2015

Propagation of Unit Location Uncertainty in Dense Storage Environments

Patrick Reilly
University of Central Florida

PROPAGATION OF UNIT LOCATION UNCERTAINTY
IN DENSE STORAGE ENVIRONMENTS

by

PATRICK J REILLY
Bachelor of Science in Industrial Engineering
University of Central Florida, 2014

A thesis submitted in partial fulfillment of the requirements
for the degree of Master of Science
in the Department of Industrial Engineering and Management Systems
in the College of Engineering and Computer Science
at the University of Central Florida
Orlando, Florida

Spring Term
2015

Major Professor: Jennifer A. Pazour

ABSTRACT

Effective space utilization is an important consideration in logistics systems and is especially important in dense storage environments. Dense storage systems provide high-space utilization; however, because not all items are immediately accessible, storage and retrieval operations often require shifting of other stored items in order to access the desired item, which results in item location uncertainty when asset tracking is insufficient. Given an initial certainty in item location, we use Markovian principles to quantify the growth of uncertainty as a function of retrieval requests and discover that the steady state probability distribution for any communicating class of storage locations approaches uniform. Using this result, an expected search time model is developed and applied to the systems analyzed. We also develop metrics that quantify and characterize uncertainty in item location to aid in understanding the nature of that uncertainty. By incorporating uncertainty into our logistics model and conducting numerical experiments, we gain valuable insights into the uncertainty problem such as the benefit of multiple item copies in reducing expected search time and the varied response to different retrieval policies in otherwise identical systems.

ACKNOWLEDGMENTS

I am greatly indebted to my graduate advisor Dr. Jennifer Pazour for her immense support and direction throughout the development of this work. I doubt I could have found elsewhere the kind of encouragement, insight, and genuine care that I have received from her. My appreciation for her involvement cannot be overstated.

I am also very grateful to my committee members, Dr. Kellie Schneider and Dr. Qipeng Zheng for their assistance and involvement throughout this process. Thank you for offering your time and insight above and beyond expectations.

A special thanks also to the Office of Naval Research for funding this research,
grant number N00014-13-1-0594.

I also give thanks to the Lord Jesus for the life he has granted me in him and for the privilege of seeing his guiding, sovereign hand at work in every stage of this research and in my life.

Commit your work to the LORD, and your plans will be established – Proverbs 16:3

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

CHAPTER ONE: INTRODUCTION

While there is no shortage of warehousing logistics problems discussed in the literature (Gu, Goetschalckx, & McGinnis, 2007), few deal with various kinds of uncertainty and none address the problem of growing uncertainty in item location. This problem, however, does appear in storage systems that are characterized as high density storage environments. In such an environment, effective space utilization is an especially important consideration. Usually the goal in operating a high density system as opposed to a more traditional system is to minimize required space and maximize the amount of inventory stored in that space. Frequently, the driver behind the level of density required is some sort of physical constraint on the system.

One developing application of high density storage is known as sea-basing (Futcher, 2003). Instead of traditional, spacious warehouses as seen in land-based systems, sea-bases consist of a network of ships that function as a maritime distribution center. Since these ships cannot easily be expanded and have limited space devoted to cargo, the storage systems are designed to maximize the inventory stored and they can be characterized as high density storage systems.

One feature of high density systems is that many items are not immediately accessible due to being located behind or under other stored items. In most traditional systems, items are either immediately accessible or can be accessed without much effort. Dense storage systems provide high-space utilization, however, because not all items are immediately accessible, storage and retrieval operations often require shifting of other stored items in order to access the desired item. This results in item location uncertainty that grows as the system operates when asset tracking is insufficient.

In this research, we explore the propagation of uncertainty in dense storage environments, seeking answers to questions about how it behaves and how, if possible, it can be controlled or manipulated. As retrievals occur in a dense storage environment, the system can be modeled and location probabilities tracked using finite-state Markov chains. Given an initial certainty in item location, we use Markovian principles to quantify the growth of uncertainty as a function of retrieval requests to discover the steady state probability distributions approached for single and multiple communication classes. Using these results, an expected search time model is developed and applied to the systems analyzed. We also develop metrics that quantify and characterize uncertainty in item location to aid in understanding the nature of that uncertainty. By incorporating uncertainty into our logistics model and conducting numerical experiments, we gain valuable insights into the uncertainty problem such as the benefit of multiple item copies in reducing expected search time and the varied response to different retrieval policies in otherwise identical systems.

Chapter 2 in this paper is a literature review on various work related to warehousing, dense storage, uncertainty, sea-basing, search theory, and Markov chains. In Chapter 3, we discuss the details of the problem of growing uncertainty in dense storage systems and Chapter 4 presents the methodology for developing the uncertainty model. The results of the model, including descriptions of how the uncertainty behaves in various systems, as well as anticipated search times are discussed in Chapter 5 along with a case study. Chapter 6, the final chapter, discusses the conclusions of the work and future research opportunities.

CHAPTER TWO: LITERATURE REVIEW

This chapter reviews literature that considers problems that are either directly related or are analogous to problems involving dense storage systems, uncertainty in item location, search theory, and simple Markov chain models. The goal is to understand the current state of academic research regarding these questions. As is discussed below, a vast majority of the literature involving dense storage systems, both manual and automated, ignores the possibility of uncertainty in terms of item location. Instead, most work assumes to have constant, perfect knowledge of where items are in the storage system and no effort, therefore, is wasted in search time.

Some work in search theory is reviewed here to address the work that has been done to solve the problem of finding an item when its location is not known with certainty. There is no published work that connects the activity of searching with the material handling operations of a dense storage environment. In most work, there is some a priori probability distribution, continuous or discrete, that gives the likelihood of a target item being located in each possible location. However, we are not aware of any work that considers how this probability distribution came to be what it is in the model or how it might develop over time. If the concepts of dense storage and uncertainty were to be connected, the result would be an understanding of changing uncertainty and thus the need for a searching methodology that address the propagation of uncertainty.

Markov chain theory is considered since it is a branch of stochastic processes, which studies how random variables change over time, and is thus an excellent candidate for measuring and analyzing the change and spread of uncertainty throughout a dense storage system. Markov chains capture the probabilities of a system entering into various possible states and the probabilities are updated over time in response to changes occurring in the system. In the work reviewed below, there is nothing that evaluates the change in uncertainty in a storage environment. Since most material handling work assumes perfect knowledge of item locations, there has been no need to apply Markov chain theory in this way. In presenting these various areas of thought, this literature review aims at identifying a research gap in terms of the application of Markov chain theory to describe the changing uncertainty of item locations in dense storage environments.

General Uncertainty

The following papers are presented to show their consideration of various types of uncertainty in a system. Some address uncertainty that is unrelated to dense storage while others demonstrate that most work involving dense storage has not incorporated uncertainty into their problems.

Brusey et al. (Brusey, Floerkemeier, Harrison, & Fletcher, 2003) give an overview of Radio Frequency Identification (RFID) technology and address two situations (a robotic storage stack and a medicine cabinet.) that illustrate how uncertainty in item location can occur. Reliability issues are addressed including two types of possible errors: false negative and false positive reads. Using a statistical filter, the authors reduce the occurrence of false reads in the two situations. The authors here are concerned particularly with uncertainty in item location that results from a malfunction in asset tracking.

Ergen et al. (Ergen, Akinci, & Sacks, 2007) integrate RFID and GPS technology to address uncertainty in identifying and locating prefabricated components in a precast storage yard. They identify high-level requirements of the model and develop a conceptual system in which components are identified via an RFID tag read by a scanner mounted on a crane. The location of the component is then determined via a GPS receiver also mounted on the crane to give the final location after an item movement. Functional requirements of the chosen technologies are specified and a physical prototype system is built. The feasibility of such a system is demonstrated through a real-world field test of the prototype system. As with the paper by Brusey et al., the objective of this work is to manage and reduce uncertainty in item location through technology based solutions. The type of system considered in this work is highly analogous to very high density storage systems and could even be classified as one since the prefabricated components occasionally require relocating in order to retrieve requested pieces.

Song and Hass et al. (Song, Haas, Caldas, Ergen, & Akinci, 2006) consider the problem of tracking industrial piping products through the supply chain. Uncertainty and imperfect visibility can be an issue so RFID is evaluated as a possible solution. Field tests are performed to determine the feasibility of implementing RFID technology into an automated tracking process. As in the work by Ergen et al., RFID is considered in response to uncertainty in the system, or in this case the supply chain. Although the goal of this work is to maintain visibility and reduce uncertainty, the application in a supply chain is mostly unrelated to the problem of uncertainty within a dense storage environment.

Zhang and Goh et al. (Zhang, Goh, & Meng, 2011) develop a mathematical model for inventory visibility (IV) as it relates to supply chain visibility (SCV). The purpose of the model is to formally conceptualize IV and offer objective, quantitative methods for evaluating performance in IV. While this paper addresses a form of uncertainty in the field of supply chain management, the focus is beyond uncertainty within storage systems, dense storage systems specifically.

Rezapour and Allen et al. (Rezapour, Allen, & Mistree, 2015) address uncertainty propagation in the supply chain and supply networks by dealing with both demand-side uncertainty, as well as supply-side uncertainty. They consider the supply-side uncertainty in terms of a multi-echelon process as opposed to a single-echelon process as is found in most of the literature. A mixed integer nonlinear model is proposed for production planning in light of uncertainty propagation and an example case study is presented from the automotive industry. This paper also is not focused on uncertainty in item location within storage systems but is instead analyzing uncertainty in the reliability of production systems and its effects on the supply chain.

Sahinidis (Sahinidis, 2004) reviews various strategies that have been presented for solving optimization problems in which decisions are made in the presence of uncertainty. Some of the reviewed methodologies include recourse-based stochastic programming, robust stochastic programming, fuzzy programming, probabilistic programming, and stochastic dynamic programming. This paper seeks to account for various optimization techniques that have been used to address the problem of uncertainty and an input into such approaches is a probability distribution function. Thus, the intention of the included papers is not to describe the uncertainty in a system or characterize its progression over time.

Warehouse Operations

Gu et al. (Gu et al., 2007) present an extensive review of work involving various warehouse operation planning problems. All of the papers discussed are grouped according to their particular research question with topics including SKU department allocation, zoning, storage location assignment, batch order picking, routing, and sorting. There are also a few papers included that touch on receiving and shipping operations. This review is helpful as it demonstrates that the problem of uncertainty in item location has yet to be considered in the warehousing literature. All of the work assumes that item locations are always known with certainty. Automated systems may not develop uncertainty in the same way as manual systems but no work seeks to quantify the uncertainty in item location or considers an element of search time that would affect the overall retrieval time. The output of these works may benefit from the consideration of uncertainty and search time, particularly how uncertainty can grow throughout the system.

Storage Systems

Bozer and White (Bozer & White, 1984) consider automated storage and retrieval systems and develop models that determine travel time. The storage and retrieval mechanism can move both horizontally and vertically simultaneously in an aisle. Expected travel times are formulated for both single and dual command cycles when the storage is randomized. The model results are presented for multiple cases in which there are different input/output points. In this work, like virtually all work involving automated storage and retrieval systems, there is an implicit assumption that item locations are known with certainty.

High Density Storage

The papers mentioned in this section all have some relationship to high density storage systems. As is seen below, these papers address some of the difficulties seen in dense storage applications including layout design and the development of operating policies. However, none consider the presence of uncertainty in item location. And none, therefore, address the problem of operating a system when uncertainty grows and propagates as a result of changes in the system.

Some of the papers are grounded in graph theory problems which are mathematically analogous to the problem of maximizing storage density given certain layout constraints. Others discuss the concept of sea-basing, giving a brief history and also describing some of the logistical challenges faced and what techniques have been used to address those issues. A few of the papers focus on the material handling techniques associated with compact, automated storage and retrieval systems. While these papers provide crucial knowledge when considering dense storage systems, none of the cited works incorporate the propagation of uncertainty in item location in these environments and demonstrate the need for such models.

Futcher (Futcher, 2003) addresses the need for sea-basing ships to be able to perform selective offloading in high density storage environments. Different configurations and densities are considered and analyzed to determine their trade-off with mean retrieval time. Two demand scenarios are used as well as two distinct retrieval rules that govern the system's response to retrieval requests. In this work, it is assumed that all of the item locations are known with certainty at all times. There is therefore no search time impacting the retrieval time. In future work, a similar analysis of various layouts and operational policies can be performed in light of the problem of uncertainty. Uncertainty in item location creates search time which in turn affects retrieval time, one of the critical metrics in this paper. While quantifying uncertainty is itself critical, the consideration of its dynamic nature is also crucial. In dense storage, knowledge of item locations can change with every operation that happens in the system.

Gue (Gue, 2006) develops models involving very high density storage systems which are distinguished as occasionally requiring movements of items that are not requested. He speaks of the impact of various levels of density saying that "less-dense storage systems confer an advantage by allowing easier access to items. For example, in single-deep pallet rack storage, workers never have to move an interfering item to gain access to a desired item. In a deep bulk-storage system a single retrieval might require several items to be moved, and this adversely affects the average retrieval time and system capacity" (Gue, 2006). Thus items may change locations several times in response to retrieval requests, even without that item being requested. In this work, an algorithm is given that maximizes storage density given a maximum lane depth. Gue also provides a proof for an upper bound on storage density for any rectangular space. Results show that, generally, in order to achieve the highest storage density, a system should be arranged with several aisles connected by a single cross-aisle. Also shown is that rectangular spaces should have lengths and widths equal to or slightly less than a multiple of $2k + 1$, where k is the maximum lane depth. This paper gives insight into some of the characteristics of very high density storage systems. No uncertainty is considered but uncertainty can easily be applied to a system that embodies some of the layouts described in this paper.

Gue and Kim (Gue & Kim, 2007) analyze high density, puzzle-based systems by developing scenarios based on a 4 x 4, single empty cell puzzle. An algorithm is developed that retrieves items from the puzzle system via the input/output point in the least number of moves. Models involving multiple empty locations are more complex and some experimental results are reported. The authors describe their contribution as twofold: "First, we develop results for puzzle-based storage systems, including an optimal algorithm for systems having a single open cell. We also consider puzzle-based systems with more than one empty cell, and show that expected retrieval time decreases with the number of empty cells. Second, we describe the relationship between storage density and expected retrieval time for both puzzle- and aisle-based systems" (Gue & Kim, 2007). Though not addressed in this paper, the puzzle-based problem is an excellent candidate for a case-study on the propagation of uncertainty throughout a dense storage system. Such a system embodies the characteristics of very high density systems, particularly the need to shift non-requested items in order to retrieve those that have been requested. The algorithm for retrieving items can be interpreted as a retrieval policy involving rules that govern the movements of items in the system. With this knowledge, the location probabilities of all items can be quantified and updated as retrievals are happening in the system. The analysis of puzzle-based systems in this work provides a starting point or framework for gaining insight into how uncertainty can grow and spread throughout a storage system.

De Koster et al. (De Koster, Le-Duc, & Yugang, 2008) analyze a three-dimensional compact automated storage and retrieval system that uses an automated crane to move pallets both vertically and horizontally, while a gravity or powered mechanism performs depth movement. Their work determines the optimal dimensions for such a system that minimize travel time. They also develop a closed-form expression for the expected retrieval time for single-command cycles and an approximation for dual-command cycles. This work also demonstrates the common assumption among work involving compact storage systems that item locations are known with certainty. Automated systems may have less of a need to deal with uncertainty as movements are likely tracked with a high degree of certainty. However, in general, the very nature of dense storage creates the possibility of uncertainty in item location. The goal of this paper is to minimize travel time in storage and retrieval operations in the system but search time is not considered a factor in the model.

Yu and De Koster (Yu & De Koster, 2012) consider storage and retrieval operations in a three-dimensional compact automated warehousing system, a type of high density storage environment. With the objective of minimizing the makespan, the authors use sequencing heuristics to sequence a block of storage and retrieval requests in a multi-deep automated storage system. A new heuristic is proposed called Percentage Priority to Retrievals with Shortest Leg (PRR-SL). PRR-SL is shown to perform better than all other current heuristics and surpasses First-Come-First-Served by between 20 and 70 percent. This work again assumes certainty in item location meaning that none of the retrieval time is spent searching. With imperfect knowledge of item locations, expected retrieval times will increase due to the additional element of search time. And that element may grow over time thus continuously altering the optimal retrieval strategy in the system.

De Castillo and Daganzo (de Castillo & Daganzo, 1993) discuss storage systems that involve the stacking of goods and operations that determine required handling effort. With an emphasis on container logistics at marine terminals, the authors develop methods for evaluating the effort required for two distinct handling policies. One strategy aims at maintaining an equal stack height in the system while the other separates containers based on their arrival time. This work is related to the problem of dense storage. Many goods are stored in a relatively tight space and this creates challenges in terms of material handling. The methods presented for handling the containers may be modified to fit different kinds of systems, including dense storage environments on naval ships. The authors do not aim to address the possibility of uncertainty in the location of the storage containers which is a problem that is seen in similar systems as demonstrated by Ergen et al. (Ergen et al., 2007). The work presented in this paper can be extended by incorporating uncertainty into the material handling policies and determining how searching impacts the optimal retrieval policies.

In the field of graph theory, Fujie (Fujie, 2003) uses a branch-and-bound algorithm to solve the maximum leaf spanning tree problem (MLSTP). In the MLSTP, given an undirected graph of n nodes, the goal is to find a spanning tree with the maximum possible number of leaves. An upper bound is presented which was found by solving a minimum tree problem. This work gives insight into how dense storage systems can be designed to maximize storage density. Each storage location can be represented by a leaf in a maximum leaf spanning tree problem and the solution containing the maximum number of leaves is equivalent to the maximum number of storage locations in a given system. This work is mentioned only because of its loose relationship to high density layouts.

Iranpour and Tung (Iranpour & Tung, 1989) developed a new approach to the optimal design of a parking lot. The model is a system of nonlinear equations and is solved by iteration for a rectangular corner lot. Input involves data that consists of combinations of both compact and standard cars. In cases where multiple layouts had the same maximum capacity, a factor called Ease is derived. Ease takes into account several variables that contribute to the overall easiness of parking in a given lot. This work has implications for dense storage system design so it is not very closely related. The methods discussed in this paper also give insight into how objects of different dimensions may interact and affect the performance of the system. In this work, the goal is to maximize capacity given that the parking spaces are identical but the objects traveling throughout the system and parking in those spaces are not identical.

Uncertainty in the location of the cars is not considered in this work, nor is it necessary since the goal is simply system design and there is no explicit possibility of searching for objects in the parking lot.

Kim and Kim (Kim & Kim, 2002) seek to determine both the optimal number of storage spaces as well as the optimal number of transfer cranes for the movement of import containers. Costs are considered in the model including space cost, investment in cranes, and operating costs of the cranes and trucks. A deterministic model is presented which aims at minimizing only the terminal operator cost and a stochastic model is presented which minimizes both the terminal operator cost as well as the cost to customers. Although these systems are not described as high density systems, this work has implications for the design and operation of dense storage systems. Determining what spaces are to be allocated for storing items is critical as is determining the optimal amount of material handling resources such as cranes and forklifts. Absent from the work is the consideration of possible search time. Uncertainty in item location is not in the scope of this paper. In adding the element of uncertainty to the model, the results of this work in terms of the number of storage spaces and the number of transfer cranes may be impacted.

Flake and Baum (Flake & Baum, 2002) consider the children's game *Rush Hour* in which cars can only move either vertically or horizontally in a given space that is largely occupied by other vehicles. The goal is to perform a series of allowed moves in order to get the target car out of the system. The authors demonstrate that determining whether or not the target car can exit the system is PSPACE-complete using a version of dual-rail reversible logic. No papers relate the *Rush Hour* problem directly to the design and operation of dense storage systems, nor do they consider uncertainty in location a factor in the problem.

Degano and Di Febbraro (Degano & Di Febbraro, 2001) consider an intermodal terminal where containers are the only means by which freight arrives and departs. Different areas of the terminal are distinguished by the type of handling operation performed in each. Since the material transportation system is partly automated, the model complexity is increased and a Petri net model is developed in which unmanned vehicles are performing the container handling activities. This work is related to material handling problems in dense storage environments. Container terminals function like larger-scale dense storage systems in that containers frequently require relocating in order to access containers that are requested for some kind of retrieval. The authors do not consider search time to be an element in the model as it is assumed that the correct locations of the containers are known with certainty throughout operation of the system. However, in practice, dense storage systems often see some level of uncertainty in item location due to the nature of the system. The output of the model may differ significantly if a portion of the material handling effort is spent searching for the correct container. Even the operating policies that govern movements in the system may need to be adjusted in response to uncertainty in the system. Also of value is the consideration of how uncertainty develops in a system. This work could be expanded to observe how uncertainty in container locations grows and spreads throughout the system as movements are happening.

Search Theory

In this section, the papers presented give an account of the perspective of the search theory literature towards the growth of uncertainty in various systems. Currently, search theory is rich in models that solve the problem of finding a target object under conditions of uncertainty in terms of its physical location. No papers have been found that apply the concepts of uncertainty and searching to dense storage environments.

However, many of the works discussed do incorporate the concept of search effort in their models. This is analogous to the effort involved in relocating items in dense storage systems. Thus, search theory has vital implications for work that analyzes the impact of uncertainty in dense storage. Generally in search theory, there is an underlying probability distribution that describes the likelihood of the target being found in particular places within the system. In search theory, this probability distribution is typically given and no work has been found that considers the origin and subsequent growth of uncertainty in the system.

De Guenin (De Guenin, 1961) develops a method for solving the search problem without a specific probability distribution but instead in the general case. As in most search theory problems, a certain amount of effort or resources is expended in searching for a desired item within a system. There is usually some underlying distribution that determines the probability of detection as searching occurs. This work develops a framework which can be applied to the searching problem for a system that has obtained any location probability distribution. This work, however, does not consider the development of the probability distribution. A more general case may be developed that captures how uncertainty propagates throughout a system in response to decisions made and actions taken. The search problem then can incorporate the dynamic nature of the uncertainty and provide a robust search methodology for a broader category of systems.

Arkin's (Arkin, 1964) work is similar to what is presented in most papers in the field of search theory. There is an a priori probability distribution associated with the system and some strategy involved which determines the resources that are spent while searching for the item. This work is not applied to a dense storage system under uncertainty. Also, this paper assumes that the probability distribution is already present with no consideration as to how such uncertainty developed. In the problem of dense storage and uncertainty of item location, a key factor is the change in the location probabilities over time.

Assaf and Zamir (Assaf & Zamir, 1985) determine optimal strategies for sequential searching which aims at locating one item among *m* boxes. Uncertainty is considered in terms of the distribution of the *n* items among the boxes and a Bayesian approach is used. Results show that the uncertainty, even when small, may cause the optimal strategy to stray far from the classical model which does not involve the uncertainty distribution. This work has some similarity to uncertainty in item location in dense storage systems. In a dense storage system there may be knowledge of the location probabilities associated with each item and the procedures discussed in this paper may impact the optimal sequential searching strategy. However, the distribution is assumed from the beginning of the problem which may not be the case in some systems. In some cases, the distribution may develop over time as the system is in operation. This work is interested in simply prescribing a search method once a probability distribution is present. A possible extension may be to consider how to search for the item among the boxes when the uncertainty distribution is changing over time and in response to which locations have previously been searched.

Beck (Beck, 1964) considers the linear search problem. A man in a vehicle is searching for another individual who is at some other unknown point. The probabilities of the second individual being at every possible location on the road are known by the first man from the beginning. The problem tries to determine when the optimal search path with the minimal expected travel distance can be achieved. The probabilities associated with the target being at each possible position are assumed a priori. The goal of this paper is not to describe how the probability distribution developed, nor are the methods directly related to dense storage systems. This problem may be extended to consider how to optimally search in an environment where the uncertainty is changing with time. The problem must take into account the possibility that the location chosen to be searched next may have an updated probability that leaves that location no longer favorable for searching.

