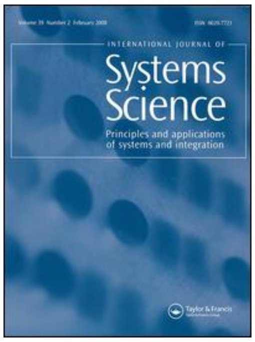

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Cooperative global optimal preview tracking control of linear multi-agent systems: an internal model approach

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ABSTRACT
This paper investigates the cooperative global optimal preview tracking problem of linear multi-agent systems under the assumption that the output of a leader is a previewable periodic signal and the topology graph contains a directed spanning tree. First, a type of distributed internal model is introduced, and the cooperative preview tracking problem is converted to a global optimal regulation problem of an augmented system. Second, an optimal controller, which can guarantee the asymptotic stability of the augmented system, is obtained by means of the standard linear quadratic optimal preview control theory. Third, on the basis of proving the existence conditions of the controller, sufficient conditions are given for the original problem to be solvable, meanwhile a cooperative global optimal controller with error integral and preview compensation is derived. Finally, the validity of theoretical results is demonstrated by a numerical simulation.

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Multi-agent system; cooperative tracking; optimal control; preview control; internal model principle; periodic signal

1. Introduction

Consensus is a basic problem in cooperative control of multi-agent systems, and its key idea is to design distributed control algorithms to make the collective behaviour reach an agreement on some variables of interest. To date, there have been fruitful results on this topic, especially the consensus in first- and second-order multi-agent systems (Hu, 2012; Olfati-Saber, & Murray, 2004).

Consensus problems were primarily investigated when there is no reference state. Considering the fact that multi-agent systems might converge onto a desired reference trajectory, consensus with a leader, also called cooperative tracking or consensus tracking, was studied. In (Li, Duan, Chen, & Huang, 2010), the synchronisation of complex networks was addressed by a distributed observer-type algorithm. Motivated by (Li et al., 2010), linear quadratic regulator (LQR) and the method of observer design were used for cooperative tracking consensus of multi-agent systems (Zhang, Lewis, & Das, 2011). Similar to Zhang et al. (2011), Riccati inequalities were utilised to investigate the synchronisation of discrete-time multi-agent systems (Hengster-Movric, You, Lewis, & Xie, 2013). Based on inverse optimality approach and partial stability, a quadratic performance criterion that can capture the topology of a graph was derived in Hengster-Movric & Lewis (2014), which also allowed for cooperative global optimal consensus and pinning control by using local control strategies. By using M-matrix theory, consensus tracking for multi-agent systems with high-order Lipschitz-type nonlinear dynamics and switching directed topologies was studied in Wen, Duan, Chen, and Yu (2014). In the presence of multiple leaders, containment control problem for high-order multi-agent systems under a directed fixed communication topology was tackled by a dynamic output approach (Wen, Zhao, Duan, Yu, & Chen, 2016).

It should be noted that the aforementioned works are only concerned with the case that leader and followers were homogeneous. For the heterogeneous scenario, a local controller and a state-estimate rule based on neighbour information were designed in Hong, Hu, and Gao (2006). Consensus problem of multi-vehicle systems with a time-varying reference state was considered in Ren (2007), where the author proposed several consensus protocols and showed necessary and sufficient conditions for reaching consensus. However, the methods of Hong et al. (2006) and Ren (2007) would not be effective for the dynamics of followers described by general linear systems. To this end, a dynamical full information distributed control scheme was devised in (Su & Huang, 2012), and the cooperative tracking problem of multi-agent systems was tackled through output regulation theory (for detailed information about output regulation theory, refer to Francis & Wonham, 1976; Huang, 2004). Furthermore, in Wang, Hong, Huang, and Jiang (2010), a consensus protocol was developed in view of the internal model principle, which can be exploited to achieve asymptotic tracking and disturbance rejection of uncertain multi-agent systems. Recently, Su, Hong, and Huang (2013) proved that the no-cycle assumption on the information graph in Wang et al. (2010) can be relaxed when all the followers had the same nominal dynamics. In addition, the containment control problem for heterogeneous linear multi-agent systems was studied based on output regulation approach (Haghshenas, Badamchizadeh, & Baradarannia, 2015). Excellent results about this type of

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problem were also given in Hong, Wang, and Jiang (2013); Li, Feng, Wang, Luo, and Guan (2014); Su and Huang, 2013; Yu and Wang (2014).

Assume that the reference signal or the disturbance signal is previewable in a fixed interval of time ahead, then the future information will be applied to improve the transient response of a closed-loop system. Such a control problem, for which the future information is available, is usually referred to as a preview control problem (see, e.g. Tomizuka, 1975). In the existing references, preview control theory is mainly developed along two directions, namely, linear quadratic (LQ) optimal preview control and H_∞ preview control. In Katayama and Hirono (1987); Katayama, Ohki, Inoue, and Kato (1985), the preview tracking problem was considered in discrete-time and continuous-time settings, respectively, and type-one servo controllers were derived by applying state augmentation technique and linear quadratic integral technique. Based on the results of Katayama et al. (1987), Liao, Tang, Liu, and Wang (2011) investigated the scenario that the reference signal and external disturbance were previewable simultaneously. In recent years, a number of significant results about the preview theory were produced by combing with other control issues. For instance, the preview control problem for linear discrete-time multi-rate systems with time-delay was addressed in Liao, Takaba, Katayama, and Katsuura (2003). Recently, the preview control theory was also extended to continuous-time linear descriptor systems, and a design method of optimal preview controller was obtained under the assumption of impulse-free (Liao, Ren, Tomizuka, & Wu, 2015). In addition, related results can also refer to Cao and Liao (2015); Wu, Liao, and Tomizuka (2016). Different from the above works, the optimal preview control problem was reformulated in infinite-dimensional time setting in Kojima and Ishijima (1999), and an optimal solution was obtained via functional-analytic technique. Pertinent works along H_∞ preview control include Katoh (2004); Kojima (2015); Kojima and Ishijima (2003); Mianzo and Peng (1999); Shaked and de Souza (1995).

Compared with the aforementioned methods, the state augmentation technique is better to deal with the optimal preview control problem. First, by taking the regulated output as a part of the augmented state vector, the controlled plants can achieve preview tracking if the closed-loop augmented system is asymptotically stable. Second, the closed-loop augmented system matrix is stable when the algebraic Riccati equation is solvable. Third, it is not difficult to incorporate the preview action into a controller.

For a cooperative tracking problem of multi-agent systems, if the output of the leader is previewable, whether the future information can be utilised to design controllers, such that the cooperative tracking performance of the closed-loop multi-agent systems is improved more effectively than that without preview information. From the above consideration, we proposed the cooperative optimal preview tracking problem of multi-agent systems. Liao, Lu, and Liu, (2016) addressed the cooperative optimal preview tracking problem of multi-agent systems first. However, if the reference signal of the above work contains unstable parts, such as the periodic signal with non-decreased amplitude, the closed-loop augmented system will be no longer asymptotically stable. As a further development on this issue, we consider a scenario that the output of the leader is a previewable periodic signal. The main contributions of this paper are threefold. First, the theory of the preview control is developed based on internal model principle, and which is successfully applied to the cooperative tracking problem. Second, the distributed internal model, together with the state augmentation technique, was utilised to convert the cooperative preview tracking problem to an optimal regulator problem of an augmented system. Thus, it will not only eliminate the impact of unstable reference signal on the asymptotic stability of the closed-loop augmented system, but also get rid of certain matrix equations needed for achieving consensus in distributed output regulation problem, e.g. Hong et al. (2013). Third, it has shown that the agent dynamic structure, communication topology and the characteristics of internal model have joint effect on the stabilisability and detectability of the augmented system.

