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## The Influence Of Virtual Manipulatives On Second Grader's Acquisition Of Place Value Concepts

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**THE INFLUENCE OF VIRTUAL MANIPULATIVES ON  
SECOND GRADER'S ACQUISITION OF PLACE VALUE CONCEPTS**

by

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A thesis submitted in partial fulfillment of the requirements  
for the degree of Master of Education in K-8 Math and Science  
in the Department of Teaching and Learning Principles  
in the College of Education  
at the University of Central Florida  
Orlando, Florida

Spring Term  
2011

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## ABSTRACT

The purpose of this study was to examine my own practice of teaching place value and the influence virtual manipulatives had, in addition to physical manipulatives, on place value understanding of my second grade students. I wanted to see how adding a base-ten computer applet might better meet the needs of all learners while also meeting the needs of today's technological classroom. Through this study, I found that both physical and virtual manipulatives helped students acquire place value concepts. I found that virtual manipulatives had features that engaged students in a way that increased their mathematical language, increased students' ability to represent more conceptual understanding of composing and decomposing numbers, and express enthusiasm towards mathematics. A pretest and posttest revealed that students' academic performance increased. While research on virtual manipulatives and mathematical achievement is fairly recent, this study offers insight to other classroom teachers and the research community.

## ACKNOWLEDGMENTS

I would like to thank the Lockheed Martin Corporation for providing an endowment that has allowed so many teachers to pursue their passion of teaching math and science. Thank you to the Lockheed Martin Academy faculty for their expertise in helping me to grow personally and professionally in my practice. Specifically, I would like to thank Dr. Juli Dixon. You have shown me that many challenges in life come our way, expected and unexpected, but with patience, goal setting, and a positive attitude, new doors open that can lead to an incredible journey!

Thank you to my co-hort. You are an amazingly talented group of teachers who have made attending classes not only intellectually challenging, but magical! A special thank you goes to Michelle Bartley. Your support, encouragement, humor, and advice helped me to survive these last two years! I am thankful we could share this journey together.

Last, I must thank my family for their patience and understanding during my endless days and nights of typing. Thank you to my mom and dad for cheering me on as I pursue my goals in education.

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## CHAPTER ONE: INTRODUCTION

### Rationale

Glance into many elementary mathematics classrooms and you will see students using hands-on manipulatives as they explore mathematical concepts and procedures. Various research studies have shown that the use of manipulatives has improved mathematical achievement and conceptual understanding (Bryant, 1992; Fuson, 1990; Reimer & Moyer, 2005; Reys, R. E., Lindquist, M. M., Lambdin, D. V., & Smith, N. L., 2007; Rosen, D., & Hoffman, J., 2009; Steen, Brooks, & Lyon, 2006). With an increased use of computer technology in today's classrooms, would similar improvement be realized from the use of virtual manipulatives?

Virtual manipulatives are pictorial representations that have the ability to be manipulated by the user via a computer in much the same way as concrete manipulatives can be physically manipulated by hand. The use of virtual manipulatives in mathematics education is a fairly recent practice (Suh, Moyer, & Heo, 2005) with few studies related to their effectiveness in learning (Steen, et al., 2006). However, as more and more educators are being empowered with technological tools in their classrooms, such as interactive whiteboards (Hwang, Su, Huang, & Dong, 2009; Mildenhall, Swan, Northcote, & Marshall, 2008), many are advancing their training and beginning to use virtual manipulatives as a means of instruction in mathematics. In a study of first graders who were given the availability to use only virtual manipulatives while learning geometry, there was a significant increase in their pre/post test scores over the control group who did not have availability to use the virtual manipulatives, but only worksheets, the textbook, and physical manipulatives. This treatment group also showed increased motivation and challenged

themselves to higher levels while using the virtual manipulative compared to the control group. (Steen, et al., 2006). Likewise, in a study of third grade students' use of virtual manipulatives to learn about fractions, Reimer & Moyer (2005) indicated a "statistically significant improvement in students' posttest scores on a test of conceptual knowledge, and a significant relationship between students' scores on the posttests of conceptual knowledge and procedural knowledge" (p. 2). This study indicated there were benefits to using virtual manipulatives in conjunction with physical manipulatives. Over half of the students in this study increased their conceptual knowledge of fractions after using the virtual manipulatives. A theme that emerged from this study indicated that practicing with the visual computer images might have enhanced students' abilities to explain and represent their thinking using pictorial models (Reimer & Moyer, 2005). Not only have virtual manipulatives been connected with an increase in student test scores, but they also have the potential to demonstrate the processes involved when children are engaged in doing mathematics. "Virtual manipulatives, used in combination with concrete manipulatives and other real world exploration, and in ways that encourage discussion and critical thinking, can make a unique and significant contribution to young children's mathematics education" (Rosen, D., & Hoffman, J., 2009, p.32). There is much more emphasis on the importance of deeper understanding of mathematics and new standards are shaping the mathematics curriculum in the United States (NCTM, 2006).

Florida adopted new mathematics standards in 2007. These Next Generation Sunshine State Standards (FLDOE, 2007) are modeled after the Curriculum Focal Points (National Council of Teachers of Mathematics, 2006). According to the focal points, one of the three areas for second grade mathematics is "Number and Operations: Developing an understanding of the

base-ten numeration system” (p. 14). In order for my second grade students to gain a deeper understanding of and fluency with place value I believe that a combination of concrete, representational, and abstract experiences are essential to build an understanding of the base-ten numeration system. By integrating virtual manipulatives into my place value unit, my hopes are that I will provide my students with an increase in their conceptual understanding of place value.

### Purpose

The purpose of my action research was to investigate the relationship between using virtual manipulatives and mathematical understanding in my second grade class and how this will inform my practice of teaching place value. I currently teach second grade and the idea of using virtual manipulatives became of interest to me during the last school year when I began using the interactive whiteboard as a way to enhance my mathematics instruction with technology. Not only did I see an instant fascination among my students but the power it held in reaching all the learning styles and levels of learning in my classroom was astonishing. When I began teaching my unit on place value I found a web site from the internet and allowed the students to simply explore using the base ten blocks to create numbers of their choice up to the hundreds place. The students were immediately engaged in mathematical language as they explained how they were moving the base ten blocks around the screen to create their numbers. Other students also chimed in with suggestions for new numbers. I still relied heavily on the physical manipulatives for my active instruction during this unit but I felt that the level of engagement increased when we used the virtual manipulatives. I was looking for this increase of motivation in my classroom that studies revealed (Reimer & Moyer, 2005; Steen, et al., 2006). I

wanted to systematically study the impact on student learning in place value while I taught using these virtual manipulative in conjunction with physical manipulatives.

### Research Question

As I began researching about both physical and virtual manipulatives to inform my teaching practice, I designed my action research to answer the following question.

How will the use of virtual manipulatives, in addition to physical manipulatives, influence the mathematical understanding of my students and inform my practice of teaching place value in my second grade class?

### Significance of the Study

With a variety of virtual manipulatives now accessible on the internet and through the mathematics textbook companies, it is crucial that research be conducted to determine the most effective ways to use this new instructional tool in the mathematical setting. This study guides teachers toward developing new ideas about how students learn place value using this new format for manipulatives. It may also aid teachers in instruction by furthering their ability to help students make the connection between concrete and symbolic representations to emphasize deeper understanding of place value.

### Conclusion

As I refine my teaching to meet the Next Generation Sunshine State Standards for Mathematics, I welcomed the challenge of integrating technology into my mathematics instruction to further student understanding of place value. Through an extensive literature review, I found research that supports the use of virtual manipulatives for mathematical

instruction. In Chapter 2, I review literature that supports using both physical and virtual manipulatives to help students move through a combination of conceptual experiences to build a deeper understanding of the base-ten numeration system. Subsequent chapters include the methodology of this study, data analysis, and conclusions made based on the data.

## **CHAPTER TWO: LITERATURE REVIEW**

### Introduction

As technology of the 21<sup>st</sup> century steadily moves into the classroom, there has been a recent surge in research on virtual manipulatives related to their effectiveness in classroom instruction and learning of mathematics. The study presented here was based on a multi-chapter unit of study on place value (Houghton Mifflin Harcourt, 2009) therefore, cognition of place value and the instructional practices used to teach it are pertinent as a primary focus. This literature review continues with research related to physical manipulatives and the recent research on virtual manipulatives as a basis for the conceptual learning of place value. To summarize this literature review, research on the advantages to and concerns about virtual manipulatives will be explored.

### Place Value

Place value can be expressed as the product of the value (location of a digit in a number) and its face value (value of the digit without regard to position); for example in the number 425 the location of the digit 2 has a value of ten, while the digit itself (face value) indicates that there are two tens. The basis for the number system today is derived from the Hindu-Arabic system on principles of grouping by ten (Sharma, 1993). It is represented by using only 10 digits (0-9) and as a result, we are able to represent an indefinite amount of numbers while only using a small number of symbols. There are four important characteristics in our numeration system: place value, base of ten, use of zero and additive property. These properties make the system efficient and contribute to the development of number sense (Reys, et al., 1997, p.180).



## Learning and Cognition of Place Value Concepts

“One of the most important arithmetical concepts to be learned by children in the early elementary grades is that of place value” (Sharma, 1990, p.3). The development of place value comes from many meaningful experiences, and research studies have concluded that instructional practices need to be clear and cover a wide range of place value learning experiences (Hiebert & Wearne, 1992; Baroody, 1990; Sharma, 1993). An essential prerequisite to meaningful understanding of place value is early and frequent hands-on counting activities, such as those that will develop grouping concepts (Hiebert & Wearne, 1992; Reys, et al., 1997; Sharma, 1993). Hiebert & Wearne’s (1992) research stated that “understanding place value involves building connections between the key ideas of place value, such as quantifying sets of objects by grouping by 10 and treating the groups as units” (p.99). Reys (1997) suggests two key ideas in the fundamental learning of place value: “explicit grouping or trading rules are defined and consistently followed, and the position of a digit determines the number being represented” (p.180).

These are the most common misconceptions students have when learning place value along with making errors such as writing 20056 for two hundred fifty-six and making the transition to the next decade or hundred when counting aloud or writing, ex: 48, 49,4010 (Baroody,1990; Reys, et al., 1997; Sharma, 1993). Many students may be able to identify the place value for numbers using rote memorization, that does not mean that have they conceptual understanding. A belief often held by teachers is that students understand place value concepts because they can place numbers correctly in a place value chart or on a mat when this actually demonstrates procedural knowledge. Students need many and varied experiences learning and

applying the face value and complete value of a digit (Reys, et al.,1997). “Most young children focus their attention on the absolute position of the number and they have difficulty in focusing on the relative positions of the number. Because of the confusion, place value is a difficult concept for most children” (Sharma, 1993, p.12).

Conceptual learning of place value supports students’ efforts to build relationships between quantities and actions on quantities that are represented physically, pictorially, verbally, and symbolically. Cognitively, building these relationships between external representations supports more associated and useful internal relationships (Hiebert & Wearne, 1992). In a study conducted by Hiebert & Wearne (1992) on the links between teaching and learning place value with understanding in first grade, 4 classes received an alternative instruction based on the authors’ theoretical framework that conceptual understanding is constructed as connections are built between representations of mathematical ideas, while the other two classes received textbook instruction. The students in these 4 classes were able to demonstrate a deeper understanding of the groupings of ten during place value instruction. It was suggested that the students in the alternative instruction class had begun connecting representations, that is the processes and products that form in one’s mind, and could make use of written notation to represent groups of ten because they had internalized certain representations and then used them to compose and decompose tens and ones to solve two-digit addition and subtraction problems mentally (Hiebert & Wearne, 1992). Research even suggests that students should learn multi digit addition and subtraction alongside place value concepts suggesting that this provides a context for motivating and supporting the development of base-ten number concepts (Baroody, 1990; Fuson, 1990; 1997).

Through research, base-ten blocks have shown that children learn place value concepts that move from concrete to abstract. Base-ten blocks also assist students in acquiring grouping and trading rules and determining the value of the number being represented (Baroody 1990; Fuson, 1990). They allow children to see and touch the ideas they are being asked to conceptualize. Base-ten blocks usually come in four sizes. There is the cube which represents a value of 1. The "long" is a block that looks like 10 of the cubes glued together; it represents a value of 10. The "flat" is a block that looks like 10 of the "longs" glued together; it represents a value of 100. Finally, to represent a value of 1,000 there is the "block"; the size of 10 of the "flats" laid on top of each other and glued together. In Fuson's (1990) study, students were successful in demonstrating meaningful multidigit addition and place-value concepts up to at least four-digit numbers using these base-ten blocks. Students used base-ten blocks that embodied the English named value system of number words and digit cards to embody the positional base-ten system of numeration. Steps in addition and subtraction of four-digit numbers were motivated by the size of the blocks and then were carried out with the blocks; each step was immediately recorded with base-ten numerals. Fuson found this process helped direct students' attention to critical features of the mathematical systems and embodiments and support the construction of links among the different systems and embodiments. In a study conducted by Fuson & Briars (1990) they found that multidigit addition and subtraction algorithms were attainable by most second graders and concluded that they achieved a greater level of conceptual understanding if they were supported with the assistance of manipulatives that "embody the relative size of the base-ten places and demonstrate the positional nature of the multi digit written marks and if the focus of such learning is understood, and not just procedural

competence” (1990, p. 204). Teachers facilitate support of place value concepts through instruction.

### Instruction of Place Value Concepts

Studies and research indicate teachers must change their method of instruction to help students reach a more profound understanding of mathematics (Baroody, 1990; NCTM, 1989; Reys, et al., 1997; Sharma, 1993). Sharma (1993) states when children are introduced to the place value concept with the help of inefficient activities such as sequential counting and premature paper and pencil activities instead of appropriate concrete materials, children learn to manipulate just the symbols rather than having an understanding of the concept. Therefore, difficulties children have learning place value may come from inefficient teaching. Place value plays a crucial role in the development of children’s comprehension of number concepts (NCTM, 1989). NCTM (2006) stated in their expectations for grades Pre-K through 2<sup>nd</sup> grade that students should:

Use multiple models to develop initial understandings of place value and the base-ten number system.

Develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers.

Connect number words and numerals to the quantities they represent, using various physical models and representations. (p.14)

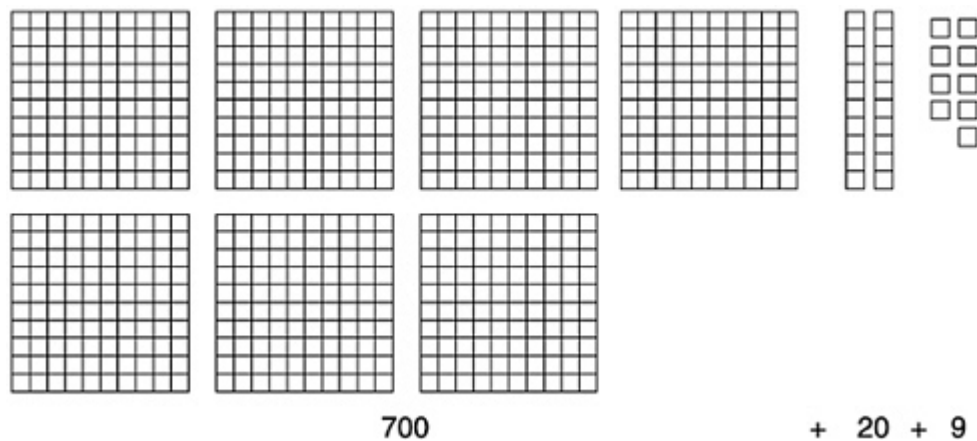
It is imperative to present place value in ways that students have the opportunity to develop mathematical competence (NCTM, 2000). Much research has been conducted on the importance of teaching place value as a foundation for mathematical competence and the significance of

teaching it conceptually (Baroody, 1990; Fuson, 1990; Fuson and Briars, 1990; Fuson et al., 1997; Reys, et al., 1997; Irons, 2002; Sharma, 1993; NCTM, 1989; 2000). Place value is the cornerstone of our number system and we use it throughout all of our work with whole numbers and later with decimals (Reys, et al., 1997). Conceptually-based frameworks should be used in the instruction for the acquisition of place value concepts. Therefore, a good teaching sequence of place value includes a range of language experiences that involve real world experiences, classroom materials, language that stresses place value, and finally the symbols of place value (Heddens, 1986; Irons, 2002). In order to lessen a student's dependence on concrete activities and bridge the gap to the abstract level, teachers need to carefully plan for a sequencing of activities for a smooth transition (Heddens, 1986). For optimization of these experiences, Sharma (1993) states that:

Concrete activities must link concrete models/materials to pictorial to abstract/written strongly and tightly. The models to be used must be both quantitative (sequential) and qualitative (visual/spatial) in nature to meet the needs of the concept representation and students with different learning personalities. The concrete experience should be recorded immediately both in representational and written form after or during the concrete activity. (p.10)

The learning approach of Concrete-Representational-Abstract (CRA), (Access Center, 2004) meets these goals. The CRA learning approach first incorporates the use of hands-on manipulatives in the concrete stage with visual, tactile, and kinesthetic experiences to establish the understanding of numbers. The concrete stage is followed by pictorial displays in the representational phase where students expand their understanding through pictorial

representations of the concrete objects. In figure 1, the number 729 is shown using pictures of base-ten blocks in the representational phase (National Research Council, 2001).



**Figure 1: Pictorial representation of 729 with pictures of base-ten blocks (NRC, 2001, p.97)**

The student then moves to the next phase of understanding the meaning of numbers through abstract reasoning of numerical symbols. Expanded notation is one example that helps students understand the meaning of each place value through symbolic representation. The number 729 expanded would be  $700 + 20 + 9 = 729$  (NRC, 2001). With CRA, students can be provided with support from more than one level at a time as they progress between the stages at their own rate of learning. “Place value concepts emerge over time as children become increasingly flexible and efficient in the use of base-ten materials. As their use of the materials becomes more automatic, they come to depend less on the manipulations of the physical materials themselves. Over time they are able to abstract their solutions with physical materials so that they can add and subtract multi-digit numbers without them” (Fuson, 1997, p.134).

## Manipulatives

Research on reform from the last 30 years indicates positive support that students' mathematical understanding will be deeper if manipulative materials are used for mathematics instruction, especially with a concept as important as place value (Clements, 1996; 1999; Heddens, 1997; Reys, et al., 1997; Sharma, 1993). Manipulatives help make connections between the students' concrete understanding of mathematical concepts and abstract mathematical ideas (Baroody, 1990; Fuson, 1990; 1997; Heddens, 1997; Irons, 2002). Using manipulative materials in teaching can help students learn how to relate real world situations to mathematics symbolism and work together cooperatively in solving problems. Furthermore, manipulatives allow students' to discuss mathematical ideas and concepts, and verbalize their mathematical thinking (Heddens, 1997; Hiebert & Wearne, 1992; Reys, et al., 1997; Sharma, 1993). Manipulatives can be classified into two types, physical and virtual.

### Physical manipulatives

Physical manipulatives are defined as materials that are physical objects that can be touched and moved by students to introduce or reinforce a mathematical concept. By appealing to several senses, physical manipulatives can be used as important tools that allow students to reach higher levels of thinking (Heddens, 1997). Historically, manipulatives have been used as early as the Romans. Examples such as the abacus, counting sticks, blocks and fingers are considered manipulatives. Examples of physical manipulatives that are used in the classroom today are Cuisenaire rods, geoboards, pattern blocks, base-ten blocks, attribute blocks, color tiles, and Unifix cubes. Physical manipulatives provide a hands-on experience. These experiences focus attention and increase motivation, improve students' attitude toward mathematics, and help

students retain information and increase scores on assessments (Bryant, 1992; Sowell, 1989; Suydam & Higgins, 1977). Suydam and Higgins concluded that using manipulative materials produces greater achievement gains than not using them in their study on Kindergarten through eighth grade activity-based learning in mathematics (Suydam & Higgins, 1977). Sowell (1989) performed a meta-analysis of 60 studies to examine the effectiveness of manipulatives used in mathematics with kindergarten through postsecondary students. Sowell concluded that the long-term use of concrete instructional materials by teachers knowledgeable in their use improved student achievement and attitudes. In a study from Bryant (1992) designed to provide teachers' mathematics instruction through the use of manipulatives with at-risk fourth through sixth graders, an increase in test scores and grades resulted. This study was designed to help teachers provide alternative instruction other than the textbook. Teacher surveys and school test scores indicated a need for better problem solving and critical thinking. Students used ready made as well as teacher made manipulatives over an eight month period. Half of the 65 at-risk students improved in the target mathematical objectives as indicated by an increase in their test scores and by their letter grades in the fourth quarter. It was noted in this study that other strategies such as peer tutoring and computers were also used as other alternative strategies and had an impact on student achievement as well (Bryant, 1992).

Simply allowing students to use manipulatives for mathematics instruction does not imply that learning will take place (Baroody, 1990; Clements, 1996). Manipulatives do not guarantee that students will understand the concept just because they are holding a manipulative. Thus, manipulatives do not always carry the meaning of the mathematical idea. In one collective case study about teachers using physical manipulatives, it was recognized that teachers needed to



emphasize the link between pedagogy and content, not just the specific use of manipulatives (Puchner, 2008). Teachers often do not embrace the benefits of using manipulatives properly in the classroom. Teachers may use physical manipulatives in the lesson introduction, but then instruct students to do the mathematics the paper and pencil way (Puchner, 2008). Sharma says (1993) children will learn to manipulate just the symbols rather than having an understanding of the concept, therefore, the use of concrete materials should not be limited to demonstrations (Sharma, 1993). It is important that students use manipulatives in meaningful ways rather than in a rigid and prescribed way that focuses on remembering rather than on thinking. Further, it is important that students come to see the two-way relationship between concrete embodiments of a mathematical concept and the notational system used to represent it. Therefore, it is important to consider the context and activity in which they are being presented and when evaluating the potential of physical representations to support learning (Clements, 1999; Manches, A., O'Malley, C., & Benford, S, 2009). It is essential to make sure students explain what they are doing and link their work with manipulatives to underlying concepts and formal skills. Students must use these manipulatives in the context of well-planned activities and ultimately reflect on their actions in order to grasp the idea.

### Virtual Manipulatives

Although relatively new, virtual manipulatives can support learning in mathematics (Clements, 1996; Clements & Sharma, 2002; Olkun, 2003; Reimer & Moyer, 2005; Steen, et al., 2006). Virtual manipulatives are pictorial representations that have the ability to be manipulated by the user via a computer in much the same way as concrete manipulatives can be physically manipulated by hand. Moyer, P. S., Bolyard, J. J., & Spikell, M. A. (2002) defined a virtual

manipulative as, “an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (p. 373). Virtual manipulatives can be found on the Web as applets, which are simply smaller versions of application programs and include representations of physical manipulatives such as pattern blocks, geoboards, or base-ten blocks. Mathematics textbooks are also now incorporating them as part of the mathematics curriculum (Houghton Mifflin Harcourt, 2009).

Although virtual manipulatives have similar characteristics as physical manipulatives, virtual manipulatives have additional enhancements that allow for increased mathematical actions. They perform specific mathematical transformations on objects on the screen. For example, instead of trading physical base-ten blocks (trade 1 ten, for 10 ones), children can break computer base-ten blocks into 10 ones by moving them from one place value section to another throughout the screen. This movement transforms the blocks to compose and decompose automatically and also links the blocks to the symbols. The number represented by the base-ten blocks is dynamically linked to the child’s actions on the computer, so when the blocks are changed, the displayed number changes as well. Such actions are more inline with mental actions we want students to learn (Clements & Sharma, 2002). Computer manipulatives can help students build on their physical experiences, tying them tightly to symbolic representations. In this way, computers help students link Sensory-Concrete and abstract knowledge so they can build Integrated-Concrete knowledge (Clements, 1996). Clements (1996) states,

Students demonstrate *sensory-concrete* knowledge when they use sensory material to make sense of an idea. For example, at early stages, children cannot count, add, or subtract meaningfully unless they have actual objects to touch. *Integrated-concrete*

knowledge is built through learning. It is knowledge that is connected in special ways.

Mathematical ideas are ultimately made integrated-concrete not by their physical or real-world characteristics but rather by how "meaningful" -- connected to other ideas and situations. *Concrete* cannot be equated simply with physical manipulatives. Computers might supply representations that are just as personally meaningful to students as real objects; that is, they might help develop integrated-concrete knowledge (p.273).

The use of virtual manipulatives has been shown to have value, may also develop more complex understandings of concepts, and impact student achievement (Olkun, 2003; Reimer & Moyer, 2005; Steen, et al., 2006). Olkun (2003) compared the effect of computer versus concrete manipulatives for the learning of two-dimensional geometry. Participants were 93 4th and 5th grade students. A pretest, treatment, and posttest experimental design was used. There were three treatment groups: computer, concrete, and control group. The computer groups solved computer-based Tangrams. The concrete group solved wooden Tangrams. The control group had no filler activity. Both computer and concrete groups improved significantly, the computer group slightly more, after the intervention. Fourth graders gained more in concrete situation, while fifth graders benefited more from the computer manipulatives. Boys and fifth graders gained more than girls and fourth graders respectively (Olkun, 2003). In a study of third grade students' use of virtual manipulatives to learn about fractions, Reimer & Moyer (2005) indicated a "statistically significant improvement in students' posttest scores on a test of conceptual knowledge, and a significant relationship between students' scores on the posttests of conceptual knowledge and procedural knowledge" (p.2). Reimer taught a fraction unit to 19 third graders. Virtual manipulatives were used in a computer lab setting for two weeks. Data were collected from

pretests and posttests of students' conceptual and procedural knowledge. Student attitude surveys were completed and results revealed that virtual manipulatives helped students learn by providing immediate and specific feedback. The survey results also showed that students found the virtual manipulatives to be faster and easier to use than paper and pencil. Virtual manipulatives were also found to enhance the students' enjoyment while learning fractions. Likewise, first graders who were given the availability to use virtual manipulatives while learning geometry had a significant increase in their pre/post test scores over the control group who did not have availability to use the virtual manipulatives (Steen, et al., 2006). This study investigated the impact of virtual manipulatives on first grade students' academic achievement as well as on student attitudes, behaviors, and interactions. Thirty-one first graders were assigned to either the control or treatment group. Both groups studied identical objectives, but the treatment group used virtual manipulatives for practice. The first graders took two different sets of pretests and posttests, one at the first grade level and one at the second grade level. The following results occurred:

On the first grade level pretest, the treatment group scored significantly lower than the control group. At posttest, following the completion of the geometry unit, the treatment group closed the gap and actually slightly outscored the control group, though not at a significant level. The post hoc analysis demonstrated a significant change within the treatment group from pretest to posttest, but no significant change within the control group. The change data also showed that the treatment group had a significantly greater overall improvement during the geometry unit. On the second grade level pretest, the treatment group scored slightly lower than the control group. At posttest, following the

completion of the geometry unit, the treatment group again surpassed the control group, but the difference was not statistically significant. The post hoc analysis demonstrated a significant change within both the treatment and control groups from pretest to posttest. The change data also showed that the treatment group had a significantly greater overall improvement (p.385)

The teacher of the treatment group reported increased instructional time, repetition of practice activities, time-on-task, and feedback from her daily notes regarding virtual manipulatives. She found that students showed increased motivation and challenged themselves to higher levels. The treatment group did overcome large gaps and had significant improvements from pretest to posttest at both grade levels. The control group only showed a significant improvement at the second grade test level. The results indicate that the use of the virtual manipulatives as an instructional tool was extremely effective for the treatment group, and perhaps more effective than the use of the traditional text activities (Steen, et al., 2006) Another study observed that virtual base-ten blocks enabled second grade students to demonstrate more sophisticated strategies and explanations of place value after using the virtual manipulatives. In addition, they note that the English Language Learners were able to demonstrate their understanding of place-value concepts even though they could not explain them verbally (Moyer, P. S., Niezgoda, D., & Stanley, J., 2005).