Beck and Beck (Beck & Beck, 1984) consider some particular variations on the general linear search problem. For the same problem in which a man is searching for another individual with knowledge of all of the possible location probabilities, the authors consider three possible scenarios involving different a priori distributions regarding where the target is located. Those distributions are: uniform on an interval, triangular around the original point, and normal about that same point. They give results for both the uniform and triangular distributions. This work gives insight into the impact of the initial probability distribution on the optimal sequential search path. Again, a possible extension is to consider a distribution that is developing in response to changes in the system.

Markov Chains

Some work involving applications of Markov chains is discussed here due to their applicability to the problem of quantifying uncertainty in dense storage systems over time. Markov chains give the probabilities of systems transitioning from one state to another when there may be several possible states. Markov models may be discrete, changing in individual steps, or continuous, changing continuously over time. They also may be either finite or infinite meaning that the number of possible states are either finite or infinite.

The papers discussed here involve the problem of determining the transition matrix of a Markov process, locating a target object such as a robot, and identifying the steady state distribution and behavior of a system. All of these problems have implications for uncertainty in dense storage systems, yet none address that problem directly.

Craig and Sendi (Craig & Sendi, 2002) develop methods to determine the maximum likelihood estimate of a Markov chain transition matrix involving the progression of a disease. They estimate the transition matrix for when the cycle length coincides with observation times, when it does not, and when the observations are performed in unequal intervals. Methods to evaluate the uncertainty of the transition matrix are discussed. This work is fairly unrelated to dense storage systems and the uncertainty there within.

Fox (Fox, 1998) addresses problems identified in the efficient and safe navigation of indoor mobile robots. Self-localization is considered specifically as well as how to better avoid collisions during self-localization. Fox uses Markov localization, a special case of Markov state estimation. This work relates the concept of uncertainty in item location and the use of Markov chains. The goal is for a robot to determine its own location rather than to maintain knowledge of the locations of items in a storage system.

Quah (Quah, 1993) considers the determinants of long-term economic growth and investigates if per capita income approaches a steady state growth path. The methods have similar implications as those involved in determining the steady state location probabilities in high density storage environments. Certain metrics that characterize the system, whether economic or storage, impact the probabilities of transitioning between states and, therefore, the nature of the steady state behavior.

Research Gaps

As shown in this chapter, there have been a few research efforts that are analogous to the problem of locating items in a dense storage system when there is uncertainty in item location. Some of the work involves dealing with uncertainty and aims at reducing or eliminating it, but very few of the papers deal with uncertainty in terms of item location. And when the uncertainty is related to the location of stored items, the uncertainty is either left unquantified or the probability distribution is assumed a priori. There is no analytical evaluation of how uncertainty can grow throughout a system in response to changes happening over time.

Other work is directly related to dense storage systems or sea-basing. Here the effort has been to design better storage systems given some objective such as maximizing storage density. None of the papers that have design implications consider uncertainty a factor in the design process. Future work should include the possibility of uncertainty as a factor in storage system design so that the system might have built-in defenses against the impact of uncertainty. A few of the papers dealing with dense storage are concerned with the optimal operation of the system, such as algorithms for retrieving items from a puzzle-based system. However, these papers do not consider uncertainty in item location as a factor in their methodology. All locations are assumed to be known with certainty and therefore no searching is necessary. Because dense storage systems often require movements of items that are not requested, the possibility of loss of knowledge of item location is a reality. Search time should be considered as an element in the methods described for designing operation policies in dense storage systems, whether in a container terminal or in a compact storage and retrieval system.

In the field of search theory, researchers do consider the need to search for targets when their location is not known with certainty. Even the search effort is evaluated in terms of expended resources such as manpower and time. However, just as in the work involving uncertainty, there is no analysis of changing uncertainty over time. The probability distributions are assumed to be present from the beginning. No work explores the growth and spread of uncertainty in the system. In dense storage, this is exactly what happens. Because items are moving frequently in response to requests, knowledge of their locations is lost over time. Future work should include search mechanisms that deal with changing uncertainty in response to the operation of the system.

Another field discussed in this chapter is Markov chain theory. Markov chains are discussed here because of their excellent applicability to the problem of uncertainty propagation throughout a system. A Markov chain can be used to easily quantify the level of uncertainty at any time in a system as it is transitioning between states. These papers mentioned are identified simply to show that Markov chains have not yet been applied directly to dense storage systems. The changing uncertainty in a dense storage environment can be captured and analyzed through the use of Markov chains and then can be used as an input into other models with different goals, such as constructing a dense storage layout, designing storage and retrieval policies, and developing searching methods to deal with the uncertainty.

CHAPTER THREE: PROBLEM STATEMENT

The objective of this research is to determine how uncertainty can permeate throughout a dense storage environment due to the unique characteristics present in a high density system. As has already been noted, a very high density storage system is one in which not every storage unit is immediately accessible for picking or retrieval. This means that in order to retrieve certain units, other units must be relocated so that the requested unit can be accessed. This characteristic makes asset tracking more difficult as there is more information that must be maintained than just what unit is being retrieved. Each retrieval may affect where the next requested unit can be found. This work aims at providing a mathematical foundation to describe how uncertainty can originate in such a system, quantifying that uncertainty, and demonstrating how it can spread throughout the system given a set of rules governing how units are rearranged during retrievals.

The model presented in this paper, though general, is applied to a certain kind of dense storage system. This work is closely related to the NAVY's maritime logistic operation known as sea-basing, which provides the operational and tactical sustainment of forces from the sea. Sea-basing requires the ability to deliver emergent requests for tailored resupply packages by selectively offloading cargo stored in holds of ships. These ships contain storage holds that can be characterized as very high density systems. These storage holds are currently operated manually with workers using forklifts to store, retrieve, and relocate pallets and containers. Each ship generates a load plan that, ideally, is followed perfectly, giving initial certainty in unit locations. Pallets and containers are packed in tightly, frequently stacked and often without aisles, in order to maximize storage capacity. Once in operation, the ships will receive orders requesting specific units and quantities to be retrieved. This need to retrieve specific units, perhaps even ones located in the most inconveniently placed location, is known as selective offloading and it is analogous to the concept of order fulfillment in the warehousing literature. This is the kind of system being analyzed in this study. The model and results give insights that are applicable to other variations of dense storage but they are identified via the analysis of systems such as those found on NAVY ships.

One aim in describing the growth of uncertainty throughout these kinds of systems is to determine the impact of a given retrieval policy on that growth. The dense storage system is assumed to have defined retrieval policies that govern how all storage units move in response to requests in the system. Since the relocation of unrequested units is the source of uncertainty under investigation, the specific rules of the retrieval policy are expected to have the greatest impact on the development of uncertainty throughout the system. This impact is evaluated by comparing the retrieval policy, in terms of the metrics defined below, with the computed location probabilities as retrievals occur in the system.

Another objective of this work is to quantify the effect of uncertainty on expected search time. With growing uncertainty in unit locations, the average time wasted in searching for a requested unit will naturally increase. This paper presents a simple model that calculates the expected search time for a particular system that has reached a steady state uncertainty distribution.

In this work, a dense storage system is modeled as a simple Markov chain in which the state space consists of the various storage locations in the system. Thus, the state of a given unit is the location it is in. The unit transitions to another state in accordance with the given retrieval policy. The inputs of the model include the number of storage locations, assumed equal to the number of storage units, and the retrieval policy. The expected search time model requires parameters that define how long specific elements of the searching process take, such as moving between adjacent pallet stacks. The uncertainty of each unit's location is quantified after each retrieval in terms of the probabilities that each unit is located in each possible storage location. A goal of the model is to identify any steady state behavior in the changing uncertainty or any patterns that give insight into the problem.

CHAPTER FOUR: METHODOLOGY

Uncertainty Propagation Model

In order to observe how uncertainty grows in a dense storage system and quantify the uncertainty at a particular time, the storage system must be defined. To do so, we denote the following sets:

- Storage Locations, L, indexed on , $l = 1,2, \ldots, |L|$
- Retrieval Requests, T, indexed on , $t = 0, 1,2, \ldots, |T|$

The set of storage locations are physical spaces used for storing the units in the dense storage system. We consider a storage system in which units are stored in unit loads (e.g., on pallets or in containers). These storage units usually contain multiple copies of a particular item (i.e., multiple cases of an item are stored on a pallet), the model receives retrieval requests in terms of the particular storage unit to be retrieved. We model such a system based on storage locations where units are mapped to the location where it originated. For example, a retrieval request is made for the unit that was initially in location 15.

Storage units may be stacked multiple units high and if they are stacked, we consider that each unit occupies a single storage location; e.g., a stack of five unit loads takes up five storage locations, not one.

The storage locations are stationary while the storage units move between different locations when retrievals occur. It is assumed that the number of storage units equals the number of locations and each location has exactly one unit assigned to it. Our model is derived from the number of locations in the dense storage system, and thus is general enough to handle storage systems that have varying physical dimensions, such as the length or width of the storage units and various storage configurations.

Assumptions

The following assumptions are made to provide the framework in which the model is based. They also provide further definition of the type of system to which the model is applied.

Time Is Related to Retrievals Made and Initial Certainty in Unit Location

We observe the system in discrete time units, specifically after each retrieval operation; we denote $t = n$ as the period prior to the $(n + 1)th$ retrieval. The initial state of the system $(t = 0)$ is assumed as one of complete certainty in that the location of each unit is known with a probability of one prior to the first retrieval. Note that time is directly connected to the number of retrievals that have been made and therefore does not express how much real time has passed with or without retrievals being made.

Random Storage

The storage system operates under a random storage policy. This means that there are no restrictions placed on which locations a particular unit can be stored in. There are also class-based systems in which certain sections or areas within the system are dedicated strictly to some particular group of items. Those items designated to a particular class, however, may be stored randomly within that class. Therefore, our models are also applicable to studying a class-based system, where storage and retrieval operations for separate classes of items are independent of one another.

This stands in contrast to a dedicated storage system in which each unit has a designated location and cannot be stored in a location designated to a different unit. A dedicated storage system, if followed perfectly, would never produce uncertainty in the location of units. Since the system begins with initial certainty and each storage unit can never change location, the probability that a unit is in its dedicated location will always be one.

Non-Depleting System

The storage system considered in the model is non-depleting, meaning that the storage units in which items are stored never leave the system. While each storage location has only one type of unit at any time, it is assumed that no retrieval request will demand the entire contents of the unit load. For example, a particular unit will be requested at the case-level, the pallet containing the requested unit will be accessed, cases will be retrieved from the pallet, and the pallet with the remaining cases will be returned to the storage system, although possibly in a new location. Therefore, the propagation of uncertainty is analyzed through a series of retrieval operations only. The addition of storage operations is an interest of future research. The number of storage units and the number of storage locations are equal. This means that all storage locations in the system contain a unit load at all times. Such an environment is applicable in sea-basing applications where a cargo ship may function as a distribution center and provide goods to land-based customers in various locations. The ship may resupply its inventory fairly infrequently. In between those stages of resupplying, the ship will receive only retrieval requests from customers. This paper is interested in the growth of uncertainty under these conditions.

Constant Uniform Demand for All Units

We assume that each unit has an independent and identically distributed demand probability for how likely the unit is to be requested. This paper considers the case of uniform demand for all units and assumes the demand is stationary.

Unit Retrieved History Is Unknown

The unit retrieval history is not kept and thus unknown. After a given number of retrievals, all that is known is the current level of uncertainty. The model is intended to quantify the uncertainty in the system after a given number of retrievals. Since the operation of the system is a Markov chain, the memoryless property of Markov chains is present, meaning that the next state of the system is dependent only on the current state and is independent of the history prior to the current state.

Retrieval Policy Is Unambiguous

We model a system that assumes retrieval policies, which govern all unit movements in the system, are unambiguous and that all possible movements are clearly defined and their probabilities are known. Our model can handle both pure strategies and probabilistic strategies, as long as the probability of using each policy and the movements associated with each policy are clearly defined. We denote a pure strategy as one in which each location has only one possible reconfiguration associated with it and it is executed with certainty when that location is accessed for a retrieval. A probabilistic strategy is one in which a retrieval from one or more locations can be executed in more than one way but the probabilities of each occurring is known.

Location Probability Matrix

One of the functions of this model is to quantify the uncertainty in unit location after any number of retrieval operations. To do this, the probabilities associated with the location of each storage unit must be expressed and also updated. The Location Probability Matrix $\boldsymbol{P_t}$ gives the probabilities of each storage unit being in each location within the system after t retrievals. Therefore, an individual entry $p_{tll'}$ in $\boldsymbol{P_t}$ is the probability that the unit originally in location l is in location l' after t retrievals.

Initially, at $t = 0$, all of the unit locations are known with certainty. In $\boldsymbol{P_t}$, the storage units are identified by the number corresponding to their initial locations. Without loss of generality, we assume the first unit starts in location one, the second in location two, and so on. Thus, prior to the first retrieval, $\boldsymbol{P_0}$ is equivalent to the identity matrix. $\boldsymbol{P_t}$ is then updated according to the number of retrievals t that have occurred based on the given retrieval policy.

Retrieval Policy

In order to update $\boldsymbol{P_t}$, the movements that may take place must be defined by what is called the retrieval policy. This policy governs all unit movements in the system. It may be a single general rule applied to the whole system or a set of steps that are particular to different areas of the system, but followed the same way for every retrieval request (i.e., a pure strategy). Alternatively, different rules can be applied with varying probabilities (i.e., a probabilistic strategy). Essentially, with the retrieval policy defined a user can, for any particular storage location, determine how all of the units in the system will or will not move if a unit is retrieved from that location. An illustrated example of a pure strategy is given below in Figures 1 and 2.

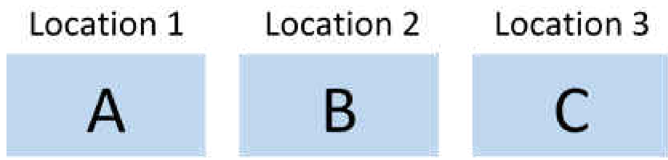

Figure 1: Retrieval Policy Example at t = 0

Units A, B, and C are occupying three locations. The policy states that any unit accessed for a retrieval moves to the end of the row (Location 3) and all units previously to the right of the retrieved item shift left to fill the gap created. Thus if A is retrieved, then according to the policy, the locations will contain the storage units as seen in Figure 2 following the retrieval.

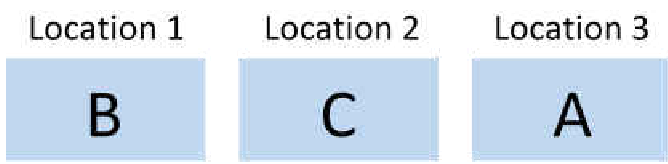

Figure 2: Retrieval Policy Example at t = 1

So the purpose of the retrieval policy is to define what will happen to the units stored in locations 2 and 3, and any others in the system, if a unit in location 1 is retrieved.

The retrieval policy can also be expressed mathematically. The policy determines what movements will take place in the system for any location from which a retrieval is made. So each location $l.$ is assigned a retrieval matrix $\boldsymbol{R}_{l.}$ that expresses the movements executed when that location is accessed, meaning that the desired unit was found in that location. Each $\boldsymbol{R}_{l.}$ is an $|L| \times |L|$ matrix and each entry $r_{l.ll'}$ in $\boldsymbol{R}_{l.}$ is the probability that the unit originating in location l moves to location l' when location $l.$ is accessed for a retrieval. The retrieval matrix $\boldsymbol{R}_{1.}$ for the example above is expressed below in Figure 3.

Location	**1**	**2**	**3**
1	0	0	1
2	1	0	0
3	0	1	0

Figure 3: Retrieval Policy Example Matrix

Movement Matrix

The retrieval policy, expressed through the $\boldsymbol{R}_{l.}$ matrices, defines all of the *possible* rearrangements of the storage units that could occur. But, individually they give no information regarding which movements are *probable*. There must be information that quantifies the probability that each storage location is accessed for a retrieval. The given demand for the units provides that information. The demand for all units can be expressed as the vector $\boldsymbol{D}$ where each value d_l is the demand for the storage unit that originated in location l.

Consider a system in which there are n locations, with each location containing one storage unit. To calculate the probability that location $l.$ is accessed for a retrieval, a vector product is taken between the $l.$th column vector of $\boldsymbol{P_t}$ (associated with location $l.$) and the demand $\boldsymbol{D}$.

$$P(\boldsymbol{R}_{l.}) = \langle \boldsymbol{p_{tl.}}, \boldsymbol{D} \rangle = \sum_{l=1}^{n} p_{tll.} d_l \qquad (1)$$

$P(\boldsymbol{R}_{l.})$ – probability that location $l.$ is accessed for a retrieval

$\boldsymbol{p_{tl.}}$ – $l.$th column vector of $\boldsymbol{P_t}$

$p_{tll.}$ – probability that unit starting in l is in location $l.$ after t retrievals

Because the demand is uniform, the likelihood of each location being accessed remains at a constant value. If the demand is constant and uniform, then $d_l = \frac{1}{n}$ for the unit that originated in each location. Thus, the probability that location $l.$ is accessed for a retrieval becomes:

$$P(\boldsymbol{R}_{l.}) = \langle \boldsymbol{p_{tl.}}, \boldsymbol{D} \rangle = \sum_{l=1}^{n} p_{tll.} d_l = \frac{1}{n} \sum_{l=1}^{n} p_{tll.} = \frac{1}{n} \qquad (2)$$

With uniform demand, the probability of any location being accessed for a retrieval remains constant and is also uniform for all locations in the system.

Now the Location Probability Matrix $\boldsymbol{P_t}$ can be altered according to the number of retrievals that have occurred and the expected movements that were executed in the system. Consider the first retrieval.

Though the specific unit requested is unknown, the *expected* changes or movements in the system can be determined. The expected movements can be expressed in the transition matrix, called the Movement Matrix $\boldsymbol{M}$. $\boldsymbol{M}$ is simply a matrix of the probabilities of each possible relocation and is defined as:

$$\boldsymbol{M} = \sum_{l.=1}^{n} P(\boldsymbol{R}_{l.}) * \boldsymbol{R}_{l.} \tag{3}$$

Because the uniform demand causes each $P(\boldsymbol{R}_{l.})$ to remain constant and uniform, the Movement Matrix becomes:

$$\boldsymbol{M} = \sum_{l.=1}^{n} \boldsymbol{P}(\boldsymbol{R}_{l.}) * \boldsymbol{R}_{l.} = \frac{1}{n} \sum_{l.=1}^{n} \boldsymbol{R}_{l.} \tag{4}$$

Thus, $\boldsymbol{M}$ remains constant because each $\boldsymbol{R}_{l.}$ remains constant. As the transition matrix in the Markov chain, $\boldsymbol{M}$, determines the likelihood of each possible state or location change in the system. The Location Probability Matrix $\boldsymbol{P_t}$ is multiplied by $\boldsymbol{M}$ for each retrieval, changing t to $t+1$, and expressing the updated uncertainty in the system. It is easy to show that $\boldsymbol{M}$ is always doubly stochastic, meaning that the values in each row, as well as in each column, sum to one. In every $\boldsymbol{R}_{l.}$ each column and row must sum to one. Since $\boldsymbol{M}$ is the expected value of these retrieval matrices, each column and row of $\boldsymbol{M}$ must also sum to one.

Markov Chain

Thus, $\boldsymbol{M}$ is a stochastic, or Markov matrix. At $t = 0$, all storage unit locations are known with certainty. After each retrieval, $\boldsymbol{P_t}$ is multiplied by $\boldsymbol{M}$ and t is increased by one. The pattern is shown below. $\boldsymbol{I}$ is the identity matrix.

$$\boldsymbol{t=0} \qquad \boldsymbol{P_0 = I} \tag{5}$$

$$\boldsymbol{t=1} \qquad \boldsymbol{P_1 = P_0 M = M} \tag{6}$$

$$\boldsymbol{t=2} \qquad \boldsymbol{P_2 = P_1 M = M^2} \tag{7}$$

$$\boldsymbol{t=3} \qquad \boldsymbol{P_3 = P_2 M = M^3} \tag{8}$$

$$\boldsymbol{t=n} \qquad \boldsymbol{P_n = P_{n-1} M = M^{n-1} M = M^n} \tag{9}$$

The ll'th entry $m_{ll'}^{(t)}$ of the matrix $\boldsymbol{M^t}$ gives the probability that the unit originally in location l will be in location l' after t retrievals.

CHAPTER FIVE: RESULTS

Steady State Location Probabilities

By defining a dense storage system with a particular retrieval policy such that the Movement Matrix $\boldsymbol{M}$ can be determined, the probability that each unit is in any location can be quantified after any number of retrievals. But another interesting question is what distribution, if distinguishable, is approached after a large number of retrievals, or when the system is in steady state. To determine the steady state probabilities of a system, one can take advantage of the Markovian properties of the model.

Assume that a particular system has n storage locations and any unit can reach any of the locations within the system. The system is then said to have only one communication class C_1 and is an irreducible Markov chain. A communication class is a finite set of states, or storage locations, in which any state within that class can reach any other state in the same class. If there is only one communication class, then that set of locations is equivalent to the set of all locations in the system.

$$C_1 = L = \{1,2, \dots, n\} \tag{10}$$

The Fundamental Theorem of Markov Chains states that for any irreducible, aperiodic, positive-recurrent Markov Chain, there exists a unique stationary distribution $\boldsymbol{\pi}$ (Puterman, 2014). $\boldsymbol{\pi}$ is a row vector that is equivalent to every row in $\boldsymbol{P_t}$ when the system is in steady state. It is the probability distribution that is approached as the number of retrievals increases to infinity.

$$\left\{\boldsymbol{\pi} \;\middle|\; \boldsymbol{\pi M} = \boldsymbol{\pi},\;\; \sum_{l'=1}^{n} \pi_{l'} = 1\right\} \tag{11}$$

$\pi_{l'}$ – steady state probability of any unit being in location l'

The conditions expressed above are derived from Markov Chain principles. The first implies that once the system has reached the steady state distribution $\boldsymbol{\pi}$, it will remain in that state, even after subsequent retrievals. The second maintains that the sum of state probabilities is one.

