Housing Prices and Multiple Employment Nodes: Is the Relationship Nonmonotonic?

Liv Osland & Gwilym Pryce

Routledge
Taylor & Francis Group

Housing Prices and Multiple Employment Nodes: Is the Relationship Nonmonotonic?

LIV OSLAND* & GWILYM PRYCE**

*Stord/Haugesund University College, Haugesund, Norway, **Department of Urban Studies, School of Social and Political Sciences, University of Glasgow, Glasgow, Scotland

ABSTRACT *Standard urban economic theory predicts that house prices will decline with distance from the central business district. Empirical results have been equivocal, however. Disjoints between theory and empirics may be due to a nonmonotonic relationship between house prices and access to employment arising from the negative externalities associated with proximity to multiple centres of employment. Based on data from Glasgow (Scotland), we use gravity-based measures of accessibility estimated using a flexible functional form that allows for nonmonotonicity. The results are thoroughly tested using recent advances in spatial econometrics. We find compelling evidence of a nonmonotonic effect in the accessibility measure and discuss the implications for planning and housing policy.*

KEY WORDS: House prices, rent gradients, employment access, externalities, gravity models, monotonic relationships

1. Introduction

This paper analyses the impact of employment nodes on surrounding house values. It is an important subject with implications that go to the heart of housing and urban policy, the effect of urban structure on human welfare, the role of planning versus laissez faire, the spatial distribution of income and the impact of inequality.

   Consider the direct implications for human welfare. If we control for house type and size, variation across space in house prices will reflect differences in the quality of life. People are willing to pay more to live in an area with good access to employment, environmental quality and security. If the impact on surrounding house prices of an employment centre is always positive, then the implication is that welfare is raised unambiguously whenever and wherever firms choose to locate. The role of planners and policy makers is minimal in such a world—the overriding imperative is to do nothing that

Correspondence Address: Liv Osland, Stord/Haugesund University College, Bjørnsonsgt 45, N-5528 Haugesund, Norway. Fax: +47 52702601; Tel.: +47 52702768; Email: liv.osland@hsh.no

will inhibit firms locating. Governments may seek to attract firms to the city but have no cause to dictate where those firms locate once they arrive (other than, perhaps, as part of a strategy to coordinate transport provision).

Even if employment location has a negative impact on human welfare—due to the congestion, noise and/or air pollution associated with commercial activity—provided these negative effects are offset by the benefits of reduced commuting time, the need for intervention remains negligible. Employment centres will still produce a non-negative welfare impact overall for surrounding residents. So there will be no compelling reason for governments to inhibit or 'zone' employment location.

Consider, however, the prospect of the negative effects of employment location outweighing the benefits of access, at least for homes in close proximity. Under this scenario, an imperative exists to minimise the number of households affected by negative externalities, which is likely to involve policy intervention that concentrates employment in particular locations so as to minimise the exposure of residents to the negative effects of production. Other things being equal, the optimal internal structure of cities will therefore look very different when the negative impacts of employment centres outweigh the positive effects of access to work. In such a world, the case for land planning becomes compelling unless we can be confident that market forces (such as those implied by agglomeration economies—e.g. Fujita & Krugman, 2004) will cause all firms to group into central locations.

Unfortunately, economic theory suggests that such an outcome is unlikely to occur without intervention. For example, Lucas & Rossi-Hansberg (LRH, 2002) show that, even when agglomeration economies are present, unfettered market forces will produce areas of mixed land use (firms and houses located in close proximity). Firm location that is suboptimal for society arises because 'there is no reason to believe that market prices— land rents and location-specific wage rates—give firms and households the right incentives for making land use decisions' (LRH, 2002, p. 1445). In the LRH model, the case for land planning arises largely from the fact that 'a firm deciding to locate has no incentive to take its [positive agglomeration] effect on other producers into account' (LRH, 2002, p. 1471). Presumably, this conclusion is reinforced considerably if firms also have no incentive to take into account the negative externality effects of their location on surrounding households.

Note that whether house prices fall monotonically or nonmonotonically with distance to employment is not the only important aspect of the relationship. The particular *shape* of the house price-distance function is also crucial. For example, employment may reduce prices of houses in close proximity, but boost prices at some intermediate distance. (Think of a water bed—depression at one point only leads to expansion elsewhere.) And if the rise in welfare at medium distance outweighs the depression at short distance then planners will be faced with the dilemma of whether to permit firm locations that will make some households better off while making others worse off (i.e. satisfy a Benthamite rule of maximising overall happiness but violate the Pareto criterion). There is an imperative, therefore, to quantify the way in which house prices change with distance to employment, holding other factors constant. Empirical estimation is frustrated considerably, however (as we shall discuss in Section 3), by the existence of multiple employment nodes of varying size.

Even if overall welfare is raised when an employment centre emerges, society will be concerned to know which households will lose out as there may be implications for

inequality. For example, firms may be drawn to low land prices, which are likely to coincide with the location of poor households. And even if the initial location decisions of firms were random, it is possible that poorer households will be sorted into living in close proximity to employment centres because of the reduction in house prices. In other words, there may be a systematic tendencies in a *laissez faire* urban economy for low income households to bear the brunt of negative employment externalities.

The sorting effect generated by the house price gradient with respect to distance to employment may have further implications of interest to planners and policy makers. For example, if employment location helps shape the geography of house prices, which in turn affects the location of low and high income households across space, then this will affect the optimal placement of political and administrative boundaries. There will be important corollaries, for example, for the tax-raising potential of different jurisdictions depending on how administrative boundaries interact with the contours of employment density. Notwithstanding the feedback effects from the location of high income households on the location of employment and the geography of house prices, the urban distribution of incomes as a result of employment location will have multi-faceted implications for human wellbeing (see the Glaeser *et al.*, 2009, summary of links between spatial inequality and various social ills, such as crime).

The effect of employment location on house prices and, in particular, whether or not the relationship is monotonic, therefore warrants careful empirical examination. We begin, in Section 2, by setting this research goal in the context of existing work, and we present the negative externality explanation of nonmonotonicity as a possible solution to an apparent conflict in the literature between theory and evidence. In Section 3 we clarify the methodological challenges associated with estimating house price gradients when employment is scattered across the urban landscape in varying density, and set out our proposed solutions. We describe our data in Section 4 and discuss spatial econometric issues and results in Section 5. We offer a brief summary and conclusion in Section 6.

2. Literature Review

The standard urban economic model (Alonso, 1964; Muth, 1969) predicts that house prices will decline with distance to the central business district (CBD). This finding mimics that of von Thünen's model of farmland; and like the von Thünen's result, it rests crucially on the assumption that total transport costs rise with distance to the CBD. The prevalence of multiple centres of employment within a city or region (Wheaton, 2004) led to the standard urban economic model being extended to incorporate polycentric employment (Papageorgiou & Casetti, 1971; Yinger, 1992). However, these multi-nodal models only confirm the basic prediction of Alonso and Muth—that the house price gradient will be negatively related to distance from an employment node. Papageorgiou & Casetti (1971), for example, find that equilibrium residential land price is at its highest in the largest employment district, and that local maxima in land prices will occur at the local employment centres. Moreover, the land price at local employment centres will decline as distance between centres declines, and equilibrium land price falls with distance to employment node.

Although neoclassical theory predicts a negative relationship between constant quality house prices and access to employment in both monocentric and polycentric urban economies, the empirical evidence has been ambiguous. Econometric studies of the house

price gradient have found it to be negative, zero or even positive. Early empirical studies tended to confirm the negative house price gradient proposition, but subsequent evidence painted a more complex picture (see review by Bartik & Smith, 1987). Heikkila *et al.* (1989), for example, found that coefficients on distance to CBDs were positive and not significant. They concluded that workplace accessibility had been overemphasised in the urban economics literature. Clapp *et al.* (2001) found that, after controlling for other factors, land values were

> negatively influenced by proximity to employment centres. Moreover, the remaining effect of distance on land values and work at home in 1990 is positive over the most distant part of our study area. This is opposite of effect of commuting in [standard urban economic] theory (p. 59).

A number of explanations for the empirical paradox have been proposed, most notably: (1) the modern city may have developed into a form where the role of distance is less important or more complex (Clapp *et al.*, 2001; McMillen, 2003; Richardson *et al.*, 1990); (2) errors in the measurement of access to work may have led to bias in empirical results; (3) submarket effects may distort the house price gradient and therefore need to be controlled for (Adair *et al.*, 2000); (4) there may exist negative externalities in close proximity to employment centres (McDonald & McMillen, 2007). The first three explanations, as we discuss below, do not provide a satisfactory account of the conflict between theory and evidence. The fourth hypothesis is promising but has not been investigated in sufficient depth to know whether the externality effect is sufficient to cause nonmonotonicity in the house price gradient.

Consider the first of these four theories. It purports to explain the contradictory empirical results as arising from fundamental changes over time in the role of distance—the negative house price gradient is simply a particular phase in urban development, not the immutable archetype to which all cities converge. Clapp *et al.* (2001) find significant changes to the land rent gradient over time. Similarly, Richardson *et al.* (1990) suggest the effect of distance to CBD may have declined—the distance coefficient for Los Angeles CBD was negative in regressions based on 1970 data, but not significantly different from zero when run on house price data from 1980 (Richardson *et al.*, 1990; see also McMillen, 2003).

This explanation is unlikely to fully account for the variety of empirical results, however. Other studies have continued to find significant negative effects of distance to CBD (Osland *et al.*, 2007, for example) and of distance to suburban employment centres (see the review by Yiu & Tam, 2004). There is also no coherent theoretical explanation for why distance to employment (as opposed to distance to CBD) has become less important. For example, Glaeser & Kohlase (2004) conclude that, whilst transportation costs for goods and raw materials have declined significantly, and hence 'should play an increasingly irrelevant role in the urban economy' (Glaeser & Kohlase, 2004, p. 209), the costs of moving people 'are certainly not disappearing, [and] should continue to be a dominating presence in the structure of urban form' (Glaeser & Kohlase, 2004, p. 209). Therefore, if multiple centres of employment are appropriately incorporated into econometric models, we should still expect the access-to-work house price gradient to be always and everywhere negative. However, this is not the case (as our own results, presented below, corroborate).

