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Investigating the relationships between preferences, gender, and high school students' geometry performance

Bhesh Mainali
University of Central Florida

INVESTIGATING THE RELATIONSHIPS BETWEEN PREFERENCES, GENDER, AND HIGH SCHOOL STUDENTS' GEOMETRY PERFORMANCE

by

BHESH RAJ MAINALI
MS Tribhuvan University, 2000
MS University of Amsterdam, 2008

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Major Professor: Erhan Selcuk Haciomeroglu

ABSTRACT

In this quantitative study, the relationships between high school students' preference for solution methods, geometry performance, task difficulty, and gender were investigated. The data was collected from 161 high school students from six different schools at a county located in central Florida in the United States. The study was conducted during the 2013–2014 school year. The participants represented a wide range in socioeconomic status, were from a range of grades (10-12), and were enrolled in different mathematics courses (Algebra 2, Geometry, Financial Algebra, and Pre-calculus). Data were collected primarily with the aid of a geometry test and a geometry questionnaire. Using a think-aloud protocol, a short interview was also conducted with some students.

For the purpose of statistical analysis, students' preferences for solution methods were quantified into numeric values, and then a visuality score was obtained for each student. Students' visuality scores ranged from -12 to +12. The visuality scores were used to assess students' preference for solution methods. A standardized test score was used to measure students' geometry performance. The data analysis indicated that the majority of students were visualizers. The statistical analysis revealed that there was not an association between preference for solution methods and students' geometry performance. The preference for solving geometry problems using either visual or nonvisual methods was not influenced by task difficulty. Students were equally likely to employ visual as well as nonvisual solution methods regardless of the task difficulty. Gender was significant in geometry performance but not in preference for solution methods. Female students' geometry performance was significantly higher than male students' geometry performance. The findings of this study suggested that instruction should be focused

on incorporating both visual and nonvisual teaching strategies in mathematics lesson activities in order to develop preference for both visual and nonvisual solution methods.

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LIST OF ACCRONYMS/ABREVIATIONS

ANOVA	Analysis of Variance
AP	Advanced Placement
CCSSM	Common Core State Standards for Mathematics
EOC	End-of-Course
FLDOE	Florida Department of Education
MANOVA	Multivariate Analysis of Variance
MANCOVA	Multivariate Analysis of Covariance
MPI	Mathematical Processing Instrument
NCTM	National Council of Teachers of Mathematics
NGSSS	Next Generation Sunshine State Standards
SAT	Scholastic Aptitude Test

CHAPTER ONE: INTRODUCTION

Preference and Performance in Mathematics

Preferences have an important implication in educational theory and practice (Stenberg & Grigorenko, 1997). In this study, preference refers to an individual's preferred and habitual approach to organizing, representing, and processing information, which subsequently affects the way in which an individual perceives and responds to ideas, events, or problems (Riding & Rayner, 1998). Preference is also called cognitive styles. Researchers have identified various types of cognitive styles, but in the domain of mathematics education, the verbalizers and visualizers continuum is the most widely accepted (Krutetskii, 1976).

Krutetskii (1976), a Russian scholar, laid a foundation for the verbalizers and visualizers continuum for the teaching and learning of mathematics. He identified two modes of thought or ways of processing mathematical information: verbal-logical and visual-pictorial. He contended that everyone is endowed with these two modes of thought. Verbalizers use verbal-logical modes of thought and visualizers employ visual-pictorial modes of thought while attempting to learn mathematical ideas and concepts or do mathematical tasks.

Krutetskii (1976) investigated the relationships between mathematical abilities and spatial abilities based on a study of several gifted students. According to Krutetskii, students can be placed in a continuum with regard to their preference for thinking and correlation between the two modes of thought. They belong to one of three categories: (a) visualizers (geometric), who have a preference for the use of visual solution methods, which involve graphic representation (i.e., figures, diagrams, and pictures); (b) verbalizers (analytic), who have a preference for the use of nonvisual solution methods, which involve algebraic, numeric, and verbal representation; and (c) harmonics (mixer), who use visual and verbal methods equally.

Based on the Krutetskii (1976) framework, several research studies have been conducted which examined the relationships between students' preferences for solution methods, and mathematical performance (Battista, 1990; Haciomeroglu, Aspinwall, & Presmeg, 2010; Krutetskii, 1976; Moses, 1977; Suwarsono, 1982). For example, researchers attempted to find the correlation between preference for solution methods and mathematical performance. What kind of solution methods an individual uses and how the method is associated with mathematical performance have been of interest to many researchers in the mathematics education field.

Krutetskii (1963) categorized low-achieving students into different categories and investigated the factors behind their poor performance. He suggested that high-level development of analytical thinking does not determine mathematical thinking; however, low development of analytical thinking does result in an incapacity for mathematics. Krutetskii (1976) further contended that there is a correlation between the ability to visualize abstract mathematical relationships and the ability to make sense of spatial geometric concepts. However, these are not the essential components that determine students' mathematical abilities. He further stated that strengths or weaknesses of analytical or visual thinking do not determine the extent of students' mathematical giftedness; however, they determine its type. A student can be mathematically capable with different correlations between verbal-logical and visual-pictorial modes of thinking. In fact, it is the correlation between the two modes of thinking—verbal-logical and visual-pictorial—that determines each student's category (analytical, geometric, and harmonic).

Krutetskii's study (1976) also revealed a correlation between the verbalizers and success in learning algebra and, similarly, between the geometric type and success in learning geometry. However, Krutetskii further contended that the classification of verbalizers and visualizers should not be regarded as a classification of thinking according to the subject relationships (school subject—algebra and geometry). In fact, the analytic cast of mind can be shown in geometry and geometric type can be shown in algebra.

Spatial ability is defined as the ability to generate, retain, retrieve, and transform well-structured visual images (Lohman, 1996). It also refers to skill in representing, transforming, generating, and recalling symbolic, non-linguistic information (Linn & Petersen, 1985). Krutetskii contended that spatial ability does not determine students' geometric performance; he documented many cases in which students who showed good spatial ability were poor in geometry performance. Moreover, he contended that a well-developed spatial ability does not imply that students will use it while attempting mathematical tasks. For example, students may be able to solve a problem by visual methods; however, they may not prefer to solve it using visual methods. Several research studies have been conducted to examine the relationships between the preferences for solution methods and spatial ability; however, they revealed that there was little or no correlation between preferences and spatial ability (Haciomeroglu, Chicken, & Dixon, 2013; Hagarty & Kozhevnikov, 1999; Kozhevnikov, Hagarty, & Mayer, 2002; Moses, 1977; Lean & Clements, 1981; Suwarsono, 1982). Presmeg (1985) also pointed out the same issues: spatial tests may be solved by using analytic solution methods, or students with good spatial ability may not prefer to use visual solution methods. Reffering the work of Wattanawha and Clements, Clements (1984) reported that mathematically gifted students had a strong preference for analytic methods (nonvisual solution methods) on space visualization tests. Therefore, for this study spatial ability will not be used to measure students' geometry performance; rather, preference for solution methods will be the focus. One of the main purposes of this study is to examine the relationship between students' preference for solution methods and their geometry performance.

Solution Methods and Preference

Research shows that visual and verbal (nonvisual) methods both predominantly used solution methods while attempting mathematical tasks (Janvier, 1987; Krutetskii, 1976; Lesh, Post, & Behr, 1987). Many researchers have investigated students' preferences for solution methods and their relationships to mathematical performance (Gorgorio, 1988; Haciomeroglu, 2012; Lean & Clements, 1981; Lowrie & Kay, 2001; Moses, 1977; Presmeg, 1986b). Various distinctions have been made between visual and nonvisual solution methods.

Presmeg (1986) stated that a visual solution method is one that involves visual imagery, with or without a diagram, even if algebraic methods are also employed. Visual imagery in solution methods involves any kind of graphic representation (diagrams, figures, and visual representations), either on paper or in the head of students. Krutetskii (1976) referred to these two methods as visual and mental solution methods. However, Suwarsono (1982) used the term mathematical visuality to describe solution methods. He stated that mathematical visuality is the degree to which someone prefers to use a visual method when attempting mathematical problems that can be solved in both visual and nonvisual ways. When students use either given diagrams and figures or they draw diagrams and figures or visualize diagrams and figures in their head while attempting mathematical tasks, it is considered to be a visual solution method.

In nonvisual solution methods (verbal), the reasoning is conducted purely on the basis of the processing or manipulation of verbal and mathematical statements, and these manipulations are performed using the rules of language and mathematics (Suwarsono, 1982). Nonvisual solution methods do not involve visual imagery (Presmeg, 1986b). Thus, algebraic, numeric, and verbal representations have fundamental roles in nonvisual solution methods. A nonvisual solution method is one that involves analytic reasoning while attempting mathematical tasks. By analytic reasoning, the researcher means employing mathematical formulae, rules, postulates, axioms, conjectures, and so forth while students attempt to solve mathematical tasks. Students do not use any kind of diagrams or figures; they do not visualize them when attempting mathematical tasks in nonvisual solution methods. Despites of using different terms for two types of solution methods, the research will used visual and nonvisual solution methods for the purpose of this study.

Several research studies examined the relationships between preferences for solution methods, gender, and mathematical performance but without conclusive findings (Battista, 1990; Fennema, 1979; Fennema & Carpenter, 1981; Galindo, 1994; Haciomeroglu, Chicken, & Dixon, 2013; Moses, 1977; Suwarsono, 1982). For example, Lowrie and Kay (2001) suggested that visual solution methods are positively correlated with mathematics performance while Lean and Clements (1981) claimed that nonvisual solution methods are positively correlated with mathematics achievement.

Preferences for solution methods are also associated with difficulty levels of the mathematics problems (Haciomeroglu, 2012; Lowrie, 2001; Lowrie & Kay, 2001). As the degree of difficulty of the mathematics problems change, students also alter their preferences. For example, students were more likely to use visual methods than nonvisual methods to solve difficult problems (Lowrie & Kay, 2001). Haciomeroglu (2012) also found that as task difficulty increased, the number of visual solution methods (correct to incorrect) increased significantly, supporting the conclusions of Lowrie and Kay (2002). On the other hand, some studies revealed that there is no significant relationship between task difficulty and preference for solution method (Lowrie, 2001).

Gender, Preference, and Mathematics Performance

The relationship between gender, preference for solution methods, and mathematical performance has been of great interest to researchers for many decades. A substantial number of research studies were done in this area and many of them revealed that generally male students outperform female students (Battista, 1990; Fennema, 1974; Fennema & Sherman, 1978; Guay & McDaniel, 1977; Maccoby & Jacklin, 1974; Matteucci & Mignani, 2011). However, several research studies that have been done in this area also assert that gender is independent of mathematical performance (Galindo, 1994; Haciomeroglu & Chicken, 2012). Similarly, The Trends in International Mathematics and Science Studies (TIMMS) also revealed inconsistent relationships between gender and geometry performance. Gender differences in geometry performance were evident in some countries; however, other countries showed no gender difference in geometry performance (Neuschmid, Barth, & Hastedt, 2008). Some studies, however, did not find relationships between gender and mathematics performance (Hall & Hoff, 1988; Penner & Paret, 2008). Hyde, Fennema, and Lamon (1990) also reported that there was no gender difference in students' arithmetic or algebra performance in elementary and middle school. Thus, findings are not conclusive regarding gender, preferences, and performance.

Gallagher and De Lisi (1994) reported gender difference both in preference for solution methods and mathematics performance. Fennema, Carpenter, Jacobs, Franke, and Levi (1998) reported that there was no gender difference in mathematics performance, but that gender difference prevailed in solution methods. On the other hand, some studies did not find gender difference in preference for solution methods and mathematics performance (Galindo, 1994; Haciomeroglu & Chicken, 2012; Haciomeroglu, Chicken, & Dixon, 2013; Lowrie & Kay, 2001). Hyde, Fennema, and Lamon (1990) indicated that there was a gender difference in arithmetic or algebra performance; male superiority in geometry was small, and the tests with mixed content showed the largest gender differences.

Representation

Representation is an important topic for this study because students employ various types of representations in their solution methods. The fact is that visualizers have preference for using graphic representations while verbalizers have preference for employing algebraic, numeric, and verbal representations. Kaput (1987b) stated that "representation and symbolization are the heart of the content of mathematics and are simultaneously at the heart of cognitions associated with mathematical activity" (p. 22). The role of representation in mathematics is supported by the National Council of Teachers of Mathematics (NCTM, 2000), which includes representation as one of the process standards. The Common Core State Standards for Mathematics (CCSM) also emphasize the role of representation. For example, the document states that students should be able to analyze functions using different representations (Council of Chief State School Officers & National Governors Association, 2010). In fact, representation acts as a tool for manipulation, communication, and conceptual understanding of mathematical ideas (Zazkis & Liljedahl, 2004). Researchers contend that representation plays an important role and its use is fundamental in teaching and learning mathematics (Arcavi, 2003; Goldin, 1987; Janvier, 1987; Kaput, 1987a; Roubicek, 2006; Zazkis & Liljedahl, 2004).

A representation is a sign or combination of signs, characters, objects, diagrams, or graphs, and it can be an actual physical product or mental process (Goldin, 2001). In fact, it may be a combination of something expressed on paper, existing in the form of physical objects, and a constructed arrangement of ideas in one's mind (Janvier, 1987). Researchers suggested various types of representational systems (Goldin, 2001; Janvier, 1987; Lesh, Post, & Behr, 1987).

Gleason and Hallett (1992) proposed the rule of three which consists of three types of representation: (a) symbolic, (b) graphic, and (c) numeric. The rule of three was modified to become the rule of four, which includes four types of representation: (a) graphic, (b) numeric, (c) algebraic, and (d) verbal. The rule of four is one of the most widely used and commonly accepted classifications of representation in mathematics education.

Rationale

Some students prefer to use visual solution methods based on the visual-pictorial thought process, while others like to use nonvisual solution methods based on the verbal-logical thought process. Some research studies focused on visual solution methods, while others emphasized nonvisual solution methods. Some research studies showed that students need to have both problem-solving skills—visual and nonvisual solution methods—for successful mathematical performance. For instance, the balance between visual and analytical reasoning ability is likely to be an important factor, particularly in geometry performance (Battista, 1990). In fact, the research findings related to preferences for solution methods and mathematical performance are not conclusive (Haciomeroglu et al., 2013). Thus, more research studies would help to find conclusive findings in this regard.

Gorgorio (1998) contended that solution methods can be shared and therefore taught, while preference is an individual trait. For instance, although students in the same class get the same instruction in problem-solving mathematics, there is much variance in their solution methods (Hegarty & Kozhevnikov, 1999). Gorgorio further stated that the study of preferences can contribute not only to the enlargement of theory but also to the solution of the actual problems of teaching mathematics. Thus, exploring the relationships between preferences and performance will provide insights and ideas to mathematics teachers, researchers, and educators when developing a mathematics curriculum, as well as planning effective instructional strategies (Galindo, 1994). Moreover, rigorous study in this area will help elucidate which solution methods students use and the difficulties they encounter when solving geometry tasks.

The correlation between gender and mathematical performance has been of great interest for researchers for many decades with several studies conducted regarding students' gender and its impact on preferences and mathematical performance. However, the related research studies indicated inconclusive findings in this area as well. For instances, some studies identified that gender is related with preference for solution methods and mathematical performance (Fennema & Sherman, 1978; Gallagher & De Lisi,1994) while other studies revealed that gender is independent of preference for solution methods and mathematical performance (Galindo, 1994; Haciomeroglu & Chicken, 2012; Haciomeroglu, Chicken, & Dixon, 2013; Lowrie & Kay, 2001). Some studies found gender difference on preference for solution methods but not on mathematical performance (Fennema, Carpenter, Jacobs, Franke, & Levi, 1998). Moreover, Fennema and Sherman (1978) stated that "in view of negative sociocultural effect of the belief that female do not do well in mathematics; authors and journal editors should be more responsible in reporting sex-related difference in mathematics achievement" (p. 202). Thus, more research studies in this area will help reach more general agreement regarding sex-related differences in mathematics achievement.

From a didactic perspective, it is important to know students' preferences for solution methods because students' preferences may stimulate teachers' awareness that students' problem-solving methods may be different from their own (Gorgorio, 1998). For instance, teaching style might be a learning obstacle for students who use problem-solving methods that are different from those of their teachers, their manuals, or their textbooks. Thus, understanding preference and performance helps in the design of course content and teaching approaches with a consideration for the differences in the learning environment (Sevimli & Delice, 2011). Moreover, comparing students' differences in geometry performance can also help us better understand how all students learn geometry (Battista, 1990). Therefore, more research studies can contribute to add more didactical and pedagogical knowledge.

The preferences for solution methods are also associated with difficulty levels of the mathematics problems (Haciomeroglu, 2012; Lowrie, 2001; Lowrie & Kay, 2001); however, research studies have shown inconclusive findings. Therefore, further studies are required to find more concrete results.

The Mathematical Processing Instrument (MPI) developed by Suwarsono (1982) has been used extensively to examine the verbalizer-visualizer continuum, preference for solution methods, and mathematical performance. However, the MPI is limited to algebraic word problems and was designed for middle school students. Thus, conducting similar study but in a different domain will provide broader perspectives. Moreover, the balance between verbal-logical and visual-pictorial processing may be a key variable in investigating students' problem-solving abilities and strategies in geometry (Battista, 1990). Thus, it is worthwhile to investigate the verbalizer-visualizer continuum for high school geometry.

Representation is fundamental in teaching and learning mathematics (NCTM, 2000). Mathematics textbooks contain wide varieties of representations; however, limited attention is given to the effects of representations. As a result, children are confused by various types of representations while learning mathematics (Dufour-Janvier, Bednarz, & Belanger, 1987). Kaput (1987) stated that there is a common tendency to undermine the role of representation in teaching and learning mathematics as well as in the mathematics curriculum. Thus, the knowledge of students' usage of different types of representations during problem solving can help in the design of suitable educational resources and the development of effective teaching strategies.

Students employ different modes of representation while attempting to solve mathematics problems and the representations may have an influence on their solution methods, because each mode of representation has different characteristics (Jane, 1996). For instance, some representations, such as graphic, are visual, while others, such as verbal representations, are nonvisual. Larkin and Simon (1987) suggested that graphic representations help learners to recognize features easily and help to make inferences directly. Moreover, pictures, diagrams, and similar visual representations can give learners access to knowledge and skills that are unavailable from less visual representations (Zhang, 1997). However, graphic representation is open to interpretation and can reveal as well as can hide necessary information (Mathai & Ramdas, 2006), which might influence students' solution methods and mathematical performance. Thus, there are controversies concerning the role of representation. In fact, most researchers contend that being able to use both visual and nonvisual representation and being able to translate between them will result in a more in-depth understanding of mathematics (De Jong & van Joolingen, 1998; Lesh et al., 1987). Further research will help to address issues pertinent to the effectiveness of representation.

Purpose of the Study

How students process mathematical information (verbal-logical or visual-pictorial) can affect their solution methods (Galindo, 1994; Haciomeroglu et al., 2013; Lowrie & Kay, 2001; Krutetskii, 1976; Moses, 1977; Suwarsono, 1982). In-depth knowledge about what kind of solution methods students prefer to use and what difficulties they encounter when solving geometry tasks can contribute not only to theoretical knowledge but also to the solution of the actual problems in learning mathematics (Gorgorio, 1998). Thus, one of the main purposes of this study is to examine the relationship between preference for solution methods and performance on geometry.

The way in which mathematical ideas are represented is fundamental to how students can understand and use those ideas; using and interpreting representations in appropriate ways are essential parts of learning and doing mathematics (NCTM, 2000). Geometry, in particular, is the study of the visualization, drawing, and construction of geometrical objects (Usiskin, 1987). Despite the fact that geometry problems may require more drawings and figures, this study also intends to analyze how students use different modes of representation while attempting to solve the problems.

An extensive number of studies have examined the verbal-visual continuum in mathematics based on Suwarsono's (1982) Mathematical Processing Instrument (MPI). The MPI was originally designed for middle school students using algebraic word problems. Thus, this study also aims to examine the verbal-visual continuum and students' preferences for solution methods in the domain of high school geometry. There can be various factors that may influence students' preferences for solution methods. For example, one could be teachers' teaching style. However, this study is focused on testing situations because a geometry test will be used to examine students' preferences for solution methods. The researcher poses the following research questions:

1. Are preferences for solution methods associated with high school students' geometry performance?
2. Are the degrees of difficulty of geometry tasks associated with students' preference for solution methods?

3. Do males and females differ in preference for solution methods and geometry performance after controlling for course assignments and grade levels?

Definition of Terms

For the purpose of the study, the following terms based on the existing literature and background will be used.

Imagery: Visual representations of things or events, available even in their absence that depicts visual and spatial information inside the head.

Nonvisual Solution Method (NSM): A solution method in which students use mathematical formulae, rules, axioms, and postulates, while attempting mathematical tasks.

Representation: A combinations of signs, characters, symbols, and any kind of diagrams and pictures that can be used to present mathematical ideas, concepts, and problems.

Spatial Ability: The ability to see, inspect, generate, retrieve, and manipulate the given visual situation.

Visual Solution Method (VSM): A solution method in which students use given diagrams and figures, or draw diagrams and figures, or visualize diagrams and figures in their mind while attempting mathematical tasks. The diagrams and figures play a dominant role while attempting mathematical tasks.

CHAPTER TWO: LITERATURE REVIEW

The literature review for this study consists of four parts: (a) a review of modes of thought and solution methods; (b) a review of imagery and spatial ability; (c) a review of preference for solutions methods, gender, and students' mathematical performance; and (d) a review of representation in mathematics education.

Modes of Thought and Solution Methods

Krutetskii (1976) conducted a comprehensive study on gifted students' cast of mind in connection with mathematical abilities. He identified two modes of processing mathematical information: verbal-logical and visual-pictorial, stating that everybody is endowed with these two components of thinking. In the context of mathematics, students attempt to solve mathematical tasks or learn mathematics with the aid of formulae, logical reasoning, and so forth, without using the visual images in the verbal-logical mode of thought, whereas they process mathematical information based on visual images in the visual-pictorial mode of thought. He further suggested that verbalizers employ the verbal-logical component while visualizers use the visual-pictorial component.

Krutetskii (1976) contended that every person has two components of thinking. He also identified two propositions: (1) the two components, the ability to visualize abstract mathematical relationship and the ability to use spatial geometry concepts, are not necessary components in the structure of mathematical ability; and (2) the presence or absence of these two components does not determine the extent of mathematical giftedness, but the components do determine its type. He contended that "A pupil can be mathematically capable with a different correlation between visual–pictorial and the verbal–logical components, but the given correlation determines what type the pupil belongs to" (p. 315).

According to Krutetskii (1976), the level and quality of schoolchildren's mathematics achievements are determined by the level of development of each thinking component and by the interrelation between these two thinking components. Based on the correlation between verbal-logical and visual-pictorial components, different structures of mathematical abilities and casts of mind are formed for successful mathematical performance. In fact, the *levels* of mathematical abilities are largely determined by a verbal-logical component, while the *types* of mathematical giftedness are determined largely by a visual-pictorial component. Moreover, in the case of the visual-pictorial component, it is not only the ability to use the component but the preference for its use that determines the type of mathematical giftedness of an individual. Krutetskii observed, from his analysis of children's thinking processes while they were attempting mathematical problems, that mathematically weak students always had a very weak verbal-logical component, whereas mathematically capable students always had a very strong verbal-logical component. He claimed that the visual-pictorial component merely affects the nature of a student's mathematical ability but not its level, because Krutetskii found some students in his study were very capable in mathematics but had very weak visual-pictorial components. Thus, he associated the preference for solution methods with the visual-pictorial component, while mathematical ability would be associated with the verbal-logical component.

Following the work of Krutetskii, Moses (1977) placed students in a continuum with regard to their preference for solution methods for solving mathematical problems. Students belong to one of the three categories: (a) analytic (a preference for manipulating words and sentences), (b) geometric (a preference for manipulating images), and (c) harmonic (a preference for using both analytic and geometric methods equally). The analytic type operates mathematical concepts and ideas easily with abstract schemes without a need for visual supports for visualizing objects or patterns in problem solving, even when a given mathematical task demands visual schemes. These students always attempt to process mathematical information via a verbal-logical approach. However, the geometric type attempts mathematical tasks with the aid of graphic representations. According to Krutetskii, the geometric type students feel a need to interpret visually an expression of an abstract mathematical relationship, and they always try to use graphic representations even when the problem can be done easily using nonvisual solution methods. Students who belong to the visualizer type process the mathematical information with the help of a visual-pictorial component. The third type is called harmonic. The majority of capable students in Krutetskii's research study belonged to the harmonic group. Students who belong to this group are successful at implementing both visual and nonvisual solution methods while solving mathematical problems. Spatial concepts are well developed in harmonic types. Krutetskii further classified the harmonic into two subtypes: abstract-harmonic and pictorial-harmonic. Both subtypes can depict mathematical relationships equally well by visual pictorial means; however, the abstract-harmonic subtype feels no need to do so and does not strive to use visual images, whereas the pictorial-harmonic subtype does feel a need and often relies on graphic schemes while attempting mathematical tasks.

Following Krutetskii's (1976) work, Suwarsono (1982) also classified students into three groups based on the preference for solution methods. He, however, slightly modified the name of the groups. Suwarsono divided students into verbalizers, visualizers, and mixers. He also called the visual method and nonvisual method of processing mathematical information what Krutetskii called the verbal-logical method (mental method) and visual-pictorial method (visual method). In fact, there are no fundamental differences between Krutetskii and Suwarsono's classification. The analytic and verbalizers are the same. Similarly, geometric and visualizers as well as harmonic and mixers are also the same. Suwarsono called visual and nonvisual methods of solving mathematical tasks what Krutetskii called visual and verbal methods. However, for the purpose of this study, the researcher uses verbal solution method (*verbalizer)*, visual solution method (*visualizer)*, and *harmonic* method (use both verbal and visual solution method).

Kozhevnikov, Hegarty, and Mayer (2002) suggested that the verbalizer-visualizer continuum needs to be revised to include two groups of visualizers. They stated that visualizers are not one homogenous group with respect to their spatial abilities. Some of them have a low spatial ability and some of them have a high spatial ability. They called these groups of students as iconic type (low spatial ability) and spatial type (high spatial ability). Kozhevnikov, Kosslyn, and Shepard (2005) even objected to the verbalizer-visualizer dichotomy. They suggested three types of groups: verbalizers, object visualizers, and spatial visualizers. Object visualizers are more accurate and faster in generating static objects, whereas spatial visualizers are good at manipulating dynamic images.

Why students solving mathematical problems prefer one solution method over another when multiple solution methods are possible could be an important field of investigation in mathematics education. In this regard, Krutetskii (1976) laid a foundation for the distinction between preferences and abilities in relation to doing mathematics tasks. He contended that ability and preference are not the same thing. For example, students might have the *ability* to solve a problem with visual methods, but they might not *prefer* to solve it by visual methods; rather, they might prefer to solve it by a verbal method. Similarly, students might have the *ability* to solve a problem by a verbal method, which does not necessarily imply that they *prefer* to solve it by the verbal method. Thus, as far as the verbal-logical and visual-pictorial frameworks are concerned, students demonstrate different preferences for solution methods while attempting mathematical tasks.

Verbal and visual are the predominantly used solution methods in the domain of mathematics. Even though researchers may have used different terms to represent these solution methods, most of the researchers, if not all, share common concepts about verbal and visual solution methods. Many researchers have investigated preferences for verbal and visual solution methods while attempting mathematical tasks (Haciomeroglu, 2012; Lean & Clements, 1981; Lowrie & Kay, 2001; Moses, 1977; Presmeg, 1986b). Various distinctions are made between verbal and visual solution methods. Presmeg (1986b) stated that:

> A visual solution method is one which involves visual imagery, with or without a diagram, as an essential part of the solution method; even if algebraic methods are also employed while verbal solution method involves no visual imagery (p. 42).