Not only have virtual manipulatives been shown to correlate with a more complex understanding and an increase in mathematical achievement, but they also have the potential to demonstrate the processes involved when children are engaged in doing mathematics. One example that Clements (1999) gives is “The computer helps students make sense of their activity

and the numbers by linking the blocks to symbols. For example, the number represented by the base-10 blocks is usually linked dynamically to the students' actions with the blocks, automatically changing the number spoken and displayed by the computer when the student changes the blocks. As a simple example, a child who has 16 single blocks might glue 10 together and then repeatedly duplicate this 10. In counting along with the computer, 26, 36, 46, and so on, the child constructs composite units of 10" (p.101). One of the most important differences between virtual manipulatives and the physical manipulative is the computer's ability to provide the student instant feedback as the student works (Moyer-Packenham, et al., 2008). In this report, Moyer-Packenham (2008) claims,

virtual manipulatives are a powerful cognitive tool for learners because they constrain the user's actions on the mathematical object in the virtual environment, directing the user to focus on the mathematics in the environment; they react to user input with visual and verbal/symbolic feedback showing the user the results of their actions on the object; and, they enforce mathematical rules of behavior (Zbiek et al., 2007). As the NCTM Technology Principle indicates, "Work with virtual manipulatives...can allow young children to extend physical experience and to develop an initial understanding of sophisticated ideas like the use of algorithms" (NCTM, 2000a, pp. 26-27). In essence, virtual manipulatives have some of the advantageous properties of several different forms of representation, as well as some additional advantages brought about by their technological properties (p.204).

Reimer and Moyer (2005) reported these benefits of feedback. When students were interviewed regarding their impressions of the virtual manipulatives, an emergent theme was their appreciation for the immediate feedback possible with the computer-based manipulatives.

Many advantages to using virtual manipulatives have been reviewed. Computers encourage students to make their knowledge explicit, which helps them build integrated-concrete knowledge. “Specific theoretically and empirically grounded advantages of using computer manipulatives” follow as outlined in research from Clements and McMillen (1996):

- They provide a manageable, clean manipulative.
- They offer flexibility.
- They can change arrangement or representation.
- They can store, and later retrieve, configurations.
- They record and replay students’ actions.
- They link the concrete and the symbolic with feedback.
- They dynamically link multiple representations.
- They change the very nature of the manipulative.
- They link the specific to the general.
- They encourage problem posing and conjecturing.
- They provide a framework for problem solving, focus attention, and increase motivation.
- They encourage and facilitate complete and precise explanations. (pp.51-53)

Students working with virtual manipulatives see the advantages of manageability as well. While using the virtual manipulatives, students find them easy to organize and easy to fit pieces together (Clements, 1999). The virtual manipulatives do not come apart until the student deliberately moves the pieces with a mouse action. Students are not worried about pieces falling to the floor, or their creation being lost by a bump of a desk (Clements, 1999). Another advantage is having this free accessibility to virtual manipulatives at home can help students continue their learning in the same format that is used at school (Moyer-Packenham, et al., 2008).

This relatively new manipulative does not come without caution. Current literature brings attention to concerns about virtual manipulatives. Barta (2002) warns that if virtual manipulatives replace physical manipulatives in classrooms, students may miss out on an important kinesthetic experience needed to bridge the concrete understanding to the abstract understanding. There needs to be a balance with frequent opportunities for students to interact with real objects (Barta 2002). Another concern about using virtual manipulatives in the classroom is that teachers will need training and time to practice before using the virtual manipulatives with students (Suh, Moyer, & Heo, 2005). In Moyer, Niezgodia, and Stanley's (2005) study, teacher's knowledge of how to use the virtual manipulatives was seen as a factor in the study's positive results. Lastly, although direct feedback can be seen as an advantage for some students, it can also be seen as a disadvantage for others. Students who do not have a basic understanding of the concept being explored may find themselves using the trial and error technique to finish each exercise (Barta, 2002). On some Internet sites, as the answer is revealed, students know they have finished the exercise. The direct feedback can also be viewed by some educators as a step too soon toward symbolic representation (Barta, 2002).

After reviewing the current literature, support for my study shows that teachers are beginning to see the advantages of using technology in the classroom and many teachers are advancing their training and beginning to use virtual manipulatives as a means of instruction in mathematics (Clements, 1999; Moyer-Packenham, et al., 2008; Suh, Moyer, & Heo, 2005). Moyer-Packenham, et al. (2008) analyzed 95 lesson summaries in which K-8 classroom teachers used virtual manipulatives. Number & Operations and Geometry were the two NCTM Standards (2000) covered in most of the lessons. The teachers used virtual geoboards, pattern blocks, base-



ten blocks, tangrams, and many other virtual manipulatives. In particular, the K-2 teachers in the study used virtual base-ten blocks, virtual money, virtual pattern blocks, and virtual tangrams the most during their lessons. Over half of the virtual manipulative lessons were preceded by lessons involving physical manipulatives. One important finding of this study was that teachers used the virtual manipulatives during the main portion of their lessons when students were learning mathematics content. Clements and Sharma (1998) state that studies have shown a combination of physical and onscreen manipulatives is more effective than either alone (Clements & Sarama, 2002).

### Summary

Using physical and virtual manipulatives together supports NCTM's view of providing opportunity for students to make connections by relating various representations of concepts or procedures to one another (NCTM, 2000). The research and review of literature that has been presented supports using both physical and virtual manipulatives as instructional tools to help students bridge their conceptual knowledge of mathematics to more abstract thinking. "Virtual manipulatives, used in combination with concrete manipulatives and other real world exploration, and in ways that encourage discussion and critical thinking, can make a unique and significant contribution to young children's mathematics education" (Rosen, D., & Hoffman, J., 2009, p. 32) While the use of these types of manipulatives have shown higher levels of thinking, achievement, focus, and enjoyment they have not come without concerns of proper use or teacher training. The following chapters discusses the methodology used to conduct research examining second grade students' understanding of place value using physical and virtual manipulatives, analysis of the data collected, and the conclusions formed from this study.

## **CHAPTER THREE: METHODOLOGY**

### Introduction

The purpose of the study was to examine the effects of virtual manipulatives along with physical manipulatives on student understanding of place value. More specifically, I conducted this study to reflect on my own teaching practices using these manipulatives and how they impact student understanding. My question is, “How will the use of virtual manipulatives, in addition to physical manipulatives, influence the mathematical understanding of my students and inform my practice of teaching place value in my second grade class?” In this chapter, I describe the design, setting, procedure, data collection, and analysis of the study.

### Design of Study

The study was designed and conducted to help me better understand how second grade students conceptualize place value. I chose to conduct Action Research because it “provides teachers with a philosophy and practice that allows them to systematically study the effects of their teaching on student learning” (Mills, 2003, p. 4). It was my goal to examine my instructional methods for teaching place value in the classroom and how I could improve my instruction, in hopes of helping my students become more proficient in place value concepts. Data were collected from a pre and posttest from the second grade curriculum of Go Math! Florida (Houghton Mifflin Harcourt, 2009), a student questionnaire, student class work both written and on the computer, informal interviews with students, mathematics journals, and observations with field notes.

## Setting

### School Setting

This research study took place in a suburban area charter school in Central Florida which provides education for students in kindergarten through fifth grade. The school specializes itself to the county based on its cooperative learning and research-based reading program and students from the entire county attend this school. The curriculum meets state and federal guidelines and takes part in both norm and criterion referenced testing. According to the 2010 demographics the student body population is 67% White-Non Hispanic, 8% Black, 15% Hispanic, and 10% Other. Seven percent of the students are serviced by exceptional education programs (Specific Learning Disabilities, Gifted, and Speech and Language). Less than <1% of the students served in the basic classrooms are instructed with ESOL strategies. Approximately 27% of the students are eligible for the Free and Reduced Lunch Program offered by the state.

### Classroom Setting

The second grade classroom where this study was conducted consisted of 9 boys and 8 girls between the ages of seven and eight. The administrative staff assigned this class to me for gender balance as well as on the basis of race, social behavior, and academic achievement levels. All academic subjects were taught to the same group of students throughout the day except reading, where students were taught based on reading level. The class had 2 computers and an interactive whiteboard to aid in instruction. The classroom is set up for and instructed through cooperative learning. All students sit at tables in groups of four. The class consisted of 17 students; 11 White-Non Hispanic, 4 Black, and 2 Hispanic. Twenty-four percent of the students receive free or reduced lunch. None of these students attended Exceptional Student Education

class, but 1 student was in the Response to Intervention process for all subjects. All students participated in the study.

## Methods

### Preliminary Action

Permission was requested and the Institutional Review Board (IRB) at the university (Appendix A) determined this study was designated as “Not Human Research”. Approval for the study was obtained from the school principal (Appendix B) and the county (Appendix C). Parental consent forms (Appendix D) were passed out to each parent at student orientation day. All parental consent forms were signed and returned the first week of school, granting permission for each student to participate in the study. During the first week of school all students were given the student assent letter. It was read aloud, reviewed, and the opportunity to ask questions was given. After all permission sources were obtained, they were kept in a secure file by the researcher.

### Procedures

To begin the study, all students were given a questionnaire that reflected their attitudes and experiences towards mathematics and counting. During the first week of data collection a pretest on place value was administered (Appendix E). The pretest was comprised of place value concepts in the ones, tens, hundreds, and thousands place from the second grade curriculum of Go Math! Florida (Houghton Mifflin Harcourt, 2009). The students in this study were instructed on topics in mathematics daily for 60 minutes using physical manipulatives and or virtual mainpulatives based on the school adopted mathematics curriculum, Go Math! Florida

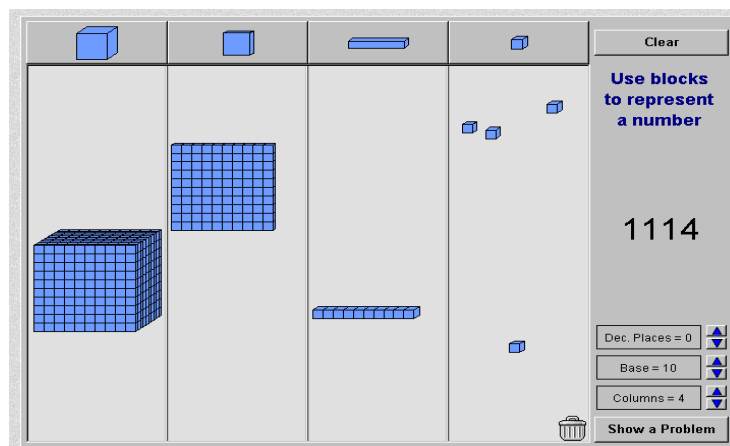
(Houghton Mifflin Harcourt, 2009). Students sat in teams of 4 and worked as partners while in the classroom. While in the lab, each student sat at an individual computer, and cooperative structures that were used in the classroom were also used in the lab. The unit on place value consisted of three chapters and was taught sequentially as presented in the teacher edition. The order of instruction can be found in Table 1.

**Table 1: Sequence of Content Addressed During Study**

Sequence of Instruction	Mathematical Content
Week One	Understanding Place Value, Expanded Form, Different Forms of Numbers, Different Ways to Show Numbers
Week Two	Even and Odd Numbers, Skip Count on a Hundred Chart, Patterns, Extended Patterns, Make Predictions for Patterns
Week Three	Hundreds, Hundreds/Tens/Ones, Place Value:3-digit Numbers, Different Forms of 3-Digit Numbers, Compare Numbers
Week Four	Compare 3-Digit Numbers, Order 3-Digit Numbers, Number Patterns, Thousands, Place Value: 4-Digit Numbers
Week Five	Different Forms of 4-Digit Numbers, Compare 4-Digit Numbers, Order 4-Digit Numbers

During and after each daily lesson, I observed and recorded both student-to-teacher and student-to-student interactions through teacher field notes. Daily lessons consisted of a problem of the day, direct instruction, guided practice focusing on the mathematical objective for the day, and independent practice time. Students were given physical base-ten blocks during the first two weeks of the study. Students were allowed exploration time before each lesson. During instruction, the students followed the lesson format of the second grade curriculum, Go Math! Florida (Houghton Mifflin Harcourt, 2009). During week 3, virtual base-ten blocks were introduced on the interactive white board for direct instruction (Mildenhall, P., Swan, P.,

Northcote, M., & Marshall, L., 2008) following the Go Math! Florida curriculum lessons. Students were introduced to the base-ten blocks applet using the textbook website provided by the mathematics textbook company (Houghton Mifflin Harcourt, 2009) it was assessible by both teacher and student under the terms of purchase. Students manipulated the pictorial versions of these on-screen blocks in a manner that was similar to moving real base-ten blocks. The second site, figure 2, was part of the National Library of Virtual Manipulatives (<http://nlvm.usu.edu>) and is free to internet users. Both programs' virtual base-ten blocks consisted of individual units, sticks (containing 10 units), flats (containing 10 sticks), and blocks (containing 10 flats) to show place value for numbers.



**Figure 2: National Library of Virtual Manipulatives base-ten blocks**

In addition to the 60 minutes of mathematics instruction, students were instructed in the computer lab two times per week for 20 minutes as an opportunity to use virtual manipulatives individually during the fourth and fifth week of the study.

The students used their mathematics journals daily to record their thinking and solve problems that the teacher or students posed. Mathematics journals were used to “see” student thinking through drawings and written word. Students used their journals to demonstrate a

reflection of partner and individual work. Periodically, focus questions posed by the teacher for this study were given and analyzed using a teacher made rubric (Appendix H).

At the end of the five week data collection period, I had a personal interview with each student at their computer station during the final week of the study. Finally, a posttest from the textbook was given to compare student responses to the pretest to determine student gains.

