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Effects Of Discussion And Writing On Student Understanding Of Mathematics Concepts

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EFFECTS OF DISCUSSION AND WRITING ON STUDENT UNDERSTANDING OF MATHEMATICS CONCEPTS

by

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ABSTRACT

For this action research project, I wanted to examine my practice of teaching mathematics. Specifically, I encouraged students to improve their communication skills during my math class through daily discussion and writing tasks. After establishing a class set of sociomathematical norms, the students solved problems provided by the *Every Day Counts: Calendar Math* program and used verbal and written formats to describe their problem solving methods and reasons. My study showed the effects of using discussion and writing to help students develop their conceptual understanding of mathematical ideas. Focus was placed on the quality of daily discussions and written tasks both at the beginning of the study and continually as the study progressed. Through daily discussions, monthly written assessments, and student interviews, the study helped to determine the importance of developing students' mathematical communication skills and building conceptual understanding of mathematical ideas.

For Michael.

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CHAPTER 1: INTRODUCTION

> What I can do, I can think about. What I can think about, I can talk about. What I can say, I can write. What I can write, I can read. The words remind me of what I did, thought, and said. I can read what I can write and what other people can write for me to read (Cited in Van de Walle, 1994, p. 33)

Rationale

Deep mathematical understanding does not happen in isolation. We cannot expect our students to sit silently, listen to lectures, and absorb any kind of useful information. "For effective learning to occur, the learner must be an active agent in the learning process and be able to reflect on this learning" (Sherin, Mendez et al. 2004). Sherin et al. also add that students must be able to work cooperatively, sharing and supporting each others' learning. According to Van de Walle (1994), students not only need to be able to think on their own and share their learning with a group, but they also need opportunities to write about their learning, embedding the learned concepts into the students' minds and providing a reflective tool for further understanding. Throughout the learning process, students are actively engaged in the mathematics they are learning and are ultimately held accountable for the concepts they have learned.

In the past, this type of mathematical instruction would have been seen as radical. A more traditional, procedural-based method of teaching math was preferred. Since I began teaching, I have seen program after program come and go in the school system that all claimed to "fix" every student's problems with learning mathematics. However, these "fix-it-all" programs did little more than to delineate prescriptive steps for completing menial mathematical tasks. Students were no longer expected to think about their mathematics learning – math simply became a matter of memorizing the steps.

For some students, this task was easy. Memorizing the steps for solving math problems required little or no effort. For others, however, the task of memorizing complex math algorithms was not only a daunting task, sometimes it seemed downright impossible. As more complex math concepts trickled their way down to the elementary level and standards were written that expected more out of students, students began to sink in the ocean of information they were expected to memorize. To make matters worse, standardized tests began emerging that required students to not only solve complex math problems, but to explain their thought processes as they did so. I witnessed first-hand as my students continually struggled with these tasks. Being a relatively new teacher, it frustrated me that I was unable to help them more effectively.

With the emergence of these standardized tests, math curriculum and standards were revised to help better prepare students. The National Council for Teachers of Mathematics (NCTM) produced four *Standards* documents – *Curriculum and Evaluation Standards* (1989), *Professional Standards* (1991), *Assessment Standards* (1995), and *Principles and Standards for School Mathematics* (2000) – that were intended to promote classroom mathematics that reflects the way mathematics is used outside the classroom. The primary goal of the *Standards* is to "shift toward classrooms as mathematics communities (NCTM, 1991, p. 3). These documents included major reforms for the teaching of mathematics, including a vision to encourage student interactions in small and large group settings as well as stressing discussion and argumentation of mathematical ideas among students.

Recently, NCTM published the *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* (NCTM, 2006). These *Focal Points* are an extension of NCTM's *Principles and Standards for School Mathematics* (NCTM, 2000) which outlines guidelines for the development of school mathematics curriculum as well as providing specific expectations for pre-K-grade 2, grades 3-5, grades 6-8, and grades 9-12. The *Focal Points* elaborate on the *Standards*, identifying "areas of emphasis within each grade from prekindergarten through grade 8" (NCTM, 2006, p. 1). This approach was designed to create a shift in mathematical instruction and help educators think about topics that are truly important in each grade level, moving away from the days of long lists of specific grade level expectations grouped under general standards. NCTM also designed the *Focal Points* in an effort to continue to emphasize the Process Standards included in the *Principles and Standards* – communication, reasoning, representation, connections, and problem solving – while simultaneously providing a "connected, coherent, ever expanding body of mathematical knowledge and ways of thinking" (NCTM, 2006, p. 1).

Mathematics instruction in the United States has been described as "a mile wide and an inch deep" (Schmidt, McKnight, and Raizen, 1997). Simply put, students were expected to learn so much at each grade level that teachers really only had time to briefly cover topics before moving on to the next. There was no time for in-depth work with any particular topic, and mathematical reasoning and problem solving were nearly forgotten altogether. What precious little time teachers were afforded was spent trying to get students to memorize mathematical rules. The *Focal Points* are an attempt to remedy the inconsistencies in mathematics standards across the county. According to NCTM, identifying a small number of key concepts at each grade level and ensuring that the concepts in the later grades build on and connect logically to what was learned in the earlier grades provides students access to "extended experience with core concepts and skills… [which] can facilitate deep understanding, mathematical fluency, and an ability to generalize" (NCTM, 2006, p. 5).

NCTM's goal for the *Standards* and *Focal Points* documents has always been to promote high-quality mathematics instruction through the use of the five Process Standards. Using these Process Standards as a vehicle for teaching mathematics, students will begin to view mathematics as much more than a slew of formulas and algorithms to memorize and regurgitate for a test. Instead, students will begin to see mathematics as a tool that will help them function when they exit school and enter the world outside of the classroom.

My goal for this study is not unlike NCTM's vision of the future of mathematics instruction. All too often I hear colleagues complain that students ascend the grade levels unprepared to meet curricular demands and expectations. One area of mathematics that just about every teacher complained that students were deficient in was number sense and operations. Basically, students had little or no concept of the place value of numbers and had difficulty in memorizing basic facts. To compensate for these deficiencies, repeated reteaching of basic skills became common practice. When questioned about the use of problem solving and mathematical communication and reasoning in the classroom, the answer was always the same – there is not enough time. Time constraints brought on by the pressures of performing well on standardized tests had all but eliminated critical thinking in the mathematics classroom. My goal for this study, then, became clear. Following in the footsteps of NCTM, I wanted to teach math in a way that would encourage students to take ownership of their understanding of mathematical concepts.

Being a writing teacher as well as a math teacher, I understand the power of writing to focus thoughts and communicate ideas. No matter what type of writing is being used, the writer has the responsibility to his audience to make his ideas clear and comprehensible. In my writing class, I often give students opportunities to discuss with a partner what they are going to write about or ideas for revisions they would like to make. After these discussions, students are better able to focus their work and produce exquisite pieces of writing. Having experienced the power of this combination – discussion and writing – in my writing class, I began to wonder if the same effect holds true for mathematics. Early explorations of teaching strategies with the purpose of helping students think more critically about mathematics concepts, as well as work in several graduate classes focused on math instruction, caused me to begin thinking about the connection between mathematical thinking, discussion, and writing.

Question

All of this led me to ask several questions. I wanted to research, "Are discussion and writing about mathematics related? How?" and "How do discussion and written explanations affect student understanding of mathematical concepts?" Specifically, I was interested in investigating the impact that communicating mathematical ideas through discussion and writing would have on fourth grade students' conceptual understanding of mathematics.

As this topic evolved, several focal points jumped out at me that I felt needed to be explored: (a) the history of the use of sociomathematical norms and discourse in mathematics classrooms, (b) various ways writing has been used in mathematics classrooms, and (c) how discussion and writing can be used jointly to enhance my own teaching practices.

Sociomathematical Norms and Discourse

Students like to talk. This is a basic fact that every teacher understands full well. Some find student talk at most an annoyance and a waste of precious learning time. Discussion, however, can be a very powerful teaching tool. The trick is to develop expectations, or norms, for discussion that will focus student talk for the purpose of enhancing their education.

Yackel (2001) explains that classroom norms relating to mathematical explanation and justification are both social and sociomathematical in nature. These norms describe the expectations and obligations that delegate interactions in the classroom. “Normative understandings of what counts as mathematically different, sophisticated, efficient and elegant are examples of sociomathematical norms. Similarly, what counts as an acceptable mathematical explanation and justification is a sociomathematical norm. The distinction between social norms and sociomathematical norms is subtle” (Yackel, 2001, pp. 6-7). In short, social norms are defined as the expectations for interactions in any subject area whereas sociomathematical norms are unique to mathematics (Cobb & Yackel, 1996).

Yackel (2001) continues by describing four classroom norms that characterize student interactions. “These include that students are expected to develop personally-meaningful solutions to problems, to explain and justify their thinking and solutions, to listen to and attempt to make sense of others’ interpretations of and solutions to problems, and ask questions and raise challenges in situations of misunderstanding or disagreement” (p. 6). The fact that students are expected not only to focus on their own mathematical reasoning, but to also consider the interpretations of others, demonstrates a shift in thinking about how students are expected to learn mathematics. The heart of real-world mathematics lies in group thinking, and since our goal as educators is to prepare our students for participation in the adult world, it seems only right that classroom activity should emulate the world beyond the classroom.

Pang (2001) argues that mathematics education reform involving the engagement of students in the social make-up of the classroom may help students develop a deeper conceptual understanding and value of mathematics in their lives. Her study illustrates that there is much more to this kind of reform than simply changing sociomathematical norms within the classroom. “Simply changing classroom social norms promotes neither students’ conceptual learning opportunities nor their social engagement toward characteristically mathematical ways of thinking and communicating” (Pang, 2001, p. 11). According to this study, social norms and sociomathematical norms must remain in focus for student interaction in mathematics to be effective.

For the purpose of my study, I understood the need to develop sociomathematical norms along with my students early on. The majority of my students have never experienced math instruction in this way. They have always been taught math using more traditional methods: textbooks, pencils, and paper. Usually, in their past experiences with mathematics, my students would participate in a teacher-directed lesson or activity (possibly involving a manipulative), and then would move into an independent assignment, typically out of the textbook or a workbook. The focus of these lessons would almost always be procedural-based – did students know how to regroup while adding, for example – with the teacher and students’ primary goal being to get the right answers. Knowing that my students were likely to have limited experience with mathematical discourse, it became apparent that the development of sociomathematical norms as part of my math instruction was essential.

Writing in Math

Writing is an interdisciplinary skill that is often overlooked in the mathematics classroom. This happens for several reasons. First, mathematics teachers who are more procedural in their instructional methods do not see a purpose for written work. Another is that many teachers have varying ideas of what writing actually is. When most mathematics teachers think “writing,” their initial thoughts centralize around essays presenting logically organized arguments or ideas. These essays tend to be prompt-based and have more language value than math value. While writing in math may include these types of formal writing, they represent only one kind of mathematical writing.

Writing to learn (Connolly, 1989) generally involves informal, less-structured forms of writing, including notes, brief explanations, and drawings to illustrate thinking processes. “Proponents of writing to learn have identified many potential benefits when students write as a regular part of their mathematics instruction” (Baxter et al., 2005, p. 120). Baxter et al. found that writing not only enhanced their students’ conceptual understanding of mathematics by providing an alternate method of expressing their understanding, but it also provided them an additional connection to the teacher.

Marilyn Burns (1995) claims that mathematical writing is not so different procedurally than writing in the language arts.

> The process of writing requires gathering, organizing, and clarifying thoughts. It demands finding out what you know and don’t know. It calls for thinking clearly. Similarly, doing mathematics depends on gathering, organizing, and clarifying thoughts, finding out what you know and don’t know, and thinking clearly. Although the final representation of a mathematical pursuit looks very different from the final product of a writing effort, the

mental journey is, at its base, the same – making sense of an idea and presenting it effectively. (p. 3)

Another reason that writing can be an effective learning tool for a mathematics classroom is that it forces students to focus on their metacognitive processes. Metacognition, or thinking about one's own thinking, is essential for students to identify the appropriate information and strategies used during problem solving. When asked to write in math class, students must deliberately think about the specific steps they took to solve a problem and justify why those steps were appropriate to the problem. In a study by Pugalee (2001), a metacognitive framework was evident in student writing about problem solving processes. "Through written accounts of problem solving processes, these students demonstrated their mathematical reasoning. The data showed students' use of metacognitive behaviors in the orientation, organization, execution, and verification phases of problem solving" (Pugalee, 2001, p. 243).

In this study, I hope to use writing not only to increase my students' metacognitive awareness and problem solving skills, but also as an assessment to measure student achievement in mathematics. Miller and Hunt (1994) found that writing could also serve as a powerful tool for teachers, providing valuable information about students' understanding and misunderstanding that would help to shape future reteaching or individual assistance. By reading and reflecting upon student writing, I will be able to make more informed decisions about what needs to be taught, when, and to whom.

Bringing Discourse and Writing Together

Mathematics is more than a simple list of algorithms for students to study, memorize, and be able to use. Deep understanding and clear communication of mathematics concepts is required for real-world applications. If our students are to have full access to the school curriculum and ultimately to opportunities in the adult world, mathematical literacy, or what the British call "numeracy," is just as important as verbal literacy (Pugalee, 1999).

David Pugalee (1999), in his article, *Constructing a Model of Mathematical Literacy,* describes the necessary components for the development of mathematical literacy. Among others, one of these components is communication. Pugalee continues to discuss the increasing use of discourse as a facilitator of the construction of mathematical knowledge. "It is powerful when individuals become engaged in the type of discourse that forces them to reason about the mathematics they are using" (Pugalee, 1999, p. 21).

The development of mathematical literacy stems not only from discourse, however. Writing is a method of communication that can also be a powerful tool in developing students' mathematical reasoning skills. Vygotsky (1987) argued that writing involves several cognitive and metacognitive processes that force the producer to be an active participant in his or her learning. Writing also serves in helping students clarify, refine, and consolidate their mathematical thinking (NCTM, 1989). The NCTM *Principles and Standards for School Mathematics* (2000) list five components for mathematical learning that all instructional programs should include: problem solving, reasoning and proof, communication, connections, and representation. For the purposes of this study, I plan to focus on the component of communication, which states that students should be able to:

- Organize and consolidate their mathematical thinking through communication;
- Communicate their mathematical thinking clearly and coherently to teachers, peers, and others;
- Analyze and evaluate the mathematical thinking and strategies of others; and

- Use the language of mathematics to express mathematical ideas precisely. (p. 63)

In this study, I will be using a combination of discourse and writing to analyze the development of student understanding of whole number concepts and operations. I believe that a balanced mathematics program requires the ability of students to not only use mathematics skills, but also communicate their thought processes to others verbally and in writing. By bringing discourse and writing together to teach mathematics, I hope to be able to increase my students' ability to solve problems, explain their solutions, and justify their reasoning using whole numbers and basic operations.

Conclusion

The purpose of this study is to analyze the effects discourse and writing have on student understanding of whole number concepts and operations. By working with students to develop sociomathematical norms (Yackel, 2001) I hope to provide students with opportunities to vocalize their thinking about mathematical concepts, listen to others' interpretations of and solutions to mathematical problems, ask questions to clarify misunderstandings, and ultimately construct meaning about mathematics through an integration of all three of these components.

Likewise, I will use writing as a communication tool that will help students focus and reflect on their understanding of mathematical concepts so that their knowledge can be applied in various situation. For the purposes of this study, writing and discourse will be used in a reciprocal way; discourse will lead into writing, and vice versa.

My plan is to require students to think, share, reflect upon, and revise their mathematical understanding of whole number concepts by placing the responsibility for learning on the student. My role in this will shift from "giver of information" to "facilitator/coach," in which I will act as an observer in times of accomplishment and as a guide through times of struggle. If we expect our students to eventually become productive members of society, we need to learn how to allow them to take control of their learning. While teachers will never be obsolete, the role of the teacher in a mathematics classroom needs to change in order for effective learning to occur.

A review of significant literature provided much information. By looking at what others had done in regards to developing conceptual understanding through mathematical communication, I was able to determine which methods I felt would be best in my implementation of this plan. While I anticipate roadblocks and struggles of my own during this journey, I am excited about the opportunities this study will provide in learning valuable information about my students and about my teaching style.

CHAPTER 2: LITERATURE REVIEW

Introduction

One of my main goals as a classroom teacher includes helping my students become autonomous learners. I want my students to be curious about the world around them and to motivate themselves to learn about that world and the role they play in it. Until recently, I had always thought that successful mathematics instruction was as simple as a student having all of his basic multiplication facts memorized or a student knowing that length multiplied by width was the formula for area of a rectangular figure. The notion of having students reason mathematically and explain their thinking using mathematically appropriate language was preposterous.

An ever-increasing body of mathematics education research has documented a shift away from the thinking that mathematics is little more than a collection of facts and formulas that students need to memorize and apply on a test. In this chapter, I have discussed the details of several studies that have concluded that students learn mathematics differently than originally thought. The following sections addressed several aspects of a cohesive mathematics learning environment, including how children must learn the language of math in order to communicate their thinking effectively, assisting students in building conceptual understanding of mathematical ideas, structuring a classroom environment that encourages and supports critical thinking about math concepts, and the relationship between talking and writing about mathematics. In addition, this chapter will also elucidate the implications of these studies on my own research.

Communicating Mathematically

Communication is essential to virtually every aspect of life. Unfortunately, not all students come to school with the same level of communication skills. Some are able to verbalize their ideas eloquently, while others struggle to form a simple sentence. Communication is especially central in the learning of mathematics. The communication standards outlined in NCTM's *Standards* documents (1989, 1991, 1995, and 2000) emphasize the importance of being able to talk about, write about, describe, and explain mathematical ideas. According to Hiebert et al. (1998), communication is a key component of developing students' mathematical understanding, and includes "talking, listening, writing, demonstrating, watching…participating in social interaction, sharing thoughts with others, and listening to others share their ideas" (p. 5).

The ability to express ideas clearly and coherently is a skill that helps students gain insight into their own thinking as well as the thinking of others and to build a conceptual knowledge base from the conglomeration of all of those ideas. Students should be able to express mathematical ideas both verbally and symbolically, learning not only to interpret the language of mathematics, but also to use it (Van de Walle, 1994). Without effective communication, math students miss prime opportunities to learn from their peers and to become familiar with the world of math that exists beyond the classroom.

Communicating mathematical ideas provides students opportunities to combine their own understanding and experiences to that of their classmates. It is also important for students to learn how to make sense of and evaluate another's methods and weigh the strengths and limitations of different approaches. Through this series of trials and revisions, students become critical thinkers about mathematics (NCTM, 2000).

Mathematical communication can take a variety of forms. The purpose of this literature review is to explore two communication formats – discussion and writing – and their influence on student understanding of mathematics. This review will also look at how discussion and writing affect students' abilities to communicate mathematically as well as the teacher's and students' roles in a classroom environment where social interaction and written explanations take center stage.