Based on the use of visual imagery, Presmeg (1986b), Suwarsono (1982), and Moses (1977) defined and explained mathematical visuality— the extent to which a person prefers to use visual imagery when attempting mathematical problems. Moses stated that *degree of visuality* refers to the extent to which the subject uses visual solution processes to solve the given mathematical problems. In fact, the visual approach involves the act of visualization, which consists of any mental constructions and/or transformation of objects or processes (Suwarsono, 1982). In general, visualizers primarily rely on graphs, pictures, or symbols. In contrast, verbalizers attempt to solve problems by relying on rules, formulas, and algorithms (Moses, 1977).

It is worthwhile to mention that visual solution methods may also use some verbal and mathematical symbols, verbal statements, and mathematical statements. The fact is that diagrams, pictures, or similar constructions need to be labeled or they require verbal description in order to communicate about the constructions. Verbal and mathematical symbols are merely the shorthand for ordinary language and mathematical language (Skemp, 1987). However, the role of diagrams and figures is significantly important, and without using diagrams and figures it is not possible to solve problems using visual solution methods, regardless of whether the answer is correct or incorrect. In summary, in visual solution methods students use given diagrams and figures, or draw diagrams and figures, or visualize diagrams and figures in their head while attempting mathematical tasks. The diagrams and figures play a dominant role in visual solution methods to find the answer while attempting mathematical tasks.

A verbal solution method is one that involves analytic reasoning while attempting mathematical tasks. Analytic reasoning implies the use of mathematical formulae, algebra, arithmetic, rules, postulates, axioms, conjectures, and so forth while attempting mathematical tasks. With this method, students do not use diagrams and figures. Suwarsono (1982) stated that in verbal solution methods, the reasoning is conducted purely on the basis of the processing or manipulation of verbal and mathematical statements and these manipulations are performed using the rules of language and mathematics. Zazkis, Dubnisky, and Dautermann (1996) stated that verbal solution methods involve an act of any mental manipulation of objects with or without the aid of symbols. Regardless of the different terms used to describe solution methods, the researcher decided to use verbal and visual solution methods for this study.

Imagery

Though imagery is not a focus of this study, it is relevant to provide a brief description about imagery because it is associated with students' preference for visual solution methods. Imagery involves students visualizing mathematics problems in their head while attempting the problems. The term *imagery* can refer to mental imagery, visual imagery, or simply imagery.

Most researchers agree that visual imagery plays an important role in mathematics because imagery enhances intuitive views and understandings in many areas of mathematics (Krutetskii, 1976; Suwarsono, 1982; Usiskin, 1987). Despite variations in definitions and interpretations noted when performing this literature review, scholars have been able to derive a common definition and meaning of imagery. Suwarsono (1982) stated that

> Clearly, several important controversies concerning the nature of imagery have not been resolved. For example, such questions as "What exactly is a visual image?" "What are the characteristics of images?" and "How are images stored in the memory?" have not been answered satisfactorily. Despite such problems, recent research enables several statements about imagery and mental images to be made with confidence, and these will form the basis for the research involving imagery in the present thesis (p. 38).

Visual imagery is defined as a mental construct depicting visual and spatial information (Presmeg, 1986a). The visual imagery occurs inside the mind in absence of objects when our sense organs (eyes, ears, tongue, nose, and skin) perceive them (Suwarsono, 1982). Further, according to Suwarsono, visual imagery is meant to be a pictorial representation, either on paper or in the mind. For example, when we read the word *cow,* we can visualize the cow in our mind as a mental image, which could be different from the actual cow we see. Thus, mental imagery is an ability to form images of things or events even in the absence of the objects or events. This means that students may use imagery while attempting geometry problems because they do not want to draw figures and diagrams; rather, they may prefer to visualize them in their head. Suwarsono further contended that even if pictorial representations are drawn on paper, visual imagery is also involved since before the pictorial representations are put on paper, students first must imagine the representational system in their mind. Imagery also refers to a representation of the visual appearance of an object, such as its shape, color, or brightness (Hegarty & Kozhevnikov, 1999). Clements (1982) defined visual imagery as creating a "picture in the mind" (p. 36) whereas Presmeg (1986b) defined it as "a mental scheme depicting visual or spatial information" (p. 297).

Individuals use a wide range of visual imagery in the teaching and learning of mathematics. In her research, Presmeg (1986a, 1986b) refined Suwarsono's Mathematical Processing Instrument (MPI). She divided research instruments into three parts: A, B, and C. Parts A and B were designed for high school students, whereas part B and C were intended for mathematics teachers. Presmeg (1986b) also conducted a study with only visual students (N = 54) while they were solving problems in algebra, trigonometry, and geometry. She found that students' use of imagery was widespread in mathematical reasoning, but students themselves were unaware of using visual imagery in their reasoning. Based on the study, Presmeg identified five different kinds of mental imagery: (a) concrete pictorial imagery; (b) pattern imagery; (c) memory images of formula; (d) Kinesthetic imagery; and (e) dynamic imagery. Following the work of Lakoff (1987), Wheatley (1998), however, differentiated imagery into rich images and images schemata. Rich images are static, fixed, and contain much visual details, whereas image schemata represent spatial relationships and can be transformed in various ways.

Presmeg (1986a, 1986b, 1992) also found that concrete pictorial imagery was the most used while the dynamic imagery was the least used during attempting mathematics tasks. Presmeg contended that the use of concrete pictorial imagery may focus the reasoning on irrelevant details that take the students' attention from the main elements in the original problem representation; however, other kinds of imagery play more positive roles. Presmeg stated that the most important role in mathematical problem solving is pattern imagery, in which concrete details are disregarded and pure relationships are portrayed. However, Clements (1981) contended that the imagery vividness factor was the most important one. The fact is that during the problem solving process, students do not necessarily stay with only one type of visual imagery; rather, they may use different types of imageries based on different types of mathematical situations and context. Thus, visual imagery is an important factor that teachers need to take into account while teaching mathematics (Bishop, 1989).

Spatial Ability

The literature on imagery indicates that imagery is an important component of spatial ability. The geometry test (research instrument) designed for this study does not require spatial ability; however, students may use spatial ability while solving geometry problems. Thus, it is worthwhile to provide a brief description about spatial ability in connection with solution methods. The researcher also sheds light as to why preference has been chosen instead spatial ability to measure students' geometry performance.

The term "spatial ability" is related to space and is derived from the literature of psychology on human abilities (McGee, 1979). Various terms such as spatial sense, spatial visualization, spatial orientation, spatial perception, spatial reasoning, and spatial structure are associated with the term spatial ability. It is not a specific mathematical ability; rather, it extends across various intellectual activities. Moses (1977) stated that spatial ability may or may not be an integral part of an individual's mathematical problem-solving process. However, Fennema (1979) argued that all mathematical tasks require some kind of spatial thinking and reasoning. Similarly, Clements and Battista (1992) also contended that geometry and spatial reasoning are strongly interrelated, and most mathematics educators seem to include spatial reasoning as part of the geometry curriculum.

Definitions of spatial ability abound. Spatial ability is the ability to perceive the essential relationships among the elements of a given visual situation and the ability to mentally manipulate one or more of these elements (Moses, 1977). It may be defined as the ability to generate, retain, retrieve, and transform well-structured visual images (Lohman, 1996). Spatial reasoning is the ability to see, inspect, and reflect on spatial objects, images, relationships, and transformations (Battista, 2007). Linn and Petersen (1985) stated that "spatial ability is the skill in representing, transforming, generating, and recalling symbolic, non-linguistic information" (p. 1482). The National Council of Teachers of Mathematics (NCTM, 1993) used the term spatial sense to refer to spatial perception or spatial visualization. It stated that spatial visualization is the ability to imagine movement or spatial displacement by mentally rotating, folding, or in some other way manipulating visual representations of objects.

Spatial ability includes mainly two components: spatial visualization and spatial orientation (McGee, 1979). However, Lohman (1996) stated that there are three major spatial factors: spatial visualization, spatial orientation, and speeded rotation. Wheatley (1998) has given different meanings and interpretations for spatial ability and spatial visualization. According to Wheatley, spatial visualization is the ability to mentally manipulate, rotate, twist, or invert pictorially presented two- and three-dimensional objects, and spatial orientation refers to understanding and operating on the relationships between the positions of the objects in the space with respect to one's own position (Clements & Battista, 1992). Carroll (1993) contended that there are five factors that impact spatial ability: spatial visualization, spatial relations, closure speed (conceal words and mutilated words in which tasks are mainly those of apprehending a spatial form), flexibility of closure (hidden figures, patterns and copying, in which the tasks are mainly those of searching a visual field to find spatial form), and perceptual speed (finding number comparison and identical pictures).

Various research studies have shown that spatial ability is positively correlated with measures of mathematical performance (Battista, 1990; Clements & Battista, 1992). However, spatial ability alone does not determine students' mathematical abilities because a student with high spatial ability may not prefer to use it while solving mathematics problems (Krutetskii, 1976). For example, students might be able to solve a problem by visual methods; however, they might not prefer to solve using visual methods. Presmeg (1985) also pointed out the same issues: spatial tests may be solved by using analytic solution methods, or students with good spatial ability may not prefer to use visual solution methods. Thus, spatial ability will not be measured in connection with geometry performance in this study. Rather, an investigation of students' preferences for solution methods is the main aim of this study. Krutetskii (1976) also contended that spatial ability does not determine students' geometric performance; he documented many cases in which students who showed good spatial ability were poor in geometry performance. Moreover, he contended that a well-developed spatial ability does not imply that students will use it while attempting mathematical tasks. For example, students may be able to solve a problem by visual methods; however, they may not prefer to solve it using visual methods. Several research studies have been conducted to examine the relationships between the preferences for solution methods and spatial ability; however, they revealed that there was little or no correlation between preferences and spatial ability (Haciomeroglu, Chicken, & Dixon, 2013; Hagarty & Kozhevnikov, 1999; Kozhevnikov, Hagarty, & Mayer, 2002; Moses, 1977; Lean & Clements, 1981; Suwarsono, 1982). Presmeg (1985) also pointed out the same issues:

spatial tests may be solved by using analytic solution methods, or students with good spatial ability (i.e., the geometric type) may not prefer to use visual solution methods. Quoting the work of Wattanawha and Clements (1982), Clements (1984) reported that mathematically gifted students had a strong preference for verbal methods on space visualization tests. Similarly, Haciomeroglu, Chicken, and Dixon (2013) found that that cognitive ability (spatial ability and analytical reasoning) did not influence students' preference for visual or verbal solution methods. Thus, for this study spatial ability will not be used to measure students' geometry performance; rather, preference for solution methods will be the focus.

Preference and Mathematical Performance

Preferences for solution methods, gender, and mathematical performance have been of great interest to researchers for several decades (Battista, 1990; Fennema & Sherman, 1978; Haciomeroglu et al., 2013; Lean & Clements, 1981; Moses, 1977; Samuels, 2010; Suwarsono, 1982). Students can choose different solution methods when a mathematical task can be solved in multiple ways by employing either a visual-pictorial or a verbal-logical mode of thought. For example, a study conducted for a nationally representative sample in the UK and the USA identified that males preferred to use visual solution methods but females preferred to used verbal solution methods (Lohman & Larkin, 2009; Strand, Deary, & Smith, 2006). In contrast, Calvin, Farnandes, Smith, Visscher, and Deary (2010) revealed that the association between preferences and educational achievement, including mathematics, were the same for both sexes, and there was no significant difference in employing solution methods based on gender. These are just two examples of the findings of the research studies that are not consistent with each other in this area. In this section, different research studies will be described that have been performed in the arena of preferences for solution methods and mathematical performance.

Moses (1977) conducted a comprehensive study with fifth-grade students (*N* =131) to measure relationships between problem-solving performance, and mathematical visuality. To measure the preferences for solution methods, she employed a problem-solving inventory, which contained 10 word problems. Only problem number seven had a diagram. Verbal representation was employed, except for the seventh problem, to present the problems. The problems were different in nature in the sense that three problems were analytic, four problems were spatial, and three problems were both analytic and spatial in nature. Her study revealed that there was no correlation between mathematical performance and preferences for solution method.

Moses's study (1977) also had some limitations. She measured the preferences for solution methods based on students' written response only, but some students may not express their solution process in their written response. Moreover, students at the primary school level may not be able to express all or some of their thinking process on paper. Thus, the Moses study is criticized by many researchers, including Lean and Clements (1981). Students' mean score was also too low for the problem-solving inventory both in pretest and the posttest. The pretest mean score was 1.9 out of a possible maximum score of 10, and 2.18 in the posttest. This finding suggests that the mathematics problems were too difficult for the fifth graders, and there could be consequences of this in the findings of the study as well.

In order to avoid Moses's limitation (1977), Suwarsono (1982) conducted a study with middle school students (*N*=112) in which he developed an instrument called the Mathematical Processing Instrument (MPI) to investigate the students' degree of preference to use visual imagery (visual and verbal solution methods) and its effects on their mathematical performance. The MPI consists of two parts. The first part includes 30 algebraic word problems, while the second part includes the written description of different possible solution methods (questionnaires). He also used verbal representation to present the 30 problems. None of these 30 problems were from geometry. Suwarsono designed the questionnaires to elude the limitation of Moses's study. The questionnaires contained various solution methods (visual, verbal, and other) for each problem. Students were asked to solve the word problems in the first part of MPI. In the second stage, students were required to choose the solution methods from the questionnaires. Beyond this, if students' methods were different from the ones that were listed in the questionnaires, the researcher instructed them to describe their solution methods. Thus, the researcher could understand the solution methods of those students who did not indicate their solution methods while attempting the word problems.

Consistent with Moses's findings, Suwarsono (1982) also found that mathematical visuality (preferences for solution methods) did not have a significant effect on mathematical performance. Students who preferred using visual solution methods in problem solving were likely to do as well as students who used verbal solution methods. In an experimental study, Pitta-Pantazi and Christou (2009) found that preference was not related to performances. Their results also corroborated Moses and Suwarsono's findings.

Lean and Clements (1981) conducted a study with foundation year engineering college students (*N*=116) in which they used a slightly modified version of Suwarsono's Mathematical Processing Instrument (MPI) in order to investigate relationships between preference for solution methods and mathematical performance. They found that preferences had significant influence on students' mathematical performance. Their study further revealed that students who employed verbal solution methods performed significantly better than the students who employed visual solution methods. They also contended that the verbalizers developed logical reasoning ability and were able to avoid unnecessary visual information. Their finding also supports the Krutetskii

(1976) thesis that spatial ability does not determine students' mathematical performance. However, their findings conflicted with those of Moses (1977, 1980) and Webb (1979), who reported that students who preferred to use visual solution methods tend to outperform those who use less visual solution methods.

Haciomeroglu, Aspinwall, and Presmeg (2009) conducted an empirical case study for calculus students to explore the relationship between mode of representation and preference for solution methods and calculus performance. Rather than using the verbal representation of MPI, the researchers used graphic representations to present the derivative problems. They found that students used visual as well as verbal solution methods to complete the given tasks, but students who used visual solution methods showed limited understanding and were not able to provide a complete answer, which contradicts the Lowrie and Kay (2001) findings. They also suggested that teachers need to incorporate both visual and nonvisual solution methods in their teaching strategies to support the successful mathematical performance of students. This study supported the Krutetskii (1976) thesis that regardless of the mode of representation used to present a problem, verbal-logical and visual-pictorial modes of mathematical processing were equally likely in student responses.

With the aid of 16 graphical calculus problems, Haciomeroglu, Aspinwall, and Presmeg (2010) investigated the relationship between students' preference for solution methods and calculus performance. Though a graphic representation was used to present the calculus problems, students translated the problems into algebraic representation based on their preferences for solution method, according to the researchers. Similar to the findings of Haciomeroglu et al., (2009), this study also concluded that both visual and verbal solution methods are essential components for successful mathematical performance. They emphasized the need for both modes of thinking—verbal and visual—to deepen students' understanding. Additionally, they contended that students need to be able to translate one mode of representation to another for successful mathematical performance.

With the help of graphic and algebraic words problems, Haciomeroglu and Chicken (2011) examined the relationships among student cognitive ability, preference for solution method, and calculus performance of high school students (*N*=169). This study revealed that students' preferences for solution methods were positively correlated with calculus performance, where the problems were presented with the aid of graphic representation; however, the preferences were not associated with calculus performance, where the problems were presented with the aid of algebraic representation. Moreover, this study also found that Suwarsono's (1982) Mathematical Processing Instrument (MPI) is not an appropriate instrument to measure preferences and calculus performance. In another similar study, Haciomeroglu, Chicken, and Dixon (2013) examined high school students' (*N*=150) preference for solution methods and calculus performance by employing a graphic-calculus test. The preference for visual solution methods was significantly correlated with calculus performance, which was not consistent with Moses (1977), Lean and Clements (1981) and Suwarsono's (1982) findings. Similar to Haciomeroglu and Chicken (2011), they also argued that the MPI, which is considered an ideal test to examine students' preference for solution methods and mathematical performance, was not an appropriate test for the calculus students. Moreover, they explained that visual schemes involved in calculus tasks may not be captured by the algebraic test.

With the help of MPI, Hegarty and Kozhevnikov (1999) investigated how visual-spatial representations affect problem-solving performance of sixth graders (*N*=33). They found that preference for visual solution methods was positively correlated with mathematical performance.

They made further distinctions among visual solution methods. They contended that there are, in fact, two types of visualizers: schematic types (representing the spatial relationships between objects and imagining spatial transformation), who are generally successful in mathematics problem solving, and pictorial types (constructing vivid and detailed visual images), who are less successful than schematic types. The distinction between two visual solution processes was further supported by Kozhevnikov, Hegarty and Mayer (2002). The researchers found that the verbalizers and visualizers were the same on all parameters except their preferences for solution methods. Verbalizers did not have any clearly marked preference for using verbal solution methods. In contrast, visualizers showed a consistent preference for using visual solution processes. They claimed that various studies (Krutetskii, 1976; Lean & Clements, 1981; Presemeg, 1986a, 1986b) did not take the two types of visualizers into account which led them not to find the relationships between preferences for visual solution methods and mathematical performance.

Similar to Suwarsono (1982), Battista (1990) examined high school students' (N=145) preferences and geometry performance. To identify solution methods and to assess geometry performance, he designed an instrument of nine geometry problems. He concluded that preferences for solution methods were not significantly correlated to geometry performance. However, preferences for verbal solution methods were positively correlated with geometry performance only for male students. Only female students who preferred to use visual solution methods (correct number of drawings) were positively correlated with geometry performance.

Ling and Ghazali (2007) examined primary school students' preferences for solution methods (N =5) and pre-algebra problems. The problems were presented with the aid of verbal and graphic representation. Students equally used visual and verbal solution methods to solve the problems. Similarly, Sevimli and Delice (2011) investigated the relationships between calculus students' preferences for solution methods and representation preference while solving mathematics problems using the modified version of MPI developed by Presmeg (1985). They concluded that the mode of representation used to present the problems affected students' preference for solution methods. Verbalizers and harmonic were observed to have similar preference tendencies. However, visualizers altered their preference based on the mode of representation used to present the problems. This study corroborated findings of Haciomeroglu, Chicken, and Dixon (2013) and Haciomeroglu and Chicken (2011). The greater variance in preferences of solution methods particularly for visualizers was consistent with the findings of the Kozhevnikov et al., study (2002). Moreover, this study also found that most of the verbalizers predominantly preferred verbal solution methods (algebraic representation).

Galindo (1994) investigated the relationships between preferences and use of technology with calculus students and calculus performance using a modified version of Suwarsono's MPI. This study revealed that students who were verbalizers obtained significantly higher scores than visualizers in the calculus section with and without the use of technology (computers and Mathematica); however, there was not a significant relationship between preference and calculus performance using graphing calculators. In a similar way, Coskun (2011) conducted a multiple case study investigating students' preference for solution methods using algebraic word problems of Suwarsono's MPI, where she compared the effects of used paper-pencil, dynamic geometry software, and calculator for students' solution methods. Her study revealed that students were able to perform better in a dynamic geometry software environment compared to a paper-pencil environment. Students' preferences for solution methods altered in the different learning environments. It appeared that the different modes of representation (i.e., graphic and algebraic) and various tools used in both studies could be an underlying reason for the disparity in findings between paper-pencil, computer technology, and graphing-calculator learning environments. This study is not directed to investigate the teaching and learning environment and its connection to preferences for solution methods; however, the different modes (graphic or algebraic) of representation students' use during solving geometry problems can affect students' preferences and performance.

Booth and Thomas (2000), Gagatsis and Elia (2004), Hart (1991), and Campbell, Collis, and Watson (1995) conducted different studies where they compared the preferences for solution methods with the mode of representations that they used to present problems. Their findings suggested that the different modes of representation influence preferences and mathematical performances. However, their study was limited to only graphic representations. They also reported that students' preference for visual solution methods was partly determined by both the level of abstraction of the visual methods and students' corresponding ability to draw or visualize figures and pictures.

Preference and Task Difficulty

One of the aims of this study is to examine the relationships between task difficulty and preferences for solution methods; however, there is a meager amount of research studies which explain the relationships between these two factors. Lowrie and Kay (2001) conducted a study with six-year-old children (*N*=112) to examine the relationships between students' preference for solution methods, task difficulty, and mathematical performance. They used the 10 easiest and the 10 most difficult problems from Suwarsono's MPI as a research instrument; however, they did not explain that how they classified problems into easy and difficult level. Their study revealed that task difficulty had a major influence on the way students solved mathematics problems. Students were more likely to use visual solution methods than nonvisual solution methods to solve the difficult problems. They also found that visual solution methods were more efficient because it helped the problem solvers organize and access relevant knowledge effectively.

Lowrie (2001) also conducted a study for a middle school students with six-year-old students (*N*=58) to investigate preferences for solution methods, preferences efficiency for solution methods, and mathematical performance with the aid of Suwarsonso's Mathematical Processing Instrument (MPI). The visuality preference for solution methods included solutions of all problems, irrespective of whether the solutions were correct or incorrect. However, the researcher took only the solution methods with correct answers into account when measuring the preference efficiency. He found that there was no significant correlation between the preference for solution method (visuality preference) and mathematical performance. In contrast, there was a significant difference between students' mathematical performance and preference efficiency. Students who predominantly used visual solution methods outperformed to students who substantially used the nonvisual solution methods. This study also revealed that there was no significant relationship between task difficulty and preference for solution method, which did not support the Lowrie and Kay (2001) and Lean and Clements (1981) findings. Lowrie and Kay used the 10 easiest and the 10 most difficult problems from the MPI, while Lowrie (2001) used 20 problems, but it is not clear which 20 problems he used from the MPI. The different ways they chose the problems might lead them not to have similar result between their studies.

Haciomeroglu (2012) conducted a study with calculus students (*N*=498) to delve into the relationship between task difficulty and solution method. Unlike the MPI, he used 14 graphic and six algebraic word problems along with the questionnaires. Similar to Lowrie and Kay (2001), Haciomeroglu also concluded that as task difficulty level increased, the number of visual solution methods (correct and incorrect) increased significantly, and the number of nonvisual methods decreased significantly for the graphic representation. For the algebraic problems, students used more nonvisual methods than visual method. However, as the level of problem difficulty increased, the number of nonvisual solution methods was significantly decreased, while the visual methods were substantially increased.

The MPI was originally developed for seventh graders (12/13 years). It can be argued that the way that six-year-old children respond to the MPI may be significantly different from the way 12-year-old students respond. The fact is that the content level of the MPI may not reflect six-year-old students' actual preferences for solution method and mathematical performance. Thus, the appropriateness of the MPI for six-year-old students could be questioned. On the other hand, Lean and Clements (1981) used the MPI for college-level students. One of the reasons for conflicting findings between Lean and Clements' and Lowrie and Kay's (2001) could be the different types of participants they had in their studies, regardless of the use of a similar instrument.

Gorgorio (1998) conducted a qualitative study to examine students' preferences and task difficulties with the help of graphic problems. The researcher found that subjects' preference for solution methods depended on task difficulty and required action. The required action is the action to be done by students to solve the given problems. The required action consists of interpretation (students have to gain meaning from given representation) and construction (students have to generate or construct new objects). The study further revealed that when the required action was of interpretation, students tended to use visual solution methods when an object was simple; when the object was difficult, students used nonvisual solution methods.

However, when the required action was of construction, students tended to use visual solution methods when an object was complex and manipulation was not suggested (a drawing was required) and use nonvisual solution methods when an object was simple or manipulation was required (students needed to build an object). One finding supported while the other contradicted findings of Lowire and Kay (2001) and Haciomeroglu (2012). Moreover, the researcher did not make distinction between simple and complex objects.

Gender, Preference, and Mathematical Performance

The relationship between gender and mathematical performance has been of great interest to researchers for many decades. A substantial number of research studies were done in this area and many of them revealed that generally male students outperform female students (Battista, 1990; Fennema, 1974; Fennema & Sherman, 1978; Guay & McDaniel, 1977; Maccoby & Jacklin, 1974; Matteucci & Mignani, 2011). However, several research studies that have been done in this area also assert that gender is independent of mathematical performance (Galindo, 1994; Haciomeroglu & Chicken, 2012). Similarly, The Trends in International Mathematics and Science Studies (TIMMS) also revealed inconsistent relationships between gender and geometry performance. Gender differences in geometry performance were evident in some countries; however, other countries showed no gender difference in geometry performance (Neuschmid, Barth, & Hastedt, 2008). Thus, there are no conclusive findings regarding gender, preferences, and performance.

Fennema and Sherman (1978) investigated sex-related differences in mathematics and related factors with middle school students (N=1320). Spatial visualization and verbal reasoning ability were two of the factors they examined. They reported that there was no significant difference between male and female students in terms of mathematics performance. However, in another similar study, Fennama and Tartre (1985) found that boys solved more problems correctly than girls.

Fennema and Carpenter (1981) conducted a study using the 1978 National Assessment of Educational Progress (NAEP) results to examine sex-related difference in mathematics performance. They found that males significantly outperformed females in the area of geometry. This study also reported that there was no significant difference in mathematical performance between male and female students ages 9 and 13; however, there was significant difference in achievement of 17-year-old male and female students. In fact, 17-year-old male students' performance exceeded that of 17-year-old female students at every cognitive level. Their findings provide very important insights for research to explore with respect to what causes the gap in achievement between male and female students as their ages increase. In a similar study, Fennema and Tartre (1985) examined the relationship between verbal logical reasoning and gender of sixth grade students (*N*=669). They concluded that students who were discrepant in verbal skills differed in the process they used to solve mathematical problems.

Battista (1990) examined high school students' gender and geometry performance. In his study, male students scored significantly higher than female students on a geometry problem solving test. The greatest difference between males' and females' geometry scores occurred for students whose nonvisual reasoning scores were much greater than their visual reasoning scores; the smallest difference occurred when the visual solution score was much greater than the nonvisual solution score. He found that males and females differed in geometry performance but not in preferences for solution methods. Similarly, Mayer and Massa (2003) also concluded that there were no significant gender differences on students' preferences for solution methods.

Haciomeroglu and Chicken (2012) conducted a study to investigate visual thinking and gender difference with high school calculus students (*N* =188). The calculus problems were presented with the help of graphic representation. Their study suggested that preference for visual thinking was a significant factor influencing male students' performance on the AP test but not for female students. However, similar to Battista's (1990) findings, students' gender did not have a significant influence on their preference for solution methods on the calculus test. They also found that a stronger preference for visual thinking was associated with higher mathematical performance, which also aligned with Battista's finding. However, the stronger preference for visual thinking and its association with higher mathematical performance was not consistent with the findings of studies by Moses (1977), Suwarsono (1982), Galindo (1994), and Lean and Clements (1981).