An outline of this paper is as follows. In Section 2, some notations and basic concepts about graph theory are offered. Cooperative global optimal preview tracking problem of multi-agent systems is formulated in Section 3. Main results are given in Section 4, which contain the construction of the augmented system, proofs of two important theories and a global optimal preview controller. In Section 5, numerical example is provided to illustrate the theoretical results. Conclusions are drawn in Section 6.

2. Preliminaries

Notations: $\mathbb{R}^{n\times m}$ and $\mathbb{C}^{n\times m}$ are the sets of $n \times m$ real and complex matrices, respectively; $\bar{\mathbb{C}}^{+}$ denotes the closed right half complex plane; for $\lambda \in \mathbb{C}$, $Re(\lambda)$ is the real part of λ; $\Lambda(A)$ is the set of all eigenvalues of matrix A; block diag$(A_1, A_2, \ldots, A_n)$ denotes a block-diagonal matrix with matrices $A_i, i = 1, 2, \ldots, n$ on its diagonal; $A \otimes B$ represents the Kronecker product of matrices A and B, we refer to Horn and Johnson (1990) for details about $A \otimes B$; $1_n \in \mathbb{R}^n$ is the column vector with all the entries be one. The notation 'iff' means 'if and only if'.

Some basic concepts about algebraic graph theory are needed in this paper. Let $\mathcal{G} = (\mathcal{V}(\mathcal{G}), \mathcal{E}(\mathcal{G}))$ be a directed graph (digraph), where $\mathcal{V}(\mathcal{G}) = \{v_1, v_2, \ldots, v_N\}$ is a set of vertices and $\mathcal{E}(\mathcal{G}) \subseteq \mathcal{V}(\mathcal{G}) \times \mathcal{V}(\mathcal{G})$ is a set of arcs. If $(v_i, v_j) \in \mathcal{E}(\mathcal{G})$, then v_i is called the parent vertex of v_j and v_j the child vertex of v_i. $\mathcal{N}_i$, called the neighbour set of v_i, contains all the vertices who treat v_i as the child vertex. A path in digraph $\mathcal{G}$ is a finite nonempty sequence $v_1e_1v_2 \cdots v_{k-1}e_{k-1}v_k$ with $e_i = (v_i, v_{i+1}) \in \mathcal{E}(\mathcal{G})$ $(i = 1, 2, \ldots, k)$. A directed tree is a digraph, where every vertex, except the root, has exactly one parent. A digraph $\mathcal{G}$ contains a directed spanning tree if there is a directed path from the root to every other vertices in $\mathcal{G}$. The adjacency matrix of digraph $\mathcal{G}$ is denoted by $W = [a_{ij}] \in \mathbb{R}^{N\times N}$, where $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}(\mathcal{G})$ and $a_{ij} = 0$ otherwise. The Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{N\times N}$ is defined as $l_{ii} = \sum_{j\neq i} a_{ij}$, $l_{ij} = -a_{ij}$ for $i \neq j$, and satisfies the property of $L1_N = \mathbf{0}$.

3. Problem formulation

Consider a general linear multi-agent system

$$\begin{cases} \dot{x}_i = Ax_i + Bu_i \\ y_i = Cx_i \end{cases}, \quad x_i(0) = x_{i0}, \quad i = 1, 2, \ldots, N \tag{1}$$

where $x_i(t) \in \mathbb{R}^n, u_i(t) \in \mathbb{R}^r, y_i(t) \in \mathbb{R}^m$ and $x_{i0} \in \mathbb{R}^n$ represent the state, the control input, the measured output and the initial state of the ith agent, respectively. A, B and C are constant matrices with appropriate dimensions.

If the above N agents are regarded as vertices, then the communication topology among them can be conveniently described by a digraph $\mathcal{G}$. In $\mathcal{G}$, vertex v_i presents the ith agent, $(v_i, v_j) \in \mathcal{E}(\mathcal{G})$ means that agent j can receive information from agent i. Assume that digraph $\mathcal{G}$ has no self-loops, i.e. $a_{ii} = 0$ in adjacency matrix W.

Denote

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{bmatrix}, \quad u(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_N(t) \end{bmatrix}, \quad y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_N(t) \end{bmatrix}$$

then $x(t) \in \mathbb{R}^{nN}$, $u(t) \in \mathbb{R}^{rN}$, $y(t) \in \mathbb{R}^{mN}$, and state equations (1) can be expressed as

$$\begin{cases} \dot{x}(t) = (I_N \otimes A)x(t) + (I_N \otimes B)u(t) \\ y(t) = (I_N \otimes C)x(t) \end{cases}, \quad x(0) = x_0 \qquad (2)$$

The kinematic of the leader labelled by v_0 is described as follows

$$\begin{cases} \dot{w}(t) = \Gamma w(t) \\ y_d(t) = F w(t) \end{cases}, \quad w(0) = w_0 \qquad (3)$$

with state vector $w(t) \in \mathbb{R}^l$, output vector $y_d(t) \in \mathbb{R}^m$. It is usually viewed as an exogenous system (or simply, exosystem), which generates the desired reference signal. To study the cooperative preview tracking problem, we are also concerned about another digraph $\bar{\mathcal{G}}$ consisting of the digraph $\mathcal{G}$ and the vertex v_0. If vertex v_i obtains information from vertex v_0, then an arc (v_0, v_i) is said to exist with weighting gain $m_i > 0$, and $m_i = 0$ otherwise. For simplicity of presentation, we denote $M = \text{diag}\{m_1, m_2, \ldots, m_N\}$ and $H = L + M$. Using $L\mathbf{1}_N = \mathbf{0}$, we can show that $H\mathbf{1}_N = M\mathbf{1}_N$.

Remark 3.1: According to (Zheng, 2002), system (3) can be modelled through the structural property of the given reference signal $y_d(t)$. In fact, $y_d(t)$ can be written in the form of components

$$y_d(t) = \begin{bmatrix} y_{d1}(t) \\ \vdots \\ y_{dm}(t) \end{bmatrix} \qquad (4)$$

Applying the Laplacian transform yields the following result

$$Y_d(s) = \begin{bmatrix} Y_{d1}(s) \\ \vdots \\ Y_{dm}(s) \end{bmatrix} = \begin{bmatrix} \frac{n_1(s)}{p_1(s)} \\ \vdots \\ \frac{n_m(s)}{p_m(s)} \end{bmatrix} \qquad (5)$$

Then $p(s)$, called the structural property of $y_d(t)$, is the least common multiple of $\{p_1(s), p_2(s), \ldots, p_m(s)\}$ and satisfy the polynomial

$$p(s) = s^l + \alpha_{l-1}s^{l-1} + \cdots + \alpha_1 s + \alpha_0 \qquad (6)$$

Then system (3) is derived based on (6), where the minimal polynomial of Γ is $p(s)$, and F is selected such that $y_d(t)$ is the output of system (3).

At the outset, some assumptions will be needed for solving the cooperative global optimal preview tracking problem.

A1: The constant term α_0 in $p(s)$ is nonzero, and $p(s)$ has no roots with negative real parts.

A2: Reference signal $y_d(t)$ is previewable, i.e. the future values of $y_d(\tau)$ are available in $\{\tau | t \le \tau \le t + l_r\}$ at each instant t, where l_r is the preview length.