Because $\boldsymbol{M}$ is always doubly stochastic, the steady state probability distribution $\boldsymbol{\pi}$ will be uniform for any irreducible, or entirely communicable, system. To demonstrate this, the first condition for $\boldsymbol{\pi}$ gives the following.

$$\sum_{l=1}^{n} \pi_l \, m_{ll'} = \pi_{l'} \;\; \forall \, l' \in L \qquad (12)$$

The uniform solution implies that all steady state probabilities in $\boldsymbol{\pi}$ are equal; thus, $\pi_l = \pi_{l'}$ for all locations. The following equation demonstrates that the uniform solution satisfies $\boldsymbol{\pi M} = \boldsymbol{\pi}$ when $\boldsymbol{M}$ is doubly stochastic.

$$\pi_{l'} = \sum_{l=1}^{n} \pi_l \, m_{ll'} = \sum_{l=1}^{n} \pi_{l'} \, m_{ll'} = \pi_{l'} \sum_{l=1}^{n} m_{ll'} = \pi_{l'} \;\; \forall \, l' \in L \qquad (13)$$

By the Fundamental Theorem of Markov Chains, the uniform solution must also be unique. Because the sum of the steady state probabilities must be one, the result is $\frac{1}{n}$ for each $\pi_{l'}$.

$$\sum_{l'=1}^{n} \pi_{l'} = n\pi_{l'} = 1 \qquad (14)$$

$$\pi_{l'} = \frac{1}{n} \; \forall \, l' \in L \qquad (15)$$

By saying that the steady state solution for an irreducible system is uniform, or $\pi_{l'} = \frac{1}{n}$, what is meant is that in steady state the probability of any particular unit being in any location l' is equal for all locations. The unit is equally likely to be anywhere in the system. This result, though not counterintuitive, is profound, demonstrating that for any set of communicating location states, knowledge dissipates with retrievals and steady state is its complete absence. There may be systems, however, in which not all of the storage locations communicate. Take for example a system of six locations, which may actually have two or more communication classes. Depending on the retrieval policy a system may be defined as follows.

$$L = C_1 \cup C_2 \qquad (16)$$

$$C_1 = \{1,2,5\}, C_2 = \{3,4,6\} \qquad (17)$$

Or

$$L = C_1 \cup C_2 \cup C_3 \quad (18)$$

$$C_1 = \{1\}, C_2 = \{2,3,5\}, C_3 = \{4,6\} \quad (19)$$

A system that has more than one communication class represents a reducible Markov process, meaning that it can be broken down into irreducible parts. Because the states have no interaction with states outside of their own class, each class behaves as a separate Markov process. Even when a system has multiple communication classes, the uniform solution holds within each class C_j. Suppose a system has n storage locations in L that are separated into γ communication classes. Let $\boldsymbol{\pi_j}$ be the steady state probability vector for communication class C_j, meaning that each location in C_j will take on $\boldsymbol{\pi_j}$ as its steady state distribution.

$$L = \bigcup_{j=1}^{\gamma} C_j, \quad \bigcap_{j=1}^{\gamma} C_j = \emptyset \quad (20)$$

$$\boldsymbol{\pi_j} = \left\{\pi_{j_1}, \pi_{j_2}, \ldots, \pi_{j_{|C_j|}}\right\} \quad j = 1,2, \ldots, \gamma \quad (21)$$

Note that here $\boldsymbol{\pi_j}$ has only as many entries as there are locations in C_j. This is important simply to demonstrate here that the uniform solution holds within each class. The vector $\boldsymbol{\pi_j}$ is only a subset of row j in $\boldsymbol{P}_\infty$ and it contains only the entries in that row which represent locations that are within communication class j. The remaining entries in row j are always zero because the probability of any unit moving to a location outside of its communication class is zero by definition. The individual entries in $\boldsymbol{\pi_j}$ are calculated the same way as $\boldsymbol{\pi}$ above except using a condensed form of the Movement Matrix $\boldsymbol{M}$. This matrix $\boldsymbol{M_j}$ only includes the rows and columns of $\boldsymbol{M}$ that are associated with class C_j; it is therefore a $|C_j| \times |C_j|$ doubly stochastic matrix. The rows and columns outside of class j can be ignored because all classes behave completely independent of each other, even when they are present in the same matrix. Here the other classes are excluded from the steady state calculations.

$$\sum_{l=1}^{|C_j|} \pi_{jl}\, m_{jll'} = \pi_{jl'} \;\; \forall\, l' \in C_j, j = 1,2, \dots, \gamma \qquad (22)$$

Applying uniformity for $\boldsymbol{\pi_j}$ provides the unique solution associated with each communication class.

$$\pi_{jl'} = \sum_{l=1}^{|C_j|} \pi_{jl}\, m_{jll'} = \sum_{l=1}^{|C_j|} \pi_{jl'}\, m_{jll'} = \pi_{jl'} \sum_{l=1}^{|C_j|} m_{jll'} = \pi_{jl'} \;\; \forall\, l' \in C_j, j = 1,2, \dots, \gamma \qquad (23)$$

$$\sum_{l'=1}^{|C_j|} \pi_{jl'} = |C_j| \pi_{jl'} = 1 \qquad (24)$$

$$\pi_{jl'} = \frac{1}{|C_j|} \;\forall\, l' \in C_j, j = 1,2, \dots, \gamma \qquad (25)$$

For any dense storage system as described in this model, the location probabilities within each communication class will approach uniform, the complete loss of location knowledge, according to the number of locations within that communication class. In steady state, the rows of the Location Probability Matrix $\boldsymbol{P_t}$ each contain the stationary distribution $\boldsymbol{\pi_j}$ according to the communication class each row belongs to. For example, in the system of the three classes, $C_1 = \{1\}, C_2 = \{2,3,5\}$, and $C_3 = \{4,6\}$, as t becomes large, $\boldsymbol{P_t}$ will approach the distribution displayed in Figure 4. The zero entries represent probabilities between locations of different communication classes.

	1	2	3	4	5	6
1	1	0	0	0	0	0
2	0	0.333	0.333	0	0.333	0
3	0	0.333	0.333	0	0.333	0
4	0	0	0	0.5	0	0.5
5	0	0.333	0.333	0	0.333	0
6	0	0	0	0.5	0	0.5

Figure 4: Three communication class steady state probability matrix

By rearranging the columns and rows into more visually distinct groups, the independence of the communication classes is more clearly seen in Figure 5.

	1	2	3	5	4	6
1	1	0	0	0	0	0
2	0	0.333	0.333	0.333	0	0
3	0	0.333	0.333	0.333	0	0
5	0	0.333	0.333	0.333	0	0
4	0	0	0	0	0.5	0.5
6	0	0	0	0	0.5	0.5

Figure 5: Probability matrix arranged by communication class

<u>Expected Search Time</u>

It has been shown that when a dense storage system with uniform demand reaches steady state, the location probabilities within each communication class approach the uniform distribution. This finding has implications for the operational efficiency of the storage system. It is obvious that any uncertainty in unit location at all can produce search time in a system. An operator may have to check several locations for a unit before finding it because of uncertainty in its location. With the location probabilities determined, the expected search time in steady state can be calculated for any unit.

Expected Locations to Search

The first step in quantifying an expected search time is determining how many locations an operator is likely to search through before locating the desired unit. This is the expected number of locations to search. The following expression gives the expected value for a system which has only one communication class.

$$E(SL_l) = \sum_{l'=1}^{n} l' * p_{\infty ll'} = \sum_{l'=1}^{n} l' * \pi_{l'} \qquad (26)$$

$E(SL_l)$ – expected number of locations to search for unit initially in l;

n – number of possible storage locations

l' – indicating the l'th storage location searched

Because in steady state the system approaches a uniform probability distribution, the probability of the requested unit being contained in any location is $\frac{1}{n}$ for all $l \in L$. Since the probability is constant, neither the sequence of searching nor the layout of the storage system affect the expected number of locations to search. Thus, the expression simplifies to the expected value of the uniform distribution with a maximum of n and minimum of one.

$$E(SL_l) = \frac{1}{n} * \sum_{l'=1}^{n} l' = \left(\frac{1}{n}\right)\left(\frac{n(n+1)}{2}\right) = \frac{n+1}{2} \qquad (27)$$

Clearly, an increase in possible locations n increases the expected number of locations that must be searched. However, this is under the assumption that there is only one location in which the unit of interest can be found. In a vast number of storage systems, there are multiple storage locations that contain copies of the same type of unit and only the first that can be located is needed. To see how multiple locations containing the same contents can affect the expected number of locations to search, the number of copies of unit i is designated as k_i. Prior to this point, the contents of each location were not considered as only the individual locations were needed in the model. With k_i, there is now some knowledge of the contents of the storage locations; there are k_i locations in the system that each contains an identical copy of storage unit i. Because the system has reached steady state, the k_i copies of unit i are equally likely to be found anywhere in the $|L|$ locations. Recalling that $|L| = n$, the probability of each location containing one of the k_i copies is $\frac{k_i}{n}$. Now the expected value expression becomes:

$$E(SL_l) = \sum_{l'=1}^{n-k_i+1} l' \frac{\binom{n-l'}{k_i-1}}{\binom{n}{k_i}} \qquad (28)$$

To explain the use of combinations for the probabilities, imagine that l' takes on a particular value while calculating the expression above. This represents the possibility that $(l' - 1)$ locations have been searched without success and the first of the k_i copies is then found in location l'. That leaves the $(k_i - 1)$ remaining copies distributed somewhere in the $(n - l')$ remaining possible locations, hence $\binom{n-l'}{k_i-1}$. Thus, the probability associated with each l' is the number of ways the remaining items can be located throughout the remaining locations divided by the number of ways all of the copies can be located throughout the entire system. The expression is then reduced to a conveniently simple form, the proof for which can be found in Appendix A.

$$E(SL_l) = \sum_{l'=1}^{n-k_i+1} l' \frac{\binom{n-l'}{k_i-1}}{\binom{n}{k_i}} = \frac{n+1}{k_i+1} \tag{29}$$

Expected Time to Search

Unlike the expression for the expected number of locations to search, the conversion into search time is highly dependent on the layout of the system and the actual searching process. For example, one system may consist of hundreds of stacks of pallets up to five pallets high while a different system may have only unstacked containers. Imagine an operator searching for an item within five pallets stacked on top of one another to determine if it is there. Then imagine the same operator searching through five unstacked containers for an item. It is easy to see that though in both cases the operator searches through five storage locations, the time spent searching will likely be very different in each scenario.

So to convert the expected locations to be searched into an expected search time, some parameters must be established that link the analytical results to the physical characteristics of different systems. Assuming a human operator, the parameters must address time spent moving between storage locations horizontally, as well as time spent checking each individual storage unit, even in stacks of pallets or containers. To estimate searching time, we assume that an operator can move between locations while searching without moving items out of the way. The proposed parameters are described here:

- T_h – Average time to move to next storage unit stack horizontally
- T_u – Average time to search a storage unit
- H – Average number of storage units in each stack of locations

These parameters must be specific to the system being analyzed and can be determined through some form of time study or historical data. Many organizations have some form of this information documented for standardization purposes. Using these parameters, the expected number of storage locations to search can be translated into search time using the expression below.

$$E(ST_l) = \frac{T_h}{H} * E(SL_l) + T_u * E(SL_l) = \left(\frac{T_h}{H} + T_u\right) E(SL_l) \qquad (30)$$

$E(ST_l)$ – expected time to search for unit initially in l

Metrics to Measure Uncertainty Behavior in Dense Storage Environments

One of the most common performance measures in the warehouse literature is the expected retrieval effort. We are aware of no literature that considers that item locations are uncertain when retrieving items. Therefore, we need to develop metrics that can measure uncertainty behavior in dense storage environments. We develop five such metrics which are applied to each location in the system and one metric that is a system measurement.

1. Probability a unit is found in location l_0 after t retrievals, where l_0 is the location that contained the unit at $t = 0$: P_{tl_0}

   This probability changes over time and is updated in $\boldsymbol{P_t}$ with each retrieval. For any given location, this measure is found in the diagonal entry for that location. Every unit originates from one location in the system. This is the probability of finding a unit in the location that it started in.

   $$P_{tl_0} = p_{tl_0 l_0} \qquad (31)$$

   $p_{tl_0 l_0}$ – probability that unit initially in l_0 is in location l_0 after t retrievals

   For clarity, P_{tl_0} is the denotation of this particular metric and $p_{tl_0 l_0}$ describes where this metric is located in $\boldsymbol{P_t}$.

2. Location stability measure for location l: σ_l

Unlike P_{tl_0}, this metric is constant over time and is a measure only of the retrieval policy as expressed in the Movement Matrix. This measure evaluates the spread and likelihood of the possible movements a unit may make from a given location. To calculate this measure for location l, take the standard deviation of the entries in row l of the Movement Matrix $\boldsymbol{M}$. More possible moves forces the probabilities to approach each other, dropping this measure, implying that it is a less stable location. The highest possible value this metric can attain occurs when one entry in the row is 1 and the rest are zeros, implying a high level of stability or predictability. This metric will be zero if all entries in the row are identical, or uniform. Such a case would imply that the movement from that location is entirely unpredictable and the location is unstable.

$$\sigma_l = \sqrt{\frac{\sum_{l'=1}^{n}(m_{ll'} - \bar{m}_l)^2}{n-1}} \qquad (32)$$

$m_{ll'}$ – ll'th entry in $\boldsymbol{M}$

$\bar{m}_l$ – mean of row l in $\boldsymbol{M}$

3. Probability a unit currently in location l will still be in l following the next retrieval: $P(\delta_0)_l$

Like σ_l, this metric is constant over time and comes from the retrieval policy. However, similarly to P_{tl_0}, this measure is retrieved from the diagonal entries of $\boldsymbol{M}$.

$$P(\delta_0)_l = m_{ll} \qquad (33)$$

When a unit enters location l, entry m_{ll} in the Movement Matrix is the probability that the unit will not move from that location with the next retrieval.

4. Number of retrievals for location l to approach steady state behavior: RSS_l

This is not a direct measure of the retrieval policy, nor does it change with each retrieval. This metric, applied to each location, is found through analytical trials to determine how many retrievals must occur before all of the location probabilities for a given unit fall within five percent of the steady state probability. This is essentially an indicator of how quickly the knowledge of a unit's location is lost. Approaching steady state more quickly means that the level of certainty of a unit's location is lost more quickly.

5. Expected number of locations to search for the unit that originated from location l: $E(SL_l)$

   This metric was defined in the Expected Search Time section. It is applicable to each location in the system once that location reaches steady state and it is a function of only the number of locations in the system and k_i.

$$E(SL_l) = \frac{n+1}{k_i+1} \quad (34)$$

6. Mean number of retrievals to steady state: μ_{RSS}

   This is a measure at the system level. For any dense storage system, it is the mean of all RSS_l for every location.

$$\mu_{RSS} = \frac{1}{|L|} \sum_{l=1}^{|L|} RSS_l \quad (35)$$

In the next section we compare these uncertainty measures with the expected retrieval effort for different system sizes, system dimensions, and training policies.

Experimental Results

We design several experiments to test the impact of the search policy, size of the system, system dimensions, and training level on different measures of uncertainty in dense storage systems. Three different search policies are used; two are pure strategies and the third is probabilistic. The first pure policy is an algorithm developed by Gue and Kim (Gue & Kim, 2007) that determines the minimum number of item movements required to retrieve a selected item from a high-density, puzzle-based storage system, assuming certainty about item locations. The second pure strategy is an alternative policy in which the initial movements of each item are in the direction opposite that of the original puzzle policy.

The third policy is a probabilistic composite intended to emulate training error in which the puzzle policy is executed 80 percent of the time and the alternative policy 20 percent of the time.

The experimental scenarios include seven alternative rectangular layouts, each with a different number of storage locations and a different ratio of length to width. Each layout has five columns of storage locations and each has an additional row from the previous layout, starting with two and ending with eight. The configurations are given below in Figure 6.

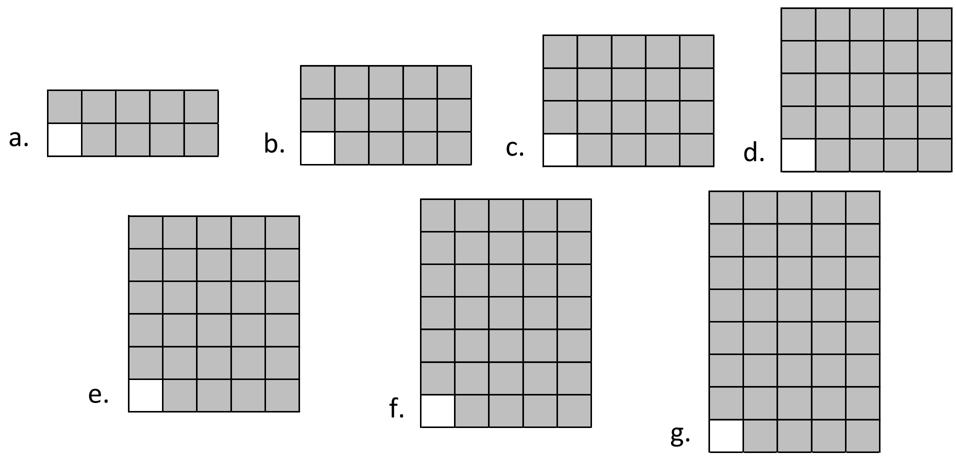

Figure 6: Experimental Layouts: a. 2 by 5 b. 3 by 5 c. 4 by 5 d. 5 by 5 e. 6 by 5 f. 7 by 5 g. 8 by 5

All three policies are used for test scenarios with each layout. The given policy is used to construct the Movement Matrix $\boldsymbol{M}$ and the Location Probability Matrix $\boldsymbol{P_t}$ is calculated for every number of retrievals until the system is sufficiently close to steady state. The movement matrix for each scenario can be found in Appendix B. Regardless of which of the three policies is used, each test configuration consists of only one communication class. Any individual scenario can be thought of either as an entire system which has only one communication class or as a single communication class within a larger system of multiple classes. According to the analytical result previously discussed, this means that for each case the location probabilities approach $\frac{1}{n}$ where n is the total number of storage locations in the given layout. However, by definition, it would take an infinite number of retrievals for the probabilities to all take on this exact value. For this reason, any location is considered to have approached steady state if the probabilities of its original contents being in each location all fall within $\frac{1}{n} \pm .05$, as stated in metric 4 above.

Retrievals to Steady State

The main output from these experiments is the number of retrievals performed until each location has approached steady state. The purpose is to describe the behavior of each individual location, or the rate at which each location approaches steady state, and whether it can be predicted using the uncertainty metrics. To do this, three of the uncertainty metrics are used: σ_l, $P(\delta_0)_l$, and RSS_l. In each test case these measures are determined for each location of the system. Table 1 gives an example result from the 3-by-5 system showing the location stability σ_l and the retrievals to steady state for each policy. The results from all scenarios are presented in Appendix C.

Table 1: 3x5 System stability and retrievals to steady state ascending by puzzle policy stability

3x5 system		Puzzle		Alternative		Training (80/20)	
Location	Distance to I/O	Stability	RSS	Stability	RSS	Stability	RSS
1	1	0.111	12	0.160	15	0.115	14
7	3	0.118	9	0.111	9	0.112	9
6	2	0.124	13	0.129	12	0.115	14
2	2	0.127	10	0.162	11	0.131	10
8	4	0.148	12	0.162	15	0.150	11
5	1	0.150	16	0.160	14	0.144	16
3	3	0.169	15	0.175	10	0.170	14
11	3	0.169	21	0.219	29	0.178	22
10	2	0.183	19	0.219	29	0.190	20
9	5	0.200	28	0.219	25	0.204	24
4	4	0.219	22	0.219	25	0.219	22
12	4	0.219	23	0.219	20	0.219	21
13	5	0.238	38	0.219	34	0.234	31
14	6	0.238	35	0.238	35	0.238	34

In the table, the results are sorted according to the location stability for the puzzle policy in ascending order. It is noticeable that in all three policies, the location stability and retrievals to steady state rise together closely and greater values tend to occur in locations that are further from the I/O point.

Figures 7 through 12 below demonstrate the relationships between the uncertainty metrics for each retrieval policy.