Haciomeroglu, Chicken, and Dixon (2013) examined high school students' (*N*=150) preferences and calculus performance by employing a calculus test. Their results suggested that gender did not have a significant effect on preferences for solution methods. Their study also revealed that visualizers and harmonics did not differ significantly with respect to their calculus scores but the verbalizers had significantly lower calculus scores than the other two groups. They also suggested that gender was not enough to predict the preference for solution methods. Galindo (1994) also reported similar results in which he noted no significant sex-related difference in preference for solution methods and calculus performance of college students. Furthermore, he also did not find interaction between gender, preference for solution methods, and calculus performance. A research study conducted by Guay and McDaniel (1977) also corroborated Galindo's findings.

Calvin, Farnandes, Smith, Visscher, and Deary (2010) compared 11-year-old students' (*N*=178599) reasoning abilities (verbal, visual, and quantitative) and their effect on educational achievement based on national standardized test scores. Their study revealed that there were no significant differences in preferences for solution methods and gender. However, girls' performances were higher than boys' in verbal and visual solution methods, whereas boys' performances were higher than girls' on quantitative reasoning. Their findings supported, as well as contradicted, some of the earlier findings reported in this area.

Kolloffel (2012) examined the relationships between preferences and mathematical performance with college students (*N*=40). The researcher experimented with two different modes of representation (graphic and verbal) as an instructional strategy. Despite the differing teaching strategies used, no correlation was observed between preferences and mathematical performance. However, participants in the verbal instruction condition obtained significantly higher posttest scores than did students in the visual instruction condition. The findings of this study contradicted the findings of various other studies, including Moses (1977). The researcher made some arguments that conflicted with several research findings. They argued that it was counterproductive to give students the opportunity to choose multiple representations, which undermined the role of multiple representations in the teaching and learning of mathematics. This study is open to criticism for several reasons. It did not mention the duration of the teaching interval and criteria of selection of students for the two environments. Moreover, one can argue about the appropriateness of selecting psychology students for participation in a mathematics study.

There are various factors which might influence students' preference for solution methods. For example, teaching styles, students' grade level, and courses they enrolled in, and so on. However, a search of the related literature particularly on effects of students' grade level and different mathematics subject they enrolled in indicated that it is likely that no research studies have been published in this area. Ben-Chaim, Lappan, and Houang (1988) examined the effects of grade level on spatial visualization. They reported that there were significant effects of grade level (grade 6, 7, and 8) on spatial visualization.

Researchers investigated different aspects of gender that attributed preference for solution strategies and mathematics performance. Some researchers identified factors such as cognitive abilities, socioeconomic status etc., underlying gender difference in mathematics (Ceci, Williams & Barnett, 2009; Wai, Cacchio, Putalaaz, & Makel, 2010), while others found that gender difference in mathematical performance was due to difference in preferred mode of processing mathematical information (Carr, Steiner, Kyser, & Biddlecomb, 2010; Lin & Peterson, 1985). For example, Carr, Steiner, Kyser, and Biddlecomb (2010) investigated different factors in conjunction with gender difference in mathematics of elementary level students. The different factors they took into account were influence of strategy use, fluency, accuracy, spatial ability, and confidence in mathematics competency. They reported that only two factors, fluency and strategy, indicated gender difference and significantly predicted mathematics competency. They further suggested that girls' preference for manipulatives used as a means of solving arithmetic problems may eventually constrain their mathematical development and skill. However, boys' preference for cognitive strategies and higher fluency may support boys' higher mathematics performance.

Gallagher and De Lisi (1994) examined the gender difference in solution strategies and mathematical performance of high school students' with the help of Scholastic Aptitude Test for Mathematics (SAT-M) problems. They classified the SAT-M problems into conventional problems and unconventional problems based on solution strategies. Conventional problems were those that could be answered only by primarily algorithmic methods. These problems were examples of routine textbook problems. Unconventional problems were those that either required the use of an atypical solution strategy, such as logical reasoning, insights or estimation. They reported that male and female students did not differ in overall mathematical performance; however, gender difference was significant for conventional problems but was not significant for unconventional problems. Female students used conventional strategies significantly more often than male students and male students used unconventional strategies significantly more often than female students. The findings of this study were partially supported by several other studies (Haciomeroglu & Chicken, 2012; Haciomeroglu, Chicken, & Dixon, 2013; Galindo, 1994).

Following the Gallagher and De Lisi (1994) study, Gallagher, De Lisi, Holst, McGillicuddy-De Lisi, Morely, and Cahalan (2000) conducted multiple studies for junior and senior high school students where they examined gender difference in solution strategies and performance with the help of multiple-choice and free response format questions. They reported that in multiple choice conditions, female students were more successful with conventional than with unconventional problems; however, in free response-response conditions male students were more successful with conventional than unconventional problems. Female students' performance was lower than male students' performance on conventional problems. They further reported that performance success rates between conventional and unconventional problems were significantly greater in the longer time condition. The timing condition did not affect significantly on gender.

Fennema, Carpenter, Jacobs, Franke, and Levi (1998) examined gender differences in young children's mathematical thinking and their solution strategies. They focused on operations of basic fact of numbers. They found that no gender difference in solving number fact, addition/subtraction, or nonroutine problems; however, gender differences were noted in solution strategies. Girls tended to use more concrete strategies such as counting and boys tended to use more abstract strategies, which was consistent with findings of Gallagher and De Lisi (1994). Similarly, a meta-analysis conducted on gender differences by Hyde, Fennema, and Lamon (1990) reported that there was no gender difference in arithmetic or algebra performance; however, males' geometry performance was slightly higher than females' geometry performance. They further found that gender difference was greatest in a test with mixed content. They also investigated students' cognitive levels, their Socio Economic Status (SES), and age regarding gender difference. Hyde, Fennema, and Lamon (1990) found that there were significant gender differences existing in students' cognitive levels, ethnicity, and age.

Representation

Representation is an important topic for this study because students' preferences for solution methods require various types of representations. Students use different types of representational systems while attempting geometry problems. Algebraic, numeric, and verbal representation are associated with nonvisual solution methods, whereas graphic representation is linked with visual solution methods. Thus, verbalizers employ particularly algebraic, numeric, and verbal representations because they prefer to use nonvisual solution methods. However, visualizers primarily utilize the graphic representation since they prefer to employ visual solution methods. Harmonic prefer to use both visual and nonvisual solution methods. Students also constantly change the modes of representation based on the nature of problems and their preferences.

Students translate one representation to another based on their preferences for solution methods (Lesh, Post, & Behr, 1987). For example, graphic representation includes pictures and diagrams. However, students who prefer to use nonvisual solution methods will translate graphic representations, for example, to algebraic representations to solve the problems. For the purpose of this study, when students employ graphic representation while attempting geometry problems, it is considered to be a visual solution method, and when they use algebraic, numeric, or verbal representation, it is pertinent to the nonvisual solution method. Thus, what kind of representation students use while attempting a geometry problem is important for this study because the use of representation is associated with visual and nonvisual solution methods. Moreover, it is also a crucial factor for the teaching and learning of mathematics (Vergnaud, 1987) and has gained significant importance in recent decades (Ozgun-Koca, 1998). Many educators, psychologists, and researchers have defined, explained, and discussed the various aspects of representation in relation to the teaching and learning of mathematics. In this section, representational systems will be briefly discussed in light of their types, nature, and translation processes.

The meaning and interpretation of representation is not consistent and uniform. Various types of definitions and descriptions are attributed to the notion of representation, particularly in the teaching and learning of mathematics (Zazkis & Liljedahl, 2004), because the meaning and interpretation of representation depends on mathematical context (Mesquita, 1998). For instance, Goldin (1998) used the term external representation; however, Lesh, Post and Behr (1987) used the term representation. Moreover, representation is a difficult concept, because it is not a static thing but a dynamic process that is associated with an individual's mathematical activities and mind (Vergnaud, 1998). In spite of the difference in naming, most of the researchers interpreted representation in a similar fashion.

Various distinctions have been made regarding the types, classifications, and nature of representation. Representation can be categorized as internal or external based upon whether the representation is formed inside the mind of an individual as mental imagery or expressed externally in the form of symbols, schemas, or graphs (Janvier, 1987). Various researchers discussed the distinction between external and internal representation (Goldin, 2001; Goldin & Shteingold, 2001; Goldin, 2003; Zhang, 1997). There is also controversy about the existence of internal representations because many scholars do not believe in the existence of internal representations (Goldin, 2003; Haciomeroglu, Aspinwall, & Presemeg, 2010). Moreover, it is very difficult to measure what's going on inside the head of an individual. Thus, this study focused only on students' external representation, therefore internal representation will not be described in this section. The term *representation* will be used for the purpose of this study instead of using external representation.

Kaput (1987) stated that mathematics is the study of the representation of one mathematical structure by another, and the focus is usually a determination of what structure is preserved in that representation. Thus several researchers have explained the nature, role, and types of representational systems. Goldin (2003) stated that representation is "A configuration of signs, characters, icons, or objects that can somehow stand for, or represent something else" (p.276). Goldin stressed the role of the configuration of signs, characters, icons, or objects in the representational system. He contended that the notion of representational system is scarcely meaningful without the configurations of signs, icons, and symbols. The symbols can be language (words and sentences).

Brinker (1996) defined representation, focusing on elementary school mathematical concepts. He stated that representation refers to students' notations and pictures, readymade drawings and fraction strips, and cuisenaire rods. Brinker's definition is more object oriented and limited to only concrete mathematical materials. In contrast, Cuoco's (2001) interpretation of representation covers a wide range of mathematics content. He affirmed that representation involves drawings, sketchings, markings, and writing algebraic equations.

Representation is classified in various categories based on nature, attributes, and modes. Janvier (1987) proposed four modes of representation: (a) verbal descriptive, (b) tablular, (c) graphic, and (d) formulaic (equation). Text, symbols, and sentences are ingredients of the verbal descriptive representation, whereas tables have a dominant role in tabular representation. Drawings, figures, and images are the main components of graphic representation. Similarly, formulas and equations are the major means of expressing mathematical ideas in formulaic representation.

Based on the existing literature and research, Lesh, Post, and Behr (1987) suggested five modes of representational systems in mathematics learning and problem solving: (a) real script model, (b) manipulative model, (c) static figural model, (d) spoken language, and (e) written symbol. The script model is experienced based in which knowledge is organized around the real world that serves as general context for interpreting and solving other kinds of problem situations. In the manipulative model, elements such as arithmetic bars, base-ten blocks, or similar manipulatives have little meaning intrinsically, but the built-in relationships and operations fit many everyday situations. The static figural model includes different types of pictures or diagrams that can be internalized as images during the teaching and learning of mathematics. The spoken languages include specialized languages and sublanguages related to domains like logic and reasoning. The written symbols refer to varieties of mathematical symbols and equations, specialized sentences and phrases, and normal English sentences and phrases.

Miura (2001) classified the representational system based on classroom activities. she stated that there are two types of representations: instructional representation and cognitive representation. Larkin and Simon (1987) also described two types of representation: sentential and diagrammatic; however, their types are different from Miura's. The sentential representation refers to the expression of problems with the help of sentences. Furthermore, Larkin and Simon stated that diagrammatic representation preserves the information about topological and geometric relations among the components of the problem, while sentential representation does not. It seems that the sentential representation is associated with nonvisual solution methods and diagrammatic representation is associated with visual solution methods. Wadsworth (2004) described different types of representational systems based on children's mental development. The different representations include deferred imitation, symbolic play, drawing, mental imagery, and spoken languages. However, according to Piaget (1926), generally there are only two types of representation: symbols (pictures, tally marks etc.,) and signs (spoken words, written language, numerals, etc.,) that play a dominant role in the learning process of children.

Palmer (1978) proposed a different view about nature and classification of representational systems. He contended that representational systems involve two related but functionally separate entities. The two related entities are representing world and represented world. The function of representing world is to reflect some or all aspects of the represented world in some fashion. In the representing–represented framework, Palmer contended that the represented world can be modeled by the representing world. In so doing, however, every characteristic of the represented world would not necessarily be reflected by the representing world. An example is provided in Figure 1.

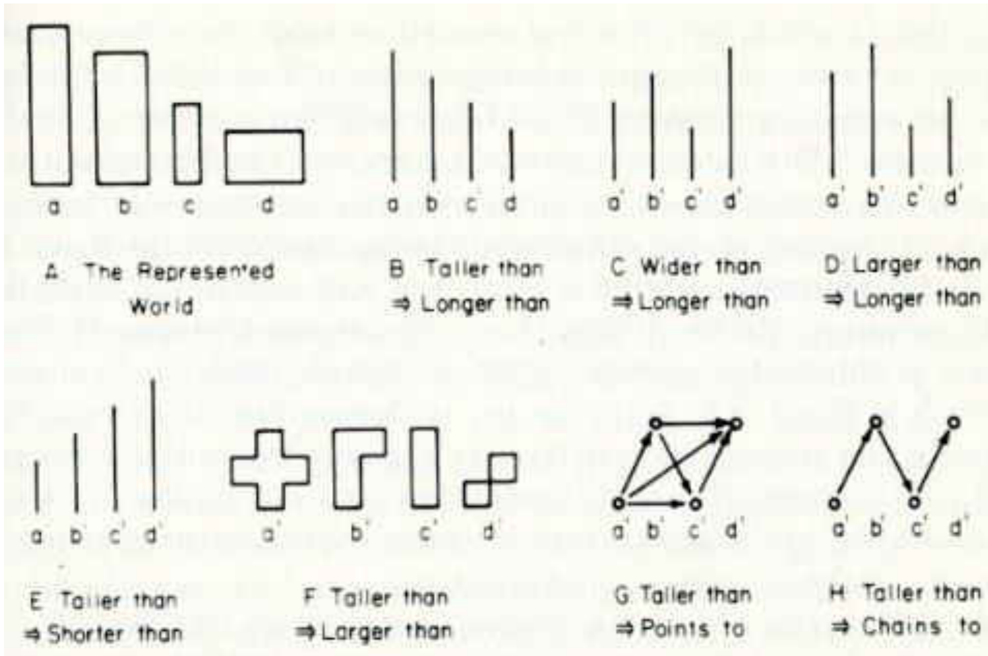


Figure 1: Examples of represented-representing world

From *Cognition and Categorization* by Palmer; E. Rosch, B. B. Lloyd, (Eds), 1978, p.263, Copyright, 1978 by Lawrence Erlbaum Associates

In this example, the represented world is the set of four rectangles as shown in Figure 2.1 (part A). The representing worlds B, C, and D show how different aspects of a same represented world can be modeled by representing worlds in different ways. Each vertical line with a different height in B is representing each rectangle of the represented world of A. World B reflects the relative height of the rectangles (a, b, c, d) of the represented world A by the relative lengths of corresponding lines (a', b', c', and d') . In fact, the representing world B models the height of rectangles in the represented world A in terms of line length; the taller the rectangle, the longer the line. However, between A and C, the wider the rectangles are in A, the taller the lines are in C. As described in the example, there must be some specific relationship or correspondence between the represented and representing worlds. In fact, all of these representing worlds in the Figure 1 are not the same. They contain some similar information about the world they represent.

The two worlds, represented and representing, consist of objects that are characterized by certain relationships that hold among them. In fact, the function of the representing world is to preserve information about the represented world as precisely as possible. Palmer (1978) further stated that there exists a correspondence (mapping) from objects in the represented world to objects in the representing world where at least some relationships in the represented world are structurally preserved in the representing world. For example, a world X is a representation of another world Y if at least some of the relations for objects of X are preserved by relations for corresponding objects in Y.

Following the represented–representing framework, Kaput (1987) classified the representation system into four broad and general categories: (a) cognitive and perceptual representation, (b) explanatory representation involving models, (c) representation within mathematics, and (d) external symbolic representation. He further explained the different types of representational systems within mathematics. Some of the common representations that Kaput explained include morphisms, generic algebraic constructions, canonical building-block constructions, approximation, feature/property isolation, and logic models. The different types of representation that Kaput described are more focused, however, on representation of abstract mathematics. Additionally, his classification is oriented to represent one mathematical concept with the aid of some sort of mathematical mappings or correspondence. Thus, for the purpose of this study, Kaput's classification of representation has limited scope because the representation he described may not be applicable to geometry.

A representation, so called *rule of the three,* includes three types of representations: symbolic, graphic, and numeric. Normally, mathematical ideas and concepts, particularly in calculus, can be presented with the help of these three types of representation (Gleason & Hallett, 1992). The rule of three, however, is not enough to grasp the various mathematical ideas and concepts. Thus, the rule of three becomes a rule of four. According to the rule of four, mathematical contents can be presented or expressed by using four modes of representation: graphic, numeric, algebraic, and verbal. The graphic representation includes pictures, diagrams, coordinate planes, and other figural representations. The numeric representation refers to displaying data or mathematical ideas and concepts in an organized fashion, possibly in an ordered list or in a table. The algebraic representation indicates the use of symbol and formula. The verbal representation includes written and spoken languages.

Following the work of Denis and Dubious (1976), Janvier (1987c) interpreted representation in three different ways: (a) representation refers to some material organization of symbols such as diagrams, graphs, schema etc., which denotes other entities or modalizes various mental processes; (b) it implies a certain organization of knowledge in the human mental system or in long-term memory; and (c) it also refers to a mental image. Janvier, however, did not make a distinction between actual material objects and mental images.

Goldin (1987) stated that representation systems consist of a collection of elements called characters or signs. He described the cognitive representation system in conjunction with mathematical problem solving, where the higher level structure and language are associated with the representational system. The higher level structures or languages include rules for forming configurations of configurations, networks of configurations, relations on the configurations, rules for assigning values to configurations, and operations on the collection of configurations.

The configuration is the set of words, characters, or symbols. He proposed a model for competence in mathematical problem solving based on five higher level languages: (a) a verbal/syntactic system, (b) a nonverbal system for imagistic, (c) a formal notation system of representation, (d) a planning language, and (e) an affective system that monitors and evaluates problem-solving progress. The main feature of this model is shown in Figure 2.

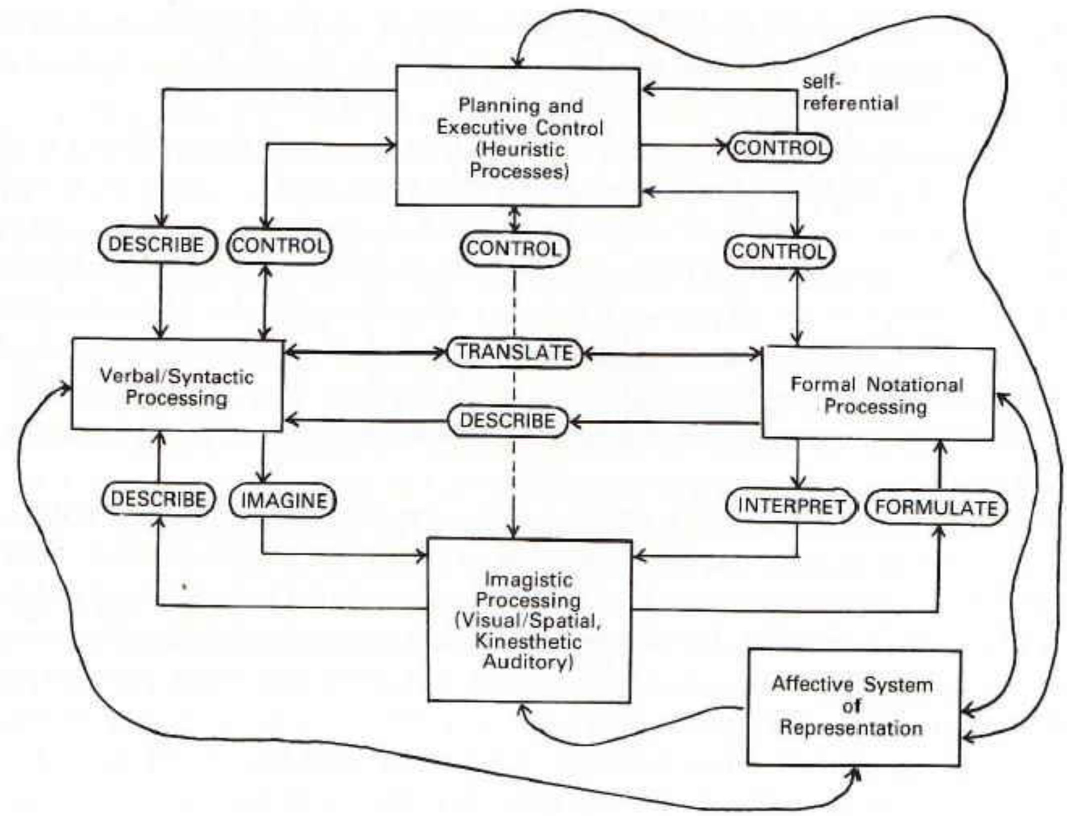


Figure 2: A model for competency in mathematical problem solving

From *Problem of Representation in the Teaching and Learning of Mathematics* by C. Janvier, 1987, p. 136, Copyright, 1987 by Lawrence Erlbaum Associates

In this model, we can see five representational systems. A verbal/syntactic system of representation can be described by means of signs, which are words and punctuation marks, together with correspondence between written and spoken words, rules for tagging by parts of speech and grammatical rules for combining words. An imagistic system of representation includes visual-spatial, kinesthetic, and auditory systems. A formal notational system includes the ability to use the notations conventionally described as the language of mathematics, and it also includes knowledge of how to represent a problem state and move from one state to another in non-standard problems. For example, it includes numeration and algebraic notations and rules for manipulating them. The planning and executive control includes four dimensions with respect to which sub-process is involved in their use. It guides problem solving, including strategic thinking, heuristics, and metacognitive capabilities. The affective representational system indicates the states of feeling that a problem solver experiences and expresses while solving a problem. Students may employ various representational systems mentioned above while doing geometry problems. For instance, students may use verbal processing and convert it into visual form by using imagistic processing, or they might convert visual (imagistic processing) forms into formula by using in the formal notational processing.

Researcher's View

The review on representational systems shows that the definitions, meanings, and interpretations of representational systems are not uniform. Moreover, disparities also exist in the categorization and classifications of representational systems. Various authors and scholars propose different ideas and concepts regarding its nature, interpretations, and classifications.

The author primarily advocates Janvier's (1987a) classification of the external representation where he classified the representation system into four classes: verbal descriptive, tabular, graphic, and formulaic. Janvier's classification of representation is similar to the rule of four. As a reminder, the rule of four includes graphic, numeric, algebraic, and verbal representations. The only distinct differences between these classifications are numeric and table. However, the rest of the modes of representation are similar. The researcher believes that tabular representation can be included in the graphic representation because graphic representation contains diagrams, figures, pictures, and also tables. Table can be considered also as a figure. Thus, for the purpose of this study, the researcher supports the rule of four representational systems. Advocating the rule of four implies participants in this study will use one or more than one mode of representation while attempting the geometry problems. Employing numeric, algebraic, and verbal representations while attempting geometry problems are considered to be nonvisual solution methods for the purpose of this study. In contrast, using a graphic representation while attempting geometry tasks will be taken as a visual solution method.

Translations between Representational Systems

Translation of geometry problems while solving from one mode of representation to another is important for this study because geometry performance also depends on students' translation (dis)abilities (Lesh, Post, & Behr, 1987). Translation ability refers to the psychological process involved in going from one mode of representation to another, for example from graphic to algebraic representation (Janvier, 1987). Most researchers agree that translation ability is very important for learning and problem solving in mathematics because translation of one mode of representation to another will provide flexibility to problem solvers while attempting mathematics problems (Doufour-Janvier, Bednarz, & Belanger, 1987; Gagatsis & Shiakalli, 2004; Hitt, 1998; Janvier, 1987; Lesh, Post, & Behr, 1987). Lesh, Post and Behr (1987) stated that:

> Good problem solvers tend to be sufficiently flexible in their use of a variety of relevant representational systems that they instinctively switch to the most convenient representation to emphasize at any given point in the solution process (p. 38).

Moreover, using different types of representation often illuminates different aspects of complex mathematical ideas or relationships (NCTM, 2000). Thus, it is important to develop skills in students so they can translate one representation to another based on the nature and situation of mathematics ideas, concepts, or tasks. Following the work of Behr, Lesh, Post, and Wachsmuth (1985), Lesh, Post, and Behr (1987) stated that translations (dis)abilities are significant factors that influence problem-solving performance, and these abilities facilitate the acquisition and use of elementary mathematical ideas. Thus, a translation process between representational systems and the ability to transfer within them is an important process for effective learning and the acquisition of successful problem-solving skills (Lesh et al., 1987).

Janvier (1987) described the translation process between the four modes of representations as shown in Figure 3. In the figure we can see that there are translations between the several modes of representations. For example, verbal representation can be translated into tabular and graphic representations, respectively, by creating a table of measurements and sketching a graph. Similarly, graphic representation can be translated into verbal by interpreting the information that is given in graphic representations.

TRANSLATION PROCESSES

To / From	Situations, Verbal Description	Tables	Graphs	Formulae
Situations, Verbal Description		Measuring	Sketching	Modelling
Tables	Reading		Plotting	Fitting
Graphs	Interpretation	Reading off		Curve fitting
Formulae	Parameter Recognition	Computing	Sketching	

Figure 3: Translation process among four modes of representations

From *Problem of Representation in the Teaching and Learning of Mathematics* by C. Janvier, 1987, p.28, Copyright, 1987 by Lawrence Erlbaum Associates

While solving problems from the geometry test (see appendix A), students may translate the problems into graphic, algebraic, numeric, or verbal representation based on their preferences for solution methods to solve the problems. Consider the following problem:

> From a ship on the sea at night, the captain can see three lighthouses and can measure the angles between them. If the captain knows the positions of the light houses from a map, can the caption determine the position of the ship (NCTM, 2000, p. 69)?

This problem can be translated into a graphic representation. In the graphic representation, the ship and the lighthouses become points in the plane. In order to solve the problem, students do not necessarily need to know about a graphic representation of the ship and the lighthouses because they might solve it by analytical reasoning using algebraic and or numeric representation, which is a nonvisual solution method. However, if a student prefers to use a visual solution method, then he/she needs the graphic representation of the lighthouse problem. In this situation, students need to be able to translate from verbal to graphic representation.

Whether students are visual or nonvisual learners, it would be useful to learn the translation process from one representation to another, which provides students with the flexibility to understand mathematical ideas and concepts effectively.

Importance of Representation

Mathematics, especially geometry, is based on the system of representation. Students employ different types of modes of representation while attempting mathematics problems. Whether students are verbalizers or visualizers, they need representation to solve mathematics problems. For example, visualizers prefer to employ graphic representation and verbalizers prefer to use algebraic representation. Moreover, mathematics teachers would hardly think of teaching geometry without using some kind of representations as pedagogical strategies. Kaput (1987b) stated that “representation and symbolization are the heart of the content of mathematics and are simultaneously at the heart of cognitions associated with mathematical activity” (p. 22). Geometry is even more a visual subject because it deals with different types of figures and diagrams, which are of fundamental importance in teaching and learning geometry (Niven, 1987). The fact is that most textbooks make use of a wide variety of representation with the goal of enhance understanding and learning of mathematics.

The National Council of Teachers of Mathematics (NCTM, 2000) described the importance of representation in the book *Principles and Standards for School Mathematics*. It states:

> Instructional programs from pre-kindergarten through grade 12 should enable all students to: (a) create and use representations to organize, record, and communicate mathematical ideas, (b) select, apply, and translate among mathematical representations

to solve problems, and (c) use representations to model and interpret physical, social, and mathematical phenomena. (p. 67)

Asli (1998) explained that the representational system has an important role in presenting problems to students and solving problems by students. Dufour-Janvier, Bednarz, and Belanger (1987) identified several reasons for tactical use of representational systems in teaching and learning mathematics (p. 110):

- Representations are an inherent part of mathematics,
- Representations are multiple concretizations of a concept,
- Representations are used locally to mitigate certain difficulties,
- Representations are intended to make mathematics more attractive and interesting.