A second explanation for the empirical ambiguity is that the accessibility variable has been incorrectly specified. One issue here is whether simple distance to CBD is an adequate proxy for accessibility. Commuting time and costs may not always correspond to Euclidean distance. However, there are reasons to believe that linear distance may be a surprisingly good approximation in large samples, implying that journey times are unlikely to explain the disjoint between the theory and evidence of house price gradients. First, there is an empirical evidence to suggest that generalised transport costs and Euclidean distances are, in fact, highly correlated. Combes & Lafourcade (2005) find that the two have a correlation coefficient $= 0.97$. Adair *et al.* (2000) find accessibility in most submarkets to be statistically insignificant despite going to considerable lengths to define a precise measure of accessibility. Second, there are theoretical grounds for believing that there are countervailing biases implied by using Euclidean distance as a measure of accessibility which are likely to cancel each other out. For example, whilst one might expect journey distances to exceed Euclidean distances in low density areas because of fewer roads, the effect will be compensated for by less congestion in those areas (Duranton & Overman, 2005, p. 1083).

A third explanation argues that submarket effects could lead to particular clusters of households having different access–space trade-offs. This explanation is also problematic. It is difficult to demonstrate that apparent submarket effects are truly at odds with the predictions of the standard urban economic model. For example, Adair *et al.* (2000) explore whether there is variation in the importance of accessibility across different submarkets in Belfast, but do not make clear how such effects can be disentangled from the consequences of polycentricity or from the standard finding that bid-rent function of the household will vary with household characteristics (Beckman, 1973; Fujita, 1989). Also, submarket boundaries are not exogenous to the urban system—they are driven, for example, by variations in school performance and social deprivation. After controlling for these factors we would still expect empirical models to yield negative house price gradients.

A fourth solution, proposed by Richardson (1977), argues that there is a missing element in the utility function of the standard urban economic model. Richardson (1977) showed that 'a positive rent gradient is feasible as a result of introducing an externality component in the determination of urban rent' (p. 62). That employment centres may be associated with significant negative externalities is not a controversial assumption—most zoning policies are premised on it (Pogodzinski & Sass, 1991). Residents located close to these centres trade-off rising transport costs against the falling externality effects of proximity to industrial or commercial activities and buildings. If the externality effect is large, the house price gradient would become nonmonotonic (McDonald & McMillen, 2007), which has significant implications for land planning and housing policy, as intimated in the introduction.

This proposition accords with the Li & Brown (1980) characterisation of 'micro-neighbourhood externalities', which they estimate through a series of distance to physical and commercial features. Unfortunately, interpretation of their results is frustrated by: (1) the fact that the size of employment centres is not measured and (2) only distance to nearest feature is computed—the effect of access to any other employment node is overlooked. This second point has the potential to introduce considerable bias. First, many households have more than one worker so the household location choice may be influenced by the need for access to a range of employment locations. Second, many

workers are required to work between two or more sites (Yiu & Tam, 2007) or visit clients scattered across many employment nodes. Third,

> the individual value of a given home and the choice of commute length are based not only on the current job site, but also on the expectation of where future jobs will be and the likelihood of both job separations and residential moves (Crane 1996, p. 342).

The transaction costs of moving home make the influence of uncertain job location, and the need to account for the effect of multiple employment nodes, all the more potent.

Although gravity-based access measures have been estimated in relation to housing prices [notably by Osland & Thorsen (O&T), 2008], we are not aware of any that consider the possibility of nonmonotonicity. Whilst the disamenity of proximity to industrial production in a traditional *monocentric city* might only affect a limited number of inner-city households located within a stone throw of the CBD, in a *polycentric city*, a very high proportion of households might live close to one or more employment centres. Our contention, therefore, is that, when combined with polycentricity, nonmonotonic distance effects could have profound effects on house price gradients. And as we argued in the introduction, house price gradients are important because they affect where households from particular income brackets choose to locate. This in turn affects likely revenue and expenditure streams from property taxes. Moreover, nonmonotonicity has important implications for the role of planners because it implies that firm location does not have an unambiguous positive effect on neighbouring residents and, as a result, the market cannot be left to allocate land in a socially optimal way.

There is an imperative, therefore, both in terms of the need to reconcile the theory and empirics of urban economics, and in terms of the implications for urban planning, to permit nonmonotonic distance effects when estimating the relationship between employment and house prices. It is also important for this nonmonotonic function to be modelled in a way that accounts for multiple employment nodes because this can multiply the number of homes affected.

3. Econometric Methodology

Whilst the direct effect of distance is not included in the utility functions of the early Alonso–Muth monocentric models, it is not a new assumption. Papageorgiou & Casetti (1971), for example, included distance to multiple centres in their objective function on the basis that the household purchases goods and services from these centres and so proximity is desirable. Crucially, however, the marginal utility of distance was assumed to be strictly negative. The possibility that the effect of distance to the employment centre (or weighted distances to multiple employment centres) will have a nonmonotonic effect on household utility opens the possibility that this effect will be represented in the valuation of constant quality house prices (or land in residential use). By removing the restriction that utility is strictly decreasing with distance, one allows for the potential negative externality effects of employment nodes as well as the positive effect of access to work.

Whilst there have been empirical studies that have looked at the externality effect of employment location in relation to house prices (Pogodzinski & Sass, 1991) they have tended among other things to overlook important spatial econometric issues and/or neglect

to account for polycentric employment structures. This gap in the literature is partly due to the methodological difficulties associated with developing a functional form that is both sufficiently general to allow for the existence of nonmonotonicity, and sufficiently parsimonious to be estimated within an econometric framework of decentralised employment. Given that it is only relatively recently that gravity-based accessibility measures have been used to capture decentralised employment location effects on house prices (O&T, 2008), it is not surprising that the possibility of nonmonotonicity in the employment–access/house–price relationship has yet to be explored. We now explain how we attempt to address this omission.

We aim to estimate a regression model that relates the price of homogenous housing at a given location i to the gravity-based access variable (Hansen, 1959), S_i, where $S_i = \sum_j L_j^\gamma v_{ij}^\theta \exp[\sigma v_{ij}]$ and, $j = 1, 2, \ldots, n$ (index variable for employment zones); $v_{ij} =$ set of distances from location i to employment zone j; $L_j =$ number of workers employed at location j; $\gamma, \theta, \sigma =$ parameters to be estimated.

S_i, thus formulated, can account for a wide variety of functional forms with respect to the effect of distance to j, whilst also permitting variation in the size of employment centre. Negative externality effects, if they exist, can be captured because the formula allows S to have a nonmonotonic relationship with distance, which will then translate into a nonmonotonic impact on house prices when included in hedonic regression model. The stock of houses is not homogenous, so we control for house variation by including a range of dwelling characteristics. Our empirical model is, therefore:

$$\ln(P) = a_0 + a_1 \sum_j L_j^\gamma v_{ij}^\theta \exp[\sigma v_{ij}] + a_2 \text{CBD} + a_3 \text{Seas\_d} + a_4 D + a_5 \text{Subm\_d}$$

$$+ a_6 \text{SPerf} + \mathbf{b} \cdot \mathbf{A} + \varepsilon, \tag{1}$$

where $P =$ observed selling price at location i, $\mathbf{A}$ is a vector of attributes of dwelling at location i, and CBD is the distance to the central business district. CBD is included to test whether there are any effects of proximity to CBD other than distance to employment (Brueckner *et al.*, 1999; O&T, 2008). To adjust for seasonal effects on sale price, seasonal dummies Seas_d are included (whether the house was sold in spring, summer, autumn or winter).

D denotes *deprivation score*. This variable represents externality effects associated with deprivation. Including this variable is potentially problematic, however, because poor people will inevitably live in lower house price areas (due to budget constraints), so deprivation may be caused by low prices, rather than causing low house prices. For this reason, we estimate the model with and without the deprivation score and other neighbourhood characteristics. Subm_d denotes the inclusion of submarket dummies based on realtor jurisdictions to account for shifts in the house price surface not captured by the other variables. SPerf denotes school performance, and has been shown to be of importance in the housing submarket literature (see for instance Goodman & Thibodeau, 1998). The variables are further commented upon in Section 4.

The estimation problem, then, is to estimate the slope parameters along with the access parameters γ, θ and σ, which we achieve using maximum likelihood (ML) methods. If one assumes monotonic distance effects on the house price gradient, then this is equivalent to imposing the restriction $\theta = 0$, and the model reduces to $S_i = \sum_j L_j^\gamma \exp[\sigma v_{ij}]$, which is similar to the O&T (2008) regression model. The two panels of Figure 1 show what
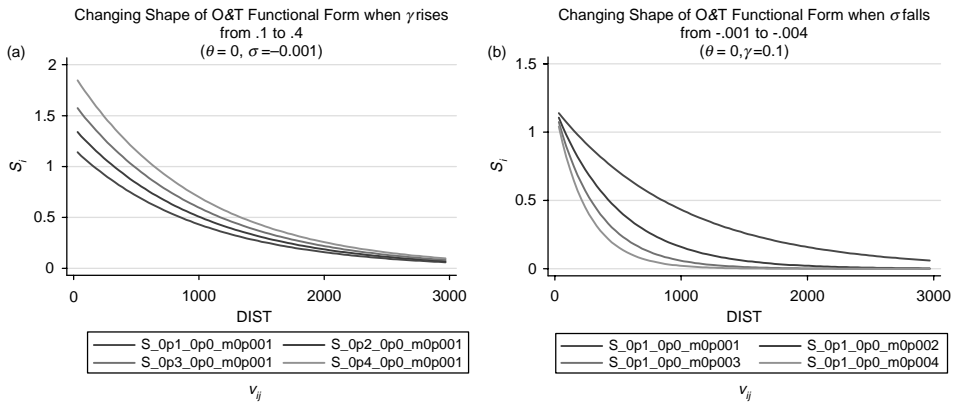
(a) Changing Shape of O&T Functional Form when γ rises from .1 to .4 ($\theta = 0$, $\sigma = -0.001$)

(b) Changing Shape of O&T Functional Form when σ falls from -.001 to -.004 ($\theta = 0$, $\gamma = 0.1$)

Figure 1. Functional form scenarios for the effect of distance to employment plotted for $\theta = 0$. *Notes*: The graphs in Figure 1 illustrate the monotonic nature of the O&T (2008) functional form for S_i, the gravity-based employment access variable. Hypothetical values of S_i are plotted on the vertical axis in both graphs. S_i is computed as follows: $S_i = \sum_j L_j^{\gamma} v_{ij}^{\theta} \exp[\sigma v_{ij}]$, where L_j = number of workers employed at location j, v_{ij} = the distance between residential location i and employment node j. In the O&T formulation, θ is held constant at zero. The graphs show how, under the O&T restriction, S_i varies monotonically with distance between i and j for different values of γ and σ. In graph (a), σ is held constant at -0.001; as γ rises from 0.1 to 0.4, the relationship between S_i and distance becomes steeper. In graph (b), γ is held constant at 0.1; as σ falls from -0.001 to -0.004 the relationship between S_i and distance becomes increasingly convex to the origin, but remains monotonic.

happens to the O&T functional form when we change the values of γ and σ, but hold θ constant at zero.