### Data Collection

I used several types of data collection to provide triangulation in the data. According to Mills (2003), using descriptive, narrative, and even nonwritten forms of data from a variety of sources to answer a question helps strengthen the qualitative research. Data collection included a pre and posttest from the second grade curriculum of Go Math! Florida (Houghton Mifflin Harcourt, 2009), a student questionnaire, student class work both written and on the computer, informal interviews with students, mathematics journals, and observations with field notes.

### Data Analysis

The pretest and posttest (Appendix F) were compared to examine any changes in the students' abilities to demonstrate place value knowledge. Content validity of the pretest and posttest was upheld by using a combination of problems from the end of the chapter tests on place value taken from the textbook series (Houghton Mifflin Harcourt, 2009). The same test was used for both the pretest and the posttest. The posttest was administered within four weeks of the initial place value instruction. I continually looked for emerging themes when I collected and analyzed the data. I coded the information into categories such as increase in mathematical language, attitudes, and processes that demonstrated an understanding of values or decomposing

and composing of numbers as I went through the data. The data were categorized into themes such as an increase or change in mathematical language, students' understanding of face value, students' understanding of composing and decomposing numbers, and attitudes towards mathematics.

### Summary

The qualitative action research model was chosen because it was best suited for my particular classroom setting and allowed me the opportunity to examine my practice of teaching place value. Using triangulation of data, I was able to collect the information needed to answer my research question and reveal the effects of virtual manipulatives on the acquisition of place value concepts with my students.



## CHAPTER FOUR: DATA ANALYSIS

### Introduction

As I began to use more technology in the classroom to enhance student learning in mathematics, I planned my action research to incorporate virtual manipulatives along with physical manipulatives to inform my teaching practice of place value (Barta, 2002; Clements & Sarama, 2002; Moyer-Packenham, et al., 2008). Since developing an understanding of the base-ten numeration system and place value concepts is one of the three big ideas in second grade based on the Next Generation Sunshine State Standards for Mathematics (FLDOE, 2007), I deemed it an appropriate subject to research. Knowing that place value is a difficult concept to understand (Sharma, 1993) and that my students needed multiple opportunities presented in a variety of contexts (NCTM, 2000; NCTM, 2006), using virtual manipulatives seemed to be a natural fit. I explored the following question through qualitative research:

How will the use of virtual manipulatives, in addition to physical manipulatives, influence the mathematical understanding of my students and inform my practice of teaching place value in my second grade class?

### What Did My Students Already Know?

Before I began the study, I wanted to find out the attitudes and beliefs my students held about mathematics, specifically, what their disposition was related to using the computer to help them learn. I created a questionnaire (Appendix E) and had all students complete it independently. The first question asked if they liked mathematics. Most of them said they liked mathematics, that it was fun or helped them learn about numbers. A few students said they did not like mathematics, that it was hard or boring. One student answered, sometimes. I was happy

to see that most of my students held a positive disposition about mathematics and that hopefully more students would come to enjoy it as well. I also wanted to know if my students were familiar with manipulatives. The questionnaire asked what types of things they have used before for counting. A wide range of answers revealed blocks, fingers, number line, popcorn, hundreds chart, candy, beans, cubes, and a ruler. I came to understand that most of my students had experience with using manipulatives. This would aid them in the introduction of base-ten blocks. Knowing if my students would enjoy using the computer to learn about numbers was important to this study. The questionnaire revealed that almost all of the students said it would be fun and one student said, “definitely”. Of the students who said they did not like mathematics, three students said they would enjoy learning about numbers using the computer. This gave me a positive indication that I had the ability to influence their attitude towards mathematics with this study.

To understand where my students stood conceptually with understanding place value, a pretest was given. The pretest consisted of eight problems that asked students to write the number that was modeled in base-ten blocks, compare numbers in their numerical form, demonstrate understanding of the value of a digit, and demonstrate understanding of groupings. The results showed that nine students scored below 75% and seven of those scored a 50% or below. Five students got more than 80% of the questions correct with two students scoring 100%. The pretest allowed me to see that most of my students did not understand groupings or the value assigned to a numeral based on its position in a number. My literature review revealed this was the most essential area I must focus my instruction on and the pretest revealed a need in this area (Hiebert & Wearne, 1992; Reys, et al., 1997; Sharma, 1993). Most of the students demonstrated

understanding in choosing a base-ten model for a given number but could not correctly write it in number form. Many students were writing 303 as 33 or 3003. My review of literature also revealed that these are common errors for students ( Baroody,1990; Reys, et al., 1997; Sharma, 1993). Knowing this information helped me focus my instruction of place value to meet my students' needs and see how virtual manipulatives played a role in helping to further their understanding.

### Understanding Values

Before beginning the required scope and sequence of lessons mandated by our school, I began by making sure that students could count numbers larger than 20, but less than 100, correctly using Unifix cubes and identify their numerical equivalent by choosing the correct card to match the amount they counted. During this time I also checked that the students understood that the ones cubes could be grouped in sets of single ones to equal ten for “quicker” counting and that students could count by tens and the remaining ones to name the total, as this was a first grade objective. Many students commented they did do this in first grade. All but one student demonstrated mastery through informal observation.

### Physical Manipulatives

When students were introduced to the base-ten physical manipulatives, only seven of the students commented that they had used them before in first grade for counting. Once it was modeled for students that ten ones equaled a ten stick, students demonstrated understanding by placing the ones next to the ten stick. Students were able to conceptualize a ten stick as a representation of the amount of ten based on their experience in first grade with groupings of ten.

Having a concrete model of a “ten” helped students understand how they could demonstrate numbers larger than ten using the ten sticks and ones instead of just ones (Heddens, 1997). I immediately saw the value these concrete models held in helping my students move from unitary counting (collections of single objects) to counting in sets (groups of ten) (Reys, et al., 1997). Comments from my field notes say, “See, 10, 20, 30, 40, 1, 2, 3, 4 equals 44”, and “4 of these ten sticks and 4 of these ones means I have 44 blocks”, demonstrated understanding and showed me that students understood the value of the blocks in front of them as an equal amount to the numerical value in the number 44. As I walked around the room and monitored student actions, I relied on student partners to point out to each other when their blocks did not match the number they were being asked to represent. One student in my class is in the Response To Intervention process and he struggles to show mastery of skills in the area of comprehension and application. I observed that my struggling student’s partner was not seeing his mistakes of counting the ten sticks by ones and not by groups of tens. An individual student conference revealed this misunderstanding. These physical manipulatives were used by him everyday with constant monitoring by me although he did not internalize the value of a ten stick.

Using physical manipulatives, students made the next transition from using concrete models to adding place value language by filling in their individual white boards with \_\_\_\_ tens + \_\_\_\_ ones = 44. Students then transferred the representation of the concrete model to the white board as a pictorial drawing. As the lessons in the textbook progressed and students practiced daily with physical manipulatives, through class work, homework, and exit slips the concrete models were beginning to help most students gain understanding of numbers and the face value of the digit in the number by first showing the number represented as physical manipulatives,

then drawing a quick picture of the manipulatives, then writing the value below each group of base-ten blocks (CRA), (Access Center, 2004; Heddens, 1986; Irons, 2002; Sharma, 1993).

Using the physical manipulatives was successful when comparing numbers. Since my students understood the face value of a number they made an immediate connection to the comparison of numbers by value. This was a very important step in place value learning. I learned through my literature review that many students focus on the absolute value of a digit when comparing numbers (Sharma, 1993). Having a physical model in front of the students helped the four students who missed this question on the pretest to gain a better understanding of the meaning of place value by having a concrete model to work with. As I sat with these students during partner and individual work time, I observed these students placing the blocks out themselves as they counted out values that each digit represented, then compared the amounts starting in the largest value. This action allowed them to “see” which number held the greatest value.

According to data collected through observations, class work, and exit slips all students but one showed success with this sequence through the thousands place as the lessons progressed. The one struggling student could represent the number correctly using the physical manipulatives, but could not consistently name the value correctly or consistently write the value out in expanded form. One example he gave me when I had a conference with him was showing me 438 correctly with manipulatives but telling me the number has a value of  $40 + 30 + 8$ . This told me he understood the first numeral to the right in a number was represented by the ones cubes, the second number to the left is represented by the ten sticks, and the third number to the left is represented by the hundred flats. I think he was procedural in his knowledge of how to represent

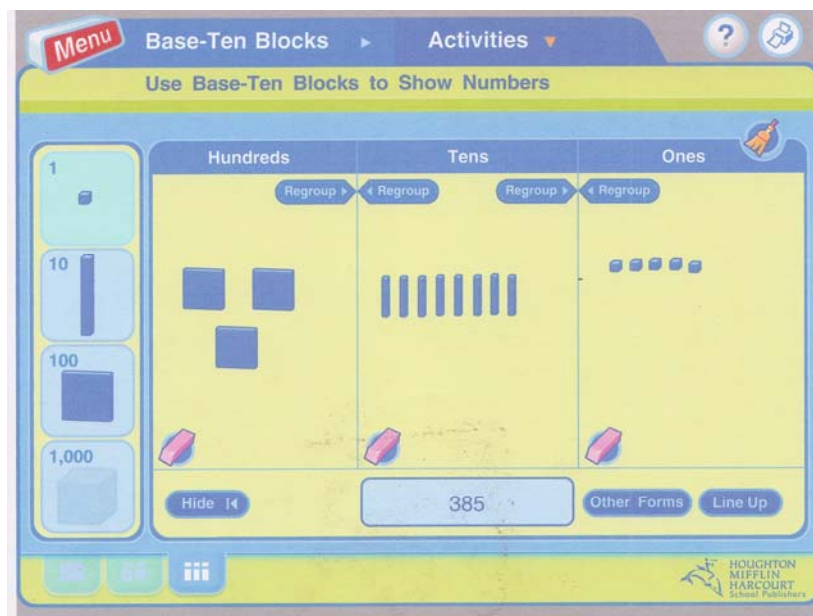
numbers with base-ten blocks. He was not internalizing the true value the ones, tens, and hundreds represented. He was still working on what the value of one hundred meant. He could count by tens to 100, but just as he initially kept counting the single ones in the ten stick and not understanding the grouping of a ten, he did not understand a grouping of one hundred.

When using physical manipulatives, students that showed difficulty on the pretest with writing the numerical value for a given amount of base-ten blocks were now showing success, especially in understanding that 0 held a place value. Having a concrete model in front of them, clarified for them that the zero in 407 meant the ten sticks were grouped as hundreds. Using place value cards to represent the model in expanded form helped the students to make the connection between the position of the digit to the value of the digit by the time we began working in the hundreds place. Observations indicated that through the sequential lessons from the textbook, and daily access to physical manipulatives, students came to understand the face value of digits according to its position in the number.

### Virtual Manipulatives

Virtual manipulatives were introduced to the whole class during week 3 by accessing the applet from the textbook company on the interactive white board. My observations indicated that students made an immediate connection to this applet when the ones, tens, and hundred blocks were presented and sorted on the place value mat screen by using the same place value language used with the physical manipulatives. “Look, there’s the one cube and ten stick,” and “It has 3 places for blocks too, just like our book,” were student comments from my field notes. All students but one had immediate success in drawing a model that represented the model on the screen and naming each value. There seemed to be no confusion whether the base-ten blocks

were on the screen or in their hands. Some students brought it to the attention of others that as they chose base-ten blocks of different values to place on the large classroom screen that the total amount of the number was visible and changed as more base-ten blocks were added or taken off. Figure 2 is an example of this student’s comment “See I am right, it says 385 and I have 3 hundreds, 8 tens, and 5 ones”.

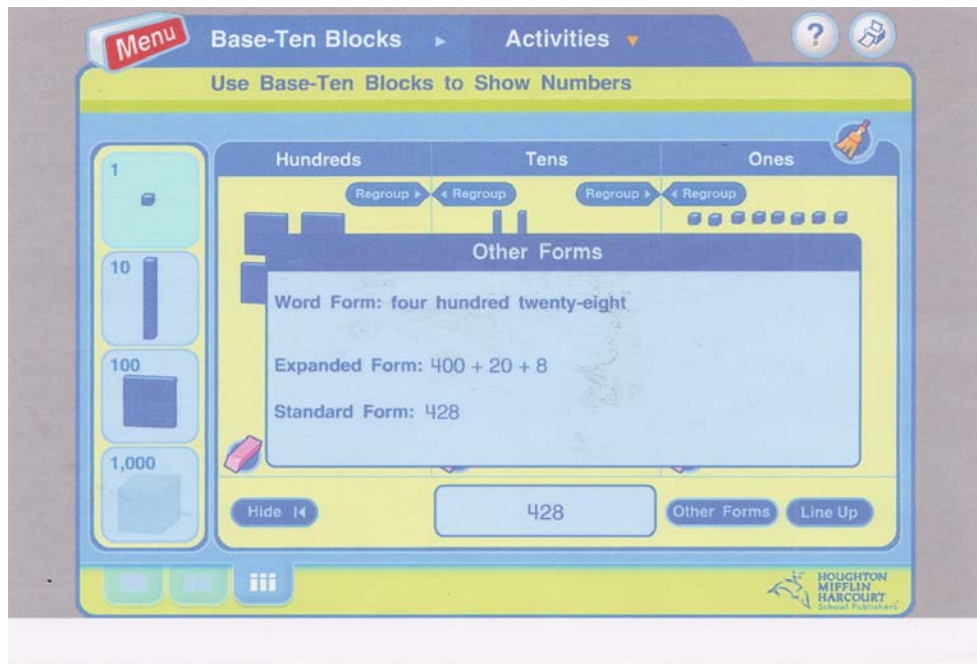


**Figure 3: Virtual base-ten blocks, Houghton Mifflin Harcourt, 2009**

This additional enhancement of increased mathematical actions (Clements & Sharma, 2002) caught the attention of my one struggling student as well and I used this as another opportunity to help in his understanding of place value using a variety of contexts (NCTM, 2000; NCTM, 2006). This immediate feedback was instrumental in his ability to understand how the amount and type of base-ten blocks he chose represented the number he was thinking about (Moyer-Packenham, et al., 2008). He began to identify the face value of the digit more consistently and successfully when using the virtual manipulatives than when using just the physical manipulatives.