Translation of one mode of representation to another is useful for students to learn because translation processes are essential tools for communication and reasoning about concepts and information in mathematics, and help to conceptualize the real world problem with the help of representations (Greeno & Hall, 1997; Vergnaud, 1987). For instance, students may use graphic representation although a geometry problem given using verbal representation Thus, one can argue that the more translation skills students possess the more they become successful in solving mathematics problems.

Summary

This chapter described various studies as they relate to students' preference for solution methods, task difficulty, mathematics performance, and their gender. There is not a consensus regarding the relationships between preference for solution methods and mathematics performance. Some studies found a significant relationship between preference and performance, while others reported no correlation between these two variables. Similarly, regarding the gender differences on preference for solution methods and mathematics performance, no studies derived the same conclusions. Some studies reported that there was a significant effect of gender on preference for solution methods and students' mathematical performance, while others found that gender differences prevail either only on preference for solution methods or mathematical performance. A majority of research studies reported that male students outperformed female students in mathematics performance. However, some studies reported that female students outperformed male students, and a few studies also found males and females did not differ in mathematics achievement.

In the domain of mathematics, four types of representation, graphic, numeric, algebraic, and verbal are employed in teaching and learning mathematics. While solving mathematics problems, students may translate problems from one mode of representation to the other based on their preference for solution methods to solve the problems. Translation ability is an important factor for learning and problem solving in mathematics because translation of one mode of representation to another will provide flexibility to problem solvers while attempting mathematics problems.

CHAPTER THREE: RESEARCH METHODOLOGY

Research Design and Method

A quantitative research design was chosen for this study. In a quantitative research design, the potential subjects are naturally embedded in a large group or setting, for example students in a class or in a school (Campbell & Stanley, 1963). Quantitative methods focus on objective measurement and numerical analysis of data collection through instruments, surveys, or polls. In a quantitative research design, the researcher answers a research questions by establishing the overall tendency of responses from individuals and notes how the tendency varies (Creswell, 2007). This study also has a causal-comparative design. This design generally involves pre-existing groups of participants, and often the variables that are examined in causal-comparative designs cannot be experimentally manipulated, for example, gender. Thus, there were no control and experimental groups in the research design.

Data Collection

Population, Sample, and Participants

Patten (2004) suggested that obtaining an unbiased sample is the main criterion when evaluating the adequacy of a sample, which can be determined by using Krejcie and Morgan’s (1970) statistical formula. Many research studies employ a convenience sampling procedure because the researchers have access to students in a school, customers of a business, or patients in a hospital (Schreiber & Asner-Self, 2011). The researcher of this study had access to certain schools. Thus, convenience sampling was employed to select participants for this study.

The researcher was working as a research assistant on the Geometry Professional Series (GPS) program for high school geometry teachers. The professional development was focused on improving the teachers’ depth of knowledge in relation to geometry topics and their real world applications. The topics were covered through discovery learning with the goal of improving the participants' depth of conceptual knowledge and providing strategies for incorporating the Standards for Mathematical Practice from the CCSSM into their mathematics lessons. Teachers were encouraged to integrate technology in their lesson activities. There were two cohorts in the GPS. There were 38 teachers in the first group in the school year of 2012-2013 and 35 in the second cohort in the school year of 2013-2014. However, during the time of this study, the first cohort completed the professional development series and cohort two was enrolled in the professional development series. Thus, the researcher only talked with most of the teachers in the second cohort about his research study and asked whether they could help to collect data for the study. And then, the researcher also communicated with teachers about the research study via emails. However, some teachers did not show interest to participate in the study. The researcher chose the first nine teachers, from six different schools, who were interested to help collect data in their classrooms for this study. The expected sample size for this study was 150 students and the number of students that had been taught by nine teachers was more than 150 students. Thus, when the researcher ensured that there were enough numbers of students, he did not have to go to other school and teachers who still were interested to help conduct this study.

The students of this study consisted of a population representative of the high school's population with respect to the proportions of compositions of gender and ethnicity. The sample consisted of 161 students whose ages ranged from 14 to 19. A total of 41% of the students were male, and 59% were female. The students also consisted of various ethnicities. Of the students, 24% were White, 37% Hispanic, 26% African American, 2.5% Asian or Pacific Islander, and 6.8% Multiracial. The breakdown for the percentage of ethnicity is listed in Table 1.

Table 1: Descriptive statistics of ethnicity

Ethnicity	*Frequency*	*Percent*
White	39	24.2
Hispanic	60	37.3
African American	41	25.5
Asian or Pacific Islander	4	2.5
Native American	1	.6
Multiracial	11	6.8
Other	3	1.9

A total of 6.8 % of the participants were between the ages of 14 and 15, 54% were between the ages of 16 and 17, and 38% were 18 and above. Eight teachers were involved from six different schools. The students were in a range of grades. Of the total students, 18.6% were from grade 10, 47.2% from grade 11, and 34.2% from grade 12. The participants were enrolled in different mathematics courses during the 2013–2014 school year. Of the total test population, 67.6% were enrolled in Algebra 2, 5% in Geometry, 19.3% in Financial Algebra, and 8.1% in Pre-calculus. Table 2 illustrates descriptive statistics of subject and grades.

Table 2: Descriptive statistics of subjects and grades

Subjects	**Students**	**Percent**	**Grades**	**Students**	**Percent**
Algebra 2	109	67.6%	Ten	30	18.6 %
Regular Geometry	8	5%	Eleven	76	47.2%
Financial Algebra	31	19.3%	Twelve	55	34.2%
Precalculus	13	8.1%			

Table 2 showed that only 5% of the students were enrolled in regular geometry. The school district made a change of course sequence so a limited number of students enrolled in geometry. In fact, in the school year of 2013-2014, a geometry course was not offered in almost all high schools from where that data was collected. Thus, there were only 5% of the total students enrolled during the time of the study.

Procedure

The data were collected from high schools at a county located in Florida in the United States. The study was conducted during the 2013–2014 school year. The geometry test and the geometry questionnaire were used to collect data for all 161 students. Upon completion of the geometry test, students were given the geometry questionnaire.

The test was conducted in a regular classroom during school time. The normal time interval of most of the classes was 52 minutes. Normally, students took a class period to complete the test. The researcher clearly described the geometry test and corresponding geometry questionnaire. The researcher also displayed an example of a geometry problem on chart paper that was solved in different ways similar to the geometry problems that were solved in the geometry questionnaire. Moreover, the researcher also explained that students were allowed to use a calculator, a ruler, scratch paper, etc., but not a reference sheet (formula sheet). (While taking the test, many participants still asked the researcher whether they were allowed to use the reference sheet.) When students finished the geometry test, the test was collected; students were then provided with the geometry questionnaires to complete. There was a variation in the time taken to complete the test. The majority of students used the entire time to work on the geometry test and the geometry questionnaire. Some students, however, finished (or gave up) the geometry test in 10-15 minutes. In general, the first 30-35 minutes were used to complete the geometry test and the remaining 15 minutes were utilized to complete the geometry questionnaire. The researcher also explained to the participants that even if they were unable to solve the geometry problems, they could still choose the solution methods that were a best fit for them from the list provided in the geometry questionnaire. Participants' demographic information relevant to this study, such as age, gender, etc., was also collected. For more information, please look at the first page of the geometry test in Appendix A.

Some classes were in a block schedule (90 minutes); others were in a regular schedule (52 minutes). Participating teachers who were in the block schedule started their lessons when students finished the test. It was noted during administering the test that all but two students were not be able to finish the geometry test and the geometry questionnaire in a regular class. In fact, the two participants did finish the test but were not able to complete the geometry questionnaire. Thus, the researcher asked them to complete the packet at home and return it to their teachers. During the test there were no time-related issues i.e., students completed the geometry test and geometry questionnaire within 52 minutes.

Instrument

A geometry test and a geometry questionnaire were used to collect quantitative data. The geometry test contained 12 geometry problems from different topics of high school geometry. Students were required to show their work while completing the geometry problems. The geometry questionnaire contained different types of solution methods of each problem in the geometry test. Upon completion of the geometry test, students were given the geometry questionnaire and asked to choose the solution methods from the list that best described the solution methods they employed to complete geometry problems. The geometry test and the geometry questionnaire were designed to measure students' preferences for solution methods as well as the geometry knowledge and skills they had already been taught. The researcher adopted the first six geometry problems from Battista's instrument; the rest of them were designed and developed based on the existing literature. For more details about the geometry test and geometry questionnaire, please see appendices A and B respectively.

Approximately 2-3 weeks after administering the geometry test, the researcher conducted short interviews to further explore students' preference for solution methods. It turned out to be difficult to conduct interviews with all participants. Moreover, since this was primarily a quantitative study, interviews for all subjects were not strictly necessary. Thus, the researcher chose only 17 students for a short interview in order to further explore the solution methods they used while solving the geometry problems. Typically, the audiotaped interview lasted 2 to 3 minutes.

Students were selected from each school to represent all schools where quantitative data were collected. From the list of names of all students who took the geometry test and the geometry questionnaire, the researcher requested participating teachers to provide the names of a couple of students for a short interview. Thus, participating teachers selected some students from their class for the interview. The researcher did not ask participating teachers how they chose their students for interview. Thus, it was not clear how participating teachers selected their students as it relates to the procedure of selection of students for the interview. In most cases, interviews were conducted in a corner of a regular classroom; however, in some cases interviews were conducted outside of the classroom, such as in a hallway or corridor of a school building.

Three problems from the geometry test— numbers 1, 4, and 8—were chosen as the basis for the interviews. A hard copy of the questions was also provided. The researcher decided to choose these three problems after preliminary analysis of the geometry test and the geometry questionnaire. Preliminary analysis of the data revealed that most of the participants chose visual solution methods while solving the geometry problems. The researcher chose three types of problems based on participants' preference for solution methods: a problem for which most students used a visual solution method, a problem for which students used visual as well as nonvisual solution methods, and a problem for which the majority of students used a nonvisual solution method.

The "think aloud" method was used to conduct interviews. Students were asked to explain their solution methods aloud so that the researcher could have the opportunity to understand their preference for solution methods. "In think aloud method the subject is asked to talk aloud, while solving a problem, and this request is repeated if necessary during the problem-solving process thus encouraging the subject to tell what he or she is thinking" (Someren, Barnard, & Sandberg, 1994, p. 25). The analysis of the audiotaped interview was carried out in a number of steps. The audiotaped interviews were carefully transcribed word by word. Glesne (2011) recommended that researcher's start a codebook soon after the data collection starts. The researcher kept track of all the data collected; however, the codebook was developed during data analysis. In fact, coding is a progressive process of sorting and defining, and defining and sorting of collected data (Glesne, 2011).

Development of Geometry Tests

Developing and designing an appropriate instrument for a research study is not easy because various aspects, such as reliability, validity, content, and standard of the items of the research instrument, are always open to comments and criticism. Even selecting a reliable instrument to measure mathematical problem-solving performance is a difficult task (Moses, 1977). Thus, the researcher has attempted to find suitable preexisting research instruments. The Mathematical Processing Instrument (MPI) developed by Suwarsono (1982) and geometry problem-solving strategies designed by Battista (1990) are the only closely related instruments for this study. The MPI, however, consists of algebra word problems and would not be an appropriate research instrument to measure students' preferences for geometry problems (Haciomeroglu et al., 2013). Battista's (1990) "Geometry Problem Solving/Strategies" was a closer fit. Battista's test consisted of 12 problems dealing with finding midpoints, determining specified distances in two and three dimensions, and so forth. The Suwarsono and Battista instruments provided very important insights and ideas useful in designing the geometry test and questionnaire for this study.

Battista's geometry instrument (problem-solving strategies) contains 12 geometry problems; however, for this study only six problems were chosen from his instrument. One of the main aims of the geometry test was to distinguish between students' preference for visual and nonvisual solution methods. Thus, if the problems from Battista's instrument clearly appeared not to have two easily accessible solution methods, they were not included in the geometry test. Similarly, another important factor for selecting only specific problems from Battista's instrument was to make sure that the potential solution methods were distinct and non-overlapping. If the problems had two solution methods but the two solution methods seemed to overlap, then the problems were not included in the geometry test.

Problem number 11 was designed based on an example provided in Common Core State Standards for Mathematics (CCSSM, 2010). Problem number 12 was chosen from a chapter of a book written by Blair and Canada (2009) and published by the National Council of Teachers of Mathematics (2009). The rest—problems 7, 8, 9, and 10—were developed and designed by the researcher. The researcher also compared the geometry problems with content standards of CCSSM. Geometry content covered by the geometry test was included in various sections of middle and high school geometry in the CCSSM. Problem one appeared to belong to grade 6 (NS-number system). Problem three (F-function), five (G-geometry), and eight (EE-expressions and equations) are closely aligned with content for 7th and 8th grades. The rest of the problems belong to high school geometry. Different content areas, such as Congruency (CO), Circle (C), Similarity, Right Triangle, and Trigonometry (SRT), Geometric Properties with Equation (GPE), and Geometric Measurement with Dimension (GMD) were covered by the geometry test which is aligned with the CCSSM. An overview of the coverage and content of the geometry test and its relation to CCSSM is provided in Table 3.

Table 3: Source of test item, content coverage, and relation to CCSSM

Problem	**Sources (taken/adapted)**	**Geometry content coverage**	**CCSSM**
1	Battista, M. T (1990)	Integers on a number line	6-NS
2	Battista, M. T (1990)	Similarity, right triangle and trigonometry	G-SRT
3	Battista, M. T (1990)	Coordinate geometry	8-F
4	Battista, M. T (1990)	Circle	G-C
5	Battista, M. T (1990)	Area and perimeter of rectangle	7-EE
6	Battista, M. T (1990)	Surface area and volume of 3D objects	G-GMD
7	Designed	Congruence: prove geometric theorem	G-CO
8	Designed	Coordinate geometry	8-EE,5
9	Designed	Transformations	G-CO
10	Designed	Coordinate geometry	G-GPE
11	CCSSM (2010)	Geometric properties with equations	G-GPE

Problem	Sources (taken/adapted)	Geometry content coverage	CCSSM
12	NCTM (2009)	Circle	G-C

The geometry test that was intended as a research instrument for the purpose of this study was an achievement test rather than an aptitude test. An achievement test is designed to measure what somebody has already learned, whereas an aptitude test is designed to determine a learner's potential for learning new information or skills (Friedenberg, 1995). The geometry test and the geometry questionnaire were designed to measure students' preferences for solution methods as well as the geometry knowledge and skills they had already been taught. However, it was not possible to include questions from each topic of high school geometry because there would have been too many questions on the test.

Another important criterion for designing and developing this test was whether the problems could be solved by using visual and nonvisual solution methods. Some topics in geometry do not lend themselves to both visual and nonvisual approaches. Thus, the researcher decided to design the questions to cover as many topics as possible from high school geometry.

Domino (2000) explained eight steps that researchers would need to think about before designing a test. They include the role of theory, practical choices, pool of items, tryouts, and refinements. Various types of tests, such as multiple-choice, true or false, and fill-in-the-blank can be designed based on the purpose and nature of the research study. In this study students were asked to solve the problems and show their work. Domino (2000) mentioned various advantages of multiple-choice items, such as the fact that they can be administered in a short interval of time, can be scored quickly and inexpensively, and can be easy to analyze; however, the researcher did not use a multiple-choice test. The fact is that this study aimed to investigate how students prefer to think and process mathematical information; thus, with the geometry test, students were required to show their work on paper while solving the problems. Doing so gave the researcher an opportunity to see students' preference for solution methods or strategies in addition to evaluating an answer as correct or incorrect.

Based on the existing literature about the mode of representation as well as theoretical and empirical evidence, verbal representations were used in the presentation of items on the geometry test. Similar to Suwarsono's MPI, the geometry test also has two parts. The first part includes 12 geometry problems suitable for high school students. The second part is a questionnaire consisting of visual, nonvisual, and other solution methods for each task. In the second part, students were asked to choose the solution method(s) from the given list that best described their solution method. If students came up with different solution methods that were not listed on the geometry questionnaire, they were asked for a description of their methods.

The researcher decided to design some of the problems of the geometry test based on the high school geometry curriculum for several reasons. First, high school students can respond to the questionnaire in more explicit ways than could students from elementary or middle school. Second, the visual solution methods include drawings and figures with and without coordinate axes, and the study of coordinate geometry is the best fit for high school students. In addition to this, research studies show that the gender difference in mathematical performance is almost unnoticeable in the primary grades; in the upper grade it becomes quite marked (Gallagher & De Lisi, 1994; Hyde, Fennema, & Lamon, 1990; Krutetskii, 1976; Steele, 2003).

Task Difficulty

The geometry test contained 12 geometry problems on various topics in high school geometry. The problems differed in level of difficulty based on whether they required few steps and simple calculations or multiple steps and rigorous thinking to solve them. Generally, test items that require more steps to solve are more difficult than test items that require fewer steps (Cheng, 2006).

One of the aims of this study was to examine the relationship between students' preference for solution methods and task difficulty. Thus, the researcher divided the geometry problems into three categories: easy, moderate, and difficult. The researcher used the following criteria to make the distinction between easy, moderate, and difficult problems. If a problem did not require many steps to solve, then it was considered easy. Students did not have to think critically, and simple calculations and formulas would be enough to solve the easy problems. Easy problems did not require using geometry theorems. Moderate problems were not as straightforward and simple as easy problems. They needed more steps and required sound knowledge to solve them. The difficult problems required more rigorous and critical thinking. Students needed to use formulae as well as geometry theorems in order to solve the difficult problems. For more detail, please see Appendixes A and B for the geometry test and questionnaire, respectively. The researcher also discussed the geometry problems with some doctoral students (mathematics education track, University of Central Florida) to determine the degree of difficulty of the problems. Based on the criteria, the researcher categorized the geometry problems as shown in Table 4.

Table 4: Classification of geometry problems

Task difficulty	Problem number
Easy	1, 8, 9, 10
Moderate	3, 4, 5, 11
Difficult	2, 6, 7, 12

Although it would have been feasible to design a test containing items from various areas of geometry, the test would then be too long and impossible to administer due to time constraints. Thus, the researcher needed to make a decision as to what types of geometry problems should be included in the test. In this regard, the following criteria were used during the development of the geometry test as a research instrument:

- The problems should be suitable for high school students.
- The geometry tasks could be equally solvable in at least two different ways: visual and nonvisual.
- The geometry test needs to include problems of varying levels of difficulty: easy, moderate, and difficult.

The geometry test included 12 items of varying degrees of difficulty. Difficulty is defined in terms of the likelihood of a correct response, not in terms of the perceived difficulty or amount of effort required (Demars, 2010). From the research standpoint, classification of the geometry problems into easy, moderate, and difficult may not be scientific, because an easy problem for one student could be difficult for another. Thus, the researcher also took students' actual work into account as well as his/her knowledge to categorize the geometry problems into easy and difficult groups.

Geometry Performance

This study is centered on students' preference for solution method, their gender, and their geometry performance. The geometry test used to collect data in this study did not cover the entire content of the high school geometry curriculum. It must be noted that this test might not assess students' actual geometry performance. Thus, the researcher decided to use students' geometry performance based on standardized test scores. The End of Course (EOC) is a standardized assessment administered for the first time in 2012 in the state of Florida where the research had been carried out.

The Florida Department of Education (FLDOE) has implemented End of Course (EOC) assessments for certain courses administered at the middle and high school levels. The EOC is part of Florida's Next Generation Sunshine State Standards (NGSSS), which is designed to measure student achievement (content knowledge and skills) for specific courses outlined in the course descriptions (Florida Department of Education, 2012). Regardless of students' enrollment in different types of geometry courses in high school, there was only a single EOC assessment for all students.

The End of Course (EOC) assessment for geometry is designed to measure students' content knowledge and skills in three areas of geometry: two-dimensional geometry, three-dimensional geometry, and trigonometry and discrete mathematics. The computerized test is administered in one 160-minute session. Students are allowed to use hand-held four-function calculators and four pages of scratch paper. Additionally, students are also allowed to use a reference sheet (formula sheet) during the assessment. For a more detail about the reference sheet, please see Appendix D.

Participants' geometry EOC scores were gathered with the help of participating teachers. It is worthwhile to mention that while most students had 2013 scores, some students had no scores more recent than 2012. In fact, 65% of students had their EOC scores from 2013. This distribution implies that some students had not taken geometry courses for two years. EOC scores for geometry were reported in two different ways between 2012 and 2013. A *T*-scale score ranging from 20 to 80 was used to report students' geometry scores in 2012; the mean was a score of 50 and the standard deviation was 10. In 2013, however, actual scores were reported, ranging from 325 to 475.

The anticipated sample size for this study was 150 high school students. As earlier explained, participating students' grades ranged from 10 to 12, and their courses also ranged from algebra to pre-calculus. Though students were in same grade levels and courses assignment during the time of this study, some of them had taken geometry a year earlier while others had taken before two years. For example, students who were enrolled in financial geometry at grade 12, some had their EOC scores from 2012 while others had from 2013. Thus, the researcher had to look for their EOC scores over the last two years.

Participants' EOC score should be in the same scale for the purpose of statistical analysis. Thus, the researcher converted participants' 2013 actual EOC scores into *T*-scale scores for consistency. The raw score can be converted into a *Z* score, and then the Z score can be converted into a *T*-scale score as follows:

$$Z = \frac{x - \mu}{\sigma}$$ (*Z* score)

$$T = (10 * Z) + 50$$ (*T*-scale score)

The statistical software SPSS was used to convert the raw score into *T*-scale score.

Scoring of the Instrument

The easier the test items, the more likely that students got correct answers and vice versa. To analyze the task difficulty, the researcher quantified participants' work by assigning numeric values to the students' work on geometry problems. The problems differed in level of difficulty—easy, moderate, and difficult—based on whether they required few steps and simple calculations or multiple steps and rigorous thinking to solve them. However, regardless of different types of problems, the researcher used only two numeric values, one for correct answer and the other for incorrect answer, for the geometry problems. Students received one point (1) for the correct answer and zero points (0) for the incorrect answer. Thus, students could receive a minimum of zero points to a maximum of 12 points on the geometry test. If students did not solve a problem or skipped it, they received zero points.

The difficulty level of each problem in the geometry task was determined by how many students were able to solve the geometry problems correctly as well as the researcher's knowledge and experience of teaching and learning mathematics. The more participants able to solve the task correctly, the easier would the problem be. For example, 26% of the total participants were able to solve problem one correctly, while only 6.8% of participants were able to solve problem two correctly. Thus, Problem 1 was deemed easier than Problem 2. Table 5 delineates the task difficulty of the geometry test, showing the percentage of students getting correct answers on the geometry test.

Table 5: Task difficulty of the geometry test

Problems	1	2	3	4	5	6	7	8	9	10	11	12
Correct (%)	26	6.8	15.5	7.4	7.4	9.3	8	37.8	33.5	28	37.8	7.4

Each participant's geometry test and geometry questionnaire were analyzed simultaneously. The researcher recorded how many geometry problems students answered correctly and incorrectly. The solution method for each participant was also recorded. The data analysis indicated that students chose the same solution methods on the geometry questionnaire that they utilized to complete the geometry problems in the test. For these students, there was consistency between the solution methods they used to solve the problems and the solution methods they chose in the geometry questionnaire. However, there were some cases in which students used one method to solve the problems but chose different methods for those problems in the questionnaire, creating an inconsistency in response between the two instruments. Some participants clearly used visual solution methods while solving the problems in the geometry test, but they chose nonvisual solution methods in the geometry questionnaire. Similarly, some students used both visual and nonvisual solution methods while solving geometry problems but they chose only one solution method in the geometry questionnaire. Other students mentioned that they just guessed the solution method from the geometry questionnaire. Moreover, some cases were noted where students chose solution method four (Other Method) in the geometry questionnaire without explaining the solution method they employed in order to solve the problems. In solution method four, students were required to explain the solution method if they came up with different types of solution methods other than those provided in the geometry questionnaire.

The researcher analyzed the geometry test and the geometry questionnaire at the same time for every participant to ensure the accuracy between the actual solution methods they used to solve the problems and solution method they chose in the geometry questionnaire. In so doing, the researcher was able to see and verify the actual solution method participants used in the geometry test and the solution method they chose in the geometry questionnaire. Examples of students' work are given in figures 4 and 5.

PROBLEM 10

Find the distance between the points *P* (−6,1) and Q (2,1).

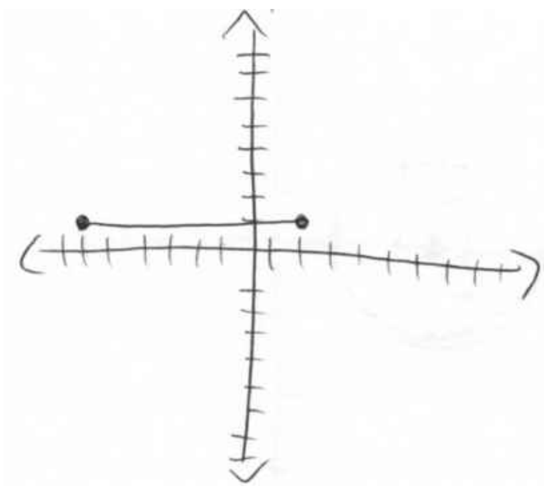


Figure 4: A visual solution of problem 10 (unedited)

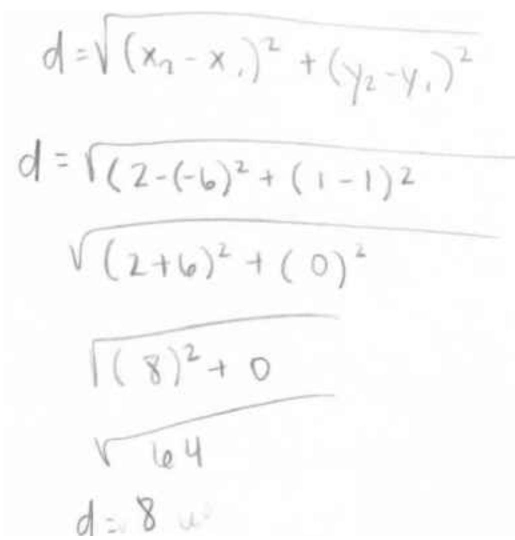


Figure 5: A nonvisual solution of problem 10 (unedited)

The researcher used the following criteria to address the various issues in connection with the students' actual solution methods for the geometry test and geometry questionnaire:

1. When students solved a problem using a visual solution method on the geometry test, for example, but chose the nonvisual solution method in the geometry questionnaire, the

researcher primarily relied on the geometry questionnaire to determine participants' preference for solution method.

2. When students solved a geometry problem but did not choose a solution method in the geometry questionnaire, their solution method was decided based on the geometry test as long as there was clear evidence as to the methods they used to solve the problems.
3. When students chose solution method four in the geometry questionnaire but did not describe their method, their solution method was determined based on the actual method they employed to solve the problems in the geometry test. If there was no clear evidence as to the solution method participants used to solve the problems, their solution methods received a score of zero.
4. When students solved a problem both ways—i.e., using both visual and nonvisual methods—their solution method was determined based on what solution method they chose from the geometry questionnaire.
5. When students solved a problem both ways—i.e., using visual and nonvisual methods—and they also chose both methods in the geometry questionnaire, their solution methods were considered harmonic.
6. When students chose solution method two (drawing) and solution method three (visualization) in the geometry questionnaire, their solution methods were considered to be visual.
7. When students mentioned in the geometry questionnaire that they simply guessed or did not know their solution method, and if there was also no clear evidence as to what solution method they employed in the geometry test, they were also placed in the

undecided group in regard to preference for solution methods and received a score of zero.

As previously stated, there are three types of students: visualizers, nonvisualizers, and harmonics. Visualizers use visual solution methods i.e., their solution methods are based purely on the diagrams, pictures, and figures. Nonvisualizers use nonvisual solution methods in which they employ arithmetic, algebra, or formulas to solve problems. Harmonic students use both visual and nonvisual solution methods.