According to a study conducted by Yackel (2000), the theory of symbolic interactionism is "useful when studying students' learning in inquiry mathematics classrooms because it emphasizes both the individual's sense-making processes and the social processes" (p. 2). During this study, Yackel examined the combined use of both social and sociomathematical norms in a variety of classrooms and their affects on student understanding of mathematical concepts. The result was that students began to listen to and make sense of others' thinking, making mathematical argumentation possible. The existence of social and sociomathematical norms in the classroom both allowed students to think and speak freely about mathematics but also provided distinct parameters for the social interactions taking place. In planning for my study, I decided to use an inquiry-based approach to mathematics instruction. Since this type of instruction may differ from other instructional methods used at my school, I hoped that developing social sociomathematical norms with my students will help to facilitate our class discussions.

In a study conducted by Steinbring (2005), mathematical communication was found to be a complex structure; it requires the combination of cognitive and social processes in a way that supports student thinking and mathematical understanding. These two systems – one mental; one social – must work closely together in order for mathematical understanding to be achieved.

"Without the participation of consciousness systems there is no communication, and without the participation in communication, there is no development of consciousness" (p. 320).

Steele (2001) discussed the implications of a sociocultural environment, in which students sharing their reasoning and listening to others share their ideas is essential to building mathematic understanding. She quoted Vygotsky (1994), who stated that individuals learn the rules of a culture by interacting with them, internalizing them, and are transformed by them as they explore the language of that culture. This idea parallels the mathematical culture in that students construct understanding and develop mathematical meaning as they learn to explain and justify their thinking using the language of mathematics. The more active students are in thinking and communicating about mathematics, the stronger their understanding of mathematical concepts becomes.

In her study, Steele (2001) analyzed the interactions between a fourth grade teacher and her students as she attempted to teach mathematics using a sociocultural approach. Steele specifically looked for characteristics of instructional strategies that most clearly represented the sociocultural perspective. The study focused mainly on the teaching practice and the reasoning behind the teacher's decisions before, during, and after each lesson. Two themes of communication emerged during the study, including ways the teacher helped students acquire and apply the language of mathematics and ways the teacher used visual representations to explain their thinking. Pirie (1998) is quoted in the discussion as saying, "[T]here are several ways of using language to communicate about mathematics. Teachers and students communicate using ordinary language, mathematics verbal language, symbolic language, visual representation, unspoken but shared assumptions, and quasi-mathematical language" (p. 412). According to Pirie, each of these forms of communication should be present in every mathematics classroom as each is justifiable. For example, a student may use an ordinary word like *middle* to describe the center of a circle, and it would be the teacher's responsibility to replace the ordinary word with the appropriate mathematical term, thereby expecting students to use the appropriate term to explain and justify their thinking henceforth.

Pirie (1998) also stated that visual representation is a powerful tool for guiding the growth of mathematical communication. If students have a concrete representation of the abstract terminology to study, then new understanding is created much more easily. In my study, I plan on using visual representations (in the form of a bulletin board display) derived from several math disciplines as a spring board to help launch students into meaningful mathematical discussions. According to Pirie, students who are learning the language of mathematics acquire communication skills more quickly with visual representations acting as a learning scaffold.

Classroom Environment

If the goal of the mathematics reform movement is to move away from traditional views of mathematics and towards developing classrooms as mathematical communities (NCTM, 1991), then it is the teacher's responsibility to create a classroom environment that supports mathematical discourse (Van de Walle, 1994) – the communication of thought by talk or writing. The *Principles and Standards for School Mathematics* (NCTM, 2000) supports the belief that elementary mathematics instruction should promote more than students' abilities to make sense of mathematics, but should also provide students with choices that help them connect the mathematics they are learning to their prior knowledge through the use of concrete materials.

The Constructivist theory supports a similar belief. For this study, I define constructivism as a student's ability to reflect on what has already been learned and connect prior knowledge to new or unfamiliar concepts. I planned to use a social constructivist approach, which involves student-teacher and student-student interactions through discussion and writing about concrete mathematical experiences. These experiences were designed to help my students make connections to previously learned math concepts and apply that knowledge to the unknown.

To help guide students through the process of making sense of mathematics through both concrete materials and abstract symbols, the Constructivist approach includes three levels of representation: concrete (enactive), representational (iconic), and abstract (symbolic) (Bruner, 1966). The concrete level includes the use of mathematical manipulatives, providing students a “hands-on” approach to mathematical learning. Students should then phase into the representational level, in which they demonstrate their understanding of mathematical concepts through the use of pictures, graphs, charts, and other mathematical representations. Finally, students move into the abstract level, in which they begin to use the symbols associated with the mathematics they are exploring to express their mathematical ideas. Students should also be able to make a connection between all three levels and use that connection, along with the appropriate mathematical vocabulary, to further their mathematical understanding.

> Mathematics is thinking, problem solving, and searching for order. The symbolism so closely associated with mathematics is only a means of recording and expressing mathematics and conveying these ideas to others. Children need to view written work with this perspective. In the same sense, children need to learn other modes of mathematical expression, including oral and written reports, drawings, graphs, and charts. Each day should include discussion and/or writing about the mathematical thinking that was going on in the classroom. No better way exists for wrestling with an idea than to

attempt to articulate it to others. Mathematical expression, therefore, is part of the process and not an end in itself. (Van de Walle, 1994, p. 8)

Constructivist vs. Traditionalist Teaching

Mathematics is traditionally taught in a lecture-based format. According to Wood (2001), the traditionally taught mathematics classroom involves “the teacher present[ing] a problem, ask[ing] for the solution, and then ask[ing] a series of questions to insure that students know a mathematical idea and the written symbols” (p. 110). In this traditional view of mathematics, students are not required to think as much as they are required to determine which mathematical rule or procedure should be used (Van de Walle, 1994).

As mentioned previously, constructivist teachers devote time to creating a classroom environment where students feel comfortable reflecting on theirs and others’ prior knowledge, and then use that knowledge to expand their thinking about new or unfamiliar concepts. Traditionalist teachers typically use teacher-centered instructional strategies that include a focus on facts and theoretical principles, few opportunities for experiential learning, and grades that are based on rote memorization and regurgitation of facts, such as multiple choice, true-false, and matching item (Wingfield & Black, 2005).

Kutz (1991) argued that teachers are neither Constructivist nor Traditionalist in nature; they tend to teach the way they were taught and use what has consistently worked in the past. Cobb, Yackel, and Wood (1992) claim that these two teaching and learning methods are in constant conflict. Learning can be viewed in two perspectives: a) as a “process in which students actively construct mathematical knowledge as they try to make sense of their world” (p. 6) or b) as a “process of apprehending or recognizing mathematical relationships presented in instructional representations" (p. 6). In other words, learning can bee seen as a social process, where students build an understanding through various experiences, or as a banking system (Freire, 1970), which depends upon the teacher's expertise of preexisting mathematical knowledge and his or her delivery of that knowledge to students.

Cobb et al. (1992) identified three dangers of the second definition of mathematical learning. First, students will have difficulty grasping mathematical relationships presented through instructional representations unless they are explicitly told what they are supposed to learn. Second, the representation provided by the teacher may not be sufficient in helping students identify the "correct" mathematical relationships. Finally, students who are taught in a more traditional manner are likely to disconnect the mathematics they learn in school from mathematics used in other settings, where problem solving and mathematical reasoning play a predominant role.

In looking at this comparison of these two teaching methods, a very basic necessity of teaching comes into focus: the classroom environment. A supportive learning environment is an important factor in a student's achievement or failure. Every teacher wants a classroom that operates smoothly and in which the students feel safe. In order to accomplish this, teachers must be willing to plan, implement, and enforce specific expectations for their students. When teachers take the time to do this, students will know what is expected of them and adapt to the methods used by the teacher (Desiderio and Mullennix, 2005). "Teachers have the ability to empower their students. They may establish roles for themselves and their students" (Egendoerfer, 2006, p. 10).

In planning for this study, I hoped that I would see a change in the way my classroom operates. In past reflections of my teaching practice, I came to realize that I had been teaching in a very traditional way – chiefly through lecture and presenting the procedural aspect of mathematics. There was very little opportunity for students to think on their own, let alone attempt to communicate their ideas to their peers. By working with my students to establish social and sociomathematical norms to help guide our math work, I hoped to in turn create a classroom environment that would encourage more student-student interactions and less teacher-directed lecture.

Classroom Interactions

According to Yackel (2000), two types of interactions can take place during mathematics instruction. The first is an intellectual interaction between teacher and students as the discussion focuses on specific aspects of mathematical problems. The other is a social interaction, as the teacher and students shift from discussing the mathematical concepts to talking about how to act and contribute to the discussion. It is due to these two types of interactions that the need for both social and sociomathematical norms is crucial to a mathematics classroom.

Social Norms

"'Norm' is a sociological construct and refers to understandings or interpretations that become normative or taken-as-shared by the group" (Yackel, 2000, p. 7). Norms describe the expectations that guide classroom interactions. According to Planas and Gorgorió (2004), social norms describe not only the way individuals are expected to behave in the classroom setting, but they also describe the collective behaviors of the group comprising the class, including the teacher. Vacha (1979) stated that social norms are created and shared by all of the members of a group. So long as a majority of the group members support and conform to the norm, it shall remain taken-as-shared during the group's interactions. In his observations of second and third grade students vying for a playing field with the intent of initiating a game of kickball, several problems ensued. Rather than fighting, for which it was commonly known would result in the involvement of the supervising teachers, the students compromised and worked out their own norms for deciding who would play on the field and when.

While in this example norms were established without the assistance or interference of a teacher, classroom norms require input from the teacher. In the classroom setting, the teacher is a member of the group, and social norms are only effective when all members of the group are supportive of them. Throughout the course of my study, I plan to first work with my students to establish the social norms necessary to guide acceptable behaviors as well as the adequate exchange of ideas within the classroom. I have identified this as an important first step in my research because I want my students to have clear expectations for their behavior and social interactions in order to maintain a high level of academic integrity.

Sociomathematical Norms

In mathematics, the specific expectations for student and teacher interactions are called sociomathematical norms. They act as guidelines for the specific exchanges between students and teachers during math class. Yackel, Cobb, and Wood (1992), in their study that involved several levels of classrooms, examined the use of instructional approaches that viewed mathematics as both an individual and a cooperative activity. During the study, the authors discussed several difficulties that underlie the representational view of the learning process, arguing that strict adherence to this view contradicts many of the goals of recent mathematics instructional reform. Instead, they offer alternatives to the representational view that help to illustrate the point that mathematical learning takes place “not between students’ internal representations and mathematical relations contained in external representations but between the students' interpretation of instructional materials and the taken-as-shared interpretations of mathematically acculturated members of the wider community" (p. 17). There for, the authors concluded that, in order for classroom interactions to remain as authentic as possible, students must be active participants in the construction of classroom norms.

Sociomathematical norms take shape when mathematical explanations and justifications become acceptable (Hershkowitz and Schwarz, 1999). This acceptability results from the comprehension and replication of explanations and justifications of mathematical solutions. No longer is the teacher expected to be the central power in the classroom. Students are now being asked to become critical thinkers about mathematics and take responsibility for their own understanding of mathematical concepts. "The establishment of new sociomathematical norms is directly related to the Constructivist approach in that students are active in their role and take responsibility for their part in mathematical discussions" (Egendoerfer, 2006, p. 10).

Sociomathematical norms play a major role in the discussions that take place within a mathematics classroom. Not only do they outline what makes mathematical explanations and justifications acceptable, but they also "deal with the actual process of making a contribution" (McClain and Cobb, 2001, p. 238) during mathematical discussions. McClain and Cobb noted the process of developing sociomathematical norms in one first-grade classroom over the course of several months. One of their goals was to develop activities that would encompass mental computation and estimation of numbers up to 100. During the study, McClain and Cobb tried to "account for the students' learning in the social context of the classroom throughout the school year" (p. 240). Two main instructional sequences were used in the study: the Patterning and Partitioning Sequence and the Structuring Numbers Sequence (McClain & Cobb, 2001 and Gravemeijer et al, 2000). The Patterning and Partitioning Sequence involved students conceptualizing and organizing a group of up to ten objects in various ways. For example, a student might represent eight items as four and four, five and three, or two less than ten depending on the situation. This sequence focused on finger patterns, spatial patterns, and mental conceptualization. The Structuring Numbers Sequence was, in short, instructional activities designed to increase student computational abilities through various thinking and composing strategies. As a result of their twelve-week study, McClain and Cobb found that sociomathematical norms are not a set of predetermined rules that should be laid out before students enter a classroom. These norms should be revisited, reflected upon, and revised if the needs of the class require it.

Williams and Baxter (1996), in a study involving a middle school classroom, sought to demonstrate that it is "both feasible and responsible to implement instructional programs that foster the acquisition of mathematical thinking and reasoning skills by students attending middle schools in economically disadvantaged communities (Silver & Lane, 1993, p. 12), as part of their work with the Quantitative Understanding: Amplifying Student Achievement and Reasoning (QUASAR) project. Through their three-year observation of one middle school classroom, they identified two types of scaffolding that would support student achievement in mathematics: analytic and social. Social scaffolding is "the scaffolding of norms for social behavior and expectations regarding discourse" (Williams and Baxter, 1996, p. 24) and is intended to support positive student interactions during discussions (Nathan and Knuth, 2003). Analytic scaffolding, on the other hand, is "the scaffolding of mathematical ideas for students" (Williams and Baxter, 1996, p. 24) and is intended to support students' learning of mathematical concepts (Nathan and Knuth, 2003).

The teacher plays an important role in the classroom, acting as a guide in the development of sociomathematical norms. By involving the students in the process of developing sociomathematical norms, teachers take on a dual role of supporting “both the student’s mathematical development and development of mathematical autonomy” (McClain and Cobb, 2001, p. 238). In this attempt to guide the development of sociomathematical norms, teachers try to also influence their students’ beliefs in what it means to learn and do mathematics. McClain and Cobb (2001) also found that the teacher’s role as a guide depended upon the teacher’s ability to be flexible, as teaching is a process that involves reflection and adaptation when expectations are challenged by unanticipated events. In other words, the teacher needs to often take on the role of a mediator between the social interactions that are typical of any classroom situation and the interactions that are mathematical in nature (Pang, 2001). “One fundamental role of the teacher in promoting communication is to create a classroom environment of mutual trust and respect in which students can critique mathematical thinking without personally criticizing their peers” (Pugalee, 2001, January, p. 297). This flexibility supports the shift in mathematical instruction towards a more student-centered classroom by presenting mathematical ideas as “emerging in a continuous process of negotiation through social interaction” (Pang, 2001, p. 1).

For example, based on the observed needs of the students, McClain and Cobb (2001) worked with first grade teacher Ms. Smith to incorporate her previously-established classroom norms into her mathematics instruction. These included:

1. The students were expected to explain and justify their reasoning. They were also expected to raise their hands to indicate that they had a contribution to make.

2. On those occasions where a student's contribution was judged to be invalid in some way by the classroom community, Ms. Smith frequently intervened to clarify that this student had acted appropriately by attempting to explain his or her thinking. Further, Ms. Smith emphasized that such situations did not warrant embarrassment.
3. The students were expected to listen and make sense of others' explanations. Ms. Smith directed students giving explanations to speak loudly enough for all to hear.
4. Ms. Smith often commented on or redescribed students' contributions, frequently notating their reasoning on the white board or overhead projector as she did so.
5. Students were expected to indicate nonunderstanding and, if possible, to pose clarifying questions to the student explaining the problem.
6. Students were expected to explain why they did not accept explanations that they considered invalid (McClain & Cobb, 2001, p. 245).

In my study, I plan to work with my students to establish sociomathematical norms for use in the classroom. This may prove to be as crucial as the development of social norms, as discussions with other teachers at my school have indicated to me that my students are entering my classroom with a purely algorithmic view of mathematics. In other words, they do not know how to think critically or talk about mathematics, merely because they have never been expected to do so. In the words of Sir Isaac Newton, I hope to be able to see further because I have stood on the shoulders of giants. By adapting the ideas presented in this chapter to my particular group of students, I hope to see a dramatic improvement in the way I teach mathematics.

Communication through Writing

The development of sociomathematical norms needs not only apply to classroom discussion of mathematical concepts. These norms may also apply to student writing about mathematical concepts as well. As stated earlier, the communication standards outlined in NCTM's *Standards* documents (1989, 1991, 1995, and 2000) emphasize the importance of being able to talk about, write about, describe, and explain mathematical ideas. However, writing often involves a much more structured utilization of language. "When students learn to use language to find out what they think they become better writers and thinkers" (Countryman, 1992. p. 11). Unfortunately, mathematical language is very rarely taught directly or assessed for correct usage (O'Shea, 2004). Little or no opportunities to write and think result in students' inability to write and think well. "When math classes, like writing classes, are focused on rote performance of academic exercises, there is equally little opportunity to think" (Rose, 1989, p. 9).

Much like literary conventions, mathematical conventions consist of a combination of words and symbols that work together to express ideas. Incorrect use of mathematical terms and symbols is "not only annoying to the reader but it can also lead to mistakes and misunderstandings" (O'Shea, 2004, p. 102). Written assignments that extend the sociomathematical norms of the classroom and require students to explain and justify their problem solving strategies have the potential to support oral discussions. When written tasks and discussion are combined, the opportunity for students to construct knowledge is increased exponentially.

In my study, I wanted to help my students make a connection between the social interactions of the classroom discussions and their ability to communicate their mathematical ideas in writing. I hoped that my students would be able to make the transition between using our social and sociomathematical norms during discussions and using the same guidelines to drive their writing. I also wanted my students to understand that while discussing, ideas are heard and responded to instantaneously. While writing explanations, however, feedback is not as readily available. It is the responsibility of the writer to be sure that all ideas are elucidated as eloquently as possible to prevent confusion when the ideas finally reach the intended audience. Mathematical language must be used appropriately and accurately, and therefore classroom assignments involving writing should be designed to support students in this endeavor.

A prime example of a writing activity that required students to use mathematical language appropriately and accurately was developed in a study conducted by Tichenor and Jewell (2001). The researchers made arrangements for 13 elementary education majors to correspond via e-mail with 21 second grade students. The college students' objective was to use questioning techniques to try to get the second graders to share their thoughts about mathematical concepts in meaningful ways in their writing. The participants benefited from the project in both expected and unexpected ways. The college students learned about effective questioning techniques and the usefulness of writing as an instructional tool in mathematics. The second graders felt as though their math and writing skills improved as a result of the project. All in all, the project provided an authentic method of incorporating writing into mathematics instruction.

In addition, writing can provide an alternative form of active participation for students who are reluctant to partake in verbal conversations (Baxter et al., 2005). With conversation, one person talks at a time while everyone else listens, forms their own thoughts on the subject, and waits for their turn to speak. During this time, some students may easily become disengaged or even allow more dominant students to do all of the work. By asking students to write prior to discussing, all students are more apt to participate and contribute to the ideas presented during the discussion (Countryman, 1992).