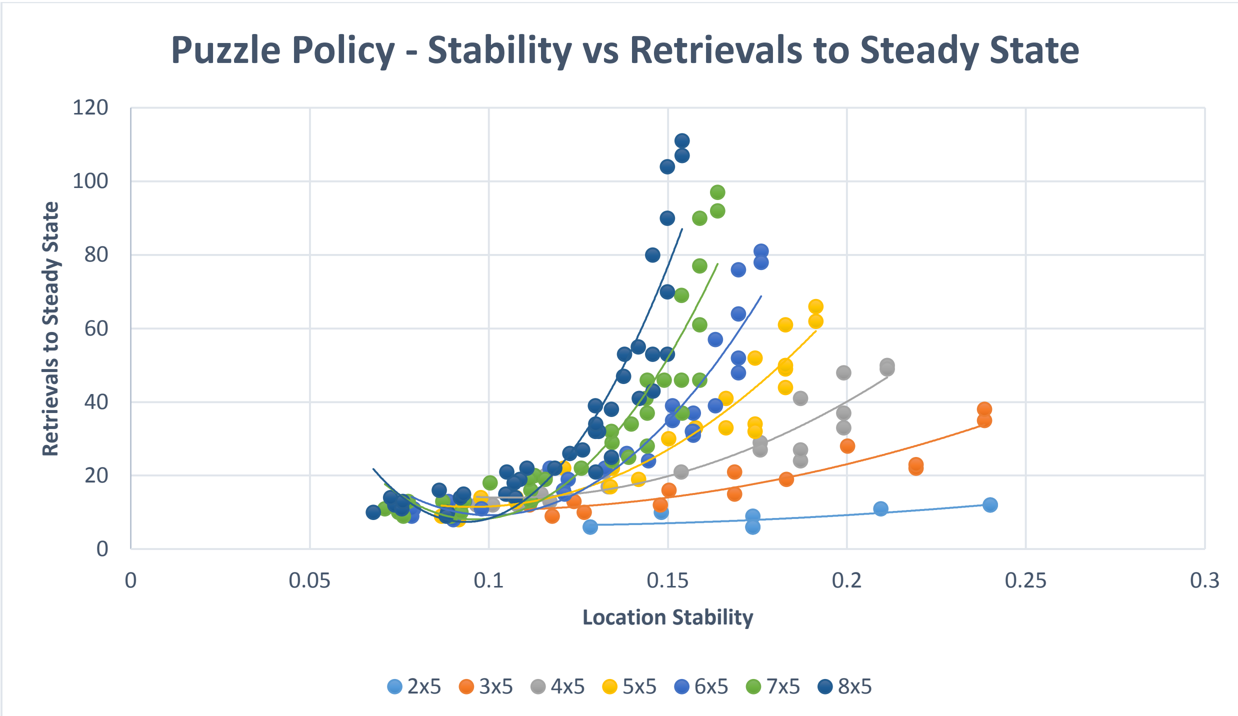

Figure 7: Puzzle Policy - Stability vs Retrievals to Steady State

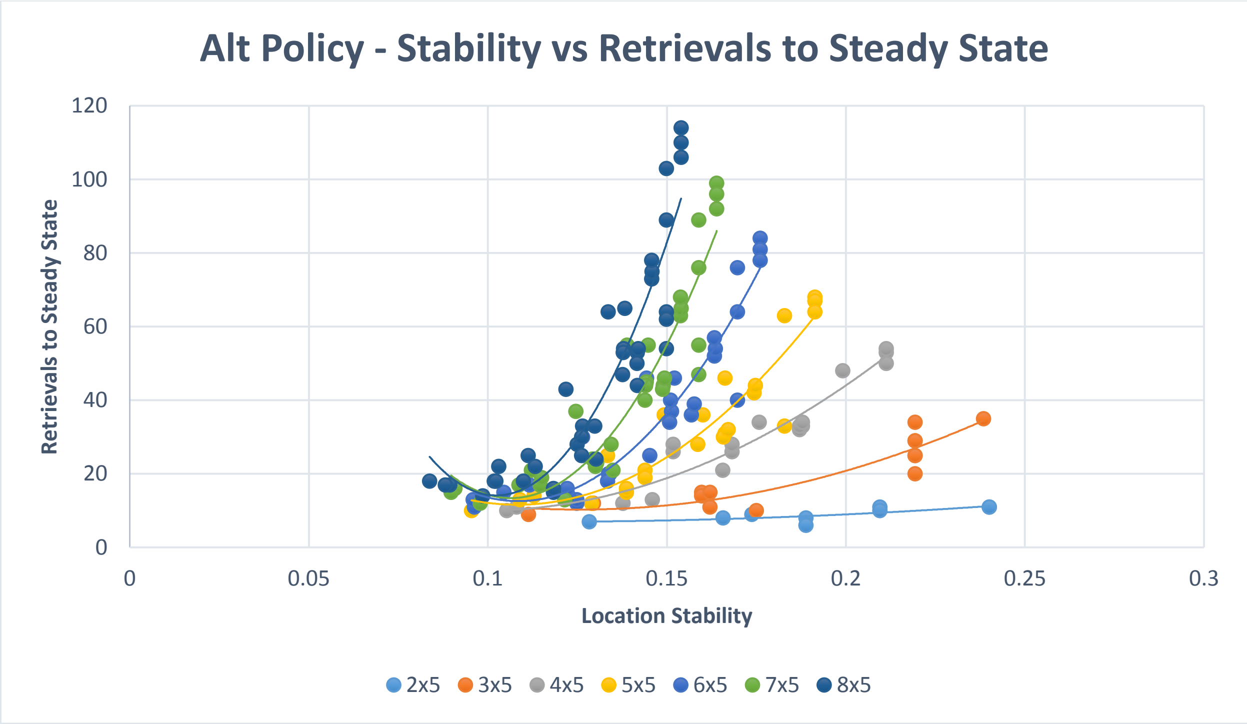

Figure 8: Alt Policy - Stability vs Retrievals to Steady State

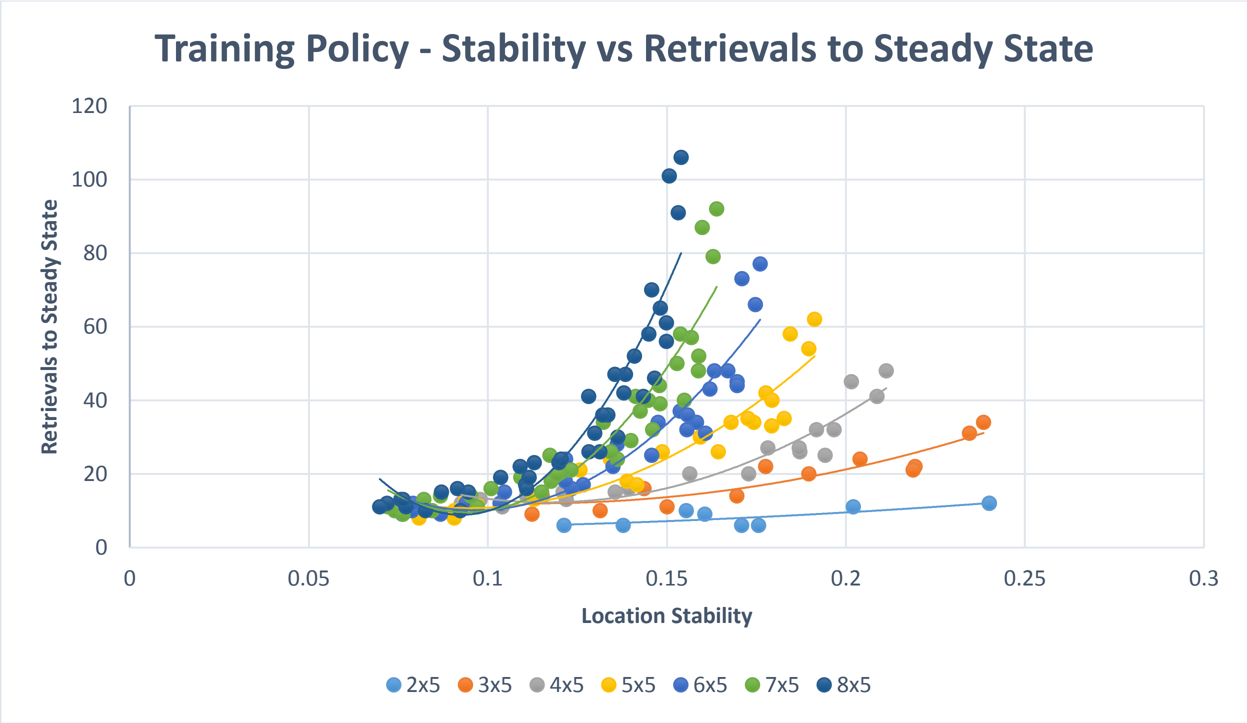

Figure 9: Training Policy - Stability vs Retrievals to Steady State

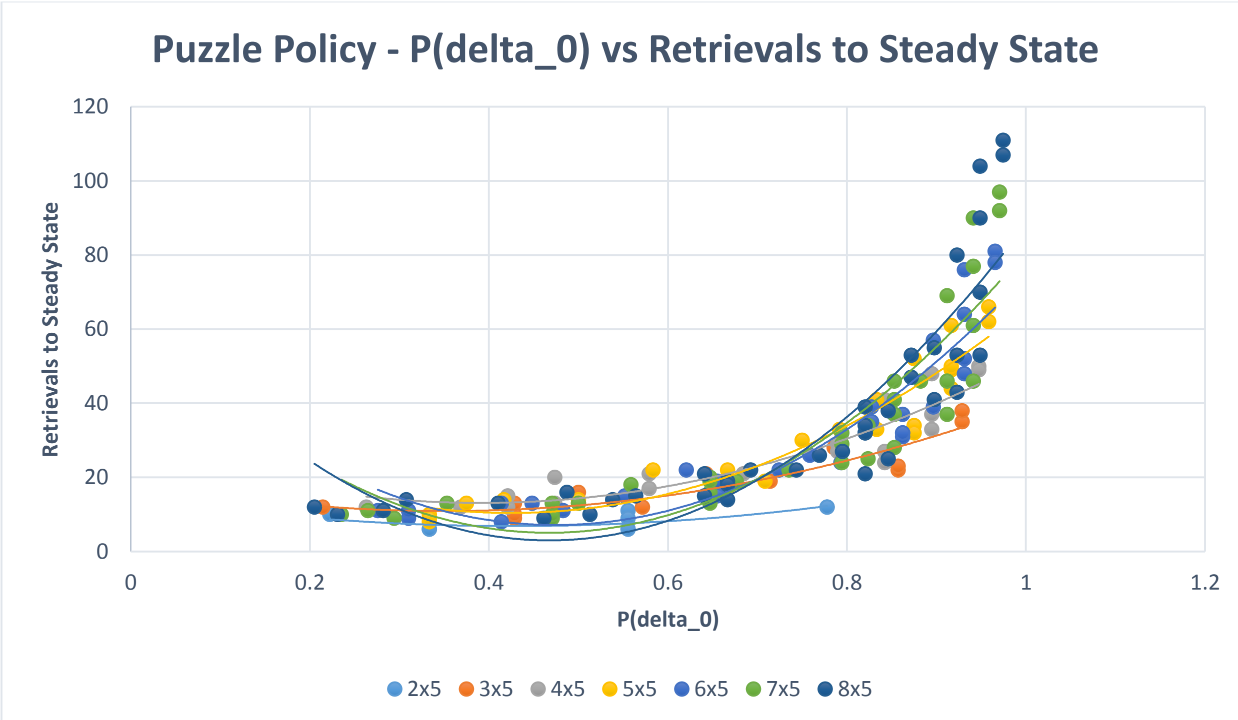

Figure 10: Puzzle Policy - P(delta_0) vs Retrievals to Steady State

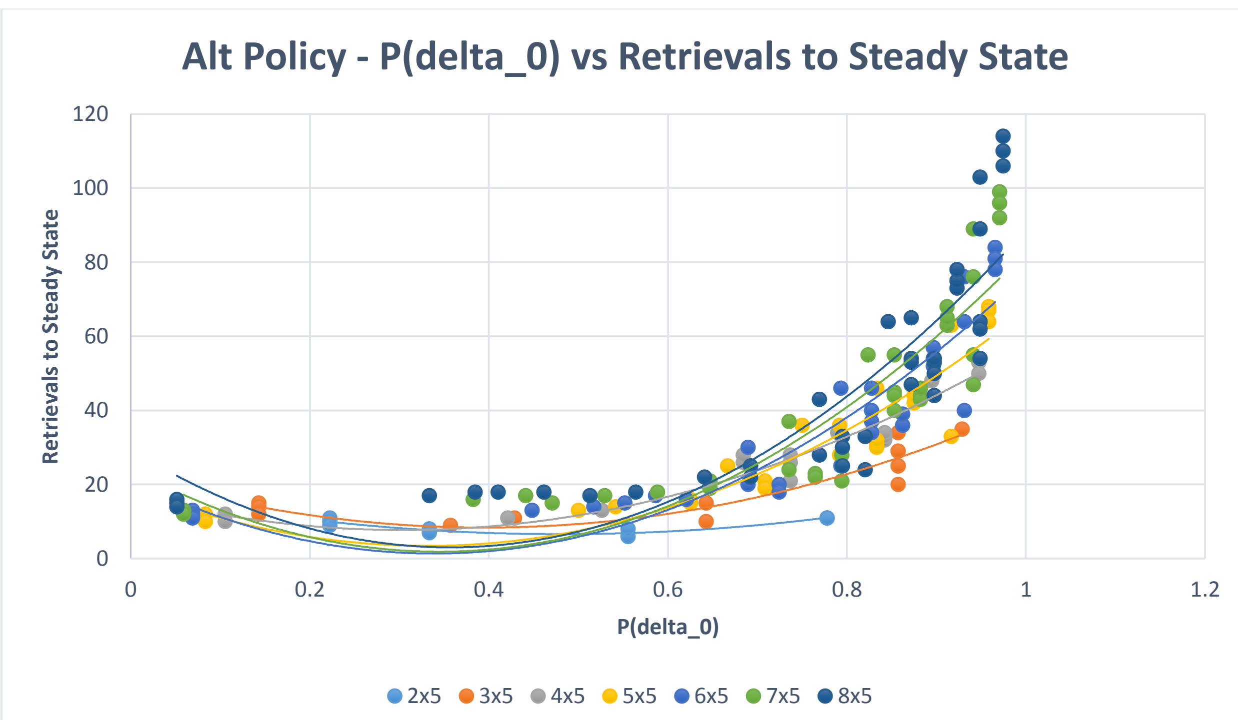

Figure 11: Alt Policy - P(delta_0) vs Retrievals to Steady State

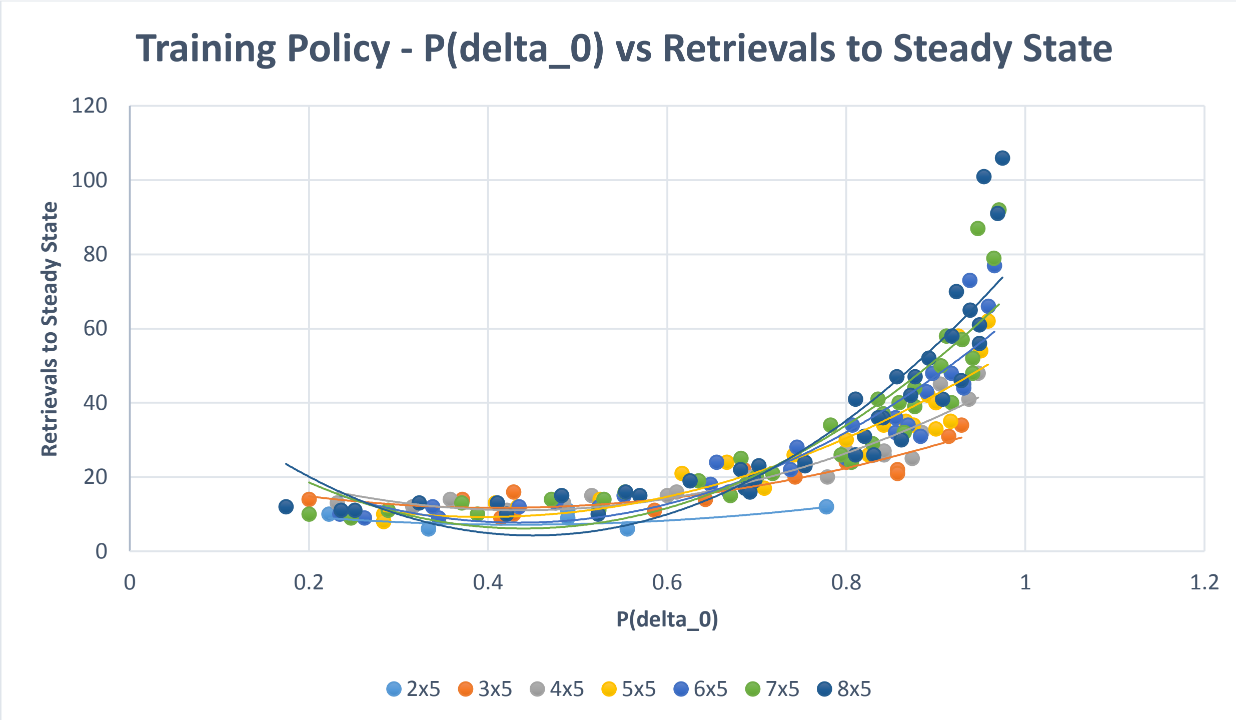

Figure 12: Training Policy - P(delta_0) vs Retrievals to Steady State

The results show that the location stability measure is more effective at predicting the behavior of uncertainty in the system than $P(\delta_0)_l$, though both have predictive value. In each scenario, the retrievals to steady state is well described by a parabolic function of location stability. However, the exact shape of each curve differs in each case. As the storage systems increase in size, the range of the location stability tends to decrease and the greater values tend to push the number of retrievals to higher values more quickly. Interestingly, all three of the policies studied here demonstrate remarkably similar relationships between the policy measures and the retrievals to steady state. Although the different policies result in different locations approaching steady state more quickly or slowly, the longer times to steady state are always found in the locations with higher stability measures. One implication of these results is that the time it takes for each item to approach steady state can be predicted directly from measures of the retrieval policy. Such knowledge can aid in assigning items to storage locations and evaluating policies prior to operation.

In each test scenario, specific trends are identified that have value in understanding the behavior of uncertainty in the system. The patterns found in all these scenarios show that locations that are closer to the I/O point tend to have:

- lower $P(\delta_0)_l$, probability of retaining the same unit with the next retrieval
- lower location stability σ_l
- lower time to approach steady state behavior, RSS_l

The first two patterns can be thought of as contributing to or producing the third. If the probability that a location will retain its current unit and that location's stability measure are low, then the unit beginning there will approach steady state more quickly. This means that the knowledge of its location is lost more quickly. The relationship between distance to the I/O point and the retrievals to steady state in the 7-by-5 system is demonstrated in Figure 13. The distance is rectilinear and is the number of spaces from the position to the I/O point, including the space representing the I/O point.

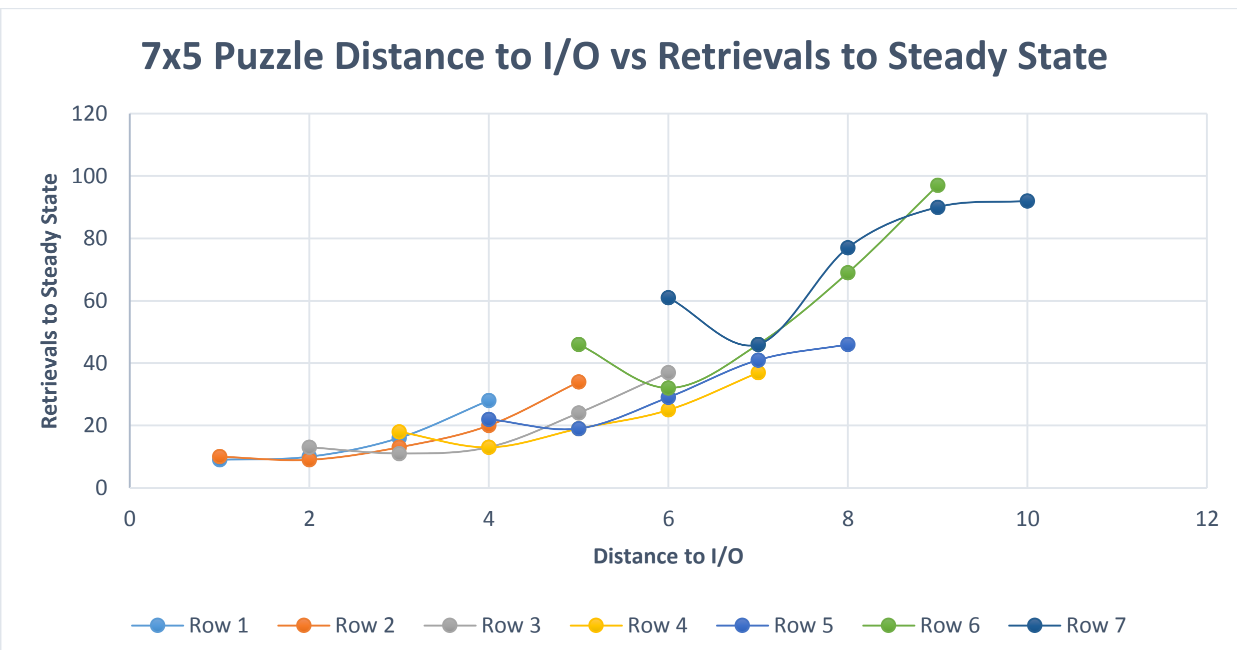

Figure 13: 7x5 Puzzle Distance to I/O vs Retrievals to Steady State

To further illustrate this feature, Figure 14 is a heat map superimposed over the 7-by-5 layout that distinguishes locations that reach steady state in relatively fewer retrievals from those that approach it more slowly. The numbers in the cells give the rank-order in which the locations reached steady state with the smaller values taking on a darker color indicating a more negative result.

19	18	21	22	23
18	14	18	20	24
9	7	13	17	18
6	4	7	11	16
4	3	4	10	16
2	1	4	8	15
	1	2	5	12

Figure 14: 7x5 uncertainty heat map by sequence in reaching steady state

Here we see the trend common to each scenario, locations closer to the I/O reaching steady state more quickly. It is important to understand the negative impact that uncertainty has on a system, and on the locations closer to the I/O point in particular, because these locations also generally have the lowest retrieval times. In much of the literature (Gu et al., 2007; Yu & De Koster, 2012) the objective is to reduce mean retrieval time via sequencing, rack design, or reducing travel distance from the I/O. However, these results indicate that while closer locations may offer shorter travel distances, in dense storage environments these locations will be more greatly impacted by uncertainty. And in most cases, the literature does not address the possibility of uncertainty in item location. Thus, uncertainty management may be an additional tool in reducing mean retrieval time by cutting down the time spent searching for items.

Mean Retrievals

It is also helpful to consider the systems more holistically in terms of the mean retrievals to steady state μ_{RSS}. The mean retrievals to steady state is the average across all locations for a given policy in a given system. Table 2 summarizes the results of these scenarios and compares μ_{RSS} with the mean number of unit moves or shifts required to retrieve a unit from each layout according to each policy.

Table 2: Mean retrieval effort (unit moves) and μ_{RSS} by retrieval policy

Scenario	Locations	Puzzle		Alternative		80/20 Training	
		Mean Retrieval Moves	μ_{RSS}	Mean Retrieval Moves	μ_{RSS}	Mean Retrieval Moves	μ_{RSS}
2x5	9	10.11	8.7	10.78	9.0	10.24	8.7
3x5	14	11.43	19.5	12.29	20.2	11.60	18.7
4x5	19	13.21	26.7	14.16	29.4	13.40	23.8
5x5	24	15.33	31.2	16.33	32.4	15.53	27.8
6x5	29	17.69	32.3	18.79	37.5	17.91	30.7
7x5	34	20.24	34.5	21.41	42.3	20.47	33.4
8x5	39	22.90	36.8	24.13	47.4	23.14	35.8

The differences in output between the three policies demonstrate that, all other things equal, different policies affect how quickly knowledge of item locations is lost. Notice that the alternative policy consistently yields greater mean retrievals to steady state than the puzzle policy, a positive result, yet also consistently has a greater expected retrieval effort, a negative result. This shows a performance tradeoff that thus far has not been addressed in the literature. These results demonstrate the possibility that a policy that is optimal in terms of retrieval effort may not be optimal in terms of uncertainty propagation and management. Incorporating uncertainty into dense storage system design offers a new dimension in which to explore performance optimization. Figure 15 illustrates the relationship between these measures for each policy.

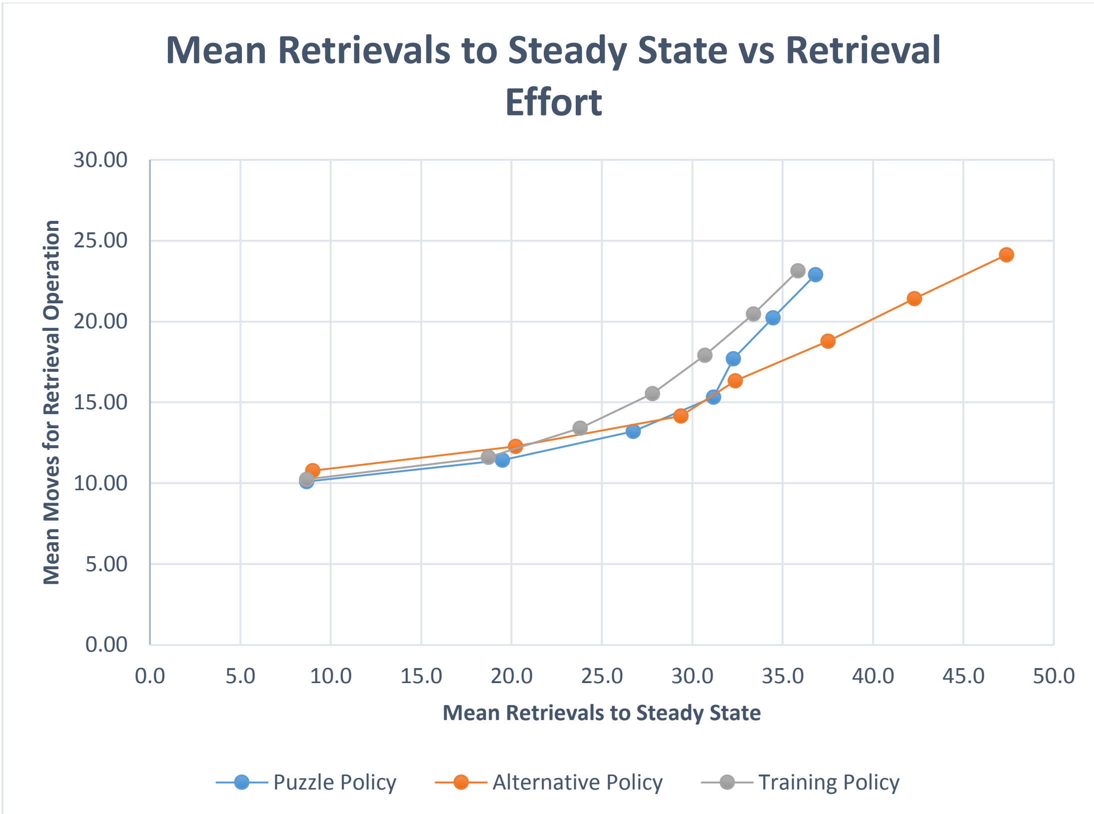

Figure 15: Mean Retrievals to Steady State vs Retrieval Effort

In order to determine what types of policies yield other differences in performance, several more policies must be tested. These results show that the retrieval policy affects how quickly the full effect of uncertainty is experienced, which is the anticipated searching effort. Figure 16 plots the size of the system against mean retrievals to approach steady state for each of the three retrieval policies.

