them 3D shapes; they call them polyhedrons; that is the mathematical name for a 3D shape. I will give you three examples. People come to me and say "Mr. John, I can't draw any shapes" but I do not grade the shape you make, OK? I want you to draw the shapes the best you can. I want you to know what the three-dimensional shapes look like and be able to tell me what they are when you see them. I will be walking around to help you

> draw a little bit, if you need it but I am not going to draw all the shapes for you; I want you to get the practice of doing it. The first shape you can have a game in, the second shape you can eat an ice cream in, and the third one you see in Egypt. So, you have gaming, food, and ancient history. Now, what makes a 3D shape is that most of them have flat surfaces which we call faces. *(The bell rang at this time indicating the end of the period).*

By reading this excerpt, it is clear to see that Mr. John was controlling the conversation in its entirety, the students were quietly writing their notes as he spoke. There was no questioning and the information was being provided to the students. In return, the students were passively receiving and recording the information provided. Mr. John had excellent classroom management and was in control of his students the whole time. The students had clear behavior expectations and the classroom environment was organized and the class was efficiently run.

Self Perceptions

Mr. John saw his role as a mathematics teacher as a facilitator of knowledge. He recognized the fact that he could have a positive influence on his students. He strived to provide his students with a positive learning environment. When asked to describe his classroom's environment, he stated that he set down his expectations during the first days of school. He said he recognized the fact that his students were at a difficult age and that for him it was very important that they felt secure enough to ask questions.

> The very first week of school to give my kids the uh, my expectations, what I expect from them, and not only what I expect from them but what I need them to expect from each other, so I try to make the classroom really, really comfortable um that not only with me that they can feel comfortable asking me any questions or but also asking questions just in general because I think that is one of the biggest fears that a lot of students have, especially at this age, is asking questions. You know, because they think they are going to feel you know maybe stupid or the kids or their peers are going to make fun of them.

Supporting Students Sharing Mathematical Thinking

For Mr. John, students' questions were of vital importance and he did all he can to ensure that they felt comfortable asking them. The importance of students' questions was evident in his response to the question regarding how he encouraged student participation.

> I love to encourage questioning and like also more important that question is uh different ways to do problems, because math is one of those subjects that there are a lot of different ways to do the same problem. So let's say what John does in class, you know he could have got the right answer doing it completely different than let's say Mary, who got the same answer too and did it completely different but they both, both their ideas work you know.

Mr. John stated that he likes to have his students share their methods because there are many ways of solving a problem and he believed this was valuable information that the students should use as an instructional tool.

> They don't only always learn from me, they can learn from each other… And so those kinds of unique ideas to solve a problem is what I like to encourage because every student sees things differently so um I love when my kids can see things differently because they teach each other.

Mr. John valued students' contributions and perceived them as an opportunity for students to learn from each other. When asked how he encouraged his students to share their mathematical thinking and methods, he stated he liked when students went to the board to explain their methods and group work.

> I like to hum do group work, a lot of group work, to try to encourage the interaction between my students with that. And in class I always try to, I try to, I know it probably doesn't seem like it, but I try not to do all of the talking and teaching, I like to have my students do a lot of talking and come up to the board and explain what they did and their ideas.

When asked how he ensured the rigor during these exchanges, he again stated that the questions were at the center of this.

> I try to always incorporate high level of questioning. Um, not just you know what's the answer but how you got it, the steps you took to get there, how you can apply that answer to other types of problems, how can you apply that math skill what they did to real word situations, so just trying to always you know keep that high level questioning there and kind of push them along, not give them the answer all of the time, because a lot of students love to wait until you give them the answers but I like to push them a little bit to let them finish it, not to just give them the answer right away, which I think a lot of

people do.

Mr. John saw a value in students struggling over the concepts they were learning. In addition, he valued the application of mathematics into real world contexts and tried to bring familiar situations into the classroom to create a link between what he taught and the real world.

Initial Analysis of Case Two: Mr. John

Mr. John was a caring teacher who perceived his students as being his own students and he worked very hard to promote a classroom environment that made students comfortable. He saw the value in his students' contributions and believes that they could learn from each other as much as they could learn from him. His classroom was run efficiently. There were set routines, like the quick group demonstrated in the initial observation, which were second nature for the students. The students asked questions and participated during the lecture and were on task. No off task behavior was observed.

Mr. John was very comfortable with the curriculum and was familiar with common mistakes that students make. He believed that students should explain their answers. However, no evidence of this was observed during the initial observation. Furthermore, the teacher dominated the conversations in the classroom. The students only participated when they were asked a direct question. These questions were usually about the answer to a problem. When students went to the board they were not expected to write their computations, explain, or justify their answers. Their final answers were enough. The teacher also dominated the conversation during this time.

Mr. John expressed that he liked when students shared their ideas and methods with each other because these represented important learning opportunities for all students. However, there was no evidence of this during the initial observation. Unfortunately, group work was not observed during the initial observation so there was no way of gauging the conversations that happened regularly in Mr. John's classroom.

Baseline for Professional Development

One of the stipulations for this study required that the teachers who participated show no evidence of students sharing their mathematical reasoning with each other. Both cases showed no evidence of this. Still, both cases were fundamentally different. Case 1, Ms. Jane, was facing many challenges as a new mathematics teacher, she was struggling with the management of her class, with the content she was teaching, and lacked methods for teaching and remediation. Case Two was different in that the teacher was experienced and was not struggling with either content or students' behavior. The discussion rendered in Chapters 1 and 2 identify three main components of SMP3, "create viable arguments and critique the reasoning of others" (NGA & CCSSO, 2010, p. 6), these being classroom environment, communication, and reasoning.

Teacher-centered Classrooms

Environment

As discussed in Chapter 1, the classroom environment of teacher-centered classrooms is characterized by the teacher standing in front of the students while they listen or take notes. There is a very structured pattern to activities in these classrooms. The problems used in these classrooms usually lack context and a spiraling practice structure is favored. In these classrooms, the students are expected to listen while the teacher instructs the lesson and afterwards, the students practice whatever concept the teacher had just explained to them by using a set of steps. In this type of classroom, getting the right answer is the goal (Boaler, 1999; Fennema et al., 1999; Gonzalez et al., 2009; Hiebert, 1998; Mullis et al., 2008; Stigler & Hiebert, 1999, 2009; Wood et al., 1993).

Both cases in this study represented a teacher-centered classroom, as both were very structured. The teachers lectured while the students took directed notes and the purpose of these notes was to give the students a set of steps for solving problems in order to make the mathematics easier. The main difference in the cases was that one teacher had deficiencies when it comes to managing the behavior of the students while the other did not. In both classrooms, students were not expected to participate other than answering direct questioning usually regarding the final answer of a problem or what step comes next.

Communication

As discussed in Chapter 2, communication is one of the central components of SMP3. Without it, the exchange of mathematical reasoning would not happen. In teacher-centered classrooms, the teacher controls the communication that happens there. Furthermore, in many instances students are strongly encouraged to be silent while the teacher demonstrates how problems are solved. Additionally, the communication among students is discouraged and there is no expectation that the students would share the reasoning behind their answers or assess their peers' logic (NCTM, 2000; Romberg & Caput, 1999; Secada et al., 1995; Schoenfeld, 1992; Steffe & Kieran, 1995; TIMSS, 2008; Von Glasserfeld, 1990, 1991; Wood et al., 1993).

The classrooms in both cases displayed the same types of communication interactions as described above. The teacher controlled the conversation and the exchange of information went from the teacher to a group of students. In Case 1, Ms. Jane struggled to maintain control of the conversation as her students talked over her. She used a method for controlling the conversation that was not effective and interrupted the instruction. In Case Two, Mr. John had total control of the conversation. In fact, for long periods during instruction, he was the only one talking. In this case, the students listened while Mr. John talked only pausing to ask brief questions or to get an opinion.

Reasoning

In teacher-centered classrooms, the steps for solving problems take the place of reasoning. There is no time for the reason-making process because of the rigorous schedule of instruction that has to be followed. Most mathematics concepts are taught as separate, unrelated themes. Moreover, there is an insignificant number of connections made among concepts and with real world situations. Reasoning is the foundation of mathematical thinking and in teacher-centered classrooms there is almost none present as the focal point of mathematics instruction is in the processes used to solve problems (Blanton & Kaput, 2005; Boaler, 2002; Hill et al., 2005; Lithner, 2000; NCTM, 2000; Phillips, 2008; Reid, 2002). As discussed in Chapter 2, procedures constitute one of the five strands of mathematics proficiency and procedures alone do not support the deep understanding that the CCSSM requires of students (NGA & CCSSO, 2010; NCR, 2000).

The classrooms in both cases had procedures and rote memorization at the heart of the instruction that the students received. There were no reason-making activities. All the steps were written for the students who were expected to copy them down and follow them implicitly. There were no conjectures, testing, and generalizing in any of these classrooms. Both teachers encouraged students to write notes and steps following the note taking style that was demonstrated in the classroom. Both teachers wanted their students to understand. However, instead of deepening the students' understanding, they were giving students steps to follow to complete procedures that were poorly understood if at all.

Teachers' Self Perceptions versus Observable Behaviors

Case One: The case of Ms. Jane

As any typical novice teacher, Ms. Jane was struggling in many aspects of her teaching from classroom management to the methods used for remediation. However, she cared for her students. She believed that her students would naturally share their mathematical thinking if they were paired or in groups. Therefore, she did not promote these kinds of conversations in her classroom. None of the paired students' conversations contained any mathematical thinking. Furthermore, in many instances, students talked across the room to their peers. Ms. Jane stated that she asked her students to "show their steps". However, this was not observed during the initial observation. Additionally, she stated that she asked her students to write reflections on what they have learned but this behavior was not observed either. These two expectations cannot be a substitute for the explanations and justifications her students need to share with each other as prescribed in SMP3. The initial interview demonstrated that Ms. Jane struggled with the mathematical content her students needed to learn and was ready to excuse them because of their behavior and their low performance in mathematics. In addition, she excused herself because of her inexperience and lack of pedagogical knowledge.

Case Two: The Case of Mr. John

Mr. John was an experienced teacher who had total control of his classroom. He was confident on what he taught his students and he cared about them. Mr. John stated that he encouraged his students to share their methods with each other. However, this was not observed. The students' responses were limited to either the final answer or a one-word answer. As we were able to see in the data for the initial observation, Mr. John controlled the conversation and the students' participation was marginal at best. Mr. John saw the value of group work and conversations, but he was focused on the social behaviors his students had during these interactions, not on the nature of his students' conversations. Additionally, he stated that he encouraged mathematical conversations among his students but this was not observed.

Standard for Mathematical Practice 3

Students' Expectations

NGA and CCSSO are very clear in the behavior expectations that SMP3 has in place in order to assess the standard. The CCSSM document specifies seven actions that students must be engaged in while demonstrating SMP3.

1. Understand and use stated assumptions, definitions, and previously established results in constructing arguments;
2. Make conjectures and build a logical progression of statements to explore the truth of the conjectures;
3. Analyze situations by breaking them into cases, and recognize and use counterexamples;
4. Justify conclusions, communicate them to others, and respond to the arguments of others;
5. Reason inductively about data, making plausible arguments that take into account the context from which the data arose;
6. Compare the effectiveness of two plausible arguments, distinguishing correct logic or reasoning from that which is flawed, and if there is a flaw in the argument explain what it is; and
7. Listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. (NGA & CCSSO, 2010, p. 6-7)

Separating these indicators into the two goals associated with SMP3, constructing mathematically sound arguments and assessing the arguments of others, can further refine these indicators. For the purposes of this study, these were the seven indicators that were used to gauge students' engagement in SMP3. Table 6 depicts the behaviors that were observed of students in each class during the initial observation. These behaviors relate to the first goal of the standard.

Table 6 - Indicators of Students Creating Viable Arguments

SMP3 Indicators	Ms. Jane's Class	Mr. John's Class
1. Understand and use stated assumptions, definitions, and previously established results in constructing arguments	Yes	Yes
2. Make conjectures and build a logical progression of statements to explore the truth of the conjectures	No	No
3. Analyze situations by breaking them into cases, and recognize and use counterexamples	No	Yes

By looking at Table 6 we can see that both Ms. Jane and Mr. John were effectively enabling their students to use different instances of the concepts and broke them down in different cases and examples. This was evidenced in the instances when both teachers were engaged in direct instruction and they were demonstrating different ways of interpreting concepts. In addition, Mr. John was facilitating his students to break down problems into smaller parts and use different examples. This was evidenced when Mr. John was going over the bell work and was talking with his students about determining the equation of a function. In both cases, the teachers were the ones in control of these situations and the students participated by answering their closed ended questions. Still, neither of the cases demonstrates the all requirements for which students must exhibit in order to engage in the first aspect of SMP3.

Table 7 depicts the indicators associated with the second goal of SMP3, which is critiquing the arguments of others, as well as the results of the initial observation by case

Table 7 - Indicators of Students Critiquing Each Other's Reasoning

SMP3 Indicators	Ms. Jane's Class	Mr. John's Class
4. Justify conclusions, communicate them to others, and respond to the arguments of others	No	No
5. Reason inductively about data, making plausible arguments that take into account the context from which the data arose	No	No
6. Compare the effectiveness of two plausible arguments, distinguishing correct logic or reasoning from that which is flawed, and if there is a flaw in the argument explain what it is	No	No
7. Listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments	No	No

In both cases, there was no evidence that the students were assessing each other's logic. As the initial observations revealed, there were no expectations that the students would justify, communicate, reason, or compare the effectiveness of any given solution method. However, Ms. Jane expressed the belief that her students would do this automatically after they were paired or grouped. Mr. John stated that he encouraged his students to share their methods because they could learn from each other. Nevertheless, none of these behaviors were evidenced during the initial observation.

Components of PD

Based on the initial observations of both participants, the PD that was offered to them focused on the following components: (1) Formative Assessment, (2) Problem Solving, (3) Improving questioning, and (4) Students Working Collaborately. These components were discussed with the participants in the context of planning and lesson delivery. Formative assessment is at the center of gauging student understanding and making instructional decisions as it is performed (Doubet, 2012; Hendrickson, 2012; Hobson, 1997; Sharkey & Murnane, 2006; Torrance & Pryor, 2001). Problems are at the heart of the mathematics classroom, they have to be worthwhile, problematic, and demonstrate real applications of the concepts they contain (Hiebert et al., 1999; NCTM, 2000). The importance of asking the right questions was discussed in Chapter 2. Teachers must plan the questions they ask and anticipate answers in order to plan for follow-up questions. It is not enough to ask questions, but the questions need to be open-ended and wide enough to elicit different answers (Hancock, 1995; Sanchez, 2013; Sharkey & Murnane, 2006). Finally, teachers must provide their students with opportunities to work together in order to share and evaluate their mathematical thinking. Still these group discussions need to be orchestrated and assessed by the teacher (Hufferd-Ackles et al., 2004; Kerslake, 1989; Walshaw & Anthony, 2008).

Teacher Participant Change During PD

The purpose of the PD used in this study was to act as a support for teachers as they attempted to implement SMP3 in their classrooms. PD must support teachers as they, in turn, support their students (Birman et al., 2000; Franke et al., 1998, 2001; Porter el al., 2001; Wei et al., 2010). Furthermore, the PD used not only focused on how to teach mathematics, it focused on how mathematics is learned (Davis & Simmt, 2006). The PD provided gave the participating teachers the opportunities to plan for and implement formative assessment as a gauge of students' understanding and for instructional decisions in their classrooms. They also planned and put into action problem solving lessons that centered on intriguing tasks for their students. Additionally, they attempted to plan and anticipate the questions they need to formulate in order to facilitate students' constructions of valid mathematical reasoning and to foster them sharing it with each other. They also planned group activities that would promote students sharing their reasoning as they work with the assigned task. The teachers were observed between days 4 and 6 of the PD sessions. Then, the teachers participated in individual informal interviews within 48 hours of the observation. The observations were video recorded and those video clips were used during the interviews that followed the observations to elicit reflections from the teachers.

Case One: Ms. Jane's Observed Change During PD

The class started as it usually did, with board work. Figure 6 shows the bell work that the students were working on that day.

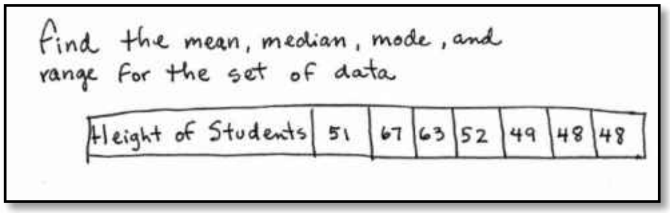


Figure 6 - Bell work used by Ms. Jane during PD

The bell work used by Ms. Jane was a review of a problem that she believed would be in the upcoming standardized test the students would be taking. The students behaved as usual. However, Ms. Jane took the time to pair them. She later stated that she decided the pairs by high-low students' abilities so they could support each other.

During the PD, Ms. Jane expressed doubts that her students would be able to respond to open-ended questions and prompts to exchange and assess each other's reasoning because of their low level of performance. However, as Ms. Jane implemented some of the questioning techniques that were presented in the PD, her students successfully interacted with each other. They were on task and behavior issues were at a minimum. One student was called to the board to find the mean of the set of data. This was the conversation that followed.

Ian	Hum… OK…, first, add all of them
Ms. Jane	OK
Ian	So, this makes 10… 11 plus 6 gives me… then 4, 8, 12… so that is 378
Ms. Jane	(to the rest of the class) Some of you might have done this in a different way. What Ian did was chunk some of the numbers together and then he added them. Now you know what he did. Did you have to do it this way?
Students	No.
Ms. Jane	Very well, you might do it differently but the answer is 378
Ian	Now I need to divide by how many kids… (Counts the number of items in the table)… 7
Ms. Jane	Why does he divide by 7? Elena?
Elena	Because that is the number of kids

Ms. Jane	Because that is the number of kids. What did she say, Aaron?
Aaron	Because that is how many there are in the table
Ms. Jane	Good job!

As we can see from the previous excerpt, Ms. Jane was incorporating some of the things she learned during the PD and finding that her students can display some of the behaviors expected of SMP3.

Ms. Jane also experimented with more open-ended questions as her students were solving the board opener. The following excerpt shows some of the questions that Ms. Jane asked at that time.

Ms Jane	OK, now what is the mode? Carlos?
Carlos	It is 48.
Ms. Jane	How do you know it is 48?
Carlos	Because its there twice.
Ms. Jane	Yes, there are two 48s. Is there two of any other number?
Carlos	No.
Ms. Jane	Then that is it. (To the class) The mode does not mean the highest number. It is the number that repeats the most. Does there need to be a mode? Ian?
Ian	No
Ms. Jane	Why not?
Ian	Because … sometimes… there is no number that repeats.
Ms. Jane	That is right!

From the observation it was clear that Ms. Jane was starting to implement some of the methods she was learning during PD. During the observation, she experimented with questioning and realized that her students could participate during class. Additionally, it was noted that the behavior of the students was more subdued and quiet than in the initial observation during these instances. Most students were focused and on task.

During this observation, there was a class-wide discussion about the meaning of range. Here, Ms. Jane was attempting to incorporate reasoning to the concept by questioning her students regarding the range of a group of data.

Ms. Jane	What is the range? Does anybody know what the range is? Yes? (pointing

	at a student)
Karl	It is like… hum… like a number… from one number to another number… that is the range
Ms. Jane	Ok, so from one number to another. You are really close. Jamal?
Jamal	You have to take the biggest number and the smallest number, then you subtract them
Ms. Jane	Yes, that is how you do it. But I want to know what it is. What is it? What does it mean? Amanda?
Amanda	It means the other numbers are in the middle.
Ms. Jane	Not exactly the middle.
Amanda	They are between both numbers
Ms. Jane	Between the…
Amanda	Between the largest and the smallest number
Ms. Jane	That is right. It tells me the number between them. That is why I subtract the lowest from the highest number.

During the initial observation, Ms. Jane was content with students repeating the steps they used to solve the problems but at this time she was trying for her students to make sense of the meaning of the concept rather than concentrating on how to compute it. However, the excerpt also shows a weakness in Ms. Jane. Her limited understanding of the concept she was teaching. Ms. Jane defined range as the "number between them", referring to the largest and smallest values in the data set. This could have caused confusion among her students.

Video Eliciting – Ms. Jane

As part of the informal interview that happened after this observation, Ms. Jane was shown a short clip of her interacting with her students. The students were discussing methods of finding the median of a set of data. At this point in the observation, Ms. Jane had reverted to her previous methods and the questioning methods that she had started during the beginning of the period had stopped. The students were not on task and Ms. Jane stopped instruction a few times to wait for them to settle down. The following were her comments after she saw herself. Ms. Jane admitted this was her first time observing her own practice.

