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THE CONCEPTUAL FIELD OF PROPORTIONAL REASONING
RESEARCHED THROUGH
THE LIVED EXPERIENCES OF NURSES

by

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A dissertation submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy Mathematics Education
in the College of Education and Human Performance
at the University of Central Florida
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Major Professor: Juli K. Dixon

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ABSTRACT

Proportional reasoning instruction is prevalent in elementary, secondary, and post-secondary schooling. The concept of proportional reasoning is used in a variety of contexts for solving real-world problems. One of these contexts is the solving of dosage calculation proportional problems in the healthcare field. On the job, nurses perform drug dosage calculations which carry fatal consequences. As a result, nursing students are required to meet minimum competencies in solving proportion problems. The goal of this research is to describe the lived experiences of nurses in connection to their use of proportional reasoning in order to impact instruction of the procedures used to solve these problems.

The research begins by clarifying and defining the conceptual field of proportional reasoning. Utilizing Vergnaud's theory of conceptual fields and synthesizing the differing organizational frameworks used in the literature on proportional reasoning, the concept is organized and explicated into three components: concepts, procedures, and situations. Through the lens of this organizational structure, data from 44 registered nurses who completed a dosage calculation proportion survey were analyzed and connected to the framework of the conceptual field of proportional reasoning. Four nurses were chosen as a focus of in-depth study based upon their procedural strategies and ability to vividly describe their experiences. These qualitative results are synthesized to describe the lived experiences of nurses related to their education and use of proportional reasoning.

Procedural strategies that are supported by textbooks, instruction, and practice are developed and defined. Descriptive statistics show the distribution of procedures

used by nurses on a five question dosage calculation survey. The most common procedures used are the nursing formula, cross products, and dimensional analysis. These procedures correspond to the predominate procedures found in nursing dosage calculation texts. Instructional implications focus on the transition between elementary and secondary multiplicative structures, the confusion between equality and proportionality, and the difficulty that like quantities present in dealing with proportions.

Key Words: proportional reasoning, dosage calculation, medication errors, nursing, conceptual fields, lived experiences, multiplicative structures

DEDICATION

I would like to dedicate this dissertation to my dad and my mom, Frank and Roberta Pearn. The topic of dosage calculations is a perfect blend of both of my parents' career interests, mathematics and nursing. My father was an electrical engineer and my mother was an occupational therapy assistant. For the many mathematical conversations that I had with my dad, for the many hours of working with my mom at the nursing home where she was employed, for the observation of lives worth modeling and the meaning that it gave to me, the fruit of those seeds that were planted ARE this dissertation. I love you both so much.

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First and foremost, I praise my God and Savior. Jesus said:

My sheep listen to my voice; I know them, and they follow me. I give them eternal life, and they shall never perish; no one will snatch them out of my hand. My father, who has given them to me, is greater than all; no one can snatch them out of my Father's hand. I and the Father are one. John10:28-30.

These powerful words sustain me through difficult trials. I have peace knowing that the Creator of the Universe will never let me go; that I am in His hand. Also, these words are a statement of the identity of Christ; that Jesus and the Father are one. These words brought the condemnation of the people on Jesus because He was declaring who He really was. Shortly after this proclamation, He was crucified. Jesus is the ultimate example of never conforming to what the world wants you to be but being who you are. I sometimes am afraid to be who I am, let alone proclaim it. To declare that I am a Christian first, a wife second, a mother third, and then a mathematics educator has not always been easy. The fear of what others will say or how it will impact my career is always before me. Even with this dissertation, I was afraid to merge the topics of mathematics education with nursing for fear that it would not be received well. But to do anything else, would not be true to myself or to my God. This topic was given to me divinely by God as I stood and prayed to him outside of Parking Lot A at UCF during my first semester in Fall 2009. I praise Him for allowing me to do this. He provided me with an amazing committee who helped to foster this vision in me. He provided me with support through some very special people. I would like to take the time to acknowledge my committee and those special people but first I need to thank God for never letting anyone snatch me from His hand. I am His and He is mine. Thank you God the Father, Jesus, and the Holy Spirit; three in one. Thank you for eternal life in you.

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Dr. Juli K. Dixon, Chair
Dr. Janet Andreasen
Dr. Debra Hunt
Dr. Erhan Selcuk Haciomeroglu

Thank you Dr. Dixon for allowing me to do a dissertation that involved such a contextualized nature. Dr. Dixon helped me to blend nursing and mathematics education with a professionalism and rigor that could be respected by both disciplines. Dr. Dixon also served the role as the chair of the mathematics education doctoral program at UCF. In this role, Dr. Dixon

provided myself and her other doctoral students with many opportunities. (By the way, UCF does stand for opportunity.) The students that Dr. Dixon brought together for our program perfectly represents diversity. Turkey, Palestine, and Panama were among the nations represented in our group. What a blessing to be able to work with such a hard-working, dedicated, and friendly group. Dr. Dixon always took the time to introduce our group, her protégés, to other professionals in mathematics education. She beamed with pride when she introduced us and I always felt proud to be her student. It is an honor to now be in the academic lineage of Dr. Dixon. She has given so much of herself. Thank you, Dr. Dixon.

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CHAPTER 1 INTRODUCTION

General Background

The concept of proportional reasoning can be found in the mathematics educational curriculum from elementary school to post-secondary education. The ability to solve proportional reasoning problems has many real world applications and therefore is an important skill for many professions and daily activities. Tournaire and Pulos (1985), in their comprehensive literature review, cite the widespread research of proportional reasoning and attribute the breadth of research to the difficulty that many people face in mastering the concept.

One area where knowledge of proportional reasoning has lifesaving meaning is in the field of healthcare. The inability of nurses, doctors, and pharmacists to solve proportional reasoning problems in the prescribing and administering of drugs has the potential to result in death for their patients. In 2000, the Institute of Medicine published a report entitled "To Err is Human" which was a nationwide summary of the available research on medication errors (Kohn, Corrigan, & Donaldson, 2000). The Institute of Medicine reported that between 44,000 and 98,000 deaths per year could be attributed to medication errors in hospitals. The Institute of Medicine continued their efforts in 2006 with a follow-up report entitled "Preventing Medication Errors: Quality Chasm Series" (Aspden, Wolcott, Bootman, & Cronenwett, 2006). In this report, it was estimated that, on average, every hospital patient is victim to one drug administration error per day. Dosage miscalculations could constitute up to 14% of the drug

administration errors which can cause serious injury and even death (Hicks, Becker, & Cousins, 2008; Segatore, Miller, & Webber, 1994).

Drug administration requires medical staff to be able to solve proportion problems associated with providing patients with the proper drug dosage. This task involves calculating the amount of medicine a patient is to receive based on a doctor's orders. For instance: if a doctor prescribes a patient to receive a mass of 225 mg of a drug, the job of the nurse is then to administer that mass of drug. Mass is usually measured by using a scale or a balance. Drugs are no longer kept in bulk to be weighed out on a scale by a pharmacist in order to be administered. The mass frequently comes in the form of a countable pill or a designated capacity of liquid. The ratio of this designated containment of mass to the quantity or the capacity is called the dose strength and is indicated on the drug label. An illustration of a drug label is shown in Figure 1. This label shows that the drug Amikacin Sulfate has a mass of 150 mg contained in every 2 mL of liquid which yields a dose strength of 150 mg per 2 mL. Therefore, a nurse needing to administer 225 mg of Amikacin Sulfate would need to use proportional reasoning to calculate the dose of 3 mL of the liquid medicine.


| | | |
|------------------------|---------------------------------------|--|
| Desired Mass: | 225 mg |  |
| Dose Strength on Hand: | $\frac{150 \text{ mg}}{2 \text{ mL}}$ | |
| Give: | 3 mL | |

Figure 1. Dose Strength of Amikacin Sulfate

Performing dosage calculation problems such as this is one of the most common mathematical applications that nurses use (Hoyles, Noss, & Pozzi, 2001). Doctors and

pharmacists are also required to be able to solve these types of problems, but nurses are considered the last line of defense; and the ability to solve these problems correctly and/or to address possible errors contributes to the determination of life or death in their patients.

The importance of this skill is evidenced by nursing preparation programs' emphasis on drug dosage calculation testing throughout the curriculum (American Association of Colleges of Nursing Organizational Leadership Network (AACN), 2006). Institutions impose strict guidelines for passing tests involving drug dosage calculations in order to determine which students will be allowed to continue in a school's nursing program. Often, nursing programs require their students to pass a dosage calculation test prior to taking clinical courses. A summary of some college's requirements are summarized in Table 1. Each of these institutions requires the student to withdraw from the course if they fail to meet the criteria (AACN, 2006). Tests were administered at the beginning of each clinical course in all cases.

Table 1
 Dosage Calculation Test Requirements at Various Institutions

| Institution | Passing Score | Number of Attempts |
|-----------------------------|---------------|---|
| University of Texas, Tyler | 90% | 2 |
| Prairie View A&M University | 94% | 3- first semester, 2- each subsequent semester |
| University of Rhode Island | 85% | Retake every 2 weeks up until the midterm |

The high stakes nature of these tests and the potential harm that miscalculation can cause point to the critical importance of these types of problems to nurses and nursing students. The pressure to obtain precise results introduces another factor common in the nursing literature: mathematics anxiety. Glaister (2007) found that 20% of nursing students in her study had mathematics anxiety and concluded that this factor must be taken into account and addressed by the educational practices of instructors. Two audiences of instructors, mathematics and nursing, need to be addressed in the research and corresponding recommendations for solving these types of proportional reasoning problems. Therefore, the types of dosage calculation problems that are utilized in this research will be referred to as Dosage Calculation Proportion Problems, DCPPs, with the hope that the term *dosage calculation* will speak to the nursing community and *proportion* will speak to the mathematics community.

Furthermore, the types of DCPPs are limited to what is called in the field of mathematics as missing value proportion problems (MVPPs). The DCPP shown in Figure 1 can be classified as such because three numbers in the proportion, 150 mg, 2 mL, and 225 mg, are given and the fourth number, 3 mL is missing. This type of DCPP is the most basic because the solution process only requires the use of one proportion and because the numbers in the problem are whole numbers. DCPPs which incorporate intravenous rates of infusion and/or are dependent upon the weight of the patient, as is common in pediatrics and critical care, present even greater challenges and will also be discussed in this research (Fleming, Brady, & Malone, 2014, Kaushal et al., 2001). A dosage calculation problem which is based upon the patient's weight is illustrated in Figure 2. The solution involves three separate calculations that utilize

proportional reasoning. Although the problems in Figure 1 and Figure 2 seem quite different, both are considered missing value proportion problems (MVPPs) and can be solved by applying the same mathematical procedures and concepts.

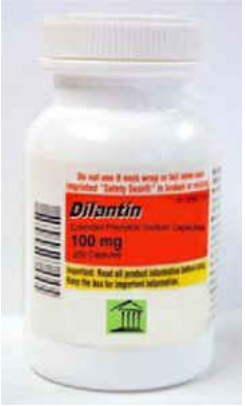
| | | |
|---|--|---|
| <p>The doctor orders Dilantin 3mg/kg for a patient weighing 146 pounds. You have Dilantin 100mg capsules on hand.</p> | | |
| Patient Weight: | 146 pounds |  |
| Desired Mass: | 3 mg for every 1 kg of patient weight | |
| Dose Strength on Hand: | 100 mg/1 capsule | |
| Give: | 2 capsules | |
| Step 1: Convert weight to kg | Step 2: Calculate the mass of the drug required for this patient | |
| $\frac{2.2 \text{ lb}}{1 \text{ kg}} = \frac{146 \text{ lb}}{x}$ $x = 66.4 \text{ kg}$ | $\frac{1 \text{ kg}}{3 \text{ mg}} = \frac{66.4 \text{ kg}}{y}$ $y = 199.1 \text{ mg}$ | $\frac{100 \text{ mg}}{1 \text{ cap}} = \frac{199.1 \text{ mg}}{z}$ $z = 1.99 \text{ or } 2 \text{ capsules}$ |

Figure 2. Critical Care DCPD and Solution

The complexity in the problem in Figure 2 has three sources. One source of difficulty is the additional numeric values. This makes the identification of proportional values more difficult than problems that explicitly give only the three values required in a MVPP. Another source of difficulty is the need to solve three separate proportion problems. First the weight must be converted, second the mass calculated, and third the dose calculated. The final source is that the values and relationships are not integers. The

complexity of these problems and the critical importance of obtaining correct solutions have facilitated the need to educate both nursing students and practicing nurses in their solution process. The focus of this research is to understand the lived experiences of nurses in connection to their solution procedures for DCPs in order to find potential areas of improvement in the instruction of proportional reasoning concepts.

Rationale of the Study

“The challenge of nurse educators is to develop teaching strategies that result in graduating nurses who have mastered nursing mathematics” (Johnson & Johnson, 2002, p. 79). The need for educating health care professionals in dosage calculations has been well defined in the literature. In 2003, the American Academy of Pediatrics issued a policy statement that outlined guidelines and recommendations to reduce the number of pediatric medication errors. The report offered recommendations for targeted populations of interest. Hospital administrators, physicians, pharmacists, and nurses were all recommended to implement specific guidelines to improve the safety of their patients. These safety guidelines specifically addressed dosage calculations as cited in Table 2.

Table 2

American Academy of Pediatrics' Recommendations (2003, p. 434)

| <i>Population</i> | <i>Recommendation</i> |
|-------------------------|--|
| Hospital Administration | Develop an educational program for all hospital and medical staff in calculating, prescribing, preparing, and administering medications for children. |
| Physicians | Confirm that the patient's weight is correct for weight-based dosages. Ensure that weight-based dose does not exceed the recommended adult dose. Ensure that calculations are correct. Write weight on each order written. |
| Pharmacist | Recheck calculations to ensure dose ordered falls within the accepted pediatric weight-based dose ranges. |
| Nurses | Check medication calculations with another professional member of the health care team. |

The guideline that was particularly pertinent to the present study was that hospital administrators were urged to develop an educational program which included the instruction of calculating medication dosages for all health care providers (American Academy of Pediatrics, 2003, p. 434). This guideline was the first one mentioned in the report and demonstrates the priority that is placed on drug dosage calculation instruction in the education of nurses and other health care providers.

The impact of drug dosage calculation errors is exemplified in the following incident which was reported in the MedMARx report of 2002 (Hicks, Cousins, & Williams, 2003). MedMARx (Quantros, 2009) is the largest database in the United

States that collects data on adverse drug events through voluntary reporting. The medication errors are self-reported by more than 400 healthcare facilities in the United States. The error described in the report details the account of a two-year-old child who was prescribed routine sedation for an outpatient computerized tomography (CT) scan of the head. The drug chloral hydrate was ordered for sedation. The dosage was to be calculated by giving 100 mg for every kilogram that the patient weighed. (This is called a weight-based calculation.) The child weighed 18 pounds. The nurse's error was in converting the child's weight from pounds to kilograms. A possible notational representation of the correct and incorrect solution to this problem is outlined in Table 3. The equivalency conversion between these units is: 2.2 pounds equals 1 kg. The nurse multiplied by 2.2 to find the weight in kilograms rather than dividing by 2.2. The nurse calculated that the patient should receive a 4 g dose. The actual dose should have been 0.8 g. The 4 g dose would have been five times the prescribed amount. The nurse, however, did not give 4 g. A safety precaution of drug administration is to list the maximum dose on the label. For this particular drug, 2 g was listed as the maximum dose; therefore the nurse gave that instead. This was still 2 ½ times the prescribed amount. The child had to spend the night in the pediatric intensive care unit to receive nebulizer treatments because he suffered significant respiratory suppression as a result of the overdose of medication.

Table 3

Possible Representation for MedMARx Medication Error

| Steps | Possible Correct Representation | Relational Calculus | Possible Incorrect Calculation |
|------------------|--|----------------------------------|-----------------------------------|
| Convert lb to kg | $\text{weight} = 18 \text{ lb} \times \frac{1 \text{ kg}}{2.2 \text{ lb}}$ | Correct: Divide 18 by 2.2 | $\text{weight} = 18 \times 2.2$ |
| | $\text{weight} = 8.18 \text{ kg}$ | Incorrect: Multiply 18 by 2.2 | $\text{weight} = 39.6 \text{ kg}$ |
| Calculate dose | $\text{dose} = 8.18 \text{ kg} \times \frac{100 \text{ mg}}{1 \text{ kg}}$ | Multiply weight by 100 | $\text{dose} = 39.6 \times 100$ |
| | $\text{dose} = 818 \text{ mg}$ | | $\text{dose} = 3960 \text{ mg}$ |
| Convert mg to g | $\frac{1000 \text{ mg}}{1 \text{ g}} = \frac{818 \text{ mg}}{x}$ | Divide dose by 1000 | $\text{dose} = 3960 \text{ mg}$ |
| | $\text{dose} = 0.818 \text{ g}$ | | $\text{dose} = 3.96 \text{ g}$ |
| | Rounded to 0.8 g | Round | Rounded to 4 g |

The mathematical representation presented by the researcher is just one possible mathematical representation of the solution process for this problem. The correct solution presented attends to the units of measurement in the problem. The representation demonstrates two common set-ups utilized in the solution process of proportion problems: dimensional analysis and equal ratios. These set-ups as well as others will be discussed later in the research. They are presented here to illustrate the

difference between procedures that are written out attending to the units of measure as opposed to ones that focus on the mathematical operations being carried out. This difference in notation is a major focus of the research on proportional reasoning as Vergnaud (1998) observed that students first solve a problem and then try to fit it to a conventional notational system. Vergnaud found it necessary to differentiate between the solution process and the notational system used. He used the term *relational calculus* to describe how the student solved the problem separate from the notational system and defined it explicitly as “the transformation and composition of relationships given in the situation” (1998, p. 264). The relational calculus for this example starts with dividing the weight in pounds by 2.2 to get the weight in kilograms. Next, the weight (in kilograms) is multiplied by 100 (milligrams per kilogram) to get the dose in milligrams. The dose in milligrams is then divided by 1000 (milligrams per gram) to convert the dose to grams. The answer is then rounded to the nearest whole number. This relational calculus with the error of multiplying by 2.2 can be seen in Table 3. The relational calculus gives only a partial view of the error. Without understanding the procedures and concepts that the nurse applied to this situation, a way of correcting the error becomes problematic.

In an effort to understand these errors, researchers have attempted to classify them according to their types. However, this classification is dependent on the field of study. Nursing and mathematics education researchers have developed different terminology and different definitions to describe the types of errors that are made. Mathematics educators generally define errors as either procedural or conceptual, whereas the terms *mathematical* and *conceptual* are commonly used in nursing

education research. This would not present a problem if there was a correspondence between similar terms, which is not the case. This disparity causes problems because research into the errors that nurses and nursing students are making in their drug dosage calculations has revealed that the most common types of errors are *conceptually based* as opposed to *mathematically based* (Arnold, 1998; Blais & Bath, 1992; Hutton, 1998; Segatore, Edge, & Miller, 1993; Weeks, Lyne, Mosely, & Torrance, 2001; Wilson, 2003). In nursing literature, a mathematical error is defined as an error in executing the computations with numbers (for example, $5 \times 7 = 30$ would be considered a mathematical error) while an error in the set-up of the problem would be classified as a conceptual error (Rice & Bell, 2005). The term conceptual as defined in nursing research is a subset of the definition given by mathematics educators. As a result, the research conclusions from the field of nursing involving the concept of proportional reasoning have not addressed what mathematics education researchers would call conceptual.

In an attempt to pursue a more developed understanding of DCPPs, nursing researchers have acknowledged that other factors may contribute to the emergence of errors. An example of efforts to focus on more than just procedures is seen in the incorporation of Polya's (1973) four stage mathematical model for solving problems into the framework for researching the calculations of drug dosaging (Huse, 2010; Wright, 2009). However, while Polya's model is related to the concept of solving DCPPs, it fails to address the underlying concept, proportional reasoning. Thus, in the present study, the concept under investigation is proportional reasoning. This inability to explicitly focus on the concept of proportional reasoning exposes a gap in the research.

Other nursing researchers have claimed to shift the research focus to concepts. The effects of computer assisted instruction on DCPD understanding has been investigated by Glaister (2005) and Weeks et al. (2001). Additionally, the relationship of student factors and program factors to the dosage calculation proficiency of nursing students was investigated by Johnson and Johnson (2002). Teaching experiment methodology was used by Gillies (2004) to look at the effect of incorporating proportion problems from everyday situations, like travel and shopping, into the instruction of DCPDs. Although these research studies have not focused on procedures, neither have they necessarily focused on concepts. Rather, they have described different situations (presentation, moderating affects, and contexts) in which the concept presents itself. These situations are a necessary part of the research but not sufficient to define the concept of proportional reasoning as applied to DCPDs. A clear construct of the procedures, concepts, and situations surrounding the mathematical concept of proportional reasoning, as exemplified in DCPDs, is needed.

This connection between concepts, procedures, and situations is encompassed in a theoretical model of understanding called conceptual fields (Vergnaud, 2009). Vergnaud defined a concept as being “altogether: a set of situations, a set of operational invariants (contained in schemes), and a set of linguistic and symbolic representations” (2009, p. 94). The theory of conceptual *fields* adds a relational aspect between multiple concepts and multiple situations. “A concept’s meaning does not come from one situation only but from a variety of situations and that reciprocally, a situation cannot be analyzed with one concept alone, but rather with several concepts forming systems” (Vergnaud, 2009, p. 86).

Applying the theory of conceptual fields to the concept of proportion and connecting it to the situation of drug dosages follows a natural progression in the research concerning DCPPs. Only one study (Hoyles et al., 2001) was discovered to address the relationship between conceptual understanding of proportions and DCPPs in nursing practice within the framework of Conceptual Fields. Hoyles et al. (2001) explicitly discusses nurses' understanding of the covariance of mass and volume as exemplifying the application of proportional reasoning to the workplace mathematics of nursing. Written tests were not used as the researchers were specifically examining the situated mathematical practice of nurses. All 30 episodes of DCPPs in their study were worked out mentally except for one. The present research specifically builds off of two research studies. First, this study more fully encompasses the idea of conceptual fields introduced by Hoyles et al. (2001) by explicating not just the relational calculus but also the set-ups used by nurses to solve DCPPs. Second, the use of everyday proportion problems as introduced by Gillies (2004) is incorporated into the study to explore connections between non-nursing proportional reasoning tasks and DCPPs.

Problem Statement

Nurses' knowledge of DCPPs needs to be developed more deeply in order to prevent errors in practice and on dosage calculation tests (Johnson & Johnson, 2002, Gillies, 2004). These errors, when made in practice, can and do cause serious harm and even death. Researchers have acknowledged the need to increase nurses' conceptual knowledge of dosage calculation proportion problems. Currently, reforms to improve the instruction in DCPPs have been centered in three areas: (a) the situations in which DCPPs and proportions are experienced; (b) the procedures used to solve them; and (c) the related concepts such as problem solving strategies and numeracy skills. Research focusing on DCPPs as centered on the concept of proportional reasoning is lacking.

The aim of this research was to investigate the lived experiences of nurses as they intersect with the concept of proportional reasoning, not from just a nursing aspect but from all areas of their lives. Building a descriptive narrative of their understanding of proportional reasoning will provide an added dimension to the literature that could not only speak to educators of nursing students but also to others who teach proportional reasoning. The research focused on the evidenced processes that nurses use to solve proportions in different situations with the aim of describing their conceptual understanding of proportion. The description of the procedures, situations, and concepts held by nurses surrounding their experiences with the concept of proportional reasoning was directed at the improvement of instruction for both nurses and general education students who need to have a deep understanding of proportions.

To facilitate this research endeavor, three research questions were developed. These questions were constructed to elicit data needed to construct a comprehensive picture of the lived experiences of nurses with proportional reasoning problems. The questions explicitly ask for information pertaining to the procedures, situations, and educational experiences that come together to form a person's concept of proportional reasoning.

Research Questions

- Lived Experiences: What are the lived experiences that nurses have with solving proportional reasoning problems on written dosage calculation tests and in nursing practice?
- Procedures: What are the procedures that nurses use to solve proportional reasoning problems on a dosage calculation survey?
- Situations: When solving proportional reasoning problems, what situational variables do nurses recognize as affecting problem difficulty and/or procedure choice: (a) numerical characteristics, (b) semantic type, (c) context, (d) presentation, and (e) student characteristics?

Definition of Terms

Conceptual Understanding – understanding that is rich in relationships and is not bound by context. Core features in superficially different pieces of information are reflected upon, recognized, and organized into a knowledge network (Hiebert & Lefevre, 1986).

Dosage Calculation Error – any mathematical error, in simulation or practice, that results in the incorrect conversion of the doctor’s orders into the amount of medication that the patient should receive.

Dosage Calculation Proportion Problem (DCPP) – any problem encountered during the course of medication administration (either in simulation or practice) that requires the use of proportional reasoning in order for the proper dose to be administered.

Medication Error – any mistake made in the medication process which includes the act of prescribing, dispensing, administering, and monitoring. (Jones, 2009)

Missing Value Proportion Problem (MVPP) – a mathematics problem where a multiplicative relationship between two quantities is defined by a ratio and then applied to a third given quantity to calculate a fourth missing value.

Procedural Knowledge - knowledge of the rules or algorithms “that prescribe step-by-step instructions to complete a task” (Hiebert & Lefevre, 1986, p. 6). This includes the knowledge of when the procedure should be properly applied.

Procedure - the combination of the notational set-up and the relational calculus utilized to solve a problem.

Proportional Reasoning - the evaluation of the multiplicative relationship between two quantities applied universally to two other quantities of the same corresponding nature and dimension.

Relational Calculus- the thinking structures that are used to plan and execute a process in which to perform a mathematical calculation.

Chapter Summary

The rationale for researching the proportional reasoning of nurses has implications to both the fields of nursing and mathematics education. In the field of nursing education, improving dosage calculation proportion problem instruction and understanding can assist nurses in preventing medication dosing errors that can have serious implications for their patients. Also, providing nursing students with quality instruction on DCPPs may help them to achieve success on dosage calculation tests which are required by many nursing programs. In the field of mathematics education, improved instruction in proportional reasoning could lead to successful application of classroom skills to contextual settings.

This research seeks to merge the fields of study combining mathematics education on proportional reasoning and nurses experiences with solving DCPPs to see where these two areas can benefit from each other. The nursing research on DCPP procedures and situations will be expanded to incorporate the concept of proportional reasoning through the theoretical framework of conceptual fields. The mathematics education research on proportional reasoning will be expanded to include the experiences of professionals who rely upon the ability to solve these problems to perform their job and to in essence, save lives. Through this framework, the focus of the research shifts to the concept of proportional reasoning while the procedures and situations become the variables for understanding the concept. When understood within the context of the conceptual field, DCPPs can then be incorporated with the other subclasses of proportion problems and not as separate knowledge.

CHAPTER 2 REVIEW OF THE LITERATURE

Introduction

This chapter provides the reader with the background necessary to understand and evaluate the research presented. A literature review serves not only to summarize but also to synthesize the previous research and theories on the topic “in a way that permits a new perspective” (Boote & Beile, 2005, p. 4). This synthesis of the literature focuses on proportional reasoning and the development of a framework to facilitate the description of a nurse’s understanding of the concept.

The literature review is divided into three main categories: (a) conceptual fields, (b) the conceptual field of proportional reasoning, and (c) nursing mathematics. A general description of conceptual fields provides the framework for the literature. The explicit conceptual field of proportions is presented under the headings of concepts, procedures, and situations. Next, the situated practice of solving DCPPs in the field of nursing is described as it relates to the mathematical concept of proportions. Since the theory of conceptual fields was the guiding framework of this research, the research connected to this theory will be presented first.

Conceptual Fields

The framework of conceptual fields that was utilized in this research is attributed to the work of Gerard Vergnaud (2009). Vergnaud described a conceptual field as being “at the same time a set of situations and a set of concepts tied together” (Vergnaud, 2009, p. 86) and added that a concept is “a set of situations, a set of operational invariants (contained in schemes), and a set of linguistic and symbolic representations” (p. 94). The components of the theory of conceptual fields therefore consist of concepts, situations, procedures and the language in which the concept is communicated. In order to clearly define the theory of conceptual fields, the distinction between two essential components of the theory, procedural and conceptual knowledge, must first be outlined. The difference between these two types of knowledge is a common theme in mathematics education and will serve as a solid foundation from which to build the theory. From this description, the other components of Vergnaud’s theory will surface with meaning and connection.

Mathematics education researchers have been trying to define, classify, and organize types of understanding, or misunderstanding, for years. In 1978, Skemp investigated what was meant in the English language by the word *understanding* in relationship to mathematics. He personally defined understanding in mathematics as knowing what to do and the reasons for doing it. He realized that there was a discrepancy between what he defined as understanding and what seemed generally acceptable as understanding. If a mathematics rule was properly applied to the correct type of problem, this was accepted as understanding. Skemp (1978) believed that this was not enough. This type of knowledge was described by Skemp as “rules without

reason” (p. 9). He believed that true understanding should be more than just the application of rules, but that a person should understand why the rule works and why it is applied to the problem. Skemp introduced the term *instrumental understanding* to describe this type of rule-based understanding. He then introduced the term *relational understanding* to define his deeper idea of what it meant to understand. (Skemp credits these terms to Stieg Mellin-Olsen of Bergen University.) This distinction led other researchers to try to classify the difference in types of mathematical knowledge.

Hiebert and Lefevre (1986) wrote about the distinction between what had been seen in the literature as *procedural knowledge and conceptual knowledge*. One of the main differences that the authors pointed out was that conceptual knowledge must be learned meaningfully, and procedural knowledge may or may not be learned with meaning. If procedures were learned with meaning, they would then be linked to conceptual knowledge. This idea is similar to Skemp’s (1978) description in his original work; however, Hiebert and Lefevre (1986) did extend the idea. They introduced several components to each of the types of understanding that will be compared to elements of Vergnaud’s theory of conceptual fields. A description of these components follows.

Hiebert and Lefevre (1986) introduced a distinction between the kinds of procedural knowledge. The first kind of procedural knowledge had to do with understanding the mathematical symbols and the standard forms of the configurations of these symbols. The second kind of procedural knowledge had to do with understanding the rules to solve mathematical problems but may not be directly associated with mathematical symbols. This second kind of knowledge was concerned

with understanding the step-by-step, algorithmic process of solving problems, whether written or mentally. This differentiation could be beneficial in describing the differences between formal and informal mathematics, where informal mathematics would still enable a person to solve a problem but not in the standard notational system. Procedural understanding was, therefore, dependent upon both understanding of symbols and algorithms.

The second distinction referred to conceptual knowledge. Rather than describing conceptual knowledge as the reason to the rules as Skemp (1978) did, Hiebert and Lefevre (1986) described conceptual knowledge as being “rich in relationships” (p. 3). Two types of relationships described the richness of this knowledge; one was a primary relationship, and the other was a reflective one. The primary relationship was confined to the context in which the information was presented but a reflective relationship was one that was formed between pieces of information that shared core features but may appear different on the surface. Depth of conceptual understanding was, therefore, dependent on whether the information could be transferred or applied to different situations.

In order to connect these ideas to Vergnaud’s theory of conceptual fields, the description of a conceptual field is reviewed. Vergnaud (2009) described a conceptual field as being “at the same time a set of situations and a set of concepts tied together” (p. 86) and added that a concept is “a set of situations, a set of operational invariants (contained in schemes), and a set of linguistic and symbolic representations” (p. 94). Vergnaud therefore incorporates the same components of Hiebert and Lefevre (1986) but emphasizes the effect that situations have on conceptual understanding by making

it part of the three essential components (concepts, procedures, and situations) of a conceptual field. Each layer of Hiebert and Lefevre’s (1986) distinction between procedural and conceptual knowledge can be captured in this model. This comparison of components can be found in Table 4 and is reviewed next.

Table 4

Comparison of Types of Knowledge to Conceptual Fields

| Components of Hiebert and Lefevre’s Procedural and Conceptual Knowledge (1986) | Components of Vergnaud’s Theory of Conceptual Fields (2009) |
|--|---|
| Procedural Knowledge: Notational system of mathematics | Set of linguistic and symbolic representations |
| Procedural Knowledge: Algorithmic processes | Set of operational invariants |
| Conceptual Knowledge: Primary relationships are context bound | Set of situations |
| Conceptual Knowledge: Reflective relationships are not context bound | |

Hiebert and Lefevre (1986) emphasized context and situations in their definition of conceptual understanding. These components can be found in Vergnaud’s set of situations. Hiebert and Lefevre’s (1986) first and second type of procedural knowledge are both covered under Vergnaud’s set of operational invariants. Hiebert and Lefevre’s (1986) first type of procedural knowledge can be associated with the symbolic

representations of Vergnaud's theory. Hiebert and Lefevre's (1986) second type of procedural knowledge can be associated with Vergnaud's set of linguistic and symbolic representations. Both of these organizational structures focus on explicating these three components, concepts, procedures, and situations as essential to describing someone's conceptual knowledge surrounding a topic.

Conceptual Field of Proportional Reasoning

Proportional reasoning as a concept cannot be described by a simple definition according to Vergnaud's theory of conceptual fields. In order to define this concept, the procedures, situations and other related concepts that are associated with it must also be considered. The literature on proportional reasoning is organized in three sections. The first section considers the concept of proportion and the term *proportional reasoning* using the framework of levels of proportional reasoning as presented by Inhelder and Piaget (1958). The second section describes the conventional procedures used to calculate the fourth value in a MVPP. A detailed definition of procedure was used to separate the literature into both the operational invariants and the notational representations commonly used to solve such problems as outlined in Vergnaud's (1980) work on multiplicative structures. The third section characterizes the different situations in which proportional reasoning is encountered. These situations not only include the contextual nature of a proportional reasoning problem but also other moderating effects that impact problem difficulty such as numerical characteristics, semantics, presentation, and student characteristics.

Concepts

Inhelder and Piaget's (1958) experiments involving flexibility of a rod, equilibrium in a balance, hauling weight on an inclined plane, the projection of shadows, and centrifugal force provide a basis for any beginning research in describing the nature of proportional reasoning. As a result of this research, Inhelder and Piaget (1958) assessed a subject's proportional reasoning skills as being in one of three levels: (a) Pre-proportional (b) Logical Proportional or (c) Full Proportional. These levels provide a framework for discussing the body of literature concerning the concept and definitions of proportional reasoning. Each level was defined by the core understanding of that level and the outwardly identifiable solution processes that are employed to solve proportional reasoning problems. This term, relational calculus, was used by Vergnaud to describe these solution processes or operational invariants free of symbolic representation. However, as these levels are described, a possible symbolic representation of the strategies used in each level is illustrated. These representations are only possible representations created by the researcher to help illustrate the mathematics involved in each level and should not be considered to be the notation that was provided by any participant in either of Inhelder and Piaget's (1958) research or any other research. In fact, Inhelder and Piaget did not use numerical quantities in their research because they wanted to focus on pure reasoning of the concept. The representations are purely for the reader's benefit to assist in the understanding of the relational calculus that marks each stage.

An overview of Inhelder and Piaget's levels and these corresponding representations is provided in Table 5. Each conceptual level of understanding will be

reviewed separately and in detail as it relates to other researchers in the field of proportional reasoning. All problems are worked using a consistent example previously stated: *A nurse needing to administer 225 mg of a drug has the drug on hand with dose strength of 150 mg per 2 mL. How many milliliters of drug will the nurse administer?*

Table 5

Concept Levels of Inhelder and Piaget's Model (1958)

| Stage | Concept | Relational Calculus | Representation |
|------------------|---|-----------------------------------|---|
| Pre-Proportional | Covariation | Scalar Decomposition/ Addition | |
| Logical | Multiplicative Relationship Applied to Another Pair | Scalar (within measure) | |
| | | Function (between measure) | |
| Proportional | Multiplicative Relationship is Universally Applied | Function (universally applied) | $f(x) = mx$ $f(x) = \left(\frac{1}{75}\right)(x)$ |

Pre-proportional Reasoning

Covariation is a concept associated with proportional reasoning. Two measures can covary additively or multiplicatively. The signature of proportional reasoning, however, is recognizing that two measures covary in a multiplicative way. An example of additive covariation is a person's age relative to another person's age. If five years were added to their ages, the interval between their ages would be the same. However, if their ages were doubled, this interval would not be the same and therefore age (time) does not covary multiplicatively. An example of multiplicative covariation is the circumference of a circle relative to its diameter. If the measurements were doubled, the ratio between the circumference and diameter would be the same. Transitioning from additive covariation to multiplicative covariation marks the pre-proportional level of proportional reasoning.

Pre-proportional reasoning is characterized by the understanding that two quantities of measure covary with one another however; the relationship is predominately marked by an additive relationship rather than a multiplicative one. Hart (1981) recognized that students at this level had developed a common strategy for dealing with proportions which she termed *building-up*. With this strategy, students multiplied or divided to generate new pairs of numbers until they found pairs that could add to the desired quantity. In the problem in Figure 3, the standard example for this research is worked out.

| | mg | mL |
|------|------|-----|
| ↓ ÷2 | 150 | 2 |
| | + 75 | + 1 |
| | 225 | ? |

Figure 3. Pre-proportional Reasoning: Scalar Decomposition

The measures of milligrams and milliliters are understood to covary. The number of milligrams is halved and therefore, the number of milliliters is also halved resulting in 75 mg contained in 1 mL. By dividing both measures (milligrams and milliliters) by 2, a *scalar* relationship is implied but does not yield a correct answer. Recognizing that 150 mg plus 75 mg equals the desired 225 mg, the milliliters are similarly added resulting in 3 mL. Because the multiplicative relationship is not identified, Vergnaud termed this relational calculus as *scalar addition* or *scalar decomposition*. The true scalar relationship of 1.5 between 150 and 225 ($150 \times 1.5 = 225$) and the function relationship of $1/75$ between 150 and 2 ($150 \times 1/75 = 2$) are not explicitly identified. These two multiplicative relationships, scalar and function, are discussed in Inhelder and Piaget's (1958) next level, Logical Proportional.

This reasoning strategy is generally accepted as marking the first steps of proportional reasoning; however some researchers do not consider procedures involving addition as representing proportional reasoning (Lamon, 2007; Lesh, Post, & Behr, 1987). Lamon (2007) argued that in order for proportional reasoning to occur, at least one of the two multiplicative relationships between four quantities in a proportion

must be recognized. Regardless of whether scalar decomposition between two quantities is considered Pre-proportional or not, under the theory of conceptual fields, it can be considered a related concept and therefore warrants inclusion in the discussion of proportional reasoning.

Logical Proportional

The second level of understanding is the Logical Proportional category. In this category, at least one of the two multiplicative relationships is recognized but not generalized to all cases. The multiplicative relationship is interpreted and used on a case-by-case basis. The identifying relationships at this level are either scalar or function. The scalar relationship is between two like quantities. It is also called *within measures* (Lamon, 2007). The quantity that defines the relationship is of the same nature or measure. This can also be called an internal relationship (Tourniaire & Pulos, 1985). In the example shown in Figure 4, the relationship between the milligrams is identified. In this case, 225 mg is one and a half times the amount of 150 mg. The relationship of times 1.5 is then applied to the milliliters.

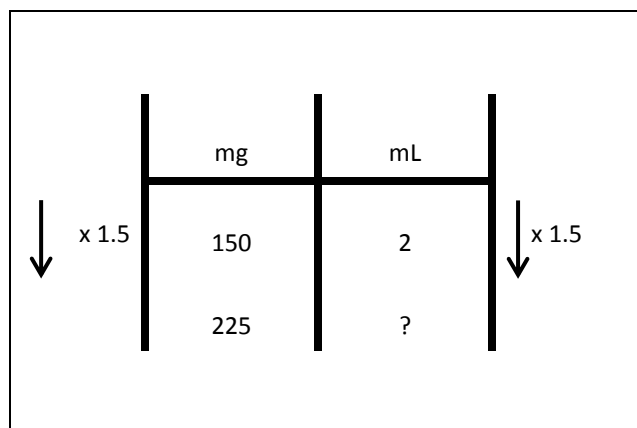


Figure 4. Logical Proportional Reasoning: Scalar

The calculation would be carried out by starting with 225, dividing by 150, and then multiplying by 2. Any calculation that carries out the operations in this order is considered to have a scalar relational calculus.

A function relationship looks at the relationship between the two measurements with unlike measures and therefore is called *between measures* (Lamon, 2007). The quantities that define the relationship are of different natures. This can also be called an external method (Tourniaire & Pulos, 1985). In the example provided in Figure 5, the function relationship is between the milligrams and the milliliters, in this case, two unlike measures. Milligrams measure mass and the milliliters measure capacity. The relationship between 150 (mg) and 2 (mL) is defined by a multiplication of $1/75$ (or a division of 75). The relationship (multiply by $1/75$) is then applied to the next ratio of milligrams to milliliters. The relationship is between the numeric values and not the actual units of measure. (150 mg divided by 75 actually equals 2 mg and not 2 mL.) Therefore, this notation would be indicative of Logical Proportional reasoning and not Full Proportional Reasoning.

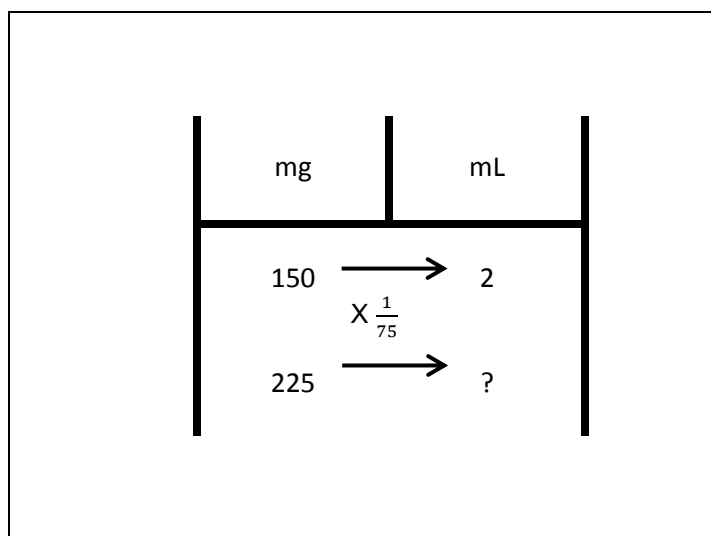


Figure 5. Logical Proportional Reasoning: Function

In the event that the quantities in a problem are all of the same unit of measurement, as could be the case in scaling problems, the function relationship is better described as the relationship between the two measurements taken from the same object, i.e., the relationship between length and width of a rectangle. This relationship is then applied to a similar rectangle. The scalar relationship would represent the multiplicative structure between values of the same dimension from different objects. These problems pose additional difficulty and are discussed in the section on situations.

Other researchers have also made a distinction between what Inhelder and Piaget (1958) termed as Logical Proportional reasoning and other types of reasoning. Baxter and Junker (2001), in their attempt to create an assessment for proportional reasoning, observed this distinction and used the term *psychological* to describe this level. Baxter and Junker (2001) described this psychological perspective as being marked by two characteristics: (a) that the values in the proportional relationship all remain as separate quantities and (b) that parallel transformations are performed in order to maintain correct proportionality. Vergnaud (1980) used the terminology, *isomorphism of measures*, to describe these parallel transformations. Figure 6 illustrates these parallel transformations with the use of parallel arrows to signify the direction of the relationship.

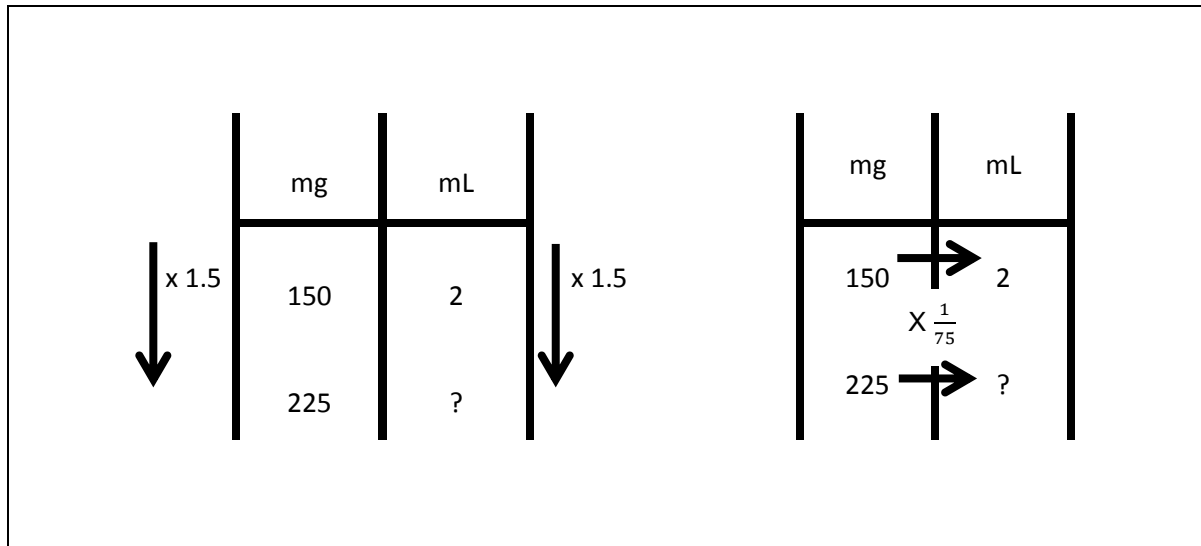


Figure 6. Parallel Lines Indicating Isomorphism of Measures

Lamon (2007) also noted the distinction in levels of understanding. She termed this level as proportional reasoning but distinguished other levels with different terminology.

Full Proportional Reasoning

Full Proportional Reasoning is attained only when it is understood that the multiplicative relationship can be universally applied (Inhelder & Piaget, 1958). This final stage of proportional reasoning is said to be attained when a student can express the relationship as a formula or rule such that when any input value will yield the output value in the desired unit of measure (Lesh et al., 1987). The ability to represent the relationship as a function, as seen in Figure 7, is considered to be evidence that the student has achieved the understanding that the relationship can be universally applied.

$$f(x) = kx$$
$$f(x) = \left(\frac{1}{75}\right)(x)$$

Figure 7. Full Proportional Reasoning: Universally Applied Function

The constant, k , in Figure 7 is called the constant of proportionality. This constant is not just understood as the relationship between two quantities, but also as a constant rate of change that can transform any x value to the corresponding $f(x)$ value (Baxter & Junker, 2001). This is tied to the concept of slope denoted by m in linear equations of the form $y = mx + b$. Proportional relationships are a special case of linear equations where $b=0$ and the slope is considered the constant of proportionality.

Baxter and Junker (2001) viewed the ability to represent a proportional relationship in terms of the constant of proportionality, k , as having achieved what they termed *mathematical* proportional reasoning. Lamon (2007) focused on this constant of proportionality, and because of this, termed this level as *proportionality* rather than *proportional* reasoning. Lamon (2007) distinguished levels of proportional reasoning through the terminology of *proportional reasoning* and *proportionality*. Vergnaud (1980) also made the distinction between Full Proportional Reasoning and the previous stage but he termed the full proportional stage of understanding as being proportional reasoning.

The term *proportional reasoning* has become a difficult term to understand with researchers attributing it to distinctly different types of reasoning. A summary of the terminology used by different researchers is found in Table 6.

Table 6
Levels of Proportional Reasoning Terminology

| Level | Marked by | Researcher | | | |
|-------|---|--------------------------|--------------------------|--------------------------|------------------------|
| | | Inhelder & Piaget (1958) | Vergnaud (1980) | Baxter and Junker (2001) | Lamon (2007) |
| 1 | Concrete Additive Relationship | Pre-proportional | Scalar Decomposition | x | x |
| 2 | Multiplicative Relationship Applied to Another Pair | Logical | Iso-Morphism of Measures | Psychological | Proportional Reasoning |
| 3 | Universally Applied | Full Proportional | Proportional Reasoning | Mathematical | Proportionality |

These levels of proportional reasoning are based on the central concept of covariation and how this relationship is applied to other values. Other researchers have proposed levels of proportional reasoning that incorporate these ideas but differ by emphasizing one particular aspect of the conceptual field of proportional reasoning: concepts, procedures, or situations. Some researchers emphasize related concepts that appear to develop alongside proportional reasoning, like quantitative reasoning (Schwartz, 1996, Smith & Thompson, 2007). Other researchers (Baxter & Junker, 2001)

distinguish levels by types of understanding: procedural or conceptual. While others (Misailidou & Williams, 2003) focus on the ability to determine appropriate situations where proportional reasoning can and cannot be applied. Modestou and Gagatsis (2010) used all three of these components to define levels of proportional reasoning, citing analogical concepts as the first level, procedural understanding as the second level, and the assessment of proportional situations as the third. This research and the influence of these components, concepts, procedures, and situations, on the development of levels of proportional reasoning are further detailed in the next section.

Influences on the Levels of Proportional Reasoning

The attainment of Full Proportional Reasoning is seen as a gradual progression through stages of understanding. Inhelder and Piaget's (1958) levels of Pre-proportional, Logical Proportional and Full Proportional Reasoning were used to outline a general idea of these levels. Researchers have differing views on how these stages should be defined and identified. These proposed levels of proportional reasoning understanding by various researchers are synthesized in Table 7. The proposed levels have been divided into categories based upon the defining character of the levels: concepts, procedures, and situations and will be discussed in detail in the following sections.

Table 7

Synthesis of Levels of Proportional Reasoning

| Highlighted Component | Concepts | Concepts | Concepts: Quantitative Reasoning | Procedures | Procedures: Relational Calculus | Situations: Context | Concepts: Analogies, Procedures, and Context |
|-----------------------|----------------------------|------------------------|----------------------------------|--------------------------|---------------------------------|--------------------------------|--|
| Researcher (date) | Inhelder and Piaget (1958) | Lamon (2007) | Research Synthesis | Baxter and Junker (2001) | Vergnaud (1980) | Misailidou and Williams (2003) | Modestou and Gagatsis (2010) |
| Level 1 | Pre-proportional | x | Scalar | x | Scalar De-composition | Familiar Contexts | Analogical Reasoning |
| Level 2 | Logical | Proportional Reasoning | Extensive | Psychological | Iso-Morphism of Measures | Familiar and Scalar Unfamiliar | Routine Proportional |
| | x | x | x | Curricular | x | x | |
| Level 3 | Full Proportional | Proportionality | Intensive | Mathematical | Proportional Reasoning | Unfamiliar Context | Meta-analogical |

Influence of Concepts: Quantitative Reasoning

Smith and Thompson (2007) emphasized the importance of quantitative reasoning in the transition between numerical reasoning and algebraic reasoning. Thompson (1994) defines quantity as those characteristics of an object that can be counted or measured, either directly or indirectly. Schwartz (1996) calls these adjectival numbers, meaning that the number has a unit of measurement which describes it, similar to an adjective describing a noun in grammar. Quantities are considered either extensive or intensive. Extensive quantities are composed of one unit of measure and can be directly counted or measured, and intensive quantities are composed of two or more measurements and cannot be directly measured (Schwartz, 1996). The quantity, *5 candies*, is an extensive quantity because it is composed of one countable unit, candies. The quantity, *5 candies per bag*, is an intensive quantity because it is composed of two countable units, candies and bags. If the quantity had been written as $\frac{5 \text{ candies}}{1 \text{ bag}}$ (fractional form) or *5 candies:1 bag* (ratio form), it would be considered a ratio of two extensive measures rather than an intensive measure. A rational number followed by a unit of measure that contains the word *per* is indicative of an intensive measure.

In addition to extensive and intensive quantities, scalar multipliers are also used in mathematics problems of quantity. For example, the number *three* in the statement, *I have three times as much candy as you* is scalar. Scalar multipliers are assumed to be devoid of unit measure. Vergnaud, however, considers scalar multipliers as special cases of intensive quantities. The number *three* could be interpreted as the relationship of two extensive quantities and could be restated as *I have 3 candies for every 1 candy*

that you have. When this is written as a ratio $\frac{3 \text{ candies}}{1 \text{ candy}}$, it converts to the intensive quantity of 3 *candies per candy* or just as the scalar multiplier of 3 because candy cancels with candy. These two quantities may seem identical but they are not. Consider the following multiplication problem involving equal grouping: *Cowtails (a caramel candy) come packaged with 5 candies in a bag. I have 3 bags. How many candies do I have?* A student may attempt to think of this problem as a scalar multiplier that could be represented as $3 \times 5 \text{ candies} = 15 \text{ candies}$ and this may be considered an acceptable representation. However, in terms of quantitative reasoning, this problem would be more accurately represented as

$$3 \text{ bags} \times \frac{5 \text{ candies}}{1 \text{ bag}} = 15 \text{ candies}$$

Schwartz (1996) and Vergnaud (1980) emphasized the precision in labeling quantities as important transitions into higher levels of thinking.

Earlier in the section on the levels of proportional reasoning, researchers seemed to agree that the difference between the Logical and Full Proportional levels of reasoning is connected to the way that the quantities are treated. If the four values of a MVPP are all considered separate, or extensive, quantities, then Logical Proportional Reasoning is attained. Only when the function relationship is defined as an intensive quantity is Full Proportional Reasoning attained. Researchers defined Full Proportional Reasoning as the ability to express proportional relationships in the form $f(x) = kx$. The constant of proportionality, k , is the intensive quantity that states the relationship between the two quantities.

As previously stated, this is a special case of the linear equation $y=mx+b$ where m is the slope or rate of change. The difference between the constant of proportionality

and slope is subtle. The constant of proportionality, k , is intensive but slope has an extensive as well as an intensive meaning. Slope can also be understood as the rise over the run of the graph. Slope, rise over run, is the ratio of two extensive quantities while slope, m , is an intensive quantity (Lobato & Thanheiser, 2002).

Howe, Nunes, and Bryant's (2010) research involved the differentiation between intensive quantities and extensive quantities within proportional reasoning problems and concluded that these two concepts are theoretically different and that these differences need to be emphasized in research. In an attempt to synthesize Howe et al.'s (2010) incorporation of intensive measure into the framework of proportional reasoning, it could be said that Pre-proportional reasoners do not attend to the intensive quantity and only think in terms of scalars, Logical Proportional reasoners think of the intensive quantity as a ratio of two extensive quantities, and Full Proportional thinkers think of it as the intensive measure that is the hallmark of proportional reasoning described by researchers.