In their study, Baxter and her colleagues wanted to examine the mathematical proficiency of low-achieving students through their writing. The study was one in a series focusing on mathematics instructional reforms with low-achieving middle school students. By observing data from classroom discussions, student journals, and interviews with the teacher, the researchers were able to obtain a complete view of classroom communication. The results of the study indicated that students who do not participate actively in mathematics discussions were just as reluctant to write about their mathematical reasoning. Just as important, these students who would not participate in whole group discussions demonstrated mathematical proficiency in a smaller groups setting, such as in pairs. After these small group discussions, students were more willing to write about the mathematical experience in their journal, showing an increasing ability to express their mathematical thinking. Simply put, writing allowed the students to express their ideas about mathematics at their own pace, using their own experiences and language in combination with mathematical language. When used in conjunction with discussion, writing also plays a role in creating a caring and cooperative classroom environment through interaction (Connolly, 1989).

Writing about mathematics has many benefits. For one, writing forces the learner to slow down their thinking, creating an awareness of the processes being used. These metacognitive behaviors are essential for effective problem solving (Pugalee, 2001, May). Through writing, students are able to keep track of their mathematical reasoning much more effectively than in conversations, which tend to be more fast-paced. Therefore, “writing can be a tool for supporting a metacognitive framework and that this process is more effective that the use of think-aloud processes" (Pugalee, 2001, January, p. 44). Pugalee (2001, May) also found that writing provides a source of information for teachers to assess their students thinking about and learning mathematical concepts. Writing assignments become a record for teachers to refer to when determining students' strengths and weaknesses and how best to address further instruction.

Charged with the instruction of such a variety or learners, teachers are constantly looking for ways to meet the needs of all learners. "Writing can revive the bored students and provide a less threatening activity for the student who is math-anxious or has a lower skill level. For the strong math student, writing is a chance to show creativity" (Birken, 1989, p. 38). Also, strong math students who run into cognitive blocks may use writing to clarify confusion and internalize concepts (Birken, 1989). Baxter et al. (2005) determined that journal writing not only enhanced students' cognitive processes, but it also served as a way for lower-achieving students to connect with the teacher on a more continual basis. Reflecting on mathematical experiences and then recording those reflections in writing enables students to show what they know and exposes what they do not know (Baxter et al., 2005). When students are given opportunities to develop control over their own learning, they feel a sense of accomplishment, which in turn improves students' attitudes toward mathematics (Powell & López, 1989).

Conclusion

A review of literature shows that students need to be active participants in their education. By learning to communicate mathematically, students can take control of their learning and become critical thinkers. Teachers have the responsibility to create a classroom environment which creates a press for conceptual understanding (Kazemi & Stipek, 2001) and supports positive social interactions between students. The verbal and written discourse in which students take part helps to construct mathematical understanding that lays a foundation for future mathematical learning.

> When students are given the opportunity to communicate about mathematics, they engage thinking skills and processes that are crucial in developing mathematical literacy…Students who are supported in their "speaking, writing, reading, and listening in mathematics reap dual benefits: they communicate to learn mathematics, and they learn to communicate mathematically" (NCTM, 2000, p. 60). Communication, then, should be a fundamental component in implementing a balanced and effective mathematics program. (Pugalee, 2001, January)

In taking a detailed look at the differences in the Constructivist and Traditionalist approaches, the benefits of the development of sociomathematical norms, and the effectiveness of using writing in the mathematics classroom, teachers can employ communication strategies that are applicable for all types of learners.

In beginning this qualitative action research project, I was interested in studying a different way of teaching mathematics than I had previously been accustomed to. With the demands laid down upon teaching by state standards and high-stakes tests, a reflection of my past teaching practices revealed that I was relying on rote, procedural-based instructional methods and drilled practice to teach mathematics. For the sake of test scores, I was ignoring the fact that my students come to me with vast experiences in mathematics and have the ability to communicate their experiences, albeit in a rudimentary manner. After experiencing great success with creating a positive learning environment through the development of social norms and classroom expectations, I was interested in trying to improve my students' mathematical communication skills through the development of sociomathematical norms and applying them to both whole-class discussions as well as individual student writings. I was also interested in redefining my role in the classroom as well as the responsibilities of my students. I hoped that the changes I planned to implement in my teaching practices would have a positive effect on the classroom environment I continually strive to create as well as on my students' learning and understanding of mathematics.

The following three chapters will describe and explain my methodology, my data analysis, and my conclusions. Convenient access to a willing group of fourth grade students provided me with many opportunities to collect useful data. In the next chapter, I will describe the data collection methods I used and the rationale behind them. My questions, "Are discussion and writing about mathematics related? How?" and "How do discussion and written explanations affect student understanding of mathematical concepts?" will be explored in greater detail in the remaining chapters.

CHAPTER 3: METHODS

In order to answer my questions, “Are discussion and writing about mathematics related? How?” and “How do discussion and written explanations affect student understanding of mathematical concepts?” I needed to provide a context for gathering data. In this chapter, I describe the setting and the methods used within my fourth grade classroom to collect appropriate information to answer my questions.

Design of Study

My goal in this process was to look closely at my practice of teaching mathematics and to determine if certain methods of teaching were effective in increasing student understanding of mathematical concepts. Therefore, I conducted an Action Research study with my students. “Action research is a qualitative study conducted in the natural classroom setting” (Egendoerfer, 2006). By using this type of research, I was able to perform the dual tasks of teacher and researcher without overloading myself or my students with work. Part of my goal was to provide narrative describing my students’ daily encounters with mathematical concepts and my observations of their use of discussion and writing during math time.

Setting

School Setting

The school is located in an average population school district in the state of Florida. Out of the 63,000 students who attend school in the district, the school houses 817 students, including the fourth largest population of low socioeconomic students in the district, making the school eligible for federal Title I funds. Free and reduced lunch is offered at the school, of which 63% of the students qualify. Of the 817 students who attend the school, 29% are African American, 3% are Asian, 25% are Hispanic, 10% are Multiracial, and 31% are Caucasian. In addition, 11% of the student population are speakers of other languages and 11% have been identified as needing special education services.

Classroom Setting

My action research study was conducted with a diverse group of 22 fourth grade students. These students, who ranged in age from nine to eleven years, were randomly placed in my class by the school administration. This random selection takes place during the summer months. Each year, teachers fill out information cards on each student, providing data such as academic standing (above, at, or below grade level), test scores, and any special services received. The boys’ cards are blue and the girls’ cards are pink. After turning them into administration, the cards are then divided into classes randomly to ensure that each teacher receives a fair mix of students.

Each student gave assent to participate in the study. During the course of the study, one student moved out of the class. The class consisted of 10 boys and 11 girls. The group was made up of 5 Caucasian students, 5 African American students, 7 Hispanic students, 1 Asian student, and 3 Multiracial students. Two students received English as a Second Language (ESL) services, 2 students received services for Emotionally Disabled (ED), 2 students received services for Speech and Language Impairment, and 12 students received free or reduced lunch.

<u>Methods</u>

Data Collection

After receiving Institutional Review Board (IRB) approval (Appendix A) and principal approval (Appendix B), I prepared a parent consent form (Appendix C). Our school holds a "Meet the Teacher" day the week before school starts, so I planned to speak to parents about the study during that event. I received 16 signatures out of 22 that day, so I sent the remaining forms home with students on the first day of school. The remaining forms were returned, giving all of my students the permission required to participate in the study and to be video or audio taped. On the first day of school I read the student assent form (Appendix D) to my students, explaining the study and answering any questions they might have had. I then asked students to complete a pretest (Appendix E) designed to measure their writing ability at the beginning of the school year. The task was also intended to help me gauge my students' ability in expressing themselves in a mathematical context. I then selected three students from my class to analyze their written explanations. I chose these students based on information I was given about their performance in mathematics in third grade. One student was high-achieving, one was an average student, and one was low-achieving. This assessment marked the beginning of my data collection.

Procedures

In order to provide my students with concrete visuals as an anchor for their discussions, I began using *Every Day Counts Calendar Math* (Gillespie & Kanter, 2005). *Every Day Counts Calendar Math* is a researched based mathematics program published by Great Source Education

Group, which is a division of Houghton-Mifflin Company. This program was designed to meet five criteria:

1. Children need to learn mathematics incrementally, giving them the opportunity to develop understanding over time;
2. Visual models help children visualize and verbalize number and geometric relationships;
3. Classroom discussion fosters the growth of language acquisition and development of reasoning. It also allows children to discover that there are many strategies for solving problems.
4. Over time, children can learn to think algebraically. Early exposure to this type of thinking will lead them to a successful future in mathematics.
5. Observing and listening to children is essential to ongoing assessment that can guide instruction (Gillespie & Kanter, 2005, p. 4).

Since the underlying purpose of the *Every Day Counts Calendar Math* program was to provide concrete, visual models intended to initiate discussion about mathematics, I felt that it would be a great structure for the purpose of my study.

The first part of the *Calendar Math* is the Calendar. This component delivers a different pattern of colors, numbers, and geometric shapes every month. As the month progresses, students develop patterning and algebraic thinking skills as they predict what the next calendar piece will look like. Another important component is the Daily Depositor, which is a place value pocket chart used to collect a certain amount of money every day of the month. Students utilize the Daily Depositor to make predictions about how much money will be collected in the month based on that month's pattern and to develop mental addition, subtraction, and multiplication skills. A third component of the *Calendar Math* program is the Counting Tape, which keeps track of the number of days of school. One new number is written on the Tape every day and the numbers are used in a study of multiples. Multiple Markers are placed on the Tape so that students can see common multiples and factors of numbers from 1-180. Finally, the Coin Counter gives students practice with counting mixed coins, determining change from purchases of one or more items, and introduces students to decimal notation in tenths and hundredths.

While these four components are staples of the program and are continuously discussed and updated for the entire school year, other components are integrated throughout various months. The Graph gives students realistic ways to collect, organize, display, and analyze data. It also serves as a tool that introduces students to probability concepts. A Fraction a Day introduces students to proper and improper fractions, mixed numbers, equivalent fractions, and simplifying fractions using a variety of methods. The fractions are shown as both a part of a whole and a part of a set. The Measurement component helps students experience the language of estimation, comparing, measuring, and conversion using customary and metric units of length, weight, and capacity. Lastly, the Clock gives students daily practice reading and setting the hands of an analog clock, understanding A.M. and P.M., and working with the concept of elapsed time. For the purposes of this study, I will focus only on the first three components of the program: Calendar, Daily Depositor, and Counting Tape.

During the first two weeks of school, as we worked through the Calendar, Daily Depositor, and Counting Tape components for the first time, I also conducted several discussions with my class about our classroom norms. The basic expectations and guidelines for classroom interactions had been a topic of discussion several times before, but now I wanted my students to think about how these guidelines applied to mathematics. As we discussed the mathematical content of the calendar display, we also took time to analyze the discussions themselves. We talked about the importance of explaining and justifying thinking in a clear way. The students expressed that clear explanations would help them understand others' ideas and that responding to various solution methods in a nonjudgmental manner would make it easier for them to take risks and participate in the discussions.

Throughout the course of these discussions, guided by my own knowledge of Yackel and Cobb's work, we established 5 social/sociomathematical norms which we called the Expectations for Math Discussions. The expectations were:

1. Students will explain and justify their thinking. They will tell what they did to solve the problem and the reason(s) for that solution method.
2. Students will share their thoughts with others using clear, precise language and using mathematical terms correctly.
3. Students will listen to others' explanations and try to make sense of their thinking.
4. Students will agree and disagree respectfully. There will be no put-downs.
5. If students are confused, they will ask the student who gave the explanation specific clarifying questions to help clear up the confusion.

Once these expectations were composed, I recorded them on chart paper and posted them in the classroom prominently so that we could refer to them during mathematical discussions. As the weeks progressed, I also began incorporating writing into my students' daily mathematical discussions. I used the written tasks as a way for my students to organize their thoughts before discussing and to have a reference point for making sense of others' explanations while the discussion took place. The data collected from this process, which will be analyzed in the next chapter, was the result of a twelve week study of my students' abilities in the areas of discussing and writing about mathematics. Table 1 shows the months of school the study took place during, what components of the calendar were discussed, what specific skills were addressed using each component, and some key terms that were used during the course of study.

Table 1: Mathematical Content Discussed During the Study

Month	Calendar	Daily Depositor	Counting Tape	Important Terms
Aug.	Multiples of 2 & 5 Square & Rectangle Horizontal Vertical	Mental Addition 1x day of school	Multiples of 2	multiples factors 2-dimensional square rectangle regroup decimal hundredths tenths
Sept.	Multiples of 2 & 5 Square & Rectangle Horizontal Vertical	Mental Addition 1x day of month	Multiples of 2	multiples factors square rectangle regroup decimal hundredths tenths
Oct.	Multiples of 3 & 6 Types of triangles 180º rotation reflection	Mental Addition 2x day of month	Multiples of 2 & 3	multiples factors double equilateral isosceles scalene right acute obtuse regroup decimal hundredths tenths
Nov.	Multiples of 4 & 8 Types of quadrilaterals 90º rotation	Mental Add. & Subtract. 10x day of month	Multiples of 2, 3, and 4	multiples factors quadrilateral square rectangle rhombus trapezoid regroup decimal hundredths tenths

The daily lesson consisted of an update of all of the components of the bulletin board set. Each day, one or two components would be discussed in detail. At first, students were only required to participate in the discussion. This was to ensure that each student could hear the appropriate mathematical language necessary to explain and justify each component and skill. Many times, especially at the beginning of the school year, I would rephrase a student’s explanation using more accurate mathematics terminology, and then explain why the language I was using was appropriate. Eventually, the students began to automatically use appropriate language when giving their explanations and justifications.

Once I was confident that my students were comfortable with the mathematical language during discussions, I began to incorporate written tasks into our daily discussion of the calendar. I asked my students to begin bringing their math journals with them to the discussion, and I would request that they respond to my questioning in writing in their journals before we engaged in the verbal discussion. I then collected journals for analysis periodically from selected students to monitor their progress.

I also placed a video camera in the classroom on the first day of school. Since a video camera is not a regular part of my classroom environment, I decided it would be easier for the students to get used to the idea of being video taped if the camera was in sight from the onset. Even though the camera was off for the first two weeks of school, it gave my students an opportunity to get accustomed to seeing it in our classroom.

I video taped selected lessons to illustrate my students’ progression with the mathematical language and skills throughout one month of the study. The first video was recorded on the first day of the month, when a new calendar was revealed and all of the components were reset. The next recording took place around the middle of the month, and a final recording was made on the last day of the month. Using these recordings, I observed the dialogue taking place during the lesson, focusing on how effectively my students were explaining and justifying their solutions, listening to others, and asking clarifying questions when they became confused.

Finally, toward the end of my data collection, I asked my students to complete a posttest (Appendix F). This assessment was designed to mirror the questions asked on the pretest and was intended to measure their mathematical writing ability at the end of the study. I also interviewed selected students (Appendix G) to gain insight into their thoughts on the instructional methods used during the study. The same students chosen for analysis from the pretest (Appendix E) were selected for the posttest analysis and the interviews.

Data Analysis

The following methods were used to collect data during this study: a pre- and post-test (Appendices E and F) focusing on student ability to write mathematical explanations, video recording during math instructional time focusing on student ability to verbalize mathematical concepts and justify solutions, student work on monthly assessments, and student interviews (Appendix G). Table 2 shows when each of these collection methods were used.

Table 2: Timeline of Data Collection Methods

Timeframe	**Collection Method**
Week 1	Pre-Test
Weeks 7, 11, 14	Monthly Writing Assessments
Weeks 11, 13, 14	Video Taping
Week 15	Post-Test, Student Interviews

Both the pre- and posttests (Appendices E and F) were created by the teacher using a Scott-Foresman Test Generator CD ROM. All of the questions on the tests were selected from a bank of items correlated to the Florida Sunshine State Standards. For each test, 3 multiple choice items were chosen to assess student ability with number concepts, the unit I planned to teach early in the school year. The last 2 items on the tests were written response questions, intended to assess my students' ability to explain and justify their problem solving strategies in writing.

The written responses on the pre- and post-tests (Appendices E and F), along with student journal responses and the responses written on the monthly assessments were analyzed using a rubric and anecdotal notes. The rubric consisted of a two-point scale. A response that was incoherent, incorrect, or missing received zero points. Responses that were partially correct received one point. For these partial responses, the student either provided a correct answer to the problem without an explanation, or the student provided an explanation that made sense but resulted in an incorrect answer. Finally, responses that were completely correct, including a correct answer to the problem combined with an acceptable explanation, received two points. I used the same rubric scores for all student writing samples in order to maintain consistency in my analysis of the data.

Video recordings were transcribed, and samples of student dialogue are included in the next chapter. When listening to the verbal explanations, I looked for several factors. First, I was listening to be sure that students were using mathematical vocabulary in a clear and concise manner. Second, I listened to the clarity of student explanations. I wanted to know if the students were improving in their ability to construct knowledge based on not only their own thoughts, but on the thoughts of others as well. Finally, I listened to myself, paying close attention to how often I interjected and in what manner. I wanted to know if I was merely giving students information or if I was guiding their thinking by asking questions and helping to clarify complex ideas. I kept anecdotal notes in my own daily journal when I noticed that I was controlling the conversation. This made me aware of my own behavior during the discussions and caused me to consciously modify my level of involvement in later discussions.

The student work was generated in a variety of ways. First, students recorded daily thoughts and questions in their math journal. They also used their journals to solve and explain math problems. Daily calendar study assisted in providing authentic problems for students to solve. These journals will be discussed in greater detail in the next chapter. A second form of student work that was collected were monthly mini-assessments that presented students will problems requiring application of the skills learned from the calendar that month. These problems involved solving the problem and explaining and justifying the solution methods. These strategies provided an immense amount of information about my students' mathematical understanding.

As a culminating activity, I selected three students from my class to interview about their feelings toward the way we learned math this year. I chose these three from the class based on pre-existing information I had been given about the students. I felt that these three students each represented a wider population of students within my classroom. A sample of interview questions (Appendix G) was created and used as a guide during each interview. The purpose of the interview was to gain insight into student interpretations of the math learning they had participated in and how they felt the instructional methods used impacted their learning.

In a thorough inductive analysis of the data collected, I was specifically looking for the quality of the verbal and written explanations concerning mathematical concepts. I was also looking for a connection between the quality of verbal explanations and the quality of written explanations. In order to maintain consistency between verbal and written responses, I tried to match the language I used during classroom discussions in the questions I asked on the monthly assessments. In addition, I looked for evidence of a shift in my students' mathematical thinking. I wanted to know if thinking, talking, and writing about mathematical ideas would begin to change the way my students thought about the way they learned math. This task marked the end of my data collection.