Interviewer	What do you think you should have done to improve that a bit?
Ms. Jane	Well, they all were so distracted. Well, that was frustrating. They are

	usually not like that.
Interviewer	I know.
Ms. Jane	I probably should have yelled… not yelled… but… you know been like OK guys, bring it back together. It is hard when people are observing you. It's like I do not want to do something like that… seriously it is not like them.
Interviewer	OK
Ms. Jane	Hum… but questioning, you know, I do not get it… I said the same thing over again, but that is not going to help them if they don't get it still.
Interviewer	Hum.
Ms. Jane	so I can maybe re-question it or go back to what we are doing. I did that like the warm up but I could have done that earlier, what did we do? You know so they kind of made that connection. Hum… earlier. Hum, but yeah there were a lot of kids fooling around, which drove me crazy. So that likes frustrates me and my voice is annoying.
Interviewer	So, how do you think you could have helped them to, for them to really get what the median was because they knew it was the middle of something.
Ms. Jane	I learned that I could have… I did, I thought I went over the definition before so maybe I could have said it again. I had it written down, so I could have said, you know, the median is the middle.
Interviewer	And what does the middle actually mean?
Ms. Jane	so when you have a number, I could have shown a number line as well so that way they could have seen, like in my number line because that helps them sometimes to visualize. Because I have had trouble with that when there are two numbers in the middle. They are like "Ms. I don't understand what is in the middle of those two numbers" and so I drew out a number line and they are like "oh… between the two", so I could have done that as well. Especially with some of my English language learners, it is more visual for them.

From the excerpt, it was evident that Ms. Jane was upset by the behavior of her students. She questioned her classroom management style and admitted that maybe she should have said something to them. Ms. Jane noticed she repeated the same thing over again and she admitted this was not helping her students. She saw some missed opportunities and she realized she could have made instruction more like it was earlier in the lesson. Ms. Jane also thought of tools, like the number line, which might have made some of her struggling students understand the concept better.

Case Two: Mr. John's Observed Change During PD

The class started as usual, the students were lining at the door and entered. There was bell work waiting for them and they sat down and started working quietly. Figure 7 shows the work that the students did at that time.

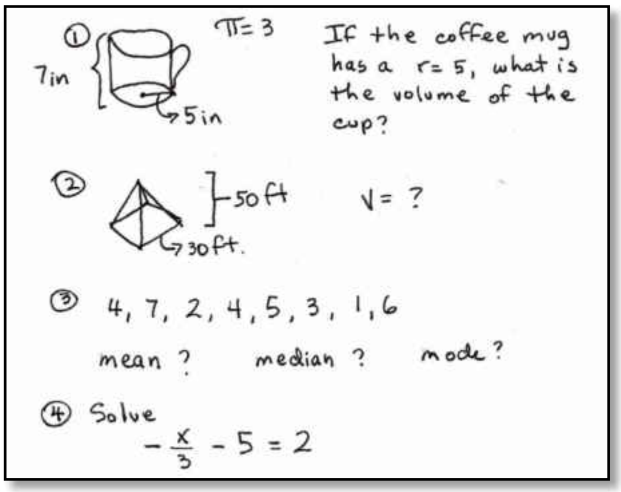


Figure 7 - Bell Work used by Mr. John During PD

Mr. John called several students to come to the board to solve the problems. The following excerpt demonstrates the conversation that happened as the students were reviewing one of the problems. This particular problem involved the volume of a square pyramid and the answer provided was found wrong by one of the students.

Mr. John	The first thing is to find the area of the square, the Base, right? Did this person do that? No, they just took the Base and multiplied if by the Height of the pyramid but we cannot do that just yet, right? Because, like Carla said, we need to find the area of the square. Luis, what do you think?
Luis	It is Base times Height
Mr. John	Base times Height, so the area of the square first because that is the area of my Base, right on the bottom. Well, Base times Height… I have my Base here (pointing to the diagram on the board) but what is the Height of the square? Carla?
Carla	30
Mr. John	How do you know that? Because I don't see it in there. (Pointing to the diagram on the board)
Carla	A square is equally four sides, so the four sides are the same.
Mr. John	That is right! She knew that all the sides are the same so I do not need that

other side because this is a square and if this is thirty (pointing at one side of the square) then, this side is thirty (pointing at another side of the square).

Mr. John was clearly trying to bring the knowledge of shapes into the foreground so students could determine the missing sides of shapes. He was focusing more on making sense of the concept than in the steps for getting the volume of the figure. He was also including his students more into the discussions by questioning them and asking them to say what their classmates have said in their own words. Mr. John was attempting to integrate some of the methods discussed during PD in his daily practice. He was asking questions that led to more class discussions and rather than stop at the right answer, he asked his students how they knew whether they were right.

This day, the students were studying the formula for the volume of cones. The following is an excerpt of what happened during that time in Mr. John's classroom

Mr. John So today we are going to find out how much ice cream we can put in this cone. The smaller the cone, the less ice cream we get, right? We want one of those big waffle cones with strawberry ice cream, right? I love chocolate. So let's find out. (puts up the notes in the doc camera for the students to copy) Lets find out how much ice cream we can put in this cone or how much volume you can fit in a cone. As you write, I want you to think about a couple of things. As you look at the formula for the volume of a cone, why do some of those parts look familiar to me? I want you to think about that as you are writing down the formula. Second thing I want you to think about is what do those letters and symbols mean in that formula. Once you finish writing it down, I want you to try the problem and see how you can pull everything together. If you don't think you can do it, I want you to try it and have confidence because we are going to do it together in a little bit.

As it can be seen in the excerpt, during direct instruction Mr. John reverted to his usual monologue while his students took notes. The students were given the formula without having a chance to make sense of what it measures. Mr. John encouraged his students to make sense of the letters and symbols in the formula. Additionally, he mentioned that it had some familiar components. However, the students were given the information and were not able to make these conjectures on their own.

Video Eliciting – Mr. John

As part of the informal interview that followed this observation, Mr. John was shown a clip of him interacting with his students. The clip showed a classroom discussion regarding cones and pyramids. Mr. John had asked the students whether pyramids were the same as cones. Then he proceeded to ask the students what kinds of shapes could make the base of a pyramid. Mr. John gave the students no time to make sense of what he was saying and asked many questions without any wait time. The students participated by giving their opinions and reasoning as they discussed the topic. As was usual in this classroom, the students were on task, the teacher circulated around the classroom as the conversation unfolded. The following transcripts show Mr. John's comment as he saw himself teaching his students.

Interviewer	So what do you think about that?
Mr. John	Uh, I barraged them with questions and didn't really give them a lot of time to really think. I asked so many questions at one time without trying to get an answer. That's what I noticed... I can understand how that would be confusing to kids. Like, wow.
Interviewer	Hum.
Mr. John	That's one thing. Um. I gave, I should have maybe asked what other shapes could be at the bottom of a pyramid and let them say rectangle… That's one thing. , Instead of me giving them that answer too.
Interviewer	OK
Mr. John	Yeah I think I gave them, I gave them the answer at the end instead of letting them come to the conclusion. It's what we talked about instead of me saying, no they're not, like having them give me the final answer.
Interviewer	So what would you change?
Mr. John	When I was asking for characteristics of each one, I definitely would just ask one question at a time instead of trying to, cause I think, I think we all do it, but for me like I have the end result in my head.
Interviewer	I see

Mr. John So I would definitely slow that down. Ask one question at a time or one characteristic and um definitely do, I notice sometimes I talk pretty quickly, maybe slow down a little bit, not just my questioning but my overall like presentation like when I am speaking just in general maybe slow it down a little bit and uh I would definitely let them come to the final answer… You know instead of me telling them.

Mr. John realized that the wait time he was giving his students to answer his questions was inadequate. In addition he realized that he was giving them the answers he wanted them to have rather than allowing them to come to their own conclusions.

Teachers' Views after PD

After the PD was concluded, the teachers shared their opinions and thoughts about their experience and what they had learned. They shared some of the challenges they had faced as they attempted to change their practice to support their students to build reasonable arguments and assess the arguments of their peers. Both teachers' agreed that their overall opinion of the PD was positive and they saw it as worthwhile. Still, they had their individual opinions.

Ms. Jane, as a first year teacher, thought that the PD was too general and that she needed more support in the form of specific directions to follow in implementing the changes that were presented in the classroom. Ms. Jane had the added pressure of learning the curriculum and dealing with establishing sound routines in the classroom. Therefore, like any other first year teacher, she needs much more support than an experienced teacher. Ms. Jane expressed that rather than the content. She was more interested in the pedagogy and the methods and how to make them work in her classroom. The following excerpt from her interview demonstrated this.

> I find it kind of hard because it's very general kind of…and I want specific things. You know what I'm saying? As a teacher, especially if you know it's hard for me to, it's kind of one of those things that you learn a lot of, you learn a lot of great information but I don't know how to specifically implement it in my classroom. As in, maybe not concepts but techniques.

Ms. Jane expressed the desire to observe an experienced teacher implementing the methods that were presented in the PD because she said it would give her a better idea of how it would look in real life and in her own classroom, with her own students.

> Because I have never really, I haven't gotten the opportunity to… observe someone who knows, you know, who specifically is… better at certain things.

Ms. Jane had a difficult time incorporating the questioning techniques mentioned during the PD and when asked if her students seemed to be more engaged when she employed these methods she stated that she noticed a little change in their behavior.

On the other hand, Mr. John had an easier time incorporating what he had learned in the classroom. He stated that he was skeptical at first because he had doubts about how to go about it, but as he implemented the changes he started to notice a change in his students.

> All of a sudden like the questioning part… we started to learn more about the questioning and how to answer, how to ask questions and what questions to really get their conversations going and then talking about math. Like I started to do it right, like more naturally, and I thought it would take me a long time to like really like consciously ask these questions but then it's basically, it kind of fit into my teaching style anyway. Because I always asked a lot of questions, but I didn't, I don't think I was asking the right ones.

In addition, he noticed some of the mistakes he was making and he strived to correct them when he saw how it impacted his students. Mr. John has really noticed a change in the talk that his students were engaged in while in groups.

> And now they're trying to talk more about math. Even my group, when I do put them in groups, they don't have to hear me so much directly, when they already start talking about math and asking questions

Mr. John had apparently started to listen to his students' conversations during group work. He stopped guiding their activities so directly, and just facilitated their learning.

Post PD Observations

As stated in Chapter 3, Phase 3 of this study entailed weeklong observations of both teachers right after the PD was finished. Additionally, the teachers wrote reflections of their experience as they supported their students to demonstrate evidence of SMP3 in their classrooms. All observations were video recorded and there were no interviews performed during this time. At the end of the week, each teacher returned his or her individual reflection. The evidence of SMP3 will be based on the presence of the seven indicators discussed earlier in this chapter. In addition, the use of formative assessment, questioning, and the use of tasks that enable students to develop their reasoning will be assessed.

Case One: The Case of Ms. Jane – The Sneaker Project

For this weeklong observation, Ms. Jane developed an activity that would take a week for her students to complete. She was going to have her students learn about statistics through the process of manufacturing sneakers, then, they were going to design their own "dream" sneaker. After that, the students would vote for their favorite designs and this data would be used to construct graphs and measure other sneaker characteristics.

Day One - Post PD Observation

The day started differently compared to how it usually did. There was no bell work working for the students. Instead, the walls of the classroom were filled with pictures of sneakers.

Figure 8 - Display of Sneakers in Ms. Jane's Class

Ms. Jane stated that they were going to talk about sneakers. This immediately sparked interest in her students. As a group, they started to mention different characteristics (like color, brand name, and design) of the sneakers they were wearing. Ms. Jane asked the students to pair with the person sitting next to them. Some students did not have anyone sitting by them, so Ms. Jane directed them to the group they were going to work with. Then, she asked the students to put the previously discussed characteristics into categories. The students were engaged and participated actively in the group activity. After they had a list of categories, the students were instructed that they would work in pairs and pick one category, like color or brand name, and collect these data regarding the sneakers that the other students were wearing. They were instructed to use a mini white board to display their data with the class. They, as a pair, had to decide how to display their data. During this time, most of the conversations in the groups concerned what kinds of sneakers their peers were wearing. There were also students directing questions to students in other groups. The following is a representation of the conversations that the students were having, not in their pairs, but all over the classroom.

Carlos	So, how many Jordan's do we have here?
Luis	There is a fake pair back there. So, there are only 2 real ones.
Carlos	(Laughing) That is two. Now, how many Addidas are there? (Running around the classroom as he tried to see how many)

At this time, some of the students were out of their seats, talking loudly with other students who were not their partners and their focus was not on constructing the display of the data but trying to identify the fake brands. The students displayed rude and unruly behavior that was ignored by Ms. Jane. The partners of those who were out of their seats just sat waiting for their partners. The following excerpt demonstrates how the pairs presented their data.

Ms. Jane	(taking and showing the pair's mini white board to the class) OK, so you decided to use…?

Trevor	Color
Ms. Jane	OK, color. And how did you do that?
Trevor	I went around and looked at the sneakers' colors
Ms. Jane	Ok, and how did you show that? How did you display it?
Trevor	I made marks
Ms. Jane	So, what are those things called?
Trevor	Tally marks.
Ms. Jane	So, they did a tally table. Who else made a tally table?

Ms. Jane took control of the presentation of the data and the students were asked closed-ended questions relating to their data collection process and their displays. Some of the students talked while Ms. Jane was presenting each pair's displays so, she stopped and waited until they finished. Most of the displays presented were lists.

After this activity, Ms. Jane led a discussion regarding how mathematics was involved in the manufacturing of sneakers. Some of the responses the students suggested were price, how much material was used, time, shipping costs, etc. While the students made their suggestions, Ms. Jane made a list of them. She informed the students that they would add more ways that mathematics is involved in the production of sneakers as they completed the activity. After that, the students were shown a couple of videos that depicted the process of making sneakers. After the videos, the students were given two articles about the manufacturing of sneakers to read. The students read for the rest of the period.

Day Two - Post PD Observation

The students entered the classroom and they were instructed to review the articles they were reading the previous day as bell work. They were to discuss the articles with their partners. The following is a picture of what the students had waiting for them instead of a bell opener.

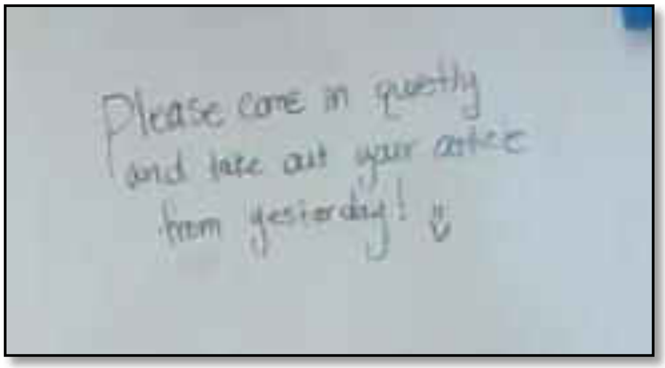


Figure 9 - Bell Opener for Ms. Jane on Day Two of Weeklong Observation

The students spent four minutes talking about the articles to each other. Then, they were to write on their mini white boards four interesting facts about the manufacturing of sneakers to share with the rest of the class. The following is a conversation that a pair of students had with Ms. Jane as they completed the given task.

Ms. Jane	Ok, so, what did we talk about yesterday? Remember?
Tomas	Types of shoes?
Ms. Jane	How many types of shoes were there?
Tomas	Low tops…
Ms. Jane	(nodding) Low tops… what else? What is the opposite of low?
Tomas	High… high tops.
Ms. Jane	Yes, high. Remember your shoes? Are they higher?
Tomas	Yes
Ms. Jane	So why don't you write that.

After the students were done with writing the four facts from the articles, the pairs were called to the front of the class to show the rest what they had come up with. The following is one of the conversations that happened as a pair of students came to share their work with the rest of the class.

Ms. Jane	Ok, Ian and Daisy, come on up. For those of you who are watching, if you are up here you would want everyone else to be quiet so give them your full attention and be a good audience, OK? So, what did you guys learn?
Daisy	That there are three shoe cuts.
Ms. Jane	Three shoe cuts, OK.
Ian	That foam pellets are supposed to be used in the middle sole.
Ms. Jane	Foam pellets… (to the class) why do you think they use foam pellets?

	Isabel?
Isabel	Me? To smooth up the bottom
Ms. Jane	Ray, what did Isabel say?
Ray	To make them comfortable…
Ms. Jane	Close. Isabel, what did you say?
Isabel	Hum..
Ms. Jane	Remember what you said? Alicia?
Alicia	To smooth the bottom
Ms. Jane	Remember to pay attention. Are there other reasons for using foam pellets?
Carlos	To keep the shape of the shoe.

After that, Ms. Jane asked several students to repeat what Carlos had said. However, none was able to repeat it. After that, the discussion centered on the cost for manufacturing sneakers. This was a whole class discussion led by Ms. Jane.

Ms. Jane	It says (referring to the article) that they have a total cost of twenty dollars for the sneakers but that includes the production labor. So, what does production labor mean? Aaron?
Aaron	Like the people they pay.
Ms. Jane	Right, so for that they say it was two dollars seventy-five cents. So, for every shoe that they make, they are using two dollars and seventy-five cents to pay this person, right? The next one is materials. What are materials? Isabel?
Isabel	Things to make the shoes.
Ms. Jane	Yes, things to make the shoes. Then it says rent equipment, what do you think that is? Alicia?
Alicia	So, if they do not have the money to buy it…
Ms. Jane	Oh yes… for renting equipment.

After this, the pairs resumed the presentation of the four facts they had learned from the articles. When they were done, Ms. Jane stated the goal of the activity the students were going to work on for the rest of the week. The following is what Ms. Jane told her class about the project and its goals.

> You guys are going to get the chance to design your own sneaker. Our goal for this unit is to be able to design a sneaker for a specific purpose using mathematical content. This is the first part we are going to be working on. After you are done, we are going to collect your sneakers and we will collect data of our favorite and display the results. You guys are going to get to vote on your favorite sneaker. The top shoe of every class all of us are going to get to vote on, on the ones that you made. Then, we are going to collect that data

and we are going to display it

Besides this, the students were told that along with their designs, they had to turn in a package that will contain the name, the price, and the type of sneaker they were designing. This package is the end product that they had to turn in for a grade. A copy of this package is provided in Appendix E. Ms. Jane continued to explain the package and answered some students' questions until the end of the period. The packets and blank sneaker designs were distributed at this time.

Day Three - Post PD Observation

The class started with a review of the prior day's lesson, specifically an explanation of the package. Instead of a board opener, there were instructions on the board for the students to take out their packets and sneaker design sheets. The following is an image of what was written on the board.

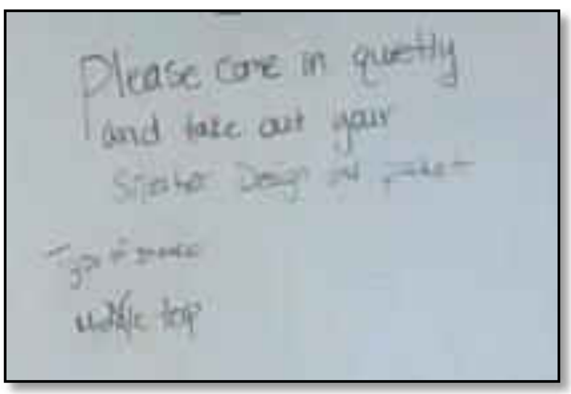


Figure 10 - Bell Opener for Ms. Jane on Day Three of Weeklong Observation

Ms. Jane reviewed the different topics they had talked about the previous day and instructed the students that this day they were going to start designing their sneakers. Additionally, she gave the students some ground rules like sharing computers and using class time wisely. The students were sitting in pairs even though this was an individual project. Most of the students got up from their seats and talked loudly. A few students were at their desks designing and coloring their sneaker design. The following depicts a conversation between a student and Ms. Jane.

Ms. Jane	Yes, Beth.
Beth	I am making a running sneaker.
Ms. Jane	That is awesome. Now, what are you going to do?
Beth	I don't know.
Ms. Jane	So, what makes your sneaker special?
Beth	It is for running.
Ms. Jane	So, you can make it whatever you want, the coloring, the bottom of the shoe, so how is it going to look like?

Most of the conversations during this day followed the same line of questioning. Students were trying to decide what colors to use, the name of the sneaker, what design should they make for the sole, and similar things. Some of the students were working. However, the majority was not and even though Ms. Jane announced that she would go around with a clipboard checking to see who was using the class time wisely or not, she did not do this. This activity continued until the end of the period.

Day Four - Post PD Observation

On this day, the students found the same message as the previous day on the board, so they got their packets and sneaker designs and continued working. Some students were on task. However, the great majority of the students were out of their seats, walking around the classroom, talking loudly, and the same students who were on the computers the day before, were on the computers again today. The nature of the students' conversations was mostly about asking to see each other's design, asking to borrow art supplies, or just friendly talk. This day was dedicated for the students to finish their projects.

Day Five - Post PD Observation

This day started the same as the rest of the week. The students were instructed to get their sneaker design and their packets. Ms. Jane stated that this day was the last day that the students were going to be working on this project and reminded them to use their time wisely. The students continued to work in the same fashion as the previous day. Some students were on task while others were walking around the classroom, talking to friends, and similar activities. At this time, some of the students who had been working the prior day were done with their project were just sitting and talking with their friends.

At this point, the researchers decided to come back an extra day to observe the final results of the activity since it was not yet observed by Day 5.

Day Six - Post PD Observation

The students did not have bell work this day either, instead, Ms. Jane started a whole class discussion on what makes a good audience. She took notes on the board as the students made their suggestions.

Ms. Jane	So, what makes a good audience? Jamal?
Jamal	Be respectful.
Ms. Jane	Be respectful. What does it mean to be respectful? What does it look like to be respectful when someone is presenting?
Jenny	Be quiet.
Ms. Jane	Be quiet. What else does it mean to be respectful when someone is presenting?
Alice	Don't say anything mean.