Influence of Concepts: Analogical Reasoning

In addition to quantitative reasoning, the concept of analogical reasoning has also been studied in connection with proportional reasoning. The connection between analogies and proportions is that they both represent relationships between relations (Goswami, 1992). "Analogy pervades all our thinking, our everyday speech and our trivial conclusions as well as artistic ways of expression and the highest scientific achievements" (Polya, 1973, p. 37). Analogies define a relationship between a pair and then that relationship is applied to another pair. Polya (1973, p.14) documents that "one

of the meanings of the Greek work “analogia” from which the word analogy originates is “proportion”. Proportions are analogies between two equal ratios; therefore, in proportional reasoning problems, the relationship between the first ratio is determined and then applied or compared to the second pair.

To further define the association between analogies and proportions, the symbolism used to represent these structures is presented. The term ratio is a broad term that denotes the multiplicative relationship of two numbers. The symbol used in ratio notation is the colon, $:$. The colon is also used in analogy contexts to represent relationship. An example of an analogy using this notation is $3:\text{triangle} :: 4:\text{quadrilateral}$ and is read ‘three is to triangle as four is to quadrilateral’. The relationship is that the number defines how many sides (or angles) are in the corresponding shape. Another analogy could be represented as $3:6$ as $7:10$ (read three is to six as seven is to ten). This relationship would be that the second number in the relationship is three more than the first. This is called an *arithmetic* analogy because it involves the operation of addition (or subtraction). Proportions can use this analogy set-up as well. For example, if the question was posed $3:6 :: 7:x$, and you were told that the analogy is also a proportion, then the value of x could not be 10 as it was in the previous example. Within the context of proportional reasoning, an analogy is always meant to represent a multiplicative relationship and is termed *geometric* analogy (Prade & Richard, 2013). Therefore, the correct value for x would be 14 because the relationship between 3 and 6 is multiply by two and 7 multiplied by 2 is 14.

Modestou and Gagatsis (2010) focused on the analogy concept in order to determine their levels of proportional reasoning. They cited and furthered the research

that showed that analogical reasoning and mathematical reasoning developed concurrently and therefore warranted its inclusion in the structuring of levels of proportional reasoning. Modestou and Gagatsis' research included not just the solving of missing value and comparison proportion problems, but also verbal and arithmetic analogies. They concluded that the ability to solve verbal and arithmetic analogies preceded the ability to solve routine proportional reasoning problems. The emphasis therefore was on the concept of analogy. Their model of proportional reasoning is illustrated in Figure 8.

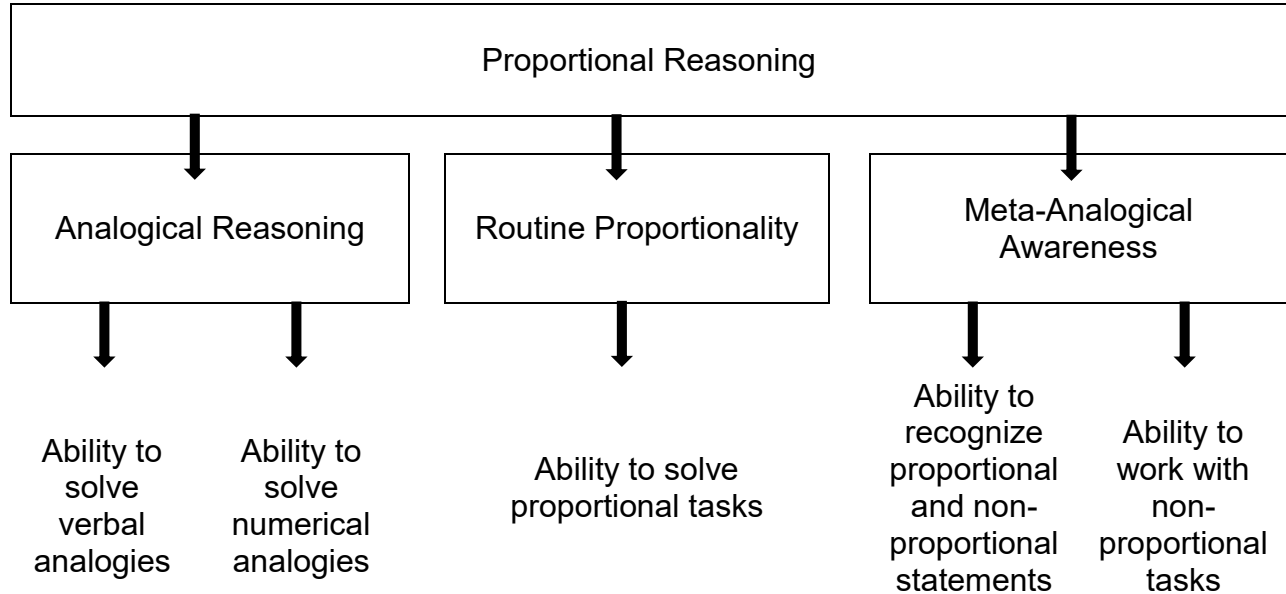


Figure 8. Levels of Proportional Reasoning Based Upon Analogy

Modestou and Gagatsis' (2010) developmental theory of proportional reasoning adds the component of analogy but it also adds the importance of procedures and contexts. These will be discussed in the next section.

Influence of Procedures

Researchers agree that there is a marked difference between both the concepts and procedures that signify each stage. The conceptual divide is between applying the relationship to another pair of values and applying the constant of proportionality universally. The procedural divide is usually marked by the ability to solve problems as equal ratios ($a/b = c/d$) or as linear functions ($y=kx$). In recent years however, the procedural divide has become complicated by instructional experiences. Researchers

have concluded that students may be able to solve routine problems with a constant of proportionality but still not understand the universality of the proportional relationship. Modestou and Gagatsis (2010) proposed a developmental theory of proportional reasoning that groups together the ability to solve problems of both forms $a/b = c/d$ and $y=kx$ in a category called routine proportionality. This is unique because solving problems of the form $y=kx$ was considered by others as Full Proportional Reasoning. Baxter and Junker (2001) also proposed this additional category into their theory and called it the curricular perspective of proportional reasoning. Both researchers concluded that the ability to solve curricular proportion problems, whether as four extensive measures or as k being a constant of proportionality, was not an appropriate determination of the attainment of proportional reasoning. This distinction between curricular familiar contexts and non-routine problems introduces the importance of context.

Influence of Contextual Situations

Misailidou and Williams (2003) in their attempt to create a diagnostic assessment for proportional reasoning classified levels of proportional understanding predominately by context combined with numerical structure. In level 1, only proportions from familiar contexts could be solved. In level 2, the context could be more difficult as long as the scalar ratio is an integer. At this level, non-integer function ratio problems could not be solved. In level 3, the context could be unfamiliar and both the scalar and function ratio problems could be solved.

Modestou and Gagatsis (2010) also emphasized the importance of context in their model but only in the final stage of Full Proportional Reasoning. They used the term *meta-analogical awareness* of proportional reasoning. The researchers described this as being able to distinguish between proportional and non-proportional tasks as well as being able to explain what elements of the problem constitute the situation as being proportional or not. This theory emphasizes the context of the task in that non-routine problems are encountered and the proportional relationship needs to be assessed. However, when Inhelder and Piaget (1958) described Full Proportional Reasoning, they too emphasized context. Inhelder and Piaget (1958) connected this final stage to Piaget's theory of cognitive development. In this theory, Full Proportional Reasoning would be housed in the formal operational stage of reasoning. With formal reasoning comes the ability to apply concepts to a variety of contexts. In other words, the formal stage could be completely devoid of any concrete objects and applied to any situation. Formal reasoning is not dependent on context and therefore, neither is Full Proportional Reasoning.

This element of context and its importance in the role of proportional reasoning can be accommodated by using the theory of conceptual fields. By explicating concepts, procedures, and situations tied to any concept, the need for extraneous vocabulary becomes obsolete. Mathematical concepts can be researched through a common lens, that of the conceptual field. The interplay between concepts and procedures applied to varying situations all come together to form an assessment of one's proportional reasoning with the goal being Full Proportional Reasoning. Therefore, Full Proportional Reasoning can be described as being able to determine *situations* where proportionality exists and being able to *calculate* the coefficient of proportionality in order to describe the linear function *relationship* between the measures and apply the relationship to other quantities. With these factors in mind, this researcher defined the concept of proportional reasoning as the evaluation of the multiplicative relationship between two covarying quantities applied universally to two other quantities of the same corresponding nature and dimension. This concept of proportional reasoning cannot be considered independent of the associated concepts, procedures and situations which facilitate proportional reasoning. The next sections address the components of procedures and situations more specifically.

Procedures

Defining proportional reasoning procedures has just as many difficulties as defining proportional reasoning concepts. In the previous section on levels of proportional reasoning, a relational calculus for solving a MVPP accompanied each of the conceptual stages of proportional reasoning: scalar decomposition (pre-

proportional), scalar and function (proportional), and linear (full proportional). The word procedure in this literature review is meant to capture what Vergnaud called the *operational invariants* combined with the *linguistic and symbolic representations* that are used in association with the concept of study. In this study, the term *set-up* will be used to identify the symbolic representations used to solve the problem. Therefore, a procedure for solving a MVPP is identified by both its *relational calculus* and its *set-up*.

Vergnaud (1998) identified five relational calculi in his research. He also detected “more than 30 different procedures” (p.194) in his work but did not define the differentiations in procedures. Rather he classified responses purely by their relational calculus. Three relational calculi were described previously, scalar addition, scalar multiplication, and function. The other two are unit-value and the rule of three which will be discussed in detail the next section. If Vergnaud differentiated between 30 procedures but classified them in only five relational calculi, what determined the classification of the 30 different procedures? The assumption is that the relational calculus was accompanied by a variety of notational representations for the set-up and solution process. These notational systems were not explicated by Vergnaud (1998). This illustrates the difficulty in classifying procedures used to solve proportional reasoning problems. The term procedure is, itself, not adequately defined in many research articles and is assumed to be understood. The researcher will attempt to provide a more robust definition of procedure.

The difference between relational calculus and set-up was identified in Weinberg’s (2002) work, *Proportional Reasoning: One Problem, Many Solutions*. The one problem in the research corresponded to the set-up of a MVPP as utilizing equal

ratios, $a/b = c/d$. The researcher showed that although the problems started with the same notation (equal ratios), three *different* solution processes were attached to it. Weinberg defined these solutions as equivalent fractions, one-step equation, and cross multiplication. All three have the set-up of equal ratios but then each has a different relational calculus: scalar, function, and rule of three (respectively). Combining the set-up of a proportional reasoning problem with its relational calculus is what will be called a procedure in this research. The relational calculus terminology and set-up terminology that are used to define these procedures is described in the next two sections.

Relational Calculus

Vergnaud (1980) based his research on the work of Inhelder and Piaget (1958). He also conducted interviews and collected student work to classify the strategies that students used in solving multiplicative structures. He classified the results into five categories of relational calculus: scalar decomposition, scalar, function, unit-value, and rule of three. The relational calculus is dependent upon the order in which the operations are performed. Each relational calculus' corresponding order of operations can be found in Table 8. In the table, the letters a, b, and c stand for the three values in a MVPP and $f(c)$ represents the missing value.

Table 8

Vergnaud's (1980) Relational Calculus for Proportion Problems

| | <table border="1" style="margin: auto;"> <tr><td>x</td><td>f(x)</td></tr> <tr><td>a</td><td>b</td></tr> <tr><td>c</td><td>f(c)</td></tr> <tr><td>1</td><td>b/a</td></tr> </table> | x | f(x) | a | b | c | f(c) | 1 | b/a | | <table border="1" style="margin: auto;"> <tr><td>mg</td><td>mL</td></tr> <tr><td>150</td><td>2</td></tr> <tr><td>225</td><td>3</td></tr> <tr><td>1</td><td>1/75</td></tr> </table> | mg | mL | 150 | 2 | 225 | 3 | 1 | 1/75 |
|---------------------|---|--|--|---|---|---|------|---|-----|--|--|----|----|-----|---|-----|---|---|------|
| x | f(x) | | | | | | | | | | | | | | | | | | |
| a | b | | | | | | | | | | | | | | | | | | |
| c | f(c) | | | | | | | | | | | | | | | | | | |
| 1 | b/a | | | | | | | | | | | | | | | | | | |
| mg | mL | | | | | | | | | | | | | | | | | | |
| 150 | 2 | | | | | | | | | | | | | | | | | | |
| 225 | 3 | | | | | | | | | | | | | | | | | | |
| 1 | 1/75 | | | | | | | | | | | | | | | | | | |
| Relational Calculus | Verbal | Notation | Numerical Example | | | | | | | | | | | | | | | | |
| Scalar | Calculate c/a and apply to b | $\frac{c}{a} * b$ | $\frac{225}{150} * 2$ | | | | | | | | | | | | | | | | |
| Function | Calculate b/a and apply to c | $\frac{b}{a} * c$ | $\frac{2}{150} * 225$ | | | | | | | | | | | | | | | | |
| Rule of Three | Calculate $b*c$ and divide by a | $\frac{b * c}{a}$ or $\frac{c * b}{a}$ | $\frac{2 * 225}{150}$ or $\frac{225 * 2}{150}$ | | | | | | | | | | | | | | | | |
| Unit Value | Calculate $f(1)$ by dividing b/a and then apply to c | $f(1) = \frac{b}{a}$ $f(c) = \frac{b}{a} * c$ | $\frac{2}{150} * 225$ | | | | | | | | | | | | | | | | |

Three of these categories are found in Inhelder and Piaget's (1958) model: scalar decomposition, scalar, and function. Scalar decomposition marks Piaget's pre-proportional level and was described in detail in that section. A notation is not presented here because the relational calculus is subjective to the pairs of numbers that are generated by the solver. Scalar and function relational calculi are used in Piaget's

Logical Proportional level. Vergnaud's research does not contain any subjects who represent proportion problems as a linear equation through the origin. The two procedures observed by Vergnaud but not by Inhelder and Piaget (1958) are termed *rule of three* and *unit value*. A possible reason for the emergence of these relational calculi is that Vergnaud used numerical quantities while Inhelder and Piaget used qualitative quantities. (This difference in research method will be discussed in the situations section of the literature review under numerical characteristics.) The existence of these two new relational calculi was confirmed by other researchers (Cramer & Post, 1993, Ercole, Frantz, and Ashline, 2011). Ercole et al. (2011) found the same relational calculi were being utilized by students but used differing terminology. Scalar decomposition was termed building-up, unit value was termed unit rate, and scalar and function strategies were grouped together as factor of change. For the purpose of this research, the author will use the terms of scalar decomposition, scalar, function, unit rate, and rule of three. The relational calculus associated with scalar decomposition, scalar, and function was discussed in the section of this research on proportional reasoning concepts. The relational calculi of the rule of three and unit rate are described next.

Rule of Three

The relational calculus associated with the rule of three does not evaluate the multiplicative relationship between either the scalar or function measures. This relational calculus is considered to be procedural rather than conceptual (Lesh et al., 1987, Cramer, Post, & Currier 1993). This could explain its absence from Inhelder and

Piaget's model, since their model was based on concepts rather than procedures. This could also explain some of the divergent descriptions on what researchers use to define proportional reasoning. Determining the relationship between quantities is not the emphasis of this relational calculus but rather, determining an order in which to carry out the operations of multiplication and division. The relational calculus begins with the measure that is missing its pair. This value is multiplied by the quantity of unlike measure and divided by the quantity of like measure. This is represented in Figure 9 using the standard example presented in this research. The visual of this relational calculus enables the viewer to see that these calculations are not parallel transformations since the arrows directing the calculation are not parallel lines.

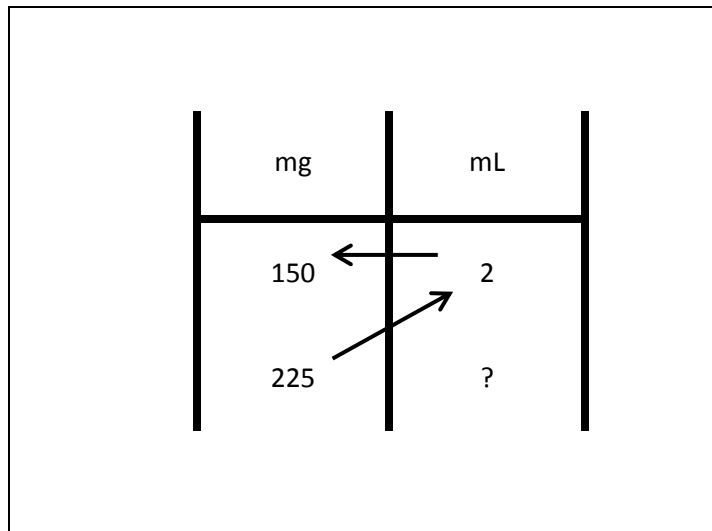


Figure 9. Rule of Three Relational Calculus

The operations for this problem would consist of starting with 225, multiplying by 2, and then dividing by 150. Because the order of the operations does not follow a parallel pattern, Baxter and Junker (2001) do not consider this relational calculus evidence of attaining psychological proportional reasoning. Nor do they indicate isomorphism of

measures in Vergnaud's (1980) model. This relational calculus is commonly tied to the cross products procedure as evidenced by Ercole et al. (2011) who used the term *cross products* rather than *the rule of three* to denote a relational calculus. The terms, *cross products* and the *rule of three* are often used synonymously. The rule of three was defined and used by Vergnaud to denote a relational calculus. In his definition, the actual set-up of the problem was not restricted. A person could use the rule of three relational calculi and make a table, use analogies, equal ratios, or even no notation at all. Cross products is specifically tied to the equal ratio set-up. Therefore, the rule of three will be considered the relational calculus which can be used with any set-up and cross products will be considered the procedure that combines the relational calculus of the rule of three and the set-up of equal ratios.

Unit Rate

The other relational calculus observed by Vergnaud (1980) was the unit value. This is also known as *unit rate* (Ercole et al., 2011) or *unitary* (Hoyles et al., 2001). The researcher will use the term *unit rate* for this relational calculus. Cramer et al. (1993) found that this strategy was the most common strategy used among students who had not yet learned cross products and also was the strategy that produced the most correct responses. The research of Vergnaud (1980) and Ercole et al. (2011), will be investigated to further describe this relational calculus.

Vergnaud described this as being a combination of function and scalar strategies even though the relational calculus is identical to the function strategy. The first step in the relational calculus for the example in Figure 10 would consist of dividing 150. The

difference between the function and the unit rate relational calculus is the interpretation of the result. With the function strategy the result is understood as the relationship between the two quantities (2 and 150), which is then applied to the third value (225). Here, the function relationship is viewed as the unit rate, or $f(1)$. The scalar relationship between the third value and 1 is always itself and therefore, $f(1)$ is multiplied by this third value.

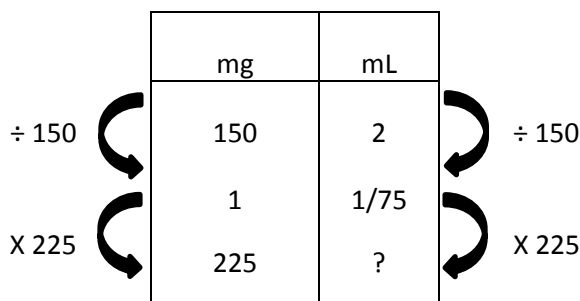


Figure 10. Unit Rate Relational Calculus

Vergnaud states that this relational calculus is “ambiguous” because of its operational similarity to the function relational calculus but the arrows denote a scalar relationship.

Ercole et al. (2011) also described a unit rate method in their research. They referred to it as the “How many for one?” strategy (p. 483) and described it as an intuitive strategy that can be used as a starting point of instruction. The inclusion of this strategy emphasized the need for students to understand that each rate could be written as two different unit rates called *dual rates* or *reciprocal rates* (p.484). This emphasis on dual rates is also cited by Cramer, Behr, and Bezuk (1989). Ercole et al. (2011) used the example of 6 apples for \$1.50. The two unit rates are \$0.25 per 1 apple and 4 apples per 1 dollar. The authors use the word per but do not consider these as intensive

quantities but rather two extensive quantities. This difference in notation between what these researchers used and the definition of intensive measure is seen in Figure 11.

| | Dual rates (Ercole et al. 2011) | Extensive Measures (Ercole et al. 2011) | Intensive Measures |
|------------------------|------------------------------------|---|---|
| 6 apples for \$1.50 | $\frac{6 \text{ apples}}{\$1.50}$ | $\frac{4 \text{ apples}}{1 \text{ dollar}}$ | $4 \frac{\text{apples}}{\text{dollar}}$ |
| | $\frac{\$1.50}{6 \text{ apples}}$ | $\frac{\$0.25}{1 \text{ apple}}$ | $0.25 \frac{\$}{\text{apple}}$ |

Figure 11. Dual Rate Notation

However, in this research, student work did not document the use of dual rates but rather the inverse properties of multiplication and division. This work is presented in Figure 12.

Problem: The Healthy Food Store sells granola by the ounce. The cost depends on the weight of the granola. Granola that weighs 8 ounces costs \$1.50. Fill in the table with the appropriate cost or weight. Explain your reasoning. (Shading indicates student writing.)

| Weight (in Ounces) | Cost (in Dollars) |
|-----------------------|----------------------|
| 6 | \$1.13 |
| 8 | \$1.50 |
| 16 | \$3.00 |
| | \$4.50 |

.1875

$$\$1.50 \div 8 = .1875$$

First step

$$6 \times .1875 = \$1.125 \text{ (1.13)}$$

$$16 \times .1875 = \$3.00$$

$$\$4.50 \div .1875 = 24$$

Figure 12. Student Unit Rate Solution from Ercole et al. (2011)

Notice how the dual rate of $5.\bar{3}$ ounces/dollar (8 ounce / \$1.50) is not used to calculate the number of ounces that can be bought for \$4.50. Instead the unit rate of 0.1875 dollars/ounce is used but instead of multiplication, division is used. Both Ercole et al. (2011) and Cramer et al. (1986) state that the unit rate method requires the solver to

decide which of the two dual rates to use. Rather, it seems that this solver, only used one of the dual rates and the decision was made whether to multiply or divide. Similarly to Vergnaud's conclusion, this strategy remains ambiguous.

In Lamon's (2012) book, *Teaching Fractions and Ratios for Understanding*, the author lists as her first requirement for characteristics of proportional thinkers that they are able to think beyond the unit rate. Lamon (2012) does not consider either the unit rate or rule of three relational calculus as identifiers of proportional reasoning. Classifying procedures by relational calculus can therefore be used to help determine what level of proportional reasoning a person has attained. When giving paper and pencil tests, the order in which values are multiplied or divided on paper can be directly interpreted to determine a relational calculus but there is other information to be gleaned from the writing on a page. Another aspect of discerning solution processes is the notational constructs that are used. These set-ups for MVPPs will be reviewed next.

Set-up

When paper and pencil tests are used as the instruments of research, the notational representations that are written on paper are clues that researchers use to infer thinking representations. Before starting with multiplication and division, the values that are related to each other proportionally are sometimes first organized by the individual. These organizational structures have not been explicitly brought together in any research that could be found by this researcher. The idea of categorizing solution strategy by set-up rather than relational calculus was unique to this research. The

importance of using set-ups to examine responses to DCPPs is that in the nursing literature, a conceptual error is determined to be a mistake in the set-up of the problem. If errors in set-ups are to be analyzed, the set-ups must first be outlined. Seven different organizational structures or set-ups, as they are referred to in this research, are compiled from the literature. Six are shown in Table 9.

Table 9

MVPP Standard Set-Ups with Research

| Set-up Name | Notational Representation | Research | | | | | | |
|----------------------------|---|--|----|-----|---|-----|---|---------------------|
| Equality of Measures | $150 \text{ mg} = 2 \text{ mL}$ $225 \text{ mg} = x$ | Ercole et al., 2011 | | | | | | |
| Ratio Table | <table border="1" style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th>mg</th> <th>mL</th> </tr> </thead> <tbody> <tr> <td>150</td> <td>2</td> </tr> <tr> <td>225</td> <td>?</td> </tr> </tbody> </table> | mg | mL | 150 | 2 | 225 | ? | Ercole et al., 2011 |
| mg | mL | | | | | | | |
| 150 | 2 | | | | | | | |
| 225 | ? | | | | | | | |
| Double Number Line Diagram | | Küchemann et al., (2014) | | | | | | |
| Analogies | $150 \text{ mg} : 2 \text{ mL} :: 225 \text{ mg} : x$ | Goswami, 1992 Modestou & Gagatsis, 2010 | | | | | | |
| Equal Ratios | $\frac{150 \text{ mg}}{2 \text{ mL}} = \frac{225 \text{ mg}}{x}$ | Weinberg, 2002 | | | | | | |
| Dimensional Analysis | $225 \text{ mg} \times \frac{2 \text{ mL}}{150 \text{ mg}} = \text{ ______ } \text{ mL}$ | Rice & Bell, 2005 | | | | | | |

Each of these set-ups will be described in the next section. The seventh set-up is known as the nursing rule and will be covered in the section of Dosage Calculation Procedures.

Equality of Measures

The term equality of measures is not found in the literature as a named set-up for proportional reasoning problems. The set-up is presented in an article in *Mathematics Teaching in the Middle School* (Ercole et al., 2011) as an intuitive notational construct that is used by a student to solve a MVPP. The problem and student's response are shown in Figure 13.

Apple Packing: Carrie is packing apples for an orchard's mail order business. It takes 3 boxes to pack 2 bushels of apples. How many boxes will she need to pack 8 bushels of apples?

Student Work:

3 boxes = 2 bushels

6 boxes = 4 bushels

9 boxes = 6 bushels

12 boxes = 8 bushels

Figure 13. Student Generated Solution from Ercole et al. (2011)

The student took the measures that compose a ratio in the problem, boxes and bushels of apples, as having an equality relationship. Instead of writing 3 boxes:2 bushels of apples, the student wrote 3 boxes = 2 bushels (of apples). The researcher did not see

this representation used in other research and therefore assumes that it has not been named. The researcher named it *equality of measures* as the construct sets two measures equal to each other.

If this set-up were used with the standard example presented in this research, the notation would look like that found in Figure 14.

| |
|---------------------------------|
| $150 \text{ mg} = 2 \text{ mL}$ |
| $225 \text{ mg} = ?$ |

Figure 14. Proposed Equality of Measures Set-up

By setting 150 mg equal to 2 mL, the implication is that if there is 2 mL of medicine then there also is a mass of 150 mg. Milligrams measures mass and milliliters measures volume. Two units that represent different measures cannot be equal. The units covary multiplicatively and therefore can be represented as a ratio but not as equalities. The equality could be a natural notational consequence of the quantities being two different attributes of the same object. The sameness of the object could be where the equality is intuitively introduced.

In the case of unit conversion, an equality relationship is accurate. For example, if a student were asked to represent the question “How many inches are in 4 feet?”, an appropriate representation would begin by stating that 1 foot equals 12 inches as seen in Figure 15.

| |
|--------------------|
| 1 foot = 12 inches |
| 4 feet = ? inches |

Figure 15. Example of Unit Conversions as Equality of Measures.

Since 1 foot does equal 12 inches, this set-up would be notationally correct and therefore would only be appropriate to use in the case where the units measure the same attribute. Whether this notation should be condoned as an appropriate set-up for mixed-measure problems should be considered by researchers. For the purpose of the current research, this researcher will consider it as acceptable.

Ratio Table

The ratio table is a table used in mathematics to keep track of quantities that covary multiplicatively. Pairs of equivalent ratios are generated by either multiplication or scalar addition until the desired ratio is found. “The ratio table is a flexible computational tool that both acts as a visual pattern to aid in operating with rational numbers and connects different notations of rational numbers” (Middleton & von den Heuvel-Panhuizen, 1995, p. 284). One of the benefits of a ratio table is its flexibility to encourage different relational calculi (Gravemeijer & van Galen, 2003, Shield & Dole,

2002). A ratio table is used in Figure 16 to show the solution for the standard example used in this research.

| | | | |
|----|-----|----|-----|
| mg | 150 | 75 | 225 |
| mL | 2 | 1 | 3 |

Figure 16. Ratio Table for Research Example

This example has a relational calculus of scalar decomposition because the quantities in the original ratio are divided by the scalar quantity of 2 to get the ratio 75 mg:1 mL and then 75 mg is added to 150 mg to get 225 mg and 1 mL is added to 2 mL to get 3 mL. The ratio table can be used with other relational calculi as well. In Figure 17, the same problem is solved using a ratio table but a unit rate relational calculus is applied.

| | | | |
|----|-----|--------------------------------|-----|
| mg | 150 | 1 | 225 |
| mL | 2 | $.01\bar{3}$ or $\frac{1}{75}$ | 3 |

Figure 17. Ratio Table Using Unit Rate Relational Calculus

Ercole et al. (2011) considers the ratio table a more structured form than the equality of measures and suggests that it could help transition into set-ups that require using a scalar or function relationship rather than scalar decomposition.

Double Number Line Diagrams

Küchemann, Hodgen, and Brown (2014) describe the recent popularity of double number line diagrams or DNL as models to solve proportional reasoning problems. The double number line diagram is a recommended way to reason about ratios and rates in the Common Core State Standards of Mathematics. (CCSS.Math.Content.8.RP.A.3). An example of this set-up is shown in Figure 18 using the standard example from this research.

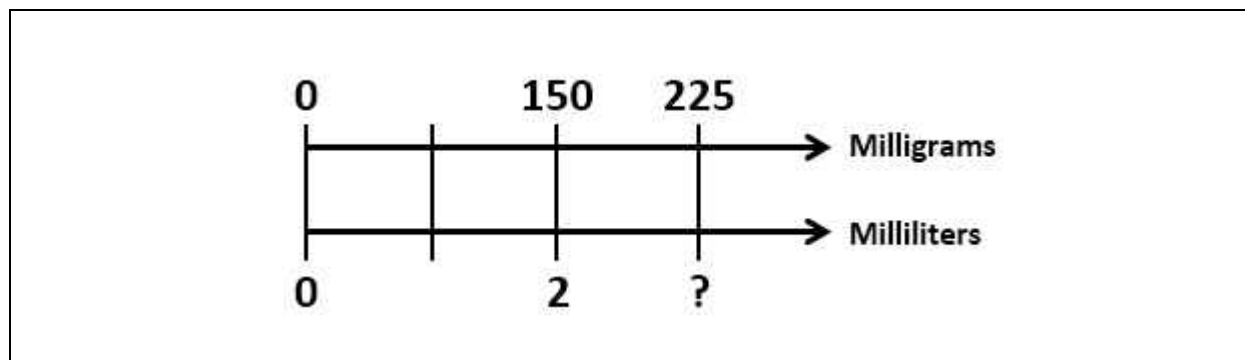


Figure 18. Double Number Line Diagram Set-up

The DNL has two number lines that begin together at zero. The scales are not the same for each number line but rather are determined by coordinated pairs. Since 150 mg corresponds to 2 mL in the example, they are drawn vertically from one another on the number lines. Additional values are located in the correct numerical order. This inclusion of magnitude makes it a more accurate visual than a ratio table. The act of coordinating units and partitioning were found to be crucial skills in working with this model (Orrill & Brown, 2012). Any of the relational calculus can be used with this representation.

Analogies

Analogical reasoning was discussed in the concept portion of this literature review as being tied to proportional reasoning. An example of a general analogy is $8:16 :: 1:2$ and is read eight is to sixteen as one is to two. If the relationship between the values in the analogy is known to be multiplicative, the double colon can be substituted with an equal sign and the colon between the values is interpreted as implying a ratio relationship. For example, $8:16 = 1:2$ is read as *the ratio of eight to sixteen is equal to the ratio of one to two*. Replacing the double colon with an equal sign is important because arithmetic analogies are not equal. For example, $3:6 :: 7:10$ (relationship is add three) cannot be rewritten as $3:6 = 7:10$ (because $3/6 = 0.5$ and $7/10 = 0.7$). Therefore, the presence of an equal sign distinguishes whether an analogy is proportional (geometrical) or not.

Solving a multiplicative analogy can be done using any of the relational calculi. The standard example for this research is presented in Figure 19.

$$150 \text{ mg} : 2 \text{ mL} = 225 \text{ mg} : x$$

Figure 19. Analogy Set-up

This analogy can be read as *the ratio of 150 mg to 2 mL is equal to the ratio of 225 mg to x*. This implies that the intensive measure of 150 mg divided by 2 mL (75 mg/mL) is equal to the intensive measure of 225 mg divided by x. The function and scalar relational calculi used with analogies is not named but the rule of three relational calculus used with the analogy set-up is called the *means and extremes* (Rice, 2002). This will be reviewed in the next section on Specific Procedures.

Equal Ratios

A common definition for proportions is equal ratios. Setting up a proportion as equal ratios is considered using the set-up of $a/b = c/d$. This is similar to the analogy set-up however the ratios are written in fractional form (a/b) rather than ratio form ($a:b$). This set-up is illustrated in Figure 20.

$$\frac{150 \text{ mg}}{2 \text{ mL}} = \frac{225 \text{ mg}}{x}$$

Figure 20. Equal Ratios Set-up

As Weinberg (2002) noted, three relational calculi (scalar, function, and rule of 3) have been associated with equal ratios and all are given names for procedures therefore these will be discussed in the next section on Specific Procedures.

Dimensional Analysis

Dimensional analysis is also known as the factor-label method, conversion factor method or unit analysis because the set-up involves the use of factors and unit labels (Rice & Bell, 2005). The factors used in the set-up are called conversion factors. A conversion factor is a ratio of two extensive quantities that are thought of as being in an equality relationship. The quantities are placed either in the numerator or the denominator, depending on their label or unit of measure. Labels (and their corresponding magnitude) are lined up so that like units can be cancelled (Reed, 2006).

The standard example for this research is interpreted as a need to convert the mass of 300 milligrams to milliliters. This set-up is illustrated in Figure 21.

$$300\text{mg} \times \frac{2\text{mL}}{150\text{mg}} = 4\text{mL}$$

Figure 21. Dimensional Analysis Set-up

The starting quantity of 300 mg is converted to 4 mL by using the conversion factor of $\frac{2\text{mL}}{150\text{mg}}$. The extensive quantities in the conversion factor, 2 mL and 150 mg, are considered to be in an equality relationship rather than a ratio. This equality relationship causes the conversion factor to be interpreted as equaling 1. If 2 mL = 150 mg, then 2 mL divided by 150 mg must equal 1 since anything divided by itself is 1. Any number of conversion factors can be multiplied to the original quantity with the idea that the conversion factor must equal 1. Therefore, since the original quantity, 300 mg, has been multiplied by 1, then it must equal 4 mL.

Dimensional analysis was not used by any of the participants in Vergnaud's (1980) research. This set-up is predominately tied to scientific applications as well as the field of nursing, therefore the research concerning its use as a set-up to solve DCPPs will be presented in the section on Dosage Calculation Procedures. In addition, the nursing rule is a set-up that is specific to the field of nursing and therefore will also be reviewed in this future section.

Specific Procedures

Using the groundwork of defining the relational calculi and set-ups that can be utilized to solve a MVPP, a definition for procedure can be created. A procedure for solving a MVPP is defined by this researcher as the unique combination of a relational calculus associated with a set-up. Five relational calculi (scalar decomposition, scalar, function, unit value, and rule of three) and six set-ups (equality of measures, ratio table, double number line, analogies, equal ratios, and dimensional analysis) have been outlined thus far in this literature review. These could combine to account for 30 strategies or procedures as they will be referred. If the option of not identifiable were included in the number of set-ups, this would then come to 35 procedures. Each of these unique combinations would be the definition of a procedure. Some of these combinations of set-ups and relational calculi have already been named in the research or in practice and are summarized in Table 10.

Table 10

Procedures for Solving Proportional Reasoning Problems

| Set-up | Relational Calculus | Procedure | Research |
|----------------------|---------------------|----------------------|-------------------|
| Equal Ratios | Scalar | Equivalent Fractions | Weinberg, 2002 |
| Equal Ratios | Rule of Three | Cross products | Weinberg, 2002 |
| Analogies | Rule of Three | Means and Extremes | Rice, 2002 |
| Dimensional Analysis | Rule of Three | Dimensional Analysis | Rice & Bell, 2005 |

The procedures of equivalent fractions, cross products, means-and-extremes and dimensional analysis all result from specific combinations of set-up and relational calculus and warrant special attention.

Equivalent Fractions

The equivalent fraction procedure is the combination of the scalar relational calculus with the equal ratios set-up. This term is used by Weinberg (2002). Cramer and Post (1993) called this the *fraction strategy*. After the equal ratios are set up as $a/b = c/d$, then “the fraction rule for equivalent fractions” is used to calculate the answer (Cramer & Post, 1993, p. 407). This fraction rule is further described as multiplying both the numerator and the denominator of the fraction (ratio) by the same number. This procedure is shown in Figure 22.

$$\frac{150 \text{ mg}}{2 \text{ mL}} = \frac{225 \text{ mg}}{x}$$
$$\frac{150 \text{ mg}}{2 \text{ mL}} \times \frac{1.5}{1.5} = \frac{225 \text{ mg}}{x}$$
$$x = 3$$

Figure 22. Equivalent Fraction Procedure

The scale factor of 1.5 could be determined by dividing 225 by 150. This scale factor is then multiplied to both 150 mg and 2 mL. Because of the multiplication of the same number to the numerator and denominator, this scale factor is equivalent to the number one, thus the equality is maintained.

Cross products

Cross products is a procedure that combines the equal ratio set-up with the rule of three relational calculus. This procedure is commonly seen in nursing DCPD literature and is therefore covered in the section on Dosage Calculation Procedures.

Means and Extremes

Using the rule of three relational calculus with an analogy is called means and extremes. A property of this analogy relationship is that the product of the means equals the product of the extremes, where the means are the inner numbers (the second and third values) and the extremes are the outer numbers (the first and fourth values). This procedure is seen in Figure 23.

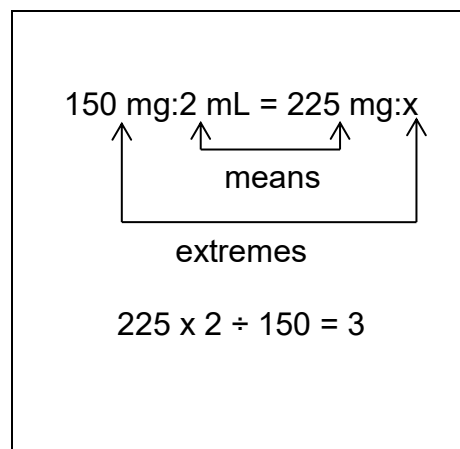


Figure 23. Means and Extremes Procedure

The means and extremes procedure uses the rule of three relational calculus but analogies can be solved using the scalar and function relational calculi as well. However, these are not named in the literature.

Situations

Proportional reasoning problems are encountered in different situations which create varying levels of difficulty. All situations are categorized as either quantitative or qualitative. Quantitative proportion problems involve numerical measures. Qualitative problems do not contain numbers but rather descriptions, such as heavier or more. The researcher decided to focus on quantitative proportions only. Studies have utilized qualitative problems to try and address reasoning and to avoid rote calculation (Inhelder & Piaget, 1958, Noelting, 1980, Cramer & Post, 1993). An example of a qualitative proportion problem can be found in Figure 24.

If Nick mixed less lemonade mix with more water than he did yesterday, his lemonade drink would taste:

- a) Stronger
- b) Weaker
- c) Exactly the same
- d) Not enough information to tell.

(Cramer & Post, 1993, p.405)

Figure 24. Qualitative Proportional Reasoning Problem

Qualitative proportional reasoning problems will not be addressed in this research since the DCPD literature does not focus on these types of problems.

Much of the research on quantitative proportional reasoning centers around two types of problems: missing value and comparison (Ben-Chaim, Fey, Fitzgerald,

Benedetto, & Miller, 1998; Fleener, 1993). MVPPs require that one of the four extensive quantities that make up two equal ratios be evaluated. The procedures used to solve MVPPs were discussed in the previous section. Comparison problems provide all the values of two ratios and require the solver to determine how a function relationship between two quantities compares to another function relationship between two other quantities of the same measure. If the ratios are not equal, the problem usually asks which one is greater (or less). Consumer best-buy problems are a familiar context for these problems. An example is illustrated in

Figure 25 where the solver is asked to calculate which is a better buy: 32 ounces for \$2.00 or 20 ounces for \$1.50.

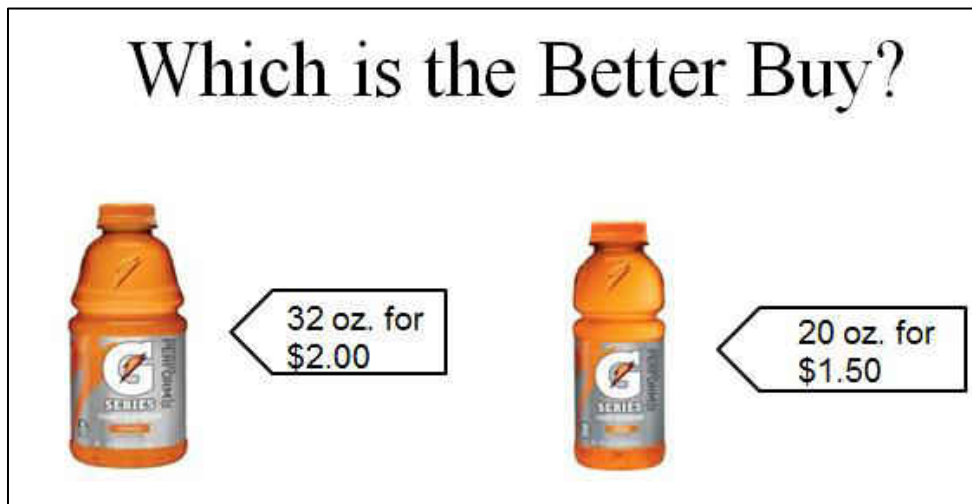


Figure 25. Comparison Problem Example

Comparison problems and MVPPs can be considered together in mathematics education research on proportional reasoning. This is not the case involving the nursing research and DCPPs. The nursing research is predominately concerned with MVPPs.

Because of this difference in research domains, this research focuses on MVPPs but introduces the best buy problem seen in

Figure 25 in an attempt to extend the nursing literature on the topic.

Although comparison problems do not use the same relational calculus as MVPPs, both types of problems are defined as having the same factors that affect problem difficulty. Baxter and Junker (2001) defined problem difficulty as a combination of four factors: contextual structure, numerical characteristics, procedure use, and conceptual understanding. Conceptual understanding and procedure use have already been discussed in this literature review. In this section, Baxter and Junker's (2001) factors of contextual structure and numerical characteristics will be discussed as well as factors presented by other researchers. In order to consistently organize this information with the theory of conceptual fields, these additional factors will be considered as the differing situations in which proportional reasoning problems present themselves. The researcher defines these situations as the moderating affects that affect problem difficulty and/or procedure choice. They include numerical characteristics, semantic type, contextual structure, presentation, and student characteristics (Vergnaud, 1988, Baxter and Junker, 2001). Each of these factors will be described in detail in the following sections and is summarized in Table 11.

Table 11

The Conceptual Field of Proportional Reasoning

| Concepts | Procedures | | Situations | | | | |
|--|---|---|--|--|---|--|--|
| (3 Levels) | Relational Calculus (5 Varieties)* | Set-up (7 Varieties)* | Numerical Characteristics* | Semantic Type* | Context | Presentation | Student Characteristics |
| <ul style="list-style-type: none"> • Pre • Logical • Full | <ul style="list-style-type: none"> • Scalar • Decomp. • Unit Rate • Scalar • Function • Rule of 3 | <ul style="list-style-type: none"> • Equality of Measures • Ratio Table • DNL • Analogies • Equal Ratios • Dimensional Analysis • The Nursing Rule | <ul style="list-style-type: none"> • Integer or Non-Integer Relationship • Discrete or Continuous Quantities | <ul style="list-style-type: none"> • Well-Chunked Measures • Associated Sets • Part-Part-Whole • Scaling | <ul style="list-style-type: none"> • Familiar • Context Bound | <ul style="list-style-type: none"> • Enactive • Iconic • Symbolic | <ul style="list-style-type: none"> • Disposition • Learning Styles |

This organizational structure is a synthesis of the research between proportional reasoning and conceptual fields and is the guiding organization structure of this research.

*Specific to quantitative missing value proportion problems

Numerical Characteristics

Four numerical characteristics that affect the solution process of MVPPs were found in the literature. They are: order of the missing number, size of the numbers, the presence of integer relationships (either scalar or function), and whether the data measurements are discrete or continuous (Tournaire & Pulos, 1985). The order of the missing number is associated with MVPPs and refers to the position of the missing number within the proportion. The size of the numbers means that larger numbers cause larger problems. Both the order of the missing number and size of the number are not a major focus of research (Tournaire & Pulos, 1985) and will not be considered here. Rather the literature on numerical characteristics focuses on the presence of integer relationships and whether the data are discrete or continuous.

Integer or Non-Integer Relationships

The presence of integer vs. non-integer relationships is found to impact procedure use (Karplus, Pulos, & Stage, 1983, Bezuk, 1988). Karplus et al. (1983) studied how the numerical structure of proportional reasoning word problems affected the chosen relational calculus used by adolescents. They presented four numerical types of problems to their research participants: 1) where both the scale factor and the function factor were integers, 2) only the scale factor is an integer, 3) only the function factor is an integer, and 4) where neither of the factors were integers. They found that students' relational calculus is affected by these factors and those students changed

their procedural use dependent on the relationship that possessed an integer relationship.

Bezuk (1988) also found that the numerical structure of proportional reasoning word problems affected the chosen relational calculus for elementary preservice and in-service teachers. She used the same numerical classifications as Karplus et al. (1983) and found that the teachers were able to flexibly use different strategies dependent on the numerical structure. Three predominate relational calculi were identified: scalar, unit rate, and rule of three. (The researchers used the terms *procedures*, *factor of change* instead of scalar and *cross products* instead of rule of three.) Each relational calculus use increased when its corresponding numerical type was represented by an integer as indicated in Table 12.

Table 12

Relational Calculus Use and Integer Relationship from Bezuk (1988)

| Integer Relationship | Unit Rate | Scalar | Rule of Three | Other | Total |
|----------------------|-----------|--------|---------------|-------|-------|
| Function Integer | 77 | 2 | 14 | 7 | 100 |
| Scalar Integer | 29 | 37 | 20 | 14 | 100 |
| Neither | 39 | 20 | 25 | 16 | 100 |

The unit rate relational calculus was used most frequently when the function relationship was an integer and the scalar relational calculus was used most frequently when the

scalar relationship was an integer. Although the unit rate relational calculus was used more frequently than the rule of three in solving problems with no integer relationships, the rule of three (cross products algorithm) had its highest usage in this category.

Discrete or Continuous Data

The presence of either discrete or continuous data in a proportional reasoning problem has been shown to have an effect on problem difficulty. Defining data as either discrete or continuous is associated with identifying the quantity as an extensive quantity. Previously in this literature review, extensive quantity was defined as either countable or measurable. Countable quantities are considered discrete and measurable quantities are considered continuous (Fleener, 1993). Karplus et al. (1983) incorporated discrete or countable data compared to continuous or measurable data in their study. Discrete quantities included pieces of gum and laps of a school track. Quasi-continuous quantities included money and time. Within the context of nursing, pills are countable or discrete while liquids are measurable or continuous.

Fleener (1993) composed a construct for examining levels of proportional reasoning and included categories of discrete and continuous quantities. Fleener (1993) and Karplus et al. (1983) found that discrete data are generally easier to use than continuous data. Fleener actually used discrete and continuous quantity as semantic classifications in her study. She placed semantic categories in a hierarchical order as follows: magnitude, discrete, continuous, consumption, ratio measure, and compensatory. This idea of categorizing problems by semantic type is considered next.

Semantic Type

Introduction to Semantic Types




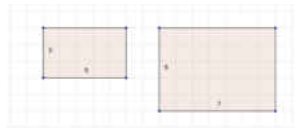
Semantics involve the categorization of word problems into problem types in order to aid in the solution process (Sherin & Fuson, 2005). Semantics and context can be confused in the literature and so a clarification needs to be made between the two. While both semantics and context are defined by the types of quantities that are used in the problem, semantics focus on the mathematical characteristics of the quantities and contexts focus on the scenario in which the quantities are encountered. Contexts are described in detail in the next section. Here the focus is on the mathematical characteristics of quantity. The terminology used to explicate these characteristics is introduced. The semantic types for multiplication and division problems are used as an introduction because of their close connection to the semantics of MVPPs.

Lamon (1993) used problem semantic type to classify the types of proportional reasoning word problems that we encounter. She classified them in four categories: well-chunked measures, associated sets, part-part-whole, and stretchers and shrinkers. These classifications were used to develop one of the assessment instruments used in this study, the Everyday Proportion Problems. Questions from this instrument will be used as examples so that the reader can become familiar with the semantic types as well as the research tool used in this study. Please refer to the actual questions in APPENDIX A.

Three other studies influenced the development of important semantic types. These are studies by Karplus et al. (1983), Shield and Dole (2002) and Ben-Chaim et

al. (1998). The comparisons of their categorization system to Lamon's (1993) can be found in Table 13.

Table 13
Research Classifications of Semantic Types

| Karplus et al. (1983) | Shield & Dole (2002) | Ben-Chaim et al. (1998) | Lamon (1993) | Everyday Proportion Problem Ratio Labels |
|-----------------------|-------------------------|-------------------------------------|--------------------------|---|
| Rates | Whole: Whole | Rate or Density Problems | Well-chunked Measures | Dollars:Ounce  |
| | | | Associated Sets | People:Eggs  |
| Ratios | Part:Part or Part:Whole | Comparisons of two parts of a whole | Part-Part-Whole | Brown Eggs: White Eggs  |
| | | Scaling Problems | Stretchers and Shrinkers | Side:Side  |

The three studies all differed from Lamon's in that they did not differentiate between associated sets and well-chunked measures. Also, Karplus et al. and Shield and Dole did not single out scaling as a separate semantic type. Ben-Chaim et al. (1998) categorization system is based upon a more scientific background incorporating the use of the word density. Karplus et al. used ratio and rate language to define their semantic types. Shield and Dole (2002) presented a system of categorization based upon their assessment of textbooks and refined the ratio and rate language into part-whole concepts. These categorizations are an important concept of proportional reasoning and the terminology will need to be introduced before the differing categorizations are presented in further detail.

Ratios are a way of representing two numbers that are in a multiplicative relationship to each other. Symbolically, ratios can be represented by either using a colon, $a:b$ or a bar, $\frac{a}{b}$. (This causes confusion because of the close connection of the bar notation to fraction operations (Shield & Dole, 2002).) The relationship between two quantities in a ratio can also be further refined. The relationships are determined using the terms part and whole depending on the context, illustrating the close connection between semantics and context. The resulting ratios could be part:whole, whole:whole, part:part, or whole:part. Part:whole ratios are more commonly known as fractions. Whole:whole ratios are more commonly known as rates. Part:part ratios have not been assigned a specific name and so they are just known as ratios. An analogy to this is that a quadrilateral that is not a parallelogram, trapezoid, rectangle, etc. is simply considered a quadrilateral. Whole:part ratios are not named because they are not the common

convention and are traditionally re-written in terms of part:whole. Karplus et al. (1983) used this categorization system but only differentiated between rates and ratios. In this study, fractions or part:whole ratios were considered as ratios.

In all categorization systems, the meaning of the ratio measure constitutes the categorization. This meaning is evaluated by the context and the labels used within the context help to guide in the evaluation of whether the values are parts or wholes. Schwartz (1996) considered numbers with labels as quantitative measures and incorporated the study of quantitative reasoning in his understanding of proportional reasoning. This research was presented in the section of the literature review on the Influences of Concepts: Quantitative Reasoning. Schwartz (1996) categorized semantic types using the terminology of intensive and extensive measure. The researcher attempts to connect their research on extensive and intensive measures to the semantic types used by Lamon. Four semantic types well-chunked measures, associated sets, part-part-whole, and scaling, will be described next. The connections between the ratio language of Karplus et al. (1983) and Shield and Dole (2002), the scientific language of Ben-Chaim et al. (1998), and the quantitative measure language of Schwartz (1996) will be incorporated. After the semantic types are presented, a possible framework for incorporating quantitative language into this structure will be presented. Examples from the Everyday Proportion Problem instrument designed for this study will be used for illustration.

Well-Chunked Measures

Well-chunked measures represent whole:whole relationships. The term *well-chunked measures* indicates that the numbers in the ratio have meaning as an intensive quantity or unit rate. The word *per* between the units, as in miles per gallon, miles per hour, or dollars per item has meaning. The example found in Figure 26 uses a dollar per ounce intensive measure.

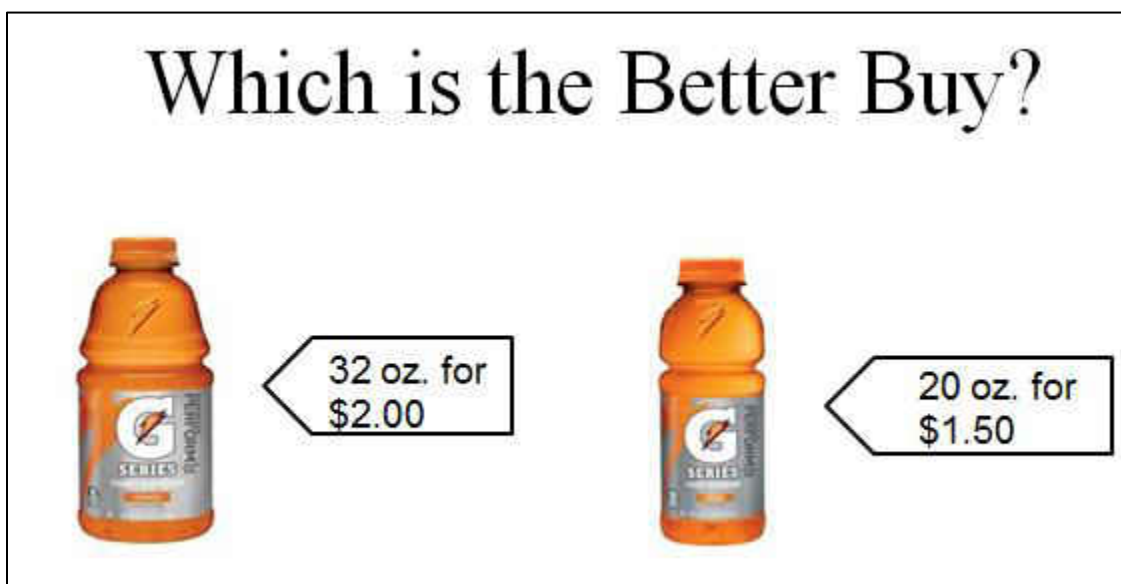


Figure 26. Well-Chunked Measures Problem Example

This example was adapted from a problem used by Ben-Chaim et al. (1998). Ben-Chaim et al. (1998) referred to both associated sets and well-chunked measures as rates or density problems. Shield and Dole (2002) did not report whether the textbooks they examined distinguished between associated sets and well-chunked measures. In their research, this semantic type was classified as whole:whole ratios.