A3: (A, B) is stabilisable and (C, A) is detectable.

A4: The digraph $\bar{\mathcal{G}}$ contains a directed spanning tree and takes vertex v_0 as its root.

Remark 3.2: The first part of assumption A1 is made to guarantee the stabilisability of the augmented system below, and the second part of assumption A1 is given for convenience because some y_{di} $(i \in \{1, 2, \cdots, N\})$ corresponding to roots of $p(s)$ with negative real parts will decay to zero. In addition, assumption A1 implies that 0 is not the eigenvalue of the matrix Γ, thus Γ is nonsingular.

Lemma 3.1 (Hong et al., 2006; Li et al., 2010)**:** *All the eigenvalues of matrix H have positive real parts iff assumption A4 holds.*

Lemma 3.2 (Zhou, Doyle, & Glover, 1996)**:** *(A, B) is stabilisable iff the matrix $[\, sI - A \;\; B \,]$ has full row rank for all $s \in \bar{\mathbb{C}}^+$; (C, A) is detectable iff the matrix $\begin{bmatrix} sI - A \\ C \end{bmatrix}$ has full column rank for any $s \in \bar{\mathbb{C}}^+$.*

For a cooperative tracking control, the regulated output of agent i is defined as

$$\xi_i(t) = y_i(t) - y_d(t), \quad i = 1, 2, \cdots, N \qquad (7)$$

The purpose of this paper is to design a global optimal preview controller for system (2) such that

$$\lim_{t \to \infty} \xi_i(t) = 0, \quad i = 1, 2, \cdots, N \qquad (8)$$

However, $y_d(t)$ and $\xi_i(t)$ are only available in the design of agent i when an arc exists from vertex v_0 to vertex v_i in $\bar{\mathcal{G}}$. To achieve the cooperative tracking, we define the local neighbourhood output error as

$$e_i(t) = \sum_{j \in \mathcal{N}_i} a_{ij}(y_i(t) - y_j(t)) + m_i \xi_i(t) \qquad (9)$$

where $a_{ij} (j \in \mathcal{N}_i)$ is described above, $m_i > 0$ only for a small subset of agents who can receive information of the leader directly.

Set

$$\xi(t) = \begin{bmatrix} \xi_1(t) \\ \xi_2(t) \\ \vdots \\ \xi_N(t) \end{bmatrix}, \qquad e(t) = \begin{bmatrix} e_1(t) \\ e_2(t) \\ \vdots \\ e_N(t) \end{bmatrix}$$

with $\xi(t) \in \mathbb{R}^{mN}$, $e(t) \in \mathbb{R}^{mN}$. Then the defined vector $e(t)$ is referred to as the global output error.

Let $y_r(t) = \mathbf{1}_N \otimes y_d(t) \in \mathbb{R}^{mN}$, thanks to the equality $H\mathbf{1}_N = M\mathbf{1}_N$, formula (9) can be expressed as a compact form

$$e(t) = (H \otimes I_m)\xi(t) \tag{10}$$

Since H is nonsingular derived from Lemma 3.1, it follows that $\lim_{t\to\infty}\xi(t) = 0$ and thereby $\lim_{t\to\infty}\xi_i(t) = 0 (i = 1, 2, \cdots, N)$ hold if and only if $\lim_{t\to\infty}e(t) = 0$ holds. In order to do this, a quadratic performance index is introduced for the multi-agent systems (1).

$$J = \int_0^\infty \left\{ \sum_{i=1}^{N} (e_i^T(t)Q_{ei}e_i(t) + \dot{u}_i^T(t)R_i\dot{u}_i(t)) \right\} dt \tag{11}$$

where $Q_{ei} \in \mathbb{R}^{m\times m}$ and $R_i \in \mathbb{R}^{r\times r}(i = 1, 2, \cdots, N)$ are positive definite matrices. Furthermore, formula (11) can be written as

$$J = \int_0^\infty (e^T(t)Q_e e(t) + \dot{u}^T(t)R\dot{u}(t))dt \tag{12}$$

with

$$\begin{aligned} Q_e &= \text{block diag}(Q_{e1}, Q_{e2}, \cdots, Q_{eN}), \\ R &= \text{block diag}(R_1, R_2, \cdots, R_N) \end{aligned} \tag{13}$$

As stated in Athans (1971; Katayama et al. (1987); Liao et al. (2011); Liao et al. (2016), by introducing the derivative of $u(t)$ into J, the integral of $e(t)$ will be incorporated in an optimal controller, which has the property of eliminating the static errors.

After these preparations, the definition of a cooperative global optimal preview tracking problem is proposed as follows:

Definition 3.1: Consider the multi-agent systems (2) and the exosystem (3) under assumptions A1–A4. The cooperative global optimal preview tracking problem is said to be solvable, if there exists an optimal preview controller such that $\lim_{t\to\infty} e(t) = 0$ for any initial conditions x_0 and w_0.

4. Design of the global optimal preview controller

4.1. The augmented system

The concept of the internal model is used to construct an augmented system together with the technique of preview control. The definition of the minimal m-copy internal model is given beforehand.

Definition 4.1 (Huang, 2004): Given any square matrix Γ, a pair of matrix (G_1, G_2) is said to incorporate a minimal m-copy internal model of the matrix Γ, if the pair admits the following form:

$$G_1 = \begin{bmatrix} \beta & 0 & \cdots & 0 \\ 0 & \beta & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \beta \end{bmatrix}_{md\times md}, \quad G_2 = \begin{bmatrix} \sigma & 0 & \cdots & 0 \\ 0 & \sigma & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma \end{bmatrix}_{md\times m} \tag{14}$$

where β is a $d \times d$ constant square matrix and σ is a $d \times 1$ constant column vector, such that

i) (β, σ) is controllable;

ii) the characteristic polynomial of β, the minimal polynomial of β and the minimal polynomial of Γ are all the same.

Remark 4.1: It follows from assumption A1 that the matrix G_1 is nonsingular, and the minimal polynomial of the matrix G_1 is equal to that of the matrix Γ.

Based on Definition 4.1, a dynamical compensator is taken as

$$\dot{v}_i(t) = G_1 v_i(t) + G_2 e_i(t), \quad i = 1, 2, \cdots, N \tag{15}$$

Denote $v(t) = [\, v_1^T(t)\ v_2^T(t) \cdots v_N^T(t)\,]^T$, then

$$\dot{v}(t) = (I_N \otimes G_1)v(t) + (I_N \otimes G_2)e(t) \tag{16}$$

Rewrite formula (10) as

$$e(t) = (H \otimes C)x(t) - (H \otimes I_m)y_r(t) \tag{17}$$

Consequently, the following holds

$$\dot{v}(t) = (I_N \otimes G_1)v(t) + (H \otimes G_2C)x(t) - (H \otimes G_2)y_r(t) \tag{18}$$

An augmented system constructed in the following depends on the dynamical compensator (18). On account of the characteristics of the quadratic performance index (12), we take the derivative of both sides of the formulae (17), (18) and the first equation in system (2) in respect of time t and acquire

$$\begin{cases} \dot{e}(t) = (H \otimes C)\dot{x}(t) - (H \otimes I_m)\dot{y}_r(t) \\ \ddot{x}(t) = (I_N \otimes A)\dot{x}(t) + (I_N \otimes B)\dot{u}(t) \\ \ddot{v}(t) = (I_N \otimes G_1)\dot{v}(t) + (H \otimes G_2C)\dot{x}(t) - (H \otimes G_2)\dot{y}_r(t) \end{cases}$$

Define a concatenated vector as

$$z(t) = \begin{bmatrix} e(t) \\ \dot{x}(t) \\ \dot{v}(t) \end{bmatrix} \in \mathbb{R}^{(m+n+md)\times N}$$

then a state equation with respect to $z(t)$ is obtained

$$\dot{z}(t) = \tilde{A}z(t) + \tilde{B}\dot{u}(t) - \tilde{D}\dot{y}_r(t) \tag{19}$$

where

$$\tilde{A} = \begin{bmatrix} 0 & H\otimes C & 0 \\ 0 & I_N\otimes A & 0 \\ 0 & H\otimes G_2 C & I_N\otimes G_1 \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} 0 \\ I_N\otimes B \\ 0 \end{bmatrix} \text{ and}$$

$$\tilde{D} = \begin{bmatrix} H\otimes I_m \\ 0 \\ H\otimes G_2 \end{bmatrix}$$

are $(m+n+md)N\times(m+n+md)N$, $(m+n+md)N\times rN$ and $(m+n+md)N\times mN$ matrices, respectively.