Figure 2 illustrates the variety of functional forms that can occur when θ is allowed to vary. The graphs illustrate the flexible nature of the functional form used in the econometric specification: how it allows both monotonicity and nonmonotonicity to occur in a distance-weighted multi-nodal setting. Panel (a) indicates a very steep monotonic distance decay effect ($\gamma = 1$, $\theta = 0.1$, $\sigma = -0.1$) which would be observed if there were no negative externalities associated with proximity to employment centres, or if the externality was dominated by transport costs at all distances. Panel (b) represents very potent but highly localised negative externality effects of employment centres ($\gamma = 1$, $\theta = 1$, $\sigma = -0.05$). Panel (c) illustrates less potent but also highly localised negative externality effects ($\gamma = 1$, $\theta = 0.1$, $\sigma = -0.005$). Panel (d) represents the scenario where negative amenity effects persist well beyond the immediate vicinity of the employment centre ($\gamma = 1$, $\theta = 2$, $\sigma = -0.005$). When S is included in a regression model of the type described by Equation (1), the range of possible functional forms is extended further by the size and magnitude of the slope coefficient a_1. For example, if $a_1 = -1$, then each of the graphs above would be vertically inverted.

To ascertain the extent of nonmonotonicity and the sensitivity of the results to model specification, we specify five different models. First, the ROT regression [representing the O&T (2008), regression] assumes that distance to employment has a monotonic effect on house prices ($\theta = 0$ in the specification of the adjusted access variable S). We include this model as a comparator as it represents the existing estimation technology in the literature (i.e. a monotonic gravity model). Second, we estimate regression R1 which
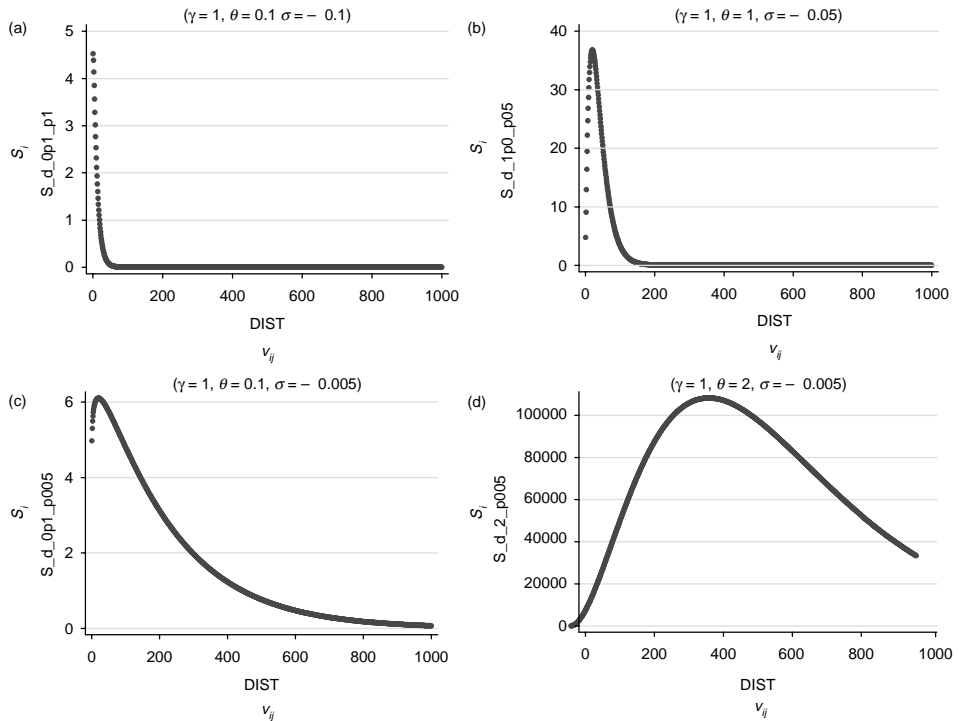
Figure 2. Functional form scenarios for the effect of distance to employment plotted for $\gamma = 1$. *Notes*: The graphs in Figure 2 illustrate the flexible nature of the functional form used in the econometric specification: how it allows both monotonicity and nonmonotonicity to occur in a distance-weighted multi-nodal setting. The graphs plot hypothetical values of S_i, the gravity-based access variable for location i, where $S_i = \sum_j L_j^\gamma v_{ij}^\theta \exp[\sigma v_{ij}]$, and L_j = number of workers employed at location j, v_{ij} = the distance between residential location i and employment node j. The graphs show how S_i varies with distance between i and j for different values of θ and σ, holding γ constant at one.

is the same as ROT except that it allows (but does not require) distance to employment to have a nonmonotonic effect (θ is permitted to vary from zero in the specification of S). Model formulation R2 excludes the CBD-variable on the basis that the traditional CBD effects of the standard urban economic model should be fully captured by the gravity variable. If CBD is included it is on the basis that there are additional 'urban attraction' effects—i.e. other than access to employment (such as the location of cultural and leisure amenities in the centre). R3 excludes the neighbourhood variables, and R4 excludes both the neighbourhood variables and the distance to CBD-variable. (These last two models are presented to illustrate the robustness of our results to exclusion/inclusion of spatial variables like distance to CBD and submarket boundaries which may be particularly likely to affect the estimates of employment access effects because of multicolinearity.)

3.1 Spatial Econometric Issues

Spatial effects (Anselin, 1988) are to be expected in cross-sectional real estate data. Such effects need to be tested and controlled for because spatial dependence in the price variable

will result in estimated parameters being biased when using the traditional ordinary least squares (OLS) estimator. Spatial autocorrelation in the residuals of an OLS regression model will give less reliable inferences, and less precision in relation to the estimated parameters (Anselin, 1988). In the econometric analysis, we follow Florax *et al.* (2003) and apply the *classical* modelling strategy, starting with relatively parsimonious model formulations and continue with more comprehensive specifications based on the results of the relevant tests. Spatial econometrics tests and estimates were obtained using the spdep (spatial dependence) package, found in the R statistical programming environment.[1]

In order to test the OLS models for spatial effects, we specify spatial weights matrices by using the *k*-nearest symmetric neighbourhood approach where distances between neighbours are allowed to vary. For $k = 1$ (for example), each observation will have at least one neighbour (see Anselin & Lozano-Gracia, 2008). As is common in spatial econometrics, row-standardized weights are used. For all estimations, we have used a sparse matrix approach (see Bivand *et al.*, 2008, p. 284).

A Moran's *I* test showed that the null hypothesis of no spatial autocorrelation could not be rejected. We then estimated the Lagrange multiplier (LM)lag statistic to test the null hypothesis of no spatial lag dependence, followed by the LM error statistic to test the null hypothesis of no significant spatial error autocorrelation. If the LM error statistics (and Robust Lagrange multiplier (RLM) error test statistics) are the largest, this indicates that spatial autocorrelation in the residual is the dominating problem (which did, in fact, turn out to be the case in our data—see Section 6) and the appropriate spatial model to use would be the spatial autoregressive error model (Florax & De Graaf, 2004):

$$\ln(P) = a_0 + a_1 \sum_j L_j^\gamma v_{ij}^\theta \exp[\sigma v_{ij}] + a_2 \text{CBD} + a_3 \text{Seas\_d} + a_4 D + a_5 \text{Subm\_d}$$

$$+ a_6 \text{SPerf} + \mathbf{b} \cdot \mathbf{A} + \varepsilon, \tag{2}$$

$$\varepsilon = \lambda W \varepsilon + u, \tag{3}$$

where W is the weights matrix and the spatial autoregressive parameter, λ is estimated jointly with the regression coefficients. Parameters inside the nonlinear adjusted accessibility variable, $S_i = \sum_j L_j^\gamma v_{ij}^\theta \exp[\sigma v_{ij}]$, are imputed values from the original ML grid-search estimation, as will be explained below. If $\lambda = 0$ the model becomes a traditional least squares regression model. ε is a $n \times 1$ vector of spatial autoregressive error terms and $u \sim N(0, \sigma^2 I)$. In a semi-logarithmic model of this kind, the coefficients in the spatial error and the OLS models are interpreted as the percentage effect of a unit change, given that the variable is continuous.

An alternative estimator-the generalized moments (GM) estimator-which is based on Kelejian & Prucha (1999)-will also be used. This method is computationally useful when the number of observations is large, and it does not assume the normality of error terms (Anselin & Le Gallo, 2006). A precise description of the differences between the two estimators in relation to the spatial error model is found in Bell & Bockstael (2000).

If the OLS estimates are significantly different to the spatial error model, the more general spatial Durbin model is advocated since this model may protect against omitted variable bias (LeSage & Pace, 2009, p. 157). A general formulation of the spatial Durbin model is:

$$P = \rho W P + X \beta_0 + \rho W X \beta_1 + \varepsilon. \tag{4}$$

This model includes a spatial lag of the dependent variable, along with the existing independent variables. In this way the spatial Durbin model can adjust for the potential of a significant spatial lag in the dependent variable and the characteristics of neighbouring houses. See for instance, Bivand (1984) for further explanations. As we explain in Section 6, it is this method of estimation which proved most appropriate given the results of the various diagnostic tests, though the OLS and spatial error models also provide good approximations.