Associated sets

Associated sets could be seen as distinct sets of numbers. The numbers in the ratio are dictated by the context as not commonly being seen. One would not normally put the two values in association with each other unless an “explicit statement in the problem indicates that rate pairs should be formed” (Lamon, 1993, p 42). In the example below in Figure 27, 8 eggs would be in a rate with 14 people.

How many people will a 12-egg recipe serve?

Recipe
Serves 14 people
Flour
Sugar
8 eggs
Vanilla

Recipe
Serves ??? people
Flour
Sugar
12 eggs
Vanilla

Figure 27. Associated Sets Proportion Problem Example

This example is adapted from problems found in research concerning associated sets (Allain, 2000, Ozgun-Koca & Altay, 2009). The quantities in the ratio, eggs and people, constitute a whole:whole relationship. Because people and eggs is not a natural or common pairing, the intensive quantity of eggs per people or people per eggs formed by this ratio is not a common unit rate and therefore the values are treated as four extensive measures.

Part-Part-Whole

Part-part-whole problems can often be interpreted using either the part:whole relationship or the part:part relationship. The example in Figure 28 provides a context for a part-part-whole problem.

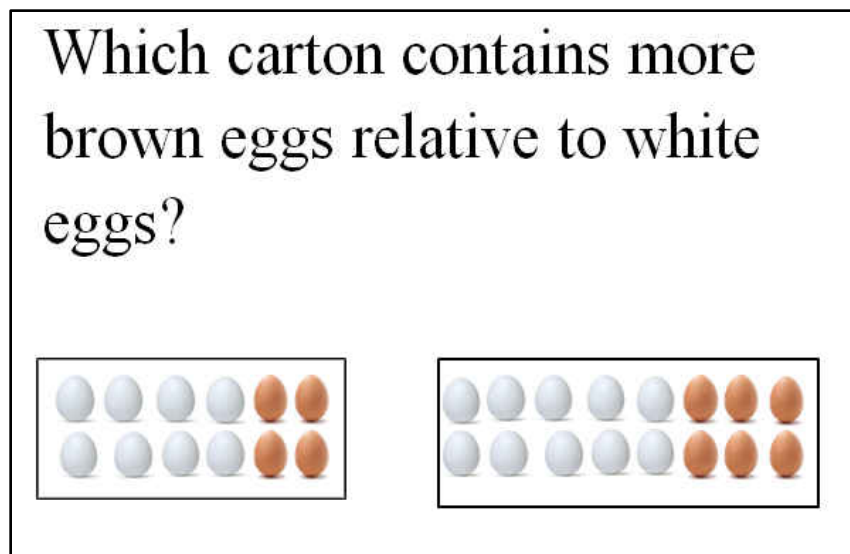


Figure 28. Part:Part or Part:Whole Problem Example

This problem was adapted from problems presented in research on part:part:whole semantics (Allain, 2000, Ozgun-Koca & Altay, 2009). Both parts of the ratio describe eggs and so are close in relationship. The problem can be solved using any one of four ratios: a part:part ratio of brown eggs:white eggs, a part:part ratio of white eggs:brown eggs, a part:whole ratio of brown eggs to the total number of eggs, or a part:whole ratio of white eggs to the total number of eggs. All of these ratio values could technically be labeled with the unit label of eggs. The resulting intensive measure would be eggs per egg. This part-part-whole relationship makes not only intensive

measure difficult to interpret but also poses problems keeping track of extensive measures since all of the extensive measures can potentially have the same label.

Scaling

Ben-Chaim et al. (1998) and Lamon included a separate category for scale measurements. An example of a scaling problem was adapted from the literature (Miyakawa & Winslow, 2009) and is found in Figure 29.

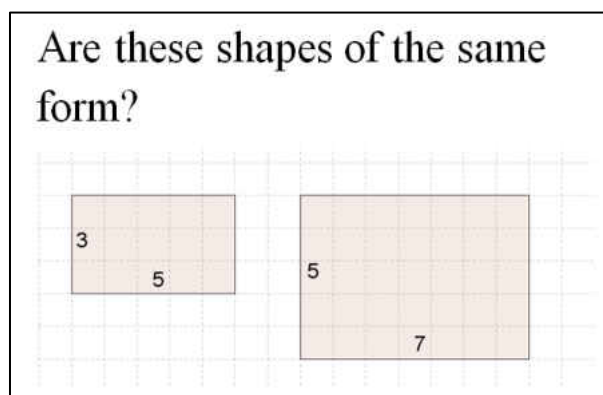


Figure 29. Scaling Problem Example

The ratio of length:width of the rectangles would be proportional if the rectangles are of the same form. The terminology of *same form* is used in the research of Miyakawa and Winslow (2009) to replace the more mathematically contextualized language of *scale* or *proportional*. This scaling problem uses the ratio of length and width which would be considered parts of the same whole and use like units of measurement. Because of this part:part relationship, Karplus et al. (1983) as well as Shield and Dole (2002) did not differentiate between ratios and scale measurement. They considered scaling problems




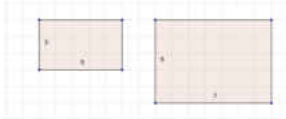
as dimensionless. These dimensionless quantities are explicitly called scalar quantities in the field of quantitative reasoning (Schwartz, 1996).

Semantic Type Synthesis

The research involving semantic types was synthesized and organized using the terminology of well-chunked measures, associated sets, part-part-whole, and scaling. These terms describe and give meaning to the relationship between the two quantities that form the ratio relationship in contextual word problems. The meaning that is given to the ratio based upon the context determines its semantic type classification. Past research classifies these relationships by the extensive quantities, whole:whole, part:part, or part:whole. This researcher conjectures that meaning can also be found in the intensive quantity formed by the ratio. These ratios and corresponding intensive quantity are presented in Table 14.

Table 14

Interpreted Intensive Quantity Semantic Type

| Semantic Classification Lamon (1993) | Everyday Proportion Problem Example | Extensive Quantities | Intensive Quantities | Interpreted Quantity |
|---|---|-----------------------|--------------------------|----------------------|
| Well-chunked Measures |  | Dollars:Ounce | Dollars per Ounce | Intensive |
| Associated Sets |  | People:Eggs | People per Egg | Extensive |
| Part-Part-Whole |  | Brown Eggs:White Eggs | Brown Eggs per White Egg | Extensive or Scalar |
| Scaling |  | Side:Side | Side per Side | Scalar |

The intensive quantity that correlates to the ratios could be interpreted in a hierarchy according to their meaning: ounces per dollar, people per egg, brown eggs per white egg, and side per side. For well-chunked measures, the intensive quantity is natural: ounces per dollar. The intensive quantity of people per egg has meaning but is not as natural. The intensive quantities for part-part-whole ratios have little meaning and in the case of scaling are reduced to scalar quantities. The subjective nature of *meaning* causes the distinction in semantic types. Perhaps it is how the ratio is

interpreted by the solver as an intensive quantity, two extensive quantities, or a scalar quantity that affects difficulty and procedure choice rather than a researcher prescribed semantic categorization. Schwartz noted that students have difficulty with intensive measures that are created by the ratio of a discrete quantity to another discrete quantity, for example, 14 people:8 eggs. When evaluating the intensive measure, a non-integer intensive quantity may not have meaning, for example, 1.75 people per egg. This could cause conflict in some minds because *people* is a discrete quantity and should be represented as an integer. Karplus et al. (1983) who differentiated semantic types by rates and ratios, underscored the role of extensive and intensive quantities by summarizing the conceptualization of proportional reasoning problems devoid of subjective meaning succinctly as:

Proportional reasoning can be conceptualized in these steps: identification of two extensive variables that are applicable, recognition of the rate of intensive variable whose constancy determines the linear function, and application of the given data and relationships to find (i) an additional value for one extensive variable (missing value problem) or (ii) comparison of two values of the intensive variable computed from the data (comparison problem). P.219

Karplus et al.'s (1983) step in the conceptualization process that requires the "recognition of the rate of intensive variable" speaks to the difficulty that semantic problems that are interpreted as extensive or scalar quantities could present. The ability to solve proportional reasoning problems without the influence of context is the benchmark of Full Proportional Reasoning. However, for students who are in the Logical Proportional level, the influence that context has on problem difficulty is a factor and is described further in the next section.

Other semantic types appear in research. Fleener (1993) described what she called, six structure types: magnitude, discrete, length, consumption, ratio measure, and compensatory. These categories mixed several components of the situations that are being explicated in this literature review. For example, Fleener (1993) classified problems with countable or discrete numbers as having the structure type of discrete. Within the framework of this research, discrete data would be considered a numerical characteristic and not a semantic type. Although Fleener's (1993) structure types were incorporated under different categories within the framework of this research, one component of her framework is utilized explicitly and that is her differentiation between contextual features of a problem. These features are described in the next section.

Context

Fleener (1993) conjectured that familiarity with a context might have a stronger impact on success rate than other variables. Fleener (1993) described context as either familiar or context bound. In an experiment involving physics compensatory problems, Fleener found that when the problem utilized a teeter-totter (or see-saw) context, the success rate jumped from 66% to 83%. This problem is reproduced in Figure 30.

A teeter totter is 10 feet long with the fulcrum exactly in the middle of the teeter totter. If George weighs twice as much as his brother Gerald, how could the boys sit to make the teeter totter balance? (Include a diagram and explain your answer.)

Figure 30. Teeter Totter Problem from Fleener (1993)

Categorizing contexts as familiar or context bound could be highly subjective. What is assumed to be familiar to one may not be familiar to all. In order to explicate this, Bayazit (2013) classified problems in three categories: socio-economic, scientific, and intra-mathematical. While this study did not focus on the impact of such contexts, it provides a framework for the classification of such contexts. Bayazit (2013) considered socio-economic contexts as being a part of student's daily life, like sports or school activities. Scientific problems were applied to the context of the sciences like physics, biology, and geography. Intra-mathematical were purely mathematical constructs. Using this categorization method, Bayazit (2013) reviewed Turkish mathematics textbooks for 6th, 7th, and 8th grade and found that 58% of the tasks used socio-economic contexts, 38% used intra-mathematical contexts, and 4% used scientific contexts.

Proportional reasoning can be applied to a variety of contexts. Inhelder and Piaget (1958) used physical experiments to investigate the proportional reasoning levels of children. These experiments included flexibility of a rod, equilibrium in a balance, hauling weight on an inclined plane, the projection of shadows, and centrifugal force. Consistency of vocabulary has also been found to be an important aspect of context.

The work of Carraher, Carraher, and Schliemann (1985) showed that street vendors performed much better on informal tests of their proportional reasoning knowledge. Scores for tests dropped from 98% to 74% to 39% as the context became successively more abstract.

Presentation

Bruner (1975, 1978) defined three modalities in which we represent mathematics: enactive, iconic, and symbolic. As cited in Weeks, et al. (2001) “The enactive mode uses representation through action. The iconic mode uses visual and mental images, and the symbolic mode uses symbols in the form of language and numbers” (p.22). Misailidou and Williams (2002) found that students experienced a 17 – 55 % improvement rate on their paint-mixture proportion problem when a picture was included. Other researchers found that the presence of physical representations or manipulatives improved student success (Wollman & Karplus, 1977).

The use of pictures could possibly cause an inversion of hierarchy of difficulty between discrete and continuous quantities. Boyer, Levine, and Huttenlocher (2008) explored differences between success rates on proportion problems as affected by discrete and continuous quantities. They found that children had greater success with continuous quantities than discrete quantities. This would seem contrary to the findings of Fleener (1993); however, the discrete and continuous quantities presented to the research participants in Boyer et al. (2008) research were presented in iconic mode (pictorial) rather than by the symbolic (numerical) mode of presentation. Participants were asked to select the bar which showed the same proportion of red to blue. Some

bars had blocks for counting while others did not. This research could also be interpreted as comparing qualitative and quantitative proportional reasoning problems since actual numbers were not used. In this case, qualitative problems were consistently ranked as being easier than quantitative problems. The research of Boyer et al. (2008) illustrates the complicated interactions of the situational variables that affect problem complexity in proportional reasoning problems.

In the case of DCPs, the need to replicate authentic contexts to assist in the transfer of knowledge from the classroom to the clinical setting is crucial (Glaister, 2005). Nursing education programs have addressed these issues. This research will be covered in the section on Dosage Calculation Situations in this literature review.

Student Characteristics

From the literature review on proportional reasoning by Tournaire and Pulos (1985) several student factors were identified that affect performance on proportional reasoning tasks. These were age, formal reasoning, M-capacity (“the number of schemes that one can attend to at one time” (p. 191)), field dependence-independence (ability to pull out essential information and disregard non-essential information), intelligence, gender, and attitude. Because student characteristics are a quality of the population used in the research, the student characteristics that are pertinent to this study will be addressed in the next section where the literature specific to nurses’ proportional reasoning skills will be presented. In accordance with the needs of this study, the researcher replaced *intelligence* with *learning styles* in the list of student characteristics.

Nursing Mathematics

Hoyles et al. (2001) researched the mathematics that nurses performed on the job. The researchers first defined the observed mathematical tasks that nurses performed on the job. The mathematics tasks were described as: drug preparation, IV infusion management, fluid monitoring, and data interpretation. Hoyles et al. (2001) concluded that the predominate type of mathematics problem solved by nurses was what this research defines as DCPPs. They therefore focused their research on these types of problems. Thirty episodes involving proportional reasoning were observed and classified by solution strategy using Vergnaud's relational calculus structures. Their research did not focus on nurse's symbolic representations for solving DCPPs as they focused exclusively with on the job skills which normally required the nurses to do mental mathematics.

DCPPs are considered a situation within the field of proportional reasoning. The concepts and procedures for solving other MVPPs all apply to this situation of DCPPs. However, since DCPPs could be considered a concept in itself, there are other concepts and procedures specific to DCPPs. These will be presented next.

Dosage Calculation Concepts

Wright (2008), a researcher in the field of nursing, summarized conceptualization with DCPPs as the ability to "extract the relevant information from the drug bottles or medication charts, set up the problem to solve, understand the answer, and recognize errors in answers" (p. 857). Blais and Bath (1992), also researchers in the field of nursing, similarly described conceptualization as having to do with the actual set up and

understanding of the solution of the problem. The conflict with mathematics education has been that the ability to both set up a problem and carry out the desired operations are collectively considered to be a part of procedural knowledge. Conceptual understanding, within the domain of mathematics education would represent an even deeper understanding as outlined in the research of Hiebert and Lefevre (1986). This type of understanding includes reflective relationships where the concept is readily applied to similar situations (Hiebert & Lefevre, 1986). In the case of DCPs, this would entail an understanding of the proportional relationships that exist between the values in the dosage calculation.

Knowledge of Specific Drugs

Hoyle et al. (2001) found that DCP solutions are highly tied to knowledge of the particular drugs. In their attempt to classify solution strategies using Vergnaud's relational calculus, they described a solution from their research that they claimed could not be classified using Vergnaud's theory. The DCP was a doctor's order for 120 mg of Amikacin with a dose strength of 100 mg:2 mL. The nurse knew that for these vials of Amikacin doubling the milligrams and moving the decimal over 2 places would result in the number of milliliters to administer. For this doctor's order, $120 \times 2 = 240$ and then move the decimal to the left 2 places to get 2.4 mL. The relational calculus for this solution is *times two and divide by 100*. This would translate into a rule of three relational calculus. Although the researchers cite this as being a functional relational calculus, the procedure was not classified as such because it was not tied to the concept of proportional reasoning but rather to knowledge of the drug Amikacin. This

researcher conjectures that this strategy should have been classified using relational calculus and that rather it is the notational or visual representation of this problem that differs. This research emphasizes the need for consistency in classification of strategies.

Indirect Measure

Lamon (2012) references indirect measurement in her book, *Teaching Fractions and Ratios for Understanding* as a concept influencing proportional reasoning. While she acknowledges that “the role of ratios and proportions in measuring quantities that cannot be measured directly, such as slope, speed, oranginess of a drink”, she only uses the term indirect measurement to “obtain measurements of physical objects....you cannot reach to measure” (p.80). The topic of similarity of shapes is presented within this context. DCPPs can be considered an application of indirect measure. The idea behind translating a doctor’s order is to convert the measure of milligrams into a volume of medicine to be dispensed. Volume is being used to measure mass. The term indirect measurement, however, in mathematics education is tied to the concept of similarity and shapes. The expanded nursing view of indirect measure more appropriately connects the concepts of proportional reasoning and indirect measure.

Indirect measure is utilized when the ability to measure the quantity in question is impossible or impractical by standard means and a constant ratio relationship is defined between the desired quantity and the measurable quantity. In geometry, the diameter of a circle can be indirectly measured by using its circumference and the ratio pi. In nursing, a particular practice, time taping, utilizes this concept of indirect measure. Time

taping is a procedure associated with IV flow rates (Pedagogy Online Learning Systems, 2014). The relationship between time and volume is what makes this an example of indirect measure. IVs can take several hours to infuse and proper infusion times need to be monitored. In the past they were monitored by the nurse. Today, IV pumps monitor these rates. When pumps are not available, nurses need to check to see that the proper amount of fluid is infusing in the correct duration of time. Prior to hanging the IV bag of solution, nurses will place a strip of tape on the bag next to the volume measurements which are usually marked off every 100 milliliters. The tape is used to draw lines where the fluid height should be every hour. The starting time is written at the top. A line is drawn where the fluid level should be after one hour. This is repeated to the bottom of the bag, where the end time of infusion is written. Most bags are already marked off in 100 mL increments; therefore, when the IV flow rate is 100 milliliters per hour, each hour corresponds to a 100 milliliter increment. An example of this is shown in Figure 31.



Figure 31. Time Taping Example

When the infusion rate is not 100 milliliters per hour, the task becomes more difficult. For example, if the rate was 120 milliliters per hour, the nurse would make a mark at 120, 240, 360, etc. milliliters. Next to each marking, they would write the one-hour increment of time after the start time. The task then becomes one of properly adding on increments of 120 milliliters and also, properly estimating the height of the fluid using the pre-marked volume markings on the IV bag. It is important to note that this procedure is not found in any of the text books on DCPs that were reviewed. Its inclusion in texts should be reconsidered as time taping could provide a real-life context

in which to develop the set-up of the double number line diagram. Also, another procedure that has been recently found in the nursing education literature utilizes the same idea of a double line diagram. This procedure termed the *syringe method* is reviewed in the next section on dosage calculation procedures.

Dosage Calculation Procedures

Introduction

The research specific to dosage calculation procedures does not emphasize the difference between set-up and relational calculus. Only one research article was found to address relational calculus, Hoyles et al. (2001). In their research, they observed 30 episodes of DCPs that were categorized by solution strategy using Vergnaud's (1980) model of scalar decomposition, scalar (multiplication), function, unit-value, and the rule of three. Eight strategies were scalar. Eight were function. Six required no computation at all because the dosage prescribed was equal to the dose strength of the medicine. Four strategies were indiscernible to the researchers. No strategies were identified as scalar decomposition or the unit-value method (Hoyles et al., 2001). Four strategies were classified with a new categorization. This new category was called *the nursing rule*. However, this study based its categorizations on relational calculi rather than procedures or set-up. This rule, which will be discussed in greater detail in the next section, traditionally follows the scalar relational calculus. Under the classification system for this study, participants who used the nursing rule could have been placed in the scalar relational calculus category. The distinction between relational calculus, set-

up, and procedure remains vague and needs to be explicated in DCCP procedure classification research.

The work of Hoyles et al. (2001) merges DCCP research with that of mathematics education research. Wright (2013) cites the work of Hoyles et al. (2001) and also attempts to infuse Vergnaud's (1980) strategies in her research article titled *How do Nurses Solve Drug Calculation Problems*. Wright (2013) categorized three strategies: the nursing rule, scalar, and syringes, citing Vergnaud for the scalar strategy. The scalar strategies were broken into three of categories of single units, doubling and halving, and relational. These strategies could have all been consistently classified using Vergnaud's relational calculus of scalar, function, unit-value, scalar decomposition, and rule of three (Vergnaud, 1980). No reason was given why only the terminology of *scalar* was singled out nor was any attempt made to compare the other strategies to Vergnaud's system. As in the Hoyles et al. (2001) study, these strategies were not connected to any set-up as they were not pencil and paper tasks.

DCCP instruction is concerned with both the relational calculus and the set-up which combine to make the procedure. DCCP textbooks are grounded in only three procedures for solution. These three procedures are known as cross products, the nursing rule, and dimensional analysis (Arnold, 1998, Morris, 2010). Within the framework of this research, the nursing rule and dimensional analysis are considered set-ups. The procedures bearing the same name are specifically tied to a relational calculus which will be discussed respectively. One such textbook, *Calculate with Confidence* (Morris, 2010), contains individual chapters on all three of these different strategies. The purpose of giving three different methods is to allow the student to

choose which method is easiest for the student to use (Morris, 2010). Another popular book, *Henke's Med-Math: Dosage Calculation, Preparation and Administration* (Buchholz, 2012) presents dosage calculations using only cross products and the nursing rule. Dimensional analysis is addressed in the appendix of this text and references another book distributed by the same publisher that deals only in dimensional analysis. However, problems are not solved within this text using dimensional analysis. Some texts use only one method of calculation, and this method is usually part of the title. For example: *Dosage Calculations: A Ratio-Proportion Approach* by Pickar (2006), *Clinical Calculations Made Easy: Solving Problems Using Dimensional Analysis* by Craig (2011) and *Medical Dosage Calculations: A Dimensional Analysis Approach* by Olsen, Giangrasso, and Shrimpton (2012). Craig (2011) found that using one standardized method reduces frustration and calculation errors and advocates for the dimensional analysis set-up.

These three procedures of cross products, the nursing rule, and dimensional analysis will be described in detail pertaining to their use in the context of nursing and DCPD instruction. In addition to these three procedures and their relevance to the DCPD literature, a specific connection to Wright's (2013) *syringes* procedure will be made.

Cross products

In this procedure, the definition of proportion as two equal ratios is used to set-up the problem and a relational calculus of the rule of three is used to solve it. A variable can replace the missing value. The two numbers that are located diagonal from one

another are multiplied and their product is divided by the value diagonal from the variable. The standard example for this research is shown in Figure 32.

$$\frac{150 \text{ mg}}{2 \text{ mL}} = \frac{225 \text{ mg}}{x}$$
$$(225)(2) = 150 x$$
$$450 \div 150 = x$$
$$3 = x$$

Figure 32. Cross Products Procedure

Loops are frequently drawn around the values that are diagonal from each other to help remember which numbers to multiply. Conceptually, this procedure (as well as its connected relational calculus) is poorly understood (Lesh et al., 1987). Cramer et al. (1993) show that the cross products rule has no physical referent. The multiplication of contrasting elements has no meaning and, consequently, makes the rule conceptually impossible to follow. For our example, $225 \text{ mg} \times 2 \text{ mL}$ would equal $450 \text{ mg} \cdot \text{mL}$ which has no meaning. This relational calculus is neither function nor scalar since neither of these relationships is evaluated. Dosage calculation textbooks often use the phrase, ratio-proportion method, when referring to the method of cross products. In this research, the term for the set-up is equal ratios and the term for the procedure is cross products.

The Nursing Rule

The nursing rule is a formula used to calculate drug dosages and is known as the “mantra” of a nurse (Hoyles et al., 2001). Calling it a mantra is essentially giving it similar characteristics as a mnemonic device. Mnemonic devices in mathematics education have served well in recalling procedures but have not been shown to assist with conceptual understanding (Hiebert & Wearne, 1986). This procedural mantra has some variations, but one form of the formula is “what you desire (d) divided by what you have (h) times the quantity (q) is what you give” or in algebraic form: $d/h * q$. An example using this strategy is worked out in Table 15.

Table 15

The Nursing Rule

| Name: | Formula | Set Up: | Relational Calculus: |
|---------------------|---|---|-------------------------|
| The Nursing Formula | $\frac{\textit{desired}}{\textit{have}} \times \textit{quantity} = \textit{give}$ | $\frac{225 \textit{ mg}}{150 \textit{ mg}} \times 2 \textit{ mL} = \textit{Give}$ | $225 \div 150 \times 2$ |

The order in the nursing formula matches precisely with the actions that a nurse takes to administer a drug (Hoyles et al., 2001). First, the dose ordered is noted (d). Then the nurse would identify the amount of drug (h) contained in the unit of measurement of the medication on hand. After dividing, the quantity of the unit of measurement (q) would be multiplied to calculate the amount of medicine to give. This formula utilizes a scalar relational calculus since a ratio is formed between the units of like measure.

Dimensional Analysis

Greenfield, Whelan, and Cohn (2006) described the use of different teaching strategies as conceptual models and recommended a standardization of these conceptual models in nursing programs. For the purpose of the present study, these different strategies are referred to as instructional strategies (rather than conceptual models) to avoid confusion with other meanings of the word, conceptual, in this literature review. Several researchers have reported findings that support using dimensional analysis as an instructional strategy to improve student outcomes (Greenfield et al., 2006; Rice & Bell, 2005, Johnson & Johnson, 2002, Arnold, 1998).

Some of the reasons for the use of dimensional analysis being advocated have been that it reduces the need to memorize formulas and enhances accuracy (Greenfield et al., 2006, Rice & Bell, 2005). Johnson and Johnson (2002) use dimensional analysis “consistently across the curriculum” (p.82) in their nursing program because it is a procedure that can be used for all of the nursing calculations: IV drip rates, weight-based dosages, oral dosages, and conversions. Dimensional analysis, also referred to as stoichiometry in chemistry, is used in solving chemistry problems. The application of this knowledge to dosage calculations could facilitate connections between chemistry and dosage calculations mathematics problems (Rice & Bell, 2005).

Rice and Bell (2005) also determined that the strategy helped to improve confidence in nurses’ dosage calculation results. They used quotes from participants that illustrated this confidence. One particular quote expressed the relief this strategy provided one of the participants from her mathematics anxiety:

“It has given me freedom from anxiety and stress related to fear associated with making a medication error. Now I can concentrate on enhancing my knowledge of medications and interventions.” (Rice & Bell, 2005, research participant quote, p. 317)

Syringe Method

Wright (2009) describes in her research a visual method for solving DCPPs that involves syringes. In 2009, Wright called this *visualization*. In a more recent article (Wright, 2013), she calls this procedure *syringes*. The syringe that is used to measure volume is used to visualize the mass, usually in milligrams, as determined by the dose strength. Using the standard example for this research, a nurse would visualize 150 mg being located at the 2 mL mark on the syringe. From there, the nurse would use scalar

decomposition strategies to find where 225 mg would be on the syringe. This procedure was found to be used in a research study that did not involve paper and pencil tests. No notation was attached to this strategy. However, this procedure could easily translate into a ratio table, equality of measures, or double number line diagram set-up. The visual of the syringe is shown in Figure 33 along with the possible set-up notations for comparison.

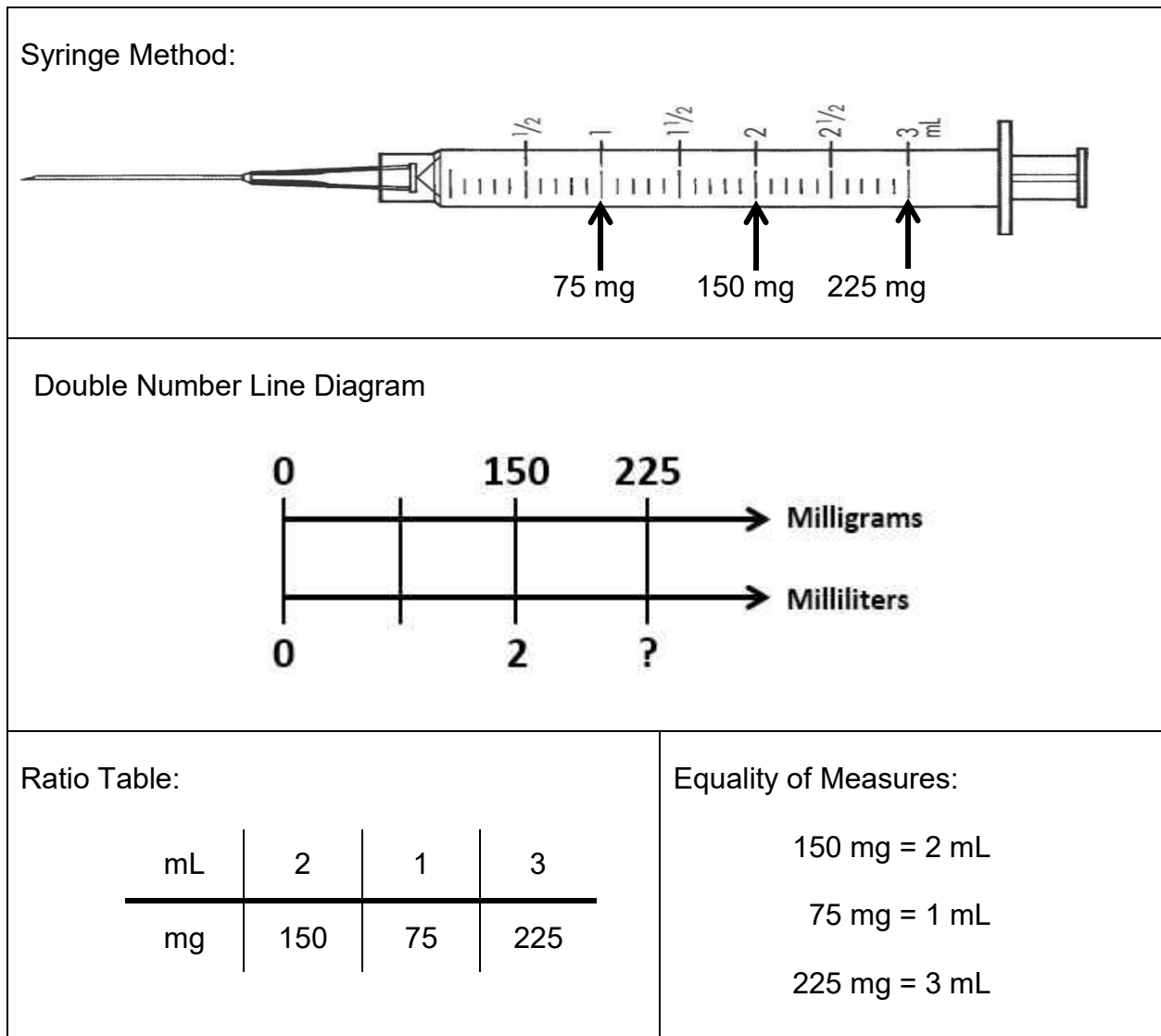


Figure 33. Syringe Visualization Procedure with Possible Set-ups

The short-coming with the ratio table and equality of measures set-ups is that in some cases, the values are not in order. For example, in the ratio table in Figure 33 the numbers go down and then up (2,1,3 mL) rather than in order (1,2,3 mL). Mathematics education literature however, has a parallel to the syringe visualization procedure. The double number line diagram is a generalizable tool that can be used in the situation of dosages as well as other applications.

Procedures for Problems Requiring Multiple Steps

Some DCPs require multiple steps. A common type of problem that requires multiple steps is dosage calculations involving drip rates. All of the set-ups can be applied to this problem however the nursing rule needs to be modified to accommodate these extra steps. This new formula is called the drip rate formula. Fleming, Brady, and Malone (2014) found that the drip rate formula was used between 25% and 50% of the time on each of the drip rate problems in their study. The drip rate formula is dependent upon the type of tubing being used to deliver the drip. The size of the drop that the tubing delivers is called the drop factor and measures the number of drops contained in one milliliter of solution. The drip rate formula is calculated by taking the total volume to be infused times the drop factor divided by the total time in minutes. An example problem and a sample solution using three different set-ups are shown in Figure 34.

Problem: 450 mL of D5NS is to be administered intravenously over 3 hours. The IV set delivers 15 drops/mL. How many drops/min will it take to deliver the prescribed dose?

Equal Ratios

$$\frac{15 \text{ drops}}{1 \text{ mL}} = \frac{x}{450 \text{ mL}}$$

$$x = 6750 \text{ drops}$$

$$\frac{1 \text{ hour}}{60 \text{ mi}} = \frac{3 \text{ hours}}{y}$$

$$y = 180 \text{ min}$$

$$\frac{6750 \text{ drops}}{180 \text{ min}}$$

$$45 \frac{\text{drops}}{\text{min}}$$

Dimensional Analysis

$$\frac{450 \text{ mL}}{3 \text{ hours}} \times \frac{1 \text{ hour}}{60 \text{ min}} \times \frac{15 \text{ drops}}{1 \text{ mL}}$$

$$45 \text{ drops/min}$$

Drip Rate Formula

$$3 \text{ hours} \times \frac{60 \text{ minutes}}{1 \text{ hour}}$$

$$3 \text{ hours} = 180 \text{ minutes}$$

$$\frac{\text{Total Volume} \times \text{Drop Factor}}{\text{Total Min}}$$

$$\frac{450 \text{ mL} \times 15 \frac{\text{drops}}{\text{mL}}}{180 \text{ min}}$$

$$45 \text{ drops/min}$$

Figure 34. Common Procedures for Solving IV Drip Rate Problems

Koohestani and Baghcheghi (2010) compared test scores between two instructional groups, one using the drip rate formula and the other dimensional analysis. Initial results showed no difference between the two groups, however, a posttest 3 months after instruction showed significant better scores in the dimensional analysis group.

A strategy used in connection with the drip rate formula is to use the drop factor constant. This formula requires that the drip rate be calculated in milliliters per hour. After this is done, the hours are converted to 60 minutes using the ratio of 1 hour:60

minutes. Most IV tubing is calibrated to 60 drops/mL, 20 drops/mL, or 15 drops/mL. The IV tubing calibration is known as the drop factor. All of the drop factors, 60, 20, and 15, are factors of 60. Since the conversion of hours to minutes involves a factor of 60, the value of the drop factor and the value of 60 minutes will always be reducible. If you have 60 drops/mL tubing, then the 60's will cancel and your milliliters per hour will equal your drops per minute. When using 20 drops/mL tubing, the 60 and 20 cancel and a division of three remains. For 15 drops/mL, the reduction between 15 and 60 yields four. These values of one, three, and four that remain are called the drop factor constants. Once these are calculated, the only step left is to divide the mL/hr by either one, three, or four. This is worked out in Figure 35.

Problem: 450 mL of D5NS is to be administered intravenously over 3 hours. The IV set delivers 15 drops/mL. How many drops/min will it take to deliver the prescribed dose?

| <u>Drop Factor</u> | <u>Drop Factor Constant</u> | <u>Procedure</u> |
|--------------------|-----------------------------|--------------------|
| 60 drops/mL | 1 | $150 \div 1 = 150$ |
| 20 drops/mL | 3 | $150 \div 3 = 50$ |
| 15 drops/mL | 4 | $150 \div 4 = 38$ |

Figure 35. Drop Factor Constant IV Flow Rate Problem

The drop factor constant procedure illustrates how the context of proportional reasoning problems impacts the solution process. Although problems like this can be solved using generalized procedures for proportional reasoning, nurses have developed their own strategies for solution that are specific to the context. The specific strategies are signaled by the context of IV flow rates but are implemented because of the numerical

quantities known to be in these problems. Because the tubing drop factor is always a factor of 60, this procedure works.

Dosage Calculation Situations

Under the conceptual field of proportional reasoning, DCPPs would fall under the situation of solving proportion problems within the context of medical administration. Therefore, DCPPs would be a contextual variable. However, since DCPPs are being considered a concept in themselves in this portion of the literature review, the situations that specifically affect DCPP problem difficulty will be described in this section. The general situations that affect problem difficulty and/or procedure choice were defined in the literature to be: numerical characteristics, semantic types, context, presentation, and student characteristics.

DCPPs can be categorized semantically as being well-chunked measures; however, Fleming et al. (2014) further classified problems as metric conversions, tablet dosages, fluid dosages and IV drip rates. After administering a DCPP test to 124 newly hired nurses, they found that tablet dosages were correct 81% of the time, fluid dosages were 69%, then metric conversions at 65%, and finally IV drip rates at 37%. The average score on the dosage calculation test was a 60%. These different problem types and their corresponding levels of difficulty can be related to the classification system used for the conceptual field of proportional reasoning. Tablets are discrete quantities while fluids are continuous quantities. Because discrete quantity proportions have been found easier to solve than continuous quantity proportions (Fleener, 1993, Karplus et

al.1983), it would follow that tablet problems would be easier than fluid problems. IV drip rates require multiple steps and have a higher chance of error.

These situations are similar for DCPPs as they are for general proportional reasoning problems with the exception of student characteristic. Nurses are the population of interest and therefore the research on the characteristics of nurses that affect problem difficulty will be reviewed here.

Johnson and Johnson (2002) defined the student characteristics that were found to affect success in solving DCPPs as basic math skills, perceived self-efficacy, learning styles, anxiety, and motivation. Nursing is a career where mathematics has life-altering consequences and yet, nursing students have been found to score significantly lower on mathematics tests than other majors (Pozehl, 1996). These poor test results are connected to mathematics anxiety and self-confidence in the literature (Bull, 2009, Andrew, Salamonson, & Halcomb, 2009). Researchers recommend that mathematics anxiety, self-efficacy, and learning styles all be taken into account when planning mathematics instruction for nursing students. Andrew et al. recommend assessing self-efficacy of nursing students and created such a test to be used in nursing programs to predict student performance.

Some researchers have found that certain instructional approaches promote confidence. Gillies (2004) found that using a problem-solving approach to teaching dosage calculations gave students greater confidence in their ability to solve DCPPs correctly. This problem-solving approach was marked by students exploring intuitive procedures and then developing procedures for solving DCPPs as opposed to being given formulas. Wright (2012) found that “there is some indication that student nurses

do have a preferred learning style for drug calculations skills which could influence their ability to access and use specific teaching strategies” (p. 722). While learning styles have a wide variety of meaning in literature, the use of the term here refers to visual, auditory, and kinesthetic learning styles. These styles are “recognized by most nurse educators” (Blevins, 2014, p. 59).

Summary

Synthesis of the literature on conceptual fields, proportional reasoning, and nursing mathematics resulted in the creation of four tools which can be used to guide study design and analysis of research. First, Vergnaud’s (2009) Theory of Conceptual Fields was used to develop an organizational structure for the Conceptual Field of Proportional Reasoning found in Table 11. Within this framework of concepts, procedures, and situations, three additional guiding structures were created; one for each category: concepts: Synthesis of Levels of Proportional Reasoning, procedures:, MVPP Set-up Identification Guide and situations: Interpreted Intensive Quantity Semantic Type.

The concepts connected to proportional reasoning were explicated and resulted in the Synthesis of Levels of Proportional Reasoning found in Table 7. The concepts of analogical reasoning and quantitative reasoning were added to the concept of covariation as major influences in proportional reasoning. The role of context and procedure use were also seen as contributors to the determination of the attainment of proportional reasoning which confirms Vergnaud’s theory that a concept cannot be studied in isolation but rather in connection with the other concepts, procedures, and

situations which surround it. This organizational structure effectively assisted in the explication of the broad definition of proportional reasoning: the evaluation of the multiplicative relationship between two covarying quantities applied universally to two other quantities of the same corresponding nature and dimension.

A more robust definition of procedure was developed in order to assist in the classification of MVPP solutions. The explication of set-ups and relational calculus from existing literature were presented. The MVPP Set-up Identification Guide found in Table 18 was designed specifically to be used in this research study as an instrument for clearly categorizing notations used in solving DCPs. This was necessitated by past research focusing on interview and observation to classify relational calculus. This literature synthesis provides a standardized way for classifying paper and pencil solutions.

The situations in which proportional reasoning occurs combined research on semantic type and quantity ((Lamon, 2007, Schwartz, 1996, Karplus et al., 1983) to create the Interpreted Intensive Quantity Semantic Type scale. This hierarchical meaning scale for the intensive relationship created between two extensive measures corresponds to Lamon's (2007) descriptive semantic categories. The scale illustrates the difficulty in interpreting ratios as intensive measures when the problem semantics translate into other types of measures (extensive and scalar). Utilizing the terminology of quantitative reasoning yielded a more generalizable system for describing solution strategies and problem difficulty associated with semantic type. The use of quantitative reasoning terminology could possibly be connected more readily to levels of proportional reasoning.

The context specific concepts, procedures, and situations of the field of nursing were outlined. The concepts of common drug dosages and indirect measurement were illustrated to show their connection to solving DCPPs. The nursing rule which is a formula used to calculate DCPPs was described as a set-up and connected to the literature on proportional reasoning as traditionally using a scalar relational calculus. The additional situations of DCPP problem types and characteristics of nurses as solvers of DCPPs were included. Specifically, the learning styles and mathematics anxiety of nurses was addressed as a student factor affecting problem difficulty.

The organization of the concept of proportional reasoning through the framework of conceptual fields provides a lens through which the analysis of an individual's proportional reasoning can be viewed. By explicating the concepts, procedures, and situations connected with proportional reasoning, a coherent analysis of the lived experiences of nurses involving proportional reasoning can commence with a clear understanding of each of the terms presented within the framework. The next chapter will outline the methodology used to research this concept utilizing and referencing many of the terms and structures outlined in this literature review.

CHAPTER 3 METHODOLOGY

Introduction

The ultimate goal of this research is to inform instructional practices of proportional reasoning. In order to study this concept, two important research design variables had to be decided upon: what is the population of interest and what research methodology will be utilized. In choosing a population of interest, the researcher sought a population where the concept of proportional reasoning had a consequential impact on the lives of the participants so that they would find value in the research and might find benefit from participation. Nurses, whose knowledge of this concept enables them to administer correct dosages of life altering medications to their patients, seemed like a natural choice for a population who would find meaning in the mathematics.

Research design choice depends on two factors: the type of data, whether quantitative or qualitative, that is to be acquired and the intellectual discipline in which the topic is categorized. The choice to do qualitative research came from the researcher's desire to investigate the general topic of proportions in an attempt to identify more specific quantitative question for future research. This choice to use hermeneutic phenomenology came from the desire to use a methodology previously used in the disciplines of mathematics and nursing education (Ajjawi & Higgs, 2007).

Broad research questions were designed to investigate the concept of proportional reasoning within the context of nursing. These questions were refined as the researcher sought out a conceptual framework in which to organize the information. Once the construct of conceptual fields was chosen as this guiding framework, research

questions were reworded to merge the methodology with the conceptual framework. The three components of the theory of conceptual fields, concepts, procedures, and situations, are used to provide the organizational structure of these questions. What resulted are the following research questions:

Research Questions

- Lived Experiences: What are the lived experiences that nurses have with solving proportional reasoning problems on written dosage calculation tests and in nursing practice?
- Procedures: What are the procedures that nurses use to solve proportional reasoning problems on a dosage calculation survey?
- Situations: When solving proportional reasoning problems, what situational variables do nurses recognize as affecting problem difficulty and/or procedure choice: (a) numerical characteristics, (b) semantic type, (c) context, (d) presentation, and (e) student characteristics?

Hermeneutic Phenomenology

The design for this research is a hermeneutic phenomenological non-experimental design (van Manen, 1990). Hermeneutic phenomenology is a qualitative research methodology that focuses on the targeted phenomenon through the lived experiences of individuals. The term hermeneutic originates from the Greek. Hermes is the Greek god who is the messenger between the gods and the mortals. As Hermes delivered the words of the people to the gods, so too hermeneutic phenomenology attempts to interpret the lived experiences of people so as to impact knowledge of the phenomenon as known to the educators, policy makers, or stake holders in the field.

In *Mathematics as an Educational Task*, Freudenthal (1973) examined the difference between educating mathematicians and non-mathematicians. Despite the fact that nursing has been considered a STEM (science, technology, engineering, and mathematics) career, many of the students who enter the field do not consider themselves mathematicians or mathematically minded (Hyde, Fennema, & Lamon, 1990). The mathematics instruction of these students should, therefore, be constructed from the viewpoint of a non-mathematician. In order to teach the non-mathematician, Freudenthal (1973) did not look for connections within the framework of mathematics but within the “lived through reality of the learner” because “for the non-mathematician the relations within the lived-through reality are incomparably more momentous” (p.77).

The phenomenon of study in this research is proportional reasoning. Many mathematics problems, including problems involving proportional reasoning, have been found to be contextually bound (Misailidou & Williams, 2003, Ben-Chaim et al., 1998, Fleener, 1993, Carraher, Carraher, & Schliemann, 1985). Ajjawi and Higgs (2007) wrote

“These phenomena cannot maintain their essential and embedded features if reduced or measured as in quantitative research” (p. 614) and “Attempting to isolate or measure reasoning (and communication in clinical practice), as specific, a-contextual processes ignores the complexity, reality, and cons” (p. 614). Proportional reasoning problems impact many occupations, including nursing (Hoyles, Noss, & Pozzi, 2001). By studying the intersection of the life experiences of nurses in relation to their education and experiences in solving mathematics problems that require proportional reasoning, the broader idea of instruction of proportional reasoning tasks for all students is anticipated to be better understood.

This research focuses on the study of human science rather than natural science. Vergnaud (1979) supported the notion of understanding the meaning of a mathematical concept for a person “through all aspects of behavior, and especially action in problem-solving and not only through the symbols by which the subject tries to represent things” (p. 268). These observable expressions of internal representations are the means by which researchers gain an understanding of knowledge, and the models of understanding are derived directly from the words and actions of the participants in the study (Steffe & Thompson, 2000).

Participants Selection

Purposive sampling for qualitative research provides a means to obtain participants who can offer specific information. Because the researcher desired to give a rich description of the lives of nurses, it was essential to find participants who were willing to communicate their experiences in detail. In order for the research to have an

impact on instruction of proportions, it was also important to select participants who had different ways of thinking about and relating to the mathematics problems that they faced on the job. This difference could possibly be identified by the mathematical procedures participants used to solve the problems. After receiving approval for the study from the Institutional Review Board of the University of Central Florida (APPENDIX B), the researcher initiated the study. To begin the selection process and to gain this preliminary information, the researcher distributed a survey that addressed the following components: (a) demographic information, (b) an answer to writing prompt, and (c) five drug dosage calculation problems. This survey can be found in APPENDIX C.

Distributing the surveys and soliciting participation proved to be a difficult task. Nurses were chosen as the population because of their use of proportional reasoning on the job. Initially, one hospital had agreed to participation and agreed to allow nurses to take part in the research during work hours. The IRB process was lengthy because nurses, as employees of the hospital, were considered a vulnerable population. Difficulty arose when hospital administrators would not allow for the solicitation of scheduled survey times because they did not want nurses to feel obligated to take the survey. Without scheduled times, nurses were expected to initiate the desire to participate on their own. At the time of survey administration, no nurses attended. Because of the limited access to the population, other options had to be sought out.

The researcher had access to a student nursing population at a college but distributing the surveys at one particular institution did not seem reasonable because institutions may have common dosage calculation instructional practices, and the single

protocol known to participants may have limited the study. Also, nurses rather than nursing students were desired for their on-the-job experience in solving DCPPs. Therefore, in order to identify participants that would be from diverse workplace environments and experiences, a snowball technique was used to distribute 100 surveys over a two-week period of time. The 44 nurses who responded to the survey were considered *respondents*.

Snowball sampling is an informal technique of sampling where one subject gives the researcher the name of another subject and then that subject gives the name of another (Atkinson & Flint, 2001). Using the snowball technique, the researcher asked acquaintances for assistance. Friends and colleagues were asked if they knew any nurses or if they knew anyone who knew any nurses, and would they be willing to help distribute surveys. Those who agreed were given two surveys for every nurse they knew, and they were asked to (a) distribute one survey to the nurse they knew and (b) have that nurse pass the other survey to another nurse. The surveys were distributed to people who lived local to the researcher in order to make subsequent face-to-face interviews more accessible. The incentives provided to nurses for their participation were a four color pen, a calculator, and five dollars. The surveys could be returned anonymously; however, respondents who were not opposed to being contacted for further study participation were asked to write their names and provide their contact information. Survey respondents who provided contact information were then eligible to be participants in the interview stage of the research. Surveys were collected over a six-week period of time before participant selection began.

Purposive sampling was then used to select participants from the 44 respondents. In order to select participants with diverse ways of thinking, stratified categories were formed from which to choose one participant. Respondents were placed in categories based upon their solution set-ups. Each solution from the five-question dosage calculation survey was classified based upon the notational set-up. These classifications were equality of measures, ratio table, double number line, analogies, equal ratios, dimensional analysis, the nursing rule, no work, and not identifiable. After each question was classified, surveys that had three or more problems solved using the same set-up were identified and categorized as having a predominate set-up. These categories were the same as the classifications for the individual solutions with the addition of a category for no predominate set-up. The criteria of three set-ups was chosen specifically to assist with the identification of a category of respondents who did *not* have a predominate set-up. This meant that if someone did not have three similar set-ups that they must have used at least three different set-ups since there were five problems. The purpose behind this guideline was to find a participant who used varied set-ups to solve problems.

The researcher executed this categorization process on three separate occasions to check for consistency of classification. The researcher's categorization was cross checked by another professional in the field of mathematics education to validate the identification of the set-up and no discrepancies were found. The surveys were then separated indicating those participants who were willing to be contacted for further participation and those who were not. Anonymous surveys were eliminated from further consideration.

The selection of a participant from each set-up classification was further refined by how consistently they used their preferred set-up. Surveys in each category were ranked by how many questions they answered using their set-up of choice. Once the surveys were ranked, the researcher further refined the selection process by reviewing the writing prompt responses. Top respondents for each set-up were considered by their consistency in set-up choice as well as their ability and willingness to provide details in the writing prompt. Taking these two factors into account, the top three (if applicable) respondents in each category were listed. One potential participant from each category was then contacted by email or telephone and invited to participate in the study. APPENDIX D contains a copy of the invitation letter used. After four days of no response, a follow-up email or phone call was made. After a week, the next highest ranked person on the list was contacted. This process resulted in the agreed participation of four nurses representing the categories of equality of measures, dimensional analysis, the nursing rule and no predominate set-up. The participant selection procedures are summarized in Table 16.

Table 16

Participant Selection Procedures

| Step | Task | Description | Data Collection Document |
|------|--------------------------------|--|---|
| 1 | Distribute Surveys | 100 DCPD surveys were distributed over a 6 week period of time using snowball sampling | DCPD survey (Appendix C) |
| 2 | Collect Surveys | 44 surveys were returned and responses were categorized and ranked by predominate set-up and writing prompt detail. | MVPP Set-up Identification Guide (Table 18) |
| 3 | Contact Potential Participants | Invitation to participate letters were sent out via email to desired participants. After a week, the next person on the list was contacted. 4 agreed to participate. | Invitation to Participate (Appendix D) |

Participant selection was facilitated by the use of the DCPD survey and marked the first phase of the research. The details surrounding the selection of individual participants are provided in Chapter 4. The second phase of the research consisted of collecting data from the four participants. These data collection procedures will be reviewed in the next section.

Participant Data Collection Procedures

Once the four participants were selected, individual meetings were scheduled to begin the data collection process. The data collection procedures for this phase were based upon van Manen’s (1990) suggestions for collecting experiential descriptions from participants. These procedures are: (a) using protocol writing to capture the lived-experience descriptions, (b) interviewing the personal life story, and (c) keeping logs as

sources of lived experiences. Each of these methods was incorporated into the design to gain as much insight as possible into the experiences that nurses had solving proportional reasoning problems in different contexts. The data were collected from participants over the course of 4 meetings. The meetings were structured by researcher developed protocols which are described in the next section. Table 17 provides descriptions of each step of the participant data collection procedure. The amount of money provided to participants as incentive for attendance at each meeting is also found in Table 17.

Table 17

Participant Data Collection Procedures

| Step | Task | Description | Data Collection Document | Incentive Provided |
|------|--------------|--|---|------------------------------|
| 1 | Meeting 1 | Greet and discuss research participation. Sign Informed Consent. Use Interview I protocol to discuss survey responses. Give participants a journal and explain procedures for journal writing. | Informed Consent (Appendix E) Interview I Protocol (Appendix F) | \$80 |
| 2 | Meeting 2 | Use Interview II protocol to discuss the Everyday Proportion Problems. Collect journal writings. | Interview II Protocol (Appendix G) Everyday Proportion Problems (Appendix A) | \$80 meeting \$60 journal |
| 3 | Meeting 3 | Use Interview III protocol to discuss mathematics used on the job and review journal writings. | Interview III Protocol (Appendix H) | \$40 meeting |
| 4 | Analyze Data | The researcher analyzed data through the lens of hermeneutic phenomenology. | (See Data Analysis Procedures) | - |
| 5 | Meeting 4 | Discuss research conclusions. Member checking. | | \$40 meeting |

Table 17 provides the names of the data collection instruments that were used in each step. These instruments and the procedures for their use are described in full detail in the next section.

Instruments of Data Collection

In this section, the data collection instruments are described in detail along with the procedures for their use. Copies of the actual instruments used in this research are located in the appendices.

Dosage Calculation Proportion Problem Survey

The survey's main purpose was to assist in the selection of the research participants. As described earlier, the participants were to be selected by their demographic diversity, their ability to describe their experiences in detail, and their set-up choices for dosage calculation problems. Therefore, in order to check these qualities, the survey consisted of three parts: demographic information, writing prompt, and DCPPs. Each part of the survey is explained in detail and a copy of the actual survey can be found in APPENDIX C.