For the purpose of statistical analysis, students' preferences for solution methods were also quantified into numeric values. To recap, students belong to one of three categories: (a) visualizers, who have a preference for the use of visual solution methods, including graphic representation (i.e. figures, diagrams, and pictures); (b) nonvisualizers (verbalizers), who have a preference for the use of nonvisual solution methods, which involve algebraic, numeric, and verbal representation; and (c) harmonic students, who use visual and verbal methods equally. Students' visuality score can be determined by their preferences for solution methods (i.e. how many geometry problems they solved using visual, nonvisual, or both methods). The visuality score was determined by adding students' visual, nonvisual, and harmonic scores.

Researchers used different scoring systems to measure visual and nonvisual solution methods and visual scores (Haciomeroglu & Chicken, 2011; Lean & Clements, 1981; Moses, 1977; Suwarsono, 1982). For example, Suwarsono gave plus two (+2) for visual solutions with the correct answer, plus one (+1) for visual solution with the incorrect answer, and zero (0) for no answer. He gave minus one (-1) for nonvisual solutions with the incorrect answer, minus two (-2) for nonvisual solutions with the correct answer. Similarly, Haciomeroglu and Chicken (2011) gave a score of zero (0) for nonvisual solution methods, two (2) for visual solution methods, and one (1) for using both methods. Moses (1977) gave a zero (0) for nonvisual solution method, one (1) for the solution method where both methods were manifested, and two (2) for visual solution methods.

The underlying reason for using different numeric values by different researchers was to differentiate the types of students based on the solution methods students used. In fact, the associated numeric values do not express the quantity; rather, they help to discriminate students' preferences for solution methods. Thus, for the purpose of this study, students were given a score one (1) for the visual solution method and negative one (-1) for the nonvisual solution method. A score of zero (0) was given if students did not choose their solution methods, chose both methods, or could not determine the solution methods they used. Students were placed into the harmonic group if they used both visual and nonvisual solution methods when completing the geometry test and the questionnaire and they also received a score of zero (0). Thus, for 12 items, an individual could obtain a 'nonvisual-visual' score ranging from -12 to +12.

An Overview of the Geometry Problems

The geometry test and the geometry questionnaire were designed to measure students' preferences for solution methods. The first six geometry problems were taken directly from Battista's (1990) research instrument (geometry problem solving/strategies); the remaining six problems were designed based on different criteria as stated earlier. The analysis of the data revealed that the geometry test appeared to be a difficult one. However, not all the problems were of the same degree of difficulty, with some easier than others. A brief overview of each problem is described below.

Problem 1

Problem one was directly taken from Battista's instrument. Originally, the researcher categorized this problem as being in the easy group. This problem deals with the concepts of integers in a number line. The introduction of the concept of integers is part of the middle and high school curriculum. However, this problem did not appear to be quite as easy as anticipated before conducting the study. The majority of the participants used visual solution methods to solve this problem. Though drawing a number line would be enough for this problem, the majority of the visualizers drew coordinate axes and tried to find the answer in terms of Cartesian coordinates, which led them to wrong answers. An example is given in figure 6**.** It must be remembered that the participants were from grade 10, 11, and 12, and coordinate geometry is one of the major topics in high school geometry, which might be a reason why so many students drew coordinate axes rather than a number line. Moreover, it had been a while since participants had learned the concept of integers in middle school. Therefore, they might have forgotten the concepts of integers that had they learned in middle school.

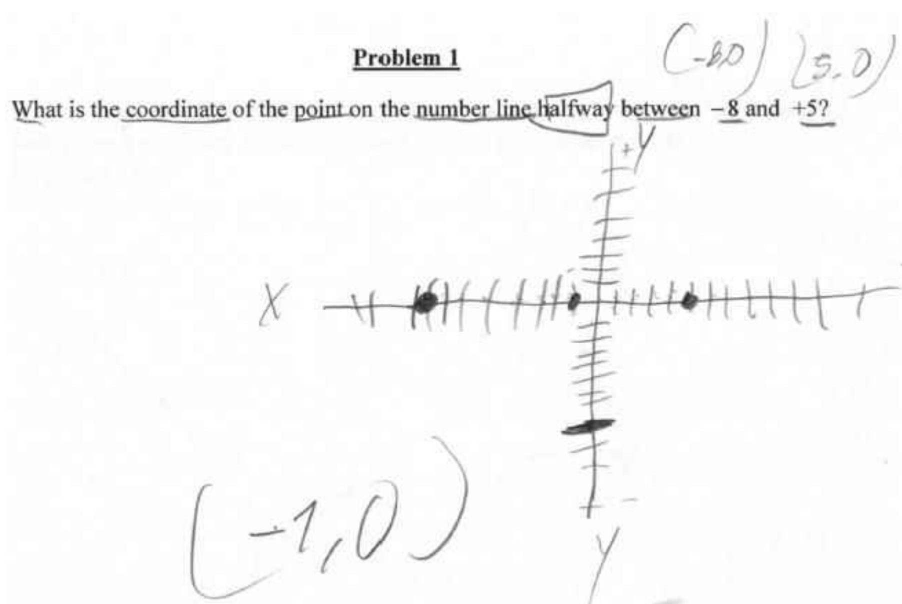


Figure 6: Example of use of coordinate axes

Problem 2

This problem was also adapted from Battista's instrument. The analysis of the participants' work unveiled that this was a difficult problem as anticipated. The majority of the participants used visual solution methods to solve this problem. Only very few students used the Pythagorean Theorem to solve this problem. Many of them directly used the numeric value given in the problem and used numeric ratios to solve it.

Problem 3

This problem was moderately difficult. The majority of the participants used visual solution methods to solve it. It appeared that most of the students were able to plot the given points on the coordinate system appropriately. However, all four points seemed to lie on the same straight line on the rough sketch. When students plotted the points on scratch paper, it was natural that the scaling of the coordinate system might not have been precise enough, which might have led students to the wrong conclusion regardless of whether they plotted the points correctly. The researcher believes that if these four points had been distinctly apart from each other, more students, particularly those in the visualizer group, could have done this problem correctly. An example of a participant's work is given in figure 7.

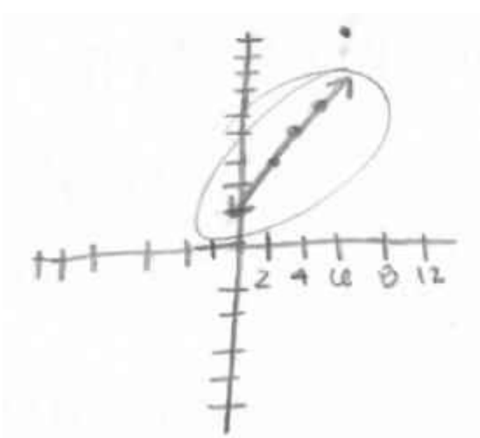


Figure 7: Example of student's work (unedited)

Problem 4

Problem 4 was adapted from Battista's (1990) research instrument. The researcher categorized it as a moderately difficult problem, a designation that was supported by the analysis of participants' work. Students used both visual and nonvisual solution methods equally to complete this problem.

Problem 5

Participants were asked to find the perimeter of a swimming pool in this problem, which was placed into the moderately difficult problem group. The majority of students used visual solution methods to find the answer to this problem. This problem also appeared to be a difficult one.

Problem 6

Problem 6 was the only problem for which graphic as well as verbal representations were used to present the problem. The majority of the participants used visual solution methods. In spite of the presence of a diagram, some students employed nonvisual solution methods. Participants' work indicated that this was a difficult problem.

Problem 7

Problem 7 was designed based on a geometry theorem. The majority of students used visual solution methods for this problem. An analysis of participants' work indicated that this was one of the most difficult problems of the geometry test.

Problem 8

Compared to the rest of the problems, the majority of students used nonvisual solution methods to solve problem 8. Moreover, many participants asked during the test whether they were allowed to use a reference sheet because they are allowed to use a references sheet while taking quizzes and the End of Course (EOC) assessments. The researcher believed that if participants were provided with the reference sheet, there would have been even more students using a nonvisual solution method. Problem 8 appeared to be a relatively easy problem compared to other problems in the geometry test.

Problem 9

This problem was designed based on transformation geometry. The majority of students used a visual solution method to solve this problem. This problem also appeared to be in the easy category. Very few students used a formula to complete this problem. Similar to problem 8, if students were provided with the reference sheet, there would have been even more students using a nonvisual solution method.

Problem 10

The researcher anticipated that this would be the easiest problem of the geometry test; however, that appeared not to be the case. Only 28% of students were able to do this problem correctly. Compared to other problems, this problem still belonged in the easy category. Like in problem 8, many participants asked whether they were allowed to use a reference sheet for this one, because they are allowed to use a reference sheet during quizzes and on the End of Course (EOC) assessments. It must be noted that many students mentioned that they did not know the distance formula. Furthermore, they explained that if they had known the distance formula, they could have done it. Providing formulae might be useful to solve this problem, particularly for nonvisual students.

Problem 11

Problem 11 was designed based on the high school geometry standard (Expressing Geometric properties with Equations G-GPE) of the Common Core State Standards for

Mathematics (CCSSM). Most of the participants used visual solution methods to solve this problem. Participants' work revealed that it was easier to solve this problem with a visual solution method. In order to solve this problem in a nonvisual way, participants were required to know the standard equation of the circle. It seemed that nonvisual solution methods required more steps and information.

The data also revealed that this problem was at a moderate level of difficulty. It is worthwhile to mention that Problem 11 was the only *yes/no* problem. Students could choose simply yes or no without doing any mathematics. Quite a few students chose their answer without showing any work in this problem. The researcher had to accept the participants' responses regardless of whether they showed their work or not.

Problem 12

This problem was adapted from a book published by the National Council of Teachers of Mathematics (2009). All participants used visual solution methods to solve this problem and many chose visual solution methods in the geometry questionnaire, too. However, a few students actually chose nonvisual solution methods in the geometry questionnaire, even though they had selected a visual solution method on this problem on the test. The analysis of participants' work indicated that this was a difficult problem. Many participants did not attempt this problem and mentioned that they had no idea how to solve it.

Reliability and Validity of the Instrument

Validity and reliability are important factors for research studies. Internal validity refers to the process of controlling variables within the study to ensure that the instrument examines what it is intended to measure (Shadish, Cook, & Campbell, 2002). With respect to internal validity, the researcher wanted to measure whether the test results truly indicated what they were supposed to measure: the students' preference for solution methods.

Campbell and Stanley (1963) stated that there are variables that can jeopardize the internal and external validity of any research instrument. However, the researcher tried to minimize the validity threats as much as possible. The researcher was concerned about the various issues pertaining to the design and development of the geometry test. Does the test measure whatever it is supposed to measure in a consistent way? Are the questions well posed? Are the questions too difficult or too easy? Do the questions discriminate between higher and lower mathematical performance? Are the outcomes significant? In short, the researcher was concerned with the reliability and validity of the geometry test. An instrument (test) is said to be reliable if it yields a consistent result (Patten, 2004). Similarly, an instrument is valid to the extent that it measures what it is designed to measure and accurately performs the function it is supposed to perform.

The internal consistency method of "coefficient alpha" also known as Cronbach's alpha was chosen to determine the reliability the geometry questionnaire. This method is based on the principle that sets of scores can be correlated to determine reliability. For example, to determine the amount of variance, the test scores determine true differences among students. A Cronbach's alpha coefficient between 0.7 and 1 is a widely accepted indicator of the reliability of an instrument (Wiersma, 2000).

One of the main purposes of the geometry test and geometry questionnaire was to measure students' preference for solution methods, as shown by their choice of solution methods when solving geometry problems. Thus, the reliability of the geometry questionnaire must be measured, because the questionnaire was a primary instrument used to measure students' preference for solution methods. As explained earlier, positive one (+1) was assigned for a visual solution method, negative one (-1) for a nonvisual solution method, and zero (0) was assigned when the solution methods were undecided.

The reliability analysis was conducted to examine the reliability scale of the 12 items of the geometry questionnaire. The analysis indicated that Cronbach's Alpha value is 0.675. The reliability analysis also indicated that the Cronbach's Alpha could be improved from 0.675 to 0.682 by removing problem six. Table 6 delineates the reliability scale of the geometry questionnaire.

Table 6: Reliability scale

Cronbach's Alpha	Cronbach's Alpha Based on Standardized Items	N of Items
.675	.690	12

As explained earlier, many participants asked during the test whether they were allowed to use a reference sheet because they are allowed to use a references sheet while taking quizzes and the End of Course (EOC) assessments. Providing a reference sheet might have raised the validity and reliability of the geometry test.

An instrument is valid to the extent that it measures what it is designed to measure and accurately performs the function it is supposed to perform. The geometry test and the geometry questionnaire were designed to measure students' preference for solution methods. The researcher was concerned particularly about the function of the geometry questionnaire: did it measure what it was supposed measure? To ensure the validity of the geometry test and geometry questionnaire, a short interview was also conducted with 17 students. The analysis of the interviews indicated that the geometry test and the geometry questionnaire reflected students' preference for solution methods. This further validated the aim of the geometry test and the geometry questionnaire in connection with assessing students' preference for solution methods.

CHAPTER FOUR: DATA ANALYSIS AND RESULTS

Data Analysis

The purpose of this study was to investigate the relationship between preference for solution methods, geometry performance, task difficulty, and gender. The tools were a geometry test and a geometry questionnaire. A short interview was also used to collect data. This chapter presents the analysis of data, organized around the following research questions:

1. Are preferences for solution methods associated with high school students' geometry performance?
2. Are degrees of difficulty of geometry tasks associated with students' preference for solution methods?
3. Do males and females differ in preference for solution methods and geometry performance after controlling for course assignments and grade levels?

Analysis of the geometry test and geometry questionnaire revealed that a majority of the students used visual solution methods in order to solve the geometry problems. In the population of subjects, 5% of the total students were nonvisual and 91% were visual. However, the percentages of visual and nonvisual students were different for each geometry problem. Table 7 illustrates the percentages of visual and nonvisual students for each problem.

Table 7: Descriptive statistics of visuality

Problems	*Visual students (%)*	*Nonvisual students (%)*	*Mean visuality score*
1	84.47	11.1	.74
2	81.36	10.55	.71
3	76.39	13.66	.63
4	62.11	28.57	.34

Problems	*Visual students (%)*	*Nonvisual students (%)*	*Mean visuality score*
5	81.36	12.42	.70
6	75.15	17.39	.58
7	72.04	14.28	.58
8	43.47	47.88	-.02
9	62.73	18.63	.46
10	67.08	22.36	.43
11	77.08	7.45	.70
12	70.18	11.18	.60

Interview

Seventeen students were selected from each school to represent all schools where quantitative data were collected. From the list of names of all students who took the geometry test and the geometry questionnaire, the researcher requested participating teachers to provide the names of a couple of students for a short interview. Thus, participating teachers selected some students from their class for the interview. The researcher did not ask participating teachers how they chose their students for interview. Thus, it was not clear how participating teachers selected their students as it relates to the procedure of selection of students for the interview. In most cases, interviews were conducted in a corner of a regular classroom; however, in some cases interviews were conducted outside of the classroom, such as in a hallway or corridor of a school building.

In order to verify further about students' preference for solution methods, the researcher cross checked between the actual solution methods students employed to complete the geometry problems and the solution methods they explained during the interview. In doing so, the researcher analyzed and compared qualitative and quantitative data of the 17 students who took part in the interview. The comparative analysis between quantitative and qualitative data revealed that the solution methods that participants used in the geometry test and the one they explained during the interview were the same. The preference for the solution method of only one student was found to be inconsistent between quantitative and qualitative data. In fact, this particular student explained during the interview that she considered herself a harmonic; however, she appeared to be a visualizer based on the quantitative analysis. Moreover, slight variations were also found between the actual solution methods utilized during the test and those they explained during the interview.

The comparative analysis revealed that one student was found to be a nonvisualizer, one harmonic, and the rest visualizers. The student who was a nonvisualizer during the interview used nonvisual solution methods to solve almost all of the geometry problems in the geometry test. Similarly, students who were harmonic during the interview used visual as well as nonvisual solution methods while solving geometry problems. Visualizers primarily used visual solution methods for the most of the geometry problems.

Table 8 delineates a comparison of participants' preference for solution methods between quantitative and qualitative data. QT and QL indicate that data come from quantitative and qualitative study. One (1), negative one (-1), and zero (0) respectively indicate visual, nonvisual, and harmonic solution methods. Visuality is the sum of visual and nonvisual scores for all of the problems on the geometry test and geometry questionnaire.

Table 8: Comparison between quantitative and qualitative data

Students	Question 1		Question 4		Question 8		Question 5	Visuality
	QT	*QL*	*QT*	*QL*	*QT*	*QL*		
1	1	Visual	-1	Visual	1	Visual	Visualizer	4
2	1	Visual	1	Visual	1	Visual	Visualizer	12
3	1	Visual	1	Visual	1	Visual	Visualizer	12
4	1	Visual	1	Visual	1	Visual	Visualizer	12
5	-1	Nonvisual	-1	Nonvisual	-1	Nonvisual	Nonvisulizer	-10
6	1	Visual	1	Visual	0	Nonvisual	Visualizer	7
7	1	Visual	1	Visual	1	Visual	Visualizer	7
8	1	Visual	1	Visual	1	Visual	Visualizer	12
9	1	Visual	1	Visual	-1	Visual	Visualizer	8
10	1	Visual	1	Visual	1	Visual	Visualizer	9
11	1	Visual	-1	Mixer	-1	Nonvisual	Harmonic	0
12	1	Visual	1	Visual	-1	Nonvisual	Visualizer	8
13	1	Visual	1	Visual	-1	Visual	Visualizer	8
14	1	Visual	1	Visual	1	Mixer	Visualizer	10
15	1	Visual	1	Visual	-1	Nonvisual	Visualizer	8
16	1	Visual	-1	Visual	-1	Visual	Harmonic	8
17	1	Visual	-1	Visual	1	Visual	Visualizer	8

One of the underlying reasons for conducting the interviews was to assess whether the geometry test and the geometry questionnaire truly gathered the relevant data regarding students' preference for solution methods. The qualitative data indicated that the geometry test and the geometry questionnaire reflected students' preference for solution methods, which further validated the aim of the instruments in with regard to assessing students' preference for solution methods.

Results of the Statistical Analysis

Research Question one: preference and geometry performance

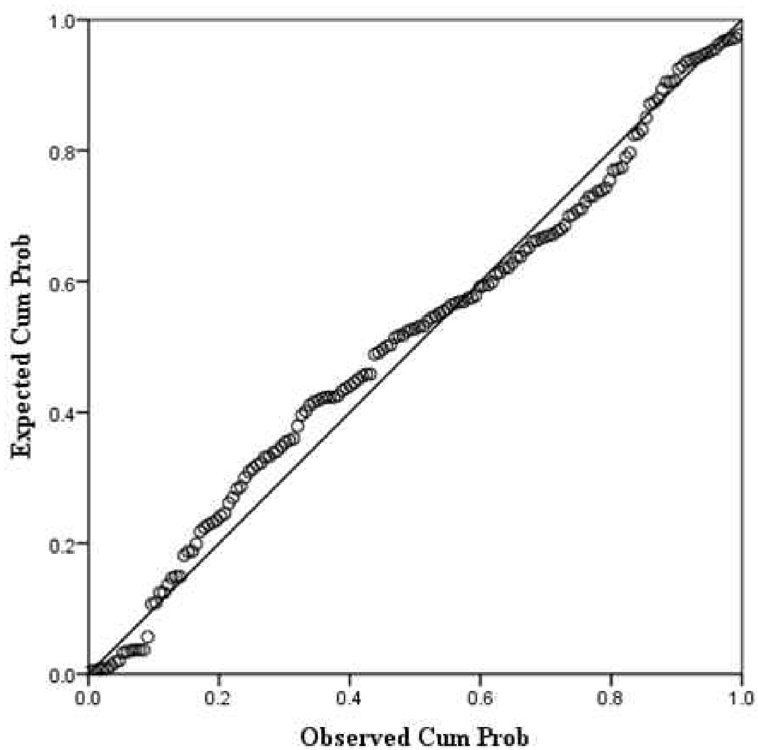
Are preferences for solution methods associated with high school students' geometry performance?

A simple linear regression can be used to explore the relationships between two variables by predicting the effect of one variable on the other (Lomax, 2007). Students' preference for solution methods was measured in terms of their visuality score which ranged from -12 to +12. Categorizing students into two different groups, visualizers and nonvisualizers, will eliminate the variances in visuality scores. For example, two different students with visual scores of 2 and 12 respectively belong to a same group (visualizers); however, there could be significant variance between these two students regardless of where they are from the same group. One of the important advantages of using a regression model is that it takes all the variances into account. Thus, the simple linear regression model was used to explore the relationships between students' preference for solution methods and their geometry performance.

Students' geometry performance was measured by the End-of-Course (EOC) assessment. The End of Course (EOC) is a standardized assessment administered Florida Department of Education. It is designed to measure students' content knowledge and skills in high school geometry course. The different geometry topics such as two-dimensional geometry, three-dimensional geometry, and trigonometry and discrete mathematics were covered by the EOC assessment. The geometry test designed for this study was not a standard test and it did not cover all topics of high school geometry. Thus, the researcher decided not to use the geometry test score to measure students' geometry performance.

All assumptions for simple regression analysis were satisfied. The residual statistics indicated that there was no issue regarding the assumption of the homogeneity of variance. The histogram and *P-P* plot indicated a normal distribution. Table 9 shows the normal *P-P* plot of regression standardized residual statistics. Similarly, the scatter plot showed that there were no systematic patterns between students' visuality score and geometry performance.

Table 9: The normal *P-P* Plot of regression standardized residual



Students' performance (EOC score) was a dependent variable, whereas preference for solution methods (visuality score) was an independent (predicator) variable. A simple regression analysis was used to test if the students' preference for solution methods significantly predicted students' geometry performance. The results of the regression analysis indicated that preference for solution methods explained only 1.1% variance ($R^2 = 0.011$, $F = 1.702$, $df = 1,159$, $p > 0.05$). Table 10 illustrates the summary of the regression model.

Table 10: Regression model summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate
1	.103[a]	.011	.004	11.070

a. Predictors: (Constant) Visuality

b. Dependent Variable: Performance

The ANOVA summary in Table 11 indicated that the visuality did not predict a significant a proportion of the total variance in the geometry scores (F (1, 8) = 1.702, $p > 0.05$).

Table 11: ANOVA summary

Model		Sum of Squares	df	Mean Square	F	Sig.
	Regression	208.500	1	208.500	1.702	.194[b]
1	Residual	19483.166	159	122.536		
	Total	19691.666	160			

a. Dependent Variable: Performance

b. Predictors: (Constant), Visuality

The coefficient Table 12 indicated that the unstandardized slope (0.280) and the standardized slope (0.103) were not significantly different from zero (t =1.304, $p > 0.05$). Thus, students' preference for solution methods was shown to be not a statistically significant predictor of students' geometry performance, which implied that there was not a significant relationship between preference for solution methods and geometry performance.

Table 12: Coefficient table

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95.0% Confidence Interval for B	
		B	Std. Error	Beta			Lower Bound	Upper Bound
1	(Constant)	45.397	1.634		27.785	.000	42.170	48.624
	Visuality	.280	.215	.103	1.304	.194	-.144	.705

a. Dependent Variable: Performance

As a follow up to the simple linear regression, a multiple regression analysis was used to test if the students' preference for solution methods, grades, and mathematics courses assignment (subjects) significantly predicted students' geometry performance. All assumptions of the multiple regression analysis were examined and satisfied the requirement. The multicollinearity was checked by a tolerance and variation inflation factor. The tolerance statistics for each variable was greater than 0.1.

Students' grade levels composite was a statistically significant predictor ($F_1(2,158) = 56.53$, $p < .001$) explaining approximately 41.7% of the variance in geometry performance. Similarly, the linear composite of grade level and mathematics courses was a statistically significant predictor ($F_2(2,158) = 6.91$, $p < .001$) explaining approximately 48.6% of the variance in geometry performance. The addition of subjects increased the explained variance in geometry performance by 6.9%. Furthermore, the linear composite of grades, subjects, and preference for solution methods was not a statistically significant predictor ($F_3(2,158) = 2.17$, $p > .05$) explaining approximately 49.3% of the variance in geometry performance. The addition of preference for solution methods increased the explained variance in geometry performance by only 0.07%. A summary of multiple regression analysis is given in Tables 13 and 14. The preference for solution methods including subjects and grades did not predict students' geometry performance.

The multiple regression analysis indicated that model one seemed to be the best model. The different models are given in the Table 13. In the model three, preference for solution methods explained only 0.7% variance which implies that preference for solution methods did not predict students' geometry performance. The variance explained by preference for solution methods in multiple regression analysis was less than that of the simple linear regression.

Table 13: Regression model summary

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics				
					R Square Change	F Change	df1	df2	Sig. F Change
1	.646[a]	.417	.410	8.523	.417	56.534	2	158	.000
2	.697[b]	.486	.469	8.082	.069	6.911	3	155	.000
3	.702[c]	.493	.473	8.051	.007	2.173	1	154	.143

a. Predictors: (Constant), Grade 11, Grade 10

b. Predictors: (Constant), Grade 11, Grade 10, RegularGeo, Financial Algebra, Algebra2

c. Predictors: (Constant), Grade 11, Grade 10, RegularGeo, Financial Algebra, Algebra2, Visuality

d. Dependent Variable: Performance

A summary of regression coefficients is presented in Table 14 indicating that only four variables (algebra 2, geometry, financial algebra, and grade 10) of the six variables significantly contributed to the model. Preference for solution methods (visuality) did not contribute to the regression model.

Table 14: Regression coefficients summary

Model		Unstandardized Coefficients		Standardized Coefficients	t	Sig.	Collinearity Statistics	
		B	Std. Error	Beta			Tolerance	VIF
1	(Constant)	40.681	1.139		35.718	.000		
	Grade 10	20.421	1.928	.719	10.589	**.000**	.800	1.250
	Grade 11	5.823	1.505	.263	3.869	**.000**	.800	1.250
2	(Constant)	49.747	2.401		20.718	.000		
	Grade 10	17.480	2.553	.615	6.848	.000	.411	2.435
	Grade 11	3.111	2.287	.140	1.360	.176	.312	3.209
	Algebra2	-6.563	2.645	-.277	-2.481	.014	.265	3.770
	RegularGeo	-15.622	3.732	-.307	-4.186	.000	.617	1.621
	FinancialAlg	-10.569	2.785	-.377	-3.795	.000	.336	2.973
3	(Constant)	48.421	2.555		18.948	.000		
	Grade 10	17.220	2.549	.606	6.755	**.000**	.409	2.447
	Grade 11	3.134	2.279	.141	1.376	.171	.312	3.209
	Algebra2	-6.658	2.636	-.282	-2.526	**.013**	.265	3.773
	RegularGeo	-16.041	3.729	-.315	-4.302	**.000**	.613	1.631
	FinancialAlg	-10.813	2.780	-.386	-3.890	**.000**	.335	2.983
	Visuality	.233	.158	.085	1.474	.143	.981	1.019

a. Dependent Variable: Performance

Research Question Two: Task difficulty

Are the degrees of difficulty of the geometry tasks associated with students' preference for solution methods?

Visuality and task difficulty were the two variables for research question two. The mean visual score of each problem was calculated for all participants. It was the sum of visual score of each problem of all students divided by the total number of students. The researcher divided the geometry problem into three groups: easy, moderate, and difficult while developing and designing the geometry test. However, the task difficulty was also assessed based on students' actual work on the geometry test. The task difficulty of each problem was determined by dividing the total number of correct answers produced by the total number of students. The easier the geometry problem, the more likely were student to get correct answers and vice versa. The mean visual score and level of difficulty for each problem is given in Table 15.