Summary

The qualitative research methodology provided an appropriate setting for me to study and describe the effects of using discussion and writing on my students' conceptual understanding of mathematics. The action research we participated in as a class, along with the *Every Day Counts Calendar Math* program, supplied the setting needed to help identify these effects.

Elucidation of these data will be provided in Chapter Four, Data Analysis. A detailed analysis of the data will describe our daily discussions, provide examples of student writing, and demonstrate my students' ever-evolving conceptual understanding of mathematics.

CHAPTER 4: DATA ANALYSIS

Introduction

In the planning stages of my action research project, I considered past students and their mathematical abilities. I asked myself what I wanted to improve about my teaching that would also reach the needs of my students. I realized that most of the math instruction at my school, due mainly to the pressures of getting the students to score well on the test, had devolved into memorizing mathematical facts and formulas. Students, no matter how adept they were at solving math problems, chiefly had little rhyme or reason to the methods they were using to solve those problems, other than that was way they had been taught. Critical thinking skills had all but been eliminated from studying math for the sake of better test scores. I then decided to examine my own teaching practices and modify what I was doing in my classroom in order to help students not only prepare for taking the test, but also to aid them in thinking about mathematics more critically, which included providing explanations for their thinking that were mathematically sound. I used my research questions, “Are discussion and writing about mathematics related? How?” and “How do discussion and written explanations affect student understanding of mathematical concepts?” to guide the design of my action research project.

It was my original intention to restructure my normal math instruction time to meet the needs of this project. However, I came across a program that provided me with exactly the structure I needed to accomplish my goals. *Every Day Counts Calendar Math,* as I described in the pervious chapter, gave me everything I was looking for to both change my teaching practices and incorporate discussion and writing into my daily math routine. The components of this program allowed for daily interaction with mathematical concepts while also providing students opportunities to practice solving real-world problems, record their own explanations of their mental math strategies, and discuss and compare their solutions with their peers.

In this chapter, I will discuss patterns I noticed during class discussions and in my students' daily journaling. First, I inquired about student feelings about math and their expectations of my class. Next, I implemented the Calendar Math program and monitored my students' abilities with explaining and justifying their problem solving strategies. During this time, I studied my students' progress with the skills presented through the Calendar Math program in depth over the course of one month. Finally, I reflected on and responded to what I observed, analyzing my students' work both during class discussions and in their written classwork. The results of my study show that this method of teaching had a profound impact on both my teaching and my students' learning of mathematics.

Student Work Samples

Pre-Test Assessment

During the first week of class, I gave my students an assessment titled *Whole Number Concepts and Operations* (Appendix E). This assessment was intended to serve two purposes. One, I wanted to gain a better understanding of my students' abilities in working with numbers in various contexts. This was to be accomplished in the form of a series of multiple choice questions dealing with number concepts, including comparing and ordering large numbers, analyzing number patterns, estimation, and basic mathematical computations. Two, I wanted to gain a better understanding of their ability to explain and justify their problem solving methods in writing. Therefore, I included two open response questions, for which a portion of the score would come from a written explanation and justification of the solution methods.

On the whole, most of the students performed very well with the multiple choice questions, both of which asked the students to compare numbers with digits in the thousands place or greater. I did not notice very many students struggling with the concept of comparing and ordering numbers, indicating to me that they had at the very least a basic understanding of place value concepts. Because of this, I decided to use the pretest to analyze my students' ability to explain their mathematical ideas in writing. In looking at the 2 written response questions, however, I noticed that their writing ability was lacking.

The first open response question required students to look at a table that was partially completed and determine an addition rule and a multiplication rule that could be used to complete it. The initial table is shown below (Figure 1).

In	3	4	5	6	7
Out	9	?	?	?	?

Figure 1 – Input-Output Table Problem

About half the class could determine an appropriate addition or multiplication rule for the table. The students correctly identified the addition rule as being "add six" and the multiplication rule as being "multiply by three", and were also able to apply those rules and complete the table correctly. However, identifying the rules and completing the table was only part of the question. Upon looking for the written explanation and justification that should have accompanied each completed table, I found that not a single student had included one.

Next, I looked at the second open response question. This question (Figure 2) asks students to estimate the distance a pilot flies between three cities and then calculate an exact distance. In addition, the students were required to explain and justify their solutions.

A pilot flies from Los Angeles to Dallas to Chicago and then back to Los Angeles. What is the total distance? Estimate first and then calculate the answer.

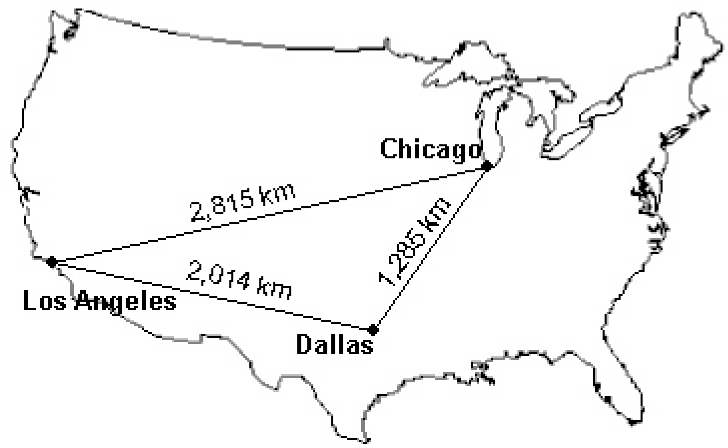


Figure 2 – Distance Problem

When I started analyzing student responses, the first thing I looked for was the estimation strategy. What I found with almost every student were varying estimations of the distance flown in the problem. Since the problem did not give specific directions about which place value the students should use to do their estimating, several different estimations were acceptable, as long as the student was able to support their solution with an explanation. Unfortunately, as with the Table Problem, not a single student provided an explanation or justification for their estimation strategies. On top of that, most students did not even record their thought processes for the estimation – only a final estimated distance was recorded in the space provided on the test form.

The second part of the problem asked students to calculate an exact distance. More students – about 75% - showed the work necessary for adding the three distances together. Out of these students, most of them were able to come up with the correct sum. The most common problem for those who did not have the correct sum was simple addition miscalculations. Once again, though, there were no explanations or justifications for the solution methods.

These two problems provided me a world of information about my students' mathematical abilities. In the Table Problem, I learned that my students had some sense of algebraic thinking and could apply basic computational algorithms. However, when required to use those computational skills with larger numbers, as in the Distance Problem, some students had difficulty applying their knowledge. Had the explanations and justifications been included, they might have given me more information about the students' thinking while solving these problems. Since they were not, I can only surmise what the issues might have been. First, it was possible that the students who had difficulty lacked conceptual knowledge about place value in the base ten number system. Another possibility is that they worked too quickly to get to the answer, resulting in careless mistakes.

Regardless of the reason for the computational errors, the complete lack of written explanations was troublesome, yet not surprising. Knowing that my students had learned math in the past as a series of facts and formulas, I had already prepared myself for the reality that they would not be able to explain and justify their problem solving methods. But was it simply the fact that they had never been expected to explain their mathematical thinking, and therefore lacked the words to do so, that prevented them from writing their thoughts on the assessment? Or could it be that they truly lacked the conceptual knowledge behind the strategies they were using to be able to explain what they were doing and why? Whatever the reason, I knew that I was going to need more research in order to come to a conclusion.

Classroom Discussions

In order to help my students become better writers of mathematics, I knew that I had to first help them become better talkers of mathematics. Several authors (Pugalee, 2001; Baxter, Woodward, and Olson, 2005; Yackel, 2001; and Ediger, 2006) have written about the distinct connection between student talk and student writing in mathematics: if student talk about mathematics is encouraged and developed, then the language of mathematics becomes more natural when students are asked to write about mathematics. Therefore, I made it my goal to make classroom discussion a primary focus during my daily math instruction.

Daily discussions were focused around the *Every Day Counts: Calendar Math* bulletin board display in my classroom. This program provides students with daily mathematical situations which require them to think mathematically and explain and justify their thinking. The beginning of each month introduced new concepts to be studied and ownership of those concepts were gradually released to the students over the course of the month. The photograph below (Figure 3) illustrates what the Calendar and Daily Depositor components of the bulletin board display looked like at the beginning of a month.

Figure 3 – Calendar Math, Beginning of November

The calendar pieces are flipped backwards with the intention of helping the students look for patterns and being able to predict the continuation of a pattern. The Daily Depositor begins every month completely empty, and students use mental math strategies to add a certain amount to the chart each day of the month.

The initial discussion about the calendar display was largely dependent upon teacher-student interactions. The following dialogue illustrates the discussion about the display on the first day of the month.

Mr. R.: “As we have discussed in previous months, we cannot make a prediction about the calendar pattern because we do not see a pattern yet. Therefore, I will begin the month by flipping over the first calendar piece.”

I move to the calendar and flip the first piece on the display.

Mr. R.: “What do you see?”

Student 1: “I see an orange square.”

Mr. R.: “How do you know it is a square?”

Student 1: “Because it has four sides that are all the same.”

Student 2: “I know it is a square because it has four square corners.”

Mr. R.: “Can anyone clarify what is meant by ‘four sides that are all the same’ and ‘four square corners’?”

Student 3: “I remember that we talked about squares back in September. A square is a four-sided figure, and all of the sides are equal in length.”

Mr. R.: “Excellent! In math, when we see two things are the same, one word we can use to describe them is that they are *equal*.”

Student 4: “Oh! I remember, too! Squares have four vertices that form right angles.”

Mr. R.: “I like that you used the word *vertices* instead of *corners*. A vertex is the mathematically correct term for a corner, or a place where two line segments meet. What do you mean by *right angles*?”

Student 4: “Right angles are angles that measure ninety degrees.”

Mr. R.: “Right. So, we see a 4-sided figure that looks like all of the sides are equal in length and all of the angles are right angles. Is there any way we could prove that?”

There is a moment of silence as students ponder this question.

Student 5: “We could measure each side using a ruler.”

Mr. R.: “Good! We can use a ruler to measure the length of each side.”

Another moment of silence as a student is chosen to measure each side of the figure.

Student 6: “Each side measures 3 inches. That means they are all the same…I mean, *equal*…length.”

Mr. R.: “Very good use of vocabulary! Since every side of the figure measured 3 inches, we can describe the sides as being equal in length. How can we prove if the angles are really right angles?”

More silence as the student think.

Student 7: “We could take a piece of paper and match up the corners and see if they are the same.”

Mr. R.: “Why would you solve the problem in that way?”

Student 7: “Because the corners of a piece of paper look like they are the same as the corners of the square.”

Mr. R.: “Okay, we could solve the problem that way. But how do you know that the vertices of a piece of paper are really right angles? Would that give us the information we would need to really prove that the vertices of this figure are right angles?”

Several students shake their heads. The students are now silent.

Mr. R.: "There is a special tool that people use to measure angles. Raise your hand if you have ever heard of a *protractor*."

A few students raise their hands. Mr. R. holds up a protractor for students to see.

Mr. R.: "This is a protractor. It is used to measure angles."

Mr. R. demonstrates how to use a protractor as students watch. At the end of the demonstration, every angle on the figure is shown to measure ninety degrees.

Mr. R.: "Now that we have proven that all of the sides are equal in length and the angles are right angles, what can we officially call this figure?"

Student 8: "A square."

Mr. R. records the observations made by the class on an untitled chart that would serve as a record of student discoveries throughout the month.

During the month of November, the pattern on the calendar concentrated on various types of quadrilaterals as well as multiples of 4 and 8. Over the next two weeks, during discussions that took place between recording dates, I noticed a shift in my students' verbal explanations. The class began to take more of an ownership of the mathematical language represented in the calendar display, using that language to explain, justify, and argue their ideas. Figure 4 portrays what the calendar looked like at mid-month.

Figure 4 – Calendar at Mid-November

Mr. R.: “Today is Friday, November 16th. Look at the pattern we have been building with the calendar. What quadrilateral will be on today’s calendar piece?”

Student 1: “I think that today’s calendar piece will be an isosceles trapezoid because yesterday’s piece was a rhombus and we always have a trapezoid after a rhombus.”

Student 2: “I agree that today will be an isosceles trapezoid, but I figured it out in a different way. I think that it will be an isosceles trapezoid because we always have a trapezoid on a multiple of four, but it looks like the isosceles trapezoids will only show up on days that are multiples of eight, and 16 is a multiple of 8."

Other students nod their heads in agreement.

Mr. R.: "So, if it is true that the isosceles trapezoids appear on days that are multiples of 8, when should the next isosceles trapezoid appear??

Student 3: "The next multiple of 8 after 16 is 24, because 8 more than 16 are 24. So the next isosceles trapezoid will appear on November 24th."

Similar shifts in discussion ability were also noted during Daily Depositor. Figures 5 and 6 illustrate what the Daily Depositor looked like at mid-month.

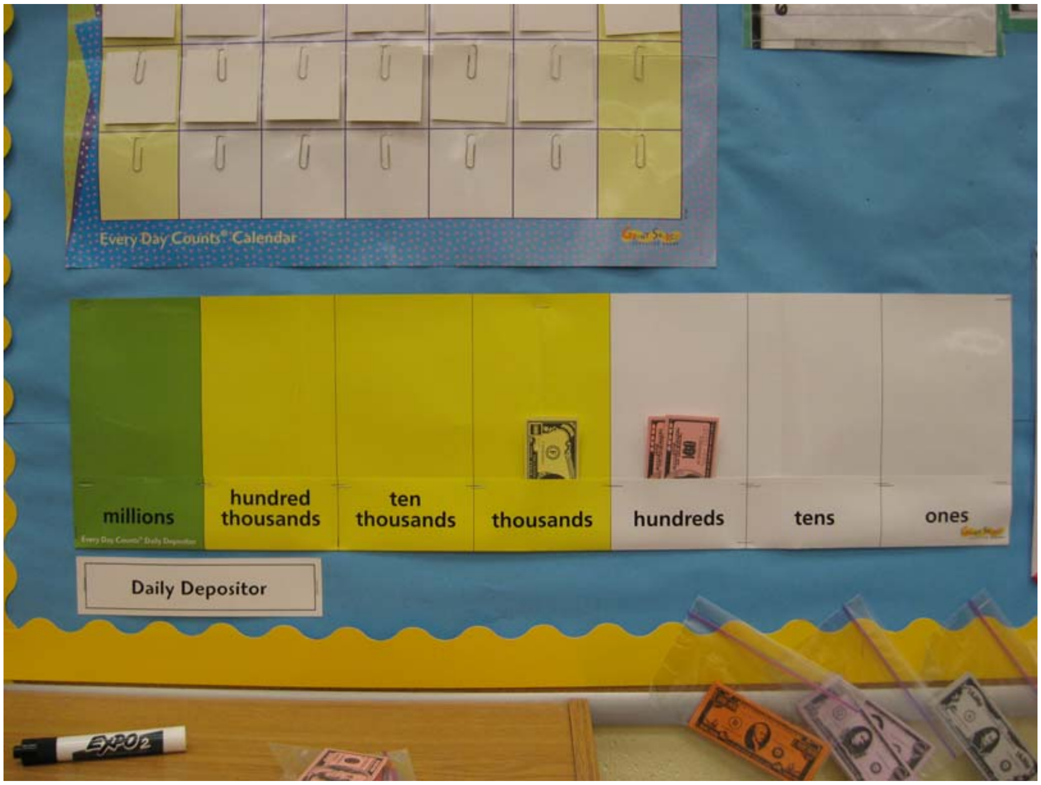


Figure 5 – Daily Depositor at Mid-November

Figure 6 – Calendar Math – Mid-November

Mr. R.:	"After we added money to our chart yesterday, we had $1,200. If today is November 16th, and we continue to multiply the date times $10, how much money will we add to the chart today?"
Student 4:	"Today we will add $160, because everyday we add $10 more than the day before. Yesterday we added $150, so today we will add $160."
Mr. R.:	"That was a great mental math strategy. Did anyone solve it in a different way?"

Student 5: "I solved it by breaking apart the numbers before multiplying. I remembered that 10 times 10 equal 100, and that 10 times 6 equal 60. Then I added 100 plus 60 and got 160."

Mr. R.: "How did you know you could break apart the number 16 in that way?"

Student 5: "I saw that 16 has a 6 in the ones place, giving it a value of 6, and a 1 in the tens place, giving it a value of 10. Since 10 plus 6 equal 16, I knew that if I multiplied both parts times 10 and then added their products, I would get the product of 16 times 10."

Mr. R.: "So if we add $160 from today to the $1,200 in the chart, how much money will we have altogether?"

Students are silent as they apply mental math strategies to solve the problem.

Student 6: "We would have $1,360. I figured it out by adding each place value together. In the ones place, zero plus zero is zero. In the tens, zero tens plus 6 tens is 6 tens. There aren't enough tens to regroup, so I just added the hundreds next. 2 hundreds plus 1 hundred is 3 hundreds. There also aren't enough hundreds to regroup, so I moved on to the thousands. 1 thousand plus zero thousands is 1 thousand. Then I put all the digits together and got $1,360."

Student 7: "I got $1,360, too, but I did it differently. I added $1,200 plus $100, which is $1,300. Then I added the leftover $60, giving me $1,360."

These verbal explanations show that students' conceptual understanding of numbers had increased tremendously. They demonstrated an enhanced mastery of mathematical vocabulary, using the language of mathematics appropriately most of the time in support of their solution methods. The dialogue also shows that students are using several different problem solving methods, all of which are valid. The students who were able to see the reasoning behind a given explanation and recognize that their solution was different demonstrated that they were listening to and trying to understand their classmates' thinking. Some students liked to talk and did so as often as they could during classroom discussions, even though they were reluctant to write. Other students would write their explanations effortlessly, but would not participate in the discussions. Others still fell somewhere in between those two extremes. While I found that a range of students existed within my classroom, every student was able to demonstrate improvement in their mathematical thinking and communication one way or another.

The quality of student explanations continued to improve over the course of the month. The following photographs (Figures 7, 8, and 9) were taken on the last day of the month.

Figure 7 – Calendar at the End of November

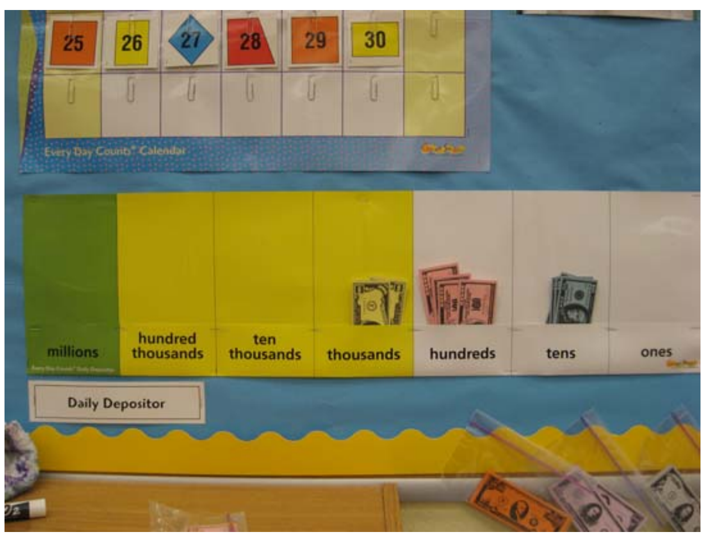


Figure 8 – Daily Depositor at the End of November

Figure 9 – Calendar Math – End of November

On the last day of each month, we did not discuss the elements of the bulletin board as a class. Instead, we quickly updated the display to finish off the month, and then the students completed an end-of-the-month review assessment designed to evaluate the students' ability to respond to mathematical problems in writing. In the next section, I will provide a progression of student responses to various problems during the first three months of school.