4. Data

Our data are for dwellings in Glasgow, Scotland. Because individuals living in Glasgow may commute to employment outside the city, we allow the initial employment catchment to cover a much greater area (distances were computed from dwellings in Glasgow to all Scottish employment data zones, and then restricted to a commuting radius of 60 km). As argued in O&T (2008) successful estimations of the spatial interaction variables probably require data referring to a connected labour market area, rather than just an urban area as in Adair *et al.* (2000). Given that the Norwegian region studied in O&T (2008) was highly atypical of most European or American metropolitan areas, the application to a larger, industrial city like Glasgow is of particular interest in terms of extending our understanding of how gravity-based estimation approaches perform in different contexts. To what extent do the O&T (2008) results on the role of urban attraction and labour market accessibility in determining housing prices hold in a larger urban metropolitan area? Glasgow is a particularly useful test-bed for such research given that it is has become a well-established location for house price and urban structure research (see, for example, Pryce, 2011; Pryce & Gibb, 2006).

We include four types of variables: *dwelling type* (house or flat, conversion, detached, semi-detached, detached-bungalow, detached villa), *internal characteristics* (traditional construction, bay window, number of bedrooms, number of public rooms (i.e. rooms other than bedrooms, bathrooms and kitchens), *en suite* bathrooms, gas central heating, whether the property needs upgrading, whether the property is described as luxurious), *external characteristics* (plot measured in acres, whether there is a garden, garage, parking and notable views), and *neighbourhood variables* [social deprivation, realtor jurisdiction (taken as an indicator of submarket), school performance]. We also include seasonal dummies and distance to CBD to capture the urban attraction effect—the benefits of locating near the centre of the city other than access to employment (e.g. shopping facilities and other amenities). Finally, we include our adjusted employment accessibility measure S which is a weighted measure of distance to each employment node that allows for distance to employment to have a nonmonotonic functional form due, for example, to negative externalities associated with living close to an employment node. More details on the computation of S are provided below.

The house price data comprise 6269 dwelling transactions in Glasgow in 2007 (see summary statistics in Table 1) and were supplied by Glasgow Solicitors Property Centre, a consortium of over 200 real estate agents across the Strathclyde city region. This is a large dataset with relatively dense spatial distribution—the maximum distance for any observation from the centre is about 30 km. For more details on the Glasgow housing market (including analysis of average selling prices, types of houses, differences in landscape, neighbourhood characteristics and access to a range of amenities that appear in

Table 1. Descriptive statistics.

	Variable name	Mean	Standard deviation
	Selling price (£)	139 850.00	75 714.17
Type of dwelling	House	0.2792	0.4486
	Conversion	0.0182	0.1336
	Detached bungalow	0.0227	0.1488
	Semi detached bungalow	0.0188	0.1359
	Detached villa	0.0638	0.2444
Internal characteristics and size	Traditional	0.1050	0.3065
	Bay	0.1916	0.3936
	Bedrooms	2.2450	0.8900
	Public rooms	1.2770	0.5735
	Ensuite	0.0518	0.2217
	Gas central heating	0.6168	0.4862
	Needs upgrading	0.0193	0.1376
	Luxury	0.0284	0.1661
External characteristics and size	Plot measured in acres	0.0016	0.0399
	Garden	0.7212	0.4485
	Garage	0.2346	0.4238
	Parking	0.1099	0.3128
	Views	0.0526	0.2233
Season	Spring	0.2823	0.4502
	Summer	0.2747	0.4460
	Autumn	0.2337	0.4232
Neighbourhood/submarket variables	Deprivation	5.6220	2.3930
	School performance	35.7866	22.8057
	Eend_d	0.0861	0.2806
	Sside_d	0.1905	0.3927
	Ngla_d	0.0322	0.1766
Distance to CBD	Distance to CBD (km)	8.4988	6.3382

these submarkets), see Pryce & Gibb (2006). Employment location data were supplied by Annual Business Inquiry (ABI) at datazone level, and deprivation data were provided by Scottish Homes. The school performance measure was computed as the percentage of pupils gaining 5 or more awards at level 5 or above (3 year average for the period 2000–2002) in the secondary school nearest to dwelling i, using examination results data supplied by the Scottish Government.

4.1 Creating the Adjusted Accessibility Variable, S

As noted in the introduction, a major obstacle to testing for nonmonotonicity in a multiple employment node framework is the methodological challenge of developing a flexible functional form for the employment access measure. We experimented with a variety of non-linear functions and found $S_i = \sum_j L_j^\gamma v_{ij}^\theta \exp[\sigma v_{ij}]$ to be the most satisfactory in terms of its elegance and flexibility. Flexibility comes at a price, however, particularly in the context of gravity-based estimation. To illustrate, consider the 6269 dwellings (all within 30 km distance from Glasgow Centre) and the 6501 employment zones (covering most of Scotland) in our data. Even a very simple exponential specification of S without flexibility

in the functional form would require the calculation of more than 40 million distances ($6269 \times 6501 = 40\,754\,769$). If we introduce an additional non-linear parameter, such as θ, and then use ML estimation to grid search over say a 100 possible values of that parameter, the S variable has to be recomputed 100 times for each value of that parameter, raising the number of computations to 4 billion. Adding another two parameters and searching over 100 values of each would increase the number of computations by a factor of 10 000 to 40 trillion. To reduce the number of computations, we preceded the computations with an initial grid-search using large increments in the parameters to identify plausible ranges of each, and then conducted a more refined search within the most promising intervals. Each grid-search entailed finding the parameter values that maximised the log likelihood of the regression function described in (1). The entire process was repeated for each of the five regression specifications: ROT, R1, R2, R3 and R4.

The results obtained at optimum values of γ, θ, and σ are displayed in Table 2 for the five different model alternatives. It is clear from the results that when θ is allowed to vary (regressions R1, R2, R3 and R4), rather than being held constant at zero (ROT), it tends towards a value substantially greater than zero (our estimates range from 3.2 to 5.0), indicating a strong degree of nonmonotonicity. Model R1 is the best performing model ($\theta = 4.6$). Note that the models ROT and R1 are nested variants since, if $\theta = 0$, then model R1 reduces to model ROT. As a result, we can test the significance of θ by performing the likelihood ratio (LR) test,[2] where ROT and R1 are the restricted and unrestricted models, respectively. The estimated LR test statistics is 204.4, which exceeds the 5 per cent critical value of the χ^2 distribution $= 3.84$ by a massive margin (our test statistic is more than 50 times greater than the 5 per cent critical value, implying a significance value of 2.3E-46, or less than a one in a billion trillion trillion trillion [10^{45}] chance of falsely rejecting the null hypotheses of $\theta = 0$). This result provides very strong evidence for nonmonotonicity in the effect on house prices of distance to employment. Further evidence of this result is presented in Section 5.

To illustrate the implications of our findings, Figure 3 plots the functional form of S at the optimised values of parameters from R1 ($\theta = 4.6$, $\gamma = 0.1$, $\sigma = -0.007$). The graph is plotted for the case where the number of employees, L, at node j, is 100. Translating the relationship between distance and S into the relationship between distance and log house price entails a simple linear scaling (to obtain the effect on log house price, simply multiply S by 16.165, the R1 coefficient on S). It is clear, therefore, that the nonmonotonic relationship between S and distance will translate into a nonmonotonic relationship between house prices and distance. The implication of Figure 3 is that the estimated externality effect is so pronounced that it more than counteracts any positive travel benefit associated with living close to an employment node. The distance at which S is maximised will also correspond to the distance at which the house price effect is maximised. For example, it can be seen from Figure 3 that S, the benefit of locating near employment net of the externality effect, peaks at approximately 650 m from an employment node (computed assuming there is but one employment node with 100 employees, and everything else is held constant). Beyond 2 km, the net benefit of proximity to employment falls to zero. Note that, for simplicity, Figure 3 is plotted for the strictly hypothetical case of a single employment node. In reality, houses will be located in varying degrees of proximity to many employment nodes of varying sizes, so the actual value of S at a particular residential location i will typically be considerably higher than the peak of 0.14 in Figure 3. Since our functional form permits multiple employment locations, this is accounted for in the

Table 2. Results from the ML estimations and OLS regressions.

		ROT	R1	R2	R3	R4
		Neighbourhood variables + CBD	Neighbourhood variables + CBD	Neighbourhood variables + no CBD	No neighbourhood variables + CBD	No neighbourhood variables + no CBD
Type of dwelling	House	0.2098 (15.96)	0.2197 (16.77)	0.2172 (16.68)	0.2257 (16.54)	0.2248 (16.72)
	Conversion	0.4182 (14.99)	0.3695 (14.10)	0.3635 (14.02)	0.387 (14.14)	0.3856 (14.28)
	Detached bungalow	0.3029 (10.77)	0.3158 (11.11)	0.3152 (11)	0.3625 (11.58)	0.3626 (11.55)
	Semi-detached bungalow	0.1597 (7.43)	0.1641 (7.64)	0.1633 (7.4)	0.1802 (7.80)	0.1809 (7.73)
	Detached villa	0.1283 (6.95)	0.1381 (7.59)	0.1342 (7.44)	0.1359 (7.15)	0.1328 (7.1)
Internal characteristics and size	Traditional	0.0762 (4.05)	0.0719 (3.89)	0.0684 (3.67)	(0.0719) (3.81)	0.0696 (3.65)
	Bay	0.1293 (13.02)	0.1095 (11.5)	0.1099 (11.55)	0.0997 (9.74)	0.1009 (9.88)
	Bedrooms	0.1880 (25.53)	0.1850 (25.49)	0.1855 (25.58)	0.1827 (24.39)	0.1836 (24.57)
	Public rooms	0.1667 (15.63)	0.1635 (15.63)	0.1627 (15.54)	0.1733 (16.02)	0.1724 (15.89)
	Ensuite	0.1139 (7.19)	0.1097 (7.15)	0.1088 (7.12)	0.1107 (6.76)	0.1099 (6.73)
	Gas central heating	0.0437 (5.08)	0.0416 (4.93)	0.0419 (4.95)	0.0401 (4.49)	0.0398 (4.47)
	Needs upgrading	−0.1085 (−4.3)	−0.1108 (−4.38)	−0.1135 (−4.54)	−0.1276 (−4.63)	−0.1291 (−4.72)
	Luxury	0.1346 (5.70)	0.1359 (5.98)	0.1360 (6.00)	0.1692 (7.15)	0.1684 (7.13)
External characteristics and size	Plot measured in acres	0.3360 (3.41)	0.3363 (3.63)	0.3209 (3.60)	0.3146 (3.45)	0.3039 (3.40)
	Garden	0.0449 (4.18)	0.0458 (4.41)	0.0454 (4.36)	0.0280 (2.65)	0.0279 (2.66)
	Garage	0.0971 (8.59)	0.0991 (8.8)	0.0990 (8.72)	0.1093 (9.15)	0.1087 (9.07)
	Parking	0.0339 (2.05)	0.0369 (2.25)	0.0352 (2.14)	0.0496 (2.94)	0.0472 (2.79)
	Views	0.0706 (3.01)	0.0685 (2.97)	0.0669 (2.91)	0.0860 (3.6)	0.0851 (3.57)
Season	Spring	0.0399 (3.84)	0.0362 (3.53)	0.0365 (3.55)	0.0392 (3.58)	0.0391 (3.56)
	Summer	0.0567 (5.56)	0.0539 (5.37)	0.0543 (5.39)	0.0569 (5.34)	0.0568 (5.32)
	Autumn	0.0468 (4.05)	0.0436 (3.82)	0.0424 (3.71)	0.0499 (4.18)	0.0491 (4.10)
Neighbourhood/submarket variables	Deprivation	−0.0173 (−6.35)	−0.0171 (−6.29)	−0.0220 (−9.34)	—	—
	Eend_d	−0.1945 (−14.67)	−0.1587 (−11.81)	−0.1343 (−10.38)	—	—
	Sside_d	−0.1352 (−13.51)	−0.1388 (−14.18)	−0.1299 (−13.58)	—	—
	Ngla_d	−0.2562 (−12.57)	−0.2119 (−10.08)	−0.1795 (−8.85)	—	—
	School performance	0.2189 (9.35)	0.2209 (9.54)	0.2233 (9.64)	—	—
Distance to CBD	Distance to CBD	−0.0100 (−8.85)	−0.0058 (−4.79)	—	−0.0026 (−2.71)	—