When the students were brought to the computer lab for their independent practice during week four and five the second website, National Library of Virtual Manipulatives (NLVM), was accessed because students could have easy access to this site at home via my class webpage (Moyer-Packenham, et al., 2008). This independent practice time allowed all students the opportunity to choose base-ten block values of their own choice to create numbers up to the thousands place. On the first visit to the lab, the thousands place had not been introduced yet but many students concluded that the large cube on the screen must represent the thousands place because they could see that it had rows of hundred flats in it just as the hundred flat had rows of ten sticks. It was exciting to see self discovery of this concept using the knowledge they had gained in class! My field notes indicated to me that immediate feedback of the numeral being represented as a base-ten block was being chosen was a key factor in the success of understanding if the value of the blocks they chose matched the number they were trying to represent. In class, they relied on their partner or myself to tell them if they were correct, but in the lab the results were an immediate action of their placement and choice of base-ten blocks (Moyer-Packenham, et al., 2008). My field notes and exit slips indicated this type of instruction was useful for all students in understanding the face value of a digit in a number to the thousands place while representing the same number in expanded form. My struggling student became more independently successful. He continued to use the textbook applet in the lab and began to label his pictorial drawing correctly on a consistent basis because he could use the screen tab “other forms” to check his understanding of the expanded notation value, see figure 4. The NLVM site did not have this option. The textbook applet allowed this student to get continued reinforcement of the value of his number and it was represented to him in a variety of forms.





**Figure 4: Virtual base-ten blocks with other forms, Houghton Mifflin Harcourt, 2009**

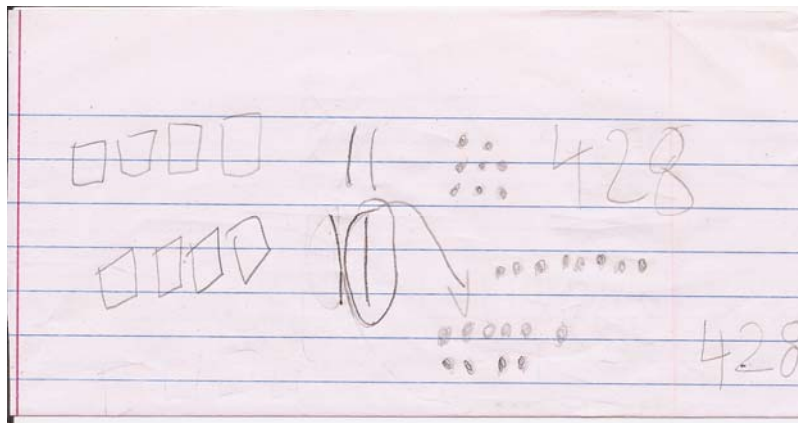
### Composing and Decomposing Numbers

NCTM (2006) stated in their expectations for grades Pre-K through 2<sup>nd</sup> grade that students should “develop a sense of whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers” (p.14). Knowing from my literature review that explicit grouping or trading rules needed to be defined and consistently followed, I provided a variety of opportunities for students to practice representing numbers in a multiple of ways. This is new to my practice of teaching place value (Baroody, 1990; NCTM,1989; Reys, et al., 1997; Sharma, 1993) and I found that using both physical and virtual manipulatives played an import role in student understanding of place value relationships.

## Physical Manipulatives

Using physical manipulatives enhanced my students' abilities to represent numbers in a variety of ways. Now that students had become familiar and successful with representing a number using physical manipulatives and demonstrated an understanding of the value of the number and the groupings each place value held, I wanted to expose them to the trading rules so that they would become efficient with representing numbers in a variety of ways. I knew this was an important prerequisite to multi-digit addition and subtraction with regrouping (Baroody, 1990; Fuson, 1990; Fuson & Briars, 1990). Manipulatives allowed the students to easily demonstrate the same number by simply trading a ten stick for the same amount of ones to show the number in a different way. Example: 47 is equal to 4 tens and 7 ones which is equal to 3 tens and 17 ones. This concrete model made it real for my students. They saw the results of their actions (Heddens, 1997). In the initial stage of this learning sequence, students had great success with trading one ten stick. As more ten sticks were traded however, students lost count of the number of ones they were counting as they traded the ten sticks. Comments like, "I think this is 40 ones," and "this is a lot to count out," indicated to me they were frustrated. One student commented, "I showed 87 with 8 tens and 7 ones, then showed it as 7 tens and 17 ones, then I showed it as 6 tens and 27 ones, if I want to show it as 5 tens and 37 ones do I have to keep counting out all of these blocks or can I just draw it with quick pictures?" This student conversation revealed that he may be frustrated or he was moving away from the use of physical manipulatives to support his understanding. As the lessons progressed, more and more students chose to draw quick pictures instead of using the physical manipulatives.

As I analyzed student drawings (figure 5), I realized they understood the decomposing of numbers because in their quick pictures they were drawing a circle around each group of ten that was traded for a ten stick. I noticed that some students miscounted their physical cubes and had only nine ones for some of their ten sticks but still counted it as a ten when giving me a total of all their tens and ones. This told me that even though they miscounted the number of cubes, mentally they knew the group represented a set of ten.



**Figure 5: A drawing representing decomposing**

As students moved to decomposing 3-digit numbers, they were just as successful in trading hundreds for tens. Being able to physically count 10 ten sticks while counting by tens, and then saying “100” as they moved the hundred flat away, quickly bridged the more abstract understanding of regrouping. I did not observe the frustrations with counting out ten sticks as I did with the ones cubes. My observations while I walked around revealed the same pattern of miscounting a large amount of ten sticks or not counting the correct amount of ten sticks for the same number of hundred flats they were trading. An example would be when one student showed “40” tens for 400 but only had 39 tens in front of her. I believed that most of the

miscounting came from too many blocks on the table, another student possibly taking from their group of blocks, or blocks dropping off the table. In their drawings, when these same instances of miscounting occurred, the students quickly corrected their mistakes when I asked them to count it back to me.

### Virtual Manipulatives

I knew from last year's introduction of these virtual manipulatives, that students became very excited when the blocks "broke apart" or "came together" when I moved them from one place value section to the next. But this year I wanted my students to discover it themselves. I wanted them to explain what they saw and why it was happening (Heddens, 1997; Hiebert & Wearne, 1992; Reys, et al., 1997; Sharma, 1993). They were already introduced to composing and decomposing numbers with physical manipulatives but I wanted to see if they would make the connection to it happening on the screen for themselves and how it would enhance their ability to express numbers in different ways. On the first day in the lab using the NLVM site, I asked the students to create the number 58. When I asked the students to show another way you could represent the same number, it didn't take long for the students to see that their actions with physical manipulatives could be replicated with the virtual manipulatives. As soon as the students moved the mouse with a ten stick over to the ones column, the ten stick broke up into ten ones (Clements, 1999). The students were fascinated! Just about all of them decided they were going to show me the number 58 as 58 ones! A student then asked "Can I try it with the hundreds?" I asked, "What do you think will happen?" Several students were already moving a hundred flat over to the tens column as soon as the student asked the question and many of them were able to tell me that it broke apart into ten sticks. I asked how many ten sticks it broke into

and I saw some students counting the sticks on the screen, but most of the students shouted out “ten!” This indicated to me that many of the students transferred the meaning of decomposing numbers with physical manipulatives to the visual in front of them. They immediately understood the rule for groupings in a different format (NCTM, 2000; NCTM, 2006). The students also figured out through my questioning that if they highlighted ten ones it would compose back to create a ten stick. With this program, the students were required to move the ten stick back over to the tens column, simulating regrouping. Later, the students found out the textbook applet moved it for them when they pushed the button “regroup” These actions closely mirrored the types of thinking required of students at the abstract level (Clements & Sharma, 2002). I noticed the word “regroup” was being used now instead of the word “traded” with the physical manipulatives. The types of questions and comments I heard from the students demonstrated they were thinking about their actions and they also expressed them to their peers. These are just a sample of what I recorded in my field notes.

“Look, I moved my one hundred block to the tens and then all my tens over so that I would have 100 ones.”

“Watch, this is how I moved my ten stick to the ones, and now back again. See I still have the same amount.”

“Watch me make one thousand, watch it break into 10 hundreds.”

“This is so cool how ten ones makes a ten stick, ten ten sticks make a hundred, ten hundreds make a thousand, and I bet ten thousands makes a million!

These types of questions and comments were not heard while working with the physical manipulatives. They gave me more insight into what my students understood (Clements and

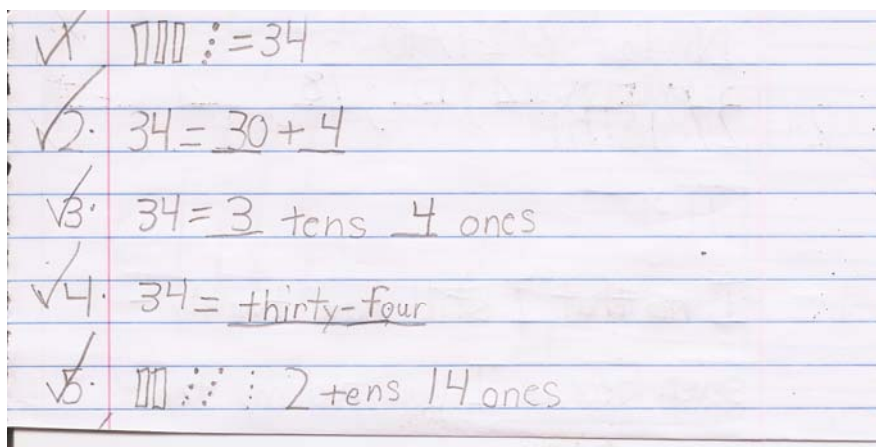
McMillen, 1996) The last question prompted me to give a mini lesson on the thousands place. Simply clicking on the thousand cube ten times confirmed for this student the value would be ten thousand and not a million. He responded, “Oh, I didn’t know that’s what you call it.” I wonder why this question didn’t come up with physical manipulatives, and would my demonstration of gathering ten, thousand cubes be as powerful to this student as it was for him to make the blocks increase in number as he counted and clicked?

Using virtual manipulatives helped my struggling student relate his actions of composing a ten stick or a hundred flat to his ability to correctly count out ten ones or ten ten sticks. This is where many of his mistakes came from when working with the physical manipulatives and the computer would not allow him to compose the number he wanted to without correctly highlighting ten of the desired blocks. This was sometimes overlooked by himself or his partner when working with the physical manipulatives. This action alone was a continual confirmation for him that our place value system was made from groupings of ten. The virtual manipulatives were also beneficial for him because they provided independent practice. When he made a mistake, the computer wouldn’t let him follow through with his intent. This allowed for purposeful learning that he was creating himself (Clements, 1999; Clements & Sharma, 2002; Moyer-Packenham, et al., 2008).

### Journals

Daily use of mathematics journals provided another piece of evidence for me to see my students’ emerging mathematical understanding of place value concepts (Mills, 2003). Mathematics journals have been a part of my methods in teaching for many years as a means of analyzing student mathematical learning. I value the individual understanding they demonstrate

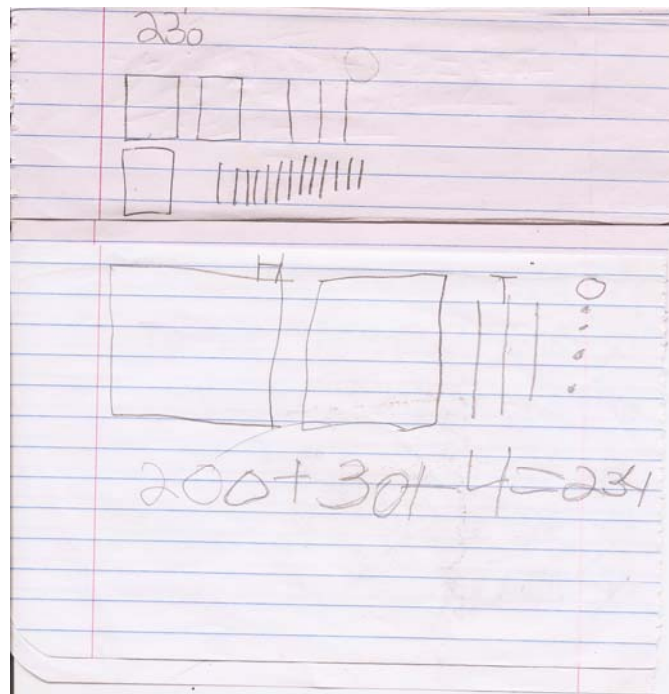
of each student's thinking. Through my literature review, I learned the importance of helping students gain abstract understanding by using the Concrete-Representational-Abstract (CRA) method. Using their journal after demonstrating a number with a concrete model allowed students to express their understanding in pictures. This was important to help students bridge the gap towards more abstract thinking as they progressed in their place value learning (Access Center, 2004). Figure 6 is an example of a student who was already transferring her knowledge of place value into demonstrating how numbers can be represented in different ways.



**Figure 6: Representing numbers different ways**

It was week 2 and this student had only been introduced to the base-ten manipulatives for one week and was using a representational model without using the physical manipulatives first. This journal sample came from an exit question after a lesson. Most of the students used the base-ten blocks with their white boards and expanded notation cards before representing it in their journal. Most of my students did not have the understanding of the manipulation of numbers as this student did in the beginning of the study. Without this journal piece, I might not have had this opportunity to see her thinking and see that she was ready to begin using and representing numbers at a more abstract level.

All students but one were able to accurately and consistently draw a picture of their model and record the value whether it was the physical manipulatives in front of them or the virtual manipulatives on the screen. Virtual manipulatives played a bigger role in the labeling of their drawings. Figure 7 shows how more students labeled their pictures with more forms of the number after using the virtual manipulatives than with the physical manipulatives. Their drawings more closely resembled the models they were viewing on the computer than they did using the physical manipulatives. Most students would draw out the hundreds, tens, and ones columns, and label under their drawings the correct number of hundred flats, ten sticks, and ones cubes they saw on the screen and then wrote that it equaled the number.