Progression of Written Responses

In order to show an improvement in the writing ability of my students, I selected three students to focus on throughout the study. As stated previously, one student was high-achieving in mathematics based on third-grade performance, on student was average, and one student was low-achieving. I analyzed the work of these three students for all of the written tasks: the pretest (Appendix E), each monthly assessment, the posttest (Appendix F), and the student interview. This section presents the analysis of each of the monthly assessments.

September

In this task, students were asked to solve math problems based on our daily discussions, explaining and justifying their thinking in writing. The first question was, "What is 37 + 6? Show your work and explain how you solved the problem." In this question, I wanted to assess my students' ability to add with regrouping, and the explanation would give me information about their conceptual understanding of place value and multi-digit addition.

One student wrote, "37 + 6 = 43 Because 6 + 7 = 13 add that ten to 30 equals 40 then add 3 to 40 equals 43" This response clearly shows that the student understands the concept of place value and its connection to multi-digit addition. By adding the ones place of both addends, the student found the sum to be greater than 10, indicating a need to regroup. The student then added the 10 in 13 to the 30 in 37, totaling 40. The 3 ones remaining were then added to 40, giving a total sum of 43. Although justification for this method was not provided, the explanation shows an understanding of the concepts.

Another student wrote, “43. 37 + 6 = 43. 7 + 6 = 13 so I carryied the one and added 1 + 3 = 4.” While this response shows that the student understands the process of addition, an understanding of place value concepts is lacking. It was apparent to me after reading this response that this student needed more work on place value concepts and their use in mathematical computations.

A third student solved the problem a little differently. “43, because 7 + 3 = 10 +3 = 13, drag the ten to the tens place. 30 + 10 = 40, 40 + 3 = 43.” This response shows that the student initially broke the 6 into 3 + 3, and then added the 3s separately. This method tells me that the student was thinking in terms of tens and saw that 3 more than 37 were 40. The remaining 3, combined with 40, were 43.

The second question I asked on the assessment was, “How do you know that 19 is not a multiple of 2? Explain your answer.”

One student responded, “19 is not a multiple of 2 because there would be 2 groups of 9 with 1 left over.”

Another student wrote, “I know that 19 is not a multiple of 2 because you can’t split it into 2 equal groups.”

Yet another student wrote, “I know that 19 is not a multiple of 2 because you can’t split 19 into 2 equal groups, there will be 10 in one group and 9 in the other.”

Similar responses to this question were given by every student. Overall, I noticed that the students had a grasp of the concept of multiples of two, indicated by their responses that 19 cannot be divided into two equal groups. Our discussions during the month had centered around identifying multiples of two based on a number’s divisibility into two equal groups. We then used hearts to mark the multiples of two that we identified as a class on a number line at the front of the classroom, a portion of which is shown in Figure 10. Figure 11 also shows the students' identification of all of the multiples of 2 found on a hundreds chart.

Figure 10 – Counting Tape with Multiple Markers

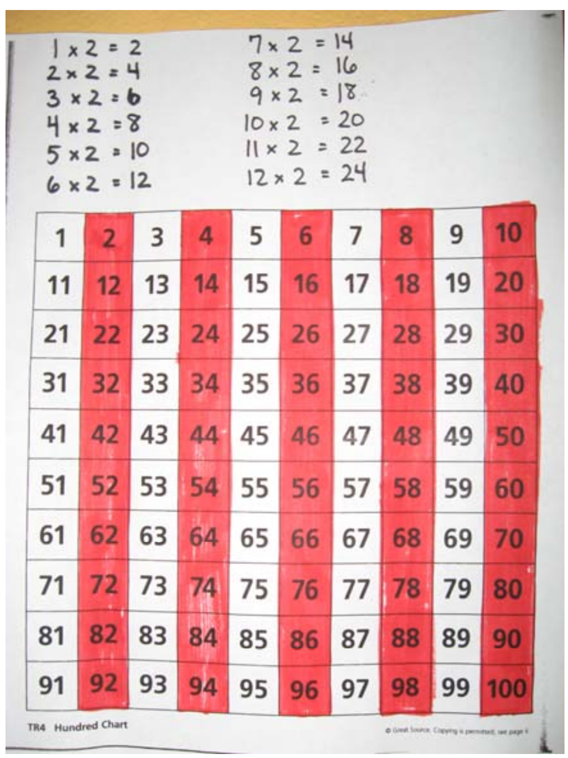


Figure 11 – Hundreds Chart with Multiples of 2 Identified

October

During the month of October, new concepts were introduced. One concept we explored using the bulletin board display was doubling numbers. To add money to the Daily Depositor, the students were asked to double the date and then add the result to the total. On the end of the month assessment, I asked the students to solve the following problem.

"What is 26 doubled? Explain how you found your solution."

Responses to the question varied, showing a wide array of problem solving methods. Here are a few examples:

"26 doubled is 52. because 6 + 6 = 12 carrey the one over the 2 tens and you get 52." This response illustrates an understanding of the doubling process, but only partially explains the process from a conceptual standpoint. The student is not applying place value concepts ("carrey the 1" instead of "regroup the 10"), indicating that the student's thinking is still dependent upon the process of adding multi-digit numbers.

"26 doubled is 52 because 26 doubled means 26 x 2. 26 x 2 = 52." This response shows an explanation for how the student came to translate the word *doubled* into mathematical symbols (*doubled* means *times 2*). This tells me that the student is making connections between mathematical words and symbols and is able to apply knowledge of both.

"26 doubled is 52. I figured this out by adding 6 + 6 = 12 and 20 + 20 = 40, then I did 12 + 40 = 52." This response shows that the student has a complete understanding of what it means to *double* a number. The student literally doubled each part of the number. 6 doubled is 12 and 2 tens doubled is 4 tens, or 40. The student then combined the two doubled parts and found the sum. This is an excellent mental math strategy that would prove useful when doing any kind of multiplication.

The next question I asked on the end of the month assessment once again dealt with the concept of multiples. Using the number line (see Figure 14) the students were responsible for identifying the multiples of 3, or finding numbers that were evenly divisible by 3, and marking them with a triangle. They also were asked to find all of the multiples of 3 on a hundreds chart (Figure 16). As they did this, they began to notice a pattern emerging – numbers like 6, 12, 18, and 24 were multiples of both 2 and 3. In other words, these numbers could be divided into two equal groups and divided into three equal groups. Because of these discussions, I wanted to assess their explanations for the following question:

"Name a number that is both a multiple of 2 and a multiple of 3. Explain how you know the number you chose is a multiple of both 2 and 3."

Below are a few of the responses to the question.

"6 is a multiple of 2 and 3. It is because 3 + 3 = 6 and 2 + 2 + 2 = 6."

"12 because it can be split into 2 equal groups of 6. 12 can be split into 3 equal groups of 4."

"6 is a multiple of 3 and 2 because 2 x 3 = 6 and 3 x 2 = 6."

Most of the responses for this question used numbers lower than 20, and all of the explanations were similar. The reasoning demonstrated in the responses show that students are becoming more comfortable with the concept of groups of numbers and how factors and multiples, along with addition and multiplication, are related. Although I would have liked students to extend their thinking to larger numbers, the question did not require such extension. However, the students' responses show a clear understanding of the concept of multiples and the relationships between numbers.

Once again, the last day of the month was also spent identifying the multiples of 3 on a hundreds chart (Figure 12). Just as with the multiples of 2, students began to notice patterns emerging on the hundreds chart as well as on the counting tape (Figure 14), to which multiple markers in the shape of triangles were used to locate the multiples of 3. As these activities were completed, students noted that every other multiple of 3 was also a multiple of 2, demonstrating a growing understanding of the relationship between the two numbers.

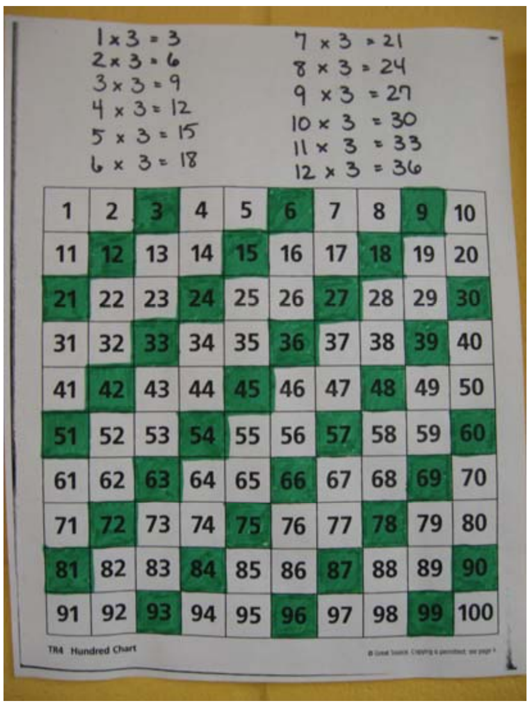


Figure 12 – Hundreds Chart with Multiples of 3 Identified

November

Learning from the previous month, I made sure to ask a more specific question about my students' understanding of multiples during the month of November. Throughout the month, we studied multiples of 4 and 8, although the main focus was on multiples of 4. On the end of the month assessment, I asked the following question:

"What is the first multiple of 4 greater than 30? How do you know it is a multiple of 4?"

The question was designed to not only assess student understanding of multiples of 4, but also to se if they could apply their understanding to numbers greater than 20. Here are a few of the responses.

"32 because you can multiple this → 4 x 8 = 32."

"The first multiple of 4 greater than 30 is 32 because if you count by fours you go: 4, 8, 12, 16, 20, 24, 28, 32."

"The first multiple of 4 that is greater than 30 is 32. I know this because if there were 32 pieces of pie and share it with 4 people each person will get 8."

The responses given to this question were very similar from the previous month. They continued to demonstrate understanding of multiples as groups of the same number. The differences were in the operations used to illustrate that understanding. From the first response, we can see a direct approach through multiplication. The second response shows that the student is still using a skip counting method but is still able to come to the same result. Finally, the third response demonstrates a work-backward approach, starting with 32 pieces and dividing them equally among four groups. All three methods are valid and indicate the students continue to build their understanding of number relationships.

Finally, as in previous months, the students were given an opportunity to identify the multiples of 4 on a hundreds chart (Figure 13) and on the counting tape (Figure 14). The multiples of 4 were indicated on the counting tape using squares as markers. Once again, students took note of the patterns emerging, observing that every other multiple of 2 was also a multiple of 4, and that every third multiple of 4 was also a multiple of 3, continuing their understanding of the relationships between the numbers.

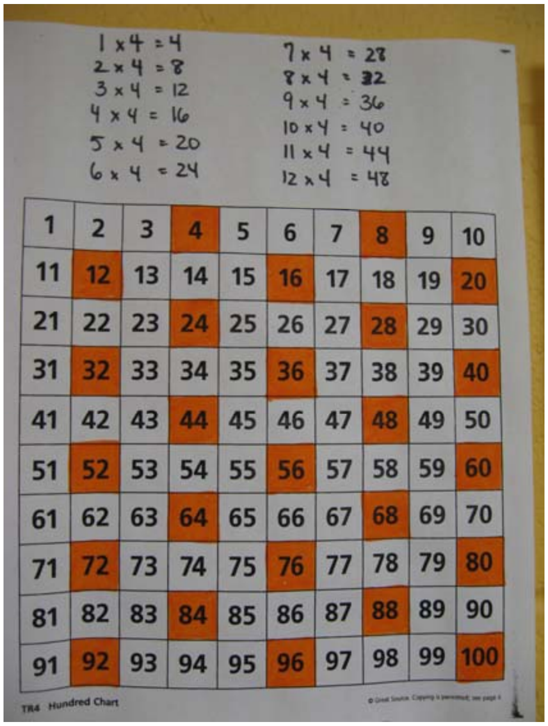


Figure 13 – Hundreds Chart with Multiples of 4 Identified

Another concept we explored as a class during the month of November was multiplying numbers by 10. As part of the Daily Depositor, the students multiplied each date by 10 and added that amount of money to the chart. To assess their mental math strategies, I asked the following question:

"What is 34 x 10? Explain your solution."

As expected, a variety of solutions resulted in the student responses. These examples are a small selection of the responses received.

“340 because you can count by tens 34 times and it will equal 340.”

“34 x 10 = 340 because 30 x 10 = 300, 4 x 10 = 40, and 300 + 40 = 340.”

“34 x 10 = 340 because I doubled 34 and got 68. I added 68 five times and got 340. 68 + 68 + 68 + 68 + 68 = 340.”

I chose the first two responses because they represented the majority of the solution methods used. On the whole, students either counted by tens or used the distributive property by breaking apart 34 into 30 and 4 and multiplying both parts by ten and adding them together. These solution methods show that students understand multiplying larger numbers (2-digit by 2-digit) without the use of the algorithm. The third response was unique among the class. This student in particular demonstrated that numbers can be thought about in various ways. Instead of breaking apart 34, the student broke apart 10 into 5 x 2. He then noticed that he could distribute 34 across the equation, providing an opportunity to apply a previously learned skill – doubling. Although still faced with the problem of 68 x 5, the student then applied another previously learned concept – multiplication as repeated addition. By applying both of these concepts, the student was able to come to the desired answer – 340 – albeit by an entirely original solution method.

Summary

As I have illustrated throughout this section, and as Steele (2001) stated, representation of ideas can be a useful tool in helping students verbalize their thoughts about mathematical concepts. Although I used a pre-existing math program as the basis of my research, the program was designed in a way that allowed for classroom discussion to progress naturally. The Teacher's Guide provided in the *Every Day Counts Calendar Math* kit gave suggestions for probing students' thinking, but its primary purpose was to present mathematical concepts in a context that would facilitate students as they transitioned from using ordinary language to using mathematical language.

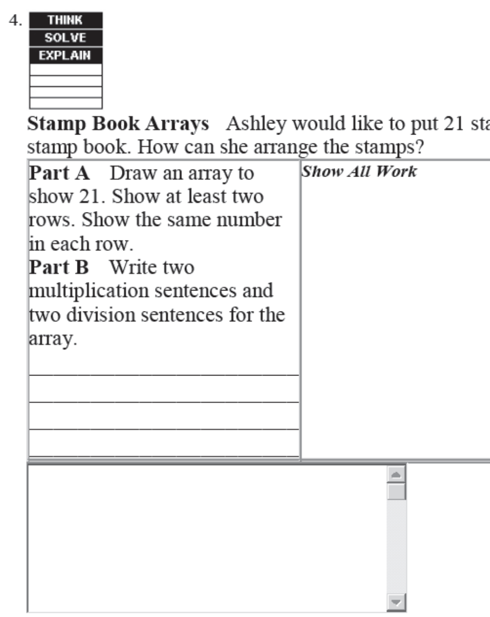
In addition, the calendar math allowed for students to examine the multifaceted nature of mathematics, providing several opportunities throughout each month for students to make connections and construct relationships between various mathematical disciplines. For example, numbers became much more than objects to be added, subtracted, multiplied, and divided. Instead, my students began to see numbers all around them: in geometry, measurement, and in the collection and analysis of data. Even though their mathematical thinking is still far from perfect, I feel that the instructional approaches used in conjunction with the calendar math have guided my students to thinking more critically about their mathematics education.

Post-Test Assessment

The last piece of data that I collected was in the form of a post-test (Appendix F). This assessment, which mirrored the pre-test, was meant to help me compare my students' writing ability from the beginning to the end of the study. I did not score the written responses in the same way that I had for the pre-test; the results of the pre-test showed that their ability to express their mathematical ideas in writing was practically non-existent. Instead, I used the post-test to see if there had been any change in the amount of written explanations the students were providing along with their problem solving methods. While multiple choice questions were still a part of the assessment, I did not focus on them as much as in the pre-test.

During the pre-test, the students had solved the problems but included no written responses. On the post-test, I noticed an increase in the amount of written explanations provided with the solutions. The students demonstrated that they had the ability to express their mathematical ideas in writing, even if their solution method or final answer was not completely correct.

The first open response question that was asked on the assessment presented a situation where a girl named Ashley wanted to arrange 21 stamps in a stamp book. The students were them asked to arrange the stamps into an array using at least two rows and write two multiplication sentences and two division sentences for the array. I selected two student responses that were representative of the responses provided by the entire class.


4. THINK SOLVE EXPLAIN

Stamp Book Arrays Ashley would like to put 21 stamps in a stamp book. How can she arrange the stamps?

Part A Draw an array to show 21. Show at least two rows. Show the same number in each row. **Part B** Write two multiplication sentences and two division sentences for the array.	***Show All Work***

Figure 14 – Array Problem

In my analysis of this problem, I wanted to see if the students not only could follow the directions given, but I also wanted to see if they would include a written explanation with their work.