Table 2. *Continued*

		ROT	R1	R2	R3	R4
Access to employment	Accessibility (S_j)	17.1911 (18.46)	16.165 (23.92)	36.3766 (28.48)	13.4276 (30.04)	10.7741 (33.83)
Constant	Constant term	10.894 (339.04)	10.813 (336.00)	10.763 (347.70)	10.681 (373.45)	10.645 (426.96)
Parameters	γ (p1)	0.2311	0.100	0.100	0.100	0.100
	θ (p2)	0.000 (by assumption)	4.600	3.200	5.000	3.400
	σ (p3)	−0.0027	−0.007	−0.005	−0.007	−0.005
	N	6269	6269	6269	6269	6269
	R2 (adj)	0.596	0.609	0.607	0.570	0.570
	AIC	3157.812	2953.435	2957.569	2955.833	2961.008
	LL	−1548.906	−1446.717	−1462.587	−1740.038	−1744.128

Notes: Robust *t*-statistics are found in parentheses of the OLS regressions. LL denotes log-likelihood values. AIC represents Akaike's information criterion. ROT gives the results using the monotonic accessibility measure based on O&T (2008), which implies that $\theta = 0$ in $S_j = \sum_{j} L_j^\gamma v_j^\theta \exp[\sigma v_j]$. The other models have been estimated without imposing this restriction on θ.
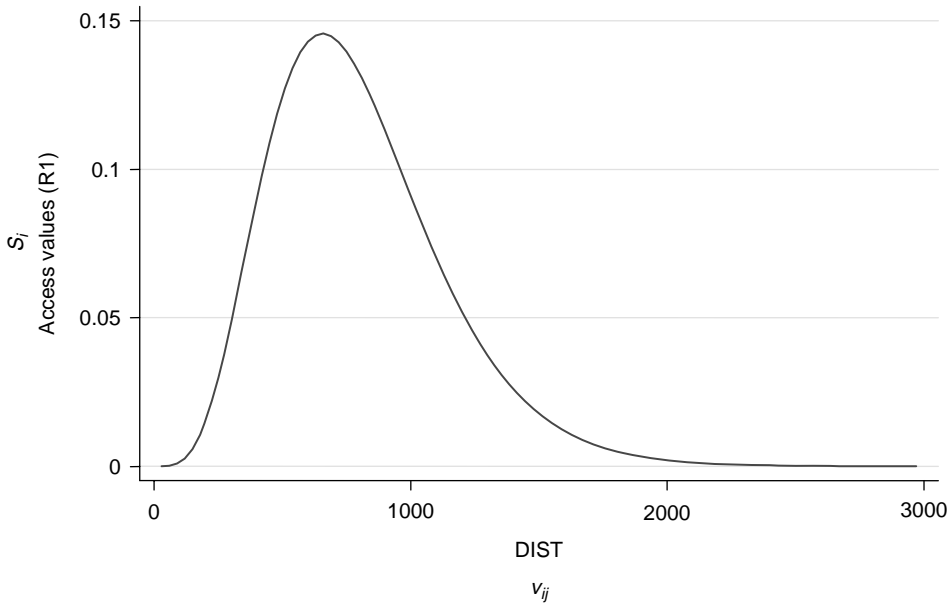
Figure 3. The nonmonotonic effect of distance to employment plotted using parameter values derived from regression R1. *Notes*: Figure 3 plots the graph of S_i against v_{ij}, where S_i is the gravity-based measure of access to employment at residential location i, and v_{ij} is distance from dwelling i to employment node j. The graph is plotted for the case where the number of employees, L, at node j, is 100. Distance is measured along the horizontal axis and is computed as distance in metres to this single employment node. The access variable, $S_i = \sum_j L_j^\gamma v_j^\theta \exp[\sigma v_j]$, is plotted on the vertical axis at the estimated values of its parameters ($\gamma = 0.1$, $\theta = 4.6$, $\sigma = -0.007$), derived from a ML grid-search on preferred regression, R1. To obtain the effect on log house price at a given distance, the values of S_i plotted here need to be multiplied by 16.165, the R1 coefficient on S_i.

estimated regressions. We now subject the results to further scrutiny to ensure they are not made spurious by spatial effects.

5. Spatial Econometrics and Results for Control Variables

In Table 3 we report test results for spatial effects using weights so that $k = 22$. Using neighbourhood structures with lower/higher numbers of neighbours leads to the same conclusions as those implied by the tests presented in Table 3.

   The OLS models are nested within the spatial autoregressive error models, so comparisons of corresponding spatial error models are straightforward. In all cases, the spatial error models give a higher log-likelihood value than the equivalent OLS models found in Table 2. The spatial autoregressive parameter, λ, is positive; takes substantial value and is clearly significant. Access to employment is significant in all models (as is the urban attraction variable, distance to CBD). A standard LR test to compare R1 and ROT shows that the inclusion of the nonmonotonic distance effect in the adjusted labour market accessibility variable is clearly significant. The value of the LR test statistic is 35.12 using the results from the spatial error models. Model R1 is also significantly better than model R2, and the value of the test statistic is 3.938. The critical value of the $\chi^2(1)$ is 3.84 at the 5 per cent significance level.

Table 3. Tests for spatial effects.

	ROT ($k = 22$)	R1 ($k = 22$)	R2 ($k = 22$)	R3 ($k = 22$)	R4 ($k = 22$)
Moran's I	0.218 (0.000)	0.204 (0.000)	0.208 (0.000)	0.266 (0.000)	0.267 (0.000)
Z	64.405	60.380	61.409	77.690	77.794
LM error	3950.771 (0.000)	3468.008 (0.000)	3606.409 (0.000)	5892.827 (0.000)	5937.264 (0.000)
LM lag	2377.238 (0.000)	1944.862 (0.000)	1924.857 (0.000)	3222.395 (0.000)	3104.295 (0.000)
RLM error	1886.480 (0.000)	1771.359 (0.000)	1903.561 (0.000)	3014.466 (0.000)	3149.523 (0.000)
RLM lag	312.948 (0.000)	248.213 (0.000)	222.001 (0.000)	344.033 (0.000)	316.554 (0.000)

Notes: $Z = (I - E(I))/(V(I)^{1/2})$. I is the estimated Moran's I statistics, $E(I)$ is the mean and $V(I)$ is the variance, given the null hypothesis of no spatial effects. The null hypothesis is rejected if the estimated Z is larger than 1.64. The LM-tests asymptotically follow a $\chi^2(1)$ distribution under the null hypothesis. *P*-values follow in parentheses. $k =$ number of neighbours in the spatial weight matrices. $k = 22$ is the neighbourhood structure that gives the highest log-likelihood value for the spatial error model, R1.

In the spatial error models (Table 4) the ML-estimator is used. The results from the GM estimator are at least as good as those from the ML-estimator, and are only presented for model R1, the preferred model. None of the above-mentioned conclusions are altered when using weights with $k < 22$. In Table 5 this is documented using results for R1 and $k = 5$. For the sake of completeness, we have also included results from a spatial lag model (Anselin, 1988).

A remaining issue relates to the impact of excluded and/or misspecified variables. If the variation in the estimated coefficients is large between the spatial error model and the OLS models, this could be a sign of misspecification (LeSage & Pace, 2009). It could also indicate that neither the OLS model nor the spatial error model is representative of the true data generating process (Pace & LeSage, 2008). To gauge the magnitude of such effects, we measure the differences between the OLS coefficients and those in the spatial error model ($k = 22$); divided by the mean value of pairs of coefficients. Although all the coefficients have the same sign, there are relatively large differences between pairs of coefficients for some of the neighbourhood/submarket variables. The most important variable in the present study is labour market accessibility. For this coefficient the variation is 33 per cent. We conducted a spatial Hausman test and rejected the null hypothesis (significance $= 0.000$) that the OLS estimates and the estimates from the spatial error model are not significantly different from each other (LeSage & Pace, 2009). Following LeSage & Pace (2009, p. 157) we proceeded to estimate the spatial Durbin model (4) and found that $k = 5$ gives the highest log-likelihood value. Results from the estimated spatial Durbin model are presented in Table 5. Note that the log-likelihood-value is higher than the corresponding spatial error model, which suggests that the spatial Durbin model is the most appropriate for our data.