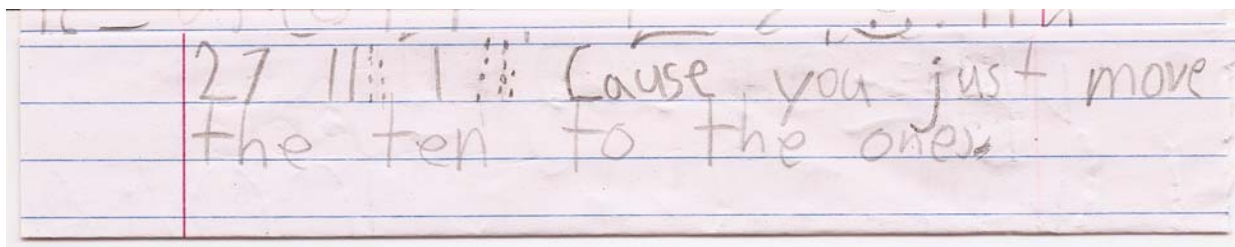


**Figure 7: Comparison of drawing after physical manipulatives and after using virtual manipulatives**



When using the physical manipulatives, students simply drew a quick picture and wrote the corresponding numerical value. I felt the virtual manipulative screen enhanced their visual images as indicated by their drawings (Clements and McMillen,1996).

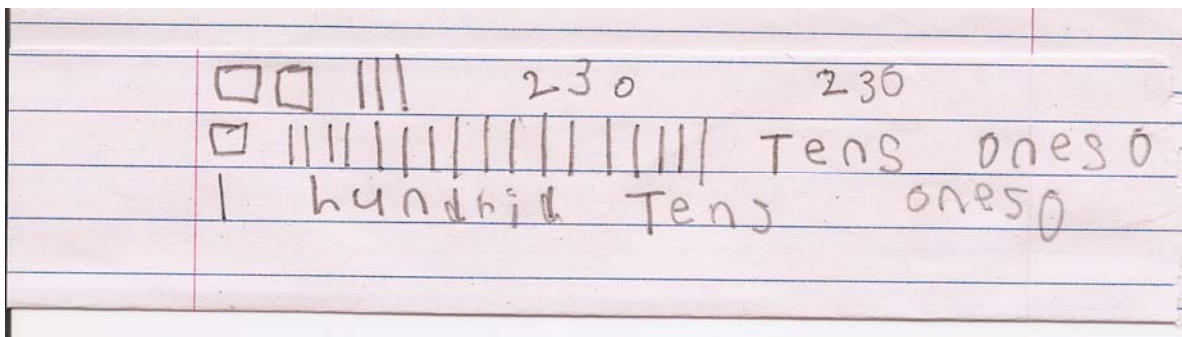
All of the students had success with composing and decomposing numbers using the virtual manipulatives. Their pictorial drawings and use of language began to show more details and understanding as the students interacted with the virtual manipulatives (Clements and McMillen,1996). My field notes indicated that the amount of conversation between the students increased as well when using the virtual manipulatives. I used mathematical journals to check for understanding of decomposing of numbers and to see if there were any misconceptions I might have missed during class. Student journals were scored on a rubric (appendix H). I asked specific focus questions (appendix G) since adding virtual manipulatives and a focus on composing and decomposing numbers, after reviewing the literature, was new to my practice. After the first week of journaling the average score of a journal entry was 2.5 out of a possible 4 when using the physical manipulatives. Figure 8 demonstrates how students were applying place value language to their actions with the physical manipulatives.



**Figure 8: Journal entry after using pyhsical manipulatives**

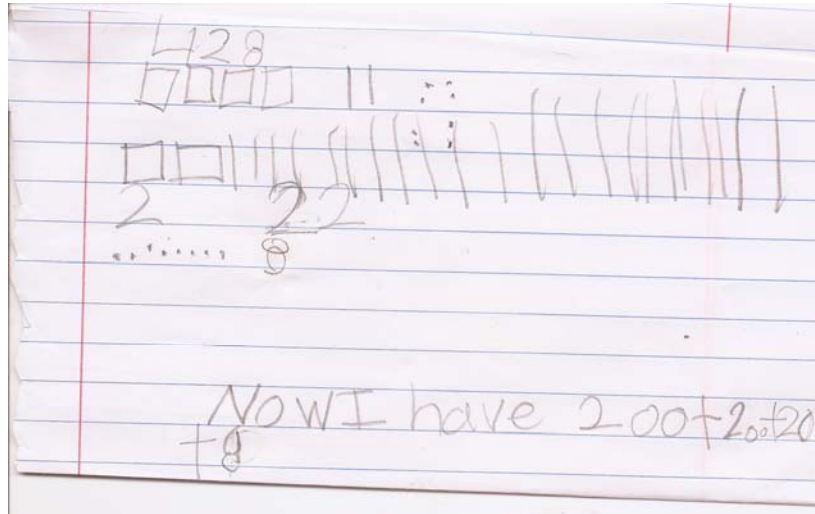
Five of the seventeen students had no language to support their understanding and all students but one could represent the number 27 in a different way using the physical manipulatives. The second journal entry during week 3 showed all students' ability to use language to support their

understanding of decomposing the number 230 using physical manipulatives with an average score of 3. Figure 9 demonstrates the struggling student's growing knowledge of place value. He had daily practice and reinforcement with physical manipulatives and expanded notation cards. Conversations provided data that I could tell he understood the trading rules but when left to represent them on his own, he was still not clear on how many tens he should use when he traded his hundred flat. This showed me he still needed a variety of learning experiences with groupings (Reys, et al., 1997; Sharma, 1993).



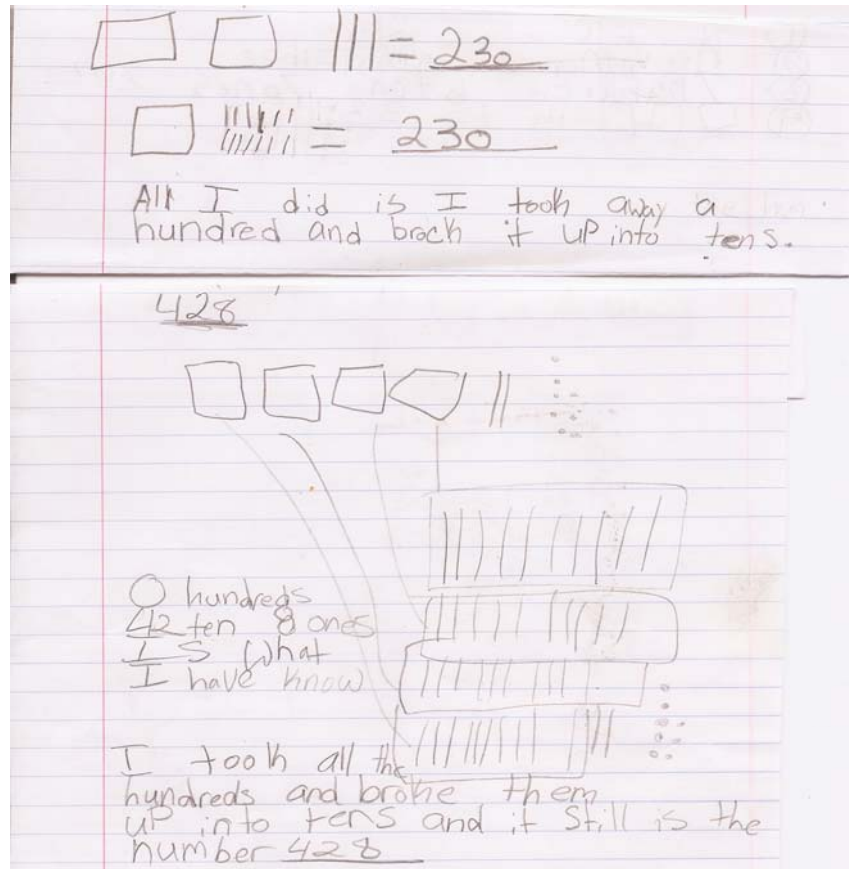
**Figure 9: A student's understanding of groupings.**

During week 4, when the virtual manipulatives were used, the journal entries averaged a 3 with all students scoring a 2 or higher and the language was changing. Students were showing a variety of thinking strategies of how they know they still have the same number just represented in a different way (Clements and McMillen, 1996). Figure 10 comes from a student who scored a 3, but was now showing her understanding of groupings.



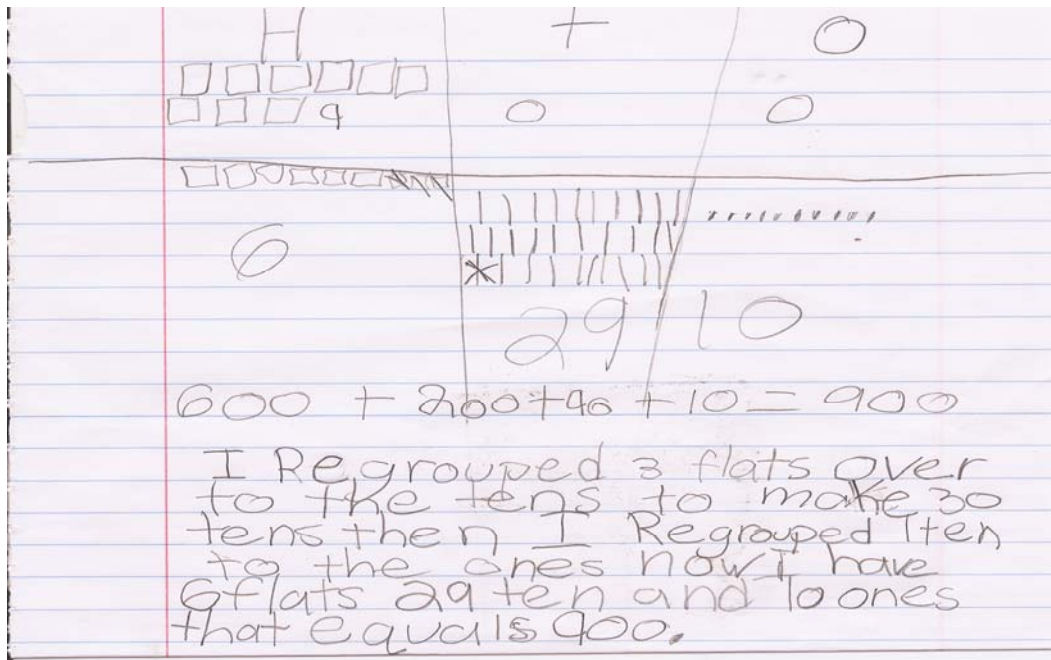
**Figure 10: A student's understanding of groupings while using virtual manipulatives.**

She explained to me while moving the virtual manipulatives around the screen that when she regrouped 2 hundred flats to the tens, they break apart but they are still a value of 200. Her entry shows her ability to see hundreds as groups of tens (22 under her ten sticks) and as a value of 200 (her expanded notation  $200+200+20+8$ .) The next journal entries, Figure 11, are from a student who scored a 3 on the first two entries and now scored a 4 because she was adding the details to her language to demonstrate her thinking. She is a high student and often performs well and the use of virtual manipulatives helped her increase her ability to represent different ways of thinking about numbers within the context of place value (NCTM, 2000, NCTM, 2006).



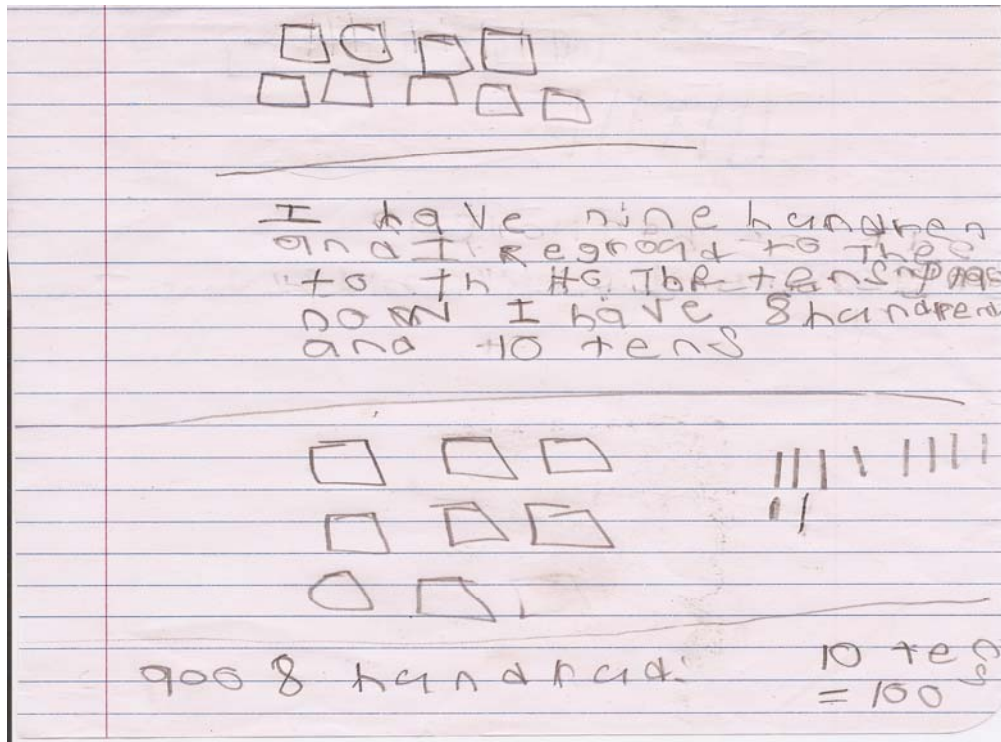
**Figure 11: A journal entry showing one student's thinking using both physical and virtual manipulatives.**

During the final week of this study, the average score of their journal entries was a 3.5 with only one student scoring a 2. When analyzing their journal entries, I found that more students were using language that supported their actions and images from the screen. Students were using the word regrouped instead of traded. Ten of the seventeen students represented their drawing exactly like the screen showing the different place value sections. Figure 12 shows one student's use of both the language that was being formulated as well as the illustrations that were being produced after working with the virtual manipulatives.