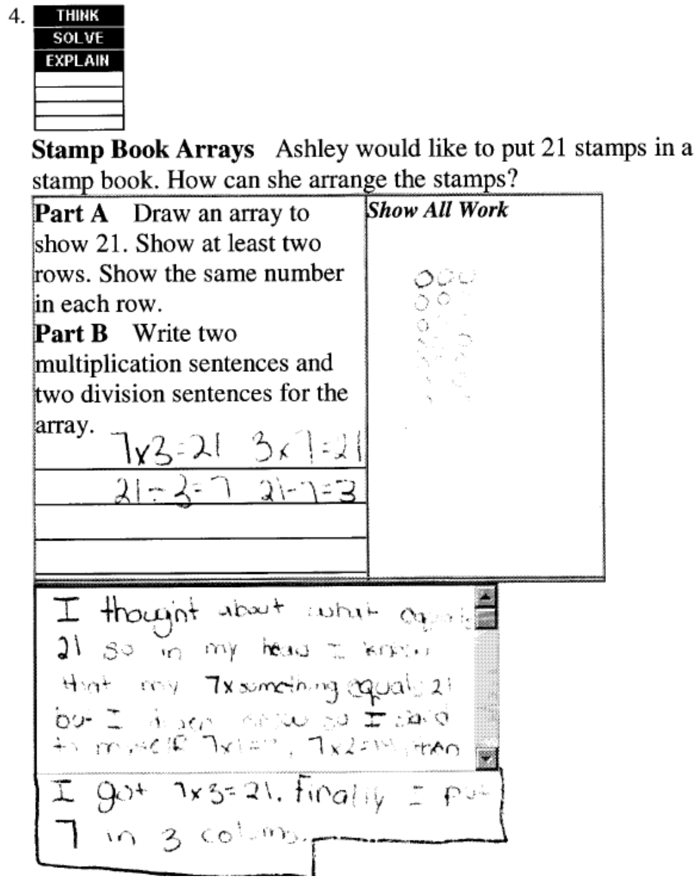
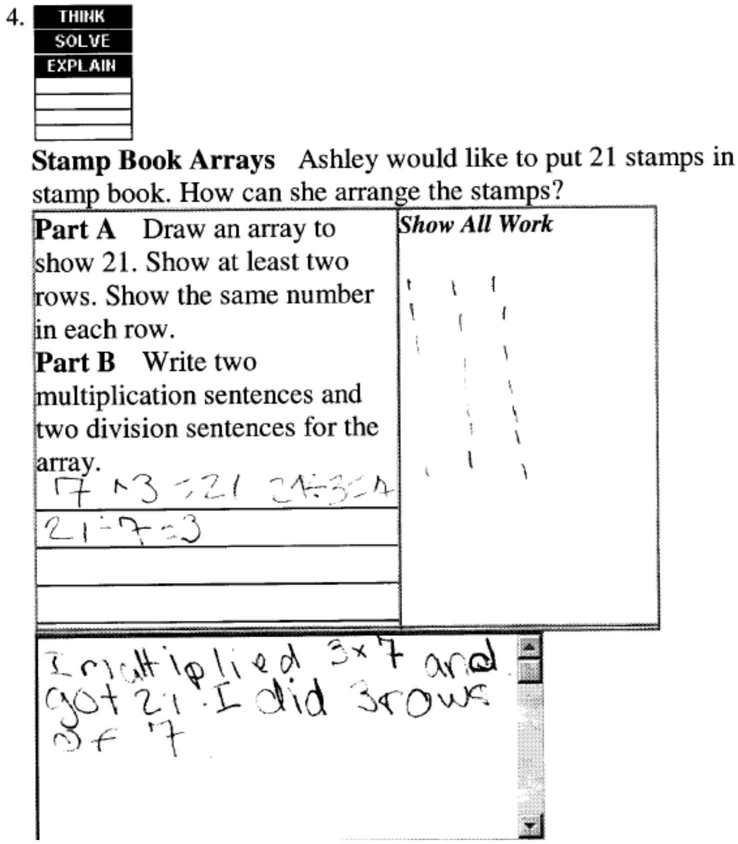


Figure 15 – Student 1 Solves the Array Problem

The student's work shows that she understands the relationships between arrays, multiplication, and division. However, her written explanation, which very clearly depicts her thinking as she solved this problem, shows that she also has a clear sense of what it means to multiply. She recognized that 7 was a factor of 21 ("7 x something equals 21"), and then used a skip counting method to determine the compatible factor, concluding that 7 stamps put into 3 columns would help to solves Ashley's problem.


4. THINK SOLVE EXPLAIN

Stamp Book Arrays Ashley would like to put 21 stamps in stamp book. How can she arrange the stamps?

Part A Draw an array to show 21. Show at least two rows. Show the same number in each row.
Part B Write two multiplication sentences and two division sentences for the array.

7 x 3 = 21 21÷3=7
21÷7=3

Show All Work

I multiplied 3x7 and got 21. I did 3 rows of 7

Figure 16 – Student 2 Solves the Array Problem

This student also shows that he understand the relationship between arrays, multiplication, and division. His explanation illustrates what he did clearly and precisely, although elaboration on these steps is not provided. It seems that the student is still struggling with explaining *why* the solution method was chosen, not just *how* it was implemented.

The second question asked on the post-test required the students to find a way to convert a time period of 4 days into minutes. The question also provided a key, informing the students that there are 24 hours in 1 day and 60 minutes in 1 hour.

5. **THINK**
SOLVE
EXPLAIN

Bird Watching Jennifer promised her friend that she woulc
watch her parakeet while she was on vacation for 4 days.

How many minutes are in 4 days?
1 day = 24 hours 1 hour = 60 minutes

Show All Work

Answer: __________ minutes

On the lines below, explain how you found the number of minutes.

Figure 17: Days Problem

In my analysis of this problem, I was specifically looking for two things. First, I was very interested in seeing what strategies the students used to solve the problem. We had solved problems similar to this during our class discussions, but none were as complex as this. Second, I was looking again for evidence of coherent mathematical thinking, provided in the form of a written explanation of the solution method.

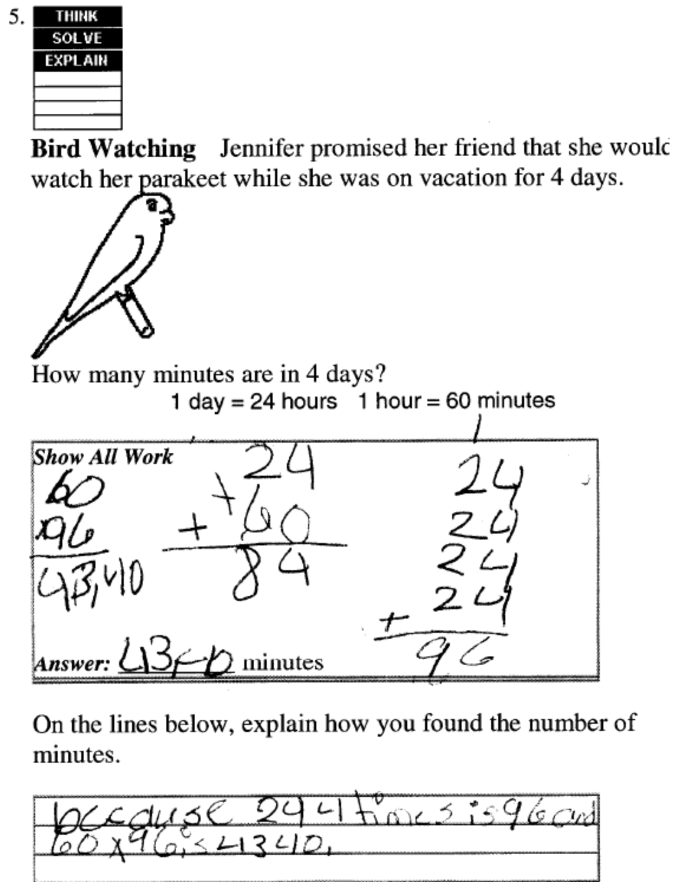
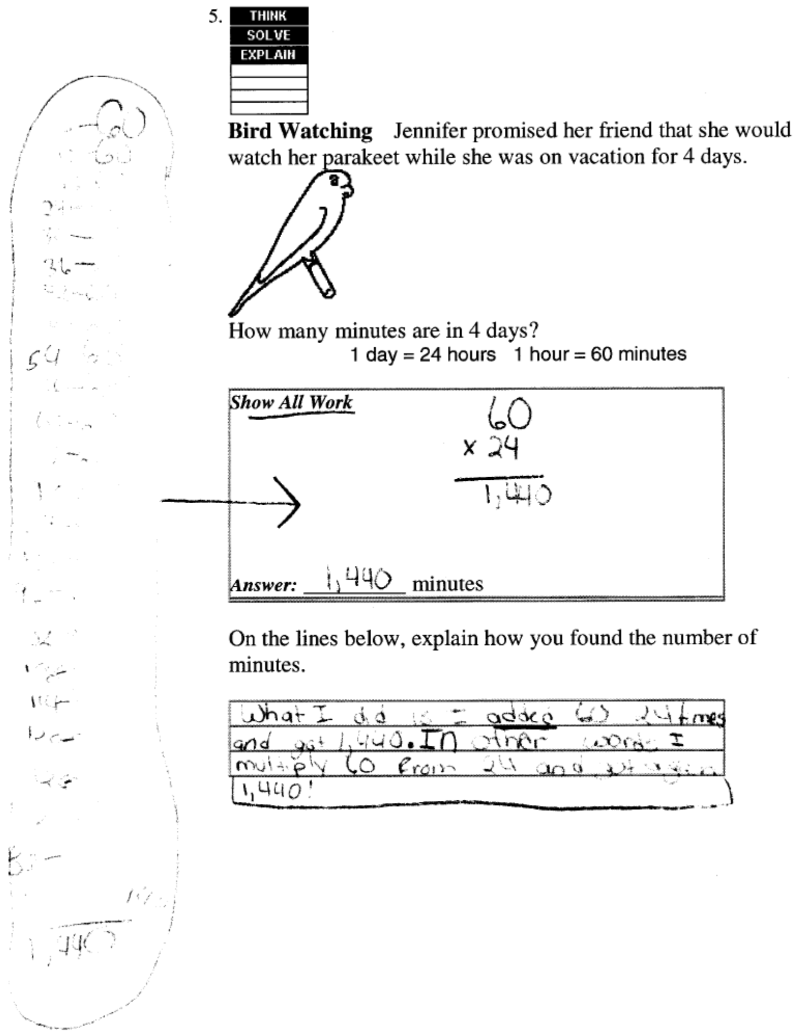


Figure 18: Student 1 Solves the Days Problem

This student illustrates an effective strategy, yet seems to be struggling with explaining her thinking. From the work she has shown, I can tell that she understands four days means four groups of 24 hours. She accurately uses repeated addition to find the number of hours in four days. She then demonstrates understanding that, if there are 60 minutes in each of the 96 hours in four days, the final result can be found by multiplying 60 x 96. It is interesting to note that the student used repeated addition for the first conversion, but switched to using multiplication for the second. Perhaps she recognized that adding 96 sixty times (or vice versa) would take far too long. Although she did not multiply correctly, her solution method illustrates that she understands the relationship between minutes, hours, and days. The justification of her work summarizes what she did, but once again does not go farther to explain why she chose those methods to solve the problem.


5. THINK SOLVE EXPLAIN

Bird Watching Jennifer promised her friend that she would watch her parakeet while she was on vacation for 4 days.

How many minutes are in 4 days?
1 day = 24 hours 1 hour = 60 minutes

Show All Work

60
x 24
1,440

Answer: 1,440 minutes

On the lines below, explain how you found the number of minutes.

What I did is I added 60 24 times and got 1,440. In other words I multiply 60 from 24 and got again 1,440!

Figure 19: Student 2 Solves the Days Problem

This response also demonstrates an understanding of the relationship between minutes, hours, and days, as well as the relationship between addition and multiplication. She begins solving the problem by adding 60 minutes 24 times, resulting in 1,440. She then transfers that work to the work box by showing that 60 x 24 = 1,440. Her explanation is effective in describing exactly what she did. Once more, the explanation stops short. It does not explain why those steps were taken. Had the student elaborated and continued to explain why she chose to multiply 60 by 24, perhaps she would have noticed that her calculations only gave her the number of minutes in one day. She only solved part of the problem. Still, her response shows at least a partial conceptual understanding of the relationships included in the problem.

Student Interviews

Once my collection of student work was complete, I wanted to hear my students' thoughts about the methods we used to learn math. I decided to interview several students in order to see if there had been any connection between the discussions that took place around the calendar and the quality of their writing. I chose these three students based on pre-existing information I had been given about the students. These students were also chosen specifically to see if there had been a shift in their thinking about mathematics from the initial journal response until the time of the interview. A sample list of questions (Appendix G) was used to guide the interviews. These questions were designed to help the students explain their thinking while revealing their opinions about the discussions and writing that took place during our math class.

The first student that I interviewed was a very high-functioning student. He always demonstrated original thinking during discussions and easily identified several different solution methods to many of the problems we worked on. He had also scored very high on the Math FCAT test in third grade and had been described by his third grade teacher as “very good” in math. The following dialogue took place during our interview.

Mr. R.: “Do you like math?”

Student 1: “Yes.”

Mr. R.: “Why?”

Student 1: “I really like working with numbers and having challenges.”

Mr. R.: “What do you mean by challenges?”

Student 1: “I mean more difficult problems.”

Mr. R.: “Have you ever learned math by talking, listening, and writing about math problems?

Student 1: “Sometimes, in my third grade class…I think we were getting into fractions. We talked about them and she taught us multiplication at the same time.”

Mr. R.: “Can you explain how she did that?”

Student 1: “Like if we were learning what zero-sevenths meant, she would show us that zero groups of 7 equal zero, not seven. Then we would get something for getting the right answer.”

Mr. R.: “Did you like learning math in this way this year?”

Student 1: “Yeah…we’ll mainly know all of our multiplication facts, twos through nines, all the way up to 180.”

Mr. R.: “Was there anything else you liked about learning math in this way?”

Student 1: (*slight pause)* "I'm not really sure, I just thought it made it more interesting."

Mr. R.: "Do you think it is important for students to talk about their math ideas and listen to the ideas of others?"

Student 1: "Yes, you get to get inside their minds and see what they are actually thinking. That really helped some of the more challenging problems make more sense."

Mr. R.: "Is there anything else you would like to share with me about our math learning this year?"

Student 1: "Well, I really liked playing games and doing the fun activities – that made learning math more fun. In second and third grade we did a lot more book work, and we didn't really talk about our ideas."

It is apparent from this interview that the student still has a limited view of mathematics. Despite his mature problem solving abilities, his mathematical learning seems to remain fixed upon learning basic facts. Also, it is clear that he considers a situation when the teacher explains an idea and students then mimic that idea, receiving rewards for correct answers, a discussion. In addition, although he apparently enjoyed the type of discussion we took place in this year, his thoughts about writing about mathematical ideas are missing from his interpretations of the math learning he has done over the past three years. This could mean that he did not find writing mathematical explanations useful, or perhaps he simply enjoyed the student-student interaction more. It is clear, however, that he preferred hands-on math activities and math games as opposed to book work.

The next student I interviewed scored at an average level in math in third grade and had no apparent difficulties in learning math, according to her third grade teacher. During classroom discussions, she tended to solve problems in simple ways, but was always willing to expand her thinking by questioning and commenting on other students' mathematical ideas. Here is a sample of our conversation.

Mr. R.: "Do you think you are good at math?"

Student 2: "Yes. My family and teachers have always said I was good at math."

Mr. R.: "How do *you* think you are good at math?"

Student 2: "Because I feel very confident when I get the right answers."

Mr. R.: "Has talking about math ideas with your classmates helped you?"

Student 2: "Mostly. It helps me to hear someone else's explanation of a problem when I am confused. Sometimes they even have a better solution than I had, and that helps me learn."

Mr. R.: "Have you ever learned math by talking, listening, and writing about math problems?"

Student 2: "In the past, we used fraction models and tools to help with measurement. My third grade teacher also did something similar to the counting tape, where we had to find the multiples of each number. We didn't do any writing, though. Mostly we just talked."

Mr. R.: “Has discussing your math ideas helped you explain your thinking in writing?”

Student 2: “Yes. Talking with others about my math ideas helped me organize my thoughts when I had to write my explanations. That made it easier to do.”

Mr. R.: “What is the teacher’s job in a math class?”

Student 2: “The teacher should give us a problem and then let us solve it and explain our solutions. Also, the teacher should help students when they are confused by asking questions and making some more difficult ideas more clear.”

Mr. R.: “What is the student’s job in a math class?”

Student 2: “Students should work to find various ways to solve problems. They should listen to the ways that other people solved the same problem. That way you can have lots of different ways to solve the same problem.”

It is evident from this interview that this student flourished in the environment of discourse. She had never written explanations before, yet revealed that both the talking and the writing helped her understand mathematical concepts. It seems as though her mathematics education in the past included some problem solving strategies, hands-on activities, and discussion of mathematical ideas. Although she claimed that her confidence is build when she gets the right answers, it is evident that she is more concerned with the process of solving problems when learning mathematics.

The last student I chose to interview scored in the below-average range on the third grade test and had been identified by her third grade teacher as below grade level in mathematics. During class discussions, she was typically very quiet, listening very closely to others' explanations but very reluctant to offer her own ideas. Her writing, although rudimentary, had improved drastically since the beginning of the study. Here is our conversation.

Mr. R.: "Do you like math?"

Student 3: "I like some math. Some of it is fun, but some of it is boring, too."

Mr. R.: "What makes math fun to learn?"

Student 3: "Playing games makes math easy. I also like doing the hands-on activities, like when we filled the boxes with cubes to measure the volume. That made it easier to understand."

Mr. R.: "Have you ever learned math by talking, listening, and writing about math ideas?"

Student 3: "Yes, my third grade teacher had us talk a lot. She would give us a problem and we would try to solve it. When we gave our answers, she always asked us how we got the answer. If we did it right, we got a prize."

Mr. R.: "Has talking about math ideas with your classmates helped you?"

Student 3: "Yes. I really liked talking about the calendar. Hearing everyone's ideas explained in different ways helped my understanding. I also liked that we didn't have to get the right answer – all we had to do was explain the math that we did. When I understood the math, it was easier to get the right answer."

Mr. R.: "Do you think it is important for you to talk about your math ideas and listen to others talk about their math ideas?"

Student 3: "Yes. I think it helps to get you thinking about math in different ways. And, if you don't get the right answer the first time, it helps to see it done in different ways so it can help you find a better way to solve the problem."

It is evident from this student's responses that she is a hands-on learner. She enjoys learning math in an exploratory way, whether she is learning through games or a manipulative-based activity. Her comment about her third grade teacher giving prizes for getting the right answers tells me that this student has not experienced true discourse. Perhaps the teacher was expecting the students to provide a specific response in regards to the solution method and was not open to allowing the students to solve the problems in ways that made sense to them. This also comes to light in the student's revelation that she appreciated not having to worry about getting the right answer. By taking the pressure off of being correct, she was able to focus more on the math learning that was taking place. Listening to other students explain their ideas gave her an opportunity to find and correct any mathematical errors she may have made without the pressure of being reprimanded for solving the problem differently than the method taught.

Her comment about understanding the math also is a testament to the power of conceptual understanding. By understanding the math, the student was able to come to the correct answer more often. This tells me that, for this student, there is certainly a big difference between the process of problem solving and the product of solving problems.

Summary

The data I collected revealed a wealth of information about the building of conceptual knowledge through dialogue and writing. Students were able to utilize the visual tools provided in the *Calendar Math* to find the appropriate words needed to explain and justify their mathematical thinking. Their daily discussions about the calendar math components helped them to improve the quality of their written work during math class. As they became more comfortable explaining and justifying their ideas during class discussions and interacting with their peers, the students established specific sociomathematical norms. As the study progressed, the students began expecting certain factors to drive their discussions. First, students expected that their peers thought about the explanations and tried to make sense of them. As a result of this, any student should have been able to reiterate another student's explanation if asked to do so. Second, solution methods were expected to be able to be followed and replicated, resulting in the same answer, to prove that the solution method was valid. Finally, the students began to take ownership of their confusion by asking specific clarifying questions. It became routine that if an explanation was confusing it was nothing to be ashamed of. Confusion meant that the explanation was not clear enough and needed to be revised. Confusion eventually became a welcome part of the learning process, not something to be avoided.