The conversation continued until there was a list of behaviors associated with being respectful when someone else was presenting. After that, the students were given instructions on how to complete the presentation critique (which was included in the packet the students were working on, Appendix E) they have to do for each of their classmates. Ms. Jane indicates that the presentations will start. The following is a transcript of one of the presentations.

Ms. Jane	Ok, Ian, come on up. Ok Ian, tell us about your sneaker.
Ian	My sneaker is the Lebron X
(At this time, most of the students are talking, not paying attention to the presenter)	
Ms. Jane	OK, being a good audience means you are not talking and if you have a question, what do you do? Alice?
Alice	How do you spell "Ian"?
Ms. Jane	I – A – N. Ok, Ian, continue… we will be a good audience.
Ian	The X on the name is like ten. They cost a lot to make.
Ms. Jane	What price did you pick?
Ian	One hundred and fifty dollars
Ms. Jane	OK.
Ian	They weigh about ten pounds, they come in every size, but the one in the picture is a 8… not a 9. The length is 2. The material you want to make your sneaker's sole is plastic, the outside is leather because I wanted materials that last. The reasons to buy my sneaker is that they are real, they are comfy, and because they are for basketball.
Ms. Jane	Does anybody have a question for Ian?
David	What about the requirements? Does it have everything it needs to have?
Carlos	Like the weight, the name of the shoe, the price.
Ms. Jane	He had all of those. Are there any other questions for Ian? Ok, thank you Ian.

The rest of the presentations continue in the same manner. Ms. Jane interrupts the presentations to remind the students about being a good audience. By the end of the period, not all the students have presented and this will be continued the following day. The researchers decide to come to observe the conclusion of the activity.

Day Seven - Post PD Observation

The class started with Ms. Jane reviewing how a good audience behaves when someone is talking. Additionally, there were some questions that the students need to answer written on the board but the students were not instructed to start working on these. Figure 11 depicts what was written on the board this day.

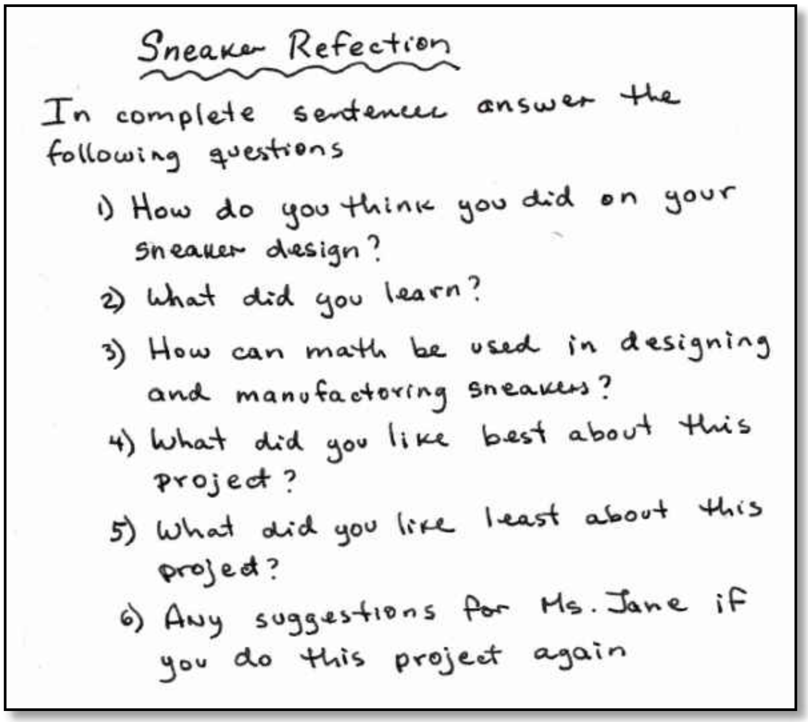


Figure 11 - Questions on Ms. Jane's Board Day Seven of Weeklong Observation

The presentations started in the same manner as the day before. However, today one of the English Language Learners of the classroom was presenting. Ms. Jane was relying on one of the bilingual students in the class to translate for this student. The student who was translating was one of the worst behaved in the class and he changes the meaning of what the student was saying to make it funny. For example, the student said in Spanish "my shoes are high tops" when he was describing his shoes but the student, who translated, said "my shoes are ten inches tall." The researcher is fluent in Spanish and could comprehend what was said. Most of the students in the class were laughing because they understood what was going on. Ms. Jane did not understand what was going on and the presentations continued.

Not all the students have gone –Ms. Jane kept asking who has not gone to show their sneaker to the class. The questions the students asked of those who present their sneaker were very repetitive: “Does it come in any other colors?” “Can I find it in Walmart?” “Can you change them?” The students clearly made fun of the store selection. All the students who went after that said that their sneakers were only available at Foot Locker. The students do not actually present their work with their own words. Ms. Jane dominates the conversation by asking one word answer questions, such as “what is the price of your sneaker?” or “what color is it?”, to the students who were presenting. Eventually, Ms. Jane seemingly noticed that the students were making fun of the presenters and told the class not to ask where they could buy their shoes anymore.

One of the students was picking up the presentation critiques while the teacher instructed the students to clear their desks and go to the front of the room. While they were at the front, the teacher laid out the students’ sneakers on their desks. She explains that they were going to vote for a sneaker, not their own sneaker, and that they could only vote once. The students were given a sticky note and were told to put the sticky note on the desk that has the sneaker for which they wanted to vote for. The students went to the front and the 3 sneakers with the most votes were separated. Ms. Jane says that these were the shoes they were going to vote for again. She explained that she had selected two or three sneakers for each class period and that they will collect data on these sneakers.

The students were asked to answer the questions that were on the board (above) and they started working. A few students were working. However, there were 8 who were not writing anything and were asking when the period was going to be over. While this was happening, Ms. Jane moved around trying to motivate the students to answer the questions in complete sentences. After a few minutes, students were all packed up and waited for the bell to go to the next period.

This was the conclusion of the Sneaker Project.

Originally, Ms. Jane was going to be observed for a period of five days. However, the researcher decided to add two additional observations for two reasons. First, Ms. Jane stated that the project was going to last seven days and she wanted to be observed for the duration of the entire assignment. Second, the researcher had not been able to observe Ms. Jane's students engaged in significant mathematics concepts that would allow them to engage in SMP3.

Case Two: The Case of Mr. John – Integer Addition

For this weeklong observation, Mr. John focused on his students making sense of integer addition. He had planned group work and other activities to develop his students' understanding of this concept.

Day One - Post PD Observation

This day started the usual way, with the students lined outside the door and with bell work waiting for them. In the bell work, Mr. John asked his students to come up with rules that should be followed during group work. Additionally, he informed his students that they were going to do a lot of group work in the coming days. The students seemed relaxed in the classroom and were free to share their thinking and rules with each other. After a whole class discussion, the students came up with the rules they were going to follow as they worked in groups. The rules were (1) all will work together, (2) all will work at the same pace, (3) no complaining about the group you get, (4) no off topic conversations allowed, and (5) concentrate on what you are doing. After this, the students took a test on unrelated content. Therefore, the observation for this day finished at this time.

Day Two - Post PD Observation

This day started like any other, with bell work. This work was written on the board and the students started working on it as soon as they walked into the classroom. Figure 12 shows the bell work that was used this day.

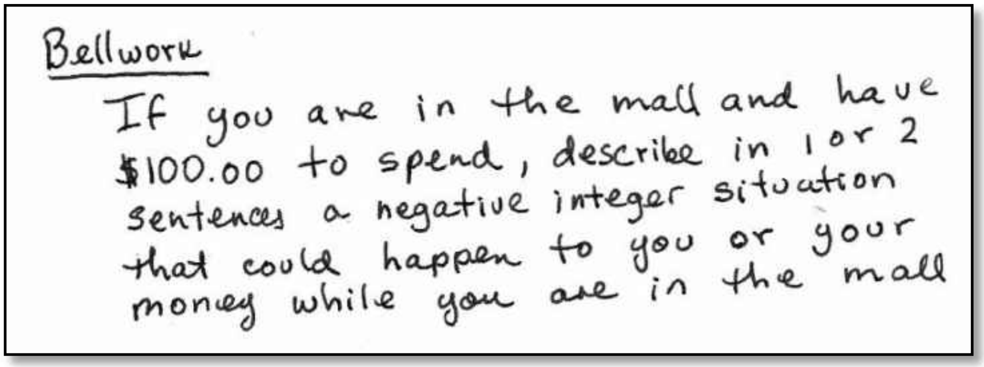


Figure 12 - Bell Opener for Mr. John Day Two of Weeklong Observation

Mr. John's students worked diligently to answer the question. Meanwhile, Mr. John moved around them making sure everyone was on task and answering individual questions. The following is one of the conversations sparked by the bell work.

Mr. John	Ok, Kenneth?
Kenneth	I am at the mall with a hundred dollars and I get robbed.
Mr. John	Ok, so what will the negative integer be?
Kenneth	-100
Mr. John	-100 because you got robbed.
Kenneth	Yes
Mr. John	Good job!

Like this student, others shared their ideas. Mr. John's goal was that his students think of real life situations in which negative integers will be used. After this activity, the students were asked to form their groups. They do this efficiently and silently. Mr. John had in place a grouping routine in his classroom; he would say "quick groups" and the students would immediately pair with the person next to them. However, this time he said "quick groups times two" and the pair of students got together with two another pair of students who were next to them. The instructions for the group activity were displayed in the classroom. The following is a picture of those directions.

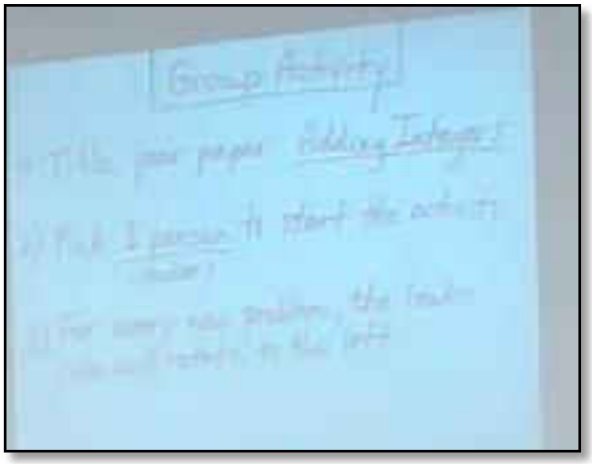


Figure 13 - Mr. John Day Two of Weeklong Observation

The teacher gave instructions and made it clear that he was not going to help the students this time. The groups were instructed to come up with their own way of adding integers. Mr. John gave the students double sided counters (red and white). The students had to use the chips to solve the problems; in addition, they were instructed to create a representation of the problem (drawing was up to them). The only thing that the teacher told the students was that the red side of the chip represents negative and that one red and one white chip cancel each other. This was his attempt for scaffolding the students in the use of the chips. It was apparent that this might be the first time students have used the chips. Mr. John reminds students of the rules they came up with working in groups. Additionally, he stated that he will not be showing them how to solve the problems and that he expects everyone to come up with their own solution method.

The students were given the following problems to solve:

1. -4+3
2. -3+6

Students worked in their groups and seem to be engaged. Some groups finish faster than others, so they were idle for a bit while everyone else was working. The following is one of the conversations the students had within their group.

Tony	This gets me confused
Kelly	This is three and this is six.
Clara	This has to be negative six, so you need to turn them around so they are red
Kelly	You are right, it is negative three.
Tony	I know what to do but I am confused.
Clara	The three red ones cancel out these white ones.
Kelly	Yeah.
Tony	I am not sure, the chips really confuse me.
Kelly	So, let's do some practice problems. If we had negative six and we had four (using the chips), then what do we have?
Tony	Negative two… oh
Clara	So if we had positive six and we have negative two, what do we have?
Tony	Four… oh I see…
Kelly	Suppose I have four bottles of Gatorade, if someone drank three of them, how many would I have left?
Tony	One
Kelly	Yes, but those bottles that someone drank are negative because I am taking them away from what I had. So, I have four bottles, positive four, and someone drinks three, negative three, I would have positive one left over (she demonstrates with the chips as she gives her example)
Tony	I get it… you are smart.
Kelly	You are smart too.

During this observation, all the groups were engaged in either the activity or in making sense of the computations they were learning. After the groups were done with the two problems, Mr. John directs a whole class discussion about the methods different groups used to solve the problems.

Mr. John	Veronica, what answer did your group come up with for the first problem?
Veronica	Positive one.
Mr. John	How many people agree with the answer of positive one? (Looking around) so we got another group that got positive one. Ok, Dustin, what did you guys get?
Dustin	Negative one.
Mr. John	OK, Dustin's group got negative one. How many people got negative one? (Looking around) We got three groups with negative one and two groups

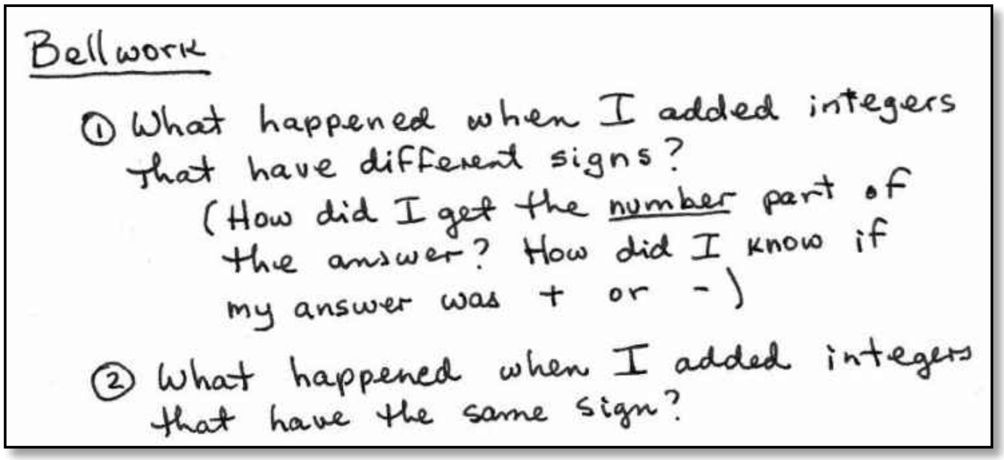
	with positive one. OK, Carlie, how did your group come up with positive one as the answer? I would like you to show me.
Carlie	We used four red chips and three white chips.
Mr. John	Why do you have 4 red chips?
Carlie	Because we have negative four
Mr. John	Why do you have three white chips?
Carlie	Because it is positive three.
Mr. John	Ok, so what did you do now?
Carlie	We stacked them, one red and one white. We had one red chip left over. So we got negative one.
Mr. John	Excellent job! Now, I want to know something. How am I able to cancel out one red and one white chip? Luis, what do you think?
Luis	Because if you have a positive one and a negative one you get zero.
Mr. John	Excellent. That is exactly why we are able to cancel them out. Can anybody tell me what he just said? Kent?
Kent	He said that if you have one positive and one negative you get zero.
Mr. John	That is right.

All the problems used throughout the period were solved first by the individual groups using the chips. Then, the whole group discussed the answers and the methods used to solve the problems.

During this day, for the first time, students took control of their learning, the classroom was centered about problem solving and building mathematical reasoning. Mr. John was facilitating his students to learn rather than teaching them the steps to solve the problems. The students were helping each other understand the concept of integer addition by using the double-sided chips and other examples. Mr. John used the questioning techniques he learned in the PD to engage his students in mathematical talk. The students continued to work in this manner until the end of the period.

Day Three - Post PD Observation

The routine of the day was consistent with what has been observed in the past. The students entered and started working on their bell work immediately. The work today consisted of answering two questions that were on the board. Figure 14 is a depiction of the work the students were doing at this time.


Bellwork

① What happened when I added integers that have different signs?
(How did I get the number part of the answer? How did I know if my answer was + or −)

② What happened when I added integers that have the same sign?

Figure 14 - Bell Work for Mr. John Day Three of Weeklong Observation

The conversation that followed the students completing this task was designed for them to start building generalizations based on their observations from the previous day. The following is one of the conversations that the whole group had with Mr. John as they collaborated together to answer the questions.

Mr. John	So, answering my first question, what happened when I added integers that had different signs? For example, how did I get the number part of the answer? To make this easier, what was the first problem we did yesterday? Carla?
Carla	- 4 + 3
Mr. John	And, what was the answer?
Carla	-1
Mr. John	(after writing the problem and its answer on the board) How can I take the number part of this answer, the one, and get it without using the chips? Carlie, what do you think?
Carlie	By looking at the three and finding what number I need to add to get to four.
Mr. John	So, I add those numbers to get four? So, who can add to what she just said? Lets try a different idea, what has somebody else come up with? How do you think I can get the 1, the number part of my answer?
Luis	By subtracting
Mr. John	By subtracting what?
Luis	- 4 – 3
Mr. John	Would that be the same problem if I do that? Or would that change the problem altogether?
Luis	I am not sure.
Mr. John	Ok, so let's think about that. I have negative four minus three… does that keep the problem the same or does that change it? Let's take a vote. How

> many people think that negative four minus three would change the problem? Ok, so three people agree with this. The fact is that changing this positive to a negative would change the problem, but I liked what Luis said because it is almost to where I want it to be. We do not change this to a minus, but we do have to subtract something to get the number part of your answer. What do you need to subtract?

At this point, Mr. John realized that many students think that -4 – 3 was the same as -4 + 3, so he stopped and regrouped. He focused their attention on just the sign of the answer.

Mr. John	Ask yourself, which one is bigger (pointing at – 4 and 3)
Vanessa	Four is bigger
Mr. John	OK, so four is bigger. So what sign is with the four?
David	Negative
Mr. John	Negative, so your answer is going to be negative. That is going to be part of our steps.

Even though the students started to build some reasoning, Mr. John gave them the steps to add integers. By the end of the lesson, Mr. John gave his students a couple of more steps like "ignore the signs and add them" and "keep the signs they both have" (referring to addition of integers with the same sign). In his effort to support his students, it seemed possible that Mr. John did the conjecturing for his students.

Day Four - Post PD Observation

The class started as it usually does with the bell work. Students, for the most part, were looking in their papers for the steps to answer the questions. Mr. John gave students a choice: either use representation of double sided chips or using the steps the he came up with on the previous day. Figure 15 shows the bell work used on that day.

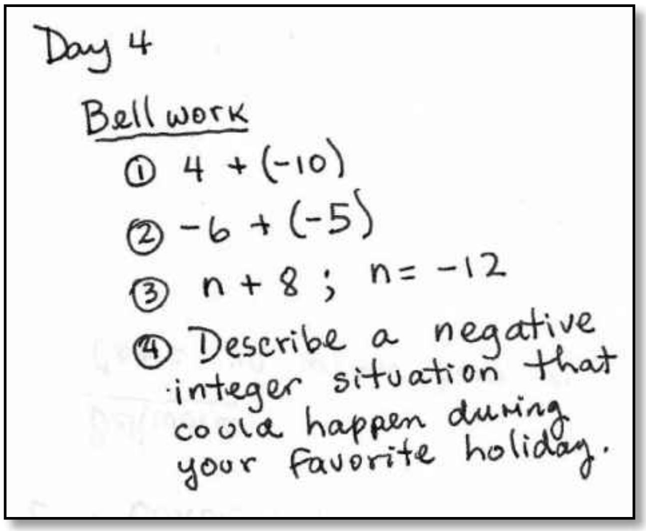


Figure 15 - Bell Work for Mr. John Day Four of Weeklong Observation

The students worked individually and silently while answering the bell work. Some of the students volunteered to write the answers on the board and they started discussing their answers for the work. The following is one of the conversations that happened at this time.

Mr. John	Ok for number one, we have an answer of positive six. How many people agree with that? (Counting those who raise their hands) OK, we have 5 people who agree. So, how many people disagree? Most of the students raise their hands. Ok, Veronica, what answer did you get?
Vanessa	Negative fourteen,
Mr. John	How many people agree with that one? (Nobody puts his or her hand up) OK… who disagrees with the answer, that is you got something different than positive six or negative fourteen. OK, Daisy, what did you get?
Daisy	I got negative six.
Mr. John	Ok, Daisy got negative six, how many people agree with that? (Most of the students put their hands up). OK, wow… so let's see who is right. So, who can tell me what the rule is when adding integers with different signs?
Carlie	Subtract the numbers and keep the sign of the higher number.
Mr. John	Very good! Who can say what she just said? Veronica?
Veronica	She said to use the sign of the larger number and then subtract the numbers.
Mr. John	Very good. So, let's see what happens here, which number is bigger, ten or four?
Kayla	Ten
Mr. John	Ten, so is ten positive or negative?
Kayla	Negative
Mr. John	So, what is my answer going to be?
Kayla	Negative

Mr. John It is going to be negative. So that right there is half of the problem. So remember to keep the sign of the bigger number. So, now, subtract the numbers. What is the answer?

Luis Six

Mr. John That is right, ignore the signs. Then you have ten minus four is six. We write the negative sign for the ten, and we get negative six.

The students continued to review the problems in the bell work in the same manner and after they were done, the students were instructed to form quick groups. Mr. John then introduced subtraction of integers. He encourages the students to use the double-sided chips but tells them to find a way other than cancelling to show what happens to the double-sided chips. The example given to the students was 4 – 3. The following excerpt is a conversation that happened between two members of one group.

Luis So, here I have four and here I have three (pointing to his representation). I marked them to cross them out, so I x them… and x the whole three out.

Carlie You have to mark the three circles too.