Demographic information

The first page of the survey stated, "Please fill out this questionnaire ONLY if you are currently working in the field of nursing." The term "nurse" has a broad definition. Everyone from a Certified Nursing Assistant (CNA) to a Doctor of Nursing Practice (DNP) could be considered a nurse. Nurses do not necessarily have to have a college degree. Typically, nurses are considered to be those that have passed the National Council Licensure Examination for Registered Nurses (NCLEX-RN). In order to be eligible to take the NCLEX-RN, one must complete an associate's degree program, a

bachelor's degree program, or an accredited nurse diploma program. Diploma or certificate nurses complete programs that are usually affiliated with a hospital. Because the surveys were being distributed by non-researchers, the researcher did not want these distributors to have to determine who would be eligible for participation and who would not. This decision was to be determined by the researcher based upon the responses to the demographic information. Part one of the survey consisted of five demographic questions. These questions included:

1. What type of nurse are you?
2. Which of the following best describes the type of institution that you received your highest degree from?
3. Which of the following best describes the highest nursing degree you have?
4. Which mode of instruction best describes the one that your institution used to teach dosage calculations?
5. Check all of the mathematics courses that you have taken and passed either at the college level or high school.

The demographic information collected served the purpose of determining if the respondents were suitable for further research participation by identifying that they were actually registered nurses with either a nursing diploma or nursing degree.

Writing prompt

The writing prompt is the second portion of the survey. A full sheet of paper was provided for respondents to answer the following prompt:

Please write a direct account of your personal experiences learning the mathematics that is essential for drug dosage calculation, as you lived through it. Please describe any classes or instruction that you have participated in that has contributed to this knowledge. If possible, describe a particular example or incident from your mathematics/nursing experience. You may use the back side of this packet or attach additional pages if necessary.

This writing prompt was adapted from a generalized hermeneutic phenomenology methodology prompt provided by van Manen (1990, p. 65) which read, "Please provide a direct account of your personal experiences with (research topic) as you lived through it." The purpose of this writing prompt was to assist in the choosing of participants for the interviews by revealing the respondents' ability and willingness to provide detail. Responses to the writing prompt were also included in the data analysis of some of the research questions.

Survey DCPPs

The dosage calculation proportion problems used for this research came from a dissertation by Huse (2010). Other proportion tests were considered including the Bindler-Bayne Test (Serembus, 2000), but these did not represent current assessment practices, specifically, the incorporation of visuals in the test to achieve a more realistic context (Glaister, 2005). Huse (2010) performed tests to ensure the reliability and the content, concurrent, and criterion-related validity of the instrument and concluded that the test was reliable and valid. The original test included 15 questions. Only five of the questions were used in order to increase participation by limiting the amount of time required to take the survey. Since the surveys in this present qualitative research were not being used to quantify mathematics ability, tests of reliability and validity were not

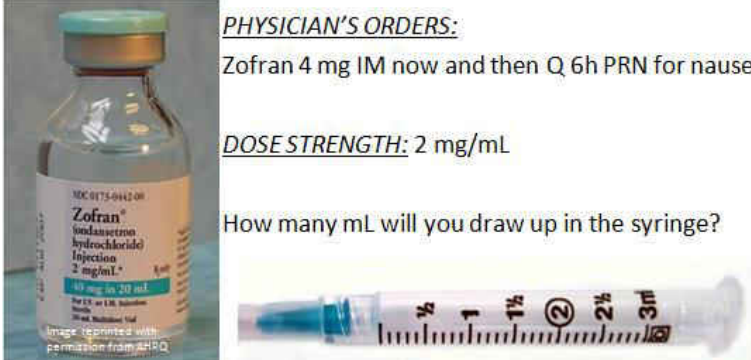
performed. The establishment of credibility in qualitative research corresponds to validity in quantitative research (Sampson, 2012). Therefore, the revised five-problem survey was reviewed by three nurse administrators at a local hospital and they confirmed the survey's credibility. Each problem is discussed in detail in the next section. Because surveys were to be categorized by the set-up used by respondents, a sixth item was added to the survey to assist with the classification of set-ups and procedures. This survey served the purpose of selecting participants for the hermeneutic portion of the research. Four respondents continued on to the next phase of research.

The five questions used for the DCPD survey in this research were chosen from Huse's (2010) research with special consideration. The first three problems specifically address numerical characteristics that affect proportional reasoning problem difficulty: the presence of integer or non-integer number relationships and discrete or continuous data. DCPD 1 and DCPD 2 both utilized continuous (liquid) measure however; DCPD 2 required the evaluation of a rational number relationship where DCPD 1 is an integer relationship. DCPD 3 incorporated the use of discrete data in the form of countable tablets. These problems and details are provided in Figure 36.

DCPP Survey Problem

Numerical Characteristics

Question 1



PHYSICIAN'S ORDERS:
Zofran 4 mg IM now and then Q 6h PRN for nausea.

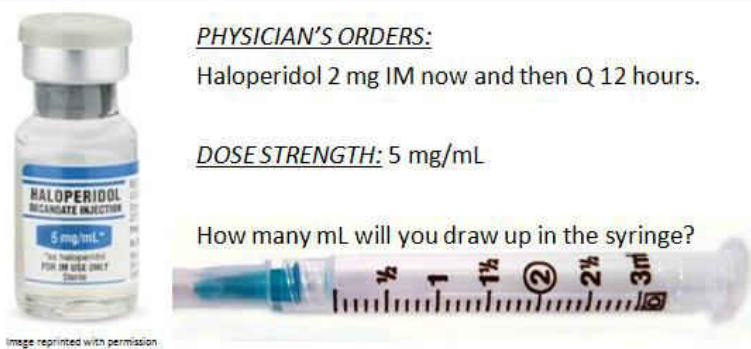
DOSE STRENGTH: 2 mg/mL

How many mL will you draw up in the syringe?

The image shows a vial of Zofran (ondansetron hydrochloride) injection with a concentration of 2 mg/mL. Next to it is a syringe with markings from 0.5 mL to 3 mL. The number 2 is circled on the syringe, indicating the volume to be drawn up.

Integer
Continuous

Question 2



PHYSICIAN'S ORDERS:
Haloperidol 2 mg IM now and then Q 12 hours.

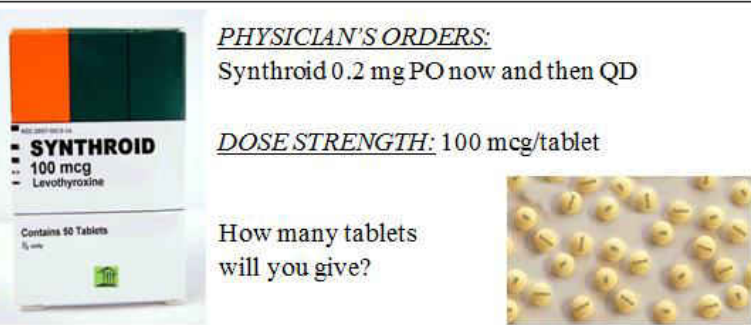
DOSE STRENGTH: 5 mg/mL

How many mL will you draw up in the syringe?

The image shows a vial of Haloperidol decanoate injection with a concentration of 5 mg/mL. Next to it is a syringe with markings from 0.5 mL to 3 mL. The number 0.4 is circled on the syringe, indicating the volume to be drawn up.

Non-Integer
Continuous

Question 3



PHYSICIAN'S ORDERS:
Synthroid 0.2 mg PO now and then QD

DOSE STRENGTH: 100 mcg/tablet


How many tablets will you give?

The image shows a box of Synthroid (levothyroxine) tablets with a strength of 100 mcg. Next to it is a pile of yellow, round tablets.

Integer
Discrete

Figure 36. Numerical Characteristics of DCPP 1, 2, and 3

DCPP 4, seen in Figure 37, requires a multi-step procedure. The patient's weight is given in pounds but the physician's order is in kilograms so a conversion needs to take place involving the weight. The weight needs to then be converted to a corresponding mass of medicine, in milligrams. Last, the milligrams need to be converted to a number of tablets to be given.



PHYSICIAN'S ORDERS:

Dilantin 3 mg/kg PO (Patient Weight = 146 pounds)

How many capsules will you give?





Figure 37. DCPP 4

DCPP 5, seen in Figure 38, also requires a multi-step procedure however, the context of this problem, IV drug infusion rates, is connected to a specific drip rate formula for some nurses. This procedure is reviewed in the literature review under Dosage Calculation Procedures. This problem was included on the DCPP survey to

generate observations about consistency of procedure choice rather than difficulty.



PHYSICIAN'S ORDERS:
D5NS at 500 mL over 3 hrs intravenously.
IV set delivers 15 gtts/mL.

a.) How many gtts/min will it take to deliver the prescribed dose?

b.) How many mL/hr will you set the IV infusion pump?




Figure 38. DCP5 5

DCPP 6 served to assist in the categorization of set-up and procedure. Participants were asked to choose the strategy that best described the way that they solved DCP5 1. The name of the strategy and a possible solution process were displayed. The options are displayed in Figure 39.

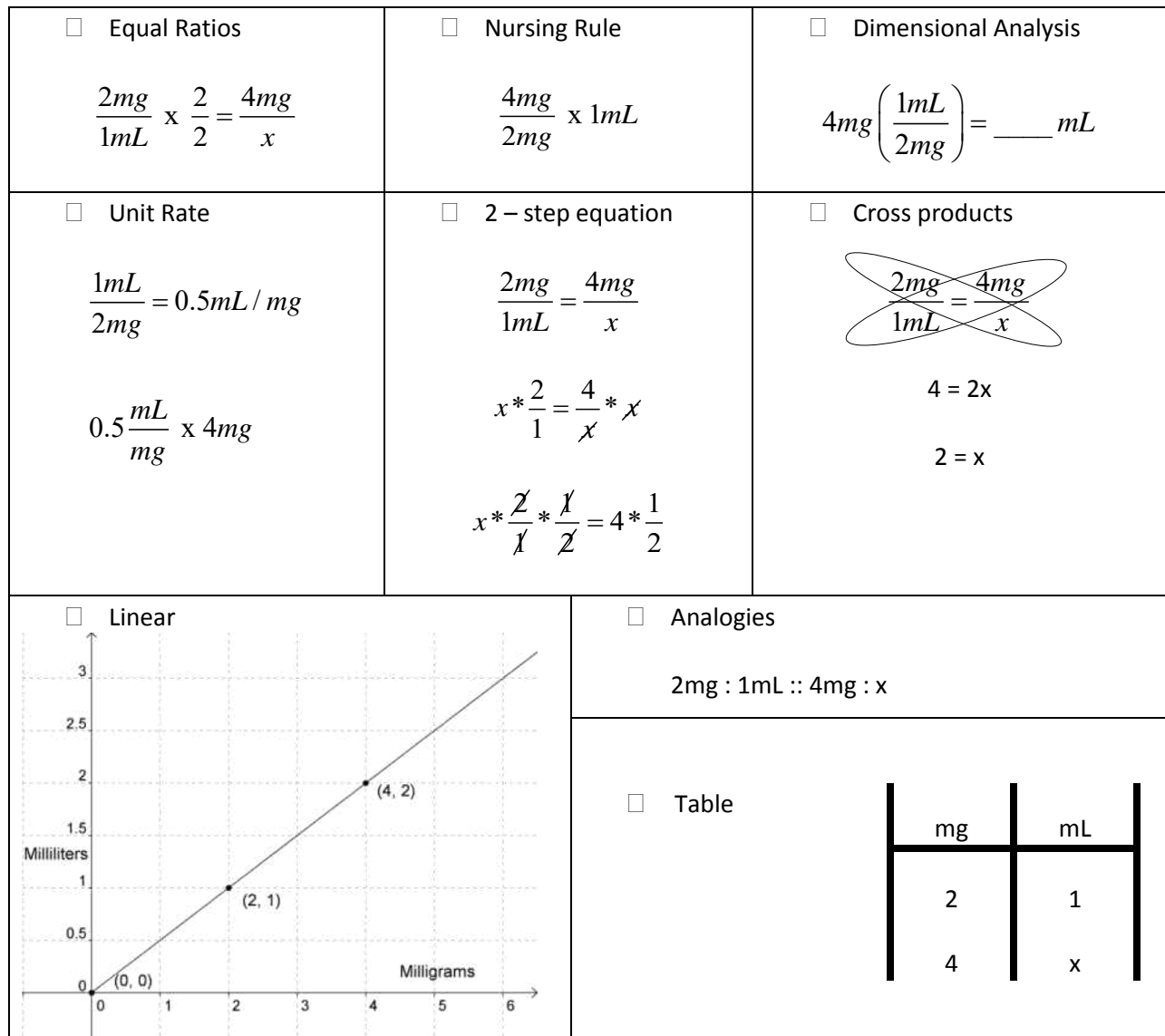


Figure 39. DCPD 6

The researcher used personal knowledge of past dosage calculation test data as well as researched procedures to compile a list of procedures from which respondents could choose. Four of the seven set-ups described in the literature review were included: the nursing rule, dimensional analysis, table, and equal ratios. Despite the inclusion of equality of measures in the list of set-ups, the researcher chose not to

include this as a choice for respondents because this set-up was only found to be documented in one research article and was unnamed in this article (Ercole et al, 2011). Equal ratios were further separated into the three different procedures: equal ratios, 2-step equations, and cross products that were listed in the research of Weinberg (2002).

Two other procedures were named: unit rate and linear. The unit rate procedure was notated by first calculating the unit rate and then multiplying. This was included because of its existence in the literature (Verghnaud, 1988, Ercole et al., 2011, and Fleener, 1993). The linear strategy was represented by a line and called linear instead of using the equation $y = mx$. This procedure is not seen in the literature as being used to solve MVPPs but was included because of its link to Full Proportional Reasoning. These procedural choices were also used to facilitate conversations with research participants and since the linear graph could be used to identify Full Proportional Reasoning, participant reactions to it would be documented.

Everyday Proportion Problems

The four participants who agreed to enter into the next phase of research were asked to complete three interviews and a writing journal. Each interview corresponded to the three different contexts in which a nurse might experience proportional reasoning problems: (a) on tests, (b) on the job, and (c) in everyday contexts. The Everyday Proportion Problems found in APPENDIX A were designed to facilitate discussion during the interview concerning situations that affect problem difficulty and procedure choice. Wedege (2010) defined everyday mathematical knowledge as knowledge that is either acquired or necessary in people's everyday life. The problems were not given

as a test but as a form of interview discussion in which problems were discussed. The focus of this research is not on errors but on thinking; thus, each problem was discussed until a correct solution was reached.

The guiding research for the inclusion of everyday contexts is the work of Gillies (2004). Gillies used a teaching experiment methodology to compare two instructional applications: formula vs. problem-solving. She described the two instructional strategies as follows:

The formula approach involved providing students with the relevant formula for each problem type, demonstrating its use, and then working through practice problems. The problem-solving approach sought to explore students' existing problem-solving skills through sheets of 'everyday problems'. The problems were designed to parallel typical drug calculation problems but were set in everyday contexts. Through class discussion students were encouraged to suggest different approaches that might be used for solving the problems. After working through each sheet of everyday problems in this way, students then applied their preferred techniques to the corresponding set of drug dosage problems. (p.258)

The idea of using parallel problems from everyday contexts made sense from a pedagogical standpoint of basing instruction off of what students already know and merited inclusion in the research design. Being able to directly compare and contrast two parallel problems: one from everyday context and one from DCPD context could possibly generate quality conversations as they had for Gillies.

A problem using identical numbers and ratio type was designed to match DCPD 5. The only difference was the context. The context of travel was used because the rates of miles per hour and miles per gallon could be considered everyday contexts. In order to fit the numbers in the problem however, miles per gallon needed to be changed to gallons per mile in order to keep the numerical structure similar. Participants were

made aware of this difference in the problem if it was not immediately identified. The resulting problem was created and is illustrated in Figure 40 along with DCP5 for the reader to compare structural similarities. A possible solution is also provided using the dimensional analysis set-up to further illustrate these similarities.


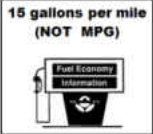


| Airplane Problem | DCPP 5 | | | | | | | | |
|---|--------------------|--|--------------------|---------|----------------|-----------|--------------------------------------|--|---|
|  <p>How many gallons of gas does the plane use per minute?</p> <table border="1" data-bbox="224 825 557 957"> <thead> <tr> <th colspan="2">Flight Information</th> </tr> </thead> <tbody> <tr> <td>Duration of Flight</td> <td>3 hours</td> </tr> <tr> <td>Distance Flown</td> <td>500 miles</td> </tr> <tr> <td colspan="2">Airplane travels at a constant speed</td> </tr> </tbody> </table>  $\frac{500 \text{ miles}}{3 \text{ hours}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} \times \frac{15 \text{ gallons}}{1 \text{ mile}}$ | Flight Information | | Duration of Flight | 3 hours | Distance Flown | 500 miles | Airplane travels at a constant speed | |  <p><u>PHYSICIAN'S ORDERS:</u> D5NS at 500 mL over 3 hrs intravenously. IV set delivers 15 gtts/mL.</p> <p>a.) How many gtts/min will it take to deliver the prescribed dose?</p>  $\frac{500 \text{ mL}}{3 \text{ hours}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} \times \frac{15 \text{ gtts}}{1 \text{ mL}}$ |
| Flight Information | | | | | | | | | |
| Duration of Flight | 3 hours | | | | | | | | |
| Distance Flown | 500 miles | | | | | | | | |
| Airplane travels at a constant speed | | | | | | | | | |

Figure 40. Structurally Similar Problems and Solutions

Other situations were considered in the construction of the Everyday Proportion Problems. While DCPs mainly focus on missing value proportion problems, comparison problems were incorporated into the Everyday Proportion Problems. The situations of numerical structure, semantic type, and context that were found to affect problem difficulty in the literature review were also represented. A summary of the

Everyday Proportion Problems and their corresponding situations are included in Figure 41.

| Problem | Problem Type | Numerical Structure | Semantic Type | Context |
|--|----------------|--|--------------------------|----------------|
|  Dollars:Ounces | Comparison | Non- Integer Continuous | Well-Chunked Measures | Consumer |
|  People:Eggs | Missing Number | Non-Integer Discrete | Associated Sets | Cooking |
|  Length:Width | Comparison | Non-Integer Continuous | Scaling | Scale Drawings |
|  Brown Eggs: White Eggs | Comparison | Function Integers Discrete | Part-Part-Whole | Consumer |
|  Airplane | Missing number | Corresponds to DCPP 5 Continuous | Well-chunked measures | Travel |

Figure 41. Descriptions for Everyday Proportion Problems

For numerical structure, both the type of relationship and type of data were varied. Problems involving both integer and non-integer relationships were used as well as problems involving discrete and continuous data. All four of the semantic types from the literature review were utilized as well as varying contextual features. All problems incorporated an iconic presentation. An attempt was made to incorporate all the situation types presented in the literature review. Consideration of combinational effects of variables was not considered since the research study is qualitative.

Log

Participants were requested to keep an on-the-job log in which they recorded the mathematics that they utilized during their workday. Participants were asked to return to the same three questions every day. They were:

- What mathematics did you use on the job today?
- What instructional techniques in your past helped you to perform these mathematical tasks?
- How did you feel about doing mathematics on the job today? Can you describe an instance where you had a feeling of success or failure?

Participants were also asked to write a descriptive story concerning their experiences with mathematics instruction within the context of nursing. Van Manen (1990) referred to this type of writing as protocol writing. The word, protocol, comes from the Greek language and is the “generating of original texts on which the researcher can work” (p. 63). Some researchers do not choose to use writing because of the participant’s dislike for it or inability to do so. Others prefer the flow of an inviting

interview. The purpose of the writing protocol, according to van Manen, is to invoke a highly reflective attitude within participants. The participants in an interpretive phenomenological study are not just participants but researchers as well. Through their reflective thought, they will be able to summarize their own experiences and be able to describe their own mathematical understanding (van Manen, 1990).

Interviews

Vergnaud (1980) suggested that when an educational topic is being researched the researcher should “go more deeply into the understanding of a specific concept” through the use of carefully planned interviews (p. 192). The interview gives participants the opportunity to express the thinking as proportion problems are solved in different situation. Van Manen (1990) suggested that even though a word-for-word interview protocol is not called for, the importance of being securely grounded in the orientation of one’s research question will prevent the interview from straying from the topic of interest. Van Manen suggested taking the questioning back to the “level of a concrete experience” (p.68) if an interview seems not to be producing the types of descriptions that are desired. Asking the participant to give an example or to explicitly describe what a situation was like are examples of prompts that can help get the interview back on course.

After obtaining informed consent (APPENDIX E), three interviews were conducted with each participant. Each interview protocol was developed with the intent to elicit information about an area of the nurses’ lives where they have used proportional reasoning. Each interview is titled by a specific situation: (a) DCPPs on Tests (b)

Everyday Proportion Problems and (c) Mathematics on the Job. Respondents were encouraged to connect proportional reasoning experiences within these three contexts. The description, purpose, and key questions for each of these interviews are described in detail in the following sections. The interview protocols used in the three interviews are contained in APPENDIX F, APPENDIX G, and APPENDIX H. In order to leave time for reflection by the researcher, each of the three interviews was scheduled at least one week apart. Reflective writing is a significant part of hermeneutic phenomenology since the researcher is a part of the research. Between interviews, the researcher reflected on the meaning of the previous interview and engaged in a process of creating a text describing the phenomenon. At the next interview, the researcher and participant engaged in a “hermeneutic conversation” (van Manen, 1990, p. 99) to clarify meanings. The major components of each of these interviews are explained next.

Interview I: DCPD on Tests

Interview I was entitled DCPD on Tests and was a two-hour interview in which participants’ responses on the survey were discussed. The interview was divided in two parts. The first part emphasized the demographic information and response to the writing prompt. The second part emphasized participants’ responses to the dosage calculation proportion problems and their experiences solving those types of problems. The protocol for this interview can be found in APPENDIX F.

Interview II: Everyday Proportion Problems

Interview II was entitled Everyday Proportion Problems and was another two-hour interview which involved the participants' solving five mathematics problems from the Everyday Proportion Problems which the researcher created (APPENDIX A). The problems were designed to reflect the differing situations that affect difficulty and procedure choice. Two sets of interview questions were constructed around the administration of the Everyday Proportion Problems. One set was asked during or after each problem the participant solved and was constructed in order to elicit thinking in action. The second set was asked after all five of the problems were solved and the purpose was to assist in constructing a more general conversation on the topic of proportional reasoning as a whole and the participants' experiences with these types of problems. The protocol used for the second interview is contained in APPENDIX G.

Interview III: Mathematics on the Job

Interview III was entitled Mathematics on the Job. This interview focused on the the mathematics that was experienced by participants on the job and their protocol journal writings. The journals were collected prior to this interview in order to formulate specific questions. General questions for this interview are located in APPENDIX H. Portions from the participant's writing were cited by the researcher, and follow-up questions related to their descriptions were asked in order to better understand their experiences.

Data Analysis Procedures

The data analysis process consisted of two parts: an analysis of the surveys completed by respondents and analysis of the information provided solely by the four participants. Therefore, this section of the research is separated into two categories: procedures for working with respondent data and procedures for working with participant data. The survey data analysis procedures focus on how the researcher used these data to assist in participant selection. The participant data analysis procedures explain the procedures used in extracting themes in the participant's lived experiences.

Each survey had a research ID number written on it before distribution. The numbers served as the research identification number for each respondent. All data from the respondents were coded using these numbers. The numbers range from 1 to 100 but only 44 surveys were returned. The selected participants were given research pseudonyms as well as research ID numbers.

Procedures for Working with Respondent Data

Demographic Information

Demographic information from the DCPD survey was coded and entered into SPSS in order to create frequency tables for the data. This process was completed at two different times in two different files and then compared for inconsistencies. Any inconsistencies were investigated for verification.

Writing Prompt

The responses to the DCPP Survey writing prompt were compiled into one Word document that cited the respondent number corresponding to each response. After reading through the compiled document twice, themes were created. Each response was read again, this time sentence by sentence. Each sentence or group of sentences pertaining to a specific theme was placed in a table under that theme with the exact quote and the respondent number. Some new themes emerged as this process took place. The document was read again, sentence by sentence to check for the new themes that were created. This same procedure was performed again at a separate time to ensure consistency of placement of statements in theme categories.

DCPPs

Respondent's procedures were classified by the set-up of the solution. The seven set-ups of equality of measures, ratio table, DNL, analogies, equal ratios, dimensional analysis, and the nursing rule were used to classify each solution provided on the DCPPs. Only obvious set-ups were labeled. All other strategies were labeled as other. The MVPP Standard Set-ups that were presented in the literature review are summarized in Table 18 with the name of each set-up, the standardized example used to illustrate the set-up and a brief description of how to identify the set-up.

Table 18

MVPP Set-up Identification Guide

| Set-up Name | Notational Representation | Description | | | | | | |
|----------------------------|---|---|----|-----|---|-----|---|-------------------------------|
| Equality of Measures | $150 \text{ mg} = 2 \text{ mL}$ $225 \text{ mg} = x$ | Function ratio is set up in an equality | | | | | | |
| Ratio Table | <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td style="padding: 5px;">mg</td> <td style="padding: 5px;">mL</td> </tr> <tr> <td style="padding: 5px;">150</td> <td style="padding: 5px;">2</td> </tr> <tr> <td style="padding: 5px;">225</td> <td style="padding: 5px;">?</td> </tr> </table> | mg | mL | 150 | 2 | 225 | ? | A two column table is formed. |
| mg | mL | | | | | | | |
| 150 | 2 | | | | | | | |
| 225 | ? | | | | | | | |
| Double Number Line Diagram | | Two parallel number lines with corresponding values | | | | | | |
| Analogies | $150 \text{ mg} : 2 \text{ mL} :: 225 \text{ mg} : x$ | Equal ratios using ratio notation. | | | | | | |
| Equal Ratios | $\frac{150 \text{ mg}}{2 \text{ mL}} = \frac{225 \text{ mg}}{x}$ | Equal ratios using fraction notation. | | | | | | |
| Dimensional Analysis | $225 \text{ mg} \times \frac{2 \text{ mL}}{150 \text{ mg}} = \text{ ______ } \text{ mL}$ | Multiplication by the function ratio using extensive measures | | | | | | |
| The Nursing Rule | $\frac{225 \text{ mg}}{150 \text{ mg}} \times 2 \text{ mL}$ | Multiplication of the scalar ratio using extensive measures | | | | | | |

This guideline was used to classify each solution.

The development of the MVPP Set-ups came from both the literature and the respondent's written answers to the DCPD questions. The original intention was to use the categories listed with DCPD 6 on the survey to classify responses, however, this

system proved inconsistent. When the categorization process was repeated by the researcher on separate occasions to check for inconsistencies, many solutions were classified differently. Therefore, a new strategy for classification needed to be established. A re-examination of the literature through the lens of respondent's data led to a back and forth process of matching written responses to documented research. The result was the establishment of the MVPP Set-ups instrument to explicate the classification process. Explicit notational guidelines were established in the identification of set-ups so that consistent results could be achieved. With this system, the researcher was able to consistently categorize responses on three separate occasions without any discrepancies. All of the answers to the DCPs that respondents gave are provided in APPENDIX I. The results of the analysis are displayed in

APPENDIX J.

In addition to the use of the MVPP Set-up Identification Guide, each response to the DCPs on the survey was categorized individually and then holistically. The process for this involved first categorizing the strategies for each problem. DCP 1 was categorized for each respondent, then DCP 2, etc. Holistic categorization was characterized by taking a respondent's survey and categorizing all of the problems for that respondent at one time. This was particularly helpful, because this process incorporated the respondents' answers to DCP 6 which asked them to categorize their solution strategy for DCP 1. This question was included on the survey explicitly for that purpose. Examples of this process are outlined in the Presentation of Themes and Data. After each solution was reviewed, surveys were labeled as having a predominate set-up if at least three problems were labeled with the same set-up. These predominate set-up classifications were then used to categorize respondents into stratified groups from which one participant was selected.

Procedures for Working with Participant Data

The participants for this research consisted of four nurses who had been selected for further investigation based upon their responses to the DCP Survey. Each participant was asked to complete three interviews and a four-day writing log, in addition to their initial DCP Survey. Each participant completed the three interviews with the exception of one participant who was unable to complete the last interview. Each interview was transcribed by an outside agency. The transcripts were then checked for accuracy by the researcher who listened to the recorded audio and read the

transcript to make any necessary corrections. This was done at three different times. The participants also completed a four-day writing log where they wrote about the mathematics that they encountered on the job. Part of the interview protocol was to collect these writing logs prior to the last interview so that the researcher could clarify questions about the participants' writing. The participants' original answers to the DCPP were also part of the data that were analyzed.

Three levels of data analysis were used to isolate themes: wholistic, selective, and detailed (van Manen, 1990). Each level is like dialing in on a microscope; the process starts at looking at the whole body of text, next essential sentences are pulled from clusters of text, and then each word and sentence is considered. Wholistic data analysis is also known as sententious because the data are taken as a whole and the researcher formulates a sentence to summarize the meaning. After each interview, the researcher attempted to summarize the phenomenon with a single sentence. After the interview process was complete, a sentence or phrase was constructed to describe the lived experience of each nurse (van Manen, 1990). These summaries were critical in creating the participant's narratives which are found in the next chapter.

Using the next step of analysis, the selective approach, the researcher read through each transcribed interview while listening to the audio a total of three times for each interview. Text that brought about the essence of the phenomenon or revealed significant descriptions, were selected and highlighted. Finally, the detailed approach was used to read through each sentence and look for meaning in individual sentences. After each sentence, the researcher asked herself, "What does this sentence reveal about the conceptual field of proportions for a nurse?" (van Manen, 1990).

Thematic analysis of the data was undertaken to develop structures of meaning from the data. Sentence and clusters of sentences from the wholistic and selective approaches were cut out from hardcopy transcripts of the interviews. These direct quotes from the transcripts were color coded to indicate which research participant provided the quote. Each piece of data was read individually while keeping the phenomenon of proportional reasoning in mind and the researcher reflected whether it should be considered as necessary or descriptive. After the data were filtered, an attempt was made to give a label to the remaining data. These labeled data were then clustered into themes (Moustakas, 1994). Themes were written as descriptive sentences on a single piece of paper and reflected upon by the researcher for several days. The reflection process allowed the researcher to relate personal experiences to the extracted themes. The researcher then re-read the literature review on the conceptual field of proportional reasoning with the data themes in mind. Themes were rewritten to extend or clarify the literature of previous research. The three levels of data analysis provided the results reported in the following chapters. The data came together to provide a detailed description of the lived experience of each participant while also providing themes that can inform pedagogy.

Summary

To deeply understand a concept, the theory of conceptual fields asserts that the concepts must be examined through the interconnections of the concepts, procedures, and situations making up the system. Dosage calculation proportion problems have connections to both the field of nursing and mathematics. Many of the nursing aspects

of the calculations have been studied, as have many of the mathematical aspects, but very few have merged the fields with equal balance. A sample of four respondents, who were carefully selected from returned surveys based upon their DCPD set-up choice and writing prompt response, agreed to take part in this research endeavor along with the researcher. This chapter served the purpose of explicating the methodological procedures used to discover the lived experiences of nurses in relation to dosage calculation proportion problems. These lived experiences were captured through the use of protocol writing, proportional reasoning problems, and interviews. Research that contributed to the creation of writing prompts, problem selection, and interview questions was presented. The procedures for analyzing the collected data included a detailed account of the reduction of data into themes. These lived experiences and themes will be presented next.

CHAPTER 4 RESEARCH PARTICIPANTS

Introduction

The methodology utilized for this research is a hermeneutic phenomenology. A major difference between hermeneutic phenomenology and other phenomenology methodologies is that the research analysis is not bracketed (van Manen, 1990). Bracketing is done when a researcher puts aside their own experiences or connection to a phenomenon in order to not influence the descriptions (Wojnar & Swanson, 2007). In a hermeneutic phenomenology, the researcher becomes a part of the research by allowing the meaning structures to be filtered through her own personal lens to help create meaning. The researcher needs to reflect on her own experiences with the phenomenon of study and merge it with the meaning of the participants. The meanings are co-created between the researcher and the participants (van Manen, 1990). This chapter serves to provide descriptions of the research participants based on the survey respondents, the participants, and the researcher. What follows is a description of the participant selection process and a description of each of the research participants.

A narrative for each participant was created with the intent of highlighting individual lived experiences. These experiences served to inform the reader of the positionality of the participant. The ideas of concept, procedure, and situations may be woven into the narrative but are not explicated. The next chapter serves the purpose of extracting common themes in an attempt to answer the research questions. These narratives serve the purpose of providing rich descriptions of individual lived experiences of nurses.

Survey Respondents and Participant Selection

A total of 44 out of the 100 distributed surveys were returned over a period of six weeks. Demographic information for the professional characteristics of the respondents was self-reported and is summarized in Table 19. Respondents indicated both the type of nurse that they were and the highest degree that they earned.

Table 19

Respondent Demographic Characteristics

| Characteristic | <i>f</i> | % |
|------------------------------------|----------|------|
| Nurse type | | |
| Licensed practical nurse | 4 | 9.1 |
| Registered nurse | 39 | 88.6 |
| Advanced practice registered nurse | 1 | 2.3 |
| Total | 44 | 100 |
| Degree | | |
| Certificate/Diploma | 13 | 29.5 |
| Associates degree | 9 | 20.5 |
| Bachelor's degree | 14 | 31.8 |
| Master's degree | 8 | 18.2 |
| Total | 44 | 100 |

The 44 surveys were placed in categories based upon the solution set-up utilized in the solving of DCPs. These categories were predetermined as a result of the literature review. They are: equality of measures, ratio table, double number line, analogies, equal ratios, dimensional analysis and the nursing rule. Three other categories were created for classification purposes. They are: not identifiable, no work, and no predominate set-up. The not identifiable and no work categories consisted of respondents who had three responses that were not classified or not answered. The no

predominate set-up category was made up of respondents who used varied set-ups. Of the 44 returned surveys, 23 were returned anonymously and therefore, those respondents could not be considered as participants. The remaining 21 respondents provided either an email or phone number as contact information. Table 20 contains a summary of the results of the initial analysis of the survey data.

Table 20

Respondents' Predominate Set-ups and Signature Cross Tabulation

| Set-up | Signature | | Total |
|-----------------------|-----------|----|-------|
| | Yes | No | |
| Equality of Measures | 4 | 0 | 4 |
| Ratio Table | 0 | 0 | 0 |
| Double Number Line | 0 | 0 | 0 |
| Analogies | 0 | 0 | 0 |
| Equal ratios | 5 | 2 | 7 |
| Dimensional analysis | 5 | 2 | 7 |
| Nursing rule | 5 | 4 | 9 |
| Not Identifiable | 1 | 11 | 12 |
| No work | 0 | 1 | 1 |
| No Predominate Set-up | 1 | 3 | 4 |
| Total | 21 | 23 | 44 |

The selection of a participant from each set-up group was further refined by how consistently they used their preferred set-up. Surveys in each category were ranked by how many questions they answered using their set-up of choice. Once the surveys were ranked, the researcher further refined the selection process by looking at respondents' writing prompt responses. Top respondents for each procedure were considered for their consistency in procedure choice as well as their ability and willingness to answer the writing prompt. This factor helped to identify participants who would be more able and willing to provide rich details. Taking these two factors into account, one potential participant from each category was then contacted by email or telephone and invited to participate in the study. APPENDIX D contains a copy of the invitation letter used. After four days of no response, a follow-up email or phone call was made. After a week, the next highest ranked person on the list was contacted. This process resulted in the agreed participation of research participants under the classifications of nursing rule, dimensional analysis, equality of measures, and no predominate set-up. These participants were all first choices. Also, three respondents were contacted to represent the equal ratio set-up and one from the not identifiable category but none agreed to participate. The demographic information provided by the four selected research participants can be found in Table 21.

Table 21

Participant Demographic Characteristics

| Characteristics | Jackie | Cathy | Rachel | Katie |
|------------------------------|--------------|----------------------|--|----------------------|
| Set-up | Nursing rule | Dimensional Analysis | No Predominate Set-up | Equality of Measures |
| Highest Degree | Master's | Associate's | Associate's | Master's |
| Type of College | Traditional | Community | Community | Traditional |
| Highest Level of Mathematics | Calculus | Algebra I | Algebra II | Algebra II |
| Instructional Mode | N/A | Taught with theory | Clinical, On-line, credited course, tutoring | Clinical |

These data were collected from the survey and not from interviews. Set-up was coded by the researcher and all other responses were selected from a list of choices by the participant with the exception of two responses. Jackie did not provide a response to the instructional mode of her DCPPI instruction. She later indicated in the interview that she could not remember how she was taught DCPPIs. Cathy wrote that her institution taught DCPPIs with theory.

Researcher as Data Analysis Instrument

In descriptive phenomenology, the researcher presents a section on positionality (1) for the reader: to allow the reader to assess for biases within the writing and (2) for the researcher: to ensure that the researcher reflects on potential biases in an attempt to make him or her conscious of the need to prevent distortion of data. With hermeneutic phenomenology, the positionality serves to provide the reader with information about the lens through which the information was seen. "The self is not some kind of virus which contaminates the research. On the contrary, the self is the research tool, and thus intimately connected to the methods we deploy" (Cousin, 2010, p. 10). This research merges the fields of mathematics and nursing education in regards to proportional reasoning. The narrative that follows will provide insight into the researcher's connection with this area of research in order to understand the lens through which the data were viewed. The researcher will use a change of voice for this section to assist with transparency.

I have always considered myself a mathematician and have had a love for problem solving and creating algorithms for processes. Sitting in my elementary mathematics classes and longing for a teacher who shared in my enthusiasm and passion for the topic, I decided to teach mathematics. I have had a direct path toward mathematics education ever since. In high school, I took electives in mathematics rather than taking study halls in order to learn more of my favorite topic. In college, I declared my major upon admission and completed my course work in the prescribed four years. I received a teaching job immediately upon graduation and became a middle school mathematics teacher at age 21.

As a middle school teacher, I had the opportunity to teach 7th and 8th grade mathematics. The mathematical content of these grades relied heavily upon rational number and proportional reasoning. During my tenure as a middle school teacher, I had the opportunity to select a textbook for my 8th grade mathematics class. I chose the book, *Mathematics Across the Curriculum* (The Ohio MATH Project, Inc., 1991), which caused some controversy among my colleagues due the text not containing sufficient examples for drill and practice. While I agreed with this assessment, I found it rich with real life applications and meaning. These qualities, to me, were more important, as I could easily create drill and practice questions, while designing application problems was more challenging. I became rather unpopular due to this choice of text and it was consequently dropped after I left the school. Choosing this text helped me to realize the controversies in mathematics education and the need to incorporate contexts and applications to make the mathematics meaningful to the students.

After teaching middle school mathematics for six years, I resigned in order to be a full-time mother to my two children, as being their primary care giver was important to me. I had another child four years later. I always enjoyed parenting and was even a foster parent for three years. I relished the opportunity to be involved in my children's education. I home schooled my biological children for middle school and I was able to participate in creating individual educational plans for my foster children. Among the many lessons this experience taught me was the importance of listening to the learner's questions closely in order to identify the potential misconceptions. In my formal role as a teacher, I probably had listened to the voice of my middle school students, but it took my own daughter's frustration to realize how upsetting it can be to have mathematical

misconceptions. The topic she struggled with, of course, was proportions. She was solving a proportion problem that looked something like this: $\frac{25}{35} = \frac{10}{x}$ and she needed help. I just started talking and explaining without listening to her. After a while, with tears in her eyes she said, “But the x is in the bottom, mom. Why is it in the bottom?” I realized that I was simply telling her how to follow a procedure, while she wanted to understand the concepts. Her frustration helped me later when I was again teaching classes of students to recognize that same voice in them. The voice that says, “Don’t just tell me how to do it, I want to understand it.”

Those students that I am referring to were not middle school students, however, but rather college students. These students struggled with the same issues. After only one year of being home and not working full time outside of the home, I took on employment as an adjunct instructor of mathematics at a college. The mathematics course that I was to teach for the next 10 years was an applied mathematics course in the field of nursing. Having no prior nursing knowledge, I knew that in order to understand the context of the mathematics, I was going to need the assistance of other nurses. While I frequently sought advice of my family members and friends who were nurses, I found the greatest source of assistance in my non-traditional students who had experience working in a health care related field and were just beginning their traditional nursing education. They were able to explain to me the realities of the mathematics that they used on the job. Owing to the input of the experienced nurses’, I was able to modify my instruction. The textbook that I utilized endorsed three different methods of solution, namely cross products, dimensional analysis, and the nursing rule. The nursing rule was new to me. Having a purely mathematical background, I believed that

learning a separate formula (the nursing rule) was not an efficient use of instructional time, as I felt that approaching the problems from a more generalizable procedure would be beneficial. After speaking to many of my students who were in the health care field, they confirmed that many of the nurses with whom they worked only used the nursing rule; thus, it was important for them to understand the formula in order to be able to communicate solution strategies to these coworkers. This also made me realize that for me to communicate clearly, it was necessary to appreciate the view of mathematics held by the population I was teaching, not solely from the perspective of a mathematician. I needed to listen to the voices of the nursing students.

My efforts in teaching the course, Mathematics for Healthcare Professionals, enabled me to achieve the Adjunct of the Year Award for my work. Achieving this award validated my extra effort and stimulated my interest in the subject. I decided to pursue my Ph.D. in mathematics education. This dissertation is the product of that interest.

As well as being able to research my mathematics education interests during my PhD work, I was also able to teach a mathematics content course for elementary education majors as a graduate teaching assistant at the university I attended. The similarities between the courses, Mathematics for Healthcare Professionals and Mathematics for Elementary Education, became apparent. The goal of both courses was to help students revisit mathematical content that had been taught during their elementary and secondary schooling with a constant reflection and re-examination of how this content intersects with their specific careers of nursing or teaching. The opposition towards both courses also became apparent. Reading Wu's (2009) article

“What’s Sophisticated about Elementary Mathematics” helped me understand that these types of courses may not be seen by others as being college-worthy, because the topics covered are seen as being prerequisites for college acceptance. The argument for such courses is that although the topics are deemed elementary, the depth of knowledge of these topics is not something that has been previously taught and needs to be covered at the college level.

My experiences teaching Mathematics for Healthcare Professionals and Mathematics for Elementary Education gave me insight into the many difficulties students had in translating prior content knowledge into usable mathematics for their future occupations. Instructors were required to have a background in both mathematics and education. I connected this idea to nursing and wondered about requiring a Mathematics for Healthcare Professionals course also to be taught by an instructor who had credentials in both mathematics and nursing. Although I do not idealistically meet these credentials, I have learned a great deal about the nursing aspect of the course through my years of experience teaching DCPPs. Believing that my past experiences could perhaps impact others, I decided to pursue a line of research in the mathematics of nurses. Thus, it was through the lens of a mathematics educator who has observed and respects the culture of nurses, that I viewed the data collection and analysis processes in the present study. The findings of this research were made possible by four nurses who shared with me their unique experiences with proportional reasoning problems. A brief narrative for each participant will be provided that highlights their unique qualities in connection to this research.

Lived Experiences of Research Participants

Katie's Narrative: Equality of Measures

Katie's caring personality was expressed in many ways. Her experiences in over 35 years of nursing have provided her with the expertise she needs to educate both her patients and other nurses. Working her way up the ranks as a diploma nurse, she worked in an intensive care unit for ten years. She enjoys caring for the physical needs of her patients as well as educating them about their conditions. She became involved with medical auditing and the business side of her profession. She went back to school and received her Legal Nurse Certificate. During this time, she also worked in the home health care setting as an agency nurse. Her specialty area of nursing is critical care and cardiology. In 2008, she completed her bachelor's degree in nursing. She is currently continuing her education and hopes to become a nurse practitioner. Here is what she shared about her desire to be a nurse practitioner:

(As a nurse practitioner) I actually have a chance to help people learn about their lives and I can teach nurses better too because now I have more, I guess, authority to teach on a higher level. I'm more educated so I can say, "Well, this is why you do what you do." Because nurses tend to get into tasks. It's a very weird, weird occupation. It's kind of a little blue collar, a little white collar. Nurses like to toss in and follow the way they're told and sometimes we, we've got to stop with some of that thinking, they've got to say, "If this, then that." Because, it's hard because they're not really allowed to do that. It used to be the doctors were really smart and we were just women following through orders and that became nursing. As nurses become smart and more educated, the system still wants us to do that. Nursing recognizes that we're smart enough to do more than that, but we're still told not to think outside on our own. And so I found that I wanted to.

Katie is a person that is concerned with the reasons as well as the procedures.

The answer to the question “why?” was an issue in the forefront of her mind.

Like I just love geometry because I always like to figure out how to put things together, and that’s kind of medication, what’s the whole. But if, if you don’t want to (reason), you give one pill and that’s fine, but I think some people you have to understand why you’re doing what you’re doing.

Katie’s confidence in solving DCPPs is tied to her knowledge of normal dose ranges.

You have to know your parameters. Think about if an answer seems reasonable. That is what they teach us too. If something seems unreasonable, then you might be incorrect in your calculations. Like if I’m grabbing five vials because they are 0.5 milliliters, (I need to ask) “why did they unit dose it the way they did?” So they encourage you to think that way.

Katie is knowledgeable about errors concerning DCPPs. She knew of a nurse who had a patient die because the doctor ordered the wrong amount of Digoxin and she did not catch the error. “A friend gave 2.5 of dig because that is what the doctor ordered. But it was supposed to be .25. The patient died. But if you are opening 10 bottles of medicine, something’s wrong.”

She did not have confidence in her mathematical skills. “I really feel very weak in some of the mathematics nowadays. I mean, because I don’t use it consistently”... so she tries to work things out using reasoning. “I think you lose it if you don’t use it.” “Yeah and it makes it worse because then if you do have to do a calculation, it’s like, oh now wait a minute, I don’t know how to do that anymore.” “But I struggle, you can see, I obviously struggle. I do get it but it takes me awhile.” “If I just break them down into smaller, manageable units.”

Because Katie has been in the nursing field for over 35 years, she was able to provide a detailed description of the evolution of DCPs in nursing through the years. She noted the differences between medication administrations past and present. Four important differences were mentioned. The first is that dosages were not calculated by weight in the past but now dosages are calculated in micrograms per kilogram. She said this practice started in the 80s. Second, nurses were responsible for mixing medications in the past. Today, the pharmacy handles this. Third, dosages were not supplied in unit dosages. Nurses had to draw up the desired amount in syringes. Now, unit dose syringes are available. "If the patient got 5 meds there were 5 pill bottles in the drawer and I would take them out and give whatever medication it is and now it's all prepackaged individual pills, vials." Fourth, before IV infusion pumps, nurses would time tape IV bags to monitor IV infusion rates.

She described the time taping practice in detail.

Well the time tapes, the time tapes, we used to have it would be like in increments. So let's say if the IV rate would be at 75 an hour, they would have to 75 an hour the same length. And the same color all the way down the tape. So then you would kind of figure out, okay, so every, every green line is 75 mls so you would see that and then make it a thing, and then every 100 mls would say, you know red and a line, and then, you know, but then you kind of have to. Like sometimes you just have to pull out your own tape and stick it down and start – so it was like when you had 60 ml or sometimes. Well 60 mls an hour was a little bit, you know, we wanted everybody at 100 because it was easy. Yeah. But I mean at 60 it's kind of like, okay, 60, 120, 180, 240. And so would you make up your own tape for that then. Well we had to because we didn't have any other way to do it. You know, but then the machines came along.

The use of machines for IV drip rates affected her ability to solve these once routine problems. DCP 5 on the Survey was an IV rate problem and Katie stated,

This one I really had to think about because I kind of don't do it anymore. So I knew that I needed to give ... 500 divided by 3, so I had to give 166 milliliters per hour for three hours. Right off the bat. Like as soon as you see that, like that's kind of what you do (change the rate to mL per hour). I can think in hour increments because it's easier time and we used to have time tapes so we could figure out how much to give in an hour. So then IV sets, 15 drops ml, so I had to multiply, oh, I don't remember what I did. So I knew that that was how many milliliters an hour. 160 milliliters. So I wanted to figure out it per minute. So I divided by 60, so I have 27.7 milliliters a minute. And then, okay, so yeah, so I wanted to know 15 drops per milliliter so, I don't know, is it right? 7×15 so I had 41.55 drops per minute.

When asked to check her work, she stated, "Right. So I didn't carry – I made an error but I still got the right answer." When asked how she knew to multiply or divide, she stated,

Well I'm trying to narrow it here, to milliliters, but then I'm increasing it by 15 because I know – This would be 2.7 milliliters per minute, but if there's 15 drops, I have to, so this is, this is one, like I know I've narrowed it down to 2.77 milliliters, so I've got to make it bigger somehow.

Katie spoke about the importance of explicitly teaching dosage calculations to nursing students. She did not feel that it would be fair to expect nursing students to automatically apply their past experiences with proportional reasoning to dosage calculations. "I don't think people think in partials. People think in wholes. So when you add a dimension to that, you need to give people a tool to use it."

Cathy's Narrative: Dimensional Analysis

Cathy was eager to share her passion for the need to be proficient at solving DCPPs. Cathy recently (within the last 5 years) graduated from a community college

with an associate's degree in nursing. She is a registered nurse and works in the home health care setting. In this setting, Cathy is frequently on her own in carrying out the doctor's orders for her patients.

You are on your own and there's going to be unforeseen things that you have to deal with...When you are in the home care setting, you are the driver of the car. You are responsible. It is you and only you. You have nobody else to go to. You don't have pharmacy on hand and you do so much more care.

Cathy shared many testimonies of how she was able to use mathematics in dealing with these unforeseen events. She took pride in her ability to problem solve and use her mathematics to help her patients. Cathy attributes her confidence in solving DCPPs to the instruction that she received from her college. Her instructor's exclusively taught DCPPs using dimensional analysis. She was told, "stick with dimensional analysis, don't go any other way or you'll get confused."

Cathy followed that direction and was selected as a participant for this research study because of her consistent use of dimensional analysis to solve DCPPs. She proudly stated, "Once you get dimensional analysis, you are good for the most part." In order to "get" dimensional analysis, Cathy had to work hard. "It took me until almost my third semester to really get my head around that. To really, really, fully understand it." Her understanding of this procedure gives her confidence in her work setting.

So much happens all at once. It's the end of your shift, you are just getting back from the doctor's office, unloading and unpacking the car, getting settled, you are plugging in all of your equipment, mom went out to the pharmacy to get the new meds, you are documenting, updating the MAR chart, checking all the calculations. This is why it is good to be proficient in math. And this is why when all else fails, and I can't figure it out in my head, I'm like, I got to do my dimensional analysis. And I have to line it all up. I'm very much a concrete learner. I work through things that way and I need to visualize and see things right in front of me. (When doing a problem) you know it's right. I got to figure out

a way to get this on top and this on the bottom. And that really, I'm telling you, that is what I rely on.

Although Cathy has confidence in her ability to solve routine DCPD problems, she admits that her initial reaction when faced with a non-routine mathematics problem on the job is panic. Cathy spent a short time working in a nursing home and she shared how this setting was filled with stressful situations.

There is so much going on and when you have to stop and do a math problem, it causes you to panic. We are so over-loaded with work. You have this crisis over here, Mr. So-and-so is peeing all over the floor and hanging off his chair almost falling, and you have this one over here trying to escape through the door. So you're talking madhouse and you're in this situation where you haven't done math in how long, and I don't know how I ever resolved it. You panic. I mean you panic.

She described this in contrast to DCPDs that were common to her and that she routinely solved.... "But if I am fluent at it, I can be very calm. I can be the opposite." This same disposition could be seen in her solving of the Everyday Proportion Problems. Initially she seemed nervous as she took in the components of the problems, but then she would calm herself down by breaking the problem into pieces. She was very verbal while working out the problems.

She was calm and persistent in her solutions on the Everyday Proportion Problems as she took each problem in and said, "Okay, I got to get my head around this first." She became frustrated with the scaling problem and the Brown Eggs:White Eggs because she could not apply her dimensional analysis strategy. The scaling problem was not labeled with any unit of measure and in frustration "You know what, it's funny. I have to call it something. I have to have a word; I have to have a label." She was

unable to give the quantities a label and continued to work with them numerically. She thought of this problem using additive strategies saying that “the interval is not even.”

Cathy had difficulty with the Brown Eggs:White Eggs because she treated the word *more* as meaning subtraction. She calculated that the 12 egg container was filled $\frac{1}{3}$ with brown eggs and $\frac{2}{3}$ with white eggs. She then said, “Now wait; oh jeez, now I got to go a step further. Then I have to subtract. $\frac{2}{3} - \frac{1}{3}$ equals $\frac{1}{3}$. So there is $\frac{1}{3}$ more white eggs than brown.” She used an additive relationship rather than multiplicative. There is two times the amount of white eggs not $\frac{1}{3}$.

She did not like the rectangle problem or the Brown Eggs:White Eggs problem because they involved “proportional comparing.” She said that they were different because in the other problems, “you had all of your information. You had everything you needed to work it out. All your numbers were there and all of your labels were already there.”

Jackie's Narrative: The Nursing Rule

Jackie came into this research from the viewpoint of a student, nurse, and teacher. Jackie held a Master's degree in nursing (M.S.N.) and had 20 years of experience in the field of nursing. Most of her nursing career was spent working in an intensive care unit. She had worked in four hospitals. Jackie, however, was working as a clinical instructor at the time of interview. She had 22 years of teaching experience in nursing. She had taught both clinical and traditional courses. Jackie was strongly connected to the topic of DCPs because she herself had formally taught nurses how to do DCPs, in particular, IV flow rates. She volunteered to formally teach IV flow rates. "They said, 'who wants to teach this' and so I volunteered to teach the IV math. I taught it for fifteen years or so."

Jackie's confidence in teaching and doing dosage calculations was evident. Jackie described herself as liking math and she attributes this positive disposition to a particular instance in her education when a teacher took the time to validate her confusion on a particular topic. Jackie shared this experience in her own words:

I remember in eighth grade where we were solving some kind of equation... and I was not getting it. I was sitting there thinking, "I am not getting it" and the teacher called on me and I obviously did not know it. But, do you know what? He was not known for being real student friendly, but he took the time to explain it and then all of a sudden, I seemed to get math all over. I got all of algebra. It was all interesting to me. Because that teacher took that little bit of extra time in a classroom of thirty people to realize that I had not gotten it and I was not the kind of student who was going to go to a teacher. He took that little extra time. I got it and I liked math ever since.