Similar to Katayama et al. (1985); Katayama et al. (1987); Liao et al. (2016), we take the observation equation as

$$e(t) = \tilde{C}z(t), \quad \tilde{C} = \begin{bmatrix} I & 0 & 0 \end{bmatrix} \in \mathbb{R}^{mN\times(m+n+md)N} \tag{20}$$

then (19), together with (20), is referred to as the augmented system

$$\begin{cases} \dot{z}(t) = \tilde{A}z(t) + \tilde{B}\dot{u}(t) - \tilde{D}\dot{y}_r(t) \\ e(t) = \tilde{C}z(t) \end{cases} \tag{21}$$

Remark 4.2: The reason why an augmented system is constructed based on the internal model is that if the multi-agent systems are restricted to track a periodic signal in (Liao et al., 2016), the asymptotic stability of the closed-loop augmented system will not be guaranteed. This is because the periodic signal does not satisfy Assumption 3.1 of Liao et al. (2016). However, according to Zheng (2002), the essence of the dynamical compensator (15) lies in the fact that it can cancel the unstable components of the reference signal. Hence, when the closed-loop system matrix of the augmented system (21) is stable, $\lim_{t\to\infty} z(t) = 0$ and thus $\lim_{t\to\infty} e(t) = 0$ hold.

4.2. Optimal controller of the augmented system

Based on the relevant variables in (21), we represent the quadratic performance index (12) as follows:

$$J = \int_0^\infty (z^T(t)Qz(t) + \dot{u}^T(t)R\dot{u}(t))dt \tag{22}$$

where

$$Q = \begin{bmatrix} Q_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

is a $(m+n+md)N\times(m+n+md)N$ matrix.

To ensure the uniqueness and existence of the solution of the algebraic Riccati equation, a weighting term $\int_0^\infty (\sum_{i=1}^N \dot{v}_i^T(t)Q_{vi}\dot{v}_i(t))dt$ is added with respect to the dynamical compensator to (15), where Q_{vi} is a $md\times md$ positive definite matrix. Then

$$\tilde{J} = J + \int_0^\infty \left(\sum_{i=1}^N \dot{v}_i^T(t)Q_{vi}\dot{v}_i(t)\right)dt$$
$$= J + \int_0^\infty (\dot{v}^T(t)Q_v\dot{v}(t))dt$$
$$= \int_0^\infty (z^T(t)Qz(t) + \dot{u}^T(t)R\dot{u}(t))dt + \int_0^\infty (z^T(t)\hat{Q}z(t))dt$$
$$= \int_0^\infty (z^T(t)\tilde{Q}z(t) + \dot{u}^T(t)R\dot{u}(t))dt \tag{23}$$

with

$$\hat{Q} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Q_v \end{bmatrix}, \quad Q_v = \begin{bmatrix} Q_{v1} & 0 & \cdots & 0 \\ 0 & Q_{v2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & Q_{vN} \end{bmatrix},$$

$$\tilde{Q} = \begin{bmatrix} Q_e & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Q_v \end{bmatrix}$$

Finally, the cooperative preview tracking problem is converted into an optimal regulator problem. Next, the optimal control input $\dot{u}(t)$ will be determined that can minimise $\tilde{J}$ under system (21). By employing the results of Katayama et al. (1987); Liao et al. (2011); Liao et al. (2016), the following theorem can immediately hold.

Theorem 4.1: *Suppose that $(\tilde{A}, \tilde{B})$ is stabilisable and $(\tilde{Q}^{1/2}, \tilde{A})$ is detectable, then the optimal control input of system (21) that minimises quadratic performance index (23) is given by*

$$\dot{u}(t) = -R^{-1}\tilde{B}^T Pz(t) - R^{-1}\tilde{B}^T g(t) \tag{24}$$

where

$$g(t) = \int_0^{l_r} \exp\left(\sigma\tilde{A}_c^T\right) P\tilde{D}\dot{y}_r(t+\sigma)d\sigma \tag{25}$$

and P is a $(m+n+md)N\times(m+n+md)N$ positive semi-defined matrix, which satisfies the following algebraic Riccati equation

$$\tilde{A}^T P + P\tilde{A} - P\tilde{B}R^{-1}\tilde{B}^T P + \tilde{Q} = 0 \tag{26}$$

Moreover, $\tilde{A}_c$ is a stable matrix defined as follows:

$$\tilde{A}_c = \tilde{A} - \tilde{B}R^{-1}\tilde{B}^T P \tag{27}$$

Proof. The proof is elementary and is referred to Katayama et al. (1987); Liao et al. (2011). ■

4.3. Discussion of the sufficient conditions

To ensure the establishment of Theorem 4.1, the stabilisability of $(\tilde{A}, \tilde{B})$ and the detectability of $(\tilde{Q}^{1/2}, \tilde{A})$ will be demonstrated depending on assumptions A1, A3 and A4.

First, $(\tilde{A}, \tilde{B})$ will be shown stabilisable.

Theorem 4.2: *Under assumption A3, $(\tilde{A}, \tilde{B})$ is stabilisable, if*

i) (A, B) is stabilisable;
ii) for any $s\in\Lambda(G_1)\cup\{0\}$, the following condition holds:

$$\text{rank}\begin{bmatrix} sI-A & B \\ -C & 0 \end{bmatrix} = n+m \qquad \text{(full row rank)} \tag{28}$$

Proof. According to Lemma 3.2, $(\tilde{A}, \tilde{B})$ is stabilisable iff there exist any $s \in \bar{\mathbb{C}}^+$ such that

$$\operatorname{rank}\left[sI - \tilde{A}\ \tilde{B}\right] = (m+n+md)N \qquad \text{(full row rank)}$$

■

Denote $V(s) = [sI - \tilde{A}\ \tilde{B}]$, by the structure of $\tilde{A}$ and $\tilde{B}$, $V(s)$ has the following form:

$$V(s) = \begin{bmatrix} sI_{mN} & -H\otimes C & 0 & 0 \\ 0 & sI_{nN} - I_N \otimes A & 0 & I_N \otimes B \\ 0 & -H \otimes G_2 C & sI_{mdN} - I_N \otimes G_1 & 0 \end{bmatrix}$$