Luis Yes (and he proceeds to do just that)

The following is a picture of what their representation looked at the time they were done with it.

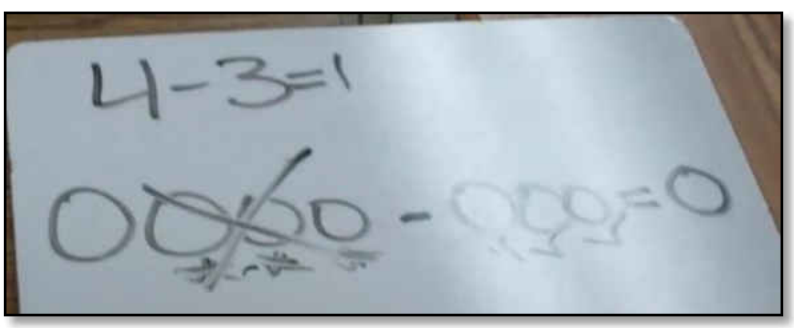


Figure 16 - Representation of Integer Subtraction Constructed by One of the Groups

They used this picture to justify their answer to this problem. Mr. John continued with a whole class discussion regarding the representations that the different groups used to represent the subtraction. The next problem that Mr. John gave his class is -3 – (-1). He asked them to use the same kind of representation to show this operation. Some of the students struggled with the concept of subtracting negative numbers. Figures 17 depicts of one of the groups' representations.

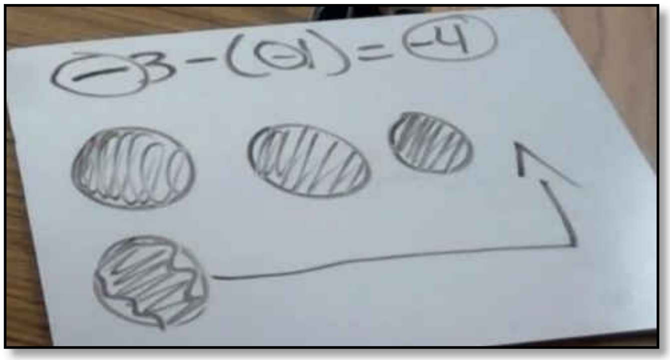


Figure 17 - Representation of the Subtraction of Negative Integers

Figure 18 is a picture of another group's representation depicting the same problem as the previous one.

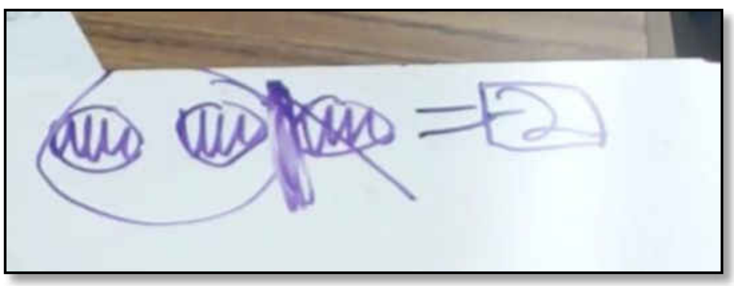


Figure 18 - Representation of the Subtraction of Negative Numbers

The following excerpt is the whole class discussion that happened after the groups shared their representations with each other.

Mr. John	Ok, let's see what we got. I see some good arguments here. I see a few negative twos and a few negative fours. We have two different answers. Let's try this together. How many chips should I have to start this problem with? Matt?
Matt	Three shaded chips.
Mr. John	Three shaded chips. Why?
Matt	Because it is negative.
Mr. John	Right, because it is negative. OK, so we got three negative chips… what am I doing now? To show the subtraction? Talia, what do you think?
Talia	Subtract negative one.
Mr. John	So, how can I show this with my chips?

Talia	Cross one out.
Mr. John	Ok, Talia said I can cross out one of my negative chips, can I do that? (some students put their hands up) Yes, I can do that because when I cross something out its telling I am doing what kind of problem?
Luis	Subtraction
Mr. John	That is right!

The process of solving problems in groups, sharing the representations, and having whole class discussions about the representations continued until the period was over.

Day Five - Post PD Observation

Class started with the bell work as usual. The students were asked to show their work with drawings (double sided chips). Mr. John took a couple of minutes to explain his expectations. Figure 19 depicts the bell work for this day.

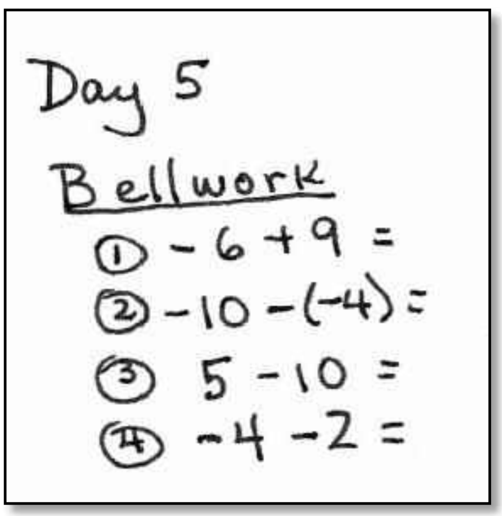


Figure 19 - Bell Work for Mr. John Day Five of Weeklong Observation

The students worked silently and diligently to finish their work in the time allotted. While the students worked, Mr. John takes care of routine tasks (attendance roster). He gauged how many students were done by asking them and allowed an extra minute to finish. He realized that the majority of the students were still working to finish the problems. Mr. John circulated amongst the desks, stopping here and there to ask and answer questions. Mostly, he was looking at the students' work and he stopped to ask when he sees something that calls his attention. When the time was done, the students got into their quick groups and were instructed to compare their answers to see whether they would agree or not. The students did as they were asked. In the meantime, the teacher went around and stopped at every group to reinforce the group work rules and to explain his expectations of their work. After they were done, one of the groups went and represented their solving method on the board. They were asked to explain their reasoning. The following excerpt depicts one of the groups justifying their answer.

> Mr. John — OK, for group number 1, I will go with the fellows in the back.
>
> (The students got up and drew their representation on the board; six shaded chips and nine blank chips. They drew a line through each pair of shaded and not shaded chips. They write the number three as their answer)
>
> Mr. John — So, the fellows in the back got positive three for their answer. Explain it to me.
>
> Luis — So we drew the circles.
>
> Mr. John — What do the circles mean?
>
> Luis — The regular ones without shade are positive and the shaded are negative.
>
> Mr. John — OK.
>
> Luis — Then we… we cancelled them, one negative and one positive makes zero...
>
> Mr. John — Sounds good.
>
> Luis — So after we cancelled all we could, we had 3 positive left over.
>
> Mr. John — That is great! You guys did a good job.

The rest of the problems in the bell work were reviewed in the same manner. Then, Mr. John instructed his students to take out their notebooks and open to the page they were using the prior day.

> Mr. John — I am going to show you guys a little trick to use that is going to help you subtract without using pictures. So I want you to draw a line underneath what you did yesterday and write 'subtracting integers'. I am going to give you a little trick. To give you that I want to go back when we divided fractions. So, this is going way back; we are switching gears and going back. What was it that we did when we divide fractions? Matt?
>
> Matt — We same, change, flip

Mr. John	We same, change, flip it. Right? So, we kept the same fraction, we changed to multiplication, and we used the reciprocal of our second fraction in the problem. What did that do to the division problem? Did we divide anything after we same, change, flip it? No? I see heads shaking. So, what did it turn the division into? Beth.
Beth	Multiplication
Mr. John	Multiplication. We know that multiplication is easier than division, so we changed into multiplication and we just have to multiply across. Same thing here. We are going to take out the difficult, the subtraction and I am going to give you three words to remember just like in division of fractions. The three words are same, change, and opposite.

At this point in the weeklong lesson is when Mr. John gives the students the shortcut they need to perform the computation efficiently. There were just a few more minutes left in the period and Mr. John used them to show the students an example of this method. He could not finish because the bell rang and announced they would continue next week. This was the end of Mr. John's weeklong observation. The researcher decided not to observe the end of the unit as the students were observed working with significant mathematics and because Mr. John had given them a shortcut to the operation.

Students Constructing Viable Arguments and Critiquing the Reasoning of Others

During the weeklong observation, Ms. Jane's students did not construct any reasoning that backed up their answers. However, Mr. John's students started to build explanations based on solid mathematical logic. The arguments used by the students were simple but effective in conveying their understanding of integer addition and subtraction. By using the indicators of SMP3, we can see what Mr. John's students were able to accomplish. The students were not constantly demonstrating all indicators associated with building reasoning. However, Mr. John was able to support his students as they began to construct their mathematical reasoning. Table 8 shows what the students were able to accomplish during the weeklong observations.

Table 8 - Indicators for Students Creating Viable Arguments in Mr. John's Class

SMP3 Indicators/Creating Viable Arguments	Mr. John's Class
Understand and use stated assumptions, definitions, and previously established results in constructing arguments	Yes
Make conjectures and build a logical progression of statements to explore the truth of their conjectures	Yes
Analyze situations by breaking them into cases, and can recognize and use counterexamples	Yes

As far as sharing reasoning with each other, Ms. Jane did not support her students to assess each others' logic. On the other hand, Mr. John's students were not as successful as when they created viable arguments. Nevertheless, his students justified their conclusions and communicated them to others. However, they were not given opportunities to respond to the arguments of others. The farthest they went here was to repeat the reasoning of others in their own words. The abilities to reason inductively and assess reasoning were emerging at best. This was recorded in the conversations that happened within the groups as the students were trying to make sense of integer addition and subtraction. However, the ability to compare the effectiveness of two plausible arguments was not present at all. Table 9 shows the indicators that students demonstrated during the weeklong observation.

Table 9 - Indicators for Students Assessing Others' Arguments in Mr. John's Class

SMP3 Indicators/Assessing Others' Reasoning	Mr. John's Class
Justify their conclusions, communicate them to others, and respond to the arguments of others	Yes
Reason inductively about data, making plausible arguments that take into account the context from which the data arose	Emerging
Compare the effectiveness of two plausible arguments, distinguishing correct logic or reasoning from that which is flawed, and if there is a flaw in the argument explain what it is	No
Listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments	Emerging

Teacher's Perceptions for Weeklong Observation

As part of the observations that followed the completion of PD, the teachers were asked to reflect about each of the days they were observed. They were given writing prompts to facilitate them to express their thoughts and rating scales in order for rate the engagement of their students in SMP3. In addition, the teachers were given a Likert scale that they could use to rate their own ability to engage their students in SMP3 (Appendix F).

Case One: The Case of Ms. Jane

For her weeklong observation, Ms. Jane designed an activity that she called the "Sneaker Project." At the end of this project, the students had designed a sneaker for a specific purpose using mathematical concepts, collected and organized data of their favorite sneakers, and displayed this data in different ways. Ms. Jane stated that the focus of this project was to have the students collect, organize, and display data in the real world.

Day One - Post PD Observation

Learning Goal

Ms. Jane stated that the goal for the day's lesson was to "gain the students' attention and learn about the math used in making sneakers." When prompted to respond to how her students reacted to this lesson in comparison to other lessons she stated that her students were excited about the project.

> I felt at first the students were hard to focus but were excited about the topic – which usually does not happen; they also had the opportunity to move around and work cooperatively together which was a struggle but I think will get better with practice.

Ms. Jane also reflected on the overall success of this lesson and she thought it was fine but that she felt her students struggled with the questions she asked in the beginning of the lesson.

> I thought it went OK, when beginning with the students having an open ended question kind of threw them off; I did not specifically tell them exactly what to do, I think this will improve with practice – letting them struggle a little bit but not getting too frustrated.

SMP3

Ms. Jane rated her students' ability to create viable arguments as "poor" (circled "2"). She gave them this rating because she indicated she could have asked better questions that allowed them to explain their methods. Additionally, she said she did not give them adequate wait time to answer her questions. When rating her students' ability to critique the arguments of others, she stated that she was not sure whether they did anything to demonstrate this ability.

> I did not create much of a way for them to do this kind of (activity) when working with their partner and deciding what to do with their data.

Day Two - Post PD Observation

Learning Goal

Ms. Jane stated the learning goal for this day as "learning about the different materials of sneakers and learn the project outcome to design sneaker." When asked how her students reacted to this lesson compared to other lessons, she replied:

> They seemed to be more engaged, had a little trouble with sharing and reading an article in math but overall seemed more on task and interested in the topic.

When asked to reflect on the overall success of the day's lesson she stated that:

> I thought it went OK; could have done a better job of exploring partner work (each person could have a job – one writing other speaking), more accountability.

SMP 3

Ms. Jane was asked to rate her students' ability to construct viable arguments and she rated them as "poor" (circled "2"). Ms. Jane stated that her students demonstrated this ability during the whole class discussion. Specially, when they were talking about the costs of production and retail prices of sneakers. One specific change she stated she could make was to have questions for the students to ask their partners as they discussed the articles in their group.

Ms. Jane was also asked to rate her students' ability to critique the reasoning of others. She again rated their ability as "poor" (circled "2"). She stated that her students demonstrated this ability when they were grouped. She admits that they could have done a better job at it but she believes they will improve as they have more experiences doing group work.

Day Three - Post PD Observation

Learning Goal

For today, Ms. Jane described the learning goal as having her students "designing a sneaker for a specific purpose using mathematical concepts." When prompted to comment about her students' reaction to that day's lesson she stated that they were eager to finish their design and that they felt enthusiastic about the lesson. When asked about the overall success of the lesson, she stated that

> It went pretty well; most students were engaged in the lesson and seemed to be on task. A few had trouble starting but once going, worked well.

SMP3

This day, Ms. Jane rated her students' ability to construct viable arguments as "poor" (circled "2"). She stated that her students were able to answer "why" when asked the reasoning behind picking a sales price for their sneaker. Ms. Jane rated her students' ability to assess each others' reasoning as "average" because

> They did pretty well telling others what they thought and their reasoning for it.

Day Four - Post PD Observation

Learning Goal

Ms. Jane listed the learning goal of this day the same as the prior day as her students continued to work on their sneaker design. When she commented about the overall success of the lesson, she stated that she feels her students are paying more attention to details and taking care of doing a good job.

> They are more excited to get to work – very critical of themselves and want to be perfect. Before they were not as focused on the details and perfection but they are taking more time and not rushing through the project.

SMP3

Ms. Jane was asked to rate her students ability to construct viable arguments and again she rated them as "poor" (circled "2"). She stated that her students showed this ability when they explained how they were designing and the manufacturing plan for their sneaker.

> One student was talking about the investment he wants to make and how he needs commercials, etc. to make it (the sneaker) popular for people to buy.

This day, Ms. Jane rated her students' ability to critique the reasoning of others as "poor" (circled "2") because while in their groups, the students talked about the details of their sneakers.

> When working together each makes arguments about why they want to design their shoe and what materials they want to use.

Day Five - Post PD Observation

Learning Goal

The learning goal for this day was the same as the previous day. When asked how her students reacted to her lesson, she thought they were on task and motivated to finish with their project.

> They were more engaged and interested in actually completing the project. They wanted to come to class and work, also working on it at home and other classes.

When commenting on the overall success of the lesson, Ms. Jane stated that most students enjoyed the work and that they were on task. Additionally, she stated that some students were so motivated they actually finished early.

SMP3

Ms. Jane rated the ability of her students to construct viable arguments as “poor” (circled “2”). She stated that her students talked about the prices they wanted to sell their sneakers at and the reasons behind their choices. Ms. Jane also rated her students as “poor” for their ability to assess each others’ reasoning. She stated that the students talked about their opinions regarding each other’s sneakers and what they would do.

Case Two: The Case of Mr. John

For the weeklong observation, Mr. John decided to teach integer operations. His goal was that his students make sense of integers, what they mean, make connections with real life situations, and to be able to create context that reflect integers. Since Mr. John had scheduled a test on Day 1 of the observation, his reflections span Days 2 to 5.

Day Two - Post PD Observation

Learning Goal

For this day, Mr. John had the goal that his students would be able to understand integer addition. When asked about how his students responded to the lesson, Mr. John stated that he expected his students to be uncomfortable because it was the first time they were going to learn on their own.

> My students did not feel comfortable struggling and having to find the answers on their own, but I think they got a better understanding this way.

Mr. John commented in the overall success of the lesson by stating that he saw many light bulb moments when his students realized their mistakes. He regretted not having more time to discuss all the problems. Nevertheless, he is already planning for the following school year.

> For next year I might give them all of the problems at one time and discuss all at one time at the end.

SMP3

Mr. John rated his students' ability to create viable arguments as "poor" (circled "2"). Still, he thought that during the group activity, his students were able to use representation and had the tools to be able to create viable arguments. As far as critiquing the arguments of others, Mr. John rated his students as "average" (circled "3"). He said that during the discussion, he tried to focus on those problems that students got wrong to use them as learning opportunities. Additionally, he stated that the process of seeing and assessing these errors gave his students ways to be able to critique the reasoning of others.

Day Three - Post PD Observation

Learning Goals

Mr. John had set the goal that his students would be able to add integers with 100% accuracy. This was an extension to the lesson of the previous day. He seemed disappointed by the way his students responded to the lesson when compared to previous lessons.

> Students did not respond well at all today during the lesson. Today my students seemed tired and confused compared to yesterday and the days before.

In reflecting of the lesson overall, Mr. John did not think it was successful because he wanted his students to extend what they learned the day before and come up with generalizations from those previous experiences and this did not happen naturally. He stated that he felt like he needed to hone in his questioning skills.

SMP3

Mr. John's rating of his students' ability to create viable arguments was "very poor" (circled 1). He blamed his questions because he could not get his students where he wanted them to be.

> I was trying to get my students to come up with viable arguments to verify the steps that we would come up with in class, but that did not happen. I think it was my questioning that needed to change and be better so my kids would understand where I was going with the lesson.

Mr. John rated his students' ability to critique each other's reasoning as "poor" (circled "2") because even though he was attempting for them to assess each other's steps with the knowledge they had acquired the day before, they were unable to do this effectively.

Day Four - Post PD Observation

Learning Goal

This day's goal was that students would understand integer subtraction. Mr. John believes that his students showed a marked improvement from the previous day. Moreover, he stated that they were more engaged.

> Students responded a lot better today than they did yesterday and better that the students [I had] last year at this time of the school year. They were more engaged and understood the concept better than previous lessons.

Mr. John believed that the lesson was effective and stated that his students worked better in smaller groups. Additionally, he was aware that his students were not feeling as comfortable as usual because they were struggling to make sense of the concepts they were learning. Mr. John hoped that by putting in place the expectation that students will struggle with learning from the beginning of the school year, his future students will deal with the frustration in a more positive way.

SMP3

Today, Mr. John rated his students' ability to create viable arguments as "good" (circled "4"). He attributed this improvement to the fact that he listened to his students as they worked in their groups.

> I saw them talking about how to show subtraction on their white boards and also how different subtraction was than addition.

Mr. John rated his students' ability to critique each others' reasoning as "good." He stated that he observed students critiquing each other's arguments using the representations in their white boards. In addition, he said that those students who were wrong took their peers critiques with a positive attitude and actually fixed their mistakes.

> I observed students, while in their groups, critiquing their partners with positive comments if their idea was unclear or did not make sense… they were learning from their peers.

Day Five - Post PD Observation

Learning Goal

Today was the last day of the weeklong observation and Mr. John wanted to extend the concept of addition of integers into the concept of subtraction of two negative integers. He felt that his students responded well to the lesson. However, he was a bit disappointed because they did not perform the way he expected during the whole class discussion.

> When I came up to groups and saw what they were doing, it seems that they grasped what was going on, but the class discussion did not reflect that.

When asked about the overall success of the lesson, Mr. John stated that he felt grouping the students was very successful because their representations were good and helped the students explain their reasoning.

> The class discussion at the end of the lesson did not go so well and it was clear that students were struggling putting everything together. For next year I am definitely going to put problems in context for them first, then introduce the chips.

SMP3

Today, Mr. John rated his students' ability to create viable arguments as "good" (circled "4") mainly because of their use of representations to support their mathematical reasoning.

> Since students were used to working with the chips, they seemed to have a smooth transition with constructing arguments with subtraction of integers.

Mr. John observed that his students engaged in evaluating their peers' logic when they were working in groups. However, he did not see his students do this during whole class discussion. Therefore, he rated their ability to critique the reasoning of others as "poor" (circled "2").

Mr. John's Final Reflection

Mr. John provided the researchers with a final reflection where he discussed the importance of receiving support during this time where the expectations of students will change because of the CCSSM and the SMPs. Specifically, he reflected about SMP3 and how simple questioning methods, using formative assessment, and letting students struggle would help students create viable arguments and support the reasoning of others.

> I have seen my children expand their knowledge of math through different questioning techniques I have used in my classroom and have seen them better relate mathematical topics to their own lives… with these questions come conversations that students have with one another about the math topics which create a wonderful and dynamic environment where young people begin to flourish and gain confidence in their mathematical skills.

Mr. John believed that "teacher-centered" teaching was not what his students needed to be successful and that these traditional methods will not yield the higher results that everyone wants. Mr. John stated that before the PD he was hoping to become a better teacher but that instead he became a better facilitator of knowledge for his students.

Teacher Classroom Observations Four Weeks After PD

As discussed in Chapter 3, Phase 4 of the study consisted in observing the participating teachers four weeks after the PD was completed. These observations were conducted during the same class periods that were observed before. The observations were video recorded. However, the teachers were not interviewed because the researchers just wanted to gauge the students' engagement on SMP3 and determine whether the teachers were able to implement the methods they had learned. The teachers were notified of this observation on the last day of the weeklong observation.