Interestingly, Jackie did not remember the exact mathematics topic that was being presented. Rather, her connection to mathematics was because a teacher had taken time to value and help her.

Jackie frequently switched from teacher voice to student voice during our interview because of her experiences with teaching DCPPs. She would answer the interview question in terms of instruction, “we always tell the students” this or that, and then she would personalize it and describe her way of thinking about the DCPP problems. Jackie was chosen for participation in this study because of her strong and consistent use of the nursing rule, but she also demonstrated that she could solve each of the problems using the cross products procedure. Jackie spoke extensively on the other types of strategies used to solve DCPPs. She acknowledged that there were several ways to solve these types of problems but was impressed with the number of other ways that were suggested on the survey for DCPP 6. She reviewed each one with interest, seemingly thinking from a pedagogical stand point. When reviewing the other options for solution, she felt like the table would be a good set up to use stating, “I kind of liked your table. I thought that was a good idea. It comes up as a graphic, as a visual, that might be how I see it. It’s a nice visual picture I think.” Jackie’s genuine interest and intrigue in these different procedures was evident.

Jackie did not approach the everyday contexts with the same amount of confidence as she did the DCPPs. DCPPs are *her* everyday contexts. The contexts presented were not familiar to her and this speaks to the arbitrary use of classifying contexts. “Medications and titrations, I can do. That’s okay. I don’t claim to be a grocery expert.” Context familiarity is personal and powerful. Jackie’s disposition toward an

unfamiliar problem changed when she could relate it to her nursing calculations and nursing experience. Initially, she expressed her reserve in solving the People:Eggs problem stating that, “this is the type of problem I never solve.... this looks bad.... (laughing)... I have no idea and I really don’t care.” But after solving the problem successfully and with some prompting, she was able to make comparisons to this type of problem to the DCPPs that she solves on a daily basis. She recognized that she could have solved this problem by applying the nursing rule. “I could have set it up as a desired over have times quantity, which is that frequent calculation that we give students.” She also related the fact that she had to find the information on the recipe card to being similar to what they do in nursing.

(This is) interesting. Just like in nursing, you have to read the label. When we teach students math for med, we give them all the information. What truly stumps them (nursing students) is being on the floor and saying not only do you have to do the math, but you have to go find the stuff, you have to find the med cart; you have to find the key.

These two connections to nursing lead her to rank the People:Eggs problem as the easiest of the Everyday Proportion Problems even though at first it seemed to cause her anxiety.

The airplane problem served the purpose of comparing solution procedures to DCPPs because of its parallel structure to DCPP 5. Jackie’s solutions for both of these problems are shown side by side in Figure 42 so that one can compare the solution to a problem that only differed by context. Notice that she solved both problems correctly, 41 gal/min and 42 drops/min. (Problem answers differed because of rounding.)

| Jackie's Solution to the Airplane Problem | Jackie's Solution to DCP5 5 |
|--|---|
| <p>15 gal. 500 miles <hr/>7500 gallons needed</p> <p>3 hrs. x 60 min = 180 min.</p> <p>41 gal./min</p> | <p>Step 1 - find drip rate 500 mL ÷ 3 = 166.666666 167 mL/hr.</p> <p>For IV infusion, cannot divide gtt's → depends on infusion device</p> <p>Step 2</p> $\frac{V \times D}{T \text{ (min)}}$ $\frac{167 \times 15}{60 \times 4}$ <p>41.75 or 42 gtt's/minute</p> |

Figure 42. Jackie's strategies for similar problems.

Jackie's solution to the Airplane Problem, seen on the left of Figure 42, does not utilize any of the traditional set-ups that Jackie had experienced while teaching DCPs. The process was reduced to arithmetic operations without the appropriate unit labels.

Jackie's solution to DCP5, seen on the right of Figure 42, shows a short-cut version of the drip rate formula. Jackie used the drip rate constant of 4 to quickly calculate her desired drops per minute.

Although Jackie impressively solved the airplane problem quickly and correctly, she stated, "I am not confident about it at all, to be honest. I think this one was hard."

She was able to correctly relate the problem to rate problems in nursing saying that:

This is definitely a multiple step one. I see that there are multiple steps in which are not unusual in that, I might ask a student, to do that in a med that is delivered in milligrams and the dosage is ordered in micrograms, so it would have to change, just like I have been changing here.

But even after this realization, she did not feel comfortable in her solution and did not feel like it would be fair to ask a nurse a question like this on a nursing test. Jackie's familiarity with the types of quantities in the problem assisted in her solution process but not having a formula that she could apply to her solution caused her to lack confidence in her solution. Whereas, she was able to apply the nursing rule to the People:Eggs problem, she was not able to apply the drip rate formula to a problem with similar quantities but different context. This is not surprising as the drip rate formula uses the highly specific term of *drop factor* in its verbalization and the nursing rule uses more generalizable terms of *desired*, *have*, and *quantity*.

The two questions on the Everyday Proportion Problems that were most difficult for her to solve were Brown Eggs:White Eggs and the Length:Width. Jackie had an easier time solving rate problems (whether associated sets or well-chunked) than she did part-part-whole (Brown Eggs:White Eggs) and scaling problems. Both of these problems contained a semantic type where the units of measure are similar. Jackie described this difference very well and said that the problem was that they were "comparing two things that are similar but on two different scales. Something I am not used to doing."

Rachel's Narrative: No Predominate Set-up

Rachel is an energetic, enthusiastic nurse with so much to share. She was eager to share her vast knowledge of what it means to be a nurse and it quickly became apparent that Rachel's strategies for solving DCPs were heavily tied to these experiences. Rachel has extreme confidence in her nursing skills and spoke about the complexities of nursing with ease. Rachel's comments pointed to her comfort level with the mathematics on the DCP Survey: "This is all basic stuff." "I do this every day." "It's like kindergarten." "This one is easy." These comments supported the relevancy of the DCP survey used in this research.

Rachel worked her way up the ranks in nursing, starting as a licensed practical nurse (LPN). She worked as an LPN for five years while she went to school to get her associate's degree in order to become a registered nurse. Her confidence in her dosage calculation skills comes from her extensive knowledge of the drugs and their attached protocols. She was familiar with every drug on the dosage calculation survey. She knew what the drug was used for, the common dose strengths of the drug, and the normal dose ranges. In discussing the DCP Survey, she would always begin discussing the problem by describing the drug and what it was used for. For DCP 1, she stated:

Yeah, Zofran, that is for nausea. A lot of my patients say it works really good. They have it in tablets as well, but you can't go over 8 milligrams in eight hours. It is not recommended. So you (in the DCP problem) want to do 4 milligrams. (Thinks.) You could do 4 milligrams like every eight hours. Six hours would be good but you don't want to go over 8 milligrams every six to eight hours. And so, I mean, this problem is kind of cheap because I know when I give 8 milligrams that is always 4 milliliters, so 4 milligrams would be 2 milliliters. I know this

because I do this every day. Well, every day that I work. Someone is always nauseous.

She was able to share detailed information like this for every problem on the DCPP Survey.

She was chosen as a participant because her solutions to the DCPP problems did not show any predominate set-up. She used analogies to solve DCPP 1 and DCPP 2, her solutions for DCPP 3 and 4 were classified as not identifiable, and she used the nursing rule to solve DCPP 5. Rachel could be considered to be a flexible thinker because of her varied use of set-ups dependent on the problem. Her diverse problem solving strategies made her the perfect selection for this solution category.

Rachel used unique language to go along with her strategies. She did not use the term *analogies* to describe her procedure for DCPP 1 and 2; she used the word *dots*. She said, "I use the dots." Her unique mathematics vocabulary was also illustrated in other areas. She used the term *wormies* to describe loops drawn beneath a number to indicate a shift in the decimal place. She described the cross products strategy as calculating "in a heart". An illustration of these symbols is shown in Figure 43.

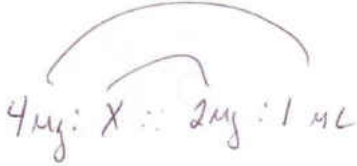
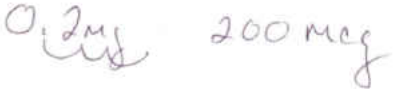
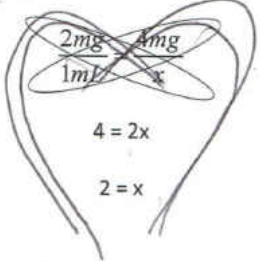
| Dots | Wormies | Calculating in a heart |
|---|--|---|
|  |  |  |

Figure 43. Rachel’s Unique Symbols and Terminology

When discussing her unique mathematics terminology, she said. “I just like it simple”. This attitude was conveyed when she discussed how she came to solve simple DCPPs using *the dots*.

So we got together, a group of girls, and we all showed each other. We do this ratio thing... the dots (the analogy procedure). That’s how we do it cuz when I was in school they did this long drawn out thing and by the time you get to the end, you were like, what has transpired? And this was so much easier and every time, you got the same answer. No matter how you slice it and so I use it, even now.”

Rachel did not have even a hint of mathematics anxiety. She was very confident in solving any mathematics problem and approached each problem with a problem-solving attitude. However, she was not always confident in her answers. She categorized the Brown Eggs:White Eggs problem as being the most difficult. This problem took her by far the most amount of time to solve and yet she displayed continual persistence until she achieved an answer. “Look, I just went from this answer to this answer to the same – Yeah, I’m real confident.” She went on to say, “I think you are trying to trick me.” But she was determined to solve the problem. “All right. I am going to figure this out, but I’m gonna have to burn some brain cells on this one.”

The Airplane problem was ranked second in difficulty but she still did not express any negative emotion toward the problem. “Yeah, the only thing was the airplane one; I had to really think for a second. Like it didn’t make me anxious. It was like, now wait a minute, you know, put your thinking cap on.

She shared an experience that also illustrated her confidence.

My girlfriend’s daughter, she is in tenth grade now and they are doing this stuff and the girl couldn’t do it, and I’m sitting there, and I’m like, all right, I’ll figure it out, because, math I can do.

Rachel did not see similarities between any of the questions from the Everyday Proportion Problems and the DCPD Survey. Even for the People:Eggs problem, which was the only everyday problem for which she used analogies in her solution process, she did not see the similarities. She was unfamiliar with the contexts, saying that she never does problems like this. She also did not use ratios in any of the problems except for the Brown Eggs:White Eggs problem. She viewed proportional reasoning problems in terms of multiplication and division being applied in the proper order.

Summary

Specific details about the individual participants and their personal lives were presented so that the reader could create a lens from which to view participant data. These data are infused into the themes contained in the next chapter. As this is not a case study, themes are not organized by participants. However, each participant’s responses will be highlighted under their predominate category of solution set-up. A brief conclusion made by the research about each of the participants

understanding of proportional reasoning is given in Chapter 6. This present chapter served the purpose of introducing you to the research participants whose lived experiences shaped the answers to the developed research questions and the conclusions that are to follow.

CHAPTER 5 PRESENTATION OF THEMES AND DATA

Introduction

This chapter serves to present the data provided by the respondents and participants in an attempt to answer the specific research questions. Each of the three research questions will be presented in its own section. The themes from each question will be provided.

Research Question 1: Lived Experience

What are the lived experiences that nurses have with solving proportional reasoning problems on written dosage calculation tests and in nursing practice?

This section of Chapter 5 focuses primarily on the four participants, Jackie, Cathy, Rachel, and Katie, who agreed to further research participation. However, some data given by other respondents are included when they pertain to a theme that emerged in the lived experiences of the participants. Data from respondents are provided with the associated respondent numbers. The lived experiences are categorized using the contexts of dosage calculation tests and nursing practice.

Dosage Calculation Tests

Data concerning dosage calculation tests were collected from two sources: the DCPD Survey writing prompt and Interview I with the four participants which was comprised of Jackie, Cathy, Rachel, and Katie. In both sources, experiences with

taking dosage calculation tests were cited as having occurred (a) in nursing school, (b) upon hiring for a nursing job, and (c) at regular intervals during employment.

In Nursing School

Six respondents (5, 13, 19, 37, 41, 44) wrote about their experiences taking DCPD tests in nursing school. Tests occurred prior to admission, prior to clinical experiences, and throughout coursework. Respondent 37 stated that the nursing students in her program “were tested in this (DCPD) knowledge both in written and practical (clinical) form.” Tests were given in both written and oral formats. Oral tests were administered in the clinical setting.

Jackie shared her experiences as an educator and confirmed that she tested her nursing students within the clinical setting. She did not necessarily check their procedures although they were required to write down their processes. She explained:

On most math tests, you grade on the correct answer and the correct label. You don't necessarily grade them on the process, which sometimes can be scary when you get that odd student who gets the right answer with some weird math.

Cathy talked about the tests that she had to take at nursing school. She said, “They really just merged the math in with whatever you were doing at that time.” She indicated that there were approximately five mathematics problems on every test she took, but that these problems did not count toward her grade. She also recalled having approximately 10 mathematics problems on her NCLEX exam.

Out of all the nursing mathematics tests, the ones with the highest stakes occurred at the beginning of the program and before students could enter the clinical setting. “When we started nursing school, there was a short course with a test during

our orientation. We had two chances to pass the test or we had to withdraw and re-apply for admission,” shared Respondent 5. Adding to this, Respondent 44 shared a similar requirement of her nursing program, “It was considered a required course in which the test must be passed in order to remain in the nursing program. The course was given in the first semester; however, a test must be passed at the beginning of each semester.” The practice of continual testing throughout the program was experienced by others. Rachel recalled that she had weekly mathematics tests in school, and students were expected to score 100% on the tests. She stated that, “When you are dealing with people’s lives, you can’t make a calculation error. The pressure was on.”

Mathematics testing was also used to measure the readiness of nursing students before they could enter the clinical field. Cathy took tests before clinical courses and was expected to earn an 80% or higher in order to be able to move on to the clinical setting. Students had two opportunities to pass the test. Those who did not pass were out of the program. A similar requirement was confirmed by Respondent 19, “Then we took a test. If we didn’t pass, we couldn’t move on to dispensing meds in the clinical setting.”

At Time of Hire

Two respondents from the surveys and two participants also mentioned the presence of DCPD tests required at the time of hire. Respondent 18 stated that, “Medication exams are often given at time of employment or in specialty areas.” Respondent 9 discussed this from the aspect of the test administrator, stating that, “I

can't tell you how many nurses do poorly in our med test required on hire, in the calculation portion of the test, due to basic math and formula set-up."

Rachel said that she has taken written math tests on interviews for new jobs as well. She described this process.

You have to get x amount right before you get the job, or if not, they make you repeat it or do a little class through them or something. You know, they can't put a liability on the floor.

Cathy also confirmed this protocol for having to take a mathematics test before being hired.

On the Job

The presence of retesting during employment was only cited by one survey respondent and by one participant, Katie. This testing did not carry the stress or the high stakes of other DCPD tests. Respondent 17 stated that she was "retested yearly in the course of my work." Katie stated that at the hospital where she was currently employed, nurses were given an on-line critical care mathematics test every two years. She described the testing process.

Usually what happens is a bunch of nurses get together and help each other take the test. Because, I mean, that's kind of how nurses do things anyway. If we don't know the answer, then you go to the next person and you kind of figure out your dosages together.

The shared experience of working together to solve DCPDs was continued as a major theme in nursing practice as will be described in the next section.

Nursing Practice

Three common themes emerged from the lived experiences of nurses within the nursing practice: trust but verify dosages, pick a procedure and stick with it, and know common dosages.

Trust but Verify Dosages.

All of the participants and five respondents made mention of the need to double check your calculations with other professionals. Respondent 23 stated that, “We always double checked with another nurse and if there was ever a question, we checked with our nursing supervisor.” Respondent 40 stated that, “While at my current position, I still double check many calculations especially pediatric dosages. I also help my trainees with calculations and make sure the calculations are correct before allowing the med to be given.”

Jackie was asked specifically if there was a stigma attached to asking for help with dosage calculation. She discussed her experiences with the accepted and even mandated practice of double checking dosages with other professionals.

I have never run into that. I was working evening shift one time and the person for night shift had an emergency and asked for a double; so I worked a double. Now, I don't normally work doubles. Because I am not really good, and the Heparin drips were different, and I was so scared that I would mix them up that every time the blood was drawn, I had to recalculate. I would say to somebody, ‘would you just double check this for me?’ At 4 am, working a double shift, I don't want to make an error. And I have never. In many institutions it is a policy that if you are giving certain drugs, that you have another nurse check the dosage. With a narcotic count, there is always two nurses that count. So, I have never run into the stigma and I have worked in about four different facilities.

Jackie added:

I always say 'trust but verify'. You can run that simple math problem. If you can't, you know, have a peer over here to maybe help you with that. You can call the pharmacist and say, 'could you do that one more time'.

Rachel discussed her ability to easily ask for a double check from another professional.

And you know where I work, and I'm very comfortable, if I'm not, you know, if I'm not 100%, I'm going to get double checked behind me. Either I will have a colleague make sure they get the same answer or I'll call Pharmacy – 'Hey what's up', you know, 'what did you get on this?'

Rachel even described how she reviewed the drug orders with her patients before administering drugs.

When I see high doses, I always verify. (Asking the patient) 'You take that at home? You take 100 milligrams at home?' You know, just to make sure that they're on the same page with me. Because unfortunately, you know, stuff happens.

Katie confirmed that she also would check calculations with others if she were unsure of her results. She indicated that she knew of a person who personally administered a lethal dose of a drug and because of that she triple checks her drug dosages.

I probably put extra steps (into the solution) but if I make it make sense then I can avoid it becoming a problem. I don't trust anything. I'm not a very, like faith, person anymore. You know, because I don't want to make a med error. One time, one of my girlfriends went to nursing school and she and a girl started together at (*a certain hospital*) and, one of the girls, the doctor ordered 2.5 of Digoxin and she gave 2.5 of Digoxin and the patient died. It was supposed to be 0.25, so it was ten times the dose. A ten times error. And so she gave it, and it was like she was equally liable. Because she needs to figure it out. If you open 10 bottles of medicine, something's wrong. You know two bottles might be okay.

Like I grabbed three bottles of medication because the one thing, with the protamine the other day, the way it was listed in the Accudose, it listed the milligrams per ml. I knew I needed 25 milligrams, but the way that they had it listed versus what was on the vial was different. Well I grabbed three vials because it said something like 10 milligrams per ml. So I needed three, because I needed two and one-half. But the bottle itself had 50 milligrams in it. (She needed two and a half milliliters not two and a half vials.) So that's why I always triple check my medicine. So I look at the vial and I'm like, I thought three was a lot. I didn't give the med very often. But, you know, you never know what dose is going to be needed. But nurses are really meticulous that way. We really are. I watch all of my nurses. Really, really look at those things before they give the calculation. They really sit down and they'll go with each other and talk it out with each other. Be really careful. Because you don't want to make a med error. Med errors are scary.

Both Jackie and Cathy discussed double checking not only saves lives but saves their jobs. They both realized that their jobs and futures were in trouble if a mistake was made. Jackie shared her feelings about double checking:

Now the pharmacy mixes them all, and they label them like heck, and you got to really look at those anymore. The pharmacists are good and they calculate and they give you the sheets with them so that you know how much you going to cue, but I can tell you and I see as a teacher, pharmacists make mistakes and the bottom line is us, and it always rolls downhill. Oh yeah, the pharmacist made a mistake, but the nurses should have caught it because the nurse is the one at the bedside.

Cathy provided a descriptive experience about an incident that she had where she was unsure of a medication order and had to work with the medical intern on call to determine the proper dosage.

When you work the weekend shift like I did so often, you often, when you have to put a call into the doctor, you're not getting a doctor, you're getting an intern. And you end up figuring out real fast that you, as the new nurse, have to be smarter than the intern. It makes the nurse; it makes her have to say "okay, I'm the professional here. This is my license". So, anyhow in that instance it was a cancer patient and her blood pressure was dropping. This is a little more vivid.

And she was really fragile. She was not doing well. And she came; I think she had just come back to us from chemo. And her pressures were dropping, so it was an emergent situation really and I called the doctor, and of course I got an intern, and he decided to give her fluids and, um, I'm like okay. I mean this doctor wasn't even giving me the, how you say an order. You say an order (and) you get the route, the time, the patient, dose. So I had to ask him everything. "So what are we giving? How much?" Be sure to ask. So I knew the route was IV and over what amount of time. Mind you, okay, normal saline is given for everything. It's fluid; you're dehydrated; you need fluids. And he's like, "um, well, darn it, I left my book in my car". He's like, "I don't know; what do you think?" I'm like, "oh my goodness, are you kidding me?" I'm like okay, I'm sitting there trying to think – okay it is normal, what do we normally give for a quick bolus of fluid? I did work in the ER in clinical and I'm trying to scan my brain and I'm like what – like over a bolus – because bolus is like, anything that is bolus is usually like 15 minutes, a half hour or 15 minutes. I know that much. So, I'm like, "What? For like a half hour?" and he's like, "um you can". No confidence. Oh my gosh, okay, "how about an hour? Maybe an hour?" He said, "If you're comfortable with that". I was like, okay, this isn't working here. So I knew it was one liter, and he was trying to tell me 15 minutes initially. Now mind you, this lady was 90 lbs. You think a liter – what do you think a liter of fluid is going to do to her in 15 minutes. Right? Right, pretty much anybody can figure that out. So, I think we decided over, I don't know 45 minutes, to kind of come in between. But then I'm trying to think if I had access to any resource on that and I reviewed it with another nurse. Yeah. But, you know, I had to at that point decide that that amount was okay because if I didn't, I would have had to call him back up and had a verbal approval to have it redone... Yeah that's really scary.

Cathy's experience echoed Jackie's, Rachel's, and Katie's experiences. The need to verify drug dosages is essential and expected. The possibility of making an error presents scary consequences for both the patient and the nurse.

Pick a Procedure and Stick with It.

Many of the nurses talked about their personal ways of learning and what worked for them. Jackie, Cathy, and Katie all shared stories about the importance of drug

calculation skills in high-pressure situations. Each one spoke of how their strategy use gave them confidence. Jackie articulated her belief that nurses should select one procedural strategy and stick with it. Cathy's confidence in her calculations comes from her faith in her dimensional analysis procedure. Katie spoke of how her mental math strategies became automatic when in a crisis situation. Their stories are provided.

As an experienced nurse, Jackie volunteered to assist with the mathematics instruction of her clinical nursing students. The following transcript records her instructional strategies of DCPPs in her voice as the teacher.

The students had already had basic drug calculation and they learned the desired over have (the nursing rule) and felt comfortable with that, whereas when I worked, I felt a little more comfortable with have over have equals need over need (equal ratios). But I wanted to also be adaptive to students and piggyback on what they already learned, so um, I used that but the other caveat is that I told the students three ways: proportions (equal ratios), desired over have (the nursing rule), and I even threw in a little dimensional analysis, but it was real little because it does not really work for my brain really well, but with some students it does. I would be very careful telling the students to, 'pick one and stick with it.' If this one works, stick with it. If you don't like what I am saying, put your hands over your ears and go 'la, la, la', because you'll become very confused if you try and do all three. For the most part that worked. Although every once in a while I had a student come up and say, you solved it all those different ways and it got me confused during the test. Pick one way and stick with it for the purposes of this. I really try to use that K.I.S.S. method, that Keep It Short and Simple, because when you get on the floor, that is what you need. You need to be able to figure out quickly and you need to be able to make it through. Double check and go. Because when that patient comes out of open heart surgery with four different scripts ... granted, you have cheat sheets and pharmacy has probably done a lot of the calculations... but you need to trust, but verify. Those people could make an error. That was my whole reason for trying to at least to show them two solid ways and pick what is right for your brain.

Cathy emphasized how using dimensional analysis to calculate DCPs on the job gave her confidence. Cathy works as a home health nurse for young patients. She describes how the stress of her job requires her to be quick with her calculations.

Okay so when we finished up with this doctor visit, like I said it took about 3 ½ hours, mother dropped us off, myself and my client, and she went to get the prescriptions filled, because she needed the antibiotic right away. Well it was summer and it was humid. She has no air conditioning in her car. You know, those kinds of things you're thinking about. So there you are getting back. You know that you have meds to give pretty quickly because you have the regular meds that are due. You have an hour window before and after the meds are prescribed. Um, and not only that but guess what – it's the end of my shift. You can't just dump this all on the next nurse. So, right, you're unloading everything, you're stabilizing the child, you're making sure their temperature didn't go up because they're out in the heat. And make sure their oxygen level is good and hooking everything back up again because everything's powered by batteries. So this pertains to the math portion because as we all know you need to be quick. It's why it is good to be proficient in math. And this is why when all else fails, and I can't figure it out in my head, I'm like, I got to do my dimensional analysis and I have to line it all up. I'm very much a concrete learner. I work through things that way and I need to visualize and see things right in front of me.

Katie talked about how experience really helped her to feel comfortable and confident in her calculations. She used a scalar decomposition relational calculus so that she could work out solutions in her head.

They're (the other nurses on the unit) like, 'well, how do you figure out the protamine'. And I'm like, 'you know, okay, we need to give this much protamine' and they're like, they have a big bottle this big in the code cart and they're like – 'how much protamine is that'? So there's 10 milligrams in a milliliter and I need 25, so, 2 ½, one plus one plus one half. So, you know, you do it quickly but, it's funny, like you do it quickly in your head when you're in a code.

These nurses all found procedures that worked for them. They had confidence in their procedures and using them in emergency and stressful situations. Connected to

these procedures was a familiarity with common drug dosages that assisted them in their assessment of reasonableness of an answer. This theme of knowing common dosages is described next.

Know Common Dosages.

Jackie, Rachel, and Katie shared how being familiar with common drug dosages affected their work. Jackie discussed the automaticity of drug dosage administration when a nurse works in the same unit consistently. She was asked to comment on whether she believed that nurses actually did mathematical calculations in their head when they were checking the drug dosages.

We all work in different areas and most everything comes in a pill, maybe once in a while. . . for instance, I worked in the CCU for years and years. . . we gave the same drugs over and over again and the same dosages over and over again. And they came the same way over and over again. You just pull them out and you know there are two, and we automatically do the math in our head because we know that Tetracycline comes in 250 mg and the doctor ordered 500 mg, so we don't sit down and write that. Do they do the math in their heads? I can. After a while. It is automatic. Now, if I worked in CCU and I went to OB, I would have to think through that math again. Personally and professionally rethink through that math and have to probably do it a little bit.

Katie shared an experience where this familiarity and repetition on the unit caused problems when a substitute drug was ordered.

Like we used to give Pentothal to do cardio versions so that people sleep; they'll be asleep for a minute or two. We want them to be asleep long enough to do the shock, but the Pentothal, we can't get it. It's really expensive. It's hard to get. There are a lot of meds they're not making. So we had to change to Brevital. So we had to do a whole other (calculation) because we used to give 100, 200 of Pentothal, but Brevital, we will give anywhere from 25 to 60 micrograms versus

100 to 150 (micrograms of Pentothal). So all of us were really kind of crazy figuring it out. You had to switch it in your head.

While misplaced automaticity of drug administration negatively affected Katie in this instance, she shared how her knowledge of the drug Integrilin once helped her to catch a pharmacy error once for one of her patients. “The patient was ordered 22,500 mg but the maximum dose is 22,000 or something like that.” She recognized that the pharmacy had done it wrong. “I know that our maximum dose that you want to give is this regardless of how many kilograms your patient is.”

Rachel talked about the confusion that surrounds some drug dosages that have wide normal dose ranges.

Like Seroquel, (it can be prescribe from) 12.5 up to 300 (milligrams). Unfortunately, it is such a wide range like that. So you have to know. It depends on the diagnosis. Why is the patient getting the higher milligram versus the lower milligram? And it can even be just the move of a decimal point unfortunately. 12.5 to 125. Something as simple like that. So there are errors that unfortunately happen. Seroquel is also an antipsychotic but for like an older person, as a mood stabilizer, a lower dose is recommended, for like a dementia patient, just to keep them calm, 12.5 maybe twice a day. But a person who is psychotic and is having an episode or has severe mental health issues. They're the ones that are going to get the 300 a day like two or three times a day. So, unfortunately if you don't know that –It could be a bad thing. Yeah. Just stuff like that. Even like Lopressor, how it increases your heart rate – I mean lowers your heart rate, it works for your heart, it is a beta-blocker, and it is supposed to keep the blood pressure and heart rate under control, but you know, some people are like on a maintenance dose, maybe 12.5, 25, but if you come in and you're giving them 50 to 100, if you make a mistake you're going to drop their heart rate. ...So you really, you really have to know the condition of your patient. Why am I giving this drug? What is the purpose of it? Or even, are (they) supposed to get it once a day versus twice a day? I mean that happened before. I had a lady, and her heart rate was like in the 40s, and we're like, why is her heart rate so high [*sic*]– they had her on a high dose of the metoprolol instead of the lower dose.

Jackie, Rachel, and Katie all shared detailed descriptions on how being familiar with specific drugs impacted their practice. Years of practice and repetition provided these nurses with an added layer of knowledge to apply to their dosage calculation problems. Rachel summarized the importance of knowing specific drugs when she said, “The biggest thing is with, on the job, is knowing what the most common dose is.”

Research Question 2: Procedures

What are the procedures that nurses use to solve proportional reasoning problems on a dosage calculation survey?

After an analysis of the surveys that were returned by mail, the frequency of each set-up was tallied and the results are provided in Table 22. The set-ups of the nursing rule, dimensional analysis, and equal ratios were the three predominate set-ups. These data support the literature in that these set-ups correspond to the three commonly taught procedures: the nursing rule, dimensional analysis, and cross products. The set-up of the equality of measures, which was not common in the literature, was identified as the fourth most frequently used.

Table 22

Set-ups Used in Solving DCPs

| Category | Occurrences | Overall Classification |
|-----------------------|-------------|------------------------|
| Nursing rule | 46 | 9 |
| Dimensional Analysis | 39 | 7 |
| Equal Ratios | 37 | 7 |
| Equality of Measures | 21 | 4 |
| Analogy | 2 | 0 |
| Ratio Table | 0 | 0 |
| DNL | 0 | 0 |
| Not identifiable | 64 | 12 |
| No work | 11 | 1 |
| No predominate set-up | - | 4 |
| TOTAL | 220 | 44 |

Actual responses were scanned from the surveys to illustrate written procedures and are found in APPENDIX I. Selected responses pertaining to each set-up will be presented in the same order that they appear in the literature review: equality of measures, ratio table, double number line, analogies, equal ratios, dimensional analysis, and the nursing rule. If a set-up was used by a respondent who was also selected as a participant, her answer was highlighted. The three additional categories of no work, not identifiable, and no predominate set-up will also be discussed.

Equality of Measures: Katie

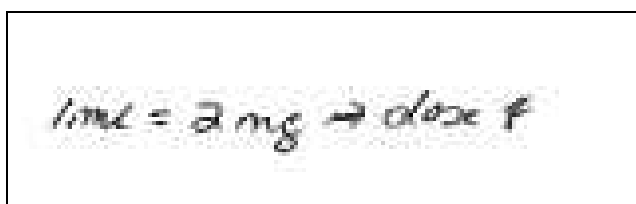
A total of 21 responses were classified as using the equality of measures. This represents 10% of the data. Additionally, four respondents were classified as using equality of measure as their predominate set-up (Respondent 3, 9, 21, 27). Respondent 3's work is provided in Figure 44 to illustrate this set-up.

| DCPP | Respondent 3 |
|------|--|
| 1 | 2mg/ml $\text{how many ml} = 4\text{mg}$ $2\text{mg} = 1\text{ ml}$ $4\text{mg} = 2\text{ ml}$ |
| 2 | $5\text{mg} = 1\text{ ml}$ $2\text{mg} = \frac{4}{10}\text{ ml}$ $\frac{1}{10}\text{ ml} = 0.5\text{mg on syringe}$ |
| 3 | $100\text{ mcg} = 1\text{ tablet} = 0.1\text{ mg}$ $200\text{ mcg} = 2\text{ tablets} = 0.2\text{ mg}$ |

Figure 44. Respondent 3's Equality of Measures Set-up

The presence of a set of equal signs and/or omission of ratio or fraction notation aided in the identification of this set-up. This set up consists of setting one extensive quantity equal to another extensive quantity rather than placing the extensive quantities in a ratio relationship. Katie was chosen for participation in the research study because of her use of this set-up. Her solutions are described next.

Katie began DCP 1 by setting 1 mL equal to 2 mg. Katie's solution for DCP 1 is provided in Figure 45.



The image shows a handwritten equation inside a rectangular box. The text is written in cursive and reads "1 mL = 2 mg -> dose P".

Figure 45. Katie's Response for DCP 1

This relationship between 2 mg and 1 mL in DCP 1 is called the dose strength and is usually written as a ratio. A ratio can be written using ratio notation, 2 mg:1 mL, or in fractional form, $\frac{2\text{mg}}{1\text{mL}}$. With the equality of measures set-up, the measures are written as an equality, 2 mg = 1 mL. When describing her solution process, Katie did not use the equality relationship in her speech, instead she said, "I have 2 milligrams per ml". When asked about why she wrote 1 mL = 2 mg, Katie stated, "2 milligrams is 1 milliliter". She seemed to flexibly view the dose strength as an equality and as a ratio. She proceeded to double both the mass and volume to find that 4 mg would equal 2 mL. The relational calculus that Katie used with her set-up was scalar decomposition. When asked if she knew how to set this up using a formula, she said that she did not.

Katie applied the same scalar decomposition relational calculus to DCP2. Her response is illustrated in Figure 46.

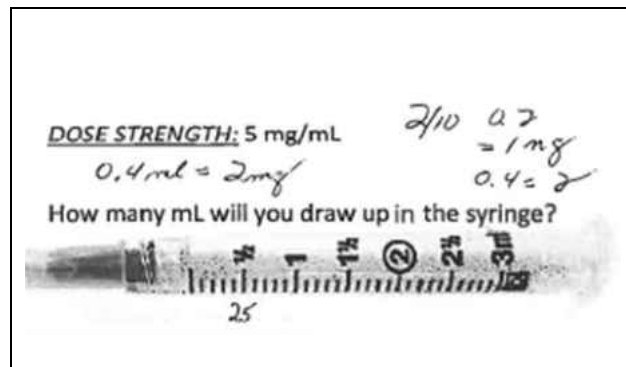


Figure 46. Katie's Response to DCP2

The numbers in this problem were not integers and so she had to do some creative building up in order to get “nice” numbers. Her written notation may be difficult to follow because Katie used the syringe to help her keep track of her equalities.

All right, so, Haldol is 2 IM (intramuscular) now. So 5 milligrams in a milliliter, so I know I need to give 2 milligrams, so I'm going to be giving less than half because there is less than 2.5 at the half. (She marks this on the syringe) So you know that it's less than half of .5 milliliters. But what I end up doing is, is because 2 and 5 aren't nice, I'll make it 10 milligramsit's kind of hard. So 5 milligrams is 1 milliliter. I'll make it 10 milligrams over 1, like, equals, so 10 milligrams per ... Like I would just try and figure out where 2 milligrams is, it's kind of hard, so two tenths. So 0.4 mL would be 2 milligrams. . . I had to get it down to tenths.

In order to clarify her response, the researcher wrote out what these steps looked like symbolically in order in Figure 47.

| |
|-----------------------------------|
| $5 \text{ mg} = 1 \text{ mL}$ |
| $2.5 \text{ mg} = 0.5 \text{ mL}$ |
| $10 \text{ mg} = 2 \text{ mL}$ |
| $1 \text{ mg} = 0.2 \text{ mL}$ |
| $2 \text{ mg} = 0.4 \text{ mL}$ |

Figure 47. Katie's Steps for DCPP 2

Katie's procedure was clearly tied to the use of the syringe. She shows equality of measures in a unique way by actually labeling the syringe with milligrams when she wrote 2.5 (milligrams) under the 0.5 mL on the syringe. This response came closest to being labeled as a double number line diagram but because only 1 corresponding set of values were indicated on the model, it was not.

Respondent 9 and 21 were both classified as using equality of measures for their set-up but it is obvious that their relational calculus as seen in Figure 48 was that of the rule of three.

| Respondent 9 | Respondent 21 |
|--|---|
| <p>I would draw up 2ml</p> <p>If $\frac{2\text{mg}}{4\text{mg}} = \frac{1\text{ml}}{x}$ cross multiply</p> $4 = \frac{2x}{2}$ $2 = x$ | $\frac{2\text{mg}}{4\text{mg}} = \frac{1\text{ml}}{x}$ $2 \frac{x}{4} = x = 2 \text{ ml}$ |
| <p>If $\frac{5\text{mg}}{2\text{mg}} = \frac{1\text{ml}}{x}$ cross multiply</p> $\frac{5x}{5} = \frac{2}{5} \frac{4}{2.0}$ $x = 0.4 \text{ ml}$ | $\frac{5\text{mg}}{2\text{mg}} = \frac{1\text{ml}}{x}$ $5x \cdot 2 = x \cdot 0.4 \text{ ml}$ $5 \sqrt{2.0}$ |
| <p>First you had to convert mg to mcg</p> <p>$1\text{mg} = 1000\text{mcg}$</p> <p>$2\text{mg} = 2000\text{mcg}$</p> <p>If $\frac{100\text{mcg}}{200\text{mcg}} = \frac{1\text{tab}}{x}$ Give 2 tablets</p> $200 = 100x$ $2 = x$ | <p>$62\text{mg} = 200\text{mcg}$</p> <p>$\frac{100\text{mcg}}{200\text{mcg}} = \frac{1\text{tab}}{x}$</p> $100 \frac{x}{200} = 2\text{tablets}$ |

Figure 48. Equality of Measures Set-up with a Rule of Three Relational Calculus

Because of the use of the rule of three relational calculus, as well as the presence of the fraction bar, both Respondent 9 and Respondent 21's answers could possibly have been interpreted as equal ratios. The decision to classify these responses as equality of measures came from the evidence that the values were used as equal measures rather than equal ratios. The presence of a second set of equal signs was the strongest indicator that each of the measures was thought of as being equal to another rather than the values being in a ratio relationship. The presence of the additional equal sign used by Respondent 9 and 21 is highlighted in Figure 49.

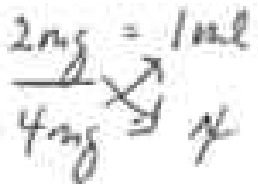
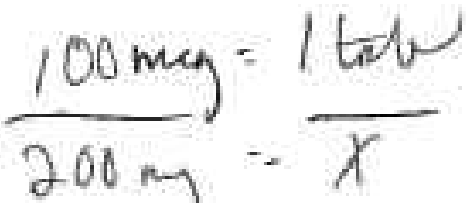
| Respondent 9 | Respondent 21 |
|--|---|
|  |  |

Figure 49. Presence of Two Equal Signs

Respondent 9's inconsistent use of the fraction bar also assisted in this classification.

One respondent did not give any indication to her relational calculus but instead stated, "No math calculation required". This response is shown in Figure 50 and was classified as equality of measures.

No math ^{calculation} required.

2mg = 1 ml.
 4ml = 2 ml.

Figure 50. Respondent 18's Equality of Measures Set-up for DCP 1

This response illustrates the usefulness of classification based upon set-up rather than relational calculus. (Note that 4mg was mislabeled as 4mL.)

Equality of measures was not listed as a choice of procedure for DCP 6 for reasons described in the literature review. Therefore, this set-up was not confirmed by respondents. Five of the six respondents (Respondent 2, 3, 18, 21, and 27) whose set-up for DCP 1 was classified as equality of measures choose analogies for DCP 6. All five set-ups contain an equality relationship between the mass and volume of the drug and are found in Figure 51.

| Respondent 2 | Respondent 3 | Respondent 18 | Respondent 21 | Respondent 27 |
|-------------------------------------|------------------------------|-------------------------------|---|---------------|
| $\frac{2mg}{4mg} = \frac{1ml}{2ml}$ | $2mg = 1 ml$ $4mg = 3 ml$ | $2mg = 1 ml.$ $4ml = 2 ml$ | $\frac{2mg}{4mg} = \frac{1ml}{x}$ $2 \frac{x}{4} = x \cdot 2 ml$ | $1ml = 2mg$ |

Figure 51. Equality of Measure Responses Interpreted as Analogies

These solutions were not classified as analogies because they did not use the traditional set-up which is marked by the use of a colon and double colon rather than equal signs. This set-up is described after ratio table and double number line.

Ratio Table and Double Number Line Diagram

A table such as the one displayed in Figure 52 was not used by any of the respondents.

Table

| mg | mL |
|----|----|
| 2 | 1 |
| 4 | x |

Figure 52. Table Response for DCP 6

Also, the double number line diagram was not used. Katie's response to DCP 2, seen in Figure 53, was considered as possibly being a double number line diagram.

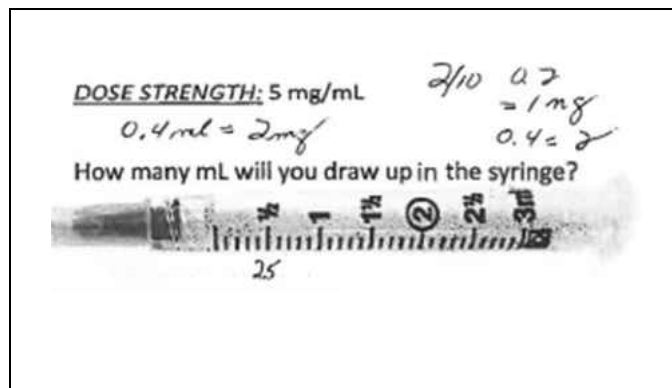


Figure 53. Katie's Response to DCP 2

This set-up was classified as equality of measures because of the presence of the equal signs and also wholistically; Katie's other responses were equality of measures. The use of the syringe as part of the calculation process was unique to Katie.

Analogies: Rachel

Rachel is the only respondent to use analogies on her survey. She used this strategy to solve DCPD 1 and 2. She also recorded it as her choice of strategy. This strategy is identifiable by the use of the colon and double colon as seen in Rachel's responses in Figure 54 and Figure 55 which will be described in further detail. Notice however, Rachel used the double colon between ratios but she could have used an equal sign.

Rachel used the means and extremes procedure with her analogy set-up although she did not know the formal name for it. Rachel's response for DCPD 1 can be seen in Figure 54. She described her set-up during our interview. "Four milligrams, oh, so I have 2 milligrams and 1 milliliter and I need 4. So if 2 is in 1, how many is in 4? So 4 is to x as 2 is to 1." When asked why she put the 4 milligrams first in her set up, she stated, "I just put that first because that is what I want. That's just the way I do it." After deeper discussion about this problem, Rachel mentioned that this problem was linked to common drug dosages.

$$4\text{mg} : X :: 2\text{mg} : 1\text{ mL}$$

$$\frac{4 \times 1}{2} \quad \frac{2x}{2}$$

2 mL's
2/4

Figure 54. Rachel's Analogy Set-up for DCP 1

Rachel's description for solving problem DCP 2 is similar. She mentioned familiarity with the drugs and their corresponding dosages, having used them in practice. She at first mixed up the positioning of the five and the two. But her knowledge of the drug prevented her from making the error.

Five milligrams along with 1 ml. That's kind of a high dose IV. Oh, okay, yeah 2 mg. So 2 mg is nice. All right 2 mg is to x, the milliliters that I want, and this is what I have here. 5 mg is to 1 mL.

When asked what the lines that she drew were for, she responded,

That is telling me the 2 – that's just the way I learned it, the 2 goes over here with the 1 and the two middles go together. So I don't get my numbers mixed up. So I don't do the 2 and the 5 or the x and the 1. The two middles go together and the two outer go together. Even though I should know that, it is just the way I've been doing it for years and years and years.

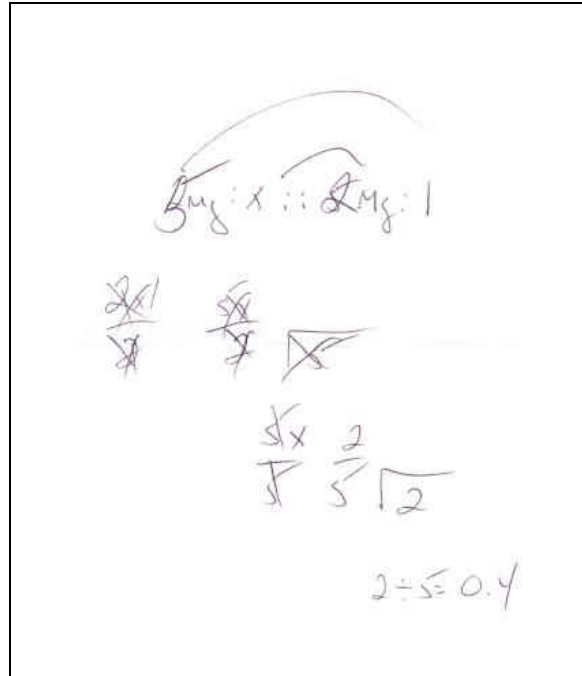


Figure 55. Rachel's Analogy Set-up for DCPP 2

Rachel's description is consistent with the means and extremes procedure outlined in the literature review.

Rachel's set-ups for DCPP 1 and 2 were the only responses on the surveys to be classified as analogies. However, a total of six respondents chose analogies for their procedure for DCPP 6. One of these was Rachel; the other five were all respondents whose set-up was classified as equality of measures.

Equal Ratios

A total of 37 responses were categorized as representing equal ratios, and seven survey participants were categorized as predominantly using this set-up. This set-up was explicitly identified by two ratios equal to each other and using a variable for the unknown quantity. This set-up is the only one of the three predominant strategies that utilized a variable. The top three candidates for this strategy were contacted and invited to participate in the next phase, but all three declined. One respondent who used equal ratios in all of her set-ups was Respondent 11. The work for the first three DCPs for Respondent 11 can be found in Figure 56.

| DCPP1 | DCPP 2 | DCPP 3 |
|--|--|---|
| $\frac{2\text{mg}}{1\text{ml}} = \frac{4\text{mg}}{x}$ | $\frac{5\text{mg}}{1\text{ml}} = \frac{2\text{mg}}{x}$ | $\frac{0.1\text{mg}}{1\text{tab}} = \frac{0.2\text{mg}}{x\text{ tabs}}$ |

Figure 56. Respondent 11's Equal Ratio Set-up

Without any extra work or interview, these set-ups in Figure 56 could not be further classified as one of the procedures associated with the equal ratio set-up. All three of the procedures associated with equal ratios were used as choices on DCP 6 where respondents had to choose their strategy choice.

In DCP4, multiple steps were required. As shown in Figure 56, Respondent 11 was unique in that she used cross products in the conversion from pounds to kilograms. Step three of Respondent 11's procedure could have been completed with cross products but instead the answer was found either mentally or on a calculator with no set-up shown. However, in Figure 57, Respondent 11's answer could be further refined as the cross products procedure because of the work shown.

Figure 57 shows handwritten mathematical work for DCP4, consisting of three numbered steps:

- Step 1: A proportion is set up: $\frac{1 \text{ Kg}}{2.2 \text{ lb}} = \frac{x \text{ Kg}}{146 \text{ lb}}$. Below this, the equation $\frac{2.2x}{2.2} = \frac{146}{2.2}$ is written. The result $x = 66.4 \text{ Kg}$ is circled.
- Step 2: A second proportion is set up: $\frac{66.4 \text{ Kg}}{x \text{ mg}} = \frac{1 \text{ Kg}}{3 \text{ mg}}$. The result 199.2 mg is circled.
- Step 3: The final answer is written as "2 capsules".

Figure 57. Respondent 11's Cross Products Procedure for DCP4

For DCP5, Respondent 11 again used the cross products procedure for multiple steps. This work is shown in Figure 58. In step one, notice that the respondent set up an equal ratio and stuck to her procedure even though the time is only one hour. She could have just divided to find a unit rate. But then notice in step two, the last step that the respondent began to set up equal ratios but then scribbled out the one. Perhaps she used a unit rate strategy after setting up the first step.

| Step 1 | Step 2 |
|--|--|
| $\frac{500 \text{ ml}}{3 \text{ hr}} = \frac{x \text{ ml}}{1 \text{ hr}}$ $\frac{3x}{3} = \frac{500}{3}$ $x = 167 \text{ ml/hr}$ | $\frac{167 \text{ ml}}{x} = \frac{1 \text{ ml}}{15 \text{ gtt}}$ $x = 2505 \text{ gtt/hr}$ $\frac{2505}{60} = \frac{x}{\text{min}}$ $x = 41.75 \text{ gtt/min}$ <hr/> 42 gtt/min |

Figure 58. Respondent 11's Cross products Procedure for DCP 5

Notice also in Figure 58 that Respondent 11 used 167 milliliters rather than 167 milliliter per hour to set up her equal ratios in step two. She dropped the *per hour* to convert to drops but then at the end of the calculation re-inserted the *per hour*. Other shorthand versions of the set-up were found. These are shown in Figure 59. Respondent 39 omitted the equal sign in her ratios. Some respondents eliminated some or all of the labels for the quantities being used. The set-up shown for Respondent 30 in Figure 59 demonstrates this omission.

| Variation | Respondent | Set-up |
|----------------------|------------|-------------------------------------|
| Omit the equal sign | 88 | $\frac{4mg}{x} \quad \frac{2mg}{1}$ |
| Omit the unit labels | 68 | $\frac{4}{x} = \frac{2mg}{1mL}$ |

Figure 59. Variations of the Ratio Set-up

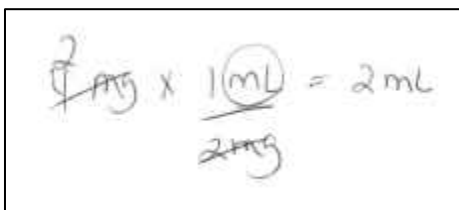
Katie, who used equality of measures over other procedures, recognized the cross products strategy but didn't use it because she couldn't remember how to use it. "I actually like that. I think that's a pretty good way to do it. I just forget about them."

Dimensional Analysis: Cathy

A total of 39 responses were categorized as representing dimensional analysis and seven survey participants were categorized as predominantly using this set-up. This set-up was identified by starting with the desired mass of the drug multiplied by a ratio which represented the dose strength of the medication. This set-up was usually accompanied by a relational calculus of the rule of three and was therefore considered a procedure in itself. This procedure can be associated with canceling out units of measure before performing the multiplication and division. Cathy was selected as a

research participant because of her consistent use of this strategy. Cathy described her solution strategy for DCP 1, which is shown in Figure 60.

That's the strength (2mg/1 mL), all right. So this is what the doctor wants (4mg). So what you want is always on top. Milliliters. So to me you want to cross this out (mg). And you have to line it up to where you can cross it out to get just milliliters. See, this is why I'm not a math teacher. This is the way my mind works. So I have to take care of the labels so to speak. I focus on the labels first, as far as solving. And then you do the math. If it's even you can multiply it. If it wasn't even, you would multiply and then divide. You get rid of what you can. What do they call that – like factoring? I don't know. Just canceling I guess. You get rid of what you can. And I always, my little brain, I always have to circle what I'm looking for. Because when you get more complicated problems for me it really helps. Because you got so much going on, the problems get so big, and you get lost.



A handwritten equation showing a dimensional analysis setup. The equation is $4 \text{ mg} \times \frac{1 \text{ mL}}{2 \text{ mg}} = 2 \text{ mL}$. The 'mg' in the numerator of the fraction is circled, and the 'mg' in the denominator is crossed out with a horizontal line. The '4' in the numerator is also circled. The result is '2 mL'.

Figure 60. Cathy's Dimensional Analysis Set-up for DCP 1

The consistency of her work is evident in Figure 61. All five of Cathy's procedures are presented together to see the common steps of crossing out units and circling remaining units.

| DCPP Survey Question | Cathy's Procedures |
|----------------------|--|
| 1 | $2 \text{ mg} \times \frac{1 \text{ mL}}{2 \text{ mg}} = 2 \text{ mL}$ |
| 2 | $2 \text{ mg} \times \frac{1 \text{ mL}}{5 \text{ mg}} = \frac{2}{5}$ |
| 3 | $2 \text{ mg} \times \frac{1000 \text{ mg}}{1 \text{ mg}} \times \frac{1 \text{ tab}}{1000 \text{ mg}}$ |
| 4 | $146 \text{ lbs} \times \frac{1 \text{ kg}}{2.2 \text{ lbs}} = 66.34 \text{ kg} \times 3 \text{ mg} = 199.02 \text{ mg} \times \frac{1 \text{ cap}}{100 \text{ mg}} = 1.99 = 2 \text{ caps}$ |
| 5 | $\frac{500 \text{ mg}}{3 \text{ hrs}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{15 \text{ sets}}{\text{hr}} =$ |

Figure 61. Cathy's Dimensional Analysis Procedure

Variations on the dimensional analysis procedure became more evident when the problems involved multiple steps like DCP 3, 4 and 5. The procedure can be carried out by separating the steps like Cathy did for DCP 4 or by setting up a single

equation like Cathy did for DCPD 3 and 5. Respondent 32 and 44 also chose to split up this steps and their responses are shown in Figure 62.

| Respondent | DCPD | Two-Step procedure |
|------------|------|--|
| 32 | 5 | $\frac{3 \text{ hrs}}{1} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 180 \text{ min (in 3hrs)}$ $\frac{500 \text{ pt}}{180 \text{ mins}} \cdot \frac{15 \text{ gtt}}{1 \text{ pt}} = 41.6 \text{ gtt/min}$ |
| 44 | 3 | $.2 \text{ mg} \times \frac{100 \text{ mg}}{\text{mg}} = 20 \text{ mg} \times \frac{1 \text{ tab}}{100 \text{ mg}} = 2 \text{ tab}$ |

Figure 62. Variations of the Multi-step Dimensional Analysis Set-up

Other variations in the dimensional analysis procedure were evident. The factor-label version of dimensional analysis was represented in one survey as seen in Figure 63. Notice that the respondent began the set-up by writing the unit being sought. The first ratio used contains that desired unit in the numerator. Factors were then chosen based upon the cancellation of the previous unit.