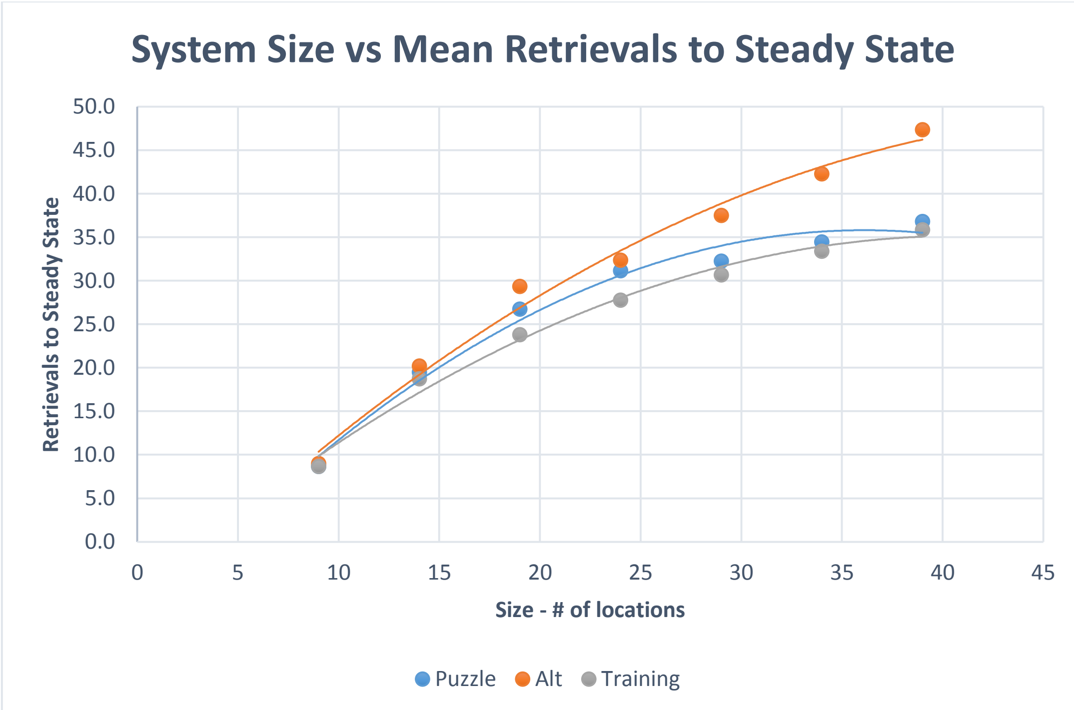

Figure 16: System Size vs Mean Retrievals to Steady State

In the figure, each curve has seven data points, each representing the output of one scenario for the retrieval policy associated with that curve. As expected, the mean number of retrievals to approach steady state increases as the total number of locations increases. Although each policy has a different effect, the overall trends are similar, meaning that the size of the system is a fair indicator of the mean retrievals to steady state.

Expected Locations to Search

Table 3 gives the expected locations to search $E(SL_l)$ for each scenario. It is clear from these results that increasing the number of item copies in the system greatly reduces the expected locations to search in each scenario, although the benefit is less substantial in the smaller systems.

Table 3: Expected Locations to Search by Scenario

Scenario	# of Locations	E(SL)				
		k = 1	k = 2	k = 3	k = 4	k = 5
2x5	9	5.0	3.3	2.5	2.0	1.7
3x5	14	7.5	5.0	3.8	3.0	2.5
4x5	19	10.0	6.7	5.0	4.0	3.3
5x5	24	12.5	8.3	6.3	5.0	4.2
6x5	29	15.0	10.0	7.5	6.0	5.0
7x5	34	17.5	11.7	8.8	7.0	5.8
8x5	39	20.0	13.3	10.0	8.0	6.7

Figure 17 plots the size of the system against the expected number of locations to search in steady state for various values of k_i.

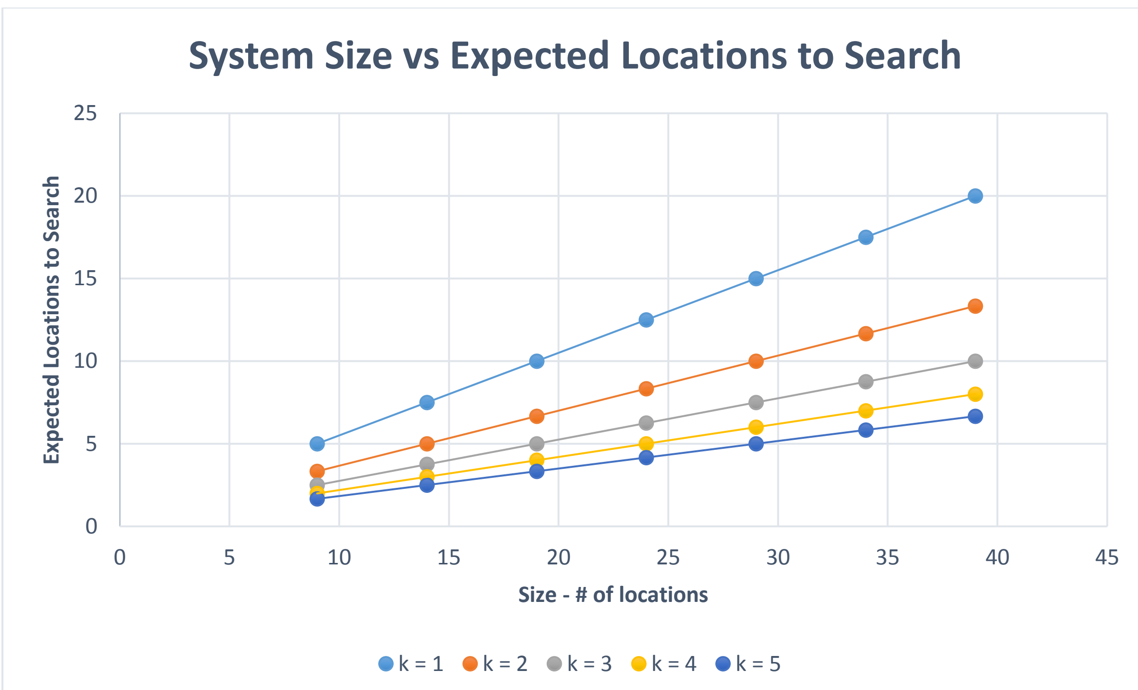

Figure 17: System Size vs Expected Locations to Search

Remember that this relationship was determined analytically and discussed above. Here the model results demonstrate a tradeoff between conflicting values. The scenarios that have the lowest expected search efforts, a positive result, also have the lowest mean retrievals to steady state, meaning that steady state is reached faster, or knowledge of the unit locations is lost more quickly. Some practical insights follow from these results:

- In large systems, multiple unit copies reduce expected search time resulting from uncertainty
- Location knowledge is maintained longer in large communication classes and lost more quickly in small communication classes
- Small communication classes are preferred when the number of item copies is small
- The retrieval policy affects propagation of uncertainty in otherwise identical systems

13DX Case Study

Uncertainty Growth

An additional case study was performed for a real dense storage environment: stow location 13DX, deck 3 Ammunition on the USNS Sacagawea. All three policies were applied to a subsection of the system consisting of 27 storage locations and an I/O point. Figure 18 shows the layout of the section studied.

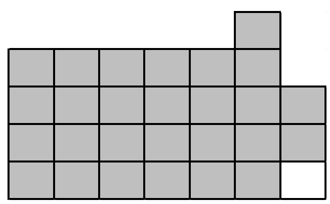

Figure 18: 27-location subsection of 13DX storage system

Table 4 gives the retrievals to steady state for each location according to each retrieval policy.

Table 4: Retrievals to Steady State by Location for each Policy

13DX Retrievals to Steady State			
Location	Puzzle	Alternative	Training (80/20)
1	9	15	10
2	11	15	12
3	14	18	14
4	17	19	16
5	33	34	32
6	47	45	45
7	10	14	10
8	10	14	10
9	10	14	10
10	11	17	12
11	16	17	16
12	27	37	24
13	34	47	37
14	13	30	15
15	12	26	13
16	17	30	19
17	22	28	23
18	29	32	28
19	53	52	44
20	70	75	67
21	46	62	50
22	36	35	37
23	28	42	31
24	45	56	43
25	76	69	61
26	72	71	71
27	76	77	76

Figures 19 and 20 demonstrate the relationships between the uncertainty metrics for each retrieval policy.

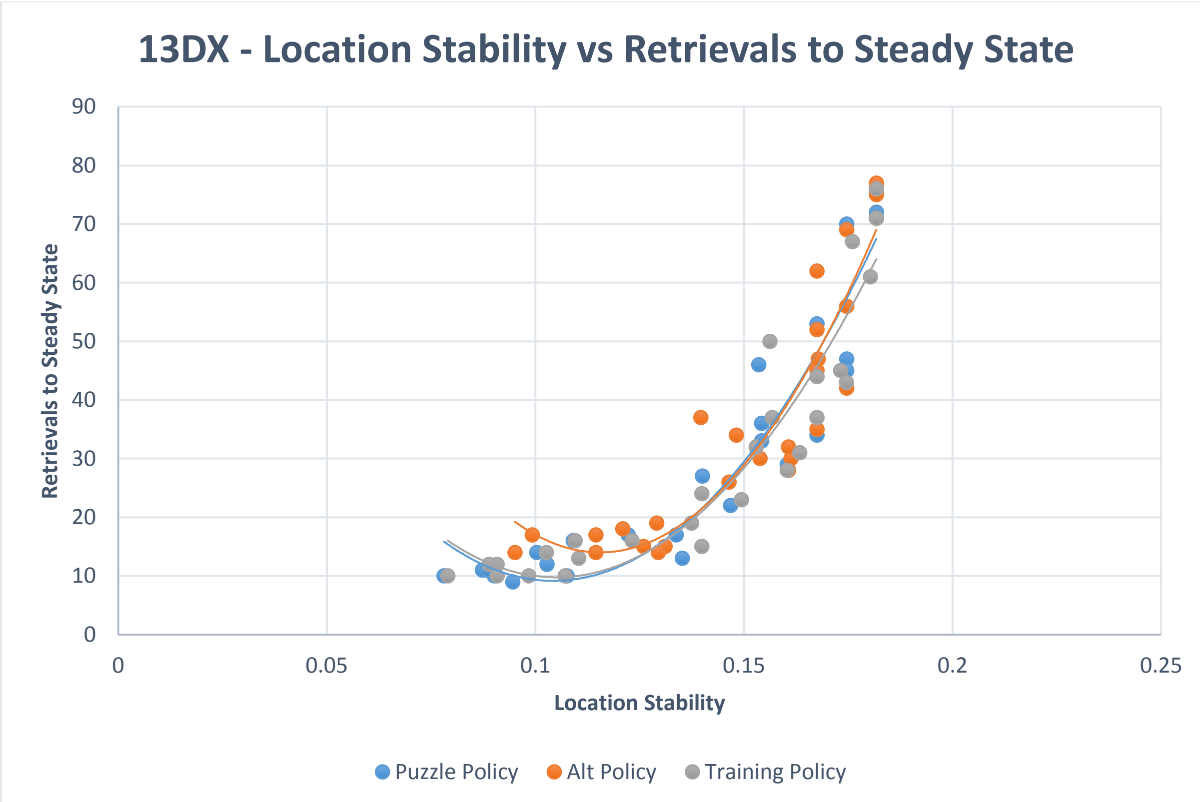

Figure 19: 13DX - Location Stability vs Retrievals to Steady State

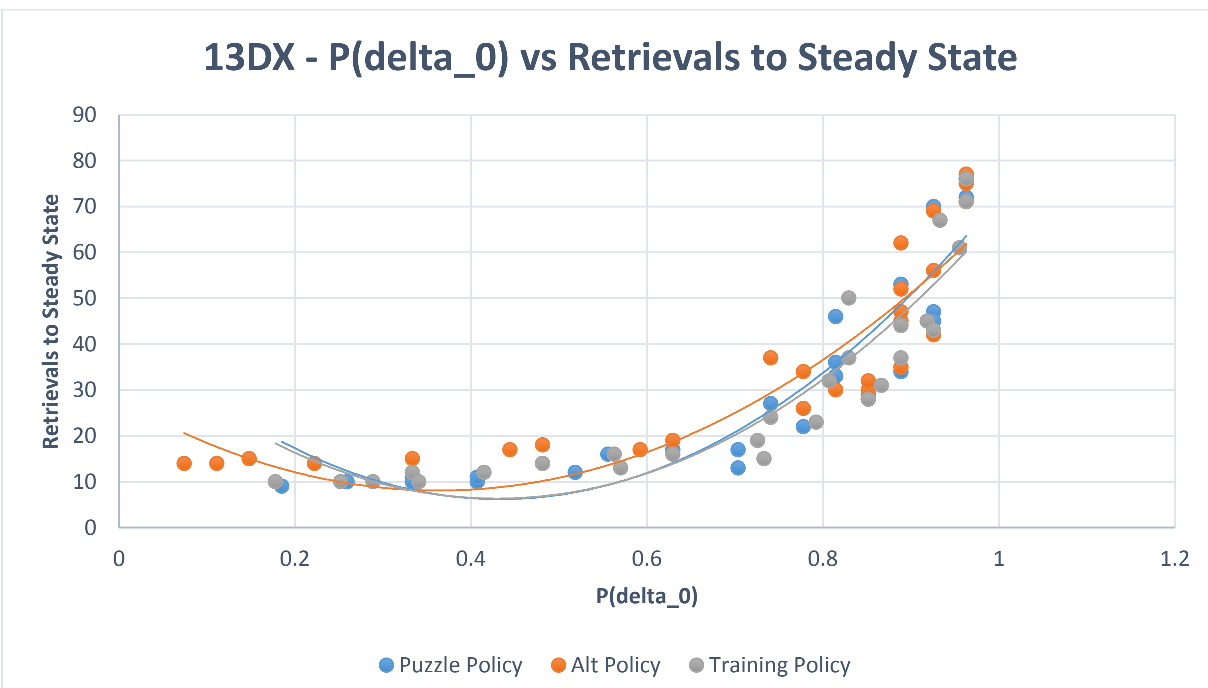

Figure 20: 13DX - P(delta_0) vs Retrievals to Steady State

From these results, it is clear that uncertainty in this stowage hold behaves in much the same way as in the other scenarios. No two test cases have identical output but all scenarios demonstrate the same characteristics: higher retrievals to steady state occur in locations with higher stability, $P(\delta_0)_l$, and increasing numbers of communicating storage locations.

Expected Search Time

The expected search time model was also applied in this system layout. For the sake of demonstration, we assume that now each of the 27 floor spaces contains a stack of two pallets so there is a vertical dimension introduced, and there are 54 total storage locations. For any location in this system that has reached steady state, Table 5 gives both $E(SL_l)$ and $E(ST_l)$ for k_i values one through five and parameter values $T_h = 10\ seconds$, $T_u = 30\ seconds$, and $H = 2\ pallets\ per\ stack$.

Table 5: Expected Locations to Search and Search Time for 13DX

k_i	$E(SL_l)$	$E(ST_l)$ min
1	27.50	16.04
2	18.33	10.69
3	13.75	8.02
4	11.00	6.42
5	9.17	5.35

The results are an example of the impact that uncertainty can have on a dense storage system. In this communicating set of 54 storage locations in steady state, an average of 16.04 minutes will be wasted searching for the singular requested unit. For any method described in the literature for reducing retrieval time in a dense storage system, this model says that much time should be added so that the retrieval time includes the time spent searching for the right unit.

CHAPTER SIX: CONCLUSIONS AND FUTURE RESEARCH

In conclusion, we developed a mathematical model which describes the behavior of location uncertainty in dense storage systems. The storage systems were modeled as finite-state Markov chain processes in which the storage locations are the states occupied by the units in the system. It was discovered that for any non-depleting, communicating set of storage locations, the steady state location probability distribution is uniform across those locations. This result was then used to develop a model that estimates the mean search time in a system in steady state given the expected number of locations that must be searched to find the requested unit.

The focus on location uncertainty in this work also adds a new dimension to the current warehousing literature. No known work has addressed the possibility of changing location uncertainty in a warehouse logistics context. By defining metrics to measure uncertainty in these dense storage systems, we have demonstrated the trade-off between the impact of uncertainty and storage density and retrieval time, two metrics already of great interest in the literature.

The metrics defined in this model also aid in evaluating various dense storage systems and can be used to make recommendations in their design. These metrics quantify certain characteristics of the chosen retrieval policy and can predict the behavior of uncertainty in a system over time. Thus, they can be used to mitigate the impact of uncertainty through system design. Some of the insights into uncertainty gained through the analysis of these metrics include:

- The specific retrieval policy does affect the propagation of uncertainty in a system
- Distinct communication classes behave independently and each approaches a uniform steady state distribution
- Search time in steady state can be affected through use of communication classes and unit copies
- Greater location stability σ_l and $P(\delta_0)_l$ delay loss of location knowledge; increase RSS_l
- Easier to retrieve units tend to have lower values of σ_l, $P(\delta_0)_l$, and RSS_l, a performance tradeoff

While this work offers a novel approach to uncertainty in dense storage environments, there remains much possibility for future development. Future work should include a depleting system model which incorporates storage operations in addition to retrievals. Such a model would need to account for varying numbers of storage units and a changing layout over time. Another addition to this work is to expand the model to include non-uniform demand distributions for the storage units. An interesting future model could also feature non-stationary retrieval policies that are adapted strategically in response to the current state of the system. This would be a Markov decision process where the decision variables determine the retrieval policy at a given time. Our hope is that this work, along with future developments, will offer a new perspective of uncertainty in the warehousing literature and produce valuable practical insights into storage system design and operation.

APPENDIX A EXPECTED SEARCH LOCATIONS PROOF

$$E(SL_l) = \sum_{l'=1}^{n-k_i+1} l' \frac{\binom{n-l'}{k_i-1}}{\binom{n}{k_i}} = \frac{1}{\binom{n}{k_i}} \sum_{l'=1}^{n-k_i+1} l' \binom{n-l'}{k_i-1} \qquad (36)$$

Equation 37 is the numerator from Equation 36.

$$\sum_{l'=1}^{n-k_i+1} l' \binom{n-l'}{k_i-1} \qquad (37)$$

$$= \binom{n-1}{k_i-1} + 2\binom{n-2}{k_i-1} + 3\binom{n-3}{k_i-1} + \cdots + (n-k_i+1)\binom{k_i-1}{k_i-1}$$

Equation 37 is expanded to $n - k_i + 1$ lines to gain a visual perspective of the expression.

$$= \binom{n-1}{k_i-1} + \binom{n-2}{k_i-1} + \binom{n-3}{k_i-1} + \cdots + \binom{k_i-1}{k_i-1} \qquad Eq. 37\ line\ 1$$

$$+ \binom{n-2}{k_i-1} + \binom{n-3}{k_i-1} + \cdots + \binom{k_i-1}{k_i-1} \qquad Eq. 37\ line\ 2$$

$$+ \binom{n-3}{k_i-1} + \cdots + \binom{k_i-1}{k_i-1} \qquad Eq. 37\ line\ 3$$

$$\vdots \qquad \vdots$$

$$+ \binom{k_i-1}{k_i-1} \qquad Eq. 37\ line\ n-k+1$$

Equation 38 is derived from Pascal's Formula.

$$\binom{n-1}{k_i-1} = \binom{n}{k_i} - \binom{n-1}{k_i} \qquad (38)$$

Express each term in Equation 37, line by line, using Equation 38. All terms cancel out except the first and the last, which equals zero.

$$= \binom{n}{k_i} - \binom{n-1}{k_i} + \binom{n-1}{k_i} - \binom{n-2}{k_i} + \binom{n-2}{k_i} - \cdots - \binom{k_i}{k_i} + \binom{k_i}{k_i} - \binom{k_i-1}{k_i} = \binom{n}{k_i} \qquad Eq. 37\ line\ 1$$

$$= \binom{n-1}{k_i} - \binom{n-2}{k_i} + \binom{n-2}{k_i} - \cdots - \binom{k_i}{k_i} + \binom{k_i}{k_i} - \binom{k_i-1}{k_i} = \binom{n-1}{k_i} \qquad Eq. 37\ line\ 2$$

$$= \binom{n-2}{k_i} - \binom{n-3}{k_i} + \binom{n-3}{k_i} - \cdots - \binom{k_i}{k_i} + \binom{k_i}{k_i} - \binom{k_i-1}{k_i} = \binom{n-2}{k_i} \qquad Eq. 37\ line\ 3$$

$$\vdots \qquad \vdots$$

$$= \binom{k_i}{k_i} - \binom{k_i-1}{k_i} = \binom{k_i}{k_i} \qquad Eq. 37\ line\ n-k+1$$

Adding the expressions from each line, Equation 37 becomes

$$= \binom{n}{k_i} + \binom{n-1}{k_i} + \binom{n-2}{k_i} + \cdots + \binom{k_i+1}{k_i} + \binom{k_i}{k_i} \qquad Eq. 37$$

Use Equation 38 again for each term in Equation 37. The result gives the final reduced expression for Equation 37.

$$= \binom{n+1}{k_i+1} - \binom{n}{k_i+1} + \binom{n}{k_i+1} - \binom{n-1}{k_i+1} + \cdots + \binom{k_i+1}{k_i+1} + \binom{k_i}{k_i+1} = \binom{n+1}{k_i+1} \qquad Eq. 37$$

Substituting Equation 37 into Equation 36 completes the proof of the expected value expression.

$$\frac{\binom{n+1}{k_i+1}}{\binom{n}{k_i}} = \frac{n+1}{k_i+1} \qquad (39)$$

APPENDIX B MOVEMENT MATRICES

	1	2	3	4	5	6	7	8	9
1	0.222	0.111	0.000	0.000	0.444	0.000	0.222	0.000	0.000
2	0.111	0.333	0.111	0.000	0.000	0.000	0.333	0.111	0.000
3	0.111	0.000	0.556	0.111	0.000	0.000	0.000	0.222	0.000
4	0.111	0.000	0.000	0.778	0.000	0.000	0.000	0.000	0.111
5	0.000	0.000	0.000	0.000	0.556	0.444	0.000	0.000	0.000
6	0.111	0.222	0.000	0.000	0.000	0.556	0.111	0.000	0.000
7	0.111	0.333	0.111	0.000	0.000	0.000	0.333	0.111	0.000
8	0.111	0.000	0.222	0.000	0.000	0.000	0.000	0.556	0.111
9	0.111	0.000	0.000	0.111	0.000	0.000	0.000	0.000	0.778

Figure 21: Puzzle 2x5 Movement Matrix

	1	2	3	4	5	6	7	8	9
1	0.222	0.000	0.000	0.000	0.667	0.111	0.000	0.000	0.000
2	0.111	0.333	0.000	0.000	0.000	0.000	0.556	0.000	0.000
3	0.111	0.000	0.556	0.000	0.000	0.000	0.000	0.333	0.000
4	0.111	0.000	0.000	0.778	0.000	0.000	0.000	0.000	0.111
5	0.111	0.000	0.000	0.000	0.222	0.667	0.000	0.000	0.000
6	0.000	0.556	0.000	0.000	0.111	0.222	0.111	0.000	0.000
7	0.111	0.111	0.333	0.000	0.000	0.000	0.333	0.111	0.000
8	0.111	0.000	0.111	0.111	0.000	0.000	0.000	0.556	0.111
9	0.111	0.000	0.000	0.111	0.000	0.000	0.000	0.000	0.778

Figure 22:Alternative 2x5 Movement Matrix

	1	2	3	4	5	6	7	8	9
1	0.222	0.089	0.000	0.000	0.489	0.022	0.178	0.000	0.000
2	0.111	0.333	0.089	0.000	0.000	0.000	0.378	0.089	0.000
3	0.111	0.000	0.556	0.089	0.000	0.000	0.000	0.244	0.000
4	0.111	0.000	0.000	0.778	0.000	0.000	0.000	0.000	0.111
5	0.022	0.000	0.000	0.000	0.489	0.489	0.000	0.000	0.000
6	0.089	0.289	0.000	0.000	0.022	0.489	0.111	0.000	0.000
7	0.111	0.289	0.156	0.000	0.000	0.000	0.333	0.111	0.000
8	0.111	0.000	0.200	0.022	0.000	0.000	0.000	0.556	0.111
9	0.111	0.000	0.000	0.111	0.000	0.000	0.000	0.000	0.778