Let T be a transformation matrix such that

$$S = THT^{-1} = \begin{bmatrix} \Xi_1 & & & \\ & \Xi_2 & & \\ & & \ddots & \\ & & & \Xi_k \end{bmatrix},$$

$$\Xi_i = \begin{bmatrix} \lambda_i & 1 & & \\ & \lambda_i & \ddots & \\ & & \ddots & 1 \\ & & & \lambda_i \end{bmatrix} \in \mathbb{R}^{n_i \times n_i}$$

where $\sum_{i=1}^{k} n_i = N$ and λ_i, $i = 1, 2, \cdots, k \le N$, are eigenvalues of H and have positive real parts under assumption A4,

Taking the invertible matrices

$$P = \begin{bmatrix} T\otimes I_m & 0 & 0 \\ 0 & T\otimes I_n & 0 \\ 0 & 0 & T\otimes I_{md} \end{bmatrix},$$

$$Q = \begin{bmatrix} T^{-1}\otimes I_m & 0 & 0 & 0 \\ 0 & T^{-1}\otimes I_n & 0 & 0 \\ 0 & 0 & T^{-1}\otimes I_{md} & 0 \\ 0 & 0 & 0 & T^{-1}\otimes I_r \end{bmatrix}$$

then $V(s)$ is transformed into

$$\begin{aligned} \bar{V}(s) &= PV(s)Q \\ &= \begin{bmatrix} sI_{mN} & -S\otimes C & 0 & 0 \\ 0 & sI_{nN} - I_N\otimes A & 0 & I_N \otimes B \\ 0 & -S\otimes G_2C & sI_{mdN} - I_N\otimes G_1 & 0 \end{bmatrix} \end{aligned}$$

Noting that the elements of $\bar{V}(s)$ are either block diagonal or block upper-triangular, some elementary row and column operations (this is elementary transformation) give

$$\bar{V}(s) \to \bar{V}'(s) = \begin{bmatrix} M_1(s) & & & \\ & M_2(s) & & \\ & & \ddots & \\ & & & M_k(s) \end{bmatrix}$$

where

$$M_i(s) = \begin{bmatrix} sI_{n_i}\otimes I_m & -\Xi_i \otimes C & 0 & 0 \\ 0 & I_{n_i}\otimes (sI_n - A) & 0 & I_{n_i}\otimes B \\ 0 & -\Xi_i\otimes G_2C & I_{n_i}\otimes (sI_{md} - G_1) & 0 \end{bmatrix}, \quad i = 1, 2, \cdots, k$$

Because of the fact that the elementary transformation does not change the rank of a matrix, the matrix $V(s)$ has full row rank iff the matrices $M_i(s)$, $i = 1, 2, \cdots, k$, have full row rank. Furthermore, performing the elementary transformation on the matrix $M_i(s)$ yields

$$M_i(s) \to \bar{M}_i(s) = \begin{bmatrix} \bar{M}(s) & \bar{C} & & \\ & \bar{M}(s) & \ddots & \\ & & \ddots & \bar{C} \\ & & & \bar{M}(s) \end{bmatrix}, \quad i = 1, 2, \cdots, k$$

with

$$\bar{M}(s) = \begin{bmatrix} sI_m & -\lambda_i C & 0 & 0 \\ 0 & sI_n - A & 0 & B \\ 0 & -\lambda_i G_2 C & sI_{md} - G_1 & 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} 0 & -C & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -G_2C & 0 & 0 \end{bmatrix}$$

From the structure of the matrices $\bar{M}_i(s)$, $i = 1, 2, \cdots, k$, if the matrix $\bar{M}(s)$ has full row rank, then the matrices $\bar{M}_i(s)$, $i = 1, 2, \cdots, k$, have full row rank and so does the matrix $V(s)$. Therefore, when the conditionsâ…°and â…± hold, it suffices to prove that the matrix $\bar{M}(s)$ is of full row rank for any $s \in \bar{\mathbb{C}}^+$.

Firstly, for any $s \in \{s | s \notin \Lambda(G_1) \cup \{0\}, s \in \bar{\mathbb{C}}^+\}$, $\bar{M}(s)$ will be proved to have full row rank. Since the conditionâ…°holds, $\operatorname{rank}[\,sI - A\ B\,] = n$ for all $s \in \bar{\mathbb{C}}^+$. Alternatively, it follows from $s \notin \Lambda(G_1) \cup \{0\}$ that the matrices $sI - G_1$ and sI_m have full row rank, i.e. $\operatorname{rank}(sI - G_1) = md$ and $\operatorname{rank}(sI_m) = m$. Thus $\bar{M}(s)$ has full row rank for any $s \in \{s | s \notin \Lambda(G_1) \cup \{0\}, s \in \bar{\mathbb{C}}^+\}$.

Next, $\bar{M}(s)$ will be demonstrated to have full row rank for any $s \in \Lambda(G_1)$. Write $\bar{M}(s) = \bar{M}_1(s)\bar{M}_2(s)$, where

$$\bar{M}_1(s) = \begin{bmatrix} I_m & 0 & \lambda_i I_m & 0 \\ 0 & I_n & 0 & 0 \\ 0 & 0 & \lambda_i G_2 & sI_{md} - G_1 \end{bmatrix},$$

$$\bar{M}_2(s) = \begin{bmatrix} sI_m & 0 & 0 & 0 \\ 0 & sI_n - A & 0 & B \\ 0 & -C & 0 & 0 \\ 0 & 0 & I_{md} & 0 \end{bmatrix}$$

Since (G_1, G_2) is controllable and $Re(\lambda_i) > 0$, $\bar{M}_1(s)$ has full row rank for any $s \in \mathbb{C}$. Also, since G_1 satisfies Remark 4.1, i.e. $0 \notin \Lambda(G_1)$, under the condition â…±, it is easy to verify that $\bar{M}_2(s)$ has full row rank for all $s \in \Lambda(G_1)$. Consequently, by Sylvester's inequality (Huang, 2004; Zheng, 2002), the following holds

$$\begin{aligned} m + n + md &= (m+n+md) + (m+n+m+md) \\ &\quad - (m+n+m+md) \\ &\le \operatorname{rank}\bar{M}(s) \le \min\{m+n+md,\ m+n+m+md\} \\ &= m+n+md, \quad s \in \Lambda(G_1) \end{aligned}$$

Finally, $\bar{M}(s)$ will be demonstrated to have full row rank for $s = 0$. In this case, rank$\bar{M}(s)$ degenerates to

$$\text{rank}\bar{M}(0) = \text{rank}\begin{bmatrix} -\lambda_i C & 0 & 0 \\ -A & 0 & B \\ -\lambda_i G_2 C & -G_1 & 0 \end{bmatrix}$$

$\lambda_i, i = 1, 2, \cdots, k \le N$, are known to have positive real parts, thus

$$\text{rank}\bar{M}(0) = \text{rank}\begin{bmatrix} \lambda_i I & 0 & 0 \\ 0 & I & 0 \\ \lambda_i G_2 & 0 & -I \end{bmatrix}\begin{bmatrix} -C & 0 & 0 \\ -A & 0 & B \\ 0 & G_1 & 0 \end{bmatrix}$$

$$= \text{rank}\begin{bmatrix} -C & 0 & 0 \\ -A & 0 & B \\ 0 & G_1 & 0 \end{bmatrix}$$

Because of the non-singularity of the matrix G_1, it follows from the above formula that $\bar{M}(0)$ has full row rank when the condition â…± holds.

Based on the above argument, $\bar{M}(s)$ has been proved to have full row rank for all $s \in \bar{\mathbb{C}}^+$ under the conditions â…°and â…±. Hence the pair $(\tilde{A}, \tilde{B})$ is stabilisable.