Table 15: Mean visual score and task difficulty

Problems	1	2	3	4	5	6	7	8	9	10	11	12
Correct (%)	26	6.8	15.5	7.4	7.4	9.3	8	37.8	33.5	28	37.8	7.4
Visuality	.74	.71	.63	.34	.70	.58	.58	-.02	.46	.43	.70	.60

Analysis of students' work revealed that (Table 15) problems 1, 8, 9, 10, and 11 appeared to be relatively easy tasks, and problems 2, 4, 5, 6, 7, and 12 seemed to be relatively difficult tasks. Only the problem 3 appeared to be a medium-difficult problem compared to the rest of the problems. The difficulty level of geometry task did not fall into three categories: easy, medium, and difficult as it was anticipated when the test was designed and developed. The researcher, therefore, decided not to divide problems into three categories; rather, used the degree of difficulty as it was they appeared when students solved problems.

Visuality and degree of difficulties were the two variables for the research question two. The association between task difficulty level and preference for solution methods were examined using a Pearson's product-moment correlation coefficient. One of the advantages of using Pearson's correlation coefficient is that students' visuality scores and numeric values of task difficulty can be used directly i.e., dividing the problems into three groups (easy, medium, and difficult) is not necessary.

The Pearson's product-moment correlation coefficient indicated that there was not a significant correlation between task difficulty and preference for solution methods ($r = -.385$ $n = 12$, $p > .05$). The summary of the analysis is shown in Table 16. The result indicated that there is negative correlation between task difficulty and preference for solution methods.

Table 16: Summary of correlation analysis

		Visuality	Difficulty
Visuality	Pearson Correlation	1	-.385
	Sig. (2-tailed)		.216
	N	12	12
Difficulty	Pearson Correlation	-.385	1
	Sig. (2-tailed)	.216	
	N	12	12

The geometry test contained 12 problems. Students were expected to show their work to while solving the geometry problems. However, students could find answer of problem 11 without showing any work because the problem 11 was yes-*no* question in nature. Thus, the problem 11 was somewhat different than rest of the problems in which students simply can choose their answer *yes* or *no*. Analysis of the test also indicated that many students did not show their work; instead they simply chose their answer in the problem 11. This might be a reason that problem 11 appeared to be easier than it was anticipated. Thus, the researcher also examined the association task difficulty and preference for solution methods excluding the problem 11.

The Pearson's product-moment correlation coefficient indicated that there was not a significant correlation between task difficulty and preference for solution methods (r = -.578 n =11, p > .05). However, the p value was very close to the alpha level of 0.05. The summary of the analysis is shown in Table 17. The result indicated that there is negative correlation between task difficulty and preference for solution methods. The negative correlation indicated that as task difficulty increases the visuality decreases, which implies that students tend to use visual solution methods for more difficult task. However, correlation was not significant.

Table 17: Summary of correlation analysis

		Visuality	Difficulty
Visuality	Pearson Correlation	1	-.578
	Sig. (2-tailed)		.063
	N	11	11
Difficulty	Pearson Correlation	-.578	1
	Sig. (2-tailed)	.063	
	N	11	11

The number of variables in the Pearson's product-moment correlation coefficient was comparatively low because there were only 12 variables (geometry problems). Thus, the researcher also conducted Spearman's rank correlation coefficient to examine the correlation between task difficulty and preference for solution methods. However, the analysis indicated that still there was not a significant correlation between task difficulty and preference for solution methods.

Research Question Three: Gender, preference, and performance

Do males and females differ in preference for solution methods and geometry performance after controlling for course assignments and grade levels?

Multivariate Analysis of Covariance (MANCOVA) can be used to examine effects of various covariates on independent variables. The researcher can incorporate one or more covariates into MANCOVA, and inclusion of several variables helps to reduce error variance (Mertler & Vannatta, 2005). MANCOVA also helps to control the effects of various covariates and provides more accurate results that researcher aims to find. As an extension of further investigation of effects of gender on preference for solution method and geometry performance, the researcher decided to conduct MANCOVA.

Grades and subjects were taken as covariates for MANCOVA analysis because grade and subject were significantly correlated with preference for solution methods and geometry performance. Correlation between performance, visuality, subject, and grade is given in Table 18. Subject-performance of students is given in Table 19

.

Table 18: Summary of correlation analysis

		Performance	Visuality	Subject	Grade
Performance	Pearson Correlation	1	.103	-.203**	-.617**
	Sig. (2-tailed)		.194	.010	.000
	N	161	161	161	160
Visuality	Pearson Correlation	.103	1	.008	-.019
	Sig. (2-tailed)	.194		.923	.812
	N	161	161	161	160
Subject	Pearson Correlation	-.203**	.008	1	.583**
	Sig. (2-tailed)	.010	.923		.000
	N	161	161	161	160
Grade	Pearson Correlation	-.617**	-.019	.583**	1
	Sig. (2-tailed)	.000	.812	.000	
	N	160	160	160	160

**. Correlation is significant at the 0.01 level (2-tailed).

Table 19: Descriptive statistics of subject-performance

	N	Mean	Std. Deviation
Algebra 2	109	49.70	9.942
Regular geometry	8	34.13	12.552
Financial Algebra	31	39.28	9.395
PreCalculus	13	53.15	8.214
Total	161	47.20	11.094

MANCOVA rests on some basic assumptions. The following assumptions were checked:

- Testing for homogeneity of regression slopes: The correlation between covariates and dependent variables did not differ across independent variable (gender).
- Independence of covariates: There was not a significant difference in subject scores (F (1,158 = 0.007), p = .935) or grade (F (1,158) =3.026, p = .084)).
- Correlation between covariates and dependent variables: There was a significant correlation between performance, grade, and subject. There was not a significant correlation between preference, grade, and subject, and none of them had a correlation coefficient greater than 0.7.

Multivariate Analysis of Covariance (MANCOVA) was conducted to determine the effects of gender on preference for solution methods and geometry performance while controlling the effects of subjects and grades. Subjects (four categories) and grades (three categories) were categorical variables. Thus, the categorical variables were dummy coded to convert them into bivariate measures.

Homoscedasticity is the assumption that variability in scores for one continuous dependent variable is roughly the same at all values of another continuous variable. Box's M test of equality of variance-covariance matrices was used to assess the homoscedasticity. Box's M = 50.97 with F (3, 1333792.95) = 2.85, p = .036 revealed that the assumption of equality of covariance matrices across the cells was not met, indicating that the null hypothesis of equal covariance matrices was rejected. Similarly, the assumption of linearity was also not satisfied. Since the homoscedasticity assumption was not satisfied and group sample sizes were unequal, Pillar's Trace was selected to report the analysis.

The statistical analysis showed that gender was significant in determining the combined test results in preference for solution methods and geometry performance (F (2,153) = 4.08, p < .05, Pillar's Trace = .051, η^2 = .051). The combined covariates did not significantly influence the gender difference on preference for solution methods and geometry performance. Table 20 illustrates the summary of the multivariate test. After controlling the covariates, the effect size reduced from 9.2% to 5.1%. The MANCOVA analysis indicated that the covariates did not significantly influence the gender difference in preference for solution methods and geometry performance.

Table 20*:* Summary of multivariate test

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared
Intercept	Pillai's Trace	.702	180.125[b]	2.000	153.000	.000	.702
	Wilks' Lambda	.298	180.125[b]	2.000	153.000	.000	.702
	Hotelling's Trace	2.355	180.125[b]	2.000	153.000	.000	.702
	Roy's Largest Root	2.355	180.125[b]	2.000	153.000	.000	.702

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared
Gr10	Pillai's Trace	.298	32.523[b]	2.000	153.000	**.000**	**.298**
	Wilks' Lambda	.702	32.523[b]	2.000	153.000	.000	.298
	Hotelling's Trace	.425	32.523[b]	2.000	153.000	.000	.298
	Roy's Largest Root	.425	32.523[b]	2.000	153.000	.000	.298
Gr12	Pillai's Trace	.015	1.173[b]	2.000	153.000	.312	.015
	Wilks' Lambda	.985	1.173[b]	2.000	153.000	.312	.015
	Hotelling's Trace	.015	1.173[b]	2.000	153.000	.312	.015
	Roy's Largest Root	.015	1.173[b]	2.000	153.000	.312	.015
Algebra2	Pillai's Trace	.030	2.334[b]	2.000	153.000	.100	.030
	Wilks' Lambda	.970	2.334[b]	2.000	153.000	.100	.030
	Hotelling's Trace	.031	2.334[b]	2.000	153.000	.100	.030
	Roy's Largest Root	.031	2.334[b]	2.000	153.000	.100	.030
RegularGe	Pillai's Trace	.096	8.105[b]	2.000	153.000	**.000**	**.096**
	Wilks' Lambda	.904	8.105[b]	2.000	153.000	.000	.096
	Hotelling's Trace	.106	8.105[b]	2.000	153.000	.000	.096
	Roy's Largest Root	.106	8.105[b]	2.000	153.000	.000	.096
FinancialAlg	Pillai's Trace	.065	5.327[b]	2.000	153.000	**.006**	**.065**
	Wilks' Lambda	.935	5.327[b]	2.000	153.000	.006	.065
	Hotelling's Trace	.070	5.327[b]	2.000	153.000	.006	.065
	Roy's Largest Root	.070	5.327[b]	2.000	153.000	.006	.065

Effect		Value	F	Hypothesis df	Error df	Sig.	Partial Eta Squared
Gender	Pillai's Trace	**.051**	**4.080[b]**	**2.000**	**153.000**	**.019**	**.051**
	Wilks' Lambda	.949	4.080[b]	2.000	153.000	.019	.051
	Hotelling's Trace	.053	4.080[b]	2.000	153.000	.019	.051
	Roy's Largest Root	.053	4.080[b]	2.000	153.000	.019	.051

a. Design: Intercept + Gr10 + Gr12 + Algebra2 + RegularGe + FinancialAlg + Gender

b. Exact statistic

The univariate analysis indicated that gender was a significant in geometry performance (F_1 (2,154) =8.127, $p < .001$, $\eta^2 = 0.051$) but not significant in preference for solution methods (F_2 (2,154) = .004, $p < .05$, $\eta^2 = 0.00$, $p > .05$) after controlling the effect of covariates. None of the covariates had significant effects in gender difference on students' preference for solution method. However, the covariates grade 10 and subjects (geometry, algebra, and financial geometry) had significant gender effects on students' geometry performance. Table 21 summarizes the univariate analysis.

Table 21: Summary of univariate analysis

Source	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Corrected Model	Visuality	54.787[a]	6	9.131	.541	.776	.021
	Performance	10070.453[b]	6	1678.409	26.865	.000	.511
Intercept	Visuality	235.166	1	235.166	13.936	.000	.083
	Performance	22532.586	1	22532.586	360.663	.000	.701
Gr10	Visuality	35.535	1	35.535	2.106	.149	.013

Source	Dependent Variable	Type III Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
	Performance	**4077.468**	**1**	**4077.468**	**65.265**	**.000**	**.298**
Gr12	Visuality	4.485	1	4.485	.266	.607	.002
	Performance	117.715	1	117.715	1.884	.172	.012
Algebra2	Visuality	4.280	1	4.280	.254	.615	.002
	Performance	**257.440**	**1**	**257.440**	**4.121**	**.044**	**.026**
RegularGe	Visuality	12.199	1	12.199	.723	.397	.005
	Performance	**908.450**	**1**	**908.450**	**14.541**	**.000**	**.086**
FinancialAlg	Visuality	6.406	1	6.406	.380	.539	.002
	Performance	**606.356**	**1**	**606.356**	**9.706**	**.002**	**.059**
Gender	Visuality	.062	1	.062	.004	.952	.000
	Performance	**507.721**	**1**	**507.721**	**8.127**	**.005**	**.050**
Error	Visuality	2598.642	154	16.874			
	Performance	9621.213	154	62.475			
Total	Visuality	9307.000	161				
	Performance	378355.144	161				
Corrected Total	Visuality	2653.429	160				
	Performance	19691.666	160				

a. R Squared = .021 (Adjusted R Squared = -.018)

b. R Squared = .511 (Adjusted R Squared = .492)

The MANCOVA indicated that the students' grades level and courses they enrolled in did not covariate students' gender. Thus, students' grade levels and courses they enrolled in were eliminated, and MANOVA was used to compare males and females within preference for solution methods (visuality) and their geometry performance. The statistical analysis showed that gender was significant in determining the combined test results in preference for solution methods and geometry performance (F (2,158) = 7.985, p < .001, Pillar's Trace = .092). The test between-subject effects indicated that gender was a significant factor in geometry performance (F_1 (2,158) =15.895, p < 0.001, η^2 = 0.091) but not significant in preference for solution methods (F_2 (2,158) = 0.00, η^2 = 0.00, p > .05).

The statistical analysis indicated that an effect of gender was significant in students' geometry performance but not in preference for solution methods. To investigate further the gender differences in geometry performance, an independent sample *t* was conducted. Table 22 delineates descriptive statistics of gender and geometry performance.

Table 22: Descriptive statistics

	Gender	N	Mean	Std. Deviation	Std. Error Mean
Performance	Male	66	43.20	12.233	1.506
	Female	95	49.98	9.326	.957

According to Leven's test, the homogeneity of variances assumption was not satisfied (F = 6.06, p = .015). The independent *t* test indicated that geometry performance was statistically significantly different (t (115.10) = -3.80, p < .001) between male and female students. Female students' geometry performance (M = 49.98, SD = 9.32) was significantly higher than male students' geometry performance (M = 43.20, SD = 12.33). The effect size was measured by using

Cohen's *d*. The effect size was 0.623, implying a medium effect size. Table 23 summarizes the results of the independent *t* test.

Table 23: Result of independent t test

		Levene's Test for Equality of Variances		t-test for Equality of Means				
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference
Performance	Equal variances assumed	6.066	.015	-3.987	159	.000	-6.779	1.700
	Equal variances not assumed			-3.800	115.109	.000	-6.779	1.784

Summary of the Statistical Analysis

The analysis of data unveiled that about 90% of students were found to be visualizers while nonvisualizers and harmonic students consisted of only 9%. A simple linear regression analysis was conducted to test if the students' preference for solution methods significantly predicted students' geometry performance. Analysis indicated that students' preference for solution methods was not associated with students' geometry performance. There was not a significant relationship between task difficulty and preference for solution methods. The direction of the difference in visuality between easy and difficult tasks indicated that preference for solution methods chosen was independent to tasks difficulty. The statistical analysis showed that gender was significant in determining the combined results of preference for solution methods and geometry performance. The test between-subject effects, however, indicated that gender was significant only in geometry performance but not significant in preference for solution methods. Geometry performance of female students was statistically significantly higher than that of male students.

CHAPTER FIVE: SUMMARY, DISCUSSION, AND RECOMMENDATIONS

Summary of the Study

The purpose of this study was to examine the relationships between preference for solution methods, task difficulty, geometry performance, and gender. The data were collected during the 2013-2014 school year from six different high schools at a county located in Florida within the United States. High school students who took the geometry test were enrolled in various mathematics courses at the time of the study.

A geometry test and a geometry questionnaire were used to collect data from all 161 students. Upon completion of the geometry test, students were given the geometry questionnaire and asked to choose the solution methods from the list that best described the solution methods they employed to complete the geometry problems. Students were allowed to use a calculator, ruler, scratch paper, etc., but not a reference sheet (formula sheet). The test was conducted in a regular classroom during school time. The normal time interval of the classes was 52 minutes. There was a variation in time to complete the geometry test. The majority of students used the entire time to work on the geometry test and the geometry questionnaire. However, some students finished the geometry test in 10-15 minutes. In general, the first 30/35 minutes were used to complete the geometry test and the remaining 15 minutes were utilized to finish the geometry questionnaire.

A short interview (2 to 3 minutes) was also conducted with 17 students. Using an audiotaped think-aloud protocol, four questions were asked during the interview. The students were presented with each problem and asked to think aloud. A hard-copy of the questions was also provided. Three questions were aimed to further explore which solution methods students used in solving geometry problems. However, the fourth question was directed to understand how students think of themselves: as visualizers or nonvisualizers. The comparative analysis between quantitative and qualitative data revealed that the solution methods that students used in the geometry questionnaire and the one they explained during the interview appeared to be consistent.

For the purpose of statistical analysis, students' preferences for solution methods were quantified into numeric values, and visuality score was obtained for each student. Students were given a score of +1 for the visual solution method and a score of -1 for the nonvisual solution method. If students did not choose their solution methods, chose both methods, or could not determine the solution methods they used, then a score of 0 was given. Thus, for twelve items, an individual could obtain a 'nonvisual-visual' score ranging from -12 to +12.

A simple linear regression analysis was conducted to test if the students' preference for solution methods significantly predicted students' geometry performance. The results of the regression analysis indicated that preference for solution methods explained only 1.1% variance. Thus, students' preference was not shown to be a statistically significant predictor of geometry performance.

There was not a significant relationship between task difficulty and preference for solution methods. The direction of the difference in visuality between easy and difficult tasks indicated that preference for solution methods chosen was independent to tasks difficulty. Thus, the result indicated that preference for visual or nonvisual solution methods for the geometry problems was not influenced by tasks' difficulty such that students were equally likely to employ visual and nonvisual solution methods for difficult and easy tasks.

The statistical analysis showed that gender was significant in determining the combined results of preference for solution methods and geometry performance. The test between-subject effects, however, indicated that gender was significant only in geometry performance but not significant in preference for solution methods. Geometry performance of female students was statistically significantly higher than that of male students.

Discussion

The purpose of this study was to investigate the relationship between preference for solution methods, geometry performance, tasks difficulty, and gender. Under this section, the results and findings of this study are discussed in connection with related literature. The discussion is centered on the following research questions:

1. Are preferences for solution methods associate with high school students' geometry performance?
2. Are the degrees of difficulty of geometry tasks associated with students' preference for solution methods?
3. Do males and females differ in preference for solution methods and geometry performance after controlling for course assignments and grade levels?

This study revealed that the preference for solution methods did not correlate with mathematical performance, in particular geometry performance. This is consistent with previous research studies (Galindo, 1994; Hegarty & Kozhevnikov, 1999; Lowrie, 2001; Moses, 1977; Suwarsono, 1982). The findings of this study indicated that students who prefer to use visual solution methods in solving geometry problems were likely to do as well as students who used nonvisual solution methods. However, some studies found a correlation between the visual solution method and mathematical performance (Battista, 1990; Bremigan, 2005; Ferrini-Mundy, 1987; Haciomeroglu, Chicken, & Dixon, 2013).

Findings of this study are inconsistent with Battista's (1990) study. One explanation of inconsistency in the findings of this study with those of other studies including Battista's involves the use of different types of mathematical tasks to measure students' preference for solution methods. Though six problems were employed from Battista's instrument, the geometry test and the geometry questionnaire were not the same as his instrument. In fact, half of the geometry problems were presented using some kind of geometric figures in Battista's instrument; however, only one problem was presented with the help of a diagram in the geometry test. Thus, these two tests were different in terms of employment of representation to present geometry problems, and that might explicate the inconsistencies in the findings between these two studies.

Students were enrolled in different types of mathematics courses at the time of this study. Thus, there was a distinct variation in terms of the mathematics courses that participating students had taken, which could have influenced students' preferences for solution methods and geometry performance. Limiting the study to a specific group of students could have provided different results. If this study had been given to different groups of geometry students—for example, regular, standard, or honor students—the results and findings might have been different.

The geometry test appeared to be difficult for the students because the majority of them were not able to solve the problems correctly. Easier geometry problems could have helped students to express their preference for solution methods in a clearer way. If the problems were easier, the findings of this study could have been different. Moreover, a convenient sampling was used to choose the population sample of this study. A much larger randomized sample from a larger population might yield a different result. Beyond this, the reliability scale of the geometry questionnaire is low (0.68). One explanation is that the low reliability scale could have affected the relationships between preference for solution methods and mathematics performance.

Students' geometry EOC scores were gathered from the year 2012 and 2013. In fact, 35% and 65% of the students had their EOC score respectively from the years 2012 and 2013. Only 5% of students were enrolled in a geometry course at the time of this study (2014). This distribution implies that some students had not taken geometry courses for two years. Thus, it seemed that students might have forgotten different rules and formulae that they had learned a year or two before and could not perform as well as they were expected. This might be one of the critical factors that resulted in no significant relationships between students' preference for solution methods and their geometry performance.

As stated earlier, many students asked whether they were allowed to use a reference sheet because the students were accustomed to using a reference sheet during quizzes, tests, and End-of-Course (EOC) assessments. Many students clearly informed the researcher that if they were allowed to use the reference sheet, they would have used formulae instead of diagrams and figures. Allowing students to use the reference sheet could have influenced students' preference for solution methods and indeed its relations with geometry performance. The researcher also observed during the test that many students quickly finished the test (or gave up), which could have also influenced students' preference for solution methods. If students were allowed to use a reference sheet, the number of nonvisual solution methods could have increased, which could influence their preference as well as performance on geometry tasks.

Not only the nature of mathematical tasks, but the mathematics-content areas were also different between this study and with other studies (Bremigan, 2005; Ferrini-Mundy, 1987; Haciomeroglu, Chicken, & Dixon, 2013). Bremigan (2005) focused on calculus emphasizing the role of visual representation. Ferrini-Mundy (1987) and Haciomeroglu, Chicken, and Dixon (2013) conducted studies focused on calculus and used primarily a graphic representation to present the derivative and antiderivative problems. However, verbal representation was used to design the geometry test and the geometry questionnaire in this study. The representations employed to present mathematics problems vary greatly among these studies. It is an important factor for teaching and learning mathematics. The fact is that mathematical ideas can be taught and learned in an effective way by utilizing suitable modes of representation (Goldin, 1987; Kaput, 1987; Janvier, 1987). Moreover, the ways in which mathematical ideas and problems are represented is fundamental to how students can understand and use those ideas, using and interpreting representation in appropriate ways (NCTM, 2000). For example, sketching diagrams (graphs) in high school geometry might not be as important and necessary as in college calculus. Therefore, different modes of representation influence students' mathematical thinking and problem solving skills (Campbell, Collis, & Watson, 1995) and they influence students' preference for visual and nonvisual solution methods (Haciomeroglu, 2012).

Translation ability is an important factor for problem solving in mathematics because translation of one mode of representation to another will provide flexibility to problem solvers while attempting mathematics problems (Doufour-Janvier, Bednarz, & Belanger, 1987; Janvier, 1987; Lesh, Post, & Behr, 1987). Thus, the role of representation as well as students' ability to translate mathematics problem from one mode of representation to another might be contributing factors to the inconsistency between the findings of this study compared with other studies. Beyond this, there are several factors such as students' socioeconomic status, age, grade, number of mathematics courses taken etc., which can influence the relationship between preference for solution methods and students' geometry performance. For example, this study also found that the different mathematics courses that students enrolled in and their grades were significantly related with students' geometry performance.

This study also examined the relationships between task difficulty and preference for solution methods and found that there was not a significant correlation between task difficulty and preference for solution methods. Results indicated that preference for solution methods for the geometry problems in either visual or nonvisual solution methods were not influenced by task difficulty such that students were equally likely to employ visual as well as nonvisual solution methods for difficult tasks. This is not consistent with the findings of Lowrie and Kay (2001) and Haciomeroglu (2012), who reported that task difficulty had an influence on students' preference for solution methods. As task difficulty increased, the number of visual solution methods also increased significantly. However, this finding supported the findings of Lowrie (2001), who found that there was not a significant relationship between task difficulty and preference for solution methods.

The reliability scale of the geometry test was low (0.68). Moreover, the sample size in this study was also small and significance level of the p value was close to the cut-off point of 0.05 when problem 11 was eliminated. Thus, if the sample size of this study would be larger, the result could have been changed, which might result in inconsistency with Lowrie and Kay (2001) and Haciomeroglu's (2012) findings.

Lowrie (2001) used the MPI to assess students' preference for solution methods. He used a three-point Likert scale survey to determine task difficulty. Students were asked to indicate whether they felt that the question on the MPI they had completed/attempted was easy, moderate, or difficult to solve. The task difficulty in both studies was based on students' response and work. This could also be a factor which might result in consistency with Lowrie's (2001) findings.

The findings of this study regarding gender difference in preference for solution methods and mathematics performance are consistent with some studies and contradict several other studies. This study found that there was a significant effect of gender only in geometry performance but not in the preference for solution methods. Female students outperformed male students in geometry performance. The findings of this study are partially consistent with Battista's (1990) findings. Battista reported that male and female students differed in geometry performance (males outperformed females), but not in their solution strategies. He suggested that there is a fundamental difference in the relative roles of spatial visualization and logical reasoning played in males' and females' geometry achievement. Moreover, he contended that discrepancy between spatial visualization and logical reasoning also influenced students' solution strategies. Male students' spatial visualization was negatively correlated with using drawing strategy and the reverse was true for female students.

Similarly, the finding of this study are partially consistent with Gallagher and De Lisi (1994), who reported gender difference both in preference for solution methods and mathematics performance. Fennema, Carpenter, Jacobs, Franke, and Levi (1998) reported that there was no gender difference in mathematics performance, but that gender difference prevailed in solution methods. On the other hand, while some studies did not find gender difference in preference for solution methods and mathematics performance (Galindo, 1994; Haciomeroglu & Chicken, 2012; Haciomeroglu, Chicken, & Dixon, 2013; Lowrie & Kay, 2001). Hyde, Fennema, and Lamon (1990) indicated that there was a gender difference in arithmetic or algebra performance;

male superiority in geometry was small, and the tests with mixed content showed the largest gender differences.

The nature and content of mathematics problems might influence gender difference in preference and mathematics performance not only in different areas in mathematics but also within the same area of mathematics. Calculus problems may need more sketching and graphing, algebraic problems may require more computational work, and geometry problems might need more figures. Thus, the instruments used to measure students' mathematics performance varied greatly. This research study used a geometry test, a geometry questionnaire, and students' geometry End-of-Course (EOC) scores, Galindo used the modified version of MPI, quizzes, and exam, Haciomeroglu, Chicken, and Dixon used AP calculus score, and Gallagher and De Lisi (1994) used SAT score. The different areas of mathematics and the nature of mathematics problems could have supported or contradicted the findings of this study with other studies.

According to the research, computational problems versus word problems and algebra versus geometry problems have significant influence on students' mathematics performance (Gallagher & De Lisi 1994). In contrast to the geometry questionnaire used in this study, Gallagher and De Lisi used the conventional (algorithmic methods) and unconventional (atypical solution strategies) problems for high school students to measure solution methods. Fennema, Carpenter, Jacobs, Franke, and Levi (1998) conducted interviews and administered tests simultaneously to assess first graders' solution methods and performance on number facts, addition and subtraction problems. According to Gallagher, De Lisi, Holst, Mcgillicuddy-De Lisi, Morely, and Cahalan (2000) female students were more successful with conventional problems than with unconventional problems, but male students' performance did not vary significantly with problem type. However, male students were more successful with conventional problems than unconventional problems. Solving geometry problems may be significantly different than completing arithmetic or algebra problems because the nature of mathematics problems might contribute to gender difference (Meyer, 1989). For example, there was no gender difference in arithmetic or algebra problems; however, gender difference was found in geometry (Hyde, Fennema, & Lamon, 1990).

The findings of this study were also consistent with the findings of Calvin, Fernandes, Smith, Visscher, and Deary (2010); Felson and Trudeau (1991); and Lawton (1997), who found that female students' performance was significantly higher than male students' performance. However, this is not consistent with some of the previous research studies (Battista, 1990; Fennema, 1974; Fennema & Sherman, 1978; Guay & McDaniel, 1977; Maccoby & Jacklin, 1974; Matteucci & Mignani, 2011), who reported that male students outperformed female students in mathematics performance. Some studies, however, did not find relationships between gender and mathematics performance (Hall & Hoff, 1988; Penner & Paret, 2008). Hyde, Fennema, and Lamon (1990) also reported that there was no gender difference in students' arithmetic or algebra performance in elementary and middle school.