Case One: The Case of Ms. Jane

Ms. Jane started her class without bell work. Instead, she distributed pieces of paper and instructed the students to fold their papers in three sections as they were preparing to make KLW charts. A KLW chart is a tool that encourages students to think about what they know (K), what they want to learn (L) and what they have learned (W). This tool can be used in any content class and with any kind of student. Figure 20 is a picture of what Ms. Jane had written on the board for her students.

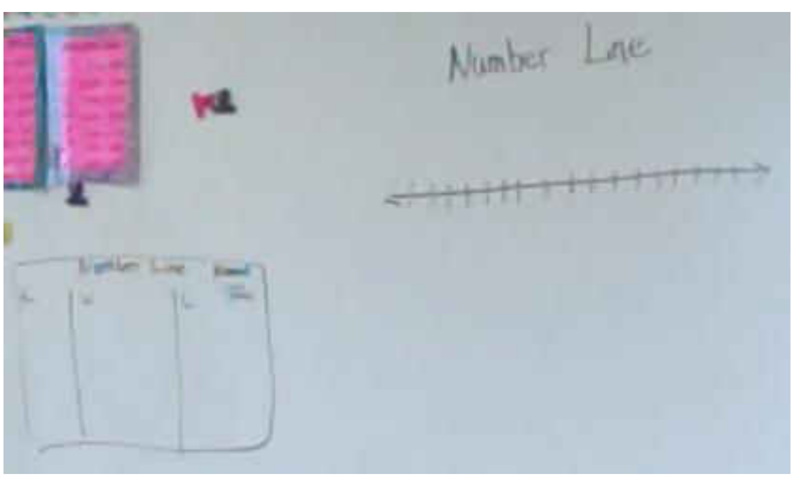


Figure 20 - Ms. Jane's Board on the Last Day of Observation

The students were asked to pair with the person seated instructed to fill the K and W parts of their chart. Ms. Jane explained what each part meant, so the students worked on writing what they knew and what they wanted to know about number lines. The students shared their answers on the chart with each other after the time given was over. A class discussion starts about the number line. The teacher asked many questions during the whole-class conversation that followed. The students answered her questions regarding the number line. The following is an excerpt of one of the conversations that happened at this time.

Ms. Jane	Ok, so, raise your hand if you know something about a number line. Anything, anything. Yes?
David	It involves… people use it for several reasons.
Ms. Jane	For several reasons, can you name one of them?
David	No… counting?
Ms. Jane	So, he says to count. Carla?
Carla	It can have negative or positive.
Ms. Jane	It can have negative and positive numbers, both kinds of numbers. What else do you know, Aaron?
Aaron	I know that you use for numbers.
Ms. Jane	Its meant for numbers, right. What else do you know about a number line? Luis?
Luis	It involves lines.
Ms. Jane	It has lines. What else, Sonia?
Sonia	You can use it for add or subtract.
Ms. Jane	You can use it for addition and subtraction. David?
David	There is a zero in the middle.
Ms. Jane	There is a zero in the middle. So, what are some things you would like to know about number lines?

The discussion continued in the same manner. Some of the things the students wanted to know included knowing how to use a number line, knowing why they needed to learn about it, why there was a zero in the middle, and similar questions. At this time, Ms. Jane started having a whole class discussion to help her students make sense of the number line.

Ms. Jane	Have you ever lost money?
Aaron	I lost twenty dollars.

Ms. Jane	Aaron, how can I put that on a number line? Losing twenty dollars?
Aaron	Negative twenty.
Ms. Jane	Alright, so I can use a negative twenty… where do I put that Aaron?
Aaron	Put zero in the middle and negative twenty to the left.
Ms. Jane	(Writing the zero and negative twenty immediately to the left) So, you are saying the lost twenty. So, he went from zero to twenty. Are you skip counting?
Aaron	No.
Ms. Jane	Ok… I guess you could if you wanted to but what comes after zero?
Aaron	One.
Ms. Jane	Oh… what kind of one? (No answer) A negative one.

After that, Ms. Jane asked if anyone had ever found money and had a similar discussion with positive integers. Once that discussion was over, the students were asked to draw a number line and to draw their house where zero goes. The teacher labels the side to the right as EAST and the side to the left of the zero as WEST. The students were asked to draw two places to the east and the west of their “homes.” Figure 21 shows what was on the board at this time.

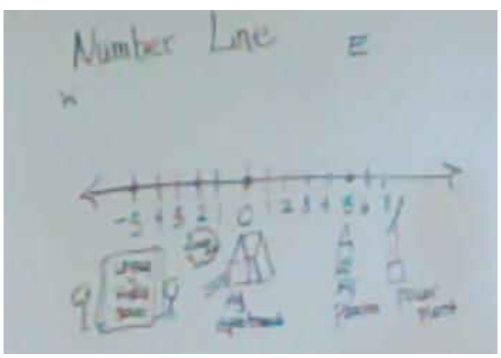


Figure 21 - Number Line Activity Used by Ms. Jane During Last Observation

The students were instructed to exchange their “maps” with their partners and each would take turns in directing the other to either east or west of their ‘house’ by using the number line to measure the distance. The students had some difficulty understanding what they were supposed to do and she had to repeat the instructions a few times. Most of the groups were not doing the activity, some students talked over the rest of the class or turned away from their partner to talk to their friends and two students had their heads down not participating at all. After this, Ms. Jane starts explaining what absolute value is.

Ms. Jane   So, we are going to talk about absolute value, OK? The symbol looks like this (writing || on the board), it is like a touchdown, two signs, and it will give you the absolute value. So, let's look at two. Absolute value is the distance from zero. So, let's look at our two, what is the distance from two to zero? How many spaces?

David   Two.

Ms. Jane   Two! So, the absolute value of two is two. But what about negative 2? Who can give me the definition of absolute value? What did I say the definition of absolute value was?

David   It's the distance.

Ms. Jane   Yes, so if the distance from two to zero is two, what about the negative two? What is the distance from negative two to zero? How many spaces? Aaron?

Aaron   Two.

Ms. Jane   Two. So what is the absolute value of negative two?

Aaron   Two.

Ms. Jane   So, what would be the absolute value of negative one thousand two hundred and seventy five?

Aaron   Negative one thousand two hundred and seventy five.

Ms. Jane   Negative? It does not matter how far it is from each side, it is still the same to zero. So, the absolute value is one thousand two hundred and seventy five. It does not matter if it is negative, it is just the value it is. OK?

After that discussion, the students were asked to write in their KWL chart what they have learned today about number lines. The teacher collected the charts and gave the students a worksheet to do and assigned it as homework if they do not finish.

Analysis of Ms. Jane's final observation

Ms. Jane tried to incorporate reason making by providing students with the opportunity to fill out their KWL charts. However, she was not successful in developing understanding in her students. The activity she used had the potential to cause more confusion because she instructed the students to draw locations to the left of their house (where negative numbers usually are placed) and to mark how many miles they were located from their house, even though miles cannot be measured in negative numbers. The trajectory she was trying to follow was clear, Ms. Jane wanted her students to understand absolute value. Unfortunately, her students struggled with the concepts. There was no evidence of student engagement in SMP3 during this lesson. The conversation was dominated by Ms. Jane. Furthermore, she had reverted to her old methods and had trouble keeping the students interested and on task.

Case Two: The Case of Mr. John

On this day, Mr. John did not have a board opener. The students immediately formed groups and started working on work they had from the previous day. They were doing group work. Mr. John gave each group a set of problems. Each set represents a different mathematics topic and each has 20 questions. The students were instructed to select a leader for each of their groups and that person was in charge of giving the rest of the group the problems, one at a time. This time, the students did not switching the leader role as they have done before. The students were to work together, one problem at a time. In addition, the students could use their class notes and each other's knowledge to solve the problems.

The conversations that happened that day focused on the mathematics associated with the problems the students were working on. The following is one of the conversations heard this day. This particular group was starting to work on the problems dealing with decimal multiplication.

Becky	OK, now number 6; multiplication 3.04 times 0.6
Maria	Did you say 0.86?
Becky	Just 0.6.
Maria	Oh, OK. I'm sorry.
	(The students work silently)
Becky	I got 1.824
Maria	I got 1.8240.
Carlos	You did?
Maria	I added a zero
Matt	You were not supposed to add a zero.
Becky	It doesn't matter. The zero at the end or at the beginning does not matter.
Carlos	It doesn't?

Becky	No.
Maria	I think it matters because you are adding an extra thousand, so it does matters.
Matt	I think it does.
Becky	Ok, so 1.824 or 0.1824?
Carlos	1.824
Maria	No, because its four (referring to the number of digits after the decimal point in the factors) and if you don't have that then the point in front, so it is 0.1…
Carlos	No… see if you…
Becky	…6 times 4, leave the 4 carry the 2… 6 times 3 is 18, put all zeros; zero, zero, zero, zero, so you have 18
Carlos	Then you do 'race track'
Becky	And there are four… one, two, three, four… and the decimal is right here… one, two, three. So it is 1.824

The students were working collaboratively in order to answer the questions. One of the students had the misconception that the number of digits after the decimal point in the product had to match the number of digits in the factors but the students worked together to solve the problem and got the right answer. The rest of the students were engaged in similar conversations and continued to work until Mr. John collected their work and they got ready to be dismissed.

Analysis of Mr. John's final Observation

Mr. John used collaborative learning and set expectations for students to explain and justify their answers in order to facilitate their engagement in SMP3. Even though the conversations witnessed this day did not demonstrate all the indicators associated with SMP3, there seemed to be emerging behaviors that with time might develop into students' engagement in SMP3. Table 10 shows the indicators shown by Mr. John's student on the day of this observation.

Table 10 - Indicators of Students Creating Viable Arguments at Final Observation

SMP3 Indicators	Mr. John's Class
Understand and use stated assumptions, definitions, and previously established results in constructing arguments	Yes
Make conjectures and build a logical progression of statements to explore the truth of their conjectures	Partially Emerging
Analyze situations by breaking them into cases, and can recognize and use counterexamples	Yes

As demonstrated in the conversation excerpt, students were using what they already knew of decimal multiplication and broke the problem down into pieces to help one of their peers understand how to compute the answer of the problem. The student who was explaining the process of multiplication built a logical progression. However, there were no conjectures made at this time. Table 11 shows the indicators related to critiquing others' reasoning that Mr. John's students demonstrated during this observation.

Table 11 - Indicators of Students Critiquing the Arguments of Others at Final Observation

SMP3 Indicators	Mr. John's Class
Justify their conclusions, communicate them to others, and respond to the arguments of others	Emerging
Reason inductively about data, making plausible arguments that take into account the context from which the data arose	No
Compare the effectiveness of two plausible arguments, distinguishing correct logic or reasoning from that which is flawed, and if there is a flaw in the argument explain what it is	No
Listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments	Emerging

As demonstrated in the conversation that was previously presented, the students attempted to justify their conclusions, not with reasoning, but by following the steps they knew. However, they were successful in communicating their ideas to each other. Moreover, all the participants listened to the argument put forth but did not debate whether it made sense or ask any questions. During this conversation there was no evidence of the students reasoning by analyzing the context of the problem and they did not compare any solution methods.

Conclusion

This chapter has described the data that were collected during the study. Moreover, the initial analysis of the data has been laid out and the indicators used to determine whether the students were engaged in SMP3 were explained. The teachers' self perceptions have been explained and the reality that exists in their classroom has been described. The next chapter further analyzes the data by relating it to the research questions associated with this study. The implications will be described and the suggestions for further research will be explore.

CHAPTER 5: ANALYSIS, DISCUSSION, & IMPLICATIONS

Introduction

This study investigated the ways teachers' influence can affect their students' performance as they engage in SMP3, "create viable arguments and critique the reasoning of others." As it has been discussed in Chapter 2, the two main components to this standard are communication and reasoning. For communication to occur in the way that it is prescribed in the standard, teachers must provide the students with the environment and instances that would foster communication. Moreover, teachers must provide students with the context with which they will be able to construct the sound mathematical reasoning that they will share with each other. The ability to create mathematical logic to back their answer up will also allow students to be able to evaluate the reasonableness of their peers' logic (NCTM, 2000). The following questions guided this research:

1. What are teachers' self-perceptions regarding how they encourage students to share mathematical reasoning?
2. What are teachers actualized roles regarding how they encourage students to share mathematical reasoning?
3. How do teachers' perceived and actualized roles related to how they encourage students to share mathematical reasoning differ?
4. How do teachers influence students' discourse in ways that enable them to express their mathematical reasoning in the classroom?
5. How do teachers influence students' abilities to critically assess the mathematical reasoning of others in the classroom?

This chapter contains a cross case analysis that addresses each of the research questions in order to focus on the findings. Following the analysis, the implications of the study will be discussed, including how the findings inform the kind of support teachers might need as they implement SMP3 in their classroom and the limitations associated with this research. Additionally, recommendations will include suggestions in order to increase the body of knowledge associated with the topic of this research.

Cross-Case Analysis

This study consisted of 4 distinct stages for data collection. Stage 1 of data collection involved an initial observation followed by an interview. This stage had the goals of informing the researcher about perceived and actualized teacher roles regarding how they encourage their students to share mathematical reasoning and to establish a baseline for the PD that was to follow in stage two. Stage two of data collection included the delivery of the PD, which consisted of seven 1-hour sessions and two classroom observations, one of which was performed after session 3 and the other after session 5 of PD, as the teachers were attempting to implement the methods they were learning. Video Eliciting Research (VER) was used during these interviews, which happened a day after the observations, as a reflection point for the teachers. Stage 3 of data collection consisted of a weeklong observation post PD and interviews. Additionally, teacher reflections were collected during this phase. Finally, Stage 4 consisted of one observation 4 weeks after the PD was administered. In all observation instances, the teachers were observed for one class period, approximately 55 minutes.

Stage 1 of data collection included a classroom observation and interview for each of the participants. These two artifacts informed the researcher about the classroom environment, the communication and interactions that occurred within the classroom. The initial observations were performed in order to gauge the actualized roles of the teachers. The initial interviews were performed immediately after the initial observation in order not to lead or influence the teachers' performance during the observations. The interviews informed the researcher regarding the self-perceptions that the teachers had regarding the way they supported their students' mathematical discourse. Both, observations and interviews, helped to form a picture of the teachers' actualized and perceived roles as related to their students explaining and justifying their answers. This information was paramount in the design of the PD used during stage Two.

Stage two of data collection included the delivery of the PD that was provided to the teachers and two classroom observations and two interviews. The PD was taught in seven 1-hour sessions over the course of four weeks. The PD used was developed by the MARS Shell Institute Center for Mathematics Education, the University of Nottingham, and the University of California at Berkeley as described in Chapter 3. In addition to the PD materials, instructional videos were used as supplemental instruction. The PD presented the teachers with the methods and pedagogy they needed in order to start establishing a classroom environment that was conducive to discourse and for them to support their students as they began to explain and justify their answers. The PD was rich in methods and pedagogy for teaching that could be applied to any mathematics topics. The videos presented allowed the teachers to observe a master teacher as she employed the same techniques taught in the PD within the mathematics classroom as she enabled the students to construct and evaluate mathematical reasoning.

Stage 3 of data collection entailed a week-long observation of the teachers immediately after the PD was finished. These observations enabled the researcher to note the ways in which teachers encouraged their students as they were engaged in SMP3 and the ways that teachers could be supported during this time. At this stage, the teachers were asked to write journals that had prompts that facilitated them sharing their experiences, frustrations, successes, and thoughts regarding the implementation of new expectations to their students.

Stage 4 comprised of one classroom observation 4 weeks after the PD was finished. This observation further informed the way teachers influence their students to explain their answers with sound mathematical reasoning and to evaluate their peers' mathematical arguments. Moreover, this observation informed the ways teachers could be supported as they implemented the changes associated with meeting SMP3 in their classrooms. Likewise, it gave the researcher a snapshot of whether the changes started during PD were still present in the classroom.

As the findings of all stages of data collection of the study came together there were four themes that were identified. These themes are (a) self-perceived vs. actualized teacher roles, (b) ways teachers influence students to construct viable arguments, (c) ways teachers influence students to evaluate each other's reasoning, and (d) ways to support teachers to enable their students' engagement in SMP3.

Self–Perceived vs. Actualized Teacher Roles

Pre-Professional Development

The initial classroom observations and interviews provided the researcher with a snap shot of the perceived teacher roles and of the reality in their classrooms. The teachers expressed different self-perceived ideas regarding how they encouraged their students to share their mathematical thinking. Ms. Jane expressed that eliciting students' mathematical reasoning was one of the areas in which she struggled because she experienced difficulties in maintaining ongoing expectations of students' explanations. Still, she indicated that most of the explanations she was able to elicit from them were in written form and centered on the steps the students follow to solve problems rather than the mathematical reasoning behind them. When asked how she ensured rigor in her students' explanations, Ms. Jane expressed a lack of knowledge on how to scaffold mathematics instruction for the students. On the other hand, Mr. John stated that he asks his students to explain their answers because for him it is important that they learn multiple methods for problem solving from each other. He stated he likes to have students go to the board and solve the problems in front of the class and explain their methods. He explained that he likes to incorporate collaboration into his instruction. When asked about ensuring rigor, Mr. John stated that he likes asking high-level questions that would encourage higher level thinking from his students. Both teachers agreed on the fact that group work was essential to enable their students to share their mathematical reasoning and that they believed that by grouping the students they were providing them with the opportunities to do so. Also, they stated that they valued students' input and conversations.

The initial observations yielded results that for the most part did not match the statements given by the teachers. In the case of Ms. Jane, there was no observable evidence of her eliciting the reasoning, neither oral nor written, from her students. Furthermore, during the observation, the conversations that the students were having were monitored and none concerned mathematics, other than comparing and sharing answers. During the lecture, Ms. Jane asked many close-ended questions from her students but did not allow them enough wait time to process the information she was asking from them. For the most part, Ms. Jane focused on the steps necessary to solve the problems and she encouraged her students to copy down the steps in their math notebook for future reference. Ms. Jane's focus was on her students getting the right answer but the reasoning for those answers was not elicited in any way. Similarly, Mr. John focused on the answer and no reasoning was asked from the students. Despite having the students solve problems in front of the class, they accomplished this task silently and just wrote the answer for each problem on the board. Mr. John was the one who explained the different problems for the students. In fact, his students did not share their methods with each other. Mr. John dominated most of the conversation that happened in the classroom and, just like Ms. Jane, asked closed-ended questions that his students could answer with either a numerical answer or with a few words. Mr. John did not elicit the reasoning of his students either. Nevertheless, he did demonstrate different solving methods as he taught. During direct instruction, Mr. John did not ask any questions and directed the students to copy the class notes as he wrote them on the overhead projector. The conversations between students in Mr. John's class were similar to those in Ms. Jane's class.

The self perceived and actualized roles that teachers had when motivating students to share their mathematical thinking and reasoning were, at the time of the initial observations and interviews, different. The teachers perceived themselves asking for the reasoning behind the answers from the students but this did not happen in either of the classrooms. The teachers believed that providing their students with group work would enable them to share their methods and mathematical reasoning. However, the teachers did not listen to the conversations that were happening there. Thus, the teachers did not support their students as they thought they did.

During Professional Development

As part of the study, the teachers were observed and interviewed after sessions 3 and 5 of the PD were delivered. During these interviews, VER was used to allow the teachers to observe themselves as they taught their students. The use of these videos provided the teachers with the opportunity to critique their own practice and to give them a broader lens that might allow them to compare their actualized roles with their self-perceptions. Moreover, this experience provided the teachers with opportunities to reflect on their own as they compared the reality versus their own perceptions.

During this observation, Ms. Jane's students were learning about different measures of center in a set of data. At that time, Ms. Jane was starting to incorporate some of the methods she had learned during PD. However, she was not able to support her students all the way through the lesson. When presented with the video, Ms. Jane was confounded by the behaviors of her students and by the fact that she did not recall how their behavior was in reality. She had thought that she was enabling their understanding by leading a class discussion when in fact she was just repeating the same statement several times. At this point she realized that it did not matter how many times she reiterated the same point if the students did not understand it. She realized that opportunities were missed to create connections between the concept being taught and real world situations and representations that would have made it possible for students to understand the concept fully. In addition, Ms. Jane realized that the questions that she was asking during that particular time were not open ended like she had previously done in the earlier part of the lesson. In particular, she realized that she asked the same close-ended question several times.

Mr. John was also observed during PD as previously described. During this observation he was teaching his students about 3D shapes. The clip he was shown was of him leading a class discussion on the differences between cones and pyramids. Mr. John was attempting to use the formative assessment methods that were discussed during PD but when he saw himself, he felt like he asked too many questions without giving the students enough wait time. In addition, he was confounded at the fact that he was giving his students the answers to his own questions. He felt that this might cause confusion for his students and he thought that this was an area he could improve. Furthermore, he noticed some missed opportunities, especially when he did not mention different examples or representations.

In both cases, the teachers had perceived their practice to be different from actuality. Nevertheless, the experience of viewing themselves as they taught was valuable because both realized their shortcomings and this motivated them to be aware of their own behavior in the classroom as they supported their students' engagement in SMP3. Additionally, the discrepancies between their self-perception and their actual roles in the classroom seemed to be lessening as the teachers became more aware of their own functions in their classrooms.