The results on the estimated parameters from the spatial Durbin model (Table 5) cannot be interpreted as partial effects. For this reason we report the so-called 'spillover impacts' associated with changes in the explanatory variables. See LeSage & Fischer (2008), Kirby & LeSage (2009), and LeSage & Pace (2009) for further explanations. The computations of these effects follow LeSage & Pace (2009, p. 38). We have calculated the covariance matrix of the coefficients by way of numerical methods (LeSage & Pace, 2009, pp. 56–59). This matrix, and traces[3] of powers series of the spatial weights matrix, carried out by Monte Carlo methods,[4] were then used to calculate impact measures, and inference results (LeSage & Pace, 2009, pp. 96–104 and 114–115). The results reported in Table 6 represent scalar average measures for each variable over all the observations and are interpreted as follows. Average direct impacts measure the scalar average effect on a house price i of a change in the value of each of the explanatory variables related to that house. The indirect impacts are the average effect on house price i of a change in each of the explanatory variables relatated to all the other houses. The average total impact is the sum of the direct and indirect impacts.

All the estimated direct impacts are significant, except for some of the neighbourhood characteristics. For these variables, the indirect impacts are signficant. In almost all cases the total impacts are significant. Except for the direct effect of distance to CBD, all the effects have the same sign as the corresponding parameters from previously used estimators. When it comes to access to labour markets, all the three impacts measures are significant and the signs of the impacts are positive. The positive indirect or spillover effects are almost twice as large as the direct impacts.

Given that the results from the spatial Durbin model are unbiased under a range of different data generating processes (LeSage & Pace, 2009, pp. 157–158), we could

Table 4. Results from the spatial error models.

		ROT Neighbourhood variables + CBD	R1 Neighbourhood variables + CBD	R2 Neighbourhood variables + no CBD	R3 No neighbourhood variables + CBD	R4 No neighbourhood variables + no CBD
Type of dwelling	House	0.2045 (18.323)	0.2054 (18.406)	0.2047 (18.345)	0.2053 (18.391)	0.2046 (18.332)
	Conversion	0.2997 (10.843)	0.2967 (10.738)	0.2965 (10.739)	0.2950 (10.669)	0.2943 (10.643)
	Detached bungalow	0.2860 (11.357)	0.2851 (11.320)	0.2849 (11.312)	0.2831 (11.237)	0.2823 (11.207)
	Semi detached bungalow	0.1449 (5.432)	0.1446 (5.417)	0.1414 (5.401)	0.1503 (5.635)	0.1497 (5.607)
	Detached villa	0.1334 (7.576)	0.1343 (7.622)	0.1336 (7.589)	0.1339 (7.600)	0.1333 (7.564)
Internal characteristics and size	Traditional	0.0494 (4.101)	0.0488 (4.045)	0.0485 (4.026)	0.0474 (3.934)	0.0472 (3.903)
	Bay	0.0864 (8.468)	0.0849 (8.328)	0.0853 (8.373)	0.0847 (8.304)	0.0852 (8.351)
	Bedrooms	0.1903 (37.821)	0.1895 (37.714)	0.1897 (37.752)	0.1890 (37.583)	0.1892 (37.612)
	Public rooms	0.1389 (17.690)	0.1394 (17.761)	0.1394 (17.762)	0.1408 (17.923)	0.1407 (17.917)
	Ensuite	0.109 (6.680)	0.1078 (6.595)	0.1080 (6.615)	0.1062 (6.505)	0.1063 (6.507)
	Gas central heating	0.0465 (6.251)	0.0466 (6.262)	0.0467 (6.285)	0.0479 (6.441)	0.0478 (6.428)
	Needs upgrading	−0.0891 (−3.535)	−0.0894 (−3.541)	−0.0897 (−3.561)	−0.0884 (−3.510)	−0.0889 (−3.525)
	Luxury	0.1318 (6.349)	0.1327 (6.390)	0.1325 (6.379)	0.1355 (6.517)	0.1347 (6.498)
External characteristics and size	Plot measured in acres	0.2787 (3.244)	0.2872 (3.323)	0.2852 (3.319)	0.2816 (3.279)	0.2799 (3.248)
	Garden	0.0543 (6.100)	0.0551 (6.190)	0.0547 (6.147)	0.0526 (5.906)	0.0522 (5.869)
	Garage	0.0628 (6.259)	0.0634 (6.317)	0.0630 (6.280)	0.0645 (6.433)	0.0642 (6.406)
	Parking	0.0302 (2.623)	0.0291 (2.5321)	0.0291 (2.527)	0.0285 (2.482)	0.0283 (2.456)
	Views	0.0400 (2.555)	0.0389 (2.470)	0.0387 (2.472)	0.0403 (2.582)	0.0400 (2.553)
Season	Spring	0.0257 (2.603)	0.0257 (2.595)	0.0257 (2.593)	0.0264 (2.665)	0.0265 (2.675)
	Summer	0.0435 (4.390)	0.0417 (4.460)	0.0424 (4.456)	0.0445 (4.483)	0.0445 (4.488)
	Autumn	0.0358 (3.465)	0.0364 (3.520)	0.0365 (3.530)	0.0369 (3.577)	0.0370 (3.579)
Neighbourhood/submarket variables	Deprivation	−0.0194 (−5.042)	−0.0190 (−5.053)	−0.0212 (−5.867)	–	–
	Eend_d	−0.1825 (−3.989)	−0.1614 (−3.781)	−0.145 (−3.430)	–	–
	Sside_d	−0.0493 (−1.867)	−0.0607 (−2.363)	−0.0538 (−2.117)	–	–
	Ngla_d	−0.1857 (−4.265)	−0.1687 (−3.968)	−0.1580 (−3.745)	–	–
	School performance	0.0778 (2.056)	0.0856 (2.313)	0.0833 (2.315)	–	–
Distance to CBD	Distance to CBD	−0.0104 (−3.855)	−0.0056 (−2.119)	–	−0.0063 (−2.282)	–
Access to employment	Accessibility	7.2453 (4.953)	11.4698 (8.098)	25.4963 (9.506)	7.8771 (7.777)	6.7279 (9.697)

Table 4. *Continued*

		ROT	R1	R2	R3	R4
Constant	Constant term	11.0582 (268.1945)	10.943 (256.230)	10.890 (297.756)	10.838 (275.272)	10.768 (395.543)
Parameters	γ (p1)	0.2311	0.100	0.100	0.100	0.100
	θ (p2)	0.0	4.600	3.200	5.000	3.400
	σ (p3)	−0.0027	−0.007	−0.005	−0.007	−0.005
	λ (*p*-value)	0.7906 (0.000)	0.7634 (0.000)	0.7676 (0.000)	0.8012 (0.000)	0.804 (0.00)
	N	6269	6269	6269	6269	6269
	AIC	1812.900	1777.600	1779.700	1824.4	1826.7
	LL	−875.440	−857.788	−859.849	−886.178	−888.340

Notes: In parentheses we report *z*-statistics. LL denotes log-likelihood values and AIC represents Akaike's information criterion. λ has been estimated by using the ML estimator. In all reported models $k = 22$.

Table 5. Results for the preferred model formulation R1 using an alternative estimator, an alternative neighbourhood structure in the weights matrix of the spatial error model and two alternative spatial models.

		Spatial error model using GM (k = 22)	Spatial error model (k = 5)	Spatial lag model (k = 5)	Spatial Durbin model (k = 5)	
		Neighbourhood variables + CBD	Neighbourhood variables + CBD	Neighbourhood variables + CBD	Neighbourhood variables + CBD	
					Non-lagged variables	Lagged variables
Type of dwelling	House	0.2061 (18.374)	0.2007 (17.892)	0.1761 (15.867)	0.1925 (17.329)	−0.0698 (−3.145)
	Conversion	0.3009 (10.861)	0.3119 (11.146)	0.2997 (11.2112)	0.3014 (10.934)	−0.0131 (−0.266)
	Detached bungalow	0.2861 (11.293)	0.3029 (11.920)	0.2668 (10.573)	0.2904 (11.577)	−0.1517 (−2.989)
	Semi detached bungalow	0.1450 (5.4804)	0.1592 (5.970)	0.1541 (5.730)	0.1628 (6.175)	−0.1077 (−1.881)
	Detached villa	0.1344 (7.587)	0.1340 (7.493)	0.1435 (6.489)	0.1275 (7.218)	−0.0682 (−1.924)
Internal characteristics and size	Traditional	0.0500 (4.121)	0.0395 (3.318)	0.0588 (4.837)	0.0395 (3.309)	0.0851 (3.194)
	Bay	0.0861 (8.417)	0.0811 (7.924)	0.0972 (9.786)	0.0794 (7.805)	0.0220 (1.107)
	Bedrooms	0.1893 (37.468)	0.1907 (37.523)	0.1745 (34.979)	0.1903 (37.968)	−0.1063 (−10.335)
	Public rooms	0.1404 (17.800)	0.1310 (16.778)	0.1371 (17.454)	0.1333 (17.321)	0.0255 (1.537)
	Ensuite	0.1075 (6.544)	0.1087 (6.706)	0.1014 (6.151)	0.1053 (6.537)	−0.0703 (−2.035)
	Gas central heating	0.0460 (6.161)	0.0383 (5.193)	0.0348 (4.686)	0.0411 (5.637)	−0.0034 (−0.223)
	Needs upgrading	−0.0903 (−3.562)	−0.1010 (−4.096)	−0.1046 (−4.326)	−0.1057 (−4.274)	−0.0014 (−0.024)
	Luxury	0.1328 (6.355)	0.1030 (5.031)	0.1217 (5.745)	0.1204 (5.464)	0.1003 (2.050)
External characteristics and size	Plot measured in acres	0.2927 (3.382)	0.3028 (3.569)	0.3016 (3.425)	0.2781 (3.270)	−0.0887 (−0.439)
	Garden	0.0547 (6.116)	0.0471 (5.302)	0.0295 (3.335)	0.0455 (5.176)	−0.0283 (−1.560)
	Garage	0.0653 (6.474)	0.0676 (6.748)	0.0658 (6.512)	0.0672 (6.781)	0.0399 (1.876)
	Parking	0.0293 (2.529)	0.0088 (0.805)	0.0287 (2.489)	0.0156 (1.379)	0.0917 (3.833)
	Views	0.0400 (2.544)	0.0289 (1.841)	0.0440 (2.792)	0.0279 (1.803)	0.0998 (3.122)
Season	Spring	0.0263 (2.629)	0.0282 (2.893)	0.0304 (3.020)	0.0305 (3.125)	0.0645 (0.275)
	Summer	0.0448 (4.487)	0.0503 (5.176)	0.0529 (5.224)	0.0495 (5.056)	−0.0198 (−0.827)
	Autumn	0.0367 (3.527)	0.0391 (3.853)	0.0423 (4.019)	0.0397 (3.897)	−0.0090 (−0.371)
Neighbourhood/submarket variables	Deprivation	−0.0188 (−5.274)	−0.0183 (−5.447)	−0.0097 (−5.203)	−0.0209 (−3.234)	0.0137 (2.024)
	Eend_d	−0.1694 (−4.578)	−0.1592 (−5.883)	−0.0781 (−5.460)	−0.0146 (−0.209)	−0.0628 (−0.884)
	Sside_d	−0.0793 (−3.481)	−0.0995 (−5.316)	−0.0778 (−7.628)	0.1039 (2.818)	−0.1809 (−4.745)
	Ngla_d	−0.1799 (−4.480)	−0.1857 (−5.227)	−0.1213 (−5.646)	−0.0656 (−1.245)	−0.0398 (−0.698)
	School performance	0.1134 (3.382)	0.0020 (6.439)	0.1363 (7.967)	−0.0592 (−1.059)	0.161 (2.729)
Distance to CBD	Distance to CBD	−0.0058 (−2.568)	−0.0050 (−2.888)	−0.0051 (−6.193)	0.0046 (2.602)	−0.0503 (−2.844)
Access to employment variable	Accessibility	12.5306 (9.889)	14.7879 (13.794)	11.6558 (18.573)	4.6252 (2.1387)	2.8156 (1.233)
Constant	Constant term	10.9254 (282.583)	10.892 (326.869)	6.3870 (50.571)	5.5123 (36.083)	