Figure 23: Training 2x5 Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.214	0.143	0.000	0.000	0.357	0.071	0.214	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.071	0.429	0.143	0.000	0.000	0.000	0.286	0.071	0.000	0.000	0.000	0.000	0.000	0.000
3	0.071	0.000	0.643	0.071	0.000	0.000	0.000	0.214	0.000	0.000	0.000	0.000	0.000	0.000
4	0.071	0.000	0.000	0.857	0.000	0.000	0.000	0.000	0.071	0.000	0.000	0.000	0.000	0.000
5	0.071	0.000	0.000	0.000	0.500	0.357	0.000	0.000	0.000	0.071	0.000	0.000	0.000	0.000
6	0.071	0.214	0.000	0.000	0.000	0.429	0.071	0.000	0.000	0.000	0.214	0.000	0.000	0.000
7	0.071	0.214	0.071	0.000	0.000	0.000	0.429	0.143	0.000	0.000	0.000	0.071	0.000	0.000
8	0.071	0.000	0.143	0.000	0.000	0.000	0.000	0.571	0.143	0.000	0.000	0.000	0.071	0.000
9	0.071	0.000	0.000	0.071	0.000	0.000	0.000	0.000	0.786	0.000	0.000	0.000	0.000	0.071
10	0.000	0.000	0.000	0.000	0.071	0.143	0.000	0.000	0.000	0.714	0.071	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.071	0.000	0.000	0.000	0.000	0.214	0.643	0.071	0.000	0.000
12	0.071	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.071	0.857	0.000	0.000
13	0.071	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.929	0.000
14	0.071	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.929

Figure 24: Puzzle 3x5 Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.143	0.000	0.000	0.000	0.571	0.286	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.071	0.429	0.000	0.000	0.000	0.000	0.500	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.071	0.000	0.643	0.000	0.000	0.000	0.000	0.286	0.000	0.000	0.000	0.000	0.000	0.000
4	0.071	0.000	0.000	0.857	0.000	0.000	0.000	0.000	0.071	0.000	0.000	0.000	0.000	0.000
5	0.286	0.000	0.000	0.000	0.143	0.571	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.500	0.000	0.000	0.071	0.143	0.143	0.000	0.000	0.071	0.071	0.000	0.000	0.000
7	0.071	0.071	0.286	0.000	0.000	0.000	0.357	0.071	0.000	0.000	0.000	0.143	0.000	0.000
8	0.071	0.000	0.071	0.071	0.000	0.000	0.000	0.643	0.071	0.000	0.000	0.000	0.071	0.000
9	0.071	0.000	0.000	0.071	0.000	0.000	0.000	0.000	0.857	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.071	0.000	0.000	0.000	0.000	0.857	0.071	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.071	0.000	0.000	0.000	0.000	0.071	0.857	0.000	0.000	0.000
12	0.000	0.000	0.000	0.000	0.071	0.000	0.000	0.000	0.000	0.000	0.000	0.857	0.071	0.000
13	0.071	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.857	0.071
14	0.071	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.929

Figure 25: Alternative 3x5 Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	0.200	0.114	0.000	0.000	0.400	0.114	0.171	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.071	0.429	0.114	0.000	0.000	0.000	0.329	0.057	0.000	0.000	0.000	0.000	0.000	0.000
3	0.071	0.000	0.643	0.057	0.000	0.000	0.000	0.229	0.000	0.000	0.000	0.000	0.000	0.000
4	0.071	0.000	0.000	0.857	0.000	0.000	0.000	0.000	0.071	0.000	0.000	0.000	0.000	0.000
5	0.114	0.000	0.000	0.000	0.429	0.400	0.000	0.000	0.000	0.057	0.000	0.000	0.000	0.000
6	0.057	0.271	0.000	0.000	0.014	0.371	0.086	0.000	0.000	0.014	0.186	0.000	0.000	0.000
7	0.071	0.186	0.114	0.000	0.000	0.000	0.414	0.129	0.000	0.000	0.000	0.086	0.000	0.000
8	0.071	0.000	0.129	0.014	0.000	0.000	0.000	0.586	0.129	0.000	0.000	0.000	0.071	0.000
9	0.071	0.000	0.000	0.071	0.000	0.000	0.000	0.000	0.800	0.000	0.000	0.000	0.000	0.057
10	0.000	0.000	0.000	0.000	0.071	0.114	0.000	0.000	0.000	0.743	0.071	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.071	0.000	0.000	0.000	0.000	0.186	0.686	0.057	0.000	0.000
12	0.057	0.000	0.000	0.000	0.014	0.000	0.000	0.000	0.000	0.000	0.057	0.857	0.014	0.000
13	0.071	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.914	0.014
14	0.071	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.929

Figure 26: Training 3x5 Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	0.263	0.158	0.000	0.000	0.316	0.105	0.158	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.053	0.474	0.105	0.000	0.000	0.053	0.263	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.053	0.000	0.684	0.053	0.000	0.000	0.053	0.158	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.053	0.000	0.000	0.895	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.105	0.000	0.000	0.000	0.421	0.316	0.000	0.000	0.000	0.105	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.053	0.158	0.000	0.000	0.000	0.368	0.105	0.000	0.000	0.053	0.263	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.053	0.211	0.053	0.000	0.000	0.000	0.421	0.211	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.053	0.000	0.158	0.000	0.000	0.000	0.000	0.579	0.105	0.000	0.000	0.000	0.105	0.000	0.000	0.000	0.000	0.000	0.000
9	0.053	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.842	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.053	0.158	0.000	0.000	0.000	0.579	0.158	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.211	0.474	0.158	0.000	0.000	0.000	0.105	0.000	0.000	0.000
12	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.053	0.789	0.000	0.000	0.000	0.000	0.105	0.000	0.000
13	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.895	0.053	0.000	0.000	0.000	0.000	0.000
14	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.895	0.000	0.000	0.000	0.000	0.053
15	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.842	0.053	0.000	0.000	0.000
16	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.105	0.789	0.053	0.000	0.000
17	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.053	0.842	0.053	0.000
18	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.947	0.000
19	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.947

Figure 27: Puzzle 4x5 Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	0.105	0.000	0.000	0.000	0.474	0.421	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.053	0.526	0.000	0.000	0.000	0.000	0.421	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.053	0.000	0.737	0.000	0.000	0.000	0.000	0.211	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.053	0.000	0.000	0.842	0.000	0.000	0.000	0.053	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.421	0.000	0.000	0.000	0.105	0.474	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.421	0.000	0.000	0.053	0.105	0.158	0.000	0.000	0.211	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.053	0.053	0.211	0.000	0.000	0.000	0.421	0.053	0.000	0.000	0.000	0.211	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.053	0.000	0.053	0.053	0.000	0.000	0.000	0.684	0.105	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.000	0.000
9	0.053	0.000	0.000	0.105	0.000	0.000	0.000	0.000	0.842	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.737	0.211	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.053	0.684	0.053	0.000	0.000	0.053	0.105	0.000	0.000	0.000
12	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.737	0.158	0.000	0.000	0.000	0.053	0.000	0.000
13	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.789	0.053	0.000	0.000	0.000	0.105	0.000
14	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.947	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.842	0.053	0.000	0.000	0.000
16	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.105	0.842	0.000	0.000	0.000
17	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.947	0.000	0.000
18	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.895	0.053
19	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.947

Figure 28: Alternative 4x5 Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	0.232	0.126	0.000	0.000	0.347	0.168	0.126	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.053	0.484	0.084	0.000	0.000	0.042	0.295	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.053	0.000	0.695	0.042	0.000	0.000	0.042	0.168	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.053	0.000	0.000	0.884	0.000	0.000	0.000	0.011	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.168	0.000	0.000	0.000	0.358	0.347	0.000	0.000	0.000	0.084	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.042	0.211	0.000	0.000	0.011	0.316	0.116	0.000	0.000	0.084	0.221	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.053	0.179	0.084	0.000	0.000	0.000	0.421	0.179	0.000	0.000	0.000	0.084	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.053	0.000	0.137	0.011	0.000	0.000	0.000	0.600	0.105	0.000	0.000	0.000	0.095	0.000	0.000	0.000	0.000	0.000	0.000
9	0.053	0.000	0.000	0.063	0.000	0.000	0.000	0.000	0.842	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.053	0.126	0.000	0.000	0.000	0.611	0.168	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.179	0.516	0.137	0.000	0.000	0.011	0.105	0.000	0.000	0.000
12	0.042	0.000	0.000	0.000	0.011	0.000	0.000	0.000	0.000	0.000	0.042	0.779	0.032	0.000	0.000	0.000	0.095	0.000	0.000
13	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.874	0.053	0.000	0.000	0.000	0.021	0.000
14	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.905	0.000	0.000	0.000	0.000	0.042
15	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.042	0.011	0.000	0.000	0.000	0.842	0.053	0.000	0.000	0.000
16	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.105	0.800	0.042	0.000	0.000
17	0.000	0.000	0.000	0.000	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.042	0.863	0.042	0.000
18	0.042	0.000	0.000	0.000	0.011	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.937	0.011
19	0.053	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.947

Figure 29: Training 4x5 Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0.333	0.125	0.000	0.000	0.292	0.125	0.125	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.042	0.500	0.083	0.000	0.000	0.125	0.208	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.042	0.042	0.667	0.042	0.000	0.000	0.083	0.125	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.042	0.000	0.042	0.875	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.125	0.000	0.000	0.000	0.333	0.292	0.000	0.000	0.000	0.125	0.125	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.042	0.125	0.000	0.000	0.000	0.333	0.125	0.000	0.000	0.125	0.250	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.042	0.208	0.042	0.000	0.000	0.000	0.417	0.208	0.000	0.000	0.000	0.083	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.042	0.000	0.167	0.000	0.000	0.000	0.000	0.583	0.083	0.000	0.000	0.000	0.125	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.042	0.000	0.000	0.083	0.000	0.000	0.000	0.000	0.833	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.042	0.125	0.000	0.000	0.000	0.500	0.208	0.000	0.000	0.000	0.083	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.208	0.375	0.208	0.000	0.000	0.042	0.125	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.042	0.000	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.042	0.708	0.042	0.000	0.000	0.000	0.125	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.833	0.083	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.875	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.750	0.125	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000
16	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.125	0.667	0.083	0.000	0.000	0.000	0.083	0.000	0.000	0.000
17	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.042	0.792	0.083	0.000	0.000	0.000	0.042	0.000	0.000
18	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.917	0.000	0.000	0.000	0.000	0.042	0.000
19	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.958	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.917	0.042	0.000	0.000	0.000
21	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.042	0.875	0.042	0.000	0.000
22	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.917	0.042	0.000
23	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.917	0.042
24	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.958

Figure 30: Puzzle 5x5 Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0.083	0.000	0.000	0.000	0.375	0.542	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.042	0.625	0.000	0.000	0.000	0.000	0.333	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.042	0.000	0.792	0.000	0.000	0.000	0.000	0.167	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.042	0.000	0.000	0.875	0.000	0.000	0.000	0.042	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.542	0.000	0.000	0.000	0.083	0.375	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.333	0.000	0.000	0.042	0.083	0.167	0.000	0.000	0.333	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.042	0.042	0.167	0.000	0.000	0.000	0.500	0.042	0.000	0.000	0.000	0.208	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.042	0.000	0.042	0.042	0.000	0.000	0.000	0.750	0.083	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.042	0.000	0.000	0.083	0.000	0.000	0.000	0.000	0.875	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.625	0.333	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.042	0.542	0.083	0.000	0.000	0.167	0.125	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.708	0.167	0.000	0.000	0.000	0.083	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.792	0.042	0.000	0.000	0.000	0.125	0.000	0.000	0.000	0.000	0.000	0.000
14	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.958	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.083	0.000	0.000	0.000	0.708	0.167	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
16	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.083	0.667	0.083	0.000	0.000	0.042	0.083	0.000	0.000	0.000
17	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.833	0.042	0.000	0.000	0.000	0.083	0.000	0.000
18	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.833	0.083	0.000	0.000	0.000	0.042	0.000
19	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.917	0.000	0.000	0.000	0.000	0.042
20	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.042	0.042	0.000	0.000	0.000	0.833	0.042	0.000	0.000	0.000
21	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.125	0.833	0.000	0.000	0.000
22	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.042	0.917	0.000	0.000
23	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.958	0.000
24	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.958

Figure 31: Alternative 5x5 Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	0.283	0.100	0.000	0.000	0.308	0.208	0.100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.042	0.525	0.067	0.000	0.000	0.100	0.233	0.033	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.042	0.033	0.692	0.033	0.000	0.000	0.067	0.133	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.042	0.000	0.033	0.875	0.000	0.000	0.000	0.008	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.208	0.000	0.000	0.000	0.283	0.308	0.000	0.000	0.000	0.100	0.100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.033	0.167	0.000	0.000	0.008	0.283	0.133	0.000	0.000	0.167	0.208	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.042	0.175	0.067	0.000	0.000	0.000	0.433	0.175	0.000	0.000	0.000	0.108	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.042	0.000	0.142	0.008	0.000	0.000	0.000	0.617	0.083	0.000	0.000	0.000	0.108	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.042	0.000	0.000	0.083	0.000	0.000	0.000	0.000	0.842	0.000	0.000	0.000	0.000	0.033	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.042	0.100	0.000	0.000	0.000	0.525	0.233	0.000	0.000	0.000	0.067	0.033	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.175	0.408	0.183	0.000	0.000	0.067	0.125	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.033	0.000	0.000	0.000	0.008	0.000	0.033	0.000	0.000	0.000	0.033	0.708	0.067	0.000	0.000	0.000	0.117	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.033	0.000	0.000	0.000	0.000	0.825	0.075	0.000	0.000	0.000	0.025	0.000	0.000	0.000	0.000	0.000	0.000
14	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.033	0.000	0.000	0.000	0.000	0.892	0.000	0.000	0.000	0.000	0.033	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.033	0.017	0.000	0.000	0.000	0.742	0.133	0.000	0.000	0.000	0.033	0.000	0.000	0.000	0.000
16	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.117	0.667	0.083	0.000	0.000	0.008	0.083	0.000	0.000	0.000
17	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.033	0.800	0.075	0.000	0.000	0.000	0.050	0.000	0.000
18	0.033	0.000	0.000	0.000	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.900	0.017	0.000	0.000	0.000	0.042	0.000
19	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.950	0.000	0.000	0.000	0.000	0.008
20	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.008	0.000	0.000	0.000	0.900	0.042	0.000	0.000	0.000
21	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.058	0.867	0.033	0.000	0.000
22	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.917	0.033	0.000
23	0.000	0.000	0.000	0.000	0.042	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.925	0.033
24	0.033	0.000	0.000	0.000	0.008	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.958

Figure 32: Training 5x5 Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1	0.414	0.103	0.000	0.000	0.241	0.138	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.034	0.483	0.069	0.000	0.000	0.207	0.172	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.034	0.103	0.655	0.034	0.000	0.000	0.069	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.034	0.000	0.069	0.862	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.138	0.000	0.000	0.000	0.276	0.241	0.000	0.000	0.000	0.138	0.207	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.034	0.103	0.000	0.000	0.000	0.310	0.138	0.000	0.000	0.207	0.207	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.034	0.207	0.034	0.000	0.000	0.000	0.448	0.172	0.000	0.000	0.000	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.034	0.000	0.172	0.000	0.000	0.000	0.000	0.621	0.069	0.000	0.000	0.000	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.034	0.000	0.000	0.103	0.000	0.000	0.000	0.000	0.828	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.034	0.103	0.000	0.000	0.000	0.414	0.241	0.000	0.000	0.000	0.103	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.207	0.310	0.207	0.000	0.000	0.103	0.138	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.034	0.000	0.000	0.000	0.000	0.000	0.069	0.000	0.000	0.000	0.034	0.655	0.069	0.000	0.000	0.000	0.138	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.069	0.000	0.000	0.000	0.000	0.793	0.069	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.069	0.000	0.000	0.000	0.000	0.862	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.655	0.172	0.000	0.000	0.000	0.069	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
16	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.138	0.552	0.138	0.000	0.000	0.034	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.034	0.724	0.103	0.000	0.000	0.000	0.069	0.000	0.000	0.000	0.000	0.000	0.000	0.000
18	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.862	0.034	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000
19	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.931	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.828	0.103	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000
21	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.069	0.759	0.069	0.000	0.000	0.000	0.069	0.000	0.000	0.000
22	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.862	0.069	0.000	0.000	0.000	0.034	0.000	0.000
23	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.897	0.034	0.000	0.000	0.000	0.034	0.000
24	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.966	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.931	0.034	0.000	0.000	0.000
26	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.034	0.897	0.034	0.000	0.000
27	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.931	0.034	0.000
28	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.931	0.034
29	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.966

Figure 33: Puzzle 6x5 Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1	0.069	0.000	0.000	0.000	0.310	0.621	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.034	0.690	0.000	0.000	0.000	0.000	0.276	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.034	0.000	0.828	0.000	0.000	0.000	0.000	0.138	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.034	0.000	0.000	0.897	0.000	0.000	0.000	0.034	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.621	0.000	0.000	0.000	0.069	0.310	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.276	0.000	0.000	0.034	0.069	0.138	0.000	0.000	0.448	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.034	0.034	0.138	0.000	0.000	0.000	0.586	0.034	0.000	0.000	0.000	0.172	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.034	0.000	0.034	0.034	0.000	0.000	0.000	0.793	0.069	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.034	0.000	0.000	0.069	0.000	0.000	0.000	0.000	0.897	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.517	0.448	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.034	0.448	0.103	0.000	0.000	0.276	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.724	0.138	0.000	0.000	0.000	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.828	0.034	0.000	0.000	0.000	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.966	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.069	0.000	0.000	0.000	0.621	0.276	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
16	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.069	0.552	0.103	0.000	0.000	0.138	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.793	0.069	0.000	0.000	0.000	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000
18	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.828	0.069	0.000	0.000	0.000	0.069	0.000	0.000	0.000	0.000	0.000	0.000
19	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.931	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.034	0.069	0.000	0.000	0.000	0.724	0.138	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
21	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.103	0.690	0.069	0.000	0.000	0.034	0.069	0.000	0.000	0.000
22	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.034	0.828	0.034	0.000	0.000	0.000	0.069	0.000	0.000
23	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.897	0.034	0.000	0.000	0.000	0.034	0.000
24	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.931	0.000	0.000	0.000	0.000	0.034
25	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.034	0.034	0.000	0.000	0.000	0.862	0.034	0.000	0.000	0.000
26	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.103	0.862	0.000	0.000	0.000
27	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.034	0.931	0.000	0.000
28	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.966	0.000
29	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.966

Figure 34: Alternative 6x5 Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29
1	0.345	0.083	0.000	0.000	0.255	0.234	0.083	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.034	0.524	0.055	0.000	0.000	0.166	0.193	0.028	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.034	0.083	0.690	0.028	0.000	0.000	0.055	0.110	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.034	0.000	0.055	0.869	0.000	0.000	0.000	0.007	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.234	0.000	0.000	0.000	0.234	0.255	0.000	0.000	0.000	0.110	0.166	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.028	0.138	0.000	0.000	0.007	0.262	0.138	0.000	0.000	0.255	0.172	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.034	0.172	0.055	0.000	0.000	0.000	0.476	0.145	0.000	0.000	0.000	0.117	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.034	0.000	0.145	0.007	0.000	0.000	0.000	0.655	0.069	0.000	0.000	0.000	0.090	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.034	0.000	0.000	0.097	0.000	0.000	0.000	0.000	0.841	0.000	0.000	0.000	0.000	0.028	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.034	0.083	0.000	0.000	0.000	0.434	0.283	0.000	0.000	0.000	0.083	0.083	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.172	0.338	0.186	0.000	0.000	0.138	0.131	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.028	0.000	0.000	0.000	0.007	0.000	0.055	0.000	0.000	0.000	0.028	0.669	0.083	0.000	0.000	0.000	0.131	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.055	0.000	0.000	0.000	0.000	0.800	0.062	0.000	0.000	0.000	0.048	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.055	0.000	0.000	0.000	0.000	0.883	0.000	0.000	0.000	0.000	0.028	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.028	0.014	0.000	0.000	0.000	0.648	0.193	0.000	0.000	0.000	0.055	0.028	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
16	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.124	0.552	0.131	0.000	0.000	0.055	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.028	0.000	0.000	0.000	0.028	0.738	0.097	0.000	0.000	0.000	0.076	0.000	0.000	0.000	0.000	0.000	0.000	0.000
18	0.028	0.000	0.000	0.000	0.007	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.028	0.000	0.000	0.000	0.000	0.855	0.041	0.000	0.000	0.000	0.041	0.000	0.000	0.000	0.000	0.000	0.000
19	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.028	0.000	0.000	0.000	0.000	0.931	0.000	0.000	0.000	0.000	0.007	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.007	0.014	0.000	0.000	0.000	0.807	0.110	0.000	0.000	0.000	0.028	0.000	0.000	0.000	0.000
21	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.076	0.745	0.069	0.000	0.000	0.007	0.069	0.000	0.000	0.000
22	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.007	0.855	0.062	0.000	0.000	0.000	0.041	0.000	0.000
23	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.897	0.034	0.000	0.000	0.000	0.034	0.000
24	0.028	0.000	0.000	0.000	0.007	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.959	0.000	0.000	0.000	0.000	0.007
25	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.007	0.007	0.000	0.000	0.000	0.917	0.034	0.000	0.000	0.000
26	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.048	0.890	0.028	0.000	0.000
27	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.007	0.931	0.028	0.000
28	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.938	0.028
29	0.000	0.000	0.000	0.000	0.034	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.966

Figure 35: Training 6x5 Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
1	0.471	0.088	0.000	0.000	0.206	0.147	0.088	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.029	0.471	0.059	0.000	0.000	0.265	0.147	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.029	0.147	0.647	0.029	0.000	0.000	0.059	0.088	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.029	0.000	0.088	0.853	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.147	0.000	0.000	0.000	0.235	0.206	0.000	0.000	0.000	0.118	0.294	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.029	0.088	0.000	0.000	0.000	0.294	0.118	0.000	0.000	0.294	0.176	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.029	0.206	0.000	0.000	0.000	0.000	0.500	0.147	0.000	0.000	0.000	0.088	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.647	0.059	0.000	0.000	0.000	0.088	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.029	0.000	0.000	0.118	0.000	0.000	0.000	0.000	0.824	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.029	0.088	0.000	0.000	0.000	0.353	0.235	0.000	0.000	0.000	0.118	0.176	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.206	0.265	0.206	0.000	0.000	0.176	0.118	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.029	0.000	0.000	0.000	0.000	0.000	0.088	0.000	0.000	0.000	0.029	0.647	0.059	0.000	0.000	0.000	0.147	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.088	0.000	0.000	0.000	0.000	0.794	0.059	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.088	0.000	0.000	0.000	0.000	0.853	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.559	0.206	0.000	0.000	0.000	0.088	0.088	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
16	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.147	0.471	0.147	0.000	0.000	0.088	0.118	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.059	0.000	0.000	0.000	0.029	0.676	0.118	0.000	0.000	0.000	0.088	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
18	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.059	0.000	0.000	0.000	0.000	0.824	0.029	0.000	0.000	0.000	0.059	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
19	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.059	0.000	0.000	0.000	0.000	0.912	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.735	0.147	0.000	0.000	0.000	0.059	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
21	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.088	0.647	0.118	0.000	0.000	0.029	0.088	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.794	0.088	0.000	0.000	0.000	0.059	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.853	0.059	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000
24	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.941	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.853	0.088	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000
26	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.059	0.794	0.059	0.000	0.000	0.000	0.059	0.000	0.000	0.000
27	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.882	0.059	0.000	0.000	0.000	0.029	0.000	0.000
28	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.912	0.029	0.000	0.000	0.000	0.029	0.000
29	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.971	0.000	0.000	0.000	0.000	0.000
30	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.941	0.029	0.000	0.000	0.000
31	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.029	0.912	0.029	0.000	0.000
32	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.941	0.029	0.000
33	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.941	0.029
34	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.971