After Professional Development

Week-long observation

Right after the PD was completed, the teachers were observed for a period of one week. At that time interviews were also performed. Teachers concentrated their efforts into implementing the methods they learned during PD in order to support their students as they engaged in SMP3. Additionally, they reflected on their experiences as they enabled their students' engagement in building reasoning for their answers and facilitated their evaluation of each other's mathematical logic.

Ms. Jane decided to have a weeklong project for her students. She was going to have them learn the mathematics behind the manufacturing of athletic shoes. Ms. Jane felt like her students really enjoyed the activity and that they were engaged for the most part. However, she rated her own ability to motivate her students to create viable arguments and to enable them to critique the logic of others as "poor" in her reflections. This rating is reflective of her actualized role in orchestrating this kind of discussions during class because at the time of the observation her students were not performing any mathematics that would enable them to exhibit any of the seven indicators of SMP3.

Mr. John decided to center the focus of his week-long observation in the concept of integer addition and subtraction. To do this, he placed his students in collaborative groups and for the first time decided not to tell the students how to solve the problems but to let them struggle with the mathematics they were learning. For the first part of the week, he rated his efforts for motivating students to be engaged in SMP3 as “poor”. However, he realized that he needed to listen to his students as they worked. When he did, he stated that he was able to better enable his students to create explanations for their answers. His awareness of their conversations facilitated him to direct them through strategic questioning. As the nature of the students’ mathematical discourse changed, he perceived his ability to have improved. Therefore, as the week finished, he perceived his ability to motivate students to construct sound mathematical logic as “good”.

Both teachers were able to keep track of their roles in their classrooms because they were aware of how to support their students. Both teachers created activities that were appealing to their students and set their classrooms in groups. They gave students tools that they could use to assist them to explain their reasoning. Ms. Jane attempted to implement what she learned during the PD but the activity that she planned had no observable mathematical concept. In Mr. John’s case, he introduced double-sided chips to help his students make sense of integer subtraction but he did not show the students how to solve the problems and he let them struggle. Mr. John stated that he noticed that his students were challenged by the fact that they were not given a solution method but instead had to come up with one of their own. He created an atmosphere in which the expectation of explanations and justifications were in place for the students. Additionally, both teachers had students create the rules that they would use when having these conversations. The different ways in which the teachers attempted to change their roles and the classroom expectations were consistent with practices, such as enabling student discourse and mathematical reasoning which were presented during PD.

Ways Teachers Influence Students To Construct Viable Arguments

As discussed in Chapter 4, there are seven indicators that can be used to gauge students' engagement in SMP3. The first part of the standard has students constructing sound mathematical reasoning. The indicators related to this part of the standard are:

1. Understand and use stated assumptions, definitions, and previously established results in constructing arguments
2. Make conjectures and build a logical progression of statements to explore the truth of their conjectures
3. Analyze situations by breaking them into cases, and can recognize and use counterexamples (NGA & CCSSO, 2010, p. 6)

The researcher looked for evidence of these three behaviors to gauge whether the students were engaged in the construction of mathematical reasoning to back up their answers. During the initial observation, none of these behaviors were observed in either of the classrooms.

Week-Long Observation Immediately After PD

Ms. Jane was not able to demonstrate any of these three indicators, as her students were not engaged in any significant mathematical content. This will be further discussed in this chapter. Mr. John demonstrated the first three indicators in different degrees, from poor to emerging to good. Mr. John was able to motivate his students to use their previous knowledge when they started to build their reasoning through formative assessment and questioning techniques. Furthermore, Mr. John gave them tools to facilitate this. Mr. John also used inquiry to facilitate his students to test their conjectures as they attempted to solve problems with their own methods. Mr. John was not able to enable his students to make conjectures. However, the single most important task that Mr. John did was to listen to his students. Once he did this, he was able to better direct their conversations.

Observation Four Weeks After PD

A final observation was performed four weeks after the PD had concluded. This time the teachers were not interviewed. The observations were announced to the teachers and they were not asked to do anything in particular other than to follow their usual routine. Ms. Jane was teaching the concept of absolute value and Mr. John was doing a review of multiple concepts in preparation for a test.

Ms. Jane used a number line during direct instruction. She was attempting to use it as a tool for the students to make sense of the concept. In addition, Ms. Jane tried to use some of the questioning techniques that she had learned during the PD sessions but this was only marginally successful as the students were not engaged and were distracted. She asked questions that were closed-ended and many times she led the students' responses. The students were not observed performing any of the three behaviors that indicated they were creating mathematical reasoning to back up their answers. The students were not in groups as they worked and Ms. Jane controlled the conversation. Ms. Jane had reverted to what she was doing before the PD and this observation was almost identical to the initial observation that was performed in her classroom at the beginning of this study.

Mr. John provided a short list of problems for the students to solve in groups. The students were instructed to solve each problem as a group and the expectation that they shared the methods they used was set. The conversations that were heard within groups demonstrated that the students were in fact beginning to construct mathematical reasoning using their background knowledge and other tools that were available to them. His students were also starting to make conjectures and using counterexamples as prescribed in the three indicators of this section of the standard. Table 12 depicts each indicator and how well Mr. John's students were engaged in each individual behavior.

Table 12 - Mr. John's Students Ability to Create Viable Arguments 4 Weeks After PD

SMP3 Indicators/Creating Viable Arguments	Mr. John's Class
1. Understand and use stated assumptions, definitions, and previously established results in constructing arguments	Yes
2. Make conjectures and build a logical progression of statements to explore the truth of their conjectures	Yes
3. Analyze situations by breaking them into cases, and can recognize and use counterexamples	Yes

The ratings that appear in Table 12 were evidenced by the observations performed in Mr. John's class. The students were able to answer questions that Mr. John asked them by using prior knowledge. Additionally, the students were able to use what they knew about decimals and applied it to extending their understanding of decimal multiplication. Moreover, they were able to break the procedures associated with decimal operations into parts, and were able to explain what happened to the position of the decimal point in the product to each other.

Ways Teachers Influence Students To Evaluate Each Other's Reasoning

As discussed in Chapter 4, there are four behaviors that students should be demonstrating as they are engaged in assessing the reasoning of others which is the second part of SMP3. These four behaviors are:

1. Justify their conclusions, communicate them to others, and respond to the arguments of others
2. Reason inductively about data, making plausible arguments that take into account the context from which the data arose
3. Compare the effectiveness of two plausible arguments, distinguishing correct logic or reasoning from that which is flawed, and if there is a flaw in the argument explain what it is
4. Listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments. (NGA & CCSSO, 2010, p. 6-7)

The researcher looked for evidence of these behaviors to gauge whether the students were engaged in assessing each other's reasoning. During the initial observations, none of these behaviors were observed in either classroom.

Week-long Observation Immediately after PD

As described in the previous section, the teachers were observed for a period of one week and once four weeks after PD. Both teachers constructed activities that would allow them to motivate their students to be engaged in SMP3. As previously discussed, Ms. Jane developed a project for her students while Mr. John used the concept of integer subtraction to create group activities using manipulatives.

Ms. Jane's students were not engaged in any of the behaviors previously described, as they were not doing any significant mathematics with which to engage in those behaviors. Mr. John, on the other hand, was partially successful in having his students assess each other's reasoning. His students were able to communicate their reasoning and were able to respond to the arguments of others. Furthermore, the conversations heard during that week indicated that they were starting to make sense of each other's logic. Mr. John was able to motivate them because he started to listen to their conversations and was able to redirect them with questioning techniques and prompts.

Observation Four Weeks After PD

For the final observation, as mentioned in the previous section, Ms. Jane was teaching the concept of absolute value and Mr. John was giving his students a review for a test. In Ms. Jane's class, the students were filling a KWL chart with the information she gave them. The students were presented with a number line to attempt to make sense of the topic but Ms. Jane was the only one using it for explanations. The students were presented with closed ended questions that sometimes led to the answer. Ms. Jane did not engage her students in any of the four behaviors related with them assessing each other's mathematical logic.

On the other hand, Mr. John had planned for his students to practice for an upcoming exam. The students were divided in groups and each group was given a short list of problems to solve. While they worked in their groups, the students were communicating their reasoning to each other. However, they were only partially answering the arguments of others. The students were listening and evaluating the reasoning given within their groups. Nevertheless, they were unable to inductively make sense of the problem by using context because the problems used throughout this lesson lacked any context. Additionally, the students did not compare and contrast different arguments to evaluate if they were correct or not. Table 13 depicts the SMP3 indicators for students assessing each other's logic and how they were observed in Mr. John's class during the final observation.

Table 13 - Indicators of Students Critiquing the Arguments of Others at Final Observation

SMP3 Indicators	Mr. John's Class
1. Justify their conclusions, communicate them to others, and respond to the arguments of others	Emerging
2. Reason inductively about data, making plausible arguments that take into account the context from which the data arose	No
3. Compare the effectiveness of two plausible arguments, distinguishing correct logic or reasoning from that which is flawed, and if there is a flaw in the argument explain what it is	No
4. Listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments	Emerging

The ratings in Table 13 were evidenced by the observation performed in Mr. John's classroom. When the students were trying to make sense of the product of two decimal numbers, they were communicating the justifications of their answers but they did not assess this logic. Thus, the first indicator was emerging, but not totally present. While one student was explaining how she got the product of the two decimal factors and justifying the position of the decimal point, the other students in her group were listening. Some of the students asked some questions about her argument. However, their questions were not helpful in making sense of what she was saying. Thus, the fourth indicator was rated emerging because it was partially fulfilled. The other two indicators were not observed and the rating of 'no' was given to indicate their absence.

Ways to Support Teachers To Enable Their Students' Engagement In SMP3.

Teachers have many ways of motivating students to construct viable arguments as described in SMP3. The teachers in this study were given several tools and methods to enable their students to do this like questioning techniques, problem selection, and using inquiry during PD. As previously discussed, there are seven indicators that describe how students engaged in SMP3. Table 14 depicts each of the indicators along with the ways the teachers in the study were instructed to modify their roles in order to enable their students' engagement in SMP3.

Table 14 - Teachers' Role Modifications per Indicator

Indicator	Teacher Role
Understand and use stated assumptions, definitions, and previously established results in constructing arguments	Use similar problems Use problems that enable students to use prior knowledge Listen to students' conversations
Make conjectures and build a logical progression of statements to explore the truth of their conjectures	Provide tools (manipulatives or representations) to enable students to build a logical progression of their reasoning Provide instances for explorations and conjecture testing Ask open-ended questions Listen to students' conversations
Analyze situations by breaking them into cases, and can recognize and use counterexamples	Select problems with significant mathematics that can be broken into parts Use similar problems Use problems that enable students to use prior knowledge Listen to students' conversations
Justify their conclusions, communicate them to others and respond to the arguments of others	Provide students with instances in which they are able to communicate Make students explaining and justifying their answers an expectation Listen to students' conversations
Reason inductively about data, making plausible arguments that take into account the context from which the data arose	Select problems that contain context that can be applied to real life Provide instances for explorations and conjecture testing Ask open-ended questions Provide students with instances in which they are able to communicate Listen to students' conversations
Compare the effectiveness of two plausible arguments, distinguishing correct logic or reasoning from that which is flawed, and if there is a flaw in the argument explain it	Provide students with different arguments so they can compare/contrast them Make students explaining and justifying their answers an expectation Provide tools (manipulatives or representations) to enable students to build a logical progression of their reasoning Provide students with instances in which they are able to communicate Listen to students' conversations
Listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments	Provide students with instances in which they are able to communicate Provide an atmosphere in which students' questions are welcomed Guide student questioning Listen to students' conversations

These role modifications can work in any classroom. However, based on the results of the study, there seems to be a difference between the levels of support that a first-year mathematics teacher needs compared to what a seasoned mathematics teacher might need. This notion is supported by the literature (Cady et al., 2006; D'Amato & Quinn, 2005; Ingwalson & Thompson, 2007; Quinn & D'Amato, 2004; Merseth, 1992; Zepeda & Ponticell, 1997). Ms. Jane, a first-year mathematics teacher, did not implement the changes necessary to motivate her students' engagement in SMP3. This suggests that a first-year teacher might need more support than an experienced teacher. Novice teachers have a hard time adjusting to the demands of teaching. In fact, "a first-year teacher's job is often challenging, frequently discouraging, and sometimes devastating" (Merseth, 1992, p. 679). Ms. Jane, like many typical first year teachers, struggled with classroom management, had difficulties engaging her students, did not know how to scaffold the learning of her students, and was unsure of the curriculum she taught. To change their practice, first-year teachers must be offered professional development opportunities, should have a school environment that complements their own values, and must be given time to hone in their own teaching experience (Cady et al., 2006; Ingwalson & Thompson, 2007; Zepeda & Ponticell, 1997). It is common that new teachers have a teacher mentor for support during their first year but it is important that this support goes beyond that time. It is paramount that novice teachers have the opportunity to observe others and themselves as they teach and that they belong to a collaborative group of their peers for resources and support (D'Amato & Quinn, 2005; Ingwalson & Thompson, 2007; Quinn & D'Amato, 2004; Zepeda & Ponticell, 1997).

The degree of support that experienced teachers might need is less than a first-year teacher, as demonstrated in this study. Nevertheless, both types of teachers will benefit from PD opportunities that enable them to expand their knowledge of SMP3, of the behaviors associated with the standard, and topic specific activities that they can use. Most importantly, teachers must perceive the tasks that students perform as important and worthwhile and provide opportunities that have their students explore solution methods, test conjectures, use significant mathematics, and provide a classroom environment that promotes discourse (NCTM, 2000, 2010).

<u>Summary of the Cross–Case Analysis</u>

This study sought to answer the following questions,

1. What are teachers' self-perceptions regarding how they encourage students to share mathematical reasoning?
2. What are teachers actualized roles regarding how they encourage students to share mathematical reasoning?
3. How do teachers' perceived and actualized roles related to how they encourage students to share mathematical reasoning differ?
4. How do teachers influence students' discourse in ways that enable them to express their mathematical reasoning in the classroom?
5. How do teachers influence students' abilities to critically assess the mathematical reasoning of others in the classroom?

The results of this study demonstrated that teachers, both experienced and new, have self-perceptions regarding how they enable their students to share their mathematical reasoning. The difference between the perceived and actualized roles that teachers have in orchestrating mathematical discourse through classroom norms are considerable and these perceived roles are rooted in teacher-centered practices that are not always consistent with the higher level of understanding sought in mathematics education (Carpenter & Leher, 1999; Carpenter et al., 2000; Cobb et al., 1992; NCTM, 2000; NGA & CCSSO, 2010; NRC, 2001; Romberg & Kaput, 1999). In order to influence students to share their mathematical reasoning, teachers must provide students not only with the tools necessary for them to express their reasoning, but also with the opportunities that will enable them to construct this logic. In order to influence their students' ability to critically assess the mathematical reasoning of others, teachers must provide students with a classroom environment that is conducive for discourse and mathematical reasoning (Akkus & Hand, 2011; Fennema et al, 1999; Goos, 2004; Henningsen & Stein, 1997; Hiebert et al., 1997; Hufferd-Ackles et al., 2004; Nathan & Knuth, 2003; Reid, 2002). For students to be fully engaged in SMP3, teachers must be cognizant of the behaviors associated with this standard. Teachers must be willing to relinquish the power of the classroom to the students, as they take control of their learning. Most importantly, teachers must listen to the conversations that happen in the classroom as students talk about their solutions and the methods they have used. Teachers must create a more student-centered classroom environment which has the built-in expectation that students must back up their answers with solid mathematical reasoning and that the students must assess their peers' logic (Fennema et al., 1999; Hiebert et al., 1998). One of the most important behaviors that teachers should have as they implement changes that will support their students' engagement in SMP3 is self-monitoring and reflection (Johnston, 1994). As previously discussed, after the PD, the experienced teacher in this study, Mr. John, could better assess his role in the classroom and adjust it to better motivate his students because he was more aware of his own behavior and he had knowledge of what he needed to do in order to support his students.

Study Limitations

The first limitation of this study was in its design, case study. Since the researcher is the one that determines constructs and builds the cases that are going to explain the phenomena observed, this study was limited by its own design (Creswell, 2007). The second limitation of this study was time. The researcher acknowledges that due to the complex nature of the mathematics classroom and the interactions that can happen there, the change of fundamental teaching practices requires an extended amount of time (Ball & Forzani, 2010; Birman, 2000).

Conclusion and Implications

This study sought to find ways teachers support their students in order for them to construct viable arguments and critique the reasoning of others. This study found that there are many ways in which teachers can support their students. However, this support is dependent on several factors. These factors are (a) teaching experience, (b) teacher content and pedagogical knowledge, (c) questioning, (d) awareness of communication, (e) teacher expectations, and (f) classroom management.

The results of this study suggest that there is a difference in how teachers support their students based on their own teaching experience. Ms. Jane was a first-year mathematics teacher and during the time of the study she struggled with setting a structured classroom that promoted understanding and discourse. In her classroom, there was a lack of routines and students' expectations. The result was that in her classroom students' misbehaviors were common and this affected their ability to engage in instructional activities. Mr. John, an experienced teacher, had a classroom that was organized and had set expectations for the students. His classroom had routines in place, which promoted students' engagement. Furthermore, the students were diligently working and were no discipline issues. The classroom environment is paramount in enabling students to construct viable arguments and critique the reasoning of others.

The teachers' pedagogical and mathematical content knowledge were two factors that were not taken into consideration at the time of the study. However, both proved to be major factors on how teachers were able to support their students' engagement in SMP3. The researcher's years of field experience as a mathematics teacher, classroom observer, and assessor of teachers provided a lens that was used to evaluate, in a limited manner, the content and pedagogical knowledge that the teachers who participated in the study possessed. Mathematical content and pedagogical knowledge are essential to select problems, materials, questions, and support students' mathematical thinking (NCTM, 2000; Hiebert et al., 1997). In fact, when teachers lack these kinds of knowledge, they are unsuccessful when teaching students (Ball et al., 2008). The mathematics content and pedagogical knowledge that the teachers who participated in this study demonstrated were vastly different. Ms. Jane was unsure of the content she taught, was unable to present context that was relevant to her students, and habitually taught from the teacher's edition textbook. Furthermore, Ms. Jane did not recognize the fact that the project she had planned for her students had no mathematical content. Ms. Jane admitted that she had trouble scaffolding her students. These facts added to her inability to maintain the interest and engagement of her students are signs of a lack of mathematical content and pedagogical knowledge. Mr. John demonstrated that he had strong mathematical and pedagogical content knowledge because of the problems he selected for his students to work with, for the learning trajectories he was following, for his ability to foresee common roadblocks his students might encounter, and for his ability to present mathematical concepts in different contexts. Moreover, Mr. John was able to scaffold and provide students with meaningful mathematics content and he was also able to provide students with support as they engaged in SMP3.

The teachers in this study were observed for a period of one week after the delivery of the PD. During that time, each of the teachers selected the activities that the students were going do. Ms. Jane, who was a first-year mathematics teacher, selected to create an investigation type project for her students. Her goal was for the students to understand the mathematics behind the manufacturing of sneakers. However, the tasks that she assigned to the students, like reading articles and coloring a shoe design, did not contain any mathematics concepts. The planned period of time for this project was five days but it ended up being 7 days for two reasons, one because the teacher asked to be observed until the end of the project, the other because the researcher wanted to observe the students engaged in significant mathematics that would enable them to share and assess each others' reasoning. At the end of this activity, students were to present their sneaker design to the rest of the class and give the price the shoes would have. Unfortunately, the reasoning behind the prices were "I want everyone to be able to buy it" or "It's an expensive shoe" rather than a mathematical reason. During the presentations, the classroom environment played a major role. Since the students were misbehaving and saying rude comments to those who were presenting, the environment was not one that promoted mathematical reasoning and understanding. Ms. Jane showed with the selection of this project her lack of content knowledge and her misunderstanding of the topics her students need to learn. This is further supported by the fact that she seemingly did not realized that her project contained no mathematics. On the other hand, Mr. John provided his students with double-sided chips and enabled them to use the chips in order to build the reasoning behind their answers. The different problems chosen by Mr. John for his students had the goal of making it increasingly challenging for the students. He also decided to teach this concept as an inquiry lesson, so he let his students struggle with the concepts as they attempted to answer and explain the reasoning behind their answers. According to his final interview, Mr. John had a hard time identifying his students' mathematical struggles as they worked but he was extremely pleased with how his students were able to explain and justify integer subtraction by using the chips and their own words. The mathematical tasks assigned to students as they engage in SMP3 must contain significant mathematics in order to allow them to build reasoning and explain their answers.