Clearly, discussion and writing are related in that they both are useful in helping students communicate mathematical ideas. Additionally, talking and writing about mathematical concepts helps both the student and the teacher identify which ideas are understood and where misconceptions still exist. For many of my students, no longer was learning math only about getting the right answer. It became a discovery process, one that allowed for many different solution methods. The quality and quantity of their discussions and writing were both positively impacted by the increase in their value of the learning process. Students were able to apply the social and sociomathematical norms developed for classroom discussions in their written explanations, helping to enhance the quality of their overall communication skills. In the following chapter, I will review the key points of this study, highlight the implications of my findings, and discuss my recommendations for future studies.

CHAPTER 5: CONCLUSION

Introduction

When I began this project, I was interested to see what would happen if I provided my students with opportunities to communicate about mathematics, both verbally and in writing. Through an action research study, I was able to answer my questions, “Are discussion and writing about mathematics related? How?” and “How do discussion and written explanations affect student understanding of mathematical concepts?” In this chapter I will review the results of my data analysis, investigate some limitations and implications of the study, and make some suggestions for future studies.

Summary of Findings

After collecting data from my students’ daily work and interactions over a period of four months, I found that the discussions we engaged in every day directly and positively impacted the students’ abilities to express their mathematical ideas in writing. Over the series of months that we worked with the Calendar Math bulletin board display, I noticed an increased transfer of the language we used in discussions to the students’ writing. By this I mean that as my students began to develop their mathematical language during discussions, they demonstrated an increasing ability to apply this language to their written explanations. The students became responsible for the learning taking place in the classroom. Just as Countryman (1992) stated, the acquirement of precise mathematical language helped the students to become better writers and thinkers of mathematics.

In addition, the acquisition of the language of mathematics caused my students to shift their thinking about how mathematics is learned. Instead of focusing on the product of problem solving, discussions and writing tasks were centered on the process of problem solving. In other words, I was less concerned with my students getting the right answer as opposed to being able to explain how they reached their answer. Most often, as students explored their problem solving processes, they were able to identify mathematical mistakes and modify their strategies. Similarly to Yackel (2000) and McClain and Cobb (2001), I found that the establishment of social and sociomathematical norms helped guide the daily discussions and kept the discussions focused on the mathematical content. Also, as Powell and López (1989) noticed, the students were able to develop control over their learning, enabling them to feel accomplished, which consequently resulted in an improvement in their attitude toward mathematics.

With the assistance of the *Every Day Counts Calendar Math* bulletin board display, students were given tangible representations of mathematical concepts to help guide their conceptual development. They began speaking and writing like students of mathematics, building communication skills that moved beyond recitation of memorized facts and formulas. As students gained confidence in their ability to talk about mathematical concepts, their ability to express those ideas in writing became easier and more natural. They also learned that, although they may have solved a problem differently than someone else, their ideas were no less valid and could, in fact, help other students who did not “get it” the first time the concept was explained. Therefore, every student felt empowered in their knowledge of mathematics and their unique perspective of challenging problems. As each month began, the teacher was responsible for introducing key words and concepts that would be the focus of study for the month. In the early part of each month, the students' confidence level was low, but built as the month progressed and students were given daily opportunities to interact with the mathematics and each other. Eventually, authority over the mathematics was held almost entirely by the students; the teacher became more of an observer and guide. Despite this continual cycle of authority in the classroom between the teacher and the students, the teacher was no longer seen as the ultimate math expert, and the sharing of ideas among students made this point evident.

Implications

Many studies have illustrated the importance of communication and its relationship to building conceptual understanding of mathematical ideas (Hiebert et al, 1998; Van de Walle, 1994; Yackel, 2000; Steinbring, 2005; Steele, 2001; Vygotsky, 1994; Pirie, 1998). Several studies also explored the usefulness of visual representations in helping to build mathematical concepts (Yackel, 2000; Steele, 2001; Vygotsky, 1994; Pirie, 1998). Moreover, research has shown that conceptual understanding of mathematics flourishes in classroom environments that utilize discussion effectively (Yackel, Cobb, & Wood, 1992; Hershkowitz & Schwarz, 1999; Egendoerfer, 2006; McClain & Cobb, 2001; Williams & Baxter, 1996; Silver & Lane, 1993; Nathan & Knuth, 2003; Pang, 2001; Pugalee, 2001, January). The development of social norms (Yackel, 2000; Planas & Gorgorió, 2004; Vacha, 1979) and sociomathematical norms (Yackel, Cobb, & Wood, 1992; Hershkowitz & Schwarz, 1999; McClain & Cobb, 2001) within the classroom environment provide appropriate structures for student interaction with each other as well as with mathematical concepts.

Numerous other studies focused on the importance of writing as an expressive tool in mathematics learning (Countryman, 1992; O'Shea, 2004; Rose, 1989; Tichenor & Jewell, 2001; Baxter et al, 2005; Connolly, 1989; Pugalee, 2001, January and May; Birken, 1989; Powel & López, 1989). Writing about mathematical ideas and to explain problem solving processes helps students to organize their thoughts and expand their metacognition about the topic. By revealing what they know and what they do not know (Baxter et al, 2005), students are more apt to work through problems astutely, discovering the relationships and commonalities between mathematical concepts along the way.

If discussion and writing can both be used to help students build mathematical conceptual understanding and their ability to communicate their ideas about mathematics, then traditional instructional methods in mathematics classrooms need to be reevaluated. As times change, so do our students. Although many teachers teach the way they were taught or using methods that have been successful in the past (Kutz, 1991), those methods may not be as successful with the types of learners we have in our classrooms today. Today's workforce has become increasingly demanding, requiring its members to be able to communicate ideas, work cooperatively, and apply a wide array of knowledge in various situations. If the goal of our educational system is to prepare students to meet these demands, then we need to begin providing them with the skills that will enable them to be successful, including the ability to solve problems and communicate ideas effectively.

In my study, I created a learning environment that was different from any my students had experienced in previous classrooms at this school. Although some had experienced environments that incorporated a few of the aspects of a mathematics classroom that I have discussed, as was evident in the final interviews, I felt confident that I had created an environment that truly focused on process learning, student interaction, and verbal and written communication, all of which are important for helping students build their conceptual understanding of mathematical ideas. My findings support the research in regard to the effectiveness of using discussion and writing, constructivist approaches, and the development of an interactive classroom environment. Similarly to Pugalee (2001, January), I found communication to be an indispensible addition to my daily mathematics instructional methods.

What's more, I found that the use of discussion and writing as tools to teach mathematics also had additional benefits. Daily discussions and written tasks provided a form of alternative assessment, allowing me to “see” inside my students’ heads. Instead of assuming that they did not understand a concept based on wrong answers, I was able to guide them through their confusions and misconceptions as I listened to their explanations and as I read their writing. As Baxter et al. (2005) found, I realized that writing about mathematical ideas provided every student with an opportunity to participate in thinking about the concepts on some level. Furthermore, discussing and writing their mathematical ideas made students accountable for their work during our daily math lessons. Each student was responsible for not only inputting their own thoughts and ideas into the discussion, but they were also responsible for listening to others' explanations, thinking critically about the ideas presented, and responding to them in a mature, non-criticizing manner.

This causes me to firmly believe that it is no longer appropriate for teachers to deliver mathematical information to their students. As the expectations of student performance and learning increase, teachers need to find alternative teaching strategies that help meet those expectations. It is no longer enough to allow students to sit in neatly formed rows, passively absorbing mathematical facts and formulas. Students cannot learn to think for themselves if we continually force information into their heads.

While some of these implications may be challenging for many teachers to accept and implement, the alternative is to maintain the status quo. How can we expect changes to be made if we do not take risks? While these instructional strategies may not work in the same way for every teacher, the ultimate goal is for teachers to try new things, discover what works for them and for their students, and continually attempt to positively impact their students' academic lives.

Limitations

As teachers, we do not teach one class. We teach several students – individuals – whose needs should be taken into consideration every moment of the school day. Individual differences in student participation, motivation, mathematical background, and self-confidence all directly influenced my action research study.

Student minds are the result of their interactions with and their interpretations of the world around them. At the beginning of this research study, I knew that most of my students came from underprivileged home environments. Their parents, while supportive of education, lacked the time or energy to be as involved as necessary in the educational process. In addition, I also was aware of the types of teaching strategies being used in each of the grade levels before mine. It was largely acceptable in the primary grades at my school to teach basic facts and mathematical rules. Much of the work students had done in math at each grade level prior to entering fourth grade was in an effort to prepare them for standardized tests. Classroom discussion and writing during mathematics instructional times, although highly recommended by the administration, was viewed as too time-consuming and consequently sacrificed to make more time for test preparation. Therefore, I was prepared to encounter students who might struggle with or even completely reject the teaching strategies I wanted to implement. While I saw positive results in using discussion and writing as part of my mathematics instruction, it would be impossible to generalize my findings to other students.

Teachers are as individual as their students. Each classroom has its own personality, its own voice – a voice that is a direct representation of the teacher in charge of it. Since my class was composed of students from a variety of different classes, I knew that each student was entering my classroom having been affected by the philosophy and teaching strategies of their previous years' teachers. Every teacher has their own beliefs about how best to teach their students. Between the academic curriculum and the hidden curriculum, teachers support their own ideas and, whether consciously or unconsciously, pass those ideas to their students.

Some teachers feel that they need to be in complete control of their students every moment of the school day. I was willing to relinquish some of that control when I challenged my students to think about mathematics in a different way. I was able to do this because I believe that students can learn just as much from each other as they can from me. I did not feel that it was my job to merely impart knowledge onto my students through rote memorization and textbook work. During this process, I was forced to reflect on my teaching practices and make conscious changes that may or may not have been beneficial to my students' mathematical learning. Other teachers may have been reluctant or apprehensive about such a daunting process. The fact that teachers have varying philosophies and expectations for the students I would be teaching this year would be a limitation in this study.

Recommendations

Through my action research study, I found that students are capable of communicating mathematically, both verbally and in writing, without having to recite ideas as they were presented by a teacher. The daily anticipation of working with the calendar math seemed to help increase my students' motivation about sharing their mathematical ideas.

Based on the results of my study, I would make three recommendations for the future. 1) Teachers could be more reflective about their practices and be open to teaching strategies that incorporate constructivist approaches. 2) Students could be given more opportunities in mathematics classes to work closely with the language of mathematics and visual representations of mathematical concepts. 3) Students could be given daily opportunities to discuss and write about their mathematical ideas and solution methods.

Teacher Change

"Educational change depends on what teachers do and think – it's as simple and as complex as that" (Fullan, 2007, p. 129). As teachers are the primary decision-maker in their classrooms, it is the teacher who must manage the realities that accompany any change. It would be easy for legislators, educational researchers, and school administrators to implement changes in teaching strategies if they could change the way teachers think.

This is especially true in mathematics classrooms. If the goal is to help students build mathematical conceptual understanding then several things must take place. First, teachers must be willing to analyze, evaluate, and modify their role in the classroom. It may require teachers and students to challenge the system that is already in place. However, when teachers are willing to relinquish the idea that they are the "giver of knowledge" and provide their students opportunities to construct knowledge through interaction, discovery, and writing, positive changes will occur.

A second way that teachers can initiate change in their classrooms is by recognizing when changes need to be made. Our students are not the same types of learners as they were ten years ago and yet our classrooms look the same as they ever did. Students are not the only factor affecting the way we teach. Teachers must take into account not only educational changes, but societal changes as well. As it is part of our responsibility to help our students become successful, functioning members of society, we need to be sure that we are preparing students for the society *they* will be entering – a society that requires them to think critically, work cooperatively, and communicate ideas effectively.

By evaluating our practice and considering the various forces of change that affect our students, teachers can help to directly impact student participation in the classroom. Making students accountable for their learning is a crucial aspect of the educational system. So much emphasis has recently been placed on teacher accountability that students have become languid in their learning. Pressure is placed upon teachers to increase student achievement and, in effect, increase test scores. As a result, textbooks and scripted programs have been developed that have all but reduced the teaching and learning process to a series of unconnected skills to be memorized. “If teachers consistently place emphasis on students that can supply correct answers students will recognize the hidden agenda of the teacher. Students will supply correct answers when they can, and stay quiet when they are unsure” (Egendoerfer, 2006, p. 73).

By working to create a learning environment where student interaction and discovery is encouraged, the teacher is sending the message that students are responsible for their learning, and therefore students will become more active participants in the learning process. In this type of learning environment, students quickly learn that misunderstanding and confusion is not something to be ashamed of. Struggle becomes a welcome part of the learning process – something that is necessary in order for true learning to take place.

Mathematical Language

The acquisition of a language – any language – is indeed a difficult undertaking. The same is true about learning the language of mathematics. When we learn a new language, we often make mistakes: we use an incorrect term, phrase a statement in an awkward manner, etcetera. Mathematical language is no different. Barwell (2005) points out that ambiguity can be useful in a mathematics classroom. Although mathematics is typically seen as a precise, technical subject, student confusion can actually be helpful in the learning process.

Mathematical discourse is cyclical in nature. As new vocabulary is encountered, it may not be instantly clear to the learner. By providing repeated encounters with mathematical vocabulary, exploring them in numerous, varied contexts, students' understanding of these terms, along with their experience as part of a discursive environment, become more complex over time (Barwell, 2005).

Students can explore the language of mathematics through concrete and visual representations of mathematical ideas. These representations act as springboards into the world of mathematics for most students. The struggle students experience as they attempt to analyze and describe what they observe provides a setting of mathematical argumentation (Yackel, 2001; Morgan, 2005) that contributes to a student's overall understanding of mathematical concepts.

By understanding the nature of language acquisition and applying that to mathematics classrooms, students can begin to communicate mathematical ideas in new and unique ways. Consistent, repeated exploration of mathematical ideas and vocabulary, through the use of concrete and visual representations, is a key component to helping students build their conceptual understanding of complex mathematical ideas. In a classroom environment that supports mathematical discourse, students and teachers can work together to find new ways to use language to do mathematics.

Verbal and Written Communication

"Giving students opportunities to develop skills in communicating mathematically should be a natural outgrowth of a well-balanced mathematics program" (Pugalee, 2001, January). My study supports this claim. My students began to build their understanding of the mathematical ideas represented on the calendar math bulletin board display through daily dialogue and writing.

At first, my students had a difficult time adjusting to my expectation that they explain and justify their own ideas regarding mathematical concepts. It seemed as though they were stuck in the assumption that learning math was only about getting the right answer. As they began to trust their own knowledge about mathematics and think critically about the others' ideas, their communication skills improved. As was seen in the transcripts of our calendar discussions, my students began to realize how the language of mathematics was useful in helping them communicate their thought processes.

Something I would recommend to teachers who are considering implementing constructivist teaching approaches in their classroom is to make the change slowly. Change can be overwhelming, even for the most experienced teachers. I suggest making the change to using discussion to build conceptual knowledge during one part of the whole school day. That way the entire process does not become too much to handle on top of all of the other teaching responsibilities. Once some small successes have been achieved, choose another part of the school day to implement the change. This might take months, or even years, but the overall effects of the change will provide an educational experience our students deserve.

Summary

Education is a living entity – always moving, always changing. As teachers, we need to pinpoint what is most important in order to provide the best experience for every child that enters our classrooms. During this study, I made it a priority to incorporate discussion and writing about mathematics as part of my instructional practices. In the process, I learned that it is perfectly acceptable for me to step aside, allowing my students to elucidate their own thoughts and confusions during math class. Not only did this help me identify which areas my students were struggling with, but it also provided a vehicle for helping other students hear different perspectives about the same problem or topic. This, in turn, helped my students learn to communicate their mathematical ideas clearly and precisely, while simultaneously building their conceptual understanding of mathematics. From this experience, I learned that I have the ability to impact my students' education either positively or negatively. If I continue to set priorities for my classroom based on my philosophies and beliefs, I can make decisions that will benefit all of my students. More than anything, I want to be the type of teacher who constantly acts with the best interests of my students at heart. By encouraging my students to believe in themselves and become active members of the classroom community, I feel confident that they will be successful in all aspects of life.

APPENDIX A: INSTITUTIONAL REVIEW BOARD (IRB) APPROVAL

University of Central Florida Institutional Review Board
Office of Research & Commercialization
12201 Research Parkway, Suite 501
Orlando, Florida 32826-3246
Telephone: 407-823-2901, 407-882-2901 or 407-882-2276
www.research.ucf.edu/compliance/irb.html

Notice of Expedited Initial Review and Approval

From : **UCF Institutional Review Board**
FWA00000351, Exp. 5/07/10, IRB00001138

To : **Joseph Roicki**

Date : **July 19, 2007**

IRB Number: **SBE-07-05076**

Study Title: **Effects of Discussion and Writing on Student Understanding of Whole Number Concepts and Operations**

Dear Researcher:

Your research protocol noted above was approved by **expedited** review by the UCF IRB Chair on 7/19/2007. **The expiration date is 7/18/2008.** Your study was determined to be minimal risk for human subjects and expeditable per federal regulations, 45 CFR 46.110. The category for which this study qualifies as expeditable research is as follows:

7. Research on individual or group characteristics or behavior (including, but not limited to, research on perception, cognition, motivation, identity, language, communication, cultural beliefs or practices, and social behavior) or research employing survey, interview, oral history, focus group, program evaluation, human factors evaluation, or quality assurance methodologies.

The IRB has approved a **consent procedure which requires participants to sign consent forms.** Use of the approved, stamped consent document(s) is required. Only approved investigators (or other approved key study personnel) may solicit consent for research participation. Subjects or their representatives must receive a copy of the consent form(s).

All data, which may include signed consent form documents, must be retained in a locked file cabinet for a minimum of three years (six if HIPAA applies) past the completion of this research. Any links to the identification of participants should be maintained on a password-protected computer if electronic information is used. Additional requirements may be imposed by your funding agency, your department, or other entities. Access to data is limited to authorized individuals listed as key study personnel.

To continue this research beyond the expiration date, a Continuing Review Form must be submitted 2 – 4 weeks prior to the expiration date. Advise the IRB if you receive a subpoena for the release of this information, or if a breach of confidentiality occurs. Also report any unanticipated problems or serious adverse events (within 5 working days). Do not make changes to the protocol methodology or consent form before obtaining IRB approval. Changes can be submitted for IRB review using the Addendum/Modification Request Form. An Addendum/Modification Request Form **cannot** be used to extend the approval period of a study. All forms may be completed and submitted online at http://iris.research.ucf.edu .

Failure to provide a continuing review report could lead to study suspension, a loss of funding and/or publication possibilities, or reporting of noncompliance to sponsors or funding agencies. The IRB maintains the authority under 45 CFR 46.110(e) to observe or have a third party observe the consent process and the research.