Table 5. *Continued*

Parameters	Spatial error model using GM (k = 22) Neighbourhood variables + CBD	Spatial error model (k = 5) Neighbourhood variables + CBD	Spatial lag model (k = 5) Neighbourhood variables + CBD	Spatial Durbin model (k = 5) Non-lagged variables	Spatial Durbin model (k = 5) Lagged variables
γ (p1)	0.100	0.100	0.100	0.100	
θ (p2)	4.600	4.600	4.600	4.600	
σ (p3)	−0.007	−0.007	−0.007	−0.007	
λ (or ρ) < br > (p-value)	0.6999 (0.000)	0.529 (0.000)	0.3872 (0.000)		0.4800 (0.000)
N	6269	6269	6269		6269
AIC	1789.8	1828.1	1872.0		1581.0
LL	−863.919	−883.026	−905.012		−731.490

Notes: The first column display results using the GM-estimator in accordance with Kelejian & Prucha (1999). The second column displays results from the spatial error model using weights so that $k = 5$. In the third the spatial lag model is shown. In the fourth and fifth columns the results from the spatial Durbin model is documented. For this model the parameters of the spatially non-lagged independent variables appear to the left and the parameters related to the lagged independent variables appear to the right. z-values are given in parentheses. LL denotes log-likelihood values and AIC represents Akaike's information criterion.

Table 6. Average direct, indirect and total impacts from the spatial Durbin specification of R1.

	Variable name	Average direct impact	Average indirect impact	Average total impact
Type of dwelling	House	0.1951 (0.000)	0.0409 (0.335)	0.2360 (0.000)
	Conversion	0.3164 (0.000)	0.2380 (0.004)	0.5543 (0.000)
	Detached bungalow	0.2890 (0.000)	−0.0223 (0.7974)	0.2667 (0.0095)
	Semi detached bungalow	0.1594 (0.000)	−0.0535 (0.6041)	0.1059 (0.3644)
	Detached villa	0.1267 (0.000)	−0.0127 (0.8258)	0.1139 (0.0800)
Internal characteristics and size	Traditional	0.0513 (0.000)	0.1883 (0.000)	0.2396 (0.000)
	Bay	0.0863 (0.000)	0.1088 (0.052)	0.1951 (0.000)
	Bedrooms	0.1886 (0.000)	−0.0270 (0.1541)	0.1616 (0.000)
	Public rooms	0.1435 (0.000)	0.1619 (0.000)	0.3054 (0.000)
	Ensuite	0.1030 (0.000)	−0.0358 (0.5720)	0.0672 (0.3346)
	Gas central heating	0.0429 (0.000)	0.0294 (0.2713)	0.0723 (0.0171)
	Needs upgrading	−0.1116 (0.000)	−0.0942 (0.3790)	−0.2059 (0.0922)
	Luxury	0.1297 (0.000)	0.2788 (0.0013)	0.4084 (0.000)
External characteristics and size	Plot measured in acres	0.2832 (0.002)	0.0810 (0.889)	0.3642 (0.3989)
	Garden	0.0448 (0.000)	−0.0117 (0.7491)	0.0331 (0.2814)
	Garage	0.0755 (0.000)	0.1305 (0.001)	0.2060 (0.000)
	Parking	0.0270 (0.0254)	0.1795 (0.000)	0.2064 (0.000)
	Views	0.0408 (0.004)	0.2048 (0.000)	0.2456 (0.000)
Season	Spring	0.0329 (0.004)	0.0381 (0.4194)	0.0710 (0.177)
	Summer	0.0500 (0.000)	0.0072 (0.861)	0.0572 (0.250)
	Autumn	0.0408 (0.000)	0.0182 (0.747)	0.0590 (0.288)
Neighbourhood/ submarket variables	Deprivation	−0.0205 (0.001)	−0.0067 (0.315)	−0.0138 (0.000)
	Eend_d	−0.0226 (0.751)	−0.1263 (0.074)	−0.1489 (0.000)
	Sside_d	−0.0890 (0.014)	−0.2369 (0.000)	−0.1480 (0.000)
	Ngla_d	−0.0738 (0.143)	−0.1289 (0.025)	−0.2027 (0.000)
	School performance	−0.0004 (0.517)	0.0024 (0.000)	0.0020 (0.000)
Distance to CBD	Distance to CBD	0.0426 (0.014)	−0.0512 (0.004)	−0.0086 (0.000)
Access to employment	Accessibility	5.1998 (0.016)	9.1089 (0.000)	14.3087 (0.000)

Note: p-values in parentheses.

compare the results for the accessibility parameter from this model with the results from the other estimators. When comparing the average total impact with the parameter found in the spatial error model ($k = 5$), the difference in estimated parameter is 3 per cent. It is 12 per cent in comparisons with the OLS model. The 95 per cent quantiles for the point estimate of the total average effect are [11.293; 16.904]. Based on this, the results from the spatial error model can be used as an approximation of the average total impact of changes in accessibility on house prices, whereas the results from the OLS model can represent an approximate upper bound to this effect.

In terms of the coefficients on the other variables in the preferred model (R1) included as controls, they are all as anticipated. Houses, particularly detached and semi-detached bungalows and large houses converted to flats, are worth more than purpose-built flats. Traditional dwellings and luxury properties earn a premium, as do those with bay windows, *en suite* bathrooms or gas central heating. Properties in need of upgrading are worth significantly less than those that are not. Properties with more bedrooms and public rooms are worth more; as are those with large plot sizes, garage, parking or notable views. Properties in deprived areas are worth significantly less, while those close to schools with good exam results tend to have a significantly higher price. The realtor-based submarket dummies for the East End, West End, South Side and North Glasgow were all highly significant. These findings were generally stable across the five regression models and are consistent with those reported in the existing hedonic literature (see, for example, Adair *et al.*, 2000; Osland, 2010; Pryce, 2011).

In summary, after accounting for spatial effects, our results remain unambiguous. Regardless of estimation method, number of neighbours included in the weights matrix and choice of spatial model, the labour market accessibility variable, S, with a nonmonotonic functional form, remains highly statistically significant. Coefficients on other factors included in the model as control variables were generally stable, plausible and statistically significant.

6. Conclusion

We have argued that the apparent conflict between urban economic theory and the empirical evidence with respect to the slope of the house price gradient is best accounted for by allowing for a local externality effect. We have also argued that such effects need to be incorporated into a model that allows for multiple employment nodes. This is important because the effect of local employment externalities is likely to be magnified in a multiple-employment-centre environment (as opposed to a monocentric one) because concentric rings of local externality emanating from multiple employment nodes will cover a much greater proportion of the urban area and potentially overlap. Note that by allowing for both nonmonotonicity and multiple employment nodes, undulations and plateaus in the house price surface can arise without the addition of more elaborate and *ad hoc* irregularities to the von-Thünen–Alonso–Muth model (such as submarkets, variable transport provision, school performance etc.). Crucially, however, for such a degree of complexity to emerge from local externalities alone, the effect would have to be sufficiently potent to cause nonmonotonicity—house prices actually falling as one approaches an employment node—otherwise one would simply observe shallow slopes and this would fail to explain why some studies have found positive house price gradients.

We have also argued that reconciling theory and empirics in this field are of more than academic interest. If the negative effects on house prices of production externalities outweigh the benefits of short commuting times then the case for planning is significantly strengthened. Firm location ceases to be a marginal policy issue because there is an imperative to minimise exposure of residents to the negative effects that occur at close proximity to employment nodes. Maximising social welfare requires government intervention because firms have no obvious market-based incentives to take into account the negative externality effects of their location on surrounding households. Restrictions would then be needed on where commercial and residential construction takes place in order

to separate land use. However, if the relationship between employment density and constant quality house price is strictly monotonic—if the effect of firm location on the quality of life of surrounding residents is unambiguously positive—then the case for land planning is far less compelling. Employment centres in such a world are always beneficial, so it would not matter where they are located. House price gradients are also important because they affect where households from particular income brackets choose to locate. This in turn affects likely revenue and expenditure streams from property taxes.

The overwhelming conclusion of the empirical models presented here is that the relationship between distance to employment and house price is indeed nonmonotonic: house prices rise initially as one moves further away from an employment node, and then decline. The implication of our results is not that all other explanations for positive price gradients should be disregarded; merely that more complex and *ad hoc* factors (such as submarket effects) are not needed for flat or positive house price gradients to occur. Our results also imply that the ambiguous role for planning implicit in the standard urban economic models (both the monocentric and polycentric versions) is incorrect. There are both sound conceptual and empirical reasons to expect positive land rent gradients over certain distances to employment centres. The fact that such gradients occur in polycentric urban economies is important because a much larger proportion of households are potentially affected by local negative externalities (if all production is concentrated in the CBD then only inner-city residents would be affected). So our findings underscore the importance of using gravity models of multiple employment nodes when testing for nonmonotonic effects.