One of the most powerful tools that allows teachers to steer conversations and increase student learning is questioning (Black & William, 1998). Questioning is indispensable for meaningful discourse. Classroom discourse entails students creating their own ideas and exchanging them. As they exchange mathematical ideas, the differences within those ideas become apparent. These differences are used as discussion points that give way to reasoning (Wood, 1999). Teachers can use questioning to direct and enable students to communicate their ideas to each other and to examine those differences. In this study, the questioning techniques provided to the teachers were specific to mathematics. However, the teachers' ability to ask the right questions at the right time was dependent on their content knowledge, timing, and listening to their students. Additionally, a clear knowledge of the students' learning trajectory is necessary for taking those conversations to where they need to go. Ms. Jane asked closed-ended questions for the most part. She attempted to ask her students more probing questions but she was unable to maintain this for any consistent length of time. During her week-long observation, the questions that she was asking her students lacked in mathematical content. In her final interview, Ms. Jane expressed frustration because she wanted PD that was specific to her. This comment demonstrated an inability to synthesize the information given and with applying it to her own teaching practices. This might be due to her lack of mathematics teaching experience and her lack of content and pedagogical knowledge. Mr. John was able to ask questions that probed his students' knowledge and allowed them to build mathematical reasoning. He encountered some roadblocks along the way. For example, when he started listening to his students' comments during peer work, he realized they were not where he wanted them to be. Therefore, he used questioning to steer them towards the goal of building complete explanations and justifications of their answers. Mr. John saw value in asking questions. According to his interview after PD, he saw a dramatic change in his students and he attributed this to the conversations that were caused by the questions he had asked. In order for teachers to support the engagement of their students in SMP3, they must be familiar with questioning schemes that allow for open-ended answers and that enable students to construct and assess mathematical reasoning.

In order to be able to evaluate students' ability to construct and assess mathematical reasoning, teachers must listen to what the students are saying. In the initial interviews, both teachers expressed that if they grouped the students they would talk about the mathematics they were doing. However, they were not listening to the conversations the students were engaged in. Most of the time, these conversations were about subjects other than mathematics with the exception of exchanging answers. Ms. Jane was partially successful in listening to her students' conversations because a few times during and after PD she was able to determine they were off task. One pivotal moment for her was when she had the opportunity to view herself teaching. She was upset at the time but this gave her the opportunity to actually listen to the students without the burden of trying to keep the order in the classroom. New teachers have a very difficult time during their first year teaching. The task of teaching can become so overwhelming that sometimes the focus is on controlling students' behavior rather than what is really occurring in classrooms. Mr. John, the experienced teacher, had an easier time listening to his students' conversations. Since he did not have to spend any energy controlling his class, he was able to listen to the students' conversations and was able to direct classroom talk to where it needed to go. This experience allowed Mr. John to shift his views and he was able to see his role as a teacher morph into one of facilitator. Teachers must listen to the conversations students are having as they explain and justify their answers in order to guide them and to enable them to build reasoning based in mathematical concepts (Blanton & Kaput, 2005; Forman & Ansell, 2001; Hadjioannou, 2007; Herbel-Einsenman & Otten, 2011).

Classroom management was not accounted for when planning the PD. The kind of support that teachers are able to give their students as they engage in SMP3 depends on their classroom management. The effect that poor classroom management had over students' behaviors cannot be ignored. Ms. Jane struggled with classroom management. Her classroom was chaotic at times because her students had no set rules to follow and they were often off task. Ms. Jane stopped class often to deal with misbehaving students. On the other hand, Mr. John, the experienced teacher, had created a positive environment in his classroom. He had routines in place and clear expectations of behaviors. His students were well behaved and were engaged in all activities he had for them and they knew the expectations he had of them. Creating a positive and nurturing classroom is vital for students' engagement in SMP3. Students are able to construct arguments and critique others when their classroom environment promotes understanding and where there are set routines and expectations.

Potential Study Implications

The results of this study have potential implications in the ways teachers support students as they are engaged in SMP3. The position of the researcher is that the shift noticed in the teachers' roles did affect the ways their students could explain and justify their answers. The first implication of this study is that the support the students receive from their teachers depends on their teaching experience, student expectations, and content knowledge. The second implication is that teachers will need varying degrees of continuous support as they implement the changes necessary to have a classroom that allows students to be engaged in SMP3. The third implication is that teachers must create a classroom environment in which students feel comfortable and are able to exchange ideas, test hypothesis, and conduct experiments.

The support that students get from their teachers is directly related to what the teacher knows and his or her experience. Teachers must be able to build and assess mathematical reasoning in order to have their students do the same thing. The results of this study indicate that more mathematically proficient and experienced teachers will be better prepared to support their students as they engage in SMP3. The types of support teachers give to their students might vary as the given mathematical tasks and learning trajectories change. However, it seems more likely that a knowledgeable and seasoned teacher will provide support to his or her students' need. Conversely, teachers will have a harder time providing their students with appropriate support to engage them in SMP3 if they are inexperienced and lack in mathematical content and pedagogical knowledge. Moreover, the ability to have efficient routines and classroom management in place will make the teacher role in orchestrating the kind of communication that has to happen as students are engaged in SMP3 much easier. Ms. Jane, an inexperienced teacher, had many classroom management issues that impeded her ability to focus on the mathematics she was teaching her students as well as her supporting them as they engaged in SMP3. On the other hand, Mr. John, who had excellent classroom management and set routines in place for his students, was able to concentrate his efforts in supporting his students as they attempted to build mathematical reasoning.

In order to enable teachers to support their students as they engage in SMP3, they must receive ongoing support that will allow them to exchange ideas, ask questions, and share experiences. The support that experienced teachers require will be much different to what new teachers might need. As the results of this study show, the novice teacher felt like the PD and support provided were inadequate for her because she could not translate what she had learned into her practice. However, the experienced teacher had no problem doing just this. It is, therefore, important to provide more in-depth support for novice teachers. This support should include opportunities for them to observe experienced teachers teach, the ability to video record themselves as they teach so they can see what they are doing without the burden of trying to control the class, and participation in collaborative planning sessions that will help them tailor what they learn during PD to their own classrooms.

The classroom environment is of paramount importance in motivating students' engagement in SMP3. The results of the study show that a classroom in which there are set rules and routines is more productive than one in which there are management issues. A classroom where the environment is in turmoil, where there are no stated teacher's expectations, where students do not respect each other's opinions, and where students are constantly distracted, is not an ideal place in which to implement SMP3. A classroom that promotes understanding is one that has students follow specified rules and routines, it is a place where students feel valued and safe, where there are a set of behaviors that must be followed, and where students are free to communicate their reasoning without fear of being ridiculed (Fennema, Sowder, & Carpenter, 1999). The impact that the right environment has on the engagement of the students in SMP3 is great and teachers must insure it is the kind of environment that will foster the behaviors teachers want their students to demonstrate.

The need for mathematics content is paramount to students' engagement in SMP3, as they need mathematics concepts in order to build mathematical reasoning. Ms. Jane's lack of content knowledge for teaching mathematics did not let her realize that the sneaker project that she planned for her students did not, in fact, contain any mathematics. This finding is important and it might be worthy of further investigation.

The disparity noted in the two cases investigated in this study was due to the differences in the participants themselves. The teachers involved in this study had very different teaching styles, teaching experience, mathematical content and pedagogical knowledge, and expectations. These characteristics gave this study rich results and allowed the researcher to conclude that novice teachers need more support than experienced teachers. This diversity of teachers' characteristic should be taken into account in subsequent research as this can inform the support that different types of teachers might need.

Conclusion

This study looked at the kinds of support that teachers give their students as they make the transition to the CCSSM and the SMPs, more specifically to SMP3. It also looked into the kinds of support teachers might need as they change their practice to enable their students to construct viable arguments and assess each other's reasoning. Those needs are different, depending on the teacher's characteristics and classroom environment. The trend in mathematics education literature is to support a more student-centered classroom where the teacher is a facilitator of the knowledge. In fact, it supports the notion of moving away from teacher-centered methods and advocating the development of mathematical proficiency through reasoning. In order to make fundamental changes to the basic structure of a teacher-centered classroom, especially where communication and reasoning need to be reinforced, the teacher will need support that is specific to his or her needs. This support needs to be ongoing over time. This study found that these specific teachers could modify their classroom environments and their practice in order to support their students' engagement in SMP3. Furthermore, it found that the way teachers support their students depend on their experience, content and pedagogical knowledge, and classroom management style.

APPENDIX A:
INTERVIEW PROTOCOL

Interview Protocol

Thank you for agreeing to talk to me. I must get your permission to tape this conversation. The recording will be erased once it is transcribed. Your name will not appear in the transcriptions. The digital copy of the transcription of this interview will be kept in a secure place. Do I have your permission to record this conversation?

[*Turn digital recorder on*]

Interview Questions

1. How would you describe the environment in your classroom?
2. Do you perceive a value to allowing students to talk in the classroom? Why or Why not?
3. How do you encourage students to share their mathematical thinking with each other?
4. How do you encourage students to participate in classroom discussions?
5. Do you know what the Common Core State Standards for Mathematics are?
6. If yes, how would you describe them? If no, no question will be asked.
7. Do you know what the Standards for Mathematical Practice are?
8. If yes, how would you describe them? If no, no questions asked.
9. Do you know what a learning community is?
10. If yes, do you have one here? How often do you participate in it?
11. Is there anything you would like to add?

Thank you so much for your time.

[*Turn digital recorder off*]

APPENDIX B: INSIDE THE CLASSROOM TEACHER INTERVIEW PROTOCOL

Inside the Classroom
Teacher Interview Protocol

I appreciate your letting me observe your class. I have some questions I'd like to ask you related to this lesson. Would you mind if I taped the interview? It will help me stay focused on our conversation and it will ensure I have an accurate record of what we discussed.

Preliminary

If applicable, ask:
What is the name/title of this course?

What class period was this?

If applicable, ask:
Can I have a copy of the instructional materials you used for this lesson? [Specify what you would like to have copies of, if necessary.]

A. Learning Goals

1. I'd like to know a bit more about the students in this class.
   Tell me about the ability levels of students in this class.
   How do they compare to students in the school as a whole?

   Are there any students with special needs in this class?
   Are there any students for whom English is not their first language?
   Are there any students with learning disabilities?

2. Is student absenteeism or mobility a problem for you in this class?

3. Please help me understand where this lesson fits in the sequence of the unit you are working on. What have the students experienced prior to today's lesson?

4. What was the specific purpose of today's lesson?

5. How do you feel about how the lesson played out?
   What do you think the students gained from today's lesson?

6. What is the next step for this class in this unit?

B. Content/Topic

7. What led you to teach the mathematics/science topics/concepts/skills in this lesson?

   (Use the following probes, as needed, so you can assess the extent of importance of each of these influences:)

   Is it included in the state/district curriculum/course of study?
   If yes, or previously implied: How important was that in your decision to teach this topic?

   Is it included in a state/district mathematics/science assessment? What are the consequences if students don't do well on the test?
   If yes, or previously implied: How important were these tests in your decision to teach this topic?

   Is it included in an assigned textbook or program designated for this class?
   If yes, or previously implied: How important was that in your decision to teach this topic?

C. Resources Used to Design the Lesson

8. What resources did you use to plan this lesson?
   (Be sure to get details on sources of materials and activities.)
   (If teacher developed materials, SKIP to part D.)

9. Were these resources/materials/activities designated for this class/course or did you choose to use them yourself?

10. What do you like about these resources/materials/activities?
    (*Compared to what the district designated for the class/course, if applicable.*)
    What do you not like?

11. a. *If the lesson was based on one resource/material:*
    Did you plan this lesson essentially as it was organized in [name of resource/material] or did you modify it in important ways?

11. b. *If the lesson was based on more than one resource/material:*
    Did you plan this lesson essentially as it was organized in any one of these resources/materials?
    If yes:
    Did you modify it in important ways?

12. *If modified:*
    Can you describe the modifications you made and your reasons for making them?

D. The Teacher

13. How do you feel about teaching this topic?
    Do you enjoy it?
    How well prepared to you feel to guide student learning of this content?
    What opportunities have you had to learn about this particular content area?
        (Probe for professional development opportunities.)
    How did you become involved in these professional development opportunities?
    Were they required or encouraged by the district?
    How helpful were they?

14. How do you feel about teaching with this pedagogy?
    How comfortable do you feel using the instructional strategies involved in teaching this lesson?
    What opportunities have you had to learn about using these strategies?
        (Probe for professional development opportunities.)
    How did you become involved in these professional development opportunities?
    Were they required or encouraged by the district?
    How helpful were they?

15. How many years have you been teaching prior to this year?
    Have you taught this lesson before?
    If yes: How different was today from how you have taught it previously?

    Is there anything about this particular group of students that led you to plan this lesson this way?

16. *If applicable ask:*
    I noticed there was another adult in the classroom. Who was that and what was his/her role?

E. Context

17. Sometimes schools and districts make it easier for teachers to teach science/mathematics well, and sometimes they get in the way.
    What about your teaching situation influenced your planning of this lesson?

PROBES:
    Did the facilities and available equipment and supplies have any influence on your choice of this lesson or how you taught it?
    Were there any problems in getting the materials you needed for this lesson?

18. Sometimes other people in the school and district can influence your planning of a lesson. Did your principal have any influence on your choice of this lesson or how you taught it?

Other teachers in the school?
Parents/community?
School board?
District administration?
Anyone else?

Thank you for your time. If I have any additional questions or need clarification, how and when is it best to contact you?

APPENDIX C:
IRB APPROVAL LETTER

University of Central Florida

University of Central Florida Institutional Review Board
Office of Research & Commercialization
12201 Research Parkway, Suite 501
Orlando, Florida 32826-3246
Telephone: 407-823-2901 or 407-882-2276
www.research.ucf.edu/compliance/irb.html

Approval of Human Research

From: **UCF Institutional Review Board #1**
FWA00000351, IRB00001138

To: **Tashana D. Howse** and Co-PI: **Mercedes Sotillo**

Date: **March 01, 2013**

Dear Researcher:

On 3/1/2013, the IRB approved the following to human participant research until 2/28/2014 inclusive:

Type of Review:	UCF Initial Review Submission Form
Project Title:	A Case Study: Constructing Viable Arguments and Critiquing the Reasoning of Others in a Mathematics Classroom
Investigator:	Tashana D Howse
IRB Number:	SBE-13-09132
Funding Agency:	
Grant Title:	
Research ID:	N/A

The scientific merit of the research was considered during the IRB review. The Continuing Review Application must be submitted 30days prior to the expiration date for studies that were previously expedited, and 60 days prior to the expiration date for research that was previously reviewed at a convened meeting. Do not make changes to the study (i.e., protocol, methodology, consent form, personnel, site, etc.) before obtaining IRB approval. A Modification Form **cannot** be used to extend the approval period of a study. All forms may be completed and submitted online at https://iris.research.ucf.edu .

If continuing review approval is not granted before the expiration date of 2/28/2014, approval of this research expires on that date. When you have completed your research, please submit a Study Closure request in iRIS so that IRB records will be accurate.

Use of the approved, stamped consent document(s) is required. The new form supersedes all previous versions, which are now invalid for further use. Only approved investigators (or other approved key study personnel) may solicit consent for research participation. Participants or their representatives must receive a copy of the consent form(s).

In the conduct of this research, you are responsible to follow the requirements of the Investigator Manual.

On behalf of Sophia Dziegielewski, Ph.D., L.C.S.W., UCF IRB Chair, this letter is signed by:

[%electronic_signature%]
IRB Coordinator

APPENDIX D:
ORANGE COUNTY PUBLIC SCHOOLS RESEARCH REQUEST FORM

Submit this form and a copy of your proposal to:
Accountability, Research, and Assessment
P.O. Box 271
Orlando, FL 32802-0271

Orange County Public Schools

RESEARCH REQUEST FORM

RECEIVED JAN 07 2013

Your research proposal should include: Project Title; Purpose and Research Problem; Instruments; Procedures and Proposed Data Analysis

Requester's Name: Mercedes Sotillo / Tashana Howse   Date: Dec. 18, 2012

Address: Home [redacted]   Phone [redacted]

Business: N/A   Phone

Project Director or Advisor: Dr. Juli Dixon   Phone [redacted]

Address [redacted]

Degree Sought: (check one)
- ☐ Associate
- ☒ Doctorate
- ☐ Bachelor's
- ☐ None
- ☐ Master's
- ☐ Specialist

Project Title: A case study: Constructing Viable Arguments and Critiquing the Reasoning of Others in a Mathematics Classroom

ESTIMATED INVOLVEMENT

PERSONNEL/CENTERS	NUMBER	AMOUNT OF TIME (DAYS, HOURS, ETC.)	SPECIFY/DESCRIBE GRADES, SCHOOLS, SPECIAL NEEDS, ETC.
Students	N/A		
Teachers	2	See attached	[redacted]
Administrators	N/A		
Schools/Centers	N/A		
Others (specify)	N/A		

Specify possible benefits to students/school system: See attached

ASSURANCE

Using the proposed procedures and instrument, I hereby agree to conduct research in accordance with the policies of the Orange County Public Schools. Deviations from the approved procedures shall be cleared through the Senior Director of Accountability, Research, and Assessment. Reports and materials shall be supplied as specified.

Requester's Signature

Approval Granted: ☒ Yes ☐ No   Date: 2-20-13

Must have pri[illegible] permission

Signature of the Senior Director for Accountability, Research, and Assessment [redacted]

NOTE TO REQUESTER: When seeking approval at the school level, a copy of this form, signed by the Senior Director, Accountability, Research, and Assessment, should be shown to the school principal.

Reference School Board Policy GCS, p. 249

FORM ID #GB0103/23-1/1FY REV 1/04

APPENDIX E:
STUDENT PACKET FOR SNEAKER PROJECT

Name:
Period:

Sneaker Design

Name of sneaker:

Price of sneaker:

When picking a price remember how much it will cost to manufacture

Why you picked this price:

Sneaker dimensions:

Width:

Height:

Length:

Materials you want to use to make your sneakers' sole:

Materials you want to use to make your sneakers' middle:

Materials you want to use to make your sneakers' middle:

Why did you choose to use these materials:

Name:
Period:

3 Reasons why I should buy your sneaker:

1.

2.

3.

Name:
Period:

Data Collection Questions

1. Which way did you use to organize your data? Why did you choose this way?

2. Which two ways did you use to display your data? Why did you choose these ways?

3. Find the mean, median, mode, and range of the data. **SHOW ALL WORK!!**

   Mean:

   Median:

   Mode:

   Range:

Name:
Period:

Displaying and collecting Data

There are many ways to collect and display data. We have taken a pool of our favorite sneakers created and our data is displayed around the room.

With your partner I would like you to choose:

One way to organize your data

Examples:

- Frequency tables
- Tally chart

2 ways to display your data

Examples:

- Line plots
- Bar graph
- Circle graphs
- Line graph

Make sure you don't forget to provide titles, labels, and keys.

Student Name: ______________________________

Date: __________________ Period: ______

Sneaker Presentation

1. Name of Student Presenting: ______________________________
2. Name of Sneaker: ______________________________
3. Did the student include all the requirements? ______________
4. What is your favorite thing about the sneaker?

____

____

____

5. What would you do to improve the sneaker?

____

____

____

Sneaker Presentation

1. Name of Student Presenting: ______________________________
2. Name of Sneaker: ______________________________
3. Did the student include all the requirements? ________________
4. What is your favorite thing about the sneaker?

____

____

____

5. What would you do to improve the sneaker?

____

____

____

APPENDIX F: JOURNAL WRITING PROMPTS FOR WEEKLONG OBSERVATIONS

Journal Writing Prompt

Lesson focus:

Describe the learning goal(s)?

Describe the CRT instructional strategies that you used in today's lesson.

How did your students respond to this lesson in comparison to lessons that you have taught before the professional development?

Reflect on the overall success of the lesson today. What went well? What did not go as well? How would you improve this lesson?

Directions: In the table below, please rate on a scale from 1 to 5, where 5 indicate "Excellent", 4 indicate "Good", 3 indicate "Average", 2 indicate "Poor" and 1 indicates "Very Poor". In addition, please provide a description for each question.

Questions	Circle one
Rate your students' ability to construct viable arguments.	Very Poor **1 2 3 4 5** Excellent
During instruction, describe how your students engaged in constructing viable arguments?	
Rate your students' ability to critique the reasoning of others?	Very Poor **1 2 3 4 5** Excellent
During instruction, describe how your students engaged in critiquing the reasoning of others?	
Rate your ability to use CRT to support student engagement?	Very Poor **1 2 3 4 5** Excellent
Describe how you feel about your ability to use CRT to support student engagement?	

REFERENCES

Achieve. (2010). Comparing the Common core state standards in mathematics the NCTM's Curriculum Focal Points. In *Achieving the Common Core.* Retrieved from http://www.achieve.org/achievingcommoncore

Akkus, R. & Hand, B. (2011). Examining teachers' struggles as they attempt to implement dialogical interaction as part of promoting mathematical reasoning within their classrooms. *International Journal of science and Mathematics Education, (9),* 975-998.

Alexander, K. & Pallas, A. (1984). Curriculum reform and school performance: An evaluation of the "New Basics". *American Journal of Education, 92*(4), 391-420.

Amit, M. & Fried, M. (2005). Authority and authority relations in mathematics education: A view from an 8th grade classroom. *Educational Studies in Mathematics, 58*(2), 145-168.

Aronson, E. (1997). Back to the future: Retrospective review of Leon Festinger's "A Theory of Cognitive Dissonance". *The American Journal of Psychology, 110*(1), 127-137.

Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal, 90*(4), 449-466.

Ball, D. L., Hill, H., & Bass, H. (1999). Knowing mathematics for teaching. *American Educator Fall 1990*, 14-22/43-46. doi: 10.1.1.152.9220

Ball, D. L. & Rowan, B. (2004). Introduction: Measuring instruction. *The Elementary School Journal, 105*(1), 3-10.

Ball, D. L. Hill, H. & Hyman, B. (2005). Knowing mathematics to teach: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, 29(1), pp. 14-17, 20-22, 43-46.