There are various factors, such as students' Socioeconomic Status (SES), ethnicity, grade and age, number of mathematics courses students had taken, confidence in learning mathematics, mathematics content etc., which could have contributed to (in)consistency in the findings regarding gender differences in mathematics performance between this study and various other studies. For example, white students outperformed Hispanic students, and greater difference between males and females was noted in mathematics performance in Hispanic students than in White students (Moore & Smith, 1987). Similarly, confidence in learning mathematics is an effective factor related to mathematics achievement (Tartre & Fennema, 1995).

In contrast to findings of the majority of research studies, that males outperformed females, this study revealed that females outperformed males. One of the reasons could be the change in the trends of people's perception of mathematics. People believe that mathematics is considered to be a male-dominant subject. Students' mathematics achievement is also related to their attitude (Childs, 2013). Particularly, girls believed that mathematics is less useful for them and were less confident in their ability to do mathematics (Fennema & Sherman, 1978). During the last couple of decades, people's perception of mathematics as a male-dominant subject might have changed. Parents might have particularly encouraged their daughters to enroll in more mathematics courses. Female students' perception might have also changed as it relates to ability to do mathematics. Due to a change in the perception of parents as well as female students, female students might have higher geometry performance than males. However, more research studies need to be conducted in this area.

Researchers investigated different aspects of gender that attributed differences in mathematics performance. Some researchers identified factors such as cognitive abilities, socioeconomic status etc., underlying gender difference in mathematics (Ceci, Williams, & Barnett, 2009; Wai, Cacchio, Putalaaz, & Makel, 2010) while others found that gender difference in mathematical performance was due to difference in preferred mode of processing mathematical information (Carr, Steiner, Kyser, Biddlecomb, 2010; Lin & Peterson, 1985). For example, Carr, Steiner, Kyser, and Biddlecomb (2010) investigated different factors in conjunction with gender differences in mathematics performance of students in elementary school. They reported that only two factors, fluency and strategy, indicated gender differences and significantly predicted mathematics competency. Similarly, Meyer (1989) reported that even the nature of mathematics problems can cause gender difference because he found that gender difference was slightly higher in problem-solving tasks than in skill-level tasks. In fact, there are several factors that seem caused gender difference in preference for solution methods and mathematics performance.

Carr, Steiner, Kyser, and Biddlecomb (2010) suggested that there might be some theories that explain gender differences in mathematics performance, but no single theory can be used to explain gender difference in mathematics because there can be various factors that contribute to the gender differences in preference for solution methods and mathematics performance. It is obvious that the various factors such as influences of parents and their educational backgrounds, students' motivational factors, instructional strategy, teachers' visuality, students' demography, location of schools, and so forth could have influenced gender differences in preference for solution methods and mathematics performance.

During the interviews, students clearly explained the solution methods they used during the test or that they would use if they were required to do the problems. The researcher also tried to explore why students wanted to use diagrams and pictures over the rules and formulae, or vice-versa. Most students simply replied that they (dis)liked to use diagrams or formulae, but they were not able to explain clearly why they liked to use one solution method over the other. Qualitative research studies such as case studies, phenomenological studies, or ethnographic studies can provide more insights on why students prefer to use a specific solution method while solving mathematics problems and how it relates to gender and mathematics performance.

Implications for Teaching

This study found that the majority of students preferred to use visual solution methods. Moreover, results of statistical analysis indicated that as the geometry problems become more difficult, students tended to use visual solution methods. However, from problem-solving methods and mathematical performance perspectives, it is essential for students to develop both solution methods because some problems are easier to solve using visual solution methods over nonvisual solution methods and vice-versa. Thus, the development of only one-sided preferred mode of mathematical processing results in narrow mathematical development for students because they do not have an opportunity to see mathematics problems from the other perspective. Similar to the recommendation made by Haciomeroglu, Chicken, and Dixon (2013), Haciomeroglu, Aspinwall, and Presmeg (2010), and Clements (2014), instruction needs to focus on students' development of balance in their knowledge and skills between visual and nonvisual solution methods. In fact, students who use only (non)visual solution methods may have a limited understanding, and will not be able to provide a complete answer.

This study also unveiled that about 90% of students were found to be visualizers while nonvisualizers and harmonic students consisted of only 9%. Because students had a strong preference for visual solution methods, either more emphasis on nonvisual solution methods seems to be in order in lesson activities or high school geometry books may need to include lesson activities that are more non-visually oriented. To be proficient in mathematics, students are encouraged to develop preference for both solution methods: visual and nonvisual.

Some mathematics problems can be done in an easier way when they are solved with a (non)visual solution method. For example, when students used visual solution method to solve the problem number 3 of the geometry test, the majority of them got an incorrect answer. However, when students used nonvisual solution methods, the majority of them got the correct answer. Similarly, when students used visual solution methods to solve the problem number 11, the majority of them got correct answer. However, when they used the nonvisual solution method, the majority of them got an incorrect answer. Thus, based on the nature of mathematics problems, one specific solution method to solve mathematics problems can be more useful over the other solution methods. Thus, it is equally important to develop both visual and nonvisual preference for solution methods in order to be a successful learner and performer of mathematics.

Nonvisual teachers might over-emphasize rote memorization of mathematics rules and formulae for success in mathematics whereas visual teachers might be over reliant on figures and diagrams to assist their students to learn mathematics. In doing so, teachers inhibit students' opportunity learning mathematics employing visual as well as nonvisual solution methods. Teachers might be unaware of the fact that they are over reliant on only one instructional strategy, which might lead their students to develop preference for using only visual or nonvisual solution methods. Thus, it is suggested that instruction should be focused on incorporating both visual and nonvisual teaching strategies in mathematics lesson activities.

Limitations

This study had some limitations. The sample size was relatively small, and the students were not randomly selected. Moreover, only 17 students were interviewed and the researcher did not observe the classes. The participating teachers were not interviewed. The instructional strategies that participating teachers have been using in the classroom would be helpful to further explore and explicate the relationships between gender, preference for solution methods and mathematics performance.

The researcher intended to pilot the geometry test. However, due to time constraints and for some other reasons, piloting the geometry test was not feasible. Piloting the geometry test would have provided more insights and ideas to make the test better for actual study, which might yielded different results in this study. Moreover, the geometry test appeared to be difficult. If the problems were easier, the findings of this study could have been different.

The results concerning preference for solution methods for this study were primarily based on the geometry questionnaire and the geometry test. However, the researcher also conducted a short interview with 17 students to explore their preference for solution methods. Conducting similar interviews for the entire sample of participants might have provided more accurate and comprehensive results regarding preference for solution methods.

Another limitation of this study is that it was impossible to know whether students were putting their full effort into solving the geometry problems. Some students may have been randomly guessing answers and randomly choosing solution methods. This approach would reduce the validity of the results of this study and could affect the findings of this study.

As explained earlier, the participating teachers of this study participated in geometry professional series where they were encouraged to use various teaching learning materials, including technology. Thus, mathematics teachers from the participating schools might have used various types of mathematical resources such as manipulatives, dynamic geometry software, and so forth. Integrating technology and various mathematical resources in the lesson activities might have encouraged their students to solve the mathematics problems using more visual solution methods. This could be a reason that a majority of students were visualizers in this study. If participating teachers were not participated in the geometry professional series, the findings of this study could have been different.

Recommendations for Future Research

The sample in this study was students in grades 9 through 12, enrolled in different courses: algebra 2, regular geometry, pre-calculus, and financial algebra. In future studies researchers could look at students' geometry performance and preference for solution methods, limiting the sample to a particular group of students, for example groups of geometry students. Beyond this, researchers could conduct similar studies on a specific topic from high school geometry. Similar studies can also be conducted in other branches of mathematics such as algebra.

About 90% of students were found to be visualizers. There could be different factors why a majority of students preferred to use visual solution methods. For example, instructional strategies and technology-integrated lesson activities could have influenced students' preference for solution methods. Beyond this, even teachers' preference of instructional strategies might have affected students' preferred mode of processing mathematical information. Thus, researchers could further investigate various factors in conjunction with students' preference for solution methods. Including the quantitative research, the researcher recommends conducting more qualitative studies to delve deeper regarding students' preference for solution methods, gender difference, and mathematics performance. The qualitative studies would be helpful to find why students prefer to use one solution method over the other and how they develop one-sided preference for solving mathematics problems.

The geometry test did not cover the entire content of a high school geometry curriculum. Thus, the results and findings reported in this study could have been different if the geometry test had been designed based on different geometry topics other than those used in this study. Rather than trying to cover different topics, researchers could investigate students' preference for solution methods focusing on a specific geometry topic, which might lead to a more general conclusion.

Mathematics is considered to be a male-dominant subject (Fennema & Sherman, 1977). Noddings (1998) posed a question as to why males outperformed females; is it because females are simply less interested than males in mathematics. However, female students outperformed male students in this study. The findings of this study could be important and interesting from a gender-issue perspective. The sample size of this study was small, and the findings of this study may not be generalized. Therefore, more research studies need to be conducted with greater sample size in various content areas of mathematics to further examine the findings of this study.

The researcher has also tried to explore why students like to use one solution method over the others. Students were not able to explain clearly why they liked to use one solution method over the others. Qualitative research studies such as case studies, phenomenological studies, or ethnographic studies can help to explore students' preference for solution methods and its relationships with mathematical performance and gender.

APPENDIX A: GEOMETRY TEST

Geometry Test

First of all, I would like to thank you for taking part in this research study. Please do your best on the test; however, you will not receive a grade for it.

1. *Name*: ______________________________
2. Circle to indicate appropriate:

Gender: Male Female

Ethnicity: White African American Asian or Pacific Islander

Hispanic Multiracial Native American Other

Your Age _________ *Grade* _____

The geometry test contains 12 items.

On each page, there is a problem that you are asked to try to solve. Complete the problem to the best of your ability. Show your work. <u>Be sure to place your answer in the answer box provided on the page.</u>

Problem 1

What is the coordinate of the point on the number line halfway between -8 and $+5$?

Answer

Problem 2

A wire is stretched tightly from the top of a 60 foot tall pole to the top of a 10 foot pole. Both poles are standing vertically in level ground. If the poles are 100 feet apart, how long is the wire?

Answer

Problem 3

Which three of the points (2,6), (3,8), (4,12), and (6,18), lie on a straight line?

Answer

Problem 4

When the circumference of a circle is decreased from 200 inches to 150 inches, by how many inches is the diameter decreased?

Answer

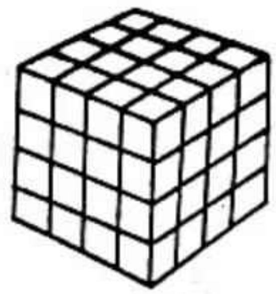
Problem 5

David and Lisa have a rectangular swimming pool that measures 10 feet by 7 feet. A cement walkway 8 feet wide boarders the pool on all sides. (Thus, the walkway forms a rectangular region surrounding the pool). If David and Lisa want to erect a fence to enclose the pool and walkway, how many feet of fencing will they need?

Answer

Problem 6

Sixty-four identical cubes are arranged to form the larger cube depicted below. If the entire outside surface of the large cube is painted, how many of the smaller cubes will have no painted faces?



Answer

Problem 7

Points $A(1,1)$, $B(7,1)$, and $C(3,5)$ are the vertices of the ΔABC. Find the length of the midsegment $\overline{DE}$ by connecting the midpoints of sides $\overline{AC}$ and $\overline{BC}$.

Answer

Problem 8

Find the slope of the line segment joining the points $A(1,1)$ and $B(3,2)$.

Answer

Problem 9

ΔABC with vertices A (4,3), B (2,1), and C (6,2) is reflected about the X axis, where $\Delta A'B'C'$ is the image of ΔABC. A' (x,y), B' (2,-1), and C' (6,-2) are the vertices of the $\Delta A'B'C'$. Find the coordinates of the point A'.

Answer

Problem 10

Find the distance between the points $P(-6,1)$ and $Q(2,1)$.

Answer

Problem 11

Does the point (5,3) lie on the circle centered at the point (5,0) with a radius of 3 units?

Answer

Problem 12

What is the maximum number of points of intersection are possible between a circle and a square?

Answer

APPENDIX B: GEOMETRY QUESTIONNAIRE

Geometry Questionnaire

How did you solve it?

Although there are correct answers to the problems, there are no <u>correct</u> ways to think about solving the problems. So be sure that you accurately indicate the type of thinking <u>you</u> used in attempting the problem.

It does not matter whether you completed the solution or not; whether your answer is right or wrong. If your solution is similar to one of the methods provided in the list, please choose the method that <u>**best explains**</u> how you solved the problem, even if other methods are considered.

Please put a tick mark (✓) in the appropriate box.

*Name*____

PROBLEM 1

What is the coordinate of the point on the number line halfway between −8 and +5?

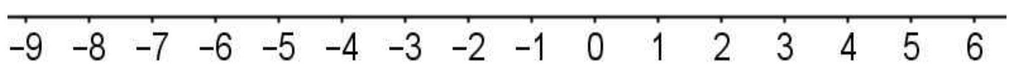
☐ **Solution Method 1**

I did not draw a diagram or try to visualize the situation. I tried to logically deduce the answer based on a careful analysis of the given information using arithmetic, algebra, or formula. For instance, I added −8 and 5 and divided by 2.

$$\frac{-8+5}{2} = \frac{-3}{2} = -1.5$$

☐ **Solution Method 2**

I drew a diagram. I was then able to figure out the answer using the diagram. For instance, I counted halfway from the point −8 towards the left and halfway from the point 5 towards the right in the diagram. Thus, the coordinate is -1.5.



☐ **Solution Method 3**

I did not draw a diagram, but I tried to visualize the situation. I was then able to figure out the answer.

☐ **Other Method**

I did not use any of the above methods. I attempted the problem in the following way:

PROBLEM 2

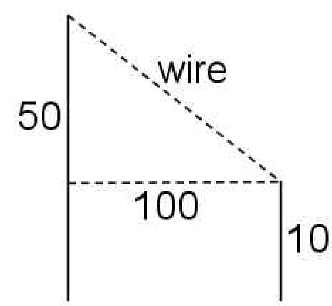
A wire is stretched tightly from a top of a 60 foot tall pole to the top of a 10 foot pole. Both poles are standing vertically in level ground. If the poles are 100 feet apart, how long is the wire?

☐ **Solution Method 1**

I did not draw a diagram or try to visualize the situation. I tried to logically deduce the answer based on a careful analysis of the given information using arithmetic, algebra, or formula. For instance, I used Pythagorean Theorem to find the length of the wire.
The difference in the height between the two poles is 50 feet.
Distance between the two poles is 100 feet.
Using Pythagorean Theorem
Length of the wire = $\sqrt{50^2 + 100^2} = \sqrt{2500 + 10000} = \sqrt{12500} = 50\sqrt{5}$ feet

☐ **Solution Method 2**

I drew a diagram. I was then able to figure out the answer using the diagram. For instance, I used Pythagorean Theorem to determine the length of the wire.



Length of the wire = $\sqrt{50^2 + 100^2} = \sqrt{2500 + 10000} = \sqrt{12500} = 50\sqrt{5}$ feet

☐ **Solution Method 3**

I did not draw a diagram, but I tried to visualize the situation. I was then able to figure out the answer.

☐ **Other Method**

I did not use any of the above methods. I attempted the problem in the following way:

PROBLEM 3

Which three of the points (2,6), (3,8), (4,12), and (6,18) lie on a straight line?

☐ **Solution Method 1:**

I did not draw a diagram or try to visualize the situation. I tried to logically deduce the answer based on a careful analysis of the given information using arithmetic, algebra, or formula. For instance, I used the slope formula to calculate the slope between two given points, $m = \frac{y_2 - y_1}{x_2 - x_1}$. I then figured out points with same slope lie in a straight line.

Slope of the line segment joining the points (6,18) and (4,12)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12-18}{4-6} = \frac{-6}{-2} = 3$$

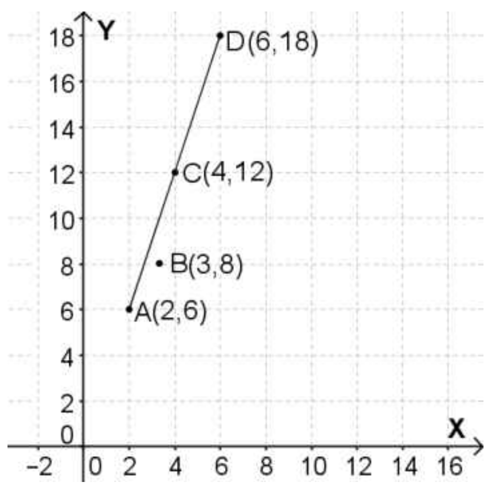
Slope of the line segment joining the points (2,6) and (4,12)

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12-6}{4-2} = \frac{6}{2} = 3$$

Thus, the points (2,6), (6,18), and (4,12) lie on a straight line.

☐ **Solution Method 2:**

I drew a diagram. I was then able to figure out the answer using the diagram. For instance, I drew a line to determine which points were on the same line.



☐ **Solution Method 3**

I did not draw a diagram but I tried to visualize the situation. I was then able to figure out the answer.

☐ **Other Method**

I did not use any of the above methods. I attempted the problem in the following way:

PROBLEM 4

When the circumference of a circle is decreased from 200 inches to 150 inches, by how many inches is the diameter decreased?

☐ **Solution Method 1**

I did not draw a diagram or try to visualize the situation. I tried to logically deduce the answer based on a careful analysis of the given information using arithmetic, algebra or formula. For instance, I used the formula $C = \pi d$ where C is circumference and d is the diameter of a circle.

$C_1 = \pi d_1$ $\qquad$ $C_2 = \pi d_2$

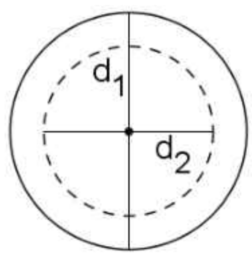
$200 = \pi d_1$ $\qquad$ $150 = \pi d_1$

$\frac{200}{\pi} = d_1$ $\qquad$ $\frac{150}{\pi} = d_2$

Decrease in diameter, $d_1 - d_2 = \frac{200}{\pi} - \frac{150}{\pi} = \frac{50}{\pi}$ = 15.9 inches

☐ **Solution Method 2**

I drew a diagram. I was then able to figure out the answer using the diagram. I then, used algebra and formula to find the answer.



$C_1 = \pi d_1$ $\qquad$ $C_2 = \pi d_2$

$200 = \pi d_1$ $\qquad$ $150 = \pi d_1$

$\frac{200}{\pi} = d_1$ $\qquad$ $\frac{150}{\pi} = d_2$

Decrease in diameter, $d_1 - d_2 = \frac{200}{\pi} - \frac{150}{\pi} = \frac{50}{\pi}$ = 15.9 inches

☐ **Solution Method 3**

I did not draw a diagram but I tried to visualize the situation. I was then able to figure out the answer.

☐ **Other Method**

I did not use any of the above methods. I attempted the problem in the following way:

PROBLEM 5

David and Lisa have a rectangular swimming pool that measure 10 feet by 7 feet. A cement walkway 8 feet wide boarders the pool in all sides. If David and Lisa want to erect a fence to enclose the pool and walkway, how many feet of fencing will they need?

☐ **Solution Method 1**

I did not draw a diagram or try to visualize the situation. I tried to logically deduce the answer based on a careful analysis of the given information using arithmetic, algebra, or formula. For instance, I used the formula $2(l+b)$ to find the perimeter. I added the width of cement walkway to the length and breadth of the swimming pool.

Now, dimensions of swimming pool including the walkway are,

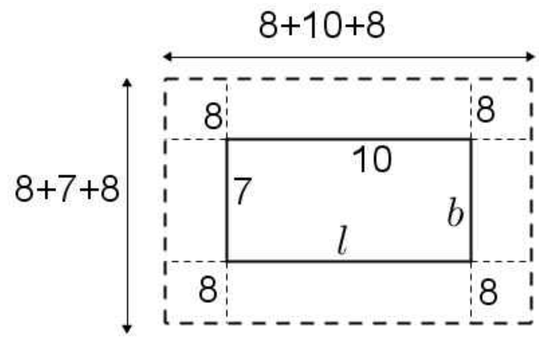
Length (l) = (10+8+8) = 26

Breadth (b) = 7+8+8 = 23

Length of fence = $2(l+b) = 2(26+23) = 2(49) = 98$ feet.

☐ **Solution Method 2:**

I drew a diagram representing the situation. I was then able to figure out the answer using the diagram. For instance, I used formula $2(l+b)$ to find the length of the fence. Length of fence = $2(l+b) = 2(26+23) = 2(49) = 98$ feet.



☐ **Solution Method 3**

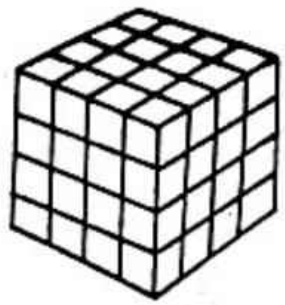
I did not draw a diagram but I tried to visualize the situation. I was then able to figure out the answer.

☐ **Other Method**

I did not use any of the above methods. I attempted the problem in the following way:

PROBLEM 6

Sixty-four identical cubes are arranged to form the larger cube depicted below. If the entire outside surface of the large cube is painted, how many of the smaller cubes will have no painted faces?



☐ **Solution Method 1**
I did not draw a diagram or try to visualize the situation. I tried to logically deduce the answer based on a careful analysis of the given information using arithmetic, algebra, or formula. For instance, I used the formula $V = n^3$ to find the length of the side of the big cube. $64 = n^3 \Rightarrow 4 = n$, Each face of large cube has 4 • 4 = 16 small cubes with painted faces. I take off one small cube from each side from the large cube and subtracted it from the side of the large cube (4-2 =2). Number of small cubes that do not lie on the faces of large cube is 2 • 2 • 2 = 8 small cubes. Thus, 8 small cubes will have no pained faces.

☐ **Solution Method 2**
I used the diagram to help me count the small cubes on the outside of the large cube because all of these cubes would be painted. I then subtracted this number form the total number of small cubes. Total number of small cubes with painted faces is 56. Total number of cubes is 64.
Thus, number of small cubes with no painted faces = $64 - 56 = 8$

☐ **Solution Method 3**
I tried to visualize how many small cubes were on the inside of the large cube because these cube would not be painted.

☐ **Other Method**
I did not use any of the above methods. I attempted the problem in the following way:

PROBLEM 7

Points A (1,1), B (7,1), and C (3,5) are the vertices of the ΔABC. Find the length of the midsegment $\overline{DE}$ by connecting the midpoints of sides $\overline{AC}$ and $\overline{BC}$.

☐ **Solution Method 1**

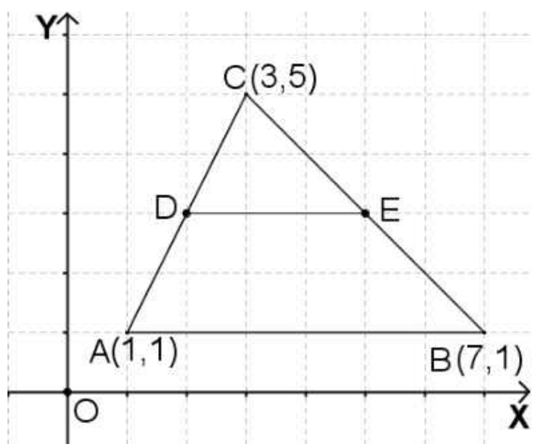
I did not draw a diagram or try to visualize the situation. I tried to logically deduce the answer based on a careful analysis of the given information using arithmetic, algebra, or formula. For instance, I figured out the length of the side $\overline{AB}$ by using distance formula.

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(7-1)^2 + (1-1)^2} = \sqrt{(6)^2 + (0)^2} = 6$ units

I know from the midsegment theorem, segment joining midpoints of two sides of a triangle is parallel to the third side and half of its length. Thus, length of $\overline{DE}$ is 3 units.

☐ **Solution Method 2**

I drew a diagram representing the situation. I was then able to figure out the answer using the diagram. I simply counted distance from the point D to the point E on the coordinate axes which is 3 units.



☐ **Solution Method 3**

I did not draw a diagram but I tried to visualize the situation. I was then able to figure out the answer.

☐ **Other Method**

I did not use any of the above methods. I attempted the problem in the following way:

PROBLEM 8

Find the slope of the line segment joining the points A (1,1) and B (3,2).

☐ **Solution Method 1**

I did not draw a diagram or try to visualize the situation. I tried to logically deduce the answer based on a careful analysis of the given information using arithmetic, algebra, or formula. For instance, I used the slope formula.

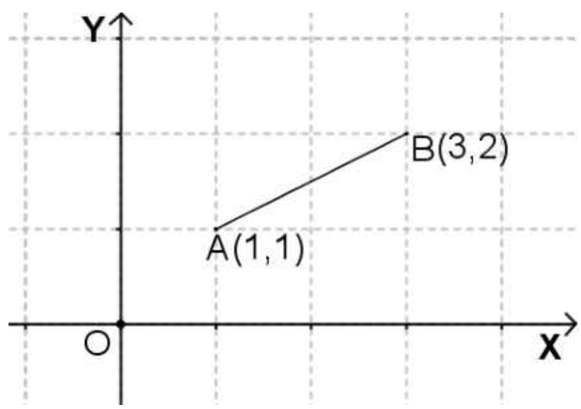
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-1}{3-1} = \frac{1}{2}$$

☐ **Solution Method 2:**

I drew a diagram representing the situation. I was then able to figure out the answer using the diagram. I calculated the ratio of rise over run between the two points on the coordinate axes.

Rise = 1 units, Run = 2 units.

Slope, $m = \frac{rise}{run} = \frac{1}{2}$



☐ **Solution Method**

I did not draw a diagram but I tried to visualize the situation. I was then able to figure out the answer.

☐ **Other Method**

I did not use any of the above methods. I attempted the problem in the following way:

PROBLEM 9

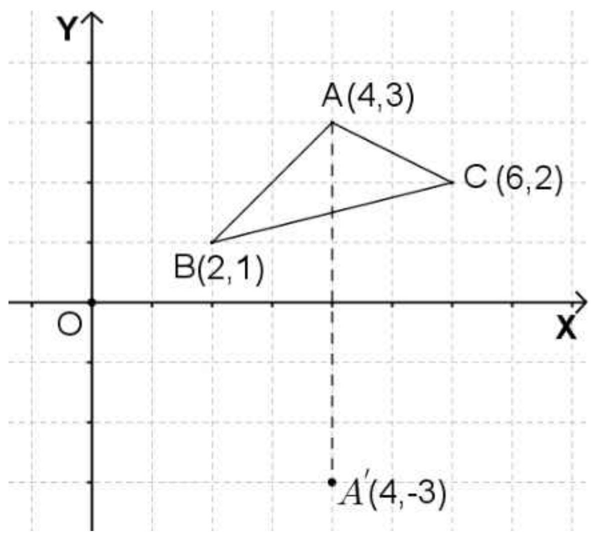
ΔABC with vertices A (4,3), B (2,1), and C (6,2) is reflected about the X axis, where $\Delta A'B'C'$ is the image of the ΔABC. A' (x, y), B' (2,-1), and C' (6,-2) are the vertices of the image $A'B'C'$. Find the coordinates of the point A'.

☐ **Solution Method 1**
I did not draw a diagram or try to visualize the situation. I tried to logically deduce the answer based on a careful analysis of the given information using arithmetic, algebra, or formula. For instance, I used formula for the reflection of a point about the *x* axis.

$P(x, y) \rightarrow P^1(x, -y)$

Using this formula the coordinates of the point A' is (4,-3).

☐ **Solution Method 2**
I drew a diagram representing the situation. I was then able to figure out the answer using the diagram. I drew the triangle *ABC* on the coordinate axes. I calculated the distance from the point A to the *x* axis. And, I then reflected the point *A* to the below about the *x* axis as a same distance from the point A to the *x* axis. The coordinates of the point A′ is (4,-3).