Whilst the paper has offered a number of innovations, including: (1) development of a flexible functional form that allows for nonmonotonicity; (2) estimation of this functional form in the context of a gravity-based accessibility measure to capture polycentric employment and (3) estimation using the spatial Durbin model with detailed comparison of a variety of other models), there are many important avenues for future development. For example, we did not decompose employment into industrial classification or by gender, partly because of data limitations. A more nuanced approach would be achieved if the relationship was estimated separately for different categories of employment. A second area for further work is to experiment with a variety of measures of access. Though we cited evidence that Euclidean distance was probably a good proxy for travelling time, further research needs to be done to establish whether and how the shape of the house price gradient varies across different measures. Thirdly, one might conjecture that the apparent variation in house price gradients across the cities observed in the literature could be the result of different patterns of employment location interacting with local externality effects. In other words, even if the econometric relationship between house prices and our adjusted measure of access remained entirely constant across cities, variations in the pattern of employment would cause house price surfaces to differ considerably. The approach needs to be applied to urban economies of varying scale and type to ascertain the extent to which nonmonotonicity is a general finding.

Acknowledgements

Notes

[1] R is available at: http://www.R-project.org.
[2] The LR test statistic is computed as twice the difference in the log-likelihoods: LR = 2(log-likelihood of the unrestricted model—log-likelihood of the restricted model), which has a χ^2 distribution (see Wooldridge, 2006, p. 588).
[3] The trace of a square matrix is the sum of its diagonal elements.
[4] Monte Carlo methods are a range of statistical techniques for deriving repeated hypothetical samples that incorporate the kind of random variation one might observe in real life but in a controlled way. This confers a distinct advantage over using real data—using hypothetical samples that allows one to control the data environment and hence better understand how a particular procedure behaves. Monte Carlo simulation has become a standard tool for statisticians (see Robert & Casella, 2004 for an overview of different methods and related techniques).

References

Adair, A., McGreal, S., Smyth, A., Cooper, J. & Ryley, T. (2000) House prices and accessibility: The testing of relationships within the Belfast urban area, *Housing Studies*, 15, pp. 699–716.

Alonso, W. (1964) *Location and Land Use. Toward a General Theory of Land* (Cambridge, MA: Harvard University Press).

Anselin, L. (1988) *Spatial Econometrics: Methods and Models* (London: Kluwer).

Anselin, L. & Le Gallo, J. (2006) Interpolation in spatial hedonic models, *Spatial Economic Analysis*, 1, pp. 32–52.

Anselin, L. & Lozano-Gracia, N. (2008) Errors in variables and spatial effects in hedonic house price models of ambient air quality, *Empirical Economics*, 3, pp. 5–34.

Bartik, T. J. & Smith, V. K. (1987) Urban amenities and public policy, in: *Handbook of Regional and Urban Economics*, Vol. 2, pp. 1207–1254 (Amsterdam: Elsevier).

Beckman, M. J. (1973) Equilibrium models of residential land use, *Regional and Urban Economics*, 3, pp. 361–368.

Bell, K. P. & Bockstael, N. E. (2000) Applying the generalized-moments estimation approach to spatial problems involving microlevel data, *The Review of Economics and Statistics*, 82, pp. 72–82.

Bivand, R. (1984) Regression modelling with spatial dependence: An application of some class selection and estimation methods, *Geographical Analysis*, 16(1), pp. 25–37.

Bivand, R., Pebesma, E. J. & Gómez-Rubio, V. (2008) *Applied Spatial Analysis* (New York: Springer).

Brueckner, J. K., Jacques-Francoise, T. & Zenou, Y. (1999) Why is central Paris rich and downtown Detroit poor? An amenity-based theory, *European Economic Review*, 43, pp. 91–107.

Clapp, J. M., Rodriguez, M. & Pace, R. K. (2001) Residential land values and the decentralization of jobs, *Journal of Real Estate Finance and Economics*, 22, pp. 43–61.

Combes, P. -P. & Lafourcade, M. (2005) Transport costs: Measures, determinants, and regional policy implications for France, *Journal of Economic Geography*, 5(3), pp. 319–349.

Crane, R. (1996) The influence of uncertain job location on urban form and the journey to work, *Journal of Urban Economics*, 37, pp. 342–356.

Duranton, G. & Overman, H. G. (2005) Testing for localisation using micro-geographic data, *Review of Economic Studies*, 72, pp. 1077–1106.

Florax, R. J. G. M. & De Graaf, T. (2004) The performance of diagnostic tests for spatial dependence in linear regression models: A meta analysis of simulation studies, in: L. Anselin, R. J. G. M. Florax & S. J. Rey (Eds) *Advances in Spatial Econometrics. Methodology, Tools and Applications*, Chapter 2, pp. 111–135 (Berlin: Springer).

Florax, R. J. G. M., Folmer, H. & Rey, S. J. (2003) Specification searches in spatial econometrics: The relevance of Hendry's methodology, *Regional Science and Urban Economics*, 22, pp. 557–579.

Fujita, M. (1989) *Urban Economic Theory* (Cambridge: Cambridge University Press).

Fujita, M. & Krugman, P. (2004) The new economic geography: Past, present and the future, *Papers in Regional Science*, 83(1), pp. 139–164.

Glaeser, E. L. & Kohlase, J. E. (2004) Cities, regions and the decline of transport costs, *Papers in Regional Science*, 83, pp. 197–228.

Glaeser, E. L., Resseger, M. & Tobio, K. (2009) Inequality in Cities, *Journal of Regional Science*, 49(4), pp. 617–646.

Goodman, A. C. & Thibodeau, T. G. (1998) Housing market segmentation, *Journal of Housing Economics*, 7, pp. 121–143.

Hansen, W. G. (1959) How accessibility shapes land use, *Journal of the American Institute of Planners*, 25, pp. 73–76.

Heikkila, E., Gordon, P., Kim, J. I., Peiser, R. B. & Richardson, H. W. (1989) What happened to the CBD-distance gradient?: Land values in a polycentric city, *Environment and Planning A*, 21, pp. 221–232.

Kelejian, H. H. & Prucha, I. R. (1999) A generalized moments estimator for the autoregressive parameter in a spatial model, *International Economic Review*, 40(2), pp. 509–533.

Kirby, D. K. & LeSage, J. (2009) Changes in commuting to work times over the 1990 to 2000 period, *Regional Science and Urban Ecnomics*, 39, pp. 460–471.

LeSage, J. & Fischer, M. M. (2008) Spatial growth regressions: Model specification, estimation and interpretation, *Spatial Economic Analysis*, 3, pp. 275–304.

LeSage, J. & Pace, R. K. (2009) *Introduction to Spatial Econometrics* (New York: CRC Press).

Li, M. M. & Brown, H. J. (1980) Micro-neighbourhood externalities and hedonic prices, *Land Economics*, 56, pp. 125–140.

Lucas, R. E. & Rossi-Hansberg, E. (2002) On the internal structure of cities, *Econometrica*, 70(4), pp. 1445–1476.

McDonald, J. F. & McMillen, D. P. (2007) Using the monocentric city model, in: *Urban Economics and Real Estate. Theory and Policy*, Chapter 6 (Oxford: Blackwell Publishing).

McMillen, D. P. (2003) The return of centralization to Chicago: Using repeat sales to identify changes in house price distance gradients, *Regional Science and Urban Economics*, 33, pp. 287–304.

Muth, R. (1969) *Cities and Housing* (Chicago, IL: University of Chicago Press).

Osland, L. (2010) An application of spatial econometrics in relation to hedonic house price modelling, *Journal of Real Estate Research*, 32(3), pp. 289–320.

Osland, L. & Thorsen, I. (2008) Effects on housing prices of urban attraction and labor market accessibility, *Environment and Planning A*, 40(10), pp. 2490–2509.

Osland, L., Thorsen, I. & Gitlesen, J. P. (2007) Housing price gradients in a geography with one dominating center, *Journal of Real Estate Research*, 29, pp. 321–346.

Pace, K. & LeSage, J. P. (2008) A spatial Hausman test, *Economic Letters*, 101, pp. 282–284.

Papageorgiou, G. J. & Casetti, E. (1971) Spatial equilibrium residential land values in a multicentric setting, *Journal of Regional Science*, 11, pp. 385–389.

Pogodzinski, J. M. & Sass, T. R. (1991) Measuring the effects of municipal zoning regulations: A survey, *Urban Studies*, 28(4), pp. 597–621.

Pryce, G. (2011) Bidding conventions and the degree of over-pricing in the market for houses, *Urban Studies*, 48(4), pp. 765–791.

Pryce, G. & Gibb, K. (2006) Submarket dynamics of time to sale, *Real Estate Economics*, 34(3), pp. 377–415.

Richardson, H. W. (1977) On the possibility of positive rent gradients, *Journal of Urban Economics*, 4, pp. 60–68.

Richardson, H. W., Gordon, P., Jun, M. -J., Heikkila, H., Reiser, P. & Dale-Johnson, D. (1990) Residential property values, the CBD, and multiple nodes: Further analysis, *Environment and Planning A*, 22, pp. 829–833.

Robert, C. P. & Casella, G. (2004) *Monte Carlo Statistical Methods, Springer Texts in Statistics*, 2nd ed (New York: Springer).

Wheaton, W. C. (2004) Commuting, congestion, and employment dispersal in cities with mixed land use, *Journal of Urban Economics*, 55, pp. 417–438.

Wooldridge, J. M. (2006) *Introductory Econometrics: A modern Approach*, 3rd ed (Mason, OH: Thomson).

Yinger, J. (1992) Urban models with more than one employment center, *Journal of Urban Economics*, 31(2), pp. 181–205.

Yiu, C. Y. & Tam, C. S. (2004) A review of recent empirical studies of property price gradients, *Journal of Real Estate Literature*, 12, pp. 307–322.

Yiu, C. Y. & Tam, C. S. (2007) Housing price gradient with two workplaces—empirical study in Hong Kong, *Regional Science and Urban Economics*, 37, pp. 413–429.