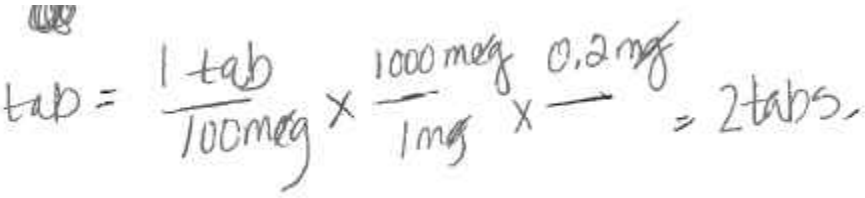
| Respondent | Strategy |
|------------|--|
| 43 |  <p>Handwritten calculation: $1 \text{ tab} = \frac{1 \text{ tab}}{100 \text{ mg}} \times \frac{1000 \text{ mg}}{1 \text{ mg}} \times \frac{0.2 \text{ mg}}{1} = 2 \text{ tabs.}$</p> |

Figure 63. Factor-Label Variation of the Dimensional Analysis Set-up

Cathy gave detailed descriptions and tried to incorporate other nursing protocols in her descriptions. In the next problem, Cathy explained the difference between liquid capacity and mass. She also used the syringe to show how the mL would be drawn up into the syringe. Cathy explained her procedure for DCP2, shown in Figure 64, in detail.

All right this is the same thing, right? Okay so I knew that he's looking for 2 milligrams and you need to set it up so that I can get rid of milligrams because we want milliliters. Because another – another thing is, is, you can't draw up milligrams into a syringe. It's always going to be the volume of the milliliters. So here you have an example, like I said, you can't divide here. You got rid of your milligrams. Circle my milliliters; that's what I'm looking for. And I can't take 2 and 5. So you multiply across and now you've got your fraction. And divide. You always need the 0 in front of the decimal point (0.4 mL not .4 mL) so they're the same. So looking at your syringe, when you see your syringe, I mean you see, um, you know, 0.1 milliliter, 0.2 milliliter, 0.3 milliliter, here.

Handwritten work for DCPP 2:

$$2 \text{ mg} \times \frac{1 \text{ mL}}{5 \text{ mg}} = \frac{2}{5}$$

$$5 \overline{) 2.00}$$

$$\begin{array}{r} .40 \\ 5 \overline{) 2.00} \\ \underline{-20} \\ 00 \end{array}$$

0.4 mL

Figure 64. Cathy's Dimensional Analysis Set-up for DCPP 2

DCPP 3 required multiple dimensional analysis steps. Cathy explained her set-up and process of solving in great detail. She explains what she does and why she does it. She uses cancellation of both the units and the numbers to assist with calculating the dose mentally. She circles her desired unit to assist her. She also states aloud the questions that she is asking herself as she goes through the problem. Cathy's work is displayed in Figure 65. She explained her thinking in the following way.

All right. So, first I put 0.2 because that's what is called for. And then I'm looking for tabs (tablets) so I know that tabs is going to be at the top. Because that's what I want. So I know this in my head, you know what I mean? I know where it comes out. This is the beginning and this is the end. It took me to like the third semester. And that's how you always start, you look at, what is the beginning and what's the end and then you fill in the middle. Then it is like, all right, how do I get to the end? The beginning and the end and I have to get everything else. So of course you need to get rid of the milligrams and your micrograms. I have to resort to my conversion. Pretty simple. I know 1000 micrograms are in 1 milligram, and because milligrams are here, I know that 1000 micrograms goes on top because I know I want to cancel those milligrams. And then, from there,

because tabs are on the top, I know that I have to have micrograms on the bottom to get rid of micrograms. So I do that, get rid of them, and I circle this. Okay. Then I see what I can get rid of and I know that I can get rid of two zeroes to make it smaller. In case we don't have a calculator. And then you do your multiplication across. 10×0.2 . Um, so now this has this extra step in the middle. So you multiply straight across the top; multiply and then divide it.

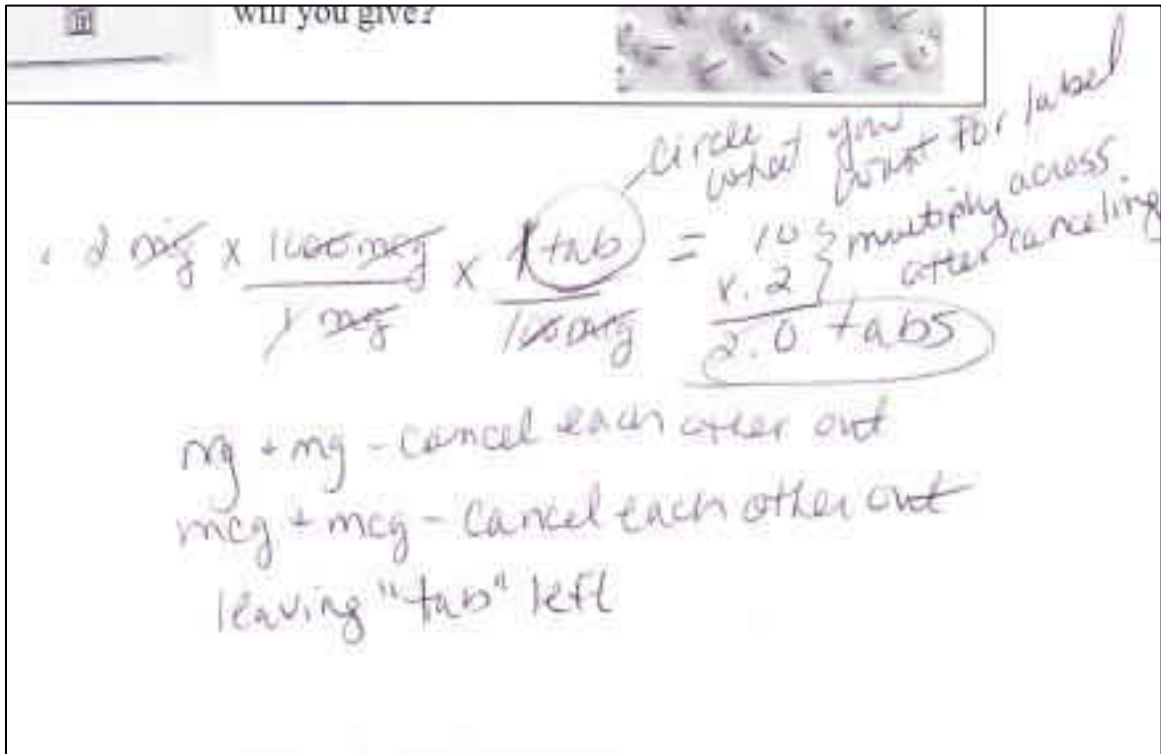


Figure 65. Cathy's Dimensional Analysis Procedure for DCPP 3

Cathy's work in solving DCPP 4 is displayed in Figure 66. This problem involved a conversion from pounds to kilograms. Cathy noticed that she had not carried through in the use of her dimensional analysis strategy, and this confused her when she reviewed her answer. She wanted to use mg/kg but instead wrote in the multiplication by 3 milligrams.

$$146 \text{ lbs} \times \frac{1 \text{ kg}}{2.2 \text{ lbs}} = 66.34 \text{ kg} \times 3 \text{ mg} = 199.02 \text{ mg} \times \frac{1 \text{ cup}}{100 \text{ mg}} = 1.99 = 2 \text{ cups}$$

Figure 66. Cathy's Dimensional Analysis Set-up for DCPP 4

The difference in notation was attributed to semantic type by Cathy who noted that sometimes these problems give safe dose ranges. In this case, the answer could be found by multiplying both the low dose and the high dose to the weight. The association of this type of problem to multiplication prevented her from using her dimensional analysis set-up.

Dimensional analysis is usually associated with the relational calculus of the rule of three. The means and extremes procedure and the cross products procedure also use the rule of three relational calculus. The next set-up to be described, the nursing rule is usually associated with a scalar relational calculus.

The Nursing Rule: Jackie

A total of 46 responses were categorized as representing the nursing rule and 9 survey participants were categorized as predominantly using this strategy. This reflects 21% of the respondents. Many respondents wrote the actual wording of the formula that they used on their DCPP Survey. Variations in the wording are shown in Figure 67. The words *ordered* or *desired* are used in the numerator. The words *available* or *have* are used in the denominator. The factor by which this ratio is multiplied is called the *quantity* or *amount* or the actual unit of measure is used.

| Respondent 13 | Respondent 22 | Respondent 29 |
|--|--|--|
| $\frac{\text{order}}{\text{available}} \times \text{amt} \quad \frac{4}{2} \times 1$ | $\frac{\text{Now, we take the:}}{\text{Have}} \frac{\text{Desired}}{\text{Have}} \times \text{amt.}$ | $\frac{D}{H} \times Q = X$ <p>Formula Method</p> |

Figure 67. Nursing Rule Wording Variations from Surveys

This strategy is identifiable even when the words are not present because a ratio is first formed from the like quantities and then multiplied by the third. The relational calculus usually associated with this procedure is scalar. Jackie was chosen as a participant for her consistent use of this strategy. She described her process for solving DCPD 1 using her teacher voice. The solution is displayed in Figure 68.

When you set this up, in the numerator, the tags need to be the same and. . . . If you have milligrams up here, you have to have milligrams over here. If you have milliliters over here, you have to have milliliters over here. And the labels in the numerator and the denominator are obviously the same. And then I would show them (the students) how it (the mg) would cancel out and . . . you are left with milliliters; so the answer must be milliliters.

$$\frac{4 \text{ mg} \times 1 \text{ mL}}{2 \text{ mg}} = 2 \text{ mL}$$

Figure 68. Jackie's Nursing Rule Set-up for DCPD 1

The consistency of her work is evident in Figure 69. Through the use of wholistic categorization, some set-ups were classified as being the nursing rule because of previous work shown on the survey. For example, Jackie omitted the unit labels in DCPP 4. Because of her previous answers, however, this was classified as the nursing rule.

| DCPP | Jackie's Nursing Rule Set-up |
|------|---|
| 1 | $\frac{4 \text{ mg}}{2 \text{ mg}} \times 1 \text{ mL} = 2 \text{ mL}$ |
| 2 | $\frac{2 \text{ mg}}{5 \text{ mg}} \times 1 \text{ mL} = 0.4 \text{ mL}$ |
| 3 | $\frac{200 \text{ mcg}}{100 \text{ mcg}} \times 1 \text{ tab} = 2 \text{ tabs}$ |
| 4 | $\frac{200}{100} \times 1 = 2 \text{ capsules}$ |

Figure 69. Jackie's Nursing Rule Set-up

Other nurses used this shorthand method notation throughout their survey.

Variations in the nursing formula are shown in Figure 70. These examples illustrate how other nurses omitted unit labels and the value of one as the quantity.

| Description | Nurse's Work | Respondent |
|-------------------------------|---|------------|
| Omit the value of 1 for mL | $\frac{4\text{mg}}{2\text{mg}} \times \text{ml} = 2\text{ml}$ | 73 |
| Omit the 1 and its unit label | $\frac{200\text{mcg}}{100\text{mcg}} = 2 \text{ tabs}$ | 61 |

Figure 70. Nursing Rule Notational Variations

Jackie used the nursing rule, which she called “desired over have”, to solve all of her problems on the test, but she also demonstrated that she could solve the problem using equal ratios. Part of Jackie’s experience was in training clinical nurses in dosage calculations. Jackie called the procedure involving cross products, “proportions.” She indicated that she preferred to use the nursing rule over cross products because it was “quick.” She stated that, “. . . this is not a hard formula. This is very basic, simple math that gives you the answer quickly.”

Jackie was aware of the difficulties that are present with using the nursing rule. When using the nursing rule, the units in the dose strength and the physician’s orders

need to be the same. This is not always the case, such as observed in DCP 3 in Figure 71. Jackie described this process.

This involves transferring milligrams into micrograms. So, in my brain...notice that the dose strength (100 mcg/1 tab) and the physicians order (0.2mg), notice that you have two different labels (mcg and mg). As I said here, my suggestion is to change it to all one strength before you do it, because you obviously cannot set that up in any kind of math before you do that. I always go down to the smaller unit. (Converts 0.2 mg to 200 mcg.) But then it all narrows down to remember the desired over have. These (the beginning ratio) have to have the same labels. This is the odd guy out or the tablet.

make changes all to mg or mcg i use this no.

$$0.2 \text{ mg} = 0.200 \rightarrow 200 \text{ mcg}$$

$\frac{\text{Des.}}{\text{Hav.}} \times Q$

$$\frac{200 \text{ mcg}}{100 \text{ mcg}} \times 1 \text{ tab} = 2 \text{ tabs}$$

Figure 71. Jackie's Nursing Rule Set-up for DCP 3

Jackie used the nursing rule for weight-based operations, as well, as seen in Figure 72. Jackie did not use any of the specific set-ups for solving DCPs in the first steps of this multi-step process. Also when calculating the mass of the drug to be administered, she again did not use any of the specific set-ups. She stated, "I know to use the numbers that I have figured out" and to multiply.

Step 1: Change lbs \rightarrow Kg
 $140 \div 2.2 = 66.3636 \Rightarrow 66.4$ (✓ instructions for tenths or hundredths place)

$$\begin{array}{r} 66.4 \text{ Kg} \\ \times 3 \text{ mg/Kg} \\ \hline 199.2 \end{array} \Rightarrow 200 \text{ mg} \quad \text{Kg "cancel" out}$$

$$\frac{D}{H} \times Q$$

$$\frac{200}{100} \times 1 = 2 \text{ capsules}$$

Figure 72. Jackie's Nursing Rule Set-up for DCP 4

Jackie stayed consistent with her use of formulas for DCP 5 and correctly switched from using the nursing rule to the drip rate formula. A total of eight respondents used the drip rate formula. The drip rate formula is calculated by multiplying the volume to be infused by the drop factor and then dividing by the total time in minutes. This formula was written out on two surveys as volume times calibration divided by minutes and can be seen in Figure 73. The term *calibration* was used instead of drop factor. The drop factor of the tubing can be correctly interpreted as a calibration since it determines the size of the drop that the tubing will release.

| Jackie | Respondent 13 | Respondent 33 |
|--------------------------------------|---|--|
| $\frac{V \times D}{T \text{ (min)}}$ | $\frac{\text{Vol} \times \text{Cal}}{\text{minutes}} = \text{gtts/min}$ | $\frac{\text{Volume} \times \text{calibration}}{\text{Minutes}}$ |
| $\frac{167 \times 15}{60 \times 4}$ | $\frac{500 \times 15}{180}$ | $\frac{500 \text{ mL} \times 15 \text{ gtt/s/mL}}{180 \text{ min}} = 7500$ |

Figure 73. Drip Rate Formula Wording Variations with Solutions

Jackie's complete procedure for solving DCP5 5 is shown in Figure 74. Notice that Jackie reduced the 500 milliliters infused in 180 minutes to 167 milliliters infused in 60 minutes. Jackie also used the drop factor constant of 4 when performing the calculations in DCP5 5. The drop factor constant for this problem was calculated by simplifying the multiplication of 15 and division by 60 to a division of 4. The process of obtaining a drop factor constant is further explained in Chapter 2 of this document under the section Procedures for Problems Requiring Multiple Steps. Jackie was the only respondent to use the drop factor constant method.

Step 1 - Find hourly rate
 $500 \text{ mL} \div 3 = 166.666 \text{ or } 167 \text{ mL/hr.}$

For IV infusion, cannot divide gts } \rightarrow depends on infusion device

Step 2 $\frac{V \times D}{T \text{ (min)}}$

$$\frac{167 \times 15}{60}$$

41.75 or 42 gts/minute

Figure 74. Jackie's Drip Rate Formula for DCP 5

During the interviews, the participants were asked if they were familiar with the nursing rules. Katie and Rachel had never heard of it. Cathy had heard of the formulas but did not use them.

Other Categories

In addition to the set-ups, two other classifications were created to accommodate respondent solutions. These categories were: not identifiable and no work. These categories were created specifically to separate the remaining responses between those who did not do anything and those who tried something. When whole surveys were classified by their predominate set-up, another category needed to be created for surveys that did not have three or more solutions of the same set-up. This category was named *no predominate set-up*. Twelve surveys were categorized as *not identifiable*, 1 survey as *no work*, and 4 surveys as *no predominate set-up*. Of the 17 surveys in these three categories, only two indicated that the respondent would be willing to participate in further study. The number of surveys with signatures indicating further participation is found in Table 20, however, it is reproduced in Table 23 using percentage notation.

Table 23

Percentage of Signed Surveys Categorized by Set-up

| Set-up | Signature | |
|-----------------------|-----------|------|
| | % Yes | % No |
| Equality of Measures | 100 | 0 |
| Equal ratios | 71 | 29 |
| Dimensional analysis | 67 | 33 |
| Nursing rule | 56 | 44 |
| Not Identifiable | 8 | 92 |
| No work | 0 | 100 |
| No Predominate Set-up | 25 | 75 |
| Total | 48 | 52 |

Utilizing percentages, the values in Table 23 show that these three categories (no work, not identifiable, and no predominate set-up) are the only three where the percentage of no signatures outweighed the yes signatures. These three categories will be discussed in further detail.

No work

The question that was most frequently not answered was DCP 5. No respondent skipped all of the questions. Table 24 displays the frequency of no responses for DCP 1 through 5.

Table 24

No Response Frequency for DCPD 1-5

| Questions | No Response |
|-----------|-------------|
| 1 | 2 |
| 2 | 1 |
| 3 | 3 |
| 4 | 0 |
| 5 | 5 |

DCPD 5 on the DCPD survey was an IV infusion problem with multiple steps and this could account for the higher frequency of no responses. Only one survey had more three or more responses that were not answered. This respondent did not sign to be contacted for further participation.

Not Identifiable

Approximately 30% of the individual answers to the survey DCPD questions were unclassified, meaning that they did not fit into any of the prescribed set-ups listed in the literature. This is the highest percentage of any category. Twelve surveys were categorized as predominately not having an identifiable set-up.

Most of the solutions that were classified as having a not identifiable set-up consisted of multiplication and division. Sometimes the multiplication and division were accompanied by units of measure and sometimes they were written without regard to the unit of measure. Labels may have been written at the beginning and the end but were not used within the calculation itself as seen in Figure 75.

| Respondent 20 DCPP 4 | Respondent 26 DCPP 5 |
|---|--|
| $146 \div 2.2 = 66.36$ $\begin{array}{r} 66.36 \\ \times 3 \\ \hline 199.08 \end{array} = 2 \text{ capsules}$ | $60 \text{ minutes} \times 3 = 180$ $\text{multiply } 500 \times 15 = 7500$ $7500 \div 180 = 41.6$ <p>either 41 or 42 gtt/</p> |

Figure 75. Multiplication and Division without Units

Some respondents did try to include units in their calculations. Respondent 2 and Respondent 41 provided responses that included units but no set-up was associated with these units. These are illustrated in Figure 76.

| Respondent 2 DCPP 4 | Respondent 41 DCPP 5 |
|--|---|
| $146 \text{ lbs.} \div 2.2 \text{ Kg} = 66.36 \text{ Kg}$ $\begin{array}{r} 66.36 \\ \times 3 \\ \hline 199.08 \text{ mg} \end{array}$ <p>100mg/cap</p> <p><u>2 cap.</u></p> | $500 \times 15 = 7500 \text{ gtt}$ $\sqrt[3]{500} = 16.7 \text{ mL/hr}$ $60 \overline{)167} = 2.78 \text{ mL/min}$ $15 \text{ gtt/min} \times 2.78 \text{ mL/min}$ 41.7 gtt/min |

Figure 76. Calculations with Units

The units of measure for the solutions in Figure 76 were written with many of the numbers but did not seem to be used as a means of assisting in the calculation process.

Not all of the responses that were categorized as not identifiable used multiplication and division in their notation. Figure 77 displays a traditional scalar decomposition relational calculus used by Respondent 1.

| Respondent 1 |
|---|
| $\begin{array}{r} 2\text{mg/ml} \\ + 2\text{mg/ml} \\ \hline \text{Zofran } 4\text{mg} = 1\text{ml} + 1\text{ml} = \end{array}$ |

Figure 77. Respondent 1's Scalar Decomposition Relational Calculus

This response is categorized as not identifiable because scalar decomposition is a relational calculus and this research utilized the set-up to classify solutions. Other respondents who appeared to use a scalar decomposition relational calculus may have been classified as using equality of measures. This classification was considered. Wholistic classification was implemented and found that this respondent did not answer DCPP 3, provided an incorrect answer for DCPP 4, and answered equal ratios for DCPP 6. These inconsistent results led the researcher to believe that Respondent 1 did not have a particular set-up in mind.

Respondent 1's scalar decomposition strategy was one exception to the not identifiable classification. The view that these problems could be solved by simple multiplication or division as opposed to a proportional reasoning procedure appeared to be the overwhelming reason for the not identifiable classification. This view was supported by written responses to the survey writing prompt as well as verbal responses in participant interviews. Respondent 20 stated on her survey, "I do work in a cancer center and we have to recalculate continuous infusion chemo pumps and rate of particular infusions at times, but it usually is simple multiplication and division." Judy also echoed this idea that much of the nursing mathematics was "basic multiplication, division, and formula set-up." Rachel summarized this category best when she said, "It's all just multiplication and division no matter how you slice it."

Rachel's responses for DCP3 and DCP4 represent set-ups that were not identifiable. Her response for DCP3 is presented in Figure 78 and her description of this solution follows.

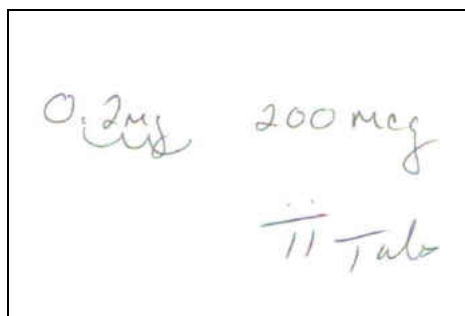


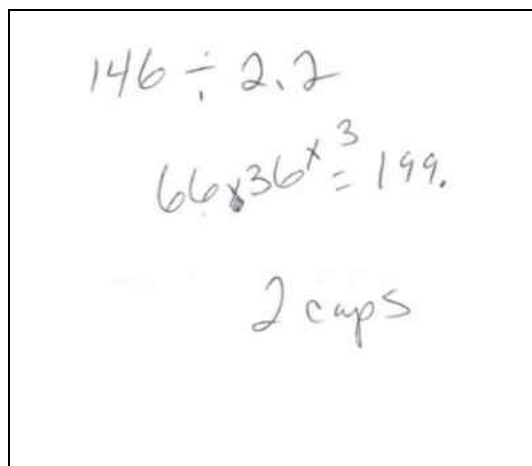
Figure 78. Rachel's Response for DCP3

Rachel explained her conversion from milligrams to micrograms. She used what she called "wormies" or loops under the values. "This keeps me on track ... I have to ... Just to keep it right." She was asked how she knew which way to go with her

“wormies”. She said, “Because it’s smaller ...The micrograms are smaller than milligrams...So you go to the right as it is smaller.” Once Rachel got the 200 micrograms, she stopped doing work and just wrote her answer of 2 tabs (written using roman numerals). When asked why she wrote her answer without any work, she said,

Well because it’s right there. 100 micrograms is one tablet, right? So you need two tablets. Because 100 plus 100 is 200. So one tablet is 100, you know, two tablets is 200. That was easy ... That’s why you don’t see any work.

The idea that certain procedures are automatic and do not require traditional set-ups was echoed in DCPP 4 by Rachel as well as others. Rachel’s work for DCPP 4 is found in Figure 79 and her description follows.



The image shows a rectangular box containing handwritten mathematical work. At the top, it says $146 \div 2.2$. Below that, it says $66 \times 36^3 = 199.$. At the bottom, it says "2 caps".

Figure 79. Rachel's Response for DCPP 4

This one is a little different. I kind of cheated– I think I kind of cheated on this one. Because I know like kilograms is less than pounds, I know that, the number is going to be less. But I forgot to times 2.2, to divide 2.2, but then I remembered, you know, it’s less so I’m going to divide. But 2.2 – That’s a standard equation like pi – 3.14. You just know that. And you know it by because it needs to be smaller. Right because I know kilograms is smaller than pounds. Then I times it by the 3, see? Times 3, 199, so 1 capsule – how many

capsules, why did I say 2 capsules? Oh because my answer 199 – so I had to round to 200 right? One capsule is 100, two capsules 200. Because one capsule wouldn't be enough so you got to give two.

Rachel's lack of set-up caused her to have to re-think how she solved this problem.

This lack of set-up was found to be prominent in DCP4, especially in converting pounds to kilograms. Thirty-two or 76% of the 44 respondents did not use a documented set-up for this conversion. In addition, Jackie and Katie both made similar comments to Rachel's concerning this conversion. These are presented next.

Jackie was asked why she wrote pounds and kilograms with an arrow between them and she stated, "I wrote that because you gave me the weight in pounds, and because I am in the metric system, I know that I want everything in the metric system, not the apothecary system." Next, she was asked to explain why she divided and how did she know to divide rather than to multiply. Her response was, "I know the number is different, again, I have been a nurse too long, so I know that is what you do and I also know that kilograms is a smaller number than pounds." Katie's explanation for the conversion of pounds to kilograms in DCP4 was similar but rather than using the words "smaller number", she incorrectly used the term "lighter" to describe the relationship of kilograms to pounds.

It's kilograms and I have pounds. So I have to go with my 146 and divide it by 2.2. So I can figure out how many kilograms. Because kilograms I'd rather – because, kilograms are lighter.

This relationship, however stated, indicated that the desired operation was division. It is unclear if proportional reasoning was used in acquiring this answer. Cathy was the only participant to use a set-up for this conversion. She consistently used dimensional

analysis and applied this set-up to the conversion of pounds to kilograms. When describing her procedure, Cathy did make a similar statement in her explanation when she said she “always” needs to “remember kilograms is smaller than pounds”.

The lack of set-up for converting pounds to kilogram however, did not mean that DCP4 was classified as not identifiable. DCP4 required a multi-step conversion. Three conversions needed to take place: weight, mass, and volume. Although all three of these conversions could have been solved using the same set-up, many respondents’ solutions used a variety of set-ups. Examples of responses that used different set-ups for different steps are found in Figure 80. Each of the three steps is categorized.

| Respondent | Set-up for each conversion | Response to DCPD 4 |
|------------|--|---|
| 8 | Division Multiplication Nursing Rule | $146 \text{ lbs} \div 2.2 = 66.4 \text{ kg}$ $\begin{array}{r} 66.4 \\ \times 3 \\ \hline 199.2 \text{ mg} \end{array}$ $\frac{199 \text{ mg}}{100 \text{ mg}} \times 1 \text{ cap}$ |
| 24 | Equal Ratios Multiplication Division | $\frac{146 \text{ lb}}{2 \text{ kg}} = \frac{1 \text{ lb}}{2.2 \text{ kg}}$ $66.36 \text{ kg} \times 3 \text{ mg/kg} = 199 \text{ mg}$ $199 / 100 = 2 \text{ caps (round up)}$ |
| 25 | Division Multiplication Equal Ratios | $\frac{146 \text{ lbs}}{2.2} = 66.4 \text{ kilo}$ $66.4 \times 3 = 199.2 \text{ mg total}$ $\frac{199.2}{x} = \frac{100}{1}$ $\frac{100x}{100} = \frac{199.2}{100}$ $x = 199.2 \text{ or } 2 \text{ tabs}$ |
| 31 | No Work Multiplication Nursing Rule | $\begin{array}{r} 66.4 \text{ Kg} \\ \times 3 \text{ mg/Kg} \\ \hline 199.2 \end{array} \rightarrow 200 \text{ mg} \quad \text{Kg "cancel" out}$ $\frac{D}{H} \times Q$ $\frac{200}{100} \times 1 = 2 \text{ capsules}$ |

Figure 80. Different Set-ups for Different Steps

Various set-ups posed a problem when trying to classify DCP4. In the case where only one proportional set-up was identified, the set-up was classified as such. When two or more were identifiable, the one used in the last step was used for the purpose of classification. For instance, in Figure 80, Respondent 8's solution was categorized as the nursing rule. Despite the researcher's decision to classify a solution's set-up as long as at least one step displayed a distinguishable set-up, 45% of the 44 responses for DCP4 were classified as not identifiable. This means that none of the three steps used a formal set-up. This was a higher percentage than for any other problem on the DCP4 survey.

No Predominate Set-up: Rachel

Overall, only four surveys (9%) were classified as having no predominant set-up. This means that they did not use the same set-up on three or more of the five DCP4 problems. Rachel's survey was placed in this category. Rachel's responses to the five DCP4 questions were classified respectively as: analogy, analogy, not identifiable, not identifiable, and the nursing rule. Because she did not use the same set-up for three solutions, Rachel was categorized as having no predominate set-up. Rachel was the only respondent who agreed to participation from this category.

The creation of this category was explicitly to find a participant who could be said to represent a flexible thinker. Because of Rachel's use of varied strategies, descriptions of her responses are located in various sections. A description of Rachel's overall thinking is provided in her narrative in Chapter 4. Her individual solutions are found in their corresponding set-up sections. Rachel's unidentifiable set-ups for DCP4

3 and DCP 4 were presented in the previous section. Her analogy solutions for DCP 1 and DCP 2 were presented in the analogy portion of this chapter. For DCP 5, Rachel used the nursing rule for drip rates which is fully described in Chapter 2: Procedures for Problems Requiring Multiple Steps. Her diverse solution strategies seemed to be tied to context. The importance of context and other situational variables surrounding DCP's is addressed in the next section pertaining to Research Question 3.

Research Question 3: Situations

When solving proportional reasoning problems, what situational variables do nurses recognize as affecting problem difficulty and/or procedure choice: (a) numerical characteristics, (b) semantic type, (c) context, (d) presentation, and (e) student characteristics?

Participants shared their thoughts regarding various aspects of their problem solving on the questions from the Everyday Proportion Problems. Table 25 displays their self-ratings regarding their perceived difficulty of the problems.

Table 25

Participants' Self-rated Problem Difficulty

| Problems | Jackie | Cathy | Rachel | Katie | TOTAL |
|-----------------------|--------|-------|--------|-------|-------|
| Dollars:Ounces | 2 | 1 | 1 | 2 | 6 |
| People:Eggs | 1 | 3 | 2 | 1 | 7 |
| Length:Width | 5 | 4 | 3 | 5 | 17 |
| Brown Eggs:White Eggs | 5 | 5 | 5 | 4 | 19 |
| Airplane | 5 | 2 | 4 | 3 | 14 |

In order to simplify language, any rating of a one or two will be called easy and any rating of a four or five will be called difficult. If the totals in Table 25 are considered in this same light, these data could be generalized to say that the Dollars:Ounce and People:Eggs would be considered easy and Length:Width, and Brown Eggs:White Eggs would be considered difficult. This section will present the situational aspects that participants commented on as having affected their perceived difficulty and/or

procedure use. While the Everyday Proportion Problems are emphasized in this section, some statements are included concerning the DCPD Survey as well.

Numerical characteristics

The numerical characteristics of concern in this study were identified to be integer or non-integer relationships and discrete or continuous quantities. These two characteristics will be presented together as the ties between these two characteristics are associated. Cathy's struggle with the People:Eggs problem which utilized a non-integer relationship with discrete quantities exemplifies this. The intensive relationship between 14 people and 8 eggs is 1.75 people/egg. Cathy recognized the difficulty in interpreting this non-integer relationship of 1.75 people/egg.

One point seven five...Which you can't...You can do kids...1.75...So, it's really only going to be 1... Because you can't – You gotta round it...Because you can't have a 0.75 person ...: Now this is like algebra here... I don't get it. I don't get the algebra...Okay, yeah, where was I. How many people can I ... One. Okay. All right if 1 egg per – I don't know, maybe they'll come out even with the 12, I don't know...Okay so one egg feeds 1.75 people.

Cathy recognized that it was impossible to have 1.75 people in the People:Eggs problem because people is a discrete quantity. She decided to leave the relationship as a non-integer relationship and continue on in her calculations. She then multiplied 4 eggs by 1.75 people/egg and concluded that the four extra eggs would feed 7 more people. She had enough eggs to feed 21 people total. Cathy was pleased that the answer was an integer. She questioned if a non-integer relationship was possible with discrete quantities. Rather than resolve this, she continued in her calculations and waited to see if her answer made sense.

The lack of integer relationships in the People:Eggs problem caused Katie to use a guess and check procedure on the Everyday Proportion Problems. Katie was trying to “write an equation” to represent the People:Eggs problem but she could not. She did try to get a unit rate and multiply by the number of eggs, but she could not figure out whether to divide 8 by 14 or 14 by 8. Her strategy was to try it both ways and see which answer was more reasonable. Her decision to divide or multiply was also based on reasonableness. According to her interview, if she needed a number to be smaller she divided and if she needed it to be higher, she multiplied.

DCPP 2 used an integer function relationship of 5mg/mL but Katie used a scalar decomposition relational calculus to solve her DCPPs. The desired amount to give was 2 mg. She did not switch to a function relational calculus but stated, “I do it this way because 2 and 5 aren’t nice.” The presence of this non-integer did not make the problem unsolvable for Katie; however she had great difficulty in explaining her answer. Her complete solution for this problem is recorded under Research Question 2: Equality of Measures.

Jackie clearly interpreted discrete quantities as countable and continuous quantities as measureable. She remarked how pills are “readily countable” while liquids need to be measured in a cup. Jackie commented on how the presence of non-integer answers to DCPPs with pills can cause problems when administering drugs because many times the nurses assume that the answers will be integers.

Chances are you have a decimal problem and its one. There are a couple of drugs that you will give maybe a larger quantity to get their blood level up but then you give them 1 a day or 2 a day so and I’ll tell you nurses tend to get lazy. We tend to get lazy when we see that and then they throw something at you and ya have to stop. And that’s where the nurses make mistakes.

She also attributed the countable quality of money to being one of the attributes that made money problems easier to work with.

But in a grocery situation I am not turned off by halves and quarters, just from the point of view of is that I am going to start kind of rounding now. Ya know, it is going to be approximate because groceries don't deal with, I mean when your handing your money over you are not dealing with quarters of pennies and that kind of stuff.

These numerical characteristics did not seem to affect procedure choice but certainly played a role in the difficulty of the problem. A situation that affected both procedure choice and problem difficulty was semantic type. This situation will be described next.

Semantic Type

Participants who recognized the semantic similarities between DCPPs and Everyday Proportion Problems utilized their predominant set-up to solve those problems. Two of the Everyday Proportion Problems closely matched the semantics of DCPPs. These two problems were People:Eggs and the Airplane problem. Jackie recognized the similarity between the People:Eggs problem and Cathy recognized the similarity to the Airplane problem. Their responses are described here.

The People:Eggs most closely matched the semantics of a single step DCPP because it is a well-chunked MVPP. Jackie recognized this similarity and rated this problem as being the easiest. She was the only respondent to classify this problem as a proportion problem. She used equal ratios to solve the People:Eggs problem. She stated, "Ok well I, I would set it up as a desire ... as a ratio proportion because I like that particular one. So I would set it up as 8 eggs over 14 people as 12 eggs over X." Notice

that Jackie started to say desired over have. She later commented on this and said that this problem was “parallel” to a DCP; “I could have set it up as a desired over have times quantity (the nursing rule), which is that frequent calculation you would give to students.”

The Airplane problem also provided an opportunity for participants to make semantic connections because the numerical quantities were the same as DCP 5. DCP 5 was a multi-step well-chunked MVPP involving IV drip rates. The highlighted difference between the problems was the context. Although Jackie recognized the semantic similarities between the People:Eggs problem and DCPPs, she did not recognize the similarities between the airplane problem and DCP 5. She stated, “If I had to do a problem like this in nursing, (laughing) I would be suicidal.” The semantic similarity was noticed by Cathy who was the only participant to rank this problem as being easy. When asked which one of the Everyday Proportion Problems was most like a DCP, she chose the airplane problem. She said, “This is your basic ... I mean that you’re technically using this every day in nursing.” When questioned if she uses miles per gallon in nursing every day, she said,

No, but, you’re using an amount of something to turn it into an amount of something else. Units to units. I mean because essentially when a doctor prescribes a medication he has to give you an amount. And you have to have an amount or you can’t give it. You’re converting it.

By recognizing the semantic similarity in these two problems, Cathy was able to apply her knowledge of proportional reasoning and dimensional analysis to solve this problem. She was not affected by the context. Her step-by-step procedure is provided in Figure 81.

| Language | Writing |
|---|---|
| Okay. How many gallons of gas does a plane use per minute? What is this (15 gallons per mile)? Do I need this? Gallons per mile? Not miles per gallon. All right. Hmm. Just a minute. | $\frac{15 \text{ gallons}}{1 \text{ mile}}$ |
| So I want gallons per minute. | $\frac{15 \text{ gallons}}{1 \text{ mile}} \quad \frac{\text{gallons}}{\text{minute}}$ |
| That's what I want. And so.... Hold on. Okay, it just takes me a lot of writing. Okay so I want minutes. So 500 miles every 3 hours. | $\frac{15 \text{ gallons}}{1 \text{ mile}} \times \frac{500 \text{ miles}}{3 \text{ hours}} \quad \frac{\text{gallons}}{\text{minute}}$ |
| Get rid of that. Get rid of that. | $\frac{15 \text{ gallons}}{1 \text{ mile}} \times \frac{500 \text{ miles}}{3 \text{ hours}} \quad \frac{\text{gallons}}{\text{minute}}$ |
| So I got gallons, now I need minutes. | $\frac{15 \text{ gallons}}{1 \text{ mile}} \times \frac{500 \text{ miles}}{3 \text{ hours}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} \quad \frac{\text{gallons}}{\text{minute}}$ |
| Get rid of that (miles). Get rid of that (hours). | $\frac{15 \text{ gallons}}{1 \text{ mile}} \times \frac{500 \text{ miles}}{3 \text{ hours}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} \quad \frac{\text{gallons}}{\text{minute}}$ |
| That's right. Okay. I got my gallons and I got my minutes. | $\frac{15 \text{ gallons}}{1 \text{ mile}} \times \frac{500 \text{ miles}}{3 \text{ hours}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} \quad \frac{\text{gallons}}{\text{minute}}$ |
| Okay, so I'm not even going to do that math; I have a calculator and I'm allowed to use it. So $500 \times 15 = 7500$ over 60×3 is 180, right? | $\frac{15 \text{ gallons}}{1 \text{ mile}} \times \frac{500 \text{ miles}}{3 \text{ hours}} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = \frac{7500}{180} \frac{\text{gallons}}{\text{minute}}$ |
| And then I'm taking 7500 divided by 180 and I'm going to say 41.7 gallons per minute. That was the easiest problem on the test. | $41.7 \frac{\text{gallons}}{\text{minute}}$ |

Figure 81. Cathy's Airplane Problem Solution

Cathy was confident in her answer. She said, "I look at this and I know what to do because I've been trained. You know that is just like your dimensional analysis and you know what to do. I am very familiar with that type of problem."

Interestingly, Cathy did not make a connection between the People:Eggs problem and dosage calculations. She felt that this problem was difficult because you had to find the information on the sheet; it wasn't given to you. She used a unit rate relational calculus to solve this problem. She calculated that the recipe fed 1.75 people per egg. She referred to this unit rate as her base. Once she had that she said she could figure it out. Since she had 4 extra eggs, she could feed seven more people, because "4 x 1.75 people came to 7 people" She was not deterred by the fact that the unit rate was a continuous number rather than an integer. She focused on her labels.

Katie also recognized the similarity between DCP5 and the airplane problem. "This is like when you have mics (meaning micrograms) per kilogram because you've got a three part thing... This isn't bad; it just took a lot of steps ... I could come to an answer pretty good." Katie solved both of these problems using multiplication and division.

In order to elicit additional comments concerning the semantics of other problems, participants were asked to describe each of the Everyday Proportion Problems. Participant responses are found in Figure 82. The researcher's intended semantic type is also recorded in Figure 82.






| Problem | Researcher's Description | Jackie | Cathy | Rachel | Katie |
|---|---|------------------------------------|--------------------------------------|---|-------------------------|
|  Dollars:Ounces | Comparison Well-Chunked Measures | Division | Comparing Division | “It’s all just basic math. No matter how I sliced it, I was multiplying or dividing.” | Unit Rate |
|  People:Eggs | MVPP Associated Sets | Ratio-proportion | Missing number Algebraic | | Word Problem Algebra |
|  Length:Width | Comparison Scaling | Geometry Ratio-Proportion | Comparing Geometry Proportions | | Geometry Proportion |
|  Brown Eggs: White Eggs | Comparison Part-Part-Whole | Comparing size Ratio-proportion | Proportional fractions | | (no response) |
|  Airplane | Multiple Step- MVPP Well-Chunked Measures | Multi-step Converting | Converting Units | | Word Problem Algebra |

Figure 82. Participants' Descriptions of the Problems

This research focused on MVPPs; however, comparison proportion problems were introduced into the Everyday Proportion Problems. These problems were designed to represent each of the semantic types described in the literature review: Well-Chunked, Associated Sets, Part-Part-Whole, or Scaling.

Figure 82 shows that Jackie and Rachel considered the Dollars:Ounce problem a division problem rather than a well-chunked comparison proportion problem. Cathy also described this as a division problem but accurately included the description of a comparison problem. This reduction of a problem's semantic type to its operations for solution was most consistently used by Rachel. Rachel did not have much to say when asked to describe the Everyday Proportion Problems. Her response is recorded in Figure 82, "It's all just basic math. No matter how I sliced it, I was multiplying or dividing." She realized that they were all similar but could not find a consistent way of thinking of them.

There has to be an easier way because this doesn't seem like a hard problem. It doesn't seem hard, but I don't know why it was hard for me. Maybe because of the way it looks. I don't know. ...Like, what kind of formula is that? Like, do all of these equate to some kind of formula? Each different one?

Rachel was struggling to find a way to think of these in a generalized way rather than having to figure out which numbers to multiply and divide each time. She did not apply the analogy set-up that she used on DCP1 1 and DCP1 2 to any of these problems. All the problems were classified by their mathematical operation.

As well as using the mathematical operations to classify problems, the use of mathematical subjects to classify problems was also evidenced. Both Cathy and Katie classified the People:Eggs problem as an algebra problem. Katie also classified the

Airplane problem as an algebra problem. In connecting these problems with a mathematical subject, references were made to their difficulty. Cathy stated for the People:Eggs problem, “Now this is like algebra here. I don’t get it. I don’t get the algebra.” Katie stated for the airplane problem, “That’s where algebra comes in and that’s why I don’t remember.”

Similarly, Jackie, Cathy, and Katie classified the Length:Width problem as a geometry problem. Jackie attached this classification with her difficulty with geometry. She stated, “I hate to tell you how poorly I did at geometry. I just didn’t get it.” Katie however created a positive connection between this problem and geometry. She stated that, “I have a very good spatial concept.” Interestingly, Katie also connected DCPPs to geometry.

In addition to the general classification of algebra and geometry, the categorization of proportions was used six times. The problem most classified as a proportion was the Length:Width problem which was classified as proportion by three participants: Jackie, Cathy, and Katie. Proportional shapes seemed to be directly linked to scale. Cathy connected the Brown Eggs:White Eggs to proportions as soon as she read the problem. “More brown eggs relative to white eggs. Relative. Relative. Okay. This is like scale I think – out of proportion!” Jackie made a similar statement as soon as she looked at this problem, “Ok. Which carton contains more brown eggs relative to white eggs? Relative? Ya know. It’s that old ratio proportion I somewhat can feel like I have a grasp on it.”

Jackie also classified the People:Eggs problem as a ratio-proportion. This is interesting because although Jackie’s set-up was termed equal ratios by the researcher,

Jackie consistently called it ratio-proportion. Her name for her strategy matched her name for the type of problem. This is similar to participants naming problems by the operation that they used to solve it: i.e. multiplication and division as described earlier.

Using the term *proportion* seemed to be connected with problem difficulty as five of the six times it was used to describe a problem, it was classified by participants as being difficult. However, the difficulty could have also been attributed to them being comparison problems. Although Kathy did not explicitly classify the Brown Eggs:White Eggs Problem and the Length:Width Problem as comparison problems, she was able to comment on their semantic similarity. She stated that “These two were in common because you had all of your information.” She realized that they were not MVPPs. When asked if she could use dimensional analysis for these problems, she said, “No. No. I don’t think we can.”

Context

All four of the research participants commented on how the context of the Everyday Proportion Problems affected their perceived difficulty. Jackie, Cathy, and Rachel remarked how they were much better with nursing mathematics than with the mathematics presented in the Everyday Proportion Problems. Jackie’s reaction to the Dollars:Ounces problem was, “I am not usually very good at grocery questions. Let me tell you that. I am a lot better at nursing math. It is my everyday life.”

Cathy also believed that the problems on the DCPD Survey were easier than the Everyday Proportion Problems. “A lot more. It took me more than double the time! I mean on the dosage calculations, I boom, boom, boomed the first couple. Which I still

didn't boom, boom, boom on the easiest ones (of the Everyday Proportion Problems)."

Rachel shared Jackie and Cathy's view by saying, "(we) never do math like this. Never. Never. Never. It's (nursing math is) easy math. You're not thinking this hard."

Overall, the Dollars:Ounces problem was ranked as being the easiest. Its lack of difficulty was attributed to its everyday context. Cathy stated, "I look at that and I say, 'Oh, you divide.' I just know what to do." Katie stated that, "This one isn't bad because I do it at the grocery store." Rachel stated, "This I use every day when I go to stores". The People:Eggs problem was also attributed as being easy because of its context. Katie stated that, "This is the easiest because I double recipes all the time."

Cathy noted that she didn't like the Length:Width problem, because it did not have a context. The question asked if the rectangles were of the same form. She struggled with understanding what the question was asking. The researcher asked her if they were drawn to scale. When that didn't help, the researcher asked her to imagine that they were both photos and said, "Like when you have a picture and you want to blow it up." After a context was provided, she was better able to understand the problem. She stated, "I have to call it something. I like the whole picture concept." Before the additional context was provided, Cathy was using additive strategies to solve this problem as evidenced by her statement, "The intervals are not even. They're not the same." After being provided with the enriched context of enlarging a photo, she tried to use a multiplicative strategy but switched to using exponents; "So if you would double – if this was a picture, a 3 x 5, and you would double it, (it) would become a ...9 x 25." Cathy's choice of procedure was affected by her interpretation of the context.

Katie also commented on the lack of context of the scale problem. She was familiar with scale, and she related it to a context of making a flag and “you have to keep your flag in proportions. You have to multiply if you want to make a bigger flag.” She continued to refer to her created flag context as she thought through the problem.

Presentation

Katie was the one who most used the visual representations that accompanied the problems. Katie used the syringes to write down corresponding masses to volumes. When solving the rectangle problem, she answered correctly right away just by looking at the rectangles; “This one is narrower where this one is fatter.” The visual presentation for the Brown Eggs:White Eggs problem caused Katie some confusion. “This one really kind of threw me off a little bit, because my spatial thing was really off base because I couldn’t figure it out spatially so I was thinking it’s just a dozen and a half but it’s not a dozen and a half its 16.”

Rachel did not care for the visual with the Brown Eggs:White Eggs problem either. She stated, “Numbers ... I’m good. Numbers are concrete. This picture threw me off. I couldn’t relate to this picture.” The idea of having pictures on a test was foreign to Rachel as she never recalled talking a DCPP test that had pictures, “only words.”

Student Characteristics

Mathematics Anxiety

Jackie confirmed that many nursing students exhibited mathematics anxiety. This anxiety was shown to affect their ability to solve problems in the clinical setting. Jackie stated that she had seen a lot of it and shared an experience she had with a student in her clinical course.

Her math anxiety was so high that before she opened the drawer and pulled out the packet (of medicine); she started doing the math problem. But you can't do that. You have to know what you have (on hand) first. Her anxiety for math was getting in the way of thinking through what she was doing.

In order to address this anxiety, Jackie advises her students to pick a strategy and stick with it. This remedy for anxiety is further explained in Chapter 5 in the themes for DCPPs solved in nursing practice under the heading Pick a Procedure and Stick with It.

Individual participant attitudes about mathematics and their comments concerning mathematics anxiety are recorded in their Lived Experience descriptions in Chapter 4. However, all of the participants shared a common idea that mathematics problems cause initial anxiety. These comments are summarized here.

Cathy used the word "panic" to describe how she felt when she realizes that she is going to have to do mathematics on the job. She quickly regains herself and solves the problems but that initial reaction is always panic. Katie discussed her confidence level doing mathematics problems. She said, "I always freak at first. I always freak. And then I figure, once I stop and just say, 'Oh I'll figure it out, then I do fine.'" When discussing the Everyday Proportion Problems, Rachel similarly said,

Well only, if I'm not ... If I'm not confident. Just like anything else, you know...if you're not confident in something you're going to be anxious or apprehensive, right? ... Like, it didn't make me anxious. At first I was like, now wait a minute, you know, put you're thinking cap on."

Rachel, as well as the others, did not let the anxiety prevent them from solving problems. The impact of this reaction to solution procedures and difficulty is infused into the other situations in the conceptual field of proportional reasoning.

Styles of Learning

Jackie spoke frequently of the different styles learning of individual nurses. Jackie used the phrase, "what works for your brain" to talk about the different ways that nurses learn. She talked about her personal choice not to use dimensional analysis to solve problems because "it doesn't really work for my brain. I could never figure out which went in the numerator and which went in the denominator." Her teaching experience had shown her that different students respond differently to different mathematical procedures.

Cathy considered herself a concrete learner and said that she preferred dimensional analysis because "I have to line it (the units) all up. I need to visualize it and see things right in front of me." When she explained her dimensional analysis strategy, she again mentioned that "you have to line it up to where you can cross it out to get just milliliters. This is the way my mind works. I have to take care of the labels." She mentioned several times that she was "visual" and needed to see the units lined up or in the right place. "I'm telling you, I'm visual. It's like I can feel it."

Rachel categorized herself as being a visual learner. She also mentioned that she preferred to work with numbers rather than units of measure and so she considered herself to be a concrete learner because of this.

Katie also spoke about styles of learning. She recognized and labeled herself as a visual learner. She also noted that nurses have different ways of solving problems.

I had another nurse the other day, I had a terrible code the other day, and I was so exasperated with her because she wasn't thinking and my husband's like, "Katie she's never going to think like you're thinking." You know, and I have to remember that. It's kind of like –why wasn't she doing this? He's like "because she's never seen it and she doesn't know to do that." We have to remember that sometimes.

Learning styles have precise definitions in education literature and can be attached to formalized terminology. The term *visual* is one such formalized term. Cathy, Rachel, and Katie all used the term *visual* to describe their learning style. All four participants made statements about different styles of learning even though they were not specifically asked about this topic.

Summary

This chapter presented themes and data from the research on the lived experiences of nurses surrounding solving proportional reasoning problems in different contexts, the procedures that were used to solve them, and the situations that affect problem difficulty. The themes from the lived experiences were extracted using hermeneutic phenomenology data analysis procedures. The theory of conceptual fields

was used to organize the presentation of these data and themes surrounding procedures and situations. These findings are summarized here.

The descriptive writings and interviews revealed the lived experiences of nurses concerning solving dosage calculation problems on tests and in practice. The existence of high stakes DCPD testing of nursing students and nurses was confirmed. Three themes emerged that provided insight to the ways that nurses cope with the pressures of solving these crucial calculations: develop a procedure, verify answers, and be familiar with common drug dosages. The emphasis on common drug dosages was viewed as being just as important, if not more important, than the ability to perform mathematical calculations on the job.

The procedures and situations associated with nurses' proportional reasoning were organized using the structure of the theory of conceptual fields that was outlined and developed in the literature review. The procedural components of the conceptual field of proportional reasoning outlined by the researcher (See Table 11) were all evidenced in data with the exception of the set-up of a ratio table and double number line diagram. The use of problem set-up to classify respondent solutions yielded consistent results however, the large group of unclassified solutions suggests that the unit rate relational calculus was camouflaged by respondents using multiplication and division notation.

Situational components outlined in the conceptual field of proportional reasoning proposed by the researcher were confirmed as influencing procedure choice and problem difficulty. This confirmation of their influence on an individual's ability to solve proportional reasoning problems validates their existence within the framework and also

implies their needed attention when considering pedagogical decisions within the classrooms where proportional reasoning is taught.

These data and themes provided the necessary information needed to answer the research questions which in turn provided interesting comparisons to the cited literature. The results of the analysis of these findings produced several conclusions which will be presented in the next chapter.

CHAPTER 6 CONCLUSIONS AND IMPLICATIONS

Introduction: Tools for Research

The wherewithal of nurses to successfully solve DCPPs in nursing school and in practice is justifiably important because of its impact on the lives of patients and the jobs of the nurses. Tools for explicating and analyzing this ability were developed by the researcher from the synthesis of the literature and data. The Conceptual Field of Proportional Reasoning found in Table 11 served well as the guiding framework for the organization of the literature and data analysis. The detailing of the concepts, procedures, and situations within this framework provided an efficient means to organize data. This tool could provide other researchers with a comprehensive lens from which further research can be carried out. Other tools that were developed were the Synthesis of Levels of Proportional Reasoning found in Table 7, the MVPP Set-up Identification Guide found in Table 18, and the Interpreted Intensive Quantity Semantic Type found in Table 14. Each of the tools played a role in the interpretation of the data and the formulation of the conclusions of this research. These conclusions are presented next. Where possible, a question is posed that stems specifically from these conclusions in order to help advance the research in the area of the conceptual field of proportional reasoning. These questions will be summarized after the conclusion section is complete.

Conclusions and Questions

The following conclusions and questions are a product of the lived experiences of four unique nurses. Each nurse who participated in this study offered themselves to the service of assisting educators in the explication of proportional reasoning understanding. All four of the nurses confirmed the importance of this research. A brief conclusion about each participant is provided here. Katie used equality of measures to solve her DCPs. Her procedure was not fully developed and was marked more by reasoning rather than process.