Before proving the detectability of the pair $(\tilde{Q}^{1/2}, \tilde{A})$, the following lemma is required.

Lemma 4.1: *Under assumption A4, $(H \otimes C, I_N \otimes A)$ is detectable if and only if (C, A) is detectable.*

Proof. *See the Reference (Liao et al., 2016).*

Theorem 4.3: *Under assumption A4, $(\tilde{Q}^{1/2}, \tilde{A})$ is detectable if and only if (C, A) is detectable.*

Proof. *According to Lemma 3.2, $(\tilde{Q}^{1/2}, \tilde{A})$ is detectable iff the following matrix*

$$\begin{bmatrix} \tilde{Q}^{1/2} \\ sI - \tilde{A} \end{bmatrix} = \begin{bmatrix} Q_e^{1/2} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & Q_v^{1/2} \\ sI_{mN} & -H \otimes C & 0 \\ 0 & sI_{nN} - I_N \otimes A & 0 \\ 0 & -H \otimes G_2 C & sI_{mdN} - I_N \otimes G_1 \end{bmatrix} \tag{29}$$

has full column rank for any $s \in \bar{\mathbb{C}}^+$.

Noting that Q_e and Q_v are positive definite matrices, an elementary matrix is selected as follows:

$$U = \begin{bmatrix} (Q_e^{1/2})^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \\ 0 & 0 & (Q_v^{1/2})^{-1} & 0 & 0 & 0 \\ -s(Q_e^{1/2})^{-1} & 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & I & 0 \\ s(I_N \otimes G_2)(Q_e^{1/2})^{-1} & 0 & -(sI_{mdN} - I_N \otimes G_1)(Q_v^{1/2})^{-1} & -I_N \otimes G_2 & 0 & I \end{bmatrix}$$

Pre-multiplying formula (29) by U gives

$$U\begin{bmatrix} \tilde{Q}^{1/2} \\ sI - \tilde{A} \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & I \\ 0 & -H \otimes C & 0 \\ 0 & sI_{nN} - I_N \otimes A & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

By using the fact that the elementary transformation does not change the rank of a matrix once again, we have $rank\begin{bmatrix} \tilde{Q}^{1/2} \\ sI - \tilde{A} \end{bmatrix} = rank\left(U\begin{bmatrix} \tilde{Q}^{1/2} \\ sI - \tilde{A} \end{bmatrix}\right)$, which indicates that $\begin{bmatrix} \tilde{Q}^{1/2} \\ sI - \tilde{A} \end{bmatrix}$ has full column rank iff $\begin{bmatrix} H \otimes C \\ sI_{nN} - I_N \otimes A \end{bmatrix}$ has full column rank for any $s \in \bar{\mathbb{C}}^+$. That is to say, $(\tilde{Q}^{1/2}, \tilde{A})$ is detectable iff $(H \otimes C, I_N \otimes A)$ is detectable. Then, in view of Lemma 4.1 and the transitivity between the propositions, the conclusion is obtained.

Remark 4.3: It is observed from the above theorems that A4 is a significant assumption, which not only, together with the structure properties of the individual agent's dynamics and the internal model, affects the stabilisability of $(\tilde{A}, \tilde{B})$ and the detectability of $(\tilde{Q}^{1/2}, \tilde{A})$, but also can be used to achieve the consensus of the multi-agent systems in most cases (see, e.g. Li et al., 2010; Ren, 2007; Zhang et al., 2011). Apart from assumption A4, we remark that assumption A1 plays a key role in assuring the stabilisability of $(\tilde{A}, \tilde{B})$.

4.4. Global optimal preview controller of the original system

Based on the discussion of the above theorems, we give the global optimal preview controller of the multi-agent systems (2).

Theorem 4.4: *Suppose that*

(a) Assumptions A1–A4 hold;
(b) Q_{ei}, Q_{vi} and R_i, $i = 1, 2, \cdots, N$, are positive definite matrices;
(c) the rank condition (28) holds;
(d) the dynamical compensator (15) incorporates a minimal m-copy internal model of the matrix Γ.

Then the cooperative global optimal preview tracking problems are solvable, and the corresponding global optimal preview controller is given by

$$u(t) = -K_e \int_0^t e(\sigma)d\sigma - K_x[x(t) - x_0] - K_v v(t) - R^{-1}\tilde{B}^T f(t) \tag{30}$$

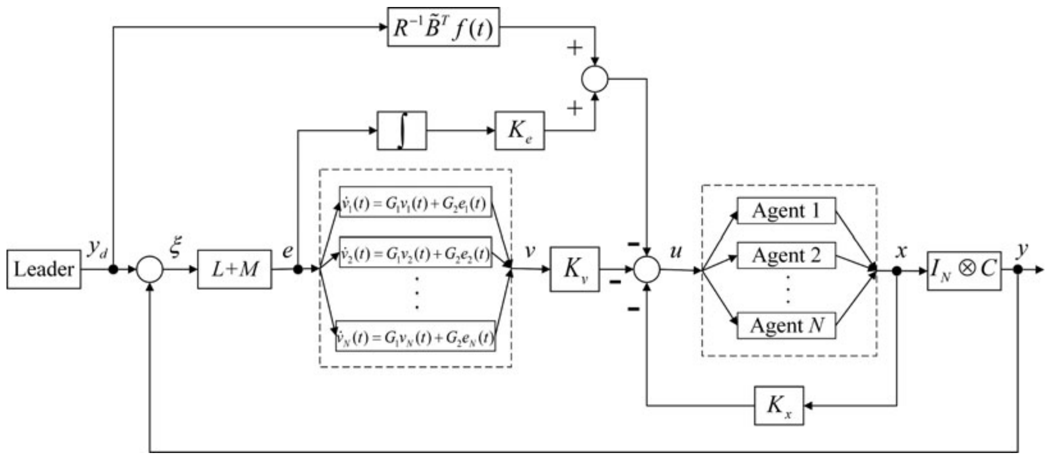

Figure 1. Block diagram of optimal preview control system based on internal model.

with $u(t) = 0$, $v(t) = 0$ and $y_r(t) = 0$ for $t \leq 0$, and $K_e = R^{-1}\tilde{B}^T P_e$, $K_x = R^{-1}\tilde{B}^T P_x$, $K_v = R^{-1}\tilde{B}^T P_v$, $P = [\,P_e\ P_x\ P_v\,]$. Moreover, $f(t)$ is defined as

$$f(t) = -\int_0^{l_r} \exp\left(\sigma \tilde{A}_c^T\right) P\tilde{D}y_r(t+\sigma)d\sigma \tag{31}$$

where the matrices P and $\tilde{A}_c$ satisfy formulae (26) and (27), respectively.

Proof. The conclusion comes from Theorem 4.1–4.3 and Katayama et al. (1987); Liao et al. (2011); Liao et al. (2016); Zheng (2002). ■

Remark 4.4: In (30), $-K_e\int_0^t e(\sigma)d\sigma$ represents the integrator, having the ability to eliminate the steady-state error. $-K_x x(t)$ is the state feedback that can make the coefficient matrix of the closed-loop system stable. $-K_v v(t)$ is the dynamical compensation based on internal model, which helps get rid of the unstable component of the reference signal $y_d(t)$. $-R^{-1}\tilde{B}^T f(t)$ is the preview feed-forward compensation with ability of improving the tracking performance, such as the tracking accuracy, tracking speed, as well as the adjusting time. In addition, $K_x x_0$ is the compensation of initial values. The structure of the optimal preview controller (30) is depicted in Figure 1.