Ball, D. L. & Forzani, F. (2010). Building a common core for learning to teach and connecting professional learning to practice. *American Educator, 35*(2), 17-21, 38-39.

Birman, B., Desimone, L., Porter, A., & Garet, M. (2000). Designing professional development that works. *Educational Leadership, 57*(8), 28-33.

Black, P. & William, D. (1998). Inside the black box: Standards through classroom assessment. *The Phi Delta Kappan, 80*(2), 139-144.

Blanton, M. & Kaput, J. (2005). Characterizing a classroom practice that promotes algebraic reasoning. *Journal for Research in Mathematics Education*, 36(5), 412-446.

Boaler, J. (1999). Participation, knowledge and beliefs: a community perspective on mathematics learning. *Educational Studies in Mathematics, 40*(3), 259-281.

Boostrom, R. (1991). The nature and functions of classroom rules. *Curriculum Inquiry, 21*(2), 193-216.

Bowers, J., Cobb, P., & McClain, K. (1999). The evolution of mathematical practices: A case study. *Cognition and Instruction, 17*(1), 25-64.

Brody, C. (1998). The significance of teacher beliefs for professional development and cooperative learning. In C. Brody and N. Davison (Eds.) *Professional Development for Cooperative Learning: Issues and Approaches*, pp. 25-48. Albany, NY: University of New York Press.

Boyd, D., Grossman, P., Lankford, H, Loeb, S., & Wyckoff, J. (2009). teacher preparation and student achievement. *Educational Evaluation and Policy Analysis, 31*(4), 416-440.

Burton, L. (1984). Mathematical thinking: The struggle for meaning. *Journal for Research in Mathematics Education, 15*(1), 35-49.

Carpenter, T., Fennema, E., Franke, M., Levi, L., Empson, S. (2000). *Cognitively guided instruction: A research-based teacher professional development program for elementary school mathematics.* Research Report. Available at: http://www.wcer.wisc.edu/ncisla

Carmichael, S., Martino, G., Porter-Magee, K., & Wilson, S. (2010). *The state of the standards – and the common core - in 2010*. Retrieved from the Thomas Fordham Institute website at http://www.edexcellence.net/publications-issues/standards-testing-accountability.html

Carpenter, T. & Lehrer, R. (1999) Teaching and learning mathematics with understanding. In E. Fennema & T. Romberg (Eds.) *Mathematics Classrooms that Promote Understanding*, 19-32. Mahwah, NJ: Lawrence Erlbaum Associates Publishers.

Cobb, P., Wood, T., Yackel, E., & McNeal, B. (1992). Characteristics of classroom mathematics traditions: An interactional analysis. *American Educational Research Journal, 29*(3), 573-604.

Cornelius, L. & Herrenkohl, R. (2004). Power in the classroom: How the classroom environment shapes students' relationships with each other and with concepts. *Cognition and Instruction, 22*(4), 467-498.

Creswell, J. (2007). *Qualitative Inquiry & Research Design: Choosing Among Five Approaches*. (2nd Ed.). Thousand Oaks, CA: Sage Publications.

Crockett, M. (2007). The relationship between teaching and learning: Examining Japanese and US professional development. *Journal of Curriculum Studies, 39*(5), 609-621,

Darling-Hammond, L. (1999). *Teacher quality and student achievement: A review of state policy evidence*. Center for the Study of Teaching and Policy. Available at: http://www.politicalscience.uncc.edu

Darling-Hammond, L. (2010). America's commitment to equity will determine our future. *The Phi Delta Kappan, 91*(4), 8-14.

Daro, P., Mosher, F., & Corcoran, T. (2011). *Learning trajectories in mathematics: A foundation for standards, curriculum, assessment, and instruction*. Research report. Consortium for Policy Research in Education. Available at http://engageny.org/wp-content/uploads/2011/07/learning-trajectories-in-math_ccii-report.pdf

Davis, B. & Simmt, E. (2006). Mathematics-for-teaching: An ongoing investigation of the mathematics that teachers (need to) know. *Educational Studies in Mathematics, 61*(3), 293-319.

Dekker, R. & Elshout-Mohr, M. (2004). Teacher interventions aimed at mathematical level raising during collaborative learning. *Educational Studies in Mathematics, 56*(1), 39-65.

Denmark, T. & Kepner, H. (1980). Basic skills in mathematics: A survey. *Journal for Research in Mathematics Education, 11*(2), 104-123.

Dixon, J. K., Andreasen, J. & Stephan, M. (2009a). Establishing social and sociomathematical norms in an undergraduate mathematics content course for prospective teachers: The role of the instructor. *AMTE Monograph Series: Scholarly Practices and Inquiry in the Preparation of Mathematics Teachers*, 6, 43-66.

Dixon, J. K., Egendoerfer, L., & Clements, T. (2009b). Do they really need to raise their hands? Challenging a traditional social norm in a second grade mathematics classroom. *Teaching and Teacher Education, 25*, 1067-1076.

Doubet, K. (2012). Formative assessment jump-starts a middle grades differentiation initiative: A school focuses on formative assessment to support its efforts to differentiate instruction. *Middle School Journal, 43*(3), 32-38.

Ehrenberg, R., Goldhaber, D., & Brewer, D. (1995). Do teachers' race, gender, and ethnicity matter? Evidence from the National Educational Longitudinal Study of 1988. *Industrial and Labor Relations Review, 48*(3), 457-561.

Fennema, E., Sowder, J., & Carpenter, T. (1999). Creating classrooms that promote understanding. In E. Fennema and T. Romberg (Eds.) *Mathematics classrooms that promote understanding* (pp.185-199). Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Fetterman, D. (2010). *Ethnography*. (3rd Ed.). Thousand Oaks, CA: Sage Publications.

Florida Department of Education (2013). State report of schools, grade 6, mathematics. Available at: http://fcat.fldoe.org/mediapacket/2013/default.asp#FCAT2

Forman, E. & Ansell, E. (2001). The multiple voices of a mathematics classroom community. *Educational Studies in Mathematics*, 46(1/3), 115-142.

Franke, M., Carpenter, T., Fennema, E., Ansell, E., & Behrend, J. (1998). Understanding teachers' self sustaining, generative change in the context of professional development. *Teaching and Teacher Education, 14*(1), 67-80.

Franke, M. (2001). capturing teachers' generative change: a follow up study of professional development in mathematics. *American Educational Research Journal, 38*(3), 653-689.

Fuson, K. (2011). *Math expressions common core*. Houghton Mifflin Harcourt. Available at http://www.hmheducation.com/assets/pdf/mathexpressions/02_52821_MX_2013_CSSNF_LR.pdf

Gavin, K. & Moylan, K. (2012). 7 steps to high-end learning. *Teaching Children Mathematics, 19*(3), 184-192.

Gay, G. (2000). *Culturally responsive teaching: theory, research, & practice*. New York: Teacher College Press

Glesne, C. (2011). *Becoming Qualitative Researchers: An Introduction*. (4th Ed.). Boston, MA: Pearson.

Gonzales, P., Williams, P., Joselyn, L., Roey, S., Kastberg, D., & Brenwald, S. (2009). *Highlights from TIMSS 2007: Mathematics and science achievement of U. S. fourth- and eighth-grade students in an international context*. Available at http://nces.ed.gov/timss/results07.asp

Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education, 35*(4), 258-291.

Hadjioannou, X. (2007). Bringing the background to the foreground: What do classrooms that support authentic discussions look like? *American Educational Research Journal*, 44(2), 370-399.

Hancock, C. L. (1995) Enhancing mathematics learning with open-ended questions. *The Mathematics Teacher, 88*(6), 496-499.

Hendrickson, K. A. (2012). Learning from Finland: Formative assessment. *The Mathematics teacher, 105*(7), 488-489.

Henningsen, M. & Stein, M. (1997). Mathematical tasks and student cognition: Classroom based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28*(5), 524-549.

Herbel-Einsenmann, B. & Otten, S. (2011). Mapping mathematics in classroom discourse. *Journal for Research in Mathematics Education*, 42(5), 451-485.

Hiebert, J. (1984). Children's mathematics learning: The struggle to link form and understanding. *The Elementary School Journal, 84*(5), 496-513.

Hiebert, J., Carpenter, T., Fennema, E., Fuson, K., Wearne, D., Murray, H., Olivier, A., & Human, P. (1997). *Making Sense: teaching and learning mathematics with understanding.* Portsmouth, NH: Heinemann.

Hiebert, J., Morris, A., Berk, D., & Jansen, A. (2007). Preparing teachers to learn from teaching. *Journal of Teacher Education, 58*(1), 47-61.

Hiebert, J. & Morris, A. (2009). Building a knowledge base for teacher education: An experience in k-8 mathematics teacher preparation. *The Elementary School Journal, 109*(5), 475-490.

Hill, H. & Ball, D.L. (2004a). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education, 35*(5), 330-351.

Hill, H., Schilling, S., & Ball, D. L. (2004b). Developing measures of teachers' mathematical knowledge for teaching. *The Elementary School Journal, 105*(1), 11-30.

Hill, H., Rowan, B., & Ball, D. L. (2005). Effects of teacher's mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.

Hill, H., Blunk, M., Charalambos, C., Lewis, J., Phelps, G., Sleep, L., & Ball, D. L. (2008). Mathematical knowledge for teaching and the mathematical quality of instruction: An exploratory study. *Cognition and Instruction, 26,* 430-511. doi: 10.1080/07370000802177235

Hill, H. & Ball, D. L. (2009). The curious – and crucial – case of mathematical knowledge for teaching. *The Phi Delta Kappan, 91* (2), 68-71.

Hobson, E. H. (1997). Forms and functions of formative assessment. *The Clearing House, 71*(2), 68-70.

Horizon Research Inc. (2000). Classroom observation and analytic protocol. Retrieved from http://www.horizon-research.com/insidetheclassroom/instruments/obs.php

Howse, T. D. (2013). A case study: Exploring the relationship between culturally responsive teaching and a mathematical practice of the Common Core State Standards. (Unpublished doctoral Dissertation) University of Central Florida, Orlando, Florida.

Hufferd-Ackles, K., Fuson, K., & Sherin, M. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education*, 35(2), 81-116.

Kerslake, D. (1989) Collaborative learning in mathematics. *Mathematics in School, 18*(1), 58-83.

Kline, M. (1973). *Why johnny can't add: The failure of the new math*. New York, NY: St. Martin's Press.

Kraus, W. (1978). Back to basics: friend or foe? *The Mathematics Teacher, 71*(3), 218-224.

Lambdin, D. V. & Walcott, C. (2007). Changes through the years: connections between psychological learning theories and the school mathematics curriculum. In W. G. Martin, M. E. Strutchens, and P. C. Elliott (Eds.) *The learning of Mathematics, Sixty-ninth Yearbook*. Reston, VA: NCTM.

Leher, R. & Franke, M. (1992). Applying personal construct psychology to the study of teachers' knowledge of fractions. *Journal for Research in Mathematics Education, 23*(3), 223-241.

Lin, C. (2009). A comparison study of web-based and traditional instruction on preservice teachers' knowledge of fractions. *Contemporary Issues in Technology and Teacher Education, 9*(3). Retrieved from http://citejournal.org/vol9/iss3/mathematics/article1.cfm

Lipin, M. (1999). *Knowing and teaching elementary mathematics: teachers' understanding of fundamental mathematics in China and the United States.* Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Lithner, J. (2000). Mathematical reasoning in task solving. *Educational Studies in Mathematics*, 41(2), 165-190.

Lo, J., Wheatley, G., & Smith, A. (1994). The participation, beliefs, and development of arithmetic meaning of a third-grade students in mathematics class discussions. *Journal for Research in Mathematics Education, 25*(1), 30-49.

Mehan, H. (1979). "What time is it, Denise?" Asking known information questions in classroom discourse. *Theory into Practice, 18*(4), 285-294.

Mehan, H. (1987). Language and power in organizational process. *Discourse Processes, 10*(4), 291-301.

Merriam, S. (1998). *Qualitative research and case study applications in education*. San Francisco, CA: Jossey-Bass.

Mewborn, D. (2003). Teaching, teachers' knowledge, and their professional development. In J. Kilpatrick, G. Martin, & D. Schifter (Eds.) *A Research Companion to Principles and Standards for School Mathematics*, 45-52. Reston, VA: NCTM.

Morris, A. (2010). Factors affecting pre-service teachers' evaluations of the validity of students' mathematical arguments in classroom contexts. *Cognition and Instruction, 25*(4), 479-522.

Mullens, J., Murnane, R., & Willett, J. (1996). The contribution of training and subject matter knowledge to teaching effectiveness: A multilevel analysis of longitudinal evidence from Belize. *Comparative Education Review, 40*(2), 139-157.

Mullis, I.V.S., Martin, M.O., & Foy, P. (with Olson, J.F., Preuschoff, C., Erberber, E., Arora, A., & Galia, J.). (2008). *TIMSS 2007 International mathematics report: Findings from IEA's trends in international mathematics and science study at the fourth and eighth grades*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College. Retrieved from http://timss.bc.edu/TIMSS2007/PDF/TIMSS2007_InternationalMathematicsReport.pdf

Nathan, M. & Knuth, E. (2003). A study of whole classroom mathematical discourse and teacher change. *Cognition and Instruction, 21*(2), 175-207.

National Council of Teachers of Mathematics. (1970). *A history of mathematics education in the United States and Canada*. Washington, DC: NCTM.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence.* Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2010). *Making it happen: A guide to interpreting and implementing common core state standards for mathematics*. Reston, VA: NCTM.

National Governors Association (NGA) & Council of Chief State Schools Officers. (2010). *Common core standards for mathematics*. Retrieved from http://www.corestandards.org/

National Research Council. (2001). *Adding it up: Helping children learn mathematics.* J. Kilpatrick, J. Swafford, and B. Findell (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. Washington, DC: National Academy Press.

Nemser, S. F. (1983). Learning to teach. In L. Shulman & B. Findell (Eds.) *Handbook of Teaching and Policy* (pp. 150-170). New York, NY: Longman.

Pace, J. & Hemmings, A. (2007). Understanding authority in classrooms: A review of theory, ideology, and research. *Review of Educational Research, 77* (1), 4-27.

Phillips, R. (2008). Motivating prospective elementary school teaches to learn mathematics by focusing upon children's mathematical thinking. *Issues in Teacher Education, 17*(2), 7-26.

Porter, A. (2002). Measuring the content of instruction: Uses in research and practice. *Educational Researcher*, *31*(7), 3-14.

Porter, A., Garet, M., Desimone, L., Yoon, K., Birman, B., & The American Institutes for Research. (2000). Does professional development change teaching practice? Executive summary. *The American Institutes for Research.*

Powell, K. & Kalina, C. (2009). Cognitive and social constructivism: Developing tools for an effective classroom. *Education*, *130*(2), 241-250.

RAND Mathematics Study Panel. (2003). *Mathematical proficiency for all students: towards a strategic research and development program in mathematics education*. Santa Monica, CA: RAND.

Reid, D. (2002). Conjectures and refutations in grade 5 mathematics. *Journal for Research in Mathematics Education, 33*(1), 5-29.

Richardson, J., Ball, D., & Moses, B. (2009). Equity and mathematics: an interview with Deborah Ball and Bob Moses. *The Phi Delta Kappan*, *91*(2), 54-59.

Romberg, T. & Kaput, J. (1999). Mathematics worth teaching, mathematic worth understanding. In E. Fennema & T. Romberg (Eds.) *Mathematics Classrooms that Promote Understanding* (pp. 19-32). Mahwah, NJ: Lawrence Erlbaum Associates Publishers.

Rowan, B., Chiang, F., & Miller, R. (1997). Using research on employees' performance to study the effects of teachers on students' achievement. *Sociology of Education, 70*(4), 256-284.

Sanchez, W. (2013). Open-ended questions and the process standards. *The Mathematics Teacher 107*(3), 206-211.

Saxe, G., Gearhart, M., & Seltzer, M. (1999). Relations between classroom practices and student learning in the domain of fractions. *Cognition and Instruction, 17*(1), 1-24.

Schoenfeld, A. H. (1980). Teaching problem-solving skills. *The American Mathematical Monthly* , *87*(10), 794-805

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook forResearch on Mathematics Teaching and Learning* (pp. 334-370). New York, NY: MacMillan

Secada, W., Foster, S., & Adajian, L. (1995). Intellectual content of reformed classrooms. *National Center for Research in Mathematical Sciences Education Newsletter, 4*(1), 3-8.

Shafer M. & Romberg, T. (1999). Assessment in classrooms that promote understanding. In E. Fennema & T. Romberg, (Eds.) *Mathematics Classrooms That Promote Understanding* (pp. 159-184). Mahwah, NJ: Lawrence Erlbaum Associates, Publishers.

Sharkey, N. S. & Murnane, R. J. (2006). Though choices in designing a formative assessment system. *American Journal of Education, 112* (4), 572-588.

Shell Center for Mathematical Education, U. C. at Berkeley, & The University of Nottingham. (2012). *Mathematics assessment resource service*. Available at http://map.mathshell.org/materials/index.php

Shulman, L. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Shulman, L. & Shulman, J. (2004). How and what teachers learn: a shifting perspective. *Curriculum Studies, 36*(2), 257-271.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching, 77*, 20-26.

Smith, T., Desimone, L., & Ueno, K. (2005). "Highly qualified" to do what? The relationship between NCLB teacher quality mandates and the use of reform-oriented instruction in middle school mathematics. *Educational Evaluation and Policy Analysis, 27*(1), 75-109.

Stake, R. (2000). Case studies. In N. Denzin and Y. Lincoln (Eds.) *Handbook of Qualitative Research*, 2nd. Ed. (pp. 435-454). Thousand Oaks, CA: Sage Publications.

Steffe, L. & Kieren, T. (1994). Radical constructivism and mathematics education. *Journal for Research in Mathematics Education,* 25(6), 711-733.

Stigler, J. W. & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom.* New York, NY: Free Press.

Stigler, J. W. & Hiebert, J. (2009). Closing the teaching gap. *The Phi Delta Kappan, 91*(3),32-37.

Stylianides, A. & Ball, D. (2008). Understanding and describing mathematical knowledge for teaching: knowledge about proof for engaging students in the activity of proving. *Journal of Mathematics Teacher Education, 11*(4), 307-332.

Suzuka, K., Sleep, L., Ball, D., Bass, H., Lewis, J., & Hoover Thames, M. (2009). Designing and using tasks to teach mathematical knowledge for teaching. *AMTE Monograph*, 6, 7-23.

Thompson, P. & Saldanha, L. (2003). Fractions and multiplicative reasoning. In J. Kilpatrick, W. G. Martin, & Schifter, D. *A Research Companion to Principles and Standards for School Mathematics* (pp.95-113). Reston, VA: NCTM.

Torrance, H. & Pryor, J. (2001). Developing formative assessment in the classroom: Using action research to explore and modify theory. *British Educational Research Journal, 27*(5), 615-631.

Turner, J. & Meyer, D. (2004). A classroom perspective on the principle of moderate challenge in mathematics. *The Journal of Educational Research*, *97*(6), 311-318.

Tutak, F.A., Bondy, E., and Adams, T. L. (2011). Critical pedagogy for critical mathematics education*. International Journal of Mathematical Education in Science and Technology*, *42*(1), 65-74.

Van Bramer, J. (2004). Conversation as a model of instructional interaction. *Literacy Teaching and Learning, 8*(1), 19-46.

Von Glasersfeld, E. (1990). An exposition of constructivism: Why some like it radical. constructivist views on the teaching and learning of mathematics. *Journal for Research in Mathematics Education Monograph, 4*, 19-29, 195-210.

Von Glasersfeld, E. (1991). *Radical constructivism in mathematics education*. Norwell, MA: Kluwer Academic Publishers.

Walshaw, M. & Anthony, G. (2008) The teacher's role in classroom discourse: A review of recent research into mathematics classrooms. *Review of Educational Research, 78*(3), 516-551.

Waring, H. Z. (2008). Using explicit positive assessment in the language classroom: IRF, feedback, and learning opportunities. *The Modern Language Journal, 92*(4), 577-594.

Wei, R., Darling-Hammond, L., Adamson, F., & National Staff Development Council. (2010). Professional development in the United States: Trends and challenges, phase II of a three-phase study. Technical Report. *National Staff Development Council.* Retrieved from http://www.learningforward.org/news/NSDCstudytechnicalreport2010.pdf

Wood, T., Cobb, P., Yackel, E. (1993). The nature of whole class discussion. In rethinking elementary school mathematics: Insights and issues. *Journal for Research in Mathematics Education Monograph*, 55-68.

Wood, T. (1999). Creating context for argument in mathematics class. *Journal for Research in Mathematics Education, 30*(2), 171-191.

Yackel, E., Cobb, P., & Wood, T. (1993a). The relationship of individual children's mathematical conceptual development to small-group interactions. In rethinking elementary school mathematics: Insights and issues. *Journal for Research in Mathematics Education Monograph*, 45-54.

Yackel, E., Cobb, P., & Wood, T. (1993b). Developing a basis for mathematical communication within small groups. In rethinking elementary school mathematics: Insights and issues. *Journal for Research in Mathematics Education Monograph*, 33-44.

Yin, R. (2003). Case study research: Design and method. (3rd Ed.). Thousand Oaks, CA: Sage Publications.

Zaslavsky, O. (2005). Seizing the opportunity to create uncertainty in learning mathematics. *Educational Studies in Mathematics, 60*(3), 297-321.