Figure 36: Puzzle 7x5 Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
1	0.059	0.000	0.000	0.000	0.265	0.676	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.029	0.735	0.000	0.000	0.000	0.000	0.235	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.029	0.000	0.853	0.000	0.000	0.000	0.000	0.118	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.029	0.000	0.000	0.912	0.000	0.000	0.000	0.029	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.676	0.000	0.000	0.000	0.059	0.265	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.235	0.000	0.000	0.029	0.059	0.118	0.000	0.000	0.529	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.029	0.029	0.118	0.000	0.000	0.000	0.647	0.029	0.000	0.000	0.000	0.147	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.029	0.000	0.029	0.029	0.000	0.000	0.000	0.824	0.059	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.029	0.000	0.000	0.059	0.000	0.000	0.000	0.000	0.912	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.441	0.529	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.029	0.382	0.088	0.000	0.000	0.382	0.088	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.765	0.118	0.000	0.000	0.000	0.088	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.853	0.029	0.000	0.000	0.000	0.088	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.971	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.059	0.000	0.000	0.000	0.529	0.382	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
16	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.059	0.471	0.118	0.000	0.000	0.235	0.088	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.794	0.059	0.000	0.000	0.000	0.118	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
18	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.853	0.059	0.000	0.000	0.000	0.059	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
19	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.941	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.029	0.059	0.000	0.000	0.000	0.647	0.235	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
21	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.088	0.588	0.088	0.000	0.000	0.118	0.088	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.029	0.794	0.059	0.000	0.000	0.000	0.088	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.882	0.029	0.000	0.000	0.000	0.059	0.000	0.000	0.000	0.000	0.000	0.000
24	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.941	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.029	0.059	0.000	0.000	0.000	0.765	0.118	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
26	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.088	0.735	0.059	0.000	0.000	0.029	0.059	0.000	0.000	0.000
27	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.029	0.853	0.029	0.000	0.000	0.000	0.059	0.000	0.000
28	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.912	0.029	0.000	0.000	0.000	0.029	0.000
29	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.941	0.000	0.000	0.000	0.000	0.029
30	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.029	0.029	0.000	0.000	0.000	0.882	0.029	0.000	0.000	0.000
31	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.088	0.882	0.000	0.000	0.000
32	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.029	0.941	0.000	0.000
33	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.971	0.000
34	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.971

Figure 37: Alternative 7x5 Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
1	0.388	0.071	0.000	0.000	0.218	0.253	0.071	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.029	0.524	0.047	0.000	0.000	0.212	0.165	0.024	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.029	0.118	0.688	0.024	0.000	0.000	0.047	0.094	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.029	0.000	0.071	0.865	0.000	0.000	0.000	0.006	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.253	0.000	0.000	0.000	0.200	0.218	0.000	0.000	0.000	0.094	0.235	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.024	0.118	0.000	0.000	0.006	0.247	0.118	0.000	0.000	0.341	0.147	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.029	0.171	0.047	0.000	0.000	0.000	0.529	0.124	0.000	0.000	0.000	0.100	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.029	0.000	0.147	0.006	0.000	0.000	0.000	0.682	0.059	0.000	0.000	0.000	0.076	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.029	0.000	0.000	0.106	0.000	0.000	0.000	0.000	0.841	0.000	0.000	0.000	0.000	0.024	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.029	0.071	0.000	0.000	0.000	0.371	0.294	0.000	0.000	0.000	0.094	0.141	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.171	0.288	0.182	0.000	0.000	0.218	0.112	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.024	0.000	0.000	0.000	0.006	0.000	0.071	0.000	0.000	0.000	0.024	0.671	0.071	0.000	0.000	0.000	0.135	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.071	0.000	0.000	0.000	0.000	0.806	0.053	0.000	0.000	0.000	0.041	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.071	0.000	0.000	0.000	0.000	0.876	0.000	0.000	0.000	0.000	0.024	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.024	0.012	0.000	0.000	0.000	0.553	0.241	0.000	0.000	0.000	0.071	0.071	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
16	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.129	0.471	0.141	0.000	0.000	0.118	0.112	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.047	0.000	0.000	0.000	0.024	0.700	0.106	0.000	0.000	0.000	0.094	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
18	0.024	0.000	0.000	0.000	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.047	0.000	0.000	0.000	0.000	0.829	0.035	0.000	0.000	0.000	0.059	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
19	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.047	0.000	0.000	0.000	0.000	0.918	0.000	0.000	0.000	0.000	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.012	0.000	0.000	0.000	0.718	0.165	0.000	0.000	0.000	0.047	0.024	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
21	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.088	0.635	0.112	0.000	0.000	0.047	0.088	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.024	0.000	0.000	0.000	0.006	0.794	0.082	0.000	0.000	0.000	0.065	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.024	0.000	0.000	0.000	0.000	0.859	0.053	0.000	0.000	0.000	0.035	0.000	0.000	0.000	0.000	0.000	0.000
24	0.024	0.000	0.000	0.000	0.006	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.024	0.000	0.000	0.000	0.000	0.941	0.000	0.000	0.000	0.000	0.006	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.012	0.000	0.000	0.000	0.835	0.094	0.000	0.000	0.000	0.024	0.000	0.000	0.000	0.000
26	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.065	0.782	0.059	0.000	0.000	0.006	0.059	0.000	0.000	0.000
27	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.876	0.053	0.000	0.000	0.000	0.035	0.000	0.000
28	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.912	0.029	0.000	0.000	0.000	0.029	0.000
29	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.965	0.000	0.000	0.000	0.000	0.006
30	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.006	0.000	0.000	0.000	0.929	0.029	0.000	0.000	0.000
31	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.041	0.906	0.024	0.000	0.000
32	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.006	0.941	0.024	0.000
33	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.947	0.024
34	0.000	0.000	0.000	0.000	0.029	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.971

Figure 38: Training 7x5 Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
1	0.513	0.077	0.000	0.000	0.179	0.154	0.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.026	0.462	0.051	0.000	0.000	0.308	0.128	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.026	0.179	0.641	0.026	0.000	0.000	0.051	0.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.026	0.000	0.103	0.846	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.154	0.000	0.000	0.000	0.205	0.179	0.000	0.000	0.000	0.103	0.359	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.026	0.077	0.000	0.000	0.000	0.282	0.103	0.000	0.000	0.359	0.154	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.026	0.205	0.026	0.000	0.000	0.000	0.538	0.128	0.000	0.000	0.000	0.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.026	0.000	0.179	0.000	0.000	0.000	0.000	0.667	0.051	0.000	0.000	0.000	0.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.026	0.000	0.000	0.128	0.000	0.000	0.000	0.000	0.821	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.026	0.077	0.000	0.000	0.000	0.308	0.231	0.000	0.000	0.000	0.103	0.256	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.205	0.231	0.179	0.000	0.000	0.256	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.026	0.000	0.000	0.000	0.000	0.000	0.103	0.000	0.000	0.000	0.026	0.667	0.051	0.000	0.000	0.000	0.128	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.103	0.000	0.000	0.000	0.000	0.795	0.051	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.103	0.000	0.000	0.000	0.000	0.846	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.487	0.205	0.000	0.000	0.000	0.103	0.154	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
16	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.154	0.410	0.154	0.000	0.000	0.154	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.077	0.000	0.000	0.000	0.026	0.667	0.103	0.000	0.000	0.000	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
18	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.077	0.000	0.000	0.000	0.000	0.821	0.026	0.000	0.000	0.000	0.051	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
19	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.077	0.000	0.000	0.000	0.000	0.897	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.641	0.179	0.000	0.000	0.000	0.077	0.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
21	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.103	0.564	0.128	0.000	0.000	0.077	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.051	0.000	0.000	0.000	0.000	0.744	0.103	0.000	0.000	0.000	0.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.051	0.000	0.000	0.000	0.000	0.821	0.051	0.000	0.000	0.000	0.051	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.051	0.000	0.000	0.000	0.000	0.923	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.769	0.128	0.000	0.000	0.000	0.051	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
26	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.077	0.692	0.103	0.000	0.000	0.026	0.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
27	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.821	0.077	0.000	0.000	0.000	0.051	0.000	0.000	0.000	0.000	0.000	0.000	0.000
28	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.872	0.051	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000
29	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.949	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
30	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.872	0.077	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000
31	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.051	0.821	0.051	0.000	0.000	0.000	0.051	0.000	0.000	0.000
32	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.897	0.051	0.000	0.000	0.000	0.026	0.000	0.000
33	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.923	0.026	0.000	0.000	0.000	0.026	0.000
34	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.974	0.000	0.000	0.000	0.000	0.000
35	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.949	0.026	0.000	0.000	0.000
36	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.026	0.923	0.026	0.000	0.000
37	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.949	0.026	0.000
38	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.949	0.026
39	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.974

Figure 39: Puzzle 8x5 Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
1	0.051	0.000	0.000	0.000	0.231	0.718	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.026	0.769	0.000	0.000	0.000	0.000	0.205	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.026	0.000	0.872	0.000	0.000	0.000	0.000	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.026	0.000	0.000	0.923	0.000	0.000	0.000	0.026	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.718	0.000	0.000	0.000	0.051	0.231	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.000	0.205	0.000	0.000	0.026	0.051	0.103	0.000	0.000	0.590	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.026	0.026	0.103	0.000	0.000	0.000	0.692	0.026	0.000	0.000	0.000	0.128	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.026	0.000	0.026	0.026	0.000	0.000	0.000	0.846	0.051	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.026	0.000	0.000	0.051	0.000	0.000	0.000	0.000	0.923	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.385	0.590	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.026	0.333	0.077	0.000	0.000	0.462	0.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.795	0.103	0.000	0.000	0.000	0.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.872	0.026	0.000	0.000	0.000	0.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.974	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.051	0.000	0.000	0.000	0.462	0.462	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
16	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.051	0.410	0.103	0.000	0.000	0.333	0.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.821	0.051	0.000	0.000	0.000	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
18	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.872	0.051	0.000	0.000	0.000	0.051	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
19	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.949	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.026	0.051	0.000	0.000	0.000	0.564	0.333	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
21	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.077	0.513	0.103	0.000	0.000	0.205	0.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.026	0.795	0.051	0.000	0.000	0.000	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.897	0.026	0.000	0.000	0.000	0.051	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.949	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.026	0.051	0.000	0.000	0.000	0.692	0.205	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
26	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.077	0.641	0.077	0.000	0.000	0.103	0.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
27	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.026	0.821	0.051	0.000	0.000	0.000	0.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000
28	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.897	0.026	0.000	0.000	0.000	0.051	0.000	0.000	0.000	0.000	0.000	0.000
29	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.949	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000
30	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.026	0.051	0.000	0.000	0.000	0.795	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
31	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.077	0.769	0.051	0.000	0.000	0.026	0.051	0.000	0.000	0.000
32	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.026	0.872	0.026	0.000	0.000	0.000	0.051	0.000	0.000
33	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.923	0.026	0.000	0.000	0.000	0.026	0.000
34	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.949	0.000	0.000	0.000	0.000	0.026
35	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.026	0.026	0.000	0.000	0.000	0.897	0.026	0.000	0.000	0.000
36	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.077	0.897	0.000	0.000	0.000
37	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.026	0.949	0.000	0.000
38	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.974	0.000
39	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.974

Figure 40: Alternative 8x5 Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39
1	0.421	0.062	0.000	0.000	0.190	0.267	0.062	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.026	0.523	0.041	0.000	0.000	0.246	0.144	0.021	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.026	0.144	0.687	0.021	0.000	0.000	0.041	0.082	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.026	0.000	0.082	0.862	0.000	0.000	0.000	0.005	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.267	0.000	0.000	0.000	0.174	0.190	0.000	0.000	0.000	0.082	0.287	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.021	0.103	0.000	0.000	0.005	0.236	0.103	0.000	0.000	0.405	0.128	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.026	0.169	0.041	0.000	0.000	0.000	0.569	0.108	0.000	0.000	0.000	0.087	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.026	0.000	0.149	0.005	0.000	0.000	0.000	0.703	0.051	0.000	0.000	0.000	0.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.026	0.000	0.000	0.113	0.000	0.000	0.000	0.000	0.841	0.000	0.000	0.000	0.000	0.021	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.000	0.000	0.000	0.000	0.026	0.062	0.000	0.000	0.000	0.323	0.303	0.000	0.000	0.000	0.082	0.205	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.169	0.251	0.159	0.000	0.000	0.297	0.097	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.021	0.000	0.000	0.000	0.005	0.000	0.082	0.000	0.000	0.000	0.021	0.692	0.062	0.000	0.000	0.000	0.118	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.082	0.000	0.000	0.000	0.000	0.810	0.046	0.000	0.000	0.000	0.036	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.082	0.000	0.000	0.000	0.000	0.872	0.000	0.000	0.000	0.000	0.021	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.021	0.010	0.000	0.000	0.000	0.482	0.256	0.000	0.000	0.000	0.082	0.123	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
16	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.133	0.410	0.144	0.000	0.000	0.190	0.097	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
17	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.062	0.000	0.000	0.000	0.021	0.697	0.092	0.000	0.000	0.000	0.103	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
18	0.021	0.000	0.000	0.000	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.062	0.000	0.000	0.000	0.000	0.831	0.031	0.000	0.000	0.000	0.051	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
19	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.062	0.000	0.000	0.000	0.000	0.908	0.000	0.000	0.000	0.000	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
20	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.005	0.010	0.000	0.000	0.000	0.626	0.210	0.000	0.000	0.000	0.062	0.062	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
21	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.097	0.554	0.123	0.000	0.000	0.103	0.097	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
22	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.041	0.000	0.000	0.000	0.005	0.754	0.092	0.000	0.000	0.000	0.082	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
23	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.041	0.000	0.000	0.000	0.000	0.836	0.046	0.000	0.000	0.000	0.051	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
24	0.021	0.000	0.000	0.000	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.041	0.000	0.000	0.000	0.000	0.928	0.000	0.000	0.000	0.000	0.005	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
25	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.005	0.010	0.000	0.000	0.000	0.754	0.144	0.000	0.000	0.000	0.041	0.021	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
26	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.077	0.682	0.097	0.000	0.000	0.041	0.077	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
27	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.021	0.000	0.000	0.000	0.005	0.821	0.072	0.000	0.000	0.000	0.056	0.000	0.000	0.000	0.000	0.000	0.000	0.000
28	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.021	0.000	0.000	0.000	0.000	0.877	0.046	0.000	0.000	0.000	0.031	0.000	0.000	0.000	0.000	0.000	0.000
29	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.021	0.000	0.000	0.000	0.000	0.949	0.000	0.000	0.000	0.000	0.005	0.000	0.000	0.000	0.000	0.000
30	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.005	0.010	0.000	0.000	0.000	0.856	0.082	0.000	0.000	0.000	0.021	0.000	0.000	0.000	0.000
31	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.056	0.810	0.051	0.000	0.000	0.005	0.051	0.000	0.000	0.000
32	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.005	0.892	0.046	0.000	0.000	0.000	0.031	0.000	0.000
33	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.923	0.026	0.000	0.000	0.000	0.026	0.000
34	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.969	0.000	0.000	0.000	0.000	0.005
35	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.005	0.005	0.000	0.000	0.000	0.938	0.026	0.000	0.000	0.000
36	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.036	0.918	0.021	0.000	0.000
37	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.005	0.949	0.021	0.000
38	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.954	0.021
39	0.000	0.000	0.000	0.000	0.026	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.974

Figure 41: Training 8x5 Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	0.185	0.074	0.000	0.000	0.000	0.000	0.407	0.074	0.259	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.037	0.333	0.111	0.000	0.000	0.000	0.000	0.037	0.259	0.222	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.037	0.000	0.481	0.111	0.000	0.000	0.000	0.000	0.037	0.222	0.111	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.037	0.000	0.000	0.630	0.074	0.000	0.000	0.000	0.000	0.037	0.185	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.037	0.000	0.000	0.000	0.815	0.037	0.000	0.000	0.000	0.000	0.000	0.111	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.037	0.000	0.000	0.000	0.000	0.926	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.074	0.000	0.000	0.000	0.000	0.000	0.407	0.407	0.000	0.000	0.000	0.000	0.000	0.111	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.037	0.296	0.000	0.000	0.000	0.000	0.000	0.333	0.148	0.000	0.000	0.000	0.000	0.000	0.185	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.037	0.222	0.222	0.000	0.000	0.000	0.000	0.000	0.259	0.111	0.000	0.000	0.000	0.000	0.000	0.148	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.037	0.000	0.185	0.111	0.000	0.000	0.000	0.000	0.000	0.407	0.148	0.000	0.000	0.000	0.000	0.000	0.111	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.037	0.000	0.000	0.148	0.037	0.000	0.000	0.000	0.000	0.000	0.556	0.111	0.000	0.000	0.000	0.000	0.000	0.111	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.037	0.000	0.000	0.000	0.074	0.000	0.000	0.000	0.000	0.000	0.000	0.741	0.074	0.000	0.000	0.000	0.000	0.000	0.074	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	0.037	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.889	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.148	0.000	0.000	0.000	0.000	0.000	0.704	0.111	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.074	0.000	0.000	0.000	0.000	0.037	0.000	0.037	0.000	0.000	0.000	0.000	0.185	0.519	0.074	0.000	0.000	0.000	0.000	0.074	0.000	0.000	0.000	0.000	0.000	0.000
16	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.111	0.704	0.074	0.000	0.000	0.000	0.000	0.074	0.000	0.000	0.000	0.000	0.000
17	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.074	0.778	0.037	0.000	0.000	0.000	0.000	0.074	0.000	0.000	0.000	0.000
18	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.852	0.037	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000
19	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.889	0.037	0.000	0.000	0.000	0.000	0.037	0.000	0.000
20	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.926	0.000	0.000	0.000	0.000	0.000	0.037	0.000
21	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.074	0.000	0.000	0.000	0.000	0.000	0.815	0.037	0.000	0.000	0.000	0.000	0.037
22	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.111	0.815	0.037	0.000	0.000	0.000	0.000
23	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.074	0.852	0.037	0.000	0.000	0.000
24	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.926	0.000	0.000	0.000
25	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.963	0.000	0.000
26	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.963	0.000
27	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.963

Figure 42: Puzzle 13DX Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	0.148	0.000	0.000	0.000	0.000	0.000	0.667	0.185	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.037	0.333	0.000	0.000	0.000	0.000	0.000	0.037	0.593	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.037	0.000	0.481	0.000	0.000	0.000	0.000	0.000	0.037	0.444	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.037	0.000	0.000	0.630	0.000	0.000	0.000	0.000	0.000	0.037	0.296	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.037	0.000	0.000	0.000	0.778	0.000	0.000	0.000	0.000	0.000	0.037	0.148	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.037	0.000	0.000	0.000	0.000	0.889	0.000	0.000	0.000	0.000	0.000	0.037	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.185	0.000	0.000	0.000	0.000	0.000	0.074	0.667	0.000	0.000	0.000	0.000	0.000	0.074	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.000	0.593	0.000	0.000	0.000	0.000	0.037	0.111	0.111	0.000	0.000	0.000	0.000	0.037	0.111	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.037	0.074	0.444	0.000	0.000	0.000	0.000	0.000	0.222	0.074	0.000	0.000	0.000	0.000	0.000	0.148	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.037	0.000	0.074	0.296	0.000	0.000	0.000	0.000	0.000	0.444	0.074	0.000	0.000	0.000	0.000	0.000	0.074	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.037	0.000	0.000	0.074	0.148	0.000	0.000	0.000	0.000	0.000	0.593	0.074	0.000	0.000	0.000	0.000	0.000	0.074	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.037	0.000	0.000	0.000	0.074	0.037	0.000	0.000	0.000	0.000	0.000	0.741	0.074	0.000	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	0.037	0.000	0.000	0.000	0.000	0.074	0.000	0.000	0.000	0.000	0.000	0.000	0.889	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.852	0.111	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.000	0.037	0.000	0.000	0.000	0.000	0.037	0.778	0.037	0.000	0.000	0.000	0.000	0.074	0.000	0.000	0.000	0.000	0.000	0.000
16	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.815	0.074	0.000	0.000	0.000	0.000	0.074	0.000	0.000	0.000	0.000	0.000
17	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.852	0.074	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000	0.000
18	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.852	0.074	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000
19	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.889	0.037	0.000	0.000	0.000	0.000	0.037	0.000	0.000
20	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.963	0.000	0.000	0.000	0.000	0.000	0.000	0.000
21	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.889	0.037	0.000	0.000	0.000	0.000	0.037
22	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.889	0.037	0.000	0.000	0.000	0.000
23	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.926	0.037	0.000	0.000	0.000
24	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.926	0.037	0.000	0.000
25	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.926	0.037	0.000
26	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.963	0.000
27	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.963

Figure 43: Alternative 13DX Movement Matrix

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
1	0.178	0.059	0.000	0.000	0.000	0.000	0.459	0.096	0.207	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.037	0.333	0.089	0.000	0.000	0.000	0.000	0.037	0.326	0.178	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.037	0.000	0.481	0.089	0.000	0.000	0.000	0.000	0.037	0.267	0.089	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
4	0.037	0.000	0.000	0.630	0.059	0.000	0.000	0.000	0.000	0.037	0.207	0.030	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
5	0.037	0.000	0.000	0.000	0.807	0.030	0.000	0.000	0.000	0.000	0.007	0.119	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
6	0.037	0.000	0.000	0.000	0.000	0.919	0.000	0.000	0.000	0.000	0.000	0.007	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
7	0.096	0.000	0.000	0.000	0.000	0.000	0.341	0.459	0.000	0.000	0.000	0.000	0.000	0.104	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
8	0.030	0.356	0.000	0.000	0.000	0.000	0.007	0.289	0.141	0.000	0.000	0.000	0.000	0.007	0.170	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
9	0.037	0.193	0.267	0.000	0.000	0.000	0.000	0.000	0.252	0.104	0.000	0.000	0.000	0.000	0.000	0.148	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
10	0.037	0.000	0.163	0.148	0.000	0.000	0.000	0.000	0.000	0.415	0.133	0.000	0.000	0.000	0.000	0.000	0.104	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
11	0.037	0.000	0.000	0.133	0.059	0.000	0.000	0.000	0.000	0.000	0.563	0.104	0.000	0.000	0.000	0.000	0.000	0.104	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
12	0.037	0.000	0.000	0.000	0.074	0.007	0.000	0.000	0.000	0.000	0.000	0.741	0.074	0.000	0.000	0.000	0.000	0.000	0.067	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
13	0.037	0.000	0.000	0.000	0.000	0.044	0.000	0.000	0.000	0.000	0.000	0.000	0.889	0.000	0.000	0.000	0.000	0.000	0.000	0.030	0.000	0.000	0.000	0.000	0.000	0.000	0.000
14	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.119	0.000	0.000	0.000	0.000	0.000	0.733	0.111	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
15	0.000	0.059	0.000	0.000	0.000	0.000	0.037	0.000	0.037	0.000	0.000	0.000	0.000	0.156	0.570	0.067	0.000	0.000	0.000	0.000	0.074	0.000	0.000	0.000	0.000	0.000	0.000
16	0.030	0.000	0.000	0.000	0.000	0.000	0.007	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.089	0.726	0.074	0.000	0.000	0.000	0.000	0.074	0.000	0.000	0.000	0.000	0.000
17	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.059	0.793	0.044	0.000	0.000	0.000	0.000	0.067	0.000	0.000	0.000	0.000
18	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.030	0.852	0.044	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000
19	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.889	0.037	0.000	0.000	0.000	0.000	0.037	0.000	0.000
20	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.933	0.000	0.000	0.000	0.000	0.000	0.030	0.000
21	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.059	0.000	0.000	0.000	0.000	0.000	0.830	0.037	0.000	0.000	0.000	0.000	0.037
22	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.096	0.830	0.037	0.000	0.000	0.000	0.000
23	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.059	0.867	0.037	0.000	0.000	0.000
24	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.030	0.926	0.007	0.000	0.000
25	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.956	0.007	0.000
26	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.963	0.000
27	0.000	0.000	0.000	0.000	0.000	0.000	0.037	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.963