**Figure 12: Virtual manipulative actions supports place value language and regrouping.**

My one struggling student was now modeling this format as well and he was demonstrating groupings on a more consistent basis. Figure 13 shows the growth he made in understanding place value.



**Figure 13: Student growth**

He initially wrote that he regrouped a hundred to the hundreds place but when I asked him to repeat his actions on the screen, he said “oh, I mean the tens place”, and changed his writing to the best of his ability on his journal paper.

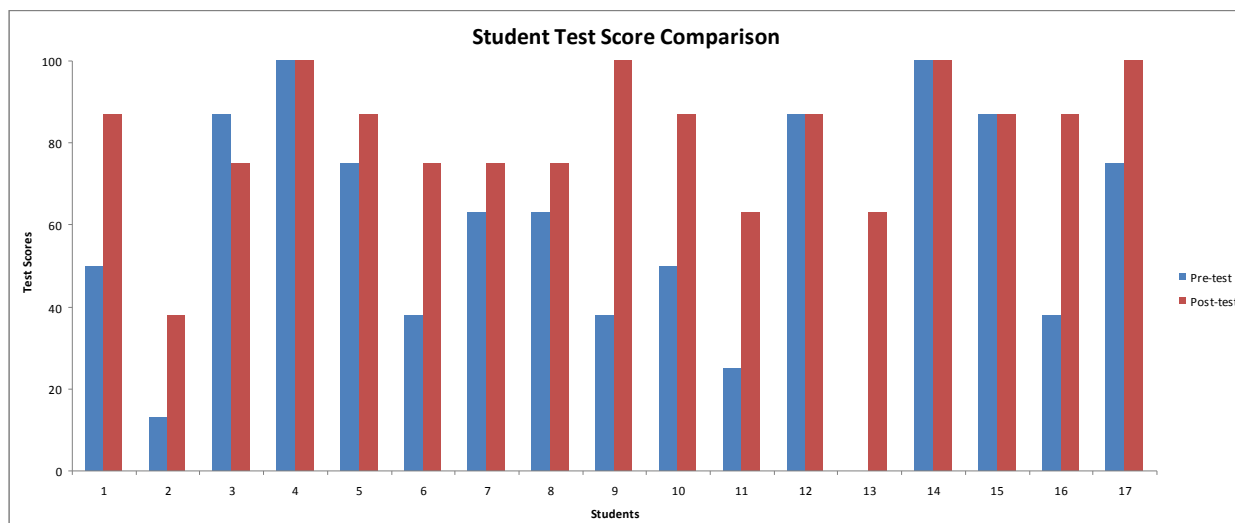
Overall, the use of journals provided insight into my students’ thinking and see how representing what they were viewing, physical or virtual manipulatives, played a role in moving my students towards more abstract thinking with place value concepts.

### Posttest

After about 5 weeks of exploring, learning, examining, and practicing place value concepts using both physical and virtual manipulatives, it was time for the posttest. The posttest had a more positive outcome than the pretest. Twelve students made gains and four students showed no difference between their pretest and posttest scores. One student missed a question on

the posttest in which he got correct on the pretest. This resulted in a decrease in student 3's score.

Figure 14 shows a comparison of the pre and posttest scores. Students were randomly given a number to protect their confidentiality.



*Figure 14: Comparison of pretest and posttest scores*

The range in scores from the pretest to the posttest was high for most students, demonstrating growth that took place. Only 3 students scored below 70% and of the remaining 14 students, 4 students scored 100%. From the three students who did not make enough gains to be proficient with place value concepts in this study, one of the students came in during mid study and did not complete the pretest. One of the students' made a gain of 38% and consistently missed the same answers on the pretest which helped me continue to focus on her areas of need. The third student, my struggling student, was experiencing difficulty demonstrating learning gains in all areas of academics and was being screened for language difficulties. This helped me understand why he was better able to demonstrate his learning when a task was given orally, but did not comprehend what was being asked of him when he read a written test.

Student gains were revealed in two areas in which I focused my instruction. The pretest allowed me to see that most of my students did not understand groupings or the value assigned to a numeral based on its position in a number. My literature review revealed this was the most essential area I must focus my instruction on and the pretest revealed a need in this area. I analyzed the questions from the test in which these concepts were focused on and 100% of students were able to write a number for a model in which zero held a place, and 100% of the students represented the value each digit held through the hundreds place. Half of the students still needed more practice with the thousands place.

The post test also gave me insight into how my students relied on virtual manipulatives when it came to demonstrating their knowledge of groupings. During the posttest, all students had access to either physical manipulatives at their table or virtual manipulatives at 3 student computer stations in my classroom. Ten students chose to complete the entire posttest using virtual manipulatives, one being my struggling student. When students answered the last question on the posttest on groupings in the thousands, twelve students chose to use the virtual manipulatives to answer this question, and eight of them answered it correctly. This question informed me that with only 58% of my students understanding groupings of hundreds, I still needed to focus instruction in this area.

#### Ease of Materials and Attitudes

When the virtual manipulatives were introduced during the middle of the study, an attitude change took place in my students. Engagement and motivation were reoccurring elements in many of the studies reviewed in the literature for this study. My observations, field notes, and journal entries showed students were more actively engaged by their student-to-



student talk in the lab. They were motivated to do their best by demonstrating their thinking in their mathematical journal. Students wanted to use their journal more often to show me the numbers they were creating and regrouping using the virtual manipulatives, in particular, my struggling student. Finally, their comments, “This is fun!”, “Yeah, computer time!” and actions of shouts and cheers showed sheer excitement when working with the interactive whiteboard or computer during mathematics time (Reimer & Moyer, 2005). When using the physical manipulatives, students were initially interested when they were introduced (Bryant, 1992; Sowell, 1989), but some students began to use the manipulatives as building blocks during lessons. When using the virtual manipulatives, moans and groans came from my students when our mathematics class came to an end. On the last day in the lab I stopped by each student’s station and asked each one if they enjoyed mathematics. Each student made a positive comment towards mathematics and I specifically asked “why”, to those few students who said they did not like mathematics on the initial questionnaire. Responses from their journal are recorded exactly as students wrote them with the exception of spelling.

I think using a computer is fun, there are so many things to do with it.

I like creating numbers, really big numbers!

I like seeing the numbers break apart and come together again, that’s called regrouping.

I like using the computer, it’s fun to move things around the screen.

I was pleased to see that I helped shape these students’ views of mathematics in a more positive way by allowing them access to the computer.

When the students used the virtual manipulatives, there were no materials to be passed out or shared (Clements and McMillen, 1996). Students were able to immediately begin working

on the computer and were in control of all the “pieces” (Clements, 1999). I observed on several occasions when miscounting occurred with physical manipulatives that it might have been due to sharing of materials, materials being misplaced, or materials being pushed from the students’ work area. This did not happen when working with the virtual manipulatives.

### Summary

Data were collected from a pre and posttest from the second grade curriculum of Go Math! Florida (Houghton Mifflin Harcourt, 2009), a student questionnaire, student class work both written and on the computer, informal interviews with students, mathematics journals, and observations with field notes. The data revealed that students came to second grade with a variety of experiences using manipulatives for mathematical learning and had some knowledge of place value concepts but that there was a need to learn the face value of digits and the groupings of our base-ten number system. While both physical and virtual manipulatives helped students acquire place value concepts, themes emerged that revealed virtual manipulatives had features that engaged students in a way that increased their mathematical language, increased students’ ability to represent a more conceptual understanding of composing and decomposing numbers, and express enthusiasm towards mathematics. Finally, the pre and posttest scores revealed that after manipulative use, students’ scores increased. The final chapter of this study explains the results of the study, implications for those involved with education, limitations, recommendations, and suggestions for teachers who want to use virtual manipulatives.

## CHAPTER FIVE: CONCLUSION

### Introduction

I went into this action research study with great enthusiasm that I was implementing technology into my mathematics instruction. As I implemented my action research, I investigated what would happen if I changed my teaching practice of place value and added virtual manipulatives (Baroody, 1990; NCTM,1989; Reys, et al., 1997; Sharma, 1993). My research question was:

How will the use of virtual manipulatives, in addition to physical manipulatives, influence the mathematical understanding of my students and inform my practice of teaching place value in my second grade class?

In this chapter I review the results of my study. I also discuss implications, limitations, and recommendations for further research.

### Results

After conducting this action research study, I found that virtual manipulatives, in addition to physical manipulatives, influenced student acquisition of place value concepts. Studies and research indicate teachers must change their methods of instruction to help students reach a more profound understanding of mathematics (Baroody, 1990; NCTM,1989; Reys, et al., 1997; Sharma, 1993). I found that when I added virtual manipulatives to my practice of teaching place value (Barta, 2002; Clements & Sarama, 2002; Moyer-Packenham, et al., 2008), mathematical language increased. Students showed a more in depth knowledge of thinking strategies and showed how they can represent numbers in more than just one way. This was revealed through student journals as students increased their mathematical language and applied terms such as

regroup to their writing after using the virtual manipulatives. Along with another study, I also found that practicing with the visual computer images might have enhanced students' abilities to explain and represent their thinking using pictorial models (Reimer & Moyer, 2005). Their drawings more closely resembled the models and actions they were viewing on the computer than they did using the physical manipulatives. Adding virtual manipulatives to my practice of place value also increased students' ability to represent more conceptual understanding of composing and decomposing numbers as evidenced by the variety of ways in which they were representing a number in a variety of ways. The enhancements that the virtual base-ten blocks offered allowed students to "see" regrouping and interact with it in a way that is more inline with mental actions we want students to learn (Clements & Sharma, 2002). The virtual manipulatives helped students build on their physical experiences, tying them tightly to symbolic representations, also evidenced in their journals and increase in posttest scores. (Clements & Sharma, 2002). In comparison to other studies, I also found that the use of base-ten blocks helped students learn place value concepts that move from concrete to abstract and assisted students in acquiring grouping and trading rules and determining the value of the number being represented as evidenced in their language, drawings, and posttest scores (Baroody 1990; Fuson, 1997). This was accomplished as they moved and touched the blocks and saw the results of their actions. The use of expanded notation cards connected the base-ten blocks to representations of their numerical values. Having students draw quick pictures furthered their understanding through another form of representation. Having students use a journal helped them formulate understanding through the use of mathematic language. Finally, the actions of the virtual base-ten blocks closely mirrored the actions of composing and decomposing numbers that we want

students to do mentally. “Virtual manipulatives, used in combination with concrete manipulatives and other real world exploration, and in ways that encourage discussion and critical thinking, can make a unique and significant contribution to young children’s mathematics education” (Rosen, D., Hoffman, J., 2009, p.32).

Furthermore, I learned through Moyer-Packenham, et al., (2003) that one of the most important differences between virtual manipulatives and the physical manipulative is the computer’s ability to provide the student instant feedback as the student works. This was true in my study and because of this instant feedback the virtual base-ten blocks were self correcting which allowed me more time to work with individual students in their areas of need. I felt my students were more on task and engaged and that I could spend quality time helping each student acquire place value concepts at their individual level of learning because of this feature. This feedback was instrumental in my struggling student’s growth. I observed how his understanding of values and grouping increased as we worked together. When he worked independently, the computer continued to motivate him when his thinking was correct and when it was not, allowed him the opportunity to manipulate the blocks in a manner that let him see how to arrive at the desired results. I believe this student made notable gains during this study although they did not reveal themselves through his posttest.

As I refine my teaching to meet the Next Generation Sunshine State Standards for Mathematics, I am pleased that I took on the challenge of integrating technology into my mathematics instruction to further student understanding of place value. Using virtual manipulatives benefited me as a teacher in many ways. Keeping young children motivated to use hands-on materials the correct way for an entire lesson can frustrate even the most veteran

teachers. It is difficult for many young children to understand “their space” at a table with others causing manipulatives to get mixed up, fall, or be used by other students. This created confusion and frustration with the students and stopped the momentum of the lessons. Having to sort out which blocks were being used for the lessons was time consuming (Clements, 1999). Having a central bag for each table helped, but storage was an issue as well in my small classroom. With virtual manipulatives, I did not have any of these issues. In fact, virtual manipulatives provided the students with unlimited access to base-ten blocks. I only had to make sure I had internet access. Most importantly, it allowed me more time to work with individual students in their areas of need and increased positive attitudes about mathematics in my students.

### Implications

Since data showed that using virtual manipulatives in addition to physical manipulatives helped students make gains in the acquisition of place value concepts, perhaps more teachers should consider adding virtual manipulatives to their mathematics instruction.

NCTM (2006) stated in their expectations for grades Pre-K through 2<sup>nd</sup> grade that students should:

Use multiple models to develop initial understandings of place value and the base-ten number system.

Develop a sense of whole number and represent and use them in flexible ways, including relating, composing, and decomposing numbers.

Connect number words and numerals to the quantities they represent, using various physical models and representations (p.14).

It is imperative to present place value in ways that students have the opportunity to develop mathematical competence (NCTM, 2000). Using base-ten virtual manipulatives accomplished this for my students. Using physical and virtual manipulatives together supports NCTM's view of providing opportunity for students to make connections by relating various representations of concepts or procedures to one another (NCTM, 2000). This study gives valuable insight to teachers who will be considering adding virtual manipulatives to their mathematical instruction in the coming years.