- Katie desired to have a more structured procedure. Her ability to reason and perhaps her lack of locked in procedure seemed to assist her in solving non-nursing proportional reasoning problems as she was the only one to successfully answer all of the Everyday Proportion Problems.
- Cathy's consistent use of dimensional analysis resulted in confidence and proficiency in solving problems involving whole:whole semantics in and out of the context of nursing. However, she struggled with solving part:part and part:whole problems in which she could not apply her procedure.
- Jackie offered perspective from both an experienced nurse as well as a nurse educator. She used the nursing rule or as she stated, "desired over have". Jackie did not immediately apply this procedure to other proportional reasoning problems outside the context of nursing. Jackie's confidence was tied to the context of nursing rather than her procedure. She felt that any procedure was fine but advised her students to "picked a procedure and stick with it."

- Rachel had great confidence. She viewed proportional reasoning problems as multiplying and dividing in the correct order and focused on relational calculus over set-ups which was evidenced by her being categorized as having no predominate set-up. She based the evaluation of whether her answer was correct on the reasonableness of her answer. In the nursing setting, her experience gave her confidence in her solutions.

The study of these four diverse participants enabled the researcher to make several conclusions concerning the conceptual field of proportional reasoning and its instruction.

These conclusions along with follow-up questions are presented next.

Set-ups are Associated with Concepts

The definition of concept is a central theme of this research. In the nursing literature, conceptual errors on DCPPs were described as being mistakes in the set-up of the problem (Rice & Bell, 2005). However, conceptual errors in the field of mathematics are tied to the ability to generalize a concept and apply it to semantically similar situations regardless of context (Hiebert & Lefevre, 1986). One emphasizes procedure and one emphasizes situations. The merging of the research of nursing DCPPs and mathematics education proportional reasoning initiated the search and development of a system of understanding that would accommodate both of these views. The Theory of Conceptual Fields of Proportional Reasoning is such a system. By incorporating these two views into one model, data from this research show that both the chosen set-up of a proportional reasoning problem and the ability to identify varying proportional reasoning situations are affected by a person's conceptual knowledge of proportions. The situational components will be discussed later but the set-ups will be discussed here.

The literature review combined with this research defines the set-ups that nurses use to solve DCPPs. These set-ups are equality of measures, ratio tables, double number line, analogies, equal ratios, dimensional analysis, and the nursing rule. The nursing definition of set-up as explicitly leading to conceptualization is supported by this research, although only as part of the conceptual field rather than in its entirety. The errors that nurses make in their set-ups of DCPPs could point to explicit misunderstandings of the concepts related to proportional reasoning. The correct use of a set-up could indicate a connected concept that the nurse has attached to

proportional reasoning. The evidence presented in this research points to different set-ups emphasizing different conceptual aspects of proportional reasoning. Therefore, unless all of the set-ups are understood, certain conceptual aspects of proportional reasoning could remain undeveloped in the minds of the learner. More importantly, since set-ups have been attached to levels of proportional reasoning, a hierarchical attribute could be applied to these set-ups, which in turn, could help identify the level of understanding that a student possesses. Also, if a hierarchy of set-ups could be established, instructional sequences could reflect this order. As students increased in their understanding of proportional reasoning, they would continue with progressively more complex concepts associated with more complex set-ups until they reached the pinnacle which is presumed to be a linear equation set-up. Once students have reached the highest level they could identify conceptual and situational aspects of the problem to highlight and use any of the set-ups to accommodate their solution from equality of measures to linear.

With this new insight, these set-ups can and should be linked to more specific conceptual aspects of proportional reasoning if they are to be used to describe conceptual misconceptions of nurses. For example, Katie who used the equality of measure set-up relied on her understanding of the concept of indirect measure to solve her DCPs. Her focus on this particular aspect of proportional reasoning could perhaps limit her understanding. Her choice to not use other set-ups could also point to misconceptions about other concepts related to proportional reasoning, such as covariation and rates of change. This research provides a clearer description of four of the seven set-ups (equality of measures, analogies, dimensional analysis, and the

nursing rule while not more fully describing ratio tables, double number line (DNL), and equal ratios) by examining the lived experiences of nurses who use these four set-ups. Conjectures can be made concerning the link between these set-ups and more specific proportional reasoning concepts in an attempt to explicate student misunderstanding based upon set-up use. The researcher presents these conjectured relationships between set-up and associated concepts in Table 26.

Table 26

Conjectured Set-up and Associated Concepts

| Set-up | Concept Concentration |
|----------------------------|--|
| Equality of Measures | Indirect Measurement |
| Ratio Table | Covariation |
| Double Number Line Diagram | Indirect Measure Unit rates |
| Analogy | Analogical Reasoning and Relationships |
| Equal Ratios | Quantitative Reasoning - Extensive Measures Fractions |
| Dimensional Analysis | Cancellation of Units Indirect Measure |
| Nursing Rule | Procedural Emphasis |
| Linear | Intensive Measure – rate of change, slope Algebraic Reasoning |

These connections between specific set-up and concepts will be further detailed in the conclusions that follow, however, an overarching question can be asked of the connection between concept and procedure...Could a learning trajectory including focus on each of the set-ups benefit learners? Does a proposed trajectory of equality of

measures, ratio tables, DNL, analogies, equal ratios, and then dimensional analysis align with students' development of proportional reasoning?

Equality of Measures Linked to Indirect Measurement

The set-up of equality of measures was confirmed as an intuitive strategy in the literature (Ercole et al., 2011) and in this study. The set-up was found to be unnamed in the literature. The researcher named it equality of measures because of the presence of the equal sign rather than ratio or fraction notation. Ten percent of the solutions in this research were classified as using this set-up. This set-up, having not previously been named, needs to be researched. Based on this study, a starting point could be that this set-up appeared to be associated with the concept of indirect measurement. Katie, who consistently used the equality of measures set-up, seemed to grasp the concept of indirect measurement and apply it to her solution procedure. She used the syringe (a capacity measurement tool) as a tool for measuring mass when she marked it with milligrams next to the corresponding milliliters. Katie consistently used the dose strength of medications as an equality to indirectly measure the desired dose. Katie also preferred geometry concepts which could also point to her ability to relate to indirect measurement problems since length measurements are more commonly associated with indirect measurement in mathematics education research (Lamon, 2012). Katie's ease of using indirect measurement on syringes and time tapes, in combination with her positive disposition toward geometry topics led to this question for further study: Does the equality of measures set-up support a student's understanding of the general

meaning of indirect measurement (as meaning more than simply length measurement) and proportional reasoning?

. Katie did not memorize formulas or procedures. She applied reasoning strategies to her solutions and was comfortable, flexible, and accurate with using this strategy. However, the literature shows that using an equality of measures construct is not mathematically sound in the case of quantities of different measure. This set-up is only acceptable for converting same units of measure. The following topic should be researched further: In what ways, if any, might setting different dimensions equal to each other ($150 \text{ mg} = 2 \text{ mL}$) be accepted as an appropriate set-up when a ratio set-up is mathematically appropriate?

Benefits and Drawbacks of Dimensional Analysis in Solving DCPPs

The literature review and data from this research pose arguments for the acceptance of teaching only one procedure. Some researchers support the idea of teaching only one procedure in the solving of DCPPs (Johnson & Johnson, 2002). The idea of choosing one strategy and sticking with it was supported by Jackie. Several researchers have reported findings that support using dimensional analysis as an instructional strategy to improve student outcomes (Greenfield et al., 2006; Rice & Bell, 2005, Johnson & Johnson, 2002, Arnold, 1998). Rice and Bell (2002) found a statistically significant increase in confidence in solving DCPPs after nursing students were shown how to use dimensional analysis. Cathy's narrative supports this increase in confidence. Cathy was the only participant who used a traditional set-up from pounds to kilograms, that set up was dimensional analysis. Dimensional analysis translates well to solving multi-step

problems as well (Johnson & Johnson, 2002). However, Cathy demonstrated difficulty with applying the dimensional analysis set-up to comparison problems in the Everyday Proportion Problems segment of this study. Research on the dimensional analysis procedure used in solving DCPPs could be extended from focusing on the benefits to include conjectured drawbacks. This led to the question: In what ways might the predominant use of the dimensional analysis procedure for MVPPs inhibit the ability to solve comparison proportion problems?

Analogical Reasoning is Missing

In recent research regarding early procedures for solving MVPPs, equality of measures and ratio tables are used as organizational structures (Ercole et al, 2011). Absent from this literature is the application of analogical thinking and constructs. Five out of the six respondents who used the equality of measures set-up indicated that they used the analogy set-up most closely represented their solution process. This attests to a possible connection between the equality of measures set-up and analogical reasoning that students may be applying to situations. Students begin to use analogies early in primary school to represent relationships. Nurses could be using analogical reasoning to bridge their intuitive set-up of equality of measures (and perhaps ratio tables) to a more sophisticated concept of proportion. From a notational standpoint, analogies could be a convenient link between the intuitive notation of equality of measures (and ratio table) and the more sophisticated ratio notation of equal ratios. With analogical reasoning being connected to proportional reasoning by research (Modestou and Gagatsis, 2010), the following research topic question is posed: How

are analogy representations and analogical thinking currently being utilized in proportional reasoning education in the United States?

Quantitative Reasoning Connects Arithmetic to Proportional Reasoning

Unit rates seemingly were not focused on in this research, however the most interesting research questions that stem from this research could be from this area. Unit rate is a relational calculus and not a set-up and therefore was not identified in the solution processes of DCPs in this research. In the literature, this relational calculus did not meet with consensus and the thinking that is connected to it is still not explicated (Vergnaud, 1980, Ercole, 2011). However, unit rates are conjectured to be the culprit behind the majority of set-ups being unclassified. The unit rate relational calculus could be hidden in the disguise of “just multiplication and division” where the numerical aspects of the quantity are multiplied without units of measure. Three of the participants and 76% of the respondents did not use a proportional reasoning set-up for converting from pounds to kilograms. Three participants specifically commented that they just divided because they knew kilograms were smaller. The interpretation of 2.2 as a nominal quantity or a scalar multiplier without unit rather than a unit rate conversion of 2.2 lb/kg was common.

The connection can be made between equal group multiplication and the unit rate relational calculus. Students who have found success solving equal grouping multiplication problems with the notation seen in Figure 83 may not understand or see the need to provide units of measure for their quantities. Because of the automaticity of dealing with unit rate multiplication in the elementary grades, the researcher conjectures

that students do not make the transition from thinking of equal group multiplication to the more appropriate concept of proportional reasoning.

| <u>Representation</u> | <u>Correct</u> |
|--|----------------|
| $3 \times 5 = 15$ candies | No |
| 3×5 candies = 15 candies | No |
| 3 bags \times 5 = 15 candies | No |
| 3 bags \times 5 candies = 15 candies | No |
| 3 bags $\times \frac{5 \text{ candies}}{1 \text{ bag}} = 15$ candies | Yes |

Figure 83. Equal Groups Multiplication Problem and Solution

This incorrect use of adjectival quantity can very naturally lead to nurses who automatically divide to convert pounds to kilograms using nominal quantities as seen in Figure 84.

| |
|--|
| $146 \text{ pounds} \div 2.2 = 66.4 \text{ kilograms}$ |
|--|

Figure 84. Incorrect Representation of Unit Conversion

Respondents described a decision process of dividing to get a smaller number and multiplying to get a larger number as seen in Figure 85.

| | | |
|-------------------------------------|----|-----------------------------|
| <u>Decide which one is correct?</u> | | |
| $146 \text{ lb} \div 2.2$ | or | $146 \text{ lb} \times 2.2$ |

Figure 85. Operational Decision

With this set-up, the units of measure are not used in the determination of a correct procedure. With traditional proportional reasoning set-ups, units of measurement can be used to confirm correct positioning of quantities as seen in Figure 86.

| Equality of Measures | Analogies | Equal Ratios | Dimensional Analysis |
|---|--|--|---|
| $2.2 \text{ lb} = 1 \text{ kg}$ $146 \text{ lb} = ?$ | $2.2 \text{ lb} : 1 \text{ kg} = 146 \text{ lb} : x$ | $\frac{2.2 \text{ lb}}{1 \text{ kg}} = \frac{146 \text{ lb}}{x}$ | $146 \text{ lb} \times \frac{1 \text{ kg}}{2.2 \text{ lb}}$ |

Figure 86. Set-ups With Units of Measure

Schwartz (1996) summarizes this deficit in the conclusions of his own research.

This summary is presented here in his own words so that it can be expounded upon.

I believe that the focus of an arithmetic curriculum, and indeed all required school mathematics, should be on its use as a set of tools for modeling the world around us, for analyzing these models, for making inferences and drawing conclusions from them and for communication with others. If this is deemed to be a reasonable set of goals for a school arithmetic curriculum, perhaps it is time to think of replacing present school arithmetic, which is largely the arithmetic of manipulation of nominal quantity, with the arithmetic of modeling and problem posing and solving with adjectival (extensive and intensive) quantity (p.41).

While Schwartz's conclusion is well worded, a possible adaptation is suggested. The adaptation is to the curriculum band at which this conclusion is made. Understanding that the carrying of adjectival units of measure may be procedurally and conceptually too advanced for the arithmetic curriculum, the initial primary focus of any instruction on proportional reasoning would be to bridge the concepts associated with the nominal quantities of arithmetic over to the adjectival quantities so powerfully at play in the concept of proportional reasoning. A follow up question: In what ways might an explication of the meaning of scalar, extensive, and intensive quantity and the correct use of adjectival quantities when performing equal grouping operations increase conceptual understanding of proportional reasoning problems?

Intensive Quantity and Semantic Type Linked to Problem Difficulty

The semantic types used in this data were adopted from Lamon (2007). Looking at the difficulty ratings of the problems, participants viewed the well-chunked and associated sets problems were seen as easier than part:part:whole and scaling problems. Karplus et al. (1983) groups together well-chunked and associated sets as

rates and part:part:whole and scalar as ratios. This research proposed a semantic categorization equal to Lamon's (2007) structure however, the focus is on explicating the meaning of the intensive measure created by the ratio quantities in the problem with well-chunked having the most meaningful intensive measure and scalar having the least meaningful intensive measure.

A connection was made between the ability to categorize the semantic type of a problem and its perceived difficulty. Cathy recognized the semantic type of the airplane problem and found this one to be second easiest. She classified it as converting "units to units" so her dimensional analysis strategy was appropriate. Her dimensional analysis set-up was connected to the presence of well-chunked ratios. She was unable to see the People:Eggs problem as requiring a dimensional analysis set-up. The People:Eggs problem used associated set ratio measures rather than well-chunked. Cathy did not recognize that the associated set ratio of People:Eggs could be used in the same way as a well-chunked ratio like miles:gallon. Explicating these ratios could help her to see the similarities in structure between problems. This lead to the following question: In what ways might students benefit from curricular exercises where they practice writing the extensive measures of a ratio as intensive quantities and discussing their meaning in order to categorize problems by semantic type?

Nursing Procedures Lack Function Intensive Quantities

A striking observation of the three predominate DCCP strategies is that none of them use a function relational calculus for solution. The nursing rule uses a scalar relational calculus and both cross products and dimensional analysis use a rule of three

relational calculus. With the identification of the function relationship being the prime indicator of Full Proportional Reasoning, this relationship could be an interesting source of further study. What are the consequences of nursing strategies for solving DCPs not utilizing the function intensive measure?

Confidence Tied to Proportional Reasoning Set-ups

Of the 44 surveys that were returned, 21 responded that they would be interested in being contacted for further research. The data in Table 20 (from the survey participant section) shows that when these data were broken down by set-up, the majority of respondents agreed to participation in all categories except for respondents that were unclassified. Equality of Measures had the highest percentage of respondents agreeing to participate at 100% while the unclassified category had the least at 17%. This could speak to the lack of confidence that respondents had in their responses or their ability to communicate those responses. The conclusion could be made that of the nurses who responded, those who lacked structured set-ups also lacked confidence in their ability to communicate those results.

Rachel who was chosen for participation in this study because she did not have a consistent strategy choice struggled to find common links between the DCP problems and the Everyday Proportion Problems. She viewed them all as multiplication and division problems. She was never anxious about solving the problems but she was not confident in solving the Everyday Proportion Problems. Cathy could be said to represent the opposite of Rachel. Cathy was very confident in her dimensional analysis set-up. She applied this strategy to the airplane problem and was the only participant to

rank this problem as being easy. She was unable to apply dimensional analysis to the comparison problems of Length:Width and Brown Eggs:White Eggs and ranked both of these as being difficult.

Summary of Questions

The aim of this research was to describe the conceptual field of proportional reasoning through the lived experiences of nurses. The purpose of this hermeneutic endeavor was to inform instruction. However, qualitative research does not speak to generalizations. Therefore, one of the goals of this research was to inform and inspire further qualitative or quantitative studies on the topic that could speak to more generalizable conclusions. The questions that were posed under each of the categories of conclusions are summarized in Table 27.

Table 27

Summary of Questions for Future Research

| Topic: | Question: |
|-----------------------------|---|
| Set-Ups | Could a learning trajectory of learning all the set-ups benefit learners? Does a proposed trajectory of equality of measures, ratio tables, DNL, analogies, equal ratios, and then dimensional analysis align with student's development of proportional reasoning? |
| Equality of Measures | Does the equality of measures set-up support a student's understanding of the general meaning of indirect measurement (as meaning more than just length measurement) and proportional reasoning? |
| Equality of Measures | In what ways, if any, might setting different dimensions equal to each other (150 mg = 2 mL) be accepted as an appropriate set-up when a ratio set-up is mathematically appropriate? |
| Analogies | How are analogy representations and analogical thinking currently being utilized in proportional reasoning education in the United States? |
| Quantitative Reasoning | Would an explication of the meaning of scalar, extensive, and intensive quantity and the correct use of adjectival quantities when performing equal grouping operations increase conceptual understanding of proportional reasoning problems? Does an explicit instructional trajectory and connection of multiplication of equal groups, multiplication with unit rates, and multiplication of non-unit rates increase the ability of students to successfully solve MVPPs? Should multiplication and division without units be accepted as correct in the teaching of proportional reasoning? |
| Intensive and Semantic Type | In what ways might students benefit from curricular exercises where they practice writing the extensive measures of a ratio as intensive quantities and discussing their meaning in order to categorize problems by semantic type? |
| Nursing Procedures | What are the consequences of nursing procedures for solving DCPPs not utilizing the function intensive measure? |
| Dimensional Analysis | In what ways might the predominant use of the dimensional analysis procedure for MVPPs inhibit the ability to solve comparison proportion problems? |
| Confidence | Is there a correlation between choice of set-up and confidence in solving missing value proportional reasoning problems? |

Limitations of This Research

As this was a lived experience methodology, observing the nurses on the job would have potentially provided an added dimension of validity to this research. Because nurses as employees of a hospital are considered to be a vulnerable population, the direct observation of nurses proved to be difficult for this study. Nurses would be considered a vulnerable population because any evidence of unsatisfactory mathematical performance could impact their jobs. In order to replace this lost source of data, the participants were asked to keep journals of their actual experiences on the job. These data taken from this source did not have an impact on any of the common themes. As these data did not have a high impact on the study, the conclusion of the researcher was that this limitation did not cause a substantial loss to the study.

The conceptual field of proportional reasoning was investigated using a limited section of the population of people who use proportional reasoning in their occupations and daily lives. This research aimed to impact the instruction of proportional reasoning at all levels, elementary, secondary, and post-secondary. Using participants from such a specialized group, nurses, hindered this ability. The lived experience aspect of this research which takes into account the lived experiences of the researcher was an important dimension to the research that allowed the researcher to speak to other populations of proportional reasoning users. Qualitative research does not speak to generalizations. One of the goals of this research was to inform and inspire further quantitative studies on the topic that could speak to more generalizable conclusions.

Possible Revisions for Future Research

After any task, reflection upon that task can yield many thoughts as to what could have been done differently. In this section, the researcher outlines several aspects of the research that may benefit from revision.

Choices for strategies

In an attempt to replicate or further this study, much revision could be made to DCPP 6. DCPP 6 was included in the survey to assist with categorizing written responses. Solution choices for this problem were developed early in the research before the decision was made to categorize results by set-up. The solution choices should have aligned with the seven set-ups from the MVPP Set-up Identification Guide (Table 18) developed in this research. In addition to these seven set-ups, the inclusion of equal group multiplication or division would be included in an attempt to classify the large group of solutions that were unable to be classified by this research. For DCPP 1, this could look like $4\text{mg} \div 2\text{mg/mL} = 2\text{ mL}$.

Other changes could also be made to DCPP 6 to enhance the research. First, a question like DCPP 6 could be included after each question (rather than just after DCPP 1) as a means to identify flexible thinkers who use different strategies or notation dependent upon the situation. Another idea would be to develop solution choices that describe respondents thinking using verbal language rather than notational mathematics. For instance, rather than using analogy notation, a verbal description of the relationship between the quantities could be written out. The language used in this research used by participants could be used as a starting point for these descriptions.

For example, Rachel's verbalization for DCPD 1 could be adapted as a possible choice to read "I have 2 milligrams in 1 milliliter and I need 4 milligrams. So if 2 is in 1, how many is in 4? Since I divide 2 by 2 to get 1, then I divide 4 by 2 to get 2 milliliters." Verbal choices as opposed to notational choices, might also help respondents to explain their thinking verbally by rewording choices that are similar to their thinking. These revisions could facilitate efforts to explicate and categorize thinking.

Problem type

The goal of the research was to investigate the conceptual field of proportional reasoning. Two types of quantitative proportional reasoning problems encountered in research are missing value and comparison. None of the problems on the DCPD survey were comparison problems. This is not to say that nurses do not encounter comparison problems on the job. The researcher was influenced by her experiences with dosage calculation tests that have always contained only MVPPs. Comparison proportion problems using the context of nursing could have been incorporated into this study to make connections to solutions to the Everyday Proportion Problems.

As well as including a comparison problem on the DCPD survey, another change that has to do with problem type would be to balance the number of missing value problems and comparison problems between the Everyday Proportion Problems and the DCPD Survey. The Everyday Proportion Problems included three comparison problems, although the dosage calculation test did not include any of these types of problems. Specifically, the Length:Width problem could have easily been made analogous to one of the DCPDs. This would provide for more opportunity for comparisons and perhaps given more meaning to that specific problem.

Context of Everyday Proportion Problems

In addition to changing the Length:Width problem to a MVPP, perhaps another adaption to this problem would be to change its context. The Everyday Proportion Problems created by the researcher elicited abundant conversation surrounding the concept of proportions, however, the difficulty with the Length:Width problem seemed distracting. In order to better reflect an *everyday* context rather than a mathematical one, photographs could be used instead of just rectangles. One of the participants actually created this context for herself in order to better understand the problem.

Also, the use of two problems that used eggs caused some confusion in terminology. An egg used in the People:Eggs problem that used the context of cooking provides the opportunity to use a discrete quantity rather than a measurable quantity like cups, spoons, or ounces. However, the Brown Eggs:White Eggs could have been changed to another discrete quantity of food, perhaps a bag of apples that has both red and green apples. Also, visually representing the eggs in two rows as they are traditionally packaged caused Katie to miscount the eggs as 18 rather than 16 because she was looking at them as “a dozen and a half”. This presented unnecessary difficulty while it did help to emphasize the importance of context.

Presentation

All the problems from both the DCPD Survey and Everyday Proportion Problems were presented in the iconic and symbolic mode. Inhelder and Piaget (1958) based their research on enactive modes of research. Nurses’ proportional reasoning may be highly tied to this enactive mode since they manipulate syringes, IV bags, and other

scaled objects in their calculations throughout the day. The incorporation of a problem in the Everyday Proportion Problems utilizing the enactive mode could provide informative data. Also, demonstrating liquid capacity on a syringe while reviewing the DCPD problems could have been easily achieved and used to observe nurses understanding of indirect measurement.

Log

The writing prompts failed to engage participants in the writing of narratives; instead respondents focused on jotting down mathematics problems and solutions. In order to elicit a truly reflective mode and accountability for the journal writing, a research design could center on the logs as being the primary source of data. Different prompts for each day could be written to assist with motivation for writing.

Summary and Recommendations

Proportional reasoning is reasoning about quantity. This type of reasoning is situated between but intertwined with the arithmetic reasoning of elementary school and the algebraic reasoning of secondary school. Both arithmetic reasoning and algebraic reasoning are commonly notated by procedures that are devoid of quantity (number and unit as one). At the elementary level, proportional reasoning problems involving unit rates are viewed as scalar multipliers in order to avoid complicated fraction notation.

($3 \text{ bags} \times \frac{5 \text{ candies}}{1 \text{ bag}} = 15 \text{ candies}$ is reduced to $3 \times 5 = 15 \text{ candies}$.) In this study, 75% of respondents and participants used this type of solution process to convert pounds to

kilograms. At the secondary level, intensive quantities are generalized and replaced by standard forms of algebraic expression. Algebraic concepts such as slope are notated using function representations involving x 's and y 's rather than quantity notation (candy and bags). No one in this study used the algebraic standard form of direct variation, $y=kx$. What lies in the middle is proportional reasoning.

The procedures for dealing with proportional quantities, double number lines, dimensional analysis, equal ratios, etc, morph and grow into the many types of notational systems created to handle increasingly sophisticated proportional reasoning tasks. These notational systems can become highly personalized to individuals or groups of individuals as seen with the creation of the nursing rule in the field of dosage calculation proportion problems. These notational systems are affected by a person's lived experiences. This study presented seven transitional set-ups used to notate and handle these problems. The transition to the pinnacle of the generally accepted function notation and the critical type of proportional reasoning needed to get there is, however, mostly unidentifiable by name. Hidden between the impressive titles of arithmetic and algebraic reasoning, proportional reasoning goes unnoticed or is washed away with the entrapping of fraction operations. Defining, identifying, and declaring proportional reasoning situations within the curriculum and in everyday life should become as easy as labeling a problem as involving multiplication or algebra. Perhaps the first step in the process is to replace the title of Pre-Algebra with Proportional Reasoning so that the mathematical lived experiences of nurses and all students could at least incorporate the term proportional reasoning. For without the knowledge of when a task requires

proportional reasoning, Full Proportional Reasoning cannot be attained (Misailidou & Williams, 2003, Modestou & Gagatsis, 2010).

In terms of necessary knowledge, proficiency with proportional reasoning is imperative in the work of a nurse. With the “trust but verify” theme at the core of handling dosage calculations on the job, nurses should be able to justify their solutions using both reasoning and commonly accepted notation. The ability to solve proportional reasoning problems should not be limited to an idea of just multiplication and division that is ill-suited to handle the rigor of justifying operations involving intensive quantities. Understanding the methods of others, as well as being able to explain one’s own methods, for solving DCPPs should be the standard. The author recommends that schools of nursing provide their students with an in-depth study of proportional reasoning that would equip nurses with the level of proficiency in DCPPs needed to not just solve, but to verify and justify solutions, while using varied notational representations.

The product and implication of this research is a strong beginning to an understandable framework for the conceptual field of proportional reasoning that can be built upon and used by educators and researchers to assess and communicate levels of understanding. The concepts, set-ups, relational calculus, and situations of this framework were supported by literature and/or participant validation. The comprehensive quality of this conceptual field of proportional reasoning is presented with the intention to be exhaustive but structured. The hope is that it will be used to instructionally shape and evaluatively understand the lived experiences of students involving proportional reasoning.

APPENDIX A
EVERYDAY PROPORTION PROBLEMS

Which is the Better Buy?

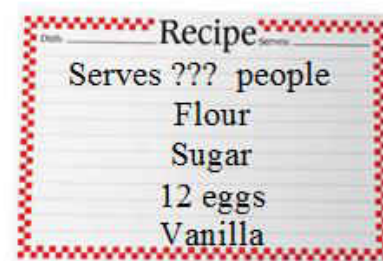
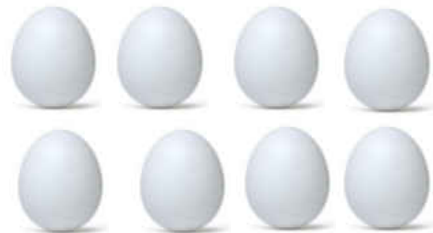


32 oz. for
\$2.00

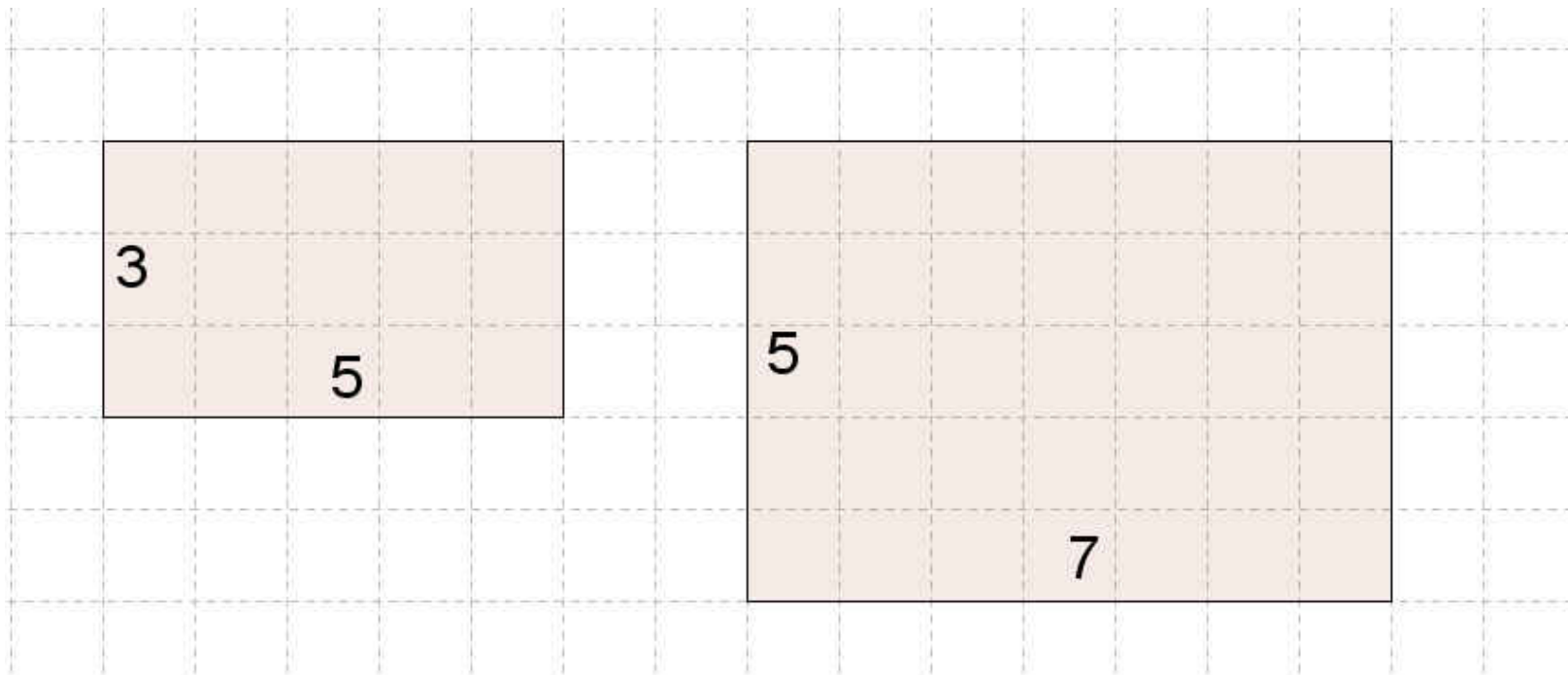


20 oz. for
\$1.50

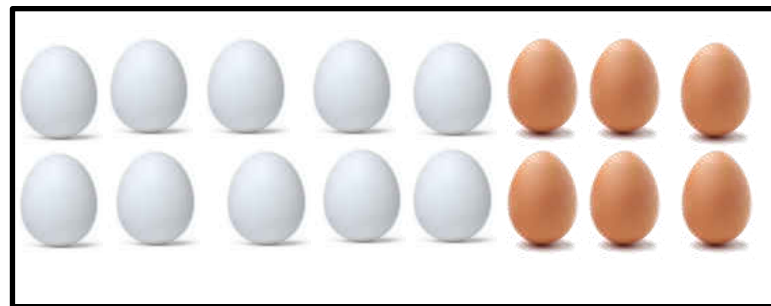
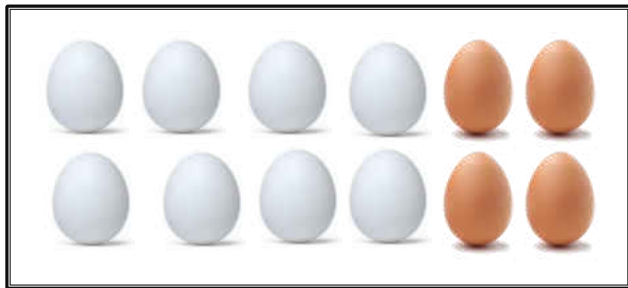
How many people will a 12-egg recipe serve?



Are these shapes of the same form?



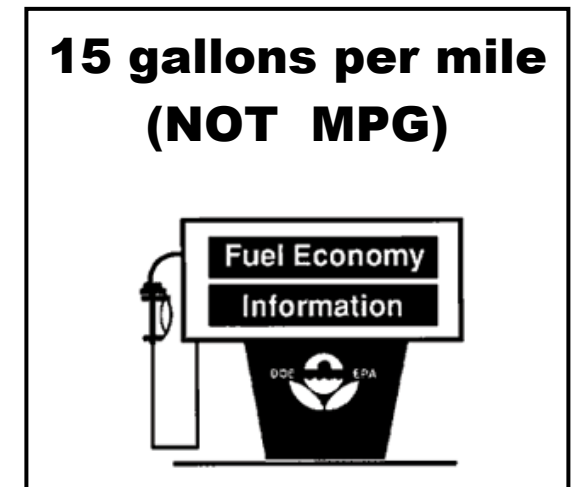
Which carton contains more brown eggs relative to white eggs?





How many gallons of gas does the plane use per minute?

| Flight Information | |
|--------------------------------------|-----------|
| Duration of Flight | 3 hours |
| Distance Flown | 500 miles |
| Airplane travels at a constant speed | |



APPENDIX B
IRB APPROVAL



University of Central Florida Institutional Review Board
 Office of Research & Commercialization
 12201 Research Parkway, Suite 501
 Orlando, Florida 32826-3246
 Telephone: 407-823-2901 or 407-882-2276
www.research.ucf.edu/compliance/irb.html

Approval of Human Research

From: **UCF Institutional Review Board #1**
FWA00000351, IRB00001138
 To: **Deana L. Deichert**
 Date: **August 12, 2013**

Dear Researcher:

On 8/12/2013 the IRB approved the following human participant research until 8/11/2014 inclusive:

Type of Review: Submission Response for IRB Continuing Review Application Form
 Project Title: The Lived Experiences of Pediatric Nurses in Solving Dosage Calculation Problems Involving Proportional Reasoning: Investigating Mathematics Instruction and Nursing Practice
 Investigator: Deana L. Deichert
 IRB Number: SBE-12-08595
 Funding Agency:
 Grant Title:
 Research ID: N/A

The scientific merit of the research was considered during the IRB review. The Continuing Review Application must be submitted 30 days prior to the expiration date for studies that were previously expedited, and 60 days prior to the expiration date for research that was previously reviewed at a convened meeting. Do not make changes to the study (i.e., protocol, methodology, consent form, personnel, site, etc.) before obtaining IRB approval. A Modification Form **cannot** be used to extend the approval period of a study. All forms may be completed and submitted online at <https://iris.research.ucf.edu>.

If continuing review approval is not granted before the expiration date of 8/11/2014, approval of this research expires on that date. When you have completed your research, please submit a Study Closure request in iRIS so that IRB records will be accurate.

Use of the approved, stamped consent document(s) is required. The new form supersedes all previous versions, which are now invalid for further use. Only approved investigators (or other approved key study personnel) may solicit consent for research participation. Participants or their representatives must receive a copy of the consent form(s).

In the conduct of this research, you are responsible to follow the requirements of the Investigator Manual. On behalf of Sophia Dziegielewski, Ph.D., L.C.S.W., UCF IRB Chair, this letter is signed by:
 Signature applied by Patria Davis on 08/12/2013 02:51:19 PM EDT

IRB Coordinator

APPENDIX C
DCPP SURVEY

Please fill out this questionnaire ONLY if you are currently working in the field of nursing.

DEMOGRAPHIC INFORMATION

Instructions: Place an in the box that **BEST** answers the question. Answer each question to the best of your ability. You may write in additional information if necessary.

1) **What type of nurse are you?**

- | | |
|------------------------------|--|
| <input type="checkbox"/> CNA | <input type="checkbox"/> RN |
| <input type="checkbox"/> LPN | <input type="checkbox"/> APRN |
| <input type="checkbox"/> LVN | <input type="checkbox"/> OTHER. Please Describe: _____ |

2) **Which of the following best describes the type of institution that you received your highest degree from?**

- | | |
|--|--|
| <input type="checkbox"/> Community College | <input type="checkbox"/> Teaching Hospital |
| <input type="checkbox"/> Traditional college | <input type="checkbox"/> On-Line College |
| <input type="checkbox"/> OTHER. Please Describe: _____ | |

3) **Which of the following best describes the highest nursing degree you have?**

- | | |
|--|--|
| <input type="checkbox"/> Nursing Certificate/diploma | <input type="checkbox"/> Master's |
| <input type="checkbox"/> Associate's | <input type="checkbox"/> Doctorate |
| <input type="checkbox"/> Bachelor's | <input type="checkbox"/> OTHER. Please Describe: _____ |

4) **What mode of instruction best describes the one that your institution used to teach dosage calculations:**

- | | |
|---|--|
| <input type="checkbox"/> On-line course | <input type="checkbox"/> Tutoring sessions/seminars |
| <input type="checkbox"/> Clinical setting | <input type="checkbox"/> No instruction was offered |
| <input type="checkbox"/> A credited dosage calculation course | <input type="checkbox"/> OTHER. Please Describe: _____ |

5) **Check all of the mathematics courses that you have taken and passed either at the college level or high school.**

- | | |
|--------------------------------------|---------------------------------------|
| <input type="checkbox"/> Pre-algebra | <input type="checkbox"/> Geometry |
| <input type="checkbox"/> Algebra I | <input type="checkbox"/> Trigonometry |
| <input type="checkbox"/> Algebra II | <input type="checkbox"/> Calculus |

You are finished with the demographic information. Thank you for taking the time to provide this information and thank you for your continued work in the field of nursing. **Please continue on to the writing prompt on the next page.**

WRITING PROMPT

*Please write a direct account of your **personal experiences** learning the mathematics that is essential for drug dosage calculation, as you lived through it. Please describe any classes or instruction that you have participated in that has contributed to this knowledge. If possible, describe a particular example or incident from your mathematics/nursing experience. You may use the back side of this packet or attach additional pages if necessary.*

You are finished with the writing prompt. Thank you for taking the time to participate in this beneficial research. **Please continue on to the dosage calculation problems on the next page.**

Dosage Calculation Problems

Situation: You have a student nurse who is interning at your place of employment and you have been assigned as their mentor. Before allowing the intern to assist you with the administration of drugs, you want to refresh their memory on how to calculate drug dosages. For the five situations that follow, write down how you would set up each problem to remind the intern of how to solve these problems. You do **NOT** have to calculate the answer. The answers can be found on the last page of this questionnaire. You may look at them at any time. You may use a calculator at any time. Remember, this is not a test. The researchers will only be looking at your suggested way of solving the problem.



The physician's orders are shown with each question along with an image of the medication. The appropriate equipment to administer these types of medications is also shown. You do not have to use these in your explanation. They are just there to help you visualize the problem.

Questions copied with permission from: Jaclynn Huse, PhD, RN, CNE

Question 1



PHYSICIAN'S ORDERS:

Zofran 4 mg IM now and then Q 6h PRN for nausea.

DOSE STRENGTH: 2 mg/mL

How many mL will you draw up in the syringe?



Question 2



PHYSICIAN'S ORDERS:

Haloperidol 2 mg IM now and then Q 12 hours.

DOSE STRENGTH: 5 mg/mL

How many mL will you draw up in the syringe?

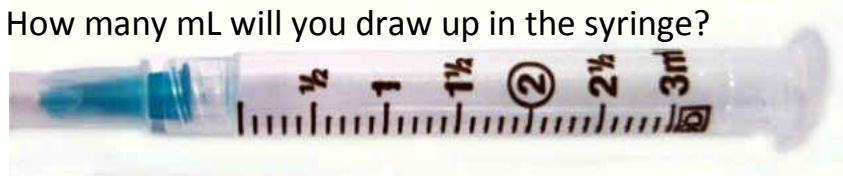




Image reprinted with permission from Bedford Labs.

Question 3

| | |
|---|--|
|  | <p><u><i>PHYSICIAN'S ORDERS:</i></u></p> <p>Synthroid 0.2 mg PO now and then QD</p> <p><i>DOSE STRENGTH:</i> 100 mcg/tablet</p> <p>How many tablets will you give?</p>  |
|---|--|

Question 4



PHYSICIAN'S ORDERS:

Dilantin 3 mg/kg PO (Patient Weight = 146 pounds)

How many capsules will you give?



Question 5



PHYSICIAN'S ORDERS:

D5NS at 500 mL over 3 hrs intravenously.

IV set delivers 15 gtts/mL.

a.) How many gtts/min will it take to deliver the prescribed dose?

b.) How many mL/hr will you set the IV infusion pump?



a.) How many drops/min will it take to deliver the prescribed dose?

b.) How many mL/hr will you set the IV infusion pump?

QUESTION 6

Place a check mark next to the representation that **best** identifies the way in which you set-up this problem (which is question #1 from this dosage calculation questionnaire). Please only select one.



PHYSICIAN'S ORDERS:

Zofran 4 mg IM now and then Q 6h PRN nausea.

DOSE STRENGTH: 2 mg/mL

| | | | | | | | | |
|--|---|---|----|----|---|---|---|---|
| <input type="checkbox"/> Equal Ratios $\frac{2mg}{1mL} \times \frac{2}{2} = \frac{4mg}{x}$ | <input type="checkbox"/> Nursing Rule $\frac{4mg}{2mg} \times 1mL$ | <input type="checkbox"/> Dimensional Analysis $4mg \left(\frac{1mL}{2mg} \right) = \text{---} mL$ | | | | | | |
| <input type="checkbox"/> Unit Rate $\frac{1mL}{2mg} = 0.5mL / mg$ $0.5 \frac{mL}{mg} \times 4mg$ | <input type="checkbox"/> 2 – step equation $\frac{2mg}{1mL} = \frac{4mg}{x}$ $x * \frac{2}{1} = \frac{4}{x} * x$ $x * \frac{2}{1} * \frac{1}{2} = 4 * \frac{1}{2}$ | <input type="checkbox"/> Cross Products $\frac{2mg}{1mL} = \frac{4mg}{x}$ $4 = 2x$ $2 = x$ | | | | | | |
| <input type="checkbox"/> Linear | <input type="checkbox"/> Analogies $2mg : 1mL :: 4mg : x$ | | | | | | | |
| <input type="checkbox"/> Table <table border="1" style="margin-left: auto; margin-right: auto; border-collapse: collapse;"> <tr> <td style="padding: 5px;">mg</td> <td style="padding: 5px;">mL</td> </tr> <tr> <td style="padding: 5px; text-align: center;">2</td> <td style="padding: 5px; text-align: center;">1</td> </tr> <tr> <td style="padding: 5px; text-align: center;">4</td> <td style="padding: 5px; text-align: center;">x</td> </tr> </table> | | | mg | mL | 2 | 1 | 4 | x |
| mg | mL | | | | | | | |
| 2 | 1 | | | | | | | |
| 4 | x | | | | | | | |

ANSWERS:

1.) 2 mL

2.) 0.4 mL

3.) 2 tablets

4.) 2 capsules

5.) a.) 166.7 mL/hr

b.) 41.6 gtt/min, round to either 41 gtt/min or 42 gtt/min depending upon hospital protocol.

YOU ARE FINISHED. Thank you for taking the time to fill out this questionnaire. You can hand in your questionnaire and you may leave. Please keep your calculator and your pen as a small token of our thanks for assisting in this beneficial research. **THANK YOU.**

Continue WRITING PROMPT here is you need more room.

| |
|--|
| |
|--|

You do not need to fill in the below information if you do not want to.

Research Number: _____

First and Last Name (print) _____

Email address: _____

Or

Cell Phone Number: _____

Please read the following:

If you are interested in participating in future research pertaining to medical dosage calculations, please provide the information above in order to be eligible. Research participants will receive a stipend of \$40/hour. Research sessions will be conducted at a convenient public restaurant. Even though you are supplying your name, your questionnaire answers will remain CONFIDENTIAL. The only person who will have access to your information is the Principal Investigator who is not affiliated with any hospital.

What you should know about a research study:

- A research study is something you volunteer for.
- You should take part in a study only because you want to.
- You can choose not to take part in a research study.
- Whatever you decide it will not be held against you.

If you do not desire to be considered for the second phase of this research study, please do not write your name on this sheet. You may leave this page blank. By writing in your name and contact information, you are granting permission to be contacted for a future research project. You are not agreeing to participate, only to be contacted.

APPENDIX D
INVITATION TO PARTICIPATE



Hi **NURSES NAME**,

My name is Deana Deichert. I wanted to thank you for participating in the Dosage Calculation Questionnaire that you recently filled out and mailed to me. I was excited to have so many nurses help out with my research. Thank you so much.

This questionnaire served two purposes:

- 1) Provide data on the educational background of nurses and their evidenced types of procedures used to solve dosage calculation problems.
- 2) Select nurses to participate in a study on dosage calculation proportion problems based upon the strategies that they used to solve problems.

You were chosen out of **(number of participants who take the dosage calculation questionnaire, not to exceed 100 nurses)** to participate in this research because of your solution strategies on the dosage calculation questionnaire. Only **(between 3 and 11)** other nurses have been chosen to participate in this study. You were specially chosen because of your unique qualities and therefore, your cooperation in this study would be greatly appreciated. Your participation in this study is entirely voluntary. You may stop participating at any time during the study without reason. You will be compensated at a rate of \$40 per hour for your time for a total of around \$300.

Please read the description of the expectations for your participation in this study which is attached to this email. After reading through this document, please feel free to email me with any questions that you may have. If you decide that you would be willing to help in this research, you can respond to this email or call me to set up a date, time, and location for our interviews.

Thank you for considering being a partner with me in this research project.

Sincerely,

Deana L. Deichert
ddeichert@knights.ucf.edu
484-300-9596
Doctoral Candidate in Mathematics Education
University of Central Florida

APPENDIX E
INFORMED CONSENT



**The Lived Experiences of Nurses in Solving Dosage Calculation Problems Involving Proportional Reasoning:
Investigating Mathematics Instruction and Nursing Practice**

Informed Consent

Principal Investigator(s): Deana L. Deichert, M.Ed
Faculty Supervisor: Juli K. Dixon, PhD

Introduction:

You are being invited to take part in a research study which will include between 3 and 11 nurses. You have been asked to take part in this research study because of the methods that you used to solve problems on your recent dosage calculation questionnaire. You were not chosen based upon your ability to solve such problems. You must be 18 years of age or older to be included in the research study.

The person doing this research is Deana L. Deichert who is a doctoral student at the University of Central Florida. Because the researcher is a doctoral student, she is being guided by Juli K. Dixon, her UCF faculty supervisor.

This consent form may contain words you do not understand. The principal investigator will discuss the informed consent and study with you. Please ask the principal investigator to explain any words or information you do not clearly understand. You may take home an unsigned copy of this consent form to think about or discuss with family or friends before making your decision.

In this consent form, “you” always refers to the subject (what a person who is enrolled in research is often called). If you are a legally authorized representative, please remember that “you” refers to the study subject.

What you should know about a research study:

- Someone will explain this research study to you.
- A research study is something you volunteer for.
- Whether or not you take part is up to you.
- You should take part in this study only because you want to.
- You can choose not to take part in the research study.

- You can agree to take part now and later change your mind.
- Whatever you decide it will not be held against you.
- Feel free to ask all the questions you want before you decide.

Purpose of the research study: The purpose of this study is to describe the lived experiences of nurses involving medication dosage calculations.

What you will be asked to do in the study: Study sessions will be conducted as:

- 1.) **Interview I:** This interview will consist of a review of your responses to the dosage calculation questionnaire that you filled out. This will last no more than 2 hours. This interview will be audio taped. Any writings will be documented using a Livescribe pen and may be used in the research publication. You will receive \$80 cash upon completion of the interview. You will also be given a journal at this time.
- 2.) **Journal Writing:** You will be asked to write in a journal for a period of 10 minutes each day that you are working. The duration of time that you will be required to keep the journal will be exactly six work days. A journal and instructions will be provided for you. You will be asked to describe in detail a mathematical task that you performed during the course of your shift that day. You will receive \$60 cash upon completion of the journal writings. You will be asked to turn in the journal at the time of Interview II. You will receive the journal back for you to keep after Interview III.
- 3.) **Interview II:** During this interview you will be asked to solve 5 problems that are considered to be everyday mathematics problems. You will be asked to solve these problems with and without the use of a calculator. These interviews will be audio taped. Any writings will be documented using a Livescribe pen and may be used in the research publication. This interview will last no more than 2 hours. You will receive \$80 cash upon completion of the interview. Your journal will be collected at this time and you will receive your journal compensation as well.
- 4.) **Interview III:** This interview will consist of two main portions: 1) a description of your drug dosage preparation procedures and 2) review of your journal writings. These interviews will be audio taped. Any writings will be documented using a Livescribe pen and may be used in the research publication. This interview will last no more than 1 hour. You will receive \$40 cash upon completion of the interview.
- 5.) **Focus Group:** The Focus Group session will include a gathering of all of the participants in order to summarize the research experience. This session will be videotaped. This interview will last no more than 1 hour. Participants will receive \$40 cash upon completion of the interview.

Location: All meetings and interviews with the exception of the Focus Group will take place at a public location like a restaurant that is mutually agreeable to both you and me. This location is not to be more than five miles from your work or home. If you are not able to attend the Focus Group session because of distance, a private meeting will be arranged.

Time required: We expect that you will be in this research study for 3 to 6 weeks. The total time required should be approximately 7 hours.

Audio or video taping: You will be audio taped during this study. If you do not want to be audio taped, you will not be able to be in the study. Discuss this with the principal investigator. If you are audio taped, the tape will be kept in a locked, safe place.

You will be video-taped during the focus group portion of this study. If you do not want to be video-taped, you can still participate in the research study as long as you agree to be audio taped instead. Discuss this with the researcher or a research team member. If you are video taped, the tape will be kept in a locked, safe place.

Risks: There is minimal risk to you as a participant. Your individual data will be kept confidential although there is always a potential for a breach of confidentiality. Any personal information given to the researcher, including but not limited to dosage calculation skills, will remain confidential. You may become anxious if you do not know the answer to a question.

Benefits:

We cannot promise any benefits to you or others from your taking part in this research. However, possible benefits include improved dosage calculation skills which could lead to higher quality service to your patients and perhaps greater confidence in your mathematics abilities.

Compensation or payment:

If you complete the entire study, you will be paid a total of \$300.00. If you withdraw from the study early, you will be paid for the study visits you complete: \$80.00 for the first interview, \$60 for the journal writings, \$80 for the second interview, \$40 for the third interview, and \$40 for the final focus group. All payments will be made in cash following the successful completion of each study visit.

Subject Cost to take part in the research study

The only anticipated cost to the subject would be travel costs. All interviews and study sessions will be scheduled with the cooperation of the subject with preference being to either prior or after each subjects work shift.

Confidentiality: Pseudonyms will be used to safeguard your identity. The key for the pseudonyms will be kept separate from the data collected on a password protected USB drive. The name of the hospital will not be used in any publications, only a description of the hospital.

Study contact for questions about the study or to report a problem: If you have questions, concerns, or complaints, or think the research has hurt you, talk to Deana Deichert, Doctoral Candidate at the University of Central Florida (484) 300-9596 or by email at ddeichert@knights.ucf.edu or Dr. Juli K. Dixon, Faculty Supervisor, by email at juli.dixon@ucf.edu.

IRB contact about your rights in the study or to report a complaint: Research at the University of Central Florida involving human subjects is carried out under the oversight of the Institutional Review Board (UCF IRB). This research has been reviewed and approved by the IRB. For information about the rights of people who take part in research, please contact: Institutional Review Board, University of Central Florida, Office of Research & Commercialization, 12201 Research Parkway, Suite 501, Orlando, FL 32826-3246 or by telephone at (407) 823-2901. You may also talk to them for any of the following:

- Your questions, concerns, or complaints are not being answered by the research team.
- You cannot reach the research team.
- You want to talk to someone besides the research team.
- You want to get information or provide input about this research.

Withdrawing from the study:

If you decide to leave the study, contact the principal investigator so that the she can continue the study with a different subject. The person in charge of the research study can remove you from the research study without your approval. Possible reasons for removal include failure to show up to scheduled study sessions.

PLEASE KEEP THIS DOCUMENT FOR YOUR RECORDS IN CASE YOU HAVE ANY QUESTIONS.

APPENDIX F
INTERVIEW I: DCPD ON TESTS PROTOCOL

Part I: Demographic Information and Writing Prompt

1.) In your writing prompt, you wrote this “*quote from participant’s writing*”.

Possible follow-up questions include:

- a. Could you explicitly describe this to me?
 - b. Can you describe a particular experience that would help me to understand this better?
 - c. How did this make you feel? Why do you think it made you feel that way?
 - d. Can you describe an example of this?
- 2.) What was the highest level of mathematics that you received prior to going to college? Describe how this mathematics prepared you for the mathematics that was required of you in your nursing program. Can you describe a particular instance where you learned something similar or related to dosage calculations during this time in your education?
- 3.) You did/did not mention your college experiences. Could you take a moment and describe for me how you feel like your college education helped you to understand how to do these problems? Could you describe your dosage calculation experiences at the college level, prior to becoming a nurse?
- 4.) You did/did not mention your on the job experiences with dosage calculations. Could you take a moment and describe for me how you feel like your work as a nurse has affected your understanding of these types of problems?
- 5.) What experiences, if any, changed how you do your dosage calculation problems on the job as compared to how you do them on a written test? Do you do these calculations differently on the job than how you do them on tests?