Remark 4.5: Theorem 4.4 gives the sufficient conditions and the optimal preview controller for multi-agent systems to track the previewable periodic signal accurately. It is worth mentioning that the cooperative optimal preview tracking of linear multi-agent systems under switching topologies is interesting and of great significance. But the methodology and theory proposed in the case of fixed communication topology cannot be directly extended to those of the switching topologies. Our further work will focus on the case with switching topologies like jointly connected topologies.

5. Numerical simulation

In order to verify the effectiveness of the theoretical results, the design method proposed in this paper is applied to vehicle systems cited from Liu and Lunze (2014).

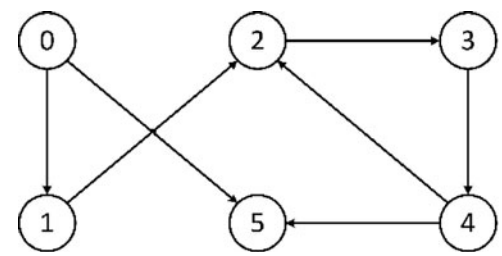

Figure 2. Communication topology among agents and leader.

Consider a group of five agents with a leader, the communication topology among them is given in Figure 2, which obviously satisfies assumption A4. Let $a_{ij} = 1$ if vertex v_j is a neighbour of vertex v_i, $i, j = 1, 2, \cdots, 5$, and $m_i = 1$ if vertex v_j can receive information from vertex v_0.

It is observed from the figure that the matrix H associated with digraph $\bar{\mathcal{G}}$ is given by

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -1 & 2 & 0 & -1 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 2 \end{bmatrix}$$

For convenience of study, we will not consider the external disturbance in the original model. The dynamics of each vehicle is described as follows:

$$\begin{cases} \begin{pmatrix} \dot{y}_i(t) \\ \dot{x}_{ai}(t) \end{pmatrix} = \begin{bmatrix} 0 & \mathrm{c}_a^T \\ 0 & \mathrm{A}_a \end{bmatrix} \begin{pmatrix} y_i(t) \\ \mathrm{x}_{ai}(t) \end{pmatrix} + \begin{bmatrix} 0 \\ \mathrm{b}_a \end{bmatrix} u_i(t) \\ y_i(t) = [\,1\ 0\,] \begin{pmatrix} y_i(t) \\ \mathrm{x}_{ai}(t) \end{pmatrix} \end{cases}, \quad i = 1, 2, 3, 4, 5$$

with

$$\mathrm{A}_a = \begin{bmatrix} -\dfrac{c+k_P}{m} & -\dfrac{k_I}{m} \\ 1 & 0 \end{bmatrix}, \quad \mathrm{b}_a = \begin{bmatrix} \dfrac{k_P}{m} \\ -1 \end{bmatrix}, \quad \mathrm{c}_a = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

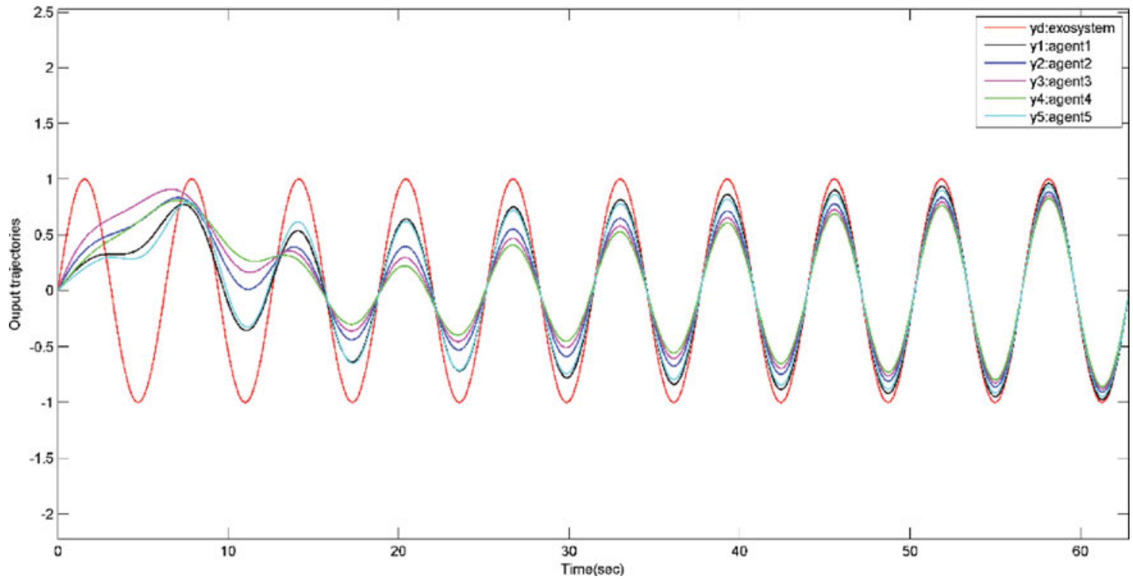

Figure 3. Output trajectories of five vehicles for $l_r = 0(s)$.

where $y_i(t)$ represents the position of ith vehicle, and the parameters utilised in the simulation are taken as

$$c = 200, \quad k_P = 70, \quad k_I = 20, \quad m = 1000$$

Let the output of the leader be $y_d(t) = \sin(t)$, optimal preview controllers will be designed for five agents such that they can track the leader without static errors. According to Remark 3.1, we take the Laplacian transformation for $y_d(t)$ and obtain

$$\bar{Y}_d(s) = \mathcal{L}[\sin(t)] = \frac{1}{s^2 + 1}$$

Based on formulae (3)–(7), the kinematics of the exosystem (vertex υ_0) is described as

$$\begin{cases} \dot{w}(t) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} w(t) \\ y_d(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} w(t) \end{cases}, \quad w(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

where $\Gamma = \left[\begin{smallmatrix} 0 & 1 \\ -1 & 0 \end{smallmatrix}\right]$ and $F = [1 \ 0]$.

From Definition 4.1 and the above kinematics, the dynamical compensator of the reference signal $y_d(t)$ is derived as follows:

$$\dot{\upsilon}_i(t) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \upsilon_i + \begin{bmatrix} 0 \\ 1 \end{bmatrix} e_i, \quad i = 1, 2, \cdots, N$$

After constructing the augmented system that incorporates the dynamical compensator, the cooperative preview tracking problems of the multi-agent systems are transformed into an optimal regulation problem of the augmented system.

Choose the weighting matrices $\tilde{Q}$ and R associated with the quadratic performance index (25) as

$$\tilde{Q} = \begin{bmatrix} Q_e & 0 & 0 \\ 0 & 0_{15\times 15} & 0 \\ 0 & 0 & Q_\upsilon \end{bmatrix},$$

$$R = \begin{bmatrix} 0.67 & 0 & 0 & 0 & 0 \\ 0 & 0.93 & 0 & 0 & 0 \\ 0 & 0 & 0.80 & 0 & 0 \\ 0 & 0 & 0 & 1.14 & 0 \\ 0 & 0 & 0 & 0 & 0.89 \end{bmatrix}$$

where

$$Q_e = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}, \quad Q_\upsilon = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 6 \end{bmatrix} \otimes I_2$$

By applying the Lemma 3.2, it can be proved that each individual system is controllable and observable, and therefore is also stabilisable and detectable. Moreover, let $p(s) = s^2 + 1 = 0$, then $s = \pm \mathrm{i}$. Apparently, the rank condition (28) holds for any $s \in \{\pm i, 0\}$ via some routine computations.