### Limitations

The results found in this study cannot be generalized to all other classroom populations. One factor to consider is the population of my study. The population of students involved in this study was not large enough to make conclusive assumptions. My students generally come from homes that support their students' learning. These children may have an advantage over other students who do not have the same level of parental support. All of my students had access to a home computer. Another factor to consider is the time frame in which this study took place. I had to abide by my school's scope and sequence of the textbook and was required to move on to other skills. If I had been able to devote more classroom instruction time specifically to virtual manipulatives, I believe the students would have benefited more. Research says that place value concepts emerge over time (Fuson, et al., 1997)

### Recommendations

Educators need to have access to research based information on how best to teach students. This study was informing and leads to additional questions such as: Is it just as

beneficial for the students to have them use only virtual manipulatives? Would introducing virtual manipulatives at the beginning of the study along with physical manipulatives produce the same results in mathematical language usage? What would the results be if the order of this study were reversed (virtual first, then physical manipulatives)?

If I were to do this study again, I would want to look into researching how composing and decomposing numbers using virtual manipulatives helps students acquire a deeper understanding of addition and subtraction with regrouping. Research suggests that students should learn multi digit addition and subtraction alongside place value concepts suggesting that this provides a context for motivating and supporting the development of base-ten number concepts (Baroody, 1990; Fuson, 1990; 1997).

I would also change some of my data collection techniques. Specifically, I would use video to capture the students' rich discussions. I found that even though students' written language in their journals was providing more insight into their thinking, it was not the same as their oral conversations and comments.

### Summary

As I look for ways to improve my teaching practice and meet the needs of all my students for the 21<sup>st</sup> century, incorporating technology into my mathematics practice was a natural fit. I wanted to know how the use of virtual manipulatives when used with physical manipulatives would influence my students' acquisition of place value concepts. I wanted to see how adding this computer applet might better meet the needs of all learners while also meeting the needs of today's technological classroom. Through this study, I found that both physical and virtual manipulatives helped students acquire place value concepts. I found that virtual manipulatives



had features that engaged students in a way that increased their mathematical language, increased students' ability to represent more conceptual understanding of composing and decomposing numbers, and express enthusiasm towards mathematics.

This action research helped me reflect on my practice of teaching place value by focusing on my instruction based on the research of others, and my students had positive attitudes and achievement outcomes. Most importantly, it is important that all students have opportunities to use technology in appropriate ways so that they have access to interesting and important mathematical ideas.

## **APPENDIX A: INSTITUTIONAL REVIEW BOARD (IRB) APPROVAL**



University of Central Florida Institutional Review Board  
Office of Research & Commercialization  
12201 Research Parkway, Suite 501  
Orlando, Florida 32826-3246  
Telephone: 407-823-2901, 407-882-2012 or 407-882-2276  
[www.research.ucf.edu/compliance/irb.html](http://www.research.ucf.edu/compliance/irb.html)

## NOT HUMAN RESEARCH DETERMINATION

From : **UCF Institutional Review Board #1**  
**FWA00000351, IRB00001138**

To : **Kay D. Jolicoeur**

Date : **August 12, 2010**

Dear Researcher:

On 8/12/2010 the IRB determined that the following proposed activity is not human research as defined by DHHS regulations at 45 CFR 46 or FDA regulations at 21 CFR 50/56:

Type of Review: Not Human Research Determination  
Project Title: The Influence of Virtual Manipulatives on Second  
Graders' Acquisition of Place Value Concepts.  
Investigator: Kay D Jolicoeur  
IRB ID: SBE-10-07058  
Funding Agency:  
Grant Title:  
Research ID: N/A

University of Central Florida IRB review and approval is not required. This determination applies only to the activities described in the IRB submission and does not apply should any changes be made. If changes are to be made and there are questions about whether these activities are research involving human subjects, please contact the IRB office to discuss the proposed changes.

On behalf of the IRB Chair, Joseph Bielitzki, DVM, this letter is signed by:

Signature applied by Joanne Muratori on 08/12/2010 02:04:00 PM EDT

IRB Coordinator

## **APPENDIX B: PRINCIPAL APPROVAL**

August 7, 2010

**Informed Consent to Conduct Research**

Dear IRB Coordinator,

Kay Jolicoeur, a second grade teacher at [REDACTED] in [REDACTED] County, has informed me of her intentions to conduct an action research project in her classroom titled "The Influence of Virtual Manipulatives on Second Graders' Acquisition of Place Value Concepts". I am writing this letter to give my consent to allow her to conduct the research with her class for the 2010-2011 school year.

I have reviewed the parental consent letter as well as the student assent letter.

I authorize Mrs. Jolicoeur to conduct her proposed research project with her students beginning in August of 2010.

Sincerely,

---

Principal

## **APPENDIX C: COUNTY APPROVAL**

BILL VOGEL, Ed.D.  
*Superintendent*

**Educational Support Center**

~~1000 Lake Mary Drive~~  
~~Longwood, Florida 32773-1125~~  
Phone: (407) 320-0000  
Fax: (407) 320-0381

**SCHOOL BOARD**

SANDRA ROBINSON  
*Chairman*

JEANNE MORRIS  
*Vice Chairman*

DIANE BAUER  
*Board Member*

SYLVIA POND  
*Board Member*

DEDE SCHAFFNER  
*Board Member*

August 16, 2010

Ms. Kay Jolicoeur  
~~18 Sheperd Trail~~  
~~Longwood, FL 32750~~

Dear Ms. Jolicoeur,

I am in receipt of the proposal and supplemental information that you submitted for permission to conduct research in the ~~Seminole~~ County Public Schools. After review of these documents, it has been determined that you are granted permission to conduct the study described in these documents under the conditions described herein at ~~Choices in Learning~~.

Please forward a summary of your project to my office upon completion.  
Good Luck!

Sincerely,

*Anna-Marie Cote*

Anna-Marie Cote, Ed.D.  
Deputy Superintendent  
Instructional Excellence and Equity

AMC/jr  
cc: Janet Kearney

## **APPENDIX D: PARENTAL CONSENT**





## Informed Consent to Participate in Research Study

Dear Parent or Guardian,

August 2010

My name is Kay Jolicoeur and I will be your child's second grade teacher this year. I am excited about the varied learning experiences your child will be exposed to this year, particularly in Math.

In addition to my responsibilities as your child's teacher, I am also a graduate student in the K-8 Math and Science Education at the University of Central Florida. I will be conducting research in the classroom for my Master's thesis this August- October 2010 and I invite your child to participate in this study. The goal of my research is to study the effects of using physical manipulatives (base-ten blocks) and virtual manipulatives (a computer version of the same base-ten blocks) on student understanding of place value. No risks of any kind are anticipated for the students in this study. Instead, I believe the students will benefit from varied experiences with mathematic manipulatives and from learning about the research process.

With your permission, I will use data collected from a pre-test and a post-test, math journals, interviews, and audio taped discussions during math class. The information gathered from these assessments will be kept confidential. All students will be given will be given a number to correspond to their name to protect confidentiality. All data, including audio will be kept under lock and key, and the audio will be destroyed at the completion of the study.

Your child's participation is encouraged although voluntary. Your child will not be compensated, such as extra credit given, and participation will not affect your child's grades in any way. If you choose not to allow your child to participate in the study, your child's classroom experience will not differ. I will still require them to complete all necessary math coursework and to participate in the lessons; however no data will be collected based on their work.

If you have any questions about this study, you may contact me at \_\_\_\_\_.  
Thank you for your interest and consideration of this study.

Sincerely,

Kay Jolicoeur  
Second Grade Teacher

Contact Information-UCF Faculty Advisor  
Juli K. Dixon, Ph.D.  
Professor, Mathematics Education  
University of Central Florida  
4000 Central Florida Blvd.  
Orlando, FL 32816-1250  
407-823-4140

**KEEP THIS LETTER FOR YOUR RECORDS. PLEASE TEAR OFF THE FOLLOWING PAGE AND RETURN IT TO YOUR CHILD'S TEACHER by \_\_\_\_\_.**

**Second Grade Physical/Virtual Mathematics Manipulative Study  
Jolicoeur**

**Researcher: Kay**

**PARTICIPANT CERTIFICATION:**

If you agree to allow your child to participate in this study please sign where indicated, then tear off this section and return it to your child's teacher by \_\_\_\_\_. Keep the consent letter of information for your records.

I have read this Consent and Authorization form. I have had the opportunity to ask, and I have received answers to, any questions I had regarding the study and the use and disclosure of information about my first grade child for the study.

**I give permission** for my second grade child, \_\_\_\_\_, to take part in this study as a research participant.

\_\_\_\_\_

Parent/Guardian Signature

\_\_\_\_\_

Date

## **APPENDIX E: INITIAL QUESTIONNAIRE**

## Questionnaire

1. Do you like math? \_\_\_\_\_

\_\_\_\_\_

2. Do you like to count numbers? \_\_\_\_\_

\_\_\_\_\_

3. What are some things you have used before to help you count numbers? \_\_\_\_\_

\_\_\_\_\_

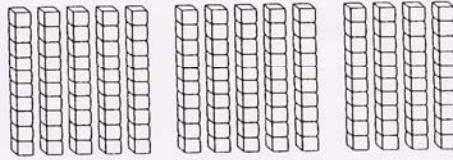
4. Would you enjoy using the computer to help you count numbers? \_\_\_\_\_

\_\_\_\_\_

## **APPENDIX F: PRETEST AND POSTTEST**

Name \_\_\_\_\_

1. Count the tens.



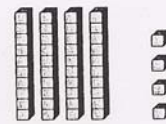
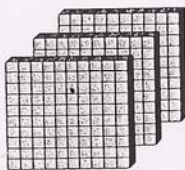
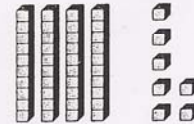
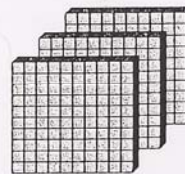
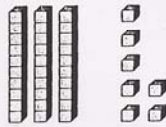
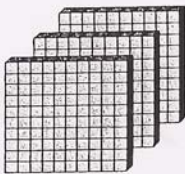
What number is modeled?  
\_\_\_\_\_

2. The number of children eating in the cafeteria is modeled below.

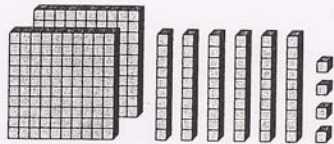


How many children are eating in the cafeteria?  
\_\_\_\_\_

3. Richard has 347 rocks in his collection. Circle the model that shows the number 347.



4. James has two hundred sixty-four paper clips.



Circle one way to describe the number.

$200 + 600 + 4$      $200 + 60 + 4$      $20 + 60 + 4$      $2 + 6 + 4$

P

5. Last weekend, 1,203 children visited the aquarium.  
Which digit is in the thousands place in 1,203?

\_\_\_\_\_

Jake has 1,735 songs in his music library.  
Keri has 1,727 songs in her music library.  
Write  $>$ ,  $<$ , or  $=$  in the circle below to make  
the comparison true.



1,727 ○ 1,735

7. There are 3,648 people watching a parade. Circle a way to write  
the number 3,648.

$3,000 + 60 + 40 + 80$

$3,000 + 600 + 40 + 8$

$300 + 600 + 40 + 8$

$3 + 6 + 4 + 8$

8. Tina has 4,000 sheets of paper. How many stacks of 100 sheets  
can she make?

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## **APPENDIX G: FOCUS QUESTIONS FOR MATH JOURNAL**



## Focus Questions for Math Journal

### Week 2

Using the base-ten manipulatives, show how you would represent 27. Draw your model in your math journal. Explain in writing what you did and how you know you have 27.

### Week 3

Using the base-ten manipulatives, show how you would represent 230. Draw your model in your math journal. What is another way to show 230 using base ten-blocks? Explain in writing what you did and how you know you have 230.

### Week 4

Using virtual manipulatives, show how you would represent 428. Draw your model in your math journal. What is another way to show 428 using virtual manipulatives? Explain in writing what you did and how you know you have 428.

### Week 5

Using virtual manipulatives, show how you would represent 900. Draw your model in your math journal. What is another way to show 900 using virtual manipulatives? Explain in writing what you did and how you know you have 900.

## **APPENDIX H: MATH JOURNAL RUBRIC**

Score Level	Mathematical Knowledge	Strategic Knowledge	Explanation
4	<ul style="list-style-type: none"> <li>• Answer is correct and labeled correctly.</li> <li>• Math terms are used correctly.</li> </ul>	<ul style="list-style-type: none"> <li>• Shows all the steps used to solve the problem.</li> <li>• Completely shows pictures, diagrams, models, or computation if used.</li> </ul>	<ul style="list-style-type: none"> <li>• Writes <i>what</i> I did and <i>why</i> I did it.</li> <li>• If a drawing is used, can explain all of it in writing.</li> </ul>
3	<ul style="list-style-type: none"> <li>• Most math terms are correct.</li> <li>• Answer may have minor errors in computation.</li> </ul>	<ul style="list-style-type: none"> <li>• Shows a reasonable plan and most of the steps used to solve the problem.</li> </ul>	<ul style="list-style-type: none"> <li>• Writes mostly about <i>what</i> I did.</li> <li>• Writes a little about <i>why</i> I did it.</li> <li>• If a drawing is used, can explain most of it in writing.</li> </ul>
2	<ul style="list-style-type: none"> <li>• Know how to do parts of the problem, but makes major errors in computation and gets a wrong answer.</li> <li>• Gives a wrong answer or only part of the answer.</li> </ul>	<ul style="list-style-type: none"> <li>• Shows some of the steps, but plan is not clear.</li> </ul>	<ul style="list-style-type: none"> <li>• Writes some about <i>what</i> I did or <i>why</i> I did it but not both.</li> <li>• If a drawing is used, can explain some of it in writing.</li> </ul>
1	<ul style="list-style-type: none"> <li>• Tries to do the problem, but does not understand it.</li> </ul>	<ul style="list-style-type: none"> <li>• Shows a plan that is not reasonable.</li> <li>• Shows almost none of the steps used to solve the problem.</li> <li>• May include unnecessary information.</li> </ul>	<ul style="list-style-type: none"> <li>• Writes or draws something that does not go with the answer.</li> <li>• Writes an answer that is not clear.</li> </ul>

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