Figure 44: Training 13DX Movement Matrix

APPENDIX C: EXPERIMENTAL RESULTS

Table 6: Alternative 2x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.210	0.222	11
2	0.189	0.333	8
3	0.189	0.556	6
4	0.240	0.778	11
5	0.210	0.222	10
6	0.174	0.222	9
7	0.128	0.333	7
8	0.166	0.556	8
9	0.240	0.778	11

Table 7: Puzzle 2x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.148	0.222	10
2	0.128	0.333	6
3	0.174	0.556	6
4	0.240	0.778	12
5	0.210	0.556	11
6	0.174	0.556	9
7	0.128	0.333	6
8	0.174	0.556	6
9	0.240	0.778	12

Table 8: Training 2x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.155	0.222	10
2	0.138	0.333	6
3	0.176	0.556	6
4	0.240	0.778	12
5	0.202	0.489	11
6	0.161	0.489	9
7	0.121	0.333	6
8	0.171	0.556	6
9	0.240	0.778	12

Table 9: Alternative 3x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.160	0.143	15
2	0.162	0.429	11
3	0.175	0.643	10
4	0.219	0.857	25
5	0.160	0.143	14
6	0.129	0.143	12
7	0.111	0.357	9
8	0.162	0.643	15
9	0.219	0.857	25
10	0.219	0.857	29
11	0.219	0.857	29
12	0.219	0.857	20
13	0.219	0.857	34
14	0.238	0.929	35

Table 10: Puzzle 3x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.111	0.214	12
2	0.127	0.429	10
3	0.169	0.643	15
4	0.219	0.857	22
5	0.150	0.500	16
6	0.124	0.429	13
7	0.118	0.429	9
8	0.148	0.571	12
9	0.200	0.786	28
10	0.183	0.714	19
11	0.169	0.643	21
12	0.219	0.857	23
13	0.238	0.929	38
14	0.238	0.929	35

Table 11: Training 3x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.115	0.200	14
2	0.131	0.429	10
3	0.170	0.643	14
4	0.219	0.857	22
5	0.144	0.429	16
6	0.115	0.371	14
7	0.112	0.414	9
8	0.150	0.586	11
9	0.204	0.800	24
10	0.190	0.743	20
11	0.178	0.686	22
12	0.219	0.857	21
13	0.234	0.914	31
14	0.238	0.929	34

Table 12: Alternative 4x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.138	0.105	12
2	0.146	0.526	13
3	0.168	0.737	26
4	0.187	0.842	32
5	0.138	0.105	12
6	0.105	0.105	10
7	0.108	0.421	11
8	0.152	0.684	26
9	0.188	0.842	33
10	0.168	0.737	28
11	0.152	0.684	28
12	0.166	0.737	21
13	0.176	0.789	34
14	0.211	0.947	53
15	0.187	0.842	33
16	0.188	0.842	34
17	0.211	0.947	54
18	0.199	0.895	48
19	0.211	0.947	50

Table 13: Puzzle 4x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.097	0.263	12
2	0.117	0.474	13
3	0.154	0.684	21
4	0.199	0.895	33
5	0.115	0.421	15
6	0.101	0.368	12
7	0.108	0.421	12
8	0.132	0.579	21
9	0.187	0.842	24
10	0.133	0.579	17
11	0.116	0.474	20
12	0.176	0.789	29
13	0.199	0.895	37
14	0.199	0.895	48
15	0.187	0.842	27
16	0.176	0.789	27
17	0.187	0.842	41
18	0.211	0.947	50
19	0.211	0.947	49

Table 14: Training 4x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.098	0.232	13
2	0.122	0.484	13
3	0.156	0.695	20
4	0.197	0.884	32
5	0.111	0.358	14
6	0.093	0.316	12
7	0.104	0.421	11
8	0.136	0.600	15
9	0.187	0.842	26
10	0.139	0.611	16
11	0.121	0.516	15
12	0.173	0.779	20
13	0.194	0.874	25
14	0.201	0.905	45
15	0.187	0.842	27
16	0.178	0.800	27
17	0.192	0.863	32
18	0.209	0.937	41
19	0.211	0.947	48

Table 15: Alternative 5x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.129	0.083	12
2	0.139	0.625	15
3	0.160	0.792	36
4	0.174	0.875	42
5	0.129	0.083	12
6	0.095	0.083	10
7	0.109	0.500	13
8	0.149	0.750	36
9	0.175	0.875	44
10	0.139	0.625	16
11	0.113	0.542	14
12	0.144	0.708	21
13	0.159	0.792	28
14	0.191	0.958	68
15	0.144	0.708	19
16	0.133	0.667	25
17	0.166	0.833	31
18	0.166	0.833	46
19	0.183	0.917	63
20	0.166	0.833	30
21	0.167	0.833	32
22	0.183	0.917	33
23	0.191	0.958	67
24	0.191	0.958	64

Table 16: Puzzle 5x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.092	0.333	8
2	0.108	0.500	13
3	0.134	0.667	17
4	0.174	0.875	34
5	0.092	0.333	10
6	0.087	0.333	9
7	0.098	0.417	14
8	0.121	0.583	22
9	0.166	0.833	33
10	0.108	0.500	14
11	0.092	0.375	13
12	0.142	0.708	19
13	0.166	0.833	41
14	0.174	0.875	52
15	0.150	0.750	30
16	0.134	0.667	22
17	0.158	0.792	33
18	0.183	0.917	49
19	0.191	0.958	66
20	0.183	0.917	44
21	0.174	0.875	32
22	0.183	0.917	50
23	0.183	0.917	61
24	0.191	0.958	62

Table 17: Training 5x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.091	0.283	8
2	0.113	0.525	13
3	0.139	0.692	18
4	0.174	0.875	34
5	0.091	0.283	10
6	0.081	0.283	8
7	0.097	0.433	12
8	0.126	0.617	21
9	0.168	0.842	34
10	0.113	0.525	14
11	0.094	0.408	13
12	0.142	0.708	17
13	0.164	0.825	26
14	0.178	0.892	42
15	0.149	0.742	26
16	0.134	0.667	24
17	0.159	0.800	30
18	0.179	0.900	33
19	0.190	0.950	54
20	0.179	0.900	40
21	0.173	0.867	35
22	0.183	0.917	35
23	0.184	0.925	58
24	0.191	0.958	62

Table 18: Alternative 6x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.125	0.069	12
2	0.134	0.690	20
3	0.152	0.828	46
4	0.163	0.897	52
5	0.125	0.069	13
6	0.096	0.069	11
7	0.112	0.586	17
8	0.144	0.793	46
9	0.164	0.897	54
10	0.123	0.517	14
11	0.096	0.448	13
12	0.134	0.724	20
13	0.151	0.828	37
14	0.176	0.966	84
15	0.122	0.621	16
16	0.104	0.552	15
17	0.145	0.793	25
18	0.151	0.828	40
19	0.170	0.931	64
20	0.133	0.724	18
21	0.126	0.690	30
22	0.151	0.828	34
23	0.163	0.897	57
24	0.170	0.931	76
25	0.157	0.862	36
26	0.158	0.862	39
27	0.170	0.931	40
28	0.176	0.966	81
29	0.176	0.966	78

Table 19: Puzzle 6x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.090	0.414	8
2	0.098	0.483	11
3	0.121	0.655	16
4	0.157	0.862	31
5	0.079	0.276	11
6	0.078	0.310	9
7	0.093	0.448	13
8	0.117	0.621	22
9	0.151	0.828	35
10	0.089	0.414	13
11	0.078	0.310	11
12	0.121	0.655	15
13	0.145	0.793	24
14	0.157	0.862	32
15	0.122	0.655	19
16	0.105	0.552	15
17	0.132	0.724	22
18	0.157	0.862	32
19	0.170	0.931	48
20	0.151	0.828	39
21	0.139	0.759	26
22	0.157	0.862	37
23	0.163	0.897	57
24	0.176	0.966	81
25	0.170	0.931	52
26	0.163	0.897	39
27	0.170	0.931	64
28	0.170	0.931	76
29	0.176	0.966	78

Table 20: Training 6x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.087	0.345	9
2	0.103	0.524	12
3	0.127	0.690	17
4	0.158	0.869	34
5	0.079	0.234	10
6	0.076	0.262	9
7	0.095	0.476	13
8	0.122	0.655	24
9	0.154	0.841	37
10	0.094	0.434	12
11	0.079	0.338	12
12	0.123	0.669	16
13	0.146	0.800	25
14	0.161	0.883	31
15	0.122	0.648	18
16	0.105	0.552	15
17	0.135	0.738	22
18	0.156	0.855	32
19	0.170	0.931	45
20	0.148	0.807	34
21	0.136	0.745	28
22	0.156	0.855	36
23	0.163	0.897	48
24	0.175	0.959	66
25	0.167	0.917	48
26	0.162	0.890	43
27	0.170	0.931	44
28	0.171	0.938	73
29	0.176	0.966	77

Table 21: Alternative 7x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.121	0.059	13
2	0.129	0.735	24
3	0.145	0.853	55
4	0.154	0.912	63
5	0.121	0.059	13
6	0.098	0.059	12
7	0.112	0.647	21
8	0.139	0.824	55
9	0.154	0.912	65
10	0.115	0.441	17
11	0.091	0.382	16
12	0.130	0.765	23
13	0.144	0.853	45
14	0.164	0.971	99
15	0.109	0.529	17
16	0.090	0.471	15
17	0.135	0.794	21
18	0.144	0.853	44
19	0.159	0.941	55
20	0.115	0.647	19
21	0.102	0.588	18
22	0.134	0.794	28
23	0.149	0.882	44
24	0.159	0.941	76
25	0.130	0.765	22
26	0.125	0.735	37
27	0.144	0.853	40
28	0.154	0.912	68
29	0.159	0.941	89
30	0.149	0.882	43
31	0.149	0.882	46
32	0.159	0.941	47
33	0.164	0.971	96
34	0.164	0.971	92

Table 22: Puzzle 7x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.089	0.471	9
2	0.092	0.471	10
3	0.112	0.647	16
4	0.144	0.853	28
5	0.075	0.235	10
6	0.076	0.294	9
7	0.093	0.500	13
8	0.113	0.647	20
9	0.140	0.824	34
10	0.077	0.353	13
11	0.071	0.265	11
12	0.112	0.647	13
13	0.134	0.794	24
14	0.144	0.853	37
15	0.100	0.559	18
16	0.087	0.471	13
17	0.116	0.676	19
18	0.139	0.824	25
19	0.154	0.912	37
20	0.126	0.735	22
21	0.111	0.647	19
22	0.134	0.794	29
23	0.144	0.853	41
24	0.159	0.941	46
25	0.144	0.853	46
26	0.134	0.794	32
27	0.149	0.882	46
28	0.154	0.912	69
29	0.164	0.971	97
30	0.159	0.941	61
31	0.154	0.912	46
32	0.159	0.941	77
33	0.159	0.941	90
34	0.164	0.971	92

Table 23: Training 7x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.084	0.388	10
2	0.097	0.524	11
3	0.118	0.688	18
4	0.146	0.865	32
5	0.074	0.200	10
6	0.076	0.247	9
7	0.095	0.529	14
8	0.117	0.682	25
9	0.143	0.841	37
10	0.082	0.371	13
11	0.072	0.288	11
12	0.115	0.671	15
13	0.136	0.806	24
14	0.148	0.876	39
15	0.101	0.553	16
16	0.087	0.471	14
17	0.119	0.700	20
18	0.140	0.829	29
19	0.155	0.918	40
20	0.123	0.718	21
21	0.109	0.635	19
22	0.134	0.794	26
23	0.145	0.859	40
24	0.159	0.941	48
25	0.141	0.835	41
26	0.132	0.782	34
27	0.148	0.876	44
28	0.154	0.912	58
29	0.163	0.965	79
30	0.157	0.929	57
31	0.153	0.906	50
32	0.159	0.941	52
33	0.160	0.947	87
34	0.164	0.971	92

Table 24: Alternative 8x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.118	0.051	15
2	0.125	0.769	28
3	0.138	0.872	65
4	0.146	0.923	73
5	0.118	0.051	16
6	0.099	0.051	14
7	0.111	0.692	25
8	0.134	0.846	64
9	0.146	0.923	75
10	0.110	0.385	18
11	0.089	0.333	17
12	0.126	0.795	33
13	0.138	0.872	54
14	0.154	0.974	114
15	0.102	0.462	18
16	0.084	0.410	18
17	0.130	0.821	24
18	0.138	0.872	53
19	0.150	0.949	64
20	0.102	0.564	18
21	0.088	0.513	17
22	0.126	0.795	25
23	0.142	0.897	44
24	0.150	0.949	62
25	0.113	0.692	22
26	0.103	0.641	22
27	0.130	0.821	33
28	0.142	0.897	53
29	0.150	0.949	89
30	0.126	0.795	30
31	0.122	0.769	43
32	0.138	0.872	47
33	0.146	0.923	78
34	0.150	0.949	103
35	0.142	0.897	50
36	0.142	0.897	54
37	0.150	0.949	54
38	0.154	0.974	110
39	0.154	0.974	106

Table 25: Puzzle 8x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.088	0.513	10
2	0.088	0.462	9
3	0.105	0.641	15
4	0.134	0.846	25
5	0.074	0.205	12
6	0.076	0.282	11
7	0.092	0.538	14
8	0.109	0.667	19
9	0.131	0.821	32
10	0.073	0.308	14
11	0.068	0.231	10
12	0.107	0.667	14
13	0.126	0.795	27
14	0.134	0.846	38
15	0.086	0.487	16
16	0.076	0.410	13
17	0.107	0.667	18
18	0.130	0.821	21
19	0.142	0.897	41
20	0.105	0.641	21
21	0.093	0.564	15
22	0.118	0.744	22
23	0.130	0.821	32
24	0.146	0.923	43
25	0.123	0.769	26
26	0.111	0.692	22
27	0.130	0.821	34
28	0.138	0.872	47
29	0.150	0.949	53
30	0.138	0.872	53
31	0.130	0.821	39
32	0.142	0.897	55
33	0.146	0.923	80
34	0.154	0.974	111
35	0.150	0.949	70
36	0.146	0.923	53
37	0.150	0.949	90
38	0.150	0.949	104
39	0.154	0.974	107

Table 26: Training 8x5 Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.083	0.421	10
2	0.092	0.523	10
3	0.111	0.687	17
4	0.136	0.862	30
5	0.072	0.174	12
6	0.077	0.236	11
7	0.095	0.569	15
8	0.113	0.703	23
9	0.134	0.841	36
10	0.076	0.323	13
11	0.070	0.251	11
12	0.111	0.692	16
13	0.128	0.810	26
14	0.138	0.872	42
15	0.087	0.482	15
16	0.076	0.410	13
17	0.111	0.697	19
18	0.131	0.831	26
19	0.143	0.908	41
20	0.104	0.626	19
21	0.091	0.554	16
22	0.120	0.754	23
23	0.132	0.836	36
24	0.147	0.928	46
25	0.120	0.754	24
26	0.109	0.682	22
27	0.130	0.821	31
28	0.138	0.877	47
29	0.150	0.949	56
30	0.135	0.856	47
31	0.128	0.810	41
32	0.141	0.892	52
33	0.146	0.923	70
34	0.153	0.969	91
35	0.148	0.938	65
36	0.145	0.918	58
37	0.150	0.949	61
38	0.151	0.954	101
39	0.154	0.974	106

Table 27: Alternative 13DX Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.131	0.148	15
2	0.126	0.333	15
3	0.121	0.481	18
4	0.129	0.630	19
5	0.148	0.778	34
6	0.167	0.889	45
7	0.129	0.074	14
8	0.114	0.111	14
9	0.095	0.222	14
10	0.099	0.444	17
11	0.114	0.593	17
12	0.140	0.741	37
13	0.168	0.889	47
14	0.161	0.852	30
15	0.146	0.778	26
16	0.154	0.815	30
17	0.161	0.852	28
18	0.161	0.852	32
19	0.167	0.889	52
20	0.182	0.963	75
21	0.167	0.889	62
22	0.167	0.889	35
23	0.175	0.926	42
24	0.175	0.926	56
25	0.175	0.926	69
26	0.182	0.963	71
27	0.182	0.963	77

Table 28: Puzzle 13DX Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.095	0.185	9
2	0.087	0.333	11
3	0.100	0.481	14
4	0.122	0.630	17
5	0.154	0.815	33
6	0.175	0.926	47
7	0.108	0.407	10
8	0.090	0.333	10
9	0.078	0.259	10
10	0.088	0.407	11
11	0.109	0.556	16
12	0.140	0.741	27
13	0.167	0.889	34
14	0.135	0.704	13
15	0.103	0.519	12
16	0.134	0.704	17
17	0.147	0.778	22
18	0.160	0.852	29
19	0.167	0.889	53
20	0.175	0.926	70
21	0.154	0.815	46
22	0.154	0.815	36
23	0.161	0.852	28
24	0.175	0.926	45
25	0.182	0.963	76
26	0.182	0.963	72
27	0.182	0.963	76

Table 29: Training 13DX Results

Location	Stability	P(Delta_0)	Retrievals to SS
1	0.098	0.178	10
2	0.091	0.333	12
3	0.103	0.481	14
4	0.123	0.630	16
5	0.153	0.807	32
6	0.173	0.919	45
7	0.107	0.341	10
8	0.091	0.289	10
9	0.079	0.252	10
10	0.089	0.415	12
11	0.110	0.563	16
12	0.140	0.741	24
13	0.167	0.889	37
14	0.140	0.733	15
15	0.110	0.570	13
16	0.137	0.726	19
17	0.149	0.793	23
18	0.160	0.852	28
19	0.167	0.889	44
20	0.176	0.933	67
21	0.156	0.830	50
22	0.157	0.830	37
23	0.163	0.867	31
24	0.175	0.926	43
25	0.180	0.956	61
26	0.182	0.963	71
27	0.182	0.963	76

LIST OF REFERENCES

Arkin, V. I. (1964). Uniformly optimal strategies in search problems. *Theory of Probability & Its Applications, 9*(4), 674-680.

Assaf, D., & Zamir, S. (1985). Optimal sequential search: a Bayesian approach. *The Annals of Statistics*, 1213-1221.

Beck, A. (1964). On the linear search problem. *Israel Journal of Mathematics, 2*(4), 221-228.

Beck, A., & Beck, M. (1984). Son of the linear search problem. *Israel Journal of Mathematics, 48*(2-3), 109-122.

Bozer, Y. A., & White, J. A. (1984). Travel-time models for automated storage/retrieval systems. *IIE Transactions, 16*(4), 329-338.

Brusey, J., Floerkemeier, C., Harrison, M., & Fletcher, M. (2003). *Reasoning about uncertainty in location identification with RFID.* Paper presented at the Workshop on Reasoning with Uncertainty in Robotics at IJCAI.

Craig, B. A., & Sendi, P. P. (2002). Estimation of the transition matrix of a discrete-time Markov chain. *Health economics, 11*(1), 33-42.

de Castillo, B., & Daganzo, C. F. (1993). Handling strategies for import containers at marine terminals. *Transportation Research Part B: Methodological, 27*(2), 151-166.

De Guenin, J. (1961). Optimum distribution of effort: An extension of the Koopman basic theory. *Operations Research, 9*(1), 1-7.

De Koster, R. B., Le-Duc, T., & Yugang, Y. (2008). Optimal storage rack design for a 3-dimensional compact AS/RS. *International journal of production research, 46*(6), 1495-1514.

Degano, C., & Di Febbraro, A. (2001). *Modelling automated material handling in intermodal terminals.* Paper presented at the Advanced Intelligent Mechatronics, 2001. Proceedings. 2001 IEEE/ASME International Conference on.

Ergen, E., Akinci, B., & Sacks, R. (2007). Tracking and locating components in a precast storage yard utilizing radio frequency identification technology and GPS. *Automation in construction, 16*(3), 354-367.

Flake, G. W., & Baum, E. B. (2002). < i> Rush Hour</i> is PSPACE-complete, or “Why you should generously tip parking lot attendants”. *Theoretical Computer Science, 270*(1), 895-911.

Fox, D. (1998). *Markov localization-a probabilistic framework for mobile robot localization and navigation.* Citeseer.

Fujie, T. (2003). An exact algorithm for the maximum leaf spanning tree problem. *Computers & Operations Research, 30*(13), 1931-1944.

Futcher, F. W. (2003). *Selective Offload Capability Simulation (SOCS): An Analysis of High Density Storage Configurations.* NAVAL POSTGRADUATE SCHOOL.

Gu, J., Goetschalckx, M., & McGinnis, L. F. (2007). Research on warehouse operation: A comprehensive review. *European journal of operational research, 177*(1), 1-21.

Gue, K. R. (2006). Very high density storage systems. *IIE Transactions, 38*(1), 79-90.

Gue, K. R., & Kim, B. S. (2007). Puzzle-based storage systems. *Naval Research Logistics (NRL), 54*(5), 556-567.

Iranpour, R., & Tung, D. (1989). Methodology for optimal design of a parking lot. *Journal of transportation engineering, 115*(2), 139-160.

Kim, K. H., & Kim, H. B. (2002). The optimal sizing of the storage space and handling facilities for import containers. *Transportation Research Part B: Methodological, 36*(9), 821-835.

Puterman, M. L. (2014). *Markov decision processes: discrete stochastic dynamic programming*: John Wiley & Sons.

Quah, D. (1993). Empirical cross-section dynamics in economic growth. *European Economic Review, 37*(2), 426-434.

Rezapour, S., Allen, J. K., & Mistree, F. (2015). Uncertainty propagation in a supply chain or supply network. *Transportation Research Part E: Logistics and Transportation Review, 73*, 185-206.

Sahinidis, N. V. (2004). Optimization under uncertainty: state-of-the-art and opportunities. *Computers & Chemical Engineering, 28*(6), 971-983.

Song, J., Haas, C. T., Caldas, C., Ergen, E., & Akinci, B. (2006). Automating the task of tracking the delivery and receipt of fabricated pipe spools in industrial projects. *Automation in construction, 15*(2), 166-177.

Yu, Y., & De Koster, R. B. (2012). Sequencing heuristics for storing and retrieving unit loads in 3D compact automated warehousing systems. *IIE Transactions, 44*(2), 69-87.

Zhang, A. N., Goh, M., & Meng, F. (2011). Conceptual modelling for supply chain inventory visibility. *International Journal of Production Economics, 133*(2), 578-585.