On behalf of Tracy Dietz, Ph.D., UCF IRB Chair, this letter is signed by:

Signature applied by Janice Turchin on 07/19/2007 10:48:28 AM EDT

Janice M Turchin

IRB Coordinator

APPENDIX B: PRINCIPAL APPROVAL

Informed Consent to Conduct Research

June 18, 2007

Dear Mr. Turner,

This coming school year, I will be conducting an action research project in my classroom on the effects of discussion and writing on student understanding of whole number concepts and operations. This study is a part of my Master's program in K-8 Math and Science Education at the University of Central Florida. You are being asked to give your consent to allow me to conduct my research with my class for the 2007-2008 school year.

During my study, I will be video/audio taping whole class and small group discussions that take place during my math block (1:20-2:40, M, Tu, Th, F and 12:05-12:50, W). I will also video/audio tape one-on-one interviews with selected students regarding their participation in the discussions. Student work containing written explanations of problem solving strategies and mathematical reasoning will be collected and analyzed as well.

I do not anticipate any risks to my students during this study. Any connection between my students and the data I will collect will be destroyed upon completion of the study, and student names will not be used anywhere in my thesis. I will be obtaining informed consent from my students' parents allowing their children to participate in the study and to be video/audio taped, as well as assent from my students that they understand the purpose of the study and my expectations of them during the process. Participation in the study is not mandatory, and parents will have direct contact with me and my faculty supervisor at all times. Extra credit will not be given for participation and student math grades will not be affected in anyway.

If you have any concerns or questions about the study, you may also contact my faculty supervisor, Dr. Enrique Ortiz, at (407)823-5222. Thank you for your support in this process.

Sincerely,

Joseph Roicki

My signature below indicates that I have read and understand the details of this study and that I give consent for Mr. Joseph Roicki to conduct research with his 4th grade class during the 2007-2008 school year.

Greg Turner _______________ 6/18/07

Mr. Greg Turner
Principal, Wicklow Elementary School

Date

UCF University of Central Florida IRB
IRB NUMBER: SBE-07-05076
IRB APPROVAL DATE: 7/19/2007
IRB EXPIRATION DATE: 7/18/2008

APPENDIX C: PARENTAL CONSENT

August, 2007

Dear Parent or Guardian,

My name is Joseph Roicki and I am delighted to be your child's teacher this year! I have spent time this summer planning for the coming school year, and I am confident that your child will have a positive learning experience in my class. In addition to my responsibilities as your child's teacher, I am also a graduate student in K-8 Math and Science Education at the University of Central Florida. I am currently planning a research project for my Master's thesis that will take place within my classroom from August until sometime in December. The goal of my research is to study the effects of using discussion and writing on student understanding of whole number concepts and operations.

The data I will collect for this investigation will come from your child's daily work in my class. I will video/audio tape your child taking part in whole class and small group discussions about their mathematical thinking and problem solving methods. I may also interview your child regarding his or her participation in class discussions. These interviews will also be video/audio-taped. Video/audio taping is completely voluntary, and your child does not have to answer any question he or she does not wish to answer. Compensation will not be provided. I would be happy to share the results of the study with you and your child.

While the data provided by your child will be used in my thesis, his or her name and identity will be kept confidential in a locked place. The purpose of this study is to analyze my teaching practices, not assess your student's mathematical ability. I do not anticipate any risks to your child during the course of the study; these are teaching methods I would be using regardless of the study. Upon completion of the project, any connection between your child and the data collected will be destroyed.

If you have any questions regarding this study, you may contact me (407-320-1267), or my faculty advisor, Dr. Enrique Ortiz (407-823-5222). You may withdraw consent at any time. Questions or concerns about participant's rights may be directed to the Office of Research & Commercialization, 12201 Research Parkway, Suite 501, Orlando, FL 32826-3246. The hours of operation are Monday through Friday, 8:00 a.m. – 5:00 p.m. at (407)823-3299.

Participation in any part of this study is completely voluntary and will not negatively affect your child's grades in any way. Students who are not given consent to participate in the study will still participate in classroom activities and their work will not be considered as data. Please sign on the line below indicating that you understand what is being asked of your child as a participant of this study and that you give consent for your child to participate in the study. You and your child have the right to withdraw consent for participation at any time without consequence. Please also check the box indicating that you give consent for your child to be video/audio-taped during class and interview sessions. By signing below, you certify that you are at least 18 years of age.

Name of child (Printed) ____

☐ I give consent for my student to participate in this study.
☐ I give consent for my child to be video/audio-taped during class time and interviews.
☐ I would like more information about this study.

Name of Parent/Guardian (Printed)	Name of Researcher (Printed)
Name of Parent/Guardian (Signed)	Name of Researcher (Signed)
Date	Date

UCF University of Central Florida IRB
IRB NUMBER: SBE-07-05076
IRB APPROVAL DATE: 7/19/2007
IRB EXPIRATION DATE: 7/18/2008

APPENDIX D: STUDENT ASSENT

Student Assent to Participate in Research Study

August 20, 2007

Dear student,

This year I would like to video and audio tape our class. I will use this to learn more about our class discussions. I would like to ask you some questions about your work in math class as well. You don't have to do this if you don't want to. You may choose to not answer any questions I ask. Your name will not appear anywhere in the results of the study.

Sincerely,

Mr. Roicki

By signing below, I am saying that would like to participate in this study. I have asked questions and they have been answered.

______________________________________ ______________

Student Name Date

APPENDIX E: WHOLE NUMBER CONCEPTS AND OPERATIONS PRETEST

Name: ______________________________ **Date:** __________

Whole Number Concepts and Operations Pretest

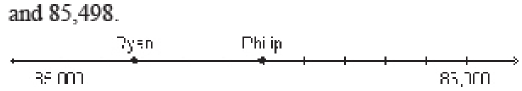
Multiple Choice
Identify the letter of the choice that best completes the statement or answers the question.

1. The chart gives the diameters of Earth and the five closest planets.

DIAMETER OF THE CLOSEST PLANETS	
Planet	**Diameter (in kilometers)**
Mercury	4,880
Venus	12,100
Earth	12,756
Mars	6,794
Jupiter	142,800
Saturn	120,660

Which of these statements is NOT true about the diameter of Mars?
a. It is less than the diameter of Venus.
b. It is less than the diameter of Earth.
c. It is less than the diameter of Saturn.
d. It is greater than the diameter of Jupiter.

2. Ryan, Joe, and Philip are playing a board game. Ryan has 85,183 points while Philip has 85,498 points. Joe's score is between 85,183 and 85,498.



Which of the following scores could NOT be Joe's score?
a. 85,249 b. 85,359 c. 85,495 d. 85,066

Response Questions
Solve each problem below. Show all of your work in the space provided and explain your solutions.

3. THINK SOLVE EXPLAIN

Look at the table below. There is more than one way to complete it.

In	3	4	5	6	7
Out	9	?	?	?	?

What is an addition rule that would work for the table?

Complete the table below using your addition rule.

In	3	4	5	6	7
Out	9				

What is a multiplication rule that would work for the table?

Complete the table below using your multiplication rule.

In	3	4	5	6	7
Out	9				

4. THINK
SOLVE
EXPLAIN

A pilot flies from Los Angeles to Dallas to Chicago and then back to Los Angeles. What is the total distance? Estimate first and then calculate the answer.

Show All Work

Estimated distance: _________ kilometers

Answer: kilometers

APPENDIX F: WHOLE NUMBER CONCEPTS AND OPERATIONS POSTTEST

Name: ____ **Date:** ___________

Whole Number Concepts and Operations Posttest

Multiple Choice
Identify the letter of the choice that best completes the statement or answers the question.

1. Which number is marked by the dot on the number line?

a. 9,750   b. 9,708   c. 9,780   d. 9,758

2. During one day of trading on the New York Stock Exchange, about 936,100,000 trades occurred. What is the value of the 3 in 936,100,000?

a. 30 hundred   b. 30 million   c. 30 thousand   d. 30 ones

3. Tappers Dance Club held their yearly recital. There were 24 dancers altogether. They were in 3 lines. Which number sentence does NOT help you find how many dancers were in each line?

a. $3 \times 8 = 24$ b. $24 \div 3 = 8$ c. $8 \times 3 = 24$ d. $21 \div 3 = 7$

1

Response Questions
Solve each problem below. Show all of your work in the space provided and explain your solutions.

4. THINK
SOLVE
EXPLAIN

Stamp Book Arrays Ashley would like to put 21 stamps in a stamp book. How can she arrange the stamps?

Part A Draw an array to show 21. Show at least two rows. Show the same number in each row. **Part B** Write two multiplication sentences and two division sentences for the array. ____________________ ____________________ ____________________ ____________________	***Show All Work***

2

5. THINK
SOLVE
EXPLAIN

Bird Watching Jennifer promised her friend that she would watch her parakeet while she was on vacation for 4 days.

How many minutes are in 4 days?

1 day = 24 hours 1 hour = 60 minutes

Show All Work
Answer: __________ minutes

On the lines below, explain how you found the number of minutes.

APPENDIX G: SAMPLE STUDENT INTERVIEW QUESTIONS

Sample Student Interview Questions

I am going to ask you some questions. I'd like you to tell me as much as you can about what I ask. Remember, there are no wrong answers. You may choose to not answer any of the questions.

- Do you like math?
- Are you good at math? Why do you think so?
- Have you ever learned math by talking, listening, and writing about math problems?
- Did you like learning math in this way? Why or why not?
- Has talking about math with your classmates helped you? How?
- Has discussing math ideas helped you explain your thinking in writing? How?
- Did you talk more in whole group or small group discussions?
- Which setting, whole group or small group, did you prefer? Why?
- Was there anything you did not like about the discussions we had in math class?
- Do think it is important for you to talk about your math ideas? Why or why not?
- Do you think it is important to listen to others' math ideas? Why or why not?
- Do you like to hear how others have solved math problems? Why or why not?
- What is the teacher's job in math class?
- What is the student's job in math class?
- Is there anything else you would like to share with me?

APPENDIX H: PERMISSION FROM PUBLISHER TO INCLUDE COPYRIGHTED IMAGES

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December 17, 2007

Joseph Roicki
Wicklow Elementary School
100 Placid Lake Drive
Sanford, FL 32773
FX: (407) 320-1215

Dear Mr. Roicki:

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Sincerely,

Karin Huang
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REFERENCES

Barwell, R. (2005). Ambiguity in the mathematics classroom. *Language and Education, 19*(2), 118-126.

Baxter, J., Woodward, J., & Olson, D. Writing in mathematics: an alternative form of communication for academically low achieving students. *Learning Disabilities Research and Practice, 20*(2), 119-195.

Birken, M. (1989). Using writing to assist learning in college mathematics classes. In P. Connolly and T. Vilardi (Eds.), *Writing to learn mathematics and science* (pp. 33-47). New York: Teacher College Press.

Bruner, J. (1966). *Toward a theory of instruction.* Cambridge, MA: Harvard University Press.

Burns, M. (1995). *Writing in math class.* Sausalito, CA: Math Solutions Publications.

Cobb, P., Yackel, E., & Wood, T. (1992). A constructivist alternative to the representational view of mind in mathematics education. *Journal for Research in Mathematics Education, 23*(1), 2-33.

Connolly, P. (1989). Writing and the ecology of learning. In P. Connolly and T. Vilardi (Eds.), *Writing to learn mathematics and science* (pp. 1-14). New York: Teacher College Press.

Countryman, Joan. (1992). *Writing to learn mathematics.* Portsmouth: Heinemann.

Desiderio, M. & Mullennix, C. (2005). Two behavior management systems, one classroom: can elementary students adapt? *The Educational Forum, 69*(4), 383-391.

Egendoerfer, L. (2006). Challenging a traditional social norm in a second grade mathematics classroom. A thesis submitted in partial fulfillment of the requirements of Master of Education in the Department of Teaching and Learning Principles. Orlando, FL: University of Central Florida.

Freire, Paulo. (1970). *Pedagogy of the oppressed.* New York: Herder and Herder.

Fullan, M. (2007). *The new meaning of educational change.* New York: Teachers College Press.

Gillespie, J. G. & Kanter, P. F. (2005). *Every day counts: Calendar math teacher's guide grade 4.* Wilmington, MA: Great Source Education Group.

Gravemeijer, K., Cobb, P., Bowers, J., & Whitenack, J. (2000). Symbolizing, modeling, and instructional design. In Cobb, P., Yackel, E., & McClain, K. (Eds.), *Mathematizing and communicating in mathematics classrooms: Perspectives on discourse, tools, and instructional design.* Mahwah, NJ: Lawrence Erlbaum Associates, 225-274.

Hiebert, J., Carpenter, T.P., Fennema, E., Fuson, K.C., Wearne, D., Murray, H., Olivier, A., & Human, P. (1998). *Making sense: Teaching and learning mathematics with understanding.* Portsmouth, NH: Heinemann.

Hershkowitz, R. & Schwarz, B. (1999). The emergent perspective in rich learning environments: some roles of tools and activities in the construction of sociomathematical norms. *Educational Studies in Mathematics, 39*, 149-166.

Kazemi, E. & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary classrooms. *The Elementary School Journal, 102*(1), 59-80.

Kutz, R. E. (1991). *Teaching elementary mathematics.* Boston, MA: Allyn and Bacon.

McClain, K. & Cobb, P. (2001). An analysis of development of sociomathematical norms in one first-grade classroom. *Journal for Research in Mathematical Education, 32*(3), 236-266.

Miller, L. D., & Hunt, N. P. (1994). Professional development through action research. In D. B. Aichele & A. F. Coxford (Eds.), *Professional development for teachers of mathematics*. Reston, VA: National Council for Teachers of Mathematics, 296-303.

Morgan, C. (2005). Words, definitions and concepts in discourses of mathematics, teaching and learning. *Language and Education, 19*(2), 103-117.

Nathan, M. & Knuth, E. (2003). A study of whole class mathematical discourse and teacher change. *Cognition and Instruction, 21*(2), 175-207.

National Council for Teachers of Mathematics. (1989). Curriculum and evaluation standards. Reston, VA: National Council for Teachers of Mathematics.

National Council for Teachers of Mathematics. (1991). Professional standards. Reston, VA: National Council for Teachers of Mathematics.

National Council for Teachers of Mathematics. (1995). Assessment standards. Reston, VA: National Council for Teachers of Mathematics.

National Council for Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA: National Council for Teachers of Mathematics.

National Council for Teachers of Mathematics. (2006). Curriculum focal points for prekindergarten through grade 8 mathematics. Reston, VA: National Council for Teachers of Mathematics.

O'Shea, J. (2004). Encouraging good mathematical writing. *International Journal of Mathematical Education in Science and Technology, 37*(1), 101-103.

Pang, J. (2001). Challenges of reform: utility of mathematical reform. Paper presented at the Annual Meeting of the American Educational Research Association, Seattle, WA.

Pirie, S. E. B. (1998). Crossing the gulf between thought and symbol: language as (slippery) stepping stones. In H. Steinbring, M. G. Bartolini Bussi, & A. Sierpinska (Eds.), *Language and communication in the mathematics classroom.* Reston, VA: National Council for Teachers of Mathematics, 7-29.

Planas, N. & Gorgorió, N. (2004). Reconstructing norms. In Chick, H. L. & Vincent, J. L. (Eds.), *Proceedings of the 29th conference of the international group for the psychology of mathematics education, 3,* Melbourne: PME, 65-72.

Powell, A. B. & López, J. A. (1989). Writing as a vehicle to learn mathematics: A case study. In Connolly, P. & Vilardi, T. (Eds.), *Writing to learn mathematics and science* (pp. 157-177). New York: Teacher College Press.

Pugalee, D. (1999). Constructing a model of mathematics literacy. *The Clearing House, 73*(1), 19-22.

Pugalee, D. (2001, January). Using communication to develop students' mathematical literacy. *Mathematics Teaching in the Middle School, 6*(5), 296-299.

Pugalee, D. (2001, May). Writing, mathematics, and metacognition: looking for connections through students' work in mathematical problem solving. *School Science and Mathematics, 101*(5), 236-245.

Rose, B. (1989). Writing and mathematics: Theory and practice. In Connolly, P. & Vilardi, T. (Eds.), *Writing to learn mathematics and science* (pp. 15-30). New York: Teacher College Press.

Sherin, M., Mendez, E., & Louis, D. (2004). A discipline apart: the challenges of 'Fostering a Community of Learners' in a mathematics classroom. *Journal of Curriculum Studies, 36*(2), 207-232.

Silver, E. A. & Lane, S. (1993). *Balancing considerations of equity, content quality, and technical excellence in designing, validating, and implementing performance assessments in the context of mathematics instructional reform: The experience of* *the QUASAR project*. Pittsburg, PA: Learning Research and Development Center, University of Pittsburg.

Steele, D. F. (2001). Using sociocultural theory to teach mathematics: a vygotskian perspective. *School Science and Mathematics, 101*(8), 404-416.

Steinbring, H. (2005). Analyzing mathematical teaching-learning situations – the interplay of communicational and epistemological constraints. *Educational Studies in Mathematics, 59*, 313-324.

Tichenor, M. S., & Jewell, M. J. (2001). Using e-mail to write about math. *The Educational Forum 65*(4), 300-308.

Vacha, E. (1979). Classroom norms. In Vacha, E., McDonald, W., Coburn, J., & Black, H., *Improving classroom social climate teacher's handbook.* New York: Holt, Rinehart, and Winston, 181-185.

Van de Walle, J. A. (1994). *Elementary school mathematics: Teaching developmentally.* White Plains, NY: Longman.

Vygotsky, L. S. (1987). Thinking and speech. In Reiber, R. W. & Carton, A. S. (Eds.), *The collected works of L. S. Vygotsky.* New York: Plenum Press, 39-243.

Vygotsky, L. S. (1994). *Thought and language.* Cambridge, MA: M.I.T Press.

Williams, S. R. & Baxter, J. A., (1996). Dilemmas of discourse-oriented teaching in one middle school mathematics classroom. *The Elementary School Journal, 97*(1), 21-38.

Wingfield, S. S. & Black, G. S. (2005). Active versus passive course designs: the impact on student outcomes. *Journal of Education for Business, 81*(2), 119.

Wood, T. (2001). Teaching differently: creating opportunities for learning mathematics. *Theory into Practice, 40*(2), 110-117.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*, 458-477.

Yackel, E. (2000, July 31-August 6). Creating a mathematics classroom environment that fosters the development of mathematical argumentation. Paper prepared for working group 1: Mathematics education in pre and primary school, of the ninth International Congress of Mathematical Education. Retrieved February 1, 2008, from http://www.nku.edu/~sheffield/eyackel.html.

Yackel, E. (2001). Explanation, justification, and argumentation in mathematics classrooms. Paper presented at the International Group for the Psychology of Mathematics Education, Utrecht, The Netherlands.