PART II: Dosage Calculation Proportion Problems

- 1.) How do you feel overall about your responses on this test?
- 2.) How did taking this test/particular problem make you feel? And why?
- 3.) For this problem, you solved it like this (*show them their own work*).
 - a. Can you explain your thinking for me?
 - b. Where did you learn how to solve this problem that way?
 - c. Do you know if this method of solving goes by a particular name? Do you know the name?
 - d. Why did you write this this way?
 - e. What work did you do in your head and what work did you do on the calculator?
 - f. Did you find this particular problem easy or hard? Why?
- 4.) Why do you think you solved this problem differently from the way you solved this problem?
- 5.) How are these two problems similar or different?
- 6.) Which of these problems do you feel is more difficult? Why?
- 7.) Do you feel like this test was representative of the types of problems that nurses should know how to solve? Why or why not?
- 8.) How do you feel about taking tests like this?
- 9.) What do you think employers/colleges/high schools could do to help nurses understand This mathematics better.

APPENDIX G
INTERVIEW II: EVERYDAY PROPORTION PROBLEM PROTOCOL

Part I: Possible probing questions during the solution process are:

1. Do you understand what the task is about?
2. *If no:*
 - a. What does the question ask?
 - b. What information is given to you?
 - c. *Have the participant write those answers down. If they are exhibiting extreme signs of confusion, skip down to the later questions.*
3. How did you arrive at that answer?
4. Does the strategy you used to solve this problem have a name?
5. If you had to give it a name, what would you call it?
6. How can you check your answer to be sure it makes sense?
7. Was the way that you checked your answer the same as the original way you solved the problem?
8. Why do you think you used a different strategy when checking?
9. Does the strategy that you used to check your answer have a name?
10. If you had to give it a name, what would you call it?
11. Did you find this particular problem easy or hard? Why?
12. Where did you learn to solve problems like this?
13. Do you ever solve problems that are anything like this on the job? Can you describe this for me?
14. Do you ever use the mathematical strategies that you used (call them by the name given) on the job? Can you describe this for me?

Part II: Questions about all five tasks that you just completed.

1. How did doing these tasks make you feel? And why?
2. Why do you think you solved this problem differently from the way you solved this problem?
3. How are these five problems similar or different?
4. Which of these problems do you feel is the most difficult? The easiest? Why?
5. Do you feel that understanding how to do these problems could benefit you in doing your dosage calculation problems? Why or why not?
6. How would you feel if you were required to answer questions like these on a nursing mathematics test?
7. What do you think employers/colleges/high schools could do to help nurses understand this mathematics better?
8. Describe for me how you think these tasks are similar/different from those on your dosage calculation questionnaire?

APPENDIX H
INTERVIEW III: MATHEMATICS ON THE JOB PROTOCOL

Part I: On-the-job Mathematics Questions.

1. Suppose you are preparing your medications for the day for you patients. What would I see happening? What would be going on? Walk me through what this would look like.
2. What are some of the thoughts that go through your head as you are getting your medications together?
3. Describe for me a time when you had to dig out a calculator in here for a dosage check.
4. So now I want to take you back to before you were a nurse. What types of experiences did you have in your schooling that prepared you for this part of your job?
5. Looking back on the drug dosage questionnaire (show nurse a blank questionnaire), what do you think of these types of drug dosage questions? Prompts: Do you think they are fair? Accurate? Worthwhile? How do you think they could be improved?

Part II: Journal Entries.

- 1.) Could you explain to me how this made you feel?
- 2.) Could you give an example of what you mean by this?
- 3.) Could you describe this for me to help me understand it better?
- 4.) What in your educational background may have caused you to react this way? Or prepared you for this?
- 5.) During this time, you have had to spend a great deal of time reflecting on the mathematics of your job. What affect has all of this mathematical reflection had on you? What things did you come to realize as a result of this reflection that you might not have thought about in the past?
- 6.) Is there anything that you think I should know about your drug administration routine that you would like to add or clarify?
- 7.) What in your educational background may have caused you to react this way? Or prepared you for this?
- 8.) During this time, you have had to spend a great deal of time reflecting on the mathematics of your job. What affect has all of this mathematical reflection had on you? What things did you come to realize as a result of this reflection that you might not have thought about in the past?
- 9.) Is there anything that you think I should know about your drug administration routine that you would like to add or clarify?

APPENDIX I
RESPONSES TO DCPs

| | Respondent 1 | Respondent 2 |
|-------------|--|--|
| Question 1 | $\begin{array}{r} 2\text{mg/ml} \\ + 2\text{mg/ml} \\ \hline \text{Zofran } 4\text{mg} = 1\text{ml} + 1\text{ml} = \end{array}$ | $\begin{array}{r} 2\text{mg} = 1\text{ml} \\ 4\text{mg} = 2\text{ml} \end{array}$ |
| Question 2 | $\begin{array}{r} 0 \times 4 \\ \hline 5 \overline{) 20} \\ 20 \\ \hline 0 \end{array}$ | $\begin{array}{r} 0.4 \\ 5 \overline{) 2} \\ 20 \\ \hline 0 \end{array}$ |
| Question 3 | Nothing | $\begin{array}{l} 100\text{ mcg} = 1\text{ tab.} \\ 2 \times 1000 = 200\text{ mcg.} \\ \quad \quad \quad = 2\text{ tab.} \end{array}$ |
| Question 4 | $2.2 \div 146 = 66.4 \times 3$ | $\begin{array}{l} 146\text{ lbs.} + 2.2\text{ Kg} = 166.36\text{ Kg} \\ \quad \quad \quad \times 3 \\ \quad \quad \quad \hline 100\text{mg/cap} \\ \quad \quad \quad = 2\text{ cap.} \end{array}$ |
| Question 5a | $\begin{array}{l} 500 \times 15 = 7500 \\ 3 \times 60 = 180 \\ 3\text{hr} \div 7500 = 2500 \\ 60\text{min} \div 2500 = 41.6 \end{array}$ | $\begin{array}{l} 166.7\text{ ml/hr} \times 15 = 2500.5 \\ \quad \quad \quad \text{gts/hr} \\ \quad \quad \quad \div 60 = \\ \quad \quad \quad 41.675 \\ \quad \quad \quad = 42 \\ \quad \quad \quad \text{gts/min} \end{array}$ |
| Question 5b | Nothing | $500 \div 3 = 166.66 \text{ (ml/hr)}$ |
| Question 6 | Equal Ratios | Analogies |

| | Respondent 3 | Respondent 4 |
|-------------|---|--|
| Question 1 | <p>Remind intern that this is multi dose vid first!</p> <p>4mg 2mg/ml how many ml = 4mg 2mg = 1 ml 4mg = 2 ml</p> | $\frac{4 \text{ mg}}{2 \text{ mg}} \times 1 \text{ ml}$ $4 \div 2 = 2 \text{ ml}$ |
| Question 2 | <p>5mg = 1 ml 1/10 ml = 0.5mg on syringe 2mg = 4/10 ml</p> | $\frac{2 \text{ mg}}{5 \text{ mg}} = 1 \text{ ml}$ $\frac{.4}{20} = .4 \text{ ml}$ |
| Question 3 | <p>100 mcg = 1 tablet = 0.1 mg 200 mcg = 2 tablets = 0.2 mg</p> | <p>1mg = 0.001mg</p> $\frac{.2 \text{ mg}}{100 \text{ mcg}}$ |
| Question 4 | <p>3mg/kg = 146 lbs = 66kg 66.3636 2.2 kg/lb</p> <p>3mg x 66 kg = 200</p> | $\frac{146 \text{ lbs} = \frac{22 \text{ lbs}}{15}}{15}$ $\frac{2.2 \times 146}{2.2} = 2.2$ $x = 66.4$ $\frac{66.4 \times 3 \text{ mg}}{199.2 \text{ mg needed}} = 2 \text{ capsules}$ $\frac{3 \text{ mg}}{150 \text{ mg}} = 1 \text{ capsule} = 200 \text{ capsules}$ $x = 199.2 \text{ mg}$ |
| Question 5a | <p>2.78333 ml/min = 41.7 g/hr (approx) (42)</p> | <p>prescribed dose?</p> $\frac{166.66 (166.7)}{3 \times 1500}$ <p>166 ml/hr ÷ 60 min = 2.766 ml/min</p> $2.766 \times 15 = 41.5$ |
| Question 5b | <p>166.66 = 167 ml/hr = 500ml/3hr</p> | <p>166.7 ml/hr</p> |
| Question 6 | <p>Table</p> | <p>Nursing Rule</p> |

| | Respondent 5 | Respondent 6 |
|-------------|---|---|
| Question 1 | <p>You would draw up 2ml.</p> $\frac{2\text{mg}}{1\text{ml}} = \frac{4\text{mg}}{2\text{ml}}$ | $\frac{4\text{mg}}{2\text{mg}} \times 1\text{ml} = \underline{2\text{ml}}$ |
| Question 2 | $\frac{5\text{mg}}{1\text{ml}} = \frac{1\text{mg}}{0.2\text{ml}}$ $2\text{mg} = 0.4\text{ml}$ | <p>Dose required</p> $\frac{2\text{mg}}{5\text{mg}} \times 1\text{ml} = \underline{0.4\text{ml}}$ <p>Dose on hand</p> |
| Question 3 | <p>2 tablets</p> $0.2\text{mg} = 2000\text{ mcg} \quad 2 \text{ (100 mg tablets)} = 200\text{ mcg}$ <p>(i.e. x1000)</p> | $\frac{0.2\text{mg}}{0.1\text{mg}} \times 1\text{tab} = \underline{2\text{ tabs}}$ |
| Question 4 | <p>pt weight = 66.4 kg</p> $3\text{mg/kg} = 3 \times 66.4 = 199.2\text{ mg}$ | <p>Must be aware that 100mg = 0.1g, we need to multiply it x10 because it's much easier to give for every kg to be given.</p> $\frac{199.2}{100} = 1.992 \text{ or } 200 \text{ - nearest}$ <p>And since we need 200mg, but only have 100mg capsules we need to give 2 - 100mg capsules to equal 200mg.</p> |
| Question 5a | $3\text{ hrs} = 180\text{ minutes}$ $500\text{ ml} \div 180\text{ minutes} = 2.7\text{ ml}$ $\frac{15\text{ gts}}{1\text{ml}} \times 2.7\text{ ml} = 42\text{ gts/min}$ | $\frac{500}{180} = 2.77 \text{ or } 2.8\text{ml/min}$ <p>then to figure out how many gts/min we know 15gts=1ml so we must multiply 2.8 x 15gts to get how many gts/min.</p> $\frac{2.8\text{ml}}{15\text{gts/ml}} = 42\text{ gts/min}$ |
| Question 5b | $500\text{ml} \div 3\text{hrs} = 166.7\text{ ml/hr}$ | $\frac{500}{3} = 166.7\text{ ml/hr}$ |
| Question 6 | Equal Ratios | Nursing Rule |

| | Respondent 7 | Respondent 8 |
|-------------|--|---|
| Question 1 | $4mg \times \frac{2ml}{2mg} = 1ml$ $\frac{4 \times 1}{2} = \frac{2 \times 2}{2}$ $2ml's$ $2 \cancel{14}$ | $\frac{4mg}{2mg} \times 1ml$ |
| Question 2 | $5 \times 2 = 10$ $\frac{10}{2} = 5$ $2 \times 5 = 10$ $2 \times 5 = 10$ | $\frac{2mg}{5mg} \times 1ml$ |
| Question 3 | $0.2mg \rightarrow 200mcg$ $\frac{200mcg}{11} \text{ Tab}$ | $0.2mg \rightarrow 0.200g = 200mcg$ $\frac{200mcg}{100mcg} \times 1 \text{ tab}$ |
| Question 4 | $146 \div 2.2$ $66 \times 3 = 198$ 2 caps | $146 \text{ lbs} \div 2.2 = 66.4 \text{ kg}$ $\times \frac{3}{199.2 \text{ mg}}$ $\frac{199 \text{ mg}}{100 \text{ mg}} \times 1 \text{ cap}$ |
| Question 5a | $\frac{500 \times 15}{180 \text{ (min)}}$ 41.6 g | $\frac{500ml \times 15}{180 \text{ min}}$ |
| Question 5b | $500 \div 3$ 167 mL/hr | $\frac{500 \times 15}{60}$ |
| Question 6 | Analogy | Nursing Rule |

| | Respondent 9 | Respondent 10 |
|-------------|---|---|
| Question 1 | <p>I will draw up 2ml</p> $\frac{4 \text{ mg}}{2 \text{ ml}} = \frac{2 \text{ mg}}{x}$ $\frac{4}{2} = \frac{2}{x}$ $2 = x$ | $\frac{20 \text{ mg}}{10 \text{ ml}} = \frac{2 \text{ mg}}{x}$ 2 ml |
| Question 2 | <p>If 5mg = 1ml cross multiply</p> $\frac{5 \text{ mg}}{5} = \frac{2 \text{ mg}}{x}$ $x = 0.4 \text{ ml}$ | $\frac{5 \text{ mg/ml} \times 2 \text{ mg}}{5 \text{ mg}} = 0.4 \text{ ml}$ <p>or 5mg = 5mg</p> |
| Question 3 | <p>First you had to convert mg to mcg</p> $\frac{1 \text{ mg}}{1000 \text{ mcg}} = \frac{200 \text{ mcg}}{x}$ $x = 200 \text{ mcg}$ <p>Give 2 tablets</p> | <p>2 Tablets</p> |
| Question 4 | <p>You first need to see how many 116</p> $\frac{116 \text{ mg}}{116} = \frac{232 \text{ mg}}{x}$ $x = 2$ <p>Give 2 Caps</p> | $\frac{116}{116} = 1$ $\frac{232}{116} = 2$ <p>2 Caps</p> |
| Question 5a | <p>If 500 ml now there 1500</p> $500 = 1500 \times 2.7 \times 15 \text{ ml} = 41.6$ $\frac{15 \text{ ml}}{15} = 2.7 \text{ ml}$ $15 \times 2.7 = 41.6$ | $\frac{1500 \times 15 \text{ ml}}{15} = 1500$ $\frac{1500}{15} = 100$ |
| Question 5b | $500 \div 3 = 166.7 \text{ ml/hr}$ <p>Set pump at</p> | <p>Nothing</p> |
| Question 6 | <p>Cross Products</p> | <p>Nursing Rule</p> |

| | Respondent 11 | Respondent 12 |
|-------------|--|---|
| Question 1 | $\frac{2\text{mg}}{1\text{ml}} = \frac{4\text{mg}}{x}$ $\frac{2x}{2} = \frac{4}{2}$ $x = 2\text{ml}$ | $\frac{2\text{mg}}{4\text{mg}} \times 1\text{ml} = 2\text{ml}$ |
| Question 2 | $\frac{5\text{mg}}{1\text{ml}} = \frac{2\text{mg}}{x}$ $\frac{5x}{5} = \frac{2}{5}$ $x = 0.4\text{ml}$ | $\frac{2\text{mg}}{5\text{mg}} \times 1\text{ml} = 0.4\text{ml}$ |
| Question 3 | $100\text{mg} = 0.1\text{mg}$ $\frac{0.1\text{mg}}{1\text{tab}} = \frac{0.2\text{mg}}{x\text{ tabs}}$ $\frac{0.1x}{0.1} = \frac{0.2}{0.1}$ $x = 2\text{ tabs}$ | $\frac{0.2\text{mg}}{1} \times \frac{100\text{mg}}{1\text{mg}} \times \frac{1}{100\text{mg}} = 2\text{ tabs}$ |
| Question 4 | $\frac{2.2\text{Kg}}{1\text{lb}} = \frac{x\text{ Kg}}{146\text{ lb}}$ $\frac{2.2x}{2.2} = \frac{146}{2.2}$ $x = 66.4\text{ Kg}$ <p>② $\frac{66.4\text{ Kg}}{x\text{ mg}} = \frac{1\text{Kg}}{3\text{mg}}$ 199.2 mg 2 capsules</p> | $\frac{2.2}{146} = 66.34 = 66.4\text{ Kg}$ $\frac{3\text{mg}}{18} \times \frac{66.4\text{ Kg}}{1} \times \frac{199.2\text{ mg}}{100\text{ mg}} = 1.99 = 2\text{ cap}$ |
| Question 5a | $\frac{167\text{ ml}}{x} = \frac{1\text{ml}}{15\text{gts}}$ $x = 2505\text{ gts/hr}$ $\frac{2505}{60} = \frac{x}{1}$ $x = 41.75\text{ gts/min}$ 42 gts/min | $\frac{15\text{gts}}{1\text{hr}} \times \frac{5\text{hr}}{3\text{hr}} \times \frac{1\text{hr}}{60\text{min}} = 41.64$ 42 gts/min |
| Question 5b | $\frac{500\text{ml}}{3\text{hr}} = \frac{x\text{ ml}}{1\text{hr}}$ $\frac{3x}{3} = \frac{500}{3}$ $x = 166.7\text{ ml/hr}$ | $\frac{\text{ml}}{\text{hr}} = \frac{500\text{ml}}{3\text{hr}} = 166.7\text{ ml/hr}$ |
| Question 6 | Cross Products | Nursing Rule |

| | Respondent 13 | Respondent 14 |
|-------------|---|---|
| Question 1 | $\frac{\text{ordered}}{\text{available}} \times \text{amt} \quad \frac{4}{2} \times 1$ | $CC = \frac{1cc}{2mg} \times \frac{4mg}{1} = 2cc.$ |
| Question 2 | $\frac{\text{ord}}{\text{amt}} \times \text{amt} \quad \frac{2}{5} \times 1 \quad \frac{4}{2} \times 1$ | $CC = \frac{1cc}{5mg} \times \frac{2mg}{1}$ |
| Question 3 | $\frac{0}{1} \times \text{amt} \quad 0.2mg = 200mg \quad \frac{200}{100} \times 1 = 2$ | $\frac{1 \text{ tab}}{100mg} \times \frac{1000mg}{1mg} \times \frac{0.2mg}{1} = 2 \text{ tabs.}$ |
| Question 4 | $3mg/kg \quad kg = 2.2 \quad 2 \times 146.0 \quad 199.8g$ $\begin{array}{r} 65.3L \\ 2.2 \overline{)146.0} \\ \underline{132} \\ 140 \\ \underline{132} \\ 80 \end{array}$ | $\frac{1 \text{ Cap}}{100mg} \times \frac{3mg}{1kg} \times \frac{1kg}{2.2lb/s} \times \frac{146lb/s}{1} =$ |
| Question 5a | $\frac{\text{Vol} \times \text{Cal}}{\text{units}} = \text{gtts/min}$ $\frac{500 \times 15}{150}$ | $\frac{500ml}{3hrs} = \frac{15gtts}{1ml} \times \frac{500ml}{3hrs} \times \frac{1hr}{60min} = 41.6 \rightarrow 42 \frac{gtts}{min}$ |
| Question 5b | $\frac{\text{Vol}}{\text{Time}} \quad \frac{500ml}{3hrs}$ | $\frac{500ml}{3hrs} = \frac{166ml}{hr}$ |
| Question 6 | Nursing Rule | Unit Rate |

| | Respondent 15 | Respondent 16 |
|-------------|---|--|
| Question 1 | $2 \text{ mg} \times \frac{1 \text{ mL}}{2 \text{ mg}} = 2 \text{ mL}$ | $\frac{4 \text{ mg}}{2 \text{ mg/mL}} = 2 \text{ mL}$ |
| Question 2 | $2 \text{ mg} \times \frac{1 \text{ mL}}{5 \text{ mg}} = \frac{2}{5}$ | $\frac{5 \text{ mg/mL}}{2 \text{ mg}} = 2.5 \text{ mL}$ |
| Question 3 | $2 \text{ mg} \times \frac{1000 \text{ mg}}{1 \text{ mg}} \times \frac{1 \text{ tab}}{1000 \text{ mg}}$ | $0.2 \text{ mg} = 200 \text{ mcg}$ 2 tablets |
| Question 4 | $146 \text{ lbs} \times \frac{1 \text{ kg}}{2.2 \text{ lbs}} = 66.34 \text{ kg}$ | $\frac{146 \text{ lbs}}{2.2 \text{ kg}} = 66.4 \text{ kg} \times \frac{3 \text{ mg/kg}}{200 \text{ mg}}$ 2 capsules |
| Question 5a | $\frac{500 \text{ mL}}{3 \text{ hrs}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{15 \text{ gtt}}{\text{mL}} =$ | $166.7 \text{ mL/hr} \times 15 \text{ gtt/mL}$ 15 gtt/mL $\frac{2500.5}{60 \text{ min}} = 41.67 \text{ gtt/min.}$ |
| Question 5b | $\frac{500 \text{ mL}}{3 \text{ hrs}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{15 \text{ gtt}}{\text{mL}} =$ | $\frac{500 \text{ mL}}{3 \text{ hrs}} = 166.7 \text{ mL/hr.}$ |
| Question 6 | Dimensional Analysis | Nursing Rule |

| | Respondent 17 | Respondent 18 |
|-------------|---|---|
| Question 1 | $\frac{4 \text{ mg}}{x \text{ mL}} = \frac{2 \text{ mg}}{1 \text{ mL}}$ $2x = 4$ $x = 2 \text{ mL}$ | <p>No math required.</p> <p>2mg = 1 mL. 4 mL = 2 mL.</p> |
| Question 2 | <p>Algebra</p> $\frac{2 \text{ mg}}{y \text{ mL}} = \frac{5 \text{ mg}}{1 \text{ mL}}$ $5x = 2$ $x = 0.4 \text{ mL}$ | $\frac{4 \text{ mg}}{5 \text{ mg}} \times \frac{1 \text{ mg}}{x} = \frac{5}{2}$ $\frac{5 \text{ mg}}{\text{mL}} \times \frac{x}{2 \text{ mg}} = \frac{5}{2}$ $\frac{5 \text{ mg}}{2 \text{ mg}} \text{ mL}$ |
| Question 3 | $\frac{0.2 \text{ mg}}{1 \text{ mL}} = \frac{100 \text{ mg}}{x}$ $\frac{0.2 \text{ mg}}{1} = \frac{0.1 \text{ mg}}{x}$ $0.2 \text{ mg} = 0.1x$ $0.2 = 0.1x$ <p>Linear</p> $\frac{100 \text{ mg} 0.001 \text{ mg}}{1 \text{ mg} 0.2 \text{ mg}}$ | <p>Need to convert micrograms to milligrams</p> <p>micrograms = — milligrams.</p> |
| Question 4 | <p>for a multi-step 2 prefer 1, use</p> $\frac{1 \text{ cup } 13 \text{ mg}}{100 \text{ mg}} \left \frac{1 \text{ kg}}{2.2 \text{ lb}} \right \frac{146 \text{ lb}}{1}$ | <p>Convert pounds to kilograms - to find pt's weight in kg.</p> <p>(about 100 mg) pt wt x mg =</p> |
| Question 5a | $\frac{1 \text{ kg } 500 \text{ mL}}{60 \text{ mL}} \times \frac{1 \text{ gtt}}{1 \text{ mL}} = \frac{7500}{180} = 41.7 \text{ gtt/min}$ 41.7 gtt/min | Nothing |
| Question 5b | $\frac{60 \text{ min}}{1 \text{ hr}} \times \frac{41.7 \text{ gtt}}{1 \text{ min}} = 16 \text{ hr. } 16 \text{ hr}$ | $\frac{500 \text{ mL}}{3} = 166.6 \text{ mL/hr}$ |
| Question 6 | 2-step equation | Analogies |

| | Respondent 19 | Respondent 20 |
|-------------|--|--|
| Question 1 | $\frac{2 \text{ mg}}{1 \text{ mL}} = \frac{4 \text{ mg}}{x}$ $\frac{2x}{2} = \frac{4}{2}$ $x = 2 \text{ mL}$ | 0 mg/mL need 4 mg would draw up 2 mL |
| Question 2 | $\frac{5 \text{ mg}}{1 \text{ mL}} = \frac{2 \text{ mg}}{x}$ $\frac{5x}{2} = \frac{2}{2}$ $x = .4 \text{ mL}$ | $5 \overline{) 2.0}$ $.4 \text{ mL}$ |
| Question 3 | $.2 \text{ mg} \times 100 = \frac{200 \text{ mcg}}{100 \text{ mcg}} = 2$ | 2 tabs. |
| Question 4 | $\frac{146 \text{ lbs}}{2.2} = 66.36 \times 3 = 199 \approx \frac{200 \text{ mg}}{100 \text{ mg}} = 2$ | $146 \div 2.2 = 66.36$ $\times 3 = 199.08$ $\frac{200}{199.08} = 2 \text{ capsules}$ |
| Question 5a | $\frac{166.7 \text{ mL}}{\text{hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} \times \frac{15 \text{ ppt}}{\text{min}}$ | $500 \times 15 = 7500 \div 180 = 41.6$ |
| Question 5b | $\frac{500 \text{ mL}}{3 \text{ hrs}} = 166.67 \text{ mL/hr}$ | $500 \div 3 = 166.66 = 166.67 \text{ mL/hr}$ |
| Question 6 | Nothing | Nothing |

| | Respondent 21 | Respondent 22 |
|-------------|--|---|
| Question 1 | $\frac{2 \text{ mg}}{4 \text{ mg}} = \frac{1 \text{ mL}}{X}$ $2 \sqrt{4} \quad X = 2 \text{ mL}$ | $\frac{4 \text{ mg}}{2 \text{ mg}} \times 1 \text{ mL} = 2 \text{ mL}$ |
| Question 2 | $\frac{5 \text{ mg}}{2 \text{ mg}} = \frac{1 \text{ mL}}{X}$ $5 \times 2 = X \cdot 0.4 \text{ mL}$ $10 = 0.4 X$ $X = \frac{10}{0.4} = 25 \text{ mL}$ | $\frac{2 \text{ mg}}{5 \text{ mg}} \times 1 \text{ mL} = 0.4 \text{ mL}$ |
| Question 3 | $0.2 \text{ mg} = 200 \text{ mcg}$ $\frac{100 \text{ mcg}}{200 \text{ mcg}} = \frac{1 \text{ tab}}{X}$ $100 \sqrt{200} \quad X = 2 \text{ tabs}$ | $\frac{D}{H} \text{ tab} = \frac{0.2 \text{ mg}}{\frac{100 \text{ mcg}}{1000}} \times 1 \text{ tab} = 2 \text{ tabs}$ |
| Question 4 | $1 \text{ kg} = 2.2 \text{ lbs} = 66.3 \text{ kg}$ $2.2 \sqrt{146} \quad 66 \text{ kg}$ $3 \times 66 = 198 \text{ mg}$ $100 \text{ mg cap} = 2 \text{ caps}$ | $D: \frac{66 \text{ kg}}{110 \text{ mg}} \times 3 \text{ mg/kg} = 1.8 \text{ mg}$ $H: \frac{1.8 \text{ mg}}{100 \text{ mg}} \times 1 \text{ cap} = 1.8 = 2 \text{ caps}$ <p>must make sure + rounds up.</p> |
| Question 5a | $\frac{500 \text{ mL}}{1500 \text{ mL}} = \frac{3 \text{ hrs}}{X \text{ hrs}}$ $500 \sqrt{1500} \quad 166.6$ 166 hr | $\frac{167 \text{ mL} \times 15 \text{ g/dL}}{60 \text{ min}} = 41.75$ $41.75 = 41.42 \text{ d/dL/hr}$ $\text{Sol pump} = 42$ |
| Question 5b | $\frac{500 \text{ mL}}{3 \sqrt{1500}} = \frac{3 \text{ hrs}}{X \text{ hrs}}$ 166 hr | $\frac{500 \text{ mL}}{3 \text{ hrs}} = 167 \frac{\text{mL}}{\text{hr}}$ |
| Question 6 | Analogies | Nursing Rule |

| | Respondent 23 | Respondent 24 |
|-------------|---|--|
| Question 1 | $\frac{4 \text{ mg}}{2 \text{ mg/ml}} = X (\text{ml})$ | $\frac{4 \text{ mg}}{X} = \frac{2 \text{ mg}}{1 \text{ ml}}$ $4 \text{ mg} \times 1 \text{ ml} = 2 \text{ mg} \times X$ $4/2 = X$ $2 = X$ 2 ml |
| Question 2 | $\frac{5 \text{ mg/ml}}{2 \text{ mg}} = X (\text{ml})$ | $\frac{2 \text{ mg}}{X} = \frac{5 \text{ mg}}{1 \text{ ml}}$ |
| Question 3 | $\frac{100 \text{ mcg/tab}}{200 \text{ mcg}} = X (\text{mcg})$ | $.2 \text{ mg (convert to mcg)}$ $\times 100 = 200 \text{ mcg}$ $\frac{200 \text{ mcg}}{100 \text{ mcg}} = 2 \text{ tabs}$ |
| Question 4 | $2.2 \text{ kg} = 116$ $2.2 \overline{) 24.2} \begin{array}{r} 11 \\ 22 \\ \hline 146 \\ 132 \\ \hline 140 \\ 132 \\ \hline 80 \end{array}$ $\frac{6.6 \text{ kg}}{3 \text{ mg}} = 19.8$ $\frac{6.6 \text{ kg}}{3 \text{ mg}} = 19.8$ | $\frac{14.6 \text{ lb}}{X \text{ kg}} = \frac{11 \text{ lb}}{2.2 \text{ kg}}$ $66.36 \text{ kg} \times 3 \text{ mg/kg} = 199 \text{ mg}$ $199/100 = 2 \text{ caps (rounded up)}$ |
| Question 5a | Nothing | Nothing |
| Question 5b | Nothing | 1166 ml/hr |
| Question 6 | 2-Step Equation | Cross Products |

| | Respondent 25 | Respondent 26 |
|-------------|---|---|
| Question 1 | $\frac{2mg}{1ml} = \frac{4mg}{x}$ $2x = 4$ $x = 2ml$ | $\frac{2mg}{1ml} \rightarrow \frac{4mg}{x} = \frac{4}{2} = 2ml$ |
| Question 2 | $\frac{5mg}{1ml} = \frac{2mg}{x}$ $\frac{5x}{5} = \frac{2}{5}$ $x = 0.4ml$ | $\frac{5mg}{1} \rightarrow \frac{2mg}{x} = 5 \div \frac{4}{5} = 0.4ml$ |
| Question 3 | $0.2mg = 200mcg$ $\frac{100mcg}{1tab} = \frac{200mcg}{x}$ $\frac{100x}{100} = \frac{200}{100}$ $x = 2 \text{ tabs}$ | $100mcg = 1mg = \frac{2}{1} \text{ tablets needed}$ $1 \cdot x(x) = 2mg$ $1 \cdot x \cdot x = 2mg$ |
| Question 4 | $\frac{146 \text{ lbs}}{2.2} = 66.4 \text{ kils}$ $66.4 \times 3 = 199.2 \text{ mg tablet}$ $\frac{199.2}{x} = \frac{100}{1}$ $\frac{100x}{100} = \frac{199.2}{100}$ $x = 1.992 \text{ or } 2 \text{ tabs}$ | <p>convert 146 lbs to kilograms, then multiply 1.5, then divide by 100 = answer = 2 capsules</p> |
| Question 5a | $\frac{15gtt}{1ml} \quad \frac{167ml}{60} = 2.7ml/min$ $2.7 \times 15 = 41.7 \text{ gtt/min}$ | $60 \text{ minutes} \times 3 = 180$ $\text{multiply } 500 \times 15 = 7500$ $7500 \div 180 = 41.6$ <p>either 41 or 42 gtt/min</p> |
| Question 5b | $\frac{500}{3} = 167 \text{ mL/hr}$ | $500 \div 3 = 166.6 =$ <p>round up to 167</p> |
| Question 6 | Equal Ratios | Nothing |

| | Respondent 27 | Respondent 28 |
|-------------|---|---|
| Question 1 | <p>1ml = 2mg → dose &</p> <p>How many mL will you draw up in the syringe?</p> <p>2ml</p> | $\frac{2mg}{1ml} \times \frac{4mg}{x} = 2ml$ |
| Question 2 | <p>DOSE STRENGTH: 5 mg/mL</p> <p>0.4ml = 2mg</p> <p>2/10 0.2 = 1mg 0.4 = 2</p> | $\frac{5mg}{1ml} = \frac{2mg}{xml} = 0.25ml$ $\frac{2.5mg}{0.5ml} = \frac{5mg}{1ml}$ |
| Question 3 | <p>0.2mg = 200mcg 100mg tablet = 100,000mcg</p> <p>How many tablets will you give?</p> <p>2 tablets</p> | <p>0.2mg = 200 mcg x 100</p> $\frac{100}{1} = \frac{200}{x} = 2 \text{ tablets}$ |
| Question 4 | <p>2.37500 150 80 66 740</p> <p>63.62g x 3 190.8 = 191 2 tablets 100mg tablet → give 2</p> | <p>3mg/kg x (60kg pt) = 180mg = 2cp</p> <p>$\frac{3mg}{kg} = \frac{180mg}{60kg}$</p> |
| Question 5a | <p>166 3/500 3 20 18 20</p> <p>166ml/hr x 3 = 498 60/166ml/hr = 27.7 12.8 460 420 700</p> <p>27.7ml/min</p> <p>27.7 x 1.5 138.5 27.70 41.55</p> <p>= 41.5 gtt/min</p> | <p>166 15 830 166 60 2490 4</p> <p>166 15 165 415 gtt/min</p> <p>60 2490 240 90 60 30 5</p> |
| Question 5b | <p>ml/hr = 166ml/hr</p> | <p>166ml/hr 3/500 3 20 16</p> <p>166ml 60min</p> |
| Question 6 | <p>Analogies</p> | <p>Nursing Rule</p> |

| | Respondent 29 | Respondent 30 |
|-------------|--|---|
| Question 1 | <p>Formula Method</p> $\frac{D}{H} \times Q = X$ | $\frac{4}{x} = \frac{2mg}{1mL} \quad \frac{2x}{2} = \frac{4}{2} = 2mL$ |
| Question 2 | $\frac{D}{H} \times Q = X$ | $\frac{5mg}{1mL} = \frac{2mg}{x} \quad - \frac{5x}{5} = \frac{2}{5} \quad 0.4mL$ |
| Question 3 | $\frac{D}{H} \times Q = X$ | $0.2mg \times 100 = 20mg \quad 200mg \times \frac{1}{2} = 1$ $\frac{20mg}{x} = \frac{100mg}{1tbl} \quad 2tbl$ |
| Question 4 | $22 \overline{)14615}^{kg}$ <p>3mg/kg</p> $\frac{D}{H} \times Q = X$ | $\frac{146}{2 \cdot 2} = 66.4kg$ $\frac{1992mg}{100mg} = 20cps$ $+ 3mg$ $199.2mg$ |
| Question 5a | <p>Convert 500ml/3hr</p> $500 \div 3 =$ | $\frac{166mL}{60min} \times \frac{65gtts}{1mL}$ $4 \overline{)166} \quad 41.5$ $42gtts/min$ |
| Question 5b | $\frac{mL/hr \times gtt/min}{Time} = X$ | <p>prescribed dose?</p> $\frac{500mL}{3hr} \quad 3 \overline{)500} \quad 166mL/hr$ |
| Question 6 | Nursing Rule | Nothing |

| | Respondent 31 | Respondent 32 |
|-------------|--|---|
| Question 1 | $\frac{2\text{mg}}{1\text{mL}} = \frac{4\text{mg}}{x}$ $2x = 4$ $x = 2\text{ mL.}$ | $\frac{4\text{mg}}{1} \times \frac{1\text{mL}}{2\text{mg}} = \frac{4}{2}\text{ mL} = (2\text{ mL})$ |
| Question 2 | $\frac{2\text{mg}}{5\text{mg}} \times 1\text{mL} = 0.4\text{ mL.}$ | $\frac{2\text{mg}}{1} \times \frac{1\text{mL}}{5\text{mg}} = \frac{2}{5}\text{ mL} = (0.4\text{ mL})$ |
| Question 3 | $0.2\text{mg} = 0.200 \rightarrow 200\text{mcg}$ $\frac{D}{X} \times Q$ $\frac{200\text{mcg}}{100\text{mcg}} \times 1\text{tab} = 2\text{ tabs}$ | $\frac{0.2\text{mg}}{1} \times \frac{1\text{tab}}{0.1\text{mg}} = \frac{0.2}{0.1}\text{ tab} = (2\text{ tabs})$ |
| Question 4 | $\frac{66.4\text{ Kg}}{x} \times \frac{3\text{mg}}{1\text{Kg}}$ $199.2 \rightarrow 200\text{ mg}$ <p>Kg "cancel" out</p> $\frac{D}{H} \times Q$ $\frac{200}{100} \times 1 = 2\text{ capsules}$ | <p>Amt Needed pt weight (kg) Amt Avail</p> $\frac{3\text{mg}}{1\text{kg}} \times \frac{66.38\text{kg}}{1} \times \frac{1\text{cap}}{100\text{mg}} = \frac{199.08}{100}\text{ cap}$ $1.99 \approx (2\text{ caps})$ |
| Question 5a | $\frac{167 \times 15}{604}$ $41.75 \text{ or } 42\text{ gtt/s}$ <p>minute</p> | $\frac{3\text{hrs}}{1} \cdot \frac{60\text{min}}{1\text{hr}} = 180\text{ min (in 3hrs)}$ $\frac{500\text{gtt}}{180\text{mins}} \cdot \frac{15\text{gtts}}{1\text{gtt}} = (41.6\text{ gtt/min})$ |
| Question 5b | $500\text{mL} \div 3 = 166.666 \text{ or } 167\text{ mL/hr.}$ | $\frac{500\text{mL}}{3\text{ hrs}} = (166.7\text{ mL/hr})$ |
| Question 6 | Nursing Rule | Dimensional Analysis |

| | Respondent 33 | Respondent 34 |
|-------------|--|--|
| Question 1 | $\frac{4mg}{2mg} \times ml = 2ml$ | $2mg/ml \times 2 = 4mg$ Draw up 2cc(ml) |
| Question 2 | $\frac{2mg}{5mg} \times 1ml = 0.4ml$ | $\frac{5mg}{10.} \times \frac{2mg}{x} = \frac{20}{5x} = .4ml$ |
| Question 3 | $0.2mg = 0.2 \times 100mcg$ $\frac{200mcg}{100mcg} \times 1 tab = 2 tabs$ | comes in mgg tablets - not sure how to calculate $0.2mg = 20mcg?$ |
| Question 4 | $146 \div 2.2 = 66.4kg$ round to 66kg $3 \times 66 = \frac{198mg}{100mg} \times 1 cap = 1.98 = 2 tabs$ capsules | need to convert wt to kg (cant remember conversion @ this time) then multiply 3mg x # of kg ex if 146 = 50 kg then $50 \times 3 = 150mg$ |
| Question 5a | $\frac{500 \cancel{g} \times 15 \cancel{g} / ml}{180 \cancel{min}}$ $41.6 gtt/min$ | Nothing |
| Question 5b | $\frac{500}{3} = 166.6 \frac{ml}{hr}$ $166.7 \frac{ml}{hr}$ | Nothing |
| Question 6 | Nothing | Cross Products |

| | Respondent 35 | Respondent 36 |
|-------------|---|--|
| Question 1 | $4\text{mg} \div 2\text{mg} = 2\text{mg/ml}$ | $4\text{mg} = 2\text{mg} + 2\text{mg} = 2\text{mg} = 2\text{cc}$ |
| Question 2 | $2\text{mg} \div 5\text{mg} = .4\text{ml}$ | $5\text{mg} \div 2\text{mg}$ |
| Question 3 | $100\text{mcg} = .1\text{mg}$ $2 \times 100\text{mcg} = .2\text{mg}$ give 2-tabs | $2\text{tabs} = 200\text{mcg}$ |
| Question 4 | $146\text{lb} = 66.3\text{kg}$ 66.3 $66 \times 3\text{mg} = 198\text{mg}$ give 2 capsules | $146\text{lb} = 2.2\text{lb} = \text{kg}$ 2 ? |
| Question 5a | Nothing | Nothing |
| Question 5b | $500 \div 3 = 166.6\text{ml/hr}$ | Nothing |
| Question 6 | Nursing Rule | Nothing |

| | Respondent 37 | Respondent 38 |
|-------------|--|---|
| Question 1 | $\frac{2 \text{mg}}{1 \text{mL}} = \frac{4 \text{mg}}{x \text{mL}}$ $\frac{2x}{2} = \frac{4}{2}$ $x = 2 \text{mL}$ | $\text{ml} = \frac{1 \text{ml}}{2 \text{mg}} \times 4 \text{mg} = 2 \text{ml}$ |
| Question 2 | $\frac{5 \text{mg}}{1 \text{mL}} = \frac{2 \text{mg}}{x \text{mL}}$ $\frac{5x}{5} = \frac{2}{5}$ $x = \frac{2}{5} \text{ (or } 4/10) \text{ mL}$ | $\text{ml} = \frac{1 \text{ml}}{5 \text{mg}} \times 2 \text{mg} = 0.4 \text{ml}$ |
| Question 3 | $\frac{.1 \text{mg}}{100 \text{mcg}} = \frac{.2 \text{mg}}{x}$ $.1x = 2000$ $x = 200 \text{ mcg or } 2 \text{ tabs}$ | $\frac{100 \text{mcg}}{1 \text{tab}} = \frac{200 \text{mcg}}{x}$ $\frac{100x}{100} = \frac{200}{100}$ $1 \text{ tab} \times \frac{1000 \text{ mcg}}{1 \text{mg}} \times 0.2 \text{ mg} = 2 \text{ tab}$ |
| Question 4 | $\frac{.45 \text{kg}}{160} = \frac{x \text{kg}}{146}$ $x = 65.7 \text{ kg}$ $\frac{65.7}{x 3} \text{ (3mg/kg)}$ 197.1 mg | $\frac{100 \text{mg}}{100 \text{mg}} \times \frac{3 \text{mg}}{\text{kg}} \times 66 \text{kg} = 2 \text{ cap}$ |
| Question 5a | $\frac{500}{x 15} = 7500 \text{ total gtt}$ $\frac{3 \text{hr.}}{x 60 \text{min/hr}} = 180 \text{ min}$ | $\frac{\text{gtts}}{\text{min}} = \frac{15 \text{gtts}}{\text{ml}} \times \frac{500 \text{ml}}{3 \text{hr}} \times \frac{1 \text{hr}}{60 \text{min}}$ $41.6 \frac{\text{gtts}}{\text{min}}$ |
| Question 5b | $\frac{166 \text{ mL}}{3 \times 1500} = 3 \frac{20}{20}$ | $\frac{\text{ml}}{\text{hr}} = \frac{500 \text{ml}}{3 \text{hr}} = 166.7 \frac{\text{ml}}{\text{hr}}$ |
| Question 6 | Cross Products | Dimensional Analysis |

| | Respondent 39 | Respondent 40 |
|-------------|--|--|
| Question 1 | $\frac{4 \text{ mg}}{x} \quad \frac{2 \text{ mg}}{1}$ | $\frac{1 \text{ ml}}{2 \text{ mg}} \times \frac{4 \text{ mg}}{1} = \frac{4 \text{ ml}}{2} = 2 \text{ ml q6hr}$ |
| Question 2 | $\frac{5 \text{ mg}}{1} \quad \frac{2 \text{ mg}}{x}$ | $\frac{1 \text{ ml}}{5 \text{ mg}} \times \frac{2 \text{ mg}}{1} = \frac{2 \text{ ml}}{5} = 0.4 \text{ ml q12hr}$ |
| Question 3 | $\frac{0.2 \text{ mg}}{x \text{ mg}} = \frac{1 \text{ mg}}{1000 \text{ mcg}}$ $\frac{200 \text{ mcg}}{x \text{ cap}} \quad \frac{100 \text{ mcg}}{1 \text{ Tab}}$ | $0.2 \text{ mg} \times \frac{1000 \text{ mcg}}{1 \text{ mg}} \times \frac{1 \text{ Tab}}{100 \text{ mcg}} = \frac{200}{100}$ $= 2 \text{ Tab}$ |
| Question 4 | $2 \sqrt[2]{146 \text{ LB}} \quad 66.4 \times 3 = 199.2$ $\frac{199.2}{x} \quad \frac{100}{1}$ | $\frac{1 \text{ Cap}}{100 \text{ mg}} \times \frac{3 \text{ mg}}{\text{kg}} \times \frac{1 \text{ kg}}{2.2 \text{ lb}} \times 146 \text{ lb}$ $\frac{438}{220} = 2 \text{ cap}$ |
| Question 5a | $3 \sqrt[3]{500} \quad \text{gtt/min} = \frac{\text{vol} \times \text{gtt}}{60 \text{ min}}$ $\frac{167 \times 15}{60}$ | $\frac{15 \text{ gtt}}{\text{ml}} \times \frac{50 \text{ ml}}{3 \text{ hr}} \times \frac{1 \text{ hr}}{60 \text{ min}} =$ $\frac{7500}{180} = 41.6$ $\approx 42 \text{ gtt/min}$ |
| Question 5b | $\frac{167 \times 15}{3 \sqrt[3]{500}} = 167$ | $\frac{\text{ml}}{\text{hr}} = \frac{\text{ml}}{15 \text{ gtt}} \times \frac{42 \text{ gtt}}{\text{min}} \times \frac{60 \text{ min}}{1 \text{ hr}}$ 168 ml/hr |
| Question 6 | Dimensional Analysis | Dimensional Analysis |

| | Respondent 41 | Respondent 42 |
|-------------|---|--|
| Question 1 | $\frac{2 \text{ mg}}{1 \text{ mL}}$ $\frac{2 \text{ mg}}{1 \text{ mL}}$ $\frac{4 \text{ mg}}{2 \text{ mL}}$ <p>draw up 2 mL in the syringe</p> | 2 mL |
| Question 2 | $5 \text{ mg} \overline{) 2 \text{ mg} \cdot 0} = 4 \text{ mL}$ | 2.5 mL |
| Question 3 | $0.2 \text{ mg} \times 1000 = 200 \text{ mcg}$ $100 \text{ mcg / tab} \overline{) 200 \text{ mcg}} = 2 \text{ tab}$ | <u>2 tabs</u> |
| Question 4 | $\frac{66.3 \text{ kg}}{\times 3 \text{ mg}} = 200 \text{ mg}$ $100 \overline{) 200} = 2$ <p>give 2 capsules</p> | $146 \div 2.2 = 66.4 \text{ kg} \times 3 = 199.2 \text{ mg}$ <p>2 capsules</p> |
| Question 5a | $500 \times 15 = 7500 \text{ gts}$ $3 \overline{) 7500} = 1667 \text{ mL/hr}$ $60 \overline{) 1667} = 27.8 \text{ mL/min}$ $15 \text{ gts/min} \times 27.8 \text{ mL/min} = 41.7 \text{ gts/min}$ | 41 gts/min |
| Question 5b | $3 \overline{) 500 \text{ mL}} = 166.7 \text{ mL/h}$ | $3 \overline{) 500} = 166.6 \text{ mL/h}$ |
| Question 6 | Cross Products | Nursing Rule |

| | Respondent 43 | Respondent 44 |
|-------------|--|--|
| Question 1 | 2 mL | $4 \text{ mg} \times \frac{1 \text{ mL}}{2 \text{ mg}} = 2 \text{ mL}$ |
| Question 2 | $\frac{5 \text{ mg}}{1 \text{ mL}} \times \frac{2 \text{ mg}}{5}$ $\frac{5 \times 2}{5} = 2$ $\times 2 = 4 \text{ mL}$ | $2 \text{ mg} \times \frac{1 \text{ mL}}{5 \text{ mg}} = \text{--- mL}$ |
| Question 3 | 100mcg = 0.1mg 2 tablets | $2 \text{ mg} \times \frac{100 \text{ mcg}}{\text{mg}} = 200 \text{ mcg} \times \frac{1 \text{ tab}}{100 \text{ mcg}} = 2 \text{ tab}$ |
| Question 4 | $\frac{3 \text{ mg}}{\text{kg}} \times 66.26 \text{ kg} = 198.78$ $\frac{100 \text{ lbs} \times 1 \text{ kg}}{2.2 \text{ lb}} = 66.86 \text{ kg}$ <u>2 capsules</u> | $4 \text{ tabs } 1 \text{ kg} \times \frac{146 \text{ lb}}{2.2 \text{ lb}} =$ $\frac{3 \text{ mg}}{1 \text{ kg}} \times \frac{146 \text{ lb}}{2.2 \text{ lb}} = 198.9 \approx 200 \text{ mg}$ $200 \text{ mg} \times \frac{1 \text{ tab}}{100 \text{ mg}} = 2 \text{ caps}$ |
| Question 5a | $\frac{15 \text{ gtt}}{1 \text{ mL}} \times \frac{500 \text{ mL}}{3 \text{ hours}} \times \frac{1 \text{ hour}}{60 \text{ minutes}}$ <u>41.67 gtt/min</u> | $500 \text{ mL} \times \frac{15 \text{ gtt}}{1 \text{ mL}} \times \frac{1 \text{ gtt}}{60 \text{ min}} = \text{--- gtt/min}$ |
| Question 5b | $\frac{500 \text{ mL}}{3 \text{ hours}}$ <u>167 mL/hr</u> | $\frac{15 \text{ gtt}}{1 \text{ mL}} \times \frac{500 \text{ mL}}{\text{min}} \times \frac{1 \text{ gtt}}{\text{min}} \times \frac{60 \text{ min}}{1 \text{ hr}}$ $\frac{1 \text{ gtt}}{15 \text{ gtt}} \times 500 \text{ mL} \times \frac{1 \text{ gtt}}{\text{min}} \times \frac{60 \text{ min}}{1 \text{ hr}} = \frac{\text{mL}}{\text{hr}}$ |
| Question 6 | Cross Products | Dimensional Analysis |

APPENDIX J
CODING OF DCPD SURVEY DATA

| Research ID | 1 | 2 | 3 | 4 | 5 | 6 | Predom | sign |
|-------------|---|---|---|---|---|----------------------|--------|------|
| 1 | 1 | 1 | 0 | 1 | 1 | Equal ratios | 1 | 0 |
| 2 | 2 | 1 | 2 | 1 | 1 | Analogies | 1 | 0 |
| 3 | 2 | 2 | 2 | 1 | 1 | Table | 2 | 1 |
| 4 | 3 | 3 | 3 | 3 | 1 | Nursing rule | 3 | 0 |
| 5 | 5 | 5 | 1 | 1 | 1 | Equal ratios | 1 | 0 |
| 6 | 3 | 3 | 3 | 1 | 2 | Nursing rule | 3 | 1 |
| 7 | 4 | 4 | 1 | 1 | 3 | Analogy | 7 | 1 |
| 8 | 3 | 3 | 3 | 3 | 3 | Nursing rule | 3 | 0 |
| 9 | 2 | 2 | 2 | 2 | 2 | Cross products | 2 | 1 |
| 10 | 1 | 3 | 0 | 1 | 3 | Nursing rule | 7 | 0 |
| 11 | 5 | 5 | 5 | 5 | 5 | Cross Products | 5 | 1 |
| 12 | 3 | 3 | 6 | 3 | 6 | Nursing rule | 3 | 1 |
| 13 | 3 | 3 | 3 | 1 | 3 | Nursing rule | 3 | 1 |
| 14 | 6 | 6 | 6 | 6 | 6 | Unit rate | 6 | 1 |
| 15 | 6 | 6 | 6 | 6 | 6 | Dimensional Analysis | 6 | 1 |
| 16 | 1 | 1 | 1 | 1 | 1 | Nursing rule | 1 | 0 |
| 17 | 5 | 5 | 6 | 6 | 6 | 2-step Equations | 6 | 1 |
| 18 | 2 | 1 | 1 | 1 | 0 | Analogies | 1 | 0 |
| 19 | 5 | 5 | 3 | 3 | 6 | No response | 7 | 0 |
| 20 | 1 | 1 | 1 | 1 | 1 | No response | 1 | 1 |
| 21 | 2 | 2 | 2 | 2 | 2 | Analogies | 2 | 1 |
| 22 | 3 | 3 | 3 | 3 | 3 | Nursing Rule | 3 | 1 |
| 23 | 1 | 1 | 1 | 1 | 0 | 2-step equation | 1 | 0 |
| 24 | 5 | 5 | 3 | 5 | 0 | Cross products | 5 | 0 |
| 25 | 5 | 5 | 5 | 5 | 1 | Equal ratios | 5 | 0 |
| 26 | 5 | 5 | 1 | 1 | 1 | No response | 1 | 0 |
| 27 | 2 | 2 | 2 | 1 | 1 | Analogies | 2 | 1 |
| 28 | 5 | 5 | 5 | 1 | 1 | Nursing Rule | 5 | 1 |
| 29 | 3 | 3 | 3 | 3 | 3 | Nursing Rule | 3 | 0 |
| 30 | 5 | 5 | 5 | 3 | 6 | No response | 5 | 1 |
| 31 | 5 | 3 | 3 | 3 | 3 | Nursing rule | 3 | 1 |
| 32 | 6 | 6 | 6 | 6 | 6 | Dimensional analysis | 6 | 0 |
| 33 | 3 | 3 | 3 | 3 | 3 | No response | 3 | 0 |
| 34 | 1 | 5 | 1 | 1 | 0 | Cross products | 1 | 0 |
| 35 | 1 | 1 | 2 | 1 | 1 | Nursing rule | 1 | 0 |
| 36 | 1 | 1 | 1 | 1 | 1 | No response | 1 | 0 |
| 37 | 5 | 5 | 5 | 5 | 1 | Cross products | 5 | 1 |
| 38 | 6 | 6 | 6 | 6 | 6 | Dimensional analysis | 6 | 1 |
| 39 | 5 | 5 | 5 | 5 | 3 | Dimensional Analysis | 5 | 1 |
| 40 | 6 | 6 | 6 | 6 | 6 | Dimensional Analysis | 6 | 1 |
| 41 | 1 | 1 | 1 | 1 | 1 | Cross Products | 1 | 0 |
| 42 | 0 | 0 | 0 | 1 | 0 | Nursing Rule | 0 | 0 |
| 43 | 0 | 5 | 1 | 6 | 6 | Cross Products | 7 | 0 |
| 44 | 6 | 6 | 6 | 6 | 6 | Dimensional Analysis | 6 | 0 |

| Set-up | Code Number | Set-up | Code Number |
|----------------------|-------------|-------------------------|-------------|
| no work | 0 | Analogies | 4 |
| not identifiable | 1 | Equal Ratios | 5 |
| Equality of Measures | 2 | Dimensional Analysis | 6 |
| Nursing Rule | 3 | no predominate strategy | 7 |

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