☐ **Solution Method 3**
I did not draw a diagram but I tried to visualize the situation. I was then able to figure out the answer.

☐ **Other Method**
I did not use any of the above methods. I attempted the problem in the following way:

PROBLEM 10

Find the distance between the points $P(-6,1)$ and $Q(2,1)$.

.

☐ **Solution Method 1**

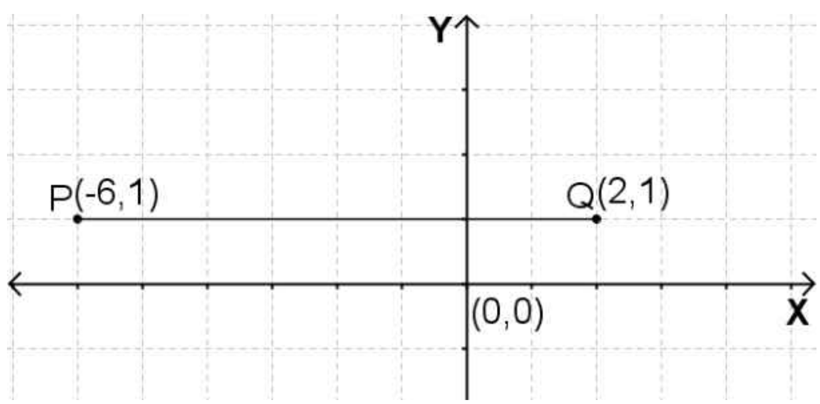
I did not draw a diagram or try to visualize the situation. I tried to logically deduce the answer based on a careful analysis of the given information using arithmetic, algebra, or formula. For instance, I used the distance formula $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Let *P* (-6, 1) be (x_1, y_1) and *Q* (2, 1) be (x_2, y_2)

$\overline{PQ} = \sqrt{(2-(-6))^2 + (1-1)^2} = \sqrt{(8)^2 + (0)^2} = 8$ units.

☐ **Solution Method 2**

I drew a diagram representing the situation. I was then able to figure out the answer using the diagram. For instance, I counted the distance between the points P and Q on the coordinate system. The distance is 8 units.



☐ **Solution Method 3**

I did not draw a diagram but I tried to visualize the situation. I was then able to figure out the answer.

☐ **Other Method**

I did not use any of the above methods. I attempted the problem in the following way:

PROBLEM 11

Does the point (5,3) lie on the circle centered at the point (5,0) with a radius of 3 units?

☐ **Solution Method 1**

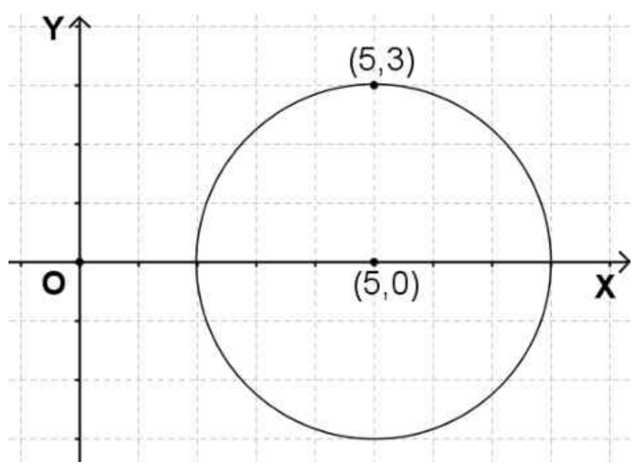
I did not draw a diagram or try to visualize the situation. I tried to logically deduce the answer based on a careful analysis of the given information using arithmetic, algebra, or formula. For instance, I used the standard equation for a circle, $(x-h)^2+(y-k)^2=r^2$.

$$(x-5)^2+(y-0)^2=3^2$$

I then plugged the point (5, 3) in the above equation of the circle to examine whether it lies on the circle. The point (5,3) lies on the circle.

☐ **Solution Method 2**

I drew a diagram representing the situation. I was then able to figure out the answer using the diagram. I drew the circle and I plotted the point (5,3) to examine whether it lies on the circle. The point (5,3) lies on circle.



☐ **Solution Method 3:**

I did not draw a diagram but I tried to visualize the situation. I was then able to figure out the answer.

.

☐ **Solution Method**

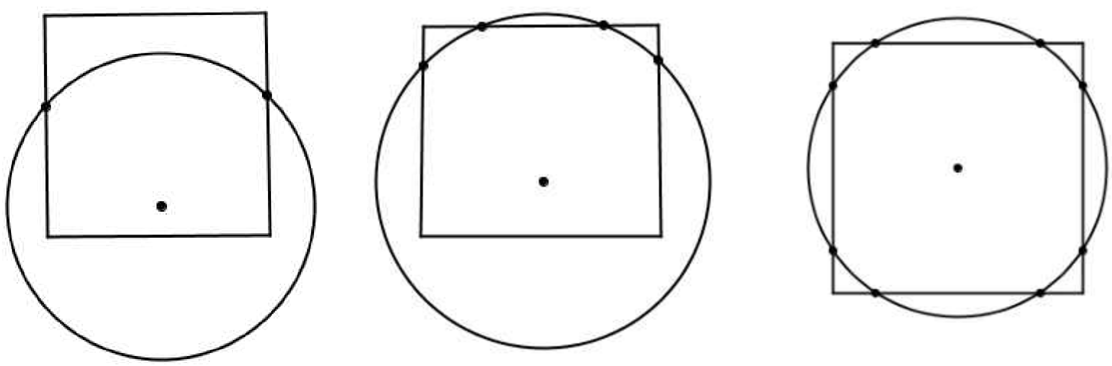
I did not use any of the above methods. I attempted the problem in the following way:

PROBLEM 12

What is the maximum numbers of points of intersection are possible between a circle and a square?

☐ **Solution Method 1**
I did not draw a diagram or try to visualize the situation. I tried to logically deduce the answer based on a careful analysis of the given information using arithmetic, algebra, or formula. For instance, a line intersects a circle at most two points. A square has four sides; thus, it intersects a circle at 8 points.

☐ **Solution Method 2**
I drew diagrams representing the situation. I was then able to figure out the answer using the diagram. I drew squares and circles intersecting each other in different possible ways. I then tried to figure it out by manipulating the squares and circles. Square intersects at 8 points to the circle.



☐ **Solution Method 3:**
I did not draw a diagram but I tried to visualize the situation. I was then able to figure out the answer.

☐ **Other Solution Method**
I did not use any of the above methods. I attempted the problem in the following way:

APPENDIX C: INTERVIEW QUESTIONS

Geometry Interview

First of all, I would like to thank you for taking part in this short interview. You will be asked some questions regarding the solution methods you used to solve geometry problems in the geometry test that you have taken recently. If you do not remember the solution methods you used that is ok. Please, try to explain how you would solve them.

Problem 1

What is the coordinate of the point on the number line halfway between -8 and $+5$?

Questions:

1. How did you solve this problem? Could you please explain?
2. If you did not solve it during the test, how would solve it.

Problem 4

When the circumference of a circle is decreased from 200 inches to 150 inches, by how many inches is the diameter decreased?

Questions:

1. How did you solve this problem? Could you please explain?
2. If you did not solve it during the test, how would solve it.

Problem 8

Find the slope of the line segment joining the points $A(1,1)$ and $B(3,2)$.

Questions:

1. How did you solve this problem? Could you please explain?
2. If you did not solve it during the test, how would solve it.

<u>**Additional Question**</u>

Visualizers are the ones whose solution method primarily is relied on drawings, pictures, and figures while solving mathematics problems. Nonvisualizers are the ones whose solution methods primarily is based on formula, arithmetic, or logical reasoning while attempt to solve mathematics problems.

What do you categorized yourself: visualizer or nonvisualizer?

APPENDIX D: INTERVIEW TRANSCRIPT

Students	***Problem 1***	***Problem 4***	***Problem 8***	***Visual/nonvisual***
1	I drew a line.	I think I solved it. I was thinking about one of the diameters like how to use it solve and then I drew a circle.	I kind of drew it.	Visualizer
2	I drew a graph and put a point -8 and 5 on the graph, and I counted evenly what I got to middle point, the middle coordinates between negative 8 and 5.	I don't know. I have to write it. I don't know. I don't remember, but I think I did it. So circumference 200….I think I drew it.	First a plotted the point A and then plotted B, and I then I used rise over run to count like slope between these two points. Sometimes I also use formula.	Visualizer

3	So, I would draw a graph and I would plots two of numbers given and I would find the midway between the lines, and that would be line halfway.	I definitely had drawn for this too. I would use references sheet if I have provided a reference sheet. I do not know definitely geometry and I do not remember the formula. I would draw it and see visually.	I drew a graph and I thought, I used formula because I can visually see rise over run and drew a graph.	Definitely visualizer.
4	I did the graph and I plotted this number and that number and coordinates, and then I plugged this number.	I drew a circle. Then it says it decreases 250 to 150. I drew a circle and then I visualized it.	This one I also drew a graph because two coordinates given A and B, and I did rise and run because this is the way we find it.	Visualizer
5	I used a formula.	I did not use drawings.	I used a formula.	Non-visualizer

6	I drew a number line.	I drew a circle.	I used formula.	More visualizer
7	I made a graph.	I will draw a circle.	I made graph.	Visualizer
8	I kind of drew a number line.	I drew a circle.	I drew a line with coordinate.	Visualizer
9	I would draw a number line.	I would draw a circle.	I would draw a coordinate line an plot it on the line	Visualizer
10	I do not know how to solve it. I think I drew on the graph paper. I used rise over run and drew on graph.	Divide 200 and 150…I drew a picture.	I would put the points where A as what it is and B and draw them.	Visualizer

11	I think what I did is graph and ...	I think what I did is I did 200 minus 150 and I got my answer like that I did divide.	I used y two minus y one over x two minus x one. I used slope formula. I did not make drawing.	I think I am kind of both. I got visualizing and I got don't.
12	I did a number line and then decided you know… in kind of between lines.	I drew out a circle.	I did the formula of y two minus y one over x two minus x one. .	I am more visual.
13	I drew a number line and counted it.	I drew the circle and subtracted it to get circumference.	I drew the line segment and I used rise over run.	I am a visualizer because I used drawings.
14	I drew a number line.	I drew a circle.	I used formula and did drawings too.	Visual

15	I just made a number line and kind of between pretty much median.	I used the circumference equation. I did not use any drawings.	I used y two minus- y one over x two minus x one. I did not use drawings.	Probably I used formula more often and I am more non-visualizer
16	I used a number line and I basically found midpoint between the two.	I used the formula of circumference of a circle and I found the one for 200 inches and I found one for 150 inches and solve the diameter because that was missing what was and I subtracted two.	I graphed the points and found the half way between the points.	Well, I think, I qualified for both categories because once I graphed actually I used formula to solve it, so like lot of them bunch of them I can do with formula and pictures also.

17	I Solved it by writing a number line between negative 8 and positive 5, and how many difference between them.	I think I would use formula. I do not use drawings.	I drew a graph. I did not use formula to solve it.	I am more a visualizer. I like to draw. I do not know why I like to draw.

APPENDIX E: REFERENCE SHEET

Algebra 1 End-of-Course and Geometry End-of-Course Assessments Reference Sheet

Area

Parallelogram	$A = bh$
Triangle	$A = \frac{1}{2}bh$
Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$
Circle	$A = \pi r^2$
Regular Polygon	$A = \frac{1}{2}aP$

KEY	
b = base	A = area
h = height	B = area of base
w = width	C = circumference
d = diameter	V = volume
r = radius	P = perimeter of base
ℓ = slant height	
a = apothem	$S.A.$ = surface area

Use 3.14 or $\frac{22}{7}$ for π.

Circumference

$C = \pi d$ or $C = 2\pi r$

		Volume/Capacity	Total Surface Area
	Rectangular Prism	$V = bwh$ or $V = Bh$	$S.A. = 2bh + 2bw + 2hw$ or $S.A. = Ph + 2B$
	Right Circular Cylinder	$V = \pi r^2 h$ or $V = Bh$	$S.A. = 2\pi rh + 2\pi r^2$ or $S.A. = 2\pi rh + 2B$
	Right Square Pyramid	$V = \frac{1}{3}Bh$	$S.A. = \frac{1}{2}P\ell + B$
	Right Circular Cone	$V = \frac{1}{3}\pi r^2 h$ or $V = \frac{1}{3}Bh$	$S.A. = \frac{1}{2}(2\pi r)\ell + B$
	Sphere	$V = \frac{4}{3}\pi r^3$	$S.A. = 4\pi r^2$

Sum of the measures of the interior angles of a polygon $= 180(n-2)$

Measure of an interior angle of a regular polygon $= \dfrac{180(n-2)}{n}$

where:

n represents the number of sides

Florida Department of Education

Algebra 1 End-of-Course and Geometry End-of-Course Assessments Reference Sheet

Slope formula

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where m = slope and (x_1, y_1) and (x_2, y_2) are points on the line

Slope-intercept form of a linear equation

$$y = mx + b$$

where m = slope and b = y-intercept

Point-slope form of a linear equation

$$y - y_1 = m(x - x_1)$$

where m = slope and (x_1, y_1) is a point on the line

Distance between two points

$P_1(x_1, y_1)$ and $P_2(x_2, y_2)$

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint between two points

$P_1(x_1, y_1)$ and $P_2(x_2, y_2)$

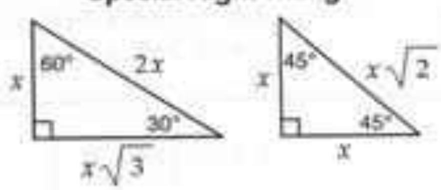
$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Quadratic formula

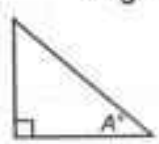
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

where a, b, and c are coefficients in an equation of the form $ax^2 + bx + c = 0$

Special Right Triangles



Trigonometric Ratios



$$\sin A° = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos A° = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan A° = \frac{\text{opposite}}{\text{adjacent}}$$

Conversions

1 yard = 3 feet
1 mile = 1,760 yards = 5,280 feet
1 acre = 43,560 square feet
1 hour = 60 minutes
1 minute = 60 seconds

1 cup = 8 fluid ounces
1 pint = 2 cups
1 quart = 2 pints
1 gallon = 4 quarts
1 pound = 16 ounces
1 ton = 2,000 pounds

1 meter = 100 centimeters = 1000 millimeters
1 kilometer = 1000 meters
1 liter = 1000 milliliters = 1000 cubic centimeters
1 gram = 1000 milligrams
1 kilogram = 1000 grams

Florida Department of Education

APPENDIX F: INSTITTIONAL REVIEW BOARD APPROVAL

University of Central Florida

University of Central Florida Institutional Review Board
Office of Research & Commercialization
12201 Research Parkway, Suite 501
Orlando, Florida 32826-3246
Telephone: 407-823-2901 or 407-882-2276
www.research.ucf.edu/compliance/irb.html

Approval of Human Research

From: **UCF Institutional Review Board #1**
FWA00000351, IRB00001138

To: **Bhesh Raj Mainali**

Date: **February 24, 2014**

Dear Researcher:

On 2/24/2014, the IRB approved the following minor modifications to human participant research until 10/24/2014 inclusive:

Type of Review:	IRB Addendum and Modification Request Form
Modification Type:	Students in [redacted] will be recruited; in addition to 350 students, 350 adults from [redacted], will also be recruited for a total of 700 study participants. A revised protocol has been uploaded; a revised Parent Consent and an Adult Consent document have been approved for use.
Project Title:	Investigating the Relationships between Preferences, Gender, and High School Students' Geometry Performance
Investigator:	Bhesh Raj Mainali
IRB Number:	SBE-13-09700
Funding Agency:	
Grant Title:	
Research ID:	N/A

The scientific merit of the research was considered during the IRB review. The Continuing Review Application must be submitted 30days prior to the expiration date for studies that were previously expedited, and 60 days prior to the expiration date for research that was previously reviewed at a convened meeting. Do not make changes to the study (i.e., protocol, methodology, consent form, personnel, site, etc.) before obtaining IRB approval. A Modification Form **cannot** be used to extend the approval period of a study. All forms may be completed and submitted online at https://iris.research.ucf.edu .

If continuing review approval is not granted before the expiration date of 10/24/2014, approval of this research expires on that date. When you have completed your research, please submit a Study Closure request in iRIS so that IRB records will be accurate.

Use of the approved, stamped consent document(s) is required. The new form supersedes all previous versions, which are now invalid for further use. Only approved investigators (or other approved key study personnel) may solicit consent for research participation. Participants or their representatives must receive a copy of the consent form(s).

In the conduct of this research, you are responsible to follow the requirements of the Investigator Manual.

On behalf of Sophia Dziegielewski, Ph.D., L.C.S.W., UCF IRB Chair, this letter is signed by:

Joanne Muratori

IRB Coordinator

Page 2 of 2

APPENDIX G: PARENT CONSENT FORM

University of Central Florida

Informed Consent form for Parents

Title: Investigating the Relationships between Preference, Gender, and High School Students' Geometry Performance

Principle Investigator: Bhesh Raj Mainali

Faculty Supervisor: Dr Erhan Selcuk Haciomeroglu, PhD

Investigational Site: [redacted]

How to Return this Consent Form: Please return this form to the school where your child goes.

Introduction: Researchers at the University of Central Florida (UCF) study many topics. To do this we need the help of people who agree to take part in a research study. Your child is being invited to take part in this research study because he or she is student in a geometry or algebra class.

The person doing this research is a PhD candidate in mathematics education track, at school of Education, University of Central Florida, Orlando, FL. This research study is conducted under the guidance of the associate professor Dr. Erhan, Selcuk Haciomeroglu, School of Education, University of Central Florida, FL. In this study, students are expected to take a geometry test followed by a geometry questionnaire in a regular classroom. The test will be an hour long.

What you should know about a research study:

- Someone will explain this research study to you.
- A research study is something you volunteer for.
- Whether or not you take part is up to you.
- You should allow your child to take part in this study only because you want to.
- You can choose not to take part in the research study.
- You can agree to take part now and later change your mind.
- Whatever you decide it will not be held against you or your child.
- Feel free to ask all the questions you want before you decide.

Purpose of the research study: The Purpose of this study is to investigate the relationships between preference, gender, and students' geometry performance.

What your child will be asked to do in the study: Your child participation will involve completing educational tests. In this study, since we will be obtaining students' demographic information and test scores (Geometry End of Course Exam scores or any other standardized test scores) that are identifiable by student name, we will ensure that we have specific permission from the schools to access those records that fall under FERPA rules.

UCF University of Central Florida IRB
IRB NUMBER: SBE-13-09703
IRB APPROVAL DATE: 2/25/2014
IRB EXPIRATION DATE: 10/24/2014

It is also possible that your child will be individually instructed or interviewed by the researchers to determine your preference for solution strategies. The tasks we will administer in the tests or the interviews will closely resemble those problems already assigned by child's instructor. We will assist you during interviews and discuss the solutions to the problems after the interviews. You or your child will not lose any benefits if your child skips questions or tasks. The tests will be administered in a regular class period. Your child will be asked verbally for his/her assent before the test. A clear and understandable explanation will be provided for child/minor whether he/she want to participate in this or not. If your child does not want to take part in the test, he/she will be allowed to engage in educational activities without disturbing other participants in the classroom. Alternatively, the cooperating teacher will make arrangement to engage him/her in relevant lesson activities if your child does not want to take part in the test.

Location: The test will be administered in a regular classroom of the school day in the school where your child goes.

Time required: I expect that your child will be engaged for an hour for the tests.

Audio or video taping: It is possible that your child will be audio taped during this study. If you do not want your child to be audio taped, your child will be able to be in the study. Discuss this with the researcher or a research team member. If your child is audio taped, the tape will be kept in a locked, safe place. The tape will be erased or destroyed 3 years after the end of the study unless the IRB advises to destroy at an earlier time.

Risks: There are minimal risks to participants to this project. This may include the unlikely breach of confidentiality. Students do not have to answer every question or complete every task. Students will not lose any benefits if they skip questions or tasks. Students do not have to answer any questions that make them feel uncomfortable.

Benefits: We cannot promise any benefits to you, your child, or others from your child taking part in this research. However, possible benefit of the study to your child is an enhanced conceptual understanding of the fundamental concepts of geometry. Moreover, the instruments that will be used in this project will provide your child and his/her teacher with information about how your child processes mathematical information or what his/her cognitive preferences are.

Compensation or payment: There is no compensation, payment or extra credit for your child's taking part in this study.

Confidentiality: I will limit your personal data collected in this study. Efforts will be made to limit your child's personal information to people who have a need to review this information. In this study, after obtaining your child's test scores (e.g., Geometry End of Course Exam), we will use a numerical code system; that is, numerical codes will replace student names to protect confidentiality of your child.

Study contact for questions about the study or to report a problem: If you have questions, concerns, or complaints, or think the research has hurt your child talk to Bhesh Raj Mainali, Graduate Student, Mathematics Education Track, College of Education, (407) 808 8806 or Dr. Erhan Haciomeroglu, Faculty Supervisor, Mathematics Education Track, College of Education, (407) 823-1366 or by email at mainalibhesh@knights.ucf.edu

IRB contact about you and your child's rights in the study or to report a complaint: Research at the University of Central Florida involving human participants is carried out under the oversight of the Institutional Review Board (UCF IRB). This research has been reviewed and approved by the IRB.

UCF University of Central Florida IRB
IRB NUMBER: SBE-13-09700
IRB APPROVAL DATE: 2/25/2014
IRB EXPIRATION DATE: 10/24/2014

For information about the rights of people who take part in research, please contact: Institutional Review Board, University of Central Florida, Office of Research & Commercialization, 12201 Research Parkway, Suite 501, Orlando, FL 32826-3246 or by telephone at (407) 823-2901. You may also talk to them for any of the following:

- Your questions, concerns, or complaints are not being answered by the research team.
- You cannot reach the research team.
- You want to talk to someone besides the research team.
- You want to get information or provide input about this research.

Withdrawing from the study: You may decide not to have your child continue in the research study at any time without it being held against you or your child. If you decide to have your child leave the research there will not be any adverse effects or harms to your child. If you decide to have your child leave the study, contact me so that I can exclude your child from the study.

Your signature below indicates your permission for the child named below to take part in this research.

DO NOT SIGN THIS FORM AFTER THE IRB EXPIRATION DATE BELOW

[redacted]
Name of participant

Signature of parent or guardian

03-18-14
Date

[redacted]
Printed name of parent or guardian

☒ Parent
☐ Guardian (See note below)

☐ Obtained

Note on permission by guardians: An individual may provide permission for a child only if that individual can provide a written document indicating that he or she is legally authorized to consent to the child's general medical care. Attach the documentation to the signed document.

University of Central Florida IRB
IRB NUMBER: SBE-13-09700
IRB APPROVAL DATE: 2/25/2014
IRB EXPIRATION DATE: 12/24/2014

APPENDIX H: RESEARCH SITE APPROVAL

BENEFIT FOR [redacted] SCHOOLS

All research approved must have a specific benefit to the students, teachers and/or administrators of [redacted] Schools. Please describe in detail how this research project directly benefits the district.

Students will get oppurtunities to take part in a geometry test where they will get chance to explore different ways that geometry problems can be solved. It also helps to enhance students' geometry understanding.

This study is anticipated to inform mathematics teachers and school administrators about the importance of the representation in teaching and learning mathematics, which will provide insights about which mode of representation could be useful for teaching and learning geometry, and its correlations to students' cognitive style and mathematical performance.

ATTACH THE FOLLOWING ITEMS TO THIS FORM:

- A copy of your IRB approval
- (2) Two copies of your approved proposal, grant, or project
- All survey and/or interview instruments

ASSURANCE

Using the proposed procedures and instrument, I hereby agree to conduct research within the policies of [redacted] Schools. Deviations from the approved procedures must be cleared through the Senior Director of Accountability, Research and Assessment. Reports and materials should be supplied when specified.

Requester's Signature ____ Date 10/28/2013

NOTE TO REQUESTER: When seeking approval at the school level, a copy of the entire Request Form, signed by the Senior Director, Accountability, Research, and Assessment, should be shown to the school principal who has the option to refuse participation depending upon any school circumstance or condition. The original Research Request Form is preferable to a faxed document.

APPROVAL STATUS

☒ **Approved:** The research request was completed in full and the research meets all [redacted] requirements. The following must be completed to meet security requirements before your research can begin:

You are only permitted to contact teachers participating in the Geometry Professional Development Series (GPS) project for participation. Sample size is not to exceed 350, and consent forms must be provided for all student participants.

Revised 9.13.13

☐ **Conditionally Approved:** The research request contains one or more elements that must be clarified or are missing. However, the request has an opportunity to be approved if the following is completed:

Please make these changes within two weeks and resubmit the entire Request Form and supporting documents.

☐ **Rejected:** The research request contains significant omissions and/or does not meet [redacted] requirements. This research request has been rejected due to the following:

Signature of the Senior Director for Accountability, Research and Assessment

Date

February 19th, 2017

Revised 9.12.13

APPENDIX I: COPYRIGHT CLEARANCE

Copyright Clearance Center

Confirmation Number: 11154603
Order Date: 01/27/2014

Customer Information

Customer: Bhesh Mainali
Account Number: 3000743773
Organization: Bhesh Mainali
Email: bhesh.mainali@ucf.edu
Phone: +1 (407)8088806
Payment Method: Invoice

This not an invoice

Order Details

Problems of representation in the teaching and learning of mathematics

Billing Status: **N/A**

Order detail ID:	64363005
ISBN:	978-0-89859-802-5
Publication Type:	Book
Publisher:	LAWRENCE ERLBAUM ASSOCIATES, INCORPORATED
Author/Editor:	JANVIER, CLAUDE ; UNIVERSITE DU QUEBEC A MONTREAL

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Requestor type	Academic institution
Format	Print, Electronic
Portion	chart/graph/table/figure
Number of charts/graphs/tables/figures	2
Title or numeric reference of the portion(s)	A model for competency in mathematical problem solving, Translation process among four modes of representations
Title of the article or chapter the portion is from	Chapter 3:Translation process in mathematics education, Chapter 12: Cognitive representational systems for mathematical problem solving
Editor of portion(s)	N/A
Author of portion(s)	N/A
Volume of serial or monograph	N/A
Page range of portion	28,136
Publication date of portion	1987
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In the following language(s)	Original language of publication

APPENDIX J: COPYRIGHT PERMISSION

Gmail by Google

Bhesh Mainali <mnarad@gmail.com>

FW: Copy right permission

1 message

Bhesh Mainali <Bhesh.Mainali@ucf.edu> Sun, Mar 16, 2014 at 7:39 AM
To: "mnarad@gmail.com" <mnarad@gmail.com>

From: Eleanor H. ROSCH [rosch@berkeley.edu]
Sent: Monday, February 10, 2014 6:49 PM
To: Bhesh Mainali
Cc: palmer@cogsci.berkeley.edu
Subject: Re: Copy right permission

You also have my permission to use Figure 9.1 from Chapter 9 in E. Rosch & B.L. Lloyd (Eds.) Cognition and Categorization, Hillsdale, N.J.: Erlbbaum.

Eleanor Rosch

On Mon, Feb 10, 2014 at 1:58 PM, Bhesh Mainali <Bhesh.Mainali@ucf.edu> wrote:

> Hello Dear Dr Rosch and Dr Palmer, Good afternoon,
>
> I am Bhesh Mainali, PhD candidate in mathematics education, University of Central Florida, working on my dissertation. I would like to use of one of the figures in my dissertation from your book " Cognition and Categorization". The figure is from the chapter 9, fundamental aspects of cognitive representation (fig 9.1 page 263). In this regard, I would like you to grant me permission to use this figure in my dissertation.
>
> Thank you very much for your help and support.
>
> Bhesh Mainali
> PhD candidate, Mathematics Education Track
> University of Central Florida, Orlando, FL
> USA

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