The above analysis indicates that conditions for Theorems 4.2 and 4.3 are satisfied, if follows from Theorem 4.1 and Remark 4.2 that the closed-loop augmented system is asymptotically stable, that is, $\lim_{t\to\infty} \xi_i(t) = 0$, $i = 1, 2, \cdots, N$. After solving the algebraic Riccati equation (26) by MATLAB, the gain matrices K_e, K_x and K_υ for the controller (30) can be obtained using

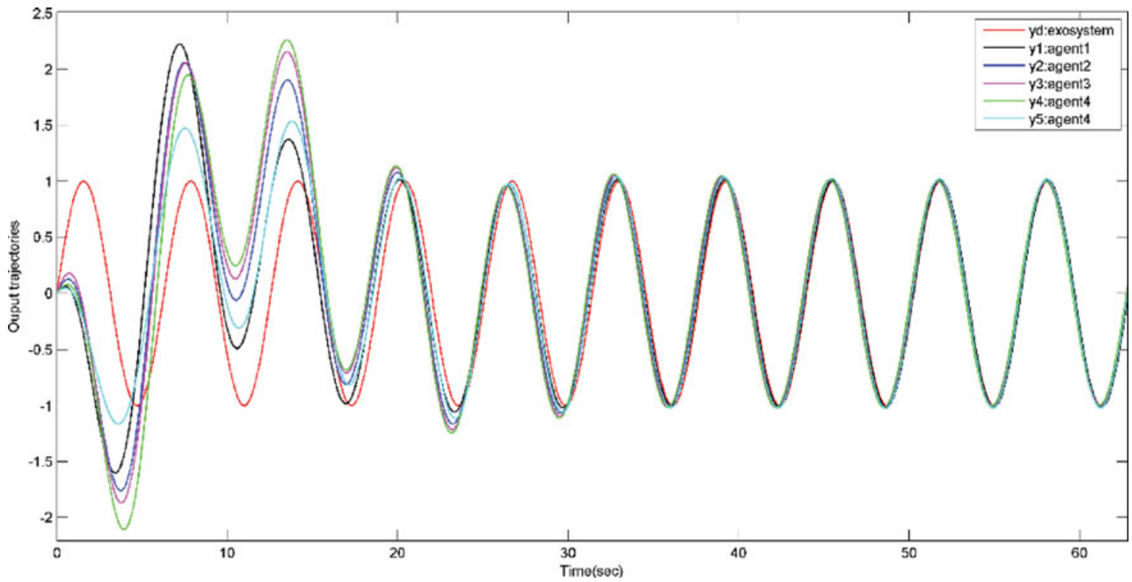

Figure 4. Output trajectories of five vehicles for $l_r = 0.05(s)$.

Theorem 4.4.

$$K_e = \begin{bmatrix} 1.03839 & -0.79998 & -0.36386 & -0.37325 & 0.03141 \\ 0.43596 & 1.26361 & -0.38525 & 0.22122 & -0.10696 \\ 0.24907 & 0.30013 & 1.27664 & -1.27022 & -0.14606 \\ 0.19304 & -0.09633 & 0.59116 & 1.70024 & -0.57409 \\ 0.09891 & 0.06403 & 0.28251 & 0.55573 & 2.01006 \end{bmatrix},$$

$$K_x = \begin{bmatrix} 12.86948 & 14.69727 & -0.26624 & -8.29728 & -5.49236 & 0.00756 & -0.62071 & -0.76399 & \cdots \\ -5.86823 & -3.95148 & 0.00582 & 21.09693 & 19.44086 & -0.26743 & -4.42864 & -3.48106 \\ -0.57418 & -0.63766 & 0.00226 & -5.07539 & -4.04050 & 0.01051 & 17.16066 & 17.11270 \\ 1.84879 & 0.87429 & 0.00101 & -5.44498 & -4.18649 & 0.01059 & -6.05548 & -4.80587 \\ 0.29425 & 0.14479 & 0.00059 & -1.30782 & -1.37523 & 0.00497 & -1.67330 & -1.67408 \end{bmatrix}$$

$$\begin{matrix} 0.00252 & 2.96232 & 1.47709 & 0.00099 & 0.36528 & 0.18878 & 0.00053 \\ 0.00867 & -6.66396 & -5.13277 & 0.01291 & -1.32108 & -1.31786 & 0.00463 \\ -0.26365 & -8.32636 & -6.83275 & 0.01910 & -1.79761 & -1.85738 & 0.00691 \\ 0.01264 & 14.33464 & 14.39639 & -0.25619 & -5.66277 & -4.10345 & 0.00887 \\ 0.00589 & -7.29703 & -5.25666 & 0.01133 & 21.85528 & 20.54674 & -0.27277 \end{matrix} \Bigg]$$

$$K_v = \begin{bmatrix} -1.77595 & -0.72093 & 2.28098 & -0.86571 & 1.42142 & 0.69771 & 0.99434 & 0.50280 & -0.07866 & 0.04889 \\ -0.64192 & -0.72686 & -2.93841 & 0.93987 & 1.57367 & -1.28287 & -0.15858 & -0.45484 & 0.23696 & 0.01981 \\ -0.31065 & -0.56365 & -0.60100 & -0.66679 & -3.21410 & -0.41671 & 2.58232 & -1.84803 & 0.31339 & 0.04427 \\ -0.10726 & -0.48449 & 0.58135 & -1.33631 & -0.88150 & -1.50733 & -2.84879 & -0.48528 & 0.86457 & -0.63557 \\ -0.06526 & -0.23265 & 0.08715 & -0.31473 & -0.38684 & -0.72989 & -0.96930 & -0.77099 & -3.22433 & 1.20680 \end{bmatrix}$$

Selecting the initial states as

$$x_1(0) = \begin{bmatrix} 0 \\ 0.2 \\ -0.2 \end{bmatrix}, \quad x_2(0) = \begin{bmatrix} 0 \\ 0.3 \\ -0.3 \end{bmatrix}, \quad x_3(0) = \begin{bmatrix} 0 \\ 0.4 \\ -0.4 \end{bmatrix},$$

$$x_4(0) = \begin{bmatrix} 0 \\ 0.2 \\ -0.2 \end{bmatrix}, \quad x_5(0) = \begin{bmatrix} 0 \\ 0.15 \\ -0.15 \end{bmatrix}$$

this implies that all the vehicles are located in the same position but with different velocities.

Figures 3 and 4 show that the output trajectories of five vehicles can finally synchronise to the output trajectory of the exosystem. Furthermore, it can be observed from above two figures that the appropriate preview length is beneficial for improving the tracking effect, especially for the tracking speed and the accurate tracking with respect to the peaks and troughs. It indicates that the design method of this paper is effective for practical issue.

6. Conclusions

In this paper, cooperative global optimal preview tracking problem of linear multi-agent systems have been investigated. With some standard assumptions, sufficient conditions are obtained for achieving preview tracking consensus by applying the preview control theory and the internal model principle, and also a cooperative global optimal preview controller of the multi-agent systems are derived.

Compared with Katayama et al. (1987); Liao et al. (2011); Liao et al. (2016), we further develop the theory of the preview control, and successfully apply it to the cooperative tracking problem. Furthermore, since the regulated output is taken as a component of the augmented state vector, the certain matrix equations, which are required in distributed output regulation problems (see, e.g. Hong et al., 2013), are not needed in this paper by directly making use of the property of the internal model. In future, one of the important research topics is to extend the present result to the linear descriptor multi-agent systems, which is more challenging due to the special structure of descriptor systems.

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